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Game Theoretical Framework for Distributed Dynamic  
Control in Smart Grids

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Doctor of Philosophy

in Electrical and Computer Engineering

by

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# Game Theoretical Framework for Distributed Dynamic Control in Smart Grids

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*To Parvin Bakhtiarpour & Mohammad Reza Forouzandehmehr  
with all my love*

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# Abstract

In the emerging smart grids, production increasingly relies on a greater number of decentralized generation sites based on renewable energy sources. The variable nature of the new renewable energy sources will require a certain form of distributed energy storage, such as batteries, flywheels, compressed air and so on to help maintain supply security. Moreover, integration of demand response programs in conjunction with distributed generation makes an economic and environmental advantage by altering end-users' normal consumption patterns in response to changes in the electricity price. These new techniques change the way we consume and produce energy also enable the possibility to reduce the greenhouse effect and improve grid stability by optimizing energy streams. In order to accommodate these technologies, solid mathematical tools are essential to ensure robust operation of heterogeneous and distributed nature of smart grids. In this context, game theory could constitute a robust framework that can address relevant and timely open problems in the emerging smart grid networks. In this dissertation, three dynamic game-theoretical approaches are proposed for distributed control of generation and storage units and demand response applications in smart grid networks.

We first study the competitive interactions between an autonomous pumped-storage hydropower plant and a thermal power plant in order to optimize power generation and storage. Each type of power plant individually tries to maximize its own profit by adjusting its strategy: both types of plants can sell their power to the market; or alternatively, the thermal-power plant can sell its power at a fixed price to the pumped-storage hydropower plant by storing the energy in the reservoir. A stochastic differential game is formulated to characterize this competition. The solutions are derived using the stochastic Hamilton-Jacobi-Bellman equations. Based on the effect of real-time pricing on users' daily demand profile, the simulation results demonstrate the properties of the proposed game and show how we can optimize consumers' electricity cost in presence of time-varying prices.

Second, we focus on controllable load types in energy-smart buildings that are associated with dynamic systems. In this regard, we propose a new demand response model based on a two-level differential game framework. At the beginning of each demand response interval, the price is decided by the upper level (aggregator, utility, or market) given the total demand of lower level users. Given the price from the upper level, the electricity usage of air conditioning unit and the battery storage charging/discharging schedules are controlled for each player (buildings that are equipped with automated load control systems and local renewable generators), in order to minimize the user's total electricity cost. The optimal user strategies are derived, and we also show that the proposed game can converge to a feedback Nash equilibrium.

Finally, the problem of distributed control of the heating, ventilation and air conditioning (HVAC) system for multiple zones in an energy-smart building is addressed. This analysis is based on the idea of satisfaction equilibrium, where the players are exclusively interested in the satisfaction of their individual constraints instead of individual performance optimization. This configuration enables a HVAC unit as a player to make stochastically stable decisions with limited information from the rest of players. To achieve satisfaction equilibrium, a learning dynamics based on trial-and-error learning is proposed. In particular, it is shown that this algorithm reaches stochastically stable states that are equilibria and maximizers of the global welfare of the corresponding game.

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# Chapter1

## Introduction and Background

### 1.1 Overview

The concept of Smart Grids [1, 2, 3, 4] refers to a network system, which is able to effectively satisfy all the new requirements and functions of a future network system by using advanced Information and Communications Technologies (ICT) technologies. The traditional electricity distribution network is a passive network that delivers electricity from the generation point to the consumption point. In future, the network system has to be changed to an active network, which is able to intelligently integrate the actions of all the users connected to it including generators and consumers, in order to efficiently deliver sustainable and secure supply. The network must be able to adapt small-scale distributed generation and enable two-way power flow inside the grid. It has to be able to support all new functions of the electricity market in order to make the operation of the network and electricity market more efficient and flexible. Figure 1.1 depicts the interaction between consumer devices with communication capabilities, energy providers, and transmission and distribution functions enabled by Smart Grid network operations. The end-use devices receive information such as price signals and respond by adjusting their operation accordingly. Consumer communication devices facilitate load aggregation and control from. The distribution part models system operation. All the subsystems interact by two-way communications through the grid operator [5]. A smart grid can be characterized to [6]:

- Enable active participation by consumers
- Accommodate all generation and storage options
- Enable new products, services, and markets
- Provide power quality for the digital economy

