A THEORETICAL AND LABORATORY STUDY OF FLUID DISPLACEMENT IN STRATIFIED SANDS

A Thesis

Presented to

the Faculty of the Cullen College of Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Petroleum Engineering

by

Robert J. Pusanik

August, 1961

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ABSTRACT

The main purpose of this Thesis is to develop a better understanding of the mechanics of fluid displacement in stratified sands. Various methods describing fluid displacement in stratified sands will be discussed and their limitations pointed out. This will be followed by a new derivation of the fluid displacement mechanics. This derivation will be rigorous enough to apply to stratified reservoirs for both miscible and immiscible displacements.

In a displacement of fluids, the viscous forces are considered to be the most significant variable; however, other factors, such as a varying porosity and displacement efficiency, also contribute to the mechanics of fluid displacements. In previous displacement methods these factors have been entirely neglected, thereby limiting their application.

As a means of verifying the derivation presented in this thesis, a mathematical model was constructed. With this model, fluid displacements with different viscosity ratios were performed, and the theoretical and experimental results were compared. For viscosity ratios greater than or equal to one, the experimental results were in close agreement with the theoretical results; for viscosity ratios less than one, however, there was a slight discrepancy between the two. This difference was attributed to the

unconsolidated sand model.

From the visual sand model and the theoretical analyses, interesting and important conclusions were observed and deducted concerning the mechanics of fluid displacements in stratified sands. Mainly, that the rate of advance of a flood front is dependent not only on its permeability but also on its porosity and displacement efficiency. Another interesting factor is that the fractional flow rate in any layer of sand will depend not only on its relative capacity but also on the relative distance of the flood front in the reservoir. The crossflow of the displacing fluid was noticed and accounted for, while the imbibition of water from the more porous beds to the less porous beds and, conversely, the transfer of oil from the tighter beds to the more porous beds - after an immiscible displacement - were observed.

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NOMENCLATURE

A cross sectional area

c compressibility

$\bar{C}$ areal sweep efficiency; ratio of the volume swept at any time to the total volume subject to invasion

C total production of the in-place and displacing fluid expressed in pore volumes

D displacement efficiency factor; ratio of the fluid displaced at any time to the total fluid present in a microscopic pore volume

f_w fraction of water in total flow rate

h net pay thickness

k absolute permeability

k_{ro} relative permeability to oil

k_{rw} relative permeability to water

L length

M mobility ratio $\frac{u_o k_{rw}}{u_w k_{ro}}$

N_p cumulative recovery

P pressure

q_i production of the in-place fluid

q_T total production of in-place and displacing fluid

r radial distance

S fractional distance advanced by displacing fluid

S_o oil saturation

S_{or} residual oil saturation

S_w water saturation

S_{wr} residual water saturation

NOMENCLATURE (Cont)

t_b	breakthrough time
u_o	viscosity of oil
u_w	viscosity of water
u_d	viscosity of displacing fluid
u_i	viscosity of in-place fluid
u_r	viscosity ratio (u_i/u_d)
Wi	water injected
Wp	water produced
WOR	water-oil ratio
V_p	pore volume
ϕ	porosity

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CHAPTER I

INTRODUCTION

Secondary recovery methods have been used for decades by the oil industry on a limited basis, but only in recent years have technological advances, coupled with the soaring costs of finding new oil fields, made large scale secondary recovery operations economically attractive.

The advances in secondary recovery have come none too soon for the petroleum industry. It currently costs an average of nearly $1.18 a barrel merely to find new oil.[1] This cost is up from 90 cents a barrel 10 years ago. Further, the chances of the industry making any spectacular new discoveries of oil in the United States are rapidly diminishing and consequently greater attention is being directed toward recovering more of the known reserve.

Although there is plenty of oil in the world at this time, crude is becoming harder and harder to find in the United States. Petroleum exploration is now being intensified in offshore areas and in remote geographical locations. The difficulty of reaching these areas and maintaining facilities adds considerably to both the cost of exploration and production.

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(1) Bibliography on Page 132.

For these reasons, most of the major oil companies are stepping up their secondary recovery operations. The end result of these accelerated operations is an increasing demand by management for predicted forecasts of current reservoir performances. These forecasts must include the ultimate hydrocarbon recovery, recovery at breakthrough or at any specific producing ratio, the amount of injection fluid needed to obtain such recoveries, payout period, profit, etc... In brief, a complete fluid displacement history is required for any secondary recovery operation.

The displacement of fluid from porous media has led to the development of many methods and theories. Perhaps the most common or well known, is the Buckley-Leverett Frontal Advance theory[2]. Since it is widely accepted in the industry as being representative of fluid displacement in a homogeneous sands, a brief summary will be given.

The Buckley-Leverett theory corresponds to a rigorous solution for two phase flow of immiscible, incompressible fluids in a system of homogeneous permeability. The method yields a continuously changing producing ratio after breakthough for simultaneous two phase flow.

According to this approach, the behavior of the flood is as follows: ahead of the displacing water front only oil is moving. At the front there is a very rapid increase in the displacing fluid phase saturation. Behind the front there is a region of continuously increasing displacing fluid phase saturation extending all the way to the injection point. At the injection point the oil saturation is at its residual value. Throughout the region of changing saturation behind the front both oil and the displacing phase flow simultaneously. This region behind the front increases with time. At the producing well only oil flows until breakthrough occurs. After breakthrough there is a very rapid increase in the production of the displacing fluid compared to the production of the oil. Following the very rapid increase in the water-oil ratio at breakthrough, there is a period of a gradually increasing water-oil ratio in the total fluid production. This period lasts until the economic limit of the well is reached.

Although this method is an excellent representation of immiscible fluid displacement in porous media, it still has one disadvantage. This disadvantage is that most reservoirs fail to conform with the main assumption - homogeneous permeability. It has been noticed that many reservoirs consist of a variation in permeability; in fact, it may be considered as an exception to the rule to find a homogeneous permeable reservoir. This reservoir characteristic has been recognized, and different techniques have been published concerning this permeability stratification.

A REVIEW OF CURRENT FLUID DISPLACEMENT CALCULATION TECHNIQUES

Stiles Method

The Stiles method[3] is an approximate procedure for making water flood calculations in cases where vertical variations of permeability must be taken into account. In this method the reservoir is imagined to be a layered system, one layer placed on top of the other (stratified). It is assumed that the permeability does not vary within a given layer, but that it can change in going from one layer to the next (see Fig. 9, Page 75). The nature of the basic fluid flow assumption is as follows: it is assumed that in each layer there is a piston-like displacement so that after breakthrough in any layer there is no more oil production from that layer. If there were no permeability variations, the above assumption would imply that there would be no gradually changing oil-water ratio after breakthrough. It is only because the front has advanced different distances in the layers of different permeabilities that there is a continually changing water-oil ratio after breakthrough. It is also assumed that there is no crossflow from one layer to another, i.e., it is imagined that there is an impermeable barrier between layers. Also, the forces due to capillary imbibition and gravity are assumed to be insignificant, and the mobility ratio $(u_o k_{rw}/u_w k_{ro})$*, the porosity, and the residual fluid saturations are assumed to be the same in each layer.

Stiles gives the recovery, expressed in barrels, as:

$$N_p = (\text{COVERAGE})\ (S_{oi} - S_{or})V_p$$

where S_{oi} is the average initial oil saturation, S_{or} the average residual oil saturation, V_p the reservoir pore volume in barrels, and COVERAGE the fraction of the reservoir swept by water.

The expression for the producing water-oil ratio (WOR) is:

$$\text{WOR} = (\text{COVERAGE}/1\text{-COVERAGE})M$$

where M is the mobility ratio.

It is noticeable that the recovery and WOR depend only on the COVERAGE, which, derived by Stiles, is:

$$\text{COVERAGE} = \frac{n}{N} + \frac{1}{k_n N} \sum_{j>n}^{N} k_j$$

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* See Table of Nomenclature, Page iv

where N is the total number of layers, n/N is the fraction of layers that have been completely swept, and k_n is the permeability of the layer that has just been completely swept.

Obviously, Stiles has assumed that the distance of advance of the flood is proportional to the permeability; this, however, is incorrect unless the mobility ratio is one, and the porosity is the same in all layers. It will be shown in a latter section (Page 42) that where the mobility ratio is not unity, the solution to the permeability stratification problem is considerably more complex than for Stiles' solution. The effects of porosity on the advance of the displacement front is very important, and cannot be disregarded. This may be proven quite easily.

Let it be assumed that there are two homogeneous beds, each having the same permeability, k_1 and k_2, but different porosities, Φ_1 and Φ_2, where $\Phi_2 < \Phi_1$; also, that the in-place and displacing fluid are the same, and there is a constant pressure differential across the entire flow system.

The fractional flow rate in layer No. 1 (q_1/q_T) is then:

$$q_1/q_T = k_1/(k_1+k_2)$$

Since $k_1 = k_2$, the fractional flow rate is just one-half of the total flow rate; therefore, the fractional flow rate in layer No. 2 is also one-half of the total flow rate. Now, according to the Stiles method, the advance of the flood front would be the same in both layers, but, layer No. 2 has a smaller pore volume than the other layer, and since the total flow rate is constant, the advance of the flood front will be greater in layer No. 2. This becomes quite important when the mobility ratio differs from unity; then the fractional flow rate depends not only upon the mobilities of the fluids, but also upon the relative distance of the advancing flood front in each layer.

In summary, the Stiles method may be employed only in stratified reservoirs with a constant porosity and whose fluids have a mobility ratio near unity. Outside of these limits, this method will prove unsatisfactory.

Dykstra-Parsons Method

The Dykstra-Parsons[4] method is similar to the Stiles method. The only exception is the calculation of the advance of the flood front in different layers. With the Dykstra-Parsons method, mobility ratios differing from unity are handled in a better way than with the Stiles method.

In this method, Dykstra and Parsons have assumed that the permeability distribution could be represented by a straight line on log probability paper when the permeability is plotted on the log scale and the per cent of the permeability exceeding each tabulated value is plotted on the probability scale. A quantity called the "permeability variation" was then defined as the median permeability minus the permeability at 84.1 cumulative per cent divided by the median permeability. Then as far as calculations are concerned, it is only necessary to calculate the permeability variation since Dykstra and Parsons cumputed curves giving the coverage as a function of the permeability variation and mobility ratio for four different water-oil ratios (see Fig. 1, Page 9). However, in order to try to make the calculations agree more closely with experimental behavior, the authors presented a correlation of the fractional recovery with the calculated recovery. This correlation was obtained from measurements on small laboratory cores. Further simplifications have been introduced[5] by calculating curves giving the fractional recovery as a function of the permeability variation and mobility ratio for the four values of the producing water-oil ratio, thus eliminating the intermediate step of finding the coverage.

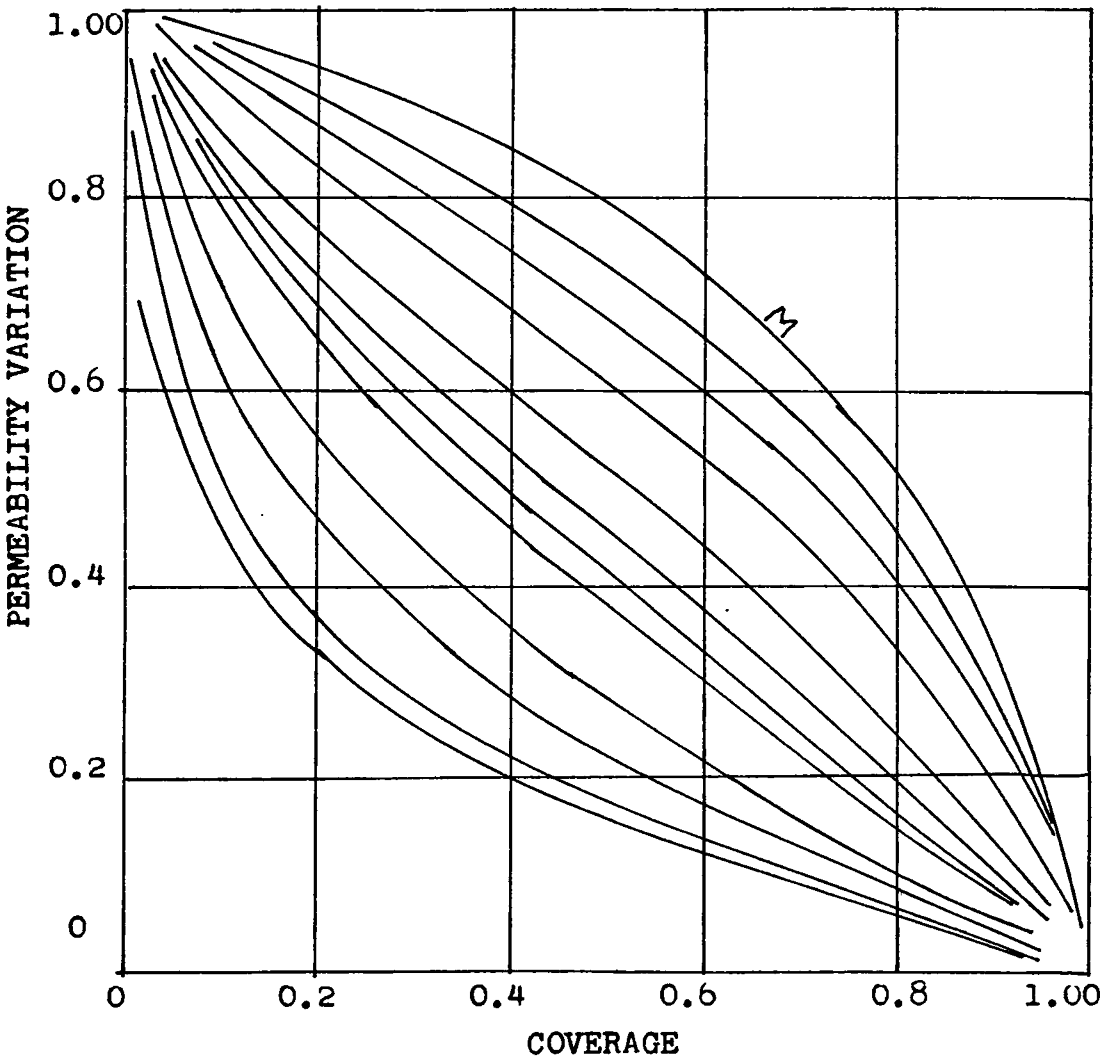


Figure 1

DYKSTRA - PARSONS METHOD
PERMEABILITY VARIATION
vs

COVERAGE

The above correlations and simplifications are based on the equation for calculating the coverage, which is:

$$\text{COVERAGE} = \frac{n+(N-n)M}{M-1} - \frac{1}{(M-1)} \sum_{j>n} \sqrt{M^2 + \frac{k_j}{k_n}(1-M^2)}$$

where N is the total number of layers arranged in order of decreasing permeability. Then, when the nth layer has broken through, all the layers with permeability greater than that of the nth layer will also have broken through. Hence the fraction of the reservoir for which the layers have been completley flooded out is $\frac{n}{N}$. The remaining layers, which have permeabilities less then the nth layer, will be only partially swept out. Thus, the COVERAGE will give the fraction of reservoir which has been invaded by water in the jth layers ($j>n$) when the nth layer has broken through.

Although this formula takes different mobility ratios into consideration, it still does not include the varying porosities of the different layers (the importance of porosity was discussed in the Stiles method.) Another interesting factor to note, is the displacement efficiency. It is obvious that the displacement efficiency of any displacing fluid is not 100% - nor is it the same in layers of different permeabilities and porosities. This too, must be included in calculations involving the advance of the flood front.

The basic fluid flow assumptions in this method are almost identical with those in the Stiles method. The Dykstra-Parson method assumes that the reservoir consists of horizontal layers packed one on top of the other with each layer having a constant uniform permeability, although the permeability may vary from one layer to the next. There is no crossflow between layers. Furthermore, since only all oil or all water is flowing, there is a piston-like displacement in each layer, so that after breakthrough of any layer no more oil is produced from that layer. It is also assumed that the mobility ratio, the porosity, and the initial and residual oil saturations are the same in each layer.

Besides the absence of a displacement efficiency and the assumptions of a constant porosity and residual fluid saturations involved in calculating the coverage, the Dykstra-Parsons method has one disadvantage which limits its application. In this method it is possible to obtain the recovery only at the four water-oil ratios for which curves were computed. Since the lowest water-oil ratio for which curves are available is one, it is necessary to extrapalate the curve to a low water-oil ratio to find the breakthrough recovery.

Although the Dykstra-Parsons method includes mobility ratios differing from unity, it still has some disadvantages which limit its application; this method is most suitable when a very rapid calculation is desired.

Hurst Method

The Hurst method(6) may be used for making water flood calculations in five-spot patterns if the water-oil mobility ratio is one. Unlike any of the other methods discussed, the Hurst method introduces areal sweep into the problem. It is this introduction of areal sweep that limits the method to its use for five-spot patterns. Actually, the Hurst method is just an application of the Stiles method to the five-spot pattern; however, due to the unique procedure in determining the displacement history - by combining the area swept with the vertical coverage - it is felt that a brief summary should be given.

In this method the fluids are assumed to move along the streamlines calculated from single-phase steady state flow. A knowledge of relative permeabilities or viscosities is not necessary since in all the calculations it is assumed that there is a piston-like displacement and that the mobility ratios are one. Through the use of a steady state pressure and streamline distributions, it is possible to derive a generalized curve which holds for all five-spot patterns. This curve makes computations by the Hurst method quite easy. Essentially what the curve gives is the coverage, i.e., the fraction of the total area swept by water vs the fraction of the area that would have been swept if breakthrough had not occurred (see Fig. 2, Page 14). The latter number may be greater than one since if breakthrough of water did not occur, the area swept would sooner or later become greater than the area of the five-spot. The area that would have been swept if breakthrough had not occurred is called the area processed. The basis of the Hurst method is, then, the coverage-area processed curve. Since the coverage is essentially proportional to the oil produced, and the area processed is essentially proportional to the water injected, it is seen that the basic coverage-area processed curve essentially gives the recovery vs the water injected. From this curve the recovery as a function of water injected or the water-oil ratio may easily be found.

In order to show how to use the areal coverage vs area processed graph ($\bar{C}$ vs Ap), the computations involving a single sand of uniform permeability will be calculated first, and then layered sands will be taken into consideration.

The recovery from a single sand is given by Hurst as:

$$N_p = \bar{C}(Soi - Sor)V_p$$

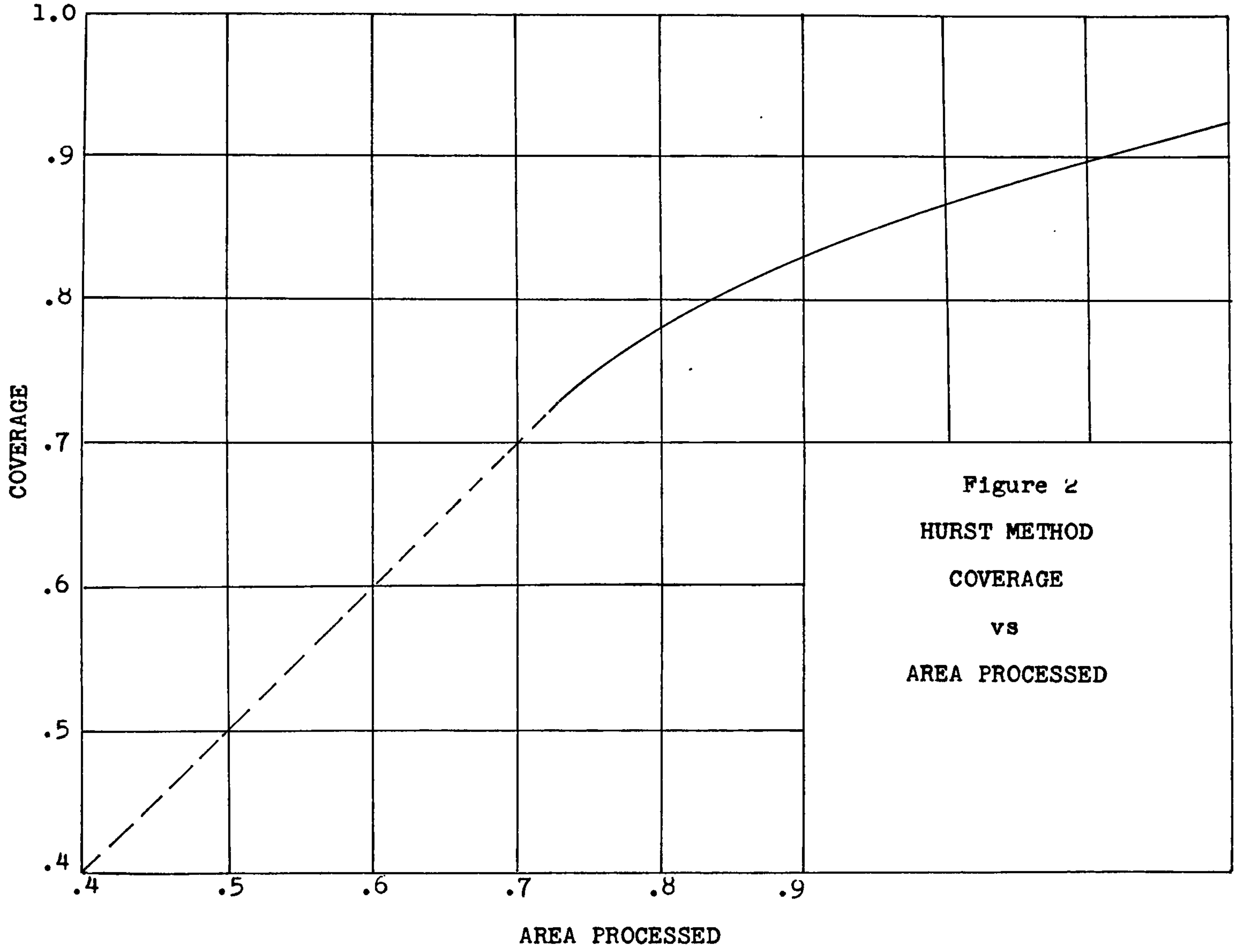


Figure 2
HURST METHOD
COVERAGE
vs
AREA PROCESSED

where C is the areal coverage.

The water injected, Wi, is:

$$Wi = Ap(Soi-Sor)V_p$$

where Ap is the area processed.

The water produced, then, must be the difference between the water injected and oil produced.

$$Wp = (Ap-\bar{\bar{C}})(Soi-Sor)V_p = Wi-Np$$

The water-oil ratio, WOR, is given by:

$$WOR = \frac{Wp}{Np}$$

In the calculations for a layered system, it will be assumed that the permeability in any layer is uniform. Furthermore, since the Hurst method assumes that the mobility ratio is unity, the ratio of the areas processed in the various layers will be the same as the corresponding permeability ratios, i.e.,

$$\frac{Api}{Apj} = \frac{k_i}{k_j}$$

It is noticed from the graph of areal coverage vs area processed that the breakthrough of any layer is:

$$(Ap)\ \text{breakthrough} = .7260$$

Thus the water injected, Wi, when the jth layer has broken through is (n layers in all):

$$Wij = \frac{0.7260(Soi-Sor)V_p}{n} \sum_{m=1}^{n} \frac{k_m}{k_j}$$

assuming that each layer has the same thickness.

The recovery, Np, when the jth layer has broken through, is:

$$Npj = \frac{(Soi-Sor)V_p}{n} \sum_{m=1}^{n} \bar{C}mj$$

$\bar{C}mj$ is the coverage of the mth layer when the jth layer has broken through. To find $\bar{C}mj$ it is necessary to use the $\bar{C}$ vs Ap graph. Now,

$$Apmj = 0.7260 \frac{k_m}{k_j}$$

Thus, with the calculated Apmj value, $\bar{C}mj$ may be found from the $\bar{C}$ vs Ap curve.

The water produced, Wp, is:

$$Wp = Wi-Np$$

and the WOR is:

$$WOR = \frac{Wi}{Np}$$

From these relations all the desired results may be found.

The Hurst method may be used for a layered system in a way quite similar to the Stiles or Dykstra-Parsons method. It is imagined that the reservoir is made up of a number of layers stacked vertically one on top of the other. In each layer the permeability is taken to be uniform, although it may vary from one layer to another. It is assumed that no crossflow occurs between the layers, i.e., it is imagined that there is an impermeable lamina between the layers. The basis for distributing the injected water between layers is that the quantity of water injected is proportional to the permeability. This follows, since it is assumed that the mobility ratio is one.

The Hurst method is useful if the displacement is taking place in a five-spot pattern when the mobility ratio of the fluids is very close to unity. If these conditions are present in the reservoir then the Hurst method is more suitable than the Stiles or Dykstra-Parsons methods, since it includes areal sweep in the calculations.

Park Jones Method

The Park Jones method[7] is a unique technique for predicting the displacement history of a reservoir. Since this method does not appear in literature, a more detailed summary will be given.

In this method, Jones defines the degree of stratification of the permeability by the following equation:

$$Kydy = Kody(1-y)^b$$

$K_y dy - K_o dy$

where y is the relative thickness measured from top to bottom of a given pay, i.e., $0 \leq y \leq 1$, where the total pay thickness is designated as unity; dy is the differential relative thickness; Kody is the highest relative differential capacity, ($K_o dy$) and Kydy is the relative differential capacity at any level ($k_y dy$) in the flow system. The exponent b is referred to as the "coefficient of stratification" and is evaluated from field data on the relative rate of production (the determination of b will be shown later). Then, by integrating the above equation, the following relationship is obtained:

$$Ko/(1+b) = K$$

where K (k_o) is the effective permeability to the in-place fluid (this value is determined in the field by the drawdown or buildup test of a well.)

