Research Notes

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Collapse of One-Dimensional Cavities in Compressible Liquids

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The collapse of a one-dimensional cavity in a compressible, inviscid liquid is treated exactly. Results of numerical computations for some specific examples are presented.

In contrast with the analogous phenomenon in three dimensions, the collapse of a one-dimensional cavity in a liquid is not followed by a rebound. The physical reason for this behavior is that, due to the geometry, there is no convergence of flow which concentrates the kinetic energy into a smaller and smaller region. As a consequence, the cavity approaches its equilibrium dimensions with a continuously diminishing velocity. In turn, the absence of high velocities stopped (or reversed) in a very short time, has the consequence that the pressure waves irradiated into the liquid are comparatively weak. Hence, although a shock wave is formed in the liquid, it appears far away from the location of the cavity, and the magnitude of the discontinuities across it is very small.

To analyze this phenomenon we consider a long (semi-infinite) tube filled with an inviscid liquid; body forces will be neglected. A small region at the closed end of the tube is occupied by a gas, the pressure P_0 of which is smaller than the pressure p_∞ of the liquid. Both fluids are initially in uniform conditions of pressure and density, and equilibrium is achieved by means of a diaphragm located at X_0, which is made to disappear instantaneously at time $t=0$. Distances are measured from the closed end of the tube.

Although (in contrast with the three-dimensional case) an incompressible treatment of the one-dimensional collapse is meaningless, an approximate description can still be obtained quite simply using the acoustical approximation. When the diaphragm is removed, the liquid acquires in a time Δt a velocity $U(0+)$ that can be estimated assuming that the mass of liquid involved has a length of the order $c\Delta t$, where c is the velocity of sound in the liquid. Indeed, by Newton's law,

$$[U(0+)/\Delta t](\Delta t\rho c)\simeq P_0-p_\infty$$

and hence

$$U(0+)\simeq(P_0-p_\infty)/\rho c.$$

If the pressure P inside the cavity is assumed uniform, it will be a function of the position X of the gas-liquid interface, so that the above equation can be extended to give the approximate law of motion of the latter:

$$U(t)=\frac{dX}{dt}=\frac{P(X)-p_\infty}{\rho c}. \tag{1}$$

This equation will provide sufficient accuracy in the majority of cases. However, it may be of interest to note that an exact solution to the problem can be obtained as follows.

As in the above approximate discussion, it is assumed that the gas can be treated as a uniform homentropic fluid; it is also assumed that its pressure–density relationship is of the polytropic form

$$PX^\gamma=P_0X_0^\gamma. \tag{2}$$

The two cases $\gamma=1$ (isothermal) and $\gamma=1.4$ (adiabatic) will be considered in detail. For the liquid, use is made of a relationship of the modified Tait form

$$(p+B)/(p_\infty+B)=(\rho/\rho_\infty)^n. \qquad (3)$$

It is well known that when the values $B=3000$ atm, $n=7$ are selected, Eq. (3) describes quite accurately the behavior of water.[1,2] From (3), the velocity of sound in the liquid can be computed:

$$c=[(p+B)/(p_\infty+B)]^{(n-1)/2n}c_\infty,$$

where $c_\infty^2=n(p_\infty+B)/\rho_\infty$. Assuming that the cavity wall maintains a plane configuration throughout the collapse, the condition on the normal stresses across it is obviously $p_{\text{liquid}}=p_{\text{gas}}$. Therefore, the equation of motion of the gas–liquid interface can readily be derived by applying the method of characteristics[3]:

$$U=\frac{dX}{dt}=\frac{2}{n-1}c_\infty\left[\left(\frac{P_0(X_0/X)^\gamma+B}{p_\infty+B}\right)^{(n-1)/2n}-1\right]. \qquad (4)$$

This is the exact counterpart of the approximate relation (1). It may be verified that the latter can be recovered carrying out an expansion in powers of (p/B) and neglecting terms of second order or higher.