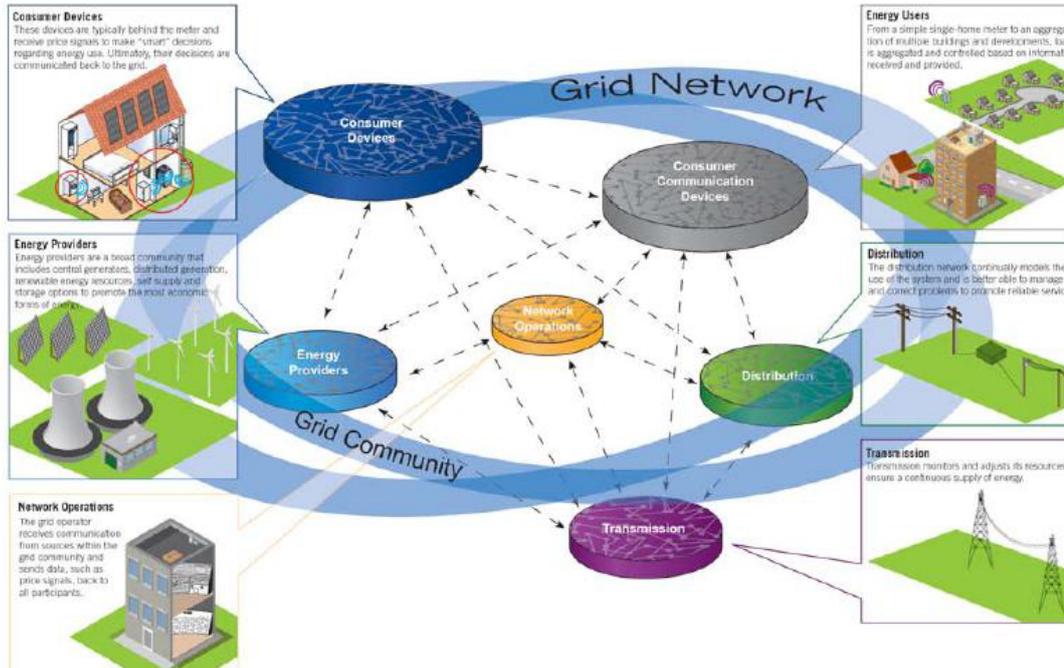


Figure 1.1: Smart grid concept [10]

- Optimize asset utilization and operate efficiently
- Anticipate and respond to system disturbances (selfheal)
- Operate resiliently against attack and natural disaster.

Achieving the vision is dependent upon participating entities and empowers consumers as well as improving reliability of the distribution system. The primary assets of a smart grid are [7]:

- Demand response (DR): communications and controls for end-use devices and systems to reduce (or, in special cases, increase) their demand for electricity at certain times.
- Distributed generation (DG): small engine or turbine generator sets, wind turbines, and solar electric systems connected at the distribution level.

- Distributed storage (DS): batteries, flywheels, super-conducting magnetic storage, and other electric and thermal storage technologies connected at the distribution level.
- Distribution/feeder automation: distribution and feeder automation expand SCADA (Supervisory Control and Data Acquisition) communications in substations and into the feeders with remotely actuated switches for reconfiguring the network, advanced protective relays with dynamic and zonal control capabilities, dynamic capacitor bank controllers, and condition-based transformer-management systems (to name a few).
- Transmission wide-area visualization and control: transmission control systems that rapidly sense and respond to disturbances.
- Electric and plug-in electric hybrid vehicles (EVs/PHEVs): the batteries in EVs represent both a new type of load that must be managed and an opportunity for them to discharge as energy storage resources to support the grid.

Investments in a number of enabling assets are also necessary to support the use of the primary assets for smart grid applications, hence the function of a smart grid.

Among these cross-cutting technologies are [8]:

- Wide-area communications networks, servers, gateways, etc.
- Smart meters could also
  - support shorter metering intervals approaching 5 minutes or less to support provision of ancillary services and distribution capacity management (rather than the hourly interval generally considered adequate for peak load management at the bulk power systems level)
  - full two-way communications including to a home-area network to communicate to smart thermostats and appliances

- instantaneously read voltage, current, and power factor to support distribution state estimation and optimized system voltage control
- Local-area home, commercial building, and industrial energy management and control systems and networks.
- Consumer information interfaces and decision support tools.
- Utility back-office systems, including billing systems.