Letting S be the fractional distance advanced by the displacing fluid in the interval at the y level, and q_y the fractional flow rate at the y level, the Darcy's equation may be written as:

$$qy = \frac{1.27AKyPdy}{\left[u_d S + u_i(1-S)\right] L}$$

q_y

Since $q_y = \frac{\Phi A}{5.615} LdS$ (porosity is assumed to be constant) y may be solved for in terms of S and the viscosity ratio ($u_r = u_i/u_d$).

$$y = 1 - \left[\frac{2u_r S + (1-u_r)S^2}{1+u_r}\right]^{1/b}$$

Now considering the displacement of water by water or oil by oil for which the viscosity of the in-place fluid u_i is equal to the viscosity u_d of the displacing fluid, then at the time of breakthrough the area under the y curve will be the amount of in-place fluid recovered (C_{bw}).

$$C_{bw} = \int_0^1 (1 - S^{1/b})dS = \frac{1}{1+b}$$

Thus C_{bw} is the breakthrough displacement factor for water. The breakthrough displacement factor C_b for an oil or gas reservoir is obtained by correcting C_{bw}

for the effect of capillary pressure.

The coefficient of stratification for a flow system $b=(1-Cbw)/C_{bw}$ is simply the ratio of the volume of remaining in-place fluid to the volume of the displaced fluid within the flow system at breakthrough time for a unit viscosity ratio.

The coefficient of stratification (b) is now determined in the following way. The type curves, y vs S, for various assumed values of b are plotted on one sheet of graph paper. The field data (values of the relative flow rate at certain intervals) are plotted on the same size sheet of transparent paper. This is done by the following relationship:

$$Sa = \frac{\Delta q}{\Delta y}$$

a

where Δq is the relative flow rate in the Δy interval and Sa, the average relative distance, plotted at the mid-level of each interval. The transparent paper is superimposed on the type curves. The y curve which best fits the mid-level points defines the coefficient of stratification.

After breakthrough the fractional content of the displacing fluid in the outflow section increases progressively from zero at the time $t = t_b$ to T at the time $t > t_b$. Jones then derived the distribution of the displacing fluid as:

$$y = 1-(1-T)S^{1/b}$$

The area under this curve,

$$Cw = \int_0^1 y dS = \frac{1+bT}{1+b} = Cbw+(1-Cbw)T$$

C_w

defines the displacement factor Cw for all viscosity ratios at any time after breakthrough. The displacement, C, for an oil or gas reservoir is Cw corrected for capillary pressure. For miscible flow, C is equal to Cw.

The relative flow rate of the in-place fluid (q_i/q_T) is derived by Jones to be:

$$q_i/q_T = (1+b)\int_T^1 (1-y)^b\, dy = (1-T)^{1+b}$$

(unit viscosity ratio)

When the viscosity ratio is other than unity, the flow rate for a fixed drawdown either increases or decreases with respect to time, depending on whether u_d is less or greater than u_i.

Now, the distribution of the displacing fluid at the time of impending breakthrough is:

$$y = 1-\left[\frac{2u_rS+(1-u_r)S^2}{1+u_r}\right]^{1/b}$$

The general solution for the breakthrough displacement factor becomes:

$$Cbw = 1-(1+u_r)^{-1/b} \int_0^1 \left[2u_r S+(1-u_r)S^2\right]^{1/b} dS$$

The breakthrough displacement factor Cb for displacement of oil or gas by water is obtained when Cbw is corrected for the capillary pressure effects.

The relative flow rate for all viscosity ratios and coefficient of stratification is given by:

$$q_1/q_T = \frac{V_b(1-T)^{1+b}}{1+(V_b-1)(1-T)^{1+b}}$$

where Vb is the viscosity ratio effect.

The reservoir barrels of displacing fluid required per reservoir barrels of in-place fluid (Cw+Cdw) is equal to the total production of both fluids. This is defined as:

$$Cw+Cdw = \frac{1-Cbw}{bVb}\left[b(V_b-1)T+(1-T)^{-b}-1\right]$$

$C_w + C_{dw}$ bV_b

The capillary-pressure effect is now determined by Jones in the following manner: consider the displacement of an undersaturated oil by water in a reservoir for which the irreducible water saturation is Sw expressed as a fraction of porosity. Then, due to the residual oil saturation, Sor, the displacement factor, C, for reservoir oil by water is less than the Cw for water by water. Jones contributes this lesser fractional displacement of oil as the capillary-pressure effect.

The total relative capacity of a rock to transmit a fluid is denoted by unity. The fractional porosity (1-Swr) is assumed to be occupied by hydrocarbons ahead of the advancing water front. The relative capacity to the reservoir oil ahead of the water front is less than unity by the amount

$$(1+b)\left[\int_0^1 (1-y)^b\,dy - \int_0^{1-Sw} (1-y)^b\,dy\right] = Swr^{(1+b)}$$

S_{wr}

This equation defines the relative capacity to water in the Swr fraction of the porosity.

Imbibition of water from larger to smaller pore sizes at the water front reduces the sectional reservoir oil content from (1-Swr) to some smaller fraction such as J. This is evaluated by assuming that the relative capacity to water in the (1-Swr) region increases from zero to Swr^{1+b} after completion of imbibition. The Sro is obtained from

$$\left[2Swr^{1+b}\right]^{1/1+b} - Swr = (2^{1/(1+b)}-1)Swr$$

In other words, the fractional pore volume which has the relative capacity $2Swr^{1+b}$ less the fractional pore volume that would be occupied by the irreducible water content, Swr, is equal to the residual oil sautration, Sro.

The mobile (displaceable) fraction of the initially in-place reservoir oil is then defined by

$$\frac{J}{1-Swr} = 1-\left[2^{1/(1+b)}-1\right] Swr$$

Jones has calculated the composition and total production versus cumulative recovery curves for three coefficients of stratification (½, 1 and 2). All that is necessary to determine the displacement history of a reservoir is to calculate the coefficient of stratification, and determine the viscosity ratio and water saturation.

The Park Jones mehtod is a vast improvement over the Stiles and Dykstra-Parsons methods. In this method, viscosity ratios differing from unity are taken into consideration, and, unlike the Dykstra-Parsons method, there is no need to extrapolate the recovery-composition curve to determine the breakthrough recovery at different viscosity ratios. Also, it is noticeable that Jones has disregarded the mobility ratio in his derivation; here, the author argues that the assumption of a constant mobility ratio in different layers is invalid, and the determining of the coefficient of stratification, b, will compensate for any reduction in permeability caused by the flow of two immiscible fluids.

The coefficient of stratification is defined as the ratio of the volume of remaining in-place fluid to the volume of the displaced fluid within the flow system at breakthrough time for a unit viscosity ratio. In other words, it is a measure of the degree of stratification of permeability within a given flow system. By incorporating this coefficient of stratification, the assumption of a constant porosity is justified since the coefficient of stratification determines the rate of advance of the displacing fluid at breakthrough for a unit viscosity ratio. However, can this coefficient of stratification also justify the absence of a displacement efficiency factor in each individual layer during a miscible viscous displacement? Investigations in literature[8,9,10,11,12] have shown that miscible displacements depend not only on the viscosity ratios but also upon the pore size and permeability of the sand. In the case of immiscible displacements the author has corrected the recovery for capillary effects by employing the average residual water sautration; but here again, the same arguements concerning the displacement efficiency factor for the individual sands are present.[13,14,15]

CHAPTER II

THEORETICAL ANALYSES

MISCIBLE DISPLACEMENT

Unit Viscosity Ratio

The following is a mathematical derivation of fluid displacement in a stratified reservoir under the assumption that the in-place and displacing fluid are miscible and have the same viscosities.

Let there exist a finite number of beds in the vertical direction where each bed has a different porosity, permeability and thickness so that the jth layer will be designated as $\bar{a}_j$, k_j and h_j where $j = 1,2,3,\ldots,n$. Now suppose these layers of beds are saturated with a fluid; then let $p_j(r,t)$ denote the pressure drop in the jth layer at a position r and a time t, i.e.,

$$p_j(r,t) = p_o - p_j(r,t)$$

where p_o is the initial pressure and $p_j(r,t)$ is the pressure in the jth layer at position r and time t.

Then from Darcy's law and the principle of Conservation of Mass:

$$1/r \frac{\partial}{\partial r}\left[\frac{r\partial p_j}{\partial r}\right] = \bar{a}_j \; u \; c/k_j \frac{\partial P_j}{\partial t}$$

must be satisfied for each layer j ($j=1, 2, 3, \ldots, n$).

At $t = 0$, the pressure is uniform throughout the entire reservoir and wqual to P_o; hence the pressure drop everywhere is zero.

Then: $P_j(r,o) = 0$ for all j and r, $r_w \leq r \leq r_e$

At $t > o$ there is a constant flow rate of the same fluid across the outer boundary (r_e). Thus:

$$\left.\frac{\partial P}{\partial r}\right]_{r=r_e} = f(q) = \text{a constant}$$

The pressure at the outer radius (r_e) is the same in all layers. Thus, if P_e denotes the pressure drop at the outer radius, then $P_j(r_e,t) = P_e$ where P_e is independent of J. Also, since

$$\left.\frac{\partial P_e}{\partial r}\right]_{r=r_e} = \text{a constant}$$

then for $t>0$, $\left.\frac{\partial P_e}{\partial t}\right]_{r=r_e} = 0$ and so $\frac{\partial P_j}{\partial t} = 0$

Since the pressure is independent of time, the diffusivity equation may now be wirtten as:

$$1/r \frac{d}{dr} \left[\frac{r dP_j}{dr}\right] = 0$$

At $t>o$ and $r = r_w$, the press re in the well bore is the same in all layers and is also independent of j. If P_w denotes the pressure drop function at the well, then

$$P_j(r_w) = P_w \text{ at } j = 1,2,3,\ldots,n.$$

Thus $$\left.\frac{dP}{dr}\right]_{r = r_w} = \text{a constant.}$$

Since the pressure gradients at the external boundary and at the well bore are equal to a constant, then at any r, the pressure drop in each layer is equal, i.e.,

$$P_1 = P_2 = P_3 = \ldots = P_n$$

<u>Solution</u>

The total production rate is constant and equal to q_T. Then, from Darcy's law, the production rate from layer j is:

$$q_j = \overset{?}{-2}\, k_j h_j r/u \frac{dP_j}{dr}$$ (a solution of the diffusivity equation). Now:

$$\sum_{j=1}^{n} 2\, k_j h_j r/u \frac{dP_j}{dr} = -q_T$$

Then $$q_j/q_T = \frac{2\ r/u\ k_j h_j dP_j/dr}{2\ r/u \sum_{j=1}^{n} k_j h_j dP_j/dr}$$

Since dP_j/dr = a constant,

$$q_j/q_T = k_j h_j / \sum_{j=1}^{n} k_j h_j$$

Now breakthrough occurs when the fluid in the most permeable bed - with the lowest storage capacity - reaches the well bore. Then the layers of beds are rearranged in order of their breakthrough, and this may be denoted by letting $j = m$ where $m = 1,2,3,\ldots,n$. Thus the first bed to break through occurs when $m = 1$. So,

$$q_1 = \frac{q_T h_1 k_1}{\sum_{j=1}^{n} h_j k_j}$$

Since this displacement occurs under the displacing steady-state conditions, the flow rate of fluid in the $m = 1$ bed is

$$q_1 = \frac{r_e^2 \Phi_1 D_1 h_1}{t_{b1}}$$

where D_1 is the displacement efficiency in the $m = 1$ bed, and t_{b1} is the time required for the displacing fluid to reach the well bore.

Solving for the breakthrough time

$$t_{b1} = \frac{r_e^2 \phi_1 D_1}{k_1 q_T} \sum_{j=1}^{n} h_j k_j$$

The cumulative production, Npm, at the time m = 1 will be:

$$Np_1 = q_T t_{b1} - q_o(t_{b1} - t_{bo})$$

where q_o and t_{bo} are equal to zero. (The reason for including q_o and t_{bo} is for convenience in expressing the general equation of Npm.) The composition of production, $(q_i/q_T)_m$, is:

$$(q_i/q_T)_1 = (q_T - q_o)/q_T$$

where q is equal to the flow rate of the in-place fluid.

Now when the fluid in the m = 2 layer reaches the well bore, this time will be:

$$t_{b2} = \frac{r_e^2 \phi_2 D_2}{k_2 q_T} \sum_{j=1}^{n} h_j k_j$$

Thus at $t = t_{b2}$, the composition of production will then be:

$$(q_i/q_T)_2 = \left[q_T - (q_o + q_1)\right]/q_T$$

The cumulative production will be:

$$Np_2 = q_T t_{b2} - \left[q_o(t_{b2} - t_{bo}) + q_1(t_{b2} - t_{b1})\right]$$

Similarly, the fluid in the next layer will have the breakthrough time of t_{b3}, where

$$t_{b3} = D_3 \bar{a}_3 \, r_e^2 \sum_{j=1}^{n} k_j h_j / k_3 q_T$$

The composition will be:

$$(q_1/q_T)_3 = \left[q_T - (q_o + q_1 + q_2)\right] / q_T$$

The cumulative production is then:

$$Np_3 = q_T t_{b3} - \left[q_o(t_{b3} - t_{bo}) + q_1(t_{b3} - t_{b1}) + q_2(t_{b3} - t_{b2})\right]$$

Now the fluid in the nth layer will have a breakthrough time of t_{bn}:

$$t_{bn} = \frac{D_n \bar{a}_n \, r_e^2}{k_n q_T} \sum_{j=1}^{n} h_j k_j$$

The composition is:

$$(q_1/q_T)_n = \left[q_T - (q_o + q_1 + q_2 + q_3 + \ldots + q_n)\right] / q_T$$

Since $\quad q_o + q_1 + q_2 + \ldots + q_n = \sum_{j=1}^{n} q_j = q_T$, then

$$q_1/q_T = 0 \text{ for } t = t_{bn}$$

However,

$$\sum_{j=0}^{n} q_j = \sum_{j=1}^{n} q_{j-1} + q_n$$

then the composition of production at the impending breakthrough of the nth layer will be:

$$(q_i/q_T)_n = \left[q_T - \sum_{j=1}^{n} q_{j-1}\right] / q_T$$

The cumulative production at the $t = t_{bn}$ is:

$$Np_n = q_T t_{bn} - \left[q_o(t_{bn}-t_{bo})+q_1(t_{bn}-t_{b1})+q_2(t_{bn}-t_{b2})+\right.$$

$$\left.q_3(t_{bn}-t_{b3})+\ldots+q_{n-1}(t_{bn}-t_{bn-1})\right]$$

since $q_n(t_{bn}-t_{bn}) = 0$. Now, the expression in the brackets may be rearranged in the following way:

$$q_o(t_{bn}-t_{bo})=q_o(t_{b1}-t_{bo})+q_o(t_{b2}-t_{b1})+q_o(t_{b3}-t_{b2})+q_o(t_{b4}-t_{b3})+\ldots+q_o(t_{bn}-t_{bn-1})$$

$$q_1(t_{bn}-t_{b1})= \qquad q_1(t_{b2}-t_{b1})+q_1(t_{b3}-t_{b2})+q_1(t_{b4}-t_{b3})+\ldots+q_1(t_{bn}-t_{bn-1})$$

$$q_2(t_{bn}-t_{b2})= \qquad q_2(t_{b3}-t_{b2})+q_2(t_{b4}-t_{b3})+\ldots+q_2(t_{bn}-t_{bn-1})$$

$$q_3(t_{bn}-t_{b3})= \qquad q_3(t_{b4}-t_{b3})+\ldots+q_3(t_{bn}-t_{bn-1})$$

$$\vdots \qquad\qquad \vdots$$

$$q_{n-1}(t_{bn}-t_{bn-1})= \qquad q_{n-1}(t_{bn}-t_{bn-1})$$

Then by adding:

$$Np_n=q_Tt_{bn}=\Big[q_o(t_{b1}-t_{bo})+(q_o+q_1)(t_{b2}-t_{b1})+(q_o+q_1+q_2)(t_{b3}-t_{b2})+(q_o+q_1+q_2+q_3)$$
$$(t_{b4}-t_{b3})+\ldots+(q_o+q_1+q_2+q_3+\ldots+q_{n-1})(t_{bn}-t_{bn-1})\Big]$$

Since

$$Sq_o = q_o$$

$$Sq_1 = q_o+q_1$$

$$Sq_2 = q_o+q_1+q_2$$

$$\vdots \qquad \vdots$$

$$Sq_{n-1} = q_o+q_1+q_2+\ldots+q_{n-1}$$

where Sq_{j-1} is the partial sums of the j-1 terms. Now the cumulative production at $t = t_{bn}$ or when $m = n$ is:

$$Np_n = q_T t_{bn} - \sum_{j=1}^{n} Sq_{j-1} \left(t_{bj}-t_{bj-1}\right)$$

Thus, the fractional recovery at any m will be:

$$Np_m = \frac{q_T t_{bm} - \sum_{j=1}^{m} Sq_{j-1}\left(t_{bj}-t_{bj-1}\right)}{Np_n}$$

Also, the composition of production at any m will be:

$$(q_i/q_T)_m = 1 - \frac{\sum_{j=1}^{m} q_{j-1}}{q_T}$$

The total production (C) of both fluids is simply the total flow rate times the breakthrough time. Thus, the total production at any m is:

$$Cm = q_T t_{bm}$$

Now, the total production expressed in pore volumes is:

$$Cm = \frac{q_T t_{bm}}{Np_n}$$

Now, by substituting the values for q_j and t_{bj}, the formulas for the cumulative recovery, composition and total production may be expressed in terms of the porosity, permeability and displacement efficiency of the different layers.

$$Np_m = \frac{\dfrac{D_m\Phi_m}{k_m} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{m} Shk_{j-1}\left[\dfrac{\Phi_j D_j}{k_j} - \dfrac{\Phi_{j-1}D_{j-1}}{k_{j-1}}\right]}{\dfrac{D_n\Phi_n}{k_n} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{n} Shk_{j-1}\left[\dfrac{\Phi_j D_j}{k_j} - \dfrac{\Phi_{j-1}D_{j-1}}{k_{j-1}}\right]}$$

$$(q_1/q_T)_m = 1 - \frac{\sum_{j=1}^{m} h_{j-1}k_{j-1}}{\sum_{j=1}^{n} h_j k_j}$$

$$\frac{\dfrac{D_m\Phi_m}{k_m} \sum_{j=1}^{n} h_j k_j}{\dfrac{D_n\Phi_n}{k_n} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{n} Shk_{j-1}\left[\dfrac{\Phi_j D_j}{k_j} - \dfrac{\Phi_{j-1}D_{j-1}}{k_{j-1}}\right]}$$

It should be noticed that the above equations are derived on the assumption of a piston-like displacement, i.e., after breakthrough in any layer, only the displacing fluid is flowing. Thus, when the second layer breaks through, the composition is just the total flow rate, minus the flow rate from the first layer that broke through, divided by the total flow rate. It is assumed that the flow rate from the first layer is 100 per cent displacing fluid. In some cases, this assumption will be invalid; thus the following correction may be applied in order to alleviate this condition.

First, it will be assumed that samples from the different layers have been taken and analyzed in the laboratory for their respective permeability, porosity and displacement values. In order to determine the displacement efficiency, the core must be saturated with the reservoir fluid and displaced by the displacing fluid. The per cent recovered at breakthrough is the displacement efficiency (D_j). Now, if a graph of % Recovered vs Pore Volumes Injected were plotted for each core from each layer, the entire displacement history of each layer will be obtained (see Fig. 3 Page37). With these curves, the following correction may be applied to the theoretical calculations.

When the second layer breaks through, the cumulative recovery will be Np_2 and the total production, C_2. From the total production, (C_2), the amount of fluid injected into the first layer may be determined from the following

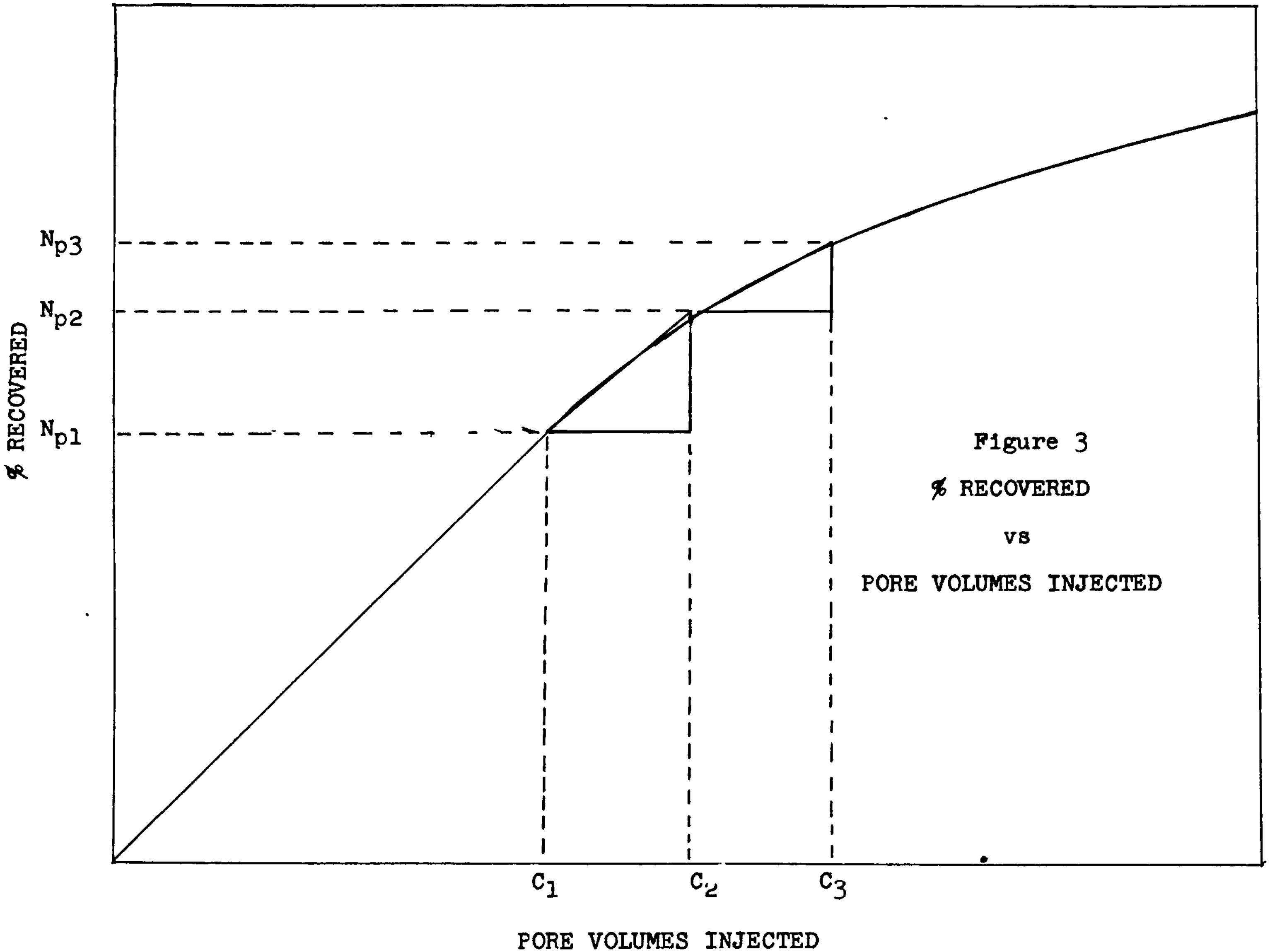


Figure 3
% RECOVERED
vs
PORE VOLUMES INJECTED

relationship:

$$q_1/q_T = \frac{k_1}{\sum_{j=1}^{n} k_j}$$

Thus, the amount of pore volumes injected into the first layer is calculated; with this value, the % Recovered may be obtained from the % Recovered vs Pore Volumes Injected graph for the first layer.

Now, the true composition of the first layer may be found since

$$\frac{\Delta C}{\Delta Np} = \frac{q_d}{q_1} = \frac{1}{(q_1/q_T)\text{ 1st layer}} - 1$$

where ΔC is the difference between the pore volumes injected in the first layer when that layer broke through and the pore volumes injected into that same layer when the second layer broke through; ΔNp is the corresponding difference on the % Recovered vs Pore Volume Injected graph.

