Reliable Route Planning for Emergency Evacuation

A Dissertation

Presented to

the Faculty of the Department of Industrial Engineering

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in Industrial Engineering

by

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To my parents

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and my wife

Priyanka

with all my love

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Abstract

Large scale evacuations are important in the wake of events such as an anticipated strike of a natural disaster or a looming military attack. Planning to evacuate people towards safe areas and effective management of the plan using limited set of resources is, therefore, an integral part of disaster management. Evacuation planning based on deterministic estimate of demand at the source nodes and capacity of the road links yield unsatisfactory result. Recent research publications are addressing the randomness associated with such events using stochastic optimization models. Models considering the inherent uncertainty associated with transportation network facilitate a robust and efficient evacuation plan.

In this dissertation, large scale network flow optimization models for both deterministic and stochastic evacuation scenarios are presented with an emphasis on coming up with an effective and reliable evacuation plan. Effective implementation of an evacuation plan in the wake of a limited set of resources demands that a minimum number of paths are selected for loading the evacuation traffic. This objective has eluded the eyes of the research community involved in evacuation planning optimization. Model, solution technique and computational results for this problem is presented that describes the complete evacuation plan comprising of paths, traffic flow and starting schedule.

Traffic scenario is often non-deterministic and assumption of a deterministic capacity for the road links would result in poor quality evacuation plan in terms of paths and time required for evacuation. Motivated by the stochastic behavior of the arc capacity, a chance constrained model for bottleneck minimization is proposed that finds the evacuation paths and the traffic flow rate on the paths within a given time bound that would result in minimum traffic congestion. Given the horizon time for evacuation, model selects the evacuation paths and finds flows on the selected paths that result in minimum congestion in the network and finds the reliability of the evacuation plan. Numerical examples are presented and we discuss the effectiveness of the stochastic models in evacuation planning. It is shown that the reliability based evacuation plan is conservative as compared to plans obtained using a deterministic model. Stochastic models guarantee that congestion can be avoided with a confidence level at the cost of increased clearance time.

Apart from the random arc capacity, in this dissertation we propose an evacuation planning model where the demand for the number of evacuees is unknown and is subject to uncertainty. Chance constrained approach is used in such situations to enforce the constraints for given level of confidence. We analyze the model for the situation when the probability distribution of the random demand is not known and only partial moments and support information is specified. A distributional robust chance constrained model is proposed for evacuation planning that guarantee the vehicle demand constraints for any probability distribution consistent with the known properties. We find a tight upper bound for the shortfall in evacuating people from the specified target in the given clearance time. Numerical experiments show that the robust approximation method of chance constraints provide excellent results as compared to solution based on approximated distribution and sampling based solution.

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Chapter 1 Introduction

1.1 Introduction

"Destruction, hence, like creation, is one of Nature's mandates." This famous quote by Marquis de Sade signifies the inevitability of destruction as one of the laws of nature. Besides nature's blow, there are numerous other major and minor incidents resulting from human deeds having short-term or long-term implications on the health and lives of people. History is replete with incidents when men were faced with life threatening events. Survival has always been the prize for only those smart few who were able to quickly plan the escape and execute the plan effectively. In this age of technological advancement, we are still faced with similar life threatening events and the rule of the game remains same: "Survival of the quickest." There has been an added rule in this game owing to large cities and huge population residing in them - "Efficient planning." In fact, efficient planning is a precursor for quick and smooth evacuation process. Survival in the time of disasters depend on either how proactive or how reactive the response is in terms of scalability and effectiveness.

When community evacuation becomes necessary in light of an approaching danger, emergency managers face a set of logistical and action timing decisions. Decisions concerning pre-positioning of personnel and material, mobilization of resources, decision on evacuation routes and schedules, distribution of humanitarian aid, communication of advisory messages, and updating of supplies, all of these are inter-dependent and become the task of prime importance. Attempts have been made to address this problem by applying the concepts of operations research (OR). Mathematical models have been designed by that attempt to mimic the real evacuation scenario. These models are then solved for optimizing the evacuation process. This dissertation focuses on applications of OR models in evacuation planning.

Emergency evacuation is the immediate and rapid movement of a population in the wake of an impending danger from an impacted geographical region towards safer destinations. Since population is primarily concentrated in cities, it becomes imperative for the disaster management personnels to have an efficient plan that could safely evacuate its residents to a shelter location and avoid the chaos. Traffic planning during evacuation is not an easy problem owing to a highly complex dynamics of evacuation resulting from behavior of people and unforeseen circumstances of the event. Mass exodus calls for effective and efficient route planning and schedule allocation considering the spatial and temporal constraints in situations of road congestion, blockage or otherwise inaccessibility due to other dangerous circumstances.

The US federal government, through FEMA, requires all states to have a comprehensive emergency operations plan. These plans guide emergency operations for all types of hazards, from natural to man-made and technological. While the general evacuation issues faced by coastal states are similar, different strategies and plans have been developed to deal with variations in population, geography, and transportation system characteristics. States also differ in the way that they delegate authority, allocate people and resources, and enforce evacuations. They seek to maximize the efficiency of their emergency operation plans within these many constraints. Evacuation orders are issued by the local authorities after analyzing the severity and possible consequences of the disruptive event. The evacuation procedure is then carried out according to the devised plan. From a logistics viewpoint, the evacuation plan model answers the following basic questions:

1. How much time would be required for evacuation?
2. What are the ideal routes that should be used for evacuation?
3. How should the traffic flow be managed within limited infrastructure?

A critical issue in evacuations, particularly during hurricanes, is timing. The earlier the evacuation order is issued, the more time residents and tourists will have to evacuate. Unfortunately, the earlier it is issued, the greater the possibility the hurricane could change course before landfall, rendering the evacuation unnecessary. Emergency management centers would not want to "cry wolf" and issue an evacuation order in situations of false alarm. Therefore, they want to wait until the last minute before making such an important decision. The time required to evacuate is estimated from a combination of clearance times and the pre-landfall hazard time (Wolshon et al. [2005]) as shown in Figure 1.1. Clearance time is the time required to configure all traffic control elements on the evacuation routes, initiate the evacuation, and clear the routes of vehicles once deteriorating conditions warrant its end. Pre-landfall hazards time is the time during which hazardous conditions exist prior to actual hurricane landfall. Hence, a nearly accurate estimate of the clearance time would arm the evacuation managers with a plan for the evacuation according to the horizon time before the danger hits the shores.

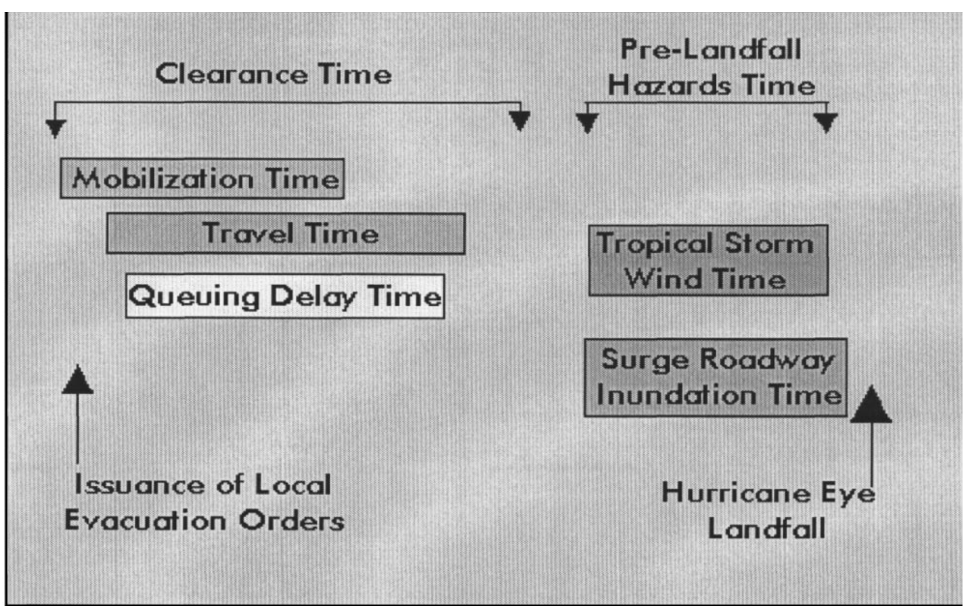


Figure 1.1: Components of evacuation time (Wolshon et al. [2005])

Clearance times are estimated using evacuation traffic models, which are dependent on data such as the population anticipated to evacuate, the number of lanes available for evacuation, and impacts from other areas that will affect the evacuation such as flooding and road closures. In an ideal scenario, the objective of a good evacuation plan is to minimize the evacuation time such that a maximum number of evacuees can be moved away from danger. In order not to overcharge the transportation infrastructure, it is essential to find an efficient set of routes to load the evacuating traffic and allocate a starting schedule and flow on those paths as per the priority. Transportation planning for evacuation is very challenging due to the huge amount of people evacuating from a large geographical region and inherent uncertainty in transportation parameters present during such rare events. The amount of time required for clearance can be significantly lengthened by en route congestion and the setup time required for complex control features (such as those required for contraflow). Managing this enormous task effectively requires a similar scale of resources, thereby, making the problem even more complex.

Network optimization approach is used in this research to address the highly complex dynamics of transportation planning during evacuation. Mathematical models trying to mimic the evacuation scenario are used and then solved for optimizing the evacuation plan. The emphasis of this research is to optimize the use of limited resources and come up with a reliable plan that account for mishaps that might occur due to inherent randomness. Our specific focus is on minimizing the number of evacuation paths and we build upon this to study the impact of uncertainty on the evacuation plan. Research presented in this thesis would provide direction in devising an evacuation plan which is efficient in terms of resource utilization and robust in terms of handling randomness.

1.2 Motivation

Evacuation planning optimization is focused on minimizing the evacuation time but the time is a surrogate parameter in evaluating a successful evacuation. In many practically motivated decision problems a number of uncertain, unforeseen or not completely known factors may play a non-negligible role thus affecting the decisions taken without considering these factors. It is therefore advisable to explicitly consider such uncertainties during the planning phase. Successfully implementing an evacuation plan with the calculated clearance time is dependent on numerous supporting factors that indirectly or directly affects the time when the network has attained the desired outcome. Motivation to investigate the reliability and efficiency of the evacuation plan under uncertainty and limitation of the resource comes from the following discussion.

1.2.1 Least number of evacuation paths

Evacuations specially with little or no-notice time produce a distinct set of challenges for those personnel involved in responding to the incidents or executing the evacuations. Such evacuation notices are impromptu and do not provide emergency managers and transportation personnel with the opportunity to prepare in immediate advance of the incident. This curtails the preparation or readiness for executing the evacuation which is critical to a successful evacuation effort. Responders will be unable to pre-activate or pre-position resources in preparation for the specific situation mandating the evacuation. Establishment of a command structure, the activation of an operations center, or the tasking and distribution of personnel and resources to manage the evacuation are absolutely necessary during such events of chaos. Moreover, there are limited number of personnel with particular skills and knowledge and also limitation on the tools to determine and monitor the status of the transportation network whose absence or shortage can significantly hinder the evacuation operation. Situation demands for a plan that use minimum resource and still is equivalently effective in carrying out the evacuation.

Utilizing least number of evacuation paths that would be able to navigate maximum number of people using minimum time would be perfect in terms of managing this huge process within the given constraint of the resources and still achieving the target. Less number of paths would result in concentrating the resources to a few selected routes and managing the operations perfectly. Effective implementation of an evacuation plan in the wake of a limited set of resources, therefore, demands that a minimum number of paths are selected for loading the evacuation traffic. This objective has eluded the eyes of the research community involved in evacuation planning optimization. To fill this important gap, a bi-objective dynamic network flow model is formulated and an evacuation plan is proposed that uses the least number of evacuation paths for complete evacuation within the minimum clearance time.

1.2.2 Minimum congestion

The relation between travel time and roadway capacity can be best explained using the link performance function. According to Bureau of Public Records (BPR), the link performance function for average travel time is given by

$$t_a(\mathcal{Q}_a, \mathcal{U}_a) = t_a^f \left[1 + \beta \left(\frac{\mathcal{Q}_a}{\mathcal{U}_a} \right)^n \right], \tag{1.1}$$

where subscript a refers to a particular link in $\mathscr{A}$; t_a^f; $\mathcal{U}_a$; t_a, respectively, are link a's free-flow travel time (which is deterministic), capacity, and travel time with flow volume $\mathcal{Q}_a$; β and n are deterministic parameters associated with the BPR travel time function for which the value of $\beta = 0.15$ minimum and $n = 4.0$ are typically used. Now consider a scenario where a fixed flow of vehicles is allocated to the link but the link capacity is subject to stochastic degradation (due to weather, accidents, driver behavior etc.). In such scenario, $\mathscr{U}_a$ is replaced by the random variable $\widetilde{\mathscr{U}_a}$. Then in (1.1), the link travel time t_a becomes a random variable with its mean and variance expressed as

$$E(\tilde{t_a}) = E\left(t_a^f\left[1+\beta\left(\frac{\mathcal{Q}_a}{\widetilde{\mathscr{U}_a}}\right)^n\right]\right) = t_a^f + \beta t_a^f E\left[\left(\frac{\mathcal{Q}_a}{\widetilde{\mathscr{U}_a}}\right)^n\right], \tag{1.2}$$

$$var(\tilde{t_a}) = E[(\tilde{t_a})^2] - E^2(\tilde{t_a}). \tag{1.3}$$

Assuming that the free flow travel time t_a^f is deterministic and constant, expressions (1.2) and (1.3) allow the calculation of the expected value $E(\tilde{t_a})$ and variance $var(\tilde{t_a})$ of link travel time which depends on the probability distribution function of link capacity $\widetilde{\mathscr{U}_a}$. Using the arc transit time as

$$t_a = E(\tilde{t_a}) + var(\tilde{t_a}), \tag{1.4}$$

the total clearance time would, therefore, increase as compared to the scenario when the free flow speed is considered with deterministic capacity. This highlights the importance of accounting for congestion probability in order to make a realistic evacuation plan.

Definition 1. Road capacity: The *Highway Capacity Manual* [2000] (HCM) defines capacity as "the maximum sustainable hourly flow rate at which persons or vehicles can reasonably be expected to traverse a point or uniform section of a lane or roadway during a given time period under prevailing roadway, environmental, traffic, and control conditions."

Much of the evacuation planning literature considers the designed capacity of a roadway link as constant at all times. Maximum flow rate or traffic volume below this capacity is considered as acceptable and volume above this is unacceptable resulting

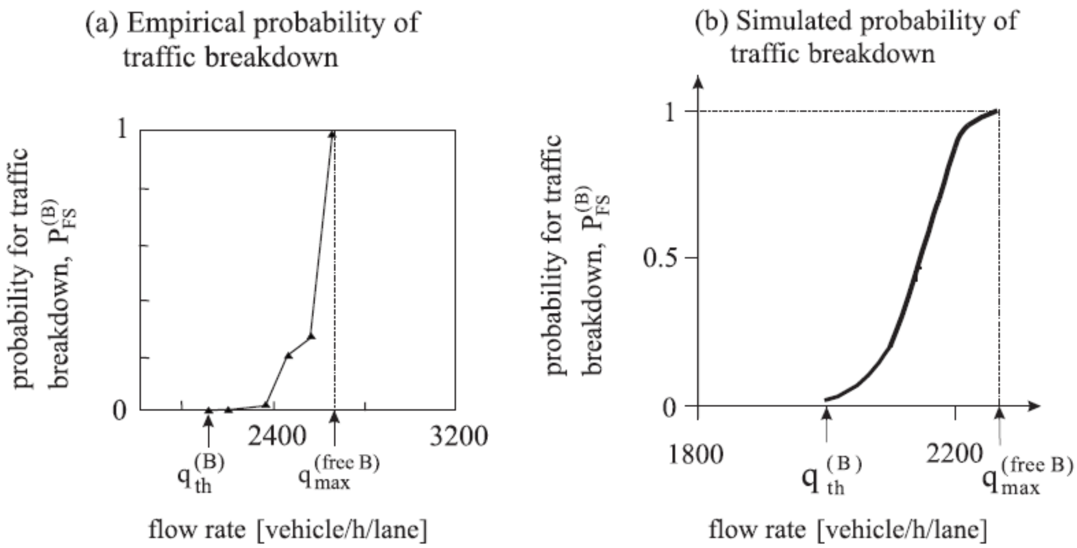


Figure 1.2: Probability of traffic breakdown taken from Kerner [2011]

in breakdown. However, it is well known from Chen et al. [2002], Lo and Tung [2003] and Persaud et al. [1998] that actual capacity of an arc representing a section of a road is not constant and is a function of volume of the vehicles present in that arc at a given time. In a realistic situation, the element of stochasticity which is locally a certain disorder in a queue of cars develops like a domino effect into a macroscopic phase transition from free to congested flow. Mahnke et al. [2005] describe this phenomenon as *traffic breakdown.* According to Kerner's three-phase traffic theory (Kerner [2011]) (see Figure 1.2), when the flow rate q of the link exceeds a certain threshold value q_{th}, then network enters a metastable state where a traffic breakdown occurs with some finite probability and this probability approaches a value of 1 when the flow exceeds the maximum volume q_{max} possible for the link. Existing evacuation planning models with an objective of minimizing the clearance time pushes maximum flow out of the network which is limited only by the arc capacity and do not consider congestion probability. Stochastically degrading capacity of the road link would result in congestion which subsequently would increase the clearance time for the network. Thus, not accounting for capacity uncertainty and congestion probability may lead to suboptimal or possible infeasible solutions in real evacuation situations. This calls for a new mathematical approach that incorporates the uncertainty inherent in the estimates of the roadway capacity to come up with a reliable evacuation plan.

1.2.3 Distributional robustness to demand uncertainty

Efficient planning of a large scale evacuation requires an accurate description of data. But evacuation is a rare event and enough data are not available to model the underlying uncertainty. Demand estimates are usually based on the judgment of individuals, creating inconsistencies in estimation methods. In several contexts, information about the past behavior can be used to compute statistical information which in turn is used to construct the expected value problem. In such problems, the uncertain parameters are assumed to take their expected value and the deterministic solution to this problem will provide decisions which work reasonably well on average. However, in the context of evacuation problem, when some unplanned events occur, it may seriously impair the effectiveness of the evacuation plan. This demands that we address uncertain input parameter by implementing robust decisions which minimize the negative impact of some real time recourse actions. Traditionally, worst case demand is assumed to solve the deterministic evacuation problem leading to high clearance time. But it is necessary to carefully balance the operational failure probability due to high demand, on one side, and minimize the clearance time for evacuation by allocating the available capacity on the other.

Robustness to demand uncertainty can be accounted for using the method of robust convex optimization. Scenario based models are sometimes used but this is unnecessary for cases when the worst-case performance have to be optimized. In this research, stochastic programming method of chance constrained programming has been applied to formulate and analyze the demand uncertainty. Consider a chance constrained program of the form

$$\text{minimize} \quad f(x) \tag{1.5}$$

$$\text{s.t. } \mathbb{P}(F(x,\tilde{\xi}) \leq 0) \geq 1-\epsilon, \tag{1.6}$$

where $\tilde{\xi}$ is the random variable with an associated probability function $\mathbb{P}$ and F is a function which describes a particular system. We fix a probability level $\epsilon \in [0,1]$ that require the constraints of the system to be satisfied with a confidence level greater that or equal to $(1-\epsilon)$. How to manage chance constraints strongly depends on what we know about the probability distribution of the uncertain parameters. In the above model, the basic assumption is that the probability distribution of the underlying random parameter is exactly known. However, in many cases, it may be very difficult to accurately identify the distribution required to solve a problem. Especially, this is more likely true when we are considering an evacuation transportation problem due to the inherent complexity and uncertainty. Typically, one has only partial information about $\mathbb{P}$, e.g. about its moments or its support. Replacing the unknown distribution $\mathbb{P}$ in (1.6) by an estimate $\hat{\mathbb{P}}$ corrupted by measurement errors may lead to over-optimistic solutions which often fail to satisfy the chance constraint under the true distribution $\mathbb{P}$. If we want to evaluate, bound or approximate the probability in the chance constraints we have to make some assumptions about the probability distribution. These assumptions affects the probability measure and consequently the optimal choice. This research, therefore, presents the evacuation planning model using the distributionally robust chance constrained setting as shown below.

$$\text{minimize} \quad f(x) \tag{1.7}$$

$$\text{s.t. } \mathbb{P}(F(x,\tilde{\xi}) \leq 0) \geq 1-\epsilon, \quad \forall \mathbb{P} \in \mathcal{P}. \tag{1.8}$$

1.3 Contribution

This research is an effort to come up with an effective evacuation plan for a large geographical region. Implementing an evacuation plan requires the mobilization of huge resources and these resources are often limited. Also, providing a reliable flow is very important in the context of emergency evacuation. This thesis, therefore, lays emphasis on finding strategies that can optimize the use of limited resources and account for the uncertainty in the optimization model.

