TWO ESSAYS ON CRUDE OIL FUTURES AND OPTIONS MARKETS

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Abstract

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This dissertation consists of two essays on crude oil futures and options markets. The first essay investigates whether aggregate risk aversion and risk premiums in the crude oil market co-vary with the level of speculation. Using crude oil futures and option data, I estimate aggregate risk aversion in the crude oil market and find that it is significantly lower after 2002, when speculative activity started to increase. Using speculation index as a state variable, risk premiums implied by the state-dependent risk aversion estimates confirm the negative correlation between speculative activity and risk premiums, and indicate that risk premiums in the crude oil market are on average lower and more volatile after 2002. These findings suggest that index-fund investors who demand commodity futures for the purpose of portfolio diversification are willing to accept lower compensation for their positions. Estimated state-dependent risk premiums have substantial predictive power for subsequent futures returns and outperform commonly used predictors.

The second essay exams the economic importance of jumps, jump risk premiums, and dynamic jump intensities in crude oil futures and options markets. Existing pricing models for crude oil options are computationally intensive due to the presence of latent state variables. Using a panel data of crude oil futures and options, I implement a class of computationally efficient discrete-time jump models. I find that jumps account for about half of the total variance in crude oil futures and options prices, and a substantial part of the risk premiums is due to jumps. Jumps are large and rare events in crude oil futures and options markets. The main role of jumps and jump risk premiums in crude oil futures and options markets is to capture excess kurtosis in the data. These findings suggest that it is critical to include jumps in pricing models for crude oil futures and options, and there is strong evidence in favor of time-varying jump intensities.

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1.1 Introduction

The commodity market has grown rapidly over the past decade and has become an increasingly important part of the financial market. For exchange-traded commodity derivatives, the Bank for International Settlements (BIS) estimates that the number of outstanding contracts increased from 13.3 millions in December 2003 to 137.4 millions in June 2013.[1] Crude oil futures and options are the most liquid commodity derivatives. In December 2011, WTI and Brent crude oil futures accounted for 51.4% of dollar value of the S&P GSCI commodity index. Understanding the risk preferences and trading activities of investors in the crude oil market is therefore of great interest. It allows us to infer relevant information about investors' expectations, and it is crucial for the purpose of pricing and risk management.

The commodity market has also witnessed structural changes over the last decade. Before the early 2000s, commodity markets were partly segmented from financial markets and from each other (Tang and Xiong, 2012). After 2002, financial institutions started considering commodities as a new asset class to strategically diversify their portfolios. Researchers ascribe this change to the crash in equity market, the negative correlation between commodity returns and stock returns documented in the literature (Greer, 2000; Gorton and Rouwenhorst, 2006; Erb and Harvey, 2006), and the emergence of new financial instruments,

[1] As a comparison, the number of outstanding contracts for exchange-traded equity index increased from 59.0 millions in December 2003 to 96.8 millions in June 2013.

such as long-only commodity index funds (LOCF) (Irwin and Sanders, 2011). According to Tang and Xiong (2012), the total value of various commodity-related instruments purchased by institutional investors increased from an estimated $15 billion in 2003 to at least $200 billion in mid-2008.

Following these structural changes, the literature has been debating if the increase in commodity index investment impacts the level of futures prices or risk premiums. Irwin and Sanders (2010) argue that there is no direct empirical link between index fund trading and commodity futures prices, and that fundamental supply and demand have determined crude oil prices; Hong and Yogo (2012) and Singleton (2011) find that speculative trading activity causes price drifts and predicts futures returns; Hamilton and Wu (2011) document significant changes in risk premiums after 2005, when speculative activity dramatically increased in the crude oil market. To the best of our knowledge, the relationship of the risk aversion of the market participants and the speculation level in the crude oil market has not been studied.

This paper investigates the relationship between the level of speculative activity and the aggregate risk aversion (or risk premiums) in the crude oil market. To motivate my main hypothesis I analyze a stylized model with one commercial hedger and one financial speculator in the crude oil futures market. From the optimal futures positions of the hedger and speculator, I find that the more risk averse a hedger is, the more short futures positions she would hold; while the more risk averse a speculator is, the less long futures positions she would hold. At equilibrium, the model suggests a negative relationship between the speculation level and aggregate risk aversion. As speculation increases, the aggregate level of risk aversion of market participants decreases, and risk premiums decrease accordingly.

The empirical investigation is motivated by this stylized model and focuses on testing the dependence of market risk aversion and risk premiums on speculative activity. Using WTI crude oil futures and option data from the Chicago Mercantile Exchange and traders' position data from the CFTC, I estimate the market risk aversion using a probability density function forecast ability test. Following Bliss and Panigirtzoglou (2004), I first assume the risk aversion parameter is stationary over time and estimate the value of risk aversion by maximizing the forecast ability of subjective density functions which are implied from risk neutral densities and the assumed utility function. The risk premium is inferred from the normalized difference between risk neutral density functions and optimal risk-adjusted physical density functions. I then run subsample estimation and estimate the market risk aversions for high and low speculation periods respectively. I find that the aggregate relative risk aversion estimated for the high speculation period is lower than that estimated for low speculation periods. Risk premiums for the high speculation periods are on average lower than those for low speculation periods as well.

To further test the relationship of market risk aversion and risk premium with traders' speculative activities, I subsequently use the speculation index as a state variable and estimate state-dependent market risk aversion. I find evidence of a negative correlation between the state-dependent market risk aversion and the speculation level. The state-dependent market risk aversion is positive on average; however, after 2002, as speculation increases, risk aversion is more volatile and decreases over time. Occasionally, the state-dependent market risk aversions are negative. With state-dependent risk aversion, implied risk premiums are more volatile and on average lower after 2002. When the market risk aversion is negative, we have negative risk premiums.

The findings on risk premiums are similar to those of Hamilton and Wu (2011), who use a very different modeling approach. My results are consistent with their interpretation that index-fund buyers who demand commodity futures for portfolio diversification are willing to accept much lower risk compensation, or would even pay a premium. Usually, commercial hedgers who hold futures positions need to hedge their price risks and would like to pay a premium to their counterparty. The financial traders who take the other side of the contract will receive this premium. However, as more and more financial institutions regard commodities as a new asset class and invest in the commodity market to diversify their portfolios, it is possible that they would want to pay a premium for their speculative positions.

To the best of my knowledge, this is the first paper to estimate the risk aversion coefficient in the crude oil market. Pan (2011) investigate investor befliefs and state price densities in the crude oil market and find that investors assign higher state prices to negative returns when there are higher dispersion of beliefs and the increase in speculation reinforces this effect. Other related literature that estimates the representative agent's degree of risk aversion is mainly focused on the equity index market (Ait-Sahalia and Lo 2000; Jackwerth 2000; Ait-Sahalia, Wang, and Yared 2001; Rosenberg and Engle 2002). This paper fills the gap by estimating the risk aversion coefficient in the crude oil market and documents the evolution of aggregate risk aversion level as the market structure changes.

This paper also contributes to the existing literature by inferring risk premiums in the crude oil market and studying their properties. Estimated state-dependent risk premiums are negatively correlated with the speculation level. I further test the ability of state-dependent risk premiums and other predictive variables, such as lagged futures returns, lagged volatility, and the speculation index to forecast subsequent futures returns. Risk premiums implied by state-dependent market risk aversion have significant explanatory power in predicting next period's futures returns, and their predictive power is higher than that of other commonly used predictors.

The rest of the paper proceeds as follows. In Section 2 I present a model with a hedger and a speculator in the crude oil futures market and develop our main hypothesis. Section 3 introduces the methodology used to estimate risk aversion and risk premiums. Section 4 discusses the data. Section 5 reports the main results and discusses the properties of the estimated state-dependent risk premiums. Section 6 concludes.

1.2 The Model

Assuming the existence of one commercial hedger and one financial speculator in the crude oil futures market, I study an equilibrium model and investigate its implications. This model builds on Duffie and Jackson (1990) who consider only one agent, the hedger, in the futures market.

1.2.1 The Hedger

The model includes a hedger (or commercial trader), who is directly exposed to the underlying crude oil commodity and uses futures to hedge the price risks.

Let $B = (B^1, \ldots, B^N)$ denote a Standard Brownian Motion in R^N which is a martingale with respect to the agent's filtered probability space. The spot price of crude oil is given by

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dB_t \tag{1.1}$$

where μ_t and σ_t are the mean and variance process of crude oil spot returns at time t, with μ_t is 1-dimentional and σ_t is $(1 \times N)$-dimensional.

Assume there are K futures contracts available for trade. The futures prices are given by a K-dimensional Ito process F_t

$$\frac{dF_t}{F_t} = m_t d_t + v_t dB_t \tag{1.2}$$

where m_t and v_t are the mean and volatility process for the futures contracts at time t, with m_t is K-dimentional and v_t is $(K \times N)$-dimensional.

The hedger's total wealth is the sum of the terminal value of a fixed portfolio of spot market assets and the terminal value of a margin account on a futures trading position. It is given by

$$dW_t^{\theta_h} = \pi_{h,t} dS_t + dX_t^{\theta_h} \tag{1.3}$$

where $\pi_{h,t}$ is the hedger's physical position in crude oil at time t and $X_t^{\theta_h}$ is the margin account with

$$X_t^{\theta_h} = \int_0^t e^{r(t-u)} \theta_{h,u} dF_u \tag{1.4}$$

where $\theta_{h,t} = (\theta_{h,t}^1, \ldots, \theta_{h,t}^K)$ is the futures position strategy of the hedger at time t. A positive number for the hedger's physical position ($\pi_{h,t} > 0$) means that crude oil is in net supply, while a negative number ($\pi_{h,t} < 0$) means crude oil is in net demand. Similarly, a positive number for the hedger's futures position ($\theta_{h,t} > 0$) represents long positions in futures and a negative one ($\theta_{h,t} < 0$) represents short positions.

The hedger's problem is

$$\max_{\theta_h} E[U(W_T^{\theta_h})] \tag{1.5}$$

Assume the hedger's utility function takes the exponential form and her relative risk aversion at time t is γ_t^h, we can solve for hedger's optimal futures position

$$\theta_{h,t} = -\frac{(v_t v_t')^{-1}}{F_t}[v_t \sigma_t' \pi_{h,t} S_t - m_t/\gamma_t^h] \tag{1.6}$$

The proof is provided in the Appendix. The hedger's optimal futures position is the same as that of the single agent model in Duffie and Jackson (1990).

1.2.2 The Speculator

Now consider a speculator in this market who trades with the hedger for financial profits. A speculator (or financial trader) is the one who is not directly engaged in trading the crude oil spot commodity and instead uses crude oil futures for the purpose of marking financial profit. The speculator does not hold the spot commodity. Her margin account is

$$X_t^{\theta_s} = \int_0^t e^{r(t-u)} \theta_{s,u} dF_u \tag{1.7}$$

where $\theta_{s,t} = (\theta_{s,t}^1, \ldots, \theta_{s,t}^K)$ is the speculator's futures position strategy at time t.

The speculator's total wealth is $dW_t^{\theta_s} = dX_t^{\theta_s}$, so her maximization problem is

$$\max_{\theta_s} E[U(W_T^{\theta_s})] \tag{1.8}$$

Similar to solving the hedger's problem, I assume exponential utilities for the speculator and denote her relative risk aversion as γ_t^s. Her optimal futures position is given by

$$\theta_{s,t} = -\frac{(v_t v_t')^{-1} m_t}{F_t \gamma_t^s} \tag{1.9}$$

1.2.3 Equilibrium

Market clearing requires $\theta_{h,t} + \theta_{s,t} = 0$. This implies

$$m_t = \frac{v_t \sigma_t' \pi_{h,t} S_t}{\frac{1}{\gamma_t^h} + \frac{1}{\gamma_t^s}} \tag{1.10}$$

Define the degree of aggregate absolute risk aversion of the representative agent, or the market risk aversion, as an average of the population degree of risk aversion. For simplicity, I assume they have equal weights[2]

$$\Gamma_t \equiv \gamma_t^h + \gamma_t^s \tag{1.11}$$

Substituting (1.10) and (1.11) back into (1.6) and (1.9), we get

$$\begin{aligned} \theta_{s,t} &= \frac{S_t (v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\gamma_t^h + \gamma_t^s} \\ &= \frac{S_t (v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\Gamma_t} \end{aligned} \tag{1.12}$$

$$\theta_{h,t} = -\frac{S_t (v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\Gamma_t} \tag{1.13}$$

Equations (1.12) and (1.13) solve equilibrium optimal positions in futures for the speculator and hedger respectively.[3] From them I obtain the following implications:

1. In equilibrium, traders' absolute positions in futures contracts are proportional to the covariance between the futures prices and spot prices, $v_t \sigma_t'$. A high covariance term indicates that futures contracts provide a good hedge, suggesting a high demand for hedging. When

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[2] This can be easily generalized to a weighted average of the risk aversion of the hedger and the speculator. This generalization does not change our main conclusions.

[3] Pareto optimal allocations are always possible in competitive economies with complete securities markets. When a market is incomplete, it typically fails to make the optimal asset allocation. A competitive equilibrium in an incomplete market is generally constrained suboptimal.

the prices of futures contracts and the spot commodity are perfectly correlated, a risk averse ($\gamma_t^h > 0$) producer ($\pi_{h,t} > 0$) would like to short futures contracts ($\theta_{h,t} < 0$) to hedge her price risk and a risk averse speculator ($\gamma_t^s > 0$) would take the other side of the contract ($\theta_{s,t} > 0$).

2. The absolute values of traders' positions in futures contracts ($|\theta_{h,t}|$ and $|\theta_{s,t}|$) are proportional to the hedger's net spot position $\pi_{h,t}$. The more physical crude oil is in net supply ($\pi_{h,t} > 0$ and $\pi_{h,t}$ increases), the more short futures positions the hedger would take for hedging purposes ($\theta_{h,t} < 0$ and $|\theta_{h,t}|$ increases), and the more long futures positions the speculator would take to offset the hedger's position ($\theta_{s,t} > 0$ and $\theta_{s,t}$ increases). On the other hand, the more physical crude oil in net demand ($\pi_{h,t} < 0$ and $|\pi_{h,t}|$ increase), the more long futures positions are required by hedgers for hedging purposes ($\theta_{h,t} > 0$ and $\theta_{h,t}$ increases), and the more short futures positions the speculator would take to offset the hedger's position ($\theta_{s,t} < 0$ and $|\theta_{s,t}|$ increases).

3. Traders' futures positions are negatively related to their level of risk aversion. The more risk averse a hedger is (γ_t^h increases), the more short futures positions she would like to hold ($\theta_{h,t} < 0$ and $|\theta_{h,t}|$ increases); while the more risk averse a speculator is (γ_t^s increases), the less long futures positions she would hold ($\theta_{s,t} > 0$ and $\theta_{s,t}$ decreases). Assume everything else equals, as the risk aversion of the speculator (γ_t^s) decreases, the market risk aversion (Γ_t) would decreases accordingly, and the speculative activities ($\theta_{s,t}$) would increase.

In the model, the level of market risk version is directly related to the market participants' trading positions. The equilibrium optimal position suggests a negative relationship between market risk aversion and speculative activity. As we observe more speculative activity in the crude oil market in recent years, we expect a lower aggregate market risk aversion.

Motivated by this model, I hypothesize that market risk aversion is state-dependent. When there is more speculation, market risk aversion is low. I will test this hypothesis in the empirical analysis below.

1.3 Estimating Market Risk Aversion

It would be interesting to estimate the risk aversion coefficients of the hedger and speculator separately, but this necessitates strong assumptions. Instead, I estimate the risk aversion coefficient of the representative agent, or the aggregate market risk aversion. I explain the methodology used to perform this estimation in 3.1. Two utility functions used in the empirical analysis are discussed in 3.2. In 3.3, I extend this methodology to allow estimation of state-dependent market risk aversion.

1.3.1 Estimating Constant Market Risk Aversion

According to asset pricing theory, the risk neutral density function is related to the objective density function by the representative investor's utility function. The representative agent's risk aversion is embedded in the utility function, given certain conditions such as complete and frictionless markets and a single asset (Ait-Sahalia and Lo, 2000; Jackwerth, 2000; Bliss and Panigirtzoglou, 2004; Christoffersen, Heston, and Jacobs, 2013). I estimate aggregate risk aversion by considering a representative agent in the crude oil market, and assuming that her wealth can be represented by the overall price level of the crude oil market.[4]

Assume that the representative agent's utility function is $U(.)$. At time t, the price of a crude oil futures contract maturing at time T, $F_{t,T}$, is given by

$$F_{t,T} = E\left[\beta \frac{U'(F_T)}{U'(F_{t,T})} F_T\right] \tag{1.14}$$

where $F_T = F_{T,T} \equiv S_T$ is the spot price at expiration, and β is the impatience factor.

[4] One may wonder if the assumption of a representative agent in the crude oil market is valid since this market is generally regarded as incomplete. Duffie (2001) shows that if the gradient of an agent's utility function at an optimal-consumption process exists and is smooth-additive, the calculations for the representative agent can be repeated for each agent, and the market level of risk aversion can be calculated using equation (2.11). A model can have a representative agent when agents differ but act in such a way that the sum of their choices is mathematically equivalent to the decision of one individual or many identical individuals.

Under the risk neutral measure,

$$F_{t,T} = E^Q \left[\exp(-r(T-t))F_T\right] \tag{1.15}$$

Defining the pricing kernel, $\xi(F_T) \equiv \frac{f^Q(F_T)}{f(F_T)}$, where $f(.)$ and $f^Q(.)$ are the physical and risk neutral density functions, we obtain

$$\xi(F_T) \equiv \frac{f^Q(F_T)}{f(F_T)} = \frac{\alpha U'(F_T)}{U'(F_{t,T})} \tag{1.16}$$

where $U'(.)$ is the marginal utility function and α is a constant.

From equation (1.16), given any two of the following three: the risk neutral density function, the physical density function, and the pricing kernel (or utility function), we can infer the third. For example, if we know the risk neutral probability density function, one can either assume pricing kernel or utility function to imply a physical density, or make an assumption on the physical density to infer the pricing kernel.

Estimating the representative agent's degree of risk aversion has a long history in the equity index market. The methodology in most studies is to separately estimate the risk neutral density from options prices and the objective (or statistical) density function from historical prices of the underlying asset. Use these two separately derived functions to infer the pricing kernel, and then draw conclusions for the implied utility function or risk aversion coefficient (Ait-Sahalia and Lo 2000; Jackwerth 2000; Ait-Sahalia, Wang, and Yared 2001; Rosenberg and Engle 2002).

Cross-sections of option prices have been widely used to estimate implied risk neutral probability density functions. These risk neutral probability density functions represent forward-looking forecasts of the distributions of prices of the underlying asset at a single point of time. The physical density function is more challenging to estimate. One cannot independently estimate a time varying statistical density from a time series of prices without imposing an a priori structure. For example, Jackwerth (2000) uses one month of daily return data and calculates 31-day, non-overlapping returns from sample. Ait-Sahalia and Lo (2000) use a relatively long series of overlapping returns to estimate the actual distribution.

Christoffersen, Heston, and Jacobs (2013) obtain a conditional density by standardizing the monthly return series by the sample mean and the conditional one-month variance on that day.

Bliss and Panigirtzoglou (2004) point out that these studies impose assumptions of stationarity on the statistical density function or the parameters of the underlying stochastic prices, which are not implied or required by the theory. They propose a different approach that assumes the risk aversion parameter is stationary over the sample period and estimates the value of risk aversion by maximizing the forecast ability of subjective PDFs, which are implied from risk neutral PDFs and the assumed utility function (or pricing kernel).

In this paper, I estimate the market risk aversion parameter by adopting Bliss and Panigirtzoglou's (2004) PDF forecast ability method. If investors are rational, their subjective density forecasts should correspond, on average, to the distribution of realizations. The risk aversion coefficient in the utility function provides a measure of the degree of risk aversion of the representative investor in the crude oil market.

I now describe the estimation procedure for market risk aversion in more detail. Option prices embed risk neutral PDFs. Breeden and Litzenberger (1978) show that the risk neutral PDF for the value of the underlying asset at option expiry, $f(S_T)$, is related to the European call price by

$$f^Q(S_T) = e^{r(T-t)}\frac{\partial^2 C(S_t, K, t, T)}{\partial K^2}|_{K=S_T} \tag{1.17}$$

where S_t is the current value of the underlying asset, K is the option strike price, and $T-t$ is the time to expiry.

In the case of the crude oil derivatives data, I use the semi-parametric approach first introduced in Ait-Sahalia and Lo (1998) and follow the implementation of Christoffersen, Heston, Jacobs (2013) and Pan (2011). The risk neutral density for the spot price at the maturity date T is given by

$$\widehat{f}^Q(F_T|F_t) = e^{r(T-t)}\frac{\partial^2 \widehat{C}(F_{t,T}, K, t, T, \sigma(K,T))}{\partial K^2}|_{K=F_T} \tag{1.18}$$

where $\sigma(K,T)$ is the Black (1976) implied volatility.

Given an estimated risk neutral density function and a utility function, the implied subjective density function is

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{\xi(F_T)}}{\int \frac{\widehat{f}^Q(x)}{\xi(x)}dx} = \frac{\frac{U'(F_t)}{\lambda U'(F_T)}\widehat{f}^Q(F_T)}{\int \frac{U'(F_t)}{\lambda U'(x)}\widehat{f}^Q(x)dx} = \frac{\frac{\widehat{f}^Q(F_T)}{U'(F_T)}}{\int \frac{\widehat{f}^Q(x)}{U'(x)}dx} \tag{1.19}$$

I then use the Berkowitz (2001) probability density function forecast ability test to estimate the market risk aversion coefficient and infer physical probability density functions, as in Bliss and Panigirtzoglou (2004). Berkowitz (2001) proposes a parametric methodology for jointly testing uniformity and independence of the density functions. He defines a transformation, z_t, of the inverse probability transformation, y_t, using the inverse of the standard normal cumulative density function, $\Phi(.)$:

$$z_t = \Phi^{-1}(y_t) = \Phi^{-1}\int_{-\infty}^{X_t} \widehat{f}_t(s)ds \tag{1.20}$$

Under the null hypothesis, $\widehat{f}_t(.) = f_t(.)$, $z_t \sim i.i.d\ N(0,1)$. Berkowitz (2001) tests the independence and standard normality of the z_t by estimating the following equation using maximum likelihood:

$$z_t - \mu = \rho(z_{t-1} - \mu) + \varepsilon_t \tag{1.21}$$

I test restrictions on the estimated parameters using a likelihood ratio test. Under the null, the parameters of this model should be: $\mu = 0, \rho = 0$, and $Var(\varepsilon_t) = 1$. Denoting the log-likelihood function as $L(\mu, \sigma^2, \rho)$, the likelihood ratio statistic is

$$LR = -2[L(0,1,0) - L(\widehat{\mu}, \widehat{\sigma}^2, \widehat{\rho}) \tag{1.22}$$

which will be distributed $\chi^2(3)$ under the null hypothesis.

1.3.2 Two Utility Functions

First, I consider the power utility function

$$U(F_T) = \frac{F_T^{1-\Gamma} - 1}{1-\Gamma} \tag{1.23}$$

where Γ is the measure of market relative risk aversion (MRRA), $\Gamma = MRRA = -\frac{F_T U''(F_T)}{U'(F_T)}$. The corresponding pricing kernel is

$$\xi(F_T) = \frac{\alpha U'(F_T)}{U'(F_{t,T})} = \alpha \left(\frac{F_T}{F_{t,T}} \right)^{-\Gamma} \tag{1.24}$$

Substituting equation (1.24) into (1.19), I get

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{F_T^{-\Gamma}}}{\int \frac{\widehat{f}^Q(x)}{x^{-\Gamma}} dx} \tag{1.25}$$

I first choose an initial value of Γ, and then maximize the forecast ability of the resulting subjective probability density functions by maximizing the p-value of the Berkowitz LR statistic with respect to Γ. The forecast ability test of physical density functions gives out an estimate of the market relative risk aversion, Γ, and the corresponding risk-adjusted physical probability density functions.

Second, I use the exponential utility function throughout this paper for comparison. The exponential utility function is given by

$$U(F_T) = -\frac{e^{-\eta F_T}}{\eta} \tag{1.26}$$

where η is the market absolute risk aversion (MARA) with $\eta = MARA == -\frac{U''(F_T)}{U'(F_T)}$.The market relative risk aversion is ηF_T. The pricing kernel under the exponential utility is

$$\xi(F_T) = \frac{\alpha U'(F_T)}{U'(F_{t,T})} = \alpha e^{-\eta(F_T - F_{t,T})} \tag{1.27}$$

Substituting equation (1.27) into (1.19), I get the physical density function as

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{e^{-\eta F_T}}}{\int \frac{\widehat{f}^Q(x)}{e^{-\eta X}} dx} \tag{1.28}$$

1.3.3 Estimating State-Dependent Market Risk Aversion

The market risk aversion estimation approach introduced in Section 3.1 assumes the risk aversion parameter is stationary over the sample period and does not allow for state dependence of market risk aversion. However, Bliss and Panigirtzoglou (2004) document that the implied relative risk aversion in the equity market is volatility-dependent. Christoffersen, Heston, and Jacobs (2013) also develop a model with a variance-dependent price kernel and find a negative variance premium. As for the crude oil market, according to the analysis in Section 2, I am interested in investigating the possibility that financialization has changed the structure of the crude oil market. Therefore I make further assumptions and allow the market risk aversion coefficient to change over time.