Solving for the composition after breakthrough in the first layer

$$(q_1/q_T)_{\text{1st layer}} = \frac{1}{\frac{\Delta C}{\Delta Np} + 1}$$

The difference in the cumulative recoveries from the first layer may be added to calculated cumulative recovery, (Np_2). The composition $(q_1/q_T)_2$ is corrected in the following way:

$$(q_1/q_T)_2 = \frac{q_T - q_o + q_1(1-q_1/q_T)\text{1st layer}}{q_T}$$

Since $$(1-q_1/q_T) = \frac{1}{1 + \frac{\Delta Np}{\Delta C}\ (\text{composition of the displacing fluid})}$$

the correction for the composition of the displacing fluid from the first layer when any layer breaks through may be designated as:

$$\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{p,1}}$$

where P refers to the slope measurements and 1 refers to the graph of the first layer. Thus, the correction for any layer will be:

$$\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{p,\ m-1}}$$

For example, when the second layer breaks through, the composition will be:

$$(q_1/q_T)_2 = \frac{1 - h_o k_o + h_1 k_1 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{11}}\right]}{\sum_{j=1}^{n} h_j k_j}$$

The third layer:

$$(q_1/q_T)_3 = \frac{1 - h_o k_o + h_1 k_1 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{21}}\right] + h_2 k_2 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{12}}\right]}{\sum_{j=1}^{n} h_j k_j}$$

The fourth layer, $(q_1/q_T)_4 =$:

$$\frac{1 - h_o k_o + h_1 k_1 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{31}}\right] + h_2 k_2 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{22}}\right] + h_3 k_3 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{13}}\right]}{\sum_{j=1}^{n} h_j k_j}$$

The nth layer, $(q_1/q_T)_n =$:

$$\frac{1 - h_o k_o + h_1 k_1 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{p,1}}\right] + h_2 k_2 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{p-1,2}}\right] + \dots + h_{n-1} k_{n-1} \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{1,n-1}}\right]}{\sum_{j=1}^{n} h_j k_j}$$

Therefore:

$$(q_1/q_T)_m = \frac{1 - \sum_{j=1}^{m} h_{j-1} k_{j-1} \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{p,m-1}}\right]}{\sum_{j=1}^{n} h_j k_j}$$

where $p = 1, 2, 3, \dots$

Thus, additional recoveries after breakthrough from any layer may be accounted for, and since the miscible displacement process is at a unit viscosity ratio, the capillary and gravity forces may be neglected. The next step in determining the displacement history would be to calculate the sweep efficiency . This may be accomplished by applying the same procedure in the Hurst method. Although the Hurst method involves only five-spot patterns, there are sweep efficiency patterns in literature[16] involving other configurations.

Viscosity Ratios Differing From One

It will be assumed that the conditions existing in the reservoir from the previous derivation will be the same with the exception of the viscosities of the displacing and in-place fluid. Gravity forces will also be neglected. For the sake of convenience, the following derivation will be performed using linear instead of the radial flow equations.

Consider first the determination of the flow-rate of the front in the jth layer. By Darcy's law:

$$q_{ij} = \frac{k_j A_j}{u_i} \frac{dp_j}{d_x} \text{ and } q_{dj} = \frac{k_j A_j}{u_d} \frac{dp_j}{d_x}$$

where q_{ij}, q_{dj}, u_i and u_d are the flow rate and viscosities of the in-place and displacing fluids respectively.

Now suppose the flood front is located at x_j, and let P_{ij} be the difference in pressure between the point x and the influx end of the layer. Then

$$q_{dj} = \frac{k_j A_j Pl_j}{u_d \quad x_j}$$

The difference in pressure between the efflux end of the layer and x_j is:

$$P_j - P_{1j}$$

where P_j is the difference in pressure between the efflux end of the jth layer and the influx end. Hence

$$q_{1j} = \frac{A_j k_j (P_j - P_{1j})}{u_1 \left[L - x_j\right]}$$

where L is the length of the jth layer.

Solving for P_j

$$Pj = \frac{u_d q_d x_j + u_1 q_{1j} (L - x_j)}{A_j k_j}$$

However, since $q_{dj} = q_{1j}$, it follows that:

$$q_j = \frac{k_j A_j P_j}{u_d x_j + u_1 (L - x_j)}$$

Letting $S_j = \frac{x_j}{L}$, and substituting:

$$q_j = \frac{k_j A_j P_j}{\left[u_1 (1 - S_j) + u_d S_j\right] L}$$

$$\text{or } q_j = \frac{k_j A_j P_j}{\left[S_j (u_d - u_1) + u_1\right] L}$$

It is obvious from the above equation, that if the pressure difference is constant, the flow rate will decrease or increase depending upon whether $u_d > u_i$ or $u_d < u_i$. Also, the change in the flow rate will depend on the relative distance, (Sj), of the flood front. Thus, if the relative rate of change of the flow rate is not the same in two adjacent layers, there will be crossflow from one layer to the other. The direction of this crossflow depends entirely on the viscosity ratio. If the viscosity ratio $(u_r = u_i/u_d)$ is less than one, then the relative rate of change of flow rate will be greater in the more permeable layer. Since the flow rate is decreasing, there will be a crossflow from the more permeable layer to the less permeable. When the viscosity ratio is greater than one, the flow rate will be increasing, hence, the crossflow is from the less permeable to the more permeable bed (actually it is the relative distance of the flood front and not the permeability that will determine the direction of crossflow; the importance of porosity in the individual layer can now be seen to be a significant factor in displacements.) Thus the problem to be solved is to calculate the relative flow rate in any layer at any time for any distance of the flood front. This may be done by imposing certain boundary conditions on the system thereby limiting the direction of flow to one direction (along the x-axis). Thus the problem will be greatly simplified by eliminating the calculation of crossflow from each layer. These boundary conditions are as follows:

At $t > o$, and for $Sj > O$:

$$Pj = P_1 = P_2 = P_3 = \ldots = P_n$$

this condition will eliminate any crossflow between layers; in order to compensate for this, further restrictions must be applied to the system. At $t > O$, and for $Sj = O$:

$$Pj \neq P_1 \neq P_2 \neq P_3 \neq \ldots \neq P_n$$

With these boundary conditions, the system may be imagined to be composed of n layers, each having its own permeability and porosity; the first boundary condition implies that there is no crossflow between layers - hence, the fluid is flowing in one direction; the second boundary condition shows that the flow rates at the influx end are not equal - thus at the influx end, a manifold is imagined to exist with n regulators for the n layers. Then, if the relative flow rate in one layer increases or decreases, it will decrease or increase in the other layers. The problem now may easily be solved.

The flow rate in the jth layer of a finite number of beds is:

$$q_j = \frac{k_j A_j P_j}{\left[Sj(u_d - u_i) + u_i\right] L}$$

since $q_j = \frac{d(\text{Vol. of displacing fluid})}{dt}$

$$q_j = D_j A_j \phi_j L \frac{dS_j}{dt}$$

where ϕ_j, Dj and Aj are the porosity, displacement efficiency and cross-sectional area of the jth layer. Therefore:

$$D_j \phi_j L \frac{dS_j}{dt} = \frac{k_j P_j}{\left[Sj(u_d - u_i) + u_i\right] L}$$

$$\text{or} \int_{Sj>0}^{1} Sj(u_d - u_i) + u_i \quad dS_j = \frac{k_j P_j}{D_j \phi_j L^2} \int_{0}^{t_b} dt$$

where t_b is the breakthrough time for the jth layer.
Integrating:

$$\left. \tfrac{1}{2} S_j^2 \, (u_d - u_i) + u_i S_j \right]_0^1 = \frac{k_j P_j}{\phi_j D_j L^2} \quad t_b$$

$$\text{or } t_b = \tfrac{1}{2} \, \frac{(u_d - u_i) \phi_m D_m L^2}{k_m P_m}$$

In order to find the relative distance of the jth layer when the nth layer has broken through, the following expression is integrated:

$$\int_0^{Sj} \left[Sj(u_d-u_1)+u_1 \right] dS_j = \frac{k_j P_j}{\bar{a}_j L^2 D_j} \int_0^{t_{bm}} dt$$

$$\tfrac{1}{2} S_j^2 \,(u_d-u_1)+u_1 S_j = \frac{\tfrac{1}{2}(u_d+u_1)\; k_j \bar{a}_m D_m}{\bar{a}_j D_j k_m}$$

$$\frac{P_j}{P_m} = 1 \text{ since } P_1 = P_2 = P_3 = \dots = P_n$$

Let $u_r = \dfrac{u_1}{u_d}$, therefore:

$$S_j^2\,(1-u_r)+u_r S_j = (1+u_r)\;\frac{k_j \bar{a}_m D_m}{\bar{a}_j D_j k_m}$$

Solving this quadratic equation:

$$S_j = \frac{u_r \pm \sqrt{u_r^2 + \dfrac{k_j \bar{a}_m D_m}{\bar{a}_j D_j k_m}\,(1-u_r^2)}}{(u_r - 1)}$$

when $j = m$ $\quad 1 = \dfrac{u_r \pm 1}{u_r - 1}$

Hence, the minus sign must be chosen. Thus when the mth layer has broken through,

$$S_j = \frac{u_r - \sqrt{u_r^2 + \frac{k_j \phi_m D_m}{\phi_j D_j k_m} (1-u_r^2)}}{(u_r - 1)}$$

gives the relative distance of advance of the flood from the jth layer.

Thus the recovery, Npm, at breakthrough is the volume of displacing fluid in the n layers when the m = 1 layer breaks through, i.e.,

$$Npm = \frac{\sum_{j=1}^{n} S_j \phi_j D_j}{\sum_{j=1}^{n} \phi_j D_j}$$

$$\text{or } Npm = \frac{\sum_{j=1}^{m-1} \phi_j D_j + \sum_{j=m}^{n} D_j \phi_j \frac{\left(u_r - \sqrt{u_r^2 + \frac{k_j \phi_m D_m}{\phi_j D_j k_m} (1-u_r^2)}\right)}{(u_r - 1)}}{\sum_{j=1}^{n} \phi_j D_j}$$

In determining the composition of production, the same procedure will be used as in the previous derivation (where

$u_r = 1$).

The flow rate in the jth layer is:

$$q_j = \frac{k_j A_j P_j}{\left[S_j(1-u_r) + u_r\right] L \ u_d}$$

Then the total flow rate from all the layers will be:

$$\frac{q_j}{q_T} = \frac{\dfrac{k_j A_j}{S_j(1-u_r) + u_r}}{\sum_{j=1}^{n} \dfrac{k_j A_j}{S_j(1-u_r) + u_r}}$$

At breakthrough, the fractional flow rate of the most permeable layer (i.e., when $j = m$) will be:

$$q_m/q_T = \frac{k_m A_m}{\sum_{j=1}^{n} \dfrac{k_j A_j}{S_j\ (1-u_r) + u_r}} \quad \text{since } S_m = 1.$$

The composition of production (q_i/q_T) will be 100 per cent at impending breakthrough; but, when the next layer breaks through, the composition will be less, owing to the influx of the displacing fluid from the previous layer that had broken through.

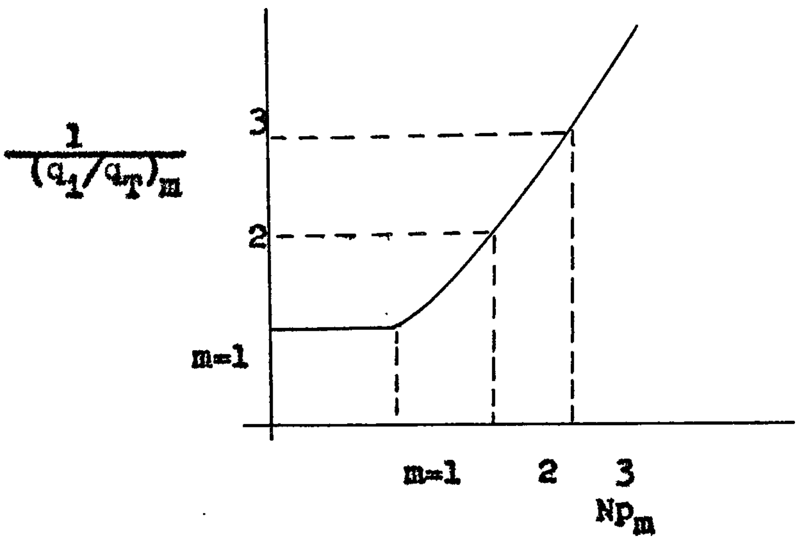
Thus the composition of production $(q_i/q_T)_m$ when any layer breaks through will be:

$$(q_i/q_T)_m = 1 - \frac{q_{m-1}}{q_T} \qquad (q_o/q_T = 0)$$

$$\text{or } (q_i/q_T)_m = \frac{1 - \sum_{j=1}^{m} k_{j-1}A_{j-1}}{\sum_{j=1}^{n} \dfrac{k_jA_j}{u_r^2 + k_j\bar{\phi}_{m-1}D_{m-1}(1-u_r^2)/\bar{\phi}_jD_jk_{m-1}}}$$

where $m = 1,2,3,\ldots,n.$

From the previous derivations the composition has been found as a function of the cumulative recovery. If the reciprocal of the composition was plotted against the recovery on rectangular coordinates, a curve would be obtained which looks something like that in the following sketch.



Now the composition is given by

$$(q_1/q_T)_m = \frac{\frac{dNp_m}{dt}}{\frac{dC_m}{dt}} = \frac{dNp_m}{dC_m}$$

where Np_m is the cumulative recovery and Cm is the cumulative total production. The question then may be asked: What does the area under the $1/(q_1/q_T)_m$ vs Np_m curve represent? The area is just:

$$\text{AREA} = \int_0^{Np_m} \frac{1}{(q_1/q_T)_m} \, dNp_m = \int_0^{Np_m} \frac{dC_m}{dNp_m} \, Np_m = C_m$$

Thus the area under the curve is just the total production produced up to the given recovery Np_m, i.e.,

$$C = \int_0^{Np_m} \frac{1}{(q_1/q_T)_m} \, dNp_m$$

However, this may be approximated:

$$C = \frac{Np_1}{(q_1/q_T)_1} + \frac{1}{(q_1/q_T)_1}(Np_2-Np_1) + \frac{1}{2}\left[\frac{1}{(q_1/q_T)_2} - \frac{1}{(q_1/q_T)_1}\right](Np_2-Np_1)$$
$$+ \frac{1}{(q_1/q_T)_2}(Np_3-Np_2) + \frac{1}{2}\left[\frac{1}{(q_1/q_T)_3} - \frac{1}{(q_1/q_T)_2}\right] + Np_3 - Np_2 + \ldots$$

Then

$$C = \frac{Np_1}{(q_1/q_T)_1} + \frac{(Np_2 - Np_1)}{\frac{1}{2}\left[(q_1/q_T)_1 + (q_1/q_T)_2\right]} + \frac{(Np_3 - Np_2)}{\frac{1}{2}\left[(q_1/q_T)_2 + (q_1/q_T)_3\right]} + \cdots$$

$$\text{Or } C = \frac{Np_1}{(q_1/q_T)_1} + \sum_{m=2}^{n} \frac{(Np_m - Np_{m-1})}{\frac{1}{2}\left[(q_1/q_T)_{m-1} + (q_1/q_T)_m\right]}$$

In correcting for the recovery after breakthrough, the same procedure concerning the correction for unit viscosity ratio may be followed.

The amount of fluid injected into the first layer when the second layer breaks through may be determined by:

$$\frac{q_1}{q_T} = \frac{k_1 A_1}{\sum_{j=1}^{n} \frac{k_j A_j}{S_j(1-u_r) + u_r}}$$

The corrected composition when the second layer breaks through will be:

$$(q_1/q_T) = \frac{1 - k_o A_o + k_1 A_1 \left[\frac{1}{1 + \left(\frac{\Delta Np}{\Delta C}\right)_{11}}\right]}{\sum_{j=1}^{n} \frac{k_j A_j}{S_j(1-u_r) + u_r}}$$

or the general form will be:

$$(q_1/q_T)_m = \frac{1 - \sum_{j=1}^{m} k_m A_m \left[\dfrac{1}{1+\left(\dfrac{\Delta Np}{\Delta C}\right)_{p,m-1}} \right]}{\sum_{j=1}^{n} \dfrac{k_j A_j}{\sqrt{u_r^2 + k_j \bar{\Phi}_{m-1} D_{m-1} (1-u_r^2)/\bar{\Phi}_j D_j k_{m-1}}}}$$

The corrected cumulative recovery (Np_m') will be:

$$Np_m' = Np_m + Np_{,m-1}$$

IMMISCIBLE DISPLACEMENT

In immiscible flow, the displacement efficiency is drastically reduced through the action of the interfacial forces between the in-place fluid, displacing fluid, and the porous media. The immiscibility of the fluids results in a residual saturation of the in-place fluid. Thus equations derived in the previous section on miscible flow will be valid for immiscible displacement, provided the displacement efficiency is corrected for this residual saturation.

The amount of in-place fluid displaced is:

$$\sum_{j=1}^{n} a_j D_j$$

If the in-place fluid is oil the displacement efficiency will be:

$$D_j = (1 - Sor_j - Sw_j)$$

where Sor_j is the residual oil saturation, and Sw_j is the water saturation of the jth layer. However, this displacement efficiency cannot be used to predict the relative distance advanced in different layers since it does not include the amount of interstitial water displaced. A previous investigation(15) had shown that the amount of interstitial water displaced in the presence of oil was about 95 per cent. This value, however, seems a little high; the determination of the displacement efficiency for the relative distance advanced will be discussed in a later section.

When a fluid is flowing in the presence of an immiscible fluid, the permeability of the porous media is somewhat reduced[17]. This reduced permeability is referred to as the relative permeability (k_r). Thus if oil is flowing in the presence of connate water, the relative oil permeability will be k_{ro}, and if water is flowing in the presence of residual oil, its relative permeability to water will be k_{rw}. These two numbers combine with the viscosity ratio to form a single number, the mobility ratio

$$M = \frac{k_{rw}u_o}{k_{ro}u_w}$$

where u_o and u_w are the viscosities of oil and water respectively.

This mobility may not be the same for all layers, but the differences may be considered to be insignificant in actual computations.

The cumulative recovery (Np_m) is:

$$Np_m = \frac{\sum_{j=1}^{m-1} \bar{\phi}_j(1-Sw_j-Sor_j) + \frac{\sum_{j=m}^{n} \bar{\phi}_j(1-Sw_j-Sor_j)\left(M-\sqrt{\frac{M^2+D_j\bar{\phi}_m D_m(1-M^2)}{\bar{\phi}_j D_j k_m}}\right)}{(M-1)}}{\sum_{j=1}^{n} \bar{\phi}_j(1-Sw_j-Sor_j)}$$

The composition:

$$(q_1/q_T)_m = \frac{1 - \sum_{j=1}^{m} k_{j-1}A_{j-1}}{\sum_{j=1}^{n} \frac{k_j A_j}{\sqrt{M^2 + k_j\bar{\phi}_{m-1}D_{m-1}(1-M^2)/\bar{\phi}_j k_{m-1}D_j}}}$$

The total production:

$$Cm = Np_m + Wp_m = \frac{Np_1}{(q_1/q_T)_1} + \sum_{m=2}^{n} \frac{2(Np_m - Np_{m-1})}{(q_1/q_T)_m + (q_1/q_T)_{m-1}}$$

The composition of production and cumulative recovery may be corrected in the same manner as the miscible displacements. The resulting equations will give a close representation of actual reservoir conditions.

CHAPTER III

DESCRIPTION OF EXPERIMENTAL EQUIPMENT AND EXPERIMENTAL PROCEDURE

LABORATORY EQUIPMENT

Sand Model

For the laboratory phase of the work, a plexi-glass core retainer was constructed. This core retainer contained six different unconsolidated sands. The permeability, porosity and other physical characteristics of each sand were measured independently; thus the displacement histories of the system were predicted for various viscosity ratios.

Core Retainer

The core retainer consisted of two sheets of plexi-glass "welded" onto two strips of the same material (see Fig. 5, Page 59). The dimensions of the void space between the sheets of plexi-glass are 6 feet by 6 inches by $\frac{1}{4}$ inch. The small thickness of the model minimized the effects due to the gravity forces during a displacement. The transparency of the plexi-glass made it possible to visualize the actual displacement process. The actual construction of the model was a very simple task; after obtaining the proper pieces of plexi-glass and assembling them in their proper position, carbon tetrachloride was "squirted" by means of an eye dropper between the surfaces

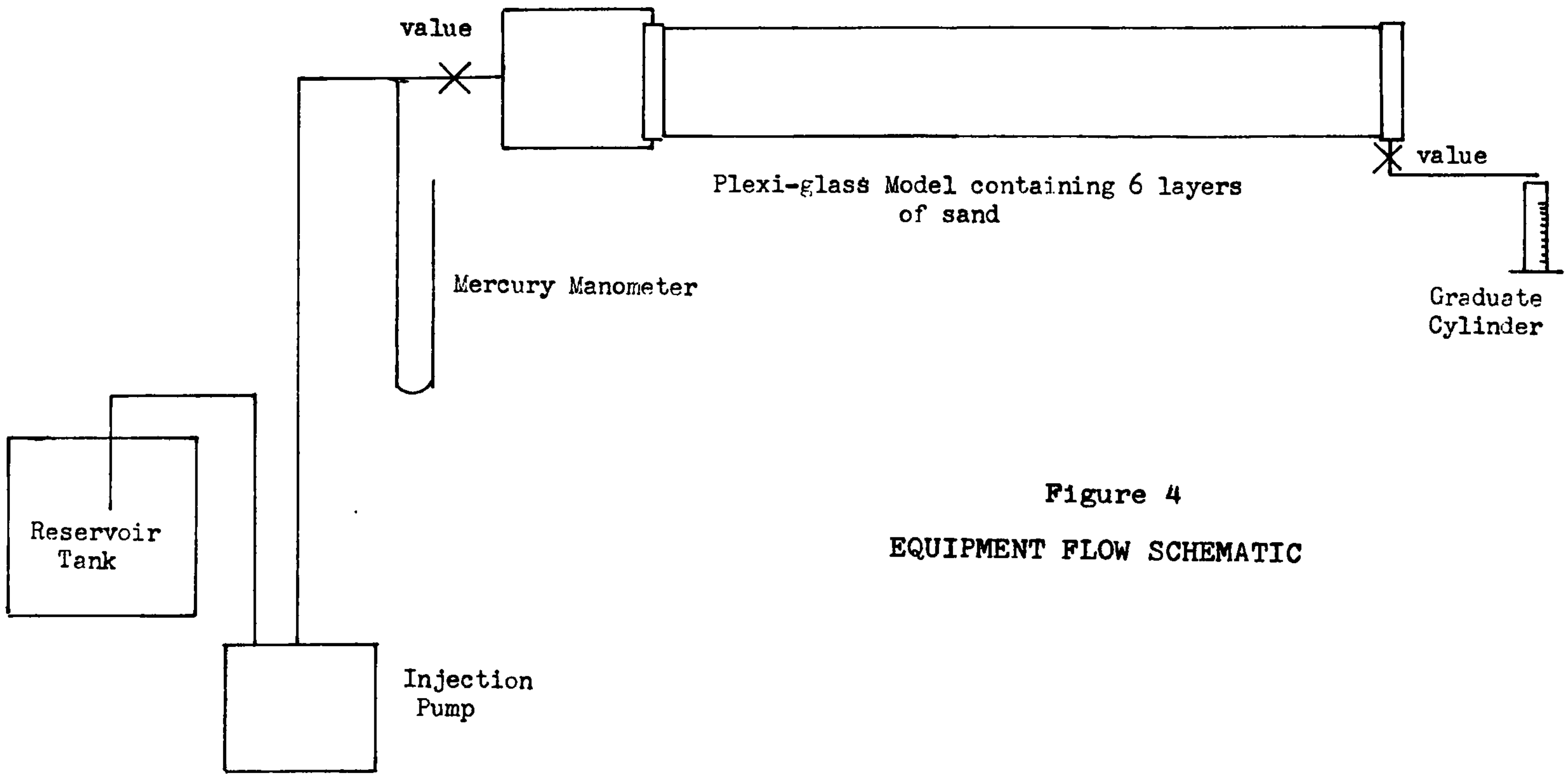
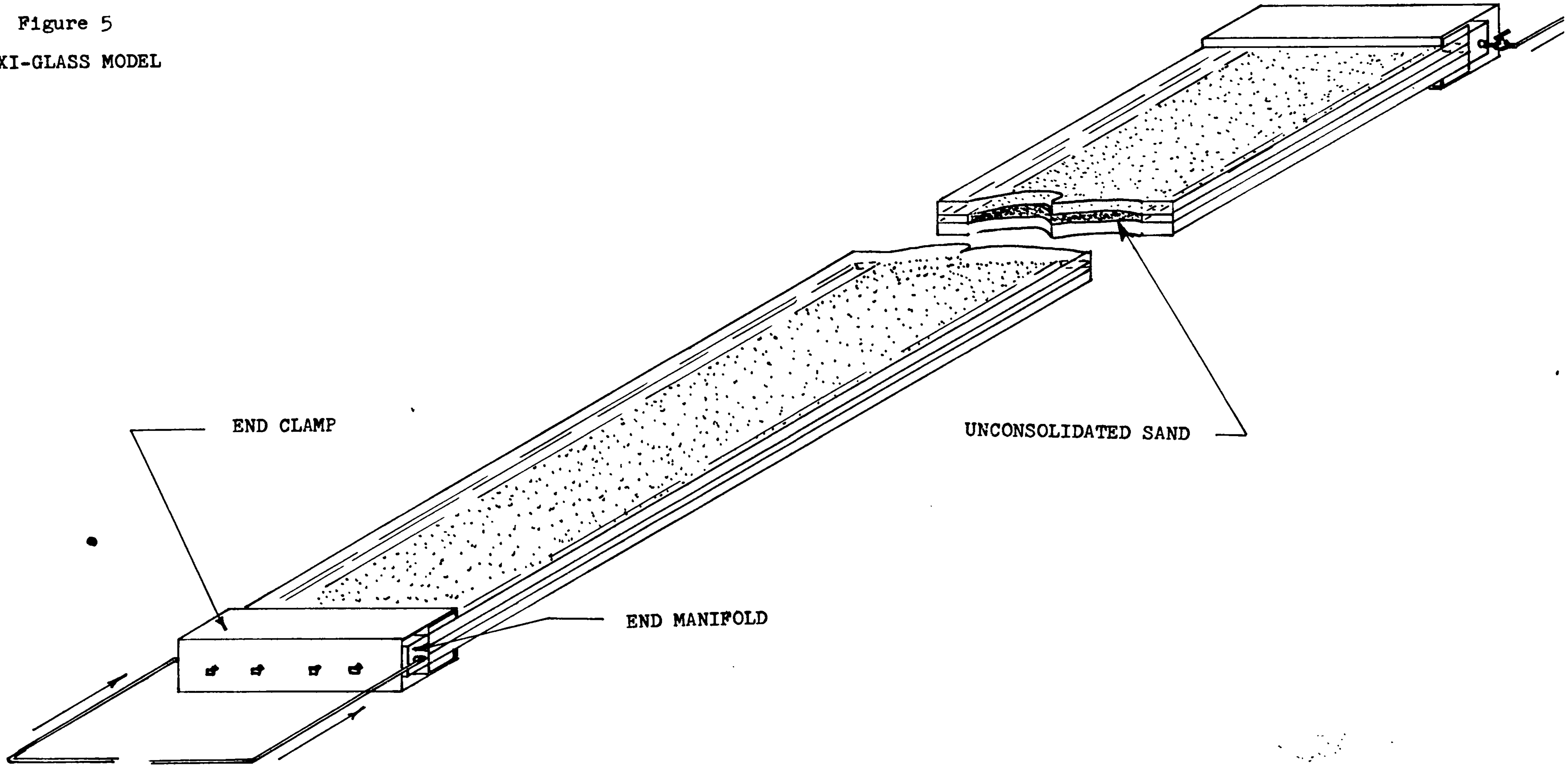


Figure 4

EQUIPMENT FLOW SCHEMATIC

Figure 5

PLEXI-GLASS MODEL



that were to be sealed. A few hours after this application, the sheets were firmly welded. Four strips (8 inches by 1 inch by ¼ inch) were welded at the ends of the plexiglass sheets in the same manner as described above; the purpose of these strips was to act as a brace for the clamps that held the end manifolds in place.