The objective operations are the benefits or applications to which smart grid assets are engaged to improve cost effectiveness, reliability, and energy efficiency of the power system. These can be summarized in broad categories including managing peak load capacity for generation, transmission, and distribution, reducing costs for wholesale operations and corresponding providing enhanced reliability/adequate reliability at less cost [9].

## 1.2 Green Smart Grids

The intelligence of a Smart Grid will facilitate greater utilization of intermittently available renewable resources such as solar and wind, from which will accrue reductions in  $CO_2$  emissions. As a result of the intermittent nature of wind and solar, their operation causes minute fluctuations in power generated as wind speeds change or clouds affect solar exposure. These fluctuations in power, if not counterbalanced in real time, can lead to frequency imbalance and disturb the stability of the electrical system [11, 12]. To smooth out the intermittency of renewable energy production, low-cost electrical energy storage will become necessary. Energy storage has been considered as a key enabler of the smart grid or future grid, which is expected to integrate a significant amount of renewable energy resources while providing fuel (i.e., electricity) to hybrid and electrical vehicles, although the cost of implementing energy



**Figure 1.2:** Features of a green grid [16].

storage is of great concern. Another major step toward achieving green grid is the energy efficiency of residential buildings which plays a significant role in addressing the climate change. In U.S. for example, 27.3 % of total greenhouse gas emission is attributed to buildings in 2003, where residential and commercial buildings account for 15.3% and 12% accordingly. For a sustainable building, the energy management units should be designed to achieve significant reductions in non-renewable resources, also minimum usage of Heating, Ventilation, and Air conditioning (HVAC) units which are accounted for the main electricity consumption factor in buildings[13].

Development and incorporation of demand response and energy-efficiency resources, deployment of smart technologies (real-time, automated, interactive technologies that optimize the physical operation of appliances and consumer devices) for metering, communications concerning grid operations and status, and distribution automation are some other approaches for developing a smart grid to help address climate change [14, 15].

### 1.3 Smart Buildings

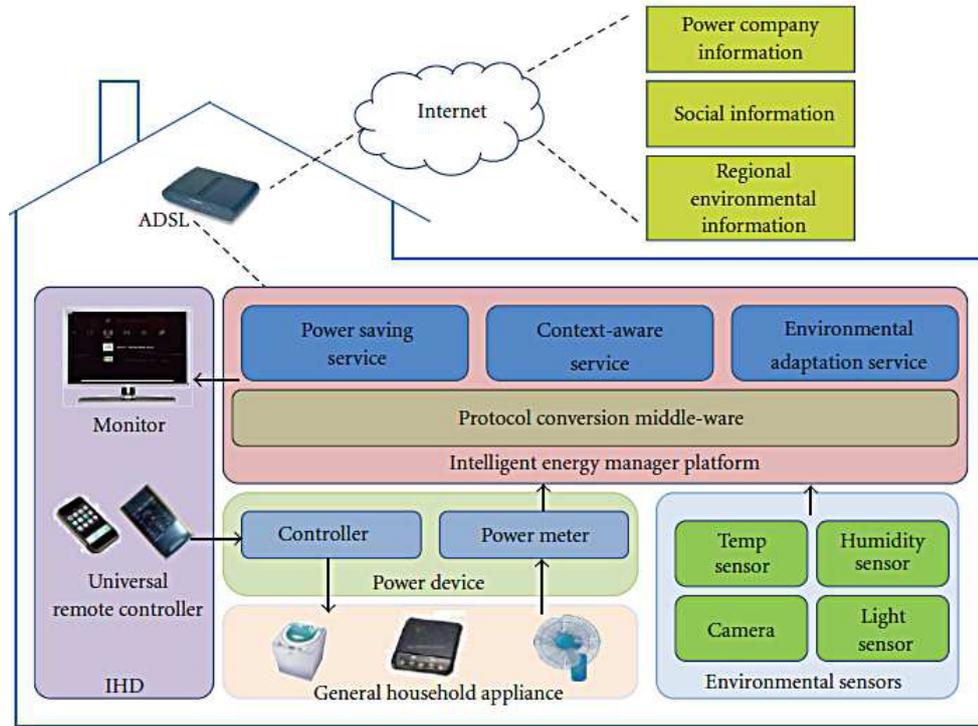
The building and the electricity grid sector are the most important sectors of interest since they are responsible for the greatest energy waste. Great energy savings can be achieved through sophisticated algorithms that control and schedule power tasks from smart devices in a dynamic manner. These processes provide the necessary foundations for demand response (DR) and self-adaptable smart grids [17, 18]. Smart devices are responsible in real time monitoring and actuation under command flow arriving from the smart grid. In general, the greatest the number of deployed smart devices is, more flexibility is given to the grid and thus more vibrant the grid operation is expected to be. In this regard, information technologies play an important role specifically with respect to to the followings [19, 20]:

