

INVESTIGATION OF SUSTAINED OSCILLATIONS
IN A FOURTH ORDER CONTROL SYSTEM WITH
AMPLITUDE SENSITIVE NONLINEARITY

A Thesis
Presented to
the Faculty of the Department
of Electrical Engineering
University of Houston

In partial fulfillment
of the Requirements for the Degree
Master of Science

by

Bala Prasad SARDA
Jan. 1970

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ABSTRACT

To investigate the nature of sustained oscillations in a fourth order control system with an Amplitude sensitive nonlinearity. The basic assumption is that the open loop linear transfer function of the system is stable.

The graphical technique for solving this problem is by substituting the nonlinearity by its equivalent describing function and then applying Nyquist stability criterion. An analytical method to solve the problem is developed.

Expressions for amplitude and frequency of sustained oscillations are derived. Regions of stability and instability are indicated. Expressions for amplitude and frequency of sustained oscillations are derived. Regions of stability and instability are indicated. Expressions for amplitude and frequency of sustained oscillations are reduced to Third and Second order control system by reducing the order of the system.

An example is presented and the results are verified.

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INTRODUCTION

One of the fundamental problems in the analysis of any physical system is the question of stability. A linear system is defined as stable if the output response to any bounded input disturbance is finite, which in simple words means that a linear system is stable if, and only if, any bounded input produces a bounded output. This implies that all the roots of the characteristic equation must be located in the left half of the S-plane. Roots that are in the right half of the S-plane give rise to transients which tend to diverge from the steady state, and the system is said to be unstable. Thus the stability of a linear feedback control system is also uniquely determined by the location of the roots of its characteristic equation.

Stability conditions for nonlinear systems have been the subject of a great deal of investigation. A nonlinear system may be completely stable or completely unstable and in addition a nonlinear system may exhibit a " Limit Cycle " or sustained finite - amplitude oscillation. The cause and effect(input and output) are no longer directly proportional, the form of the transient response to a step input is no longer independent of the amplitude of the step, and the response to an arbitrary input cannot be found as the sum of the responses to a series of step or pulse inputs. Which in simple words means that the principle of Superposition is no longer valid, and consequently

the loss of the use of the Fourier Integral. This destroys the formal relationship between the time domain and the frequency domain for nonlinear systems. Both time and frequency analyses may be made independently, but it is not possible to compute the time response from the frequency response or vice versa. In general, if the system is to be used with step and ramp inputs, a time domain analysis and synthesis seems necessary; if sinusoidal or periodic inputs are anticipated, a frequency - response analysis and design is suitable.

As a general consequence of the invalidation of the superposition principle, it may be said that the ability to extrapolate or predict is lost. Certainly knowledge of the transient response of a nonlinear system to a unit step input does not permit quantitative prediction of the response of that system to a larger or smaller step. In fact, for some systems, a change in the magnitude of the step may result in a change from a stable response to an unstable response or vice versa. Thus the study of stability in nonlinear systems requires the usual information used in linear - system studies, i.e., knowledge of the physical and mathematical characteristics of the components. In addition, however, it is necessary to know the type of forcing functions anticipated, the range of values expected for the magnitudes of these forcing functions, the types of noise and load disturbances, and any initial

conditions which may arise.

Limit Cycle: In phase - plane terminology Limit cycle is the name given to a nonlinear oscillation of fixed frequency and amplitude determined by the nonlinear properties of the system. Limit cycles are one of the most frequently encountered modes of behavior peculiar to nonlinear systems. Limit cycles are distinguished from linear oscillations in that their amplitude of oscillation is independent of initial conditions.

Limit cycles are generally defined in 3 types (shown in Figure 2)

- (1) Orbitally stable limit cycle:- Paths spiral toward the limit cycle C from both sides as t increases.
- (2) Orbitally unstable limit cycle:- Paths spiral away from C on both sides as t increases.
- (3) Orbitally semistable limit cycle:- Paths spiral toward C from one side and away from C on the other side as t increases.

Limit cycles are generally undesirable in control applications; however, at times they can be accepted if the amplitude of oscillation is small. On the other hand, limit cycles play an important role in the design of oscillators.

Types of Nonlinearities:- The nonlinearities encountered in control systems may be classified as accidental nonlinearities and intentional nonlinearities. The accidental class consists

of those nonlinearities which are inherent in the components used in the system. Intentional nonlinearities are those which are introduced deliberately, either to obtain desired performance features or for economic advantages. A well - known illustration is the introduction of a relay to provide a relay servo. A further classification may be into analytic and nonanalytic nonlinearities. Slowly varying nonlinearities, such as those which can be represented graphically by a smooth single valued curve or by a family of smooth curves, are considered analytic nonlinearities. Nonanalytic nonlinearities exhibit abrupt changes in their characteristic and are graphically represented as discontinuous or multivalued curves.

Methods of investigating Nonlinear Systems:- The first part of a nonlinear analysis is usually concerned with stability and the existence of limit cycles. If the system is not stable or seeks a limit cycle for some set of expected operating conditions, then it is always better to stop the analysis until a stabilizing modification is found or on the other hand, he might complete the analysis expecting to use the results as a guide leading to a suitable modification. Many methods are available for such analyses. No single method is applicable to all nonlinear systems because of the diversity of nonlinear phenomena, the type of information desired from the analysis, and the availability of time and equipment.

SOME OF THE MOST COMMON TYPES OF NONLINEAR ELEMENTS

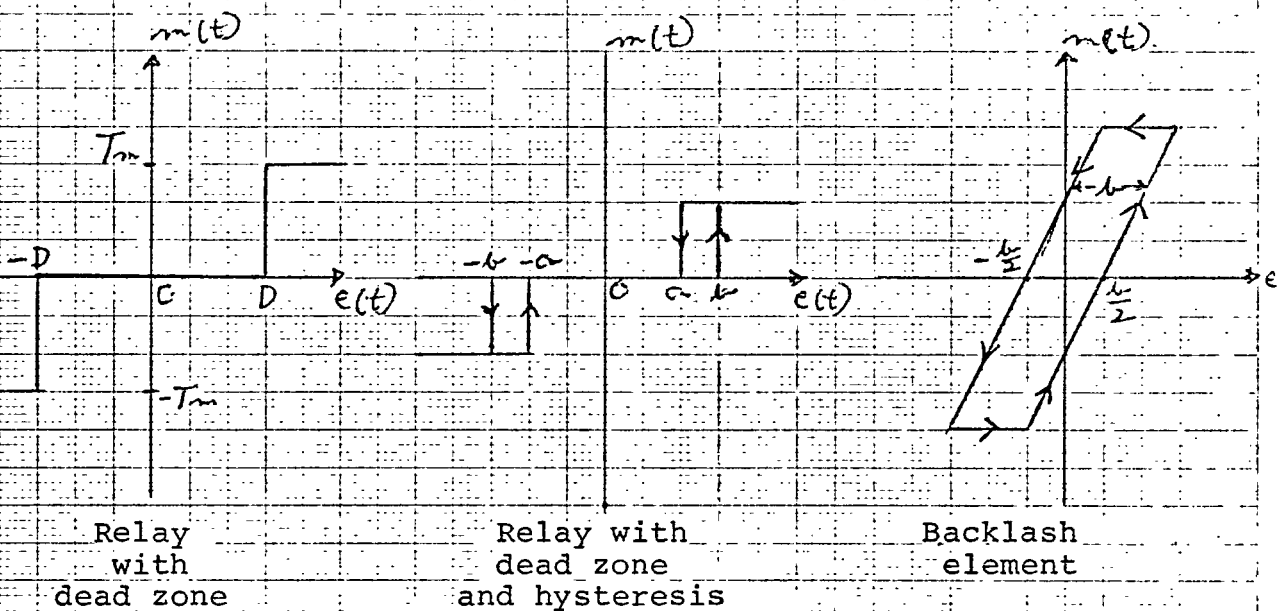
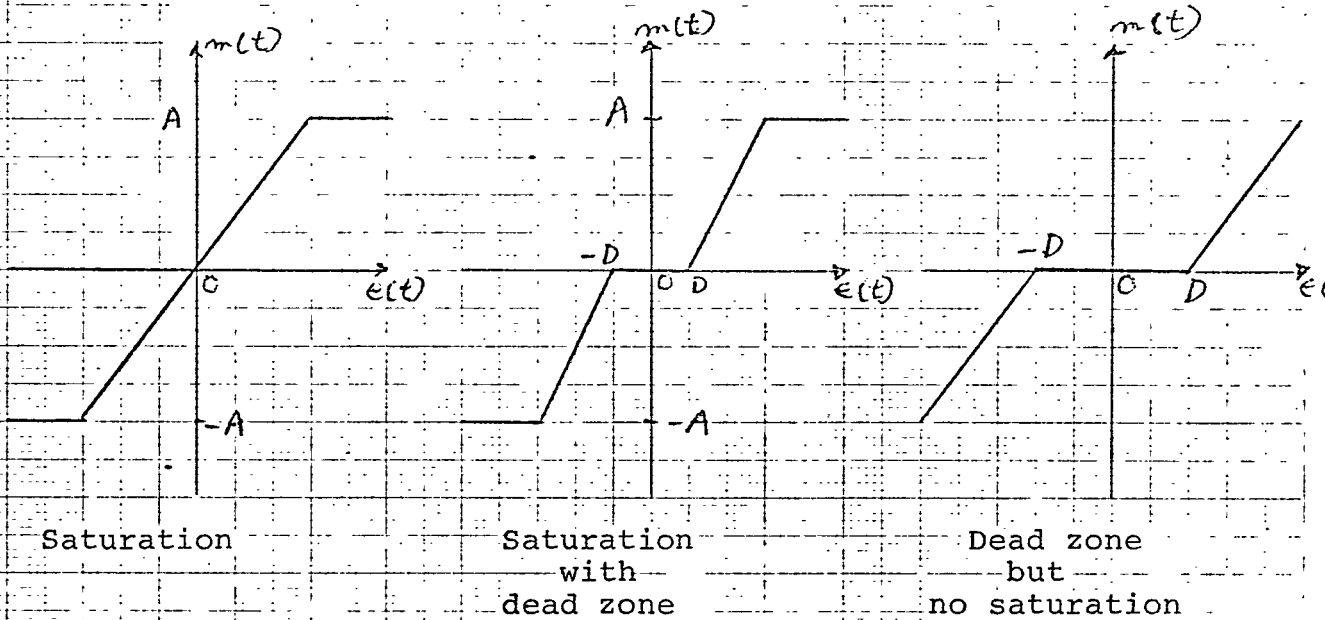
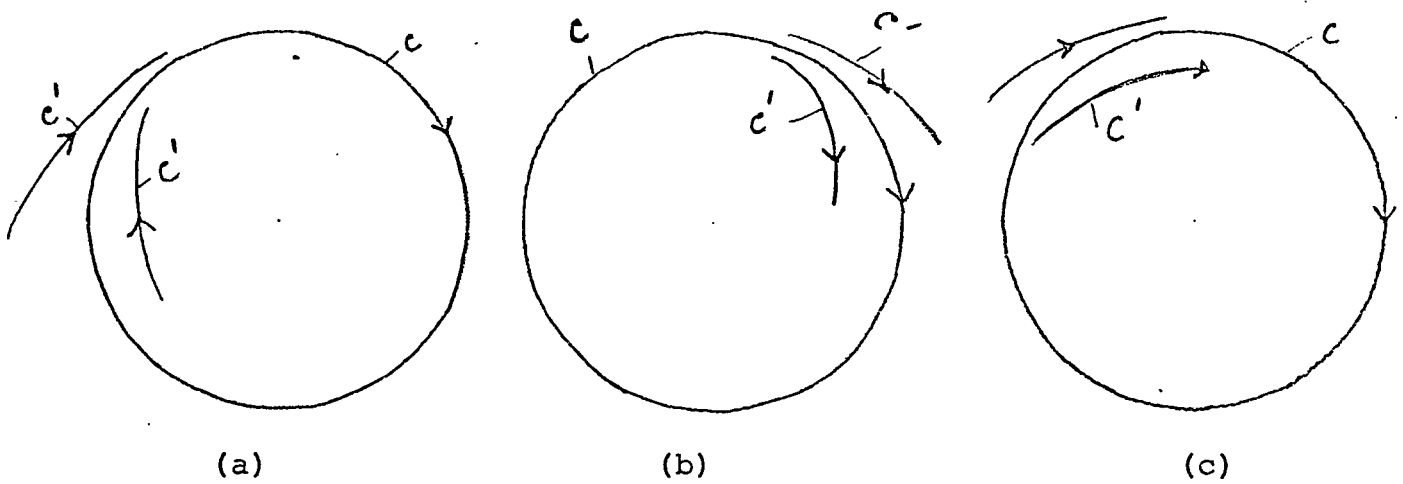


FIGURE 1.