End Manifolds

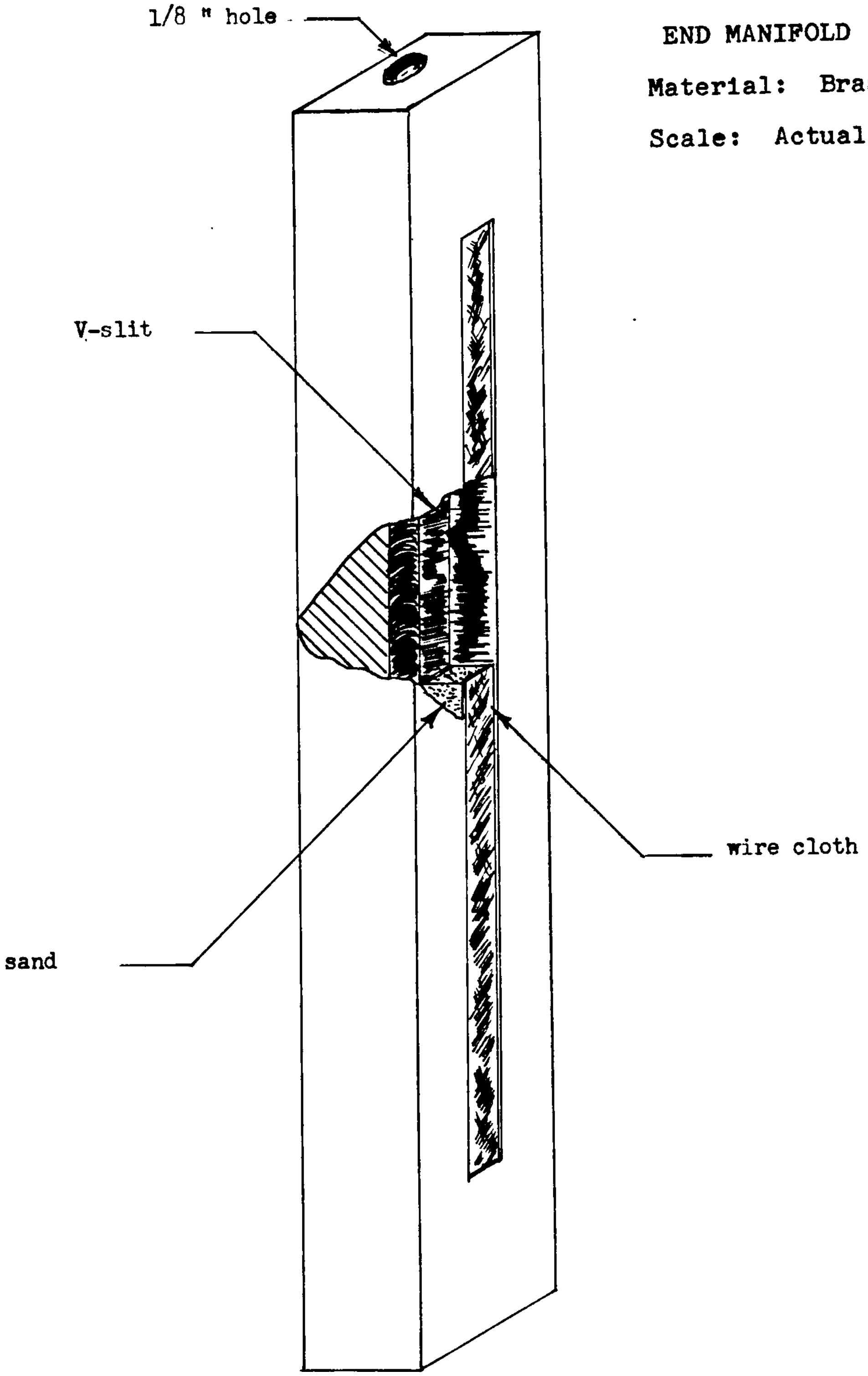
The end manifolds (see Fig. 5, Page 59 and Fig. 6, Page 61) were constructed from two solid brass bars, each being 8 inches by 1 inch by 3/4 inch. The purpose of these manifolds, was to distribute the injected flow of the displacing fluids evenly across the unconsolidated sands. The manifolds were constructed by drilling a 1/8 inch hole through the length of the bar (tapped at the ends for 1/8 inch NPT fittings). On the 1 inch face of the bar, a ¼ by 6 inch groove was milled out, approximately ¼ inch deep; the groove and the ¼ inch hole were then connected by a V-slit. A fine wire cloth was placed on top of the V-slit, and this was followed by packing the groove with 60 mesh sand. In order to insure a smooth contact between the manifold and model, another groove, 1/16 inch deep, was milled out surrounding the previous one; then a retaining wire cloth was placed over the sand, resting on the bottom of the recent drilled groove. A separate piece of brass was constructed in the shape of

Figure 6

END MANIFOLD

Material: Brass

Scale: Actual Size



the latter groove (this piece had a space milled matching the inside dimensions of the model); this separate piece of brass was inserted on top of the wire cloth and soldered under pressure until the two pieces of brass were "sweated" together; then the face of the manifold was milled to a smooth surface. A rubber gasket coated with stopcock grease was placed between the surfaces of the model and manifold face to insure a tight seal. Throughout the entire displacement runs, there were no leaks from the manifold, nor from the plexi-glass model.

End Clamps

The end clamps (see Fig. 5, Page 59) were welded pieces of steel shaped in the form of a U. The main purpose of these clamps was to hold the manifolds firmly in place. This was accomplished by four 1/8 inch machine screws which pressed the manifold against the model. These clamps were braced against the 8 inch strips previously described in the section on the core retainer.

Flow System

The flow system was assembled so as to meter the output of fluids. The schematic drawing, Fig. 4, Page 58, illustrates the layout utilized.

Reservoir Tank

The reservoir tank in which the driving fluid was stored, was a $4\frac{1}{2}$ liter pyrex glass flask. The driving fluid used in each displacement was either a water-glycerine mixture or a weak sodium-hydroxide solution with an added phenolphthalein indicator. In either case, the displacing fluid was prepared just before the displacement. For the immiscible displacement, a 0.85 specific gravity red manometer oil was used. Table 2, Page 77, shows the relationship between displacing and in-place fluids. A $\frac{1}{4}$ inch glass tubing connected the driving fluid to the displacement pump.

Injection Pump

A Milton-Roy mini injection pump was used in making the displacements. The flow rate of this pump was controlled by a screw adjustment. The Milton-Roy mini pump provided a maximum rate of 660 milliliters per hour at pressures up to 500 pounds per square inch; the range of rates used in the displacements were from 180 milliliters per hour to 48 milliliters per hour (the reason for this specific range of rates is given in the section on the determination of porosities and permeabilities).

Pressure Maintenance

A constant pressure differential was maintained over the model by means of an open end mercury manometer. This manometer was connected to the flow line between the injection pump and the influx manifold. A 10 inch capillary tube was inserted between the manometer and flow line in order to compensate for the pulsating motion of the displacing fluid caused by the positive displacement pump. The pressure reading on this manometer was stabilized at 1½ inches of mercury throughout the entire displacements.

Tubing Layout

The tubing layout is shown in schematic form in Fig. 4, Page 58. A ¼ inch saran tubing connected the injection pump with a ¼ inch swedge lock tee which branched off to the manometer and the influx end of the model. At the tee a 1/8 inch reducer connected a lucite valve with a 1/8 inch pipe tee; at the tee, two 1/8 inch brass tubings were connected to both ends of the manifold. This reduction in tubing size helped to reduce the pulsating action of the pump. By the time the displacing fluid had passed through the reservoir and sand filter in the manifold, all pulsations were completely eliminated.

Fluid Metering

Volumes of oil and water were measured in graduated cylinders or allowed to collect in larger containers on some extended runs. During miscible displacements, samples were taken at certain intervals of production and measured for composition.

EXPERIMENTAL PROCEDURE

Determination of Sand Constants

In order to predict a displacement process, it is necessary to know the physical characteristics of the sand. In this experiment the characteristics or constants of six sands had to be measured independently. The problem that arose was not the measurements of the sands, but instead, making sure that the independently measured values of the six sands were identical with the same six sands in the model. Unfortunately, to achieve this identity, a tedious procedure had to be developed. Table 1, Page 72, shows the measured values of the six sands.

Porosity

The unconsolidated sands were of Dowell washed Ottawa sand, ranging from 50 to 140 mesh. The specific gravity of the sands, measured by means of a Lechatelier specific gravity bottle, was determined to be 2.65. During the porosity determinations, it was noticed that the packing pressure had little effect upon the porosity when the sand was packed dry; the effect of different packing pressures on the permeability are shown in Fig. 8, Page 70. It is obvious, then, that it is not the packing pressure that has any appreciable effects on the porosity, but rather the confining pressure, especially if the sands are saturated with a liquid. Then by eliminating the confining pressure, any differential change in porosity will be due entirely to the change in fluid pressure. Thus the porosities were measured after the sands were subjected to a specific fluid pressure. This same fluid pressure was maintained over the sands in the model during the displacements. The procedure for determining the porosity follows: A weighed sample of unconsolidated sand was packed in a 6 inch lucite core holder. Circular wire cloth was placed at both ends in order to retain the sand. Two holes, ten centimeters apart, were drilled and tapped for 1/8 inch NPT fittings. Thus the pressure difference across the sand was measured. After subjecting the sands to a fluid pressure of 26 centimeters of Number 3 manometer fluid, the core was weighed and its fluid volume calculated. Thus, knowing the sand grain volume, the porosity was determined. These values were later checked against the measured pore volume of the model.

Permeability

The permeability of the unconsolidated sands was determined along with the porosity measurements. The layout of the apparatus is shown in schematic form in Fig. 7, Page 69. After the porosity was determined, the flow rate for various pressures was measured (see Fig. 8, Page 70). This was accomplished by moving the lucite core holder to various levels below the water reservoir. Then, plotting the flow rate as a function of the pressure, the permeability, was determined by Darcy's law from the slope of the resulting straight line, i.e.,

$$K = \frac{\Delta Q}{\Delta P}\frac{UL}{A} \text{ where } \frac{\Delta Q}{\Delta P} \text{ is the slope.}$$

Evaluating the constants and the pressure in terms of C.G.S. units, the permeabilities were then calculated.

The graph of Fig. 8, Page 70, shows the permeability for two different packing pressures. The first slope (designated by O) indicates a negligible packing pressure i.e., the sand was just poured into the lucite holder. The second slope (designated by □) indicates the most extreme packing pressure that could be applied, i.e., the sand was vigorously packed at various pouring intervals. The average difference in the two measurements of permeability is about 10 per cent. However, the ratio of permeabilities and not the absolute values is used in the actual calculations, and since the model was packed in a vertical position, the latter of the two measured values was used in the calculations. As a final check, the average permeability of the model was measured and determined to be 7.43 darcys. This value was within 1.8% of the average measured values from the six unconsolidated sands.

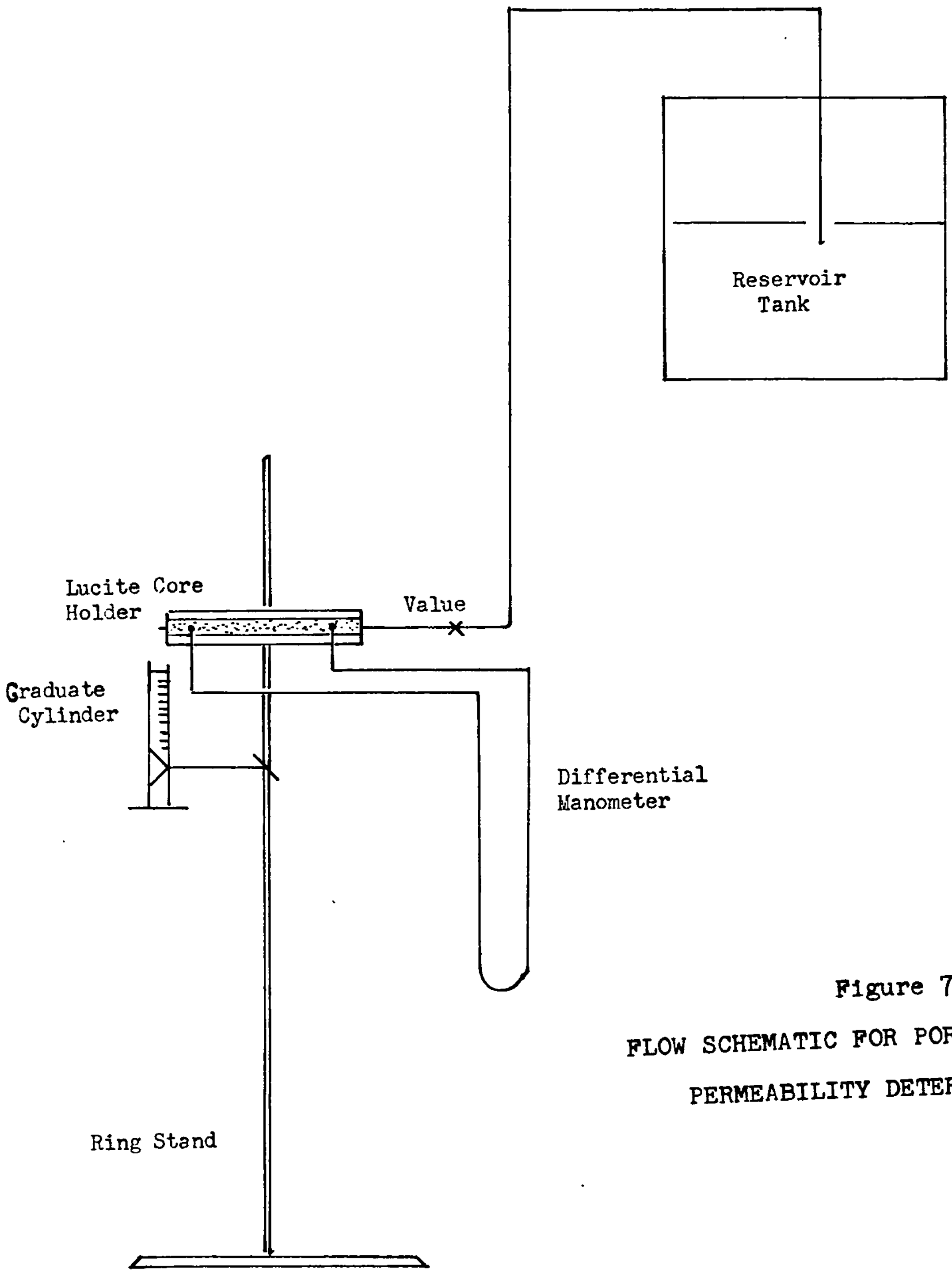


Figure 7

FLOW SCHEMATIC FOR POROSITY AND PERMEABILITY DETERMINATIONS

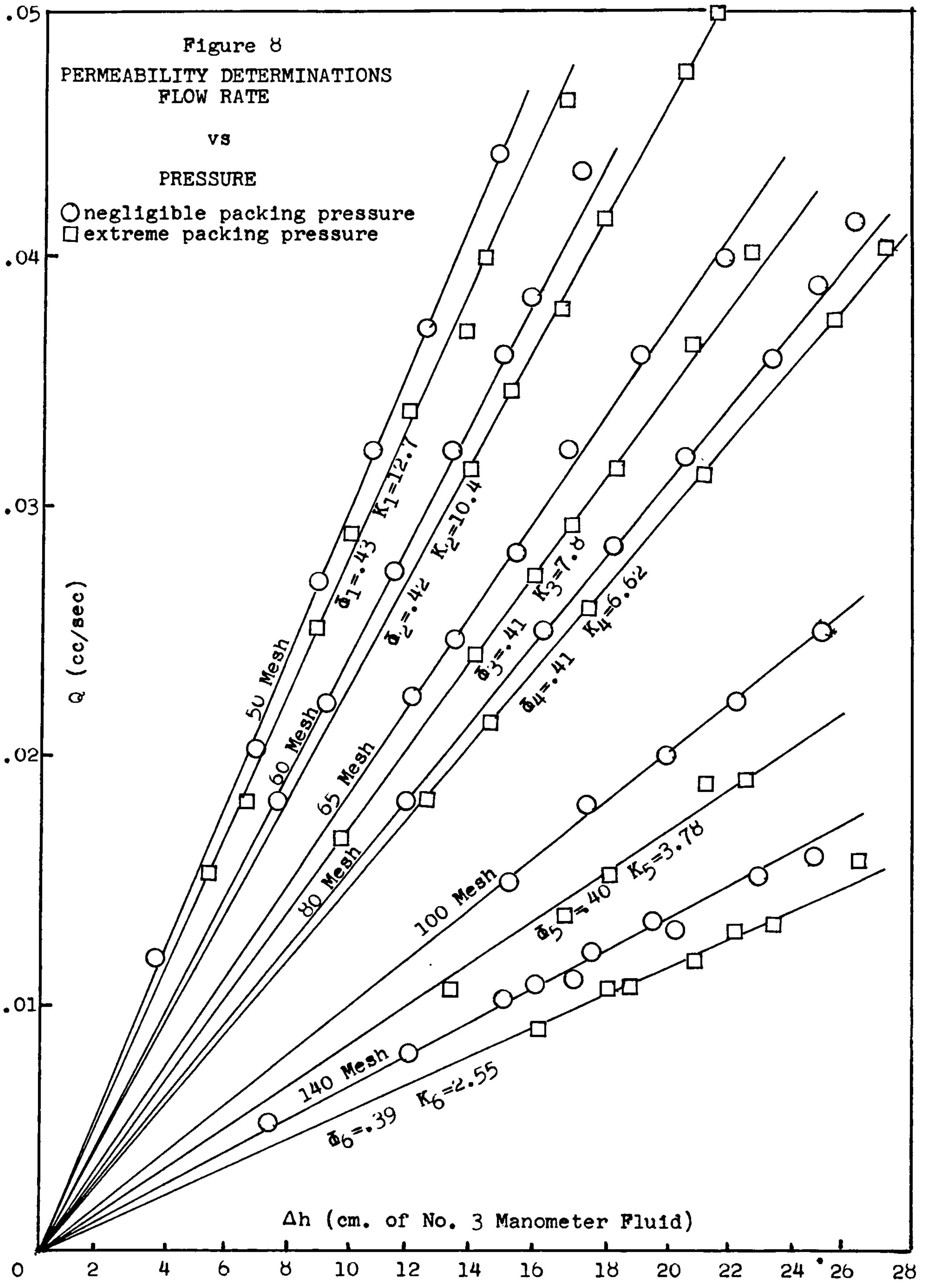


Figure 8
PERMEABILITY DETERMINATIONS
FLOW RATE
vs
PRESSURE
negligible packing pressure
extreme packing pressure
Q (cc/sec)
Δh (cm. of No. 3 Manometer Fluid)
50 Mesh
60 Mesh
65 Mesh
80 Mesh
100 Mesh
140 Mesh
δ1=.43 K1=12.7
δ2=.42 K2=10.4
δ3=.41 K3=7.8
δ4=.41 K4=6.62
δ5=.40 K5=3.78
δ6=.39 K6=2.55
.05
.04
.03
.02
.01
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28

Residual Saturations

The residual saturations were determined for each sand in the following manner: the lucite core holder was packed with sand and saturated with water; the amount of water required for this saturation was measured (see Porosity section). This water was then displaced by the 0.85 specific gravity red manometer oil; the amount of water displaced was measured and subtracted from the total amount of water originally in place. This figure, divided by the pore volume, gave the fractional water saturation for that particular sand. The red oil was then displaced by water. The amount of oil displaced, subtracted from the original amount of oil in place and divided by the pore volume of the sand gave the fractional residual oil saturation. This procedure was carried out for the six unconsolidated sands; the results are shown in Table 1, Page 72 .

Preparation of Equipment

Packing of Model

The packing of the model was probably the most difficult task in the entire experiment. The problem encountered was the even distribution of the six layers of unconsolidated sands. This was accomplished by inserting five 6-foot long brass strips (¼ inch by 1/8 inch)

TABLE 1

PHYSICAL CHARACTERISTICS OF THE UNCONSOLIDATED SANDS

Mesh	Porosity (%)	Permeability (darcys)	Residual Oil Saturation (%)	Interstitial Water Saturation (%)
50	43	12.7	25	19
60	42	10.4	22	21
65	41	7.8	18	23
80	41	6.62	17	23
100	40	3.78	15	28.5
140	39	2.55	14	30.4

inside the model. The brass strips were spaced at 1 inch intervals. Then the model was placed on its end in a vertical position; previously, an end clamp and manifold were inserted at the end of the model. The six unconsolidated sands were placed in plastic containers and weighed. The sand was then poured into its allotted space. After a portion of the model was filled, the brass strips were pulled out to a few inches below the level of the sand. This procedure was repeated until the model was packed. The remaining manifold and end clamp were inserted on the open end; the model was then evacuated for 12 hours by a vacuum pump. Afterwards, water was introduced into the model; the measured volume of water was 735 milliliters, and the weight of the packed sand was approximately 2,800 grams and 1,160 cubic centimeters respectively; the measured porosity was 41 per cent - in excellent agreement with the average individually measured values from previous tests.

Preparing Water-Glycerine Mixtures

In order to introduce various viscosity ratios for the miscible displacements, water and glycerine were mixed together. The relative viscosities of the mixtures to water were determined by the Ostwald viscosimeter. Spot checks during displacements gave identical results.

The data are shown in Table 2, Page 77.

