TWO ESSAYS ON EMPIRICAL ASSET PRICING

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TWO ESSAYS ON EMPIRICAL ASSET PRICING

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Abstract

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This dissertation consists of two essays on empirical asset pricing. The first essay examines if the idiosyncratic risk is priced. Theories such as Merton (1987) predict that idiosyncratic risk should be priced when investors do not diversify their portfolio. However, the previous literature has presented a mixed set of results of the pricing of idiosyncratic risk. We find strong evidence that idiosyncratic risk is priced differently across bull and bear markets. For the sample period from June 1946 to the end of 2010, a factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an equal-weighted monthly return of 1.59% for bull markets but -1.29% for bear markets. These evidences support the hypothesis that investors are rewarded for betting on individual stocks during bull markets and holding more diversified portfolios during bear markets.

The second essay examines the role of the limits to arbitrage in the negative effect of liquidity on subsequent stock returns. I hypothesize that if the negative effect persists because of the limits to arbitrage, the effect should be more pronounced when there are more severe limits to arbitrage. My empirical evidence supports the hypothesis. In addition, I find that the effect of the limits to arbitrage on the liquidity anomaly is not correlated to the liquidity risk.

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Chapter 1

The Pricing of Idiosyncratic Risk in Bull and Bear Markets

1.1. Introduction

In recent years, researchers have become more interested in the pricing of idiosyncratic volatilities. It appears that there are some exogenous reasons that investors hold undiversified portfolios (Goetzmann and Kumar (2007)). For example, Campbell, Lettau, Malkiel, and Xu (2001) find that firm-level volatility increased over the period 1962-1997, while the stock market as a whole has not become more volatile. This divergence between idiosyncratic risk and systematic risk gives rise to a fast-growing literature on the pricing of idiosyncratic volatility of stock returns.

Various theories predict that investors demand risk premium for bearing idiosyncratic risk. Levy (1978) shows that idiosyncratic risk affects equilibrium asset prices if investors do not hold many assets in their portfolios. Merton (1987) indicates that if investors cannot hold the market portfolio they will care about total risk (systematic risk and idiosyncratic risk). Therefore, firms with larger total variance bear larger idiosyncratic risk and require higher returns to compensate for imperfect diversification. Some empirical literature confirms Merton's prediction. Malkiel and Xu (2002) find a significantly positive relation between idiosyncratic risk and the cross section of expected returns at the firm level. Goyal and Santa-Clara (2003) find that the relationship between the value-weighted average return and the lagged equally weighted average volatility (mainly consists of idiosyncratic risk of individual stocks) is positive.

More recently, researchers such as Ang, Hodrick, Xing, and Zhang (2006) find that monthly stock returns are negatively related to the idiosyncratic volatilities by using within-month daily data to calculate idiosyncratic volatility. This negative relationship poses a challenge to the notion that under-diversified investors demand a return compensation for bearing idiosyncratic risk. Guo and Savickas (2006) also report a negative relation between aggregate stock market idiosyncratic volatility and the future quarterly stock market return.

Those negative relation findings attract much attention since they are contrary to theories and findings in the previous empirical literature. Bali and Cakici (2008) try to clarify the existence and significance of the relation. They find that many factors could change this negative relation, i.e., i) the data frequency used to calculate idiosyncratic risk, ii) the weighting scheme adopted for generating average portfolio returns, iii) the breakpoints utilized to sort stocks into quintile (or decile) portfolios, etc. They report no relation between equally-weighted portfolio returns and idiosyncratic risk. Fu (2009) indicates that the existing literature cannot identify a positive relation because the conditional idiosyncratic volatility in earlier studies does not allow the time-varying property of volatility. Using monthly data, he provides estimates of the conditional idiosyncratic volatility of stock returns based on the EGARCH model and finds a significantly positive relation. Also using monthly data, Spiegel and Wang (2005) focus on the predictive power of idiosyncratic volatility and liquidity. Their finding is that expected stock returns are increasing with the level of idiosyncratic risk and decreasing in a stock's liquidity, and the impact of conditional idiosyncratic risk is much stronger. Han and Lesmond (2010) show that by controlling for the liquidity costs on the estimation of idiosyncratic volatility, the resulting idiosyncratic volatility estimate has little pricing ability to predict future returns.

Huang, Liu, Rhee, and Zhang (2010) investigate the relation between idiosyncratic risk and expected returns with a particular interest in understanding the contrasting results between idiosyncratic risk estimated using daily data (Ang, Hodrick, Xing, and Zhang (2006), Bali and Cakici (2008), Han and Lesmond (2010)) and monthly data (Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Bali and Cakici (2008), Fu (2009)). They show that although a negative relation exists when the estimate is based on daily returns, it disappears after return reversals are controlled for. Return reversals can explain both the negative relation between value-weighted portfolio returns and idiosyncratic volatility and the insignificant relation between equal-weighted portfolio returns and idiosyncratic volatility. In contrast, there is a significantly positive relation between the conditional idiosyncratic volatility estimated from monthly data and expected returns. This relation remains robust after controlling for return reversals.

Since the literature has presented a mixed set of results of the relationship between idiosyncratic risk and expected returns, it raises an important question: What is the true empirical relation between expected return and the idiosyncratic risk? We attempt to address this question in this paper. Motivated by Kim and Zumwalt (1979) who conclude that investors expect to receive a risk premium for downside variation of returns (bear market) and pay a premium for upside variation of returns (bull market), we suspect that investors behave differently over time, especially during good times and bad times in stock market. Gervais and Odean (2001) posit that during bull markets, individual investors will attribute too much of their success to their own abilities, which makes them overconfident. Their theory implies that individual investor behavior is endogenous to market conditions, and that investing behavior is different between bull and bear market conditions. Similarly, Kim and Nofsinger (2007) test whether individuals' attitudes and preferences toward stock risk, book-to-market valuation and past returns are different between market conditions. They identify some striking differences in investing behavior between the bull and the bear market.

Those papers finding different investor behaviors during different market conditions make us wonder if the relation between idiosyncratic risk and expected returns will be different between bull and bear market conditions. It is surprising that empirical research studying relation between idiosyncratic risk and expected returns in bull versus bear markets is scant. Two possible reasons for the lack of research on the link between risk-return relation and market conditions may lie in not considering different investor behaviors during different market conditions and primarily encompassing a single market condition period. For instance, Wei and Zhang (2005) extend an additional three years from January 2000 to December 2002 to Goyal and Santa-Clara's (2003) sample period from August 1963 to December 1999 and find that the positive relationship between average returns and average volatilities found by Goyal and Santa-Clara (2003) does not show up in the extended sample period, nor in some sub-periods. The significant positive relationship found by Goyal and Santa-Clara (2003) is mainly driven by the data in the 1990s. The stock market in the U.S. experienced a prolonged boom period in the 1990s. Wei and Zhang's (2005) results make sense to us because the bull market of the 1990s was followed by a sharp decline in the next three years. Their findings motivate us to examine if the empirical relation between idiosyncratic risk and expected returns is different between good times and bad times in the U.S. stock market.

Our examination includes stocks traded on the NYSE, AMEX, and Nasdaq during the post-war period of June 1946 to December 2010. Chauvet and Potter (2000) and others state that bull market and bear market are the best way to identify good times and bad times in the stock market. Pagan and Sossounov (2003) and Gonzalez, Powell, Shi, and Wilson (2005) provide some methods to classify bull and bear market cycles. We modify some methods used in the literature to identify bull and bear markets for our research. We also compare our bull and bear market cycles to those estimated by a markov regime switching model to make sure that our calculation is accurate.

In order to estimate the idiosyncratic volatility, we adopt two asymmetric volatility models to capture the time-varying character of idiosyncratic volatility.[1] We employ the EGARCH(1,1) (exponential generalized autoregressive conditional heteroskedasticity) model of Nelson (1991), since the EGARCH series are the most widely used models for the conditional volatility of returns. And we also employ the GJR-GARCH(1,1) model by Glosten, Jagannathan, and Runkle (1993) to estimate the idiosyncratic volatility.

We find that the relation between idiosyncratic volatilities and the stock returns varies over time. Specifically, we find that there are positive relations during bull markets, while the relations are mostly negative during bear markets. The empirical evidence shows that the idiosyncratic risk is priced during good times in stock market but is not priced during bad times. More specifically, as an example, for the sample period from June 1946 to the end of 2010, a factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an equal-weighted monthly return of 1.04% when the idiosyncratic volatility is estimated with the GJR-GARCH model. For bull markets during the sample period the average return is 1.59% with a standard deviation of 0.21%. For bear markets the average return is -1.29% with a standard deviation of 0.52%. We get these numbers by calculating an Idiosyncratic Risk Factor. This Idiosyncratic Risk Factor may have important implications for equity investment strategies and portfolio management.

[1] The results using symmetric volatility models, such as GARCH model, are not reported in this paper. They are very similar to the results using asymmetric volatility models.

In this paper, we contribute to the expanding idiosyncratic risk literature by conducting empirical tests to see whether or not the relations between idiosyncratic volatility and expected stock returns are related to market conditions. Our findings are important for the following reasons. First, to our knowledge, we provide the first study to view the relation between idiosyncratic risk and expected returns under different stock market conditions. We also demonstrate the importance of market condition in analyzing the true relation between idiosyncratic risk and expected stock returns. The previous literature may be neglecting the importance of the impact of market condition on the relation between idiosyncratic risk and expected returns. We show that market condition does matter in determining the risk-return relation. During bull markets, the results confirm Merton's (1987) prediction of a positive relation between idiosyncratic risk and expected return. The finding that idiosyncratic volatility may be priced during bull markets is consistent with more risk-taking with undiversified portfolios during bull markets by Merton's argument. But our results for bear markets imply that the idiosyncratic risk is not priced in bear markets. Seasholes and Wu (2007) find that investors tend to hold more diversified portfolios during bear market. So if investors hold less diversified portfolios during bull market but more diversified portfolios during bear market, then our findings would have a good explanation and are consistent with the previous theoretical literature.

Second, we explain the mixed findings of the literature about the relation between idiosyncratic volatility and expected returns by showing that only one fifth of our sample period is bear market and four fifth is bull market. The shorter bear market period can explain why the literature doesn't find our results. And considering that more than four fifth of our sample period is bull market period which reports a positive relation between idiosyncratic volatility and expected stock returns, our findings are consistent with the literature that also uses monthly data and reports positive relationship (Malkiel and Xu (2002), Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Fu (2009), Huang, Liu, Rhee, and Zhang (2010)). Our results can also explain Wei and Zhang's (2005) finding that the positive relationship between average returns and average volatilities found by Goyal and Santa-Clara (2003) does not show up in the period January 2000 to December 2002 or in some sub-periods which are bear market periods.

Third, we find that the idiosyncratic risk factor supersedes the size factor. The phenomenon that the "size effect" no longer exists is interesting. After controlling for estimated idiosyncratic volatility, "size effect" is no longer significant. A similar but different finding is discovered in Fu (2009). Fu (2009) points out that his finding contrasts to the widely documented "size effect" that small firms have higher average returns than large firms, but supports one prediction of Merton's (1987) model that, all else equal, larger firms have higher expected returns. Merton (1987) explicitly points out that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk and investor base. While the test results in Fu (2009) lend direct support to Merton's prediction in this point, our evidence implies that after controlling for idiosyncratic risk there does no longer exist any size effect.

This paper is organized as follows. In section 1.2, we calculate bull and bear market cycles and compare them to those estimated by a markov regime switching model. In section 1.3, we estimate idiosyncratic volatility using both the EGARCH(1,1) and the GJR-GARCH(1,1) to model the relations between idiosyncratic volatilities and expected returns, and to construct an idiosyncratic risk factor. Section 1.4 describes the data used, provides all the empirical test results, and conducts related analyses. Section 1.5 contains the conclusions.

1.2. Bull and bear market cycles

1.2.1. Method of calculating bull and bear market periods

Earlier literature gives several processes to determine stock market cycles. We use the turning point dating algorithm developed by Bry and Boschan (1971) to identify troughs and peaks in stock market indices and thus indicate the starting and ending points of the bull and bear markets. Following Pagan and Sossounov (2003), we use eight-month window as the appropriate length for calculation. The eight-month window works as follows: if the index level is the highest among any eight sequential levels including itself, then we mark it as a temporary peak; if the index level is the lowest among any eight sequential levels including itself, then we mark it as a temporary trough. If there exist multiple peaks (troughs), the highest (lowest) one will be selected.

However, using only temporary troughs and peaks is not the most effective way for identifying bull and bear market periods. We modify the above method to better identify bull and bear markets in this paper. We can easily find sometimes that although a peak is higher than others around it, it could be very close to them. So some restriction should be applied to make the method more effective. Following the earliest definition of bull and bear markets by Chauvet and Potter (2000), which is also very popular in the financial press, we require there must be an absolute cumulative capital return of 20% from a trough or -20% from a peak to be an acceptable phase, i.e., requiring a 20% or -20% cumulative return after selecting temporary troughs and peaks.

After applying cumulative return requirement, we get new peaks and troughs for bull and bear market periods. We mark them as permanent peaks and troughs and use them to determine each phase and cycle. For calculating the phases of bull and bear markets, we use monthly level on the S&P Composite Index from June 1946 to December 2010. The data are obtained from the Center for Research in Security Prices (CRSP).

1.2.2. Bull and bear market cycles

We use the method mentioned above to determine bull and bear market cycles. There are eight cycles (sixteen phases) determined by our method, i.e. requiring 20% cumulative return after choosing temporary troughs and peaks. Since 20% cumulative return requirement is the most popular criteria to identify bull market and bear market in the literature, our method appears to be the most effective one for determining market cycles.

The explicit bull and bear market phases are shown in Table 1.1, and the distribution for those eight cycles (sixteen phases) is shown in Figure 1.1. Eight green (grey) phases of the curve stand for bull market periods, while eight red (dark) phases of the curve stand for bear market periods. The figure of bull and bear market distribution (Figure 1.1) clearly shows the up and down trends of the stock market. The trends during bull market periods are all going upwards, while the trends during bear market periods are going downwards. From the distribution figure, we believe that the phase distribution calculated by our method is good, presenting the good times and bad times in the stock market. We also count the total months in bear markets and in bull markets separately. Not surprisingly, there are 147 months in bear market periods and 628 months in bull market periods. The total number of bull market months is much larger than the total number of bear market months. It turns out that only less than one-fifth (147/775) of our entire sample period is in a bear market period, and more than four-fifths (628/775) is in a bull market period. Bull market periods dominate the whole sample period.

Table 1.1. Bull and bear market cycles.

This table presents eight market cycles from June 1946 to December 2010. We use the turning point dating algorithm to select temporary peaks and troughs and then apply a 20% or -20% cumulative return requirement for each phase. The corresponding time series distribution is shown in Figure 1.1.

Cycle	Bear market	Bull market
1	194606-194906	194907-196112
2	196201-196206	196207-196811
3	196812-197006	197007-197212
4	197301-197409	197410-198011
5	198012-198207	198208-198708
6	198709-198711	198712-200008
7	200009-200209	200210-200710
8	200711-200902	200903-201012

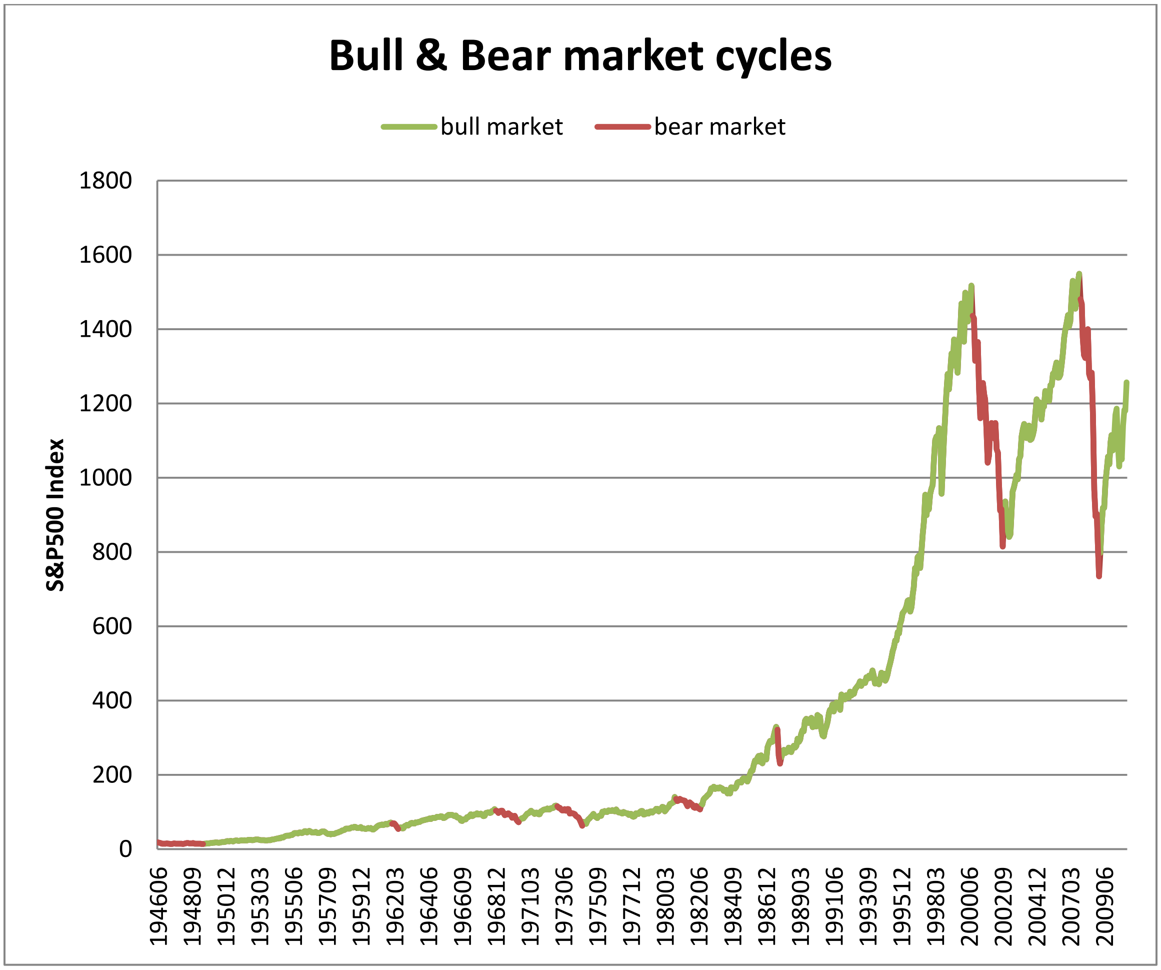


Figure 1.1. Bull and bear market distribution.

This figure presents eight bull market phases (green/grey) and eight bear market phases (red/dark) calculated by the turning point dating algorithm.

1.2.3. Markov regime switching model

The turning point dating algorithm we introduced above is an ex-post method. As an ex-post method, it is always questioned when concerning real-time implications for equity investment strategies and portfolio management. There are also some other methods to distinguish between two different states in a time series. Some of them are ex-ante methods. Markov regime switching model is the most popular one among them. [2] Therefore we would like to also identify the good times and bad times in the U.S. stock market by a two states regime switching model. We will compare the results estimated by the markov regime switching model to those calculated by the turning point dating algorithm (shown in Table 1.1 and Figure 1.1) as a robustness check.

The two states regime switching model we use is as follows:

$$\mathbf{y_t = \mu_1 + \epsilon_t \quad for\ State\ 1}$$

$$\mathbf{y_t = \mu_2 + \epsilon_t \quad for\ State\ 2}, \tag{1.1}$$

where

$$\epsilon_t \sim N(0, \sigma_1^2) \quad \text{for State 1}$$

$$\epsilon_t \sim N(0, \sigma_2^2) \quad \text{for State 2},$$

with

$$P = \begin{pmatrix} p_{1,1} & p_{2,1} \\ p_{1,2} & p_{2,2} \end{pmatrix}$$

[2] Technical details regarding markov regime switching models can be found in Hamilton (1994), Kim and Nelson (1999) and Wang (2003).

as the transition matrix, which controls the probability of a switch from state j (column j) to state i (row i). The sum of each column in P is equal to one, since they represent full probabilities of the process for each state.

This model clearly implies two different processes for the dependent variable y_t. When the state for time t is 1, the expectation of the dependent variable is μ_1 and the volatility of the innovations is σ_1^2. While when the state for time t is 2, the expectation of the dependent variable is μ_2 and the volatility of the innovations is σ_2^2. Here we will use monthly return on the S&P Composite Index from June 1946 to December 2010 as variable y_t. The higher value μ_{Bull} (μ_1 or μ_2) will be the expected return on a bull market state, which implies a positive trend for the U.S. stock market prices and consequently a positive return for y_t. The lower value μ_{Bear} (μ_2 or μ_1) measures the expected return for the bear market state, which then implies a negative trend in the U.S. stock market prices. We could expect that the bear market state is more volatile than the bull market, which means that we can expect σ_{Bear}^2 (σ_2^2 or σ_1^2) to be higher than σ_{Bull}^2 (σ_1^2 or σ_2^2).

In this paper, we estimate the two states regime switching model above using maximum likelihood method. Considering $f(y_t|S_t = j, \Theta)$ as the likelihood function for state j conditional on a set of parameters Θ, then the full log likelihood function of the model is given by:

$$\mathbf{lnL} = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} \big(f(y_t|S_t = j, \Theta)\Pr(S_t = j)\big) \tag{1.2}$$

which is a weighted average of the likelihood function in each state, where the weights are given by the state's probabilities. We use Hamilton's filter to calculate the filtered probabilities of each state $\Pr(S_t = j)$ based on the arrival of new information. Then the estimation of the model is obtained by finding the set of parameters that maximize Equation (1.2).

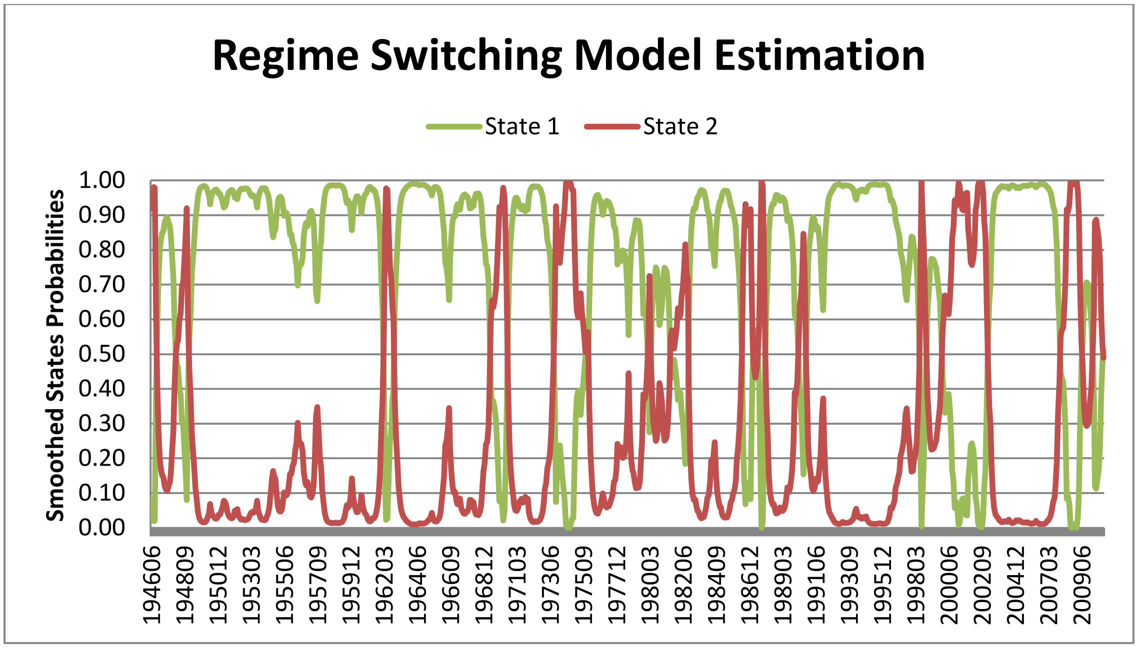
The estimated parameters and other outputs are shown in Table 1.2. $\mu_1 = 0.0104$ and $\mu_2 = -0.0047$ imply that state 1 represents a bull market state while state 2 represents a bear market state. The volatility $\sigma_2^2 = 0.0035$ is larger than $\sigma_1^2 = 0.0011$, which meets our expectation that the bear market state is more volatile than the bull market state. We also get the expected duration of the two regimes. State 1 is expected to last 26.29 time periods while state 2 lasts 9.41 time periods. This result is consistent with the finding before that bull market periods dominate the whole sample period.

As a robustness check, the most important part is to compare the estimated states probabilities distribution to bull and bear market distribution (Table 1.1 and Figure 1.1). We show the comparison in Figure 1.2. There are two figures in Figure 1.2. Figure a shows the estimated smoothed states probabilities by the two states regime switching model. Figure b shows the S&P500 return series divided into sixteen bull and bear market phases calculated by the turning point dating algorithm. After comparing the two figures, we could see that the two distributions are very similar. The bull market periods (green/grey) in figure b have very high probabilities of state 1 (green/grey) in figure a, while the bear market periods (red/dark) in figure b have very high probabilities of state 2 (red/dark) in figure a. The comparison makes our previous method of calculating bull and bear market cycles seem to be quite accurate. So we will apply the results calculated in earlier subsection (Table 1.1) to all the following empirical tests in the entire paper.

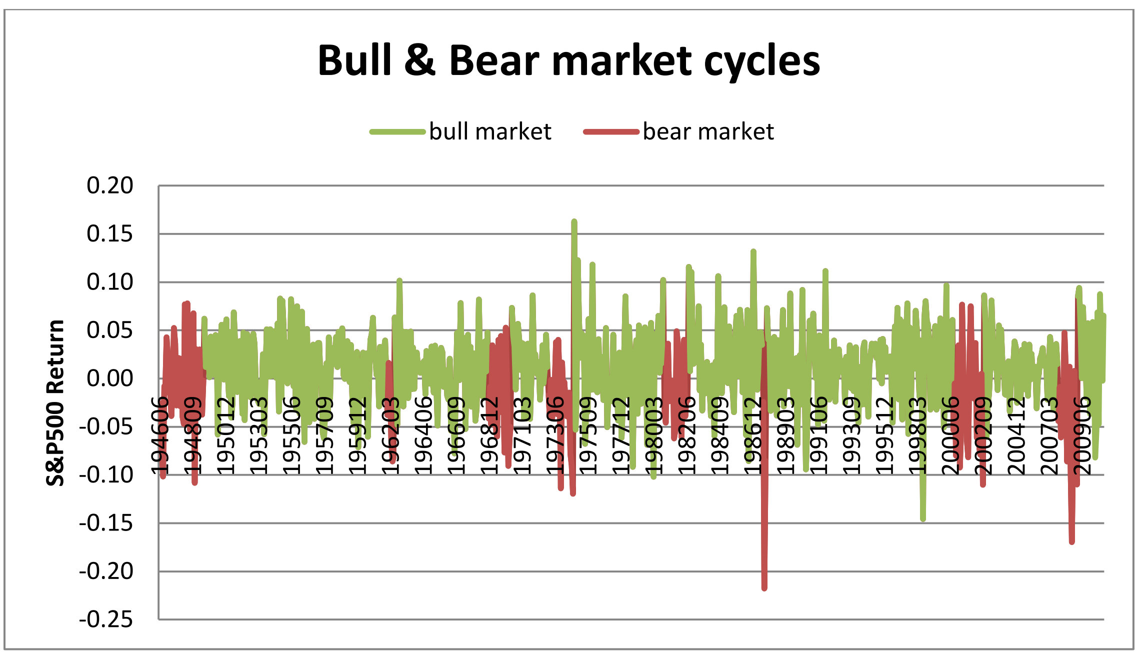
Table 1.2. Markov regime switching model estimation.

This table reports the estimated switching parameters and other output by a two states regime switching model. Dependent variable y_t is monthly return on S&P Composite Index from June 1946 to December 2010.

Final log Likelihood: 1383.7654 Number of Observations: 775 Type of Switching Model: Univariate Distribution Assumption: Normal		
	State 1	State 2
Regressors Std Error (p. value)	0.0104 0.0017 (0.00)	-0.0047 0.0098 (0.63)
Model's Variance Std Error (p. value)	0.001090 0.0002 (0.00)	0.003494 0.0008 (0.00)
Transition Probabilities Matrix	0.96 0.11 0.04 0.89	
Expected Duration of Regimes	26.29 time periods	9.41 time periods



a. Smoothed states probabilities



b. S&P500 return series

Figure 1.2. Comparison of the results by two different dating methods.

Figure a shows the estimated smoothed states probabilities by a two states regime switching model. Figure b shows the S&P500 return series divided into sixteen bull and bear market cycles calculated by the turning point dating algorithm.

1.3. Empirical approach

1.3.1. Estimation of idiosyncratic volatility

Following Fu (2009), we use in-sample monthly data to calculate idiosyncratic volatility based on the three-factor Fama-French (1993) model. The explicit form is as follows.

$$\boldsymbol{R_{it} - r_{ft} = \alpha_i + \beta_i\left(R_{mt} - r_{ft}\right) + S_i SMB_t + h_i HML_t + \varepsilon_{it}}, \tag{1.3}$$

$$\varepsilon_{it} \sim N(0, \sigma_{it}^2),$$

where R_{it} is the individual return. $R_{mt} - r_{ft}$ is the excess return on a broad market portfolio. SMB_t is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML_t is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. The residual ε_{it} is assumed to be normally distributed with mean zero and variance σ_{it}^2. To get the idiosyncratic volatilities, our objective is to estimate σ_{it}^2. We will use the asymmetric GARCH (generalized autoregressive conditional heteroskedasticity), since GARCH is the most widely used model for estimating the conditional volatility of returns.[3] We will employ both the EGARCH(1,1) model and the GJR-GARCH(1,1) model by Glosten, Jagannathan, and Runkle (1993).