- Integration with the smart grid via demand response: allows the system to manage consumption in response to supply conditions including price by, for example, selectively turning off appliances or reducing non-essential or nontime-critical services. Demand response helps balancing supply and demand by reducing the peak load and allowing increased use when production exceeds demand. Energy suppliers could avoid costly capital investments for generation capacity; and consumers would benefit from sharing the savings resulted from the lower operational cost of energy production.
- Autonomous building operations through continuous sense and respond. This enables the system to fully manage building energy consumption. This opportunity integrates the dynamic information of users activities (e.g., occupancy information), ambient conditions (e.g., weather, light), and energy supply conditions (e.g., real-time prices) while considering various building users constraints (e.g., business mission, cost, preferences, safety, comfort, convenience). The result is optimized building operation with the greatest energy efficiency.

Towards these directions, the role of the building energy management systems (BEMS) significant, due to their contributions to the continuous energy management and therefore to the achievement of the possible energy and cost savings. The BEMS are generally applied to the control of active systems, i.e., HVAC systems, while also determining their operating times. In the above efforts, the performance of the BEMS is directly related to the amount of energy consumed in the buildings and the comfort of the buildings occupants [21]. A BEMS basic model includes the following components [22]:

- Indoor sensors: Sensors that measure and record temperature, humidity, and  $CO_2$  mass in the building areas.
- Outdoor sensors: Sensors for the outdoor weather conditions such as temperature, humidity and luminance, which are essential for the efficient operation of the system.
- Controllers: This component category contains switches, contacts, valves, and actuators.
- Decision support unit: A real time decision support unit is included, with the following capabilities: 1) Interaction with the sensors for the diagnosis of the building state and therefore the formulation of the building energy profile. 2) Incorporation of intelligent system technologies to select the appropriate interventions, depending on the building requirements. 3) Communication with the buildings controllers based on the application of the decision.
- Database: It includes the database for the building energy characteristics and the knowledge database for all recording information.

An example of a BEMS is shown in Figure 1.3. This system receives information from light, humidity, temperature, and camera sensors. Moreover the real time prices



**Figure 1.3:** A building energy management system[23].

that are sending from market to the smart meters reported to the BEMS unit. Based on users constraints, cost and energy efficiency metrics the optimal control decisions will be sent to various control units in the building.

## 1.4 Motivations and Thesis Contribution

The smart grid is envisioned to be a large-scale cyber-physical system that can improve the efficiency, reliability, and robustness of power and energy grids by integrating advanced techniques from various disciplines such as power systems, control, communications, signal processing, and networking. Inherently, the smart grid is a power network composed of intelligent nodes that can operate, communicate, and interact, autonomously, in order to efficiently deliver power and electricity to their consumers. This heterogeneous nature of the smart grid motivates the adoption of

advanced techniques for overcoming the various technical challenges at different levels such as design, control, and implementation. In this respect, game theory is expected to constitute a key analytical tool in the design of the future smart grid, as well as large-scale cyber-physical systems. Game theory is a formal analytical as well as conceptual framework with a set of mathematical tools enabling the study of complex interactions among independent rational players. For several decades, game theory has been adopted in a wide number of disciplines ranging from economics and politics to psychology[28, 29]. In particular, there is a need to deploy novel models and algorithms that can capture the following characteristics of the emerging smart grid: 1- the need for distributed operation of the smart grid nodes for communication and control purposes, 2- the heterogeneous nature of the smart grid which is typically composed of a variety of nodes such as micro-grids, smart meters, appliances, and others, each of which having different capabilities and objectives, 3- the need for efficiently integrating advanced techniques from power systems, communications, and signal processing, and 4- the need for low-complexity distributed algorithms that can efficiently represent competitive or collaborative scenarios between the various entities of the smart grid. In this context, game theory could constitute a robust framework that can address many of these challenges [30]. Following are the main contributions of this dissertation:

### **Stochastic dynamic hydrothermal scheduling in smart grid networks**

We study the competitive interactions between an autonomous pumped-storage plant as an energy storage and a thermal-power plant in order to optimize power generation and storage. A stochastic dynamic game is formulated to characterize this competition. The simulation results demonstrate the properties of the proposed game and suggest how to optimize the amounts of generation in hydropower and thermal power plants over time with the fluctuations of price. The proposed framework and games can reduce the peak to average ratio and total energy generation for the thermal

plant, which helps power plant operation and reduces CO2 emission.