Different types of limit cycle



- (a) Orbitally stable limit cycle.
- (b) Orbitally unstable limit cycle.
- (c) Orbitally semistable limit cycle.

FIGURE 2.

The most general methods applicable to nonlinear control systems are as follows:--

- (1) Linear approximations.
- (2) Piecewise linear approximations.
- (3) Numerical methods.
- (4) The phase plane.
- (5) Describing Functions.
- (6) The Analog computer.

The linear approximation technique is that of applying linear analysis principles to the system while restricting the significant parameter values in the system to those values defined at a selected operating point. The analysis is valid only for operation in a restricted region about this operating point. The results are correct for sufficiently small signals and for the systems in which the degree of nonlinearity is small.

Piecewise linear methods are used for relatively large signal conditions. This technique is convenient for the systems where the nonlinearity exhibits abrupt or discontinuous characteristics and may also be used with analytic nonlinearities by approximating the nonlinear characteristics with straight-line segments. In short, the method consists in establishing a linear differential equation for each section of straight line used in the representation of the nonlinearity. This

method is very laborious if the system is of high order or if the nonlinearity requires a large number of linear segments for an accurate approximation.

Many numerical and analytical methods are available for the solution of nonlinear differential equations. Numerical methods are applicable to analytic nonlinearities but fail for discontinuous nonlinearity.

The phase - plane method of analysis is one of the most powerful analytic tool available. It is basically a graphical method. The most valuable feature of the phase - plane method is that it tends to provide insight into the physical phenomena which are due to different types of nonlinearities. The major disadvantage of the phase - plane method are its limitation to second - order systems and to step and ramp inputs.

In some cases the computer is the best analysis tool available. The nonlinearities can be simulated accurately, parameter values can be altered as desired, wider ranges of initial conditions and forcing functions may be used, and compensation may be designed on a trial - and - error basis. The major draw back being to design the experiment intelligently.

The describing function method is a very powerful tool in determining the stability of a nonlinear control system. The technique is easy to apply, it is very useful in predicting limit cycles, and the accuracy of the result improves as the

complexity of the system increases. This method attempts to extend frequency - response techniques to nonlinear system analysis and design. It is, however, quite helpful in suggesting the type and amount of compensation needed, and a qualitative prediction of transient response is possible, though not completely trustworthy. The major disadvantage is the lack of accuracy for simple systems and a practical restriction to only one nonlinear component.

The describing - function and phase - plane techniques, when used together, often complement each other in the analysis of complicated systems. The describing function might be employed to check stability and suggest compensation if needed. After compensation, a second - order approximation may be made which includes the nonlinear characteristics, and a phase - plane analysis can be made to provide significant data on the effect of the nonlinearity. The Thesis problem is solved by applying Describing Function technique. The describing function technique will be further discussed in detail in Chapter two.

The analytical Technique developed to determine existence of sustained oscillation in Fourth order control system is discussed in detail in Chapter three.

GRAPHICAL TECHNIQUE

The graphical technique of determining sustained oscillations in a Control system with nonlinear element is also called as the describing function method of determining stability. This method was first developed by Goldfarb and Kochenburger in about 1950.

Describing Function:- The describing function technique is a method to extend frequency - response techniques to nonlinear system analysis and design. The practical solution to these requirements has been to approximate the effect of the nonlinear component by using a linear " approximate transfer function" or "describing function". The formulation of describing function is based upon two conditions.

1. That the system can be separated into two sections, one linear and the other nonlinear or into several sections, each nonlinear section followed by a linear section.
2. That the linear sections following each nonlinear section are low pass filters that attenuate the harmonics to a greater degree than the fundamental component.
3. That the nonlinearity may not contain any time varying elements.

If the above three conditions are met, then the input to each nonlinear section can be assumed to be sinusoidal.

This assumption is fairly reasonable as the higher harmonics are attenuated by the linear low pass filters. The results obtained are better for higher order systems than for second order.

The describing function may be defined as the ratio of the magnitude of the fundamental term in the Fourier series for the output wave to the magnitude of the input sinusoid at a phase angle which is the angle between the two sine waves for all permissible amplitudes and frequencies of the input wave.

If $E \sin \omega t$ is the input to the nonlinear element, then the Fourier series for the signal at the output of the nonlinear component may be written.

$$O(t) = \frac{A_0}{2} + A_1 \cos \omega t + B_1 \sin \omega t + A_2 \cos 2\omega t + B_2 \sin 2\omega t \\ + A_3 \cos 3\omega t + B_3 \sin 3\omega t + \dots$$

where the coefficients of this Fourier series are given by

$$A_N = \frac{2}{\pi} \int_0^{\pi} O(t) \cos N\omega t \, d\omega t$$

$$B_N = \frac{2}{\pi} \int_0^{\pi} O(t) \sin N\omega t \, d\omega t$$

The definition of describing function requires that $A_0/2 = 0$ and A_N, B_N for $N \geq 2$ are negligible. The

coefficients of interest are A_1 and B_1 , and they define the fundamental frequency output as

$$O_1(\omega) = \sqrt{A_1^2 + B_1^2} \left[\tan^{-1} \frac{A_1}{B_1} \right]$$

Then the describing function is given by

$$N(E) = \frac{\sqrt{A_1^2 + B_1^2} \left[\tan^{-1} (A_1/B_1) \right]}{E \angle 0^\circ}$$

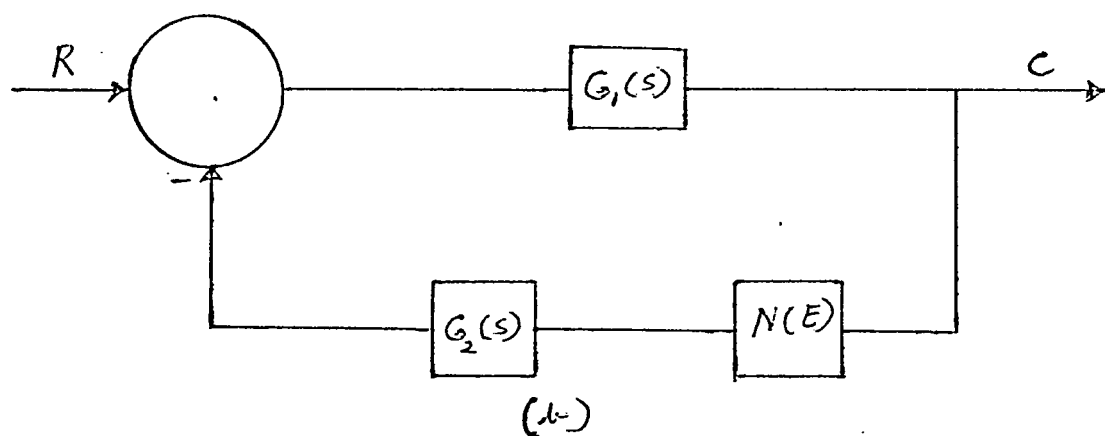
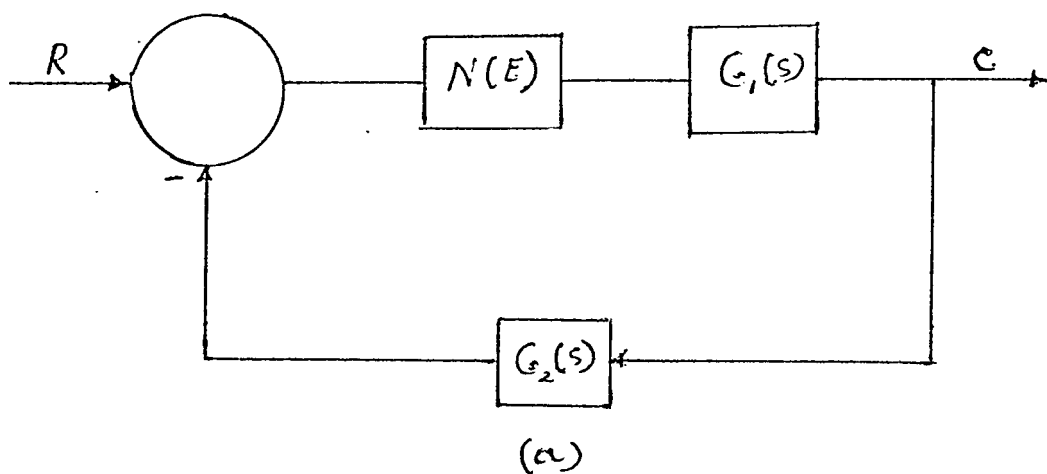
When the nonlinearity is single valued, then A_1 is also zero, and in that case

$$N(E) = \frac{B_1 \angle 0}{E}$$

The block diagram of a feedback system containing a nonlinear element is usually reducible to one of the two configuration shown in Figure 3. When the system has two or more loops but only one nonlinear component, the simplest procedure is usually that of manipulating the blocks in such a way so as to reduce it to the form of Figure 3.

Stability analysis;- Figure 4. shows the block diagram of a system which contains a nonlinearity. The describing function of the nonlinearity be $N(E)$,
Where $N(E) = g(E) + jb(E)$

For single valued nonlinearities, the imaginary part vanishes.



Block diagram for single - loop nonlinear systems
 (a) Nonlinear block in forward path (b) Nonlinear block in feedback path.

FIGURE 3.

The closed loop transfer function is

$$\frac{C(j\omega)}{R(j\omega)} = \frac{N(E)G(j\omega)}{1 + N(E)G(j\omega)}.$$

Stability can be determined by equating the characteristic equation to zero.

i.e.

$$1 + N(E)G(j\omega) = 0$$

$$G(j\omega) = \frac{-1}{N(E)}$$

This is the condition for continuous oscillations of the output.

Stability is determined by frequency response analysis using polar plot. A modification of the conventional Nyquist principle must be made in order to apply Nyquist's stability criterion to the frequency response plots. The locus of $\frac{-1}{N(E)}$ can be considered the locus of the critical point,

which for linear systems is the -1 point in the complex frequency plane. When the critical point lies to the left of the $G(j\omega)$ plot or is not enclosed by it, the poles of the control system have negative real parts. The system is then stable since any disturbances which appear tend to die out. Conversely, when the critical point lies to the right of the $G(j\omega)$ locus and is therefore enclosed by it, the poles of the control system have positive real parts and the system

is unstable. If the $G(j\omega)$ plot passes through the critical point, the system may have a sustained oscillation, which may be either stable or unstable. If a slight disturbance in amplitude or frequency occurs and the oscillation returns to its original value, the oscillation is stable. If the oscillation continues to increase or decrease, it is unstable. This is better explained from Figure 5.

Here

$$\frac{-1}{N(E)} = - \frac{\pi D}{4 T_m (D/E)} \frac{1}{\sqrt{1 - (D/E)^2}}$$

For a given value of dead zone D , $-1/N(E)$ depends entirely on the quantity E/D . In the figure 5. $-1/N(E)$ locus starts at $-\infty$ for $E/D = 1$; as the value of E/D is increased further, the critical point moves toward $-\infty$ as E/D approaches infinity. The relative position of the $G(j\omega)$ and $-1/N(E)$ plots indicate the stability condition of the system for given set of values of K and D . When the system forward gain is of a small value, K_1 , the $G(j\omega)$ locus does not intersect or enclose any portion of the $-1/N(E)$ locus and the system is stable. When $K = K_2$, the system becomes on the verge of instability. Point a indicates unstable limit cycle. When $K = K_3$, we have $G(j\omega)$ intersecting $-1/N(E)$ at two points B and C. B indicates unstable limit cycle and C indicates stable limit cycle.