The key contributions of this thesis are as follows:

1. Optimization model to find an evacuation plan that utilizes least number of evacuation paths.
2. Model that accounts for the capacity uncertainty.
3. Introduction of model and solution that minimizes the probability of congestion in a stochastic network setting.
4. Use the verification model to find the reliability of an evacuation plan in terms of congestion probability.
5. Distributional robust chance constrained optimization for evacuation planning under demand uncertainty.

1.4 Organization

This thesis is organized as follows. **Chapter 2** is a review of the optimization approach in evacuation planning with emphasis on stochastic models and also the models that aims at efficient use of limited infrastructure. **Chapter 3** develops a dynamic network flow model for minimizing the number of evacuation paths. The solution method is presented for this model and the results are discussed using an example of Houston evacuation network under the assumption of deterministic parameters. Extension of the minimum path model to a stochastic setting that account for capacity uncertainties of the links is presented in **Chapter 4** . A reliability based model is presented and the solution of the convex constrained model is discussed in this chapter. In **Chapter 5** we present a distributionally robust chance constrained model to account for demand uncertainty in the evacuation planning model. In **Chapter 6**, we conclude the dissertation with a summary of our contributions.

Chapter 2 Literature review

Evacuation planning has been a major research topic in the operation research (OR) community. The research on this problem has evolved over the years to encapsulate various aspects of a real evacuation scenario and come up with a realistic evacuation plan. Mathematical models for evacuation planning are handled using network models. Initial research on this problem done by Chalmet et al. [1982] addressed building evacuations during emergency situations. Numerous other models were developed that handled building evacuations and an exhaustive survey on these models is presented in works by Aronson [1989] and Hamacher and Tjandra [2002]. Many of the models designed for building evacuations are also applicable to regional evacuation. This section discusses the models that are applicable for both types of evacuation problems. An excellent and comprehensive reading for network optimization approach can be found in Ahuja et al. [1993]. In particular to the network models for modeling evacuation problems, the dissertation by Tjandra [2003] is a comprehensive read.

Designing an evacuation plan often involves a series of optimization problems with various objectives and constraints. Given the large number of research papers in this area, we classify the problem into broad categories to provide some structure for the rest of the chapter. Different aspects of the problem in evacuation planning can be classified as follows:

1. Deciding routes, assigning traffic and finding clearance time
2. Efficient use of limited infrastructure
3. Reliable planning in stochastic conditions

Although, we have offered this simple classification, it is found that many papers deal with problems that intersect two or more of the above categories. The models that we discuss in this chapter are classified under a common umbrella of macroscopic modeling. Macroscopic models do not consider any individual's behavior during the emergency situation but are useful to provide good lower bounds for evacuation time. Individual behaviors can be modeled using microscopic models and simulations are used for their analysis.

Since time is a decisive parameter during the evacuation, an estimate of evacuation time or clearance time is the primary information required by the evacuation planners. Most of the literature in evacuation planning is therefore centered around minimization of the clearance time. As per the FEMA report prepared jointly by DOT and DHS [2006], one of the criterion for the evaluation of an evacuation plan is based on the effective implementation and ease of managing the routes loaded with the evacuating traffic. Therefore, deciding the best routes and assigning the traffic on the selected routes based on the evacuation schedule that leaves behind minimum evacuees is an important aspect of evacuation planning.

Highway network clearance times are greatly influenced by other factors such as location of shelters, number of intersections on the selected routes, and decision on the timing for start of contraflow on certain highway stretches. Overlooking these factors may result in a build-up of traffic on certain road sections. Efficient use of the limited transportation infrastructure is therefore required. Evacuation is a rare scenario and many of the parameters that are used to come up with a prescriptive evacuation plan are random. Parameters such as number of evacuees, travel time, and link capacity cannot be considered as deterministic. Therefore, stochastic models are essential to embed the uncertainty associated with the problem and come up with a reliable evacuation plan. A detailed discussion of the optimization models and solution techniques for each of the above classification is provided in subsequent subsections.

2.1 Network models for evacuation planning

Routing evacuating traffic involves selecting a set of paths among the alternative paths between origin−destination pairs. Typically, the number of evacuees for a regional evacuation are huge and the limited number of paths can't handle all the vehicles simultaneously. Therefore, evacuees have to be grouped according to a pre-allocated schedule of evacuation. Since the objective is to evacuate in minimum time, the vehicle routing and scheduling decisions are intertwined. At the macroscopic level, evacuation routing is a "many to many" routing problem with multiple origins and multiple destinations. From the perspective of evacuation planner whose target is to evacuate the maximum number of people within a minimum time, Wardrop's traffic principle of System Optimal (SO) flow is best suited to decide the routes, schedule and traffic flow.

A precise estimate of the evacuation time is of primary importance to the evacuation planners. In this section, we will provide a detailed treatment of how the answer to this question is found and subsequently discuss the methods for deciding the routes and traffic assignment for the evacuation. The objective of finding the minimum time is modeled as a minimum cost network flow problem. Under the assumption of deterministic travel time and capacity of the arcs, the evacuation model minimizes the clearance time. Since time is a decisive parameter in such problems, dynamic networks are used instead of static ones and the evacuation planning problems are modeled under the discrete time dynamic network flow framework.

A discrete time dynamic network flow problem is a discrete time expansion of a static network flow problem. In this case we distribute the flow over a set of predetermined time periods $t = 1, 2, \cdots T$. Consider a directed static network $\mathscr{G} = (\mathscr{N}, \mathscr{A})$ with $\mathscr{N}$ and $\mathscr{A}$ as the set of nodes and arcs, respectively. A constant transit time λ_{ij} is associated with each arc $(i, j) \in \mathscr{A}$. The time expansion of $\mathscr{G}$ over a time horizon T defines the dynamic network $\mathscr{G}_T = (\mathscr{N}_T, \mathscr{A}_T)$ associated with $\mathscr{G}$. The time expansion essentially is the replication of the static network $\mathscr{G}$ at each discrete unit of time in T. Since there are multiple copies of source and sink nodes, a super source node s and sink node d is introduced to create a single source/single sink network. The time-expanded network can be treated as a static network and then any minimum cost static network flow algorithm can be applied to obtain the solution. The benefit of a time-expanded network is that it facilitates solving the flow over time problems by static flow computations. On the other hand, time-expanded networks are huge in practice and the size of the network increases linearly with the given time horizon T, and therefore, exponentially in the input size. This makes the problem difficult to solve and is proved to be a pseudo-polynomial problem in Hoppe and Tardos [1995].

A number of dynamic network flow models were discussed by Tjandra [2003] in which the objective of the models was to push for maximum flow in minimum time. Given the total number of evacuees, the capacity of the road links and the shelter destinations, the objective is typically to minimize the time for sending all the supply from source to destination for the underlying evacuation network. A mixed integer dynamic network flow model under the *SO* principle of traffic flow is used. Other variants of this problem can be obtained by changing the objective function to maximizing flow out of the network (maximum dynamic flow problem) during a given time horizon T, maximizing flow out of the network for any smaller time horizons $T' \in T$ (universal maximum flow problem) and minimizing the time horizon to clear the network (quickest flow problem). Since the problem is pseudo-polynomial because of the large network size of a time-expanded network, a number of solution techniques have been proposed. Hoppe and Tardos [1995] came up with a first polynomial time algorithm for the quickest transshipment problem and provided an integral optimum flow that can send exactly the right amount of flow out of each source and into each sink in the minimum overall time.

Heuristic algorithms have been proposed to come up with routes and schedules for the evacuation. One such algorithm is Capacity Constrained Route Planning (CCRP) proposed by Lu et al. [2005]. This algorithm divides evacuees into multiple groups and assigns a route and time schedule to each group. The CCRP algorithm employs a shortest path algorithm to find the shortest route from source nodes to destination nodes and assigns the evacuation schedule based on the available capacity on the route. The solution of the CCRP algorithm is within ten percent of the optimal evacuation time in all test cases. Another heuristic approach by Lim et al. [2012] used Evacuation Scheduling Algorithm (ESA) for the capacity constrained network flow optimization. ESA utilizes *Dijkstra's algorithm* for finding the evacuation paths and a greedy algorithm for finding the maximum flow of each path and schedule to execute the flow for each time interval.

In general, the mathematical models and the solution methods discussed in this section are only useful for finding a lower bound of evacuation time and the corresponding routes and schedules. A closer look of the transportation network (such as intersections), the dynamics of the evacuation (such as contraflow decisions, priority), and realistic assumptions (i.e., occurrences of incidents, congestion) would present a better picture and result in a realistic evacuation plan. Researchers and practitioners, therefore, consider the above factors carefully and incorporate them in the evacuation planning model. In the next two sections, we discuss the models for decision making in evacuation that takes these factors into account.

2.2 Efficient use of the critical roadway segments

As transportation infrastructure is a limited resource both in terms of directional accessibility and capacity, it is worth pursuing a routing plan that makes optimal use of this infrastructure. Lane-based routing and contraflow of traffic are two such traffic engineering tools that can be very effective for routing the evacuating traffic during a catastrophic event. Apart from these two, decisions corresponding to the location of shelters is also very important so that the traffic is not biased towards a particular shelter location and does not saturate a specific route. Cova and Johnson [2003] first modeled the lane based evacuation routing problem (ERP) as an integer extension of the min-cost flow problem. The primary objective of the model is to route vehicles to their closest evacuation zone exit. A secondary objective is to minimize the number of intersection merging-conflicts. Furthermore, the model prevents intersection crossing-conflicts.

Contraflow is emerging as an important and widely used tool to improve evacuation traffic capacity. Unlike normal traffic conditions where traffic is in both directions, evacuation events result in a traffic that is directed to a single direction. Contraflow is defined as the reversal of traffic flow in one or more of the inbound lanes of a divided highway for use in the outbound direction, with the goal of increasing capacity. The increased capacity that contraflow provides can substantially reduce clearance times. From the optimization perspective, research on contraflow concentrates on deciding the network configuration, i.e., coming up with the optimum network structure with lane directions that will result in minimum evacuation time. Heuristic solution approach was proposed by Kim and Shekhar [2005] to come up with a network configuration for contraflow. The location of shelters in a region threatened by a hurricane can greatly influence the highway network clearance time, i.e. the time needed by evacuees to escape from origin points to safe areas. Sherali et al. [1991] developed and solved a location-allocation model for determining a set of viable shelter locations from among potential alternatives, and accordingly, simultaneously prescribing a traffic diversion strategy in order to minimize the total evacuation time of automobiles from designated origins to the shelters under some emergency conditions. But the problem of finding minimum evacuation paths to address limited resource while still having the same throughput from the network has never been addressed in literature.

2.3 Stochastic Models

The routing models do not properly capture the dynamic nature of transport risk factors at the tactical level (e.g., traffic conditions, population density, and weather conditions). Moreover, most of these risk factors cannot be known *a priori* with certainty. They are both time-dependent and stochastic in nature; i.e., they are random variables with probability distribution functions that vary with time. There are numerous sources of uncertainty during evacuation and most are not easy to quantify or control. Factors such as severity of the disaster, human behavior during the evacuation, and the impacts of disasters on infrastructure are beyond our control. There are two major factors that affect evacuation planning that are being studied by the research community: the uncertain demand levels at the impact nodes and the degrading capacity of the road links during disasters. These unexpected changes in evacuation demand levels and capacity may result in significant differences in terms of the predictions of a model. It is therefore advisable to explicitly consider such uncertainties during the planning phase. This dissertation addresses both capacity uncertainty of the arcs and the uncertainty of the demand to come up with a reliable evacuation plan.

Developing *a priori* path sets for evacuation requires estimation of demand. The anticipated demand may deviate significantly from the actual number of people evacuating. If the solution consists of a set of paths, and more demand appears than was anticipated, there will be an insufficient number of paths to assign. Because the realized demand differs from the predicted demand, new paths must sometimes be calculated by sub-optimal means. Estimating a static capacity value for the links is equally difficult. As pointed out by numerous works on transportation network reliability (Chen et al. [2002], Lo and Tung [2003]), the capacity degrades as the number of vehicles on the link increases. This network has a higher probability of encountering a catastrophic event, such as extreme congestion or perhaps gridlock.

Various demand loading models have been proposed based on different regions and behavior of evacuating people. S-curve, Rayleigh distribution and sequential logit model are the widely accepted evacuation demand models. Yazici and Ozbay [2010] pointed out the uncertainty in the estimation process from the demand loading models. In his work he found that the variations in demand curve or the level of demand significantly impacted the clearance time and proposed a probabilistic model to account for uncertainties in road capacities and demand origination during evacuation. Traffic congestion occurs due to the uncertain demand and capacity realization during emergency evacuation of a geographic region and is overlooked in the evacuation planning literature. Not accounting for congestion probability at bottlenecks may not be a correct approach. For an illustration, consider the following real life example from a study by Litman [2006]: During Hurricane Rita the state's highway system in Houston became gridlocked and average travel time to Dallas were 24-36 hours, travel times to Austin were 12-18 hours and travel times to San Antonio were 10-16 hours, depending on the point of departure in Houston.

It becomes necessary to design approaches that account for demand and capacity uncertainty and develop more robust solutions that are less likely to fail under these extreme events and potentially reduce the variance of future costs. Robust optimization (RO) and chance constrained programming (CCP) are used to account for parameter uncertainty in cases when a mathematical program is formulated. The models are developed for dynamic traffic assignment (DTA) with the underlying principle of cell transmission modeling (CTM) introduced by Daganzo [1994]. The main advantage of using a stochastic programming technique is that they introduce reliability to the model.

Research focusing on uncertainty during evacuation has recently been addressed within chance constrained programming and robust optimization framework. Often, it is impossible to know the exact probability distribution of the number of evacuees. Yao et al. [2009] and Chung, Yao, Xie and Thorsen [2011] used robust optimization framework to address demand uncertainty and came up with a robust model in which no infeasibilities were allowed. Robust optimal solution can be interpreted as the solution being feasible for any realization of the uncertain data and achieving best worst case objective value. Choi et al. [1988] first addressed the building evacuation problem with arc capacity as a function of flow in incident arcs. Chance constraint programming for the traffic assignment problem is analyzed by Travis Waller and Ziliaskopoulos [2006] and the results are obtained assuming an uniform distribution for the traffic demand. Ukkusuri and Waller [2008] proposed a two stage stochastic programming with recourse model to account for demand uncertainty. They showed that not accounting for demand uncertainty explicitly provides sub-optimal solution. All these models assumed *a priori* knowledge of underlying distribution. Miller-Hooks and Sorrel [2008] proposed a noisy genetic algorithm to find the maximum expected number of evacuees who can successfully evacuate within a given egress time considering variable time and roadway capacity with known distribution functions. Stepanov and Smith [2009] has approached the stochastic evacuation as a queuing model to avoid congestion. None of the work in the evacuation literature, however, considers capacity in the context of traffic breakdown and model the problem with objective of minimizing the probability of congestion. Minimizing clearance time which is central to all the models is very much dependent on the hypothesis of fixed transit time on the arcs and the calculations can be misleading in case of the traffic jam buildup. Therefore, our study aims to model the mass evacuation with stochastic arc capacity having an objective of minimizing the network congestion.

To account for unknown distribution, Ng and Waller [2010] came up with probability bounds on travel time reliability and gave probabilistic guarantees on the evacuation plan considering uncertainty in the number of evacuees and arc capacities. Ng and Waller [2011] extend their work on capacity uncertainty by considering symmetric probability distributions for the random capacity and provided the reliability bound for a stochastic user equilibrium model. Recently, distributionally robust chance constrained approach was applied in dynamic traffic assignment (DTA) by Chung, Yao and Zhang [2011] where they used the moment information to come up with a deterministic estimate of the problem. Ben-Tal et al. [2011] applied affinely adjustable robust counterpart (AARC) method for finding the robust solution of the problem and showed improvement over the deterministic and sampling based approach.

Stochastic optimization without the knowledge of underlying distribution is recently being researched by a number of researchers. Calafiore and Ghaoui [2006] proposed a distributionally robust chance constrained method and showed that a chance constraint is second-order cone representable based on moment, support or symmetric information of the uncertainty. More generally, they showed that for $\epsilon \leq 0.5$ individual chance constraints can be converted to second-order cone constraints whenever the random vector $\tilde{\xi}$ is governed by a radial distribution. Nemirovski and Shapiro [2007] developed Bernstein approximation of the chance constrained problem which is convex and efficiently solvable and provide less conservative approximation of a chance constraint. Ben-Tal et al. [2010] proposed a soft robust optimization framework for robust optimization that relaxes the standard notion of robustness by allowing the decision maker to vary the protection level in a smooth way across the uncertainty set. Recently, Calafiore and Campi [2005], Erdoğan and Iyengar [2006] and Luedtke and Ahmed [2008] have proposed to replace the chance constraint (1.6) by a point-wise constraint that must hold at a finite number of sample points drawn randomly from the distribution $\mathbb{P}$. The advantage of this Monte Carlo approach is that no structural assumptions about $\mathbb{P}$ are needed and that the resulting approximate problem is convex. However, the drawback of such sampling based methods is that they may be computationally prohibitive to solve large problems or to solve problems for which a small violation probability ϵ is required.

2.4 Traffic Assignment Models

The ultimate aim of traffic flow is to create and implement a model which would enable vehicles to reach their destination in the shortest possible time using the maximum roadway capacity. Traffic routing and scheduling problems usually use either Wardrop's user equilibrium *(UE)* or system optimum *(SO)* traffic flow principles proposed by Wardrop [1952] to come up with the clearance time in current evacuation literatures. *SO* principle is used by the network operator trying to minimize the network-wide travel time and is based on the assumption that routes of the vehicles and traffic flow on the routes are controlled by the system. *UE* principle reflects the wish of the drivers to reach their destinations as soon as possible. When the congestion occurs on highway, it will extend the delay time in traveling through the highway and create a longer travel time. Under the user optimum assumption, the users would choose to wait until the travel time using a certain freeway is equal to the travel time using city streets, and hence equilibrium is reached.

During evacuation, how the situation will progress is uncertain and traffic breakdown occurs in the network with some probability which is not taken into account by the network travel cost optimization principles proposed by Wardrop. In this dissertation, we use network breakdown minimization *(BM)* principle proposed by Kerner [2011] to model capacity uncertainty. Rather than an explicit minimization of travel time that is the objective of *SO* and *UE*, the *BM* principle minimizes the probability of the occurrence of traffic congestion in a traffic network. Under a great enough traffic demand, the application of the *BM* principle should lead to implicit minimization of travel time in the network. Aim of the model is thus to assign link flow rates that minimizes the probability of traffic breakdown in a network during a given observation time.

2.5 Summary

Evacuation planning problem has been approached using static and dynamic models and substantial literature are published with different modeling versions of this complex planning problem. Research also targets to efficiently utilize the limited infrastructure resource but the objective of using the least number of evacuation paths has eluded the eyes of the research community involved in evacuation planning optimization. To fill this gap, a bi-objective dynamic network flow model is formulated to find the least number of evacuation paths for complete evacuation within the minimum clearance time. Further, there are very limited works on the network evaluation and design under uncertain conditions. The congestion minimization problem studied in Chapter 4 is the first attempt in literature to model the problem with the objective of congestion minimization. Chapter 5 would further contribute to this important area by coming up with modeling approach for demand uncertainty with an emphasis on distributional robustness of the evacuation plan to uncertain demand.

