ATTENUATION OF ULTRASONIC SHEAR WAVES IN COPPER

A Thesis Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

By

Dongjoo Choi Kim

December, 1973

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ABSTRACT

The attenuation of circularly polarized shear waves propagating in the [001] direction in copper was calculated for external magnetic fields up to 32 kG. The attenuation by electrons on different regions of the Fermi surface was identified. This study used the Fermi surface calculated from the studies of M. R. Halse by L. T. Wood.

CONTENTS

I - INTRODUCTION

The fact that circularly polarized shear waves could be used to obtain information about the momentum of conduction electrons in pure metals at low temperatures was proposed by Kjeldaas[1] in 1959. The circularly polarized shear waves establish circularly polarized electromagnetic fields in the metal which interact with conduction electrons. Boyd and Gavenda[2] first observed this interaction mechanism in pure copper and studied Doppler-shifted cyclotron resonance (DSCR).

The important feature of Doppler-shifted cyclotron resonance is that it predicts the existence of an absorption edge above which the shear waves are not attenuated. This absorption edge is determined by electrons which have the largest value of $m_c\bar{v}_z$ or $\partial S/\partial k_z$, where m_c is the cyclotron mass, $\bar{v}_z$ is the average drift velocity of electron in the direction of the shear wave propagation, S is the cross sectional area of the Fermi surface, and k_z is the electron wave number. For a simple Fermi surface it is easy to predict which electrons are responsible for the absorption edge. In the case of a spherical Fermi surface, the electrons at the limiting point have the maximum $m_c\bar{v}_z$ and, hence, are responsible for the absorption edge. However, with more complicated Fermi surfaces, in particular, those which have necks such as copper, silver, or gold, it is a more complicated matter.

Boyd and Gavenda reported an absorption edge in a copper which was not a result of the limiting point electrons, but rather a group of electrons in the neighborhood of an extremal value of $m_c\bar{v}_z$. This is perhaps the first indication that absorption edge can occur by electrons with k_z less than the maximum value $(k_z)_{max}$ on the Fermi surface. They reported an absorption edge by electrons with k_z=0.45 x 10^8 cm^{-1} for shear waves of frequency ν = 110 Mc/sec propagating in the 001 direction for an applied magnetic field up to 10 kG. They interpreted their experiment by considering electrons in the region I ($k_z < 0.6 \times 10^8$ cm^{-1} in figure 1). They treated the attenuation by electrons near the necks (regions II and III, $k_z >$ 0.6 x 10^8 cm^{-1}) as background noise.

Since the work by Boyd and Gavenda, Jericho and Simpson[3] have conducted similar measurements on copper and reported an absorption edge at a much higher field near 19 kG for about the same shear wave frequency used by Boyd and Gavenda. This absorption edge is possible, if one considers the electrons in the necks of Fermi surface since these electrons have higher $m_c\bar{v}_z$ or $\partial S/\partial k_z$ than the maximum value for the region I. Since there was no detailed information of $\partial S/\partial k_z$ near the neck regions, this large value of magnetic field for the absorption edge has been left unexplained.

Since the above studies with ultrasonic shear waves, Hui[4] first conducted in 1969 studies of Doppler-shifted resonance with helicon waves in copper. His results were compatible with the ultrasonic attenuation measurements by Jericho and Simpson in that he also saw structure in the helicon transmission curves at fields higher than the edge reported by Boyd and Gavenda.

The purpose of this study is to use the Fermi surface model of Halse[5] to recalculate the attenuation of shear waves by electrons in the three different regions, and to see if the attenuation by the electrons in the regions II and III can explain the attenuation curve at high fields up to 32 kG as reported by Jericho and Simpson.

II - THEORY

When an external magnetic field $\vec{B}$ is applied to a metal in the z - direction, the electrons execute an orbit with a cyclotron frequency ω_c which is given by

$$\omega_c = \frac{eB}{m_c C} \tag{1}$$

where e is the charge of an electron and c is the velocity of light. Since the electrons drift along the z - direction with an average of $\bar{v}_z$, the effective frequency ω_e in the sound field experienced by the electrons is

$$\omega_e = \omega\left(\frac{\bar{v}_z}{c_s} - 1\right) \tag{2}$$

where c_s is the velocity of the shear wave propagating through the metal. Equation (2) states the Doppler-shift effect.

In order to facilitate an energy transfer from a shear wave to conduction electrons in metal, the cyclotron frequency and the Doppler-shifted frequency must be the same, i.e.,

$$\omega_c = \omega_e \ . \tag{3}$$

For most electrons $(\bar{v}_z/c_s) \gg 1$, therefore from equations (2) and (3)

$$\frac{\omega \bar{v}_z}{c_s} \simeq \frac{eB}{m_c C} \ . \tag{4}$$

If subharmonic interactions are included, equations (2) and (3) may be rewritten as

$$\frac{q\,\bar{v}_z}{\omega_c} = 4n \pm 1 \,, \quad (n = 0, \pm 1, \pm 2, \ldots) \tag{5}$$

where q is the wave number of the shear wave equal to ω / c_s. Equation (5) has been called the Doppler-shifted cyclotron resonance condition.