### **Autonomous demand response using stochastic differential games**

We propose a two level dynamic game framework is proposed to model distributed energy management of smart residential buildings. At the beginning of each hour, the price is decided by the upper level (market) given the total demand of users in the lower level from the previous hour. At the lower level, for each player (i.e., one building), given the price from the upper level, the electricity usage of air conditioning unit and the battery storage charging and discharging scheduled are controlled in order to minimize the user total cost. Based on the effect of real-time pricing on users daily demand profile, the simulation results demonstrate the properties of the proposed game and show how we can optimize the household electricity cost in presence of time-varying prices.

### **Distributed control of HVAC systems in smart buildings**

The problem of distributed control of HVAC systems in an energy-smart building is addressed. Using tools from game theory the interaction among several autonomous HVAC units is studied and simple learning dynamics based on trial-and-error learning are proposed to achieve equilibrium. In particular, it is shown that this algorithm reaches stochastically stable states that are equilibria and maximizers of the global welfare of the corresponding game. Simulation results demonstrate that dynamic distributed control for the HVAC system can significantly increase the energy efficiency of smart buildings.

## **1.5 Thesis Organization**

The remainder of this thesis is organized as follows: In Chapter 2, An overview of analysis methods used in this dissertation is presented. The problem of grid integration of energy storages is studied for specific case of pumped-storage plant using stochastic differential game theory in Chapter 3. In Chapter 4, we proposed a game

theoretic demand response scheme for energy-smart buildings. Using this game, the optimal control decisions for controllable load to minimize the consumption cost while satisfying users' constraints are achieved. In Chapter 5, using satisfaction game theory which requires minimum information of players, we designed a scheme for distributed control of HVAC units of multiple zones in large-scale buildings. Finally in Chapter 6 we explain the future researches and conclude this thesis. Table 1.1 shows the abbreviations used in this thesis.

**Table 1.1:** Thesis abbreviation.

(BEMS)	Building Energy Management Systems
CC	Control Center
DG	Distributed Generation
DS	Distributed storage
DR	Demand Response
ECC	Energy Control Center
ED	Economic Dispatch
EV	Electric Vehicle
FTR	Financial Transmission Right
HJB	Hamilton-Jacobi-Belman Equation
HVAC	Heating, Ventilation and Air Conditioning
ICT	Information and Communications Technologies
ILP	Integer Linear Programming
ISO	Independent System Operator
LMP	Locational Marginal Price
NE	Nash Equilibrium
OPF	Optimal Power Flow
PHEV	Plug-in Hybrid Electric Vehicle
PJM	Pennsylvania, New Jersey, Maryland
PDE	Partial Differential Equation
RTU	Remote Terminal Unit
SCADA	Supervisory Control and Data Acquisition
SE	Satisfaction Equilibrium
SVD	Singular Value Decomposition

## Chapter2

### Game Theory Preliminaries

#### 2.1 Overview of Differential Games

Optimization theory deals with the case where there is only one individual, making a decision and achieving a payoff. Game theory, on the other hand, is concerned with the more complex situation where two or more individuals, or "players" are present. Each player can choose among a set of available options. His payoff, however, depends also on the choices made by all the other players [31].

Game can be static or dynamic. In static games, each player makes one choice and this completely determines the payoffs. In other relevant situations, the game takes place not instantaneously but over a whole interval of time. This leads to the study of dynamic games. The study of differential games as a special class of dynamic games, was initiated by Rufus Isaacs in the early 1950's. Basically a differential game is a mathematical model designed to solve a conflicting situation that changes with time. In differential games, there are more than one player, each having separate objective functions which each is trying to maximize and it is subjected to a set of differential equations which model the dynamic nature of the system [32].