The explicit form for the EGARCH(1,1) is as follows:

[3] We also use the symmetric GARCH (GARCH(1,1)) to estimate the idiosyncratic volatilities. The final results are very similar to those using EGARCH(1,1) model.

$$\boldsymbol{ln\sigma_{it}^2 = a_i + b_i ln\sigma_{i,t-1}^2 + c_i \left\{\theta\left(\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right| - (2/\pi)^{1/2}\right]\right\}}, \tag{1.4}$$

where $\varepsilon_{i,t-1}$ is the lagged residual and $\sigma_{i,t-1}^2$ is the lagged variance.

In tandem with the EGARCH approach, we apply the GJR-GARCH (Glosten, Jagannathan, and Runkle (1993)) model to estimate idiosyncratic volatilities. The GJR-GARCH model allows positive and negative innovations to returns to have different impacts on conditional variance. As Glosten, Jagannathan, and Runkle (1993) suggest, if most of the fluctuations in stock prices are caused by fluctuations in expected future cash flows and the riskiness of future cash flows does not change proportionally when investors revise their expectations, then unanticipated changes in stock prices and returns will be negatively related to unanticipated changes in future volatility. So the GJR-GARCH is a good way to capture the different impacts that unanticipated returns from different directions have on conditional volatility. Following Glosten, Jagannathan, and Runkle (1993), we assume that the impact of $\varepsilon_{i,t-1}^2$ on conditional variance σ_{it}^2 is different when $\varepsilon_{i,t-1}$ is positive (i.e., when the indicator or dummy variable $I_{i,t-1}$ is 1) from when $\varepsilon_{i,t-1}$ is negative (i.e., when the indicator or dummy variable $I_{i,t-1}$ is 0). So the explicit form for the GJR-GARCH(1,1) is as follows:

$$\boldsymbol{\sigma_{it}^2 = a_i + b_i\sigma_{i,t-1}^2 + c_{1i}\varepsilon_{i,t-1}^2 + c_{2i}\varepsilon_{i,t-1}^2 I_{i,t-1}}, \tag{1.5}$$

where $\varepsilon_{i,t-1}$ is the lagged residual, and $\sigma_{i,t-1}^2$ is the lagged variance. $I_{i,t-1} = 0$ if $\varepsilon_{i,t-1} \geq 0$, and $I_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$.

1.3.2. Modeling the relations between stock returns and idiosyncratic volatilities

First, following Fu (2009), we regress monthly stock returns on idiosyncratic volatilities and several firm characteristics to check the relationship between these variables. For firm characteristics, we use four control variables in our regression. They are ME, BE/ME, RET2_7 and Turnover. ME is the market value of equity at the end of June of year t. BE/ME is the book-to-market equity according to Fama and French (1993) at the end of fiscal year ending in calendar year t–1 divided by the market value of equity at the end of December of year t–1. Ret2_7 is the compound gross return from month t-7 to t-2. Turnover is the average ratio of trading volume to the number of shares outstanding of the previous 36 months as a proxy for liquidity. The explicit form is as follows:

$$\mathbf{R_{it} = \gamma_0 + \gamma_1 IVOL_{it} + \gamma_2 ME_{it} + \gamma_3 BE/ME_{it} + \gamma_4 Ret2\_7_{it} + \gamma_5 Turnover_{it} + \varepsilon_{it}}, \qquad (1.6)$$

where R_{it} is the realized individual return on stock i at time t. $IVOL_{it}$ is the idiosyncratic stock return volatility of stock i at time t based on the information at time t-1. How to estimate $IVOL_{it}$ has been stated in last subsection. γ_n is the coefficient of the corresponding variable, where n=0, 1, 2, 3, 4, 5. We will focus on the coefficient estimate γ_1 of the cross-sectional regressions. If $\gamma_1 > 0$, we will get a positive relation between stock returns and idiosyncratic volatilities, which means the idiosyncratic risk is priced; if $\gamma_1 \leq 0$, we will get a negative or an insignificant relation between those two variables, which means the idiosyncratic risk is not priced.

We run the above regression for two classifications. One is for eight bull and bear market cycles as calculated earlier in this paper. Within each market period, we run Equation (1.6) once. There are sixteen groups in total. The other is for forty-eight industries as stated in Fama and French (1997). Within each industry, we sum up all the bull market periods and all the bear market periods and run Equation (1.6) for bull and bear separately. There are ninety-six groups in total for this classification.

Second, we regress monthly stock returns on idiosyncratic volatilities and several market factors to check the relationship between those variables. We use four market factors in our regression. Mkt-r is the excess return on a broad market portfolio. SMB is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. Mom is the momentum factor. The explicit form is as follows:

$$\mathbf{R_{it} = \gamma_0 + \gamma_1 IVOL_{it} + \gamma_2 (Mkt - r)_t + \gamma_3 SMB_t + \gamma_4 HML_t + \gamma_5 Mom_t + \varepsilon_{it}}, \qquad (1.7)$$

where $\mathrm{R_{it}}$ is the realized individual return on stock i at time t. $\mathrm{IVOL_{it}}$ is the idiosyncratic stock return volatility of stock i at time t based on the information at time t-1. As what we did for Equation (1.6), we will also run Equation (1.7) for eight bull and bear market cycles (sixteen groups) and forty-eight industries (ninety-six groups), separately.

At the same time, we will apply the standard Fama and MacBeth (1973) methodology to control the cross-correlation in residuals. We will give all the coefficients by calculating the time series average of the monthly cross-sectional slope estimates and the t-statistic by calculating the average slope over Newey-West (1987) standard error.

1.3.3. Calculating idiosyncratic risk factor

To better see if idiosyncratic stock risk is priced, that is to say if stocks with high idiosyncratic risk have tended to do better than stocks with low idiosyncratic risk, we attempt to construct a factor to serve this purpose. The factor will be calculated with combinations of portfolios composed by ranked stocks.

Following the same method as the Fama-French factors are constructed, we use six portfolios formed on size and idiosyncratic volatilities to construct an Idiosyncratic Risk Factor (IRF). The portfolios are formed monthly by both value-weighted scheme and equal-weighted scheme. First, we form two portfolios on size where the monthly size breakpoint is the median market equity (ME). Then, we form three portfolios on idiosyncratic volatility with the 30th and 70th percentiles as two monthly idiosyncratic volatility breakpoints.[4] The final six portfolios are the intersections of two portfolios on size and three portfolios on idiosyncratic volatility. The six portfolios are shown as follows:

[4] We use the idiosyncratic volatility estimated by both EGARCH(1,1) model and GJR-GARCH(1,1) model.

	Median ME	
	Small High	Big High
70th idiosyncratic volatility percentile		
	Small Medium	Big Medium
30th idiosyncratic volatility percentile		
	Small Low	Big Low

Next, we use four of these six portfolios to construct IRF. IRF is the average return on the two high idiosyncratic volatility portfolios minus the average return on the two low idiosyncratic volatility portfolios. Since this measure is introduced by Fama and French, we mark it as IRF(FF).

$$\boldsymbol{IRF(FF) = 1/2(Small\ High + Big\ High) - 1/2(Small\ Low + Big\ Low)} \quad (1.8)$$

In order to make sure that the Idiosyncratic Risk Factor (IRF) is not affected by the way how we form the portfolios, we revised the Fama-French factors constructing method into another two refined measures. In refined measure one (R1), we form nine portfolios which are the intersections of three portfolios on size, instead of two portfolios in FF measure, and three portfolios on idiosyncratic volatility. The nine portfolios are shown as follows:

	30th ME percentile	70th ME percentile	
	Small High	Medium High	Big High
70th idiosyncratic volatility percentile			
	Small Medium	Medium Medium	Big Medium
30th idiosyncratic volatility percentile			
	Small Low	Medium Low	Big Low

Then IRF is calculated by the average return on the three high idiosyncratic volatility portfolios minus the average return on the three low idiosyncratic volatility portfolios, as shown in Equation (1.9). We mark this measure as IRF(R1).

$$\boldsymbol{IRF(R1)} = \mathbf{1/3}(\boldsymbol{Small\ High} + \boldsymbol{Medium\ High} + \boldsymbol{Big\ High}) - \mathbf{1/3}(\boldsymbol{Small\ Low} + \boldsymbol{Medium\ Low} + \boldsymbol{Big\ Low}) \tag{1.9}$$

In another refined measure, refined measure two (R2), we form fifteen portfolios which are the intersections of five portfolios on size and three portfolios on idiosyncratic volatility. The fifteen portfolios are shown as follows:

	20th ME	40th ME	60th ME	80th ME	
70th idiosyncratic volatility percentile	Smallest High	Smaller High	Medium High	Bigger High	Biggest High
	Smallest Medium	Smaller Medium	Medium Medium	Bigger Medium	Biggest Medium
30th idiosyncratic volatility percentile	Smallest Low	Smaller Low	Medium Low	Bigger Low	Biggest Low

Then IRF is calculated by the average return on the five high idiosyncratic volatility portfolios minus the average return on the five low idiosyncratic volatility portfolios, as shown in Equation (1.10). We mark the refined measure two as IRF(R2).

$$\boldsymbol{IRF(R2)} = \mathbf{1/5}(\boldsymbol{Smallest\ High} + \boldsymbol{Smaller\ High} + \boldsymbol{Medium\ High} + \boldsymbol{Bigger\ High} + \boldsymbol{Biggest\ High}) - \mathbf{1/5}(\boldsymbol{Smallest\ Low} + \boldsymbol{Smaller\ Low} + \boldsymbol{Medium\ Low} + \boldsymbol{Bigger\ Low} + \boldsymbol{Biggest\ Low}) \tag{1.10}$$

The Idiosyncratic Risk Factor (IRF) will be calculated on a monthly basis. So we will get a number of IRF for each month. If the IRF is positive, then on average the idiosyncratic stock risk is priced in that month; if the IRF is negative or zero, then on average the idiosyncratic stock risk is not priced in that month. Besides the monthly IRF, we will calculate the cumulative IRF as well, to finally exam if the idiosyncratic risk is priced differently under different stock market conditions.

1.4. Empirical analysis

1.4.1. The data description

For analyzing the relationship between idiosyncratic volatility and expected stock returns, we use monthly holding period stock return data. The data are obtained from the Center for Research in Security Prices (CRSP). We include stocks traded on the NYSE, AMEX, and NASDAQ during the post-war period of June 1946 to December 2010. There are a total of 775 months in our sample. The monthly Fama-French three-factor and momentum factor data are downloaded from Kenneth R. French’s Website. To avoid the inaccuracy of idiosyncratic volatility estimates caused by infrequent trading, we require a minimum of 30 trading months for each stock when CRSP reports a non-zero share volume. Table 1.3 presents the descriptive statistics summary of variables used in this paper. R is the monthly raw return. R-r is the monthly excess return, where r stands for the one-month T-bill rate. IVOL (EGARCH) is the idiosyncratic volatility estimated by the EGARCH(1,1) model while IVOL (GJR-GARCH) is the idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. Firm size is measured by ME, the market value of equity at the end of June of year t. BE/ME is the book-to-market equity according to Fama and French (1993) at the end of fiscal year ending in calendar year $t-1$ divided by the market value of equity at the end of December of year $t-1$, whose data is available from year 1950 to year 2010. In order to catch the momentum effects, Ret2_7 is the compound gross return from month t-7 to t-2. Turnover is the average ratio of trading volume to the number of shares outstanding of the previous 36 months as a proxy for liquidity. Mkt-r, SMB and HML are the Fama-French three factors while Mom is the momentum factor.

Table 1.3. Variable descriptive statistics summary.

This table reports the descriptive statistics summary of variables used in this paper. The data are obtained from CRSP and Kenneth R. French's Website. We include stocks traded on the NYSE, AMEX, and Nasdaq during the post-war period of June 1946 to December 2010. R is the monthly raw return. R-r is the monthly excess return, where r stands for the one-month T-bill rate. IVOL (EGARCH) is the idiosyncratic volatility estimated by the EGARCH(1,1) model. IVOL (GJR-GARCH) is the idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. ME is the market value of equity. BE/ME is the Book-to-market ratio. Ret2_7 is the compound gross return from month t-7 to t-2. Turnover is the average turnover rate. Mkt-r is the excess return on a broad market portfolio. SMB is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. Mom is the momentum factor. S&P is the monthly return of S&P Composite Index. S&P Level is the monthly level on S&P Composite Index.

Variables	N	Mean	Median	Std dev.	Skewness
R (%)	3,283,202	1.19	0.00	17.05	6.55
R-r (%)	3,283,202	0.79	-0.28	17.05	6.55
IVOL (EGARCH)	3,283,196	15.74	9.96	59.00	181.82
IVOL (GJR-GARCH)	3,283,170	12.57	10.21	9.94	9.96
ME (Billion)	2,929,707	1.05	0.08	7.52	26.98
BE/ME	2,380,201	0.79	0.60	86.87	149.31
Ret2_7 (%)	2,914,766	5.61	0.00	93.16	86.68
Turnover	2,416,599	0.80	0.39	2.50	85.89
Mkt-r (%)	775	0.62	1.03	4.40	-0.77
SMB (%)	775	0.16	-0.04	3.17	0.71
HML (%)	775	0.35	0.33	3.04	0.03
Mom (%)	775	0.85	0.89	4.50	-1.32
S&P (%)	775	0.62	0.87	4.20	-0.43
S&P Level	775	355.39	106.29	441.90	1.34

1.4.2. Cross-sectional correlations

We first check the simple correlations between stock returns and idiosyncratic volatilities. We check them in each bull and bear market period, which have been previously calculated and shown in Table 1.1, to see if there are any obvious differences in the correlation between stock returns and idiosyncratic volatilities under different market conditions. If there is a clear pattern in results, we will further proceed to investigate the true relationships by Fama-MacBeth regressions.

The cross-sectional correlations are presented in Table 1.4. We employ both the EGARCH(1,1) model and the GJR-GARCH(1,1) model to estimate the idiosyncratic stock return volatility. So for each bull and bear market period, there are two correlations reported. One is the correlation between stock return and idiosyncratic volatility estimated by the EGARCH(1,1) model, while the other is the correlation between stock return and idiosyncratic volatility estimated by the GJR-GARCH(1,1) model.

During bull market periods, almost all the correlations are positive. The correlations between stock returns and the idiosyncratic volatilities calculated by the EGARCH(1,1) model are all positive. The results for the GJR-GARCH(1,1) are similar but a little weaker. The correlations using the GJR-GARCH(1,1) are all positive except for the bull market period from July 1970 to December 1972. During bear market periods, almost all the correlations are negative. The correlations between stock returns and the idiosyncratic volatilities calculated by the GJR-GARCH(1,1) are all negative, while the correlations using the EGARCH(1,1) model are mixed. Six out of eight bear market periods correlations are negative, but the remaining two are positive.

Until now, it is obvious that the correlations in bull markets and bear markets follow different patterns. No matter which model we use to estimate idiosyncratic volatility, we find different pictures are drawn for the bull market and the bear market.

Wei and Zhang (2005) report that the positive relationship between average returns and average volatilities found by Goyal and Santa-Clara (2003) does not show up in the extended sample period from January 2000 to December 2002. They also conclude that the significant positive relationship found by Goyal and Santa-Clara (2003) is mainly driven by the data in the 1990s. Our results show that, for the bear market period from September 2000 to September 2002, the correlation is 0.026 and -0.043, separately. Compared with the correlation in the previous bull market period from December 1987 to August 2000, which is 0.144 and 0.097, separately, the correlation in the period from September 2000 to September 2002 is much smaller. These results are consistent with and also can explain Wei and Zhang's (2005) finding that the positive relationship between average returns and average volatilities found by Goyal and Santa-Clara (2003) does not show up in the period January 2000 to December 2002 or in some sub-periods that are bear market periods. Next, we will apply Fama-MacBeth methodology to further test the relations between stock returns and idiosyncratic volatilities to confirm our finding that the relation between stock returns and idiosyncratic volatilities varies when the U.S. stock market condition changes.

Table 1.4. Correlations between stock returns and idiosyncratic volatilities.

This table presents the correlations between stock returns and idiosyncratic volatilities in each bull and bear market period. We use both the EGARCH(1,1) model and the GJR-GARCH(1,1) model to estimate the idiosyncratic stock return volatility.

Bull market	EGARCH (1,1)	GJR-GARCH(1,1)	Bear market	EGARCH (1,1)	GJR-GARCH(1,1)
			194606-194906	-0.088	-0.146
194907-196112	0.052	0.019	196201-196206	-0.337	-0.346
196207-196811	0.208	0.121	196812-197006	-0.149	-0.148
197007-197212	0.003	-0.021	197301-197409	-0.001	-0.039
197410-198011	0.219	0.126	198012-198207	-0.060	-0.088
198208-198708	0.118	0.046	198709-198711	-0.109	-0.169
198712-200008	0.144	0.097	200009-200209	0.026	-0.043
200210-200710	0.102	0.090	200711-200902	0.048	-0.045
200903-201012	0.387	0.293			

1.4.3. Relations between stock returns and idiosyncratic volatilities for eight stock market cycles

Following Fama and MacBeth (1973), we use their regression methodology to control the cross-correlation in residuals. We first regress the stock returns on idiosyncratic volatilities and other firm characteristics for eight bull and bear market cycles. The model is a multivariate regression presented in Equation (1.6): $\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{IVOL_{it}} + \gamma_2 \mathrm{ME_{it}} + \gamma_3 \mathrm{BE/ME_{it}} + \gamma_4 \mathrm{Ret2\_7_{it}} + \gamma_5 \mathrm{Turnover_{it}} + \varepsilon_{\mathrm{it}}$. The test results for eight cycles are presented in Table 1.5 and Table 1.6, where Table 1.5 shows the regression results when using the EGARCH(1,1) model to estimate idiosyncratic volatility, while Table 1.6 shows the regression results when using the GJR-GARCH(1,1) model. Since BE/ME data is available only from year 1950, we exclude this variable from the regression for the bear market period from June 1946 to June 1949.

We find that during bull market periods, except for only one period from July 1970 to December 1972, all the coefficient estimates of idiosyncratic volatility estimated by the EGARCH(1,1) are positive and most of them are significant at the 1% level. We also pool all the bull market periods together to get a total bull time period. The coefficient estimate of idiosyncratic volatility for the total bull period is 0.0450 with 1% significance for the EGARCH(1,1). These significant positive coefficient estimates imply that the idiosyncratic risk is priced during bull market periods.

During bear market periods, the signs of coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model change to negative except one which is insignificantly positive. Similarly as for bull markets, we pool all the bear market periods together to get a total bear time period. The coefficient estimate of idiosyncratic volatility for the total bear period is -0.0036 with 1% significance for the EGARCH(1,1). These significant negative coefficient estimates imply that the idiosyncratic risk is negatively priced during bear market periods.

Moreover, we find that during bull market periods, except for two periods, all the coefficient estimates of idiosyncratic volatility estimated by the GJR-GARCH(1,1) are positive and all positive coefficients are significant at the 1% level. We also pool all the bull market periods together to get a total bull time period. The coefficient estimate of idiosyncratic volatility for the total bull period is 0.0908 for the GJR-GARCH(1,1) with 1% significance. During bear market periods, all the coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model are significantly negative. And the coefficient estimate of idiosyncratic volatility for the total bear period is -0.0668 for the GJR-GARCH(1,1) with 1% significance. These coefficient estimates of the GJR-GARCH(1,1) are consistent with those of the EGARCH(1,1) and confirm our finding that the idiosyncratic risk is priced during bull market periods but is negatively priced in bear market periods.

There is another result that is interesting. The coefficients of ME become mostly insignificant after including IVOL in the regression. Controlling for estimated idiosyncratic volatility, "size effect" is no longer significant. A similar but different finding is discovered in Fu (2009). Fu (2009) points out that his finding contrasts to the widely documented "size effect" that small firms have higher average returns than large firms, but supports one prediction of Merton's (1987) model that, all else equal, larger firms have higher expected returns. Merton (1987) explicitly points out that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk and investor base. While the test results in Fu (2009) lend direct support to Merton's prediction in this point, our evidence implies that after controlling for idiosyncratic risk there does no longer exist any size effect.

Next, we regress the stock returns on idiosyncratic volatilities and market factors for eight bull and bear market cycles. The model is a multivariate regression presented in Equation (1.7): $\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{IVOL_{it}} + \gamma_2 (\mathrm{Mkt} - \mathrm{r})_\mathrm{t} + \gamma_3 \mathrm{SMB_t} + \gamma_4 \mathrm{HML_t} + \gamma_5 \mathrm{Mom_t} + \varepsilon_{\mathrm{it}}$. The test results for eight cycles are presented in Table 1.7 and Table 1.8, where Table 1.7 shows the regression results when using the EGARCH(1,1) model to estimate idiosyncratic volatility, while Table 1.8 shows the regression results when using the GJR-GARCH(1,1) model. Since the bear market period from September 1987 to November 1987 contains only three months, less than the number of independent variables, we do a univariate regression for this bear market period.

It turns out that during bull market periods, all the coefficient estimates of idiosyncratic volatility estimated by the EGARCH(1,1) are positive and most of them are significant at the 1% level. The coefficient estimate of idiosyncratic volatility for the total bull period is 0.0195 with 1% significance. These significant positive coefficient estimates again support that the idiosyncratic risk is priced during bull market periods. During bear market periods, the signs of coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model are mixed. Six out of eight coefficient estimates are negative, but the other two are positive. The coefficient estimate of idiosyncratic volatility for the total bear period is -3.96E-8, which is very close to zero. These mixed coefficient estimates support that the idiosyncratic risk is not priced during bear market periods.

Additionally, we find that during bull market periods, except for only one period, all the coefficient estimates of idiosyncratic volatility estimated by the GJR-GARCH(1,1) are positive and all positive coefficients are significant at the 1% level. We also check the relation between stock return and idiosyncratic volatility for the total bull time period. The coefficient estimate of idiosyncratic volatility for the total bull period is 0.1039 with 1% significance for the GJR-GARCH(1,1). During bear market periods, all the coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model are significantly negative. Similarly, we check the relation between stock return and idiosyncratic volatility for total bear time period, too. The coefficient estimate of idiosyncratic volatility for the total bear period is -0.0202 with 1% significance for the GJR-GARCH(1,1). Although these coefficient estimates of the GJR-GARCH(1,1) are more negative than those of the EGARCH(1,1), both of them can support the finding that the idiosyncratic risk is priced during bull market periods but not in bear market periods.

To compare our regression results to other papers in the literature, we also conduct a univariate regression with only idiosyncratic volatility as an independent variable. Model 3 in Fu (2009) is also a univariate regression of stock returns on his estimated idiosyncratic volatility, and his coefficient estimate is 0.11. His result for his whole sample from July 1963 to December 2006 is very similar to our univariate regression results for the total bull period, which are 0.1290 for the EGARCH(1,1) and 0.1139 for the GJR-GARCH(1,1), respectively. As suggested earlier, there is one possible reason that the previous literature doesn't find our results that the relations are different under two different market conditions, because they primarily encompass a single market condition period. Based on very similar results to Fu's (2009), perhaps his sample period primarily encompasses a single market condition period. To establish that this point is true, we bring the finding to mind that bull market periods dominate the whole sample period. The shorter bear market period can explain why the literature doesn't reflect our results. And considering that more than four-fifths of our sample period is in a bull market period, which reports a positive relation between idiosyncratic volatility and expected stock returns, our findings are consistent with the literature that also uses monthly data and reports a positive relationship (Malkiel and Xu (2002), Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Fu (2009), Huang, Liu, Rhee, and Zhang (2010)). Our findings are also consistent with Chua, Goh, and Zhang (2007) who use an AR(2) model, and Diavatopoulos, Doran, and Peterson(2007) who decompose implied volatility from option prices to estimate conditional idiosyncratic volatility. Both studies conclude the positive tradeoff between idiosyncratic risk and expected return.

Table 1.5. Regressions of stock returns on idiosyncratic volatilities and firm characteristics for eight bull and bear market cycles (EGARCH).

This table reports the regression results for eight bull and bear market cycles when using the EGARCH(1,1) model to estimate idiosyncratic volatility. $R_{it} = \gamma_0 + \gamma_1 IVOL_{it} + \gamma_2 ME_{it} + \gamma_3 BE/ME_{it} + \gamma_4 Ret2\_7_{it} + \gamma_5 Turnover_{it} + \varepsilon_{it}$. ME is the market value of equity. BE/ME is the Book-to-market equity. Ret2_7 is the compound gross return from month t-7 to t-2. Turnover is the average turnover rate. The numbers in parentheses are the standard errors. The coefficients followed by * are significant at 1% level, followed by ** are significant at 5% level and followed by *** are significant at 10% level.

Bull						Bear					
	IVOL	ME	BE/ME	Ret2_7	Turnover		IVOL	ME	BE/ME	Ret2_7	Turnover
						194606-194906	-0.2533* (0.0659)	0.0009 (0.0077)		0.0962* (0.0109)	-0.0717* (0.0117)
194907-196112	0.0024 (0.0183)	0.0002 (0.0003)	0.0001 (0.0006)	0.0294* (0.0016)	-0.0009 (0.0020)	196201-196206	-0.1463*** (0.0798)	-0.0004 (0.0007)	0.0060** (0.0028)	0.0362* (0.0077)	-0.0399* (0.0078)
196207-196811	0.3207* (0.0126)	-0.0001 (0.0002)	-0.0004 (0.0008)	0.0104* (0.0014)	0.0003 (0.0013)	196812-197006	-0.1002* (0.0217)	0.0005 (0.0004)	-0.0167* (0.0017)	0.0205* (0.0028)	-0.0230* (0.0020)
197007-197212	-0.0607* (0.0125)	0.0007** (0.0003)	-0.0032* (0.0008)	-0.0259* (0.0016)	-0.0006 (0.0009)	197301-197409	-0.0607* (0.0131)	0.0008** (0.0004)	-0.0011*** (0.0006)	-0.0033 (0.0025)	-0.0064* (0.0018)
197410-198011	0.0659* (0.0038)	-0.0005** (0.0002)	-0.0017* (0.0002)	-0.0119* (0.0010)	0.0106* (0.0015)	198012-198207	-0.0070 (0.0126)	-0.0007** (0.0003)	0.0039* (0.0007)	-0.0100* (0.0019)	-0.0148* (0.0020)
198208-198708	0.0687* (0.0026)	0.0010* (0.0001)	0.0035* (0.0003)	0.0035* (0.0005)	-0.0023* (0.0005)	198709-198711	-0.0493** (0.0194)	0.0008 (0.0008)	0.0027*** (0.0014)	-0.0010 (0.0016)	-0.0075* (0.0018)
198712-200008	0.0804* (0.0012)	0.0003* (0.0000)	0.0003* (0.0000)	-0.0012* (0.0002)	0.0000 (0.0002)	200009-200209	-0.0079* (0.0021)	-0.0001* (0.0000)	0.0000 (0.0001)	0.0056* (0.0007)	-0.0115* (0.0003)
200210-200710	0.0116* (0.0007)	-0.0001* (0.0000)	0.0000 (0.0000)	0.0001 (0.0004)	-0.0006* (0.0001)	200711-200902	0.0000 (0.0026)	0.0001* (0.0000)	0.0007* (0.0002)	-0.0086* (0.0012)	-0.0009** (0.0004)
200903-201012	0.5043* (0.0072)	0.0001 (0.0001)	-0.0001 (0.0002)	-0.0134* (0.0007)	0.0030* (0.0004)						
Total bull period	0.0450* (0.0006)	0.0001* (0.0000)	0.0001* (0.0000)	-0.0012* (0.0001)	0.0013* (0.0001)	Total bear period	-0.0036* (0.0014)	0.0000 (0.0000)	0.0001** (0.0001)	0.0031* (0.0005)	-0.0065* (0.0002)

Table 1.6. Regressions of stock returns on idiosyncratic volatilities and firm characteristics for eight bull and bear market cycles (GJR-GARCH).

This table reports the regression results for eight bull and bear market cycles when using the GJR-GARCH(1,1) model to estimate idiosyncratic volatility. $\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{IVOL_{it}} + \gamma_2 \mathrm{ME_{it}} + \gamma_3 \mathrm{BE/ME_{it}} + \gamma_4 \mathrm{Ret2\_7_{it}} + \gamma_5 \mathrm{Turnover_{it}} + \varepsilon_{\mathrm{it}}$. ME is the market value of equity. BE/ME is the Book-to-market equity. Ret2_7 is the compound gross return from month t-7 to t-2. Turnover is the average turnover rate. The numbers in parentheses are the standard errors. The coefficients followed by * are significant at 1% level, followed by ** are significant at 5% level and followed by *** are significant at 10% level.