An effective energy transfer from the shear wave to electrons requires that the mean free path of the electron ℓ be long compared with the wave length of the shear wave:

$$q\ell > 1 \,. \tag{6}$$

The resonance conditions for an effective energy transfer and hence for an attenuation of the shear wave is given in the expression for the attenuation $A(k_z)$ derived by Boyd and Gavenda.

$$A(k_z) \propto \sum \int_{-(k_z)_{max}}^{(k_z)_{max}} \frac{m_c \tau \, |a_n|^2 \, dk_z}{1 + \tau^2 [q \bar{v}_z - (2n+1)\omega_c]^2} \,, \tag{7}$$

where τ is the relaxation time of the electron. The coefficients a_n are related to the interaction due to the electric field, collision drag, and the deformation potential. The exact calculation of a_n is very involved even if the Fermi surface if known. Therefore, following

Boyd and Gavenda, it is assumed that the subharmonic resonances can be neglected, and a_o varies only slowly with k_z. Then equation (7) reduces to

$$A(k_z) \propto \int_{-(k_z)_{max}}^{(k_z)_{max}} \frac{m_c \tau \, dk_z}{1 + \tau^2 [q\bar{v}_z - \omega_c]^2} \tag{8}$$

Since $\omega_c \tau > 1$, only significant contribution to the attenuation occurs around $q\bar{v}_z = \omega_c$, or when the resonance condition is satisfied.

III - CALCULATION TECHNIQUE

Harrison[6] has shown that

$$m_c \bar{v}_z = - \frac{\hbar}{2\pi} \left(\frac{\partial S}{\partial k_z} \right) \tag{9}$$

Figure 1 shows $\partial S/\partial k_z$ as a function of k_z in the [001] direction for copper. This curve was obtained by Wood[7] by using the Fermi surface parameters given by Halse. Figure 2 shows m_c/m_B as a function of k_z, where m_B is the cyclotron mass for k_z=0. For a given frequency of the shear wave and an external magnetic field, equation (8) can be integrated by using the numerical results of figures 1 and 2 to calculate the attenuation. Three regions are identified and designated as regions I, II, and III in figure 1.

The Simpson's rule was used to numerically integrate equation (8) on a computer. The values of $\partial S/\partial k_z$ and m_c/m_B were read in at $k_z = 0.00239\ \mathrm{A}^{\circ\,-1}$. Three calculations were carried out to compare with the results by Boyd and Gavenda and by Jericho and Simpson.

The first calculation was carried out for the following conditions.

$$\nu = \frac{\omega}{2\pi} = 110 \text{ Mc/sec.}$$

$$c = 3.0 \times 10^{10} \text{ cm/sec.}$$

$$c_s = 3.0 \times 10^{5} \text{ cm/sec.}$$

$$e = 4.803 \times 10^{-10} \text{ esu}$$

$\hbar$ = 1.05 x 10^{-27} erg-sec.

τ = 4.8 x 10^{-11} sec.

m_B = 1.25 x 10^{-27} gram

Substituting the above quantities into equations (8) and (9), the following expression is obtained.

$$A(k_z) \propto \int_0^{1.33} \frac{(m_c/m_B)\, dk_z}{1+0.23\left[\left(3.103\frac{\partial S}{\partial k_z}\Big/\frac{m_c}{m_B}\right)-\left(1.283\, B\Big/\frac{m_c}{m_B}\right)\right]^2} \qquad (10)$$

where B is in kG. It is noted that $(k_z)_{max}$ = 1.33 A^{o-1} is devided into 518 equal intervals. The integration in equation (10) up to the first 244 intervals is the attenuation due to electrons in region I, and between 245 and 382 due to electrons in the region II, and the rest is due to the electrons in the region III.

The second calculation was carried out for ν = 100 Mc/sec. and τ = 10^{-10} sec. Other constants are the same as above. Equation (10) is modified as

$$A(k_z) \propto \int_0^{1.33} \frac{(m_c/m_B)\, dk_z}{1+\left[\left(2.82\frac{\partial S}{\partial k_z}\Big/\frac{m_c}{m_B}\right)-\left(1.283\, B\Big/\frac{m_c}{m_B}\right)\right]^2} \qquad (11)$$

The last calculation was carried out for τ = 4.8 x 10^{-10} sec. and ν = 100 Mc/sec. and all other quantities are the same as the first calculation.

IV - DISCUSSION OF RESULTS

Figure 3 shows attenuation calculated for ν = 110 Mc/sec and $\tau = 0.48 \times 10^{-10}$ sec for B ranges up to 32 kG. The ordinate indicates only the relative magnitude of the attenuation because equation (10) only calculates a quantity which is proportional to some absolute attenuation which must be determined by experiment. There are three lines in Figure 3. The upper line indicates the total attenuation due to electrons in the whole regions I, II and III. The middle line shows the attenuation by electrons in the regions I and II, while the lower curve indicates the attenuation due to the electrons in the region I.

Figure 3 compares with the attenuation measured and calculated by Boyd and Gavenda shown in Figure 4. Both figures 3 and 4 indicate a maximum attenuation around B = 4.6 kG. A sharp drop in attenuation after the peak indicates that the resonance condition is no longer satisfied by the electrons in the region I. This is the absorption edge mentioned by Boyd and Gavenda. However, as can be seen from figure 3, attenuation due to the electrons in the regions II and III is more than just background noise as presumed by Boyd and Gavenda. In fact, the attenuation at high magnetic fields is mostly due to electrons in the regions II and III, as these electrons are more favorably satisfying the resonance condition. The reason that Boyd and Gavenda were able to explain their experimental results with the electrons in the region I is due to the fact that up to the range of magnetic field they considered (10 kG), the general shape of of attenuation curve due to the all electrons is quite similar to that due to the electrons in the region I alone.