Chapter 3 Using Least Paths for Evacuation in Minimum Time

Motivation for this chapter is to provide the emergency managers with a plan where a complete evacuation can be executed in least possible time and using the minimum number of evacuation paths. A bi-objective arc-based formulation is done for this optimization problem. The formulated model is non-linear mixed integer problem and finding an optimum solution for the model is intractable for even a moderate sized network. Therefore, a three phase solution method is proposed for this problem by decomposing the original model into three separate sub-models. The solutions of these models provide a lower bound on clearance time for complete evacuation, a solution pool of feasible paths between all origin-destination (O-D) pairs and the minimum number of paths required for evacuation in least possible time along with the starting schedules on the selected paths assuming a variable flow rate on the paths at each time interval. The proposed models are mixed integer linear problems and formulation is done for *System Optimum (SO)* traffic flow principle where the emphasis is on complete network evacuation in minimum possible clearance time without any preset priority.

For a situation when the emergency management has only limited number of resources to allocate and, therefore, they want to limit the number of paths to be used for evacuation, we provide solutions for the required clearance time and the corresponding evacuation traffic flow and schedule that is channelized through those limited number of paths. The proposed approach is able to provide emergency personnels a complete schedule and route guidance for all the major and minor intersections in a large evacuation network that can be imposed on the evacuees to safely escape from the affected regions.

3.1 Problem formulation

A dynamic network flow model has been used to mathematically represent traffic flow evolution in an evacuation network for the proposed optimization model. A dynamic network can be visualized as a static network with an additional dimension representing time, i.e., the static network is repeated for each discrete slice of time. Traffic assignment on such time-expanded networks relies upon a more aggregate representation of traffic as a series of flows that attempts to match the demand for road space with the capacity of the highway system's links and intersections at various time.

Consider a directed network $\mathcal{D} = (\mathscr{N}, \mathscr{A})$ consisting a set of nodes $\mathscr{N}$ and a set of arcs $\mathscr{A}$. For each arc $a \in \mathscr{A}$ also expressed with a pair of tail and head, i.e., (i, j), define t_{ij} as the arc transit time and $\mathscr{U}_{ij}$ as the arc capacity. Nodes in the network are categorized into source nodes ($\mathscr{N}_s$), intermediate nodes, and destination nodes ($\mathscr{N}_d$). Let $\mathcal{S}_i$ be the number of evacuees at source node $i \in \mathscr{N}_s$ and $\mathscr{C}_j$ be the capacity of destination node $j \in \mathscr{N}_d$. We assume that there are T time periods $\{0, 1, \cdots, T-1\}$ to complete transportation of evacuees from the source nodes to the destination nodes.

The optimization model is designed for minimization of objective function which is a sum of scaled value of clearance time and path counts for the number of paths to be used for evacuation. We name this model as minimum time least path (MTLP) model. In this model, for any path $p \in \mathscr{P}$, source node of path is denoted by $\mathbb{O}_p$ and the sink node of path by $\mathbb{D}_p$. There are three decision variables in MTLP model:

$f_{pt} \in \mathbb{Z}^+$: number of vehicles flowing on path $p \in \mathscr{P}$ at any discrete time $t \in T$,

$$y_{i,j,p} = \begin{cases} 1, & \text{if path } p \text{ uses arc } (i,j); \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}, \forall i \in \mathscr{N}, \forall j \in \mathscr{A}(i).$$

$$w_p = \begin{cases} 1, & \text{if the path } p \text{ is selected;} \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}.$$

The formulation of the proposed model is similar to the arc-based model (as seen in Lim and Baharnemati [2011]). The objective function (3.1) is a linear combination of two separate objectives, i.e., $z = \kappa \cdot z_a + z_b$. In the objective function, optimal value for the objective $z_a^* = min\{\sum_{p\in\mathscr{P}} w_p|\text{Constraints}\}$ corresponds to least number of selected paths w_p required for complete evacuation. Optimal $z_b^* = min\{\sum_t \sum_p t \cdot f_{pt}|\text{Constraints}\}$ corresponds to minimum total time required for complete evacuation. Since z_a and z_b are in different scales, a constant κ is multiplied with z_a to bring it to a scale similar to z_b.

$$\text{Minimize} \quad \kappa \sum_{p\in\mathscr{P}} w_p + \sum_t \sum_p t \cdot f_{pt} \tag{3.1}$$

$$\text{Subject to:} \quad \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{jip} = 1, \quad \forall p \in \mathscr{P}, i \in \mathscr{N}, i = \mathbb{O}_p, \tag{3.2}$$

$$\sum_{j|(i,j)\in\mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{jip} = 0, \quad \forall p \in \mathscr{P}, i \in \mathscr{N}, i \neq \mathbb{O}_p, i \neq \mathbb{D}_p, \tag{3.3}$$

$$\sum_{j|(i,j)\in\mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{jip} = -1, \quad \forall p \in \mathscr{P}, \forall i \in \mathscr{N}, i = \mathbb{D}_p, \tag{3.4}$$

$$\sum_{p\in\mathscr{P}} f_{pt} \cdot y_{ijp} \leq \mathscr{U}_{ij}, \quad \forall (ij) \in \mathscr{A}, \quad t \in T, \tag{3.5}$$

$$\sum_i \sum_{t\in T} f_{it} = \mathcal{S}_i, \quad \forall i \in \mathscr{N}_s, \tag{3.6}$$

$$\sum_j \sum_{t\in T} f_{it} \leq \mathscr{C}_j, \quad \forall j \in \mathscr{N}_d, \tag{3.7}$$

$$\sum_{t\in T} f_{pt} \leq M_p \cdot w_p, \quad \forall p \in \mathscr{P}, \tag{3.8}$$

$$f_{pt} \in \mathbb{Z}^+, w_p, y_{ijp} \in \{0,1\} \quad \forall p \in \mathscr{P}, \quad \forall t \in T, \quad \forall (ij) \in \mathscr{A}. \tag{3.9}$$

Path generation is done using network flow balance equations as specified by constraints (3.2)-(3.4). These set of equations are repeated for each path originating from a particular source node and ending into corresponding sink node and thus deciding the arcs to be included into the set of optimal paths. Constraint (3.5) is the capacity constraint on arc (i, j) at each time t. This constraint limits the total flow using all paths $p \in \mathscr{P}$ at any particular time t on any arc $(i, j) \in \mathscr{A}$ to the maximum capacity of that arc. Using constraint (3.6), we balance the total outgoing flow from source node $i \in \mathscr{N}_s$ over the time horizon $t \in T$ to the original supply of the source node. Similarly, constraint (3.7) limits the flow reaching at the sink nodes below or equal to the maximum holding capacity of the destination node $j \in \mathscr{N}_d$. Constraint (3.8) relates the flow on any path p over all time $t \in T$ with the path selection variable w_p. Parameter M_p is the limiting value of the flow on the selected path and is set to the original number of supply at the origin.

Observe that constraint (3.5) is quadratic making the model non-linear. To make the model linear, Reformulation-Linearization Technique (RLT) by Sherali and Tuncbilek [1992] is applied. Linearization of the model provides an advantage of using the commercially available linear optimization solvers for the problem. RLT works by introducing a new variable $x_{ijpt} = f_{pt} \cdot y_{ijp}$. Modification of the quadratic constraint (3.5) is done by replacing the 2^{nd} order non-linearity with the new variable x_{ijpt} and also adding the bounds corresponding to the new variable in the earlier formulation to come up with the new set of constraints (3.10) - (3.12).

$$\sum_{p \in \mathscr{P}} x_{ijpt} \leq \mathscr{U}_{ij}, \qquad \forall (ij) \in \mathscr{A}, \quad t \in T, \tag{3.10}$$

$$0 \leq x_{ijpt} \leq f_{pt}, \tag{3.11}$$

$$f_{pt} - M(1 - y_{ijp}) \leq x_{ijpt} \leq M \cdot y_{ijp}. \tag{3.12}$$

Variable x_{ijpt} can take a value of either zero or it can take a value of positive integer value f_{pt}. This arc-based linear model can be solved to achieve the multi-objective result. Linearization of the model required the introduction of new variables as well as a new set of constraints which makes the model very large and consequently very difficult to solve. Moreover, there is an inherent difficulty of choosing an appropriate weight κ applied to the objective z_a. The model gives an optimal solution for a very small network but it is unable to scale up for even a medium sized network. In the next section, we present a solution approach based on model decomposition that can achieve the desired multiple objectives.

3.2 Three Phase MTLP

The proposed solution approach decompose the model into three separate sub-models. These sub-models are separately solved to achieve the bi-objectives and is referred as three phase MTLP hereafter in the chapter. Decomposition is done to first find the lower bound on clearance time based on an arc-based model. Path based model is then used to find evacuation paths and the corresponding flow and schedules. To make the model description more modular, this section is divided into three different subsections each discussing their corresponding mathematical model and solution method.

3.2.1 Generation of Lower Bound on Clearance Time

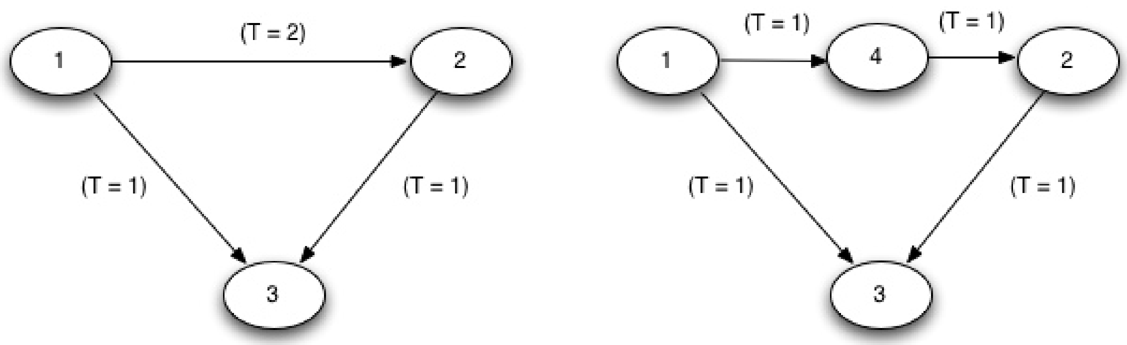
To achieve the SO objective of pushing the maximum flow towards the destination in minimum possible time, an arc-based formulation for the minimum cost dynamic network flow problem is proposed. Two integer decision variables y_{ij}^t and $x_i^t \in \mathbb{Z}^+$ are introduced for the model which we name as MET model. Integer variable x_i^t denotes the number of vehicles present at node i at ant time t and integer variable y_{ij}^t denotes the number of vehicles leaving node i towards node j at time t. The objective of *SO* dynamic network flow evacuation problem is to minimize the total travel time experienced collectively by all the users in the system. Therefore, the choice of destination node is not at the discretion of the individual vehicle but is decided by the model that results in optimum evacuation for the overall system.

For a dynamic network flow model, the travel time experienced by users in the network is equivalent to the difference between the arrival time at the destination and departure times from the source for every unit of flow within the network. Since the departure times are known and constant, they can be dropped and the arrival times can be considered as an equivalent travel time that has to be minimized. Arrival times can be determined when the flow exits the network. In order to minimize the total travel time in the network, we assign an uniformly increasing cost t to the flow expression departing the network. This will assign an increasingly higher cost to the delayed flow terminating into the destination. The objective of reducing the total travel time in the network can, therefore, be expressed as a product of cost function and flow function summed over all time for the flow exiting the network:

$$\sum_{t \in T} \sum_{(i,j) \in \mathscr{A}(\mathscr{N}_d)} t \cdot y_{ij}^{t}. \tag{3.13}$$

The network is modified such that nodes are separated by unit transit time and thus allowing to capture the precise instance until there is flow. This is achieved by introducing dummy nodes between two adjacent nodes if the arc transit time between them is more than unity, i.e., $t_{ij} > 1$. Arc capacity is kept same as the parent arc for the newly introduced arcs. The above modification is better illustrated by a sample network $\mathscr{G}_O = (\mathscr{N}_O, \mathscr{A}_O)$ as shown in Figure 3.1a. The boxed units denote the transit time on arcs and is greater than one for the arc from node 1 to node 2. The sample network $\mathscr{G}_O$ is modified by introducing a dummy node with unit arc transit time between the adjacent nodes as shown in Figure 3.1b. This results in a new graph

$\mathscr{G}_M = (\mathscr{N}_M, \mathscr{A}_M)$ with nodes separated by unit transit time. Finally, another dummy node '$\mathscr{N}_d^+$' named as "Super-sink" is added to the modified graph $\mathscr{G}_M$. Super-sink '$\mathscr{N}_d^+$' is connected with all destination nodes and is set to have infinite capacity ($\mathscr{C}_{\mathscr{N}_d^+} = \infty$) and arcs joining the sink nodes to the super-sink node allow infinite flow in zero transit time thus not limiting the super-sink node with any constraints.



(a) Original Graph: $\mathscr{G}_O = (\mathscr{N}_O, \mathscr{A}_O)$ (b) Modified Graph: $\mathscr{G}_M = (\mathscr{N}_M, \mathscr{A}_M)$

Figure 3.1: Graph modification

As in any network model, the movements of vehicles between nodes is defined by flow propagation and flow conservation equations (4.2) - (4.4). These relations decide respectively the flows y_{ij}^t between two nodes based on upstream/downstream traffic flow on the arcs and depict the evolution of the node status (i.e., the number of vehicles in each node x_i^t) over time. Considering a time-expanded network, y_{ij}^t can be visualized as the flow on transition arcs and x_i^t as the flow on holdover arcs associated only with the source nodes. Constraint (4.5) states that the total incoming flow into the super-sink node should be equal to the total supply at the start of the analysis period. The implication of this constraint is that it does not allow any withholding at the impact nodes and thereby pushing for the complete evacuation of the network. It pushes the flow towards the sink nodes which are connected to the super-sink node and thus result in flow propagation in the network. The total amount of flow, however, is determined by the objective function (4.1). Constraint (4.6) specifies that the total incoming flow into the set of sink nodes $i \in \mathscr{N}_d$ should not exceed the capacity of the sink nodes. A limiting constraint on the maximum flow possible on any arc in the arc set $(i,j) \in \mathscr{A}$ at any time $t \in T$ is expressed in constraint (4.7). Constraint (4.8) specifies the model to push for zero vehicles that are left behind at the end of the analysis period. Constraint (4.9) limits the node capacity x_i^t to its maximum capacity $\mathscr{C}_i$. It should be noted that the initial assignment period T should be high such that all traffic assigned in the network exits the network, otherwise they would be left behind and problem will not meet the constraints.

$$\text{Minimize:} \quad \sum_{(i,j)\in\mathscr{A}(\mathscr{N}_d)} \sum_{t\in T} t \cdot y_{ij}^t \tag{3.14}$$

$$\text{Subject to:} \quad x_i^0 + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^0 = \mathcal{S}_i \qquad \forall i \in \mathscr{N}, \tag{3.15}$$

$$x_i^t - x_i^{t-1} + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^t - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} = 0 \quad \forall t \in T \setminus \{0\}, \quad \forall i \in \mathscr{N}, \tag{3.16}$$

$$x_i^t - x_i^{t-1} - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^t = 0 \qquad \forall t \in T, \quad i = \mathscr{N}_d^+, \tag{3.17}$$

$$\sum_{(j,i)\in\mathscr{A}^{-1}(\mathscr{N}_d^+)} \sum_{t\in T} y_{ji}^t = \sum_{i\in\mathscr{N}_s} \mathcal{S}_i, \tag{3.18}$$

$$\sum_{t\in T\setminus\{0\}} \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} \leq \mathscr{C}_i \qquad \forall i \in \mathscr{N}_d, \tag{3.19}$$

$$y_{ij}^t \leq \mathscr{U}_{ij} \qquad \forall t \in T, \quad \forall (i,j) \in \mathscr{A}, \tag{3.20}$$

$$x_i^{|T|-1} = 0 \qquad \forall i \in \mathscr{N}, \tag{3.21}$$

$$0 \leq x_i^t \leq \mathscr{C}_i \qquad \forall t \in T, \quad \forall i \in \mathscr{N}, \tag{3.22}$$

$$y_{ij}^t \in \mathbb{Z}^+, x_i^t \in \mathbb{Z}^+. \tag{3.23}$$

Theorem 1. *The optimal solution of MET gives a lower bound for the minimum clearance time of the MTLP model, i.e.,* $z^*_{MET} \leq z^*_b$.

Proof: *MET model is a relaxation of MTLP model. If we remove from the MTLP model, the path generation constraints as well as constraint (8) for path selection, it reduces to the MET model. Therefore,* $z^*_{MET} \leq z^*_b$.

Using the above formulation, we exploit the property of unit travel time between nodes in the graph and formulate the problem as a SO minimum cost network flow problem. Separation of the nodes by unit time allows the model evolution for flow that can be determined at each time unit. Solution of the proposed MET model is used to calculate the lower bound on clearance time T required for complete evacuation.

3.2.2 Paths Set Generation

The main objective of the paths set generation model (PG) is to find a set of possible paths from a source node to a sink node. In an evacuation scenario, emergency personnels generally prefer to use the paths that are prescribed for evacuation. Situations might arise when the prescribed path is not usable or would not be able to handle the traffic surge during emergency to evacuate within safe time. Creating a pool of usable paths for evacuation would provide a viable alternative to emergency managers where they can set a priority for the paths to be used. Generation of this set of possible paths is achieved using the solution pool feature of CPLEX. The solution pool feature generates and stores multiple solutions in addition to the optimal solution to our model for path set generation.

The paths set generation model is expressed using same notations as used in earlier models. PG model is a shortest path problem that is used for finding the paths between all O-D pairs for a static network graph $\mathscr{G} = (\mathscr{N}, \mathscr{A})$. Binary decision variable y_{ij} in the model takes a value of 1 if the arc is present in the shortest path between node i and node j and takes a value of 0 if the arc is not present. Using this model, we aim to find the shortest path from source node $i \in \mathscr{N}_s$ to super sink node $\mathscr{N}_d^+$. Therefore, model objective (3.24) is to minimize the total transit time on arcs if that arc is present in the path from origin to destination. Constraint (3.25) ensures that we leave the origin by selecting an arc from the source node. Constraint (3.26) is for intermediate nodes which ensures that if we enter the node then we must leave the node as well. Constraint (3.27) ensures that we reach the destination $\mathscr{N}_d^+$. Constraints (3.28) and (3.29) limit the total number of arcs going out and coming into a node to 1 and thus eliminating the generation of cycles in the solution pool.

$$\text{Minimize:} \quad \sum_{(i,j)\in\mathscr{A}} t_{ij} \cdot y_{ij}, \tag{3.24}$$

$$\text{Subject to:} \quad \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = 1, \qquad i \in \mathscr{N}_s; \tag{3.25}$$

$$\sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = 0, \qquad \forall i \in \mathscr{N} \setminus \{\mathscr{N}_s \cup \mathscr{N}_d^+\}; \tag{3.26}$$

$$\sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = -1, \qquad \forall i \in \mathscr{N}_d^+; \tag{3.27}$$

$$\sum_{i|(i,j)\in\mathscr{A}(i)} y_{ij} \leq 1 \qquad \forall i \in \mathscr{N}, \tag{3.28}$$

$$\sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} \leq 1 \qquad \forall i \in \mathscr{N}, \tag{3.29}$$

$$y_{ij} \in \{0, 1\}. \tag{3.30}$$

The solution pool feature determines an appropriate number of paths to be populated for the solution. A user defined set of paths are selected from this solution that is generated using a solution pool relative gap of α. This relative gap allows the paths to be generated that are within $100\alpha\%$ of the incumbent solution, i.e., paths are generated with travel times ranging from least time required between the O-D pair to $100\alpha\%$ of the least travel time. Solution pool can have duplicates which we omit and make a repository of unique paths that can be considered to be used for evacuation.

3.2.3 Path Selection and Flow Generation

The path selection and flow generation (PSFG) model is used to select the best set of paths from each source node and find the flow and schedule on those paths. PSFG model is a combinatorial problem of selecting the best paths from the solution pool obtained using PG model. It should be noted that the bound on the clearance time found using MET model did not use any prior path information. Maximum flow objective of MET model pushes for the flow on arcs to the maximum arc capacity at all times. Consequently, it might not be possible to assign flows on limited path set within given time T. Algorithm 1 describes the process that we use to come up with our results.