To estimate the state-dependent market risk aversion, I assume that the market risk aversion is a linear function of a state variable x_t, which measures the level of speculative activity. For the case of the power utility function, I assume

$$\Gamma_t = a + b(x_t - \overline{x_t}) \tag{1.29}$$

where Γ_t is the state-dependent market relative risk aversion at time t. a is the average level of the overall market relative risk aversion. The slope coefficient, b, determines the variation of market relative risk aversion with the state variable x_t.

Similarly in the case of exponential utility, I assume

$$\eta = a + b(x_t - \overline{x_t}) \tag{1.30}$$

where η is the state-dependent market absolute risk aversion at time t.

I estimate the values of coefficients a and b by running the Berkowitz (2001) density forecast ability test by maximizing LR p-values for the adjusted physical density functions in the case of power and exponential utilities respectively.

1.4 Data

1.4.1 Crude Oil Futures and Options Data

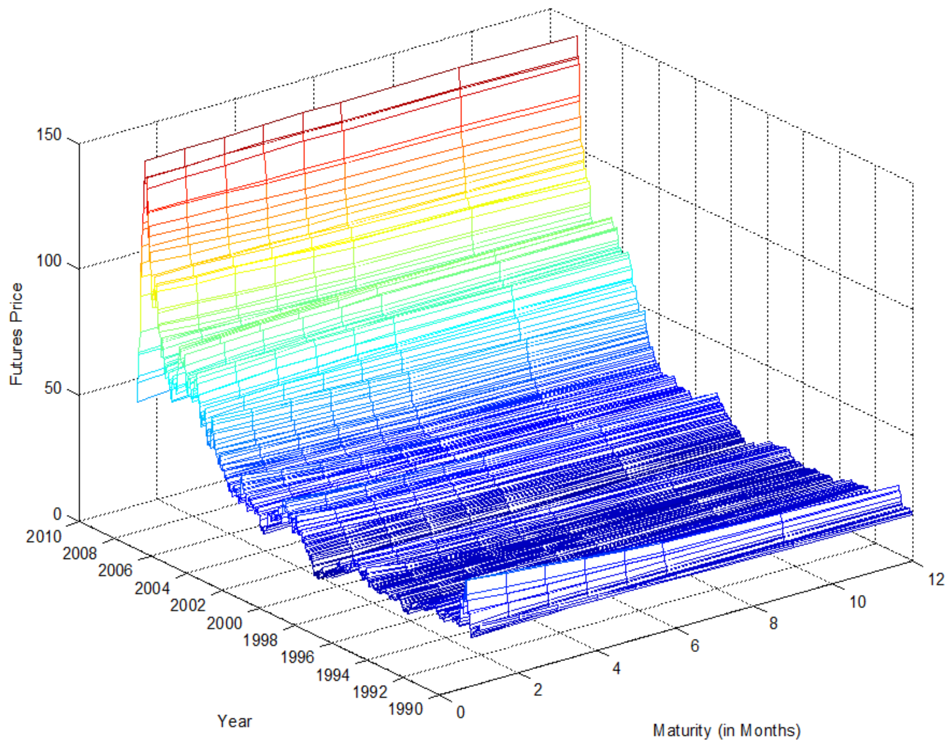
The Chicago Mercantile Exchange (CME group, formerly NYMEX) crude oil derivatives market is the world's largest and most liquid commodity derivatives market. The range of maturities covered by futures and options and the range of option strike prices are also greater than for other commodities (Trolle and Schwartz, 2009). I use a data set of WTI crude oil futures and options contracts traded on the CME from January 2nd, 1990 to December 3rd, 2008.

Futures contracts were screened based on patterns in trading activity. Open interest for futures contracts tends to peak approximately two weeks before expiration. Among futures and options with more than two weeks to expiration, the first six monthly contracts tend to be very liquid. For contracts with maturities over six months, trading activity is concentrated in the contracts expiring in March, June, September, and December. Due to these liquidity patterns, I filter the futures and options data as follows: I retain all futures contracts within six weeks or fewer days to expiration; among the remaining, I retain the first five monthly contracts (M2-M6); beyond that, I choose the first two contracts with expiration either in March, June, September or December (Q1-Q2).

Figure 1-1 plots the prices of the filtered futures contracts with maturity from one month up to one year (M1-M6 and Q1-Q2). All prices in this paper are settlement prices.[5] To avoid cluttering of the figure, only the futures term structure on Wednesdays is displayed. From Figure 1-1, we can observe that futures prices have increased dramatically since 2003 and subsequently declined after July 2008. The prices of long maturity futures contracts, e.g., Q2 futures contracts, are lower on average than that of short maturity futures contracts, e.g., M1 futures contracts. Generally speaking, the crude oil market is in backwardation, consistent with existing studies (Trolle and Schwartz, 2009; Litzenberger and Rabinowitz,

[5] The CME light, sweet crude oil futures contract trades in units of 1000 barrels. Prices are quoted in US dollars per barrel.

Figure 1-1: Futures Prices



Notes to Figure: I plot the prices of futures contracts maturing in 1, 2, 3, 4, 5, 6, 9, and 12 months (M1-M2 and Q1-Q2 futures contracts). The data spans 4,753 trading days from January 2, 1990 to December 3, 2008. To avoid cluttering the figure, I only display the futures term structures for Wednesdays.

1995). It is worth noticing that in recent years, especially after 2005, the frequency of the crude oil market being in backwardation decreases gradually. Using one month (M1) futures contracts as a proxy for the spot prices, the one year futures contracts (Q2) are strongly backwardated 70.7% of the time before January 2005 and strongly backwardated 50.3% of the time after 2005. According to the normal backwardation theory in Keynes (1930), when the market is backwardated, we expect the risk premium to be positive; while when the market is in Contango, we expect a negative risk premium.

To screen the options data, I first retain options on filtered futures contracts above. Because trading in options markets is asymmetrically concentrated in at-the-money and out-of-the-money strikes, and the spline algorithm will not accommodate duplicate strikes in the data, I discard in the money options. Options which are impossible to compute an implied volatility (usually far-away-from-the-money options quoted at their intrinsic value), or options with implied volatilities of greater than 100 percent, are also discarded. If there are fewer than five remaining usable strikes in a given cross-section, the entire cross-section is discarded. Only those options that have open interest in excess of 100 contracts and options with prices larger than 0.10 dollars are considered. In addition, I exclude those observations with Black (1976) implied volatility less than 1% or greater than 100%.

Crude oil futures contracts expire on the third business day prior to the 25th calendar day (or the business day right before it if the 25th is not a business day) of the month that precedes the delivery month. Options written on futures expire three business days prior to the expiration date of futures. A target observation date is then determined for horizons of 1, 2, 3, 4, 5, and 6 weeks; 2, 3, 4, 5, 6, and 9 months; and 1 year by subtracting the appropriate number of days (weekly horizons) or months (monthly and 1-year horizon) from the expiration date, according to Bliss and Panigirtzoglou (2004). If there are no options traded on the target observation date, the nearest options trading date is determined. If this nearest trading date differs from the target observation date by no more than 3 days for weekly horizons or 4 days for monthly and 1-year horizons, that date is substituted for the original target date. If no sufficiently close trading date exists, that expiry is excluded from the sample for that horizon. Table 1.1 reports the summary statistics of the filtered options contracts for each forecast horizon.

Table 1.1: Crude Oil Option Data Summary Statistics

Forecast Horizon	**1w**	**2w**	**3w**	**4w**	**5w**	**6w**	**2m**	**3m**	**4m**	**5m**	**6m**	**7m**	**8m**	**9m**	**1y**
No. of Cross-sections	180	212	220	226	225	226	218	215	221	215	177	173	156	135	148
No. of Contracts / Cross-section	17.19	22.82	26.84	29.81	31.13	32.34	29.45	27.59	25.46	21.05	3706	3129	2331	1769	15443
Implied Volatility	0.41	0.4	0.4	0.4	0.39	0.39	0.37	0.34	0.33	0.32	0.3	0.29	0.28	0.27	0.3
Option Price	0.29	0.4	0.49	0.57	0.65	0.71	1.38	1.61	1.81	1.82	1.97	2.07	2.13	2.3	1.99
Strike Price	55.35	55.69	55.62	54.74	53.18	52.46	52.54	47.76	48.34	44.73	43.81	44.5	42.08	41.07	42.96
Futures Price	55.02	54.78	54.83	53.9	53.05	52.55	52.98	49.52	51.18	47.54	46	47.23	44.85	43.43	45.37
Open Interest (OI)	5233	5471	5311	5011	4519	4144	3190	2323	2215	1740	1684	1779	1559	1422	1814
Volume	695	476	410	429	341	257	181	112	64	44	36	30	29	32	72
Volume / OI	0.13	0.09	0.08	0.09	0.08	0.06	0.06	0.05	0.03	0.03	0.02	0.02	0.02	0.02	0.04

Notes to Table: I report the summary statistics of filtered crude oil options contracts for forecast horizons of 1, 2, 3, 4, 5, and 6 weeks; 2, 3, 4, 5, 6, and 9 months; and 1 year. A target observation date is identified by subtracting the appropriate number of days (weekly horizons) or months (monthly and 1-year horizons) from the expiration date, according to Bliss and Panigirtzoglou (2004). If there are no options traded on the target observation date, the nearest options trading date is determined. If this nearest trading date differs from the target observation date by no more than 3 days for weekly horizons or 4 days for monthly and 1-year horizons, that date is substituted for the original target date. If no sufficiently close trading date exists, that expiry is excluded from the sample for that horizon.

This data consists of American options on crude oil futures contracts. CME has also introduced European-style crude oil options, however, the trading history is much shorter and liquidity is much lower than the American options. Since the option pricing formula is designed for European options, we have to convert the American option prices to European option prices. I convert the American option prices to European option prices using the method in Trolle and Schwartz (2009). It consists of inverting the Barone-Adesi and Whaley (1987) formula for American option prices which yields a log-normal implied volatility, from which we can subsequently obtain the European option price using the Black (1976) formula.

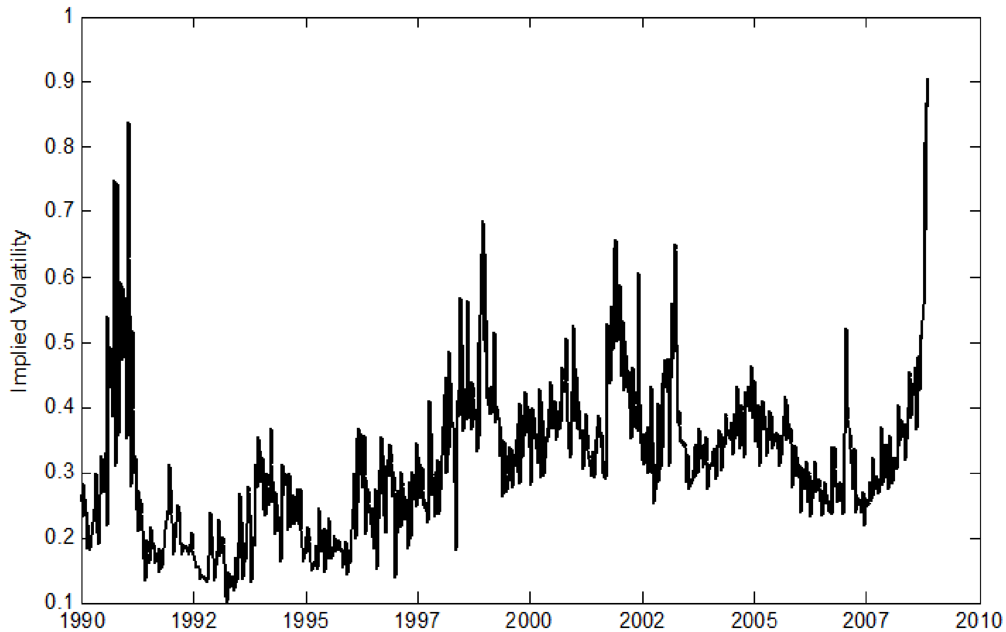
Figure 1-2 plots the implied ATM volatility of options on futures contracts maturing in four weeks. The large spikes in the option implied volatilities appear around the end of 1990 and beginning of 1991 (which is the time of the first Gulf War), the September 2001 terrorist attack, the second Gulf War in March 2003, and during the financial crisis in 2008.

1.4.2 Trading Position Data

The Commodity Futures Trading Commission (CFTC) publishes trading positions of commercial hedgers and financial speculators twice every month before September 30, 1992 and once every week since then. Futures positions can be found in the futures only Commitments of Traders (COT) report. As defined in the report, hedgers are those investors who have direct exposure to the underlying crude oil commodities and use crude oil futures for hedging purposes, and speculators are those investors who are not directly engaged in the underlying crude oil commodities but use derivatives markets for the purpose of financial profits. This definition is in accordance with that proposed in the model of Section 2 and thus can be used as a measure of the trading position of crude oil market participants. Starting in 2006, the CFTC began to report positions of traders in a finer category: commercials, managed money, commodity dealers, and others. The fundamental distinction among traders as to whether they have physical attachment in the crude oil market or not still holds (Pan, 2011).

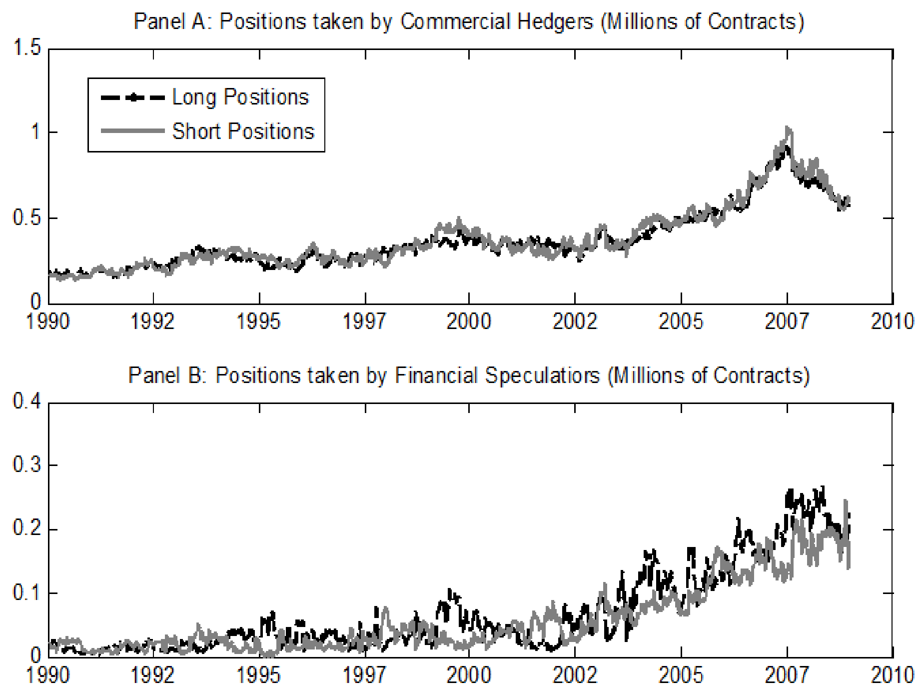
Figure 1-3 shows long and short positions taken by hedgers and speculators in the

Figure 1-2: Implied Volatility of ATM Futures Options with Maturity of One Month



Notes to Figure: I plot the implied volatility of filtered ATM options on futures contracts maturing in 1 month. The data spans 4,753 trading days from January 2, 1990 to December 3, 2008.

Figure 1-3: Traders' Futures Positions

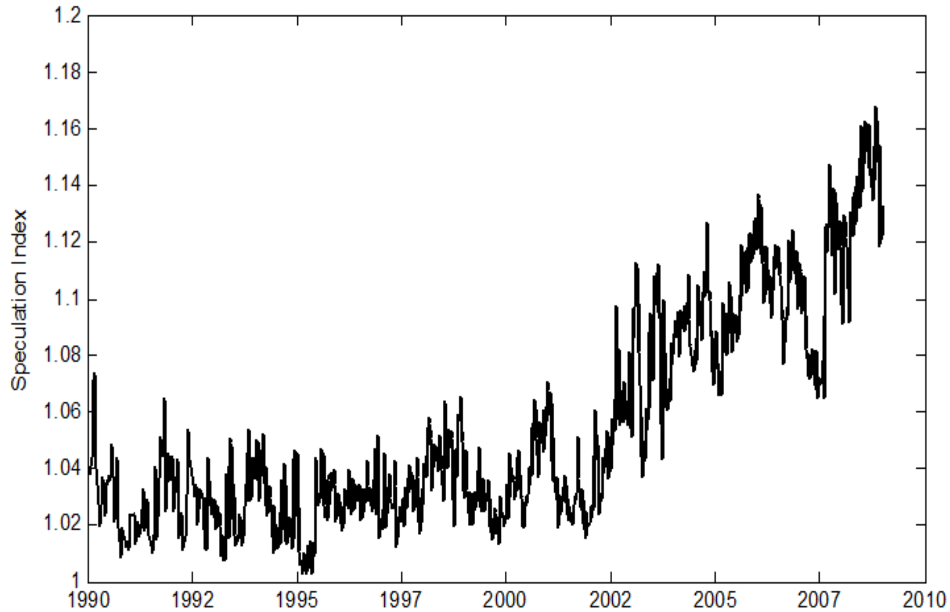


Notes to Figure: I plot futures positions taken by commercial hedgers and financial speculators from January 1990 to December 2008. Positions of hedgers and speculators are from the U. S. Commodity Futures Trading Commission's (CFTC) futures only Commitments of Traders (COT) report.

futures market, which are obtained from the CFTC futures-only Commitments of Traders (COT) report. Although participation in the futures market by hedgers and speculators has experienced steady growth from 190 onwards, the increase of positions has been faster since 2004. While both positions of hedgers and speculators increase over time, speculators take relatively more long positions than short positions. Since late 2007, positions taken by hedgers have gradually decreased while the speculators' positions kept increasing.

To measure the level of speculative activities, I use the speculation index (SI) which is designed to gauge the intensity of speculation relative to hedging (Working, 1960; Buyuksahin and Robe, 2010; Buyuksahin and Harris, 2011; Pan, 2011). If we denote SS (SL) as

Figure 1-4: Speculation Index



Notes to Figure: I plot the speculation index defined in equation (2.14).

the speculator's short (long) positions, and HS (HL) as the hedger's short (long) position, the speculation index is defined as

$$SI_t = \begin{cases} 1 + \frac{SS_t}{HL_t + HS_t}, & if \;\; HS_t \geq HL_t, \\ 1 + \frac{SL_t}{HL_t + HS_t}, & if \;\; HS_t < HL_t \end{cases} \tag{1.31}$$

The speculation index measures the extent by which speculative positions exceed the necessary level to offset hedging position. For instance, a 1.1 speculation index means that there are 10% more speculative positions than what is needed to offset the hedging demand.

Figure 1-4 plots the speculation index defined in equation (1.31) from 1990 to 2008. This time series reveals that there has been a high level of speculation in recent years. Before 2002, the speculation index was around 1.05; however it has risen steadily over time to 1.16 in 2008. It suggests that speculative activities in excess of hedging needs in the crude oil market have increased since 2002.

1.5 Estimation Results

I first estimate the market risk aversion using the probability density forecast methodology for the whole sample and report the results in 5.1. Using estimated market risk aversion, I infer the risk premiums from the risk adjusted densities and the risk neutral densities in 5.2. In 5.3, I estimate the state-dependent market risk aversion and infer state-dependent market risk premiums. I interpret the empirical findings by relating it with the model implication in Section 2 and explain the impact of the increased financialization on the risk aversion and risk premiums in 5.4. A comparison of the predictability of state-dependent risk premiums and other commonly used predictors is discussed in 5.5.

1.5.1 Estimation of Market Risk Aversion

I first fit Black (1976) implied volatilities of the cross-sectional option data at a given day as a second order polynomial function of strike price and maturity following Pan (2011). Then I construct a grid of strike prices and obtain at-the-money (ATM) Black (1976) implied volatilities from the fitted polynomial function for each maturity. With these implied volatilities, I back out call prices $\widehat{C}(F_{t,T}, K, t, T, \sigma(K,T))$ on the desired grid of strike prices, and then calculate the risk neutral density in equation (1.18) for the futures price at the maturity date T.

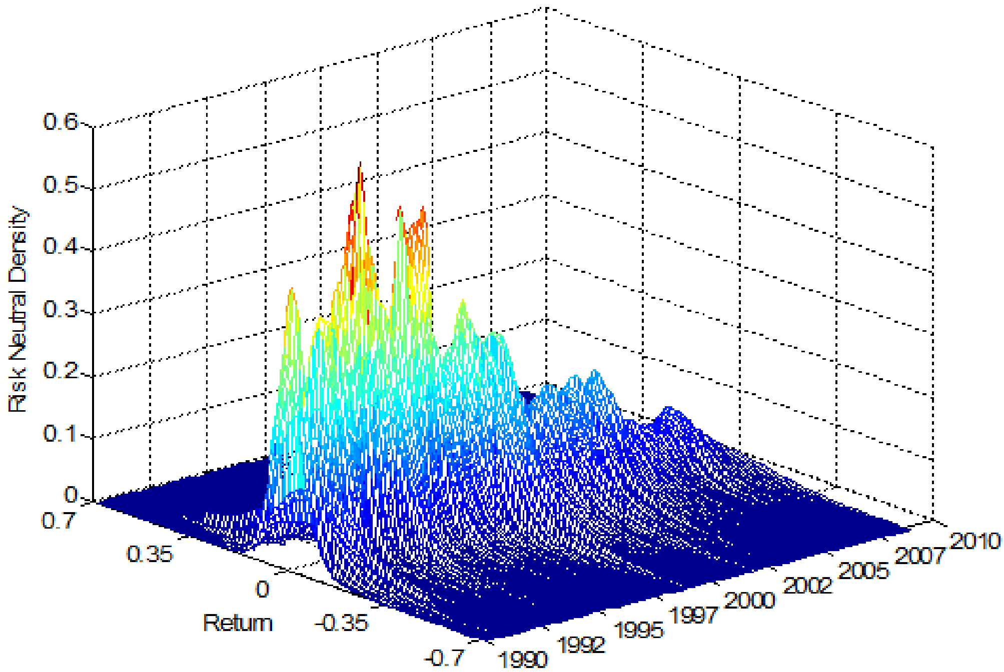
Defining futures return as $R_{t,T} = \log\left(\frac{F_T}{F_{t,T}}\right)$, where $F_{t,T}$ is the time t price of a futures contract maturing at time T, and $F_T = F_{T,T} \equiv S_T$.

The density function of futures returns over the period of $T-t$ is

$$\widehat{f}^Q(R_{t,T}|F_t) = \widehat{f}^Q(S_t \exp(u)|F_t) \times S_t \exp(u) \tag{1.32}$$

Figure 1-5 plots the risk neutral density functions of futures returns maturing in four weeks, which are calculated from options prices according to equations (1.18) and (1.32). Risk neutral density functions in Figure 1-5 show a strong pattern of stochastic volatility. The variances in the risk neutral density function path are larger for the crisis periods such

Figure 1-5: Risk Neutral Probability Density Functions of Futures Returns with Maturity of Four Weeks



Notes to Figure: I plot the risk neutral density functions of futures returns maturing in four weeks, which are calculated from options prices according to equations (1.18) and (1.32).

as late 1990 to early 1991, September 2001, March 2003, and second half of 2008. These are consistent with the implied volatility patterns plotted in Figure 1-2.

Using the risk neutral probability densities, I test the corresponding forecast ability by running the Berkowitz (2001) likelihood ratio (LR) test , as in equation (1.22). The p-values of the Berkowitz (2001) LR statistic and coefficient estimates in (1.21) for different horizons are reported in Table 1.2, Panel A. The p-values reported in Table 1.2, Panel A for the two weeks and six weeks horizons are smaller than 5%. It suggests that for two weeks and six weeks horizons, I reject the hypothesis that the risk neutral PDFs provide accurate forecasts of the futures distribution.