Figure 5 shows the attenuation calculated for $\nu = 100$ Mc/sec and $\tau = 1.0 \times 10^{-10}$ sec. The maximum attenuation still occurs at B = 4.6 kG, however, a secondary peak seems to appear at higher magnetic fields. The general shape of the attenuation curve is more sharply peaked than that shown in figure 3. The frequency and the relaxation time used for figure 5 closely approximates the condition of Jericho and Simpson's experiment. Figure 6 shows attenuation curves obtained by Jericho and Simpson for two different experiments. The relaxation time for the sample designated by Cu_{1A} is shorter by a factor of two than that for the case designated by Cu_1. Note that the secondary peak shown in figure 5 is in general agreement with the secondary peak for the sample Cu_{1A}. Even though the relative magnitudes are different, the position in magnetic field is qualitatively correct.

Figure 7 shows the result for $\tau = 5.0 \times 10^{-10}$ sec and $\nu = 100$ Mc/sec. A number of secondary peaks appear in this curve which tends to agree with the experimental results for Cu_1 by Jericho and Simpson. However, the magnitude of attenuation at these secondary peaks is not as pronounced as what was observed experimentally.

It is rather interesting to observe how sensitive the attenuation curve is to the relaxation time. This study also seems to support the fact that the secondary peaks reported by Jericho and Simpson are due to the electrons in the regions II and III. Since the calculated attenuation at high fields is not as pronounced as what has been observed experimentally, further research will be necessary to refine the expression for attenuation by equation (9). This may include consideration of attenuation by higher harmonic resonance and more realistic description of the relaxation time rather than using an average value. The values of a_n may have to be determined by a more careful comparison of theory and experiment.

V - CONCLUSIONS

The attenuation of circularly polarized shear waves by the electrons in the regions II and III is more than a background noise although the electrons in the region I are responsible for the main peak of the attenuation. Furthermore, the attenuation is mostly due to the electrons in the regions II and III of the Fermi surface at high magnetic fields.

The simple model proposed by Boyd and Gavenda for attenuation (equation (8)) is not sufficient to explain the secondary peaks in the attenuation curve observed by Jericho and Simpson. However, the general trend in the shape of attenuation curve could be constructed by choosing appropriate values of relaxation time. It is also fair to say that the secondary peaks are due to the electrons in the regions II and III.

APPENDIX I - BOYD AND GAVENDA APPROXIMATION

Boyd and Gavenda approximated the integration in equation (8) as follows: Since $(\omega_c\tau)\gg 1$, the denominator in equation (8) is very large except near the resonance condition, $\frac{\bar{q}\, v_z}{\omega_c}\simeq 1$. The half width of this resonance denominator is $(\omega_c\tau)^{-1}$ in terms of $\frac{q\, v_z}{\omega_c}$. In terms of $|\partial S/\partial k_z|$ the half width is

$$\left|\frac{\partial S}{\partial k_z}\right|_{\text{half width}} = \frac{2\pi\, m_c}{q\hbar\tau} \tag{12}$$

For a given external magnetic field B, the S/ k_z value corresponding to the resonance condition is given by

$$\left|\frac{\partial S}{\partial k_z}\right|_{\text{resonance}} = \frac{2\pi\, e B C_s}{\hbar\,\omega\, C} \tag{13}$$

Boyd and Gavenda assumed that the electrons in the region defined by $|\partial S/\partial k_z|_{\text{resonance}} \pm 1.5\,|x\, \partial S/\partial k_z|_{\text{half width}}$ contribute to the attenuation of shear wave. This region is then devided into three equal parts. Each of these parts defines a region over k_z from figure 1. Then the area under the curve of m_c/m_B in figure 2 is found for each of the three regions of k_z. The integration of equation (8) is approximated by

$$A(k_z) = 0.4 \times (m_c\Delta k_z)_1 + 0.65 \times (m_c\Delta k_z)_2 + 0.9 \times (m_c\Delta k_z) \tag{14}$$

where $(m_c \Delta k_z)_1$, $(m_c \Delta k_z)_2$, and $(m_c \Delta k_z)_3$ are areas under the curve in figure 2 for the three regions of k_z in the order of decreasing distance from the resonance condition. The factors 0.4, 0.65, and 0.9 come from the resonance denominator integrated over three equal parts of $\partial S/\partial k_z$ region.

This study used this approximation method to calculate the attenuation from figures 1 and 2. The result was essentially the same as that shown in figure 3 (or 5 and 7), which was obtained using more rigorous numerical integrations by equation (10) or (11).

APPENDIX II - ATTENUATION DUE TO SUBHARMONIC RESONANCE

In order to study the effect of subharmonic resonance on the attenuation of shear wave, the coefficients a_n in equation (7) must be known. Since a_n could not be calculated explicitly, it was treated as the same constant for all subharmonics. Then equation (7) is reduced to

$$A(k_z) \propto \sum_{n=0}^{50} \int_{-k_{z\,max}}^{+k_{z\,max}} \frac{m_c \, dk_z}{1+\tau^2\left[q\bar{v}_z-(2n+1)\omega_c\right]^2}, \tag{15}$$

after factoring out a_n from equation (7).

Integration of equation (15) gave an attenuation curve which decreased monotonically with increasing magnetic field. The rate of decreasing attenuation was a little slowed down around b = 5 kG. This exercise indicated that the coefficients in equation (7) can not be factored out, but must be evaluated correctly to obtain correct attenuation from equation (7), when one tries to include effects of subharmonic resonance on the attenuation.

