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Vulnerability Analysis of the Financial Network

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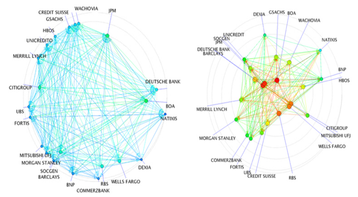
Introduction

Financial Network:
Interacting with each other trough borrowing or lending
Interconnecting indirectly through market by holding similar shares or portfolio.



Systemic Risk in Financial Network:

Motivation

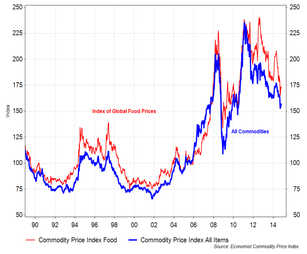
The Uncertainties

- Incomplete Information on Interbank Liabilities

$$\begin{bmatrix} 0 & l_{12} & l_{13} & \cdots & l_{1n} \\ l_{21} & 0 & l_{23} & \cdots & l_{2n} \\ l_{31} & l_{32} & 0 & \cdots & l_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 0 \end{bmatrix} \quad \begin{matrix} \sum_j \\ p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{matrix}$$

$$\sum_i \quad c_1 \quad c_2 \quad c_3 \quad \cdots \quad c_n \rightarrow \text{Known (balance sheet)}$$

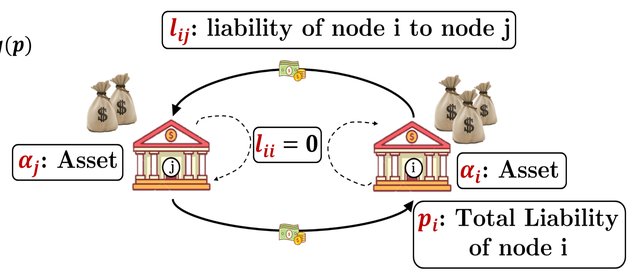
- Market Fluctuation



Eisenberg-Noe's (Clearing Agent Model)

$P = diag(p)$

$e = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$



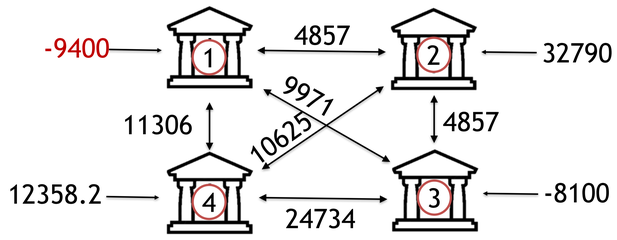
$$\max_x \ p^T x \qquad s.t.\ (P - L^T)x \le \alpha \qquad 0 \le x \le e$$

$x_i^* = 1$ Solvent
$x_i^* \in (0,1)$ Default
$x_i^* \le 0$ Bankrupted

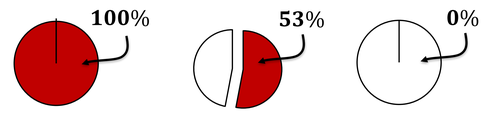
The Vulnerability of A Financial Network under A Single Shock

The impact of a single shock on the receiving node

Theorem 1 : We estimate the lower bound for the amount of negative shock under which bank i will be bankrupted.

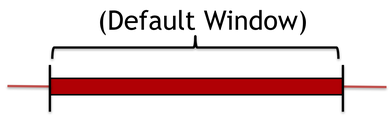


Theorem 2 : We estimate the lower bound for the amount of positive shock under which bank i will be solvent.



$\alpha_{1+} = 1781.71$ $\alpha_1 = -9400$ $\alpha_{1-} = -17894.7$

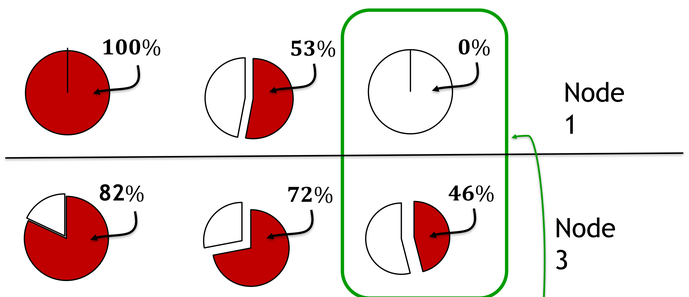
Corollary 1 : We estimate the interval under which bank i is default



$\alpha_{1-} = -17894.7$ $\alpha_{1+} = 1781.71$

The indirect impact of the shock on other nodes in the system

Theorem 3: For the case that node 1 receives the shock (s_1), we can study the sustainability of the other nodes in the system.



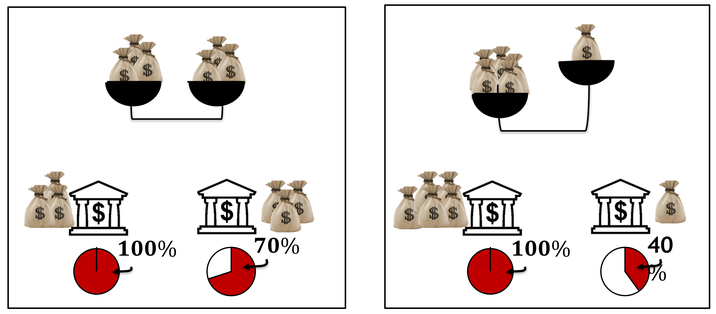
$\alpha_{1+} = 1781.71$ $\alpha_1 = -9400$ $\alpha_{1-} = -17894.7$

Node 3 is more sustainable than node 1

The Vulnerability of A Financial Network under Multiple Shocks

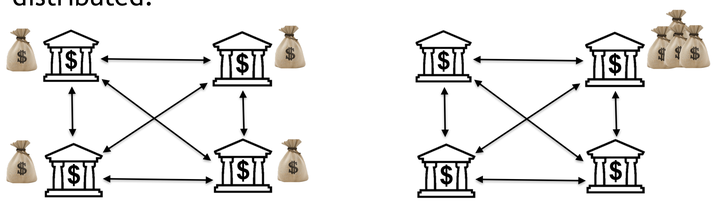
Asset inequality and stability of the financial system

Proposition 1: Suppose that the summation of the assets of one default node and another strictly solvent node remain invariant. A larger asset inequality between these two nodes will decrease the stability of the financial network.



Probability analysis on the vulnerability of a financial network

Theorem 4: For a fixed total asset we have:
(i) The network with a monopoly node has the highest probability of insolvency and is the most vulnerable one.
(ii) The system is the most stable when the assets are evenly distributed.



The Most Stable — The Least Stable

Conclusion

- (In)feasibility analysis of the financial network based on the (extended) clearing-agent model;
- Estimate the probability of default and bankruptcy of financial institute in the system;
- Identified some worst-case scenario in the financial network.

References

[1] Eisenberg, L., and Noe, T. H. (2001). Systemic risk in financial systems. Management Sci. 47(2):236-249.