Displacement Techniques

Miscible Displacement

In the first run, distilled water was displaced by a weak sodium hydroxide solution. The prepared sodium hydroxide solution had a concentration of 0.0153 normal. Phenolphthalein indicator was added to give the displacing fluid a deep violet color. Previous to the displacements, all lines leading to the model were filled with the displacing fluid; this was done by disconnecting the swedge-lock fitting in front of the 1/8 inch lucite valve. During the displacement a sharp front was noticed; no abnormalities were observed for all miscible displacements whose viscosity ratios were greater or equal to one. At breakthrough, the displacing front was drawn on the plexi-glass model. The production from the efflux end of the model was measured in graduated cylinders. After breakthrough the composition of production was determined by titrating a measured sample, at specific intervals of production, with a standard 0.01 normal hydrochloric acid solution. The floods were terminated when the displacing fluid was no longer present in the effluent or in such small quantities which could not be reduced appreciably with further flooding. For the other

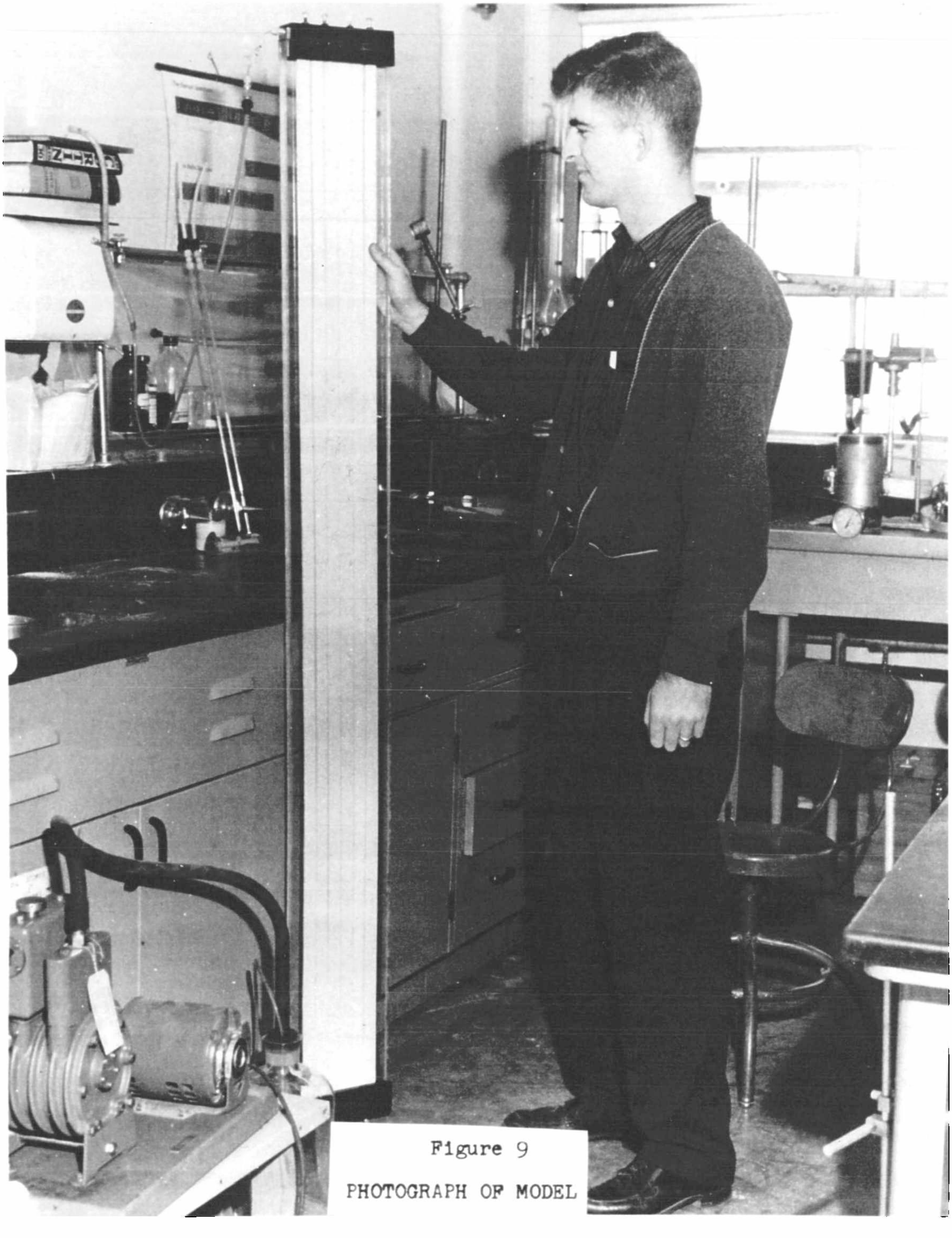


Figure 9

PHOTOGRAPH OF MODEL

miscible displacements, the pressure difference was kept constant and the flow rate was varied by adjusting the turning screw on the injection pump; otherwise, the same procedure as previously described was followed.

Immiscible Displacements

In this type of displacement, the same procedure was followed with the exception of determining the composition after breakthrough. Here the effluent was collected in graduate cylinders and its composition determined by fluid separation. In the oil by water displacement, the flood front at breakthrough could not be traced; this was due to the strong preference of the plexi-glass model for the red oil.

TABLE 2

DISPLACEMENT PROCEDURE

Order of Runs	u_r	Concentration of Displacing Fluid	In-Place Fluid
1	1	0.0153-N	-
2	½	-	0.0153-N
3	2	0.0150-N	-
4	1/3	-	0.0150-N
5	3	0.0133-N	-
6	0.275	IMMISCIBLE	SPECIFIC GRAVITY
7	3.63		-0.85

CHAPTER IV

DISCUSSION OF RESULTS

UNIT VISCOSITY RATIO

In this water by water displacement, the breakthrough was measured to be 59 per cent of the pore volume in the model. The per cent recovered, measured from the fractional area contacted by the drive at breakthrough, was found to be larger than the corresponding 59 per cent recovery. For a perfect piston-like displacement, these quantities would be equal. The area contacted in approximately 61.2 per cent of the total pore volume (see Fig. 10, Page 79). This difference in recoveries has been described in literature as a measure of the mixing zone.[14,18,19] Actually, though, this mixing zone may also be considered as a measure of the displacement efficiency. Thus, if a piston-like displacement did exist, the area contacted at breakthrough would be equal to the measured recovery, and the displacement efficiency would be 100 per cent. In this case, the recovery measured from production is 96 per cent of the recovery determined from the area contacted. Hence - instead of determining the displacement efficiency for each individual sand - the recovery, as derived in the theoretical section, will be determined for a 100 per cent displacement efficiency and then corrected by using the displacement efficiency calculated from the differences

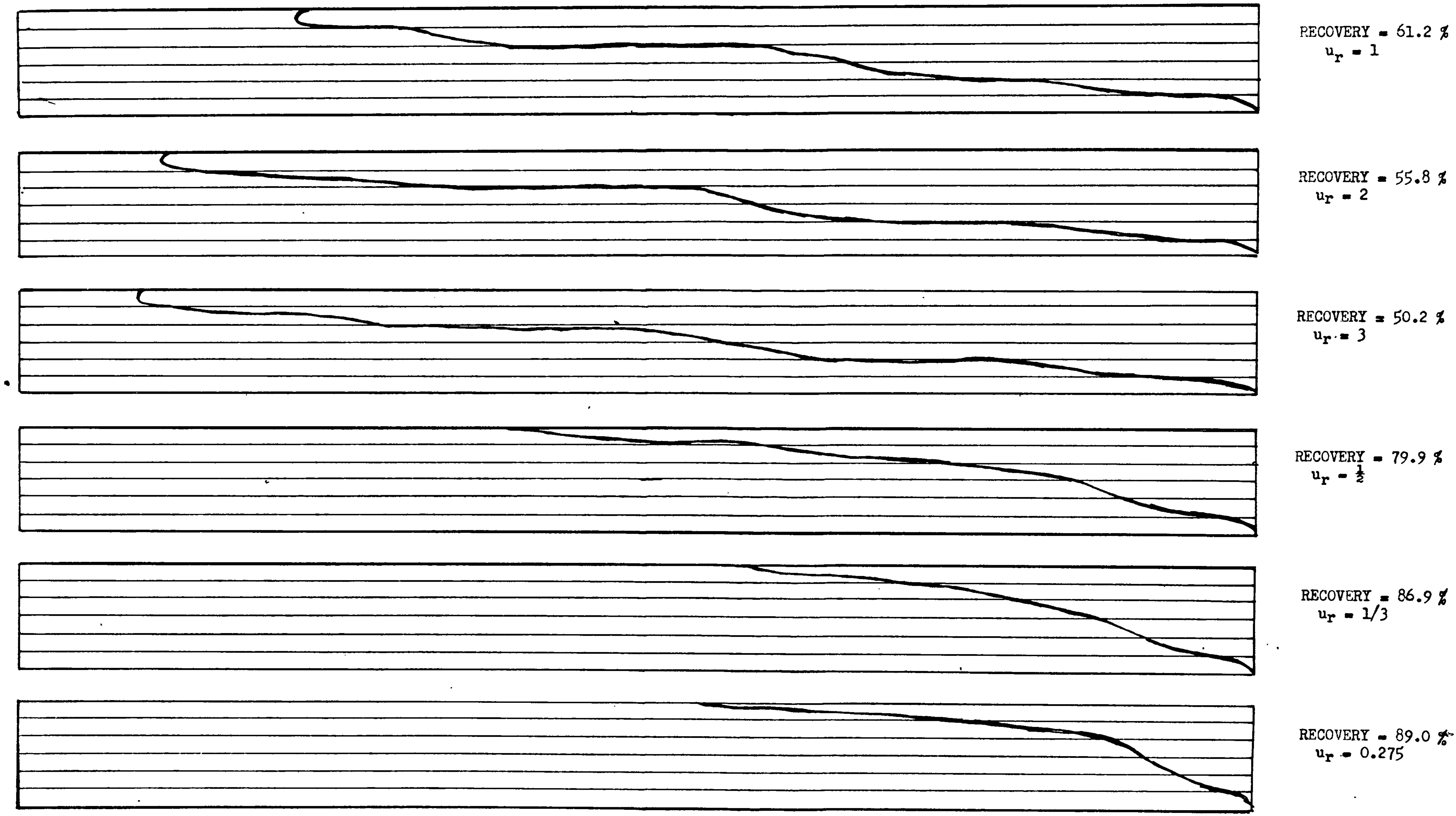


Figure 10

DISPLACEMENT FRONTS AT

BREAKTHROUGH

in recoveries.

Table 3, Page 81, shows the theoretical calculations of the cumulative recovery, total production and composition. Fig. 11, Page 82, is a comparison between the experimental and calculated recoveries. The calculated breakthrough recovery is 58 per cent of the total recovery; this is within 2 per cent of the experimental value of 59 per cent - such a difference may be considered to be insignificant.

TABLE 3

CALCULATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = 1$

$$Np_m = \left\{ \frac{\frac{\bar{a}_m}{k_m} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{m} S(hk)_{j-1} \left[\frac{\bar{a}_j}{k_j} - \frac{\bar{a}_{j-1}}{k_{j-1}} \right]}{\frac{\bar{a}_n}{k_n} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{n} S(hk)_{j-1} \left[\frac{\bar{a}_j}{k_j} - \frac{\bar{a}_{j-1}}{k_{j-1}} \right]} \right\} D$$

$$Cm = \left\{ \frac{\frac{\bar{a}_m}{k_m} \sum_{j=1}^{n} h_j k_j}{\frac{\bar{a}_n}{k_n} \sum_{j=1}^{n} h_j k_j - \sum_{j=1}^{n} S(hk)_{j-1} \left(\frac{\bar{a}_j}{k_j} - \frac{\bar{a}_{j-1}}{k_{j-1}} \right)} \right\} D$$

DATA: $\sum_{j=1}^{6} h_j k_j = 43.85$; $D = 0.96$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
m	$\bar{a}_m$	k_m	$\bar{a}_m/k_m$	$(\bar{a}_m/k_m) \sum_{j=1}^{6} h_j k_j$	$\sum_{j=1}^{m} S(h_{j-1}k_{j-1})(\bar{a}_j/k_j - \bar{a}_{j-1}/k_{j-1})$	(5)-(6)	Np(%) (7)/2.446	(D) Np(%)	C(%) D(5)/2.446
1	.43	12.7	.0339	1.48	0	1.48	60.5	58.0	.580
2	.42	10.4	.0420	1.76	.080	1.69	68.9	60.0	.690
3	.41	7.8	.0525	2.30	.364	1.936	79.2	76.0	.900
4	.41	6.62	.0619	2.70	.654	2.046	83.5	80.0	1.050
5	.40	3.78	.1060	4.62	2.314	2.306	94.5	90.5	1.810
6	.39	2.55	.1530	6.70	4.254	2.446	100.0	96.0	2.620

CALCULATION OF COMPOSITION FOR $u_r = 1$

$$(q_i/q_T)_m = 1 - \frac{\sum_{j=1}^{n} h_{j-1} k_{j-1}}{\sum_{j=1}^{n} h_j k_j}$$

m	$\sum_{j=1}^{m} (h_{j-1} k_{j-1})$	$\sum_{j=1}^{m} (h_{j-1} k_{j-1}) / \sum_{j=1}^{6} h_j k_j$	q_i/q_T (%)
L	0	0	100.0
2	12.7	.29	71.0
3	23.1	.528	47.2
4	30.9	.705	29.5
5	37.52	.86	14.0
6	41.30	.945	5.5

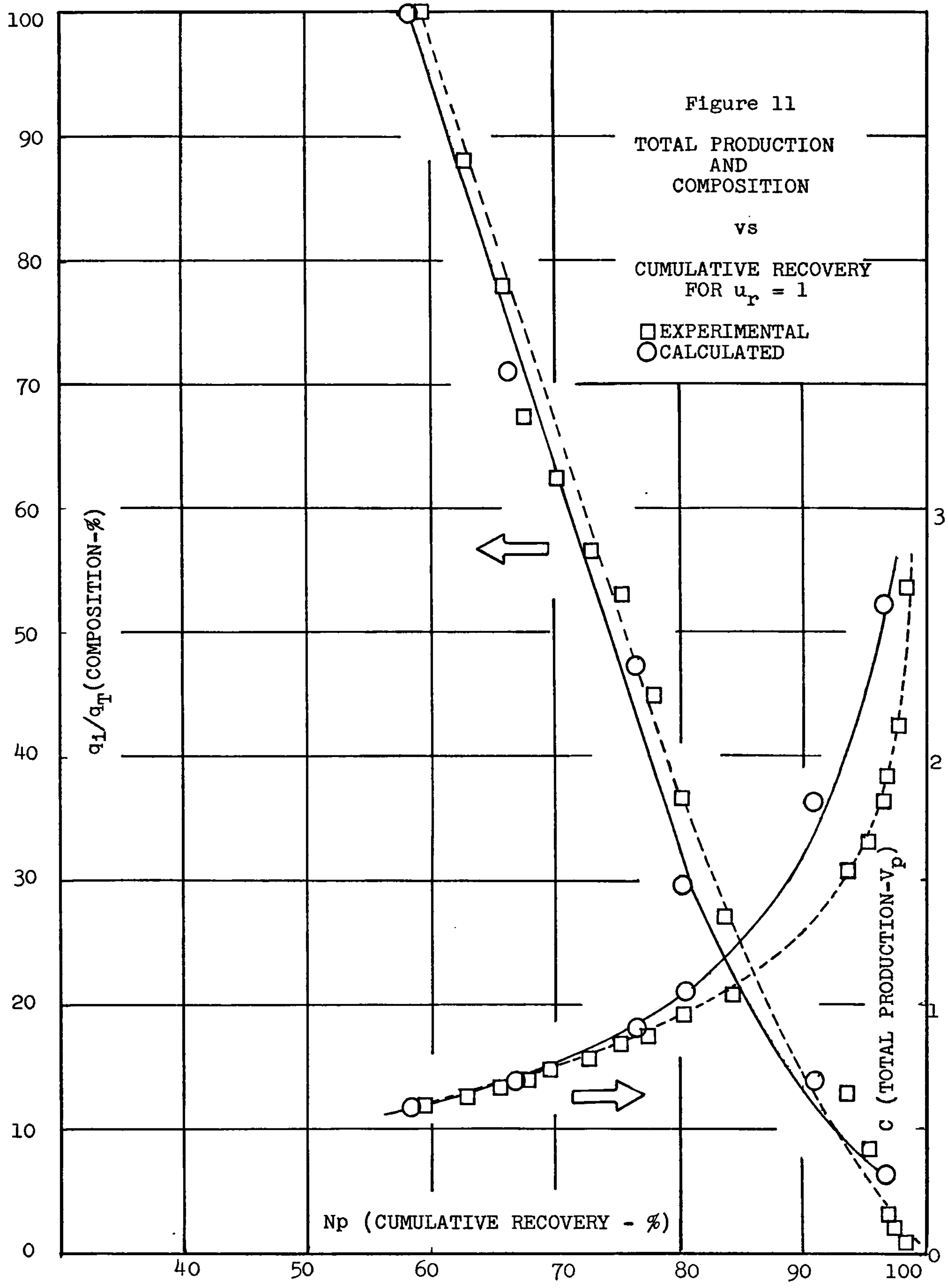


Figure 11
TOTAL PRODUCTION AND COMPOSITION vs CUMULATIVE RECOVERY FOR $u_r = 1$
EXPERIMENTAL
CALCULATED
q_i/q_T (COMPOSITION-%)
c (TOTAL PRODUCTION-V_p)
Np (CUMULATIVE RECOVERY - %)
100
90
80
70
60
50
40
30
20
10
0
40
50
60
70
80
90
100
3
2
1
0

VISCOSITY RATIOS GREATER THAN ONE

Miscible Displacements

In displacements where the in-place fluid is more viscous than the driving fluid, the recoveries will be less than the recovery from a displacement where the in-place and displacing fluid have the same viscosities. This, as discussed in the theoretical section, is due to a smaller displacement efficiency and to the unfavorable crossflow from the least to the more permeable beds. Thus, not only is less fluid being displaced from the the microscopic pore volume, but also, the coverage encountered in the system is somewhat reduced.

For a viscosity ratio equal to two, the recovery obtained from production was 51.3 per cent of the total fluid in-place; the recovery estimated from the area contacted was 55.8 per cent (see Fig. 9, Page 75). This implies that the displacement efficiency was 92.9 per cent. The recovery calculated from core data was 51.0 per cent (see Table 4, Pages 84,85). A comparison between the experimental and calculated recoveries as a function of the composition and total production is shown in Fig. 12, Page 86. Calculated values of the composition and total production are shown in Tables 5 and 6, Pages 87, 88 and 89.

TABLE 4

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 2$

$$Np_m = \left\{ \frac{\sum_{j=1}^{m-1} \bar{a}_j + \sum_{j=m}^{n} \bar{a}_j \dfrac{\left(u_r - \sqrt{u_r^2 + k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m}\right)}{(u_r-1)}}{\sum_{j=1}^{n} \bar{a}_j} \right\} D \qquad \text{where } m = 1,2,3,\ldots,6$$

DATA: $u_r = 2$, $n = 6$, $D = 0.929$, $\sum_{j=1}^{n} \bar{a}_j = 2.46$ and $c = k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m$

(1) j	(2) $k_j/\bar{a}_j$	(3) c	(4) $u_r^2 - c$	(5) $\sqrt{(4)}$	(6) $u_r-(5)$	(7) $\bar{a}_j(6)/(u_r-1)$	(8) $\sum_{j=1}^{m-1} \bar{a}_j + \sum_{j=m}^{n} (7)$	(9) $D(8)/\sum_{j=1}^{n} \bar{a}_j$ (%)
1	29.5	3.00	1.00	1.00	1.00	.430	.430	
2	24.8	2.53	1.47	1.21	.79	.332	.332	
3	19.0	1.93	2.07	1.44	.56	.230	.230	51.0
4	16.2	1.65	2.35	1.53	.47	.193	.193	
5	9.45	.963	3.04	1.74	.26	.104	.104	
6	6.52	.662	3.34	1.84	.16	.062	.062	
							1.351	
m=2								
1	-	-	-	-	-	-	.430	
2	24.8	3.00	1.00	1.00	1.00	.420	.420	
3	19.0	2.29	1.71	1.31	.69	.283	.283	59.1
4	16.2	1.96	2.04	1.43	.57	.233	.233	
5	9.45	1.14	2.86	1.69	.31	.124	.124	
6	6.52	.785	3.22	1.79	.21	.082	.082	
							1.572	
m=3								
1	-	-	-	-	-	-	.430	
2	-	-	-	-	-	-	.420	
3	19.0	3.00	1.00	1.00	1.00	.410	.410	70.2
4	16.2	2.55	1.45	1.20	.80	.328	.328	
5	9.45	1.49	2.51	1.58	.42	.168	.168	
6	6.52	1.03	2.97	1.72	.28	.109	.109	
							1.865	

TABLE 4 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.430	
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	75.0
4	16.2	3.00	1.00	1.00	1.00	.410	.410	
5	9.45	1.75	2.25	1.50	.50	.200	.200	
6	6.52	1.21	2.79	1.67	.33	.128	.128	
							1.993	
m=5								
1	-	-	-	-	-	-	.430	
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	87.0
4	-	-	-	-	-	-	.410	
5	9.45	3.00	1.00	1.00	1.00	.400	.400	
6	6.52	2.06	1.94	1.39	.61	.258	.258	
							2.308	
m=6								
1	-	-	-	-	-	-	.430	
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	92.9
4	-	-	-	-	-	-	.410	
5	-	-	-	-	-	-	.400	
6	6.52	3.00	1.00	1.00	1.00	.390	.390	
							2.460	

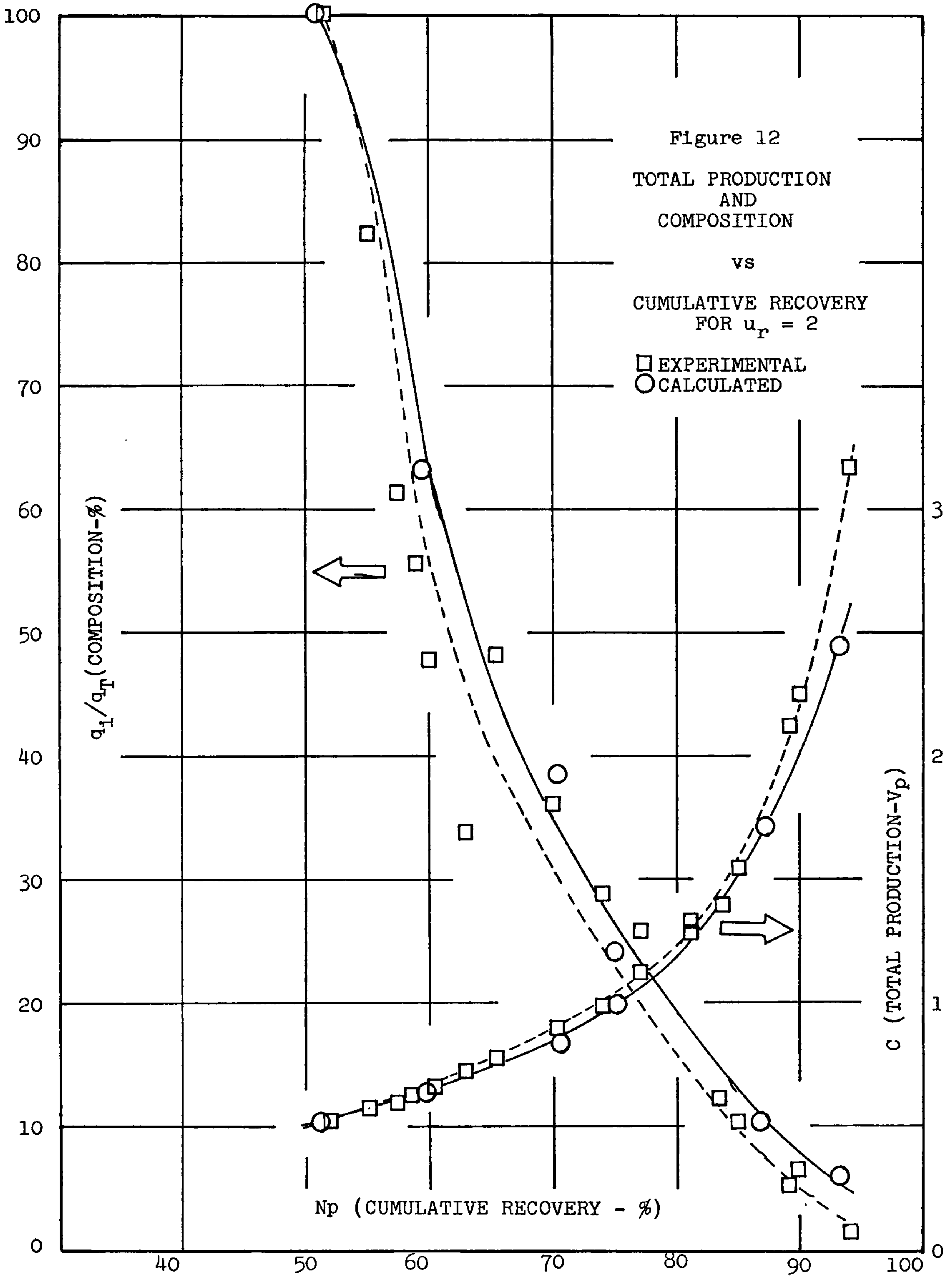


Figure 12
TOTAL PRODUCTION
AND
COMPOSITION
vs
CUMULATIVE RECOVERY
FOR u_r = 2
EXPERIMENTAL
CALCULATED
a_1/a_T (COMPOSITION-%)
C (TOTAL PRODUCTION-V_p)
Np (CUMULATIVE RECOVERY - %)
100
90
80
70
60
50
40
30
20
10
0
3
2
1
0
40
50
60
70
80
90
100

TABLE 5

CALCULATION OF COMPOSITION FOR $u_r = 2$

$$(q_i/q_T)_m = 1 - \sum_{j=1}^{m} k_{j-1} \Big/ \sum_{j=1}^{n} k_j \Big/ \sqrt{u_r^2 + k_j \bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$$

(1) j	(2) k_j	(3) $\sqrt{\dfrac{u_r^2 + k_j \bar{a}_{m-1}(1-u_r^2)}{\bar{a}_j k_{m-1}}}$	(4) (2)/(3)	(5) $\sum_{j=1}^{m} k_{j-1}$	(6) (5)/Σ(4)	(7) $(q_i/q_T)_m$ (%)
m=1						100.0
m=2						
1	12.7	1.00	12.7	12.7	.37	63.0
2	10.4	1.21	8.55			
3	7.8	1.44	5.42			
4	6.62	1.53	4.32			
5	3.78	1.74	2.17			
6	2.55	1.84	1.39			
			34.55			
m=3						
1	12.7	-	12.7	12.7	.62	38.0
2	10.4	1.00	10.4	10.4		
3	7.8	1.31	5.94	23.1		
4	6.62	1.43	4.64			
5	3.78	1.69	2.23			
6	2.55	1.79	1.42			
			37.29			

TABLE 5 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	-	12.7	12.7	.765	23.5
2	10.4	-	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	1.20	5.52	30.9		
5	3.78	1.58	2.39			
6	2.55	1.72	1.48			
			40.29			
m=5						
1	12.7	-	12.7	12.7	.90	10.0
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	1.00	6.62	6.62		
5	3.78	1.50	2.52	37.52		
6	2.55	1.67	1.53			
			41.55			
m=6						
1	12.7	-	12.7	12.7	.95	5.0
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	1.39	1.84	41.30		
			43.14			

TABLE 6

CALCULATION OF TOTAL PRODUCTION FOR $u_r = 2$

$$C = Np_1/(q_1/q_T)_1 + \sum_{m=2}^{n} 2(Np_m - Np_{m-1})/ (q_1/q_T)_m + (q_1/q_T)_{m-1}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.51	-	-	1.00	-	-	-	.51
2	.591	.51	.081	.63	1.00	.815	.0992	.6092
3	.702	.591	.111	.38	.63	.505	.221	.8302
4	.750	.702	.048	.235	.38	.3075	.156	.9822
5	.870	.750	.120	.100	.235	.1675	.72	1.7062
6	.929	.870	.059	.060	.100	.0800	.735	2.4412

Obviously, as the viscosity ratios increase, the recoveries at breakthrough decrease; thrus for ur=3, the recovery measured at breakthrough was found to be 46 per cent of the total pore volume. Similarly the recovery calculated from the area contacted by the displacing fluid was 50.2 per cent - the displacement efficiency was also less - its value: 91.2 per cent. The calculated recovery (see Table 7, Pages 91 and 92) was 47 per cent of the total pore volumes. Results of both the experimental data and calculated values are shown in Fig. 13, Page 93. Calculations involving the composition and total production are shown in Tables 8 and 9, Pages 94, 95 and 96.