Algorithm 1 Flow Generation Algorithm

```
repeat
    SolutionStatus ← Solve PSFG : f_pt, Y_p, T
    if (SolutionStatus == Infeasible) then
        T++;
    end if
until (SolutionStatus == Feasible)
```

In the flow generation algorithm, clearance time T is initialized with the lower bound obtained from the MET model and the path pool from the PG model. If the solution to PSFG is feasible, i.e., paths are found in the pool that can empty the network within T, then flow and schedule on those paths are obtained from PSFG. If the solution is infeasible, we increase the clearance time T by one unit and feed this value to PSFG model. The process is repeated until the feasible solution is obtained. Using the conservative approach of increasing the clearance time by unity, we ensure that the total evacuation is completed within minimum time using the paths available in the solution pool.

Keeping other notations the same, we introduce a binary expression δ_{pa} which is set to 1 if the path p contains an arc $a \in \mathscr{A}$. There are two decision variables for this model.

$f_{pt} \in \mathbb{Z}^+$ representing the flow on path $p \in \mathscr{P}$ at any discrete time $t \in T$.

$$y_p = \begin{cases} 1, & \text{if the path } p \text{ is selected;} \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}.$$

Objective function (3.31) of the PSFG model minimizes the total number of paths selected in the solution. These paths are selected from the solution pool of the PG model that is provided as an input. Selection of the paths is based on the criteria which ensures that all the supply at source is exhausted within the given clearance time T. The objective will thus give priority to paths that have greater capacity and lower travel time.

$$\text{Minimize} \quad \sum_{p \in \mathscr{P}} y_p \tag{3.31}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} f_{pt} \cdot \delta_{pa} \leq \mathscr{U}_a \qquad \forall a \in \mathscr{A}, \quad t \in T, \tag{3.32}$$

$$\sum_{p|\mathbb{O}_p=i} \sum_{t \in T} f_{pt} = \mathcal{S}_i \qquad \forall i \in \mathscr{N}_s, \tag{3.33}$$

$$\sum_{p|\mathbb{D}_p=j} \sum_{t \in T} f_{pt} \leq \mathscr{C}_j \qquad \forall j \in \mathscr{N}_d, \tag{3.34}$$

$$\sum_{t} f_{pt} \leq M_p \cdot y_p \qquad \forall p \in \mathscr{P}, \tag{3.35}$$

$$f_{pt} \in \mathbb{Z}^+, y_p \in \{0, 1\} \qquad \forall p \in \mathscr{P}, \quad \forall t \in T. \tag{3.36}$$

Constraint (3.32) ensure that the sum of flows for all paths p on any arc $a \in \mathscr{A}$ during any interval of time t does not exceed the maximum capacity of that arc. Constraint (3.33) guarantees that the sum of flows on path originating from the nodes in $\mathscr{N}_s$ over all time is equal to the supply at that node. This constraint could also have been greater than equal to constraint but we designed this to be a tighter constraint making the MIP problem easier to solve. Constraint (3.34) ensures that the summation of flow on paths coming into the destination over all time do not exceed the capacity $\mathscr{C}_j$ of the destination nodes $\mathscr{N}_d$. Constraint (3.35) limits the sum of all flows on a selected paths $p \in \mathscr{P}$ over all time $t \in T$ to vector $M_p = \mathcal{S}_{i|\mathbb{O}_p}$, i.e., the maximum possible supply initially present at the origin of the path. We relax the integer constraint on variable f_{pt} without observing any consequences to the result. Solution of the model will result in variable flow on each paths and also a variable flow on the same path at different time interval. This is because we have a SO objective of assigning the maximum flow for 100% evacuation within minimum possible time.

Often times emergency managers are faced with the problem where they have only a limited option of paths to be selected from the set of available paths. Also, due to resource constraints, there might be a restriction on the number of paths to be used for outgoing traffic during evacuation. We introduce a new constraint (3.38) for limiting the paths selection from each source node to the assigned limited value μ_i. Instead of the minimization objective of the total number of paths selected, we design the model to have a maximum flow objective (3.37).

$$\text{Maximize} \quad \sum_{p \in \mathscr{P}} \sum_{t \in T} f_{pt} \tag{3.37}$$

$$\text{Subject to:} \quad \sum_{p|\mathbb{O}_p = i} Y_p \leq \mu_i \qquad \forall i \in \mathscr{N}_s. \tag{3.38}$$

Using this model we assign the flow by considering only a limited number of paths from each source node. This model gives the flexibility to the emergency managers for choosing a limited number of paths for evacuation. Note that the number of paths may not be enough to evacuate desired evacuees within the given clearance time T. To account for the limitation for the number of selected paths to μ_i, clearance time for complete evacuation has to be increased.

3.3 Computational Results

We first describe the numerical results of three phase MTLP on a small network and then use this solution approach for a large evacuation network of the Greater Houston area and Galveston County, Texas. All algorithms are implemented in a C++ environment. We use CPLEX 12.3 to solve the mathematical models in the algorithms. All experiments are done on a workstation with 3.07 GHz Intel Core i7 processor having 24 GB RAM and running Ubuntu 10.04.3.

Figure 3.2 is the test network being used to illustrate the solution approach. This is the same network used by Lim et.al. Lim and Baharnemati [2011]. The test network has three impact nodes ($\mathscr{N}_s = \{1, 2, 3\}$), five intersections, and two safe nodes ($\mathscr{N}_d = \{9, 10\}$). Each arc in the network is assigned a transit time and capacity. Number of evacuees in safe nodes 1, 2, and 3 are 350, 185, and 200, respectively. The capacity of both destination nodes 9 and 10 is 750. We first provide a solution for the evacuation test network using three phase MTLP (Section 3.2). For our model compatibility, the network is modified as explained in Section 3.2.1. We provide an initial large value for T to the model as an input which should be sufficient for a complete evacuation. Note that providing a large value of T does not affect solution quality or computation time in our model at this stage. The MET model (Section 3.2.1) is then solved for the modified network to find a lower bound on evacuation time to ensure 100% evacuation. The lower bound on clearance time obtained using the MET model for the test network is $T = 28$.

Generating a set of paths from each source node is done using the PG model (Section 3.2.2). The original test network with the super-sink node is used for the PG model. A solution pool relative gap of 1 is used which implies including the shortest path between the O-D pairs and all the paths within the relative solution gap of 100% from the optimal path. This is equivalent to including all the paths with

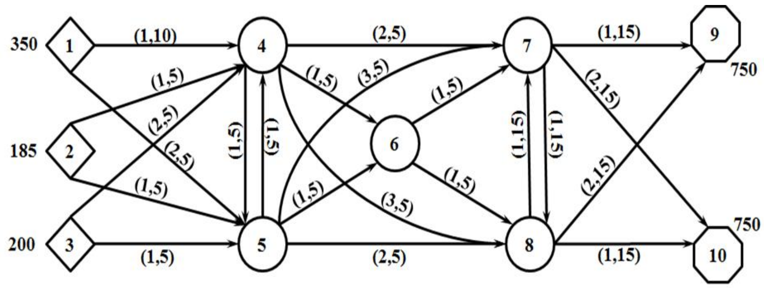


Figure 3.2: Evacuation Test Network

travel time within 10 units in the solution pool if the shortest route has a travel time of 5 units between the O-D pair. Using the solution pool method, we were able to obtain a set of 64 unique paths originating from each source node for the test network.

Decision on selecting the best path and assigning the flow and schedules for those paths is done using the flow generation algorithm (Section 3.2.3). The paths set and clearance time bound obtained earlier are fed as an input to the algorithm. PSFG is solved to find the best combinations of paths. In this test network, there is no combination of paths which is able to completely evacuate within the minimum clearance time of $T = 28$. Therefore, PSFG is solved again for time $T + 1 = 29$. The selected paths are now able to take out all the initial supply at the source nodes to the shelter or sink nodes taking minimum evacuation time of $T = 29$. In Table 3.1, we report all the selected paths, travel time between the O-D pair using the corresponding paths and the schedule of the vehicles on the selected paths. This result gives the least number of paths that is required for complete evacuation in the minimum possible time. A minimum of 8 paths are required for evacuation within a clearance time of $T = 29$ for the test network. Since the model is formulated for a variable flow rate, we observe a 0 flow on some paths during certain time intervals. A flow rate with value of 5 is obtained for all the paths whenever there is a flow present in the path.

The bi-objective MTLP model discussed in Section 2 did not result in optimal solution for the test network in Figure 3.2 when the model was run for 3 hours. We tested the bi-objective model on an even smaller network with only five nodes out of

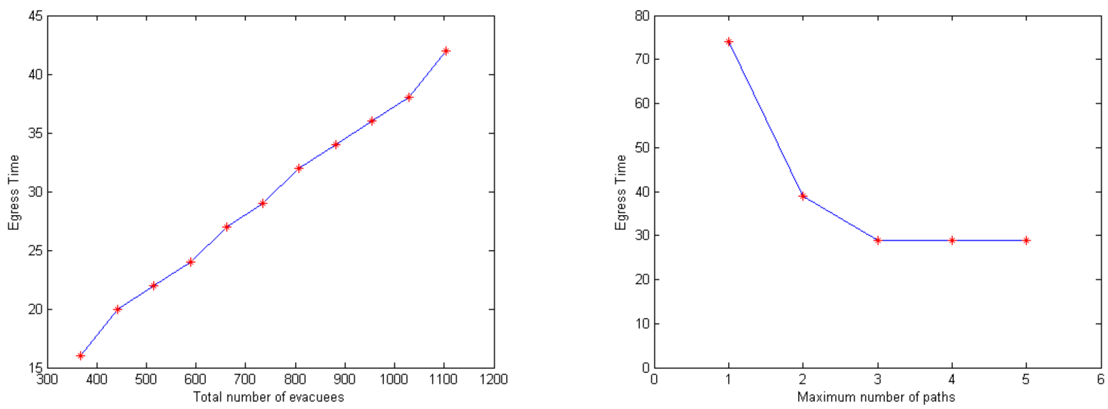
Table 3.1: Computational results for test network

Path	Travel Time	Evacuation Start Time
1-4-7-9	4	$0 - 24$
1-4-8-10	5	$0 - 16,18 - 23$
1-5-7-9	6	$0 - 22$
2-4-6-7-9	4	$0 - 24$
2-5–6-8-10	4	$0, 2, 3, 4, 7, 9 - 24$
3-4-5-6-8-10	6	$1,5,6,8 - 10,12 - 22$
3-4-8-10	6	14,17,20
3-5-8-10	4	$0 - 24$

which two were impact nodes, two safe nodes and a single intersection node. With a weight of $\kappa = 30$, we were able to get an equivalent solution for path and schedule of evacuation for both the models. The bi-objective model took a computation time of 0.483 seconds as opposed to 0.28 seconds by our proposed solution approach.

Sensitivity analysis for the variation in the initial number of evacuees was done for the test network. To find out the variation in the clearance time and the corresponding number of total paths used for the modified demand, the initial number of evacuees is varied for each source node using a step size of 10% variation. Sensitivity of the model to the demand variation is shown in Figure 3.3a. For this particular network, we observe a linear relationship of clearance time with the variation in demand at the source nodes. The number of paths used for the evacuation remains same with a value of eight for all demand scenarios for this network but this result can be different for other networks if the capacity of the arc is able to handle the reduced demand with less number of paths.

Consider a scenario where the number of evacuation paths are limited to a certain fixed value less than the minimum obtained in the results. We find that the clearance time increases during such cases. Clearance time of $T = 74$ is obtained when only 1 path is allowed from each source and $T = 39$ is obtained when 2 paths are allowed. For 3 or more paths, the clearance time obtained is equal to the minimum clearance time calculated for this particular network. Figure 3.3b shows the result for the variation in clearance time when the number of paths are limited to a certain value for each source node. As evident, the clearance time is higher when the number of paths is very low. Clearance time decreases with the increase in number of paths and approaches a constant value. This is achieved when the limit on the number of paths originating from each source node is equal to or greater than the minimum number of paths required for evacuation within the minimum calculated clearance time.



(a) Clearance time to demand variation

(b) Clearance time to path limitation

Figure 3.3: Sensitivity Analysis

For evacuation planning using a large network the evacuation network of Houston metropolitan with 42 nodes are selected of which there are total of 13 source nodes 4 destination nodes. Arcs going out from the source nodes and coming into the destination nodes are "uni-directional" and arcs connecting the intermediate nodes are bidirectional. There are totally 566,000 vehicles distributed among the source nodes to be evacuated. Network is sampled at $\tau = 30$ minutes interval, i.e., transit time which separates each pair of node apart are multiples of τ. Houston, TX, is the fourth largest city in US and is one of the most vulnerable metropolitan cities situated at the Gulf coast. Houstonians have witnessed many hurricanes and its population being subjected to evacuation multiple times. It is the largest city that have evacuated due to hurricanes. Demonstration of the evacuation planning for the Greater Houston area would be an appropriate example for large size networks (see Figure 5.1).

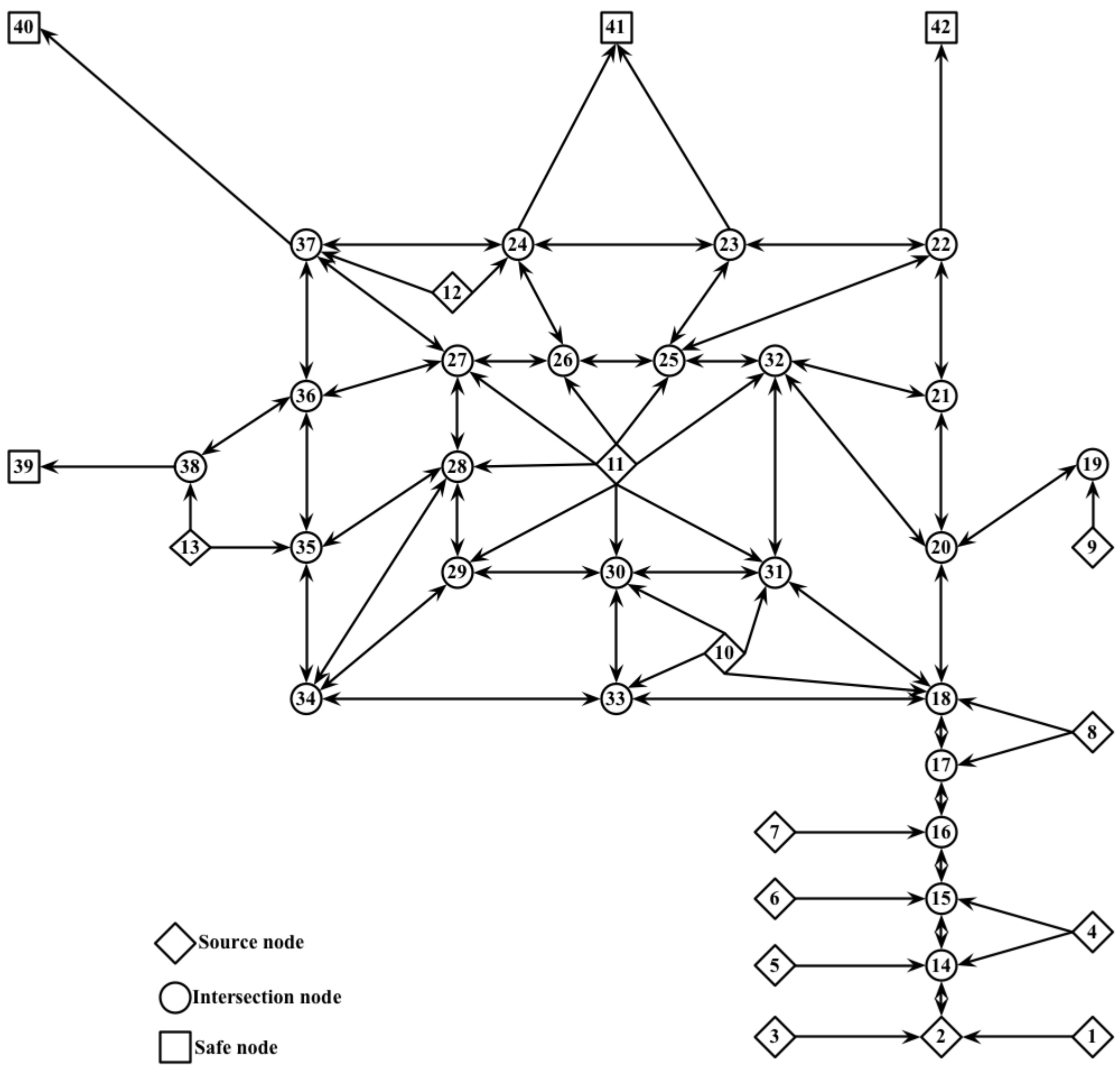


Figure 3.4: City of Houston Transportation Network

Using the three phase MTLP solution approach, we were able to find a solution for 100% evacuation using a total computation time of approximately 50 minutes. Lower bound on the egress time required for the evacuation was calculated to be 129 τ. This translates to a minimum of approximately 2 days and 17 hours required for evacuating a population of approximately 1.3 million based on the assumption that each vehicle occupancy is 2.3 persons as reported in a study by Southworth [1991]. A solution pool of a total of 1829 unique paths between all O-D pairs was generated using a relative gap of 100% from the optimal solution. PSFG algorithm was then solved to obtain the paths, their starting schedule and corresponding flow values. The algorithm resulted to a selection of a total of 16 paths from the O-D pairs that are able to completely evacuate the network within 129 τ. Complete evacuation plan for the transportation network of Houston metropolitan area can be found in Rungta et al. [2011]. Table 4.1 shows the paths selected between the O-D pairs along with their travel times and the total number of vehicles initially present at the source nodes.

Table 3.2: Computational results for Houston evacuation network

Source Node	Total Vehicles	Selected Path between O-D pair	Travel Time
1	1000	1 2 14 15 16 17 18 33 30 31 32 21 22 42	28
2	1000	2 14 15 16 17 18 31 32 20 21 22 42	26
3	1000	3 2 14 15 16 17 18 33 34 28 27 26 24 23 41	27
4	1000	4 15 16 17 18 31 32 25 26 24 23 41	24
5	1000	5 14 15 16 17 18 31 32 25 22 42	25
6	1000	6 15 16 17 18 20 32 25 22 42	24
7	35000	7 16 17 18 20 32 25 22 42	23
8	35000	8 17 18 20 21 22 42	21
9	35000	9 19 20 18 33 30 31 32 25 23 41	23
10	35000	10 18 20 21 22 42	20
11	140000	11 27 37 40 11 25 23 41	14 16
12	140000	12 37 40 12 24 41	13 15
13	140000	13 38 39 13 35 28 27 37 40	13 16

3.4 Conclusion

This chapter developed a model for finding the least number of evacuating routes to completely clear the network in minimum time. The presented model address the problem of limited resources required for executing the evacuation that has a direct effect on the clearance time and has never been addressed in the literature. The relationship between the clearance time and the number of evacuation paths is obtained by numerical experimentation and a sensitivity analysis of the solution was done for the variation in demand at origin. Numerical results show that the clearance time can't go below a particular value even when the number of evacuation paths are more than the minimum required paths. Finding the least evacuation paths, therefore, is a sensible alternative rather than loading the evacuation traffic on extra paths without observing any improvement in clearance time and engaging the limited resources.

The multi-objective formulation proved to be intractable even for a small sized network. To find the solution, a three phase solution approach was proposed that was tailored to find the multiple objectives. Numerical result for the evacuation network of Greater Houston region confirms that the solution approach is scalable to large networks. A natural extension of this work is to incorporate uncertainty in the model and study the impact of uncertainty on clearance time.

Chapter 4 Reliability Analysis under Capacity Uncertainty

Providing a reliable flow is important in the context of emergency evacuation. *A priori* analyses of envisioned evacuation paths for traffic reliability with high probability would guarantee that actual evacuation does not result in undesirable surprises. This chapter, therefore, lays emphasis on finding an evacuation plan considering variable arc capacity with known distribution function which would result in a free flowing traffic without any congestion. Our overall strategy to address capacity uncertainty and congestion minimization in the network evacuation problem is to use the capacity distribution function and thus find the traffic reliability estimate. The key contributions of this chapter are as follows: a) we model an optimization problem that minimizes the probability of congestion in a stochastic network setting; b) we find a relationship between the clearance time, number of evacuation paths and congestion probability; and c) our approach acts as a verification model for the evacuation plan and reports the reliability in terms of confidence level with which a congestion might occur in the network if the said plan is used.