From Eq. (4) it can be deduced that at the point X_e, defined by

$$P_0(X_0/X_e)^\gamma=p_\infty, \qquad (5)$$

both velocity and acceleration of the interface vanish. It is interesting to note that *this also holds if the polytropic exponent* γ *is assumed to be a function of* X, at least insofar as $d\gamma/dX$ does not diverge as $X\to X_e$. This gives confidence that the somewhat crude approximations involved in the description of the gas do not considerably alter the picture. It may be observed that both Eqs. (1) and (4) predict an infinite duration of the collapse; for example, Eq. (1) can be integrated

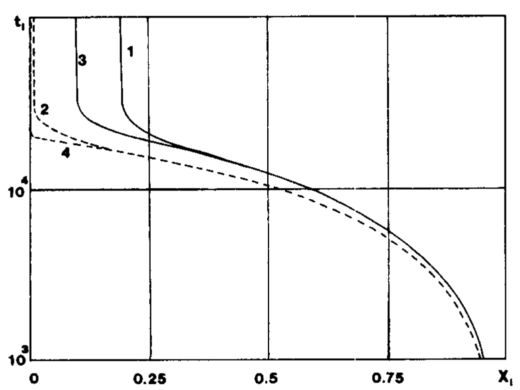

FIG. 1. Position of the gas–liquid interface as a function of time for the adiabatic (curves 1 and 2) and isothermal (curves 3 and 4) cases in terms of dimensionless variables. The ambient pressure is $p_\infty=1$ atm, the initial bubble pressure $P_0=10^{-1}$ atm (curves 1 and 3) and $P_0=10^{-3}$ atm (curves 2 and 4).

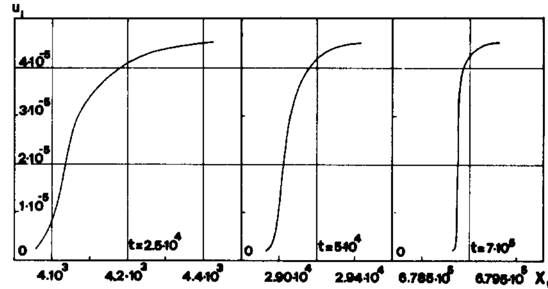

FIG. 2. Velocity profiles in the liquid at three different instants of time showing the formation of the shock wave in the case $\gamma=1$, $P_0=10^{-3}$ atm. The ambient pressure is $p_\infty=1$ atm, and dimensionless variables are used.

exactly in the isothermal case with the result

$$t=(\rho_\infty c_\infty/p_\infty)[X_0-X-X_e\log(X-X_e)/(X_0-X_e)].$$

It is evident from this expression that $t\to\infty$ as $X\to X_e=P_0X_0/p_\infty$.

Equation (4) as it stands can only be integrated numerically. The results for the motion of the cavity wall are shown in Fig. 1 in terms of the dimensionless variables

$$X_1=X/X_0, \qquad t_1=tc_\infty/X_0, \qquad u_1=u/c_\infty.$$

It is apparent that they are much more sensitive to changes in P_0 than in γ. Indeed, the effect of γ shows up essentially only near the end of the collapse through its influence on the limit value X_e in Eq. (5).

From the equation of the characteristic lines,

$$x=(c_\infty+\tfrac{1}{2}(\gamma+1)u)t+f(u), \qquad (6)$$

[where $f(u)$ is determined by requiring that this relation can be satisfied at the bubble wall[4]] it is now possible to obtain the pressure and velocity profiles in the liquid. An example is shown in Fig. 2, in which the gradual steepening of the velocity wave into a shock wave is evident. The exact location (x_s, t_s) of the formation of the latter can be obtained by considering, together with Eq. (4), the relations[4]

$$\left(\frac{\partial x}{\partial u}\right)_t=0, \qquad \left(\frac{\partial^2 x}{\partial u^2}\right)_t=0.$$