Table 1.2: Estimates of Market Risk Aversion

Forecast Horizon	**1 week**	**2 weeks**	**3 weeks**	**4 weeks**	**5 weeks**	**6 weeks**
n	181	213	221	227	226	227
Panel A. Risk Neutral Density						
p-value	0.2497	0.0407	0.4214	0.3093	0.0610	1.17E-06
μ	0.0932	0.0948	0.1407	0.1283	0.1127	0.1104
ρ	0.1018	0.1403	0.0223	0.0548	0.1113	0.2964
σ	1.0222	0.9922	1.0215	0.9747	0.9952	0.9933
Panel B. Physical Density Calculated Using Power Utility						
p-value	0.4967	0.0712	0.9679	0.8239	0.1337	4.55E-06
μ	0.0058	0.0014	0.0011	0.0028	0.0066	0.0013
ρ	0.1054	0.1796	0.0188	0.0606	0.1545	0.3339
σ	1.0262	1.0041	1.0200	1.0111	1.0086	0.9951
MRRA (Γ)	2.0262	1.1807	1.3856	1.1877	0.8759	1.0009
Panel C. Physical Density Calculated Using Exponential Utility						
p-value	0.4005	0.0615	0.9363	0.7215	0.1107	2.81E-06
μ	0.0266	0.0149	0.0077	0.0168	0.0220	0.0238
ρ	0.1124	0.1823	0.0241	0.0699	0.1579	0.3372
σ	1.0303	1.0054	1.0249	1.0161	1.0103	0.9962
MARA (Γ')	0.0409	0.0255	0.0367	0.0291	0.0196	0.0195
MRRA	1.5360	0.8839	1.2604	0.9879	0.6670	0.6620

Notes to Table: I present the results of the modified Berkowitz test of the risk neutral and subjective probability density functions to forecast the futures distribution of the prices of the underlying asset. Probability density functions are constructed by adjusting the risk neutral probability density functions using the appropriate utility function. Risk aversion parameters in the utility function were selected to maximize the Berkowitz likelihood ratio (LR) statistic. I report the p-value of the LR test for i.i.d normality of the inverse-normal transformed inverse-probability transforms of the realizations.

Assuming an initial value of Γ (for power utility function) or η (for exponential utility function), I then maximize the forecast ability of the resulting subjective probability density functions in equation (1.25) or (1.28) by maximizing the p-values of the Berkowitz LR statistic with respect to Γ (or η). This procedure gives the estimate of the Γ (or η).

Panel B and Panel C in Table 1.2 report Berkowitz LR statistic p-values and the estimates of the market risk aversion coefficients from power-utility and exponential-utility adjusted density functions. The p-values of the Berkowitz LR test of physical densities are all much higher than that of the risk neutral density functions. It means the risk adjusted physical density functions have better forecast ability than risk neutral probability density functions.

For forecast horizons of one to five weeks, all the physical probability density functions have significant forecast ability. For the six week forecast horizon, both p-values of the Berkowitz LR test implied by the power utility function and the exponential utility function are lower than 5%, suggesting that both of them reject the null hypothesis. Rejection of the Berkowitz LR test does not necessarily suggest that they cannot provide an accurate density forecast. It may attribute to the overlapping forecast period and violating the independent hypothesis. This could be verified by the large correlation coefficient, ρ. For example, the estimated correlation coefficient of the physical probability density function implied by the power utility function is 0.33, which is far apart from the null hypothesis $\rho = 0$. Similar properties hold for risk neutral density functions and physical density functions implied by the exponential utility.

For forecast horizons one to five weeks, the estimated market relative risk aversions implied by the power utility function are 2.03, 1.18, 1.39, 1.19, and 0.88, respectively. The estimates in the case of exponential utility are absolute risk aversions. For this case, the means of market relative risk aversions are also calculated and reported at the bottom of the Table 1.2. They are 1.54, 0.88, 1.26, 0.99, and 0.67, respectively for the one to five weeks forecast horizons. These numbers, on average, are lower than most of the relative risk aversion coefficients documented in the equity index market.[6] There are several possible explanations for this finding: one is that commercial hedgers have crude oil commodity as a natural hedge, thus they are relatively less risk averse than the investors in the equity market; another possible explanation is that financial investors who long crude oil futures for the purpose of portfolio diversification may require less premium to compensate their risk exposure in the crude oil market.

The process of maximizing the Berkowitz statistic with respect to the market risk aversion does not provide a measure of whether the resulting risk aversion coefficient is significantly different from zero. The process of searching for the optimal level of Γ (or η) alters the distribution of the test statistic, biasing the likelihood ratio toward unity and thus overstating the p-value. To investigate the properties of the estimation procedure and the significance of the resulting estimates, I run the bootstrap test to check the distribution of the estimates, following Bliss and Panigirtzoglou (2004).

The bootstrap test captures the impact of the actual data and potential model misspecification on the reliability of parameter estimates. I apply the bootstrap test for each option horizon with M=1,000 replications. Each replication consists of drawing with replacement a random sample of pairs of densities and associated outcomes from the original sample. Each bootstrap sample was then used to estimate the risk aversion coefficient and the p-value of the Berkowitz LR statistic.

Since bootstrapping invalidates the independence assumption underlying computation of p-values, the distribution of bootstrap p-values is uninformative. However, the distribution of risk aversion coefficients provides an indication of the sampling variation of these estimates.

Figure 1-6 gives the box plot of distributions of market risk aversion coefficients from

[6] For example, Bliss and Panigirtzoglou (2004) find that the relative risk aversion implied by power utility function using S&P 500 data are between 3.37 to 9.52; Guo and Whitelaw (2001) estimate a coefficient of relative risk aversion of 3.52; Ait-Sahalia and Lo (2000) report a value of 12.7.

bootstrap tests.[7] Figure 1-6.A plots the market relative risk aversion coefficients implied by the power utility function and Figure 1-6.B plots the market absolute risk aversion coefficients implied by the exponential utility function. From these figures, I conclude that market risk aversion estimates are significantly different from zero across all forecast horizons and for both power and exponential utilities.

Table 1.3 reports bootstrap estimation statistics. Consistent with the results in Figure 1-6, the estimated market risk aversion coefficients are significantly different from zero for all forecast horizons.

1.5.2 Implied Risk Premiums

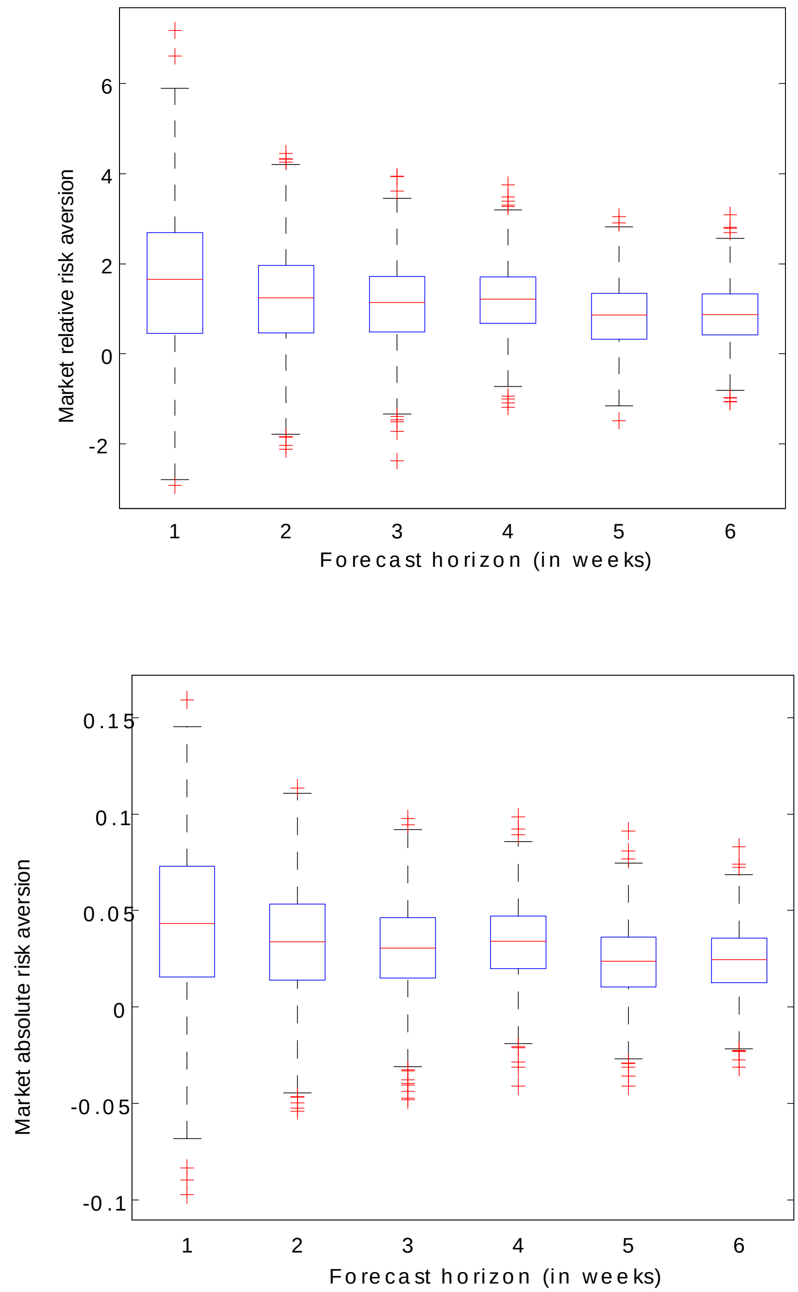
The difference between the normalized means of the risk neutral probability density function and the subjective probability density function is an approximate measure of the risk premium (Bliss and Panigirtzoglou, 2004)

$$Risk \quad Premium \ (RP)_t \approx \frac{E_t[\widehat{f}^Q(F_T)] - E_t[\widehat{f}(F_T)])}{E_t[\widehat{f}^Q(F_T)]} \tag{1.33}$$

With the estimated risk aversion parameters we can obtain the implied physical or risk-adjusted probability density functions and infer the risk premium. I calculate the risk premiums implied by power utility function and exponential utility function respectively. Figure 1-7 plots the risk premiums calculated using function (1.33) for both powr uetility function and exponential utility function for futures options expiring in four weeks. We observe large spikes in the risk premium paths in late 1990 and 2008 for both utility functions. For the risk premium implied by power utility function, we also observe spikes around late 1998, 2001, and 2003. For example, the risk premium implied by the power utility function

[7] The line dividing the box is the median, the bottom and top of the box are the 25th and 75th percentile. Points outside of the whiskers are outliers which are larger than $q3+\omega*(q3-q1)$ or smaller than $q1-\omega*(q3-q1)$, where $q1$ and $q3$ are the 25th and 75th percentiles of the sample, respectively, and ω is the maximum whisker length with a default value of 1.5. (The default of 1.5 corresponds to approximately $+/-2.7\sigma$ and 99.3 coverage if the data are normally distributed).

Figure 1-6: Distribution of Market Risk Aversion Using the Bootstrap Tests



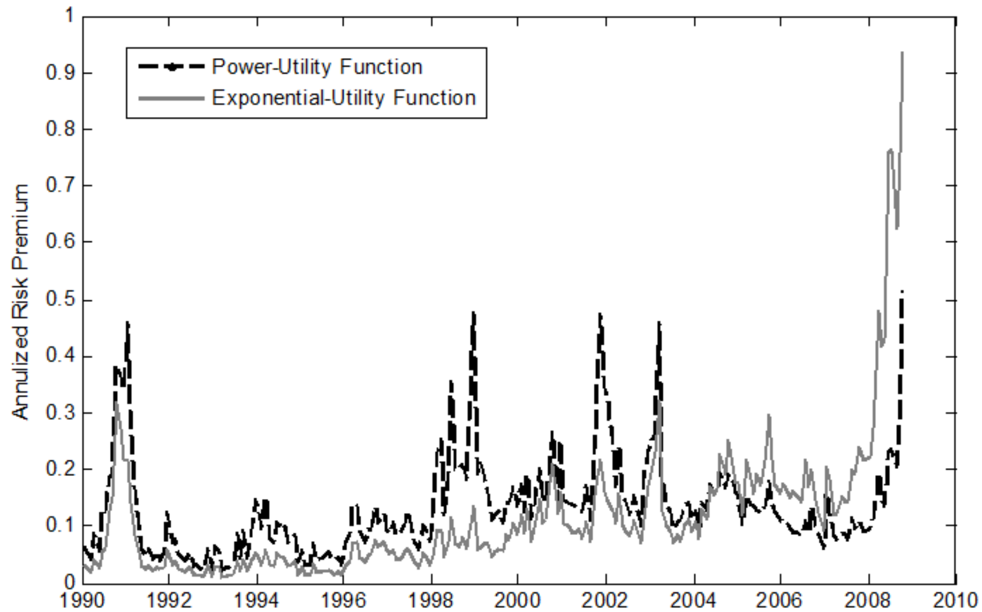
Notes to Figure: I plot of distributions of market risk aversion coefficients from bootstrap tests. Panel 6.A plots the market relative risk aversion coefficients implied by the power utility function and Panel 6.B plots the market absolute risk aversion coefficients implied by the exponential utility function.

Table 1.3: Bootstrap Estimation Results

Forecast Horizon		1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks
Panel A. Physical Density Calculated Using Power Utility							
MRRA		2.0262***	1.1807***	1.3856***	1.1877***	0.8759***	1.0009***
Bootstrap	Minimum	-2.93	-2.12	-2.38	-1.18	-1.49	-1.06
	Mean	1.58	1.23	1.09	1.20	0.84	0.86
	Median	1.66	1.24	1.13	1.21	0.84	0.86
	Maximum	7.19	4.46	3.94	3.75	3.39	3.08
	Standard deviation	1.58	1.10	0.91	0.77	0.70	0.66
Panel B. Physical Density Calculated Using Exponential Utility							
MRRA		1.5360***	0.8839***	1.2604***	0.9879***	0.6670***	0.6620***
Bootstrap	Minimum	-3.66	-1.88	-1.65	-1.39	-1.40	-1.06
	Mean	1.65	1.17	1.03	1.14	0.80	0.82
	Median	1.63	1.18	1.05	1.16	0.82	0.83
	Maximum	5.99	3.96	3.36	3.35	3.11	2.82
	Standard deviation	1.53	1.01	0.81	0.71	0.65	0.58

Notes to Table: I report the statistics of market risk aversion coefficients (MRRA) from bootstrap tests using futures options maturing in four weeks. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively.

Figure 1-7: Implied Risk Premiums with Constant Market Risk Aversion



Notes to Figure: I plot the risk premiums in the crude oil market calculated using equation (2.23).

for forecast horizon of four weeks at August 1990 is approximately 45% per year, while the risk premium implied by the exponential utility function for forecast horizon of four weeks at December 2008 is more than 90% per year. These high risk premium periods coincide with the periods of high volatility in the crude oil market, if we compare Figure 1-7 with Figure 1-3 and Figure 1-5. The risk premiums inferred using constant risk aversion are positively correlated with market volatility.

1.5.3 State-Dependent Market Risk Aversion and Risk Premiums

To estimate state-dependent market risk aversion, it is necessary to identify states in the crude oil market. Hamilton and Wu (2011) and Kang and Pan (2011) investigate the role of risk premium in the crude oil market and suggest the increased participation by speculators in the crude oil futures market may have been a factor in changing the nature

Table 1.4: Subsample Estimation Results

Subsamples	Mean Volatility	Mean Speculation	MRRA	RN LR p-value	Physical LR p-value
Low Speculation Period	0.3048	1.0261	1.6045***	0.1356	0.3297
High Speculation Period	0.3579	1.082	1.0506***	0.6438	0.8733
Jan. 1990 – Dec. 2002	0.3251	1.0355	1.2940***	0.3149	0.7377
Jan. 2003 – Dec. 2008	0.3495	1.1048	0.8503***	0.6771	0.7705
Jan. 1990 – Dec. 2004	0.3287	1.0391	1.4884***	0.2555	0.8568
Jan. 2005 – Dec. 2008	0.3419	1.1085	-0.0644**	0.6355	0.6359

Notes to Table: I report subsample estimation results of market relative risk aversion (MRRA) coefficients for forecast horizon of four weeks. physical probability density functions are implied using power utility function. The mean volatility level and mean speculation index level for each subsample period are also reported. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively. Speculation has been rising since 2002 (Figure 1-4). I therefore perform the empirical analysis using two subsamples: January 1990 to December 2002, and January 2003 to December 2008. To compare our results with those of Hamilton and Wu (2011), I also analyze another two subsamples by dividing the sample into the January 1990 to December 2004 and January 2005 to December 2008.

of risk premium. Since the speculation index introduced in equation (1.31) measures the extent the speculative positions exceeding the necessary level to offset hedging positions, I choose this index as our state variable to investigate the state dependency of the market risk aversion and risk premiums.

First I use the speculation index as a criterion to distinguish two subsamples: one is the low speculation period and the other is the high speculation period. Each period has the same number of observations. For each subsample period, I run the Berkowitz (2001) density function forecast ability test of the underlying asset prices. The market risk aversion coefficients and the Berkowitz LR p-values are reported in the first two rows in Table 1.4.

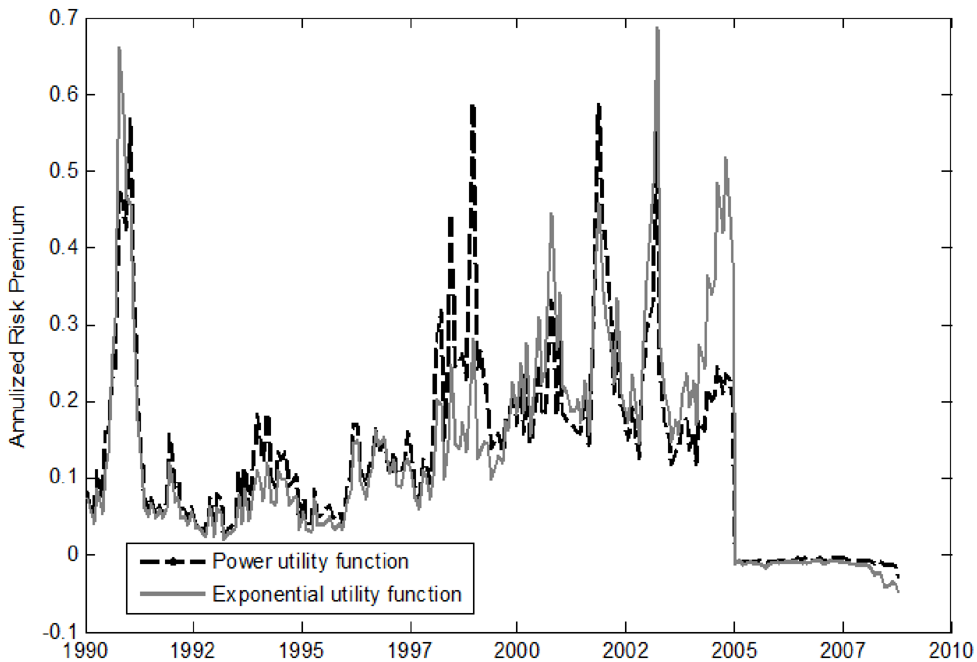
Table 1.4 presents the subsample estimation results for market risk aversion coefficients for the four-week forecast horizon. Physical probability density functions are implied using power utility function. I also report the mean volatility level and mean speculation index level for each subsample period. The high speculation period corresponds to the high volatility period. For all three tests reported in Table 1.4, the market relative risk aversion coefficients estimated for the high speculation period are all lower compared to those estimated for low speculation periods. This confirms our hypothesis that as the speculative level increases, the market risk aversion level decreases.

Using the estimation results in Table 1.4, I plot the subsample risk premium for the period of January 1990 to December 2004 and January 2005 to December 2008 in Figure 1-8. Compare to Figure 1-7, this figure shows sharp decreases in the level of risk premium after 2005. The risk premiums after 2005 are much less volatile and negative since the market risk aversion coefficient for the period of January 2005 to December 2008 is negative.

The subsample estimation in Table 1.4 divides the whole sample into two periods and estimates the market risk aversion coefficient for two sub-periods. This approach may not provide sufficient evidences to capture the state dependence of market risk aversion and risk premiums. I therefore estimate the state-dependent risk aversion coefficients defined in equations (1.29) and (1.30) for the power utility function and exponential utility function respectively. I run the Berkowitz (2001) forecast ability test by maximizing LR p-values for the risk adjusted physical density functions with respect to the values of coefficients a and b.

Refer to the level of speculation index as the state variable, x_t. Table 1.5 reports the estimation results of the coefficients a and b that determine state-dependent market risk aversion, and the corresponding p-values. Using the intuition provided by the model in Section 2, we expect the slope coefficient to be negative. The result from both power utility function and the exponential utility function shows that bs are significantly negative and confirms the negative relationship of market risk aversion and the level of the speculation index.

Figure 1-8: Risk Premiums Using Subsample Estimates



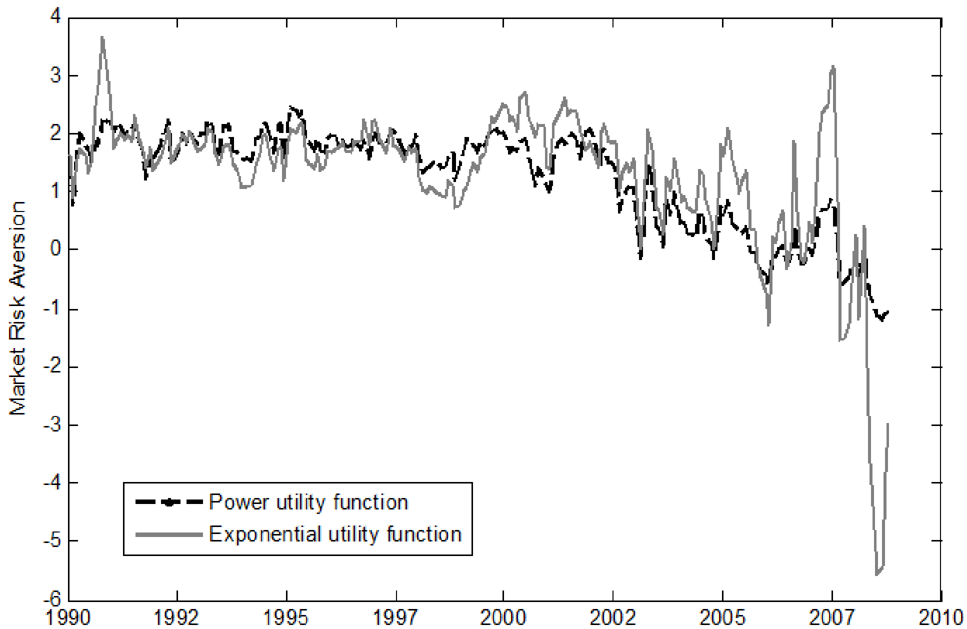
Notes to Figure: I plot risk premiums obtained using separate estimation for the periods 1990-2004 and 2005-2008. The parameter estimates are from Table 1.4.

Table 1.5: Estimation of State-Dependent Market Risk Aversion

a	b	p-value	μ	ρ	σ
Panel A. Risk Neutral Density					
		0.3093	0.1050	0.0564	1.0095
Panel B. Physical Density Calculated by Power Utility Function					
1.2603***	-24.4638*	0.8728	0.0022	0.0545	1.0066
Panel C. Physical Density Calculated by Exponential Utility Function					
0.0610***	-1.0543***	0.9683	-0.0016	0.0330	0.9955

Notes to Table: I report the estimation results for the coefficients in the state-dependent market risk aversion specification and the corresponding p-values. Results are for a forecast horizon of four weeks. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively.

Figure 1-9: State-dependent Market Risk Aversion Coefficients

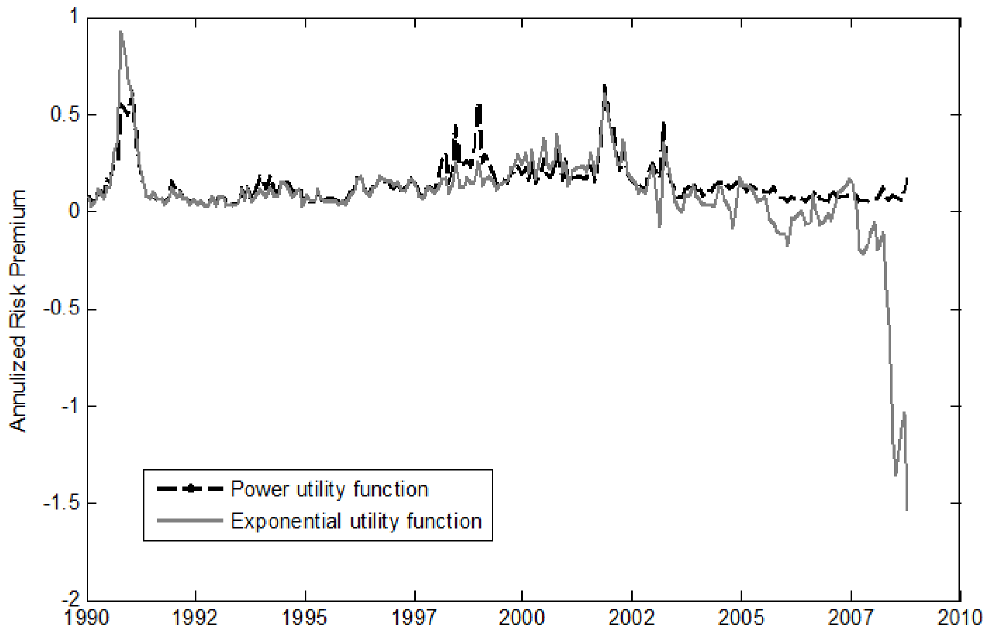


Notes to Figure: I plot the state-dependent market risk aversion coefficients calculated using equation (2.24). The parameter estimates are from Table 1.5.