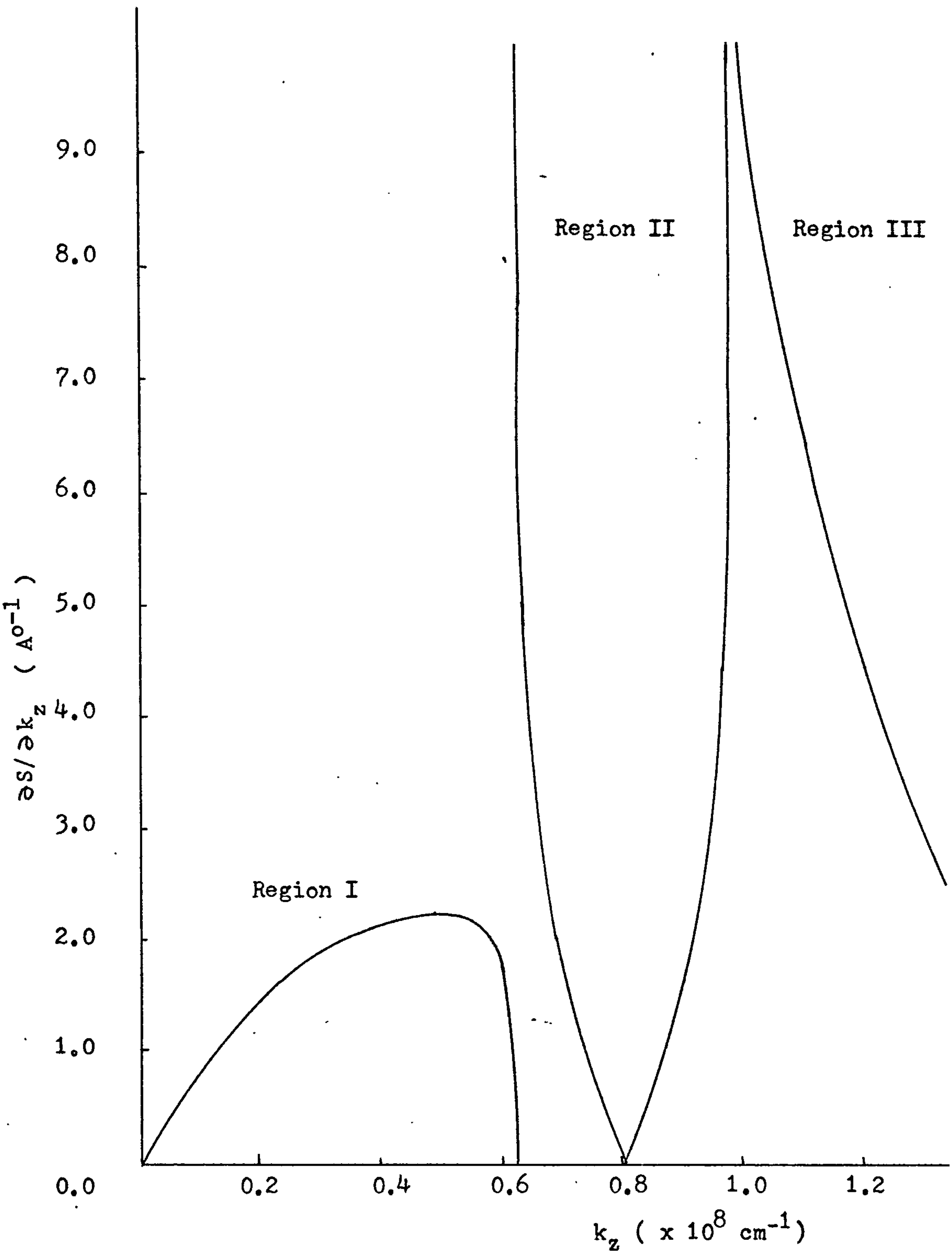

Fig. 1 - $\partial S/\partial k_z$ of the Fermi surface in the [001] direction

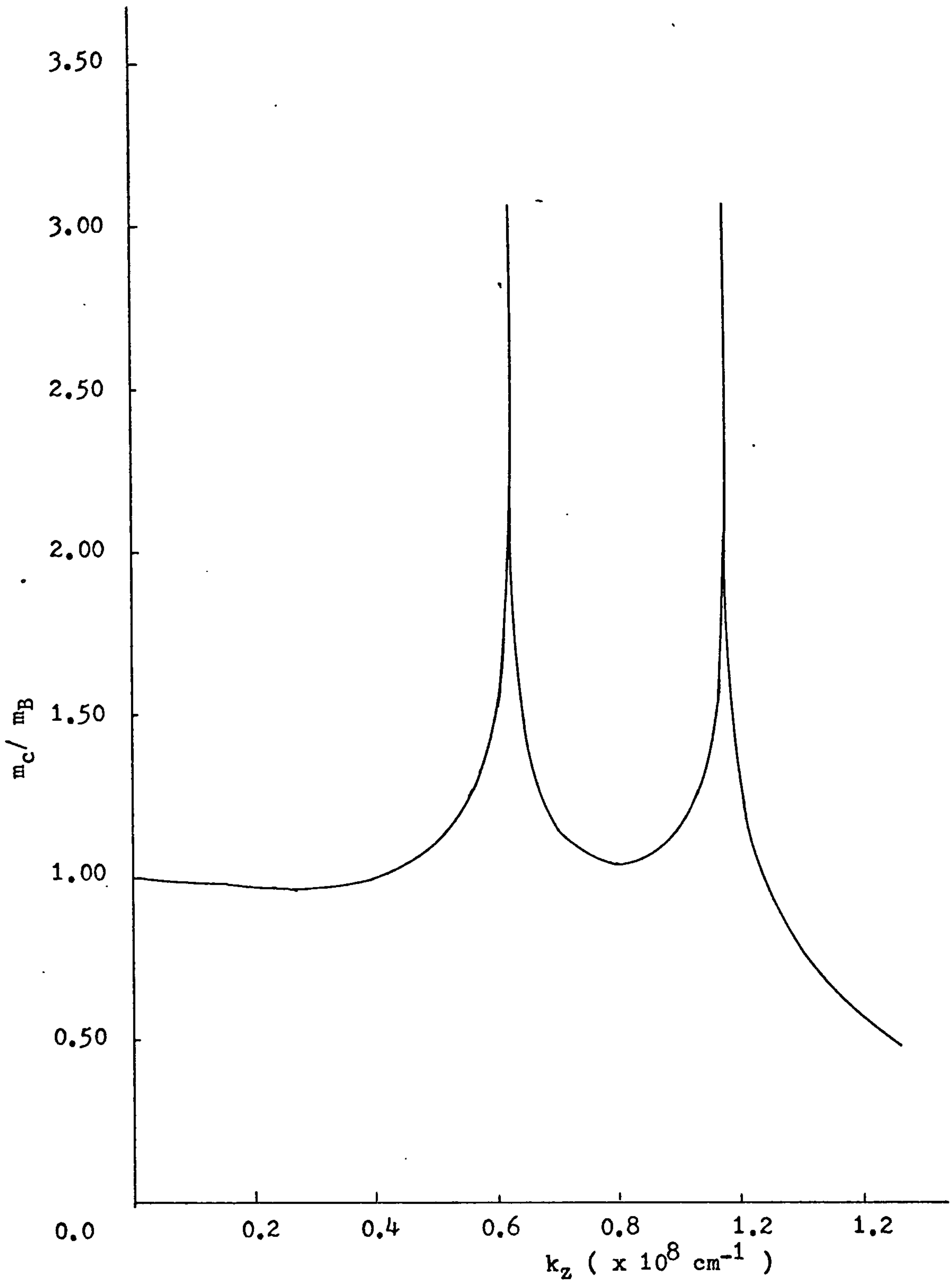