Upon substitution of (6) these become:

$$\tfrac{1}{2}(\gamma+1)\frac{dU}{dt}t_s-c_\infty-\tfrac{1}{2}(\gamma-1)U=0,$$

$$\tfrac{1}{2}(\gamma-1)U\frac{d^2U}{dt^2}-\left(\frac{dU}{dt}\right)^2+c_\infty\frac{d^2U}{dt^2}=0.$$

Now, substituting for U from Eq. (4), the characteristic on which the shock wave appears can be determined. For example, for the case to which Fig. 2 refers, one obtains $x_s=7.54\times10^5X_0$, $t_s=7.33\times10^5X_0/c_\infty$.

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Rarefied Gas Flow through a Slit*

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The transition from near free molecule flow to slow viscous continuum flow through a slit has been studied experimentally. The measurements have been compared with the results of a theoretical analysis of the expected flow through a slit with zero length in the flow direction.

Much effort has been expended in attempts to understand phenomena in the transition region between free molecule and continuum flow. Recently, Blum[1] has analyzed the transition from free molecule to slow viscous continuum flow (pressure difference small compared with average pressure) through an ideal slit (zero length in the flow direction, infinite lateral length, and a finite gap width) using both an iteration method and a variational technique. Precise data for the flow through a slit constructed so as to approximate an ideal slit have been obtained and compared with Blum's theoretical work.

The slit was constructed by mounting two razor blades over a square hole machined through the center of a flanged cylindrical mounting base. The gap width of the slit is 9.59×10^{-3} cm and the lateral length is 0.8357 cm. Optical examination indicated that the length in the flow direction is less than about 10^{-4} cm.

The method for measuring flow is a pressure difference decay technique developed by Berman and Lund.[2] In this method an initial pressure difference is developed across the slit, and the time decay of this pressure difference is followed. An analysis of such a pressure decay was carried out by Maegley,[3] who designed and built the precision flow measuring equipment used to obtain the data for the flow through a slit.

A reduced flow variable W is defined as the ratio of the mass flow through the slit to the free molecule mass flow through an orifice with the same cross-sectional area and pressure drop. Maegley's analysis gives the following expression for the reduced flow:

$$W=[4V_fV_b/(V_f+V_b)A\bar{v}t_{12}]\ln[(\Delta p)_1/(\Delta p)_2], \qquad (1)$$

where A is the cross-sectional area, $\bar{v}$ is the mean thermal speed, V_f and V_b are the equipment volumes up and downstream of the slit, respectively, Δp is a pressure difference across the slit, and t_{12} is the time required for a decay to proceed between the two pressure differences. A reduced pressure X is defined by

$$X=2h\bar{p}/\mu\bar{v}, \qquad (2)$$

where h is the gap width, μ is the dynamic viscosity, and $\bar{p}$ is the average pressure for a decay. The Knudsen number Kn, defined as the molecular mean free path λ divided by the gap width, is related to X by $\mathrm{Kn}=\pi/2X$ if the rigid sphere viscosity $\mu=\frac{1}{2}\rho\bar{v}\lambda$ is used. For equal up and downstream volumes, and isothermal conditions, the reduced flow and the reduced pressure are constant for a given decay.

Figure 1 shows the data for argon and helium flow through the slit. For argon, the slope dW/dX is of order 1 for $X<0.05$, and large changes in the flow rate with increasing X are observed. The continuum region appears to be reached at about $X=10$. The experimental continuum slope is determined by using a linear least-squares fit of the data for $X>10.5$. The slope is 0.1965 with a standard deviation of 0.0003, a value which is within 0.05% of the theoretical value of $\frac{1}{16}\pi$ determined by Roscoe.[4] The linear least-squares fit also leads to an intercept at $X=0$ of 1.165. The difference between this intercept and the free molecule flow intercept is called an apparent slip effect. The apparent slip effect for argon flow through our slit is estimated to be 16.5% of the theoretical free molecule flow through an ideal orifice.