Throughout the miscible displacements, a small degree of fingering was observed in the initial flood stages; this, however, seemed to dissipate as the flood front advanced. Other than this, there were no unusual observations.

TABLE 7

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 3$

$$Np_m = \frac{\left\{ \sum_{j=1}^{m-1} \bar{a}_j + \frac{\sum_{j=m}^{n} \bar{a}_j \left(u_r - \sqrt{u_r^2 + k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m}\right)}{(u_r-1)} \right\} D}{\sum_{j=1}^{n} \bar{a}_j}$$

DATA: $u_r=3$, $n=6$, $D=0.912$, $\sum_{j=1}^{n} \bar{a}_j = 2.46$ and $c = k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
j	$k_j/\bar{a}_j$	c	$u_r^2 - c$	$\sqrt{(4)}$	$u_r-(5)$	$\bar{a}_j(6)/(u_r-1)$	$\sum_{j=1}^{m-1}\bar{a}_j + \sum_{j=m}^{n}(7)$	$D(8)/\sum_{j=1}^{n}\bar{a}_j$ (%)
1	29.5	8.00	1.00	1.00	2.00	.430	.430	47.0
2	24.8	6.70	2.30	1.52	1.48	.310	.310	
3	19.0	5.13	3.87	1.97	1.03	.211	.211	
4	16.2	4.38	4.62	2.15	.85	.174	.174	
5	9.45	2.55	6.45	2.54	.46	.092	.092	
6	6.52	1.76	7.24	2.69	.31	.061	.061	
							1.278	
m=2								
1	-	-	-	-	-	-	.430	56.7
2	24.8	8.00	1.00	1.00	2.00	.420	.420	
3	19.0	6.12	2.88	1.69	1.31	.268	.268	
4	16.2	5.22	3.78	1.94	1.06	.217	.217	
5	9.45	3.04	5.96	2.44	.56	.112	.112	
6	6.52	2.11	6.89	2.62	.38	.074	.074	
							1.521	
m=3								
1	-	-	-	-	-	-	.430	67.9
2	-	-	-	-	-	-	.420	
3	19.0	8.00	1.00	1.00	2.00	.410	.410	
4	16.2	6.82	2.18	1.47	1.53	.313	.313	
5	9.45	3.99	5.01	2.24	.76	.152	.152	
6	6.52	2.75	6.25	2.50	.50	.098	.098	
							1.823	

TABLE 7 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.430	73.0
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	16.2	8.00	1.00	1.00	2.00	.410	.410	
5	9.45	4.67	4.33	2.07	.93	.186	.186	
6	6.52	3.22	5.78	2.40	.60	.117	.117	
							1.973	
m=5								
1	-	-	-	-	-	-	.430	85.2
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	9.45	8.00	1.00	1.00	2.00	.400	.400	
6	6.52	5.51	3.49	1.87	1.13	.220	.220	
							2.290	
m=6								
1	-	-	-	-	-	-	.430	91.2
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	-	-	-	-	-	-	.400	
6	6.52	8.00	1.00	1.00	2.00	.390	.390	
							2.460	

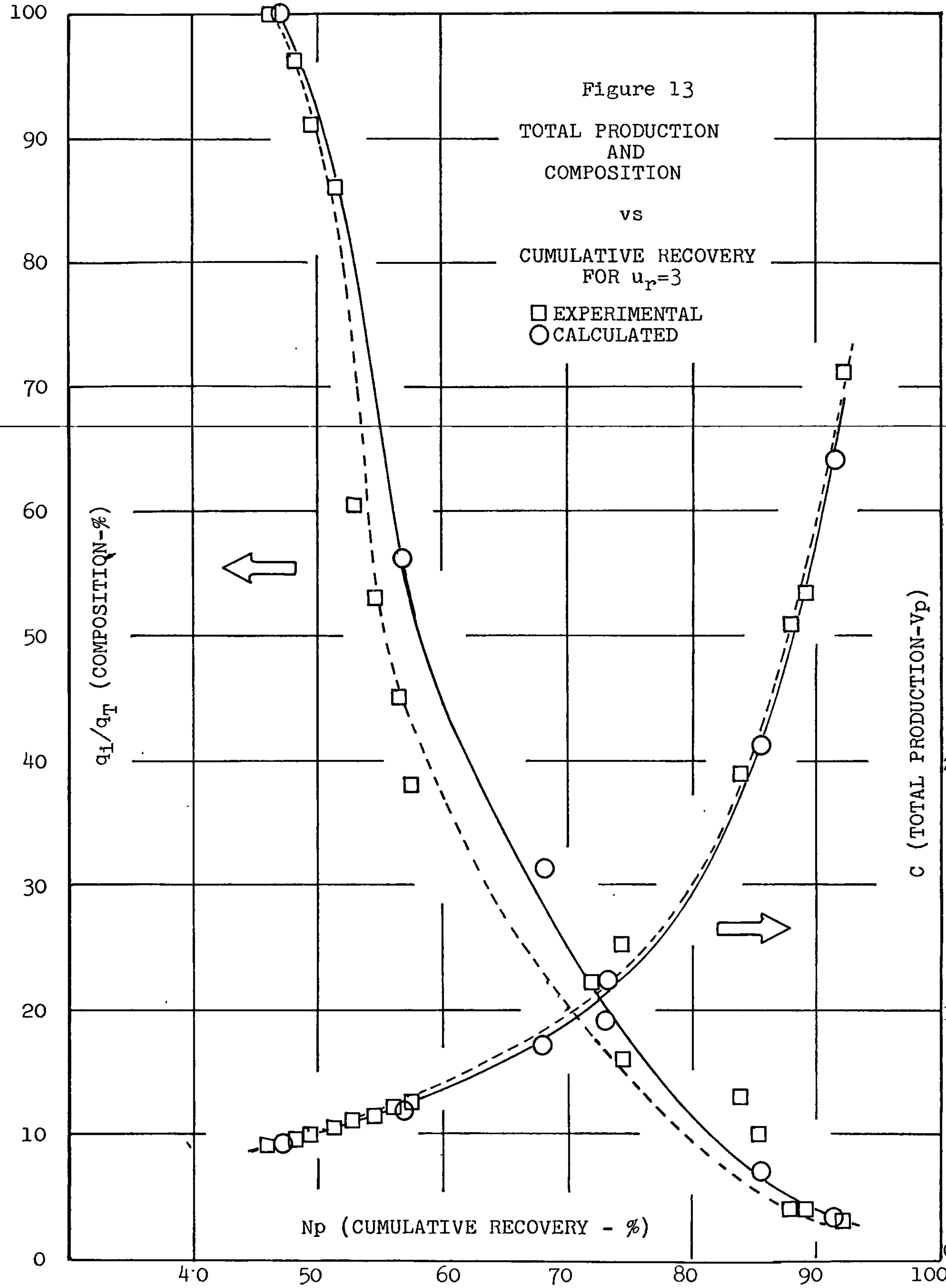


Figure 13
TOTAL PRODUCTION
AND
COMPOSITION
vs
CUMULATIVE RECOVERY
FOR u_r=3
EXPERIMENTAL
CALCULATED
q_i/q_T (COMPOSITION-%)
C (TOTAL PRODUCTION-Vp)
Np (CUMULATIVE RECOVERY - %)
100
90
80
70
60
50
40
30
20
10
0
40
50
60
70
80
90
100
3
2
1
0

TABLE 8

CALCULATION OF COMPOSITION FOR $u_r = 3$

$$(q_1/q_T)_m = 1 - \sum_{j=1}^{m} k_{j-1} \Big/ \sum_{j=1}^{n} k_j \Big/ \sqrt{u_r^2 + k_j \bar{m}_{m-1}(1-u_r^2)/\bar{m}_j k_{m-1}}$$

(1) j	(2) k_j	(3) $\sqrt{u_r^2+k_j\bar{m}_{m-1}(1-u_r^2)/\bar{m}_j k_{m-1}}$	(4) (2)/(3)	(5) $\sum_{j=1}^{m} k_{j-1}$	(6) (5)/Σ(4)	(7) $(q_1/q_T)_m$ (%)
m=1						100.0
m=2						
1	12.7	1.00	12.7	12.7	.44	56.0
2	10.4	1.52	6.83			
3	7.8	1.97	3.95			
4	6.62	2.15	3.09			
5	3.78	2.54	1.48			
6	2.55	2.69	3.95			
			28.99			
m=3						
1	12.7	-	12.7	12.7	.688	31.2
2	10.4	1.00	10.4	10.4		
				23.1		
3	7.8	1.69	4.62			
4	6.62	1.94	3.41			
5	3.78	2.44	1.54			
6	2.55	2.62	.93			
			33.65			

TABLE 8 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	-	12.7	12.7	.811	18.9
2	10.4	-	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	1.47	4.5	30.9		
5	3.78	2.24	1.69			
6	2.55	2.50	1.02			
			33.11			
m=5						
1	12.7	-	12.7	12.7	.932	6.8
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	1.00	6.62	6.62		
5	3.78	2.07	1.82	37.52		
6	2.55	2.40	1.06			
			40.40			
m=6						
1	12.7	-	12.7	12.7	96.8	3.2
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	1.87	1.36	41.30		
			42.66			

TABLE 9

CALCULATION OF TOTAL PRODUCTION FOR $u_T = 3$

$$C = Np_1/(q_1/q_T)_1 + \sum_{m=2}^{n} 2(np_m - Np_{m-1})/\ (q_1/q_T)_m + (q_1/q_T)_{m-1}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.470	-	-	-	-	-	-	.470
2	.567	.470	.097	.56	1.00	.780	.124	.594
3	.679	.567	.112	.312	.560	.436	.257	.851
4	.730	.679	.051	.189	.312	.2005	.254	1.105
5	.852	.730	.122	.068	.189	.1285	.945	2.050
6	.912	.852	.060	.032	.068	.0500	1.200	3.200

Immiscible Displacement

When oil is displaced by water in a porous media, there is a residual oil saturation due to the immiscibility of the two fluids. The fraction recovery of the oil in-place is then easily calculated - if this residual oil saturation is known. This, however, is easily calculated. But the problem that presents itself is determining the amount of interstitial water that is displaced by the advancing flood. Investigations by Brown[18] show that the residual interstitial water saturation is chiefly dependent on the oil viscosity and column length. Therefore, following this relationship, a displacement efficiency (D_j) for the advance of the flood front in any layer may be roughly approximated under the following assumption: when the flood front displaces oil in a microscopic pore volume, the amount of interstitial water left behind will be the same amount when the water displaces a miscible fluid having the same viscosity as the oil. Thus, in the previous miscible displacement (u_r=3), about 90 per cent of the in-place fluid was displaced leaving behind a 10 per cent residual saturation. Then the displacement efficiency for an oil-water u_r=3.63 will be:

$$D_j = 1\text{-}Sor_j\text{-}Sw_j\text{+}Sw_j\text{-}1\text{+}D$$

where D=10 per cent residual saturation, (1-Sor-Sw) is the fraction of oil displaced and (Sw-1+D) is the fraction of interstitial water displaced. Thus,

$$Dj = D - Sor_j$$

In determining the mobility ratio (M), the relative permeability to oil and the relative permeability to water were assumed to be equal (in this case, the assumption was verified; the relative permeabilities of the model, when all oil or all water was flowing, were almost identical).

Fig. 14, Page 99, shows the comparison between experimental data and calculated values for recovery vs composition and total production. The recovery at breakthrough was 44.5 per cent of the total recoverable oil; the calculated recovery was 47.2 per cent. At higher recoveries there is a slight discrepency between the calculated and experimental values. This was due to the red oil clinging to the surfaces of the oil-wet plexi-glass model. Thus, not only was the recovery reduced by the added residual oil saturation, but also the displacing front of water was obscured at breakthrough. Another interesting observation was the effects of capillary imbibition of water into the tighter layers. This imbibition added to the increase in recovery, especially at breakthrough. It can be noticed from the cumulative

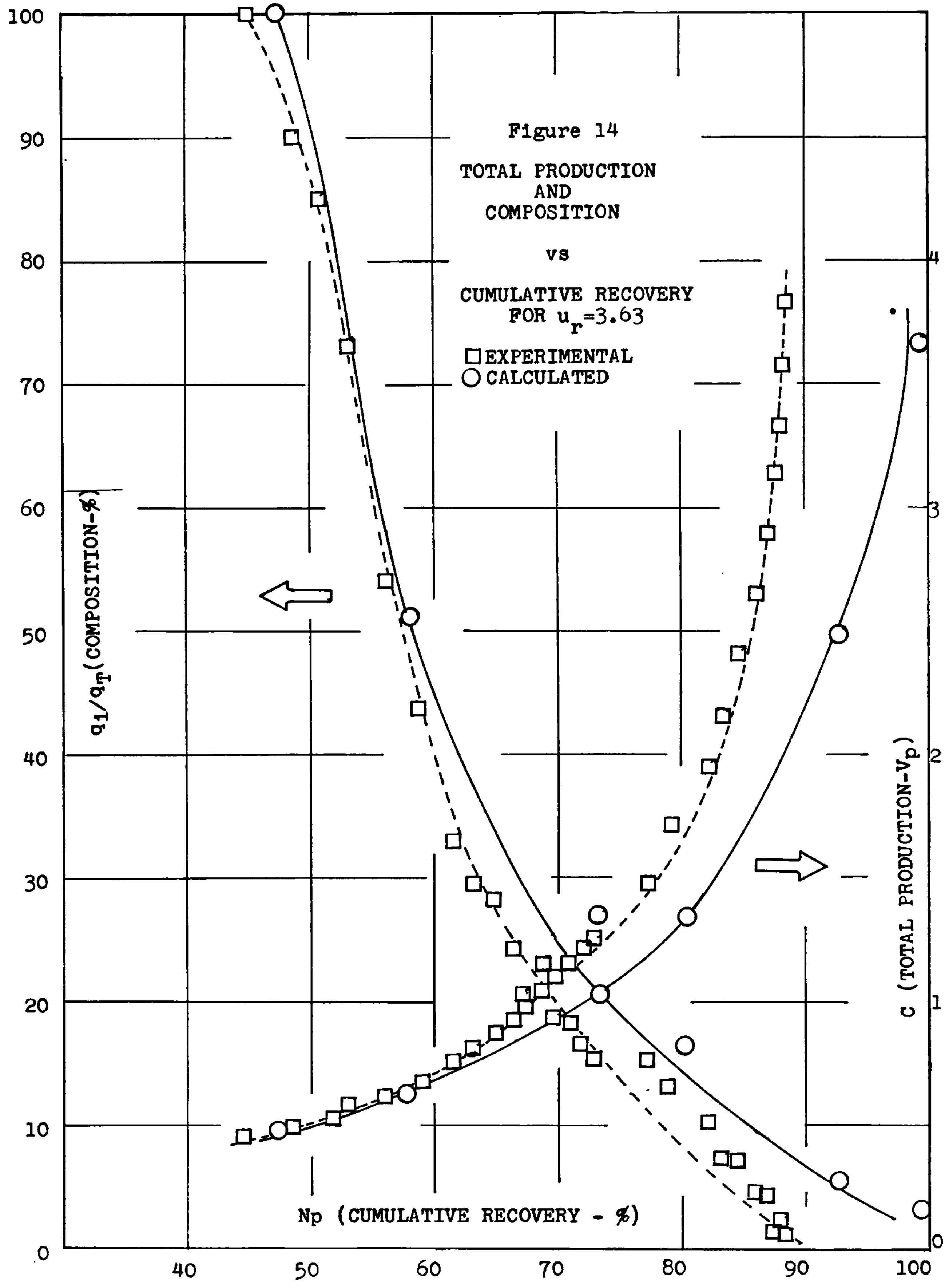


Figure 14
TOTAL PRODUCTION AND COMPOSITION vs CUMULATIVE RECOVERY FOR u_r=3.63
EXPERIMENTAL
CALCULATED
q_1/q_T(COMPOSITION-%)
C (TOTAL PRODUCTION-V_p)
Np (CUMULATIVE RECOVERY - %)
0
10
20
30
40
50
60
70
80
90
100
40
50
60
70
80
90
100
0
1
2
3
4

recovery graph that the differences between the experimental and calculated values is almost a constant for compositions below 40 per cent; above this value, the two curves seem to coincide. If there were no capillary effects the two curves would be in juxtaposition to each other at an equal interval for the entire recovery curve. Hence, the recovery due to imbibition of the water into tighter layers, may be visualized.

Calculations of recovery, composition and total production are shown in Tables 11, 12 and 13, pages 102, 103, 104, 105 and 106.

TABLE 10

DETERMINATION OF DISPLACEMENT EFFICIENCY

FOR $u_r = 3.63$

$D_j = D\text{-}Sor_j$

(1) j	(2) ϕ_j	(3) Sw_j	(4) Sor_j	(5) $\phi_j(1-Sw_j-Sor_j)$	(6) $\phi(1-Sw_j)$	(7) $D\text{-}Sor_j$
1	.43	.19	.25	.242	.348	.65
2	.42	.21	.22	.239	.332	.68
3	.41	.23	.18	.242	.315	.72
4	.41	.23	.17	.246	.315	.73
5	.40	.28.5	.15	.226	.286	.75
6	.39	.30.4	.14	.216	.272	.76
				1.411	1.868	

TABLE 11

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 3.63$

$$Np_m = \frac{\sum_{j=1}^{m-1} \bar{a}_j(1-Sw_j-Sor_j) + \dfrac{\sum_{j=m}^{n} \bar{a}_j(1-Sw_j-Sor_j)\left(u_r - \sqrt{u_r^2 + k_j\bar{a}_m D_m(1-u_r^2)/\bar{a}_j D_j k_m}\right)}{(u_r-1)}}{\sum_{j=1}^{n} \bar{a}_j(1-Sw_j-Sor_j)}$$

DATA: $u_r=3.63$, $u_r^2 = 13.2$, $\sum_{j=1}^{n} \bar{a}_j(1-Sw_j-Sor_j)=1.411$ and $c = k_j\bar{a}_m D_m(1-u_r^2)/\bar{a}_j D_j k_m$

(1) j	(2) $k_j/\bar{a}_j D_j$	(3) c	(4) u_r^2-c	(5) $\sqrt{(4)}$	(6) $u_r-(5)$	(7) $\dfrac{\bar{a}_j(j-Sw_j-Sor_j)(6)}{(u_r-1)}$	(8) $\sum_{j=1}^{m-1}\bar{a}_j(1-Sw_j-Sor_j)+\sum_{j=m}^{n}(7)$	(9) $(8)/\sum_{j=1}^{n}\bar{a}_j(1-Sw_j-Sor_j)$
1	45.4	12.20	1.00	1.00	2.63	.242	.242	47.2
2	36.3	9.82	3.38	1.84	1.79	.162	.162	
3	26.4	7.12	6.08	2.46	1.17	.107	.107	
4	22.2	5.96	7.24	2.69	.94	.088	.088	
5	12.6	3.38	9.82	3.13	.50	.043	.043	
6	8.6	2.31	10.89	3.30	.33	.027	.027	
							.669	
m=2								
1	-	-	-	-	-	-	.242	57.8
2	36.6	12.20	1.00	1.00	2.63	.239	.239	
3	26.4	8.79	4.41	2.10	1.53	.140	.140	
4	22.2	7.38	5.82	2.42	.121	.113	.113	
5	12.6	4.19	9.01	3.00	.63	.054	.054	
6	8.6	2.86	10.34	3.21	.42	.034	.034	
							.812	
m=3								
1	-	-	-	-	-	-	.242	73.3
2	-	-	-	-	-	-	.239	
3	26.4	12.20	1.00	1.00	2.63	.242	.242	
4	22.2	10.25	2.95	1.74	1.89	.179	.179	
5	12.6	5.83	7.37	2.71	.92	.079	.079	
6	8.6	3.97	9.23	3.04	.59	.043	.043	
							1.029	

TABLE 11 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.242	80.2
2	-	-	-	-	-	-	.239	
3	-	-	-	-	-	-	.242	
4	22.2	12.20	1.00	1.00	2.63	.246	.246	
5	12.6	6.92	6.28	2.50	1.13	.097	.097	
6	8.6	4.73	8.47	2.81	.82	.067	.067	
							1.133	
m=5								
1	-	-	-	-	-	-	.242	92.9
2	-	-	-	-	-	-	.239	
3	-	-	-	-	-	-	.242	
4	-	-	-	-	-	-	.246	
5	12.6	12.20	1.00	1.00	2.63	.226	.226	
6	8.6	8.32	4.88	2.21	1.41	.116	.116	
							1.311	
m=6								
1	-	-	-	-	-	-	.242	100.0
2	-	-	-	-	-	-	.239	
3	-	-	-	-	-	-	.242	
4	-	-	-	-	-	-	.246	
5	-	-	-	-	-	-	.226	
6	8.6	12.20	1.00	1.00	2.63	.216	.216	
							1.411	

TABLE 12

CALCULATION OF COMPOSITION FOR $u_r = 3.63$

$$(q_i/q_T)_m = 1 - \sum_{j=1}^{m} k_{j-1} / \sum_{j=1}^{n} k_j / \sqrt{u_r^2 + k_j \bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$$

(1) j	(2) k_j	(3) $\sqrt{u_r^2+k_j\bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$	(4) (2)/(3)	(5) $\sum_{j=1}^{m} k_{j-1}$	(6) (5)/$\sum$(4)	(7) $(q_i/q_T)_m$ (%)
m=1						100.00
m=2						
1	12.7	1.00	12.7	12.7	.49	51.0
2	10.4	1.84	5.66			
3	7.8	2.46	3.17			
4	6.62	2.69	2.46			
5	3.78	3.13	1.21			
6	2.55	3.30	.77			
			25.97			
m=3						
1	12.7	-	12.7	12.7	.732	26.8
2	10.4	1.00	10.4	10.4		
3	7.8	2.10	3.72	23.10		
4	6.62	2.42	2.74			
5	3.78	3.00	1.26			
6	2.55	3.21	.79			
			31.61			

TABLE 12 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	-	12.7	12.7	.838	16.2
2	10.4	-	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	1.74	3.81	30.9		
5	3.78	2.71	1.43			
6	2.55	3.04	.84			
			36.90			
m=5						
1	12.7	-	12.7	12.7	.945	5.5
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	1.00	6.62	6.62		
5	3.78	2.50	1.51	37.52		
6	2.55	2.81	.90			
			39.93			
m=6						
1	12.7	-	12.7	12.7	.97	3.0
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	2.21	1.15	41.30		
			42.45			

TABLE 13

CALCULATION OF TOTAL PRODUCTION FOR $u_r = 3.63$

$$C = Np_1/(q_1/q_T)_1 + \sum_{m=2}^{n} 2(Np_m - Np_{m-1})/(q_1/q_T)_m + (q_1/q_T)_{m-1}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.472	-	-	1.00	-	-	-	.472
2	.578	.472	.106	.51	1.00	.755	.140	.612
3	.733	.578	.155	.268	.51	.389	.400	1.012
4	.802	.733	.069	.162	.268	.215	.320	1.392
5	.929	.802	.127	.055	.162	.1085	1.162	2.494
6	1.000	.929	.071	.030	.055	.0425	1.165	3.659

VISCOSITY RATIOS LESS THAN ONE

When a more viscous fluid displaces a less viscous fluid there is a crossflow from the higher permeable beds to the lower permeable beds. This crossflow of displacing fluid will increase the coverage and thus increase the recovery at breakthrough. In the three displacements where the viscosity ratio is less than one, the calculated values differ from the experimental results. For a viscosity ratio equal to one-half, the measured recovery was 77 per cent (see Fig. 15, Page 110), while the calculated recovery at breakthrough was only 66.4 per cent (Table 14, Pages 114, 115). For a viscosity ratio of one-third, the difference was still greater; the values of the recovery at breakthrough for the experimental and the theoretical calculations are 86.2 per cent and 68 per cent respectively. Even for the immiscible displacement, where oil is displacing water, the discrepency is very large. After each displacement the model was thoroughly checked for any signs of sand compaction, and none was found.