Fig. 2 - Cyclotron mass normalized with respect to m_B for $k_z = 0$

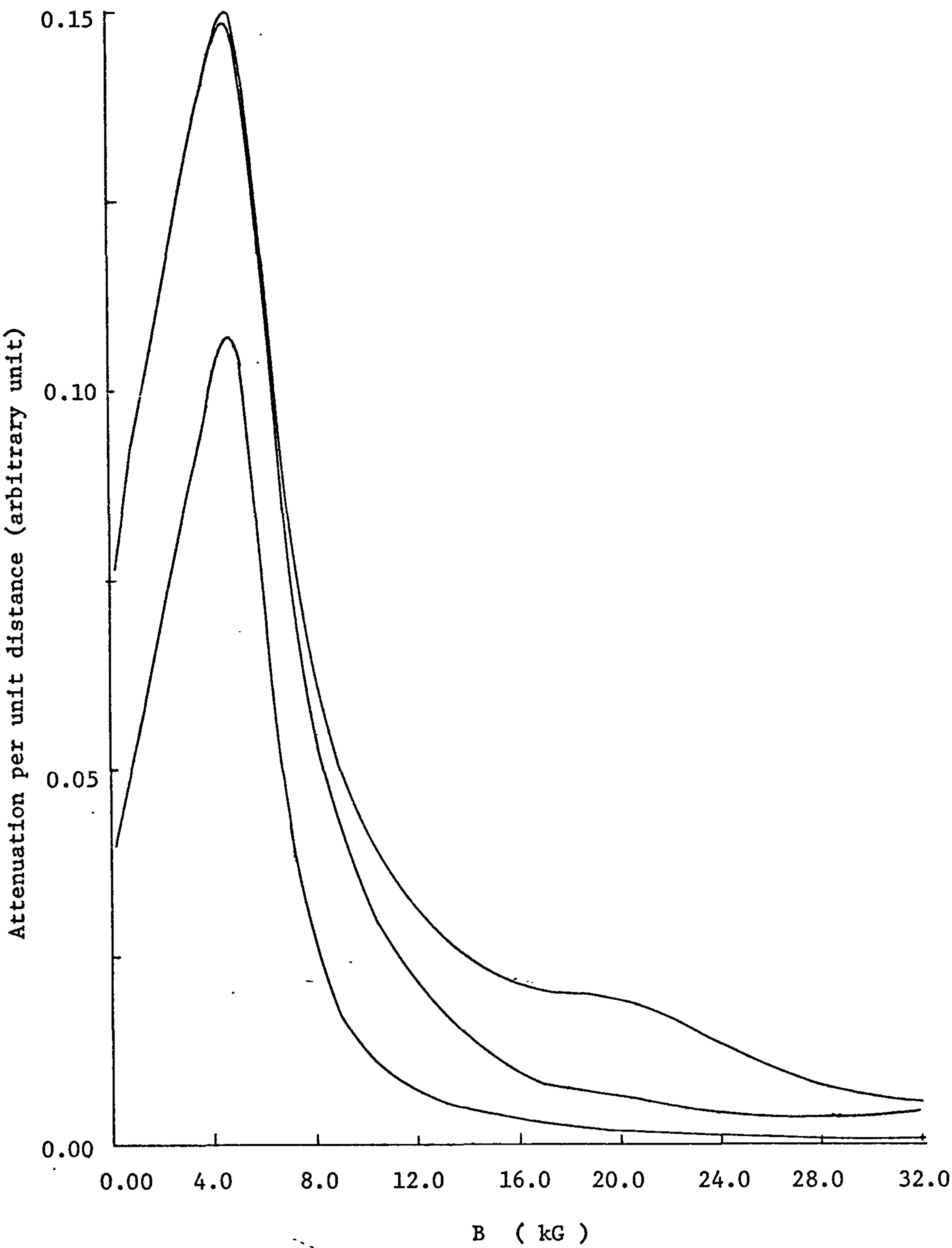

Fig. 3 - Attenuation of shear wave for ν= 110 Mc/sec. and τ= 0.48 x 10^{-10} sec.