Stability analysis of relay-type system (relay with dead zone)

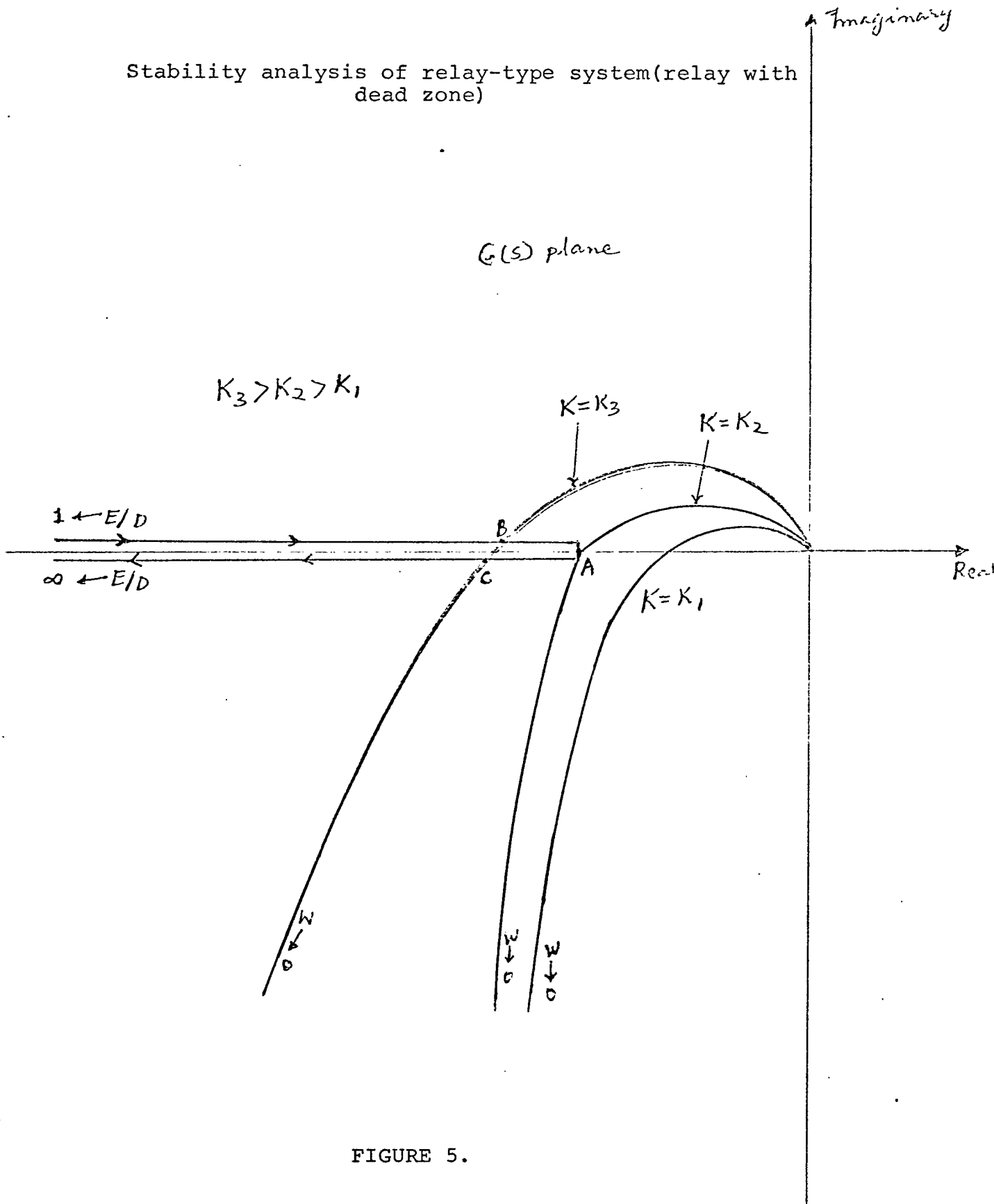


FIGURE 5.

Limitations and errors:- The describing-function method is an approximate method, since the higher harmonics generated by the nonlinearity are neglected, and thus the input to the nonlinear element is assumed sinusoidal. Therefore the describing-function method gives accurate results when these basic assumptions are justified and less accurate results when the harmonics are not negligible.

Example: The describing function for saturation is

$$G = \frac{2K}{\pi} \left(\sin^{-1} R + R \sqrt{1 - R^2} \right) \angle 0^\circ$$

Where $R = \frac{E_s}{E}$

Similarly it can be shown that the ratio of the third harmonic to the input sinusoid is

$$G_{D3} = \frac{4KR (1 - R^2)^{\frac{3}{2}}}{3\pi}$$

The relationships are plotted in figure 6. It is seen that, for small signal input, the distortion is small and the third harmonic content is negligible. But for larger signals, the amplitude of the third harmonic is one-third that of the fundamental. Thus for small input amplitudes, the linear system does not need much filtering but for larger signals filtering is very important.

Thus the basic purpose of the describing function analysis is merely a stability check. If the system is

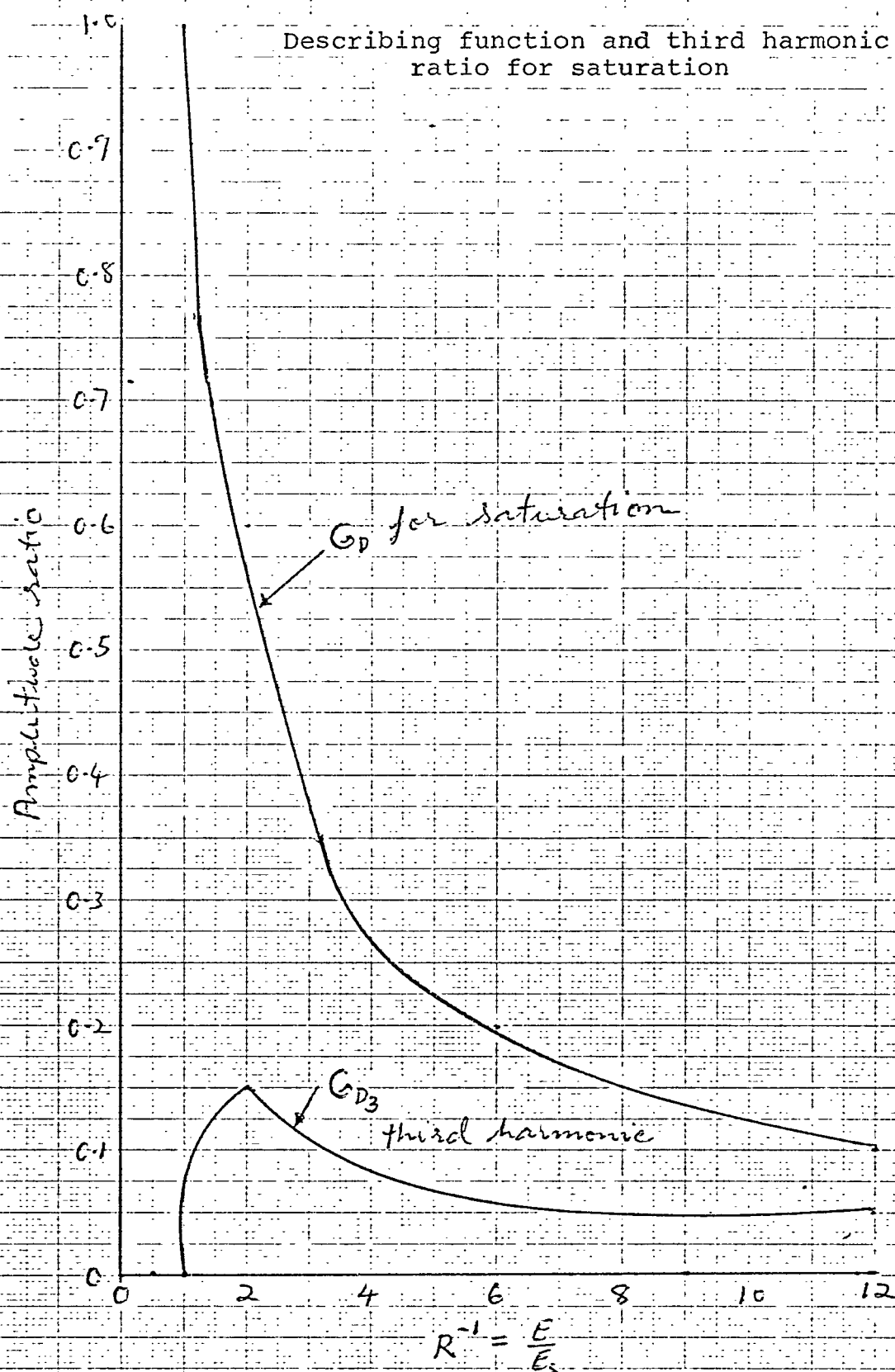


FIGURE 6.

completely unstable, no further checks are needed, but if the linear locus approaches or intersects the describing function, limit cycles may exist and this condition must be carefully checked. However, difficulty sometimes arises owing to intersections which should not exist and which predict limit cycles that do not occur in the physical systems. Another difficulty arises if, when the loci are quite close together but do not intersect, a limit cycle that has not been predicted appears in the physical systems.

Thus it may be stated in general that the describing function method is accurate when the linear system filters out the harmonics generated in the nonlinear component. Hence for higher-order systems in which there are many more open-loop poles than zeros, describing-function predictions are quite accurate.

ANALYTICAL TECHNIQUE

The Gain Margin and the Phase Margin have been generally used to define the amount of relative stability of a closed-loop linear system.

In a closed loop linear system, The magnitude of $G(j\omega)H(j\omega)$ when it crosses the negative real axis is called $(GH)_c$, and the corresponding frequency on the locus is ω_c . The Gain Margin of the system is defined as

$$\text{Gain Margin} = 20 \log_{10} \frac{1}{|(GH)_c|} \text{ db.}$$

Now for a unity feed back system,

$$\text{Gain Margin} = 20 \log_{10} \frac{1}{|(G)_c|} \text{ db.}$$

If in the Nyquist plot, the gain K is increased to the extent that, the G locus goes through the critical point so that $|(G)_c|$ equals unity, the Gain Margin is zero db.

For a second order system, the G locus does not intersect the negative real axis; Therefore $|(G)_c|$ equals zero, and the Gain Margin given is infinite ~~is inf.~~.

Gain Margin is the amount of Gain K in dbs that can be allowed to increase before the system reaches instability.

In order to define adequately the relative stability; Gain Margin is not the only essential term, Phase Margin also

PHASE MARGIN AND GAIN MARGIN

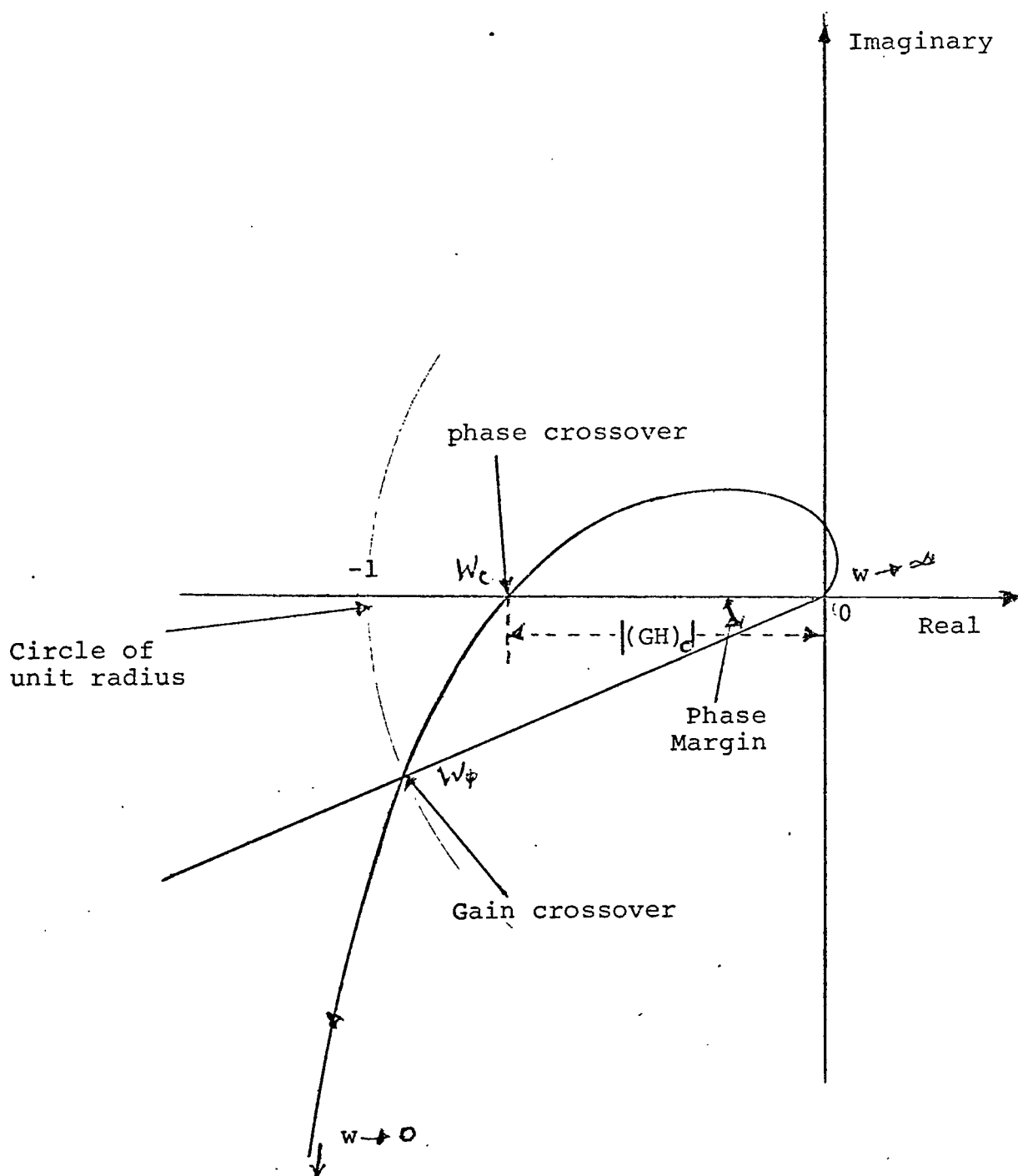


Figure 7.

has to be taken into consideration.