The rest of the chapter is structured as follows. Section 4.1 introduces the modeling approach for uncertain arc capacity within the framework of chance constrained programming. We propose a model for finding the evacuation routes and traffic flow that will result in minimum congestion in the network. The stochastic model under mathematical optimization framework is described. Section 5.3 reports the solution approach and computational results. Finally, conclusions and future research directions are discussed in Section 4.3.

4.1 Evacuation Planning under Uncertain Arc Capacity

In this section, we first describe the deterministic minimum cost network flow problem for finding the minimum clearance time. Subsequently, we introduce the model with random capacity of arcs and find the minimum clearance time for a desired reliability level. Further, we propose a path based model for finding the path reliability and determining a reliable flow on evacuation paths to be used between each origin-destination (O-D) pair.

4.1.1 Deterministic Minimum Cost Flow Evacuation Problem

A time expanded network flow model has been used to mathematically represent traffic flow evolution in an evacuation network for this optimization model. The network consists of a graph with capacities and transit times associated with the arcs. Consider a directed static network $\mathcal{D} = (\mathscr{N}, \mathscr{A})$ with $\mathscr{N}$ and $\mathscr{A}$ as the set of nodes and arcs, respectively. The time expansion of $\mathcal{D}$ over a time horizon T defines the dynamic network $\mathcal{D}_T = (\mathscr{N}_T, \mathscr{A}_T)$ associated with $\mathcal{D}$ having holdover arcs and movement arcs. Holdover arcs are virtual road sections represented on a time expanded network whose content represents the number of vehicles still remaining at the source node $\mathscr{N}_c$ and the capacity of these arcs are equal to the capacity of the source nodes. Movement arcs represent the actual road link of a traffic network at different time interval and its content represents the movements of vehicles from one node to another. The flow on the movement arcs are limited by their maximum capacity $\mathscr{U}_{ij}$. It is this capacity of movement arcs that we consider as random in the stochastic model.

We denote as $\mathscr{T}$, the set of discrete time intervals, i.e., $\mathscr{T} = \{0, 1, \ldots, T-1\}$. The time expansion essentially is the replication of the static network $\mathcal{D}$ at each discrete unit of time in $\mathscr{T}$. Since there are multiple copies of sink nodes, a super sink node $\mathscr{N}_d^+$ is introduced to create a single sink network. In the dynamic network flow model, the flow variable y_{ij}^t is the number of vehicles that leave node i at time t and reach node j at time $t + \sigma_{ij}$ where σ_{ij} represents the transit time on arc (i, j). The primary goal of the model is to find the lower bound of the clearance time for the underlying network with a given initial supply of vehicles. Therefore, we wish the network to have nodes separated by unit transit time such that it is possible to capture the precise time till there is flow in the network. We modify the network such that $\sigma_{ij} = 1$ for all the arcs in the modified network by introducing dummy nodes between the nodes having travel time greater than 1. Flow variable x_i^t represents the number of vehicles that move on holdover arcs. Let $\mathscr{N}_s$ denote the set of destination nodes, $\mathscr{C}_j$ as the total capacity of the destination node j and $\mathcal{S}_i$ as the initial demand at source node i. Arc (i, j) is also alternatively represented as arc a in this paper. Shown below is the deterministic model for finding the minimum clearance time. This is a minimum cost flow model and we name the model as MET-D.

As in any network model, the movements of vehicles between nodes is defined by flow propagation and flow conservation equations (4.2) - (4.4). These relations decide respectively the flows y_{ij}^t between two nodes based on upstream/downstream traffic flow on the arcs and depict the evolution of the node status (i.e., the number of vehicles in each node x_i^t) over time. Constraint (4.5) states that the total incoming flow into the super-sink node $\mathscr{N}_d^+$ should be equal to the total supply at the start of the analysis period. The implication of this constraint is that it does not allow any withholding at the impact nodes and thereby pushing for the complete evacuation of the network. It pushes the flow towards the sink nodes which are connected to the super-sink node and thus result in flow propagation in the network. The total

$$\text{Minimize:} \quad \sum_{(i,j)\in\mathscr{A}(\mathscr{N}_s)} \sum_{t\in\mathscr{T}} t \cdot y_{ij}^t \qquad (MET-D) \tag{4.1}$$

$$\text{Subject to:} \quad x_i^0 + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^0 = \mathcal{S}_i \qquad \forall i \in \mathscr{N}, \tag{4.2}$$

$$x_i^t - x_i^{t-1} + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^t - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} = 0 \qquad \forall t \in \mathscr{T} \setminus \{0\}, \quad \forall i \in \mathscr{N}, \tag{4.3}$$

$$x_i^t - x_i^{t-1} - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^t = 0 \qquad \forall t \in \mathscr{T}, \quad i = \mathscr{N}_d^+, \tag{4.4}$$

$$\sum_{(j,i)\in\mathscr{A}^{-1}(\mathscr{N}_d^+)} \sum_{t\in\mathscr{T}} y_{ji}^t = \sum_{i\in\mathscr{N}_c} \mathcal{S}_i, \tag{4.5}$$

$$\sum_{t\in\mathscr{T}\setminus\{0\}} \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} \leq \mathscr{C}_i \qquad \forall i \in \mathscr{N}_s, \tag{4.6}$$

$$y_{ij}^t \leq \mathscr{U}_{ij} \qquad \forall t \in \mathscr{T}, \quad \forall (i,j) \in \mathscr{A}, \tag{4.7}$$

$$x_i^{|T|-1} = 0 \qquad \forall i \in \mathscr{N}, \tag{4.8}$$

$$0 \leq x_i^t \leq \mathscr{C}_i \qquad \forall t \in \mathscr{T}, \quad \forall i \in \mathscr{N}, \tag{4.9}$$

$$y_{ij}^t \in \mathbb{Z}^+, x_i^t \in \mathbb{Z}^+. \tag{4.10}$$

amount of flow, however, is determined by the objective function (4.1). Constraint (4.6) specifies that the total incoming flow into the set of sink nodes $i \in \mathscr{N}_s$ should not exceed the capacity of the sink nodes. A limiting constraint on the maximum flow possible on any arc in the arc set $(i,j) \in \mathscr{A}$ at any time $t \in \mathscr{T}$ is expressed in constraint (4.7). Constraint (4.8) specifies the model to push for zero vehicles that are left behind at the end of the analysis period. Constraint (4.9) limits the node capacity x_i^t to its maximum capacity $\mathscr{C}_i$. It should be noted that the initial assignment period T should be high such that all traffic assigned in the network exits the network, otherwise they would be left behind and problem will not meet the constraints.

In this model, a deterministic capacity estimate of the arcs is used to limit the flow on paths. This model results in a minimum clearance time estimate along with the flows on the arcs as per the deterministic capacity. Deterministic capacity as mentioned in HCM (2000) is the maximum sustainable hourly flow rate that can be achieved repeatedly during peak periods. But the demand volume that causes breakdown varies in real traffic flow and the flow rate observed during a breakdown depends on the behavior of drivers thus making the arc capacity a random variable $\widetilde{\mathscr{U}_{ij}}$. Congestion occurs at an arc (i, j) with a finite probability p when the optimal flow y_{ij} obtained from the deterministic model exceeds the realized capacity.

When the demand on an arc exceeds its capacity, or capacity decreases to a level less than demand, then congestion occurs resulting in a bottleneck. Therefore, the clearance time calculated using deterministic capacity may not be enough to evacuate using the selected evacuation paths and the corresponding flow rate. In such situations of stochastically degrading capacity, the clearance time calculated using the deterministic model would result in an infeasible evacuation plan. Given the consequences of the deterministic capacity, our next task is to model the problem as a probabilistic constrained program and to incorporate a reliability measure for the variable arc capacity of the links.

4.1.2 Chance Constrained Model

We consider a probabilistic programming approach to model the evacuation problem assuming that the probability distribution of the random arc capacity is known. Inability of the traffic flow on a road section to meet the capacity requirement of that arc at all times is modeled using chance constrained programming. The probability level is set to a value which ensures that the flow is assigned in such a way as to meet criteria most of the time. More specifically, we find the deterministic equivalent of the variable capacity that should be used in the model such that the probability with which the flow value might exceed the capacity is bounded within a pre-specified tolerance level. This can be thought of as determining of the *a priori* traffic flow on the arcs where the future capacity variations are already accounted for.

Referring to the deterministic model, the arc capacity constraint (4.7) is modified to ensure the feasibility of capacity constraint for each arc within a reliability level ε_{ij}. The modified constraint is of the form

$$\Pr\left(y_{ij}^{t} \leq \widetilde{\mathscr{U}_{ij}}\right) \geq \varepsilon_{i,j}, \quad \forall (i,j) \in \mathscr{A}. \tag{4.11}$$

Keeping other constraints of the MET-D model same, the model with the modified arc capacity constraint is termed as MET-S. Constraint (4.11) is the individual chance constraint equivalent of the deterministic constraint (4.7) with the desired probability level imposed individually on each constraint. Parameter $\varepsilon_{ij} \in (0, 1]$ is the desired reliability level and the value of ε is set such that the optimal solution to the approximation of the chance constraint is feasible to the probabilistic constrained programming (PCP) model. Capacity uncertainty is denoted by random variable $\widetilde{\mathscr{U}_{ij}}$ and we assume that the distribution function of the random capacity is known.

Consider the cumulative distribution function (CDF) for capacity in terms of probability when the traffic volume y_{ij} on the link exceeds its capacity $\mathscr{U}_{ij}$, i.e.,

$$F_{\mathscr{U}_{ij}}(y_{ij}) = p(\mathscr{U}_{ij} \leq y_{ij}). \tag{4.12}$$

This implies that the overload probability for a single bottleneck is equal to the CDF of capacity. The probability of no congestion, i.e., $p(\mathscr{U}_{ij} > y_{ij})$, can be expressed as the complementary event:

$$P_{\mathscr{U}_{ij}}(y_{ij}) = 1 - F_{\mathscr{U}_{ij}}(y_{ij}). \tag{4.13}$$

Consider Figure 4.1 for curve fitting reproduced here from the empirical study by

Brilon et al. [2005] done using analogy of lifetime data analysis. Considering traffic breakdown as a failure event to estimate the capacity $\mathscr{U}_a$ of an arc, he concluded that when the flow rate exceeds a certain threshold value $\mathcal{Q}_a$ then there is a finite probability with which traffic volume can exceed the capacity of the link and congestion can occur. We define this saturation flow rate as the capacity of the arc. After reaching a certain upper bound of flow rate $\mathcal{Q}_{a_{max}}$ for a particular link, this probability reaches unity and it is certain that congestion would occur in that link. From his work, CDF for link capacity was found to follow *Weibull* distribution, i.e.,

$$F_{\mathscr{U}_a}(\mathcal{Q}_a) = 1 - e^{-(\frac{\mathcal{Q}_a}{\beta_a})^{\alpha}}, \tag{4.14}$$

where α and β are respectively, shape parameter and scale parameter. Shape parameter α is typically having a value in range of $9 - 15$ for a three lane road. Weibull distribution is used in this research to model the uncertainty of link capacity with stochastic traffic volume $\mathcal{Q}_a$ used as a the link capacity constraint. Other discrete or continuous distributions can also be used that better approximates the random capacity of a roadway section. For example, Siu and Lo [2008] consider road capacity following a general uniform distribution in their study.

Probabilistic constrained models are usually solved using a deterministic approximation. The difficulty in solving such models arises mainly from the fact that the chance-constraint set (4.11) may not be convex (Nemirovski and Shapiro [2007]). Proving the convexity of the chance-constraint set (4.11) would thus simplify the optimization of the problem by suitable approximations of the chance-constrained function.

Corollary 1. *The feasible region of chance constraint* (4.11) *is convex.*

Proof: *Constraint* (4.11) *is of the form* $\Pr\{Ax \leq \tilde{b}\} \geq p$*, where,* A *is a deterministic coefficient matrix and* $p \in (0, 1]$ *is given. Variable capacity vector* $\tilde{b}$ *follows Weibull*

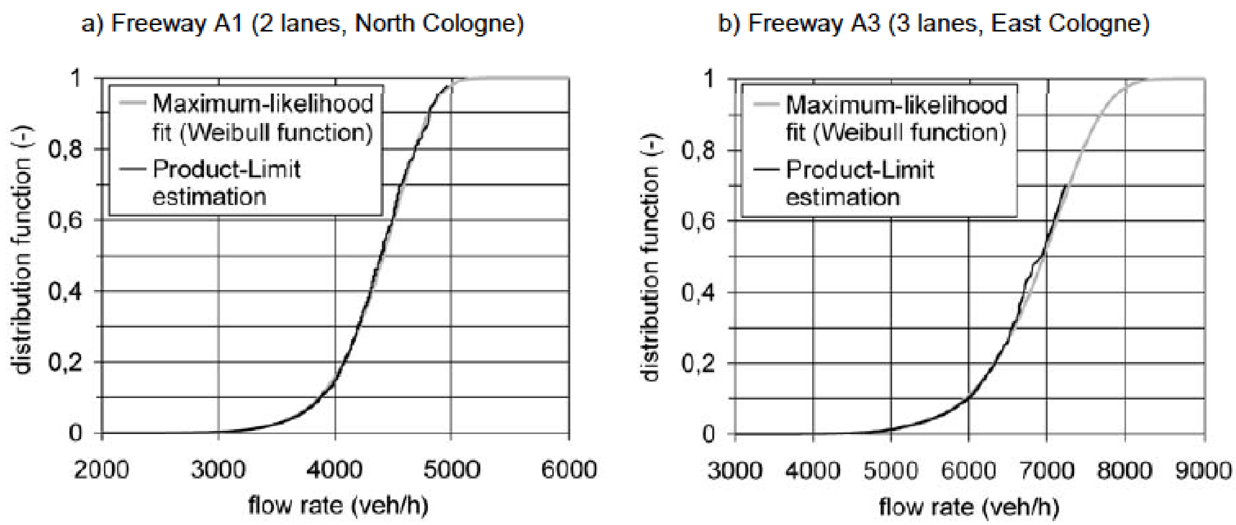


Figure 4.1: Estimated capacity distribution function (Figure source Brilon et al. [2005])

distribution and for $\alpha > 1$*, it is a log-concave distribution. In this case, according to Prékopa [1970], chance constraint feasible set* (4.11) *is convex.*

The individual chance constraint (4.11) of the form $\Pr\{Ax \leq \tilde{b}\} \geq p$ has a separable structure with random right hand side (RHS). Such probabilistic constraints can be solved using a deterministic approximation by replacing the values of random RHS with the corresponding p^{th} quantile of the distribution function. This is effectively translating the assumption on capacity uncertainty into equivalent conservative deterministic saturation flow rate that would ensure the constraint feasibility with probability ε_{ij}. More specifically, the probabilistic constraint can be re-written in its deterministic equivalent as

$$y_{ij}^{t} \leq P_{\mathscr{U}_{ij}}^{-1}(\varepsilon_{ij}), \tag{4.15}$$

where the RHS of the above equation is rounded to the nearest integer value to ensure that the capacity is an integer. The model can then be solved after substituting the deterministic equivalent of probabilistic constraint for each arc (Charnes and Cooper [1959]).

Proposition 1. The optimal objective value of the approximation of PCP upper bounds the optimal objective value of PCP.

Proof: Consider the following situation.

$$P^{-1}_{\mathscr{U}_{ij}}(\varepsilon_{ij}) \leq \widetilde{\mathscr{U}_{ij}} \tag{4.16}$$

Finding the nominal flow value based on the deterministic assumption of random capacity for the above mentioned scenario might result in under-utilization of the roadway capacity, thereby, resulting in upper bound for the objective value.

According to Proposition (1), solutions obtained using the deterministic equivalent of the probabilistic constraint are conservative. The objective value corresponding to the minimum clearance time obtained using the PCP model could be higher than the optimal objective value if everything was known and deterministic. But, the solution is certainly more reliable in the wake of uncertain arc parameters.

4.1.3 Minimum Congestion Path

The primary aim of this chapter is to address tactical preparedness concerns involving the flow on the paths and clearance time by minimizing the traffic breakdown which might occur at bottlenecks and carefully plan for evacuation considering the stochastically degrading arc capacity. Section 5.2 was devoted to find the clearance time where the reliability was set at the link level. For finding the reliability at the route level, we propose a path based model that is designed to find paths with a minimum probability level of congestion for the complete network. As opposed to arc based model, the path based model is used because it reduces the problem complexity. Moreover, the path based model finds out the evacuation routes and starting schedules for the vehicle loading in the network. The proposed path based model in this paper is based on the principle of bottleneck minimization (BM) proposed by

Kerner [2011]. We apply *BM* principle in a static network setting to incorporate the discrete uncertainty of link capacity in our model within the mathematical programming framework. Unlike other works in evacuation literature (Aronson [1989], Chiu et al. [2007], Hoppe and Tardos [1994]) that use Wardrop's *SO* or *UE* principle, we prefer *BM* principle so that congestion minimization is given a priority.

For the path based model, the underlying assumption is that a list of paths between each O-D pair is known *a priori*. Path enumeration for the evacuation network can be achieved using a shortest path model for finding a pool of unique paths between each O-D pair. Given the pool of paths, a model is designed to select evacuation routes, evaluate their reliability and find an evacuation plan that will result in minimum congestion in the network. The objective is to minimize the maximum probability of congestion that might occur in the evacuation paths during evacuation of a given number of evacuees within the time bound T. Although, the expected results obtained using this principle would be highly conservative, it can give the emergency personnel a reasonable probabilistic guarantee for smooth execution of the plan without any undue surprises.

Before moving to the formulation, we define the probability of free flow of vehicles in terms of the CDF of capacity function of the arcs constituting the path. Since a path is a sequence of arcs in series, the distribution function associated with the path would be the product of individual distribution function of each arc contributing to the path. Here, we assume that each arc is independent and do not affect other arcs. Accordingly, the free flow probability which is the compliment of congestion probability of the path can be stated as

$$P_{free}(\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_n) = \prod_{i=1}^{n} [1 - F_{\mathscr{U},i}(\mathcal{Q}_i)] = e^{-\sum_{i=1}^{n} (\frac{\mathcal{Q}_i}{\beta_i})^{\alpha}}. \tag{4.17}$$

The last equality follows from the assumption that the CDF of capacity follows a

Weibull distribution. If the capacity is assumed to follow some other distribution, then the equation can be modified accordingly.