The experimental and calculated data were then compared with other results reported in literature. In Fig. 18, Page 113, five curves were drawn showing the relationship between the mobility ratio and per cent recovered. It is noticed that the curves are in excellent agreement in the region of unit mobility ratio, but deviate at both high and low mobility ratios. The reason for his deviation is the differences in the experimental systems. In the potentiometric[19] and the electrical resistance network[20] models there is no tortuous flow through porous medium. In the x-ray Shadowgraph[21,22] techniques, actual porous models of unconsolidated sands were used. Now, the last curve is the result of a consolidated sand model. It may be noticed that the experimental data follows the same trend as the potentiometric, resistance network and x-ray Shadowgraph curves, while the calculated results are in line with the curve from the consolidated model. This brings up the question of whether there is a difference in displacements between consolidated and unconsolidated sands in the laboratory. If there is a difference, then why do the experimental and calculated results agree so well for mobility ratios greater than one? One answer is that for mobility ratios greater than one the displacing fluid and not the in-place fluid is crossflowing from the least permeable bed to the more permeable bed, and the amount of coverage that is developed is certainly less than the resulting coverage when the mobility ratio is less than one; in this case, the more viscous fluid is crossflowing from the more permeable to the least permeable beds. Thus if high recoveries are encountered, it can only imply that the displacing viscous fluid is not obeying Darcy's law, i.e., there is no tortuous flow through the porous medium. Experiments involving the comparison of consolidated and unconsolidated sands should be undertaken in the laboratory. This is probably the only way to determine if such a difference exists in a viscous displacement.

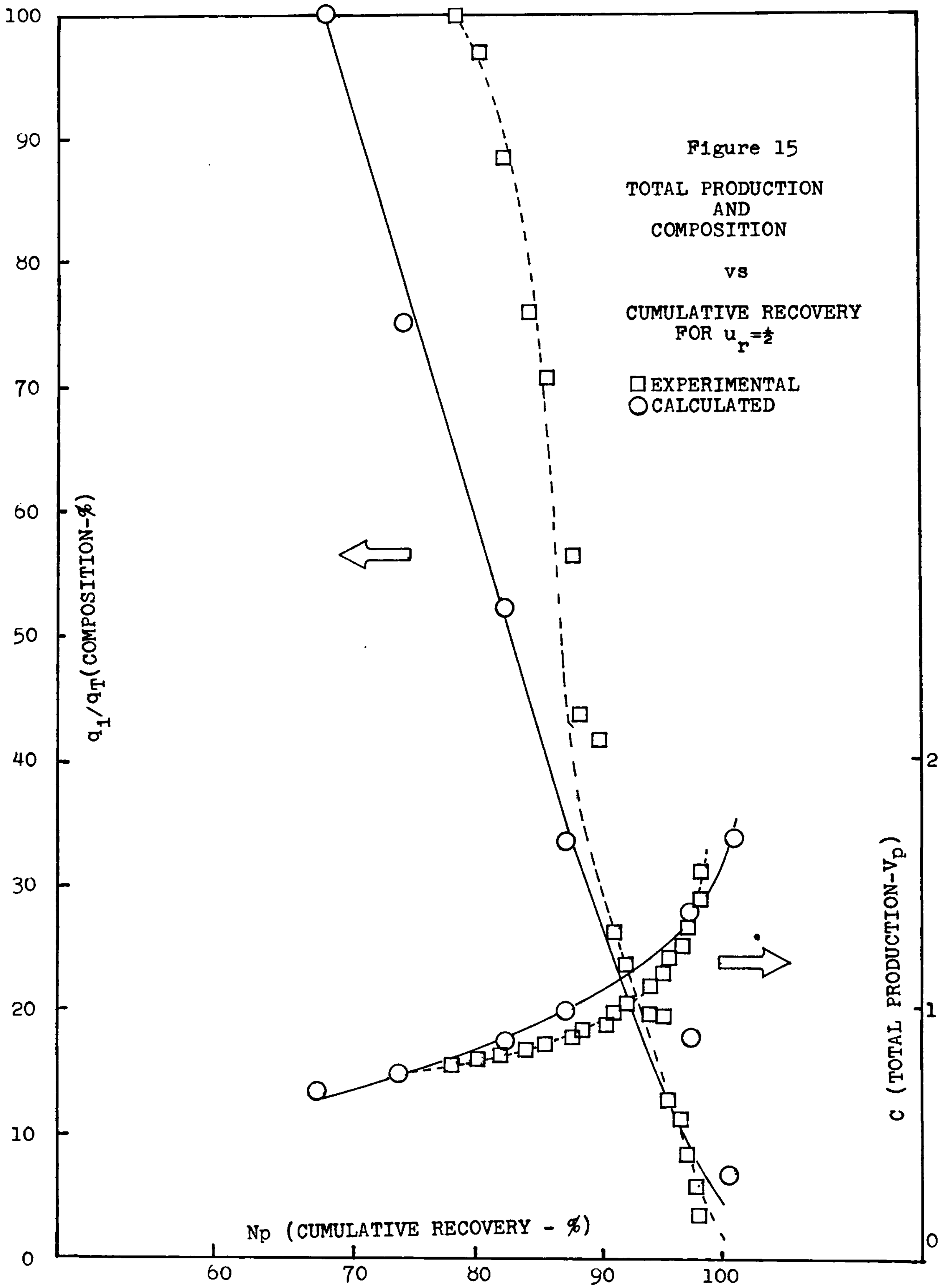


Figure 15
TOTAL PRODUCTION
AND
COMPOSITION
vs
CUMULATIVE RECOVERY
FOR $u_r = \frac{1}{2}$
EXPERIMENTAL
CALCULATED
q_1/q_T (COMPOSITION-%)
C (TOTAL PRODUCTION-V_p)
Np (CUMULATIVE RECOVERY - %)
100
90
80
70
60
50
40
30
20
10
0
60
70
80
90
100
2
1
0

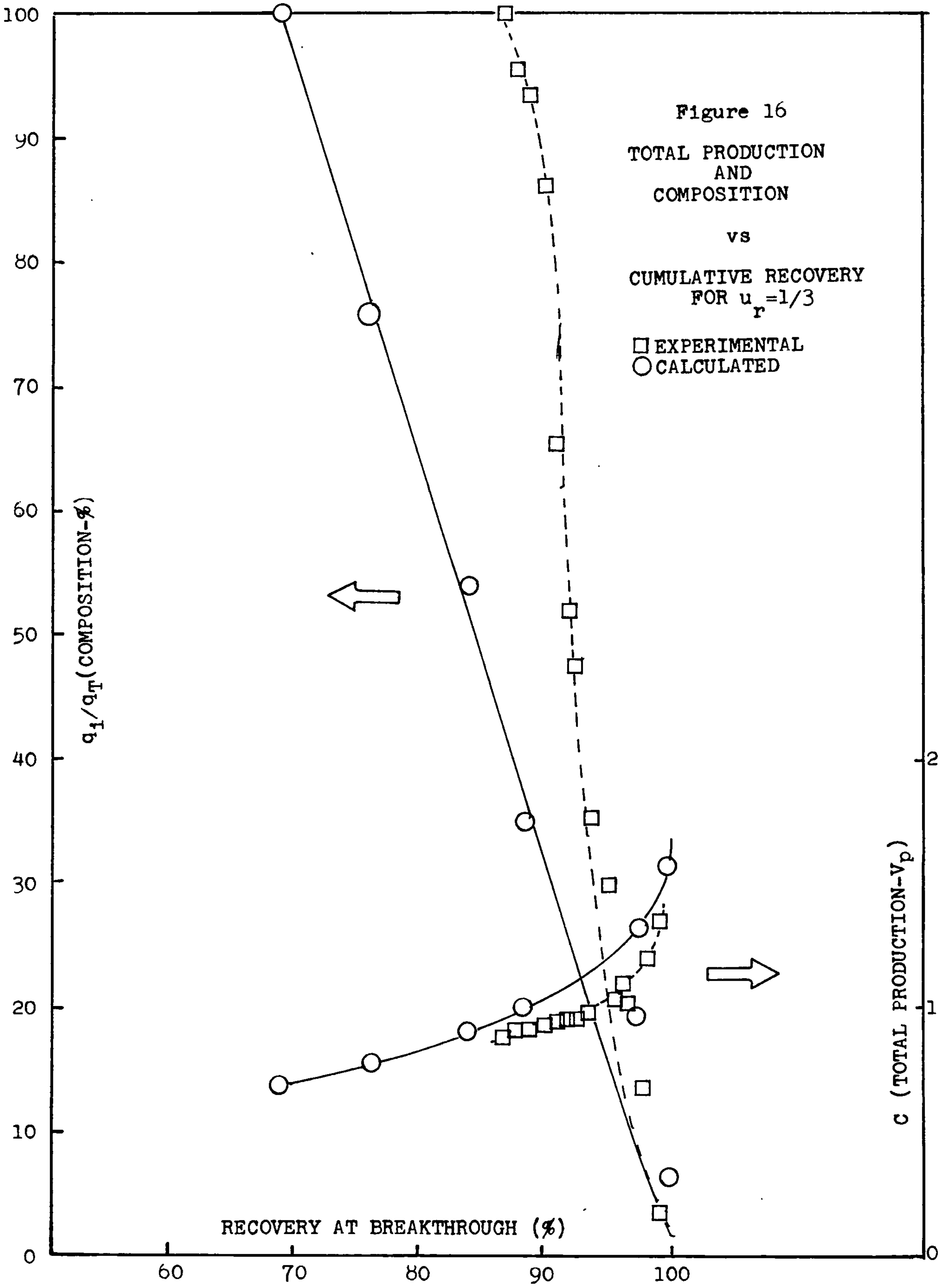


Figure 16
TOTAL PRODUCTION AND COMPOSITION vs CUMULATIVE RECOVERY FOR u_r=1/3
EXPERIMENTAL
CALCULATED
q_1/q_T(COMPOSITION-%)
C (TOTAL PRODUCTION-V_p)
RECOVERY AT BREAKTHROUGH (%)
0
10
20
30
40
50
60
70
80
90
100
60
70
80
90
100
0
1
2

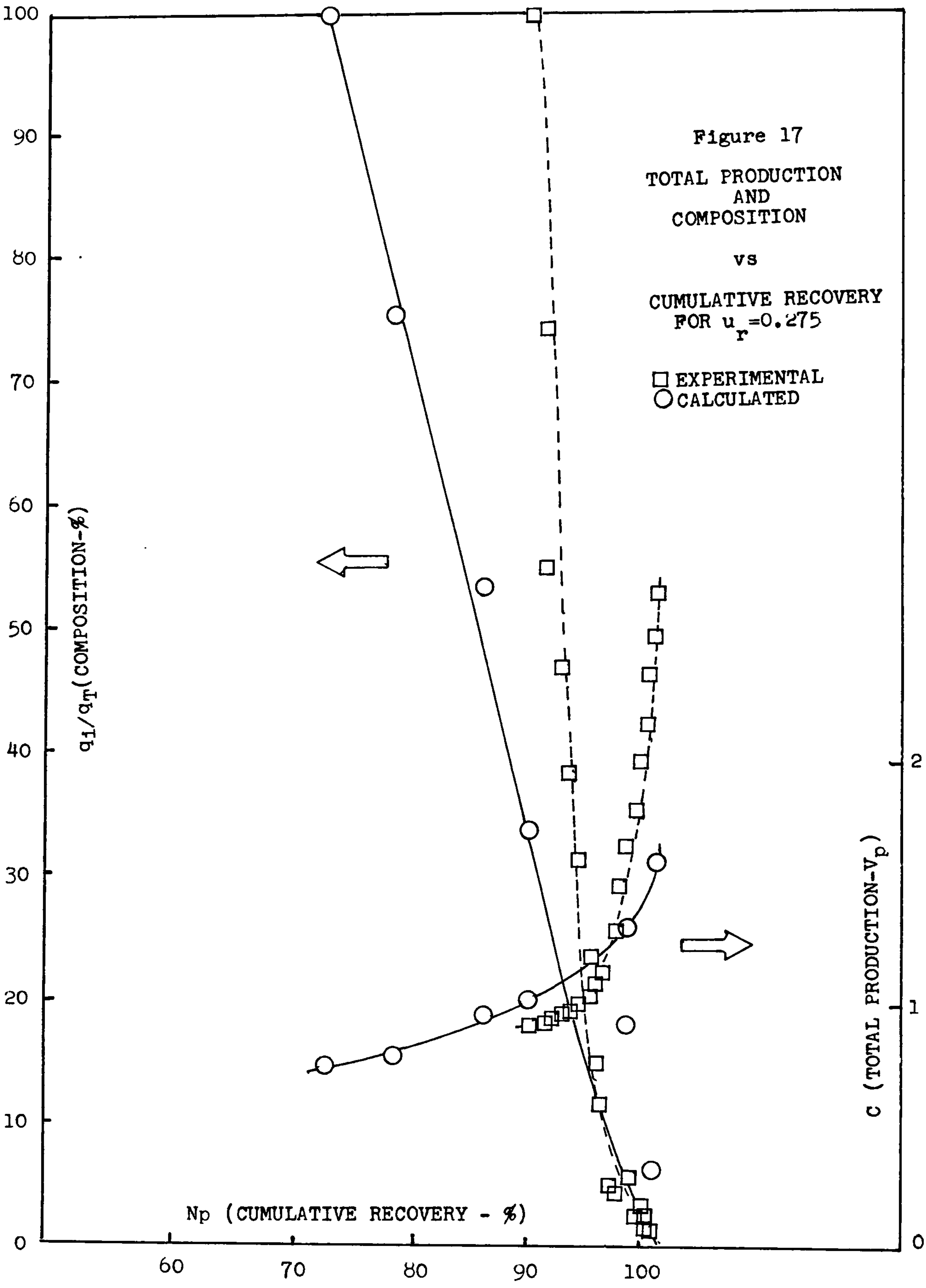


Figure 17
TOTAL PRODUCTION AND COMPOSITION
vs
CUMULATIVE RECOVERY FOR u_r=0.275
EXPERIMENTAL
CALCULATED
q_1/q_T(COMPOSITION-%)
C (TOTAL PRODUCTION-V_p)
Np (CUMULATIVE RECOVERY - %)
100
90
80
70
60
50
40
30
20
10
0
60
70
80
90
100
2
1
0

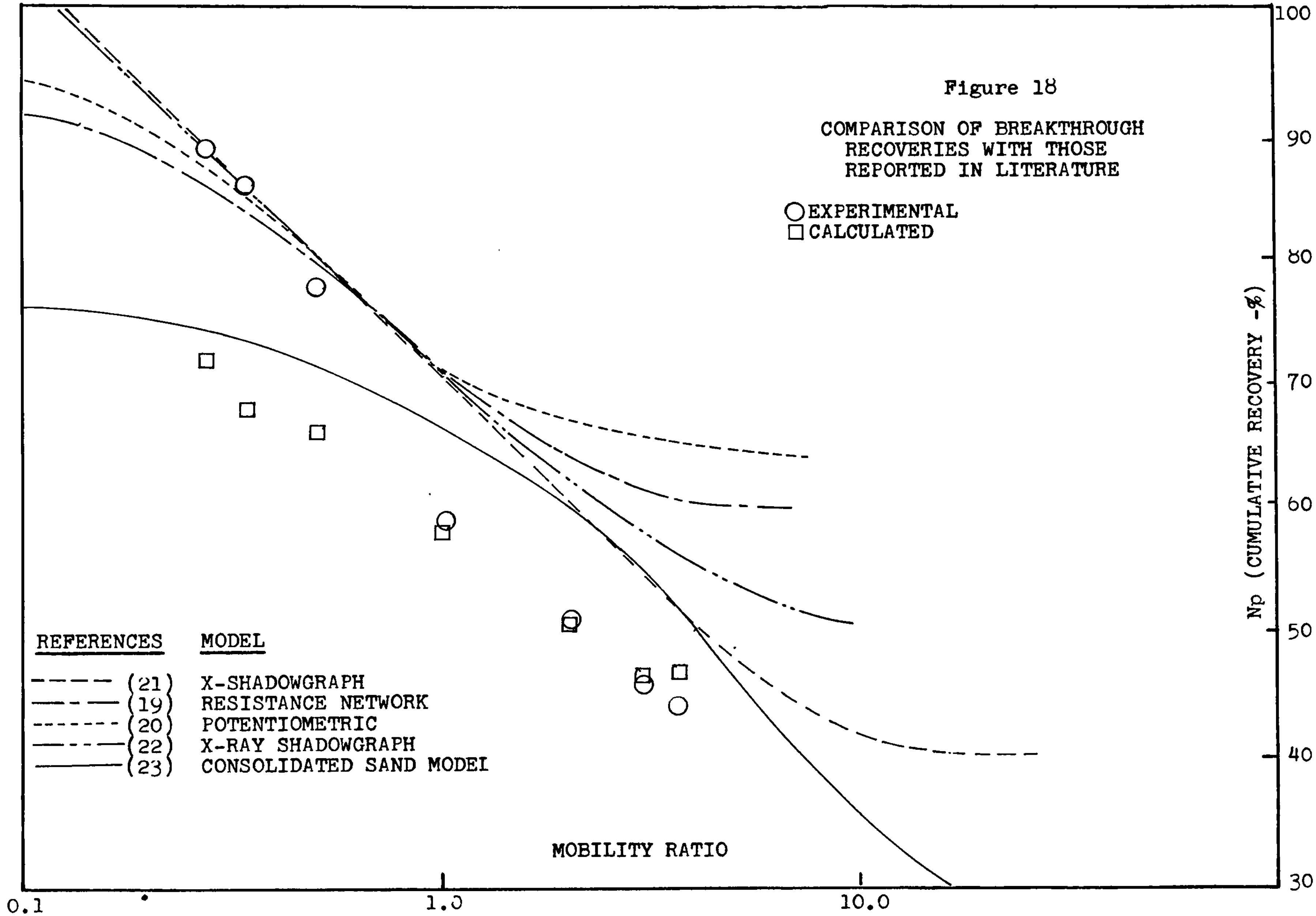


Figure 18
COMPARISON OF BREAKTHROUGH RECOVERIES WITH THOSE REPORTED IN LITERATURE
EXPERIMENTAL
CALCULATED
REFERENCES
MODEL
(21) X-SHADOWGRAPH
(19) RESISTANCE NETWORK
(20) POTENTIOMETRIC
(22) X-RAY SHADOWGRAPH
(23) CONSOLIDATED SAND MODEL
MOBILITY RATIO
0.1
1.0
10.0
Np (CUMULATIVE RECOVERY -%)
100
90
80
70
60
50
40
30

TABLE 14

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 0.5$

$$Np_m = \frac{\left\{ \sum_{j=1}^{m-1} \bar{a}_j + \frac{\sum_{j=m}^{n} \bar{a}_j \left(u_r - \sqrt{u_r^2 + k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m}\right)}{(u_r - 1)} \right\} D}{\sum_{j=1}^{n} \bar{a}_j}$$

DATA: $u_r = 0.5$, $n = 6$, $D = 1.000$, $\sum_{j=1}^{n} \bar{a}_j = 2.46$ and $c = k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m$

(1) j	(2) $k_j/\bar{a}_j$	(3) c	(4) $u_r^2 + c$	(5) $\sqrt{(4)}$	(6) $u_r - (5)$	(7) $\frac{(6)\bar{a}_j}{u_r - 1}$	(8) $\sum_{j=1}^{m-1} \bar{a}_j + \sum_{j=m}^{n} (7)$	(9) $D(8)/\sum_{j=1}^{n} \bar{a}_j$ (%)
1	29.5	.75	1.000	1.000	.500	.430	.430	66.4
2	24.8	.63	.88	.939	.439	.368	.368	
3	19.0	.483	.733	.856	.356	.292	.292	
4	16.2	.412	.671	.820	.320	.262	.262	
5	9.45	.240	.490	.700	.200	.160	.160	
6	6.52	.166	.416	.645	.145	.112	.112	
							1.624	
m=2								
1	-	-	-	-	-	-	.430	72.9
2	24.8	.75	1.000	1.000	.500	.420	.420	
3	19.0	.574	.824	.907	.407	.334	.334	
4	16.2	.490	.740	.860	.360	.296	.296	
5	9.45	.286	.536	.732	.232	.186	.186	
6	6.52	.197	.447	.668	.168	.130	.130	
							1.796	
m=3								
1	-	-	-	-	-	-	.430	81.2
2	-	-	-	-	-	-	.420	
3	19.0	.75	1.000	1.000	.500	.410	.410	
4	16.2	.64	.890	.944	.444	.364	.364	
5	9.45	.373	.623	.790	.290	.232	.232	
6	6.52	.258	.508	.712	.212	.164	.164	
							2.020	

TABLE 14 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.430	86.2
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	16.2	.75	1.000	1.000	.500	.410	.410	
5	9.45	.436	.686	.328	.328	.262	.262	
6	6.52	.302	.552	.743	.243	.190	.190	
							2.122	
m=5								
1	-	-	-	-	-	-	.430	96.2
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	9.45	.75	1.00	1.000	.500	.400	.400	
6	6.52	.518	.768	.876	.376	.292	.292	
							2.362	
m=6								
1	-	-	-	-	-	-	.430	100.0
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	-	-	-	-	-	-	.400	
6	6.52	.75	1.00	1.00	.500	.390	.390	
							2.460	

TABLE 15

CALCULATION OF COMPOSITION FOR $u_r = 0.5$

$$(q_i/q_T)_m = 1 - \sum_{j=1}^{m} k_{j-1} / \sum_{j=1}^{n} k_j / \sqrt{u_r^2 + k_j \bar{m}_{m-1}(1-u_r^2)/\bar{m}_j k_{m-1}}$$

(1) j	(2) k_j	(3) $\sqrt{u_r^2+k_j\bar{m}_{m-1}(1-u_r^2)/\bar{m}_j k_{m-1}}$	(4) (2)/(3)	(5) $\sum_{j=1}^{m} k_{j-1}$	(6) $(5)/\sum(4)$	(7) $(q_i/q_T)_m$ (%)
m=1						100.0
m=2						
1	12.7	1.00	12.7	12.7	25.2	74.8
2	10.4	.939	11.1			
3	7.8	.856	9.1			
4	6.62	.819	8.11			
5	3.78	.700	5.39			
6	2.55	.644	3.96			
			50.36			
m=3						
1	12.7	1.00	12.7	12.7	48.5	51.5
2	10.4	1.00	10.4	10.4		
3	7.8	.907	8.6	23.1		
4	6.62	.877	7.55			
5	3.78	.732	5.14			
6	2.55	.668	3.83			
			47.52			

TABLE 15 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	1.00	12.7	12.7	66.8	33.2
2	10.4	1.00	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	.944	7.01	30.9		
5	3.78	.790	4.78			
6	2.55	.712	3.58			
			46.27			
m=5						
1	12.7	1.00	12.7	12.7	82.3	17.7
2	10.4	1.00	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	1.00	6.62	6.62		
5	3.78	.828	4.57	37.52		
6	2.55	.742	3.43			
			45.52			
m=6						
1	12.7	-	12.7	12.7	.923	7.7
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	.768	3.33	41.30		
			44.63			

TABLE 16

CALCULATION OF TOTAL PRODUCTION FOR $u_r = \frac{1}{2}$

$$C = Np_1/(q_1/q_T)_m + \sum_{m=2}^{n} 2(Np_m - Np_{m-1})/ (q_1/q_T)_m + (q_1/q_T)_{m-1}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.664	-	-	1.00	-	-	-	.664
2	.729	.664	.065	.748	1.00	.874	.0742	.7382
3	.812	.729	.083	.515	.748	.632	.1310	.8692
4	.862	.812	.050	.332	.254	.392	.1175	.9376
5	.962	.862	.100	.177	.332	.254	.392	1.3737
6	1.000	.962	.038	.077	.177	.127	.299	1.6777

TABLE 17

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 0.275$

$$Np_m = \frac{\sum_{j=1}^{m-1} \bar{\Phi}_j(1-Sw_j) + \sum_{j=m}^{n} \dfrac{(1-Sw_j)\left(u_r - \sqrt{u_r^2 + k_j\bar{\Phi}_m D_m(1-u_r^2)/\bar{\Phi}_j D_j k_m}\right)}{(u_r-1)}}{\sum_{j=1}^{n} \bar{\Phi}_j(1-Sw_j)}$$

DATA: $u_r = 0.275$, $u_r^2 = 0.0756$, $\sum_{j=1}^{n} \bar{\Phi}_j(1-Sw_j) = 1.868$, $D_j = (1-Sw_j)$ and $c = k_j\bar{\Phi}_m D_m(1-u_r^2)/\bar{\Phi}_j D_j k_m$