Differential game is an extension of static noncooperative game theory by adopting the methods and models developed in optimal control theory. Optimal control theory has been developed to study the optimal solution of optimization problem of dynamic system (i.e., state evolves over time). Therefore, optimal control can be applied to game theory to obtain the equilibrium solution for rational entities with different objective or payoff functions. One major approach to solve for optimal solution in optimal control theory is the dynamic programming. This approach has been adopted in differential game in which the payoff of player depends on the state (i.e., constrained by the state) which evolves over time. The common solution concepts of

differential game are Nash equilibrium and Stackelberg solution for non-hierarchical and hierarchical structures, respectively. Using techniques in optimal control theory, these solutions can be obtained [33].

### 2.1.1 Optimal Control Problem

In optimal control, each player has an optimization problem with single objective (e.g., to maximize payoff) over a period of time. This optimization problem considers the actions of the other players to be fixed at the equilibrium.

In the standard model of control theory, the state of a system is described by a variable  $x$ . This state evolves in time, according to an Ordinary Differential Equation (ODE)[34]:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)) + \rho w \\ x(0) &= x^0,\end{aligned}\tag{2.1}$$

where  $w$  is control function. A basic problem in optimal control is to find a control function which maximizes the payoff:

$$L[u(\cdot)] = \int_0^T g(x(t), u(t))dt + h(x(T)),\tag{2.2}$$

where  $h$  is a terminal payoff, while  $g$  accounts for a running payoff.

### 2.1.2 Differential Games

Differential games are the extension of the basic optimal control problem to the situation where more than one player participate at the game, and each one of them tries to maximize his own pay. The system state  $x$  evolves through the time according

to the following ODE:

$$\begin{aligned}\dot{x}(t) &= f(x(t), u_1(t), \dots, u_i(t), \dots, u_N(t)) \\ x(0) &= x^0,\end{aligned}\tag{2.3}$$

where  $u_i$  is the control function of the player  $i$  and  $N$  is total number of players. Player  $i$  chooses his control function in a way that maximizes its payoff:

$$L_i[\mathbf{u}(\cdot)] = \int_0^T g_i(x(t), u_1(t), \dots, u_N(t))dt + h_i(x(T)).\tag{2.4}$$

The analysis of differential games relies heavily on concepts and techniques of optimal control theory. Equilibrium strategies in feedback form are best studied by looking at a system of Hamilton-Jacobi-Bellman (HJB) for the value functions of the various players, derived from the principle of dynamic programming. Dynamic programming is based on the principle of optimality. With this principle, an optimal action has the property that whatever the initial state and time are, all remaining decision must also constitute an optimal action. To achieve this principle, the solution can be obtained backwards in time. That is, we starting at all possible final states with the corresponding final times (e.g., stages). The optimal action at this final time is selected, then we proceed back one step in time and determine the optimal action again. This step is repeated until the initial time or stage is reached. The core of dynamic programming when it is applied to continuous time optimal control is a the partial differential equation (PDE) of HJB.

In order to derive the optimal control functions for each player using dynamic programming, first the value functions should be defined as follows:

$$v_i(x, t) = \max_{u(\cdot)} L_i[u(\cdot)]\tag{2.5}$$

and

$$v_i(x, t) = h_i(x). \quad (2.6)$$

For players to play the game, the available information is required. In differential game, there are three cases of available information.

- Open-loop information: With open-loop action, the players have common knowledge of the values of state variables at initial time  $t = 0$ . At this initial state, each player chooses the control variable path by taking into account the expected behavior of all other players. All players commit themselves to their action paths before the game starts.
- Close-loop information: With close-loop information, players are assumed to know the values of state variables from time 0 to  $t$ , i.e.,  $[0, t)$  without delay.
- Feedback information: At time  $t$ , players are assumed to know the values of state variables at time  $t - \epsilon$ , where  $\epsilon$  is positive and arbitrarily small. The information set at time  $t$  can be estimated from the vector of value of state variables of all players at time  $t - \epsilon$ .

At this stage, a natural assumption is that the strategies adopted by players have the feedback form:  $u_i = u_i^*(x^*)$ ; in other words, they depend only on the current state of the system, not the past history. For a Nash non-cooperative solution in feedback form, one can show that the value functions,  $v_i$ , satisfy HJB equations derived from the principle of dynamic programming.