Phase Margin is defined as the angle that the unit radius phasor makes with the negative real axis in the G plane. The Phase Margin has the significance of estimating the stability effect of changes of the parameters of the system which affect the phase of $G(S)$.

Gain crossover is the point on the plot of the transfer function at which the magnitude is unity (zero db). The frequency at gain crossover is called the phase margin frequency ω_{ϕ} .

Phase crossover is the point on the plot of the transfer function at which the phase angle is -180° . The frequency at which phase crossover occurs is called the gain-margin frequency ω_c .

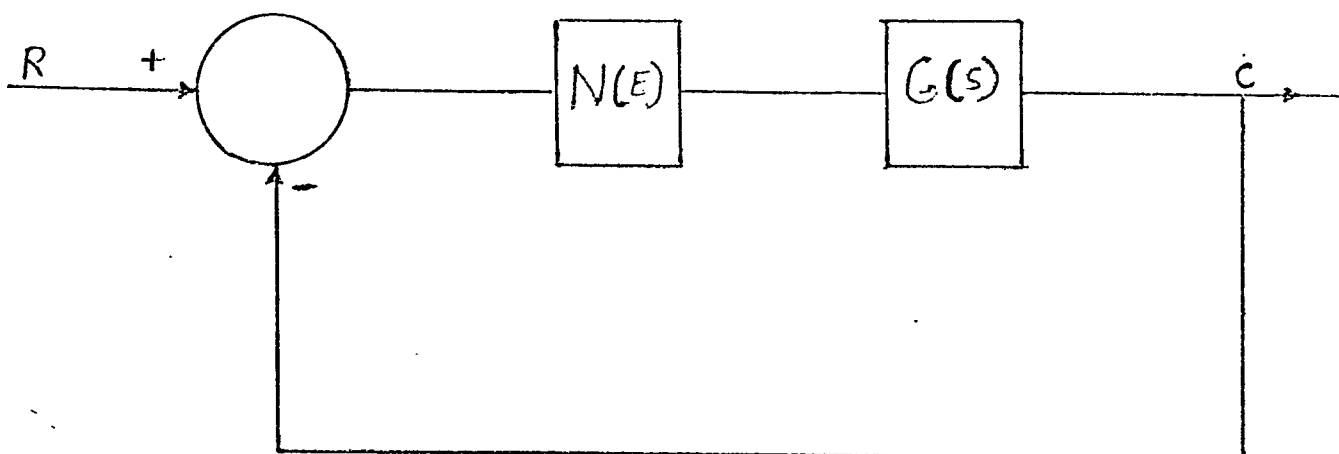
The phase margin is related to the effective damping ratio ζ of the system and therefore of the transient overshoot of the system.

Derivation of Gain Margin and Phase Margin relations:-

Considering a Fourth order Control system with amplitude sensitive Nonlinearity (shown in Fig 4.). At Limit Cycle, the Gain Margin is zero db and the Phase Margin is zero.

The open loop Linear Transfer function of the Fourth order

FOURTH ORDER CONTROL SYSTEM WITH
AMPLITUDE SENSITIVE NONLINEARITY



Where,

$$N(E) = g(E) + jb(E)$$

and

$$G(S) = \frac{K}{S(1 + ST_1)(1 + ST_2)(1 + ST_3)}$$

Figure 4

control system is

$$G(S) = \frac{K}{S(1+ST_1)(1+ST_2)(1+ST_3)} \quad (1)$$

Let $N(E)$ be the equivalent Describing Function for the Nonlinearity in Cartesian Coordinates, then

$$N(E) = g(E) + jb(E) \quad (2)$$

where, .

$g(E)$ = Real part of the describing function.

$b(E)$ = Imaginary part of the describing function.

Now equations (1) and (2) can also be written as

$$G(j\omega) = |G(j\omega)| \underline{G(j\omega)} \quad (3)$$

and

$$N(E) = |N(E)| \underline{N(E)} \quad (4)$$

When the nonlinearity is substituted by its equivalent describing function, the system reduces to a Fourth order control system. The linear section being function of frequency and the nonlinear part being function of amplitude.

Using the Gain Margin and Phase Margin principle, the equations for the existence of sustained oscillations are

The magnitude equation is

$$|N(E)| |G(j\omega)| = 1 \quad (5)$$

The phase equation is

$$\underline{N(E)} + \underline{G(j\omega)} = -180^\circ \quad (6)$$

Simplifying left hand side of equation (6)

since,

$$\underline{N(E)} = \tan^{-1} \frac{b(E)}{g(E)}$$

and

$$\underline{G(jw)} = -90^\circ - \tan^{-1}(wT1) - \tan^{-1}(wT2) - \tan^{-1}(wT3)$$

Substituting the values of $\underline{N(E)}$ and $\underline{G(jw)}$ in equation (6) we get

$$\tan^{-1} \frac{b(E)}{g(E)} - 90^\circ - \tan^{-1}(wT1) - \tan^{-1}(wT2) - \tan^{-1}(wT3) = -180^\circ$$

i.e.

$$\tan^{-1} \frac{b(E)}{g(E)} = \tan^{-1}(wT1) + \tan^{-1}(wT2) + \tan^{-1}(wT3) - 90^\circ$$

Taking tangent on both sides we have

$$\frac{b(E)}{g(E)} = \tan \left[\tan^{-1}(wT1) + \tan^{-1}(wT2) + \tan^{-1}(wT3) - 90^\circ \right]$$

Expanding the right hand side., we have

$$\begin{aligned} \frac{b(E)}{g(E)} &= \frac{\frac{wT1+wT2}{1-w^2T1T2} + \frac{wT3-X}{1+wT3X}}{1 - \left[\frac{wT1+wT2}{1-w^2T1T2} \right] \left[\frac{wT3-X}{1+wT3X} \right]} \\ \lim_{X \rightarrow \infty} &= \frac{(wT1+wT2)(1+wT3X) + (wT3-X)(1-w^2T1T2)}{(1-w^2T1T2)(1+wT3X) - (wT1+wT2)(wT3-X)} \end{aligned}$$

$$\begin{aligned}
& \frac{(wT_1 + wT_2) + \frac{(wT_3 - X)(1 - w^2 T_1 T_2)}{(1 + wT_3 X)}}{(1 - w^2 T_1 T_2) - \frac{(wT_1 + wT_2)(wT_3 - X)}{(1 + wT_3 X)}} \\
& = \frac{(wT_1 + wT_2) + \frac{\frac{wT_3}{X} - 1}{\left(\frac{1}{X} + wT_3\right)}(1 - w^2 T_1 T_2)}{(1 - w^2 T_1 T_2) - \frac{(wT_1 + wT_2)\left(\frac{wT_3}{X} - 1\right)}{\left(\frac{1}{X} + wT_3\right)}}
\end{aligned}$$

i.e.

$$\begin{aligned}
\frac{b(E)}{g(E)} &= \frac{wT_3(wT_1 + wT_2) - (1 - w^2 T_1 T_2)}{wT_3(1 - w^2 T_1 T_2) + (wT_1 + wT_2)} \\
\frac{b(E)}{g(E)} &= \frac{w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1}{wT_1 + wT_2 + wT_3 - w^3 T_1 T_2 T_3} \quad (7)
\end{aligned}$$

This equation is independent of the gain K . Since for real values of K the phase angle contribution is zero. Equation (7) satisfies the phase condition.

Now considering the magnitude conditions, we have

$$|N(E)| = \left[g^2(E) + b^2(E) \right]^{\frac{1}{2}}$$

and

$$|G(jw)| = \frac{K}{w \left[(1 + w^2 T_1^2) (1 + w^2 T_2^2) (1 + w^2 T_3^2) \right]^{\frac{1}{2}}}$$

Substituting values of $|N(E)|$ and $|G(jw)|$ in equation (5), we have

$$\frac{K \left[g^2(E) + b^2(E) \right]^{\frac{1}{2}}}{w \left[(1+w^2 T_1^2) (1+w^2 T_2^2) (1+w^2 T_3^2) \right]^{\frac{1}{2}}} = 1$$

Substituting the value of $b(E)$ from equation (7) in the above equation, we have

$$\frac{K \left[g^2(E) + g^2(E) \left[\frac{w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1}{w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3} \right]^2 \right]^{\frac{1}{2}}}{w \left[(1+w^2 T_1^2) (1+w^2 T_2^2) (1+w^2 T_3^2) \right]^{\frac{1}{2}}} = 1$$

$$\frac{K g(E) \left[1 + \left[\frac{w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1}{w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3} \right]^2 \right]^{\frac{1}{2}}}{w \left[(1+w^2 T_1^2) (1+w^2 T_2^2) (1+w^2 T_3^2) \right]^{\frac{1}{2}}} = 1$$

$$= \frac{K g(E) \left[(w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3)^2 + (w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1)^2 \right]^{\frac{1}{2}}}{(w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3) w \left[(1+w^2 T_1^2) (1+w^2 T_2^2) (1+w^2 T_3^2) \right]^{\frac{1}{2}}} = 1$$

Let $X = (w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3)^2 + (w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1)^2$
and

$$Y = (1+w^2 T_1^2) (1+w^2 T_2^2) (1+w^2 T_3^2)$$

Therefore, the above equation reduces to

$$\frac{K g(E) \left[X \right]^{\frac{1}{2}}}{w (w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3) \left[Y \right]^{\frac{1}{2}}} = 1$$

On simplification we get $X = Y$

Therefore, the above equation reduces to

$$\frac{K g(E)}{w^2 T_1 + w^2 T_2 + w^2 T_3 - w^3 T_1 T_2 T_3} = 1 \quad (8)$$

Equation (8) gives the relationship between the system parameters for the condition (5) and (6). It is independent of $b(E)$.

From equation (7), we have

$$\frac{b(E)}{g(E)} = \frac{w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1}{w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3}$$

When the nonlinearity is single valued, than the imaginary part of the describing function vanishes.

This condition implies

$$w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1 = 0$$

Solving for w^2 we obtain

$$w^2 = \frac{1}{T_1 T_2 + T_2 T_3 + T_3 T_1}$$

or

$$w = \pm \sqrt{\frac{1}{T_1 T_2 + T_2 T_3 + T_3 T_1}} \quad (9)$$

As the negative value is practically not realisable

Substituting the value of w , in equation (8) we have

$$\begin{aligned}
 & \frac{K g(E)}{\frac{T_1}{T_1 T_2 + T_2 T_3 + T_3 T_1} + \frac{T_2}{T_1 T_2 + T_2 T_3 + T_3 T_1} + \frac{T_3}{T_1 T_2 + T_2 T_3 + T_3 T_1} - \frac{T_1 T_2 T_3}{(T_1 T_2 + T_2 T_3 + T_3 T_1)}} = 1 \\
 & \frac{K g(E)}{\frac{1}{T_1 T_2 + T_2 T_3 + T_3 T_1} \left[T_1 + T_2 + T_3 - \frac{T_1 T_2 T_3}{\sqrt{T_1 T_2 + T_2 T_3 + T_3 T_1}} \right]} = 1 \quad (10)
 \end{aligned}$$

For the existence of limit cycle the above equation has to be equal to one. From design point of view the values of $g(E)_{Max}$ and gain K can be adjusted so as to make the above expression equal to one.

Substituting $T_1=1.0$, $T_2=0.5$, $T_3=0.25$, in equation (10) we get

$$K = \frac{1.847}{g(E)_{Max}}$$

For different values of $g(E)_{Max}$, the corresponding values of gain(K) are obtained and are plotted in figure 12.

$g(E)_{Max.}$	K(gain)
0.02	92.35
0.04	46.17
0.06	30.78
0.08	23.09
0.10	18.47
0.12	15.39
0.14	13.19
0.16	11.54
0.18	10.26
0.20	9.23
0.22	8.39
0.24	7.69
0.26	7.10
0.28	6.59
0.30	6.15
0.32	5.77
0.34	5.43

, Values are plotted in figure 12.

For multivalued nonlinearity.

From equation (7) we have,

$$\frac{b(E)}{g(E)} = \frac{w^2 T_1 T_2 + w^2 T_2 T_3 + w^2 T_1 T_3 - 1}{w T_1 + w T_2 + w T_3 - w^3 T_1 T_2 T_3}$$

Rearranging the terms we get

$$w^3 + w^2 \frac{g(E)}{b(E)} \frac{(T_1 T_2 + T_2 T_3 + T_1 T_3)}{T_1 T_2 T_3} - \frac{w(T_1 + T_2 + T_3)}{T_1 T_2 T_3} - \frac{g(E)}{b(E) T_1 T_2 T_3} = 0$$

Let

$$a_2 = \frac{g(E)}{b(E)} \frac{(T_1 T_2 + T_2 T_3 + T_1 T_3)}{T_1 T_2 T_3}$$

$$a_1 = - \frac{(T_1 + T_2 + T_3)}{T_1 T_2 T_3}$$

$$a_0 = - \frac{g(E)}{b(E) T_1 T_2 T_3}$$

Substituting in above equation we get

$$w^3 + w^2 a_2 + w a_1 + a_0 = 0$$

This is a third order equation in w

$$\text{Let } q = \frac{1}{3} a_1 - \frac{1}{9} a_2^2 \quad ; \quad r = \frac{1}{6} (a_1 a_2 - 3 a_0) - \frac{1}{27} a_2^3$$

When $q^3 + r^2 > 0$, one real root and a pair of complex conjugate roots.