Bull						Bear					
	IVOL	ME	BE/ME	ret2_7	turnover		IVOL	ME	BE/ME	ret2_7	turnover
						194606-194906	-0.1806* (0.0629)	0.0029 (0.0077)		0.0980* (0.0109)	-0.0736* (0.0116)
194907-196112	-0.0122 (0.0177)	0.0001 (0.0003)	0.0001 (0.0006)	0.0294* (0.0016)	-0.0002 (0.0020)	196201-196206	-0.1413*** (0.0808)	-0.0004 (0.0007)	0.0060** (0.0028)	0.0363* (0.0077)	-0.0402* (0.0079)
196207-196811	0.1996* (0.0102)	-0.0003 (0.0002)	0.0000 (0.0008)	0.0131* (0.0014)	0.0049* (0.0013)	196812-197006	-0.0871* (0.0164)	0.0005 (0.0004)	-0.0166* (0.0017)	0.0203* (0.0028)	-0.0233* (0.0019)
197007-197212	-0.0659* (0.0112)	0.0006*** (0.0003)	-0.0032* (0.0008)	-0.0259* (0.0016)	-0.0005 (0.0009)	197301-197409	-0.0968* (0.0134)	0.0007*** (0.0004)	-0.0009 (0.0006)	-0.0037 (0.0025)	-0.0054* (0.0018)
197410-198011	0.0734* (0.0056)	-0.0004** (0.0002)	-0.0017* (0.0002)	-0.0119* (0.0010)	0.0103* (0.0015)	198012-198207	-0.0269** (0.0110)	-0.0007* (0.0003)	0.0039* (0.0007)	-0.0099* (0.0019)	-0.0139* (0.0020)
198208-198708	0.0087* (0.0034)	0.0007* (0.0001)	0.0033* (0.0003)	0.0036* (0.0005)	-0.0021* (0.0005)	198709-198711	-0.1123* (0.0195)	0.0003 (0.0008)	0.0027*** (0.0014)	-0.0007 (0.0016)	-0.0075* (0.0018)
198712-200008	0.0905* (0.0020)	0.0003* (0.0000)	0.0003* (0.0000)	-0.0013* (0.0002)	-0.0001 (0.0002)	200009-200209	-0.0946* (0.0052)	-0.0002* (0.0000)	0.0001 (0.0001)	0.0051* (0.0007)	-0.0099* (0.0004)
200210-200710	0.1223* (0.0032)	0.0000 (0.0000)	0.0000*** (0.0000)	-0.0009* (0.0004)	-0.0014* (0.0001)	200711-200902	-0.0544* (0.0077)	0.0001** (0.0000)	0.0006* (0.0002)	-0.0090* (0.0012)	-0.0005 (0.0004)
200903-201012	0.3524* (0.0086)	-0.0001 (0.0001)	-0.0002 (0.0002)	-0.0127* (0.0007)	0.0029* (0.0004)						
Total bull period	0.0908* (0.0014)	0.0001* (0.0000)	0.0002* (0.0000)	-0.0015* (0.0001)	0.0008* (0.0001)	Total bear period	-0.0668* (0.0035)	-0.0000 (0.0000)	0.0001** (0.0001)	0.0029* (0.0005)	-0.0054* (0.0002)

Table 1.7. Regressions of stock returns on idiosyncratic volatilities and market factors for eight bull and bear market cycles (EGARCH).

This table reports the regression results for eight bull and bear market cycles when using the EGARCH(1,1) model to estimate idiosyncratic volatility. $R_{it} = \gamma_0 + \gamma_1 IVOL_{it} + \gamma_2 (Mkt - r)_t + \gamma_3 SMB_t + \gamma_4 HML_t + \gamma_5 Mom_t + \varepsilon_{it}$. Mkt-r is the excess return on a broad market portfolio. SMB is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. Mom is the momentum factor. The numbers in parentheses are the standard errors. The coefficients followed by * are significant at 1% level, followed by ** are significant at 5% level and followed by *** are significant at 10% level.

Bull						Bear					
	IVOL	Mkt-r	SMB	HML	Mom		IVOL	Mkt-r	SMB	HML	Mom
						194606-194906	-0.0621* (0.0140)	1.0599* (0.0103)	0.6459* (0.0328)	0.1884* (0.0199)	-0.0535* (0.0191)
194907-196112	0.0607* (0.0060)	0.9872* (0.0054)	0.6797* (0.0111)	0.2616* (0.0079)	-0.0549* (0.0077)	196201-196206	-0.3295* (0.0334)	0.9698* (0.0373)	0.7153* (0.0714)	0.2021* (0.0660)	-0.1130* (0.0419)
196207-196811	0.2177* (0.0049)	0.9073* (0.0089)	0.9732* (0.0104)	0.2532* (0.0143)	-0.0632* (0.0096)	196812-197006	-0.1030* (0.0108)	0.9773* (0.0121)	0.9989* (0.0183)	0.1447* (0.0204)	-0.1401* (0.0187)
197007-197212	0.0315* (0.0083)	0.8835* (0.0139)	1.0848* (0.0176)	0.2920* (0.0199)	-0.0277** (0.0132)	197301-197409	-0.0016 (0.0035)	0.9461* (0.0132)	0.9447* (0.0283)	0.4538* (0.0248)	-0.1353* (0.0228)
197410-198011	0.1165* (0.0034)	1.0636* (0.0066)	0.9681* (0.0109)	0.4177* (0.0113)	-0.1195* (0.0080)	198012-198207	-0.0530* (0.0098)	1.0341* (0.0208)	0.7555* (0.0274)	0.1844* (0.0268)	-0.0850* (0.0128)
198208-198708	0.0224* (0.0009)	1.0062* (0.0079)	1.0488* (0.0148)	0.1345* (0.0136)	-0.0449* (0.0093)	198709-198711	-0.0689* (0.0126)				
198712-200008	2.40E-7 (1.77E-6)	0.9014* (0.0051)	0.7675* (0.0054)	0.2650* (0.0080)	-0.2017* (0.0051)	200009-200209	0.0229* (0.0016)	0.7390* (0.0118)	0.7031* (0.0139)	0.0872* (0.0128)	-0.3265* (0.0082)
200210-200710	0.0136* (0.0006)	0.8941* (0.0094)	0.7780* (0.0108)	0.2407* (0.0116)	-0.1418* (0.0061)	200711-200902	0.0167* (0.0021)	0.9852* (0.0112)	0.4777* (0.0285)	-0.2524* (0.0165)	-0.1872* (0.0107)
200903-201012	0.0013* (0.0003)	0.6974* (0.0373)	0.5119* (0.0262)	0.0084 (0.0366)	-0.2589* (0.0125)						
Total bull period	0.0195* (0.0011)	0.9227* (0.0048)	0.7381* (0.0077)	0.0821* (0.0061)	-0.1954* (0.0041)	Total bear period	-3.96E-8 (1.51E-6)	0.9445* (0.0028)	0.8017* (0.0033)	0.2776* (0.0042)	-0.1518* (0.0026)

Table 1.8. Regressions of stock returns on idiosyncratic volatilities and market factors for eight bull and bear market cycles (GJR-GARCH).

This table reports the regression results for eight bull and bear market cycles when using the GJR-GARCH(1,1) model to estimate idiosyncratic volatility. $\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{IVOL_{it}} + \gamma_2(\mathrm{Mkt} - \mathrm{r})_\mathrm{t} + \gamma_3 \mathrm{SMB_t} + \gamma_4 \mathrm{HML_t} + \gamma_5 \mathrm{Mom_t} + \varepsilon_\mathrm{it}$. Mkt-r is the excess return on a broad market portfolio. SMB is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. Mom is the momentum factor. The numbers in parentheses are the standard errors. The coefficients followed by * are significant at 1% level, followed by ** are significant at 5% level and followed by *** are significant at 10% level.

Bull						Bear					
	IVOL	Mkt-r	SMB	HML	Mom		IVOL	Mkt-r	SMB	HML	Mom
						194606-194906	-0.1405* (0.0149)	1.0603* (0.0103)	0.6454* (0.0328)	0.1893* (0.0199)	-0.0553* (0.0191)
194907-196112	0.0286* (0.0064)	0.9873* (0.0054)	0.6804* (0.0111)	0.2622* (0.0079)	-0.0552* (0.0077)	196201-196206	-0.3254* (0.0338)	0.9699* (0.0373)	0.7167* (0.0714)	0.2014* (0.0660)	-0.1142* (0.0419)
196207-196811	0.1293* (0.0046)	0.9039* (0.0089)	0.9815* (0.0104)	0.2548* (0.0143)	-0.0645* (0.0097)	196812-197006	-0.1254* (0.0098)	0.9772* (0.0121)	0.9982* (0.0183)	0.1446* (0.0203)	-0.1405* (0.0187)
197007-197212	-0.0005 (0.0080)	0.8835* (0.0139)	1.0857* (0.0176)	0.2920* (0.0199)	-0.0276** (0.0132)	197301-197409	-0.0399* (0.0092)	0.9453* (0.0132)	0.9481* (0.0283)	0.4536* (0.0248)	-0.1335* (0.0228)
197410-198011	0.1295* (0.0046)	1.0632* (0.0066)	0.9684* (0.0109)	0.4175* (0.0113)	-0.1191* (0.0080)	198012-198207	-0.0660* (0.0089)	1.0339* (0.0208)	0.7553* (0.0274)	0.1840* (0.0268)	-0.0852* (0.0128)
198208-198708	0.0476* (0.0026)	1.0071* (0.0079)	1.0501* (0.0148)	0.1357* (0.0136)	-0.0454* (0.0093)	198709-198711	-0.1482* (0.0140)				
198712-200008	0.1115* (0.0015)	0.8995* (0.0051)	0.7664* (0.0053)	0.2679* (0.0080)	-0.2021* (0.0051)	200009-200209	-0.0182* (0.0042)	0.7386* (0.0118)	0.7046* (0.0139)	0.0919* (0.0128)	-0.3307* (0.0082)
200210-200710	0.1097* (0.0023)	0.8987* (0.0093)	0.7671* (0.0108)	0.2409* (0.0116)	-0.1358* (0.0061)	200711-200902	-0.0160* (0.0050)	0.9828* (0.0112)	0.4847* (0.0285)	-0.2601* (0.0166)	-0.1896* (0.0107)
200903-201012	0.3120* (0.0066)	0.6938* (0.0366)	0.5170* (0.0258)	0.0314 (0.0360)	-0.2382* (0.0123)						
Total bull period	0.1039* (0.0010)	0.9435* (0.0028)	0.8051* (0.0033)	0.2840* (0.0042)	-0.1516* (0.0026)	Total bear period	-0.0202* (0.0024)	0.9193* (0.0048)	0.7482* (0.0078)	0.0867* (0.0062)	-0.1967* (0.0042)

1.4.4. Relations between stock returns and idiosyncratic volatilities for forty-eight industries

For further examination and as a robustness check, we divide all the stocks into forty-eight industries as stated in Fama and French (1997). We regress the stock returns on idiosyncratic volatilities and other firm characteristics for these forty-eight industries in total bull period and total bear period, separately. The model is a multivariate regression presented in Equation (1.6): $\mathrm{R_{it}} = \gamma_0 + \gamma_1 \mathrm{IVOL_{it}} + \gamma_2 \mathrm{ME_{it}} + \gamma_3 \mathrm{BE/ME_{it}} + \gamma_4 \mathrm{Ret2\_7_{it}} + \gamma_5 \mathrm{Turnover_{it}} + \varepsilon_{it}$. To make the test results more intuitive, we put all the coefficient estimates of idiosyncratic volatility in two scatter charts.[5] One scatter chart is for coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model, while the other is for coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. The two scatter charts are shown in Figure 1.3 and Figure 1.4, respectively. Industries' names are selectively shown due to limited space.

In total bull period, all the forty-eight coefficient estimates of idiosyncratic volatility estimated by the EGARCH(1,1) model are positive and most of them are significant at the 1% level. These significant positive coefficient estimates support that the idiosyncratic risk is priced during bull market periods. In total bear period, the signs of coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model are mixed.

[5] To save space, the tables containing the test results of Equation (1.6) for forty-eight industries are not provided in this paper. The tables will be available upon request.

About half of the coefficient estimates are positive and half of them are negative. These mixed coefficient estimates support that the idiosyncratic risk is not priced during bear market periods.

Moreover, we find that during total bull period, except for four industries, all the other forty-four coefficient estimates of idiosyncratic volatility estimated by the GJR-GARCH(1,1) are positive. During total bear period, the signs of coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model are mixed. About one fourth of the coefficient estimates are positive and three fourths of them are negative. Although the distribution of the coefficient estimates of the GJR-GARCH(1,1) is a little different from the distribution of the coefficients of the EGARCH(1,1), they both confirm the finding that the idiosyncratic risk is priced during bull market periods but not during bear market periods.

The phenomenon that the “size effect” no longer exists after controlling for idiosyncratic risk is still apparent. The coefficients of ME become mostly insignificant after including IVOL in the regression. Controlling for estimated idiosyncratic volatility, “size effect” is no longer there. The evidence is consistent in forty-eight industries and lends a support to Merton’s (1987) argument that the findings of the “size effect” are due to the omitted controls for other factors such as idiosyncratic risk.

Next, we regress the stock returns on idiosyncratic volatilities and market factors for forty-eight industries in the total bull period and the total bear period, separately. The model is a multivariate regression presented in Equation (1.7): $R_{it} = \gamma_0 + \gamma_1 IVOL_{it} + \gamma_2(Mkt - r)_t + \gamma_3 SMB_t + \gamma_4 HML_t + \gamma_5 Mom_t + \varepsilon_{it}$. To make the test results more intuitive, we also put all the coefficient estimates of idiosyncratic volatility in two scatter charts.[6] One scatter chart is for coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model, while the other is for coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. The two scatter charts are shown in Figure 1.5 and Figure 1.6, respectively. Again, industries' names are selectively shown due to limited space.

It turns out that in the total bull period, all the forty-eight coefficient estimates of idiosyncratic volatility estimated by the EGARCH(1,1) are positive and most of them are significant at the 1% level. These significant positive coefficient estimates prove one more time that the idiosyncratic risk is priced during bull market periods. In the total bear period, the signs of coefficients of idiosyncratic volatility estimated by the EGARCH(1,1) model are mixed. Around sixty percent of the coefficient estimates are positive, while forty percent are negative. These mixed coefficient estimates are weaker evidence, but could still support our finding that the idiosyncratic risk is not priced during bear market periods.

Additionally, we also check the relation between stock return and idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. We find that during total bull period, except

[6] To save space, the tables containing the test results of Equation (1.7) for forty-eight industries are not provided in this paper. The tables will be available upon request.

for only two industries, all the coefficient estimates of idiosyncratic volatility estimated by the GJR-GARCH(1,1) are positive and all positive coefficients are significant at the 1% level. In total bear period, the signs of the coefficients of idiosyncratic volatility estimated by the GJR-GARCH(1,1) model are mixed. About half of the coefficient estimates are positive, while half of them are negative.

To sum up, the test results for forty-eight industries are consistent with those for eight bull and bear market cycles. The results for bull market periods all confirm Merton's (1987) prediction of a positive relation between idiosyncratic risk and expected return. However, the results for bear market periods do not. It appears that idiosyncratic risk is not priced during bear markets. According to Merton (1987) and Seasholes and Wu (2007), we provide a hypothesis to explain our findings. If investors hold less diversified portfolios during bull market, idiosyncratic risk should be priced in bull periods; if investors tend to hold more diversified portfolios during bear market, then idiosyncratic risk will not be priced in bear periods. Until now, our empirical evidences support this hypothesis.

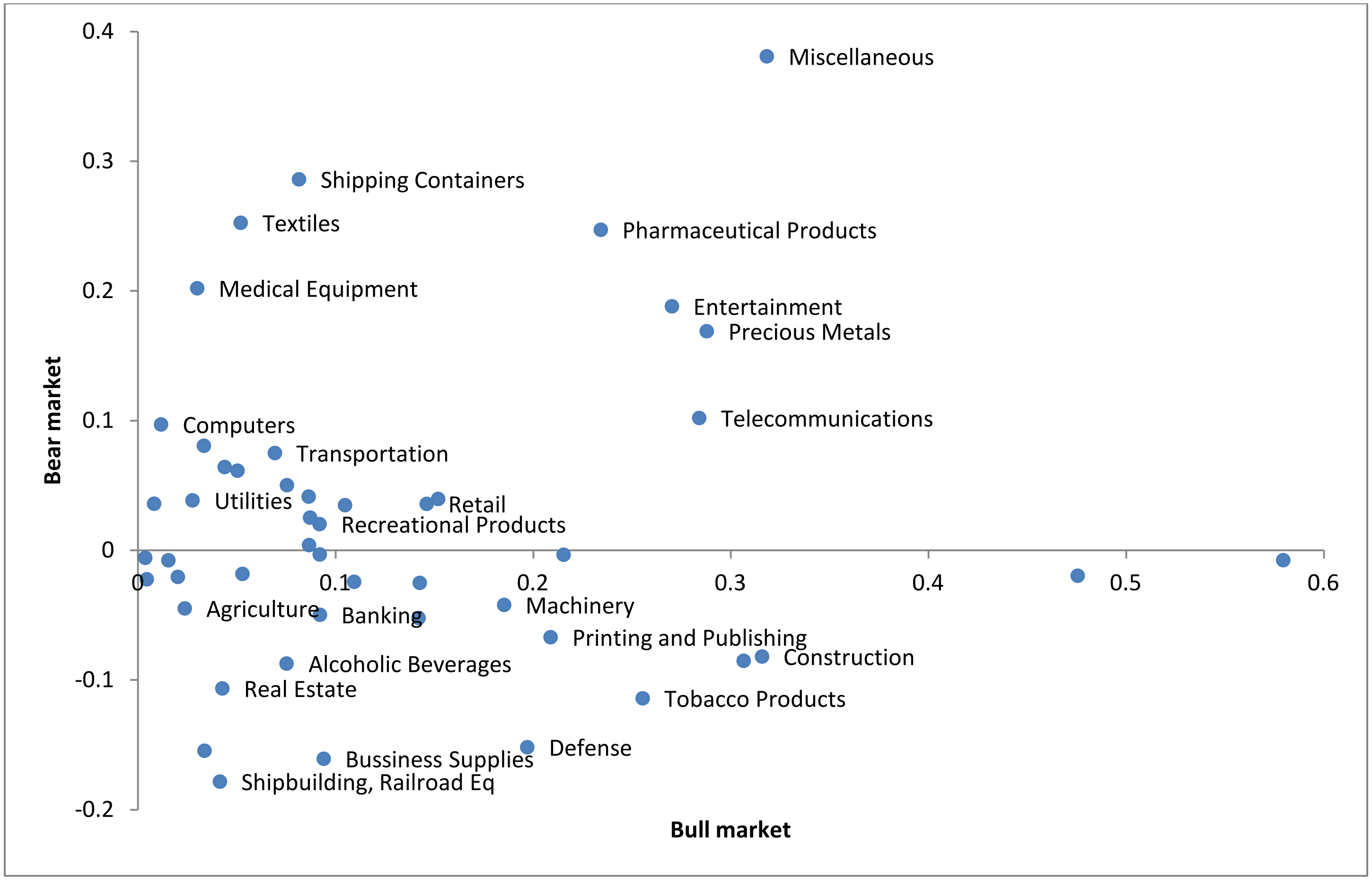


Figure 1.3. Regressions of stock returns on idiosyncratic volatilities and firm characteristics for forty-eight industries (EGARCH).

This figure reports the regression results when using idiosyncratic volatility estimated by the EGARCH(1,1) model. $R_{it} = \gamma_0 + \gamma_1 E_{t-1}[IVOL_{it}] + \gamma_2 ME_{it} + \gamma_3 BE/ME_{it} + \gamma_4 Ret2\_7_{it} + \gamma_5 Turnover_{it} + \varepsilon_{it}$. The scatter shows only the coefficients of idiosyncratic volatilities. Industries' names are selectively shown due to limited space.

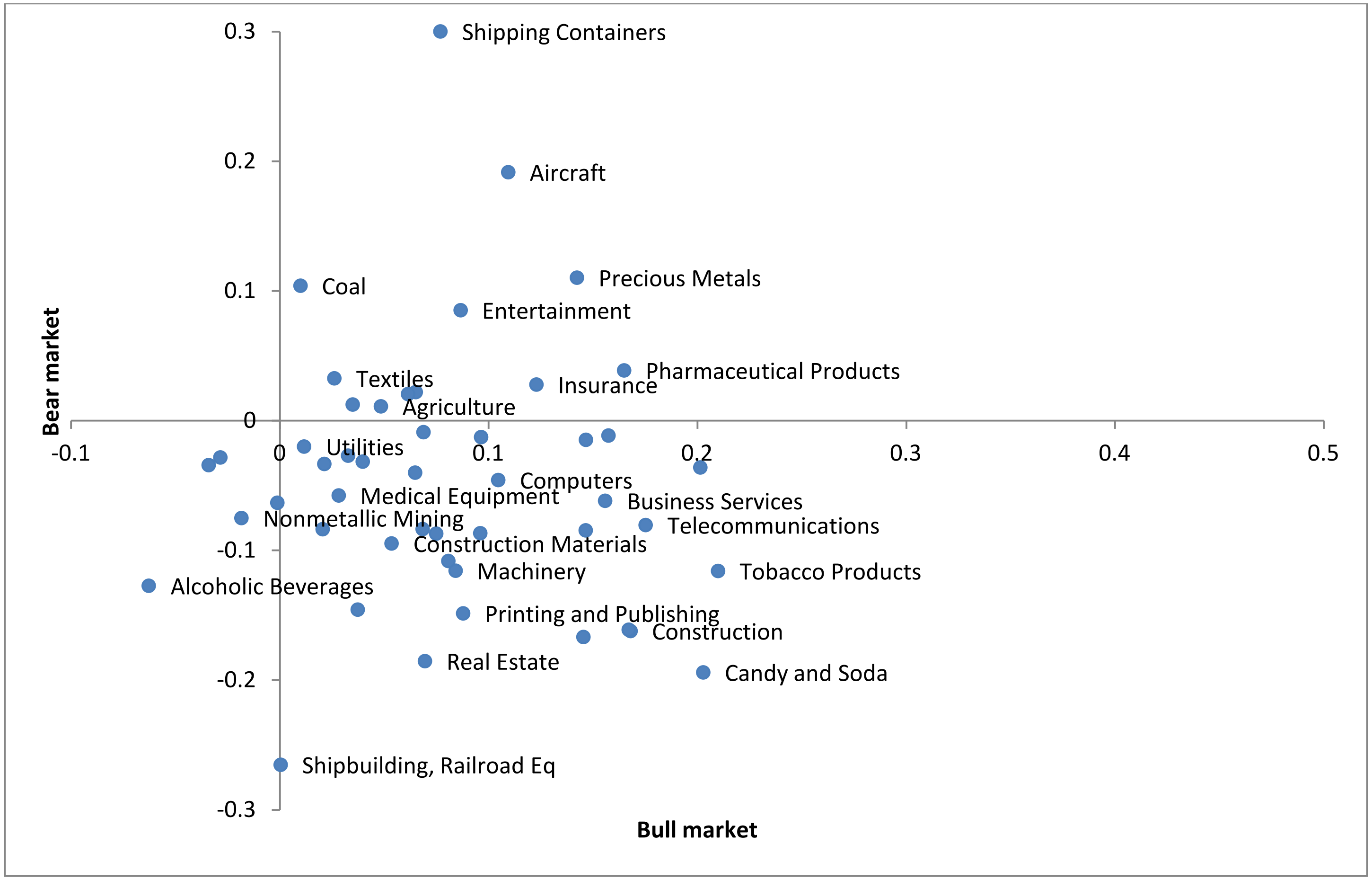


Figure 1.4. Regressions of stock returns on idiosyncratic volatilities and firm characteristics for forty-eight industries (GJR-GARCH).

This figure reports the regression results when using idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. $R_{it} = \gamma_0 + \gamma_1 E_{t-1}[IVOL_{it}] + \gamma_2 ME_{it} + \gamma_3 BE/ME_{it} + \gamma_4 Ret2\_7_{it} + \gamma_5 Turnover_{it} + \varepsilon_{it}$. The scatter shows only the coefficients of idiosyncratic volatilities. Industries' names are selectively shown due to limited space.

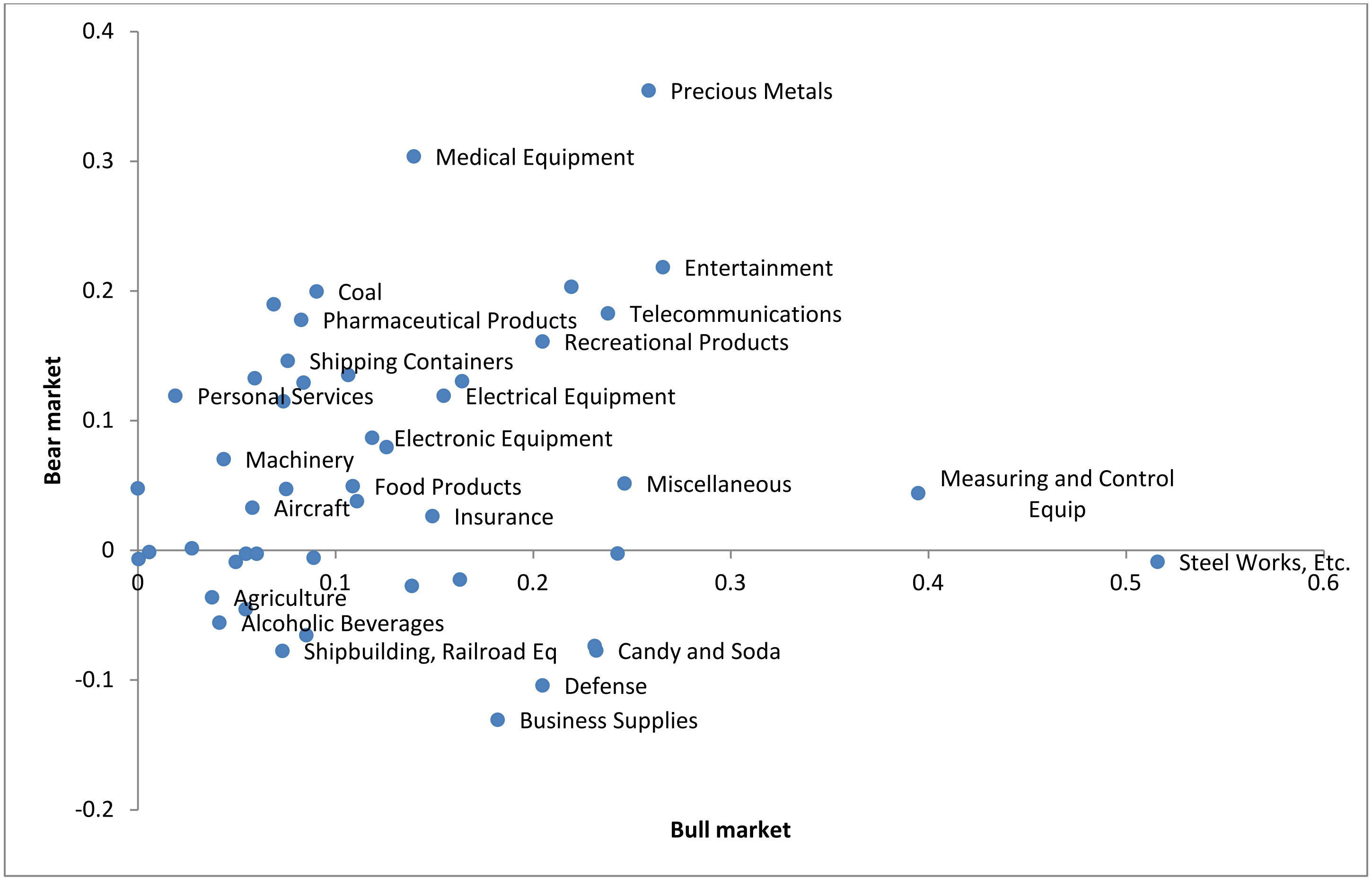


Figure 1.5. Regressions of stock returns on idiosyncratic volatilities and market factors for forty-eight industries (EGARCH).

This figure reports the regression results when using idiosyncratic volatility estimated by the EGARCH(1,1) model. $R_{it} = \gamma_0 + \gamma_1 E_{t-1}[IVOL_{it}] + \gamma_2 (Mkt - r)_t + \gamma_3 SMB_t + \gamma_4 HML_t + \gamma_5 Mom_t + \varepsilon_{it}$. The scatter shows only the coefficients of idiosyncratic volatilities. Industries' names are selectively shown due to limited space.

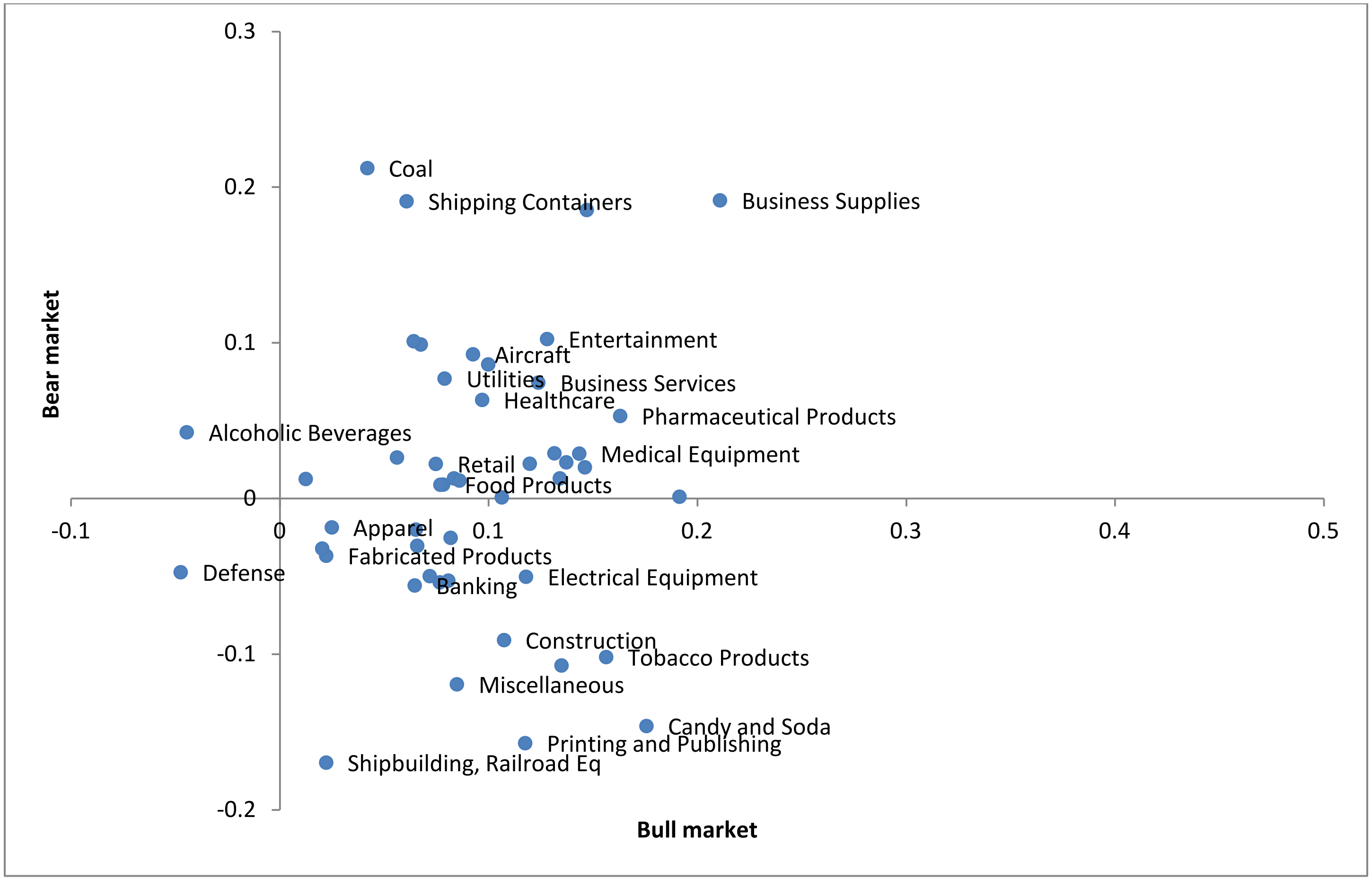


Figure 1.6. Regressions of stock returns on idiosyncratic volatilities and market factors for forty-eight industries (GJR-GARCH).