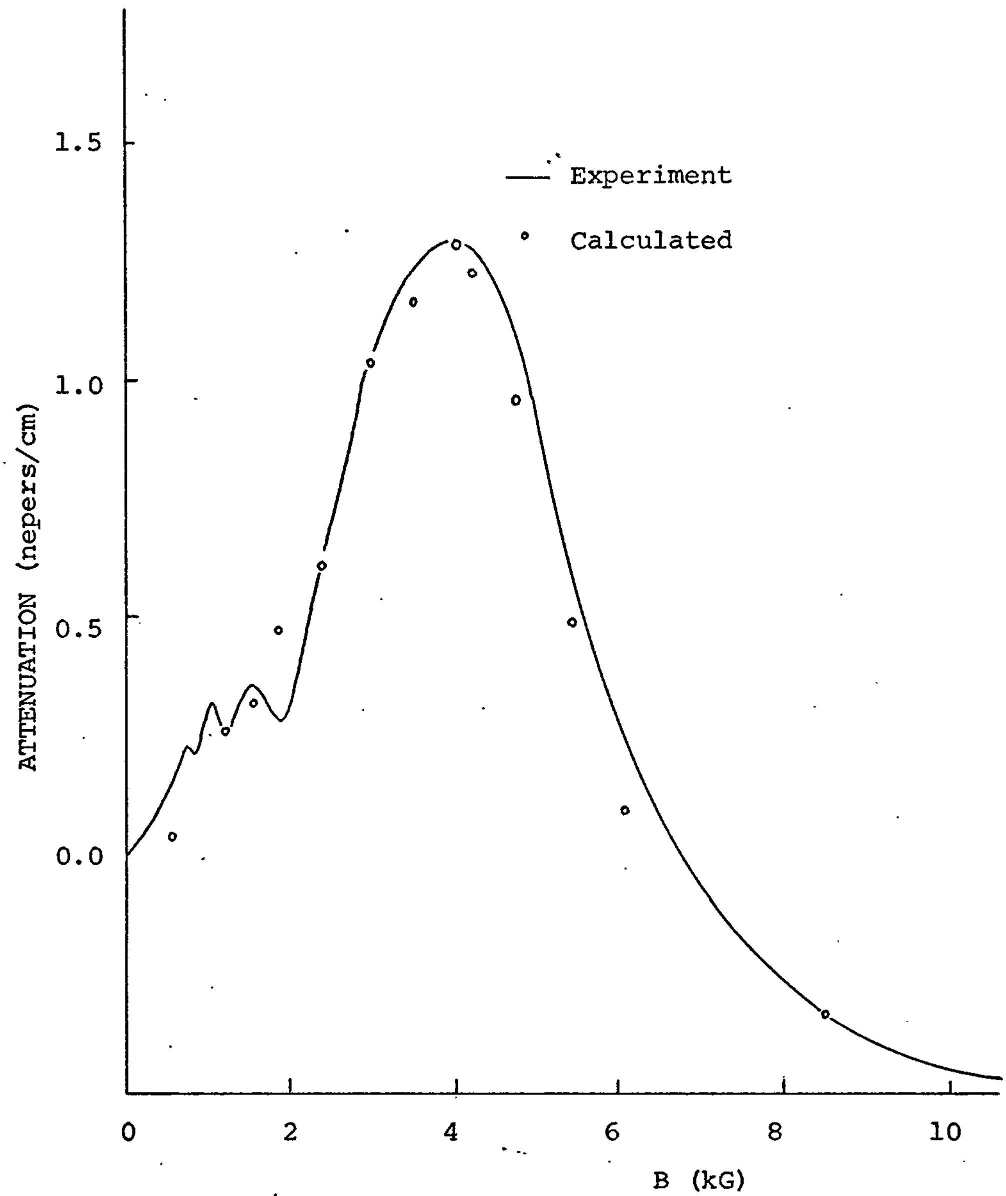

Fig. 4 Attenuation observed by Boyd and Gavenda for 110 Mc/sec.