If $q^3 + r^2 = 0$, all roots are real and at least two are equal.

If $q^3 + \lambda^2 < 0$, all roots are real (irreducible case)

Now substituting the values of $q^3 + \lambda^2$, in terms of $g(E)$, $b(E)$, T_1 , T_2 and T_3 , we get

$$q^3 + \lambda^2 = \frac{1}{108} \left[\frac{-4P}{T_1^3 T_2^3 T_3^3} - \frac{g^2(E) Q}{\lambda^2(E) T_1^4 T_2^4 T_3^4} + \frac{27g^2(E)}{\lambda^2(E) T_1^2 T_2^2 T_3^2} - \frac{16g^2(E) R}{\lambda^2(E) T_1^3 T_2^3 T_3^3} - \frac{4g^4(E) S}{\lambda^4(E) T_1^4 T_2^4 T_3^4} \right] \quad (11)$$

Where

$$P = T_1^3 + T_2^3 + T_3^3 + 3T_1 T_2^2 + 3T_1 T_3^2 + 3T_2 T_1^2 + 3T_2 T_3^2 + 3T_3 T_1^2 + 3T_3 T_2^2 + 6T_1 T_2 T_3$$

$$Q = T_1^4 T_2^2 + T_3^2 T_1^4 + T_1^2 T_2^4 + T_3^2 T_2^4 + T_2^2 T_3^4 + T_1^2 T_3^4 + 3T_1^2 T_2^2 T_3^2 + 2T_1^3 T_2^3 + 2T_2^3 T_3^3 + 2T_3^3 T_1^3 + 12T_1^2 T_2^2 T_3^2 + 8T_1^3 T_3 T_2^2 + 8T_2 T_1^3 T_3^2 + 2T_3 T_2 T_1^4 + 2T_1 T_3 T_2^4 + 2T_1 T_2 T_3^4 + 8T_2^3 T_1 T_3^2 + 8T_3 T_2^3 T_1^2 + 8T_1 T_2^2 T_3^3 + 8T_1^2 T_2 T_3^3$$

$$R = T_1^2 T_2 + T_3 T_1^2 + T_1 T_2^2 + T_2^2 T_3 + T_1 T_3^2 + T_2 T_3^2 + 3T_1 T_2$$

$$S = T_1^3 T_2^3 + T_2^3 T_3^3 + T_3^3 T_1^3 + 3T_1 T_2^3 T_3^2 + 3T_1^3 T_2 T_3^2 + 3T_1^2 T_2^3 T_3 + 3T_1^2 T_2 T_3^3 + 3T_1^3 T_2^2 T_3 + 3T_1 T_2^2 T_3^3 + 6T_1^2 T_2^2 T_3^2$$

When equation (11) is greater than zero, we have only one frequency of oscillation. When equation (11) is equal to zero, we have two values of w , i.e., two frequency of oscillation, the corresponding values of amplitude of oscillation can be obtained from equation (8).

Equation (8) and (11) when combined gives the designer an estimate of gain (K) and the maximum value of $N(E)$ for which the system is absolutely stable,

When equation (11) is greater than zero
Then

$$w = (S_1 + S_2) - \frac{a_2}{3}$$

and when equation (11) is equal to zero we have

$$w_1 = (S_1 + S_2) - \frac{a_2}{3}$$

$$w_2 = -\frac{1}{2} (S_1 + S_2) - \frac{a_2}{3}$$

The corresponding values of amplitude of oscillation can be obtained from equation (8)

where,

$$S_1 = \left[\lambda + (\lambda^3 + \lambda^2)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

$$S_2 = \left[\lambda - (\lambda^3 + \lambda^2)^{\frac{1}{2}} \right]^{\frac{1}{2}}$$

REDUCTION TO THIRD AND SECOND ORDER CONTROL SYSTEMS

When $T_3 = 0$, the conditions reduce to a Third order Control system,

In that case equation (7) and (8) will be as follows

$$\frac{b(E)}{g(E)} = \frac{\omega^2 T_1 T_2 - 1}{\omega T_1 + \omega T_2}$$

and

$$\frac{K g(E)}{\omega^2 T_1 + \omega^2 T_2} = 1$$

Which is same as obtained by Dudkikar K.L. and Nadkarni D.D.

When T_3 and T_2 are zero, the conditions reduce to a second order control system, in that case

$$\frac{b(E)}{g(E)} = \frac{-1}{\omega T_1}$$

and

$$\frac{K g(E)}{\omega^2 T_1} = 1$$

Simplifying we get,

$$\frac{g(E)}{b^2(E)} = K T_1$$

Which is same as obtained by Asim K.Sen

EXAMPLE AND RESULTS

A Q type nonlinearity is considered for the verification of the results. The nonlinearity is as shown in Fig. 8. The nonlinearity is single valued and hence, the equivalent describing function does not have imaginary component.

The values for the amplitude and frequency of oscillations are obtained by the Graphical technique derived from the plotting of the Nyquist curve and the Analytical solution using the Gain Margin and Phase margin principle.

The describing function for the nonlinearity can be derived and is

$$g(E) = \frac{b}{(a-a/N)\pi E} \left[2E(-\sin^{-1}a/NE + \sin^{-1}a/E) + \right. \\ \left. E \left(\frac{2xa/NE}{\sqrt{E^2 - a^2/N^2}} - \frac{2xa/E}{\sqrt{E^2 - a^2}} \right) + \right. \\ \left. \frac{4a}{N} \left(\frac{\sqrt{E^2 - a^2}}{E} - \frac{\sqrt{E^2 - a^2/N^2}}{E} \right) + \frac{4b}{\pi E} \frac{\sqrt{E^2 - a^2}}{E} \right]$$

From Fig. 4

The closed loop Transfer Function is

$$\frac{C(S)}{R(S)} = \frac{N(E)G(S)}{1 + N(E)G(S)}$$

Q TYPE NONLINEARITY

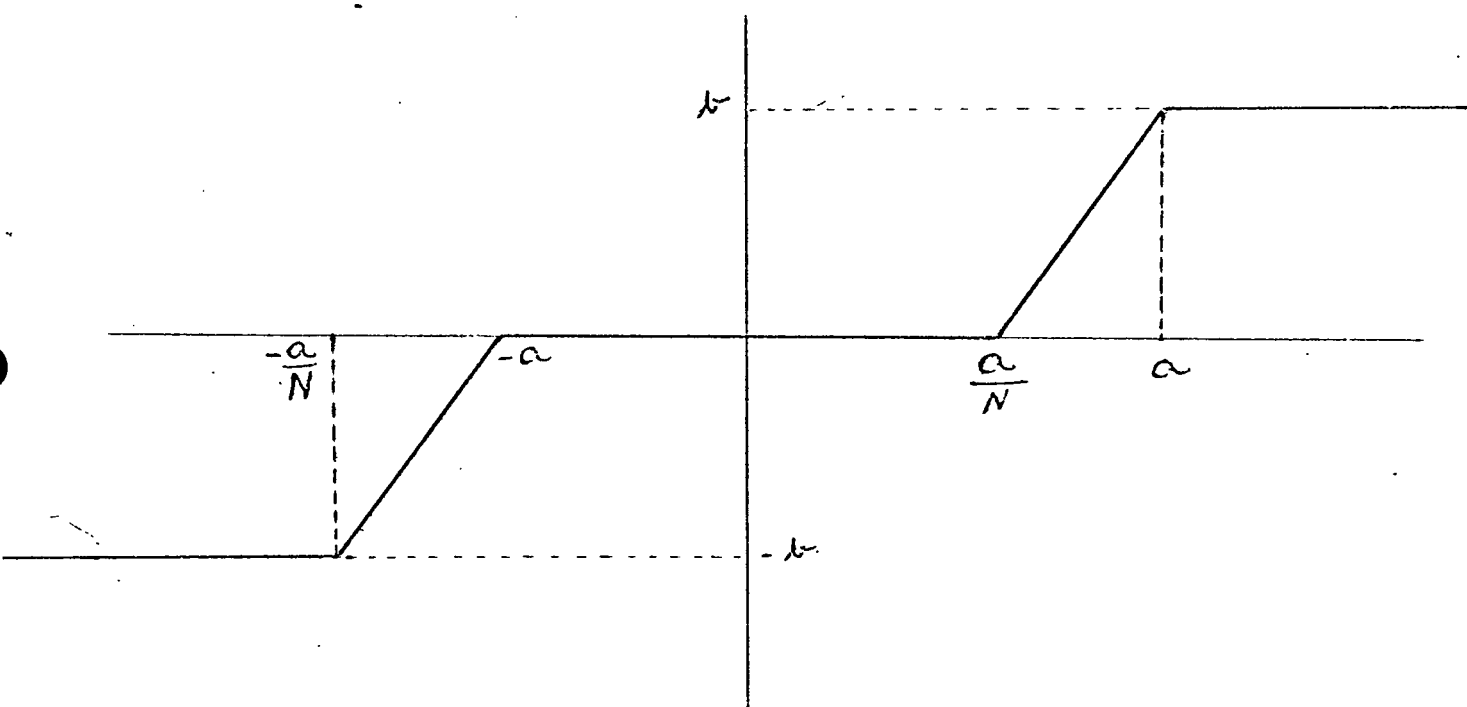


Figure 8.

(Q type nonlinearity as defined by Gibson John E., in his book, " Nonlinear Automatic Control"., Page 363.)

The characteristic equation is $1 + N(E)G(S)$

Equating this to zero we have

$$1 + N(E)G(S) = 0$$

$$N(E)G(S) = -1$$

$$G(S) = \frac{-1}{N(E)}$$

The intersection of the plot of $G(S)$ and $\frac{-1}{N(E)}$ will give the frequency and amplitude of oscillations.

Assuming

$$K = 10$$

$$T1 = 1.0$$

$$T2 = 0.5$$

$$T3 = 0.25$$

This is plotted and shown in figure 9. The values are computed with the aid of a computer program.

As the imaginary part of the describing function is zero, the locus of $\frac{-1}{N(E)}$ will be a portion of negative axis itself. The open loop linear system is stable. Therefore the $G(jw)$ plot is sufficient from $w = \text{infinite}$ to zero.

From figure 9 ., (values are obtained from computer program)

The frequency of oscillation $\underline{w = 1.0691}$ and the $|G(jw)|$

at this frequency is 5.4439.,

Since,

$$|G(jw)| = \frac{1}{|N(E)|}$$

$$5.4439 \cdot g(E) = 1$$

$$g(E) = \frac{1}{5.4439} = \underline{0.1837}$$

From ANALYTIC TECHNIQUE

We have from equation (9)

$$\begin{aligned} w &= \sqrt{\frac{1}{T_1T_2 + T_2T_3 + T_3T_1}} \\ &= \sqrt{\frac{1}{0.5 + 0.125 + 0.25}} \\ &= \sqrt{\frac{1}{0.875}} \\ &= \sqrt{1.14285} \\ &= \underline{1.0691} \end{aligned}$$

Which is exactly the same as obtained previously.

From equation (8) we have,

$$\frac{K g(E)}{w^2 T_1 + w^2 T_2 + w^2 T_3 - w^3 T_1 T_2 T_3} = 1$$

i.e.

$$\begin{aligned} g(E) &= \frac{w^2 T_1 + w^2 T_2 + w^2 T_3 - w^3 T_1 T_2 T_3}{K} \\ &= \frac{w^2 (T_1 + T_2 + T_3 - w T_1 T_2 T_3)}{K} \\ &= \frac{1.1429(1.75 - 0.133)}{10} \\ &= \frac{1.1429 \times 1.617}{10} \\ &= \underline{0.1848} \end{aligned}$$

This compares very well with the previous result and should be more accurate.

The corresponding values of E (amplitude of oscillation) is calculated from the value of $g(E) = 0.1848$. From computer programs the values of E obtained are tabulated on the next page.

N, a and b corresponds to that shown in figure 8. When $N = 1$, the nonlinearity becomes dead band with saturation.

When N becomes very large, the nonlinearity becomes only saturation. When a is very small, the nonlinearity becomes bang-bang. The higher value of the amplitude of oscillation is selected from practical point of view.