This figure reports the regression results when using idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. $R_{it} = \gamma_0 + \gamma_1 E_{t-1}[IVOL_{it}] + \gamma_2(Mkt - r)_t + \gamma_3 SMB_t + \gamma_4 HML_t + \gamma_5 Mom_t + \varepsilon_{it}$. The scatter shows only the coefficients of idiosyncratic volatilities. Industries' names are selectively shown due to limited space.

1.4.5. Portfolios analysis

The evidences from the Fama-MacBeth cross-sectional regressions suggest a positive relation between idiosyncratic volatility and average stock returns in bull market periods but a negative one in bear market periods. Next we examine the returns of portfolios formed on the sorting of idiosyncratic volatility. If individual stocks with high idiosyncratic volatility have higher returns than stocks with low idiosyncratic volatility in bull market periods, a zero-investment portfolio that is long in high idiosyncratic volatility stocks and short in low idiosyncratic volatility stocks should earn a positive return in bull market periods. But the same zero-investment portfolio will even lose money in bear market periods, since individual stocks with high idiosyncratic volatility have lower returns than stocks with low idiosyncratic volatility in bear market periods. The procedure of the portfolio-based approach is as follows. In each month, we sort IVOL (EGARCH) to form ten portfolios with an equal number of stocks. Each portfolio contains ten percent of stocks. Table 1.9 presents the descriptive statistics for these ten portfolios. Panel A, B, and C presents, respectively, the summary statistics for bull market periods, bear market periods, and the entire sample period. The third and fourth row presents, respectively, the time-series means of the value-weighted and the equal-weighted portfolio returns. The mean IVOL (EGARCH) increases from 3.64% for the first portfolio to 65.11% for the last portfolio. The idiosyncratic volatility estimated by the GJR-GARCH(1,1) model also increases monotonically across these ten portfolios.

In bull market periods, the portfolio consisting of stocks with high idiosyncratic volatility has higher returns than the portfolio consisting of low idiosyncratic volatility stocks. The value-weighted portfolio returns increase from 1.86% for the lowest idiosyncratic volatility portfolio to 12.14% for the highest idiosyncratic volatility portfolio. A hedging portfolio longing Portfolio 10 and shorting Portfolio 1 yields a statistically significant monthly return of 10.28%. The equal-weighted portfolio returns display a similar pattern, increasing from 0.39% for the lowest idiosyncratic volatility portfolio to 5.09% for the highest idiosyncratic volatility portfolio. This evidence confirms the positive relation between idiosyncratic volatility and individual stock returns in bull market periods, which means that the idiosyncratic risk is priced in bull markets. This result is consistent with the portfolio analysis findings of Fu (2009) and further confirms that firms with high idiosyncratic volatility have higher expected returns indicated by Merton (1987).

In bear market periods, there is no obvious trend of portfolio returns when idiosyncratic volatility increases. The value-weighted portfolio returns go up and down frequently when idiosyncratic volatility portfolio changes. The equal-weighted portfolio returns decrease from -0.28% for the lowest idiosyncratic volatility portfolio to -3.75% for the second highest idiosyncratic volatility portfolio (portfolio 9), and then increase to -1.35% for the highest idiosyncratic volatility portfolio. This evidence confirms the negative or insignificant relation between idiosyncratic volatility and individual stock returns in bear market periods, which implies that the idiosyncratic risk is not priced in bear markets.

For the entire sample period, the trend of portfolio returns is more like the trend in bull market periods with some noticeable differences. The value-weighted portfolio returns decrease from 1.43% for the lowest idiosyncratic volatility portfolio to 1.26% for the second lowest idiosyncratic volatility portfolio (portfolio 2), and then increase monotonically to 10.38% for the highest idiosyncratic volatility portfolio from there. The equal-weighted portfolio returns tell a different story. They increase first and then decrease slowly, but finally increase dramatically for the highest idiosyncratic volatility portfolio. The phenomenon that the value-weighted portfolio returns follow the trend of bull market periods is due to the fact that bull market periods dominate the whole sample period. Our evidence for the entire sample period weakly confirms the positive relation between idiosyncratic volatility and individual stock returns found in the literature (Malkiel and Xu (2002), Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Fu (2009), Huang, Liu, Rhee, and Zhang (2010)). However, this result contrasts sharply with the findings of AHXZ (2006) which are based on the lagged realized volatility and the findings of Guo and Savickas (2006) which are based on the future quarterly stock market returns. Recent studies by Huang, Liu, Rhee, and Zhang (2007) and Fu (2009) both suggest that the return reversal in monthly returns explains the negative results in AHXZ (2006) and Guo and Savickas (2008).

Table 1.9. Summary statistics for portfolios formed on idiosyncratic volatility.

Each month ten portfolios are formed on IVOL (EGARCH). Each portfolio consists of 10% of stocks. IVOL (EGARCH) is estimated by the EGARCH(1,1) model. IVOL (GJR-GARCH) is estimated by the GJR-GARCH(1,1) model. The third and fourth row presents, respectively, the time-series means of the value-weighted and the equal-weighted portfolio returns. Other rows show the pooled means of variables within the particular portfolio. Panel A, B, and C presents, respectively, the summary statistics for bull market periods, bear market periods, and the entire sample period.

Panel A: Bull market periods										
	Low	2	3	4	5	6	7	8	9	High
IVOL (EGARCH)	3.65	5.60	6.91	8.18	9.57	11.15	13.00	15.33	18.82	71.62
IVOL (GJR-GARCH)	3.69	5.70	7.05	8.37	9.82	11.44	13.33	15.69	19.19	31.10
VWRET	1.86	1.86	2.13	2.50	3.00	3.60	4.24	4.81	7.15	12.14
EWRET	0.39	1.41	1.49	1.54	1.53	1.57	1.45	1.48	1.75	5.09
ME (Billion)	1.01	2.29	1.95	1.34	0.85	0.57	0.38	0.25	0.17	0.10
BE/ME	0.81	0.73	1.37	0.76	0.77	0.80	0.78	0.75	0.69	0.44
Panel B: Bear market periods										
	Low	2	3	4	5	6	7	8	9	High
IVOL (EGARCH)	3.61	5.65	7.01	8.33	9.77	11.36	13.18	15.50	18.92	31.75
IVOL (GJR-GARCH)	3.56	5.68	7.07	8.44	9.93	11.58	13.47	15.78	19.14	29.84
VWRET	-1.13	-0.94	-1.07	-1.40	-1.23	-1.01	-1.66	-2.08	-0.88	3.73
EWRET	-0.28	-1.06	-1.32	-1.72	-2.10	-2.50	-3.02	-3.55	-3.75	-1.35
ME (Billion)	0.47	3.62	3.24	2.49	1.85	1.23	0.86	0.57	0.43	0.26
BE/ME	0.88	0.74	0.73	0.76	0.76	1.05	0.84	0.85	0.81	0.76
Panel C: Entire sample period										
	Low	2	3	4	5	6	7	8	9	High
IVOL (EGARCH)	3.64	5.61	6.92	8.21	9.60	11.18	13.03	15.35	18.84	65.11
IVOL (GJR-GARCH)	3.67	5.70	7.05	8.38	9.83	11.46	13.36	15.70	19.18	30.90
VWRET	1.43	1.26	1.42	1.58	1.90	2.44	2.80	3.18	5.18	10.38
EWRET	0.25	1.01	1.03	1.01	0.94	0.90	0.72	0.66	0.86	4.04
ME (Billion)	0.89	2.51	2.16	1.52	1.01	0.68	0.46	0.31	0.21	0.13
BE/ME	0.83	0.73	1.26	0.76	0.77	0.84	0.79	0.76	0.71	0.49

1.4.6. Idiosyncratic Risk Factor

From the portfolios analysis in the last subsection, we get an idea that a zero-investment portfolio that is long in high idiosyncratic volatility stocks and short in low idiosyncratic volatility stocks should earn a positive return in bull market periods, but will lose money in bear market periods. To better see if idiosyncratic risk is priced, that is to say if stocks with high idiosyncratic risk have tended to do better than stocks with low idiosyncratic risk, we attempt to construct a factor in the way Fama and French construct their factors. Our factor will be calculated with combinations of portfolios composed by ranked stocks. The specific measures are introduced in Subsection 1.3.3.

We get a number of Idiosyncratic Risk Factor (IRF) for each month. We cumulate all IRF within each bull and bear market period and the total bull and bear period. We compare the IRF under different market conditions. We call the Cumulative Idiosyncratic Risk Factor CIRF. And we also calculate the average Monthly IRF by dividing CIRF by the number of months in that period to make it easier to interpret the numbers. According to different weighting schemes and different models to estimate idiosyncratic volatility, the IRF results are reported in four separate tables, from Table 1.10 to Table 1.13.

Table 1.10 reports Cumulative IRF and Monthly IRF when using value-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model. Panel A shows IRF calculated by the Fama-French measure (FF), i.e. two size portfolios and three idiosyncratic volatility portfolios. Panel B shows IRF calculated by refined measure one (R1), i.e. three size portfolios and three idiosyncratic volatility portfolios. Panel C shows IRF calculated by refined measure two (R2), i.e. five size portfolios and three idiosyncratic volatility portfolios. Whichever measure we use to form the portfolios, we see positive Cumulative IRF and Monthly IRF in all bull market periods, including the total bull period. Taking the total bull period in Panel A as an example, a factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an average monthly return of 2.35% with a standard deviation of 0.22%. For bear market periods, most of Cumulative IRF and Monthly IRF is negative, including the total bear period. Only one bear period shows a positive Cumulative IRF and Monthly IRF but the number becomes smaller when we apply two refined measures to form the portfolios. However, these small positive numbers do not affect our conclusion that idiosyncratic risk is not priced in bear market periods. Taking the total bear period in Panel A for example, a factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an average monthly return of -0.65% with a standard deviation of 0.51%. This means if we continue following the same strategy that earns us 2.35% per month previously mentioned, in a bear market period we will lose 0.65% per month. So, our findings along with IRF itself may have important implications for equity investment strategies and portfolio management. Although there are some minor differences among the IRF results by those three measures, they are still very similar in trends. We can see this same trend from Figure 1.8. The first figure (left top) in Figure 1.8 shows CIRF for the entire sample period in Panel A, Panel B, and Panel C of Table 1.10. We can see that the curves are almost coincided with each other. These curves imply that when we use value-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model to calculate IRF, different portfolio constructing measures produce similar results.

Table 1.11 reports Cumulative IRF and Monthly IRF when using equal-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model. Panel A shows IRF calculated by Fama-French measure (FF). Panel B shows IRF calculated by refined measure one (R1). Panel C shows IRF calculated by refined measure two (R2). The IRF results in Table 1.11 follow the same pattern of those in Table 1.10. We get positive Cumulative IRF and Monthly IRF in all bull market periods, including total bull period, but get negative Cumulative IRF and Monthly IRF in bear market periods, including the total bear period. However, the IRF numbers are smaller than those using value-weighted portfolios, which means that we will get lower portfolio return if we form an equal-weighted portfolio other than a value-weighted one. Let's take the total bull period in Panel A as an example. An equal-weighted factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an average monthly return of 1.96%, lower than 2.35% for a value-weighted portfolio. In total bear period, an equal-weighted factor portfolio yields an average monthly return of -0.92%, lower than -0.65% for a value-weighted portfolio. The three curves are coincided in the first figure in Figure 1.8, but the curves in the second figure (right top) are not. The second figure in Figure 1.8 shows CIRF for the entire sample period in Panel A, Panel B, and Panel C of Table 1.11. There are some differences among the IRF results by those three measures. And we can see that the three curves are apart from each other. These apart curves imply that when we use equal-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model to calculate IRF, the portfolio constructing measure matters a little bit. Refined measure two gives the highest factor portfolio return, while Fama-French measure provides the lowest factor portfolio return.

Table 1.12 presents Cumulative IRF and Monthly IRF when using value-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. Panel A shows IRF calculated by the Fama-French measure (FF), and Panel B shows IRF calculated by the refined measure one (R1), while Panel C shows IRF calculated by the refined measure two (R2). The IRF results in Table 1.12 follow the same pattern of those in previous tables. We get positive Cumulative IRF and Monthly IRF in all bull market periods, including the total bull period, and get negative Cumulative IRF and Monthly IRF in all bear market periods, including the total bear period. The pattern in Table 1.12 is more pronounced than that in Table 1.10 and Table 1.11. All the Cumulative IRF and Monthly IRF is negative, no exception for all three measures. In addition, the IRF numbers are smaller than those in Table 1.10 using value-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model. It means that we will get lower portfolio return if we form the same portfolio but use the GJR-GARCH idiosyncratic volatility. For instance, a value-weighted factor portfolio long on stocks with high idiosyncratic volatility estimated by the GJR-GARCH(1,1) model and short on stocks with low idiosyncratic volatility yields an average monthly return of 2.02% in Fama-French measure case, lower than 2.35% for a value-weighted portfolio with idiosyncratic volatility estimated by the EGARCH(1,1) model. This rule also applies towards bear market periods, in which case a value-weighted factor portfolio yields an average monthly return of -0.85% for the GJR-GARCH idiosyncratic volatility, lower than -0.65% for the EGARCH idiosyncratic volatility. Although there are still some small differences among the IRF results by the three portfolio constructing measures, they are very similar to each other. We can see this same trend from the third figure in Figure 1.8. The third figure (left bottom) in Figure 1.8 shows CIRF for the entire sample period in Panel A, Panel B, and Panel C of Table 1.12. We can see that the curves are coincided with each other almost perfectly. These coincided curves imply that when we use value-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model to calculate IRF, the portfolio constructing measure does not matter that much. Together with the finding in Table 1.10 and the first figure in Figure 1.8, we can conclude that when we use value-weighted portfolios, no matter which model we choose to estimate idiosyncratic volatility, to calculate IRF, the three portfolio constructing measures do not differ so much.

Table 1.13 presents Cumulative IRF and Monthly IRF when using equal-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. Panel A shows IRF calculated by the Fama-French measure (FF). Panel B shows IRF calculated by the refined measure one (R1), and Panel C shows IRF calculated by the refined measure two (R2). We see positive Cumulative IRF and Monthly IRF in all bull market periods, but see negative Cumulative IRF and Monthly IRF in all bear market periods. As that in Table 1.12, all the Cumulative IRF and Monthly IRF in Table 1.13 are negative. Additionally, the IRF numbers are smaller than those in Table 1.11 using equal-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model, which means that we will get lower portfolio return if we form the same portfolio but using the GJR-GARCH idiosyncratic volatility. This finding is consistent with what we have already found for value-weighted portfolios. For instance, an equal-weighted factor portfolio long on stocks with high idiosyncratic volatility estimated by the GJR-GARCH(1,1) model and short on stocks with low idiosyncratic volatility yields an average monthly return of 1.59%, lower than 1.96% for an equal-weighted portfolio with idiosyncratic volatility estimated by the EGARCH(1,1) model. This rule also applies to bear market periods. An equal-weighted factor portfolio yields an average monthly return of -1.29% for the GJR-GARCH idiosyncratic volatility, lower than -0.92% for the EGARCH idiosyncratic volatility. As before, the last figure (right bottom) in Figure 1.8 shows CIRF for the entire sample period in Panel A, Panel B, and Panel C of Table 1.13. There are some differences among the IRF results by the three portfolio constructing measures. And we can see the differences in the figure that the three curves are apart from each other. These separate curves mean that when we use equal-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model to calculate IRF, the portfolio constructing measure matters a little bit. Refined measure two gives the highest factor portfolio return, while Fama-French measure provides the lowest factor portfolio return. This finding is consistent with what we have already found for equal-weighted portfolios with the EGARCH idiosyncratic volatility. Therefore, we conclude that when we use equal-weighted portfolios, no matter which model we choose to estimate idiosyncratic volatility, different choice of the portfolio constructing measures will affect the value of IRF. And the more size portfolios we construct to calculate IRF, the higher factor portfolio return we will get.

We put all twelve Cumulative Idiosyncratic Risk Factor (CIRF) for the entire sample period from June 1946 to December 2010 in one figure. We attempt to get a general idea that if the CIRF follows different trends when the market conditions are different. And we also try to get that if the CIRF differs from each other when we employ different techniques to calculate IRF. The whole view picture is shown in Figure 1.7. Each of the twelve curves is calculated based on one of the two models to estimate idiosyncratic volatility (the EGARCH model and the GJR-GARCH model), one of the two weighting schemes (value-weighted and equal-weighted), and one of the three portfolio constructing measures (Fama-French measure, Refined measure 1 and Refined measure 2). From Figure 1.7, we could easily see three things. First, bull market periods dominate the entire sample period. Bear market periods only account for less than twenty percent of the entire sample period. Second, the trends during bull market periods are all going upwards, while the trends during bear market periods are relatively flat or going downwards. These trends support the main finding of this paper that idiosyncratic risk is priced in bull market periods but not in bear market periods. Third, although the twelve curves have the same trend pattern, they have different values which imply different factor portfolio returns. The factor portfolios constructed using value-weighted weighting scheme with the EGARCH idiosyncratic volatility bring the highest returns, while the lowest returns are from the factor portfolios constructed using equal-weighted weighting scheme with the GJR-GARCH idiosyncratic volatility.

We have got a general idea from Figure 1.7 that the CIRF calculated by different techniques have different values. In order to dig out their variation rules, we must compare them side by side. Keeping the weighting schemes the same and the models for estimating idiosyncratic volatility the same, we could tell how the three portfolio constructing measures differ from each other. This analysis has already been done in previous paragraphs from the four figures in Figure 1.8, and we conclude that when we use value-weighted portfolios, no matter which model we choose to estimate idiosyncratic volatility, to calculate IRF, the three portfolio constructing measures do not differ so much. But when we use equal-weighted portfolios to calculate IRF, the three portfolio constructing measures do matter. And the more size portfolios we construct to calculate IRF, the higher factor portfolio return we will get.

Furthermore, keeping portfolio constructing measure and model for estimating idiosyncratic volatility the same, we could tell how the two weighting schemes differ from each other. We will give the analysis from the six figures in Figure 1.9. In each figure, keeping other techniques the same, we compare if value-weighted portfolio and equal-weighted portfolio have the same CIRF. We could get three conclusions from these six figures. First, no matter which model we choose to estimate idiosyncratic volatility and which measure to construct the portfolio, the value-weighted portfolios always get higher returns than the equal-weighted portfolios do. Second, the portfolio constructing measure will affect the difference of returns between value-weighted portfolios and equal-weighted portfolios. And the more size portfolios we construct to calculate IRF, the smaller the difference of factor portfolio returns will be. When refined measure two is applied, the return of value-weighted portfolio does not differ so much from the return of equal-weighted portfolio. Third, the models for estimating idiosyncratic volatility have nothing to do with the difference of returns between value-weighted portfolios and equal-weighted portfolios.

Last, keeping portfolio constructing measure and weighting scheme the same, we could tell how the two models for estimating idiosyncratic volatility differ from each other. We will give the analysis based on the six figures in Figure 1.10. In each figure, we compare if the EGARCH idiosyncratic volatility portfolio and the GJR-GARCH idiosyncratic volatility portfolio have the same CIRF when others are the same. And we could get two conclusions from these six figures. First, no matter which weighting scheme and which measure we apply to construct the portfolio, the EGARCH idiosyncratic volatility portfolios always get higher returns than the GJR-GARCH idiosyncratic volatility portfolios do. This finding is consistent with our previous finding from Fama-MacBeth regressions that the relation between stock returns and the GJR-GARCH idiosyncratic volatilities is more negative than the relation between stock returns and the EGARCH idiosyncratic volatilities. Second, both the weighting schemes and the portfolio constructing measures have nothing to do with the difference of returns between the EGARCH idiosyncratic volatility portfolios and the GJR-GARCH idiosyncratic volatility portfolios.

Table 1.10. Idiosyncratic Risk Factor calculated by value-weighted portfolios (EGARCH).

This table reports Cumulative IRF and Monthly IRF when using value-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model. Panel A shows IRF calculated by Fama-French measure (FF), i.e. two size portfolios and three idiosyncratic volatility portfolios. Panel B shows IRF calculated by refined measure one (R1), i.e. three size portfolios and three idiosyncratic volatility portfolios. Panel C shows IRF calculated by refined measure two (R2), i.e. five size portfolios and three idiosyncratic volatility portfolios.

Panel A: Fama-French measure (2*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-11.05	-0.30
194907-196112	80.38	0.54	196201-196206	-13.30	-2.22
196207-196811	178.79	2.32	196812-197006	-20.03	-1.05
197007-197212	29.39	0.98	197301-197409	-2.07	-0.10
197410-198011	246.52	3.33	198012-198207	-9.86	-0.49
198208-198708	124.20	2.04	198709-198711	-6.67	-2.22
198712-200008	510.28	3.34	200009-200209	-23.40	-0.94
200210-200710	181.85	2.98	200711-200902	-9.19	-0.57
200903-201012	123.75	5.63			
Total bull period	1475.16 (0.22)	2.35	Total bear period	-95.57 (0.51)	-0.65
Entire sample period	1379.58 (0.20)	1.78			
Panel B: Refined measure 1 (3*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-5.72	-0.15
194907-196112	76.72	0.51	196201-196206	-13.79	-2.30
196207-196811	181.16	2.35	196812-197006	-16.53	-0.87
197007-197212	35.00	1.17	197301-197409	2.08	0.10
197410-198011	238.34	3.22	198012-198207	-13.09	-0.65
198208-198708	134.75	2.21	198709-198711	-7.71	-2.57
198712-200008	506.44	3.31	200009-200209	-22.55	-0.90

Table 1.10. Continued.

Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
200210-200710	173.47	2.84	200711-200902	-15.37	-0.96
200903-201012	118.80	5.40			
Total bull period	1464.68 (0.21)	2.33	Total bear period	-92.67 (0.51)	-0.63
Entire sample period	1372.01 (0.20)	1.77			
	Panel C: Refined measure 2 (5*3 portfolios)				
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-6.15	-0.17
194907-196112	87.61	0.58	196201-196206	-13.12	-2.19
196207-196811	178.59	2.32	196812-197006	-13.54	-0.71
197007-197212	36.14	1.20	197301-197409	3.69	0.18
197410-198011	235.07	3.18	198012-198207	-13.71	-0.69
198208-198708	140.07	2.30	198709-198711	-9.19	-3.06
198712-200008	503.85	3.29	200009-200209	-23.58	-0.94
200210-200710	171.48	2.81	200711-200902	-16.77	-1.05
200903-201012	119.87	5.45			
Total bull period	1472.69 (0.21)	2.35	Total bear period	-92.36 (0.50)	-0.63
Entire sample period	1380.32 (0.20)	1.78			

Table 1.11. Idiosyncratic Risk factor calculated by equal-weighted portfolios (EGARCH).

This table reports Cumulative IRF and Monthly IRF when using equal-weighted portfolios with idiosyncratic volatility estimated by the EGARCH(1,1) model. Panel A shows IRF calculated by Fama-French measure (FF), i.e. two size portfolios and three idiosyncratic volatility portfolios. Panel B shows IRF calculated by refined measure one (R1), i.e. three size portfolios and three idiosyncratic volatility portfolios. Panel C shows IRF calculated by refined measure two (R2), i.e. five size portfolios and three idiosyncratic volatility portfolios.

Panel A: Fama-French measure (2*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-13.20	-0.36
194907-196112	73.92	0.49	196201-196206	-12.91	-2.15
196207-196811	158.01	2.05	196812-197006	-18.89	-0.99
197007-197212	26.67	0.89	197301-197409	-3.32	-0.16
197410-198011	199.07	2.69	198012-198207	-20.02	-1.00
198208-198708	114.41	1.88	198709-198711	-12.25	-4.08
198712-200008	402.22	2.63	200009-200209	-31.50	-1.26
200210-200710	147.32	2.42	200711-200902	-23.84	-1.49
200903-201012	110.53	5.02			
Total bull period	1232.16 (0.21)	1.96	Total bear period	-135.94 (0.52)	-0.92
Entire sample period	1096.22 (0.20)	1.41			
Panel B: Refined measure 1 (3*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-8.26	-0.22
194907-196112	79.91	0.53	196201-196206	-12.57	-2.09
196207-196811	167.93	2.18	196812-197006	-15.71	-0.83
197007-197212	33.54	1.12	197301-197409	0.28	0.01
197410-198011	210.92	2.85	198012-198207	-18.69	-0.93
198208-198708	127.74	2.09	198709-198711	-11.44	-3.81
198712-200008	453.40	2.96	200009-200209	-31.86	-1.27

Table 1.11. Continued.

Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
200210-200710	150.65	2.47	200711-200902	-22.32	-1.40
200903-201012	111.76	5.08			
Total bull period	1335.84 (0.21)	2.13	Total bear period	-120.57 (0.51)	-0.82
Entire sample period	1215.27 (0.20)	1.57			
Panel C: Refined measure 2 (5*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-5.42	-0.15
194907-196112	85.37	0.57	196201-196206	-13.72	-2.29
196207-196811	172.33	2.24	196812-197006	-10.60	-0.56
197007-197212	35.70	1.19	197301-197409	3.93	0.19
197410-198011	227.05	3.07	198012-198207	-17.58	-0.88
198208-198708	133.44	2.19	198709-198711	-10.19	-3.40
198712-200008	476.98	3.12	200009-200209	-27.01	-1.08
200210-200710	157.76	2.59	200711-200902	-18.77	-1.17
200903-201012	114.58	5.21			
Total bull period	1403.20 (0.21)	2.23	Total bear period	-99.35 (0.51)	-0.68
Entire sample period	1303.85 (0.20)	1.68			

Table 1.12. Idiosyncratic Risk factor calculated by value-weighted portfolios (GJR-GARCH).

This table reports Cumulative IRF and Monthly IRF when using value-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. Panel A shows IRF calculated by Fama-French measure (FF), i.e. two size portfolios and three idiosyncratic volatility portfolios. Panel B shows IRF calculated by refined measure one (R1), i.e. three size portfolios and three idiosyncratic volatility portfolios. Panel C shows IRF calculated by refined measure two (R2), i.e. five size portfolios and three idiosyncratic volatility portfolios.

Panel A: Fama-French measure (2*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-15.03	-0.41
194907-196112	71.23	0.47	196201-196206	-13.2	-2.20
196207-196811	156.86	2.04	196812-197006	-24.22	-1.27
197007-197212	22.6	0.75	197301-197409	-2.87	-0.14
197410-198011	215.52	2.91	198012-198207	-17.64	-0.88
198208-198708	83.18	1.36	198709-198711	-10.1	-3.37
198712-200008	425.19	2.78	200009-200209	-26.7	-1.07
200210-200710	171.88	2.82	200711-200902	-15.81	-0.99
200903-201012	121.45	5.52			
Total bull period	1267.90 (0.22)	2.02	Total bear period	-125.58 (0.52)	-0.85
Entire sample period	1142.32 (0.21)	1.47			
Panel B: Refined measure 1 (3*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-10.15	-0.27
194907-196112	71.1	0.47	196201-196206	-14.09	-2.35
196207-196811	157.52	2.05	196812-197006	-22.09	-1.16
197007-197212	26.47	0.88	197301-197409	-3.05	-0.15
197410-198011	205.04	2.77	198012-198207	-18.68	-0.93
198208-198708	87.1	1.43	198709-198711	-11.72	-3.91
198712-200008	414.4	2.71	200009-200209	-28.43	-1.14

Table 1.12. Continued.

Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
200210-200710	162.92	2.67	200711-200902	-21.66	-1.35
200903-201012	115.91	5.27			
Total bull period	1240.46 (0.21)	1.98	Total bear period	-129.88 (0.51)	-0.88
Entire sample period	1110.59 (0.20)	1.43			
	Panel C: Refined measure 2 (5*3 portfolios)				
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-11.64	-0.31
194907-196112	79.03	0.53	196201-196206	-13.14	-2.19
196207-196811	151.95	1.97	196812-197006	-19.41	-1.02
197007-197212	29.49	0.98	197301-197409	-3.74	-0.18
197410-198011	198.89	2.69	198012-198207	-20.43	-1.02
198208-198708	90.97	1.49	198709-198711	-13.23	-4.41
198712-200008	408.96	2.67	200009-200209	-34.63	-1.39
200210-200710	159.33	2.61	200711-200902	-23.28	-1.46
200903-201012	116.47	5.29			
Total bull period	1235.09 (0.21)	1.97	Total bear period	-139.49 (0.51)	-0.95
Entire sample period	1095.60 (0.20)	1.41			

Table 1.13. Idiosyncratic Risk factor calculated by equal-weighted portfolios (GJR-GARCH).