The state-dependent market relative risk aversion coefficients are plotted in Figure 1-9. From this figure, we observe that there are dramatic changes since 2002. The market risk aversion coefficient is relatively stable before 2002. After 2002, the market risk aversion coefficients are much more volatile, especially in the case of exponential utility. Since 2005, relative risk aversion coefficients are more volatile, though their averages are significantly smaller. For the period after 2007, the implied market risk aversion coefficients are negative most of the time using either power or exponential utility.

The state-dependent risk premiums calculated using the state-dependent market risk aversion are plotted in Figure 1-10. Comparing with Figure 1-7 and Figure 1-8, we see more dramatic variability in risk premiums over time when allowing for state dependence, especially in the case of exponential utility. The pattern of state-dependent risk premiums

Figure 1-10: State-dependent Risk Premiums



Notes to Figure: I plot the state-dependent risk premiums calculated using the state-dependent market risk aversion in Figure 1-9.

before 2002 is fairly similar to that calculated using a constant market risk aversion coefficient, except that the former risk premiums are larger in magnitude. After 2002, the state-dependent risk premiums on average are lower than in the first half of the sample. For some extreme periods, such as late 2005 and late 2008, risk premiums are negative. This result is consistent with the empirical findings in Hamilton and Wu (2011).

1.5.4 Interpretation of State-Dependent Market Risk Aversion and Risk Premiums

It is worth noting that the market relative risk aversion for the period of 2005-2008 in Table 1.4 is negative. And the estimated state-dependent market relative risk aversion after 2007 is negative most of the time. The negative risk aversion might be counter-intuitive at first glance. However in a commodity market with increased financialization, negative risk aversion may not be unreasonable. I interpret these findings by relating them to the stylized model in Section 2.

The implications in Section 2.3 are based on the premise that traders in the crude oil market are risk averse and crude oil futures contracts are initiated by commercial traders for hedging purposes. These properties are in line with the assumptions underlying Keynes' theory of normal backwardation. Producers of the physical commodity want to hedge their price risk by selling futures contracts. They want to offer a risk premium for this privilege, and speculators will be compensated for insuring the commercial hedgers.

However, the commodity market has been experiencing dramatic structural changes over the last decade. Many researchers ascribe them to the increased financialization of the commodity market. The financial industry has developed new instruments that allow institutions and individuals to invest in commodities, for example, long-only commodity index funds (LOCF), over the counter (OTC) swap agreements, exchange traded funds (ETF) and other products (Irwin and Sanders, 2011). These instruments provide investors with buy-side exposure from a particular index of commodities. Several studies have concluded that investors can reduce portfolio risks through investments in long-only commodity index funds (LOCF) (Gorton and Rouwenhorst, 2006; Erb and Harvey, 2006).

If a speculator invests in crude oil futures for the purpose of portfolio diversification, she may choose long positions regardless of anything happening to fundamentals and no matter what the crude oil spot position is. The more long positions a speculator holds, the less risk averse she is, and the lower the aggregate market risk aversion. To insure her long position, the speculator would like to offer, instead of receiving, a risk premium to her counterparty. Therefore she requires less premium to compensate her risk exposure in the crude oil market.

As in equation (1.12), before financialization, speculators take positions to offset the hedger's position. $\pi_{h,t}$ and $\theta_{s,t}$ should have the same signs and the risk aversion coefficient of the speculator (γ_t^s) should be positive. When the financial speculator starts to take excess speculative positions for the purpose of portfolio diversification, she would long more futures contracts than the necessary level to offset the hedger's position, ($\theta_{s,t} > 0$ and $\theta_{s,t}$ increases). The speculator's risk aversion coefficient (γ_t^s) decreases, and it reduces market risk aversion (Γ_t) accordingly. As the financial speculator's long positions keep increasing, the risk aversion coefficient of the speculator (γ_t^s) could be negative. In some extreme cases, if a speculator longs futures contracts ($\theta_{s,t} > 0$) when the aggregate spot position is in net demand ($\pi_{h,t} < 0$), the aggregate risk aversion coefficient would be negative too ($\gamma_t^s < 0$ and $\Gamma_t < 0$).[8]

A negative risk aversion coefficient does not necessarily means the speculator is risk loving. A speculator who seeks portfolio diversification cares the overall profit of her portfolio and therefore possibly has a higher risk tolerance in the crude oil futures market. I interpret the negative risk aversion coefficient as a signal of excess buying pressure from investors seeking portfolio diversification, even if the physical commodity is in net demand. This excess buying pressure in the futures market could shift the beneficiary of the risk premiums from the long side of the futures contract to the short side. Hamilton and Wu (2011) document significant changes in oil futures risk premiums since 2005. Compensation to the long position is smaller on average but much more volatile, and often significantly negative when the futures curve slopes upward. This observation is consistent with the claim that index-fund investing has become more important relative to commercial hedging in determining the structure of crude oil futures risk premium over time.

To summarize, the negative coefficients of market risk aversion may imply that speculators took more excess long positions and applied large buying pressure to the crude oil futures market. Speculators who care about their portfolio diversification would want to long the crude oil futures and pay, instead of receiving, a risk premium for their position in some extreme cases.

[8] The simple stylized model in Section 2 which considers only the crude oil futures market cannot completely explain the risk preference of a speculator who has a diversified portfolio. However, the relaitonship between risk aversion and speculative futures positions should still hold.

1.5.5 Predictive Power of the State-Dependent Risk Premiums

Since the risk premiums are obtained by maximizing the forecast ability of the probability density function of the crude oil futures prices, I would expect that they have explanatory power for predicting crude oil futures returns. In the case of state-dependent case, I would expect the risk premiums to have high predictive power because of the increased flexibility.

To test the predictive power of the state-dependent market risk premiums, I regress futures returns on the state-dependent market risk premiums. For the purpose of comparison, I also run regression of the futures returns on risk premiums calculated using constant risk aversion coefficients, as well as several other predictors used in the existing literature, such as the lagged futures returns, and the lagged ATM volatilities.

I run the regression

$$\log\left(\frac{F_T}{F_{t+1,T}}\right) = c_i + d_i y_{i,t} + \varepsilon_{i,t+1} \tag{1.34}$$

where $y_{i,t}$ is the predictor i at time t, and c_i and d_i are parameters to be estimated.

Table 1.6 reports the regression coefficients in equation (1.34) and the R-squares of the regressions. Most of the independent variables, except for the state-dependent risk premiums, have very low R-squares and insignificant coefficient estimates. The lagged futures returns has an R-squares of 1.52%, and the speculation index has a R-square of 1.63%. R-squares of the lagged ATM volatility and risk premiums calculated by single constant market risk aversion are even lower. In allowing for state dependence in risk premiums, I improve the predictability of futures returns greatly. The R-square is 2.08% in the case of the state-dependent risk premium implied by power utility function, and is 6.8% using the state-dependent risk premium implied by exponential utility. The d_i coefficients estimated using the state-dependent risk premiums implied by the power utility function and exponential utility function are both significant. One may argue that these R-squares are small. Neely, Rapach, Tu, and Zhou (2010) discuss the magnitude of R-squares in predictive regressions and conclude that because monthly returns inherently

Table 1.6: Predictive Regressions

Predictor	**c**	**d**	$\mathbf{R^2}$
Lagged Futures Return	0.0798	0.1257	0.0152
	(0.0755)	(0.0678)*	
RP Using Power Utility	0.132	-11.5722	0.0005
	(0.1406)	(35.6123)	
RP Using Exp. Utility	0.1316	-0.0149	0.0002
	(0.1949)	(0.0701)	
State-dependent RP Using Power Utility	-0.0522	1.0845	0.0208
	(0.1004)	(0.4978)**	
State-dependent RP Using Exp. Utility	-0.0884	1.3376	0.068
	(0.0859)	(0.3316)***	
Speculation Index	4.4183	-4.1079	0.0163
	(2.2506)**	(2.1365)*	
Lagged ATM Volatility	0.1246	-0.0944	0.0001
	(0.2381)	(0.6845)	

Notes to Table: I report the coefficients in predictive regression equation (2.24) and the R-square of the regression. Results are for forecast horizon of four weeks. ***, **, * represent significance levels of 1%, 5%, 10%.

contain a substantial unpredictable component, a monthly R-square of 0.5% can represent economically significant predictive power. So the state-dependent risk premiums obtained in this research have significant explanatory power in predicting crude oil futures returns.

1.6 Conclusion

I investigate if speculative activity in the crude oil market affects market risk aversion and risk premiums. Using WTI crude oil futures and option data, I estimate time varying market risk aversion by specifying the speculation level as a state variable. I find that as the speculation level increases, market risk aversion and risk premiums decrease.

When allowing for state dependence in the risk aversion coefficient, the implied risk premiums change significantly over time: they are higher during the first half of the sample period, while after 2002, when speculation increases, they are on average smaller in magnitude and more volatile. This finding is consistent with that in Hamilton and Wu (2011) and suggests that speculators who invest in the crude oil market for the purpose of portfolio diversification are willing to accept lower risk premiums for their speculative positions.

Risk premiums implied by state-dependent market risk aversion have significantly higher explanatory power in predicting subsequent futures returns, compared with several commonly used predictive variables, especially when we assume exponential utility.

Overall my findings indicate that speculative activity in the crude oil market has had a significant effect on aggregate risk aversion and the evolution of risk premiums.

Appendix

Market Participants' Optimal Positions

I first solve for the hedger's optimal position. For the hedger's margin account at time t, $X_t^{\theta_{h,t}}$, I define

$$dX_t^{\theta_{h,t}} = \alpha_t ds + \beta_t dB_t \tag{A.1.1}$$

From equation (1.4), we have $\alpha_t = rX_t^{\theta_{h,t}} + m_t\theta_{h,u}^T F_t$ and $\beta_t = \theta_{h,u}^T F_t v_t$.

Let the dynamics of the hedger's wealth process, $W_t^{\theta_{h,t}}$, be

$$dW_t^{\theta_{h,t}} = a_t ds + b_t dB_t \tag{A.1.2}$$

where $\theta_{h,t}$ is the futures strategy of the hedger at time t. From equations (1.1) – (1.3) and (A.1.1), we have $a_t = rX_t^{\theta_{h,t}} + \pi_{h,t}\mu_t S_t + m_t\theta_{h,u}^T F_t$ and $b_t = \pi_{h,t}\sigma_t S_t + \theta_{h,u}^T F_t v_t$.

Assume exponential utility function

$$U(W_t^{\theta_{h,t}}) = -\exp(-\gamma_t^h W_t^{\theta_{h,t}}) \tag{A.1.3}$$

where γ_t^h is the hedger's absolute risk aversion coefficient.

Let $Y_t^{\theta_{h,t}} = \exp(-\gamma_t^h W_t^{\theta_{h,t}})$. Apply Ito's Lemma

$$\begin{aligned} Y_{T-t}^{\theta_{h,t}} &= Y_0^{\theta_{h,t}} + \int_0^{T-t} Y_s^{\theta_{h,s}} \left[\frac{\gamma_s^{h^2}}{2} tr(b_s^T b_s) - \gamma_s^h a_s\right] ds \\ &\quad + \int_0^{T-t} Y_s^{\theta_{h,s}} \left[-s\gamma_s^h b_s\right] dB_s \end{aligned} \tag{A.1.4}$$

Thus,

$$E\left[Y_{T-t}^{\theta_{h,t}}\right] = Y_0^{\theta_{h,t}} + E\left[\int_0^{T-t} Y_s^{\theta_{h,s}} \left[\frac{\gamma_s^{h^2}}{2} tr(b_s^T b_s) - \gamma_s^h a_s\right] ds\right] \tag{A.1.5}$$

$\theta_{h,t}^*$ minimizes $\frac{\gamma_t^{h^2}}{2} tr(b_t^T b_t) - \gamma_t^h a_t$ at each time t. Taking derivative, we get

$$\theta^*_{h,t} = -\frac{(v_t v_t')^{-1}}{F_t}[v_t \sigma_t' \pi_{h,t} S_t - m_t/\gamma_t^h] \tag{A.1.6}$$

A similar derivation can be obtained for the speculator and we can have equation (1.9).

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Chapter 2

Dynamic Jump Intensities and Risk Premiums in Crude Oil Futures and Options Markets

2.1 Introduction

Crude oil is the most important commodity in international trade, and the crude oil derivatives market constitutes the most liquid commodity derivatives market. In December 2011, WTI and Brent crude oil futures accounted for 51.4% of dollar weight in the S&P GSCI commodity index.

In order to price and hedge this increasingly important commodity, it is crucial to model crude oil futures and options and to better understand their dynamics. Surprisingly though, there are relatively few studies on pricing crude oil derivatives, especially when compared with the existing literature on equity derivatives.

The extensive literature on modeling equity derivatives, which mainly focuses on index returns and options, concludes that jumps are needed to capture the higher moments of index returns. This literature includes models with stochastic volatility and jumps, as well as GARCH-style jump models.[1] The implementation of continuous-time stochastic volatility models with Poisson jumps is complex, because the likelihood function is typically not available in closed form, and therefore option pricing in the presence of jumps typically

[1] For models with stochastic volatility and jumps see Bakshi, Cao, and Chen (1997), Bates (1996, 2000, 2006), Pan (2002), Eraker (2004), Carr and Wu (2004), and Santa-Clara and Yan (2010). For GARCH-style jump models see Maheu and McCurdy (2004), Duan, Ritchken, and Sun (2006), Christoffersen, Jacobs, and Ornthanalai (2012), and Ornthanalai (2012).

relies on complex econometric methods to filter the unobserved state variables.[2] This type of estimation is computationally intensive, especially when dealing with large data sets.

The presence of jumps in crude oil futures and options seems intuitively plausible, but the computational complexity of jump models, together with the size of the data, make implementing such models for crude oil data very challenging. To overcome these computational challenges, we use discrete-time models in which the conditional variance of the normal innovation and the conditional jump intensity of a compound Poisson process are governed by GARCH-type dynamics. We empirically investigate the importance of jumps and time-varying jump intensities using an extensive panel data set of crude oil futures and option prices.

For the discrete-time models in this paper, both the conditional jump intensity and the conditional variance can be directly computed from the observed shocks using an analytical filter (Christoffersen, Jacobs, and Ornthanalai, 2012). With the analytical filter, filtering the normal component and the jump component is relatively simple and extremely fast, even when the jump intensity is time-varying. It takes less than a second to filter 38,024 futures contracts using Matlab on a standard PC. Because the variance and the jump intensity dynamics can be updated analytically, we can conveniently estimate the model using MLE estimation. Calculating the Implied Volatility Root Mean Squared Error (IVRMSE) of 283,653 option contracts takes approximately seven seconds.

We investigate the economic importance of jumps and dynamic jump intensities in the crude oil market and compare the fit of the jump models with that of a benchmark GARCH model without jumps. We study jump models with different specifications of jump intensity and conditional variance. The simplest jump model is assumed to have constant jump intensities, which is consistent with most of the existing continuous-time Stochastic-Volatility-

[2]For example, Chernov, Gallant, Ghysels, and Tauchen (2003) use an Efficient Method of Moments (EMM) based method, Pan (2002) uses the implied-state Generalized Method of Moments (GMM) technique, Eraker, Johannes, and Polson (2003), Eraker (2004), and Li, Wells, and Yu (2007) employ Markov Chain Monte Carlo (MCMC) techniques, and Trolle and Schwartz (2009) use the Extended Kalman Filter (EKF).

Jump (SVJ) literature. Our preferred model contains time-varying jump intensities and a time-varying conditional variance, but the jump intensity and conditional variance are driven by the same dynamics. This model is related to the most complex SVJ dynamics studied in the literature (see Eraker, 2004; and Santa-Clara and Yan, 2010).

We find strong evidence of the presence of jumps and dynamic jump intensities in the crude oil market. During crisis periods, when market risk is high, jumps occur more frequently. Jump models with time-varying jump intensities significantly outperform the benchmark model, and in models with time-varying jump intensities, the jump component explains a large part of the variance of the underlying futures data, regardless of the data used in estimation.

Estimates based on both futures and options data indicate the presence of relatively infrequent but large jumps, with the futures data pointing to larger and more infrequent jumps than the option data. Contrary to equity index markets, the main role of jumps and jump risk in crude oil futures markets is to capture the excess kurtosis in the data, while skewness is of second-order importance.

A number of papers in the literature on commodity derivatives contain related results. Trolle and Schwartz (2009) estimate a continuous-time stochastic volatility model using NYMEX crude oil futures and options and find evidence for two, predominantly unspanned, volatility factors. They do not consider jump processes, which have been used in other security markets to model large movements. Larsson and Nossman (2010) examine the performance of affine jump diffusion models with stochastic volatility for modeling the time series of crude oil spot prices. Their results show that stochastic volatility alone is not sufficient and jumps are an essential factor to correctly capture the time series properties of oil prices. However, they do not use panel data on futures contracts nor option prices. Hamilton and Wu (2011) model crude oil futures with an affine term structure model and document significant changes in oil futures risk premia since 2005. Pan (2011) uses options on crude oil futures to study the impact of speculation on returns. Chiarella, Kang, Sklibosios, and To (2012) document a hump-shaped volatility structure in the commodity derivatives market. To the best of our knowledge, no existing studies have implemented jump models using extensive cross-sections of crude oil futures and options.

The rest of the paper proceeds as follows. Section 2 discusses discrete-time jump pricing models for commodity futures, as well as a benchmark GARCH model. Section 3 discusses the crude oil futures and options data. In Section 4 we explain MLE estimation on futures contracts and report the estimation results. Section 5 derives the risk-neutral dynamics and the closed form option valuation formula. Section 6 presents results using options and futures jointly in estimation. Section 7 concludes.

2.2 Models for Commodity Futures Markets

In commodity futures markets, we observe futures prices for different maturities. Spot prices are much more difficult to measure for some commodities. Even if spot prices are observed, the cost of carry needs to be modeled to obtain futures prices, and the cost of carry is by definition unobservable.

Consequently, the modeling of commodity derivatives is harder than the modeling of equity options, where the underlying spot price is observable. This challenge can be addressed in several ways. A popular approach is to specify the cost of carry as a separate stochastic process, and combine it with the stochastic process for the spot price to arrive at the futures price. The parameters characterizing the spot price and the cost of carry are identified using futures and futures options data; see for instance Trolle and Schwartz (2009).[3]

However, Trolle and Schwartz (2009) correctly note that the specification of the cost of carry is of no great significance for the modeling of options on commodity futures, as the actual futures prices are used in option valuation. Because of this observation, and because the cost of carry is unobservable, we directly specify the futures return. If the specification

[3] See Casassus and Collin-Dufresne (2005) for an analysis of the most general specification for convenience yields allowed within an affine structure.

of the futures return is adequate, it implies that the implied spot return and cost of carry are adequately captured, and for our purpose there is no need to model them separately.

2.2.1 The Benchmark Model

We formulate a class of jump models for commodities markets. To provide a benchmark for these models that can capture several important stylized facts of commodity markets, we first consider a benchmark GARCH model for futures returns

$$\log \frac{F_{t+1,T}}{F_{t,T}} = (\lambda - \frac{1}{2})h_{t+1} + z_{t+1} = (\lambda - \frac{1}{2})h_{t+1} + \sqrt{h_{t+1}}\varepsilon_{t+1} \tag{2.1}$$

where $F_{t,T}$ is the time t price of the futures contract maturing at time T, z_{t+1} is an innovation which is distributed $N(0, h_{t+1})$, ε_{t+1} is distributed $N(0,1)$, h_{t+1} is the conditional variance known at time t, and λ is the market price of risk associated with the normal innovation. The conditional variance of the normal innovation h_{t+1} is governed by a GARCH$(1,1)$ process, which is specified according to Heston and Nandi (2000).

$$h_{t+1} = \omega + bh_t + a(\varepsilon_t - c\sqrt{h_t})^2 \tag{2.2}$$

GARCH models provide a convenient framework to capture stylized facts in financial markets such as conditional heteroskedasticity, volatility clustering, and mean reversion in volatility. These stylized facts are also very prominent in commodity futures markets. The GARCH dynamic in (2.2) is different from the more conventional GARCH specifications of Engle (1982) and Bollerslev (1986), and is explicitly designed to facilitate option valuation. We discuss the benefits of the specification in (2.2) in more detail below.

Consistent with other GARCH specifications, the conditional variance h_{t+1} in (2.2) is predictable conditional on information available at time t. The unconditional variance is given by

$$E[h_{t+1}] = (\omega + a)/(1 - b - ac^2) \tag{2.3}$$

where $b + ac^2$ is the variance persistence. Furthermore, given a positive estimate for a, the sign of c determines the correlation between the futures returns and the conditional variance. Equivalently, c can be thought of as controlling the skewness or asymmetry of the distribution of log returns, with a positive c resulting in a negatively skewed multi-day distribution.

2.2.2 The Cross-Section of Futures Contracts

Heston and Nandi (2000) estimate their model using S&P 500 index returns and options. For that application, it is straightforward to directly filter the conditional variance h_{t+1} from the return innovations with the GARCH model. Our empirical application is more complex, because at each time t we have multiple cross-sections of crude oil futures prices with different maturities, eight in our case. We thus have

$$\log \frac{F_{t+1,T_i}}{F_{t,T_i}} = (\lambda - \frac{1}{2})h_{i,t+1} + \sqrt{h_{i,t+1}}\varepsilon_{i,t+1} \tag{2.4}$$

where $i = 1, 2, .., 8$ and T_i represents the maturity date of the futures contracts maturing in the ith month. If the parameters for the return and variance dynamics are maturity-specific, the number of parameters in this model is very large, and it increases with the number of futures contracts. To keep the model as parsimonious as possible and limit the number of parameters, we impose the restriction that these parameters are the same for all eight maturities and assume that covariance matrix of the return innovations is a diagonal matrix, $\varepsilon_{t+1} = [\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \ldots, \varepsilon_{8,t+1}] \sim i.i.d.\ N(0, I_8)$, where I_8 is the identity matrix. This approach is consistent with the assumptions made by Trolle and Schwartz (2009), whose implementation uses the Kalman filter. Furthermore, we make the additional assumption that we filter the conditional variance using the futures contract maturing in one month. We therefore effectively have

$$h_{t+1} = \omega + bh_t + a(\varepsilon_{1,t} - c\sqrt{h_t})^2 \tag{2.5}$$

2.2.3 Commodity Futures Returns with Dynamic Jump Intensities

The futures return process in (2.1)-(2.2) provides a benchmark model that can capture several important stylized facts using a simple setup with a single normal innovation. Building on the models for index returns in Ornthanalai (2012) and Christoffersen, Jacobs, and Ornthanalai (CJO, 2012), we now formulate a much richer class of models with jumps in returns and volatilities, and with potentially time-varying jump intensities. Futures returns are given by

$$\log \frac{F_{t+1,T}}{F_{t,T}} = (\lambda_z - \frac{1}{2})h_{z,t+1} + (\lambda_y - \xi)h_{y,t+1} + z_{t+1} + y_{t+1} \tag{2.6}$$

where the z subscript in λ_z and $h_{z,t+1}$ refers to the normal component, specified as in Section 2.1. The jump component y_{t+1} is specified as a Compound Poisson process denoted as $J(h_{y,t+1}, \theta, \delta^2)$. The Compound Poisson structure assumes that the jump size is independently drawn from a normal distribution with mean θ and variance δ^2. The number of jumps n_{t+1} arriving between times t and $t+1$ is a Poisson counting process with intensity $h_{y,t+1}$. The jump component in period $t+1$ is therefore given by

$$y_{t+1} = \sum_{j=0}^{n_{t+1}} x_{t+1}^j \tag{2.7}$$

where x_{t+1}^j, $j = 0, 1, 2, \dots$ is an *i.i.d.* sequence of normally distributed random variables, $x_{t+1}^j \sim N(\theta, \delta^2)$. The conditional expectation of the number of jumps arriving over time interval $(t, t+1)$ equals the jump intensity, $E_t[n_{t+1}] = h_{y,t+1}$. The conditional mean and variance of the jump component y_{t+1} are given by $\theta h_{z,t+1}$ and $(\theta^2 + \delta^2)h_{y,t+1}$ respectively.

The conditional risk premium is given by $\gamma_{t+1} = \lambda_z h_{z,t+1} + \lambda_y h_{y,t+1}$, with λ_z and λ_y denoting the market prices of risks for the normal and jump components. The convexity adjustment terms $\frac{1}{2}h_{z,t+1}$ and $\xi h_{y,t+1} \equiv (\exp(\theta + \frac{\delta^2}{2}) - 1)h_{y,t+1}$ in (2.6) act as compensators to the normal and jump components respectively.

2.2.4 Jump Models

The most general model we investigated assumes that both the conditional variance of the normal component and the jump intensity are governed by the following extended GARCH (1, 1) dynamics.

$$h_{z,t+1} = \omega_z + b_z h_{z,t} + \frac{a_z}{h_{z,t}}(z_t - c_z h_{z,t})^2 + d_z y_t \tag{2.8}$$

$$h_{y,t+1} = \omega_y + b_y h_{y,t} + \frac{a_y}{h_{y,t}}(z_t - c_y h_{z,t})^2 + d_y y_t \tag{2.9}$$

where $\omega_z, b_z, a_z, c_z, d_z, \omega_y, b_y, a_y, c_y, d_y$ are parameters to be estimated.