(1) j	(2) $k_j/\bar{\Phi}_j D_j$	(3) c	(4) $u_r^2 + c$	(5) $\sqrt{(4)}$	(6) $u_r-(5)$	(7) $\bar{\Phi}_j(1-Sw_j)(6)/(u_r-1)$	(8) $\sum_{j=1}^{m-1}\bar{\Phi}_j(1-Sw_j)+\sum_{j=m}^{n}(7)$	(9) $(8)/\sum_{j=1}^{n}\bar{\Phi}_j(1-Sw_j)$
1	36.5	.9244	1.000	1.000	.725	.348	.348	73.2
2	31.4	.7950	.8706	.939	.664	.305	.305	
3	24.7	.6230	.6986	.834	.559	.243	.243	
4	21.0	.5310	.6066	.778	.503	.219	.219	
5	13.2	.3340	.4096	.639	.364	.143	.143	
6	9.35	.2360	.3116	.557	.282	.105	.105	
							1.363	
m=2								
1	-	-	-	-	-	-	.348	78.6
2	31.4	.9244	1.000	1.000	.725	.332	.332	
3	24.7	.7290	.8046	.896	.621	.270	.270	
4	21.0	.6200	.6956	.835	.560	.243	.243	
5	13.2	.3900	.4656	.681	.406	.160	.160	
6	9.35	.2750	.3500	.592	.317	.119	.119	
							1.472	
m=3								
1	-	-	-	-	-	-	.348	85.9
2	-	-	-	-	-	-	.332	
3	24.7	.9244	1.000	1.000	.725	.315	.315	
4	21.0	.7840	.8596	.926	.651	.283	.283	
5	13.2	.4940	.5696	.753	.478	.183	.188	
6	9.35	.2500	.4256	.652	.377	.141	.141	
							1.607	

TABLE 17 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.348	89.9
2	-	-	-	-	-	-	.332	
3	-	-	-	-	-	-	.315	
4	21.0	.9244	1.000	1.000	.725	.315	.315	
5	13.2	.5800	.6556	.810	.535	.212	.212	
6	9.35	.4120	.4876	.696	.421	.157	.157	
							1.679	
m=5								
1	-	-	-	-	-	-	.348	97.0
2	-	-	-	-	-	-	.332	
3	-	-	-	-	-	-	.315	
4	-	-	-	-	-	-	.315	
5	13.2	.9244	1.000	1.000	.725	.286	.286	
6	9.35	.6550	.7306	.854	.579	.217	.217	
							1.813	
m=6								
1	-	-	-	-	-	-	.348	100.0
2	-	-	-	-	-	-	.332	
3	-	-	-	-	-	-	.315	
4	-	-	-	-	-	-	.315	
5	-	-	-	-	-	-	.286	
6	9.35	.9244	1.000	1.000	.725	.272	.272	
							1.868	

TABLE 18

CALCULATION OF COMPOSITION FOR $u_r = 0.275$

$$(q_i/q_T)_m = 1 - \sum_{j=1}^{m} k_{j-1} \Big/ \sum_{j=1}^{n} k_j \Big/ \sqrt{u_r^2 + k_j \bar{a}_{m-1} D_{m-1}(1-u_r^2)/\bar{a}_j D_j k_m}$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
j	k_j	$\sqrt{u_r^2+k_j\bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$	(2)/(3)	$\sum_{j=1}^{m} k_{j-1}$	(5)/$\sum$(4)	$(q_i/q_T)_m$ (%)
m=1						100.0
m=2						
1	12.7	1.000	12.7	12.7	.243	75.7
2	10.4	.939	11.1			
3	7.8	.834	9.35			
4	6.62	.778	8.52			
5	3.78	.639	5.92			
6	2.55	.557	4.58			
			52.17			
m=3						
1	12.7	-	12.7	12.7	.465	53.5
2	10.4	1.000	10.4	10.4		
3	7.8	.896	8.7	23.1		
4	6.62	.835	7.92			
5	3.78	.681	5.52			
6	2.55	.592	4.30			
			49.54			

TABLE 18 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	-	12.7	12.7	.660	33.0
2	10.4	-	10.4	10.4		
3	7.8	1.00	7.8	7.8		
4	6.62	.926	7.15	30.9		
5	3.78	.753	5.00			
6	2.55	.652	3.92			
			46.97			
m=5						
1	12.7	-	12.7	12.7	.822	17.8
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	1.00	6.62	6.62		
5	3.78	.810	4.65	37.52		
6	2.55	.696	3.63			
			45.80			
m=6						
1	12.7	-	12.7	12.7	.935	6.5
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	.854	2.98	41.30		
			44.28			

TABLE 19

CALCULATION OF TOTAL PRODUCTION FOR $u_r = 0.275$

$$C_m = \frac{Np_1}{(q_1/q_T)_m} + \sum_{m=1}^{n} \frac{2(Np_m - Np_{m-1})}{(q_1/q_T)_m + (q_1/q_T)_{m-1}}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.732	-	-	-	-	-	-	.732
2	.786	.732	.054	.757	1.000	.8785	.062	.794
3	.859	.786	.073	.535	.757	.646	.113	.907
4	.899	.859	.040	.330	.535	.4325	.093	1.000
5	.970	.899	.071	.178	.330	.254	.280	1.280
6	1.000	.970	.030	.065	.178	.1215	.247	1.527

TABLE 20

CALCULATION OF CUMULATIVE RECOVERY FOR $u_r = 1/3$

$$Np_m = \frac{\sum_{j=1}^{m-1} \bar{a}_j + \sum_{j=m}^{n} \dfrac{\bar{a}_j\left(u_r - \sqrt{u_r^2 + k_j \bar{a}_m (1-u_r^2)/\bar{a}_j k_m}\right)}{(u_r-1)}}{\sum_{j=1}^{n} \bar{a}_j}$$

DATA: $u_r=1/3$, $u_r^2=0.111$, $\sum_{j=1}^{n} \bar{a}_j=2.46$ and $c = k_j\bar{a}_m(1-u_r^2)/\bar{a}_j k_m$

(1) j	(2) $k_j/\bar{a}_j$	(3) c	(4) u_r^2+c	(5) $\sqrt{(4)}$	(6) $u_r-(5)$	(7) $\frac{(\bar{a}_j)_j\ (6)}{(u_r-1)}$	(8) $\sum_{j=1}^{m-1} \bar{a}_j + \sum_{j=m}^{n} (7)$	(9) $(8)/\sum_{j=1}^{n} \bar{a}_j$
1	29.5	.889	1.000	1.000	.667	.430	.430	68.0
2	24.8	.742	.853	.924	.581	.366	.366	
3	19.0	.571	.682	.826	.493	.304	.304	
4	16.2	.486	.597	.771	.438	.270	.270	
5	9.45	.283	.394	.627	.294	.177	.177	
6	6.52	.196	.307	.554	.221	.130	.130	
							1.677	
m=2								
1	-	-	-	-	-	-	.430	75.1
2	24.8	.889	1.000	1.000	.667	.420	.420	
3	19.0	.682	.793	.890	.567	.348	.348	
4	16.2	.580	.691	.832	.499	.305	.305	
5	9.45	.339	.450	.670	.337	.202	.202	
6	6.52	.234	.345	.587	.254	.148	.148	
							1.853	
m=3								
1	-	-	-	-	-	-	.430	83.9
2	-	-	-	-	-	-	.420	
3	19.0	.889	1.000	1.000	.667	.410	.410	
4	16.2	.758	.869	.930	.597	.366	.366	
5	9.45	.440	.551	.742	.409	.245	.245	
6	6.52	.304	.415	.644	.311	.182	.182	
							2.053	

TABLE 20 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
m=4								
1	-	-	-	-	-	-	.430	87.8
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	16.2	.889	1.000	1.000	.667	.410	.410	
5	9.45	.515	.626	.791	.458	.275	.275	
6	6.52	.358	.469	.684	.351	.205	.205	
							2.150	
m=5								
1	-	-	-	-	-	-	.430	96.9
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	9.45	.889	1.000	1.000	.667	.400	.400	
6	6.52	.612	.723	.850	.517	.300	.300	
							2.370	
m=6								
1	-	-	-	-	-	-	.430	100.0
2	-	-	-	-	-	-	.420	
3	-	-	-	-	-	-	.410	
4	-	-	-	-	-	-	.410	
5	-	-	-	-	-	-	.400	
6	6.52	.889	1.000	1.000	.667	.390	.390	
							2.460	

TABLE 21

CALCULATION OF COMPOSITION FOR $u_r = 1/3$

$$(q_i/q_T)_m = 1-\sum_{j=1}^{m} k_{j-1} \Big/ \sum_{j=1}^{n} k_j \Big/ \sqrt{u_r^2 + k_j \bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$$

(1) j	(2) k_j	(3) $\sqrt{u_r^2+k_j\bar{a}_{m-1}(1-u_r^2)/\bar{a}_j k_{m-1}}$	(4) (2)/(3)	(5) $\sum_{j=1}^{m} k_{j-1}$	(6) (5)/$\sum$(4)	(7) $(q_i/q_T)_m$ (%)
m=1						100.0
m=2						
1	12.7	1.000	12.7	12.7	.243	75.7
2	10.4	.924	11.2			
3	7.8	.826	9.45			
4	6.62	.771	8.61			
5	3.78	.627	6.03			
6	2.55	.554	4.60			
			52.59			
m=3						
1	12.7	-	12.7	12.7	.465	53.5
2	10.4	1.000	10.4	10.4		
3	7.8	.890	8.75	23.1		
4	6.62	.832	8.00			
5	3.78	.670	5.62			
6	2.55	.587	4.35			
			49.82			

TABLE 21 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
m=4						
1	12.7	-	12.7	12.7	.653	34.7
2	10.4	-	10.4	10.4		
3	7.8	1.000	7.8	7.8		
4	6.62	.930	7.12	30.9		
5	3.78	.742	5.08			
6	2.55	.644	3.96			
			47.06			
m=5						
1	12.7	-	12.7	12.7	.810	19.0
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	1.000	6.62	6.62		
5	3.78	.741	5.10	37.62		
6	2.55	.648	3.72			
			46.34			
m=6						
1	12.7	-	12.7	12.7	.935	6.5
2	10.4	-	10.4	10.4		
3	7.8	-	7.8	7.8		
4	6.62	-	6.62	6.62		
5	3.78	1.00	3.78	3.78		
6	2.55	.85	3.00	41.50		
			44.30			

TABLE 22

CALCULATION OF TOTAL PRODUCTION FOR $u_r = 1/3$

$$C_m = \frac{Np_1}{(q_1/q_T)_m} \sum_{j=m}^{n} \frac{2(Np_m - Np_{m-1})}{(q_1/q_T)_m + (q_1/q_T)_{m-1}}$$

(1) m	(2) Np_m	(3) Np_{m-1}	(4) (2)-(3)	(5) $(q_1/q_T)_m$	(6) $(q_1/q_T)_{m-1}$	(7) $\frac{(5)+(6)}{2}$	(8) (4)/(7)	(9) C
1	.68	-	-	-	-	-	-	.68
2	.751	.680	.071	.757	1.000	.8785	.080	.76
3	.839	.751	.088	.535	.757	.6460	.136	.896
4	.878	.839	.039	.347	.535	.441	.088	.984
5	.969	.878	.091	.190	.347	.2685	.340	1.324
6	1.000	.969	.031	.065	.190	.1275	.243	1.567

CHAPTER V

SUMMARY AND CONCLUSIONS

The most decisive variable influencing the displacement of one fluid by another is the viscosity ratio. Although the viscous forces are important, they alone cannot predict the displacement history of a reservoir; there are other factors that must be taken into consideration. In the previous methods, there was only a correlation between the variation in vertical permeability of a reservoir and the viscosity ratios of the driving fluid and the in-place fluid; in the derivations presented in this investigation, the effects of a varying porosity and displacement efficiency, are taken into account. By means of the displacement efficiency, a method is given for approximating the recovery after a particular layer has broken through; this alleviates the assumption of a piston-like displacement. It should be pointed out that this correction was not applied to the experiment since the displacement efficiency was always greater than 90 per cent for the viscosity ratios used.

From this investigation of fluid displacement in stratified sands the following conclusions are reached:

1. The rate of advance of a flood front is dependent not only on its permeability but also upon its porosity and displacement efficiency.

2. When the viscosity ratio is equal to unity, the

fractional flow rate in any layer will depend only on its relative capacity; for other viscosity ratios, the fractional flow rate will depend not only on its relative capacity but also on the relative distance of the flood front in the reservoir.

3. For a viscosity ratio greater than one, there is a crossflow of the displacing fluid from the less permeable to the more permeable bed, and conversely, for a viscosity ratio less than one there is a crossflow of displacing fluid from the more permeable to the less permeable bed.

4. In an immiscible displacement, when water is displacing oil, there is an imbibition of water from the more permeable to the less permeable bed. Another interesting observation noticed in this displacement was in the transfer of residual oil from the tighter beds to the more porous beds after the displacement had been terminated. This phenomenon of capillary imbibition had been completely neglected in this investigation. The reason being that there is still no way possible to express capillary imbibition in an immiscible displacement.[24]

5. The Breakthrough recovery is dependent on the viscosity ratio; as the viscosity ratio decreases the recovery increases and conversely.

6. After breakthrough in a layer, the fractional flow rate will decrease in that layer if the viscosity

ratio is greater than one; for a favorable viscosity ratio, the fractional flow rate will increase.

The following areas of research resulting from this study, which require future attention, are:

1) Experiments involving the comparison between viscous displacements in unconsolidated and consolidated sands, especially for favorable viscosity ratios. This comparison will determine if such viscous displacements in unconsolidated sands are valid.

2) Experiments measuring the crossflow between two layers of sand of different permeabilities. This may be accomplished by measuring the pressure differential across each sand; such data will undoubtedly prove beneficial in viscous displacements.

3) Determining and measuring the imbibition forces of different sands in an immiscible displacement; and also, to calculate the transfer of immiscible fluids from one sand to another after a displacement.

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APPENDIX

APPENDIX TABLE OF CONTENTS

TABLE 1

DATA: DETERMINATION OF COMPOSITION
FOR $u_r = 1$

0.01N-HCl
0.0153N-NaOH

Sample #	Vol. (ml.)	Vol. of HCl	q_1/q_T(%)
(1)	6.4	1.1	88.0
(2)	6.6	2.2	78.2
(3)	6.4	3.2	67.4
(4)	6.6	3.8	62.5
(5)	6.43	4.3	56.4
(6)	6.4	4.6	53.2
(7)	6.5	5.5	45.0
(8)	6.3	6.1	36.9
(9)	6.3	7.0	27.1
(10)	6.3	8.4	13.0
(11)	6.2	8.7	8.5
(12)	6.4	9.2	6.5
(13)	6.4	9.5	3.0
(14)	6.2	9.3	2.0
(15)	6.2	9.4	1.0

TABLE 2

DATA: DETERMINATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = 1$

Sample #	Production (ml.)	Np (ml.)	C(ml.)	Np(%)	$C(V_p)$	q_i/q_T(%)
	434.0	434.0	434.0	59.0	.59	100.0
(1)	22.0 6.4	460.4	462.4	62.5	.628	88.8
(2)	21.0 6.6	483.1	490.0	65.5	.665	78.2
(3)	13.0 6.4	496.8	509.4	67.4	.692	67.4
(4)	20.0 6.6	513.9	536.0	69.9	.730	62.5
(5)	25.0 6.43	532.5	567.4	72.5	.770	56.4
(6)	28.0 6.4	551.2	601.8	75.0	.819	53.2
(7)	29.0 6.5	568.2	637.3	77.2	.865	45.0
(8)	60.0 6.3	594.7	703.6	80.0	.959	36.9
(9)	57.0 6.2	614.6	766.8	83.9	1.040	27.1
(10)	352.0 6.3	685.9	1124.1	93.2	1.53	13.0
(11)	92.0 6.2	697.2	1222.3	94.9	1.66	8.5
(12)	97.0 6.4	704.9	1325.7	96.0	1.81	6.5
(13)	84.0 6.4	709.1	1416.1	96.4	1.91	3.0
(14)	231.0 6.2	714.8	1653.3	97.2	2.24	2.0
(15)	320.0 6.2	719.6	1979.5	97.8	2.69	1.0

TABLE 3

DATA: DETERMINATION OF COMPOSITION FOR $u_r = 2$

0.01N-HCl
0.0150N-NaOH

Sample #	Vol. (ml.)	Vol. of HCl(ml.)	q_i/q_T(%)
(1)	4.8	1.3	82.0
(2)	4.6	2.7	61.0
(3)	3.6	2.4	55.5
(4)	3.8	3.0	47.5
(5)	3.8	3.8	33.5
(6)	4.3	3.4	48.0
(7)	4.7	4.4	36.0
(8)	4.3	4.6	28.5
(9)	4.9	5.5	25.5
(10)	4.8	5.3	26.5
(11)	4.9	6.5	12.0
(12)	4.5	6.1	10.0
(13)	4.0	5.7	5.0
(14)	4.7	6.6	6.5
(15)	5.0	7.4	1.0

TABLE 4

DATA: DETERMINATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = 2$

Sample #	Production (ml.)	Np (ml.)	C(ml.)	Np(%)	$C(V_p)$	q_1/q_T(%)
	378.8	378.0	378.0	51.3	.513	100.0
(1)	25.0 4.6	404.5	407.6	55.0	.552	82.0
(2)	19.0 4.6	420.8	431.2	57.2	.586	61.0
(3)	16.0 3.6	432.2	450.8	58.7	.612	55.5
(4)	13.5 3.8	441.0	468.1	60.0	.635	47.5
(5)	50.0 3.8	462.6	521.9	62.8	.709	33.5
(6)	42.5 4.3	482.5	568.7	65.5	.770	48.0
(7)	77.0 4.6	516.5	650.3	70.0	.886	36.0
(8)	76.0 4.3	542.2	730.6	73.8	.990	28.5
(9)	93.0 4.9	568.4	828.5	77.0	1.120	25.5
(10)	94.0 4.8	594.2	927.3	80.9	1.270	26.5
(11)	92.0 4.9	612.1	1024.2	83.3	1.390	12.0
(12)	94.0 4.5	622.9	1122.7	84.9	1.530	10.0
(13)	430.0 4.0	654.9	1556.7	88.9	2.110	5.0
(14)	89.0 4.7	660.2	1650.4	89.9	2.240	6.5
(15)	669.0 5.0	690.1	2324.4	93.9	3.150	1.0

TABLE 5

DATA: DETERMINATION OF COMPOSITION FOR $u_r = 3$

0.01N-HCl
0.0133N-NaOH

Sample #	Vol. (ml.)	Vol. of HCl (ml.)	q_1/q_T(%)
(1)	6.8	0.3	96.3
(2)	3.8	0.5	90.1
(3)	4.3	0.8	86.6
(4)	4.0	2.1	60.5
(5)	4.4	2.8	53.0
(6)	3.1	2.3	45.0
(7)	3.2	2.6	38.0
(8)	5.2	5.5	20.0
(9)	4.5	5.0	16.0
(10)	4.4	5.1	13.0
(11)	4.2	5.0	10.0
(12)	5.3	6.8	4.0
(13)	4.0	5.1	4.0
(14)	4.5	5.8	3.0

TABLE 6

DATA: DETERMINATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = 3$

Sample #	Production (ml.)	Np (ml.)	C(ml.)	Np(%)	$C(V_p)$	q_1/q_T(%)
	338.0	338.0	338.0	46.0	.460	100.0
(1)	9.0 6.8	353.3	353.8	48.0	.482	96.3
(2)	8.0 3.8	364.3	365.6	49.5	.497	90.1
(3)	10.5 4.3	377.3	380.4	51.2	.518	86.0
(4)	14.0 4.0	389.9	398.4	52.9	.542	60.5
(5)	15.0 4.4	400.7	417.8	54.5	.569	53.0
(6)	24.5 3.1	414.1	445.4	56.2	.607	45.0
(7)	15.0 3.2	422.2	463.6	57.5	.632	38.0
(8)	363.0 5.2	528.0	831.8	71.1	1.130	20.0
(9)	95.0 4.2	545.6	930.0	74.2	1.260	16.0
(10)	485.0 4.4	616.3	1419.4	83.7	1.930	13.0
(11)	93.0 4.2	627.4	1512.4	85.2	2.060	10.0
(12)	348.0 5.3	647.8	1865.7	87.9	2.540	4.0
(13)	92.0 4.0	651.7	1961.7	88.8	2.670	4.0
(14)	670.0 4.5	679.2	2636.2	91.8	3.560	3.0

TABLE 7

DATA: DETERMINATION OF CUMULATIVE RECOVERY, COMPOSITION & TOTAL PRODUCTION FOR $u_r = 3.63$

$V_p = 424$

C(ml.)	Np(ml.)	Np(%)	$C(V_p)$	q_i/q_T (%)
188.0	188.0	44.5	.445	100.0
207.4	205.4	48.4	.488	90.0
222.8	218.4	51.6	.522	85.0
247.2	225.0	53.0	.581	73.0
261.8	238.2	56.0	.614	53.8
287.3	249.3	58.8	.675	43.6
321.9	260.7	61.4	.759	32.9
346.3	267.9	63.1	.819	29.5
370.9	274.8	64.8	.871	28.1
395.3	280.7	66.2	.932	24.2
419.9	285.7	67.2	.989	20.3
444.5	291.3	68.5	1.045	22.8
468.9	295.8	69.5	1.100	18.4
493.4	300.3	70.8	1.160	18.3
518.8	304.5	71.8	1.220	16.5
533.6	308.3	72.6	1.260	15.3
630.6	322.8	77.2	1.485	15.0
728.6	335.8	79.0	1.710	13.3
827.1	345.8	82.2	1.950	10.1
924.1	352.8	83.2	2.117	7.2
1022.1	359.8	84.5	2.420	7.1
1122.1	364.3	86.0	2.650	4.5
1223.1	368.8	86.9	2.890	4.45
1322.6	370.1	87.5	3.130	1.3
1421.6	372.1	87.9	3.360	2.0
1520.6	373.1	88.0	3.580	1.0
1629.6	374.9	88.2	3.840	1.7

TABLE 8

DATA: DETERMINATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = \frac{1}{2}$

Sample #	Production (ml.)	Np (ml.)	C(ml.)	Np(%)	$C(V_p)$	q_1/q_T(%)
	565.0	565.0	565.0	77.0	.77	100.0
(1)	13.5 3.1	581.2	581.6	79.1	.792	97.0
(2)	12.0 3.2	595.3	596.8	81.0	.811	89.5
(3)	18.0 2.4	612.0	617.2	83.2	.839	76.1
(4)	10.0 3.0	621.5	630.2	84.6	.859	71.0
(5)	17.0 2.8	633.9	64[illegible].0	86.9	.882	56.5
(6)	17.0 3.75	644.0	669.75	87.5	.909	43.9
(7)	19.0 3.2	653.5	691.95	89.0	.942	41.8
(8)	26.5 3.0	663.3	721.45	90.2	.982	26.4
(9)	22.5 3.6	669.8	747.55	91.0	1.010	23.7
(10)	65.0 4.0	684.7	816.55	93.2	1.110	19.9
(11)	30.0 4.3	691.4	850.85	94.1	1.150	19.8
(12)	39.5 3.8	697.9	894.15	94.6	1.210	12.8
(13)	41.0 4.8	103.4	939.95	95.9	1.270	11.4
(14)	36.5 5.0	707.4	981.45	96.1	1.330	8.5
(15)	79.0 5.2	713.4	1065.65	97.1	1.450	5.9
(16)	78.0 5.4	717.1	1149.05	97.2	1.560	3.4

TABLE 9

DATA: DETERMINATION OF CUMULATIVE RECOVERY & TOTAL PRODUCTION FOR $u_r = 1/3$

Sample #	Production (ml.)	Np (ml.)	C(ml.)	Np(%)	$C(V_p)$	q_1/q_T(%)
	634.0	634.0	634.0	86.2	.862	100.0
	15.0					
(1)	3.0	641.5	652.0	87.2	.890	95.5
	6.0					
(2)	2.0	648.9	660.0	88.2	.899	93.2
	10.0					
(3)	2.4	659.9	672.4	89.5	.920	86.0
	8.0					
(4)	1.9	667.1	682.3	90.5	.930	65.0
	5.0					
(5)	1.8	670.9	689.1	91.5	.938	51.5
	10.0					
(6)	2.2	676.0	691.3	92.0	.940	47.0
	15.0					
(7)	8.4	685.8	714.7	93.2	.972	35.0
	20.0					
(8)	5.4	693.8	740.1	94.8	1.001	29.7
	50.0					
(9)	5.0	707.2	795.1	95.8	1.080	20.0
	73.0					
(10)	5.0	719.8	873.1	97.5	1.190	13.3
	105.0					
(11)	10.0	728.7	988.1	98.8	1.340	3.3

TABLE 10

DATA: DETERMINATION OF CUMULATIVE RECOVERY, COMPOSITION & TOTAL PRODUCTION FOR $u_r = 0.275$

$V_p = 565$ml.

C(ml.)	Np(ml.)	Np(%)	$C(V_p)$	q_1/q_T(%)
499.0	499.0	89.0	.890	100.0
506.0	504.2	90.48	.905	74.1
513.0	508.1	90.5	.920	55.0
519.5	511.1	91.5	.931	46.8
527.9	514.3	92.1	.950	38.1
542.1	518.7	92.8	.975	31.0
565.1	524.1	94.0	1.010	23.4
589.1	527.6	94.4	1.060	14.6
612.3	530.2	95.0	1.101	11.2
709.3	534.2	95.9	1.270	4.45
800.3	538.2	96.4	1.450	4.17
895.3	543.2	97.4	1.610	5.25
987.3	545.2	97.8	1.770	2.17
1083.3	548.2	98.1	1.960	3.13
1178.3	550.2	98.5	2.120	2.04
1270.3	551.7	98.9	2.320	1.57
1366.3	552.7	99.1	2.470	1.08
1461.3	553.7	99.5	2.630	1.04