This table reports Cumulative IRF and Monthly IRF when using equal-weighted portfolios with idiosyncratic volatility estimated by the GJR-GARCH(1,1) model. Panel A shows IRF calculated by Fama-French measure (FF), i.e. two size portfolios and three idiosyncratic volatility portfolios. Panel B shows IRF calculated by refined measure one (R1), i.e. three size portfolios and three idiosyncratic volatility portfolios. Panel C shows IRF calculated by refined measure two (R2), i.e. five size portfolios and three idiosyncratic volatility portfolios.

Panel A: Fama-French measure (2*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-20.29	-0.55
194907-196112	62.88	0.42	196201-196206	-13.68	-2.28
196207-196811	133.85	1.74	196812-197006	-24.58	-1.29
197007-197212	20.22	0.67	197301-197409	-10.17	-0.48
197410-198011	181.65	2.45	198012-198207	-26.46	-1.32
198208-198708	58.44	0.96	198709-198711	-16.00	-5.33
198712-200008	302.66	1.98	200009-200209	-48.37	-1.93
200210-200710	132.49	2.17	200711-200902	-30.70	-1.92
200903-201012	105.21	4.78			
Total bull period	997.41 (0.21)	1.59	Total bear period	-190.24 (0.52)	-1.29
Entire sample period	807.17 (0.20)	1.04			
Panel B: Refined measure 1 (3*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-13.11	-0.35
194907-196112	72.58	0.48	196201-196206	-13.44	-2.24
196207-196811	145.43	1.89	196812-197006	-20.32	-1.07
197007-197212	26.78	0.89	197301-197409	-6.03	-0.29
197410-198011	188.31	2.54	198012-198207	-25.35	-1.27
198208-198708	68.98	1.13	198709-198711	-15.76	-5.25
198712-200008	352.26	2.30	200009-200209	-47.49	-1.90

Table 1.13. Continued.

Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
200210-200710	136.24	2.23	200711-200902	-28.94	-1.81
200903-201012	106.37	4.84			
Total bull period	1096.95 (0.21)	1.75	Total bear period	-170.44 (0.51)	-1.16
Entire sample period	926.51 (0.20)	1.20			
Panel C: Refined measure 2 (5*3 portfolios)					
Bull market	Cumulative IRF	Monthly IRF	Bear market	Cumulative IRF	Monthly IRF
			194606-194906	-10.37	-0.28
194907-196112	79.54	0.53	196201-196206	-14.11	-2.35
196207-196811	149.56	1.94	196812-197006	-18.19	-0.96
197007-197212	30.83	1.03	197301-197409	-3.85	-0.18
197410-198011	195.47	2.64	198012-198207	-25.02	-1.25
198208-198708	77.87	1.28	198709-198711	-14.81	-4.94
198712-200008	376.44	2.46	200009-200209	-43.51	-1.74
200210-200710	143.75	2.36	200711-200902	-25.84	-1.62
200903-201012	109.49	4.98			
Total bull period	1162.95 (0.21)	1.85	Total bear period	-155.70 (0.51)	-1.06
Entire sample period	1007.25 (0.20)	1.30			

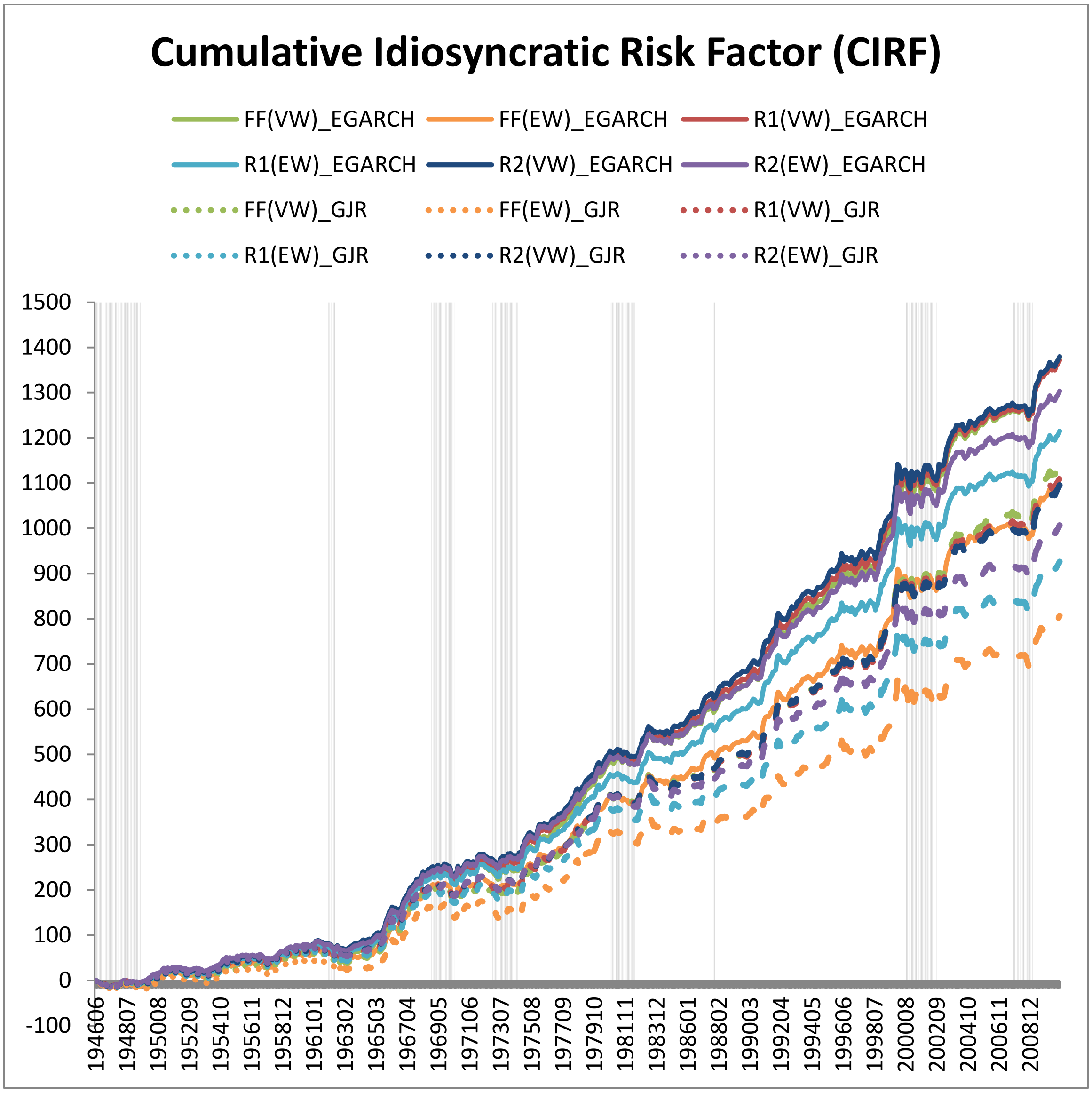


Figure 1.7. Cumulative Idiosyncratic Risk Factor.

This figure shows Cumulative Idiosyncratic Risk Factor (CIRF) by all constructing measures (FF measure, R1 measure and R2 measure), weighting schemes (value-weighted and equal-weighted), and models of estimating idiosyncratic volatility (the EGARCH model and the GJR-GARCH model). Shaded areas stand for bear market periods.

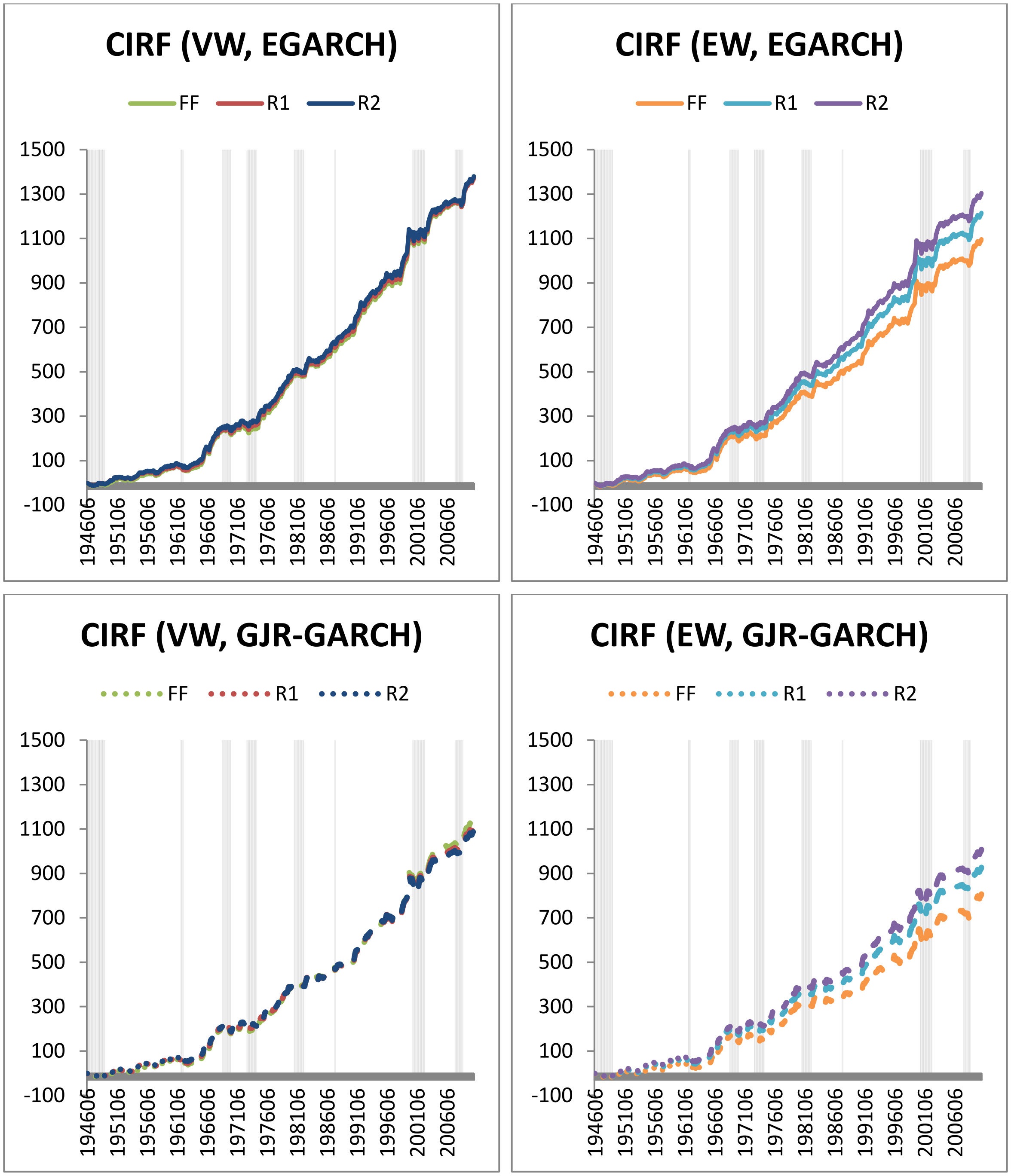


Figure 1.8. Comparison of CIRF by different factor constructing measures.

This figure compares how the three portfolio constructing measures differ when weighting scheme and model for estimating idiosyncratic volatility keeps the same. Shaded areas stand for bear market periods.

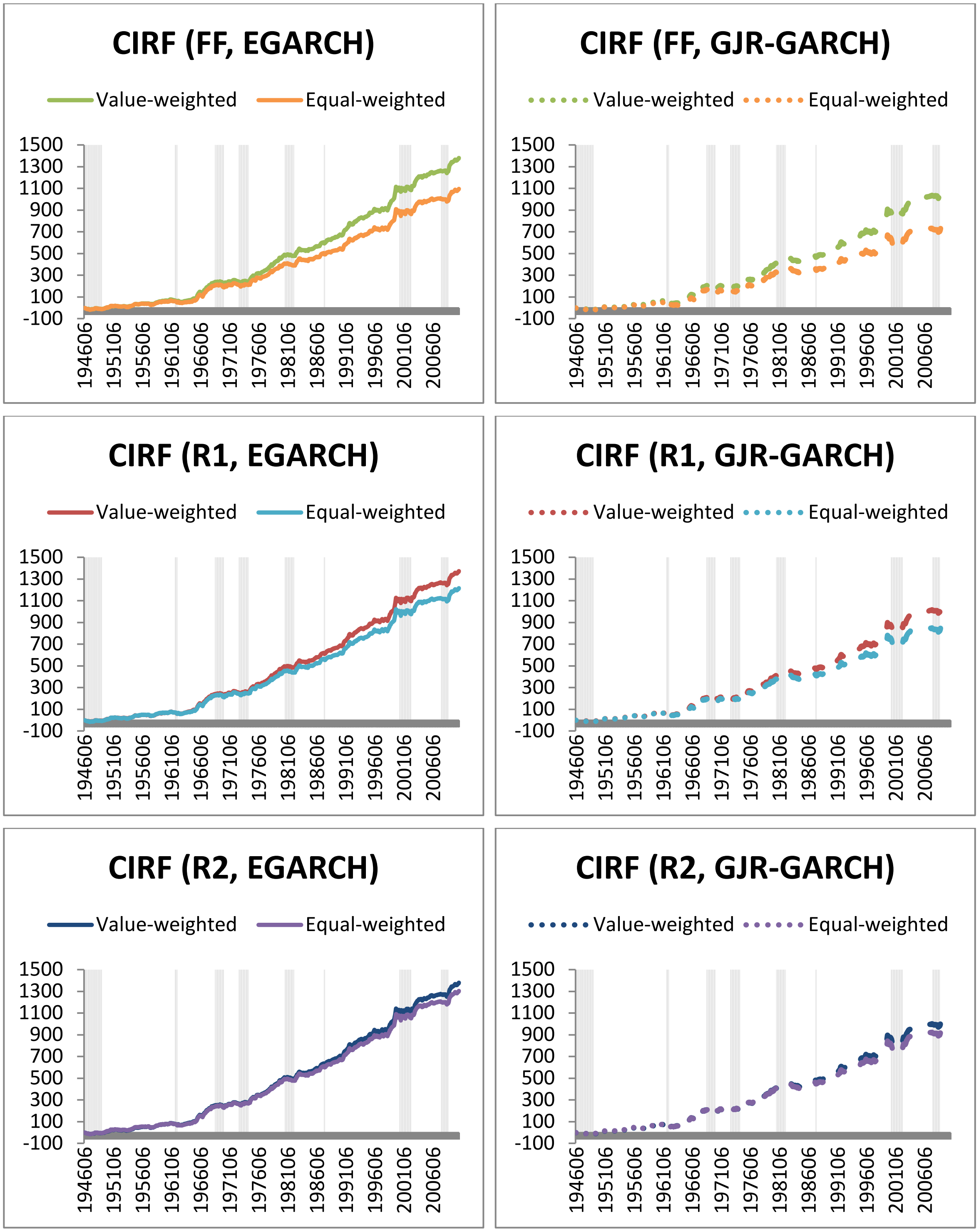


Figure 1.9. Comparison of CIRF by different weighting schemes.

This figure compares how the two weighting schemes differ when portfolio constructing measure and model for estimating idiosyncratic volatility keeps the same. Shaded areas stand for bear market periods.

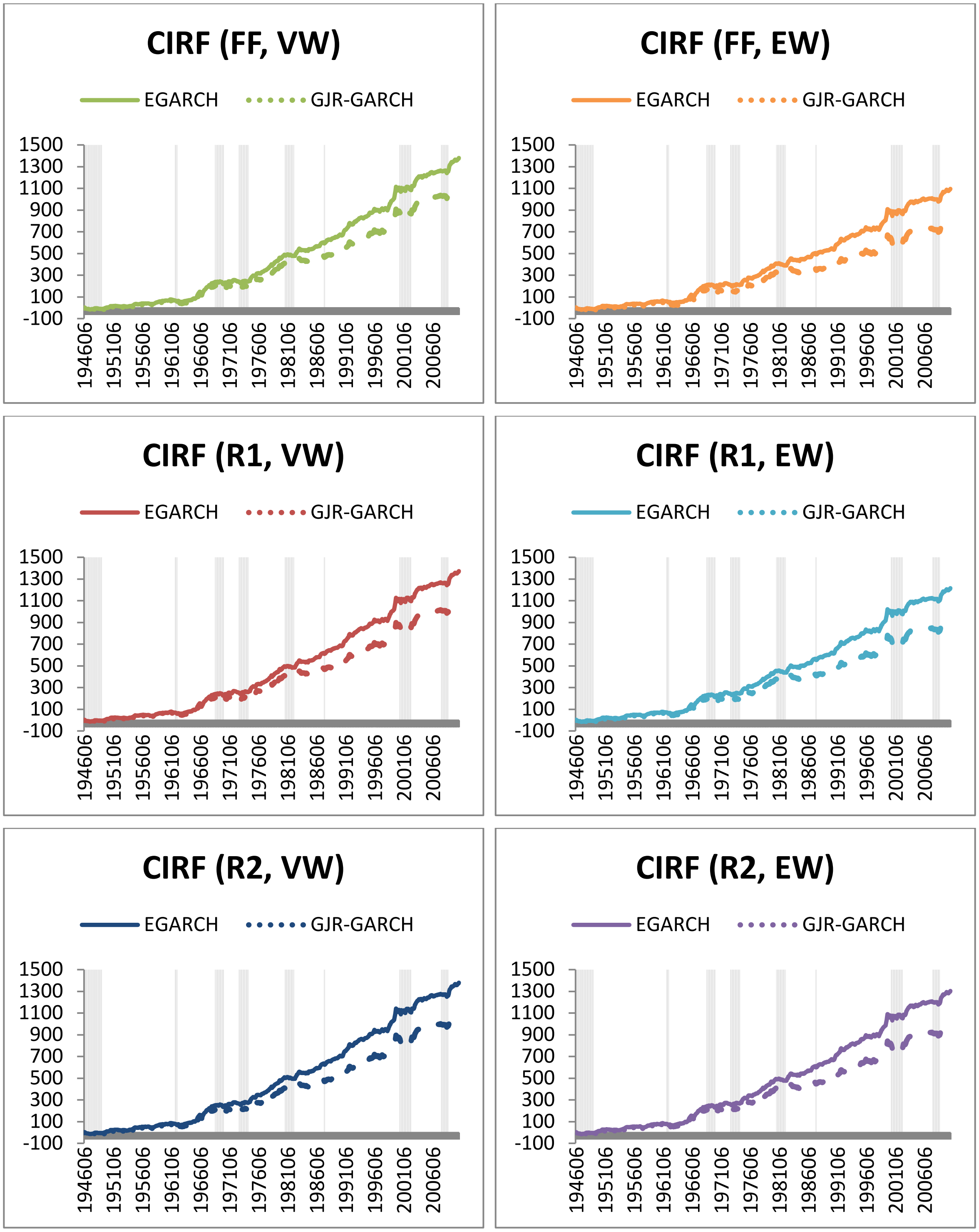


Figure 1.10. Comparison of CIRF by different models of estimating idiosyncratic volatility.

This figure compares how the two models for estimating idiosyncratic volatility differ when portfolio constructing measure and weighting scheme keeps the same. Shaded areas stand for bear market periods.

1.5. Conclusions

We examine the empirical relation between idiosyncratic volatility and expected stock returns. The literature such as Ang, Hodrick, Xing, and Zhang (2006) and others finds that monthly stock returns are negatively related to idiosyncratic volatilities; Fu (2009) and others find that the idiosyncratic volatilities are positively related to expected returns; Bali and Cakici (2008) and others report no relation between expected returns and idiosyncratic risk. We show that the relation between idiosyncratic volatilities and stock returns varies over time. Specifically, we find strong evidence that idiosyncratic risk is priced differently across bull and bear markets. We conclude that idiosyncratic risk is priced in bull markets but is not priced in bear markets. As an example, for bull markets during the sample period, a factor portfolio long on stocks with high idiosyncratic volatility and short on stocks with low idiosyncratic volatility yields an equal-weighted monthly return of 1.59% with a standard deviation of 0.21%. For bear markets the average monthly return is -1.29% with a standard deviation of 0.52%. These empirical evidences support the hypothesis that investors hold less diversified portfolios during bull market and tend to hold more diversified portfolios during bear market.

The importance of our findings lies in viewing the relation between idiosyncratic risk and expected returns under different stock market conditions. The previous literature may be neglecting the importance of the impact of market conditions on the relation between idiosyncratic risk and expected returns. Our analysis not only shows that market condition does matter in determining the risk-return relation, but also gives the direction of the relationship in bull and bear markets separately.

We find that only less than one-fifth of our sample period is in a bear market period, and more than four-fifths is in a bull market period. The fact that bull market periods dominate the whole sample period can explain why the literature doesn't find our results. Since our results for a bull market period report a positive relation between idiosyncratic volatility and expected stock returns, our findings are consistent with those literatures that also use monthly data and report a positive relationship.

We also find that the idiosyncratic risk factor supersedes the size factor. After controlling for estimated idiosyncratic volatility, "size effect" is no longer significant. Merton (1987) explicitly points out that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk and investor base. Our test results lend direct support to Merton's prediction in this point by showing that after controlling for idiosyncratic risk there does no longer exist any size effect.

Finally, we construct an Idiosyncratic Risk Factor (IRF) to see if idiosyncratic risk is priced or not. We could definitely conclude that idiosyncratic risk is priced in bull market periods but not in bear market periods from the IRF distribution. We also find that the EGARCH idiosyncratic volatility portfolios always get higher returns than the GJR-GARCH idiosyncratic volatility portfolios. The value-weighted portfolios always get higher returns than the equal-weighted portfolios. And the more size portfolios we construct to calculate IRF, the higher factor portfolio return we will get. So in summary, the factor portfolios constructed using value-weighted weighting scheme with the EGARCH idiosyncratic volatility bring the highest returns, while the lowest returns are from the factor portfolios constructed using equal-weighted weighting scheme with the GJR-GARCH idiosyncratic volatility.

To sum up again, all the IRF results, together with the Fama-MacBeth regression results, prove our finding that idiosyncratic risk is priced in bull market periods but is not priced in bear market periods. At the same time, they support our hypothesis that investors are rewarded for betting on individual stocks during bull markets and holding more diversified portfolios during bear markets, and confirm Merton's (1987) prediction that idiosyncratic risk should be priced when investors do not diversify their portfolio.

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Chapter 2

The Role of Limits to Arbitrage in the Liquidity Anomaly

2.1. Introduction

Most of the studies that investigate liquidity and asset prices, often make the argument that stocks with low liquidity level, measured by bid-ask spreads, dollar volume, etc., earn higher expected returns. Amihud and Mendelson (1986) first found that low liquidity investments are expected to produce higher returns for their holders. Theoretical work by Merton (1987) indicates that liquidity should be priced by the market.[7] After that, Amihud and Mendelson (1989), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Chordia, Subrahmanyam, and Anshuman (2001), Amihud (2002), Easley, Hvidkjaer, and O'Hara (2002), Pastor and Stambaugh (2003), Acharya and Pedersen (2004), Baker and Stein (2004), and Hasbrouck (2005), among others, using different liquidity proxies, have all confirmed that stocks with low liquidity earn

[7] Merton's (1987) paper does not directly derive any results pertaining to liquidity. However, by differentiating the stock price with respect to its supply, Spiegel and Wang (2007) generated such results and included the process in their paper.

higher subsequent risk-adjusted returns. This phenomenon is often referred to as the liquidity anomaly.

However, there are few papers discussing the reason for why the liquidity anomaly is not arbitraged away. I argue that the reason is the limits to arbitrage. There is a growing literature that examines the impact of the limits to arbitrage (Shleifer and Vishny (1997)) for the cross-section of stock returns. Several studies have examined the limits to arbitrage argument in the case of individual anomaly (e.g., Ali, Hwang and Trombley (2003) for book-to-market effect, Mendehall (2004) for post-earnings-announcement drift, Zhang (2006) for momentum effect, and Wei and Zhang (2007) for value-to-price anomaly). In this paper, I am trying to examine if the limits to arbitrage argument can explain the liquidity anomaly documented by Amihud and Mendelson (1986).

The literature on the limits to arbitrage initiated by Shleifer and Vishny (1997) argues that the arbitrage is risky and costly, and hence implementable arbitrage opportunities are limited. According to Lam and Wei (2011), since the difficulty to arbitrage varies across stocks, information should be more quickly included in the prices of stocks that are easier to arbitrage than in those that are not easy to arbitrage. Similarly, errors in stock prices due to biased expectations on future profitability should be corrected more quickly for stocks that are easy to arbitrage than for stocks that are difficult to arbitrage. Although arbitrageurs may trade against the mispricing, the correction of mispricing will take longer when the limits to arbitrage are more severe. In brief, the limits to arbitrage hinder relevant information from being included in stock prices and prevent incorrect information from being removed from stock prices. On the other hand, "Liquidity in the stock market, for example, thrives on differences of opinion about the value of a firm; information fuels the debate."[8] Therefore, in the sense that stocks with lower liquidity are harder to arbitrage, if liquidity is priced by the stock market, i.e. liquidity anomaly does exist, then the limits to arbitrage will conduce to the liquidity anomaly by means of information delay.

My main hypothesis is that if the over-performance of the stocks of low liquidity is due to the limits to arbitrage, then the over-performance should be greater when the arbitrage is more difficult to implement. In other words, the liquidity anomaly should be more pronounced among stocks with more severity of the limits to arbitrage. Furthermore, when the arbitrage is considerably easy to implement, the liquidity anomaly should be substantially reduced or even disappeared.

In this paper, I measure the limits to arbitrage using idiosyncratic risk since the idiosyncratic risk deters arbitrage activity. Idiosyncratic risk is proposed as a limit to arbitrage in several papers. Shleifer and Vishny (1997) argue that idiosyncratic risk is a large cost for risk-averse arbitrageurs who cannot hedge the idiosyncratic risk of individual stocks. Barberis and Thaler (2003) survey the literature and identify three sources of arbitrage costs: idiosyncratic risk, noise trader momentum risk, and implementation costs. They also explain idiosyncratic risk as a deterrent to arbitrage. Pontiff (2006) argues that idiosyncratic risk is the single largest impediment to market

[8] "Full disclosure; Economics focus." *The Economist* [US] 21 Feb. 2009: 75EU.

efficiency as it imposes a significant holding cost for arbitrageurs. He shows that idiosyncratic risk affects arbitrage activity even if arbitrageurs have access to a diversified portfolio and a large number of available arbitrage projects. He proves that risk-averse arbitrageurs will assign smaller portfolio weights to stocks with higher idiosyncratic risk. Cao and Han (2011) claim that trading and holding costs create the limits to arbitrage and idiosyncratic risk is the most common proxy for holding cost. Following Pontiff (1996), Wurgler and Zhuravskaya (2002), and Mashruwala, Rajgopal, and Shevlin (2006), I will use idiosyncratic stock return volatility (Ivol) to proxy for idiosyncratic risk. For robustness, I will also apply the number of institutional shareholders (Inst) as a measure of shareholder sophistication and the number of analysts following a stock (Analyst) as a measure of information uncertainty, since shareholder sophistication and information uncertainty may also influence the risk of arbitrage.

A perfectly liquid market is one where any amount of a given security can be instantaneously converted to cash and back to securities at no cost. In a less than perfect world, a liquid market is one where the transaction costs associated with this conversion are minimised (Harris (1990)). While relatively easy to define, liquidity has proved to be difficult to measure. The previous literature offers a wide variety of measurement proxies for liquidity. Aitken and Winn (1997) report that there are some 68 extant measures used in the literature suggesting that there is little agreement on the best measure to use. In this paper, I will employ the most commonly used proxy for liquidity, the Amihud (2002) measure.

Amihud (2002) develops a price impact measure that captures the "daily price response associated with one dollar of trading volume." The measure introduced by Amihud (2002) is defined as the monthly average of absolute value of return divided by dollar volume every day. Acharya and Pedersen (2004) employ the liquidity measure of Amihud (2002) to show that expected stock returns are a function of several terms: first, expected stock illiquidity and second, some covariances between stock return, stock illiquidity, market return, and market illiquidity. Sadka (2006) extends the literature on liquidity risk and argues that the measure of Amihud (2002) seems the most correlated with price impacts, among the alternative measures examined in his paper. Spiegel and Wang (2007) claim that when dollar volume is excluded from the analysis Amihud's (2002) measure does provide out of sample explanatory power for cross sectional stock returns. Goyenko, Holden, and Trzcinka (2009) conclude that the widely used Amihud (2002) measure consistently wins a majority of the effective/realized spread horseraces and hence is clearly a good proxy for price impact.

For robustness, another two measures of liquidity that will be used are turnover ratio and bid-ask spread. Turnover is the ratio of trading volume to the number of shares outstanding. By Amihud and Mendelson (1986), turnover is negatively related to illiquidity costs, and Atkins and Dyl (1997) found a strong positive relationship across stocks between the bid-ask spread and the reciprocal of the turnover ratio that measures holding period. A number of studies find that cross-sectionally, stock returns are decreasing in stock turnover, which is consistent with a negative relationship between liquidity and expected return (Haugen and Baker (1996), Datar et al. (1998), Hu (1997), Rouwenhorst (1998), Chordia et al. (2001)). The turnover ratio is also the measure used by the International Federation of Stock Exchanges (FIBV) to compare liquidity across exchanges.

The analysis of my data from 1968 to 2010 shows that the liquidity anomaly is stronger when there are more severe limits to arbitrage. In addition, the return spread between low and high liquidity stocks is mostly driven by the underperformance of high liquidity stocks. Moreover, the anomaly is very weak among stocks that have low arbitrage risk. The positive relation between the limits to arbitrage and the liquidity anomaly remains significant even after controlling for conventional factor risks or firm-characteristics exposures. I further exclude the possibilities that my results are driven by liquidity risk and conclude that the limits to arbitrage help explaining why the liquidity anomaly is not arbitraged away.

In this paper, I contribute to the expanding liquidity literature by conducting empirical tests to see whether or not the relations between liquidity and stock returns are related to the limits to arbitrage. My findings are important for the following reasons. First, I provide evidence that supports the main hypothesis that the over-performance of low liquidity stocks is more profound when the limits to arbitrage are more severe. I also find that the ease of limits to arbitrage drives away the liquidity anomaly especially when I use turnover ratio and bid-ask spread as proxies for liquidity. My findings support the main hypothesis and establish the limits to arbitrage as an important reason that the liquidity anomaly persists.