The model in (2.8)-(2.9) is very rich and flexible. It allows for jumps in volatility as well as jumps in returns. It has been shown in the index option literature that jumps in volatility are useful to explain option volatility smiles and smirks (see for example Eraker, Johannes and Polson, 2003; and Eraker, 2004). Note however that following Ornthanalai (2012), the model is designed to yield a closed-form solution for option prices, and in order to do so we have adopted a rather simple specification for jumps in volatility.

Another advantage of the model in (2.8)-(2.9) is that the normal and jump innovations, z_t and y_t, enter separately into the GARCH updating dynamics. The model therefore allows each type of innovation to impact the variance and jump intensity separately.

The specification of $h_{z,t+1}$ and $h_{y,t+1}$ in (2.8)-(2.9) therefore has substantial advantages. However, a potential problem is that the model is richly parameterized. Our empirical investigation indeed indicated that several of the model's parameters were poorly identified. We therefore investigated two nested specifications, which impose restrictions on $h_{z,t+1}$ and $h_{y,t+1}$, and greatly reduce the dimension of the parameter space.

The first nested model imposes the following restrictions on (2.8)-(2.9)

$$b_y = 0, a_y = 0, c_y = 0, d_y = 0 \tag{2.10}$$

This model maintains the normal component's GARCH dynamic, but jumps arrive at a constant rate ω_y, regardless of the level of volatility in the market.

$$h_{y,t+1} = \omega_y \tag{2.11}$$

We refer to this model as CI to reflect that the arrival rate of the jumps, or the jump intensity, is constant. In the second nested model, we assume that $h_{z,t+1}$ and $h_{y,t+1}$ are time-varying but driven by the same dynamic. The conditional jump intensity is affine in the conditional variance of the normal component

$$h_{y,t+1} = k h_{z,t+1} \tag{2.12}$$

where k is a parameter to be estimated. This specification is a special case of (2.8)-(2.9), subject to the following restrictions on the parameters of $h_{y,t+1}$

$$\omega_y = \omega_z k, b_y = b_z, a_y = a_z k, c_y = c_z, d_y = d_z k \tag{2.13}$$

We refer to this model as DI to reflect the dynamic nature of the jump intensity.

2.3 Crude Oil Futures and Options Data

We now discuss the crude oil futures and options data used in the empirical analysis, and present summary statistics.

We use a data set of Chicago Mercantile Exchange (CME group, formerly NYMEX) crude oil futures and options data. We use a sample of daily data from January 2nd, 1990 to December 3rd, 2008. The CME crude oil derivatives market is the world's largest and most liquid commodity derivatives market. The range of maturities covered by futures and options and the range of option strike prices are also greater than for other commodities (for a discussion see Trolle and Schwartz, 2009, henceforth TS).

We screen futures contracts based on patterns in trading activity. Open interest for futures contract tends to peak approximately two weeks before expiration. Among futures and options with more than two weeks to expiration, the first six monthly contracts tend to be very liquid. For contracts with maturities over six months, trading activity is concentrated in the contracts expiring in March, June, September, and December.

Following TS (2009), we therefore screen the available futures and options data according to the following procedure: discard all futures contracts with 14 or less days to expiration. Among the remaining, retain the first six monthly contracts. Furthermore, choose the first two contracts with expiration either in March, June, September or December. This procedure leaves us with eight futures contract series which we label M1, M2, M3, M4, M5, M6, Q1, and Q2.

We include the following options on these eight futures contracts. For each option maturity, we consider eleven moneyness intervals: 0.78-0.82, 0.82-0.86, 0.86-0.90, 0.90-0.94, 0.94-0.98, 0.98-1.02, 1.02-1.06, 1.06-1.10, 1.10-1.14, 1.14-1.18, and 1.18-1.22. Moneyness is defined as option strike divided by the price of the underlying futures contract. Among the options within a given moneyness interval, we select the one that is closest to the mean of the interval.

Our data consist of American options on crude oil futures contracts.[4] CME has also introduced European-style crude oil options, which are easier to analyze. However, the trading history is much shorter and liquidity is much lower than for the American options. Since the pricing formulae are designed for European options, we have to convert the American option prices to European option prices. Assuming that the price of the underlying futures contract follows a geometric Brownian motion, we can accurately price American options using the Barone-Adesi and Whaley (1987) formula. Inverting this formula yields an implied volatility, from which we can subsequently obtain the European option price using the Black (1976) formula. To minimize the effect of errors in the early exercise approximation, we use only OTM and ATM options, i.e., puts with moneyness less than one and calls with moneyness greater than one. In addition, we only consider options that have open interest in excess of 100 contracts and options with prices larger than ten cents.

This data filtering procedure yields 38,024 futures contracts and 283,653 option contracts observed over 4,753 business days. The number of futures contracts is eight on every day of the sample, while the number of option contracts is between 23 and 87.

Figure 2-1 displays the prices of the futures contracts. All prices in this paper are settlement prices.[5] To avoid cluttering the figure, we only display the futures term structure on Wednesdays. Futures prices increase dramatically between 2003 and 2007, and subsequently decline. Consistent with existing studies (Trolle and Schwartz (2009), Litzenberger

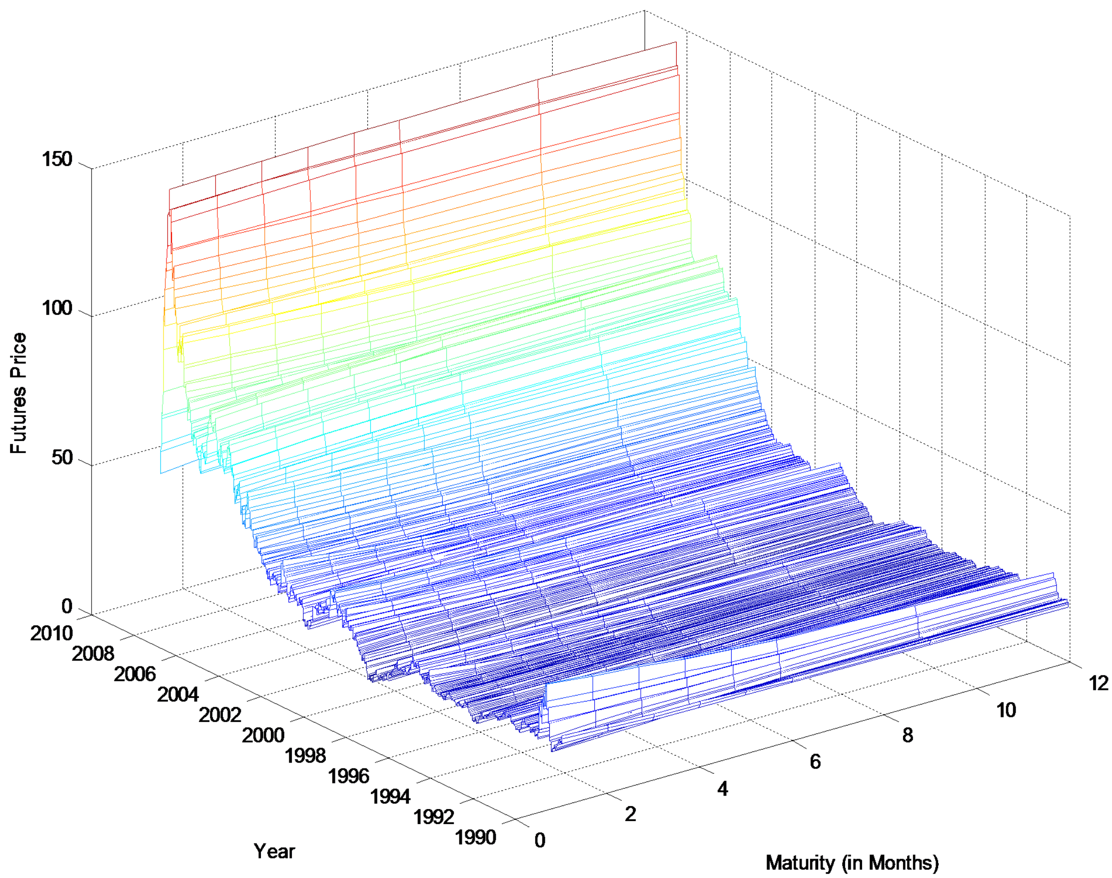
[4]Futures contracts expire on the third business day prior to the 25^{th} calendar day (or the business day right before it if the 25^{th} is not a business day) of the month that precedes the delivery month. Options written on futures expire three business days prior to the expiration date of the futures.

[5]The CME light, sweet crude oil futures contract trades in units of 1000 barrels. Prices are quoted in US dollars per barrel.

and Rabinowitz (1995)), we find that the price of long maturity futures contracts such as Q2 is lower than that of short maturity futures contracts and the crude oil market is on average in backwardation. Figure 2-2 plots the daily returns, $log(F_{t+1,T}/F_{t,T})$, for the eight (M1, M2, M3, M4, M5, M6, Q1, Q2) futures contracts, and Panel A of Table 2.1 provides summary statistics. Table 2.1 indicates that futures returns on longer maturities futures contracts, e.g., Q2 futures contracts, are less volatile than futures returns for shorter maturity contracts. However, Figure 2-2 indicates that returns of futures contracts with longer maturities seem to contain bigger spikes.

Table 1 also reports summary statistics for higher moments of the daily futures returns. On average across maturities, skewness is -0.91 and kurtosis is 14.77. The daily crude oil futures return series is thus skewed towards the left, indicating that there are more negative than positive outlying returns in the crude oil market. The skewness is smaller (in absolute value) for longer maturities, and overall the negative skewness is rather modest. In contrast, Panel A of Table 2.1 indicates that kurtosis is economically large. The return series are characterized by a distribution with tails that are significantly thicker than a normal distribution.

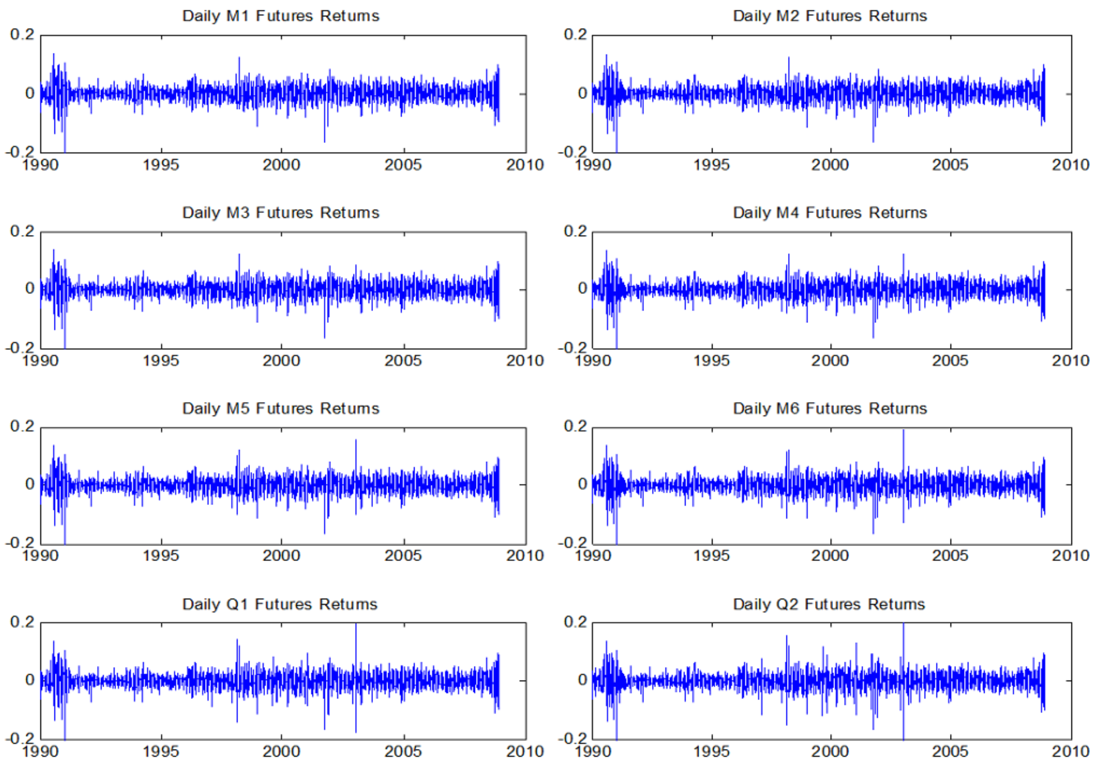
Figure 2-1: Prices of Futures Contracts



We plot the prices of the M1, M2, M3, M4, M5, M6, Q1, and Q2 futures contracts. The data spans 4,753 trading dates from January 2, 1990 to December 3, 2008. To avoid cluttering the figure, we only display the futures term structure on Wednesdays.

Figure 2-2: Daily Futures Returns

Prices of Futures Contracts



We plot daily futures returns, $logF_{t+1,T}/F_{t,T}$, on the M1, M2, M3, M5, M6, Q1, and Q2 futures contracts. The data spans 4753 trading days from January 2, 1990 to December 3, 2008.

Table 2.1: Summary Statistics

Panel A. Historical Moments of Futures Returns

	Maturity								
	M1	M2	M3	M4	M5	M6	Q1	Q2	Average
Mean	0.0494	0.042	0.0434	0.0467	0.0476	0.0508	0.0545	0.058	0.0491
Variance	32.6081	26.6954	23.1401	20.8757	19.2547	18.0024	16.6613	14.7116	21.4937
Skewness	-1.254	-1.1551	-1.0107	-0.9396	-0.8739	-0.7967	-0.7117	-0.5091	-0.9064
Kurtosis	23.3766	19.5005	15.9139	14.4642	13.2149	12.0101	10.9843	8.662	14.7658

Panel B. Number of Option Contracts

	Moneyness	Maturity								
		M1	M2	M3	M4	M5	M6	Q1	Q2	All
	0.78-0.82	443	2122	2765	2867	2790	2559	2504	1738	17788
	0.82-0.86	1087	2907	3340	3424	3293	2974	2883	2007	21915
Puts	0.86-0.90	1985	3647	3960	4065	3897	3474	3217	2335	26580
	0.90-0.94	3080	4309	4470	4262	4056	3592	3345	2434	29548
	0.94-0.98	4000	4607	4472	4190	4070	3635	3416	2559	30949
	0.98-1.02	4410	4527	4400	4116	3958	3505	3355	2482	30753
	1.02-1.06	4001	4605	4470	4259	3973	3464	3249	2337	30358
	1.06-1.10	3136	4362	4405	4155	3925	3473	3161	2153	28770
Calls	1.10-1.14	2209	3714	4196	4042	3683	3151	3116	1994	26105
	1.14-1.18	1404	3135	3482	3574	3368	2877	2877	1845	22562
	1.18-1.22	815	2547	3010	2819	2674	2367	2366	1727	18325
	All	26570	40482	42970	41773	39687	35071	33489	23611	283653

Table 2.1 (Continued)

Panel C. Average Option Prices

		Maturity								
	Moneyness	**M1**	**M2**	**M3**	**M4**	**M5**	**M6**	**Q1**	**Q2**	**All**
	0.78-0.82	0.27	0.31	0.45	0.58	0.72	0.89	1.11	1.58	0.74
	0.82-0.86	0.26	0.4	0.59	0.75	0.92	1.11	1.36	1.86	0.91
Puts	**0.86-0.90**	0.31	0.55	0.78	0.95	1.12	1.33	1.67	2.15	1.11
	0.90-0.94	0.42	0.76	1.04	1.29	1.5	1.78	2.1	2.64	1.44
	0.94-0.98	0.66	1.14	1.51	1.82	2.02	2.31	2.67	3.13	1.91
	0.98-1.02	1.08	1.69	2.09	2.43	2.64	2.96	3.36	3.9	2.52
	1.02-1.06	0.69	1.2	1.59	1.89	2.14	2.43	2.86	3.52	2.04
	1.06-1.10	0.48	0.86	1.18	1.45	1.66	1.91	2.3	2.96	1.6
Calls	**1.10-1.14**	0.37	0.68	0.91	1.13	1.3	1.53	1.83	2.51	1.28
	1.14-1.18	0.33	0.54	0.78	0.94	1.06	1.23	1.53	2.14	1.07
	1.18-1.22	0.32	0.44	0.65	0.86	0.96	1.11	1.33	1.71	0.92
	All	0.47	0.78	1.05	1.28	1.46	1.69	2.01	2.56	1.41

Panel D. Average Implied Log-Normal Volatilities

		Maturity								
	Moneyness	**M1**	**M2**	**M3**	**M4**	**M5**	**M6**	**Q1**	**Q2**	**All**
	0.78-0.82	0.57	0.44	0.4	0.37	0.36	0.34	0.32	0.3	0.39
	0.82-0.86	0.47	0.41	0.38	0.35	0.33	0.3 2	0.3	0.29	0.36
Puts	**0.86-0.90**	0.43	0.38	0.35	0.33	0.31	0.3	0.29	0.27	0.33
	0.90-0.94	0.39	0.35	0.33	0.31	0.3	0.3	0.28	0.27	0.32
	0.94-0.98	0.35	0.33	0.32	0.31	0.3	0.29	0.28	0.26	0.3
	0.98-1.02	0.33	0.33	0.32	0.31	0.29	0.29	0.27	0.26	0.3
	1.02-1.06	0.35	0.33	0.32	0.31	0.29	0.29	0.27	0.26	0.3
	1.06-1.10	0.38	0.34	0.32	0.31	0.29	0.29	0.27	0.26	0.31
Calls	**1.10-1.14**	0.41	0.36	0.33	0.31	0.3	0.29	0.27	0.26	0.32
	1.14-1.18	0.45	0.39	0.36	0.33	0.3	0.29	0.27	0.26	0.33
	1.18-1.22	0.5	0.4	0.37	0.35	0.32	0.31	0.28	0.26	0.35

We report summary statistics for crude oil futures returns and options. M1 (M2, M3, M4, M5, M6) refers to futures contracts with expiration in 1 (2, 3, 4, 5, 6) months; Q1 and Q2 refer to the next two futures contracts with expiration in either March, June, September or December. Moneyness is defined as the option strike divided by the price of the underlying futures contract. The data spans 4,753 trading days from January 2, 1990 to December 3, 2008.

Panel B of Table 2.1 lists the average number of option contracts across maturity and moneyness. The number of option contracts decreases with maturity. Among the 11 moneyness intervals, the number of option contracts is highest in the ATM interval. Panel C reports the average option prices. As expected, the average price of the option contracts increases as the maturity of underlying futures contracts increases.

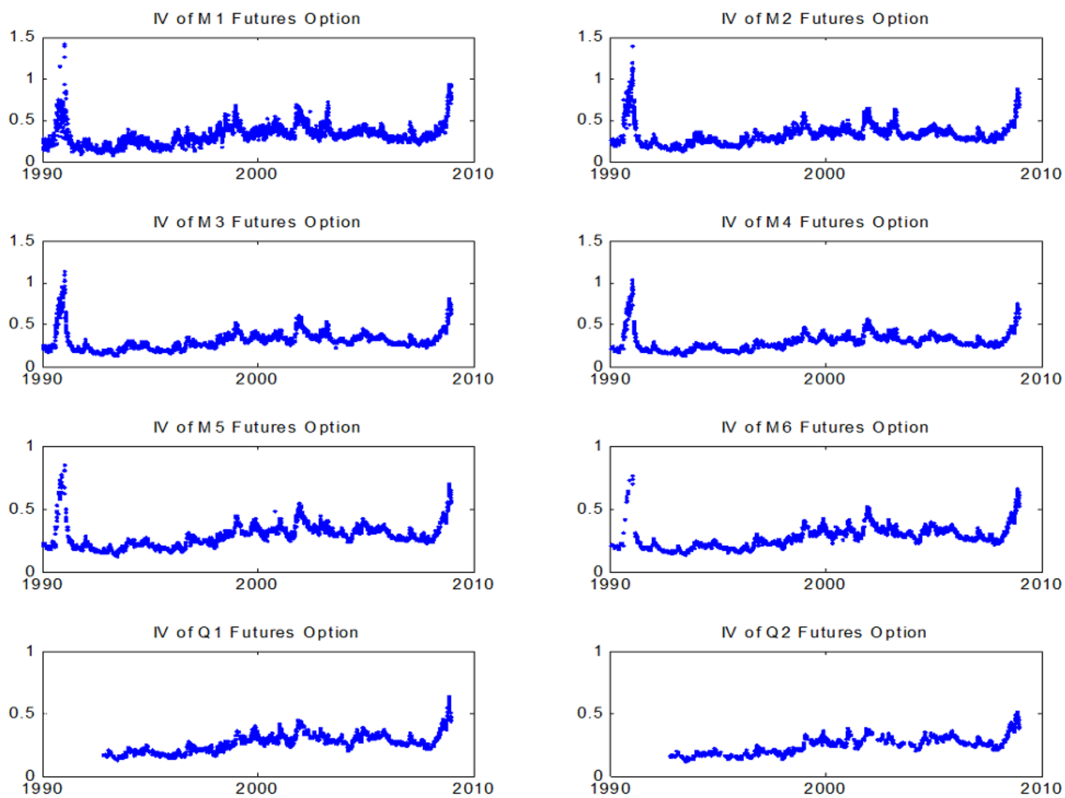
Figure 2-3 displays the ATM implied volatilities, and Panel D of Table 2.1 reports their averages by moneyness and maturity. Options with short maturities have higher implied volatilities than options with long maturities, consistent with the patterns in physical volatility documented in Panel A of Table 2.1. Large spikes in the option implied volatilities appear around the end of 1990 and beginning of 1991, at the time of the first Gulf War, around the September 2001 terrorist attack, the second Gulf War in March 2003, and during the financial crisis in 2008.

Among the eleven moneyness intervals, the average implied volatilities are lowest for ATM options. The data exhibit a smirk for some maturities, but for others it is not clear if the smirk pattern is economically significant, or if the data are instead characterized by a smile. These patterns are important with respect to the relative role of return skewness and kurtosis for characterizing the option data. Panel D of Table 1 suggests that modeling kurtosis may be more critical than capturing skewness for crude oil data. Note that the stylized facts in Panel D are informative about risk-neutral skewness and kurtosis, but that the evidence is consistent with the descriptive statistics for historical skewness and kurtosis computed from futures returns in Panel A of Table 2.1.

2.4 Evidence from Futures Prices

We first discuss how to use maximum likelihood to estimate the models using futures returns. We then present parameter estimates for the two jump models as well as for the benchmark GARCH model. Subsequently we use the parameter estimates to investigate the models' most important implications for option valuation.

Figure 2-3: ATM Implied Volatility of Futures Options



We plot ATM implied volatilities of options on the M1, M2, M3, M5, M6, Q1, and Q2 futures contracts. Implied volatilities are computed from option prices by inverting the Barone-Adesi and Whaley (1987) formula. The data spans 4,753 trading days from January 2, 1990 to December 3, 2008.

2.4.1 Maximum Likelihood Estimation using Futures Data

We estimate the model parameters using Maximum Likelihood (MLE). The likelihood function for returns depends on the normal and Compound Poisson distributions. The conditional density of the returns process in equation (2.6) with time-to-maturity T_i, given that there are $n_{t+1} = j$ jumps occurring between period t and $t+1$, is given by

$$f_t\left(R_{t+1,i}|n_{t+1} = j\right) = \frac{1}{\sqrt{2\pi(h_{z,t+1} + j\delta^2)}} \exp\left(-\frac{\left(R_{t+1,i} - \mu_{t+1} - j\theta\right)^2}{2(h_{z,t+1} + j\delta^2)}\right) \tag{2.14}$$

where $R_{t+1,i} = \log \frac{F_{t+1,T_i}}{F_{t,T_i}}$, and $\mu_{t+1} = (\lambda_z - \frac{1}{2})h_{z,t+1} + (\lambda_y - \xi)h_{y,t+1}$.

The conditional probability density of returns can be derived by summing over the number of jumps

$$f_t\left(R_{t+1,i}\right) = \sum_{j=1}^{\infty} f_t\left(R_{t+1,i}|n_{t+1} = j\right) \Pr(n_{t+1} = j) \tag{2.15}$$

where $\Pr(n_{t+1} = j) = (h_{y,t+1})^j \exp(-h_{y,t+1})/j!$ is the probability of having j jumps which is distributed as a Poisson counting process.

In estimation we assume that the conditional variance and the jump intensity are equal across maturities. This is clearly a simplifying assumption that will worsen the fit, but it is useful for the purpose of comparison with option-implied estimates. Given this assumption, we can write the log likelihood function as the summation of the log likelihoods for all eight futures contracts

$$L_{Fut} = \frac{1}{8} \sum_{i=1}^{8} \sum_{t=1}^{T-1} \log(f_t\left(R_{t+1,i}\right)) \tag{2.16}$$

When implementing maximum likelihood estimation, the summation in (2.15) must be truncated. We truncate the summation at 50 jumps per day. We have experimented with increasing the truncation limit beyond 50 and found that our results are robust.