N	a	b	E	
1.1	0.75	0.25	1.52	0.814
1.2	0.75	0.25	1.54	0.775
1.0	0.75	0.25	1.49	0.868

Assuming

$$T_1 = 1.0$$

$$T_2 = 0.5$$

$$T_3 = 0.25$$

Graphical technique

Analytical technique

K	W	g(E)	W	g(E)
5	1.0691	0.3674	1.0691	0.3696
10	1.0691	0.1837	1.0691	0.1848
15	1.0691	0.1224	1.0691	0.1232

Values for graphical technique are obtained
from computer programs.

Plot of $G(j\omega)$ for different values
of gain(K)

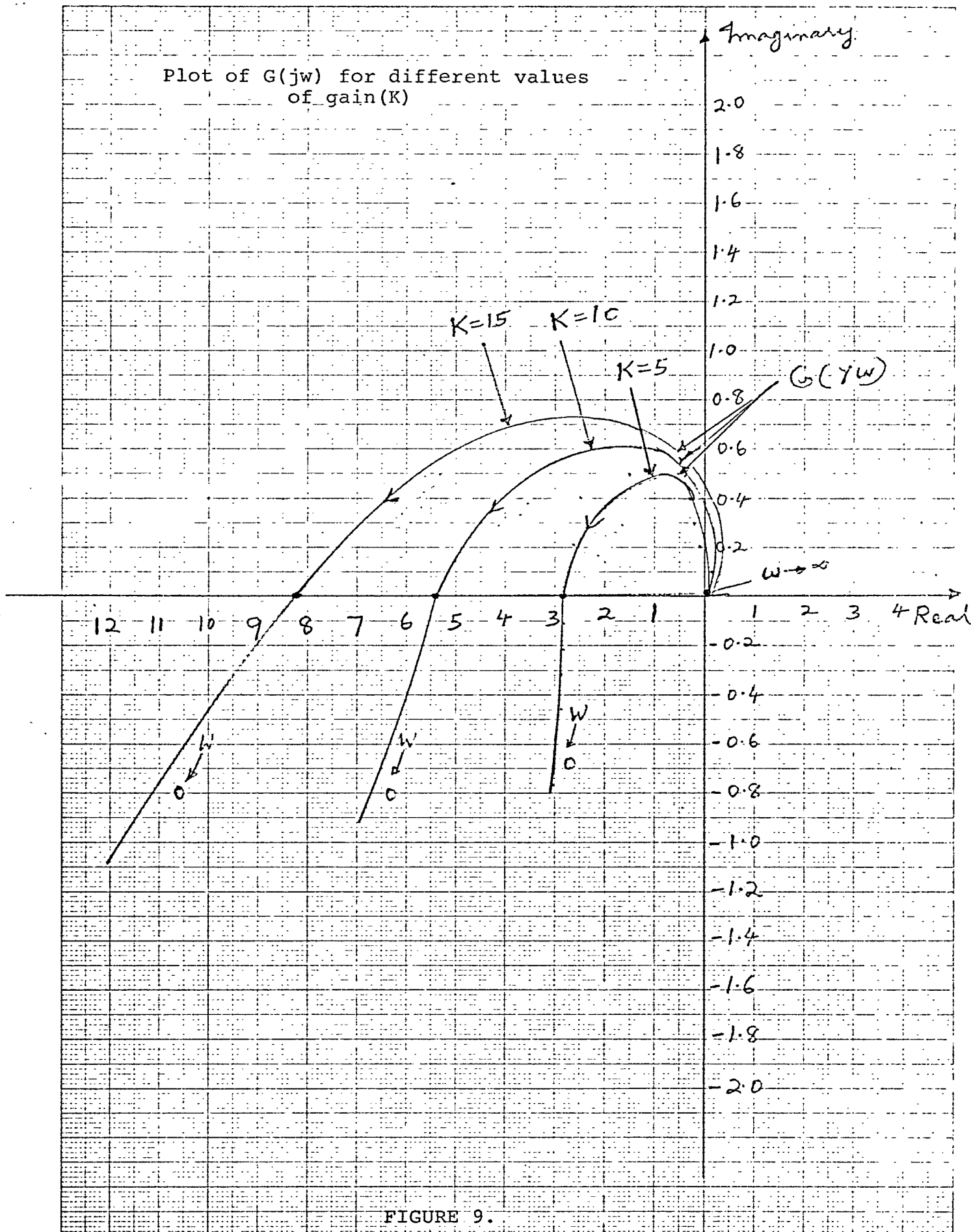


FIGURE 9.

12-49
 12-49

PLOT OF E versus $g(E)$

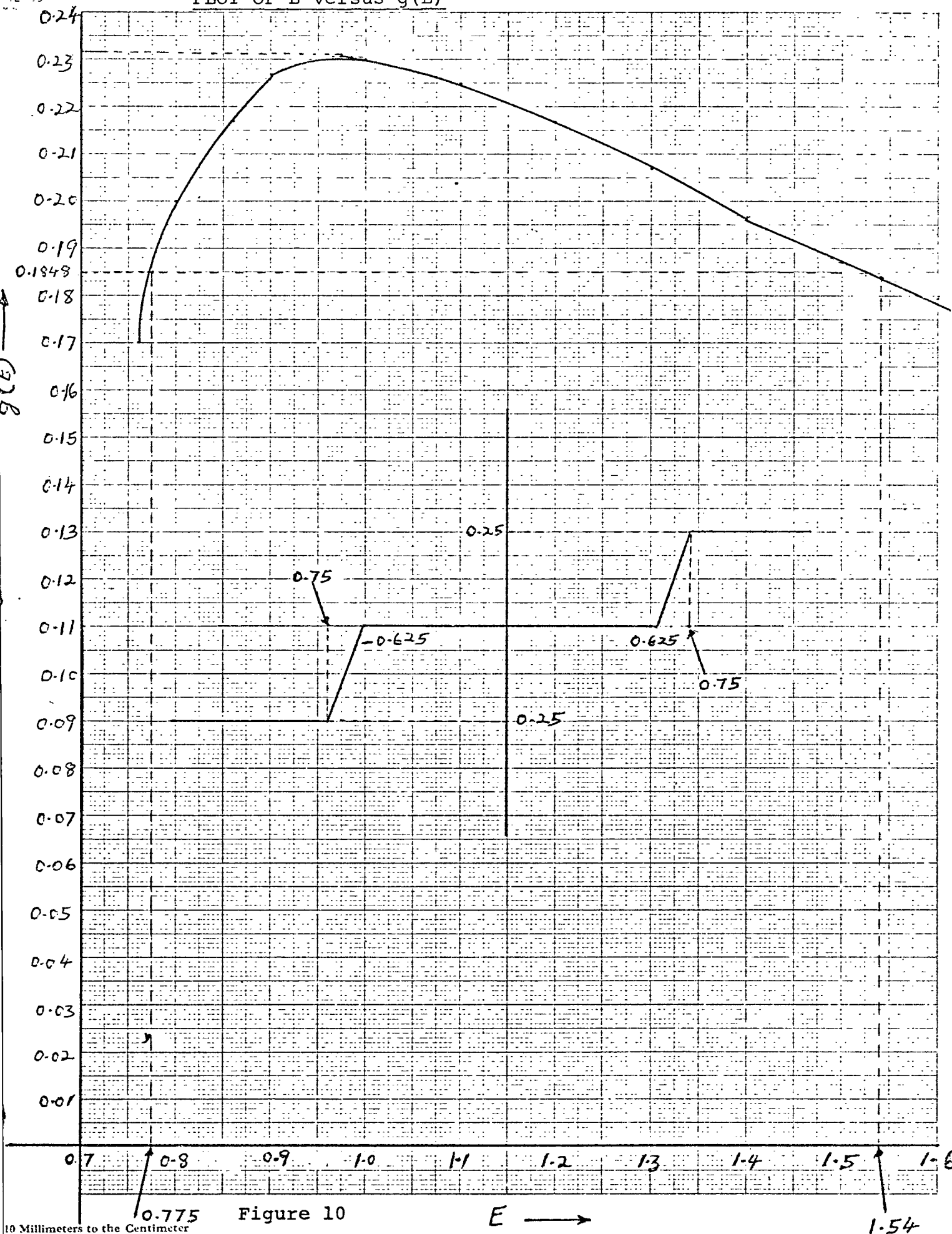
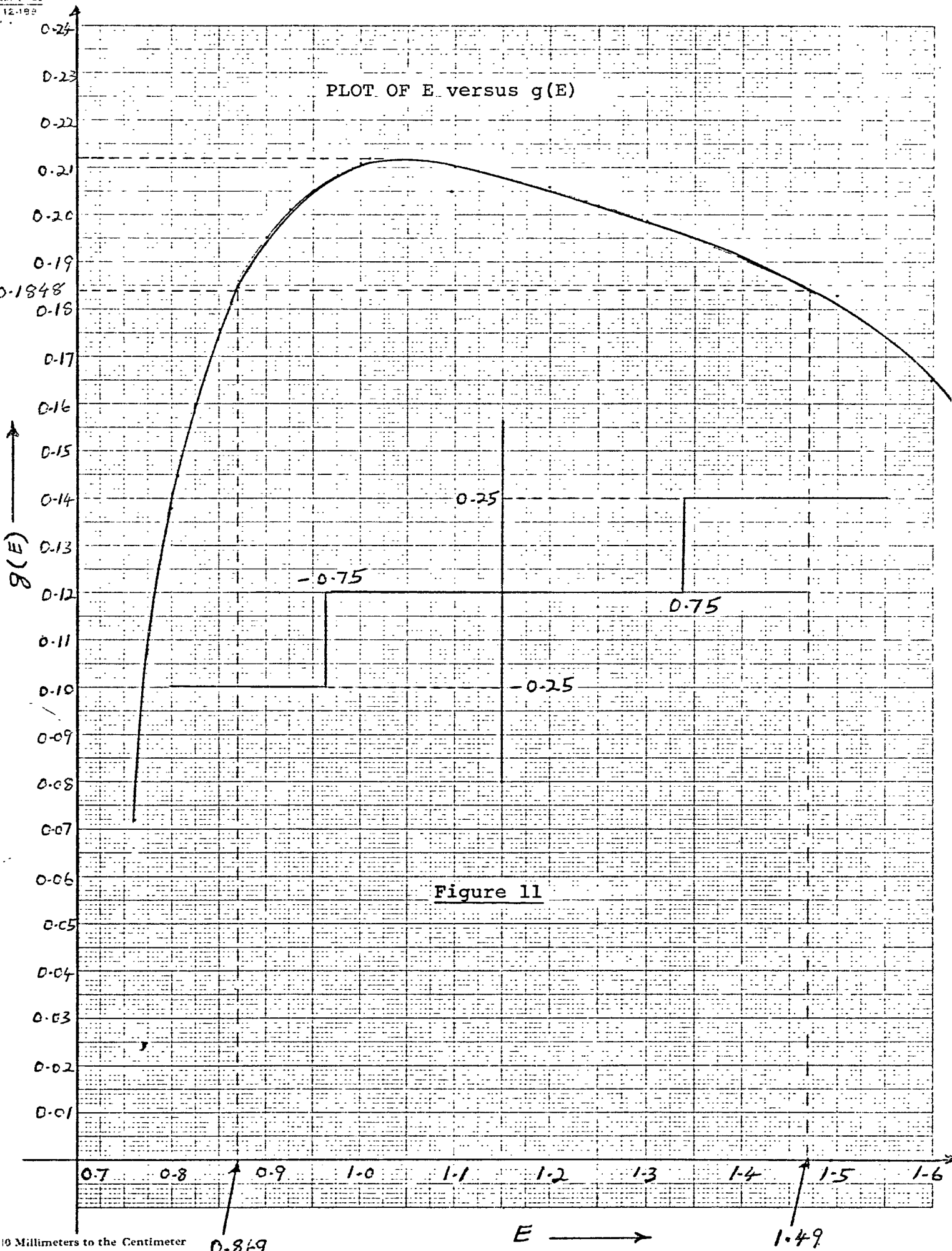