Second, most measures of the limits to arbitrage in the existing literature are associated with risks, especially systematic liquidity risk. Therefore, any attempt to use the limits to arbitrage argument to explain asset pricing anomalies are suspected to suffer from the multiple joint hypothesis problems since the limits to arbitrage may also proxy for the exposure to liquidity risk. My results clear up this suspicion, at least, in the case of the liquidity anomaly. I find that the connection between the liquidity risk exposure of the liquidity anomaly and the severity of the limits to arbitrage is not obvious. These findings help to resolve the disagreement in interpreting the results in existing studies that also use the limits to arbitrage argument to explain asset pricing anomalies.

Third, although literature claims that the impact of idiosyncratic risk can eliminate liquidity's explanatory power in stock returns, I further suggest that the limits to arbitrage, other than idiosyncratic risk itself, are the reason why the liquidity anomaly is not arbitraged away. Liquidity and idiosyncratic risk are usually allowed to compete as explanatory factors in cross sectional returns. For example, Spiegel and Wang (2007) find that while both liquidity and idiosyncratic risk play a role in determining stock returns, the impact of idiosyncratic risk is much stronger and often eliminates liquidity's explanatory power. However, my study addresses that it is not the idiosyncratic risk, but the limits to arbitrage, which fades the liquidity's explanatory power of stock returns.

The remainder of this paper proceeds as follows. The next section develops the hypotheses and describes the measures of variables and the data used. Section 2.3

investigates the role of the limits to arbitrage in the liquidity anomaly based on portfolio analysis. Section 2.4 examines the relationship between the limits to arbitrage and the liquidity anomaly using regression analysis to control for exposures to firm characteristics. Section 2.5 contains several robustness tests. Section 2.6 concludes the paper.

2.2. Empirical approach

2.2.1. Hypotheses

In this paper, I attempt to dig out the reason for why the liquidity anomaly is not arbitraged away. I argue that the reason is the limits to arbitrage. As I discussed before, in the sense that stocks with lower liquidity are harder to arbitrage, if liquidity is priced by the stock market, i.e. liquidity anomaly does exist, then the limits to arbitrage will conduce to the liquidity anomaly by means of information delay. Therefore, my main hypothesis is that if the over-performance of the stocks of low liquidity is due to the limits to arbitrage, then the over-performance should be more pronounced when there are more severe limits to arbitrage. I further wonder that although the limits to arbitrage can help to explain why the liquidity anomaly is not arbitraged away, if the limits to arbitrage fully eliminate the existence of the liquidity anomaly. Therefore, I would like to know when arbitrage is very easy to implement, if the over-performance of the stocks of low liquidity is reduced significantly or even disappeared. The above discussion leads to my first hypothesis.

Hypothesis 1: The liquidity anomaly (i.e., negative relationship between liquidity and stock returns) should be more pronounced when there are more severe limits to arbitrage. The relationship between liquidity and subsequent stock returns should be insignificant when the limits to arbitrage are very low.

As suggested, more severity of the limits to arbitrage is associated with lower liquidity, which means that stocks with more severe limits to arbitrage have higher exposure to liquidity risk. If my empirical test results support the Hypotheses 1, my next question is: is the positive relationship between the liquidity anomaly and the severity of the limits to arbitrage caused by the liquidity risk or by the liquidity per se.? This question will lead to my second hypothesis:

Hypothesis 2: The positive relationship between the limits to arbitrage and the liquidity anomaly is not due to the liquidity risk.

2.2.2. Measures of variables

In this paper, I measure the limits to arbitrage using idiosyncratic risk since the idiosyncratic risk deters arbitrage activity. Idiosyncratic risk is proposed as a limit to arbitrage in several papers as I introduced before. Following Pontiff (1996), Wurgler and Zhuravskaya (2002), and Mashruwala, Rajgopal, and Shevlin(2006), I will use idiosyncratic stock return volatility (Ivol) to proxy for idiosyncratic risk.

Following Fu (2009), I use in-sample monthly data to calculate idiosyncratic volatility based on the three-factor Fama-French (1993) model. The explicit form is as follows.

$$\boldsymbol{R_{it} - r_{ft} = \alpha_i + \beta_i(R_{mt} - r_{ft}) + S_i SMB_t + h_i HML_t + \varepsilon_{it}}, \tag{2.1}$$

$$\varepsilon_{it} \sim N(0, \sigma_{it}^2),$$

where R_{it} is the individual return. $R_{mt} - r_{ft}$ is the excess return on a broad market portfolio. SMB_t is the size factor – the difference of returns between a small stocks portfolio and a big stocks portfolio. HML_t is the value factor – the difference of returns between a high book-to-market stocks portfolio and a low book-to-market stocks portfolio. The residual ε_{it} is assumed to be normally distributed with mean zero and variance σ_{it}^2. To get the idiosyncratic volatilities, my objective is to estimate σ_{it}^2. I will employ an asymmetric GARCH (Generalized AutoRegressive Conditional Heteroskedasticity), the EGARCH(1,1) model, since EGARCH is the most widely used model for estimating the conditional volatility of returns.[9] The explicit form for the EGARCH(1,1) model is as follows:

$$\boldsymbol{ln\sigma_{it}^2 = a_i + b_i ln\sigma_{i,t-1}^2 + c_i \left\{\theta\left(\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-1}}{\sigma_{i,t-1}}\right| - (2/\pi)^{1/2}\right]\right\}}, \tag{2.2}$$

where $\varepsilon_{i,t-1}$ is the lagged residual and $\sigma_{i,t-1}^2$ is the lagged variance.

[9] I also use the GJR-GARCH(1,1) model to estimate the idiosyncratic volatilities. The final results are very similar to those using EGARCH(1,1) model.

In the robustness section, I also apply two other proxies for the limits to arbitrage. Literature shows that shareholder sophistication and information uncertainty may also influence the risk of arbitrage. Following Chen et al. (2002), Ali et al. (2003), Bartov, Radhakrishnan, and Krinsky (2000), and Bhusan (1994), I use the number of institutional investors holding a firm's shares (Inst) to proxy for shareholder sophistication. The other measure is analyst coverage (Analyst), defined as the number of analysts' estimates following a stock. Hong, Lim, and Stein (2000) show that more analyst coverage indicates lower information uncertainty. Therefore, I will use the number of analysts' estimates (Analyst) as a measure of information uncertainty.

For the liquidity proxy, I will employ the most commonly used measure in the literature, the Amihud (2002) measure. Amihud (2002) develops a price impact measure that captures the "daily price response associated with one dollar of trading volume." The measure introduced by Amihud (2002) is defined as the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Specifically, he uses the ratio:

$$\boldsymbol{Amihud_{im}} = \frac{\mathbf{1}}{\boldsymbol{D_{im}}}\sum_{\boldsymbol{d=1}}^{\boldsymbol{D_{im}}}\frac{|\boldsymbol{R_{imd}}|}{\boldsymbol{Dvolume_{imd}}}, \tag{2.3}$$

where R_{imd} is the return of stock i on day d of month m. $Dvolume_{imd}$ is the dollar volume of stock i on day d of month m. D_{im} is the number of trading days of stock i in month m. The average is calculated over all positive volume days, since the ratio is undefined for zero volume days.

In the robustness section, another two measures of liquidity will be applied. Those are turnover ratio and bid-ask spread. Turnover is the ratio of trading volume to the number of shares outstanding. A number of studies find that cross-sectionally, stock returns are decreasing in stock turnover, which is consistent with a negative relationship between liquidity and expected return (Haugen and Baker (1996), Datar et al. (1998), Hu (1997), Rouwenhorst (1998), Chordia et al. (2001)). The turnover ratio is also the measure used by the International Federation of Stock Exchanges (FIBV) to compare liquidity across exchanges. Following Chordia, Subrahmanyam, and Anshuman (2001), in this paper, the turnover ratio is calculated by the average ratio of trading volume to the number of shares outstanding of the previous 36 months.

$$\boldsymbol{Turnover_{im}} = \frac{\mathbf{1}}{\mathbf{36}}\sum_{\boldsymbol{t=m-35}}^{\boldsymbol{m}} \frac{\boldsymbol{Volume_{it}}}{\boldsymbol{Shares_{it}}}, \tag{2.4}$$

where $Volume_{it}$ is the trading volume of stock i at time t and $Shares_{it}$ is the number of shares outstanding of stock i at time t.

2.2.3. Data and variables

For analyzing the relationship between the limits to arbitrage and the liquidity anomaly, I use monthly holding period stock data. For calculating the Amihud (2002) measure as a proxy of illiquidity, I use daily stock data. The data are obtained from the Center for Research in Security Prices (CRSP). I include stocks traded on the NYSE, AMEX, and NASDAQ during the period of January 1968 to December 2010. There are a total of 516 months in my sample. The monthly Fama-French three factors and momentum factor data are downloaded from Kenneth R. French's Website. The value-weighted liquidity factor (LIQ) documented in Pastor and Stambaugh (2003) is downloaded from Wharton Research Data Services (WRDS). To avoid the inaccuracy of idiosyncratic volatility estimates caused by infrequent trading, I require a minimum of 30 trading months for each stock when CRSP reports a non-zero share volume. I also require a minimum of 15 trading days in a month for calculating Amihud's illiquidity measure.

In the robustness section, I use the number of institutional shareholders (Inst) as a measure of shareholder sophistication. The data are obtained from Thomson Reuters, with data availability beginning in March 1980. I also use the number of analysts' estimates following a stock (Analyst) as a measure of information uncertainty. The analyst coverage data are obtained from the I/B/E/S database, with data available beginning in January 1990.

In this paper, I include several firm characteristic variables. R is the monthly raw return. R-r is the monthly excess return, where r stands for the one-month T-bill rate. Ivol is the idiosyncratic volatility estimated by the EGARCH(1,1) model. Firm size is measured by the market value of equity at the end of June of year t. BM is the book-to-market equity according to Fama and French (1993) at the end of fiscal year ending in calendar year t-1 divided by the market value of equity at the end of December of year t-1. In order to catch the momentum effect, RET(-2,-7) is the compound gross return from month t-7 to month t-2. Amihud is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Turnover is the average ratio of trading volume to the number of shares outstanding of the previous 36 months as a proxy for liquidity. Mkt-r, SMB and HML are the Fama-French three factors while Mom and LIQ are the momentum factor and the liquidity factor, respectively.

2.3. Portfolio analysis

2.3.1. Summary statistics for portfolios formed on the Amihud measure

In each month, I sort the liquidity proxies to form ten portfolios with an equal number of stocks. Each portfolio contains ten percent of stocks. Table 2.1 shows the summary statistics formed by Amihud (2002) measure. The second and the third row presents, respectively, the time-series means of the value-weighted and the equal-weighted portfolio returns. Other rows show the pooled means of variables within the particular portfolio.

Low Amihud means high liquidity while high Amihud means low liquidity. It is clear that high liquidity stocks (Decile 1) have lower returns while low liquidity stocks (Decile 10) have higher returns. The value-weighted portfolio returns increase from 1.18% for the highest liquidity portfolio to 2.09% for the lowest liquidity portfolio. A hedging portfolio longing Portfolio 10 and shorting Portfolio 1 yields a statistically significant monthly return of 0.91%. The equal-weighted portfolio returns display a similar pattern, increasing from -0.24% for the highest liquidity portfolio to 1.79% for the lowest liquidity portfolio. This evidence confirms the negative relation between liquidity and individual stock returns, which means that the liquidity anomaly found by the literature does exist. High liquidity stocks also have larger market values of equity than do low liquidity stocks. This result is consistent with the portfolio analysis findings of Amihud (2002) and Spiegel and Wang (2007) that stocks with high liquidity level have larger capital sizes. Finally, compared to stocks of low liquidity, stocks of high liquidity have lower arbitrage risk, just as predicted by many theoretical models. This finding is also consistent with Spiegel and Wang (2007) who find that high idiosyncratic risk firms have low levels of liquidity.

Table 2.1. Summary statistics for portfolios formed on liquidity.

Each month ten portfolios are formed on liquidity. This table shows the summary statistics formed by Amihud (2002) measure. Each portfolio consists of 10% of stocks. The second and third row presents, respectively, the time-series means of the value-weighted and the equal-weighted portfolio returns. Other rows show the pooled means of variables within the particular portfolio.

Portfolios formed on liquidity										
	Low	2	3	4	5	6	7	8	9	High
Amihud	0.00	0.01	0.03	0.06	0.12	0.23	0.46	1.00	2.65	31.45
VWRET	1.18	1.57	1.79	1.90	1.90	2.02	2.12	2.13	2.02	2.09
EWRET	-0.24	0.44	0.82	1.01	1.11	1.36	1.52	1.65	1.64	1.79
Ivol	9.83	10.59	11.05	11.63	12.38	13.11	14.08	15.56	19.31	29.84
SIZE (Billion)	3.71	1.44	0.69	0.40	0.25	0.16	0.11	0.07	0.04	0.02
BM	0.87	0.84	0.74	0.74	0.70	0.67	0.62	0.60	0.56	0.79

2.3.2. Calculating risk-adjusted portfolio returns

To test the Hypothesis 1, i.e., the liquidity anomaly should be more pronounced when there are more severe limits to arbitrage, I sort stocks into liquidity quintiles and idiosyncratic stock return volatility (as a proxy for the limits to arbitrage) quintiles independently. I am interested in understanding how the return spreads between low-liquidity and high-liquidity portfolios (Quintile 1 - Quintile 5) vary with the severity of the limits to arbitrage.

To control for factor risks, I estimate the intercept (i.e., the risk-adjusted return) from the following four factor regression. The four factors will be the Fama and French (1993) three factors plus the Carhart (1997) momentum factor.

$$\mathbf{R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p,} \qquad (2.5)$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively.

To test the Hypothesis 2, I reexamine the relationship between the severity of the limits to arbitrage and the return spreads between low-liquidity and high-liquidity portfolios controlling for the liquidity risk. To check whether the liquidity risk exposure drives my results, I still use the same portfolios formed by liquidity quintiles and the limits to arbitrage quintiles and examine the liquidity-risk-adjusted returns of these portfolios. The liquidity-risk-adjusted returns are the estimates of the intercept from the following five factor regression:

$$\mathbf{R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p,} \tag{2.6}$$

where LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity factor from January 1968 to December 2010 is downloaded from CRSP.

2.3.3. Empirical results of Amihud as illiquidity measure and idiosyncratic risk as the limits to arbitrage measure

First, to test the Hypothesis 1, i.e., the liquidity anomaly should be more pronounced when there are more severe limits to arbitrage, I sort stocks into Amihud (as a proxy for illiquidity) quintiles and idiosyncratic stock return volatility (as a proxy for the limits to arbitrage) quintiles independently, and calculate the equally-weighted and value-weighted monthly raw stock returns for each portfolio. Amihud (2002) measure (Amihud) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Monthly idiosyncratic stock return volatility (Ivol) is estimated by Fama-French three-factor model and EGARCH(1,1) model and it proxies for arbitrage risk. The results are shown in Table 2.2 and the unit is percent. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns.

Table 2.2 shows some weak evidence that the average raw return spread between low-liquidity and high-liquidity stocks increases with the limits to arbitrage. More specifically, in panel A, the stocks of low Amihud underperform and the underperformance is monotonically more pronounced when the arbitrage risk is higher, except for the stocks with the highest idiosyncratic risk. For example, the time-series equally-weighted raw return spread between high Amihud and low Amihud stocks increases from 1.0655% per month in the lowest arbitrage risk portfolio to 2.6475% in the highest arbitrage risk portfolio. Additionally, it seems that the arbitrage risk might be priced, since the stocks with high idiosyncratic risk have higher raw returns. This finding is consistent with Merton's (1987) incomplete-information CAPM and the results documented by Carroll and Wei (1988), Spiegel and Wang (2005), and Fu (2009), but it is inconsistent with the findings of Ang, Hodrick, Xing, and Zhang (AHXZ thereafter) (2006, 2009) who find that stock returns are lower when idiosyncratic risk is higher.

The results in panel B are similar. The stocks of low Amihud underperform and the underperformance is monotonically more pronounced when the arbitrage risk is higher, except for the stocks with the lowest idiosyncratic risk. For example, the time-series value-weighted raw return spread between high Amihud and low Amihud stocks increases from 1.1040% per month in the lowest arbitrage risk portfolio to 3.2301% in the highest arbitrage risk portfolio. However, I can hardly tell if the arbitrage risk is priced or not from value-weighted raw portfolio returns, which is inconsistent with the prediction of the incomplete-information CAPM suggested by Merton (1987) and the results documented by Carroll and Wei (1988), Spiegel and Wang (2005) and Fu (2009), but it is weakly consistent with the empirical findings of AHXZ (2006, 2009).

Table 2.3 reports monthly risk-adjusted portfolio returns sorted by Amihud (2002) measure as a proxy of illiquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The risk-adjusted portfolio return shown in the table is the estimated intercept α_p from Equation (2.5). The results in Table 2.3 are very similar to those in Table 2.2. I observe that the liquidity anomaly is more profound as the arbitrage risk increases. For example, the risk-adjusted return spread between high Amihud and low Amihud portfolios increases from 1.2577% per month in the lowest arbitrage risk portfolio to 2.4904% in the highest arbitrage risk portfolio, which is a significant difference of 1.2327%. This evidence fully supports the Hypothesis 1. In addition, the return spread between low and high liquidity stocks is mostly driven by the underperformance of high liquidity stocks except for the highest arbitrage risk portfolio.

Table 2.4 shows the liquidity-risk-adjusted stock returns of the same twenty-five portfolios. Panel A reports monthly liquidity-risk-adjusted portfolio returns and Panel B reports monthly portfolio liquidity-risk-factor loadings sorted by Amihud (2002) measure (Amihud) and the proxy of the limits to arbitrage (Ivol). The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from Equation (2.6). The results in Panel A are very similar to those in Table 2.3. I still observe that the liquidity anomaly is significantly more pronounced as the limits to arbitrage increase, even adjusted for the liquidity risk. For example, the liquidity-risk-adjusted return spread between low-liquidity and high-liquidity portfolios increases from 1.2461% per month in Ivol Quintile 1 to 2.5061% in Ivol Quintile 5, which is a significant difference of 1.26%. That is to say, the liquidity risk does not drive away the positive relationship between the limits to arbitrage and the liquidity anomaly. The evidence supports the Hypothesis 2.

Table 2.2. Raw Portfolio Returns by Amihud (2002) Measure and Idiosyncratic Volatility.

This table reports the monthly raw portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns. The sample period is from January 1968 to December 2010.

Panel A: Equally-weighted raw portfolio returns						
Amihud / *Ivol*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(low)	0.9345	0.7289	0.5099	0.3515	-0.1310	1.0655
2	1.4330	1.3782	1.0095	0.6330	-0.4515	1.8845
3	1.8417	1.6682	0.9794	0.4168	-0.9929	2.8346
4	2.0746	1.9822	1.1239	0.2802	-1.3700	3.4446
5(high)	4.0897	3.7921	3.2296	2.4923	1.4422	2.6475
Panel B: Value-weighted raw portfolio returns						
Amihud / *Ivol*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(low)	1.0004	0.8672	0.7003	0.4709	-0.1036	1.1040
2	0.7498	1.1101	0.5630	0.4614	-0.1316	0.8814
3	0.4248	0.9985	0.4684	0.0651	-0.8226	1.2474
4	0.0575	0.3189	-0.2303	-0.8324	-2.1528	2.2103
5(high)	1.2726	0.2450	-0.3069	-0.8959	-1.9575	3.2301

Table 2.3. Risk-adjusted Portfolio Returns by Amihud (2002) Measure and Idiosyncratic Volatility.

This table reports the monthly risk-adjusted portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. The risk-adjusted portfolio return is the estimated intercept α_p from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively. Factor returns and the risk-free rates are from Professor Kenneth French's website. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Risk-adjusted portfolio returns						
Amihud / *Ivol*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(low)	0.1016* (7.24)	0.0088 (0.55)	-0.1493* (-8.54)	-0.2940* (-14.15)	-1.1561* (-34.01)	1.2577* (34.20)
2	0.3522* (16.71)	0.1985* (8.16)	-0.0870* (-3.21)	-0.3876* (-13.45)	-1.3372* (-35.49)	1.6894* (39.14)
3	0.7498* (21.26)	0.4847* (14.39)	-0.1489* (-4.27)	-0.6449* (-17.84)	-1.8582* (-45.44)	2.6080* (48.30)
4	1.0842* (18.17)	0.8021* (16.11)	0.0416 (0.89)	-0.7276* (-15.96)	-2.3017* (-51.00)	3.3859* (45.26)
5(high)	3.1948* (30.87)	2.7601* (21.69)	2.4693* (28.44)	1.7230* (20.78)	0.7044* (9.33)	2.4904* (19.44)
5-1	3.0932* (29.61)	2.7513* (21.46)	2.6186* (29.57)	2.0170* (23.60)	1.8605* (22.48)	1.2327* (9.25)

Table 2.4. Liquidity-Risk-Adjusted Returns by Amihud (2002) Measure and Idiosyncratic Volatility.

This table reports the monthly liquidity-risk-adjusted portfolio returns (Panel A) and the monthly portfolio liquidity-risk-factor loadings (Panel B) sorted by Amihud (2002) measure (*Amihud*) and the proxy of the limits to arbitrage (*Ivol*). Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively, from Professor Kenneth French's website. LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity risk factor is downloaded from CRSP. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Panel A: Liquidity-risk-adjusted portfolio returns						
Amihud / *Ivol*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(low)	0.0953* (6.74)	0.0065 (0.41)	-0.1618* (-9.16)	-0.3089* (-14.72)	-1.1508* (-33.51)	1.2461* (33.55)
2	0.3573* (16.79)	0.2059* (8.39)	-0.0993* (-3.64)	-0.3914* (-13.46)	-1.3351* (-35.14)	1.6924* (38.87)
3	0.7533* (21.11)	0.4860* (14.30)	-0.1613* (-4.58)	-0.6547* (-17.97)	-1.8611* (-45.13)	2.6144* (47.95)
4	1.0753* (17.84)	0.8152* (16.21)	0.0233 (0.50)	-0.7388* (-16.08)	-2.3277* (-51.15)	3.4030* (45.06)
5(high)	3.2042* (30.69)	2.7211* (21.27)	2.4621* (28.10)	1.6814* (20.08)	0.6981* (9.17)	2.5061* (19.39)
5-1	3.1089* (29.51)	2.7146* (21.05)	2.6239* (29.35)	1.9903* (23.06)	1.8489* (22.13)	1.2600* (9.37)
Panel B: Portfolio liquidity-risk-factor loadings						
Amihud / *Ivol*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(low)	1.3664* (3.51)	0.6423 (1.49)	2.6539* (5.74)	2.8982* (5.24)	-0.9694 (-1.05)	2.3358** (2.34)
2	-0.4302 (-0.43)	-1.3118** (-1.97)	2.6340* (3.58)	0.9495 (1.22)	-1.0414*** (-1.85)	0.6112 (0.53)
3	0.7317 (0.66)	2.0413** (2.07)	2.8772* (3.02)	-0.4917 (-0.54)	-0.5820 (-0.62)	1.3137 (0.91)
4	5.2584* (4.34)	2.1635*** (1.75)	3.0768** (2.42)	-2.7052** (-2.02)	1.8970 (1.20)	3.3614*** (1.69)
5(high)	8.6742* (2.59)	6.9189* (3.08)	0.9685 (0.41)	-1.9958 (-0.72)	0.8270 (0.40)	7.8472* (3.00)
5-1	7.3078** (2.16)	6.2766* (2.74)	-1.6854 (-0.71)	-4.8940*** (-1.74)	1.7964 (0.80)	5.5114** (2.36)

2.4. Regression analysis

2.4.1. The regression models

In this section, I will test my hypotheses while controlling for several important firm characteristic exposures. My multivariate regression tests are based on the following Fama-Macbeth (1973) type regressions:

$$\mathbf{R_t = \beta_0 + \beta_1 Amihud_{t-1} + \beta_2 Ivol_t + \beta_3 Amihud_{t-1} \times Ivol_t + \beta_j Controls + \varepsilon_t,} \qquad (2.7)$$

where R_t is the monthly raw stock return. $Amihud_{t-1}$ is the Amihud (2002) measure, the monthly average of absolute value of return divided by dollar volume every day, and it proxies for illiquidity. $Ivol_t$ is the monthly idiosyncratic stock return volatility estimated by Fama-French three-factor model and EGARCH(1,1) model and it proxies for arbitrage risk. Controls are SIZE, BM, and RET(-2,-7), which control for firm characteristics. SIZE is the market value of equity at the end of June of year t. Book-to-market equity (BM) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year t−1 divided by the market value of equity at the end of December of year t−1. RET(-2,-7) is the compound gross return from month t-7 to month t-2. I will give all the coefficients by calculating the time series average of the monthly cross-sectional slope estimates and the t-statistic by calculating the average slope over Newey-West (1987) standard error.

The coefficient of interest in this case is the coefficient on the interaction term β_3. My hypothesis predicts that β_3 is positive. In other words, I posit that the severity of the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. If the Hypothesis 1 is true, I also expect that the slope β_1, i.e., the liquidity effect, to be insignificant after I include the interaction term in the regressions, which means that when $Ivol_t$ equals to zero (i.e., very easy to arbitrage), there is no liquidity anomaly any more.

2.4.2. Fama-Macbeth regression results of Amihud as illiquidity measure and idiosyncratic risk as the limits to arbitrage measure

First, I use Amihud (2002) measure as a proxy for illiquidity and run Equation (2.7). The regression test results are reported in Table 2.5. The regression is estimated cross-sectionally every month between January 1968 and December 2010 (Panel A), between January 1968 and December 1989 (Panel B), and between January 1990 and December 2010 (Panel C). In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios in Panel D.[10] Model 1 is a univariate regression on Amihud measure. Model 2 controls for the arbitrage risk. Model 3 includes the interaction term that Amihud by idiosyncratic volatility. Comparing the results of Model 3 to those of Model 2 will provide a direct interpretation about if the severity of

[10] Fu (2009) argues that AHXZ's (2006) negative results are mainly based on the portfolio of the highest idiosyncratic volatility that yields a negative abnormal return in the following month.

the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. Model 4 is a multivariate regression on Amihud measure controlling for three firm characteristics without the interaction term, while Model 5 is the same regression including the interaction term. Comparing the results of Model 5 to those of Model 4 will also tell if the severity of the limits to arbitrage aggravates the liquidity anomaly.

Results in Table 2.5 show that my previous conclusions are robust. I find that the exposure to higher liquidity significantly predicts lower future stock returns. Moreover, after controlling for exposures to three firm characteristics (i.e., SIZE, BM, and RET(-2,-7)), I still find that stock returns are negatively related to stock liquidity. Most importantly, the negative effect of liquidity on future stock returns is significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected positive sign. This evidence is consistent with my previous conclusion from the portfolio analysis, and fully supports the first part of Hypothesis 1. However, although the coefficients of Amihud are much smaller after including the interaction term, the liquidity anomaly is still significant. This finding is also consistent with my previous conclusion from the portfolio analysis, but it cannot fully support the second part of Hypothesis 1 that the ease of arbitrage is able to totally drive away the liquidity anomaly.

On the other hand, the exposure to higher idiosyncratic risk predicts higher stock returns. Moreover, after controlling for exposures to several firm characteristics, I still find that stock returns are positively related to idiosyncratic risk. This paper is consistent with the literature that finds a positive relation between idiosyncratic risk and individual stock returns (Malkiel and Xu (2002), Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Fu (2009), Huang, Liu, Rhee, and Zhang (2010)). However, this result contrasts sharply with the findings of AHXZ (2006) and Guo and Savickas (2006). Recent studies by Huang, Liu, Rhee, and Zhang (2007) and Fu (2009) both suggest that the return reversal in monthly returns explains the negative results in AHXZ (2006) and Guo and Savickas (2008). There is another finding that is worth pointing out. Spiegel and Wang (2007) find that while both liquidity and idiosyncratic risk play a role in determining stock returns, the impact of idiosyncratic risk is much stronger and often eliminates liquidity's explanatory power. However, the results in Table 2.5 show that the coefficient of Amihud is still significant after controlling for idiosyncratic risk. Therefore, the impact of idiosyncratic risk, as a proxy for the limits to arbitrage, does fade the liquidity's explanatory power of stock returns, but it cannot fully eliminate the explanatory power of liquidity according to my study.

Additionally, there is another result that is interesting. The coefficients of SIZE are significantly positive after including Ivol in the regression. Controlling for estimated idiosyncratic volatility, the traditional "size effect" is reversed. A similar finding is discovered in Fu (2009). Fu (2009) points out that his finding contrasts to the widely documented "size effect" that small firms have higher average returns than large firms, but supports one prediction of Merton's (1987) model that, all else equal, larger firms have higher expected returns. Merton (1987) explicitly points out that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk and investor base. My evidence lends direct support to Merton's prediction in this point.

The rest of Table 2.5 reports the results for two subsample periods from 1968 to 1989 and from 1990 to 2010 in Panel B and Panel C, respectively. The effect of the limits to arbitrage (Ivol) seems to be weak in the earlier subsample period, but it becomes much stronger in the later subsample period. While some of the coefficient estimates are noisier in the earlier subsample period than are those in the whole sample period, all have the expected signs. Overall, the subsample results are consistent with our previous conclusions from the whole sample results. In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios in Panel D. The negative effect of liquidity on future stock returns is still significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected positive sign. This evidence is consistent with my previous conclusion, and fully supports the Hypothesis 1. However, the coefficients of Ivol become insignificant after excluding the highest idiosyncratic volatility portfolios. This finding is, to some extent, consistent with the finding in Fu (2009) that AHXZ's (2006) arguments are mainly based on the portfolio of the highest idiosyncratic volatility that yields a negative abnormal return in the following month. Also, the coefficients of SIZE become insignificant after excluding the highest idiosyncratic volatility portfolios. This evidence indirectly supports Merton's (1987) argument that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk, and suggests that the stocks with the highest idiosyncratic risk may be the reason for the existence of the "size effect".