Equations (2.8) and (2.9) indicate that we need to separately identify the two unobserved shocks z_{t+1} and y_{t+1} and filter the conditional variance $h_{z,t+1}$ and the conditional jump intensity $h_{y,t+1}$ which enter the likelihood function. The structure of the model allows us to do this using an analytical filter, which is discussed in Appendix A. Using this filter, calculating z_{t+1} and y_{t+1} is straightforward and very fast. It takes less than a second to filter 38,024 futures contracts and about seven seconds to filter 283,653 option contracts using Matlab on a standard PC.

2.4.2 Estimation Results

Table 2.2 presents the maximum likelihood parameter estimates for the GARCH benchmark model and the two proposed jump models. The results are obtained using all eight futures contracts jointly in estimation for the time period 1990-2008. For each jump model, we separate the parameters into two columns. The parameters with subscript y are reported in the column labeled "Jump". The parameters with subscript z are reported in the column labeled "Normal". Under each parameter estimate, we report its standard error calculated using the Hessian matrix. Under "Properties", we report the implied long-run risk premiums for the normal and jump components, the percent of total variance captured by the normal and the jump component, the average annual volatility, the expected number of jumps per year implied by the parameter estimates, and the log-likelihood. Some of these properties are discussed in more detail in Section 4.3 below.

The log-likelihood values of the CI and DI jump models are much higher than that of the GARCH model. To examine whether the jump models significantly improve over the GARCH model, we test the null hypothesis of no jumps. To implement this test, we use the standardized likelihood ratio test proposed by Hansen (1992, 1994). A likelihood ratio test of the null hypothesis of no jumps does not have the usual limiting chi-squared distribution because the jump parameters are unidentified under the null. Hansen's test is able to provide an upper bound to the asymptotic distribution of standardized likelihood ratio statistics, even when conventional regularity conditions (for example, due to unidentified parameters) are violated. We calculate Hansen's test for each of the CI and DI models compared with GARCH and report the standardized likelihood ratio and the corresponding simulated critical values in Table 2.3. Using Hansen's standardized LR test, we find that both jump models significantly improve over the GARCH model, suggesting that the null hypothesis

Table 2.2: MLE Estimates Using Crude Oil Futures Returns, 1990 - 2008

Parameters	**GARCH**	**CI**		**DI**	
	Normal	**Normal**	**Jump**	**Normal**	**Jump**
λ	7.20E-01	1.28E+00	1.74E-03	1.19E+00	1.08E-02
	-4.03E-03	-4.45E-03	-2.57E-05	-5.31E-04	-1.61E-05
w			2.62E-02		
			-2.74E-05		
a	1.56E-05	5.85E-06		5.03E-06	
	-7.96E-07	-2.13E-07		-3.86E-10	
b	9.60E-01	9.78E-01		9.71E-01	
	-2.01E-03	-6.71E-04		-5.56E-07	
c	2.75E+01	7.09E+00		5.50E+00	
	-2.20E-02	-1.02E-02		-2.95E-04	
d		-3.99E-04		-1.79E-03	
		-2.40E-05		-5.90E-09	
θ			-2.94E-02		-1.79E-01
			-4.66E-05		-1.04E-07
δ			1.01E-01		8.94E-02
			-7.69E-05		-5.45E-06
k					3.23E+01
					-3.72E-04
Properties					
Number of jumps / Yr		6.6		2.07	
Risk Premium(%)	7.53	7.5	1.15	7.63	2.24
% of Annual Variance	100	44.55	55.45	43.58	56.42
Ave. Annual Volatility	0.32	0.36		0.38	
Log-Likelihood	12711.72	12981.62		12988.62	

We report estimation results from MLE estimation on daily crude oil futures returns from January 2, 1990 to December 3, 2008. Columns labeled "Normal" contain estimates of the parameters governing the normal component; columns labeled "Jump" contain parameters governing the jump component. Reported in parentheses are standard errors computed using the Hessian matrix.

Table 2.3: Hansen's Standardized Likelihood Ratio test

Models	CI	DI
Hansen's standardized LR test	7.1579****	7.2829****
Simulated 20% critical value	1.3551	1.495
Simulated 10% critical value	1.7815	1.8521
Simulated 5% critical value	2.2186	2.1809
Simulated 1% critical value	2.6489	2.8726

We report Hansen's Standardized Likelihood Ratio test and the corresponding simulated critical values for the discrete-time CI and DI jump models. Under the null hypothesis there are no jumps. The log likelihoods are calculated using daily crude oil futures returns from January 2, 1990 to December 3, 2008. *, **, *** and ****represents significance at the 20%, 10%, 5% and 1% level or better.

of no jumps is rejected. These statistical tests strongly suggest that incorporating jumps in addition to dynamic volatility helps to improve model performance.

The CI model improves model fit significantly by adding a simple constant intensity jump component. For this specification, the average expected number of jumps per year is between six and seven. This number is slightly higher than what is usually found in the equity index market, see the summary table in Broadie, Chernov, and Johannes (2007). Most existing estimates in equity index markets find between one and three jumps per year. The estimate of the average jump size θ in the CI model in Table 2.2 is -0.029, which is larger than the average jump size in index returns documented in CJO (2012) for a model with constant jump intensity.

The results for the CI model indicate that allowing for state-dependent jump intensities can further improve model performance, even without increasing the number of parameters. The estimate of k is statistically significant, confirming that the arrival rate of jumps depends on the level of volatility. The mean jump size in the DI model is larger (in absolute value) than in the CI model and jumps arrive less frequently, with approximately two jumps per year. Other important model features, such as the jump variance, are similar to the CI model. Overall, the results for the DI model indicate that allowing for time-varying jump intensities can greatly improve model performance.

2.4.3 Model Implications

We now further discuss the model properties listed at the bottom of Table 2.2. We report the decomposition of the total unconditional return variance into the normal and jump components. The total unconditional return variance, σ^2, is given by

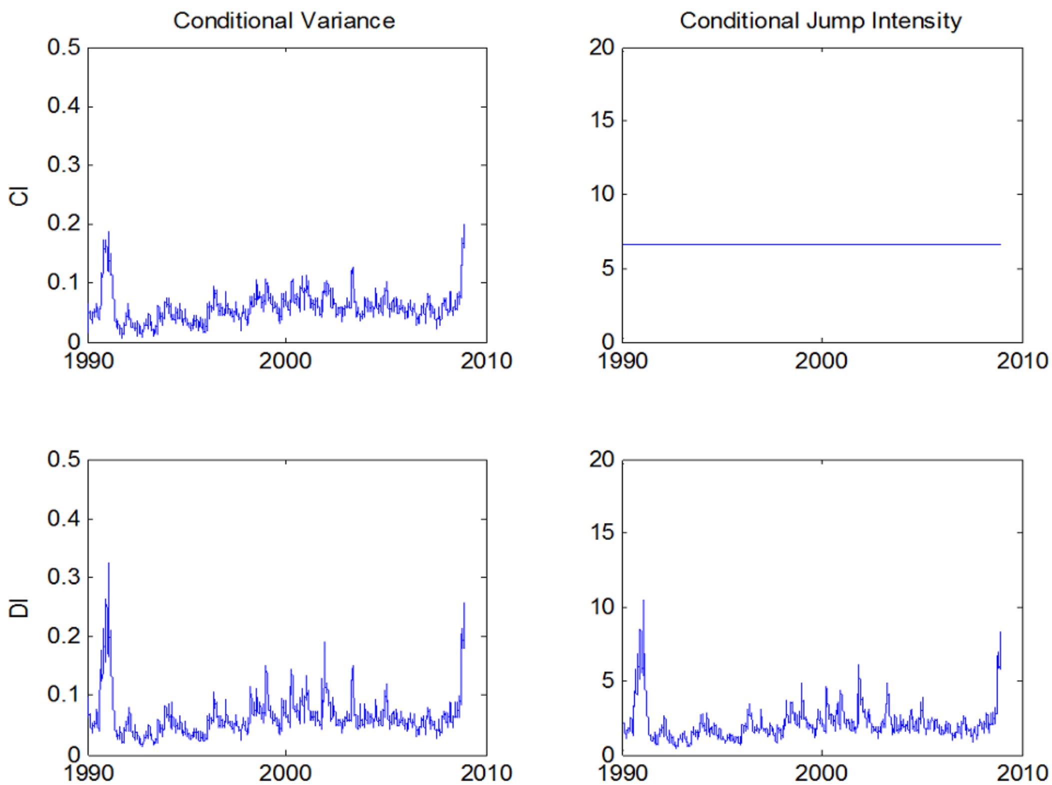
$$\sigma^2 \equiv \sigma_z^2 + (\theta^2 + \delta^2)\sigma_y^2 \tag{2.17}$$

where σ_z^2 and σ_y^2 are computed as the time series averages of $h_{z,t+1}$ and $h_{y,t+1}$. We report the normal contribution and jump contribution to the total return variance in percentages. The contribution of jumps to the total return variance is large and very similar in the CI and DI models, approximately 55%.

The average variance reported in Table 2.2 is similar in magnitude across models. The left panels in Figure 2-4 depict the time path of the conditional variance and clearly indicate that the conditional variance of the CI model is less volatile than that of the DI model. For the DI model, there is a sharp increase in the conditional variance and jump intensity during the crisis periods of 1991 and 2008.

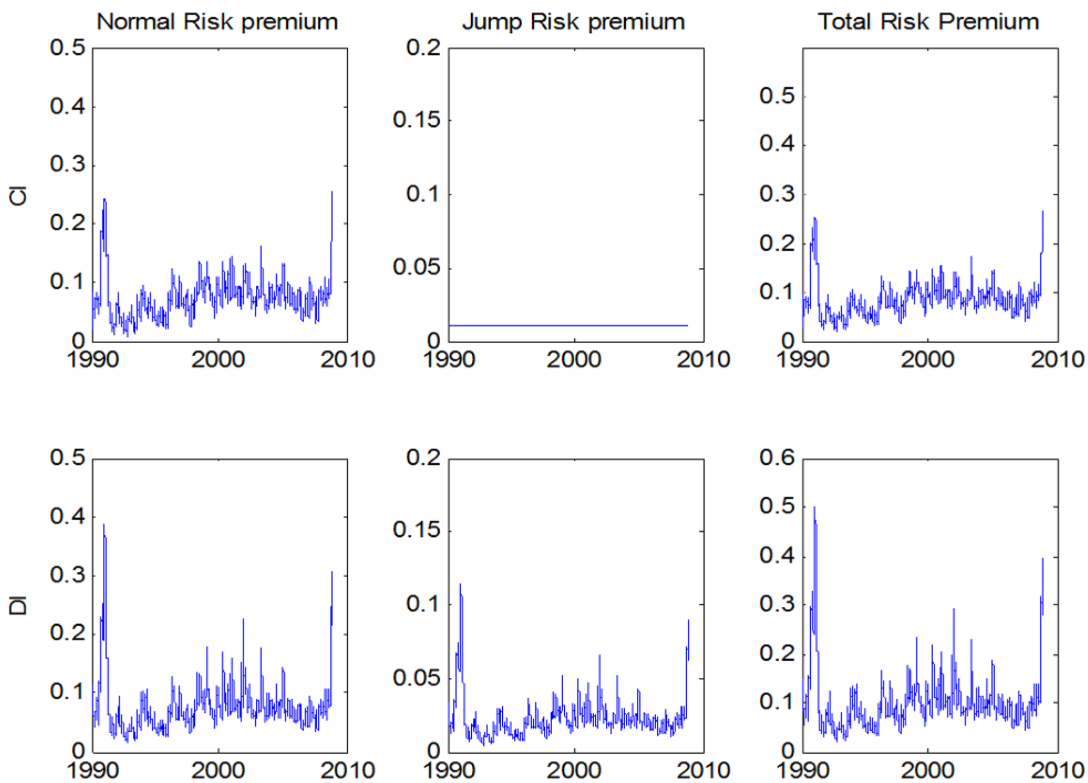
The variations in jump intensities affect the risk premiums, which are depicted in Figure 2-5. For the CI model in the first row, the jump risk premium in the middle panel is constant, and all the time variation in the total risk premium in the right panel is due to variation in the normal risk premium. In the case of the DI model, a large amount of the increase in the total risk premium in 1991 and 2008 is due to the increase in the jump risk premium. Overall, Figure 2-5 indicates that jump risk premiums are economically important, and that they represent a significant component of the total risk premium. Under "Properties" in Table 2.2 we also report the averages of the risk premiums over the sample. On average the jump risk premium contributes a significant portion of the total risk premium for both the CI and DI models, but it is larger in the case of the DI model. Allowing for time-varying jump intensities increases the importance of jumps in explaining the risk premium.

Figure 2-4: Conditional Variance and Jump Intensity Estimated Using Futures Contracts



We plot the annualized conditional variance, $h_{z,t+1}$, in the left column, and the annualized conditional jump intensity, $h_{y,t+1}$, in the right column for two jump models.

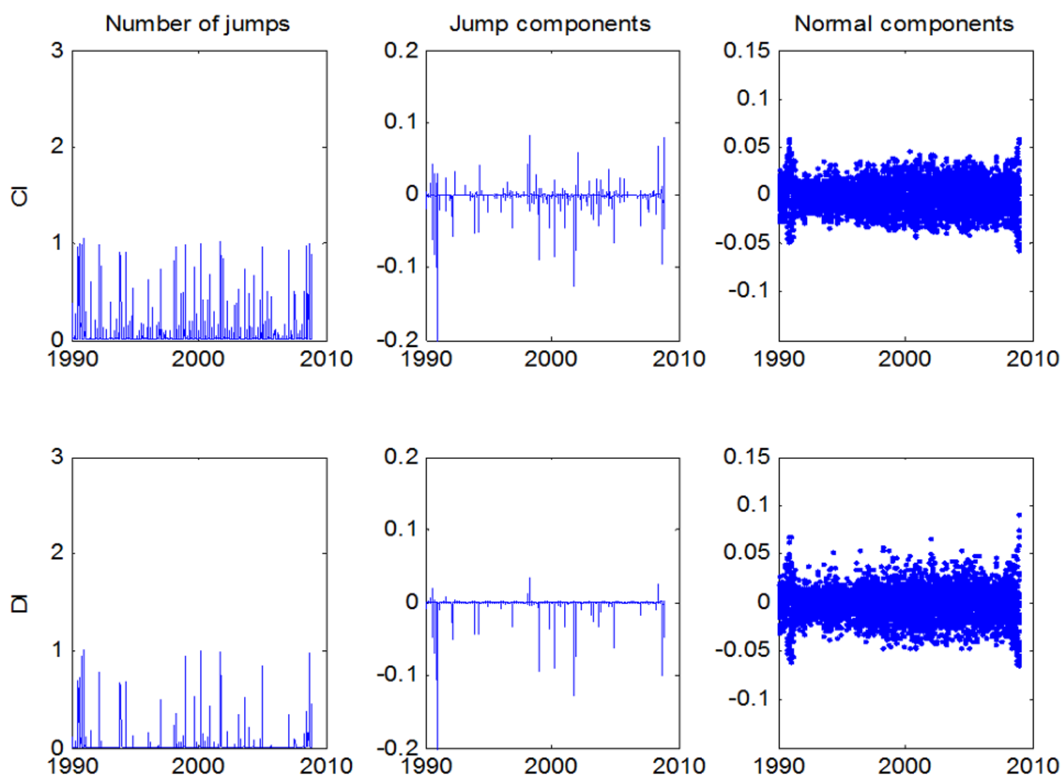
Figure 2-5: Risk Premiums Estimated Using Futures Contracts



We plot the normal risk premium, $\gamma_{z,t} \equiv \lambda_z h_{z,t}$ in the left column, the jump risk premium, $\gamma_{y,t} \equiv \lambda_y h_{y,t}$, in the middle column, and the total risk premium, $\gamma_t \equiv \lambda_z h_z + \lambda_y h_y$, in the right column, for two jump models.

Figure 2-6 applies the analytical filter to decompose futures returns into jump and normal components to infer their relative importance. The left columns depict the filtered number of jumps n_t occurring each day. The middle column contains the filtered jump component and the right column contains the filtered normal component. The heteroskedasticity in the normal component is apparent. Most of the time, the normal component dominates return innovations. However, in crises periods, such as the first Gulf War in late 1991, the jump component explains more of the movement in returns than the normal component. For the purpose of option valuation, the time path of the conditional variance is of paramount

Figure 2-6: Decomposition of Daily Futures Returns Estimated Using Futures Contracts



We plot the filtered number of jumps, n_t, in the left column, the filtered jump component, y_t, in the middle column, and the filtered standardized normal component, z_t, in the right column, for two jump models. Results are obtained using the analytical filter and the MLE estimates from Table 2.2.

importance. However, different models often yield variance paths that are nearly similar, as evidenced by Figure 2-4. It is therefore of great interest to inspect differences in the conditional third and fourth moments. We now turn to this evidence. The first four conditional moments are given by

$$E_t(R_{t+1}) \equiv \mu_{t+1} = (\lambda_z - \frac{1}{2})h_{z,t+1} + (\lambda_y - \xi)h_{y,t+1} \tag{2.18}$$

$$Var_t(R_{t+1}) = h_{z,t+1} + (\theta^2 + \delta^2)h_{y,t+1} \tag{2.19}$$

$$Skew_t(R_{t+1}) = \frac{\theta(\theta^2 + 3\delta^2)h_{y,t+1}}{(h_{z,t+1} + (\theta^2 + \delta^2)h_{y,t+1})^{\frac{3}{2}}} \tag{2.20}$$

$$Kurt_t(R_{t+1}) = 3 + \frac{(\theta^4 + 6\theta^2\delta^2 + 3\delta^4)h_{y,t+1}}{(h_{z,t+1} + (\theta^2 + \delta^2)h_{y,t+1})^2} \tag{2.21}$$

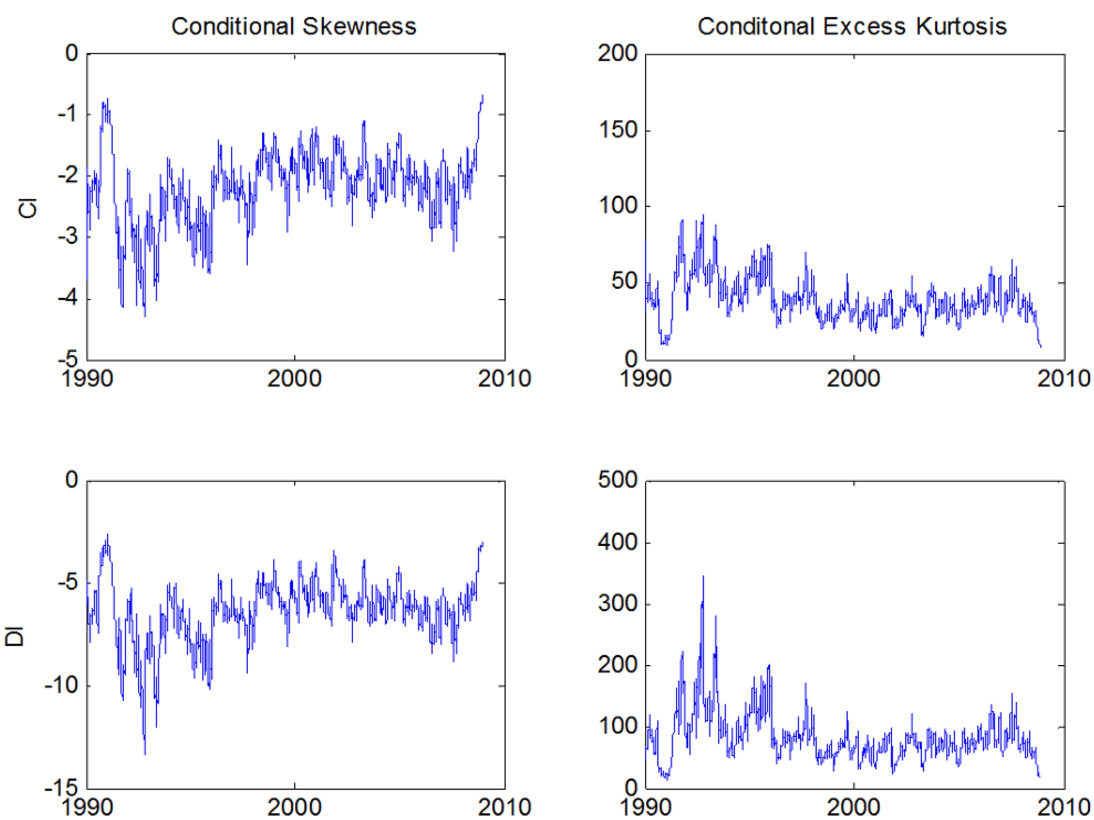
where $Skew_t(R_{t+1})$ and $Kurt_t(R_{t+1})$ are the conditional skewness and the conditional kurtosis of futures returns respectively. From equation (2.20), it is clear that in presence of jumps, when $h_{y,t+1}$ is positive, the sign of the conditional skewness depends on the sign of the mean jump size θ. Both skewness and kurtosis are critically affected by the parameters θ and δ.

Figure 2-7 plots the conditional one day ahead skewness in (2.20) and kurtosis in (2.21) for the two jump models. The estimated average jump size θ is negative for both models, and therefore the conditional one day ahead skewness is negative. Conditional skewness is relatively high compared to the unconditional values reported in Panel A of Table 2.1, especially for the DI model. Conditional excess kurtosis is also higher than the unconditional values reported in Panel A of Table 2.1, and contains more sharp peaks for the DI model. Clearly, for a given model outliers in model skewness and kurtosis are related, which is driven by the parameterization.

2.5 Option Valuation Theory for Crude Oil Futures

We first characterize the risk-neutral dynamics. Subsequently we derive the closed-form option valuation formula.

Figure 2-7: Conditional Skewness and Conditional Excess Kurtosis from Futures Contracts



We plot daily conditional skewness, in the left column, and conditional excess kurtosis, in the right column, for two jump models.

2.5.1 The Equivalent Martingale Measure and the Risk-Neutral Dynamics

The estimates obtained from futures prices in Section 4 are physical parameters. To value crude oil options, we need return dynamics under the equivalent martingale or risk-neutral measure. In a framework with compound Poisson processes, the futures price can jump to an infinite set of values in a single period, and the equivalent martingale measure is therefore not unique. We proceed by specifying the conditional Radon-Nikodym derivative:

$$\frac{\frac{dQ_{t+1}}{dP_{t+1}}}{\frac{dQ_t}{dP_t}} = \frac{\exp(\Lambda_z z_{t+1} + \Lambda_y y_{t+1})}{E_t[\exp(\Lambda_z z_{t+1} + \Lambda_y y_{t+1})]} \tag{2.22}$$

where Λ_z and Λ_y are the equivalent martingale measure (EMM) coefficients that capture the wedge between the physical and the risk-neutral measure. As in Ornthanalai (2012), this Radon-Nikodym derivative specifies a risk-neutral probability measure if and only if Λ_z and Λ_y are determined by

$$\Lambda_z + \lambda_z = 0 \tag{2.23}$$

$$\lambda_y - (\exp(\theta + \frac{\delta^2}{2}) - 1) - \exp(\Lambda_y \theta + \frac{\Lambda_y^2 \delta^2}{2})(1 - \exp((\Lambda_y + 0.5)\delta^2 + \theta)) = 0 \tag{2.24}$$

The solution for Λ_y is not analytical but it is well behaved and can reliably and efficiently be computed using a numerical approach. The futures return process under the risk-neutral dynamic then takes the form

$$\log \frac{F_{t+1,T}}{F_{t,T}} = -\frac{1}{2} h_{z,t+1} - \xi_y(1)^* h^*_{y,t+1} + z_{t+1} + y^*_{t+1} \tag{2.25}$$

with the following variance and jump intensity dynamics

$$h_{z,t+1} = \omega_z + b_z h_{z,t} + \frac{a_z}{h_{z,t}}(z_t - c^*_z h_{z,t})^2 + d_z y^*_t \tag{2.26}$$

$$h^*_{y,t+1} = \omega^*_y + b_y h^*_{y,t} + \frac{a^*_y}{h_{y,t}}(z_t - c^*_y h_{z,t})^2 + d^*_y y^*_t \tag{2.27}$$

where $h^*_{y,t+1} = h_{y,t+1}\Pi$, $\Pi = \exp(\Lambda_y \theta + \frac{\Lambda_y^2 \delta^2}{2})$, $\theta^* = \theta + \Lambda_y \delta^2$, $\xi(1)^* = \exp(\theta^* + \frac{\delta^2}{2}) - 1$, $\Lambda_z = -\lambda_z$, $\omega^*_y = \omega_y \Pi$, $a^*_y = a_y \Pi$, $c^*_z = c_z - \Lambda_z$, $c^*_y = c_y - \Lambda_z$, $d^*_y = d_y \Pi$, and $y^*_{t+1} \sim J(h^*_{y,t+1}, \theta^*, \delta^2)$.

The risk neutral dynamic for the GARCH benchmark model in Section 2.1 is a special case of (2.25)-(2.27) with $h^*_{y,t+1} = y^*_{t+1} = 0$.