Figure 10

$E \longrightarrow$

1.54



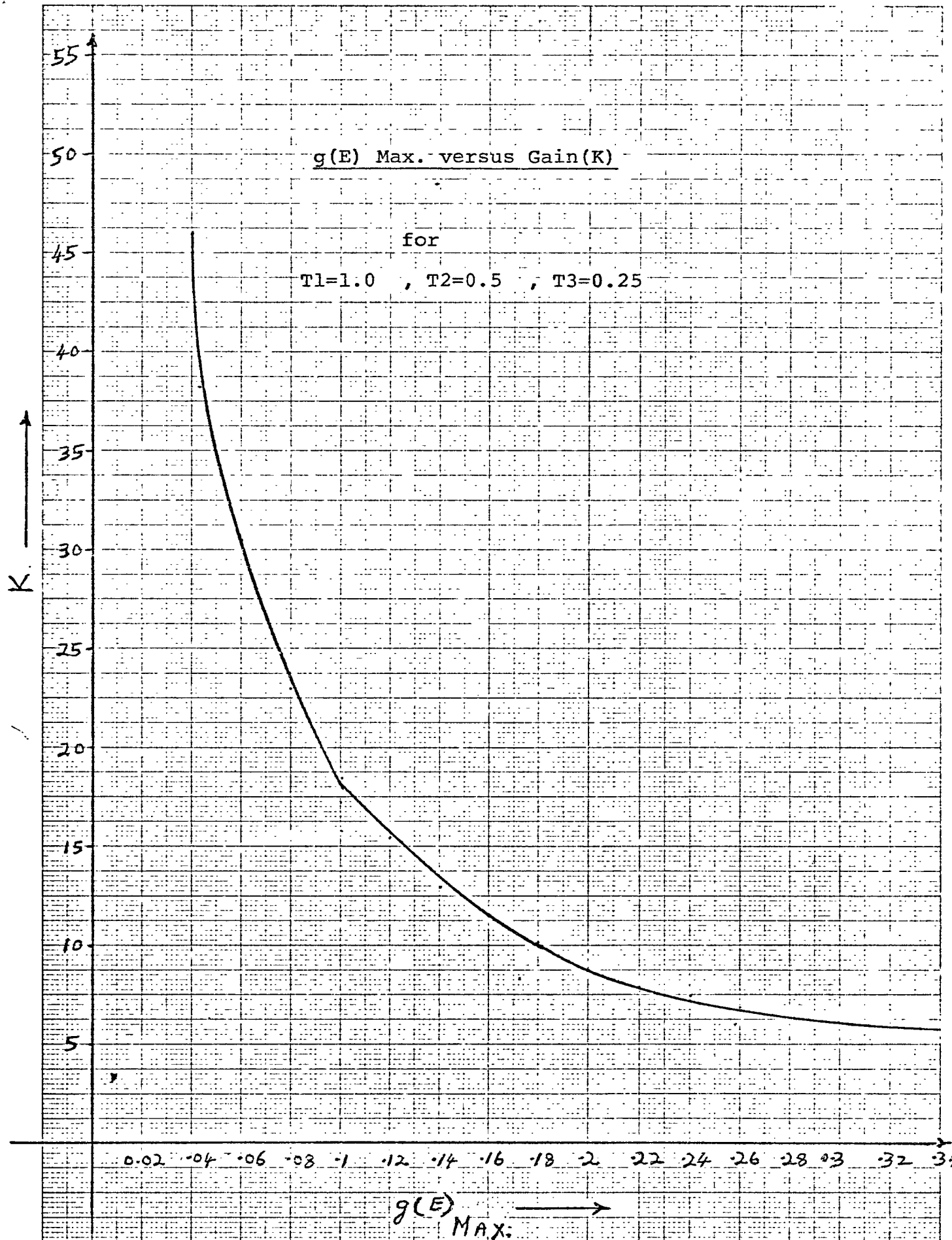


FIGURE 12

Conclusions and future developments

The analytical technique developed for determining a limit cycle in a Fourth order control system with amplitude sensitive nonlinearity provide a quick method for determining the existence of one or more limit cycle. The results developed can be reduced to determine the limit cycle for Third and second order systems also. This technique is applicable to any amplitude sensitive nonlinearity. This approach can also be applied to asymmetric nonlinearities. The graphical method of determining the existence of sustained oscillation is time consuming and laborious.

The analytical technique can be modified and used for frequency dependent nonlinearities. In this case the locus of $\frac{-1}{N(E,w)}$ for different w has to be plotted.

Aneextension of the work could be the development of the relations for the amplitude and frequency of sustained oscillations, in cases where the open loop linear system is unstable.

BIBLIOGRAPHY

1. Gibson John E., Nonlinear Automatic Control systems., McGraw-Hill Book Company, Inc; New York 1963.
2. Cosgriff Robert Lien., Nonlinear Control Systems., McGraw-Hill Book Company, Inc; New York 1958.
3. Thaler George J. & Pastel Marvin P., Analysis and design of Nonlinear Feedback Control systems., McGraw-Hill Book Company, Inc; New York 1962.
4. Sen, Asim K., A simpler stability criterion for a class of Nonlinear systems, IEEE Trans, on Automatic Control, August 1968.
5. Dudkikar, K.L. and Nadkarni, D.D., Third order Control system with Type L nonlinearity., Proceedings of the First Annual Houston conference, Houston, Texas, May 1969.
6. West, John C., Analytical Techniques for nonlinear control systems, The ELBS and the English Universities Press Ltd., London, 1966.
7. Graham Dunstan & Mcruer Duane., Analysis of Nonlinear Control systems., John Wiley & Sons, Inc., New York 1961.
8. Cunningham W.J., Introduction to Nonlinear Analysis., McGraw-Hill Book Company, Inc., New York 1958.

9. Letov Alexander M., Stability in Nonlinear Control systems., Princeton University Press, Princeton, New Jersey 1961.
10. Ku.Y.H., Analysis and Control of nonlinear systems., The Ronald Press Company, New York 1958.
11. Organick, E.L., A Fortran IV Primer., Addison Wesley, Inc., California 1967.

REFERENCES

1. Chen, Chin-Fan & Hass John I., Elements of Control system analysis., Prentice-Hall, Inc., Englewood Cliffs., N.J., 1968.
2. Hayashi Chihiro., Nonlinear oscillations in Physical systems., McGraw-Hill Book Company, Inc., New York 1964.
3. Kuo Benjamin C., Automatic Control systems., Prentice-Hall of Japan, Inc., 1964.
4. Ponzo Peter J., Some stability conditions for Linear differential Equations, IEEE Trans, on Automatic Control, December 1968, Page 721.
5. Krueger Robert J., Some necessary conditions for stability., IEEE Trans, on Automatic Control, December 1968, P. 724.
6. Bonenn A., The circle criterion and Describing Function Analysis., IEEE Trans, on Automatic Control, October 1968, Page 168.
7. Davison J.E., Application of the Describing Function Technique in a single - loop Feedback system with two Nonlinearities ., IEEE Trans, on Automatic Control, April 1968 Page 168.
8. Shirakawa H., Stability of a certain class of Third order nonlinear differential equations., IEEE Trans, on

Automatic Control, April 1968, Page 202.

9. Marchesimi G., On a bounded input-bounded output sufficient condition for Nonlinear systems., on Automatic Control, June 1968, Page 305.

10. Falb P.L., and Zames G., Multipliers with real poles and zeros; An application of a theorem on stability conditions IEEE Trans., on Automatic Control, February 1968, Page 125.

11. Duytschaever V.D., Comment on the step response of a class of Third order linear systems., IEEE Trans, on Automatic Control, February 1968, Page 134.

12. Gibson John E. & Tuteur Franz B., Control system components., McGraw-Hill Book Company, Inc, New York 1968.

13. Smith Otto J.M., Feedback control systems, McGraw-Hill Book Company, Inc., New York 1958.

14. Murphy Gordon J., Basic Automatic Control Theory, D.Van Nostrand Company, Inc., Princeton, New Jersey 1957.

15. Minorsky Nicholas, Nonlinear oscillations, D.Van Nostrand Company, Inc., Princeton, New Jersey 1962.

16. D'azzo and Houpis, Feedback control system analysis and synthesis., McGraw-Hill Book Company, New York 1966.

17. Otto J.M.Smith., Feedback control systems., McGraw-Hill Book Company, Inc., New York 1958.

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BALA PRASAD SARDA

EJECT,3)
 F:5,(DEVICE,S1)
 F:6,(DEVICE,L6)
 G5,LS

(44)

```

C      TO DETERMINE THE CORRESPONDING VALUE OF W
C      FOR WHICH IMAGINARY PART OF THE LINEAR
C      OPEN LOOP (FOURTH ORDER) TRANSFER FUNCTION
C      BECOMES ZERO
C      THIS VALUE OF W GIVES THE VALUE OF
C      SUSTAINED OSCILLATION FOR THE SYSTEM
C      K INDICATES THE GAIN OF THE LINEAR FOURTH
C      ORDER SYSTEM
C      T1,T2,T3 ARE THE TIME CONSTANTS
C      OF THE FOURTH ORDER SYSTEM
C      FOR SIMPLICITY VALUES OF T1,T2 AND T3
C      ARE TAKEN AS 1.0,0.5, AND 0.25 RESPECTIVELY
C      A TYPICAL VALUE OF K (GAIN) IS TAKEN AS 10
115  READ(5,100,END=101)K,T1,T2,T3
      WRITE(6,102)
      WRITE(6,103)K,T1,T2,T3
      WRITE(6,104)
      A=102.0
      M=0
      DO 105 I=1,1500
      M=M+1
      A=K*(W**4*T1*T2*T3-W**2*T1-W**2*T2-W**2*T3)
      B=-K*(W**3*T2*T3-W**3*T1*T3-W**3*T1*T2)
      C=(W**4*T1*T2*T3-W**2*T1-W**2*T2-W**2*T3)**2
      D=(W**3*T2*T3-W**3*T1*T3-W**3*T1*T2)**2
      P=A/C IS THE REAL PART OF THE LINEAR OPEN
      LOOP TRANSFER FUNCTION
      P=A/C
      Q=B/C IS THE IMAGINARY PART OF THE
      OPEN LOOP LINEAR TRANSFER FUNCTION
      Q=B/C
      R IS THE ABSOLUTE VALUE OF THE OPEN
      LOOP LINEAR TRANSFER FUNCTION
      R=(P**2+Q**2)**0.5
      IF(M.GT.20)GO TO 1190
      GO TO 105
1190  WRITE(6,107)
107  FORMAT(1H1,///4X,1HW,14X,4HREAL,13X,4HIMAG,10X,9HABS VALUE//)
      M=1
106  CONTINUE
      WRITE(6,108)W,P,Q,R
      IF(W.LE.1.07)GO TO 109
      IF(W.LE.2.00)GO TO 110
      IF(W.LE.5.00)GO TO 111
      IF(W.LE.10.0)GO TO 112
      W=W+10.0

```



```

      GO TO 113
109  A=-0.0001
      GO TO 113
110  A=-0.01
      GO TO 113
111  A=-0.1
      GO TO 113
112  A=-1.0
113  CONTINUE
      IF (A.LE.1.06) GO TO 114
105  CONTINUE
114  GO TO 115
100  FORMAT(15,3F10.4)
102  FORMAT(1H1)
103  FORMAT(//5X,2HK=,15,5X,3HT1=,F6.2,5X,3HT2=,F6.2,5X,3HT3=,F6.2//
104  FORMAT(//4X,1HX,14X,4HREAL,13X,4HIMAG,10X,9HABS VALUE//)
108  FORMAT(//1X,F8.5,5X,F12.6,5X,F12.6,5X,F12.6)
101  STOP
      END

```

(45)

SUBPROGRAMS

BF:PIN
BF:FR

BF:S3
BF:SX

BF:II

BF:FI

BF:SF

BF:S6

BF:F

PROGRAM ALLOCATION

152.0	K	153.0	T1	154.0	T2	155.0	T3
157.0	M	158.0	I	159.0	A	15A.0	B
15C.0	P	15D.0	Q	15E.0	R		

PROGRAM SIZE 15F

PROGRAM END

K= 5

T1=1.00

T2=0.50

T3=0.25

W	REAL	IMAG	ABS VALUE
100.00000	0.000000	0.000000	0.000000
90.00000	0.000001	0.000000	0.000001
80.00000	0.000001	0.000000	0.000001
70.00000	0.000002	0.000000	0.000002
60.00000	0.000003	0.000000	0.000003
50.00000	0.000006	0.000001	0.000006
40.00000	0.000015	0.000003	0.000016
30.00000	0.000048	0.000011	0.000049
20.00000	0.000229	0.000083	0.000244
10.00000	0.002823	0.002272	0.003624
9.00000	0.003964	0.003675	0.005405
8.00000	0.005656	0.006222	0.008408
7.00000	0.008128	0.011113	0.013768
6.00000	0.011435	0.021137	0.024031
5.00000	0.014233	0.043217	0.045500
4.90000	0.014286	0.046621	0.048760
4.80000	0.014249	0.050333	0.052311
4.70000	0.014103	0.054385	0.056183
4.60000	0.013823	0.058809	0.060412
4.50000	0.013383	0.063645	0.065037

W	REAL	IMAG	ABS VALUE
1.06967	-2.719340	0.001841	2.719341
1.06957	-2.719805	0.001544	2.719806
1.06947	-2.720268	0.001248	2.720269
1.06937	-2.720733	0.000951	2.720734
1.06927	-2.721197	0.000654	2.721197
1.06917	-2.721662	0.000358	2.721663
1.06907	-2.722127	0.000061	2.722128
1.06897	-2.722591	-0.000236	2.722591
1.06887	-2.723056	-0.000533	2.723057
1.06877	-2.723520	-0.000830	2.723520
1.06867	-2.723987	-0.001127	2.723988
1.06857	-2.724451	-0.001424	2.724452
1.06847	-2.724916	-0.001722	2.724916
1.06837	-2.725381	-0.002020	2.725382
1.06826	-2.725845	-0.002317	2.725847
1.06816	-2.726310	-0.002614	2.726312
1.06806	-2.726778	-0.002913	2.726779
1.06796	-2.727243	-0.003211	2.727245
1.06786	-2.727708	-0.003508	2.727711
1.06776	-2.728173	-0.003807	2.728177