Table 2.5. Fama-MacBeth Regression Test Results for Amihud (2002) Measure and Idiosyncratic Volatility.

This table reports the slopes of the Fama-MacBeth regression:

$$\mathrm{R_t} = \beta_0 + \beta_1 \mathrm{Amihud}_{t-1} + \beta_2 \mathrm{Ivol}_t + \beta_3 \mathrm{Amihud}_{t-1} \times \mathrm{Ivol}_t + \beta_j \mathrm{Controls} + \varepsilon_t$$

where R_t is the monthly raw stock return. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. *Controls* are *SIZE*, *BM*, and *RET(-2,-7)*, which control for firm characteristics. *SIZE* is the market value of equity at the end of June of year *t*. Book-to-market equity (*BM*) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year $t-1$ divided by the market value of equity at the end of December of year $t-1$. *RET(-2,-7)* is the compound gross return from month t-7 to t-2. The regression is estimated cross-sectionally every month between January 1968 and December 2010 (Panel A), between January 1968 and December 1989 (Panel B), or between January 1990 and December 2010 (Panel C). In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios (Panel D). The estimates reported are the time series averages of the cross-sectional slope estimates; and *t* statistics based on the Newey-West (1987) standard errors are in parentheses. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Panel A: The whole sample					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.233* (4.03)	-0.431** (-1.99)	-0.298 (-1.35)	-0.715* (-3.19)	-0.583** (-2.57)
Amihud	0.035* (4.68)	0.032* (4.03)	0.021* (3.22)	0.024* (3.63)	0.015** (2.47)
Ivol		0.126* (7.22)	0.116* (6.46)	0.129* (7.80)	0.119* (6.98)
Amihud×Ivol			0.003* (2.63)		0.004* (3.05)
SIZE				0.034* (3.37)	0.028* (2.78)
BM				0.290* (5.16)	0.299* (5.33)
RET(-2,-7)				0.320** (2.40)	0.314** (2.38)
Panel B: The period from January 1968 to December 1989					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.218* (2.88)	-0.011 (-0.04)	0.132 (0.52)	-0.370 (-1.31)	-0.229 (-0.82)
Amihud	0.045* (3.87)	0.038* (3.19)	0.031** (2.09)	0.035** (2.51)	0.022** (2.03)
Ivol		0.111* (4.07)	0.099* (3.54)	0.114* (4.50)	0.101* (3.91)
Amihud×Ivol			0.003 (1.57)		0.004** (2.02)

Table 2.5. Continued.

Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
SIZE				0.036** (1.99)	0.027 (1.51)
BM				0.377* (4.06)	0.389* (4.22)
RET(-2,-7)				0.572** (2.56)	0.563** (2.55)
Panel C: The period from January 1990 to December 2010					
Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.248* (2.81)	-0.892** (-2.58)	-0.772** (-2.16)	-1.094* (-3.14)	-0.973* (-2.71)
Amihud	0.024* (5.66)	0.024* (5.71)	0.010* (4.96)	0.010* (5.32)	0.006* (3.36)
Ivol		0.142* (6.82)	0.134* (6.24)	0.146* (7.06)	0.138* (6.46)
Amihud×Ivol			0.003* (5.10)		0.003* (5.09)
SIZE				0.033* (4.00)	0.029* (3.51)
BM				0.195* (3.39)	0.200* (3.44)
RET(-2,-7)				0.042 (0.34)	0.041 (0.33)
Panel D: Excluding the highest Idiosyncratic volatility portfolios					
Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	0.924* (3.47)	0.895* (5.42)	0.904* (5.41)	0.649* (3.92)	0.659* (3.95)
Amihud	0.057* (6.26)	0.045* (4.96)	0.026* (2.90)	0.029* (4.38)	0.026** (2.26)
Ivol		0.005 (0.19)	0.005 (0.19)	0.007 (0.25)	0.006 (0.23)
Amihud×Ivol			0.008* (3.52)		0.010* (4.90)
SIZE				0.000 (0.06)	-0.000 (-0.06)
BM				0.240* (4.02)	0.248* (4.16)
RET(-2,-7)				0.444* (3.28)	0.441* (3.26)

2.5. Robustness check

2.5.1. Empirical results of turnover ratio as the liquidity measure

Selecting turnover ratio as another liquidity measure, I sort stocks into Turnover quintiles and idiosyncratic stock return volatility (as a proxy for the limits to arbitrage) quintiles, independently, and calculate the equally-weighted and value-weighted monthly raw stock returns for each portfolio. Turnover ratio (Turnover) is the average ratio of trading volume to the number of shares outstanding of the previous 36 months and it proxies for liquidity. The results are shown in Table 2.6 and the unit is percent. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns.

Table 2.6 shows some strong evidence that the average raw return spread between low-liquidity and high-liquidity stocks increases with the limits to arbitrage. More specifically, in panel A, the stocks of low Turnover overperform and the overperformance is more pronounced when the arbitrage risk is higher. For example, the time-series equally-weighted raw return spread between low Turnover and high Turnover stocks increases from 0.064% per month in the lowest arbitrage risk portfolio to 2.231% in the highest arbitrage risk portfolio. Additionally, it seems that the arbitrage risk might be priced, since the stocks with the highest idiosyncratic risk have the highest raw returns. This finding is consistent with Merton's (1987) incomplete-information CAPM and the results documented by Carroll and Wei (1988), Spiegel and Wang (2005), and Fu (2009), but it is inconsistent with the findings of AHXZ (2006, 2009) who find that stock returns are lower when idiosyncratic risk is higher.

The results in panel B are similar. The stocks of low Turnover overperform and the overperformance is more pronounced when the arbitrage risk is higher. For example, the time-series value-weighted raw return spread between low Turnover and high Turnover stocks increases from 0.1445% per month in the lowest arbitrage risk portfolio to 2.2125% in the highest arbitrage risk portfolio. However, I can hardly tell if the arbitrage risk is priced or not from value-weighted raw portfolio returns, which is inconsistent with the prediction of the incomplete-information CAPM suggested by Merton (1987) and the results documented by Carroll and Wei (1988), Spiegel and Wang (2005) and Fu (2009), and is also inconsistent with the negative findings of AHXZ (2006, 2009).

Table 2.7 reports monthly risk-adjusted portfolio returns sorted by turnover ratio as a proxy of liquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The risk-adjusted portfolio return shown in the table is the estimated intercept α_p from Equation (2.5). The results in Table 2.7 are very similar to those in Table 2.6. I observe that the liquidity anomaly is monotonically more profound as the arbitrage risk increases. For example, the risk-adjusted return spread between low

Turnover and high Turnover portfolios increases from 0.0711% per month in the lowest arbitrage risk portfolio to 1.0269% in the highest arbitrage risk portfolio, which is a significant difference of 0.9558%. This evidence fully supports the Hypothesis 1. In addition, the return spread between low and high liquidity stocks is mostly driven by the underperformance of high liquidity stocks except for the highest arbitrage risk portfolio. The results with Turnover sorting are very similar to those with Amihud sorting, but there is some minor difference. In Table 2.7, when the arbitrage risk is low, although the liquidity anomaly is still significant at 10% level, it is quite small and the t statistic is only 1.8. This finding cannot fully support the second part of Hypothesis 1, i.e., the relationship between liquidity and stock returns should be insignificant when the limits to arbitrage are low, either. But I can at least get that when the limits to arbitrage are low, the liquidity anomaly can be mostly arbitraged away.

Table 2.8 shows the liquidity-risk-adjusted stock returns of the same twenty-five portfolios. Panel A reports monthly liquidity-risk-adjusted portfolio returns and Panel B reports monthly portfolio liquidity-risk-factor loadings sorted by the turnover ratio (Turnover) and the proxy of the limits to arbitrage (Ivol). The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from Equation (2.6). The results in Panel A are very similar to those in Table 2.7. After the liquidity risk adjusted, I still observe that the liquidity anomaly is significantly more pronounced as the limits to arbitrage increase. For example, the liquidity-risk-adjusted return spread between low-liquidity and high-liquidity portfolios increases from 0.1147% per month in Ivol Quintile 1 to 1.1463% in Ivol Quintile 5, which is a significant difference of 1.0316%. That is to say, the liquidity risk does not drive away the positive relationship between the limits to arbitrage and the liquidity anomaly. This evidence completely supports the Hypothesis 2.

Next, I use turnover ratio as a proxy for liquidity and run the following equation to get the Fama-MacBeth regression results.

$$\mathbf{R_t = \beta_0 + \beta_1 Turnover_{t-1} + \beta_2 Ivol_t + \beta_3 Turnover_{t-1} \times Ivol_t + \beta_j Controls + \varepsilon_t,} \quad (2.8)$$

where $\mathrm{Turnover_{t-1}}$ is the turnover ratio, the average ratio of trading volume to the number of shares outstanding of the previous 36 months. The regression test results are reported in Table 2.9. The regression is estimated cross-sectionally every month between January 1968 and December 2010 (Panel A), between January 1968 and December 1989 (Panel B), and between January 1990 and December 2010 (Panel C). In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios in Panel D. Model 1 is a univariate regression on Turnover ratio. Model 2 controls for the limits to arbitrage. Model 3 includes the interaction term that Turnover by idiosyncratic volatility. Comparing the results of Model 3 to those of Model 2 will provide a direct interpretation about if the severity of the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. Model 4 is a multivariate regression on Turnover ratio controlling for three firm characteristics without the interaction term, while Model 5 is the same regression including the interaction term. Comparing the results of Model 5 to those of Model 4 will also tell if the severity of the limits to arbitrage aggravates the liquidity anomaly.

Results in Table 2.9 show that my previous conclusions are robust. I find that the exposure to higher liquidity significantly predicts lower future stock returns. Moreover, after controlling for exposures to three firm characteristics (i.e., SIZE, BM, and RET(-2,-7)), I still find that stock returns are negatively related to liquidity. Most importantly, the negative effect of liquidity on future stock returns is significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected negative sign. This evidence is consistent with my previous conclusion from the portfolio analysis and the Fama-MacBeth regression of Amihud. The test results in Table 2.9 also fully support my Hypothesis 1. Furthermore, the coefficients of Turnover change their signs and become insignificant after including the interaction term, which means that the liquidity anomaly no longer exists when it is very easy to arbitrage. This finding is different from my previous conclusion from the Fama-MacBeth regression of Amihud as illiquidity measure, but fully supports the Hypothesis 1 to be true.

On the other hand, the exposure to higher idiosyncratic risk predicts higher stock returns. Moreover, after controlling for exposures to several firm characteristics, I still find that stock returns are positively related to idiosyncratic risk. This paper is consistent with the literature that finds a positive relation between idiosyncratic risk and individual stock returns (Malkiel and Xu (2002), Goyal and Santa-Clara (2003), Spiegel and Wang (2005), Fu (2009), Huang, Liu, Rhee, and Zhang (2010)). However, this result contrasts sharply with the findings of AHXZ (2006) and Guo and Savickas (2006), even though AHXZ (2006) show that the significance in the relation between idiosyncratic volatility and value-weighted returns persists even after controlling for the bid-ask spread liquidity measure. Again, the results in Table 2.9 show that the coefficient of Turnover is still significant after controlling for idiosyncratic risk. Therefore, my study is inconsistent with Spiegel and Wang (2007) who find that the impact of idiosyncratic risk is much stronger and often eliminates liquidity's explanatory power.

Additionally, the phenomenon that the "size effect" no longer exists after controlling for idiosyncratic risk is still apparent. The coefficients of SIZE are significantly positive after including Ivol in the regression. Controlling for estimated idiosyncratic volatility, the traditional "size effect" is reversed. The evidence is consistent with the Fama-MacBeth regression of Amihud and lends a support to Merton's (1987) argument that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk.

The rest of Table 2.9 reports the results for two subsample periods from 1968 to 1989 and from 1990 to 2010 in Panel B and Panel C, respectively. The effect of the limits to arbitrage (Ivol) seems to be weak in the earlier subsample period, but it becomes much stronger in the later subsample period. While some of the coefficient estimates are noisier in the earlier subsample period than are those in the whole sample period, all have the expected signs. Overall, the subsample results are consistent with our previous conclusions from the whole sample results. In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios in Panel D. The negative effect of liquidity on future stock returns is still significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected negative sign. This evidence is consistent with my previous conclusion, and fully supports the Hypothesis 1. However, all the coefficients of Ivol become insignificant after excluding the highest idiosyncratic volatility portfolios. This finding is, to some extent, consistent with the finding in Fu (2009) claiming that AHXZ's (2006) arguments are mainly based on the portfolio of the highest idiosyncratic volatility that yields a negative abnormal return in the following month. Also, the coefficients of SIZE also become insignificant after excluding the highest idiosyncratic volatility portfolios. This evidence again indirectly supports Merton's (1987) argument that the findings of the "size effect" are due to the omitted controls for other factors such as idiosyncratic risk, and suggests that the stocks with the highest idiosyncratic risk may be the reason for the existence of the "size effect".

To sum up, the results using Turnover as the liquidity measure are consistent with those using Amihud measure, and they both support the Hypothesis 1. There is only one thing that is different. I find that the liquidity anomaly does not exist among stocks with very low limits to arbitrage when I employ Turnover as the liquidity proxy, but the liquidity anomaly still exists among stocks with very low limits to arbitrage when I employ Amihud as the illiquidity proxy.

Table 2.6. Raw Portfolio Returns by Turnover Ratio and Idiosyncratic Volatility.

This table reports the monthly raw portfolio returns sorted by turnover ratio as a proxy of liquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Turnover ratio (*Turnover*) is the average ratio of trading volume to the number of shares outstanding of the previous 36 months and it proxies for liquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns. The sample period is from January 1968 to December 2010.

Panel A: Equally-weighted raw portfolio returns						
Ivol \ *Turnover*	1(low)	2	3	4	5(high)	Spread (1-5)
1(low)	1.0647	0.8873	0.6976	1.0493	1.0007	0.0640
2	1.0195	0.9295	1.0352	1.0637	0.9560	0.0635
3	1.0004	0.7287	0.8279	0.8628	0.6805	0.3199
4	1.0208	0.3662	0.4856	0.3233	0.2397	0.7811
5(high)	3.9513	2.3433	2.0499	1.4972	1.7203	2.2310
Panel B: Value-weighted raw portfolio returns						
Ivol \ *Turnover*	1(low)	2	3	4	5(high)	Spread (1-5)
1(low)	1.0254	0.9122	1.0294	1.0029	0.8809	0.1445
2	1.0300	0.6551	0.6897	0.6390	0.8939	0.1361
3	0.7391	1.3377	0.6040	0.1289	0.4520	0.2871
4	1.3054	-0.3758	0.4237	-0.7296	0.2762	1.0292
5(high)	1.8063	1.3077	2.6385	0.4932	-0.4062	2.2125

Table 2.7. Risk-adjusted Portfolio Returns by Turnover Ratio and Idiosyncratic Volatility.

This table reports the monthly risk-adjusted portfolio returns sorted by turnover ratio as a proxy of liquidity and idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Turnover ratio (*Turnover*) is the average ratio of trading volume to the number of shares outstanding of the previous 36 months and it proxies for liquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. The risk-adjusted portfolio return is the estimated intercept α_p from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively. Factor returns and the risk-free rates are from Professor Kenneth French's website. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Risk-adjusted portfolio returns						
Turnover / *Ivol*	1(low)	2	3	4	5(high)	Spread (1-5)
1(low)	-0.1119* (-8.48)	0.0267 (0.84)	0.0843* (2.74)	-0.0889** (-2.51)	-0.1830* (-4.27)	0.0711*** (1.80)
2	0.0991 (1.44)	-0.0034 (-0.05)	0.0899 (1.59)	-0.0833 (-1.47)	-0.1607* (-9.65)	0.2598* (5.68)
3	-0.1942* (-8.56)	-0.3329* (-3.55)	-0.1438 (-1.62)	-0.3340* (-4.22)	-0.4657* (-6.24)	0.2715* (5.47)
4	-0.2487* (-8.20)	-0.5777* (-4.24)	-0.8001* (-6.24)	-0.4256* (-3.62)	-0.5587* (-5.68)	0.3100* (5.06)
5(high)	3.0870* (38.84)	3.1343* (29.51)	2.6845* (19.59)	2.3476* (18.60)	2.0601* (18.81)	1.0269* (8.95)
5-1	3.1989* (44.51)	3.1076* (24.55)	2.6002* (22.64)	2.4365* (21.58)	2.2431* (22.70)	0.9558* (7.82)

Table 2.8. Liquidity-Risk-Adjusted Portfolio Returns by Turnover Ratio and Idiosyncratic Volatility.

This table reports the monthly liquidity-risk-adjusted portfolio returns (Panel A) and monthly portfolio liquidity-risk-factor loadings (Panel B) sorted by turnover ratio (*Turnover*) and the proxy of the limits to arbitrage (*Ivol*). Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Turnover ratio (*Turnover*) is the average ratio of trading volume to the number of shares outstanding of the previous 36 months and it proxies for liquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively, from Professor Kenneth French's website. LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity factor is downloaded from CRSP. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Panel A: Liquidity-risk-adjusted portfolio returns						
Turnover / *Ivol*	1(low)	2	3	4	5(high)	Spread (1-5)
1(low)	-0.1090* (-8.10)	0.0223 (0.69)	0.0669** (2.13)	-0.1238* (-3.43)	-0.2237* (-5.13)	0.1147* (2.70)
2	0.0802 (1.14)	-0.0083 (-0.13)	0.0838 (1.44)	-0.0804 (-1.39)	-0.1677* (-9.88)	0.2479* (5.37)
3	-0.2134* (-9.20)	-0.3561* (-3.74)	-0.1879** (-2.07)	-0.3310* (-4.10)	-0.5232* (-6.85)	0.3098* (6.18)
4	-0.2779* (-8.94)	-0.5870* (-4.26)	-0.8321* (-6.39)	-0.4723* (-3.94)	-0.5907* (-5.90)	0.3128* (5.05)
5(high)	3.1675* (39.28)	3.1685* (9.54)	2.6512* (9.33)	2.3210* (8.35)	2.0212* (8.51)	1.1463* (9.88)
5-1	3.2765* (45.15)	3.1462* (24.61)	2.5843* (22.22)	2.4448* (17.15)	2.2449* (22.45)	1.0316* (8.35)
Panel B: Portfolio liquidity-risk-factor loadings						
Turnover / *Ivol*	1(low)	2	3	4	5(high)	Spread (1-5)
1(low)	2.3698* (7.35)	0.5264 (1.21)	2.0514* (3.74)	1.3993*** (1.86)	0.3186 (0.32)	2.0512** (1.97)
2	0.8705 (1.52)	1.5182** (2.13)	0.1087 (0.16)	-0.6821 (-0.90)	-1.0531 (-0.99)	1.9236 (1.59)
3	2.2888* (2.80)	4.4410* (3.98)	0.0839 (0.08)	-1.4289 (-1.49)	0.8864 (0.87)	1.4024 (1.07)
4	2.8204** (2.58)	9.5888* (6.01)	0.6012 (0.40)	0.6297 (0.46)	-0.6340 (-0.53)	3.4544** (2.13)
5(high)	10.5436* (5.59)	1.7227 (0.54)	2.4628 (0.84)	-5.7960** (-2.02)	-4.0478*** (-1.70)	14.5914* (4.80)
5-1	8.1738* (4.27)	1.1963 (0.37)	0.4114 (0.14)	-7.1953** (-2.42)	-4.3664*** (-1.69)	12.5402* (3.91)

Table 2.9. Fama-MacBeth Regression Test Results for Turnover Ratio and Idiosyncratic Volatility.

This table reports the slopes of the Fama-MacBeth regression

$$\mathrm{R_t} = \beta_0 + \beta_1 \mathrm{Turnover}_{t-1} + \beta_2 \mathrm{Ivol}_t + \beta_3 \mathrm{Turnover}_{t-1} \times \mathrm{Ivol}_t + \beta_j \mathrm{Controls} + \varepsilon_t$$

where R_t is the monthly raw stock return. Turnover ratio (*Turnover*) is the average ratio of trading volume to the number of shares outstanding of the previous 36 months and it proxies for liquidity. Monthly idiosyncratic stock return volatility (*Ivol*) is estimated by the Fama-French three-factor model and the EGARCH(1,1) model and it proxies for arbitrage risk. *Controls* are *SIZE*, *BM*, and *RET(-2,-7)*, which control for firm characteristics. *SIZE* is the market value of equity at the end of June of year *t*. Book-to-market equity (*BM*) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year $t-1$ divided by the market value of equity at the end of December of year $t-1$. *RET(-2,-7)* is the compound gross return from month t-7 to t-2. The regression is estimated cross-sectionally every month between January 1968 and December 2010 (Panel A), between January 1968 and December 1989 (Panel B), or between January 1990 and December 2010 (Panel C). In order to avoid the influence of some stocks with very high idiosyncratic volatility, I also report the Fama-MacBeth regression test results excluding the highest idiosyncratic volatility portfolios (Panel D). The estimates reported are the time series averages of the cross-sectional slope estimates; and *t* statistics based on the Newey-West (1987) standard errors are in parentheses. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Panel A: The whole sample					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.334* (4.97)	-0.201 (-0.94)	-0.321 (-1.45)	-0.411*** (-1.87)	-0.518** (-2.29)
Turnover	-0.391* (-3.46)	-0.439* (-3.86)	0.058 (0.21)	-0.400* (-3.81)	0.048 (0.18)
Ivol		0.128* (7.31)	0.137* (6.53)	0.129* (7.68)	0.138* (6.80)
Turnover×Ivol			-0.041** (-1.99)		-0.037*** (-1.85)
SIZE				0.026* (2.77)	0.027* (2.94)
BM				0.186* (3.69)	0.182* (3.61)
RET(-2,-7)				0.280** (2.13)	0.282** (2.18)
Panel B: The period from January 1968 to December 1989					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.311* (3.47)	0.186 (0.73)	-0.014 (-0.05)	-0.075 (-0.27)	-0.246 (-0.88)
Turnover	-0.502* (-3.33)	-0.602* (-3.08)	0.207 (0.40)	-0.537* (-3.00)	0.182 (0.36)
Ivol		0.111* (4.06)	0.126* (3.65)	0.110* (4.30)	0.124* (3.77)
Turnover×Ivol			-0.068*** (-1.78)		-0.061 (-1.63)

Table 2.9. Continued.

Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
SIZE				0.024 (1.44)	0.024 (1.53)
BM				0.259* (3.00)	0.251* (2.92)
RET(-2,-7)				0.543** (2.42)	0.540** (2.44)
Panel C: The period from January 1990 to December 2010					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.359* (3.56)	-0.627*** (-1.82)	-0.659*** (-1.81)	-0.779** (-2.28)	-0.817** (-2.28)
Turnover	-0.270* (-2.61)	-0.260* (-2.61)	-0.106 (-0.69)	-0.250** (-2.58)	-0.099 (-0.66)
Ivol		0.147* (6.95)	0.150* (6.66)	0.150* (7.11)	0.153* (6.85)
Turnover×Ivol			-0.061* (-3.24)		-0.061* (-3.25)
SIZE				0.029* (3.72)	0.029* (3.92)
BM				0.105** (2.33)	0.106** (2.32)
RET(-2,-7)				-0.010 (-0.09)	-0.003 (-0.02)
Panel D: Excluding the highest Idiosyncratic volatility portfolios					
Variable \ Model	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	0.986* (4.21)	0.970* (5.72)	0.901* (4.93)	0.812* (4.66)	0.770* (4.17)
Turnover	-0.304* (-2.76)	-0.197*** (-1.92)	0.013 (0.04)	-0.161*** (-1.72)	-0.069 (-0.23)
Ivol		0.005 (0.21)	0.012 (0.43)	0.003 (0.14)	0.008 (0.28)
Turnover×Ivol			-0.069* (-2.71)		-0.058** (-2.29)
SIZE				-0.004 (-0.53)	-0.004 (-0.57)
BM				0.147** (2.58)	0.147** (2.57)
RET(-2,-7)				0.423* (3.16)	0.418* (3.14)

2.5.2. Empirical results of bid-ask spread as the liquidity measure

In this subsection, I choose the bid-ask spread as an alternative measure for the liquidity. I sort stocks into the bid-ask spread (BidAsk) quintiles and the idiosyncratic volatility (Ivol) quintiles, independently, and calculate the equally-weighted and value-weighted monthly raw stock returns for each portfolio. The results are shown in Table 2.10 and the unit is percent. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns.

Table 2.10 shows some strong evidence, especially in equally-weighted portfolios, that the average raw return spread between low-liquidity and high-liquidity stocks increases with the limits to arbitrage. For example, the time-series equally-weighted raw return spread between low liquidity and high liquidity stocks increases from 0.1039% per month in the lowest limits to arbitrage portfolio to 1.6204% in the highest limits to arbitrage portfolio.

Table 2.11 reports monthly risk-adjusted portfolio returns sorted by the bid-ask spread as a proxy of liquidity and the idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The risk-adjusted portfolio return shown in the table is the estimated intercept α_p from Equation (2.5). The results in Table 2.11 are very similar to those in Table 2.10. I observe that the liquidity anomaly is monotonically more profound as the limits to arbitrage increases. For example, the risk-adjusted return spread between low liquidity and high liquidity portfolios increases from 0.3107% per month in the lowest limits to arbitrage portfolio to 1.1778% in the highest limits to arbitrage portfolio, which is a significant difference of 0.8671%. This evidence fully supports the Hypothesis 1 and shows that the previous results are robust even when I change the liquidity proxy.

Table 2.12 shows the liquidity-risk-adjusted stock returns of the same twenty-five portfolios. The table reports monthly liquidity-risk-adjusted portfolio returns sorted by the bid-ask spread (BidAsk) and the proxy of the limits to arbitrage (Ivol). The liquidity-risk-adjusted portfolio return is the estimated intercept α_p from Equation (2.6). The results are very similar to those in Table 2.11. After the liquidity risk adjusted, I still observe that the liquidity anomaly is significantly more pronounced as the limits to arbitrage increase. For example, the liquidity-risk-adjusted return spread between low-liquidity and high-liquidity portfolios increases from 0.3371% per month in Ivol Quintile 1 to 1.5924% in Ivol Quintile 5, which is a significant difference of 1.2553%. That is to say, the liquidity risk does not drive away the positive relationship between the limits to arbitrage and the liquidity anomaly. This evidence completely supports the Hypothesis 2 and also shows that the previous results are robust.

Next, I use the bid-ask spread as a proxy for liquidity and run the following equation to get the Fama-MacBeth regression results.

$$\mathbf{R_t = \beta_0 + \beta_1 BidAsk_{t-1} + \beta_2 Ivol_t + \beta_3 BidAsk_{t-1} \times Ivol_t + \beta_j Controls + \varepsilon_t,} \quad (2.9)$$

The regression test results are reported in Table 2.13. The regression is estimated cross-sectionally every month between January 1968 and December 2010. As the same as before, Model 1 is a univariate regression on the bid-ask spread. Model 2 controls for the limits to arbitrage. Model 3 includes the interaction term that bid-ask spread by the idiosyncratic volatility. Comparing the results of Model 3 to those of Model 2 will provide a direct interpretation about if the severity of the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. Model 4 is a multivariate regression on the bid-ask spread controlling for three firm characteristics without the interaction term, while Model 5 is the same regression including the interaction term. Comparing the results of Model 5 to those of Model 4 will also tell if the severity of the limits to arbitrage aggravates the liquidity anomaly.

Results in Table 2.13 show that my previous conclusions are robust. I find that the exposure to higher liquidity significantly predicts lower future stock returns. Moreover, after controlling for exposures to three firm characteristics (i.e., SIZE, BM, and RET(-2,-7)), I still find that stock returns are negatively related to liquidity. Most importantly, the negative effect of liquidity on future stock returns is significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected positive sign. This evidence is consistent with my previous conclusion from the portfolio analysis and the Fama-MacBeth regression of Amihud and Ivol. Furthermore, the coefficients of BidAsk become insignificant and even change signs after including the interaction term, which means that the liquidity anomaly is totally arbitraged away by the limits to arbitrage.

To sum up, the results using the bid-ask spread (BidAsk) as the liquidity measure are consistent with those using Amihud measure, and they both support the Hypothesis 1 and 2.

Table 2.10. Raw Portfolio Returns by Bid-Ask Spread and Idiosyncratic Volatility.

This table reports the monthly raw portfolio returns sorted by the bid-ask spread as a proxy of liquidity and the idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns. The sample period is from January 1968 to December 2010.

Panel A: Equally-weighted raw portfolio returns						
BidAsk / *Ivol*	1(high *BidAsk* / low liquidity)	2	3	4	5(low *BidAsk* / high liquidity)	Spread (1-5)
1(low)	0.3892	0.0966	0.0345	0.0016	0.2853	0.1039
2	1.2537	0.6653	0.0943	0.1183	0.9260	0.3277
3	1.3229	0.2928	-0.1071	-0.4291	0.8435	0.4794
4	1.8321	0.5643	-0.2930	-0.3517	0.6366	1.1955
5(high)	4.2072	2.9641	3.1444	3.0152	2.5868	1.6204
Panel B: Value-weighted raw portfolio returns						
BidAsk / *Ivol*	1(high *BidAsk* / low liquidity)	2	3	4	5(low *BidAsk* / high liquidity)	Spread (1-5)
1(low)	1.4726	1.4745	1.3233	0.9869	1.2218	0.2508
2	-0.7784	1.5466	1.2296	1.8911	1.3959	-2.1743
3	2.3506	1.3299	1.7391	2.4051	2.0416	0.3090
4	3.7283	1.9733	1.1632	2.0542	2.7729	0.9554
5(high)	6.7238	4.6062	5.0022	5.3666	4.7120	2.0118

Table 2.11. Risk-adjusted Portfolio Returns by Bid-Ask Spread and Idiosyncratic Volatility.