2.5.2 Closed-Form Option Valuation

Under the risk-neutral measure, the generating function for the asset process in (2.25)-(2.27) takes the following form

$$f^*(\varphi; t, T) \equiv E_t^Q[F_T^{\varphi}] = F_{t,T}^{\varphi} \exp(A1(\varphi; t, T) + B1(\varphi; t, T)h_{z,t+1} + C1(\varphi; t, T)h_{y,t+1}^*) \quad (2.28)$$

Here we present the analytical solutions to the affine coefficients $A1(\varphi; t, T)$, $B1(\varphi; t, T)$, and $C1(\varphi; t, T)$. Details on the derivation are provided in Appendix B.

$$\begin{aligned} A1(\varphi; t, T) &= A1(\varphi; t+1, T) + B1(\varphi; t+1, T)\omega_z + C1(\varphi; t+1, T)\omega_y^* \\ &\quad -\frac{1}{2}\log(1 - 2B1(\varphi; t+1, T)a_z - 2C1(\varphi; t+1, T)a_y^*) \end{aligned} \quad (2.29)$$

$$\begin{aligned} B1(\varphi; t, T) &= \varphi\mu_{1z} + B1(\varphi; t+1, T)(b_z + a_z c_z^{*2}) + C1(\varphi; t+1, T)a_y^* c_y^{*2} \\ &\quad + \frac{(\varphi - 2B1(\varphi; t+1, T)a_z c_z^* - 2C1(\varphi; t+1, T)a_y^* c_y^*)^2}{2(1 - 2B1(\varphi; t+1, T)a_z - 2C1(\varphi; t+1, T)a_y^*)} \end{aligned} \quad (2.30)$$

$$C1(\varphi; t, T) = b_y C1(\varphi; t+1, T) + \varphi\mu_{1y} + \xi_y(\Xi)^* \quad (2.31)$$

where $\mu_z = -\frac{1}{2}$, $\mu_y = -\xi(1)^*$, $\xi_y(\Xi)^* = \exp(\theta^*\Xi^* + \frac{\Xi^{*2}\delta^2}{2}) - 1$, with

$$\Xi^* = \varphi + B1(\varphi; t+1, T)d_z + C1(\varphi; t+1, T)d_z^* \quad (2.32)$$

By imposing the restrictions $h_{y,t+1}^* = \omega_y^* = a_y^* = 0$, (2.28)-(2.32) reduces to the generating function for the GARCH benchmark model in Section 2.1.

With the risk neutral generating function (2.28), we can value European options using the Fourier inversion method as in Heston (1993), Heston and Nandi (2000), and Duffie, Pan and Singleton (2000). The price of a European call option on a futures contract is given by

$$\begin{aligned} CO(t, T_{co}, T, K) &= E_t^Q[\exp(-\int_t^{T_{co}} r(s)ds)(F(T_{co}, T) - K)^+] \\ &= F(t, T)(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \text{Re}[\frac{K^{-i\varphi} f^*(i\varphi + 1)}{i\varphi f^*(1)}]d\varphi) \\ &\quad - \exp(-\int_t^{T_{co}} r(s)ds)K(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \text{Re}[\frac{K^{-i\varphi} f^*(i\varphi)}{i\varphi}]d\varphi) \end{aligned} \quad (2.33)$$

where $CO(t, T_{co}, T, K)$ is the time t price of a European call option expiring at time T_{co} with strike K on a futures contract expiring at time T, and K is the strike price.

2.6 Joint Estimation Using Futures and Options Data

It is possible to use the parameter estimates in Table 2.2, obtained through MLE estimation on futures data, to compute option prices using the option valuation formulae. However, this procedure exclusively uses historical information and ignores the expectations about the future evolution of the futures prices that are embedded in option prices. It is therefore also interesting to study the models' option valuation performance by specifying a loss function based on option contracts, and matching model option values as closely as possible to observed market prices.

While such an exercise imposes considerable discipline upon the models, it has nevertheless two important drawbacks. First, if a model is richly parameterized, only fitting the option data may result in overfitting. Second, the price of risk parameters, which are some of the most economically important model parameters, cannot be reliably identified using option data only.

We therefore follow Bates (1996), who suggests that the most stringent test of an option pricing model lies in its ability to jointly fit the option data and the underlying returns. In our case, this means that we have to construct a loss function that contains a crude oil option component as well as a futures return component. We first discuss the likelihood function used to estimate the model parameters from option data. We then explain how to combine the option data with the underlying futures data and construct a joint likelihood. Subsequently we discuss the parameter estimates, and compare the most important model properties with the properties implied by the physical parameters from Table 2.2.

2.6.1 The Likelihood Function from Option Data

To obtain the fitted option prices, we first need to filter the latent state variables. For the GARCH model, it is the conditional variance; for the jump models, they are the conditional variance and conditional jump intensity. The latent state variables can be filtered using different types of information. They can be obtained from futures returns using the analytical filter, as in Section 4. Instead, we adapt Kanniainen's (2013) methodology and extract conditional variances for the GARCH, CI, and DI models from 30-day ATM implied volatilities. The methods used to extract the state variables from implied volatility for the different models are discussed in more detail in Appendix C.

We use a loss function based on implied volatilities, inverting option prices into implied volatilities. This approach uses market data that is of similar magnitude along the moneyness, maturity, and time-series dimensions, which is attractive from a statistical perspective. Option prices differ significantly along these dimensions. Define the model error

$$u_{k,t} = \sigma_{k,t} - \widehat{\sigma}_{k,t}(O_{k,t}(h_t(\theta^*))) \tag{2.34}$$

where $\sigma_{k,t}$ is the Black (1976) implied volatility of the k^{th} observed option price at time t, and $\widehat{\sigma}_{k,t}(O_{k,t}(h_t(\theta^*)))$ is the implied volatility converted from each computed option price, $O_{k,t}(h_t(\theta^*))$, using the Black (1976) formula.

Assuming normality of the implied volatility errors, $u_{k,t} \sim N(0, \sigma_u^2)$, the log-likelihood function based on options is

$$L_{Opt} = -\frac{N}{2}\ln(2\pi\sigma_u^2) - \frac{1}{2}\sum_{t,k}^{N}\frac{u_{k,t}^2}{\sigma_u^2} \tag{2.35}$$

where $N = 283,653$ is the total number of option contracts. Parameters can be estimated from options data only by maximizing L_{Opt}, but we do not proceed in this way.

2.6.2 The Joint Log Likelihood Function

The log-likelihood functions for futures and options are defined in equations (2.16) and (2.35) respectively. We maximize the weighted average of the log-likelihoods of futures and options to obtain parameter estimates for the jump models that are reflecting both the option data and the underlying futures data.

The number of option contracts in the data set is much larger than the number of futures contracts. To ensure that joint parameter estimates are not dominated by options, we assign equal weight to each log-likelihood. The resulting weighted joint log-likelihood is

$$L_{Joint} = \frac{M+N}{2}\frac{L_{Fut}}{M} + \frac{M+N}{2}\frac{L_{Opt}}{N} \tag{2.36}$$

where $M = 38,024$ is the total number of futures contracts and $N = 283,653$ is the total number of option contracts.

We report the optimized joint likelihoods as well as the corresponding implied volatility root mean squared error (IVRMSE)

$$IVRMSE = \frac{1}{T}\sqrt{\frac{1}{N}\sum_{t,k}^{N} u_{k,t}^2} \tag{2.37}$$

where $T = 4,753$ is the number of days used in our analysis.

Ideally one would fit the model directly to implied Black volatilities. However, since the optimization routine requires computing implied volatility from model prices at every function evaluation, this approach is extremely slow. We follow Trolle and Schwartz (2009) and fit the model to option prices scaled by their Black (1976) vega, that is, the sensitivities of the option prices to variations in log-normal volatilities. This approach is motivated by the approximation $\sigma_{k,t} \approx O_{k,t}/\nu_{k,t}$, where $\nu_{k,t}$ is the Black (1976) vega associated with the k^{th} observed option price at time t. This approximation has been shown to work well in existing work. Thanks to the use of the analytical filter, the quasi-closed form option valuation formula, and the use of the vega-scaled prices, the optimization problem is feasible with our large data sets.

Furthermore, to ensure reasonable model properties and to facilitate the search for optimal parameter values, we impose variance targeting. For example, for the GARCH model, instead of estimating ω, we infer it from the unconditional variance and other parameter estimates, according to equation (2.3). Variance targeting for the jump models proceeds along the same lines.

2.6.3 Empirical Estimates and Model Implications

Table 2.4 reports the parameter estimates obtained by maximizing the joint log-likelihood function in (6.3) for the GARCH model and the two jump models. At the bottom of the table, we also report the log-likelihood, the IVRMSE, and several other model properties implied by the parameters such as the long-run risk premium, the percent of total annual variance explained by the normal and the jump component, the average annual volatility, and the expected number of jumps per year. Note that unlike the parameters in Table 2.2, the parameters in Table 2.4 are risk-neutral parameters. Similar to the estimation results based on crude oil futures data in Table 2.2, the jump models outperform the benchmark GARCH model. The jump model with constant jump intensity CI has an IVRMSE of 4.11 and outperforms the GARCH model by approximately 12%. The jump model with time-varying intensities DI has an IVRMSEs of 3.70 and outperforms the GARCH model by approximately 19%. These findings confirm that incorporating jumps in addition to dynamic volatilities helps to improve model fit, and that it is important to allow for time-varying jump intensities.

Figure 2-8 reports the implied variance paths and conditional jump intensity paths based on the estimates from futures and futures options in Table 2.4. In the crisis period around the first Gulf war, the conditional variance paths in Figure 2-8 contain more pronounced spikes compared to the variance paths in Figure 2-4, which are based on estimates from returns only. This may be due to the fact that for the estimates in Table 2.4, we extract the conditional variance from implied volatility, which is more volatile than the conditional volatility filtered from futures returns.

Since option prices contain important information about the pricing kernel that cannot necessarily be inferred from the underlying futures returns dynamics, the market prices of risk for the normal and jump components and the implied risk premiums are of particular interest. The combined average risk premiums for the CI and DI models are very similar, 10.42 and 10.16 percent respectively. This is a bit larger than the ones implied by the estimates in Table 2.2. Both the normal and the jump components are economically important, and as a percent of the total risk premium, the importance of the jump risk premiums in Table 2.4 is similar to Table 2.2.

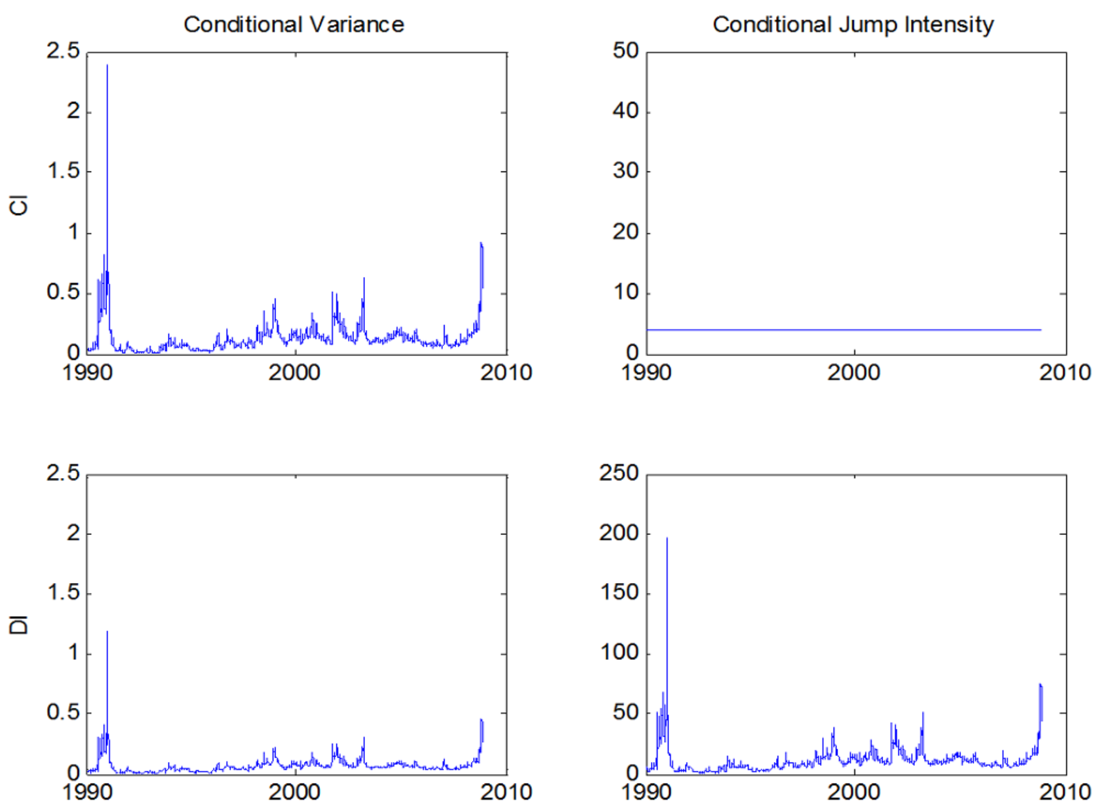
Figure 2-9 plots the resulting time variation in the conditional normal risk premium, the

Table 2.4: Joint MLE Estimates Using Crude Oil Futures and Options, 1990-2008

Parameters	**GARCH**	**CI**		**DI**	
	Normal	**Normal**	**Jump**	**Normal**	**Jump**
λ	5.83E-01	6.76E-01	4.03E-03	1.17E+00	1.86E-03
	-2.26E-04	-7.03E-04	-4.34E-06	-6.89E-04	-6.21E-07
w			1.57E-02		
			-7.18E-06		
a	2.95E-05	3.18E-05		1.10E-05	
	-1.08E-07	-3.19E-08		-1.25E-08	
b	9.63E-01	9.43E-01		8.04E-01	
	-1.28E-04	-4.81E-05		-2.74E-04	
c	2.53E+01	3.78E+01		1.30E+02	
	-7.14E-03	-3.09E-02		-9.63E-02	
d		1.44E-02		2.55E-03	
		-4.16E-06		-1.09E-06	
θ			-1.54E-03		-1.20E-03
			-9.27E-07		-7.50E-07
δ			3.97E-02		7.61E-02
			-2.09E-05		-1.29E-04
k					1.65E+02
					-2.02E-01
Properties					
Number of jumps / Yr		3.98		11.41	
Risk Premium	8.19	8.81	1.61	8.04	2.12
% of Annual Variance	100	95.4	4.6	51.08	48.92
Ave. Annual Volatility	0.37	0.37		0.37	
IV Bias	1.83	0.76		0.29	
IV RMSE	4.67	4.11		3.78	
Likelihood	318236.525	339331.74		354223.97	

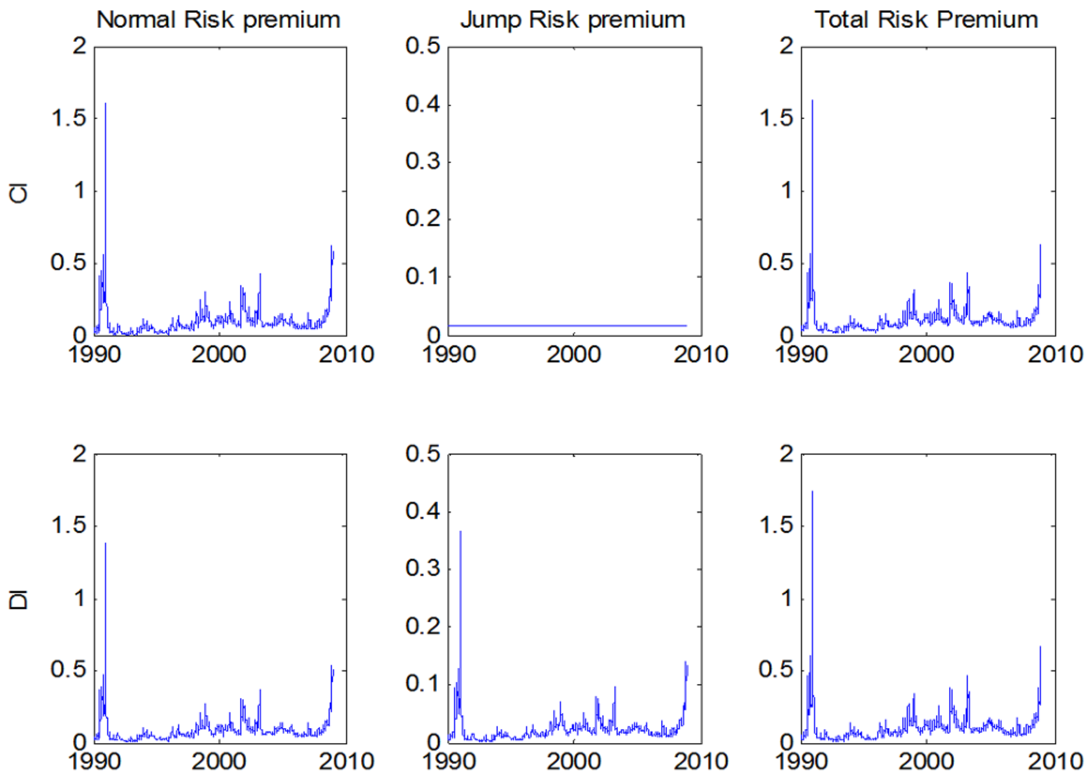
We report estimation results from MLE estimation using daily crude oil futures and options from January 2, 1990 to December 3, 2008. Reported in parentheses are standard errors computed using the Hessian matrix.

Figure 2-8: Conditional Variance and Jump Intensity Estimated Using Futures and Option Contracts



We plot the annualized conditional variance, $h_{z,t+1}$, in the left column and the annualized conditional jump intensity, $h_{y,t+1}$, in the right column for two jump models.

Figure 2-9: Risk Premiums Estimated Using Futures and Option Contracts



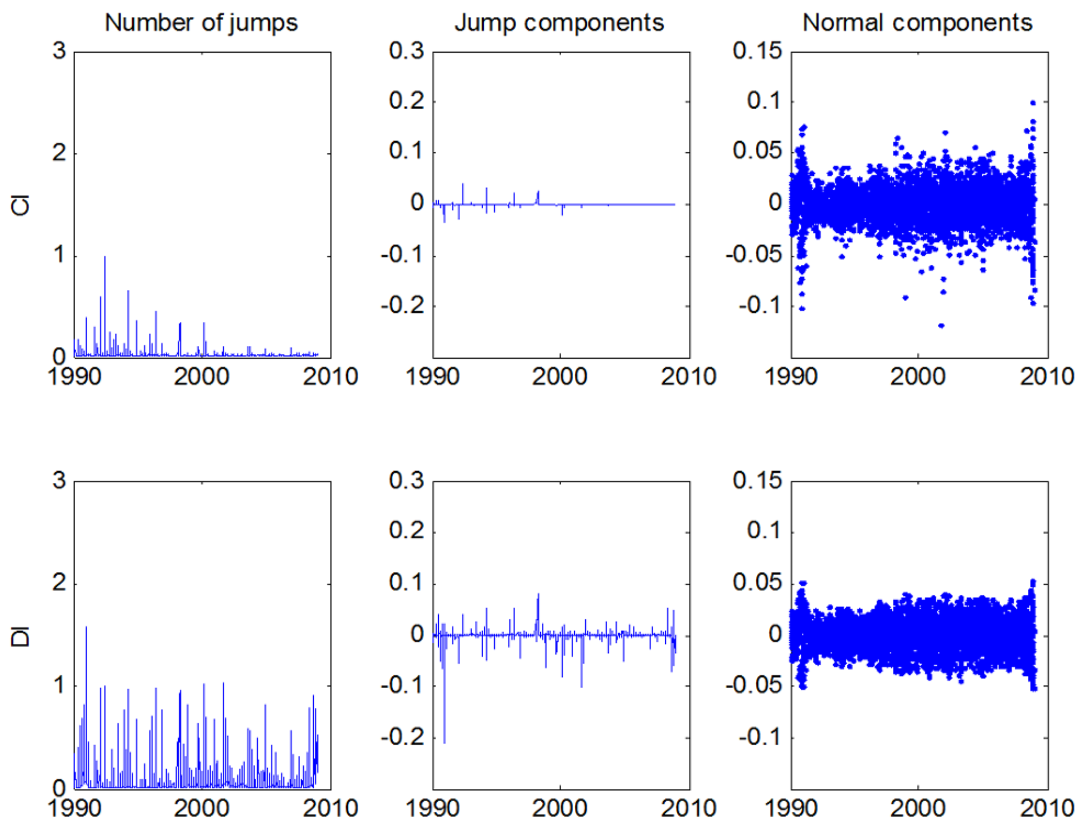
We plot the normal risk premium, $\gamma_{z,t} \equiv \lambda_z h_{z,t}$ in the left column, the jump risk premium, $\gamma_{y,t} \equiv \lambda_y h_{y,t}$, in the middle column, and the total risk premium, $\gamma_t \equiv \lambda_z h_z + \lambda_y h_y$, in the right column, for two jump models.

conditional jump risk premium, and the total risk premium. Although the CI model has a slightly higher conditional variance than the DI model, the total risk premium implied by both models is very similar, because jumps occur more frequently in the DI model.

Other properties differ between the jump models. Table 2.4 indicates that in the DI model with time-varying jump intensities, jumps explain approximately 49% of the total variance, while jumps explain 4.6% of the total variance of the CI model. Note that this finding for the CI model is at odds with the findings from futures returns in Table 2.2. For the model with time-varying jump intensities, the percentage of variance explained by the jumps is relatively similar in Tables 2.2 and 2.4. With regard to the average number of jumps, both models yield different results in Tables 2.2 and 2.4. For the CI model, futures data imply slightly more jumps, but for the DI model, futures data indicate far fewer jumps.

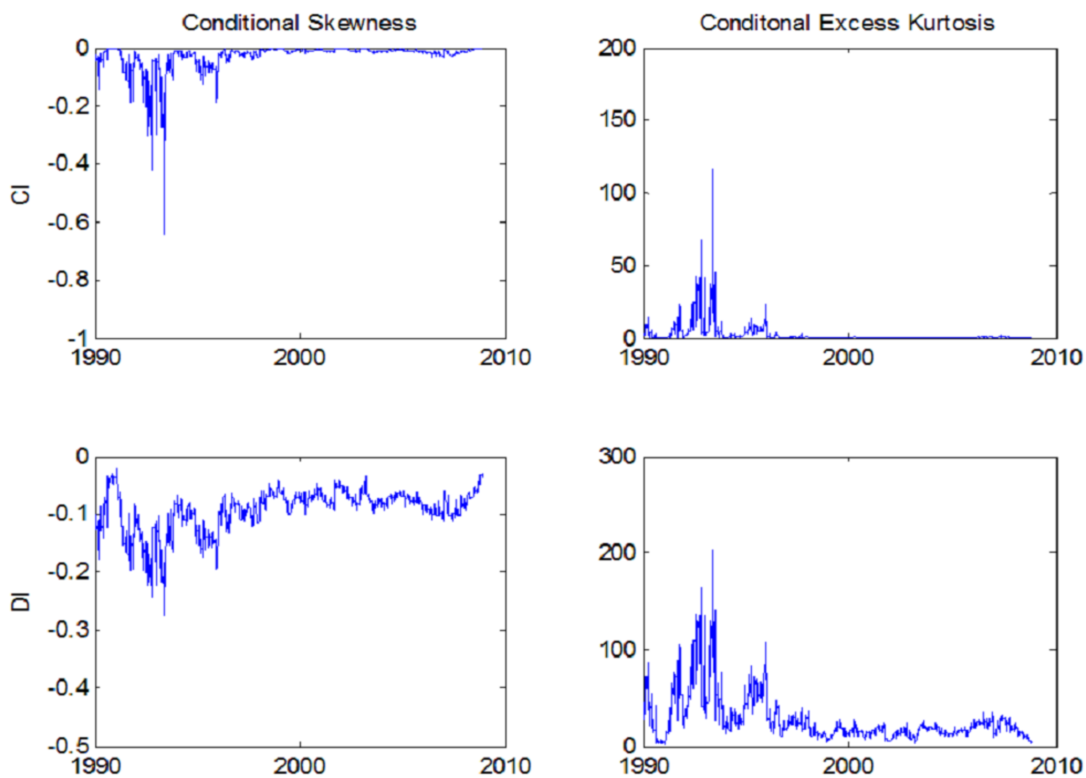
Figure 2-10 plots the sample paths of the number of jumps, as well as that of the filtered jump component and normal component. For the DI model, jumps are the dominant components of returns during the first Gulf war, and we find evidence of multiple jumps per day. Skewness and kurtosis are determined by the jump parameters. The average jump sizes, θ, for both jump models in Table 2.4 are smaller in magnitude compared to those estimated using only crude oil futures returns in Table 2.2. Consequently, the time path of conditional skewness in Figure 2-11 indicates that for both jump models, the average skewness is smaller in absolute value than that estimated from futures returns only in Table 2.7. The estimates of the variance of the jump size δ in Table 2.4 are smaller for both jump models compared to that in Table 2.2, and this determines the kurtosis estimates. Conditional excess kurtosis in the right column of Figure 2-11 is lower compared to that in Figure 2-7. Table 2.5 further investigates the differences in fit between the models. We report IVRMSE and IV bias by moneyness and maturity category. Because of space constraints, we limit ourselves to a comparison of the DI model and the benchmark GARCH model. For all moneyness and maturity categories in Panel A of Table 2.6, the average IVRMSE is significantly lower for the DI model compared to the GARCH model. We also report the Implied Volatility Bias (IV Bias) across moneyness and maturity in Panel B of Table 2.6. Again the average IV Bias is significantly lower for the DI model compared to the GARCH model. However, the IV Bias differences between the two models are much larger than the IVRMSE differences. Since RMSEs reflect model bias and variance, we conclude that the data may be rather noisy, and that some contracts are poorly fit by both models. The much improved bias for the DI model is therefore very important.