K 10

T1 1.00

T2 0.50

T3 0.25

W	REAL	IMAG	ABS VALUE
100.00000	0.000001	0.000000	0.000001
90.00000	0.000001	0.000000	0.000001
80.00000	0.000002	0.000000	0.000002
70.00000	0.000003	0.000000	0.000003
60.00000	0.000006	0.000001	0.000006
50.00000	0.000013	0.000002	0.000013
40.00000	0.000031	0.000005	0.000031
30.00000	0.000095	0.000022	0.000098
20.00000	0.000458	0.000166	0.000487
10.00000	0.005646	0.004543	0.007247
9.00000	0.007928	0.007349	0.010810
8.00000	0.011312	0.012443	0.016817
7.00000	0.016255	0.022227	0.027537
6.00000	0.022869	0.042273	0.048062
5.00000	0.028466	0.086433	0.091000
4.90000	0.028571	0.093241	0.097520
4.80000	0.028497	0.100666	0.104622
4.70000	0.028205	0.108769	0.112367
4.60000	0.028205	0.108769	0.112367
4.50000	0.026766	0.127290	0.130074

W	REAL	IMAG	ABS VALUE
1.06967	-5.438680	0.003682	5.438681
1.06947	-5.440534	0.002496	5.440534
1.06957	-5.439607	0.003088	5.439069
1.06937	-5.441465	0.001901	5.441467
1.06927	-5.442392	0.001309	5.442394
1.06917	-5.443320	0.000716	5.443322
1.06907	-5.444253	0.000123	5.444254
1.06897	-5.445181	-0.000471	5.445182
1.06887	-5.446109	-0.001065	5.446110
1.06877	-5.447038	-0.001660	5.447039
1.06867	-5.447972	-0.002255	5.447973
1.06857	-5.448901	-0.002848	5.448901
1.06847	-5.449828	-0.003444	5.449829
1.06837	-5.450758	-0.004040	5.450761
1.06826	-5.451687	-0.004634	5.451689
1.06816	-5.452616	-0.005228	5.452619
1.06806	-5.453556	-0.005826	5.453559
1.06796	-5.454487	-0.006421	5.454492
1.06786	-5.455416	-0.007017	5.455421
1.06776	-5.456347	-0.007613	5.456353

K 15

T1 1.00 T2 0.50

T3 0.25

W	REAL	IMAG	ABS VALUE
100.00000	0.000001	0.000000	0.000001
90.00000	0.000002	0.000000	0.000002
80.00000	0.000003	0.000000	0.000003
70.00000	0.000005	0.000000	0.000005
60.00000	0.000009	0.000001	0.000009
50.00000	0.000019	0.000003	0.000019
40.00000	0.000046	0.000008	0.000047
30.00000	0.000143	0.000034	0.000146
20.00000	0.000687	0.000249	0.000731
10.00000	0.008470	0.006815	0.010871
9.00000	0.011892	0.011024	0.016216
8.00000	0.016968	0.018665	0.025225
7.00000	0.024383	0.033340	0.041305
6.00000	0.034304	0.063410	0.072094
5.00000	0.042699	0.129650	0.136500
4.90000	0.042857	0.139862	0.146281
4.80000	0.042746	0.150999	0.156933
4.70000	0.042308	0.163154	0.168550
4.60000	0.041470	0.176428	0.181237
4.50000	0.040150	0.190935	0.195110

W	REAL	IMAG	ABS VALUE
1.06967	-8.158028	0.005522	8.158020
1.06957	-8.159412	0.004632	8.159411
1.06947	-8.160806	0.003744	8.160803
1.06937	-8.162195	0.002852	8.162192
1.06927	-8.163591	0.001963	8.163587
1.06917	-8.164981	0.001074	8.164979
1.06907	-8.166380	0.000184	8.166377
1.06897	-8.167774	-0.000707	8.167770
1.06887	-8.169167	-0.001598	8.169164
1.06877	-8.170562	-0.002490	8.170559
1.06867	-8.171959	-0.003382	8.171955
1.06857	-8.173354	-0.004272	8.173352
1.06847	-8.174743	-0.005165	8.174741
1.06837	-8.176143	-0.006060	8.176144
1.06826	-8.177533	-0.006951	8.177532
1.06816	-8.178927	-0.007843	8.178927
1.06806	-8.180335	-0.008739	8.180336
1.06796	-8.181728	-0.009632	8.181731
1.06786	-8.183124	-0.010525	8.183126
1.06776	-8.184517	-0.010525	8.183126
1.06776	-8.184517	-0.011420	8.184522

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BALA PRASAD SARDA

(52)

JECT,3)
 :5,(DEVICE,S1)
 :6,(DEVICE,L9)
 69,L9

C TO OBTAIN THE CORRESPONDING VALUES OF E FOR WHICH $g(E)=0.1848$
 C TO DETERMINE THE AMPLITUDE OF SUSTAINED OSCILLATIONS IN A FOURTH
 C ORDER CONTROL SYSTEM WITH AMPLITUDE SENSITIVE NON-LINEARITY
 C $A(E)$ =REAL PART OF THE D.F.
 C A=INDICATES THE START OF SATURATION
 C B=SATURATION VALUE OF THE NON-LINEARITY
 C A/N INDICATES THE END OF DEAD SPACE
 REAL N
 112 READ(5,100,END=101)N,A,B
 WRITE(6,102)
 WRITE(6,103)N,A,B
 WRITE(6,104)
 E=8.0
 N=0
 DO 105 I=1,1000
 N=N+1
 IF(N.GT.20)GO TO 998
 GO TO 999
 998 WRITE(6,997)
 997 FORMAT(1H1, '//8X,1HE,24X,'G(E)'/ //)
 N=1
 999 CONTINUE
 IF(N.EQ.1.0)GO TO 304
 $P=B*7.0*N/((A*N-A)*22.0*E)$
 C $P*Q$ =FIRST TERM OF $A(E)$
 $Q=2*E*(-ATAN(A/((N**2*E**2-A**2)**0.5))+ATAN(A/((E**2-A**2)**0.5)))$
 C $P*R$ =SECOND TERM OF $A(E)$
 $R=E*(2.0*A*((E**2*N**2-A**2)**0.5)/(N**2*E**2))$
 $1-2.0*A*((E**2-A**2)**0.5)/(E**2))$
 C $P*S$ =THIRD TERM OF $A(E)$
 $S=4.0*A*(N*((E**2-A**2)**0.5)-((E**2*N**2-A**2)**0.5))/(N**2*E$
 C U =FOURTH TERM OF $A(E)$
 304 $U=4.0*7.0*B*((E**2-A**2)**0.5)/(22.0*E**2)$
 IF(N.NE.1.0)GO TO 305
 V=U
 GO TO 306
 305 $V=P*(Q+R+S)+U$
 306 CONTINUE
 C $V=A(E)$ =REAL PART OF D.F.
 IF(E.LE.2.0)GO TO 190
 WRITE(6,301)E,V
 E=E-0.1
 GO TO 191
 190 IF(E.LE.1.05)GO TO 211
 WRITE(6,302)E,V


```

47      E=E-0.01
48      GO TO 191
49      211  WRITE(6,303)E,V
50      E=E-0.001
51      IF (E.LE.0.76)GO TO 111
52      191  CONTINUE
53      105  CONTINUE
54      111  GO TO 112
55      100  FORMAT(F5.1,2F5.2)
56      102  FORMAT(1H1)
57      103  FORMAT(//10X,2HA=,F5.1,10X,2HA=,F5.2,10X,2HB=,F5.2//)
58      104  FORMAT(//8X,1HE,24X,'G(E)')//)
59      301  FORMAT(/4X,F5.1,20X,F10.6)
60      303  FORMAT(/4X,F5.3,20X,F10.6)
61      302  FORMAT(/4X,F5.2,20X,F10.6)
62      101  STOP
63      END

```

SUBPROGRAMS

```

      BF:PIN      BF:S3      BF:FI      BF:SF      BF:S6      ATAN      B
      BF:IX

```

PROGRAM ALLOCATION

194.0	N	195.0	A	196.0	B	197.0	E
199.0	I	19A.0	P	19B.0	Q	19C.0	R
19E.0	U	19F.0	V				

PROGRAM SIZE 140

PROGRAM END

N 1.1

a 0.75

b 0.25

E	$g(E)$
8.0	0.039613
7.9	0.040111
7.8	0.0406200
7.7	0.041143
7.6	0.041680
7.5	0.042230
7.4	0.042796
7.3	0.043376
7.2	0.043973
7.1	0.044586
7.0	0.045216
6.9	0.045864
6.8	0.046531
6.7	0.047218
6.6	0.047925
6.5	0.048653
6.4	0.049404
6.3	0.050178
6.2	0.050976
6.1	0.051801

E	g(E)
1.60	0.177828
1.59	0.178665
1.58	0.179503
1.57	0.180347
1.56	0.181197
1.55	0.182049
1.54	0.182907
1.53	0.183769
1.52	0.184635
1.51	0.185505
1.50	0.186379
1.49	0.187256
1.48	0.188136
1.47	0.189021
1.46	0.189907
1.45	0.190797
1.44	0.191688
1.43	0.192583
1.42	0.193480
1.41	0.194377

E	$g(E)$
0.825	0.190721
0.824	0.190228
0.823	0.189728
0.822	0.189222
0.821	0.188705
0.820	0.188183
0.819	0.187656
0.818	0.187121
0.817	0.186575
0.816	0.186026
0.815	0.185468
0.814	0.184900
0.813	0.184325
0.812	0.183744
0.811	0.183150
0.810	0.182551
0.809	0.181943
0.808	0.181326
0.807	0.180701
0.806	0.180066

N 1.2

a 0.75

b 0.25

E	$g(E)$
8.0	0.039625
7.9	0.040123
7.8	0.040633
7.7	0.041157
7.6	0.041694
7.5	0.042245
7.4	0.042811
7.3	0.043392
7.2	0.043990
7.1	0.044603
7.0	0.046234
6.9	0.045883
6.8	0.046551
6.7	0.047339
6.6	0.047947
6.5	0.048676
6.4	0.049428
6.3	0.050203
6.2	0.051002
6.1	0.051828

E	$g(E)$
1.60	0.179503
1.59	0.180374
1.58	0.181248
1.57	0.182129
1.56	830155
1.55	0.183907
1.54	0.184804
1.53	0.185706
1.52	0.186614
1.51	0.187527
1.50	0.188445
1.49	0.189368
1.48	0.190296
1.47	0.191228
1.46	0.192165
1.45	0.193106
1.44	0.194052
1.43	0.195001
1.42	0.195954
1.41	0.196911

E	g(E)
0.785	0.191462
0.784	0.190810
0.783	0.190145
0.782	0.189470
0.781	0.188784
0.780	0.188087
0.779	0.187377
0.778	0.186655
0.777	0.185922
0.776	0.185175
0.775	0.184415
0.774	0.183641
0.773	0.182854
0.772	0.182053
0.771	0.181236
0.770	0.180406
0.769	0.179558
0.768	0.178695
0.767	0.177812
0.766	0.176915

N 1.0

a 0.75

b 0.25

E	$g(E)$
8.0	0.039598
7.9	0.040094
7.8	0.040604
7.7	0.041126
7.6	0.041662
7.5	0.042212
7.4	0.042776
7.4	0.043356
7.2	0.043952
7.1	0.044564
7.0	0.045193
6.9	0.045840
6.8	0.046506
6.7	0.048191
6.6	0.047897
6.5	0.048624
6.4	0.049373
6.3	0.050146
6.2	0.509403
6.1	0.051765

E	$g(E)$
1.60	0.175665
1.59	0.176455
1.58	0.177249
1.57	0.178047
1.56	0.178847
1.55	0.179650
1.54	0.180456
1.53	0.181265
1.52	0.182076
1.51	0.182890
1.50	0.183705
1.49	0.184523
1.48	0.185341
1.47	0.186162
1.46	0.186983
1.45	0.187805
1.44	0.188627
1.43	0.189449
1.42	0.190271
1.41	0.191092

E	$g(E)$
0.885	0.190836
0.884	0.190500
0.883	0.190159
0.882	0.189815
0.881	0.189465
0.880	0.189112
0.879	0.188753
0.878	0.188391
0.877	0.188023
0.876	0.187651
0.875	0.187275
0.874	0.186893
0.873	0.186507
0.872	0.186116
0.871	0.185720
0.870	0.185318
0.869	0.184912
0.868	0.184501
0.867	0.184084
0.866	0.183662