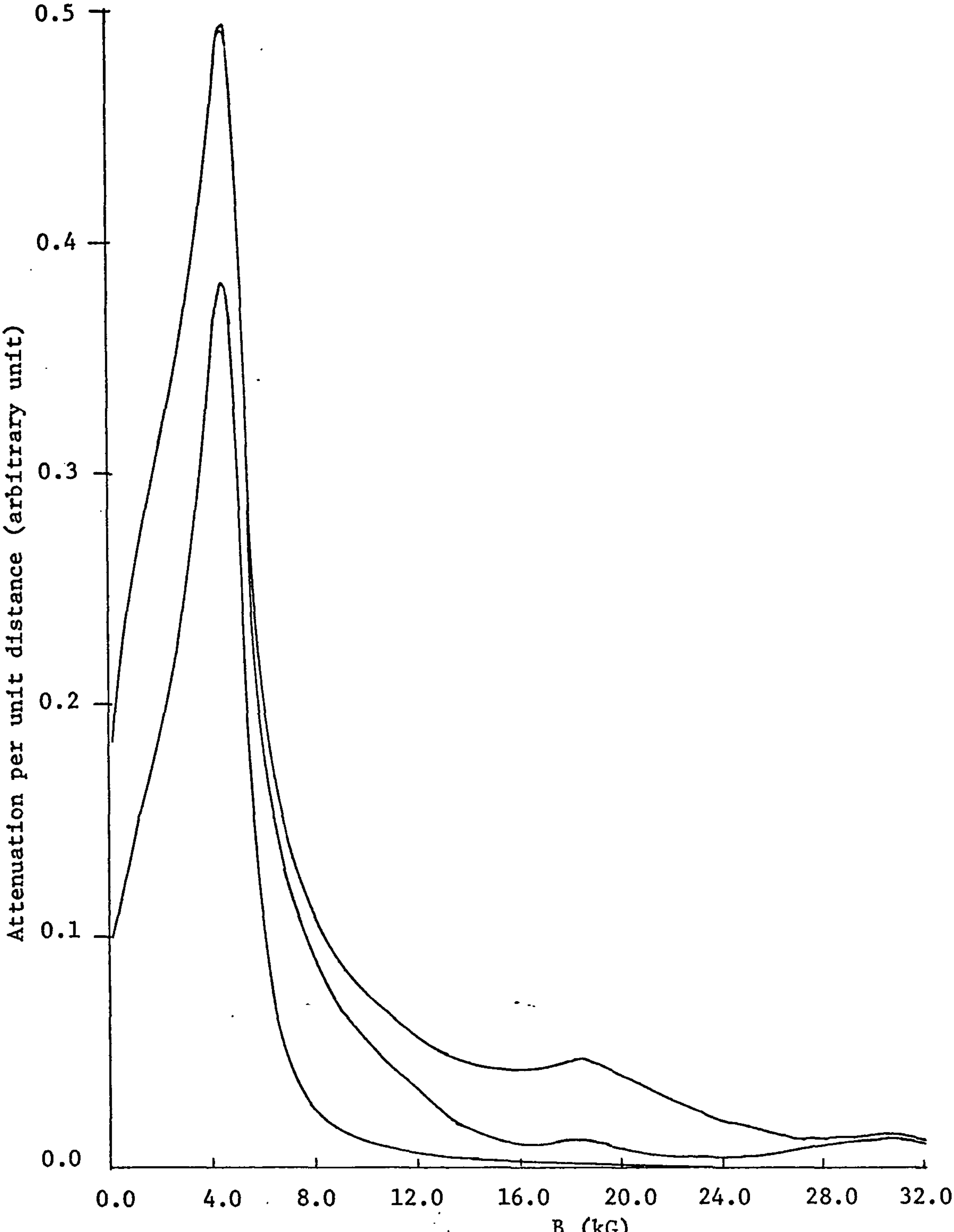

Fig. 5 - Attenuation of shear wave for ν = 100 Mc/sec. and $\tau = 1.0 \times 10^{-10}$ sec.