Figure 2-10: Decomposition of Daily Futures Returns estimated Using Futures and Option Contracts



We plot the filtered number of jumps, n_t, in the left column, the filtered jump component, y_t, in the middle column, and the filtered standardized normal component, z_t, in the right column, for two jump models. Results are obtained using the analytical filter and the joint MLE estimates of futures and options from Table 2.4.

Figure 2-11: Conditional Skewness and Excess Kurtosis from Futures and Option Contracts



We plot the daily conditional skewness, in the left column, and the conditional excess kurtosis, in the right column, for two jump models. The moments are estimated using joint MLE on futures and options data.

Table 2.5: IVRMSEs and IV Bias for Crude Oil Options by Moneyness and Maturity

Panel A. IVRMSEs for Crude Oil Options by Moneyness and Maturity

			Maturity							
	Moneyness	**Model**	**M1**	**M2**	**M3**	**M4**	**M5**	**M6**	**Q1**	**Q2**
	0.78-0.82	GARCH	5.22	4.54	4.97	5.39	5.71	5.89	6.79	6.88
		DI	4.09	3.66	3.84	3.98	3.98	4.03	4.32	4.54
	0.82-0.86	GARCH	4.1	3.67	4.41	5.09	5.65	5.91	6.82	7.65
		DI	3.52	3.13	3.54	3.84	3.9	4.05	4.29	4.74
Puts	0.86-0.90	GARCH	3.86	3.23	4.19	5.15	5.4	6.05	6.7	7.89
		DI	3.5	2.98	3.36	3.68	3.77	3.93	4.26	4.84
	0.90-0.94	GARCH	4.06	2.97	3.9	4.32	5.15	5.31	6.33	7.26
		DI	3.68	2.59	3.14	3.5	3.84	3.8	4.21	4.55
	0.94-0.98	GARCH	4.02	2.67	3.47	4.16	4.58	4.84	6.01	7.17
		DI	3.7	2.55	3.15	3.6	3.75	3.71	4.21	4.48
	0.98-1.02	GARCH	3.72	2.61	3.29	3.93	4.1	4.65	5.83	6.87
		DI	3.53	3	3.41	3.66	3.63	3.61	4.22	4.51
	1.02-1.06	GARCH	3.19	2.59	3.06	3.49	3.93	4.39	5.38	6.44
		DI	3.77	3.69	3.55	3.56	3.57	3.76	4.23	4.56
	1.06-1.10	GARCH	3.11	2.89	3.24	3.7	3.81	4.14	5.09	6.04
		DI	3.31	3.39	3.36	3.5	3.57	3.83	4.28	4.67
Calls	1.10-1.14	GARCH	3.1	3.54	3.67	3.82	3.75	4.22	4.98	6.01
		DI	2.7	3.15	3.32	3.52	3.71	3.91	4.45	5.01
	1.14-1.18	GARCH	3.69	4.14	4.26	4.17	3.9	4.33	5.08	5.83
		DI	2.75	3.27	3.58	3.76	3.73	3.93	4.9	5.44
	1.18-1.22	GARCH	4.32	4.67	4.78	4.86	4.4	4.28	4.48	5.28
		DI	3.36	3.74	3.92	4.07	3.91	3.87	4.34	5.24

Table 2.5 (Continued)

Panel B. IV Bias for Crude Oil Options by Moneyness and Maturity

	Moneynes	Model	M1	M2	M3	M4	M5	M6	Q1	Q2
			Maturity							
	0.78-0.82	GARCH	3.42	2.65	3.02	3.45	3.66	3.72	4.47	4.16
		DI	1.37	0.51	0.75	0.64	0.11	-0.56	-0.57	-0.84
	0.82-0.86	GARCH	1.96	1.92	2.46	3.08	3.61	3.79	4.61	5
		DI	1.12	0.46	0.54	0.38	0.02	-0.47	-0.65	-0.39
Puts	0.86-0.90	GARCH	1.88	1.58	2.34	3.21	3.49	4.04	4.64	5.57
		DI	1.63	0.95	0.71	0.54	0.28	-0.03	-0.28	0.19
	0.90-0.94	GARCH	2.06	1.52	2.1	2.42	3.22	3.43	4.35	5.25
		DI	1.53	0.63	0.57	0.53	0.45	0.19	0.09	0.35
	0.94-0.98	GARCH	2.03	0.89	1.18	1.99	2.66	2.99	4.08	5.29
		DI	1.81	0.65	0.39	0.3	0.46	0.41	0.55	0.86
	0.98-1.02	GARCH	1.14	0.04	0.8	1.45	2.07	2.69	3.81	4.96
		DI	-0.71	-0.88	-0.61	-0.25	0.17	0.31	0.9	1.18
	1.02-1.06	GARCH	0.45	-0.45	0.13	0.58	1.5	2.01	3.1	4.24
		DI	-2.05	-2.35	-1.52	-0.75	-0.2	0.23	0.91	1.21
	1.06-1.10	GARCH	-0.27	-1.23	-0.89	0.03	0.83	1.15	2.5	3.52
		DI	-0.98	-2.08	-1.22	-0.54	-0.06	0.22	1.1	1.41
Calls	1.10-1.14	GARCH	-1.4	-2.16	-1.61	-0.58	-0.07	0.7	2.04	3.01
		DI	-0.21	-1.24	-0.46	-0.05	0.28	0.52	1.45	1.78
	1.14-1.18	GARCH	-2.24	-2.82	-2.43	-1.76	-0.77	0.15	1.22	2.57
		DI	-0.25	-0.47	0.01	0.22	0.47	0.58	1.44	2.35
	1.18-1.22	GARCH	-2.67	-3.26	-3.09	-2.8	-1.97	-1.27	0.21	1.52
		DI	-0.12	0.24	0.49	0.15	0.06	0.06	1.06	2.21

We report the option implied volatility root mean squared errors (IVRMSE) and the implied volatility bias within each moneyness-maturity category for the GARCH and DI models. The models are estimated using daily crude oil returns and options jointly for the period January 2, 1990 to December 3, 2008. The pricing errors are defined as the difference between fitted and actual implied volatilities and reported in percentages. M1 (M2, M3, M4, M5, M6) refers to option contracts with expiration in 1 (2, 3, 4, 5, 6) months; Q1 and Q2 refers to the next two option contracts with expiration in either March, June, September or December. Moneyness is defined as the option strike divided by the price of the underlying futures contract.

Figure 2-12 illustrates the improvement in fit provided by jump models by depicting the models' ability to capture the "smiles" and "smirks" in the data. We again limit ourselves to a comparison between the benchmark GARCH model and the DI model. The green solid line shows the average log-normal volatility smiles and smirks from the data. The blue dashed line and the red dotted line show the average of the fitted "smiles" for the DI model and the GARCH model respectively. Averages are taken over a maximum of 4,753 daily observations from January 2, 1990 through December 3, 2008. Figure 2-12 clearly demonstrates that the DI model fits the implied volatility "smiles" vastly better than the GARCH model for options on all different maturity futures contracts, in line with the bias results in Panel B of Table 2.6. As maturity increases, the pricing errors of both the GARCH model and the DI model decrease, suggesting that both models fit long maturity options better.

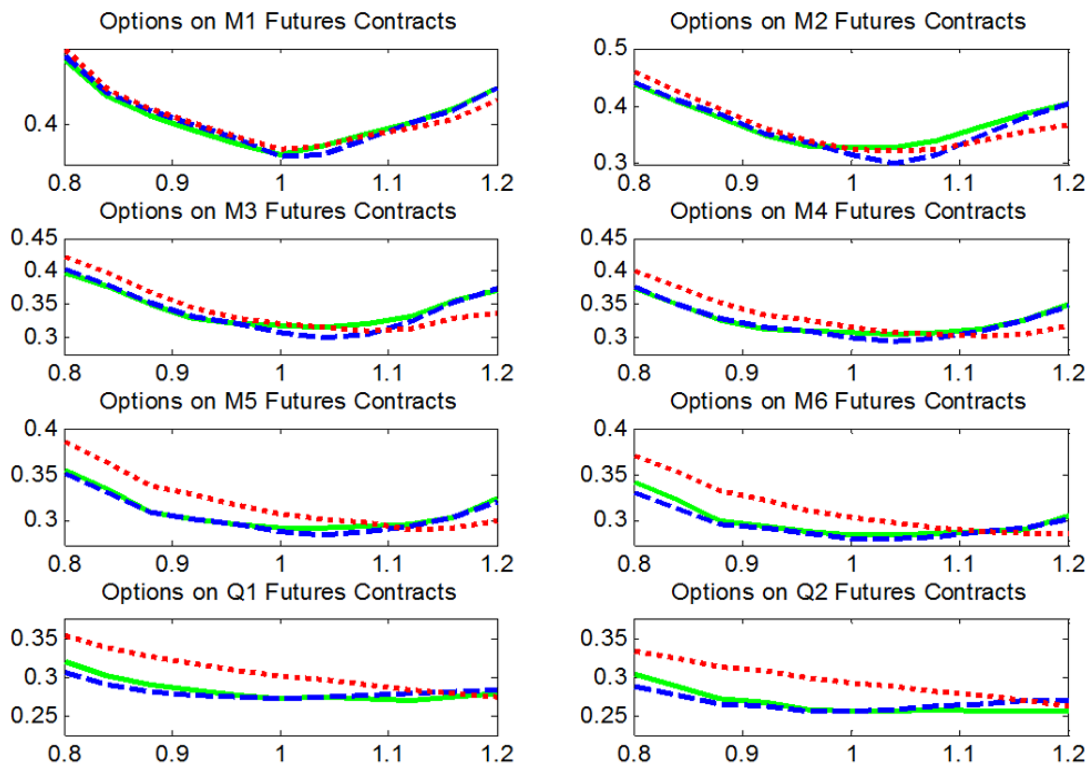
2.7 Conclusion

We estimate discrete-time jump models for CME crude oil futures and options on futures. The proposed models allow for a heteroskedastic normal innovation and a jump component. We investigate one jump model with a constant jump intensity and another model with a time-varying jump intensity, as well as a benchmark GARCH model that does not contain a jump component. All models are tractable, providing a quasi-analytical option valuation formula, and analytical results for filtering the volatility and jump intensity are available.

We find strong evidence for the presence of jumps in the crude oil derivatives market, using futures data as well as options data. Both the analysis of futures data and that based on the joint estimation of futures and options suggests jumps are relatively rare events in the crude oil market.

We find strong evidence in favor of time-varying jump intensities. Jump models with dynamic jump intensity dramatically improve model performance. This is the case whether or not futures options are used in estimation. During crisis periods, when market risk is high, jumps occur more frequently.

Figure 2-12: Average Implied Volatility Smiles and Smirks



We plot the average implied volatility across moneyness. Moneyness is defined as the option strike divided by the price of the underlying futures contract. The green solid line shows the average volatility smiles in the option data. The blue dashed line shows the average over time of the fitted smiles for the DI model. The red dotted line shows the average over time of the fitted smile for the GARCH model. Model parameters are from Table 2.4. Averages are based on 4,753 daily observations from January 2, 1990 through December 3, 2008.

Jumps account for a large part of the risk premium in crude oil market, regardless of whether jump intensities are time-varying, and regardless of whether futures or options are used in estimation. The primary benefit of modeling jumps in crude oil markets seems to be that the excess kurtosis of the distribution is modeled more adequately, whereas the modeling of skewness is a second-order effect.

Appendix

A. The Analytical Filter

Christoffersen, Jacobs, and Ornthanalai (2012) propose an analytical filter that can separately identify the normal and jump components in daily returns. We use a variation on their proposed filter. Consider the log return dynamic of

$$R_t = \mu_t + z_t + y_t \tag{A.2.1}$$

where μ_t is the first conditional moment of returns in (4.5). The filtered normal innovation can be written as

$$\widetilde{z}_t = E_t[z_t] = \sum_{j=0}^{\infty} z_t(R_t, n_t = j)\,\Pr_t(z_t, n_t = j) \tag{A.2.2}$$

where $\Pr_t(z_t, n_t = j)$ is the joint probability of z_t and $n_t = j$ given that R_t is known. If the return R_t and the number of jumps $n_t = j$ at time t are known, we can express z_t as

$$z_t(R_t, n_t = j) = \sqrt{\frac{\widetilde{h}_{z,t}}{\widetilde{h}_{z,t} + j\delta^2}}(R_t - \mu_t - j\theta) \tag{A.2.3}$$

Using Bayes' rule, the filtering density $\Pr_t(z_t, n_t = j)$ up to the normalizing constant can be written as

$$\Pr_t(z_t, n_t = j) \equiv \Pr_{t-1}(z_t, n_t = j \mid R_t) \propto \Pr_{t-1}(z_t, n_t = j)\,\Pr_t(n_t = j) \tag{A.2.4}$$

This is the ex-post inference on z_t given time t information. To compute the first term on the right hand side in (A.2.4), we express z_t as $z_t(R_t, n_t = j)$ and use the change of variable technique to obtain

$$\Pr_{t-1}(z_t \mid R_t, n_t = j) = \Pr_{t-1}(R_t, n_t = j \mid R_t, n_t = j)\sqrt{\frac{\widetilde{h}_{z,t}}{\widetilde{h}_{z,t} + j\delta^2}} \tag{A.2.5}$$

The second term in (A.2.4), $\Pr_t(n_t = j)$, is the filtering density for the number of jumps at time t, n_t. Applying Bayes' rule, it is given by

$$\Pr_t(n_t = j) \equiv \Pr_{t-1}(n_t = j \mid R_t) = \frac{f_{t-1}(R_t \mid n_t = j)\,\Pr_t(n_t = j)}{f_{t-1}(R_t)} \tag{A.2.6}$$

Using (A.2.3), (A.2.4), (A.2.5) and (A.2.6), the expected ex-post normal component of returns is

$$\begin{aligned} \widetilde{z}_t &= \sum_{j=0}^{\infty} z_t(R_t, n_t = j) \Pr_{t-1}(z_t \mid R_t, n_t = j) \Pr_t(z_t, n_t = j) \\ &= \sum_{j=0}^{\infty} \frac{\widetilde{h}_{z,t}}{\widetilde{h}_{z,t} + j\delta^2} (R_t - \mu_t - j\theta) \Pr_t(z_t, n_t = j) \end{aligned} \quad \text{(A.2.7)}$$

Therefore, the filtered jump innovation is given by

$$\widetilde{y}_t = R_t - \mu_t - \widetilde{z}_t \quad \text{(A.2.8)}$$

The filtered number of jumps is then

$$\widetilde{n}_t = \sum_{j=0}^{\infty} j \Pr_t(n_t = j) \quad \text{(A.2.9)}$$

The ex-post filter for $h_{z,t+1}$ conditional on the information set at time t is given by

$$\widetilde{h}_{z,t+1} = E\left[h_{z,t+1} \mid R_t\right] = \sum_{j=0}^{\infty} h_{z,t+1}(R_t, n_t = j) \Pr_t(h_{z,t+1}, n_t = j) \quad \text{(A.2.10)}$$

Write $h_{z,t+1} = h_{z,t+1}(R_t, n_t = j)$. From the GARCH dynamic, we have $h_{z,t+1}$ as

$$h_{z,t+1} = \omega_t + b_z \widetilde{h}_z + \frac{a_z}{\widetilde{h}_z}(z_t(R_t, n_t = j) - c_z \widetilde{h}_z)^2 + d_z(R_t - \mu_t - z_t(R_t, n_t = j) \quad \text{(A.2.11)}$$

Applying Bayes' rule again, the second term in (A.2.10), $\Pr_t(h_{z,t+1}, n_t = j)$, is given by

$$\begin{aligned} \Pr_t(h_{z,t+1}, n_t &= j) \equiv \Pr_{t-1}(h_{z,t+1}, n_t = j \mid R_t) \\ &\propto \Pr_{t-1}(h_{z,t+1} \mid R_t, n_t = j) \Pr_t(n_t = j) \end{aligned} \quad \text{(A.2.12)}$$

Using the change of variable technique, $\Pr_{t-1}(h_{z,t+1} \mid R_t, n_t = j)$ can be written as

$$\Pr_{t-1}(h_{z,t+1} \mid R_t, n_t = j) = \left| \frac{\partial h_{z,t+1}}{\partial z_t} \right| \Pr_{t-1}(z_t \mid R_t, n_t = j) \quad \text{(A.2.13)}$$

where the first term on the right hand side in (A.2.13) is

$$\frac{\partial h_{z,t+1}}{\partial z_t} = \frac{2a_z(z_t(R_t, n_t = j) - c_z \widetilde{h}_{z,t})}{\widetilde{h}_{z,t}} - d_z \quad \text{(A.2.14)}$$

The ex-post filter for $h_{y,t+1}$ conditional on the information set at time t, $h_{y,t+1} = E\left[h_{y,t+1} \mid R_t\right]$, can be obtained using a similar approach.

B. The Generating Function and the Option Valuation Formula

We solve for the coefficients $A(\varphi; t, T)$, $B(\varphi; t, T)$, and $C(\varphi; t, T)$ in equation (2.28) as in Ingersoll (1987) and Heston and Nandi (2000), utilizing the fact that the conditional moment generating function is exponential affine in the state variables $h_{z,t+1}$ and $h^*_{y,t+1}$.

Since S_T is known at time T, equation (2.28) requires the terminal condition

$$A1(\varphi; T, T) = B1(\varphi; T, T) = C1(\varphi; t, T) = 0 \tag{B.2.1}$$

Applying the law of iterated expectations to $f(\varphi; t, T)^*$, we get

$$\begin{aligned} f^*(\varphi; t, T) &= E_t^Q[f(\varphi; t+1, T)^*] \qquad \text{(B.2.2)} \\ &= S_t^\varphi E_t^Q[\exp(\varphi R_{t+1} + A1(\varphi; t+1, T) + B1(\varphi; t+1, T)h_{z,t+2} + C1(\varphi; t+1, T)h^*_{y,t+2})] \end{aligned}$$

We can rewrite the futures return process in (2.25) as

$$R_{t+1} = \mu_z h_{z,t+1} + \mu_y h^*_{y,t+1} + z_{t+1} + y^*_{t+1} \tag{B.2.3}$$

where $\mu_z = -\frac{1}{2}, \mu_y = -\xi_y(1)^*$.

Substituting the futures return process in equation (B.2.3), the conditional normal variance dynamic equation (2.26), and the conditional jump intensity dynamic equation (2.27) into (B.2.2), we get

$$\begin{aligned} f^*(\varphi; t, T) &= S_t^\varphi E_t^Q[\exp(\varphi(\mu_z h_{z,t+1} + \mu_y h^*_{y,t+1} + z_{t+1} + y^*_{t+1}) + A1(\varphi; t+1, T) \qquad \text{(B.2.4)} \\ &\quad + B1(\varphi; t+1, T)(\omega_z + b_z h_{z,t+1} + \frac{a_z}{h_{z,t+1}}(z_{t+1} - c_z h_{z,t+1})^2 + d_z y_{t+1}) \\ &\quad + C1(\varphi; t+1, T)(\omega_y^* + b_y h^*_{y,t+1} + \frac{a_y^*}{h_{z,t+1}}(z_{t+1} - c_y^* h_{z,t+1})^2 + d_y^* y^*_{t+1}))] \end{aligned}$$

After rearranging terms through completing squares and following some algebra we get

$$\begin{aligned} f^*(\varphi; t, T) &= S_t^\varphi E_t^Q[\exp(A1(\varphi; t+1, T) + B1(\varphi; t+1, T)\omega_z + C1(\varphi; t+1, T)\omega_y^* \qquad \text{(B.2.5)} \\ &\quad + (\varphi\mu_z + (b_z + a_z c_z^2)B1(\varphi; t+1, T))h_{z,t+1} + (\varphi\mu_y + (b_y + a_y^* c_y^{*2})C1(\varphi; t+1, T))h^*_{y,t+1} \\ &\quad + (a_z B1(\varphi; t+1, T) + a_y C1(\varphi; t+1, T))\frac{z^2_{t+1}}{h_{z,t+1}} \\ &\quad + (\varphi - 2a_z c_z B1(\varphi; t+1, T) - 2a_y c_y C1(\varphi; t+1, T))z_{t+1} \\ &\quad + (\varphi + d_z B1(\varphi; t+1, T) + d_y^* C1(\varphi; t+1, T))y^*_{t+1})] \end{aligned}$$

where we use the following results for normal and Poisson variables

$$E_t^Q[\exp(\alpha z_{t+1} + \beta z_{t+1}^2)] = \exp(\frac{\alpha^2 h_{z,t+1}}{2(1-2\beta h_{z,t+1})} - \frac{1}{2}\log(1-2\beta h_{z,t+1}) \qquad \text{(B.2.6)}$$

$$E_t^Q[\exp(\Xi y_{t+1}^*)] = \exp(\xi_y(\Xi)^* h_{y,t+1}) \qquad \text{(B.2.7)}$$

where $\xi_y(\Xi)^* = \exp(\theta^*\Xi^* + \frac{1}{2}\Xi^{*2}\delta^2) - 1$.

Substituting (B.2.6) and (B.2.7) into (B.2.5) and subsequently equating terms in the right hand sides of (B.2.5) and (2.28) gives the analytical solutions for the affine coefficients $A(\varphi; t, T)$, $B(\varphi; t, T)$, and $C(\varphi; t, T)$ in (2.29) and (2.30).

C. Extracting Conditional Variance from Implied ATM Volatilities

Let IV_t be the 30-day ATM implied volatility. In the absence of jumps, the relationship of IV_t with daily variances is

$$(IV_t)^2 \cong \frac{365}{30} E_t \sum_{j=1}^{30} h_{t+j} \qquad \text{(C.2.1)}$$

where $E_t(.)$ is a conditional expectation given information at time t. Using (2.2) we get

$$E_t h_{t+j} = h_t \Psi_1^j + (\omega + a)\frac{1-\Psi_1^j}{1-\Psi_1} \qquad \text{(C.2.2)}$$

where $\Psi_1 = b + 2ac$. Substituting (C.2.2) into (C.2.1), we have

$$h_t = \frac{1}{\sum_{j=1}^{30} \Psi_1^j}\left[\frac{30}{365} IV_t^2 - (\omega + a)\frac{\sum_{j=1}^{30}(1-\Psi_1^j)}{1-\Psi_1}\right] \qquad \text{(C.2.3)}$$

This method of extracting variances from implied volatility is similar to Kanniainen (2013).

For jump models, we use the relationship between implied volatilities and total variances

$$IV_t^2 \cong \frac{365}{30} E_t \sum_{j=1}^{30} \left[h_{z,t+j} + (\theta^2 + \delta^2) h_{y,t+j}\right] \qquad \text{(C.2.4)}$$

For the CI model, from equation (2.8) and constraint (2.11), we have

$$E_t h_{z,t+j} = h_{z,t+j}\Psi_2^j + (\omega_z + a_z + d_z\theta\omega_y)\frac{1-\Psi_2^j}{1-\Psi_2} \qquad \text{(C.2.5)}$$

where $\Psi_2 = b_z + a_z c_z^2$. Substituting (2.11) and (C.2.5) into (C.2.4), we get

$$h_{z,t} = \frac{1}{\sum_{j=1}^{30} \Psi_2^j}\left[\frac{30}{365} IV_t^2 - 30(\theta^2 + \delta^2)\omega_y - (\omega_z + a_z + d_z\theta\omega_y)\frac{\sum_{j=1}^{30}(1-\Psi_2^j)}{1-\Psi_2}\right] \qquad \text{(C.2.6)}$$

For the DI model, substituting the constraint (2.12) into (2.8), we have

$$E_t h_{z,t+j} = h_{z,t}\Psi_3^j + (\omega_z + a_z)\frac{1-\Psi_3^j}{1-\Psi_3} \tag{C.2.7}$$

where $\Psi_3 = b_z + a_z c_z^2 + d_z \theta k$. Substituting (C.2.7) and (2.12) into (C.2.4), we obtain

$$h_{z,t} = \frac{1}{\sum_{j=1}^{30} \Psi_3^j}\left[\frac{30}{365}\frac{IV_t^2}{(1+(\theta^2+\delta^2)k)} - (\omega_z + a_z)\frac{\sum_{j=1}^{30}(1-\Psi_3^j)}{1-\Psi_3}\right] \tag{C.2.8}$$

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