The problem is first modeled as a chance constrained problem and we call this model as MCP-J. Notations specific to the model are integer decision variables f_p that represent the flow on path $p \in \mathscr{P}$ and binary decision variables y_p which denotes the decision for path selection in the final evacuation plan. Real variable $\gamma \in (0, 1]$ represents the probability of congestion in the network. Parameter σ_p represents the travel time of path $p \in \mathscr{P}$ and sets $\mathbb{O}_p$ and $\mathbb{D}_p$ represent respectively the set of source and sink nodes. Assuming that the evacuation time T is known, the mini-max model to decide the minimum number of paths and minimize the maximum congestion among the paths can be stated as

$$\text{Minimize} \quad Z = \sum_{p \in \mathscr{P}} y_p + \gamma \qquad (MCP-J) \tag{4.18}$$

$$\text{Subject to:} \quad \Pr\left\{ \bigcap_{a \in p} \left[\sum_{p' \in \mathscr{P}} f_{p'} \cdot \delta_{p'a} \le \tilde{\mathscr{U}}_a \right] \right\} \ge 1 - \gamma, \qquad \forall p \in \mathscr{P}; \tag{4.19}$$

$$\sum_{p | \mathbb{O}_p = i} (T - \sigma_p) f_p \ge \mathscr{S}_i, \qquad \forall i \in \mathscr{N}_c; \tag{4.20}$$

$$\sum_{p | \mathbb{D}_p = j} (T - \sigma_p) f_p \le \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_s; \tag{4.21}$$

$$(T - \sigma_p) f_p \le M \cdot y_p, \qquad \forall p \in \mathscr{P}; \tag{4.22}$$

$$f_p \in \mathbb{Z}^+, \mathscr{U}_a \in \mathbb{Z}^+, y_p \in \{0, 1\}, \gamma \in (0, 1] \qquad \forall p \in \mathscr{P}, \forall a \in \mathscr{A}; \tag{4.23}$$

The deterministic model without considering stochastic arc capacity is given in Appendix. In model MCP-J, constraint (4.19) is an individual chance constraint which states that for each arc $a \in p$, the summation of flow on the arc over all the selected paths $p' \in \mathscr{P}$ is less than a random capacity with a probability of free-flow greater than variable $(1 - \gamma)$. We use the joint probability distribution for the capacity of arc in path and is given in equation (4.17). This CDF is again a

Weibull distribution having log-concave property. According to Corollary 1, a feasible set of chance constraint (4.19) is, therefore, convex and we can use a deterministic approximation of the capacity to solve the model. For solving MCP, we reformulate the model into a deterministic model which is a mixed integer non-linear program (MINLP). Specifically, constraint (4.19) is replaced by the following constraints in the MINLP model.

$$\sum_{p \in \mathscr{P}} f_p \cdot \delta_{pa} \leq \mathcal{Q}_a, \quad \forall a \in \mathscr{A}; \tag{4.24}$$

$$e^{-\sum_{a \in p} \left(\frac{\mathcal{Q}_a}{\beta_a}\right)^{\alpha}} \geq 1 - \gamma, \quad \forall p \in \mathscr{P}; \tag{4.25}$$

In the formulation, the uncertainty only affects the capacity vector $\mathscr{U}_a$ of arcs in arc capacity constraint (4.19). We define $\mathcal{Q}_a$ as an auxiliary variable whose value is determined according to the probability level $1 - \gamma$ which is in turn determined by the objective function. The only restriction on the uncertainty set $\mathcal{Q}_a$ is that $\mathcal{Q}_a \subseteq \mathbb{Z}_+^n$ (this corresponds to the requirement of non-negative integer capacities). For each arc $a \in \mathscr{A}$, the arc attribute values are random variables and are specified at the entrance to an arc. They are assumed to be static for that particular traveler until exiting the arc. This property is referred to as frozen link property by Orda and Rom [1990]. Capacity distribution functions for each arc are statistically independent assuming that arc lengths are sufficiently large. It can be assumed that the roadway capacity does not exhibit significant fluctuations for small road segments (Yazici and Ozbay [2010]). Therefore, the realization of the network is spatially independent. Everything else in the constraint matrices are assumed certain.

Congestion probability level γ and auxiliary variable $\mathcal{Q}_a$ for each arc are inter-dependent and are determined such that the objective function is minimized. Note that the model pushes for the complete evacuation within time bound T which is mathematically represented in constraint (4.20). Constraint (4.21) ensures that the capacity of the sink nodes is not exceeded at all times. Constraint (4.22) is the path selection constraint from each source node. The optimal value of objective function (4.18) can be obtained by choosing the paths that minimize the probability of maximum congestion among all the selected paths. Here paths are selected from a pool of possible evacuation paths set $\mathscr{P}$ given as an input to the model.

Path selection is a combinatorial problem and the model selects the best set of paths that can completely empty the network in time T. According to the definition of the traffic reliability, in case of overload of one section, the whole system is considered as overloaded. Therefore, for each network state, the objective function minimizes the maximum probability of congestion among all the paths. The solution will provide the reliability of the chosen paths to be used for evacuating the given number of evacuees within time T in terms of the congestion probability γ. A flow pattern for the paths corresponding to the resulting reliability is also obtained. This flow would be employed on the assumption that network state is not known to the evacuees at the time of evacuation.

Probabilistic arc capacity constraint is also formulated as an individual chance constraint with probability level assigned to each arc separately. The constraint would look like

$$\Pr\left\{\sum_{p\in\mathscr{P}} f_p \cdot \delta_{pa} \leq \tilde{\mathscr{U}}_a\right\} \geq 1-\gamma_a, \quad \forall a \in \mathscr{A}; \tag{4.26}$$

In such models, the CDF of individual arcs is used to find the deterministic estimate. This model which we term as MCP-I would yield a solution that binds the probability constraint at each arc. On the other hand, the individual chance constrained model MCP-J would result in a solution with a network wide reliability and the probability constraint is bounded for each path.

Theorem 2. *MCP-I provides a tighter bound as compared to MCP-J*

Proof: *Let n be the number of arcs in the path. Using Bonferroni's inequality, the following inequality holds true.*

$$\Pr\left\{\bigcap_{a\in p}\left[\sum_{p'\in\mathscr{P}} f_{p'}\cdot\delta_{p'a}\leq\tilde{\mathscr{U}}_a\right]\right\}\geq 1-n+\sum_{a\in p}\Pr\left\{\left[\sum_{p'\in\mathscr{P}} f_{p'}\cdot\delta_{p'a}\leq\tilde{\mathscr{U}}_a\right]\right\} \quad (4.27)$$

i.e., probability of free flow along the path is greater than the probability of free flow when calculated individually along the arcs on the path.

Using Theorem (2), it can be concluded that the solution of MCP-I would be more conservative as compared to solution obtained using model MCP-J. This simply means that the congestion probability γ would be smaller for MCP-I model but at the cost of conservative flow on the arcs and increase in clearance time for the network.

4.2 Computational Results

In this section, we report numerical results from the solution obtained for the proposed models. Models are solved on a 3.07 GHz workstation with 24 GB of memory running on Ubuntu 10.04.3 operating system.

Before undertaking an extensive set of computational results, we now put forth the following questions we wish to answer:

1. What is the probability of congestion for a deterministic evacuation route plan?
2. How different is the stochastic schedule compared to the deterministic counterpart?
3. How does the stochastic model assist in decision making?

Two numerical examples are provided for the illustration of the proposed models and to answer the questions posed. Experimental studies are conducted on the evacuation network of the Greater Houston Metropolitan area shown in Figure 5.1. The first experiment studies the impact of stochastic capacity on the clearance time and shows the variation of clearance time according to the desired reliability level. For the second experiment, we develop an evacuation plan that yields most reliable paths that should be used during evacuation with a random arc capacity.

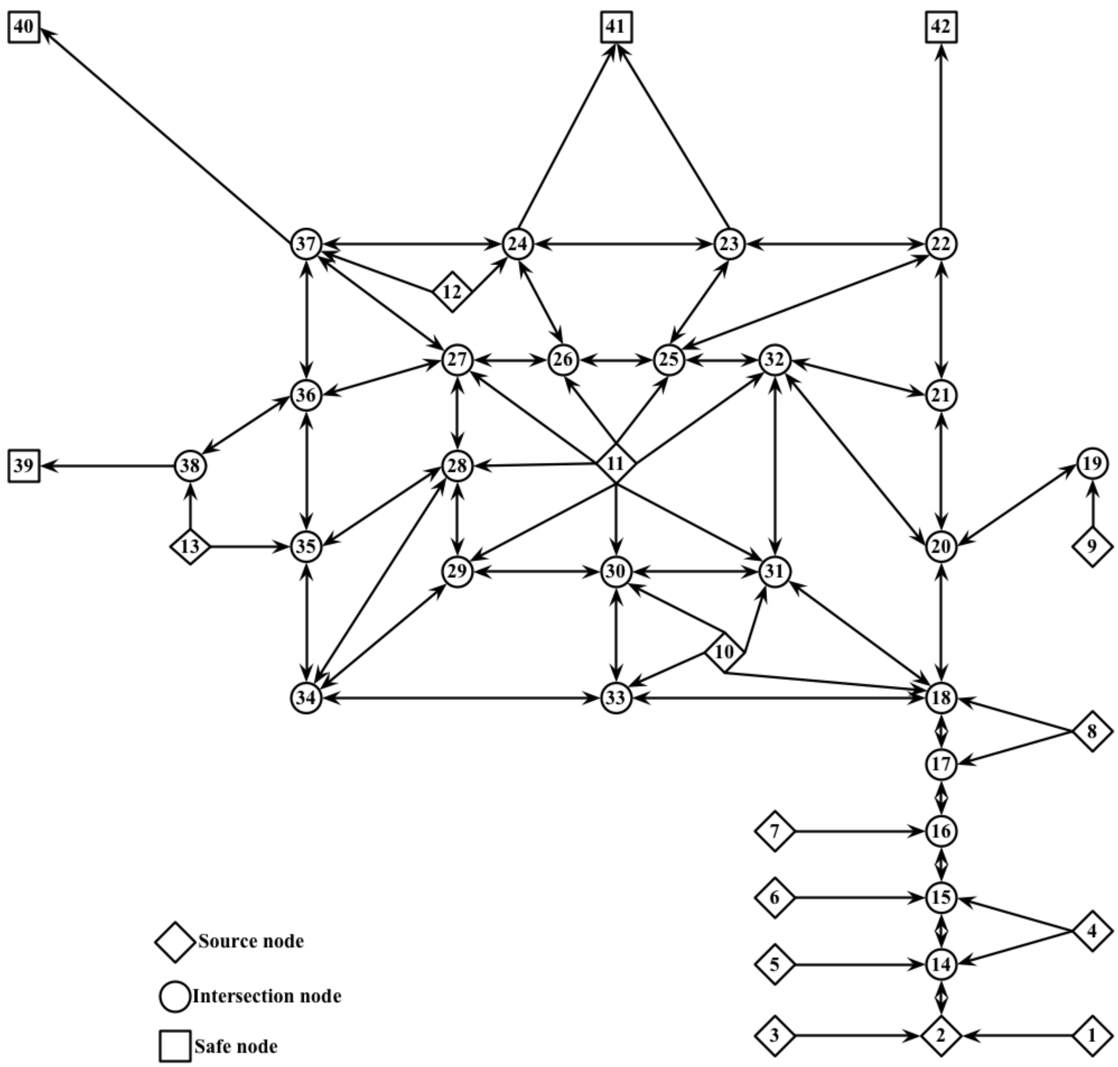


Figure 4.2: Representation of Houston evacuation network

4.2.1 Random Capacity and Bound on Clearance Time

In this section the effect of stochastic capacity on the clearance time is investigated. Stochastic programs are solved by using the deterministic approximations of the random variables in the model and the results are interpreted within a prescribed probability level on the probabilistic constraint. The evacuation network topology as shown in Figure 5.1 has multiple sources and destinations. A super-sink destination is connected to this network from all the destination nodes with arcs having infinite capacity. There are 42 nodes of which 13 are the source nodes, 4 destination nodes and the remaining nodes are intermediate nodes. Assuming that there are a total of $56,600$ evacuating vehicles present at the source nodes, analysis is done for finding the clearance time. Clearance time is first calculated assuming that arc capacities are deterministic and constant. Next, stochastically degrading capacities are considered and clearance time is found based on the desired reliability level.

For our calculation, an average value of $\alpha = 12$ is used for the shape parameter of Weibull distribution. Scale parameter β of the Weibull distribution varies as a function of different geometric and control conditions, different driver and vehicle populations and prevailing travel purposes. For an illustration purpose, we assume only two kinds of arcs for this network and the β values are set as either 52 or 104 when the maximum arc capacity is assumed to be 50 or 100, respectively. The MIP model MET-D is solved using CPLEX solver and the lower bound for the clearance time considering deterministic model was found to be $T = 128$. Results for different reliability level for the stochastic model MET-S is shown in Figure 4.3.

It can be seen that a higher reliability level results in increasing of the clearance time. By planning for a decreased capacity, the evacuation plan becomes more reliable in the sense that the probability of deviating from this plan due to degrading capacity of the road link gets smaller. This essentially shows that to ensure the desired reliability level, the network is loaded in a conservative manner so as to avoid future infeasibility of the arc capacity constraint when the flow volume might exceed the capacity and result in congestion. Planning for extreme reliability would theoretically ensure that the deviation from the resulting evacuation plan would be

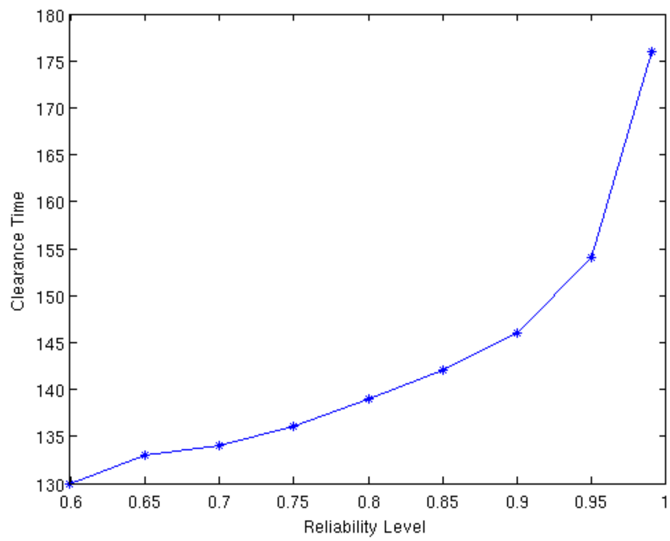


Figure 4.3: Clearance time for various values of reliability level

impossible. A judgmental decision has to be taken regarding the desired reliability level of the evacuation plan.

4.2.2 Evacuation plan with minimum congestion

We formulated an optimization model for finding an evacuation plan with minimum probability of congestion. This model would give the evacuation paths along with a fixed flow on the selected paths to be used to evacuate the vehicles from the region within time T given as an input to the model. First, consider the case when there is no uncertainty. The objective of deterministic MCP model (see Appendix) is to find the minimum number of evacuation paths. Given the clearance time T as an input, the model finds the path and the flow associated with the paths for complete evacuation. In the deterministic model, all the parameters are assumed constant and the model can be easily solved using commercially available solvers such as CPLEX. A comprehensive set of paths are provided to the model as an input parameter. MCP is a combinatorial optimization problem and an evacuation plan obtained using this deterministic model is shown in Table 4.1. This table shows that a total of 16 selected paths along with the corresponding flow that would ensure a complete evacuation within $T = 130$.

Table 4.1: Evacuation plan using the deterministic model

Source Node	Total Vehicles	Selected Path between O-D pair	Travel Time	Flow
1	100	1 2 14 15 16 17 18 31 32 25 23 41	24	1
2	100	2 14 15 16 17 18 33 30 31 32 25 22 42	27	1
3	100	3 2 14 15 16 17 18 31 32 25 23 41	24	1
4	100	4 15 16 17 18 31 32 21 22 23 41	23	1
5	100	5 14 15 16 17 18 20 21 22 42	24	1
6	100	6 15 16 17 18 20 21 22 42	23	1
7	3500	7 16 17 18 20 21 22 42	22	33
8	3500	8 18 33 34 29 28 27 36 37 40	20	32
9	3500	9 19 20 21 22 42	20	32
10	3500	10 31 32 25 22 42	20	32
11	14000	11 27 37 40	14	26
		11 25 23 41	16	97
12	14000	12 37 40	14	22
		12 24 41	15	100
13	14000	13 38 39	13	100
		13 35 36 37 40	15	20

Now consider the scenario that there is an uncertainty associated with the capacity of arcs. In such a case, one might attempt to re-solve the evacuation model with new estimates of arc capacities in real-time. However, this might not be computationally feasible for large-scale networks. Even if this is feasible, there might not be enough time to communicate the new information to the evacuees and the emergency personnels executing the plan. There is a need to incorporate capacity uncertainty in the model so that the obtained evacuation plan can be followed without modifications with a desired confidence level. Probabilistic road capacity formulation MCP-J is used in such scenarios to find the evacuation plan.

MCP-J has dual objectives that find evacuation paths and their corresponding flows by minimizing congestion within a given clearance time T. Values of Weibull parameters α and β are assumed to be same as in the last section. Distribution function of arc capacity with the typical value of α makes the MCP-J model highly non-linear. Moreover, the integral limitations for the decision variables and the combinatorial nature of the problem because of the presence of 0-1 variables make this MINLP problem computationally intractable, especially for large scale instances with more than hundreds of variables. We follow the following algorithm to solve the model.

1. Relax the binary variable and solve the resulting Non Linear Programming (NLP) subproblem of the MINLP. If $y^{(0)} = y$ is integer, stop("integer optimum found"). Else goto step 2.

2. Find an integer point $y^{(1)}$ with a Mixed Integer Program (MIP) master problem that features an augmented penalty function to find the minimum over the convex hull determined by the half-spaces at the solution $(x^{(0)}, y^{(0)})$.

3. Fix the binary variables $y = y^{(1)}$ and solve the resulting NLP. Let $(x^{(1)}, y^{(1)})$ be the corresponding solution.

4. Find an integer solution $y^{(2)}$ with a MIP master problem that corresponds to the minimization over the intersection of the convex hulls described by the half-spaces of the KKT points at $y^{(0)}$ and $y^{(1)}$.

5. Repeat steps 3 and 4 until NLP subproblems start worsening (i.e., the current NLP subproblem has an optimal objective function that is worse than the previous NLP subproblem).

Note that a similar algorithm is readily available in various solvers such as DICOPT by Grossmann et al. [2002].

To insure that the arc capacity never goes to 0 and the congestion probability does not make the model infeasible, we set up the following bounds for these variables.

$$\gamma \in [0.02, 0.99],$$

$$\mathcal{Q}_a \in [0.3\beta_a, \beta_a].$$

Given the path set and the clearance time T, three sets of experiments are performed for the model. We first present the results of each individual experiment and later a detailed discussion of the results is presented.

Case 1: Objective function is set to minimization of congestion in the network. In this case, there is no constraint on the number of paths being used for evacuation. Model MCP-J is solved for different values of T to find the minimum congestion level that can be achieved. It should be noted that the computation of the capacity approximation is done for different probability levels within the model and the final capacity that is used in the solution is obtained from the minimum probability level found by the objective function. Table 4.2 shows the result of the experiment.

Table 4.2: Minimum congestion probability attainable for clearance time T

Clearance Time	Congestion Probability	No. of paths
130	0.466	25
131	0.426	29
132	0.395	27
133	0.381	29
134	0.348	28
135	0.317	24
136	0.289	22
137	0.259	28
138	0.252	23
139	0.229	25
140	0.200	24
141	0.189	27
142	0.183	25
143	0.164	22
144	0.144	30
145	0.140	26
150	0.085	28

Case 2: Objective function is set to minimization of number of evacuation paths. Solution for this case for time $T = 130$ produced the following results.

- Number of paths = 16
- Congestion probability = 0.852

Case 3: Objective function is set to minimization of congestion probability with an added constraint of limiting the total number of paths to be used for evacuation to a value N. Solving the model for different values of N, we obtained the result shown in Table 4.3.

Table 4.3: Congestion probability attainable for clearance time T

	Number of Paths (N)				
Clearance Time	20	21	22	23	24
130	0.468	0.468	0.468	0.468	0.467
135	0.322	0.322	0.321	0.321	0.317
140	0.209	0.208	0.208	0.207	0.200
145	0.144	0.144	0.144	0.143	0.141
150	0.091	0.089	0.089	0.087	0.087

After analyzing the results from all the three experiments, it can be concluded that for achieving a congestion level below 10%, the clearance time has to be increased to 150. Even if the number of paths are increased, the congestion level cannot be reduced below a certain level for evacuation being completed within a given time. This is because of presence of bottleneck at certain sections of road which are being shared by multiple paths and cannot be avoided. For these bottleneck locations being shared by multiple paths, the flow has to be adjusted to accommodate more paths sharing the same arc and maintain the free flow with the same level of probability. Results obtained from the experiment in Case 1 provides the best bound achievable for the congestion probability within time T without any restriction on the number of evacuation paths. As seen from results in Table 4.2, even though there was no restriction on number of paths to be used, the congestion level did not go down after a certain probability level has reached for any particular input time T. This hypothesis is further proved by the experiment described in Case 3. From Table 4.3, we can see that the model achieved a comparable performance in terms of congestion probability for a given clearance time with a limited number of evacuation paths. This congestion level was achieved even though the number of evacuation paths was limited to N whereas more paths were used to achieve the same level of congestion in Case 1.