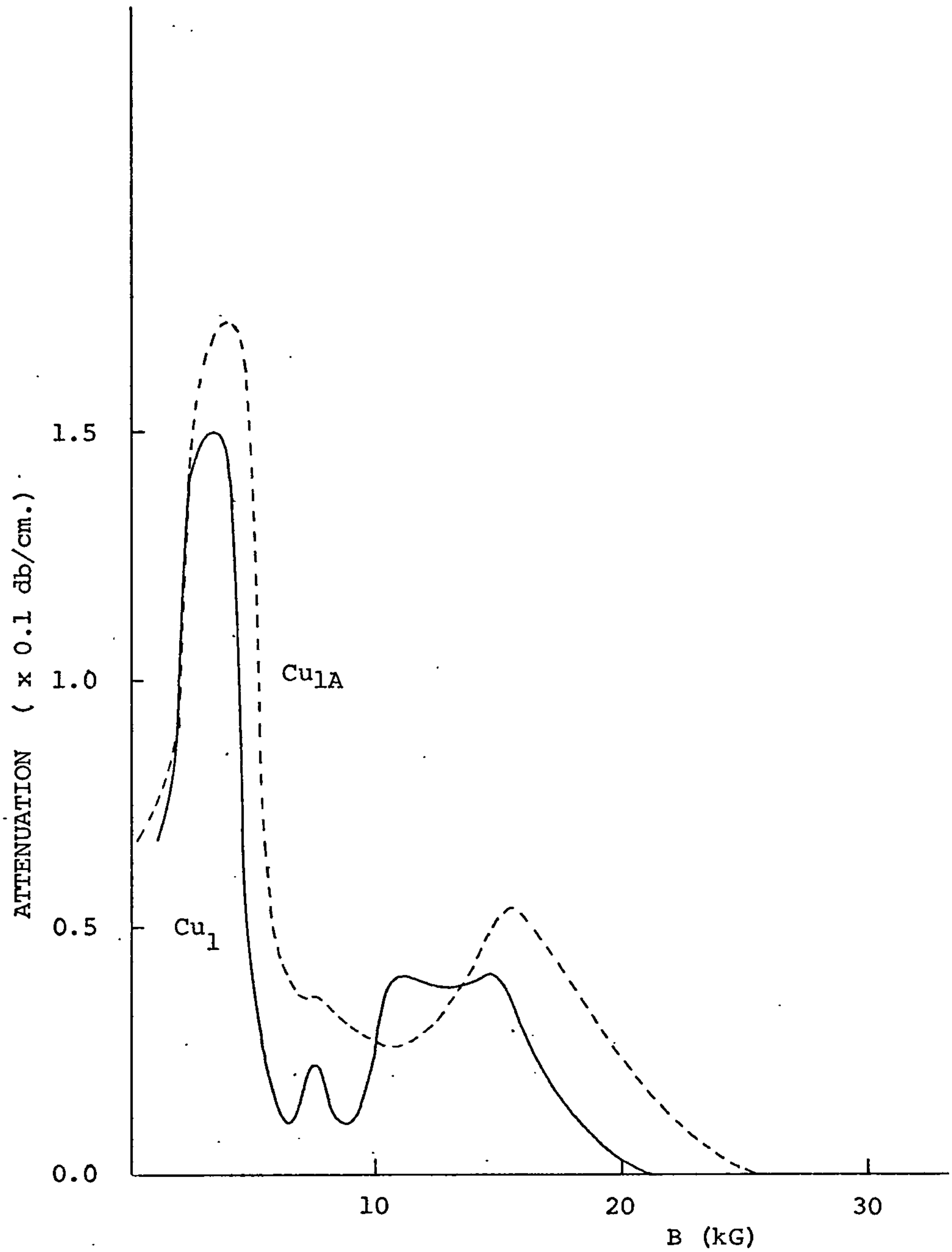

Fig. 6 Attenuation curves obtained by Jericho and Simpson for 100 Mc/sec.

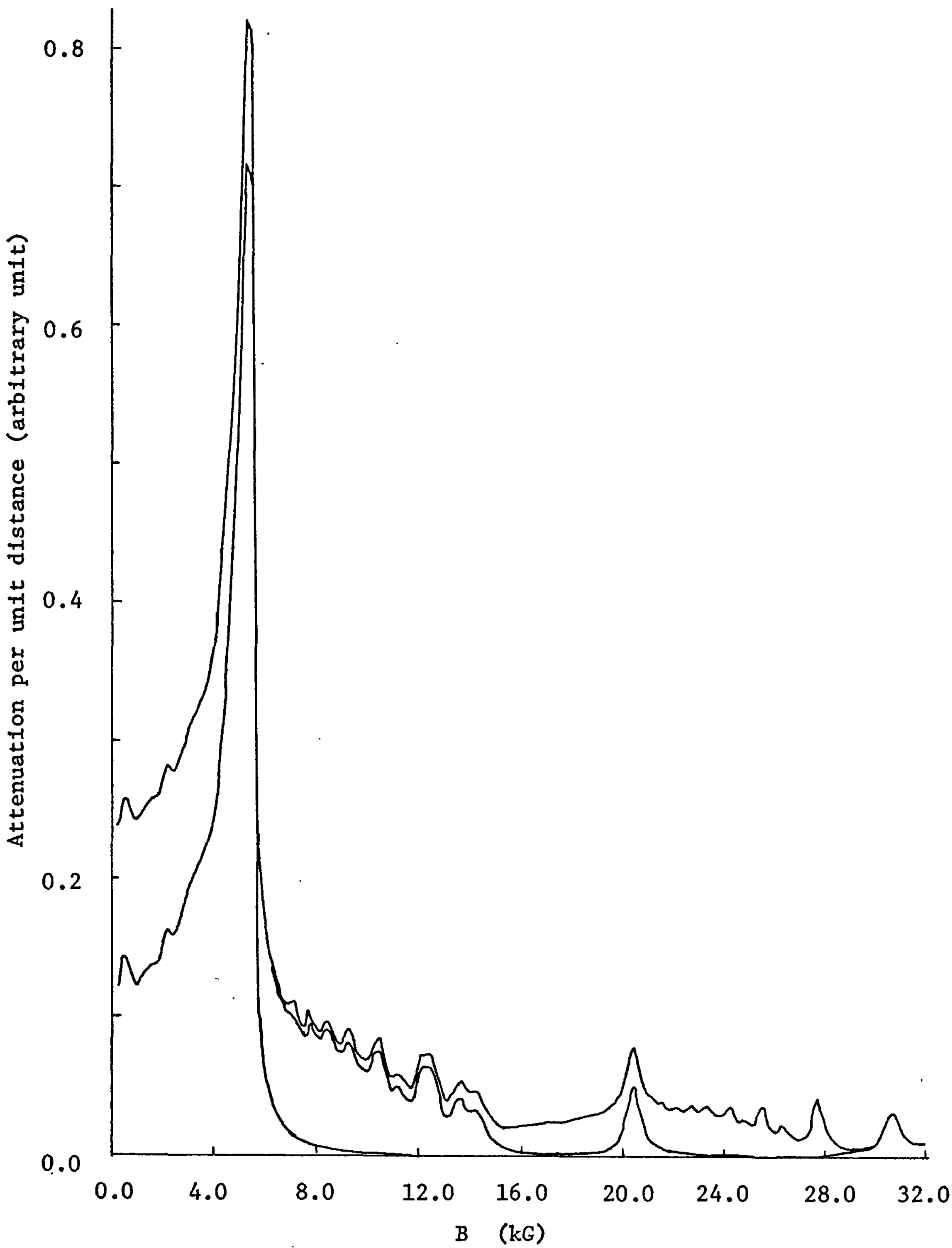

Fig. 7 - Attenuation of shear wave for ν = 100 Mc/sec. and $\tau = 4.8 \times 10^{-10}$ sec.

BIBLIOGRAPHY

1. T. Kjeldaas, Jr., Phys. Rev. 113, 1472 (1959)
2. J. R. Boyd and J. D. Gavenda, Phys. Rev. 152, 645 (1966)
3. M. H. Jericho and A. M. Simpson, Philosophical Magazine, 17, 267 (1968)
4. S. W. Hui, Phys. Rev. 185, 988 (1969)
5. M. R. Halse, Phil. Trans. Roy Soc. London, A265, 507 (1969)
6. W. A. Harrison, Phys. Rev. 118, 1190 (1960)
7. L. T. Wood, Ph.D. Dissertation, University of Texas (1968)