This table reports the monthly risk-adjusted portfolio returns sorted by the bid-ask spread as a proxy of liquidity and the idiosyncratic volatility as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The risk-adjusted portfolio return is the estimated intercept α_p from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively. Factor returns and the risk-free rates are from Professor Kenneth French's website. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Risk-adjusted portfolio returns						
BidAsk / *Ivol*	1(high *BidAsk* / low liquidity)	2	3	4	5(low *BidAsk*/ high liquidity)	Spread (1-5)
1(low)	0.2123* (4.89)	-0.2217* (-4.07)	-0.5655* (-7.93)	-0.7895* (-9.25)	-0.0984* (-10.00)	0.3107* (6.98)
2	0.3729* (5.24)	-0.2549* (-3.46)	-0.8188* (-9.81)	-0.8303* (-8.39)	-0.0495* (-3.87)	0.4224* (5.85)
3	0.3625* (3.18)	-0.6708* (-6.64)	-0.8770* (-8.15)	-1.3530* (-12.66)	-0.1240* (-7.38)	0.4865* (4.22)
4	0.6774* (3.54)	-0.4390* (-3.25)	-1.2818* (-10.00)	-1.3266* (-10.89)	-0.2659* (-11.76)	0.9433* (4.89)
5(high)	2.9062* (7.03)	1.6966* (6.60)	1.9631* (8.27)	1.6628* (8.21)	1.7284* (37.73)	1.1778* (2.83)
5-1	2.6939* (6.48)	1.9183* (7.30)	2.5286* (10.21)	2.4523* (11.16)	1.8268* (39.00)	0.8671** (2.07)

Table 2.12. Liquidity-Risk-Adjusted Returns by Bid-Ask Spread and Idiosyncratic Volatility.

This table reports the monthly liquidity-risk-adjusted portfolio returns sorted by the bid-ask spread (*BidAsk*) and the proxy of the limits to arbitrage (*Ivol*). Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively, from Professor Kenneth French's website. LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity risk factor is downloaded from CRSP. The numbers in parentheses are the t statistics. The sample period is from January 1968 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Liquidity-risk-adjusted portfolio returns						
BidAsk / *Ivol*	1(high *BidAsk* / low liquidity)	2	3	4	5(low *BidAsk* / high liquidity)	Spread (1-5)
1(low)	0.2284* (5.06)	-0.2585* (-4.38)	-0.6186* (-7.94)	-0.9600* (-9.72)	-0.1087* (-10.23)	0.3371* (7.28)
2	0.3862* (5.16)	-0.2673* (-3.35)	-0.8829* (-9.85)	-0.9455* (-8.39)	-0.0690* (-4.96)	0.4552* (5.98)
3	0.4084* (3.33)	-0.7000* (-6.39)	-0.9620* (-8.35)	-1.4739* (-12.47)	-0.1581* (-8.59)	0.5665* (4.56)
4	0.8056* (3.77)	-0.4403* (-3.03)	-1.3646* (-10.04)	-1.3754* (-10.41)	-0.3207* (-12.92)	1.1263* (5.24)
5(high)	3.4583* (7.24)	1.8810* (6.90)	2.1050* (8.49)	1.8418* (8.61)	1.8659* (37.01)	1.5924* (3.32)
5-1	3.2299* (6.73)	2.1395* (7.67)	2.7236* (10.48)	2.8018* (11.89)	1.9746* (38.34)	1.2553* (2.61)

Table 2.13. Fama-MacBeth Regression Test Results for Bid-Ask Spread and Idiosyncratic Volatility.

This table reports the slopes of the Fama-MacBeth regression:

$$R_t = \beta_0 + \beta_1 BidAsk_{t-1} + \beta_2 Ivol_t + \beta_3 BidAsk_{t-1} \times Ivol_t + \beta_j Controls + \varepsilon_t$$

where R_t is the monthly raw stock return. *Controls* are *SIZE*, *BM*, and *RET(-2,-7)*, which control for firm characteristics. *SIZE* is the market value of equity at the end of June of year *t*. Book-to-market equity (*BM*) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year $t-1$ divided by the market value of equity at the end of December of year $t-1$. *RET(-2,-7)* is the compound gross return from month t-7 to t-2. The regression is estimated cross-sectionally every month between January 1968 and December 2010. The estimates reported are the time series averages of the cross-sectional slope estimates; and *t* statistics based on the Newey-West (1987) standard errors are in parentheses. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Fama-MacBeth Regression Test Results for *BidAsk* and *Ivol*					
Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	0.125 (0.44)	-1.550* (-5.65)	-0.992** (-2.03)	-2.021* (-6.01)	-1.763* (-3.88)
BidAsk	0.734* (3.62)	1.018* (5.63)	-0.951 (-0.72)	1.267* (6.45)	-0.623 (-0.41)
Ivol		0.084* (4.77)	0.064* (4.07)	0.071* (4.44)	0.062* (3.21)
BidAsk×Ivol			0.279* (3.52)		0.275* (3.21)
SIZE				0.158 (0.13)	1.664** (2.46)
BM				0.439* (5.18)	0.488* (5.66)
RET(-2,-7)				-1.520* (-5.36)	-1.369* (-5.64)

2.5.3. Empirical results of the number of institutional shareholders as the limits to arbitrage measure

In this subsection, I choose the number of institutional shareholders as an alternative measure for the limits to arbitrage. I sort stocks into Amihud quintiles and the number of institutional shareholders (Inst) quintiles, independently, and calculate the equally-weighted and value-weighted monthly raw stock returns for each portfolio. The results are shown in Table 2.14 and the unit is percent. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns.

Table 2.14 shows some strong evidence, especially in equally-weighted portfolios, that the average raw return spread between low-liquidity and high-liquidity stocks increases with the limits to arbitrage. For example, the time-series equally-weighted raw return spread between low liquidity and high liquidity stocks increases from 0.0357% per month in the lowest limits to arbitrage portfolio to 9.1062% in the highest limits to arbitrage portfolio.

Table 2.15 reports monthly risk-adjusted portfolio returns sorted by Amihud measure as a proxy of illiquidity and the number of institutional shareholders as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. The risk-adjusted portfolio return shown in the table is the estimated intercept α_p from Equation (2.5). The results in Table 2.15 are very similar to those in Table 2.14. I observe that the liquidity anomaly is monotonically more profound as the limits to arbitrage increases. For example, the risk-adjusted return spread between low liquidity and high liquidity portfolios increases from -0.3881% per month in the lowest limits to arbitrage portfolio to 3.5803% in the highest limits to arbitrage portfolio, which is a significant difference of 3.9684%. This evidence fully supports the Hypothesis 1 and shows that the previous results are robust even when I change the limits to arbitrage proxy.

Table 2.16 shows the liquidity-risk-adjusted stock returns of the same twenty-five portfolios. The table reports monthly liquidity-risk-adjusted portfolio returns sorted by the Amihud measure (Amihud) and the proxy of the limits to arbitrage (Inst). The liquidity-risk-adjusted portfolio return is the estimated intercept α_p from Equation (2.6). The results are very similar to those in Table 2.15. After the liquidity risk adjusted, I still observe that the liquidity anomaly is significantly more pronounced as the limits to arbitrage increase. For example, the liquidity-risk-adjusted return spread between low-liquidity and high-liquidity portfolios increases from -0.4255% per month in Inst Quintile 1 to 5.5177% in Inst Quintile 5, which is a significant difference of 5.9432%. That is to say, the liquidity risk does not drive away the positive relationship between the limits to arbitrage and the liquidity anomaly. This evidence completely supports the Hypothesis 2 and also shows that the previous results are robust.

Next, I use the number of institutional shareholders as a proxy for the limits to arbitrage and run the following equation to get the Fama-MacBeth regression results.

$$\mathbf{R_t = \beta_0 + \beta_1 Amihud_{t-1} + \beta_2 Inst_t + \beta_3 Amihud_{t-1} \times Inst_t + \beta_j Controls + \varepsilon_t,} \quad (2.10)$$

The regression test results are reported in Table 2.17. The regression is estimated cross-sectionally every month between March 1980 and December 2010. As the same as before, Model 1 is a univariate regression on Amihud measure. Model 2 controls for the limits to arbitrage. Model 3 includes the interaction term that Amihud measure by the number of institutional shareholders. Comparing the results of Model 3 to those of Model 2 will provide a direct interpretation about if the severity of the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. Model 4 is a multivariate regression on Amihud measure controlling for three firm characteristics without the interaction term, while Model 5 is the same regression including the interaction term. Comparing the results of Model 5 to those of Model 4 will also tell if the severity of the limits to arbitrage aggravates the liquidity anomaly.

Results in Table 2.17 show that my previous conclusions are robust. I find that the exposure to higher liquidity significantly predicts lower future stock returns. Moreover, after controlling for exposures to three firm characteristics (i.e., SIZE, BM, and RET(-2,-7)), I still find that stock returns are negatively related to liquidity. Most importantly, the negative effect of liquidity on future stock returns is significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected positive sign. This evidence is consistent with my previous conclusion from the portfolio analysis and the Fama-MacBeth regression of Amihud and Ivol. Furthermore, the coefficients of Amihud become less significant after including the interaction term, which means that the liquidity anomaly is partially arbitraged away by the limits to arbitrage. This finding is also consistent with my previous conclusion from the Fama-MacBeth regression of Amihud and Ivol.

To sum up, the results using the number of institutional shareholders (Inst) as the limits to arbitrage measure are consistent with those using idiosyncratic volatility (Ivol), and they both support the Hypothesis 1 and 2.

Table 2.14. Raw Portfolio Returns by Amihud (2002) Measure and Investor Sophistication.

This table reports the monthly raw portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and the number of institutional shareholders as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. *Inst* is the number of institutional shareholders and it proxies for investor sophistication. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns. The sample period is from March 1980 to December 2010.

Panel A: Equally-weighted raw portfolio returns						
Amihud / *Inst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	0.6456	1.5369	1.2836	1.0825	0.6099	0.0357
2	-0.1122	1.3535	1.7022	1.1343	-0.6635	0.5513
3	-0.0919	2.0848	1.5746	0.7654	-1.0050	0.9131
4	0.3785	2.1522	1.1618	0.0470	-2.2115	2.5900
5(less)	1.0053	1.3827	0.0801	-0.4517	-8.1009	9.1062
Panel B: Value-weighted raw portfolio returns						
Amihud / *Inst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	1.8369	2.0698	1.9443	1.8518	1.6622	0.1747
2	1.5971	2.5151	2.1702	0.6636	1.0718	0.5253
3	1.1470	2.1380	2.0601	1.6034	1.1072	0.0398
4	2.7111	2.8527	2.3544	1.6191	0.3526	2.3585
5(less)	1.5556	1.9808	1.3027	1.2709	-4.7708	6.3264

Table 2.15. Risk-adjusted Portfolio Returns by Amihud (2002) Measure and Investor Sophistication.

This table reports the monthly risk-adjusted portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and the number of institutional shareholders as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. *Inst* is the number of institutional shareholders and it proxies for investor sophistication. The risk-adjusted portfolio return is the estimated intercept α_p from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively. Factor returns and the risk-free rates are from Professor Kenneth French's website. The numbers in parentheses are the t statistics. The sample period is from March 1980 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Risk-adjusted portfolio returns						
Amihud / *Inst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	-0.1579* (-4.15)	0.6402* (28.94)	0.4155* (16.27)	0.2302* (8.03)	0.2302* (12.44)	-0.3881* (-9.18)
2	-1.1210* (-9.13)	0.1919 (1.31)	0.2877 (1.23)	-0.1780 (-0.57)	-1.1735* (-7.67)	0.0525 (0.27)
3	-0.7598* (-6.28)	0.8422* (4.36)	0.4098* (3.61)	0.0061 (0.06)	-1.4500* (-10.82)	0.6902* (3.82)
4	1.0041* (5.37)	0.7249* (7.98)	0.4154* (5.59)	-0.0353 (-0.30)	-1.8677* (-5.59)	2.8718* (7.51)
5(less)	4.0453* (10.82)	0.8640 (0.53)	-0.0880 (-0.48)	0.6718* (9.84)	0.4650* (10.23)	3.5803* (9.51)
5-1	4.2032* (11.18)	0.2238 (0.14)	-0.5035* (-2.69)	0.4416* (5.96)	0.2348* (4.79)	3.9684* (10.47)

Table 2.16. Liquidity-Risk-Adjusted Returns by Amihud (2002) Measure and Investor Sophistication.

This table reports the monthly liquidity-risk-adjusted portfolio returns sorted by Amihud (2002) measure (*Amihud*) and the proxy of the limits to arbitrage (*Inst*). Stocks are sorted into quintiles based on liquidity proxy and independently into quintiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. *Inst* is the number of institutional shareholders and it proxies for investor sophistication. The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively, from Professor Kenneth French's website. LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity risk factor is downloaded from CRSP. The numbers in parentheses are the t statistics. The sample period is from March 1980 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Liquidity-risk-adjusted portfolio returns						
Amihud / *Inst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	-0.1737* (-4.09)	0.6692* (26.65)	0.4326* (15.04)	0.2403* (7.48)	0.2518* (12.21)	-0.4255* (-9.01)
2	-1.0996* (-8.91)	0.1978 (1.34)	0.3076 (1.31)	-0.1674 (-0.53)	-1.1491* (-7.50)	0.0495 (0.25)
3	-0.7491* (-6.14)	0.8404* (4.35)	0.4115* (3.62)	0.0060 (0.06)	-1.4509* (-10.88)	0.7018* (3.88)
4	1.0021* (5.35)	0.7284* (8.01)	0.4198* (5.63)	-0.0349 (-0.30)	-1.8814* (-5.74)	2.8835* (7.64)
5(less)	5.9814* (10.95)	0.8812 (0.54)	-0.0959 (-0.51)	0.6728* (9.82)	0.4637* (10.20)	5.5177* (10.07)
5-1	6.1551* (11.23)	0.2120 (0.13)	-0.5285* (-2.78)	0.4325* (5.72)	0.2119* (4.24)	5.9432* (10.80)

Table 2.17. Fama-MacBeth Regression Test Results for Amihud (2002) Measure and Investor Sophistication.

This table reports the slopes of the Fama-MacBeth regression:

$$R_t = \beta_0 + \beta_1 Amihud_{t-1} + \beta_2 Inst_t + \beta_3 Amihud_{t-1} \times Inst_t + \beta_j Controls + \varepsilon_t$$

where R_t is the monthly raw stock return. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. *Inst* is the number of institutional shareholders and it proxies for investor sophistication. *Controls* are *SIZE*, *BM*, and *RET(-2,-7)*, which control for firm characteristics. *SIZE* is the market value of equity at the end of June of year *t*. Book-to-market equity (*BM*) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year *t*−1 divided by the market value of equity at the end of December of year *t*−1. *RET(-2,-7)* is the compound gross return from month t-7 to t-2. The regression is estimated cross-sectionally every month between March 1980 and December 2010. The estimates reported are the time series averages of the cross-sectional slope estimates; and *t* statistics based on the Newey-West (1987) standard errors are in parentheses. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Fama-MacBeth Regression Test Results for *Amihud* and *Inst*					
Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	0.955** (2.30)	0.919** (2.07)	1.042** (2.33)	0.644 (1.38)	0.762 (1.63)
Amihud	0.045* (4.60)	0.044* (4.57)	0.035** (2.09)	0.044* (4.53)	0.033** (2.09)
Inst		-0.001 (-1.30)	-0.001 (-0.89)	-0.004** (-2.60)	-0.003** (-2.26)
Amihud×Inst			0.034* (3.26)		0.034* (3.33)
SIZE				-0.074* (-3.62)	-0.071* (-3.47)
BM				0.298* (2.75)	0.307* (2.90)
RET(-2,-7)				1.076* (3.40)	1.015* (3.27)

2.5.4. Empirical results of the number of analysts' estimates as the limits to arbitrage measure

In this subsection, I use analyst coverage as another measure for the limits to arbitrage. I sort stocks into Amihud quintiles and the number of analysts' estimates (as a proxy for information uncertainty) quartiles, independently, and calculate the equally-weighted and value-weighted monthly raw stock returns for each portfolio. The results are shown in Table 2.18 and the unit is percent. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns.

Table 2.18 shows that the average raw return spread between low-liquidity and high-liquidity stocks increases with the limits to arbitrage. More specifically, in panel A, the stocks of low liquidity overperform and the overperformance is more pronounced when the limits to arbitrage is higher. For example, the time-series equally-weighted raw return spread between low liquidity and high liquidity stocks increases from -0.1515% per month in the lowest limits to arbitrage portfolio to 1.9701% in the highest limits to arbitrage portfolio. The results in panel B are similar. The stocks of low liquidity overperform and the overperformance is more pronounced when the limits to arbitrage is higher.

Table 2.19 reports monthly risk-adjusted portfolio returns sorted by Amihud measure as a proxy of illiquidity and analyst coverage as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quartiles by the limits to arbitrage proxy. The risk-adjusted portfolio return shown in the table is the estimated intercept α_p from Equation (2.5). I observe that the liquidity anomaly is monotonically more profound as the limits to arbitrage increases. For example, the risk-adjusted return spread between low liquidity and high liquidity portfolios increases from 0.5188% per month in the lowest limits to arbitrage portfolio to 2.9489% in the highest limits to arbitrage portfolio, which is a significant difference of 2.4301%. This evidence fully supports the Hypothesis 1. In addition, the return spread between low and high liquidity stocks is mostly driven by the underperformance of high liquidity stocks. All the evidence shows that the previous results are robust.

Table 2.20 shows the liquidity-risk-adjusted stock returns of the same twenty portfolios. The liquidity-risk-adjusted portfolio return is the estimated intercept α_p from Equation (2.6). The results are very similar to those in Table 2.19. After the liquidity risk adjusted, I still observe that the liquidity anomaly is significantly more pronounced as the limits to arbitrage increase. For example, the liquidity-risk-adjusted return spread between low-liquidity and high-liquidity portfolios increases from 0.5730% per month in Analyst Quartile 1 to 3.4023% in Analyst Quartile 4, which is a significant difference of 2.8293%. That is to say, the liquidity risk does not drive away the positive relationship between the limits to arbitrage and the liquidity anomaly. This evidence completely supports the Hypothesis 2.

Next, I use the number of analysts' estimates as a proxy for limits to arbitrage and run the following equation to get the Fama-MacBeth regression results.

$$\mathbf{R_t = \beta_0 + \beta_1 Amihud_{t-1} + \beta_2 Analyst_t + \beta_3 Amihud_{t-1} \times Analyst_t + \beta_j Controls + \varepsilon_t,} \tag{2.11}$$

The regression test results are reported in Table 2.21. The regression is estimated cross-sectionally every month between January 1990 and December 2010. Model 1 is a univariate regression on Amihud measure. Model 2 controls for analyst coverage. Model 3 includes the interaction term that Amihud by Analyst. Comparing the results of Model 3 to those of Model 2 will provide a direct interpretation about if the severity of the limits to arbitrage aggravates the negative effect of liquidity on future stock returns. Model 4 is a multivariate regression on Amihud measure controlling for three firm characteristics without the interaction term, while Model 5 is the same regression including the interaction term. Comparing the results of Model 5 to those of Model 4 will also tell if the severity of the limits to arbitrage aggravates the liquidity anomaly.

Results in Table 2.21 show that my previous conclusions are robust. I find that the exposure to higher liquidity significantly predicts lower future stock returns. Moreover, after controlling for exposures to three firm characteristics (i.e., SIZE, BM, and RET(-2,-

7)), I still find that stock returns are negatively related to liquidity. Most importantly, the negative effect of liquidity on future stock returns is significantly stronger when the limits to arbitrage are higher as indicated by the significant coefficients on the interaction terms with an expected positive sign. This evidence is consistent with my previous conclusion from the portfolio analysis and the Fama-MacBeth regression of Amihud and Ivol. Furthermore, the coefficients of Amihud become insignificant after including the interaction term, which means that the liquidity anomaly no longer exists when it is very easy to arbitrage.

To sum up, the results using Analyst as the limits to arbitrage measure are consistent with those using idiosyncratic volatility, and they both support the Hypothesis 1 and 2. There is only one thing that is different. I find that the liquidity anomaly does not exist among stocks with very low limits to arbitrage when I employ Analyst as the limits to arbitrage proxy, but the liquidity anomaly still exists among stocks with very low limits to arbitrage when I employ Ivol as the limits to arbitrage proxy.

Table 2.18. Raw Portfolio Returns by Amihud (2002) Measure and Analyst Coverage.

This table reports the monthly raw portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and analyst coverage as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quartiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Analyst coverage (*Analyst*) is the number of analysts' estimates following a stock and it proxies for information uncertainty. Panel A shows the time series average of equally-weighted raw portfolio returns, while panel B shows the time series average of value-weighted raw portfolio returns. The sample period is from January 1990 to December 2010.

Panel A: Equally-weighted raw portfolio returns						
Amihud / *Analyst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	0.3802	1.6418	1.2988	1.0112	0.5317	-0.1515
2	3.2105	5.0635	4.9170	4.2030	1.3993	1.8112
3	0.8276	1.2121	0.9403	0.0427	-1.6050	2.4326
4(less)	0.8942	0.7925	0.1721	-0.4728	-1.0759	1.9701
Panel B: Value-weighted raw portfolio returns						
Amihud / *Analyst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	1.6708	2.1085	1.9968	1.8561	1.6473	0.0235
2	4.1303	6.6332	5.8374	4.8362	2.6917	1.4386
3	1.4739	2.2291	2.2169	1.7393	-0.9156	2.3895
4(less)	3.8939	1.8254	1.4761	1.6126	1.6921	2.2018

Table 2.19. Risk-adjusted Portfolio Returns by Amihud (2002) Measure and Analyst Coverage.

This table reports the monthly risk-adjusted portfolio returns sorted by Amihud (2002) measure as a proxy of liquidity and analyst coverage as a proxy of the limits to arbitrage. Stocks are sorted into quintiles based on liquidity proxy and independently into quartiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Analyst coverage (*Analyst*) is the number of analysts' estimates following a stock and it proxies for information uncertainty. The risk-adjusted portfolio return is the estimated intercept α_p from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively. Factor returns and the risk-free rates are from Professor Kenneth French's website. The numbers in parentheses are the t statistics. The sample period is from January 1990 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Risk-adjusted portfolio returns						
Amihud / *Analyst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	0.1583* (8.92)	0.6607* (32.05)	0.4069* (17.21)	0.1779* (6.69)	-0.3605* (-10.07)	0.5188* (12.98)
2	1.2887* (2.61)	6.6214* (2.62)	0.0154 (0.02)	0.3469 (0.45)	-0.9184 (-1.57)	2.2071* (2.88)
3	0.6694* (9.34)	0.6445* (6.27)	0.2288 (1.49)	-0.5579** (-2.40)	-2.2267* (-5.40)	2.8961* (6.92)
4(less)	0.3261** (2.24)	0.1875 (1.61)	-0.4796** (-2.28)	-1.0434* (-2.72)	-2.6228* (9.31)	2.9489* (9.30)
4-1	0.1678 (1.14)	-0.4732* (-4.00)	-0.8865* (-4.19)	-1.2213* (-3.18)	-2.2623* (-7.97)	2.4301* (7.60)

Table 2.20. Liquidity-Risk-Adjusted Returns by Amihud (2002) Measure and Analyst Coverage.

This table reports the monthly liquidity-risk-adjusted portfolio returns sorted by Amihud (2002) measure (*Amihud*) and the proxy of the limits to arbitrage (*Analyst*). Stocks are sorted into quintiles based on liquidity proxy and independently into quartiles by the limits to arbitrage proxy. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Analyst coverage (*Analyst*) is the number of analysts' estimates following a stock and it proxies for information uncertainty. The liquidity-risk-adjusted portfolio return and the portfolio liquidity-risk-factor loading are the estimated intercept α_p and the estimate of the slope coefficient $\beta_{p,LIQ}$, respectively, from the following regression:

$$R_p - r_f = \alpha_p + \beta_{p,Mkt}Mkt + \beta_{p,SMB}SMB + \beta_{p,HML}HML + \beta_{p,MOM}MOM + \beta_{p,LIQ}LIQ + \varepsilon_p$$

where R_p is the raw return on portfolio p and r_f is the risk-free rate. Mkt, SMB, HML, and MOM are returns on the market, size, book-to-market, and momentum factors, respectively, from Professor Kenneth French's website. LIQ is the liquidity risk factor documented in Pastor and Stambaugh (2003). The value-weighted liquidity risk factor is downloaded from CRSP. The numbers in parentheses are the t statistics. The sample period is from January 1990 to December 2010. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Liquidity-risk-adjusted portfolio returns						
Amihud / *Analyst*	1(high *Amihud* / low liquidity)	2	3	4	5(low *Amihud* / high liquidity)	Spread (1-5)
1(more)	0.1711* (8.74)	0.6928* (30.19)	0.4225* (16.18)	0.1801* (6.15)	-0.4019* (-10.22)	0.5730* (13.04)
2	1.1148** (2.56)	6.3527** (2.56)	0.0141 (0.02)	0.2475 (0.32)	-0.9564 (-1.63)	2.0712* (2.83)
3	0.6181* (8.30)	0.6529* (6.09)	0.2293 (1.45)	-0.6585* (-2.78)	-2.4094* (-5.81)	3.0275* (7.19)
4(less)	0.7524** (2.51)	0.2292*** (1.91)	-0.6067* (-2.84)	-1.3099* (-3.43)	-2.6499* (9.28)	3.4023* (8.22)
4-1	0.5813*** (1.94)	-0.4636* (-3.79)	-1.0292* (-4.78)	-1.4900* (-3.89)	-2.2480* (7.80)	2.8293* (6.80)

Table 2.21. Fama-MacBeth Regression Test Results for Amihud (2002) Measure and Analyst Coverage.

This table reports the slopes of the Fama-MacBeth regression:

$$R_t = \beta_0 + \beta_1 \mathrm{Amihud}_{t-1} + \beta_2 \mathrm{Analyst}_t + \beta_3 \mathrm{Amihud}_{t-1} \times \mathrm{Analyst}_t + \beta_j \mathrm{Controls} + \varepsilon_t$$

where R_t is the monthly raw stock return. Amihud (2002) measure (*Amihud*) is the monthly average of absolute value of return divided by dollar volume every day and it proxies for illiquidity. Analyst coverage (*Analyst*) is the number of analysts' estimates following a stock and it proxies for information uncertainty. *Controls* are *SIZE*, *BM*, and *RET(-2,-7)*, which control for firm characteristics. *SIZE* is the market value of equity at the end of June of year *t*. Book-to-market equity (*BM*) is the book value of equity according to Fama and French (1993) at the end of fiscal year ending in calendar year *t*−1 divided by the market value of equity at the end of December of year *t*−1. *RET(-2,-7)* is the compound gross return from month t-7 to t-2. The regression is estimated cross-sectionally every month between January 1990 and December 2010. The estimates reported are the time series averages of the cross-sectional slope estimates; and *t* statistics based on the Newey-West (1987) standard errors are in parentheses. Statistical significance at the 1%, 5%, and 10% levels are represented by *, ** and ***, respectively.

Fama-MacBeth Regression Test Results for *Amihud* and *Analyst*					
Model / Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5
Intercept	1.181* (2.86)	1.272* (2.83)	1.247* (2.78)	1.102** (2.32)	1.085** (2.31)
Amihud	0.205* (2.94)	0.205* (3.00)	0.067 (1.12)	0.302* (3.97)	0.086 (0.37)
Analyst		-0.060 (-1.01)	-0.039 (-0.59)	-0.035 (-0.66)	0.016 (0.25)
Amihud×Analyst			0.569* (3.83)		0.511* (3.01)
SIZE				-0.498 (-1.65)	-0.542 (-1.65)
BM				0.356*** (1.67)	0.318 (1.47)
RET(-2,-7)				0.073 (0.13)	-0.007 (-0.01)

2.6. Conclusions

Recent literature finds that there is a negative effect of stock liquidity on future abnormal stock returns, which is often referred to as the "liquidity anomaly". It is always interesting to examine what causes a return anomaly and why the anomaly is not arbitraged away. In this paper, I do document the reason for why the liquidity anomaly is not arbitraged away. It is the limits to arbitrage. I consider idiosyncratic volatility as a proxy of arbitrage risk, which is the most important part of the limits to arbitrage, and also consider investor sophistication and information uncertainty as two alternative proxies since they both influence the limits to arbitrage.

I find that the liquidity anomaly is stronger when the limits to arbitrage are more severe. The profitability of a strategy that utilizes the liquidity anomaly is mostly driven by the underperformance of the stocks of high liquidity stocks. Moreover, the anomaly is very weak among stocks that have low arbitrage risk, and the ease of limits to arbitrage even drives away the liquidity anomaly especially when I use turnover ratio as a proxy for liquidity or analyst coverage as a proxy for the limits to arbitrage. I further show that my results are not driven by liquidity risk. The findings are consistent with the mispricing explanation.

I also show that although the idiosyncratic risk can help to explain why the liquidity anomaly is not arbitraged away, but the real reason is the impact of the limits to arbitrage. Several robustness tests support my argument. This paper is also consistent with the literature that finds a positive relation between idiosyncratic risk and individual stock returns. In addition, I find that after controlling for idiosyncratic risk, “size effect” no longer exists. Merton (1987) explicitly points out that the findings of the “size effect” are due to the omitted controls for other factors such as idiosyncratic risk. The test results in this paper lend direct support to Merton’s prediction in this point.

In summary, all the portfolio analysis results, together with the Fama-MacBeth regression results, prove my hypothesis that the liquidity anomaly should be more pronounced when there are more severe limits to arbitrage. At the same time, they support my argument that the limits to arbitrage are an important reason that the liquidity anomaly persists, and confirm the argument by Shleifer and Vishny (1997) that when arbitrage is risky and costly, arbitrageurs stay away from engaging in arbitrage activities.

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