From the experiment of Case 2, it was found that a minimum of 16 paths are required for evacuation in clearing time $T = 130$ and this would result in a congestion probability of $\gamma = 0.85$. This congestion probability is quite high to tolerate for any practical evacuation plan and would result in heavy traffic buildup and eventually leading to require more time for evacuation. It should be noted that to achieve a traffic flow with a congestion level below a desired probability within a limited time T, the evacuation planner should consider evacuating less number of vehicles. An alternative plan would be to provide local shelters for people who are left behind.

4.3 Conclusion

In this chapter, we extended the concept of stochastic capacity in the evacuation planning problem and formulated the problem using the notion of congestion minimization in evacuation routes. Traditionally, clearance time estimates and route planning are determined considering a deterministic scenario. Only a handful of literature consider capacity as random variable and design the model to find the clearance time estimates. To capture the variation of capacity, we first formulated a probabilistic constraint for arc capacity violation in the proposed minimum cost network flow model. We assumed that the random capacity follows Weibull distribution and estimated the clearance time based on a desired reliability level. Bottlenecks are a result of traffic flow reaching the saturation point of the capacity. We explicitly considered the uncertainty of traffic jam inherent in high volume traffic that occurs in evacuation. To alleviate this problem, we used the bottleneck minimization principle and developed a model that minimizes the probability of traffic congestion for a given network state.

Numerical experiments showed that assigning traffic flow in anticipation of capacity degradation would result in a conservative plan compared to deterministic models, but such plans are more reliable. Paths and the corresponding flow that would result in minimum congestion were found using the MCP-J model. For a given network and clearance time, a state is reached where increasing the number of evacuation paths will not have any effect in decreasing the congestion probability below a certain level as the bottleneck arc is being shared by multiple paths. By providing the reliability level, stochastic model equips the evacuation planner to make probabilistic inference about the model results.

Finally, we conclude that minimization of clearance time for evacuation plan is not the primary goal that the planner should look for. Since the capacity is variable and there is a metastable region of the arc capacity in which there is a finite probability of congestion to occur, slight disturbance can cause the traffic breakdown and increase the total clearance time altogether. Therefore, an evacuation plan considering variable parameters should be used such that the tolerable level of violations of the probabilistic constraints can be inferred. The results obtained using the stochastic models are more practical considering the dynamic and uncertain nature of events during evacuation.

Future research in the direction of incorporating uncertainty in the evacuation planning model would be to consider the random demand of the number of vehicles. Owing to the non-existence of any distribution function for estimating the number of evacuees, distribution free chance-constrained models would be a better alternative to model the problem and coming up with a robust plan.

Chapter 5 Planning for Uncertain Demand

Evacuation is a rare event and the response of the people varies according to their perception of the future danger and the past experiences with similar events. It is very difficult to capture the evacuation behavior of people. Participation rate for evacuation during emergency depends on a number of factors including the nature of disaster (natural/man-made), dwelling type (permanent/mobile homes), region of impact, time of impact and perception of risk. In order to mitigate the undue consequences arising from uncertain demand, we study the problem of generating evacuation transportation plans which are robust to random outgoing demand. More specifically, we solve an evacuation traffic assignment problem with uncertain static demand assigned to each source node. Stochastic programming techniques particularly chance constrained programming is usually employed to come up with a reliable evacuation plan. Chance constrained programming usually assumes that the underlying distribution of the random variable is known.

Efficient planning of a large scale evacuation requires an accurate description of data. But evacuation is a rare event and enough data are not available to model the underlying uncertainty. Demand estimates are usually based on the judgment of individuals, creating inconsistencies in estimation methods. Moreover, the scarcity of the significant data of such events does not allow to come up with a distribution that captures the random behavior of evacuees demand. For the distributionally robust setting, the probabilistic constraint is satisfied for the set of all possible probability distributions in $\mathcal{P}$ that are consistent with the known properties of $\mathbb{P}$, such as its first and second moments or its support. In this chapter, we particularly focus on the following assumptions. (i) The demand cumulative distribution function (cdf) is unknown but only the partial distribution information such as first and second moments, etc. is available. (ii) The symmetric properties of the distribution is specified (iii) The bounds of the random parameter are known. A distributionally robust chance constrained method is used to find the deterministic approximation of the probabilistic constraint for the family of distribution $\mathcal{P}$ satisfied for the above assumptions. The key contribution of this paper towards evacuation planning literature is to consider issues related to ambiguous distribution of uncertainty and finding an evacuation plan within chance constrained programming framework that is robust to vehicle demand variations.

5.1 Problem formulation

A dynamic network flow model has been used to mathematically represent traffic flow evolution in an evacuation network for our proposed optimization model. A dynamic network can be visualized as a static network with an additional dimension representing time, i.e., the static network is repeated for each discrete slice of time. Traffic assignment on such time-expanded networks relies upon a more aggregate representation of traffic as a series of flows that attempts to match the demand for road space with the capacity of the highway system's links and intersections at various time.

Let us consider a directed network $\mathscr{G} = (\mathscr{N}, \mathscr{A})$ consisting a set of nodes $\mathscr{N}$ and a set of arcs $\mathscr{A}$. For each arc $a \in \mathscr{A}$, we define t_a as the arc transit time and C_a as the arc capacity. Nodes in the network are categorized into source nodes ($\mathscr{N}_s$), intermediate nodes, and destination nodes ($\mathscr{N}_d$). Let $\mathcal{S}_i$ be the number of evacuee at source node $i \in \mathscr{N}_s$ and $\mathscr{C}_j$ be the capacity of destination node $j \in \mathscr{N}_d$. We assume that there are T time periods $\{0, 1, \cdots, T-1\}$ to complete transportation of evacuees from the source nodes to the destination nodes.

5.1.1 Path Based Model

The optimization model is designed for minimization of objective function which is a sum of number of evacuees left behind at the end of horizon time. In this section, we present a deterministic evacuation route planning model. To formulate the problem, most of the researchers have developed a network flow optimization model that finds the flow on roads with respect to limited capacities of the roads. In short-notice emergency evacuations, evacuation paths are decided *a priori* by the authorities and the main issue is traffic control and flow management to have a smooth evacuation. Therefore, we adopt a path-based model (PBM) from Rungta et al. [2012] to select a set of paths from a paths pool and determine flow on selected evacuation paths to be used between each origin-destination (O-D) pair. The paths pool is constructed based on the available evacuation paths and if the evacuation paths are not in hand, one of the network optimization approaches can be utilized to generate evacuation paths.

Assuming limited number of evacuees (supply) at the source nodes and limited capacity at destination nodes, the goal is to evacuate maximum number of people from the evacuation zone and reach the destination within a given clearance time T. A model is designed to select a set of evacuation paths and assign flow rates to the selected paths such that the objective minimizes the total remaining evacuees at the end of time horizon which is represented as slack variable. This will in return, guarantee that the total number of evacuated people is maximized. In this model, for any path $p \in \mathscr{P}$, source node of path is denoted by $\mathcal{P}_i^+$ and the sink node of path by $\mathcal{P}_j^-$. There are two decision variables in the model,

$f_{pt} \in \mathbb{Z}^+$: Flow rate of path p at time t, $\forall p \in \mathscr{P}$

$\beta_i \in \mathbb{Z}^+$: Slack variable associated with each source node i, $\forall i \in \mathscr{N}_s$.

Using the notations in Table 5.1, the deterministic path-based model (DPBM) can be presented as follows:

$$\text{Minimize} \qquad z_{dpbm} = \sum_{i \in \mathscr{N}_s} \beta_i \qquad (DPBM) \tag{5.1}$$

$$\text{Subject to:} \qquad \sum_{p \in \mathscr{P}} \delta_{pa} f_{p(t-\theta_{pa})} \leq C_a, \qquad \forall a \in \mathscr{A}, \forall t \in \mathscr{T}, \tag{5.2}$$

$$\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i \geq \mathcal{S}_i, \qquad \forall i \in \mathscr{N}_s, \tag{5.3}$$

$$\sum_{p \in \mathscr{P}_j^-} \sum_{t \in \mathscr{T}} f_{pt} \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_d, \tag{5.4}$$

$$f_{pt} \in \mathbb{Z}_+, \qquad \forall p \in \mathscr{P}, \forall t \in \mathscr{T} \tag{5.5}$$

$$\beta_i \in \mathbb{Z}_+, \qquad \forall i \in \mathscr{N}_s. \tag{5.6}$$

Table 5.1: Notation

Notation	Description
$\mathscr{N}$	Set of all nodes
$\mathscr{N}_s$	Set of all source nodes
$\mathscr{N}_d$	Set of all destination nodes
C_a	Capacity of arc a
$\mathcal{S}_i$	Supply of source node i
$\mathscr{C}_j$	Capacity of destination node j
$\mathscr{P}_i^+$	Set of paths originating from source node i
$\mathscr{P}_j^-$	Set of paths terminated at destination node j
$\mathscr{P}$	Set of all paths

Constraint (5.2) limits the total flow on each arc to the capacity of the arc. In this constraint, parameter δ_{pa} is a binary parameter which gets value 1 if path p contains arc a and 0 otherwise. Variable $f_{p(t-\theta_{pa})}$ ensures that the flow originating at path p at time $t - \theta_{pa}$ reaches arc a after the transit time θ_{pa}. This constraint allows the simultaneous sharing of any arc by multiple paths. Constraint (5.3) is related to supply of each source node and guarantees that the sum of flows on path originating from the nodes in $\mathscr{N}_s$ over all time is equal to the supply at that node. An additional slack variable β_i is added to this term that represents evacuees who are left behind.

Constraint (5.3) together with the objective function (5.1) minimize the summation of deviation β_i from all source nodes. This objective function, in turn, maximizes the total outgoing flow from the network. Constraint (5.4) bounds the total incoming flows at each destination node to its maximum capacity. Constraints (5.5) and (5.6) reflect the non-negativity and integrality conditions.

For the model DPBM, a deterministic estimate of the demand is used on the right-hand-side of the constraint (5.3). This estimate is often based on individual assumptions and the actual demand have a finite probability of diverging from the assumed demand. The evacuation plan based on the above assumption would result in finding a minimum objective value that would not be reflective of the true number of people left behind when the actual demand is different from the estimated value. Chance constrained optimization technique has traditionally been used to address the problem and come up with an evacuation plan where the constraint is satisfied with some specified guarantee level. The constraint with random input parameter is modeled as a probabilistic constraint where the demand parameter $\mathcal{S}_i$ in constraint (5.3) is substituted by random demand $\tilde{\mathcal{S}}_i$. Such reformulation of the constraint is named as individual chance constrained model. Referring to the deterministic model, the demand constraint (5.3) is modified to limit the infeasibility of the constraint for each arc by a violation level $\varepsilon_i \in (0, 1]$. The stochastic chance constrained model with demand uncertainty can be formulated as follows:

$$\text{Minimize} \quad z_{spbm} = \sum_{i \in \mathscr{N}_s} \beta_i \qquad (SPBM) \tag{5.7}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} \theta_{pa} f_{p(t-\delta_{pa})} \leq C_a, \qquad \forall a \in \mathscr{A}, \forall t \in \mathscr{T}, \tag{5.8}$$

$$\mathbb{P}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i \geq \tilde{\mathcal{S}}_i \right) \geq 1 - \varepsilon_i, \qquad \forall i \in \mathscr{N}_s, \tag{5.9}$$

$$\sum_{p \in \mathscr{P}_j^-} \sum_{t \in \mathscr{T}} f_{pt} \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_d, \tag{5.10}$$

$$f_{pt} \in \mathbb{Z}_+, \qquad \forall p \in \mathscr{P}, \forall t \in \mathscr{T} \tag{5.11}$$

$$\beta_i \in \mathbb{Z}_+, \qquad \forall i \in \mathscr{N}_s. \tag{5.12}$$

Constraint (5.9) is the individual chance constraint equivalent of the deterministic constraint (5.3) with the desired probability level imposed individually on each constraint. Parameter $1 - \varepsilon_i \in (0, 1]$ is the desired reliability level and the value of ε is set such that the optimal solution to the approximation of the chance constraint is feasible to the probabilistic constrained programming (PCP) model. Violation of constraint (5.9) implies that the realized demand is more than the predicted demand. For modeling the chance constrained models, the basic assumption is that the probability distribution function $F_{\tilde{\mathcal{S}}_i}$ of the random parameters is known with certainty. When such is the case, and the probabilistic constraint is of the form as stated in (5.9), a deterministic approximation can be made as below that guarantees the feasibility of the constraint with reliability level of $1 - \varepsilon$.

$$\mathbb{P}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i \geq \tilde{\mathcal{S}}_i\right) = F_{\tilde{\mathcal{S}}_i}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i\right) \tag{5.13}$$

$$F_{\tilde{\mathcal{S}}_i}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i\right) \geq 1 - \varepsilon_i. \tag{5.14}$$

$$\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i\right) \geq F^{-1}_{\tilde{\mathcal{S}}_i}(1 - \varepsilon_i) \tag{5.15}$$

Replacing constraint (5.9) with constraint (5.15) would guarantee that the solution would be feasible with the confidence level of $1 - \varepsilon$.

5.2 Robust approximation of chance constraints

Assuming that the probability distribution of the random parameter is known with certainty, the standard method to solve chance constrained problems is finding a deterministic estimate based on $(1-\varepsilon)$ quantile value of the probability distribution and reformulating the model with an equivalent deterministic constraint that is tractable. But when the distribution information is ambiguous, the solution obtained using the deterministic approximation might result in infeasible solution. This will happen when the realized demand follows some other distribution as opposed to the distribution that was used for deterministic approximation. We provide a robust tractable approximation method that can be used when only the mean, variance and support information is available for the probability distribution of the random data. Assuming that $\tilde{\mathcal{S}}$ has a known component $\bar{\mathcal{S}}$ and a random component $\tilde{\xi}$, such that:

1. $\tilde{\mathcal{S}} = \bar{\mathcal{S}} + \tilde{\xi}$,
2. the mean value of $\tilde{\xi}$ is $\bar{\xi} = 0$,
3. the covariance matrix of $\tilde{\xi}$ is Σ,
4. the support of $\tilde{\xi}$ is Ξ

Consider the following robust individual chance constraint problem:

$$\text{Minimize} \qquad z = \sum_{i \in \mathcal{N}_s} \beta_i \tag{5.16}$$

$$\mathbb{P}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i \geq \tilde{\mathcal{S}}_i \right) \geq 1 - \varepsilon_i, \qquad \forall i \in \mathcal{N}_s, \forall \mathbb{P} \in \mathcal{P}. \tag{5.17}$$

This is in general a difficult convex optimization problem due to the chance constraints.

Proposition 2. Assume that the assumption is satisfied. Consider the following second order cone program for the family of distributions having given mean $\bar{\mathcal{S}}$ and covariance Σ. We denote this family with $\mathcal{P} = (\bar{\mathcal{S}}, \Sigma)$.

$$\text{Minimize} \qquad z = \sum_{i \in \mathscr{N}_s} \beta_i \tag{5.18}$$

$$\left(\sigma_i \sqrt{\frac{(1-\varepsilon_i)}{\varepsilon_i}} + \bar{\mathcal{S}} \right) - \sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} - \beta_i \leq 0 \qquad \forall i \in \mathscr{N}_s \tag{5.19}$$

then every feasible solution of (5.18) - (5.19) is feasible for (5.16) - (5.17).

Proof: Let (f, β) be a feasible solution of (5.18). We have to check that for all $i \in \mathscr{N}_s$ and $\mathbb{P} \in \mathcal{P}$ whether:

$$\mathbb{P}\left(\sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_i \geq \tilde{\mathcal{S}}_i \right) \geq 1 - \varepsilon_i. \tag{5.20}$$

Using the theorem from Calafiore and Ghaoui [2006], a constraint of the form

$$\inf_{d \sim (\hat{d}, \Gamma)} \mathbb{P}(d^T \tilde{x} \leq 0) \geq 1 - \epsilon \tag{5.21}$$

can be replaced by following equivalent convex second order cone constraint that is satisfied for all realization of the random parameter within family $d \sim (\hat{d}, \Gamma)$.

$$\kappa_\epsilon \sigma(x) + \hat{\varphi}(x) \leq 0, \quad \kappa_\epsilon = \sqrt{\frac{(1-\epsilon)}{\epsilon}}. \tag{5.22}$$

For constraint (5.17), the random parameter $\tilde{\mathcal{S}}_i$ is independent of the decision variables and thus after application of the above theorem, constraint (5.17) can be written in the form (5.19) where σ_i represents the standard deviation of the random parameter $\tilde{\mathcal{S}}_i$.

Proposition 3. Assume that the assumption is satisfied along with an additional information that the random vector ξ is symmetric about its mean. Let $\mathcal{P} = (\bar{\mathcal{S}}, \Sigma)_S$ denote the family of symmetric distributions having mean $\bar{\mathcal{S}}$ and covariance Σ. Consider the following second order cone program:

$$\text{Minimize} \qquad z = \sum_{i \in \mathscr{N}_s} \beta_i \tag{5.23}$$

$$\left(\sigma_i \sqrt{\frac{1}{2\varepsilon_i}} + \bar{\mathcal{S}}\right) - \sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} - \beta_i \leq 0 \qquad \forall i \in \mathscr{N}_s \tag{5.24}$$

then every feasible solution of (5.23) - (5.24) is feasible for (5.16) - (5.17).

Proof: Using the lemma from Calafiore and Ghaoui [2006] for robust approximation of chance constraints where the random variable have symmetric distributions, a constraint of the form

$$\inf_{d \sim (\hat{d}, \Gamma)_S} \mathbb{P}(d^T \tilde{x} \leq 0) \geq 1 - \epsilon \tag{5.25}$$

can be replaced by following equivalent convex second order cone constraint that is satisfied for all realization of the random parameter within family $d \sim (\hat{d}, \Gamma)$.

$$\kappa_\epsilon \sigma(x) + \hat{\varphi}(x) \leq 0, \quad \kappa_\epsilon = \sqrt{\frac{1}{2\epsilon}}. \tag{5.26}$$

For constraint (5.17), the random parameter $\tilde{\mathcal{S}}_i$ is independent of the decision variables and thus after application of the above theorem, constraint (5.17) can be written in the form (5.24) where σ_i represents the standard deviation of the random parameter $\tilde{\mathcal{S}}_i$.

Proposition 4. Assume that the assumption is satisfied and the support of random vector ξ_i is bounded in intervals $\xi_i \in [l_i^-, l_i^+], l_i^+ \geq 0 \geq l_i^-$. Let $\mathcal{P} = (\bar{\mathcal{S}}, L)_I$ denote the family of distributions having mean $\bar{\mathcal{S}}$ and L is a diagonal matrix containing the interval widths. Consider the following second order cone program:

$$\text{Minimize} \qquad z = \sum_{i \in \mathscr{N}_s} \beta_i \tag{5.27}$$

$$\left(\sqrt{\frac{1}{2}\ln\frac{1}{\varepsilon}}\|L\| + \bar{\mathcal{S}}\right) - \sum_{p \in \mathscr{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} - \beta_i \leq 0 \qquad \forall i \in \mathscr{N}_s \tag{5.28}$$

then every feasible solution of (5.27) - (5.28) is feasible for (5.16) - (5.17).

Proof: Using the Hoeffding tail probability inequality, it is proved in Calafiore and Ghaoui [2006] that for the chance constraints where the random variable are bounded with probability one to independent bounded intervals, a constraint of the form

$$\inf_{d \sim (\hat{d}, L)_I} \mathbb{P}(d^T \tilde{x} \leq 0) \geq 1 - \epsilon \tag{5.29}$$

can be replaced by following equivalent convex second order cone constraint that is satisfied for all realization of the random parameter within family $d \sim (\hat{d}, L)_I$.

$$\sqrt{(1/2)\ln(1/\epsilon)}\|L\tilde{x}\| + \hat{\varphi}(x) \leq 0. \tag{5.30}$$

For constraint (5.17), the random parameter $\tilde{\mathcal{S}}_i$ is independent of the decision variables and thus after application of the above theorem, constraint (5.17) can be written in the form (5.28) where $\|L\|$ represents the L_2 norm of the diagonal matrix L.

Using the Proposition as approximated value and solving the model would result in a robust solution that would be feasible for all the distributions consistent with the given information. Hereafter we name the robust approximated model as RCCP.

5.3 Computational Results

Solving the chance constrained programming without the distribution information of underlying random parameter can be tedious using the sampling approach or min-max approach. These methods can be at times intractable for large sized problems. Before undertaking an extensive set of computational results, we now put forth the following questions we wish to answer:

1. What is the feasibility of the evacuation plan obtained based on the assumed distribution?

2. How conservative is the robust approximation as compared to when the true distribution is known?

3. How does the stochastic model assist in decision making?

Experimental studies are conducted on a test evacuation network shown in Figure 5.1. In this network, nodes 1 - 3 are source nodes and nodes 9 - 10 are destination nodes. Usually demand modeling for emergency logistics is done using S-curve. Predicting the demand by estimating the parameters of S-curve using empirical or simulated data resulted in varying demand values (Lindell [2008]). Since the total demand is uncertain, modeling the constraint as chance constraint and assuming the probability distribution of the perceived demand is known, solution of the model would result in feasible evacuation plan for unexpected scenarios. Problem arises when the assumed distribution is different from the actual distribution. We perform the following numerical tests to show the feasibility of the plan.

For each source node we assume that the mean is known with value $\mu = 167$. Standard deviation of the random demand for each source node is set to be $\sigma_1 = 8.3$, $\sigma_2 = 7.5$ and $\sigma_3 = 9.1$. The path based model for minimizing the number of evacuees left behind at the end of planning horizon is used. Solution of the chance

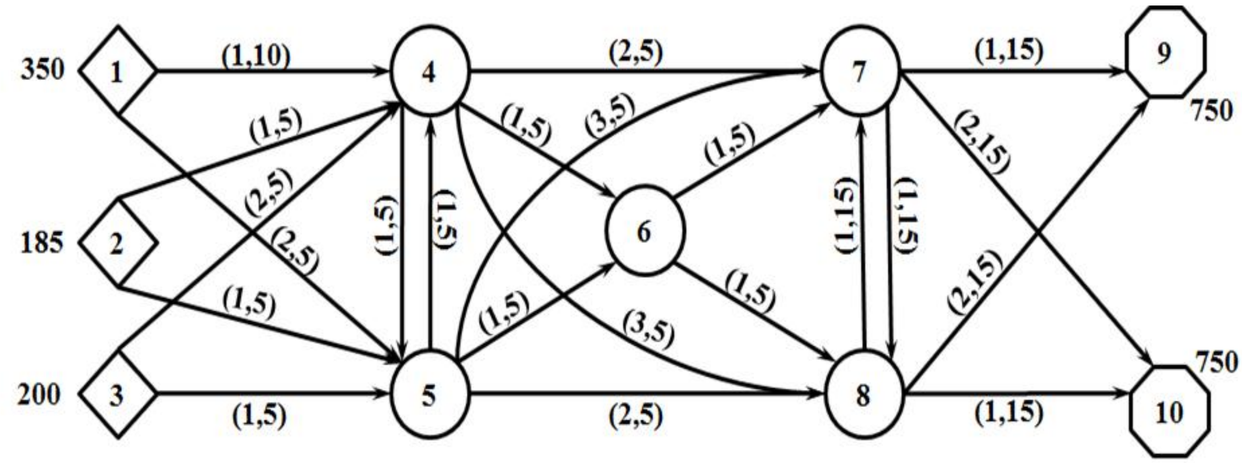


Figure 5.1: Evacuation test network

constrained model is obtained assuming that the demand distribution is following uniform distribution and beta distribution consistent with the given moment information. Specifically, the beta distribution for each source node that we use are $beta_1(2.68, 3)$, $beta_2(3, 3.45)$ and $beta_3(2.4, 2.75)$ assuming that the demand at each source node is given by $d = 160 + 15\tilde{d}$. Model is first solved for various violation levels denoted by ϵ where the approximation is done based on the information about the distribution. For example, consider the first row in Table 5.2. Here the demand for probability level 99 is obtained based on the 99% quantile value corresponding to the uniform distribution consistent with the given moment informations. Clearance time for the complete evacuation is found out for the calculated demand scenario and is reported in the second row. Next, we solve the model based on the robust approximation of the chance constraint that models the random demand. The approximation do not consider any specific distribution but is consistent with the given moment information.

To check the robustness of the evacuation plan obtained using the above methods, we generate 1000 random demands using normal distribution agreeable with the given moment information. To compare the results, we use the evacuation plan obtained from the above experiments and find out the number of people left when this evacuation plan is executed with the demand obtained from normal distribution. Since the evacuation route, path flow rate and starting schedule is deterministic, we follow a simple heuristic to make the solution meaningful. If the realized demand is less than the estimated demand then allocate demand proportionately to each selected path. If the realized demand is more than the estimated demand then the excess demand remains at the source node. Results in Table 5.2 show the estimated number of evacuating vehicles and the corresponding clearance time along with the feasible probability for the evacuation plan obtained using the model in situation of the demand following normal distribution. The solution is feasible for a particular demand scenario only if all the demand from each source node is satisfied. If the demand for any of the source node is not satisfied than the plan is infeasible for that scenario.

Table 5.2: Comparison between CCP and RCCP

		Probability level $(1-\epsilon)$						
		99	98	95	90	80	70	60
CCP(Unif)	Demand	545	544	541	537	528	520	511
	Time	23	23	23	23	23	22	22
	Feasibility	87.5%	86.2%	83.8%	78.6%	62.0%	46.2%	27.9%
CCP(Beta)	Demand	520	518	516	513	509	507	504
	Time	22	22	22	22	22	22	22
	Feasibility	46.2%	43.1%	39.6%	32.5%	24.4%	21.9%	17.2%
RCCP	Demand	750	677	611	577	552	540	534
	Time	31	28	26	24	24	23	23
	Feasibility	100%	100%	100%	99.8%	93.8%	82.7%	73.4%

From Table 5.2, we observe that when the model is solved using CDF based approach assuming either uniform distribution or beta distribution for the demand and the realized distribution is normal, the evacuation plan is having significantly lower feasibility compared to RCCP based approach. If the probability distribution used for CDF based approach is different from the underlying distribution, the feasible probability can be greatly deviated from the expected value of $1 - \epsilon$. For example, when we compare the solution for the confidence level of 99%, we expect that the chance constraint will be feasible with 99% probability. Based on the approximation using above scenarios, we find that CCP based approach is feasible only 87.5% of time for the case of assumed distribution being uniform and 46.2% for the assumed distribution being beta. This signifies the critical dependence of the solution on the exact description of the distribution.

Since the uniform distribution is more loose than normal distribution the result obtained is feasible for normal distribution but the feasibility is not good for beta distribution. This can be explained based on the fact that beta distribution with the given parameters is tighter as compared to normal distribution. It is observed that that the feasible probability depends on the probability distribution of the assumed uncertainty distribution. Solving the model using the robust approximation and using the resulting evacuation plan for simulated random demand based on normal distribution resulted in 100% feasibility. As seen, the approximated demand is significantly higher and the evacuation plan based on this inflated demand would be feasible for any realized demand obtained from the normal distribution. Ideal case would be having the perfect information of the future demand at the beginning of the planning horizon and would give a lower bound of the clearance time.

Next, we compare the results when some extra information regarding the demand distribution is available such as symmetry information and support information. In Table 5.3, we show the feasible probability of the evacuation plan when the realized demand is drawn from normal distribution and robust approximation of the chance constraint is done using the known information of the distribution.

From the Table, it can be concluded that tighter approximation can be made when we have more information about the distribution. Support information results in the tightest bound when we want the constraint to be feasible with probability of

Table 5.3: RCCP with various distribution information

		Probability level $(1-\epsilon)$						
		99	98	95	90	80	70	60
Moment	Demand	750	677	611	577	552	540	534
	Time	31	28	26	24	24	23	23
	Feasibility	100%	100%	100%	99.8%	93.8%	82.7%	73.4%
Symmetry	Demand	679	627	581	558	542	534	531
	Time	28	26	25	24	23	23	23
	Feasibility	100%	100%	99.9%	96.9%	84.7%	73.4%	67.3%
Support	Demand	647	635	618	604	588	577	566
	Time	27	27	26	26	25	25	24
	Feasibility	100%	100%	100%	99.9%	99.6%	99.5%	97.0%

99%. Information about the symmetry of the distribution significantly improves the approximation and makes it tighter resulting in an efficient evacuation plan. For all the approximations, we observe a high feasible probability for the simulated demand scenarios obtained from the normal distribution.

5.3.1 Comparison with scenario based approach

We compare the robust approximation with Monte Carlo sampling based approach. For comparison, we generate the scenarios of random data vector from uniform distribution and beta distribution. Each scenario of the random demand makes up a constraint of the problem. To ensure the feasibility of constraint with reliability of $1-\delta$, the sample size N should be according to Calafiore and Campi [2005]. Following equation represents the SA (sampling approximation) model.

$$\text{Minimize} \quad z_{sa} = (\frac{1}{\mathcal{L}}) \sum_{i \in \mathscr{N}_s} \sum_{l \in \mathcal{L}} \beta_{il} \qquad (SA) \tag{5.31}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} \theta_{pa} f_{p(t-\delta_{pa})} \leq C_a, \qquad \forall a \in \mathscr{A}, \forall t \in \mathscr{T}, \tag{5.32}$$

$$\sum_{p \in \mathcal{P}_i^+} \sum_{t \in \mathscr{T}} f_{pt} + \beta_{il} \geq \mathcal{S}_{il}, \qquad \forall i \in \mathscr{N}_s, \forall l \in \mathcal{L} \tag{5.33}$$

$$\sum_{p \in \mathcal{P}_j^-} \sum_{t \in \mathscr{T}} f_{pt} \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_d, \tag{5.34}$$

$$f_{pt} \in \mathbb{Z}_+, \qquad \forall p \in \mathscr{P}, \forall t \in \mathscr{T} \tag{5.35}$$

$$\beta_i \in \mathbb{Z}_+, \qquad \forall i \in \mathscr{N}_s. \tag{5.36}$$

For practical problems, the required sample size is too large. Therefore, for our problem we take the sample size $L = 50$. These samples are generated for each source node. In the SA model above, $\mathcal{S}_{il}$ represents the number of vehicles at source node i for scenario $l \in \mathcal{L}$. Decision variable β_{il} represents the number of vehicles left behind at source node i for sampling scenario $l \in \mathcal{L}$. The SA model is solved for flow on path p at time t and the evacuation plan is obtained. Again, to verify the performance of the evacuation plan obtained using SA model, we simulate 1000 demand from normal distribution and follow the similar procedure as in earlier experiments. Results for the mean number of vehicles left behind at each source node along with the maximum value and the feasible probability is shown in Table 5.4. Table shows the comparison of the evacuation plan obtained using the robust approximation method and the SA method when the sampling is done from uniform and beta distribution respectively, beta distribution having the same parameters as mentioned earlier.

Table 5.4: Comparison of RCCP with SA

	Uniform			Beta			RCCP		
	1	2	3	1	2	3	1	2	3
Mean	4.36	3.37	3.64	4.89	4.31	5.47	0	0	0
Maximum	11	7	15	21	17	25	0	0	0
Node Feas.	96.0%	95.3%	93.0%	71.9%	81.7%	73.5%	100%	100%	100%
N/W Feas.		84.7%			42.7%			100%	

In Table 5.4, the feasible probability of the probabilistic constraint is shown for each source node separately. For example, the constraint is feasible only 96.0% of time when the demand follows normal distribution at node 1 instead of uniform distribution. This feasibility is 71.9% at the same node when the evacuation plan was obtained assuming beta distribution and 100% when the solution is obtained from the robust approximation of the ambiguous chance constraint. In terms of network wide feasibility, the plan is feasible for 84.7% and 42.7% of time for uniform and beta distribution respectively. As compared to both demand scenarios, RCCP provides robust solution and ensures complete evacuation. RCCP penalizes the clearance time and provides a conservative solution but it outperforms other methods in terms of feasibility when the assumed demand distribution is different from the realized distribution.

While planning for evacuation, if the complete information for the demand distribution is available then the tightest approximation can be made. Proposed evacuation routes and schedule and the calculated clearance time will then be easily able to evacuate the evacuation demand. But it is more sensible to use the RCCP based approach to come up with an evacuation plan when the complete information of the distribution is not known. In such scenarios, incorrect assumption would lead to infeasibility of the plan which is not desired.

5.4 Conclusion

In this chapter, we focus on robust approximation of chance constrained problems to model the traffic demand uncertainty. Stochastic programming techniques particularly chance constrained programming is usually employed to come up with a reliable evacuation plan. Specifically, when the demand variable have an arbitrary distribution in the evacuation problem, we proposed to utilize a distribution free linear approximation technique and solve the problem. Under the assumption that we have information about the moments, support and symmetric properties of the distribution, we came up with a robust approximation of demand constraint. Numerical experiments showed that the proposed approximation is better than CDF based approach and sampling approach when the true distribution is not known. However, this approximation method is conservative and the true distribution information, if available, would result in a much better solution.

Chapter 6 Conclusions and Future Work

In this chapter, we summarize our findings in this research and we discuss the future researches that can be pursued. The introduction and literature review sections presented gave a birds eye view of OR techniques for coming up with an *a priori* evacuation plan. An overview of models was provided that targeted various aspects of evacuation problem and prepared the ground for some important and unanswered questions pertaining to the total clearance time required in wake of the infrastructure limitations and the inherent uncertainties associated with regional evacuation. In Section 6.1, we explain our findings for evacuation route planning problems and the significant impact that they can have on the emergency evacuation problem. In future work, we discuss problems and solution methods that can be explored from the current point of research.

6.1 Current findings

Eying the importance of large scale evacuations and the complexity involved in planning for such a gigantic task, the work in this thesis is directed towards aiding the emergency managers while planning to evacuate people towards safe areas and effective management of the plan using the limited set of resources. My research concentrates on developing and solving large scale network flow optimization models for both deterministic and stochastic evacuation scenarios with an emphasis on coming up with an effective and reliable evacuation plan. In this thesis, we addressed important managerial aspects of evacuation event that is lacking in prevalent models. The research aimed to propose a stochastic network flow model and come up with a realistic decision support system.

Effective implementation of an evacuation plan in the wake of a limited set of resources demands that a minimum number of paths are selected for loading the evacuation traffic. This objective is first covered in the thesis and a bi-objective dynamic network flow model was formulated to find the least number of evacuation paths for complete evacuation within the minimum clearance time. The formulation was a mixed integer nonlinear programming model (MINLP) with multiple objectives and such problems are often intractable. Therefore, a three phase solution method was proposed for this problem by decomposing the original model into three separate sub-models. The solutions of these models provided a lower bound on clearance time for complete evacuation, a solution pool of feasible paths and the minimum number of paths required for evacuation in least possible time along with the starting schedules on the selected paths assuming a variable flow rate on the paths at each time interval. The proposed models were mixed integer linear programing models and the formulation was done for *System Optimum (SO)* scenario where the emphasis was on complete network evacuation in minimum possible clearance time without any preset priority. We demonstrated that the model can handle large size networks with low computation time.

Motivated by the stochastic behavior of the arc capacity, we found the evacuation paths and the traffic flow rate on the paths during evacuation within a given time bound that resulted in minimum traffic congestion. Formulation of the resulting routing and scheduling problem was done within a static network optimization framework using the traffic principle for network breakdown minimization. This model found a reliable evacuation plan by selecting paths and flow rate on the paths that resulted in minimum congestion probability for a given network within the given evacuation time. Experiments were conducted to find the minimum clearance time required to attain a desired confidence level ensuring free flow of traffic in the network. Results reported the sensitivity of the congestion probability with respect to the evacuation time, minimum number of paths to be selected to achieve a desired reliability level, and the corresponding traffic flow on the selected paths.

We extended the stochastic model for evacuation planning to include the uncertainty in demand. It is very difficult to capture the evacuation behavior of people since it is a rare event and the response of the people varies according to their perception of the future danger and the past experiences with similar events. Moreover, the scarcity of the significant data of such events does not allow to come up with a distribution that captures the random behavior of evacuees demand. For the demand variable having an arbitrary distribution in the evacuation problem, we utilize the distribution free linear approximation techniques. Using the information about the moments, support and symmetric properties of the distribution, we come up with a robust approximation of chance constraint used to model the demand requirement during the evacuation. This approximation is robust to any distribution consistent with the given information. Numerical experiments showed its advantage over CDF based approximation method and sampling based method.

6.2 Future work

Operations research techniques applied to evacuation planning significantly contribute to decision making during unfortunate disaster events. There are still many important OR problems in evacuation planning. Guided by the motivation of this thesis, encapsulating the variability of demand and arc capacity simultaneously would entail a plan that addresses the significant uncertainty in an evacuation scenario and provide a decision support system to the evacuation managers that is effective and reliable. The current estimates for the stochastic models are conservative in nature and further research can be done to come up with more tighter estimates. Better demand estimates would certainly provide a much better evacuation plan.

Adding the zoning information for evacuation and prioritizing the evacuation plan on a zonal basis can be another important extension. This will help to streamline the currently proposed evacuation plan and it would be much more efficient and manageable. For this extension, weights need to be assigned to the source nodes in the current variable flow model according to the evacuation priority.

Access to timely and accurate traffic information during evacuations is critical to the evacuation process. Information about traffic flow rates and speeds, along with lane closures, weather conditions, incidents, and the availability of alternative routes, is needed to effectively guide evacuees. We think the focus will shift from *a priori* optimization towards a real-time adaptive decision making for several reasons, such as the availability of the necessary technology and data with the advent of Intelligent Transportation Systems (ITS) equipment. Stochastic dynamic programming would be an excellent research direction to take the advantage of the real time data and come up with an efficient evacuation plan.

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Appendices

Chapter A Reliability Analysis under Capacity Uncertainty

A.1 Deterministic MCP Model

A fixed-flow model with the objective of minimum number of paths to be used for evacuation can be formulated as follows.

$$\text{Minimize} \quad \sum_{p \in \mathscr{P}} y_p \tag{A.1}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} f_p \cdot \delta_{pa} \leq \mathcal{Q}_a, \qquad \forall a \in \mathscr{A}; \tag{A.2}$$

$$\sum_{p|\mathbb{O}_p = i} (T - \sigma_p) f_p \geq \mathcal{S}_i, \qquad \forall i \in \mathscr{N}_c; \tag{A.3}$$

$$\sum_{p|\mathcal{D}_p = j} (T - \sigma_p) f_p \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_s; \tag{A.4}$$

$$(T - \sigma_p) f_p \leq M \cdot y_p, \qquad \forall p \in \mathscr{P}; \tag{A.5}$$

$$f_p \in \mathbb{Z}^+, y_p \in \{0, 1\} \qquad \forall p \in \mathscr{P}; \tag{A.6}$$

We define the decision variables $f_p \in \mathbb{Z}^+, y_p \in \{0, 1\}, \forall p \in \mathscr{P}$ for the model. Constraint (A.2) ensure that the sum of flows for all paths p on any arc $(i, j) \in \mathscr{A}$ during any interval of time t should not exceed the maximum capacity of that arc. Constraint (A.3) guarantees that the sum of flows on path originating from the nodes in $\mathscr{N}_c$ over all time is greater than or equal to the supply at that node. This constraint generates the flow in the paths that are selected for the solution. Constraint (A.4) ensures that the summation of flow on paths coming into the destination over all time do not exceed the capacity $\mathscr{C}_j$ of the destination nodes $\mathscr{N}_s$. Constraint (A.5) limits the sum of all flows over all time $t \in T$ on any path p if the path is selected in the solution. If the path is not selected than the flow of the path at all times is set to 0. If the path is selected, the summation of the flows is limited to $M = max(\mathcal{S}_{i|\mathbb{O}_p})$, i.e., the maximum possible supply initially present at the origin of any path. Constraint (A.6) forces the variables f_p and y_p to take integer and binary values respectively. This model gives the flexibility to the emergency managers for finding the minimum numbers of paths that are required for evacuation. Using less paths for evacuation is helpful to emergency managers for efficient management of the evacuation process in wake of limited resources.