**Theorem 2.1.** [33] *The optimal solutions  $u_i^*$ ,  $i = 1, \dots, N$  lead to a feedback Nash equilibrium solution to the game, and  $x^*(t)$  is the corresponding state trajectory, if there exist suitably smooth functions  $v_i$  satisfying the following rectilinear parabolic*

partial differential equations:

$$-\frac{\partial v_i(x, t)}{\partial t} = \max_{u_i(t, x)} \left\{ \frac{\partial^2 v_i(x, t)}{\partial x^2} + \frac{\partial v_i(x, t)}{\partial x} f[t, x, U_i, U_{j \neq i}^*] + g_i[t, x, u_i, u_{j \neq i}^*] \right\}. \quad (2.7)$$

The HJB equation is usually solved backwards in time, starting from  $t = T$  and ending at  $t = 0$ . In general case, the HJB equation does not have a classical (smooth) solution. Several notions of generalized solutions have been developed to cover such situations, including viscosity solution [35], minimax solution [36]. For the special case of affine-linear quadratic game, the value function has the unique solution which should satisfy a set of first order differential equations. The closed form solution for the optimal action can be obtained for this special case.

### 2.1.3 Stochastic Differential Games

A stochastic formulation for dynamic defined in continuous time of prescribed duration involves a stochastic differential equation describes the evolution of the state as follows,

$$\begin{aligned} \dot{x}(t) &= f(x(t), u_1(t), \dots, u_i, \dots, u_N) + \rho w \\ x(0) &= x^0, \end{aligned} \quad (2.8)$$

where  $w$  represents random fluctuations modeled as Gaussian noise with zero mean and variance  $\sigma^2$ . The value functions of players for the stochastic scenario can be written as:

$$v_i(x, t) = \max_{u_i(t)} L_i = E_w \left\{ \int_0^T g_i(t) dt + h_i[x(T)] \right\}. \quad (2.9)$$

Finally for the optimal control functions can be obtained using stochastic HJB equations as follows:

$$-\frac{\partial v_i(x, t)}{\partial t} = \max_{u_i(t)} \left\{ \frac{(\rho)^2 \sigma^2}{2} \frac{\partial^2 v_i(x, t)}{\partial x^2} + \frac{\partial v_i(x, t)}{\partial x} f[t, x, u_i(t)] + g_i[t, x, u_i(t)] \right\}. \quad (2.10)$$

For special case of quadratic payoff function, the closed-form solutions can be derived. For this case the standard form of the game can be written as follows [37]:

$$\begin{aligned} \dot{x} &= f[x(t), u_1(t), \dots, u_N] + \rho w \\ &= Ax(t) + B_1 u_1(t) + B_2 u_2(t) + \dots + B_N u_N(t) + C + \rho w. \end{aligned} \quad (2.11)$$

The affine-quadratic cost function can be rewritten as follows:

$$g_i = \frac{1}{2} Q x^2 + \sum_i R_i u_i^2 + N \quad (2.12)$$

and

$$h_i = \frac{1}{2} Q^f x^2. \quad (2.13)$$

Finally, for the value function, we have:

$$\begin{aligned} v_i(x, t) &= \max_{u_i(t)} L \\ &= \max_{u_i(t)} E_w \left\{ \int_0^T \mu[x(t), u_i(t)(t)] dt + h[x(T)] \right\}. \end{aligned} \quad (2.14)$$

According to [33], the value function for an affine linear quadratic problem has a unique solution for  $v_i(t)$ :

$$v_i[t] = \frac{1}{2} T_i(t) X(t)^2 + x(t) \zeta_i(t) + \xi_i(t) + m_i(t), \quad (2.15)$$

where  $T_i(t)$  satisfies the following Riccati differential equations

$$\frac{dT_i}{dt} + 2T_iF_i + Q_i + \frac{T_i^2B_i^2}{R_i} = 0, \quad (2.16)$$

$$T_i(T) = Q_i^f, \quad (2.17)$$

and

$$F_i = A - \frac{T_iB_i^2}{R_i}. \quad (2.18)$$

$\zeta_i$  and  $m_i$  can be obtained from the following differential equations, respectively:

$$\frac{d\zeta_i}{dt} + \frac{F_i\zeta_i + T_i\zeta_iB_i^2}{R_i} + T_iB_i = 0, \quad (2.19)$$

$$\zeta_i(T) = 0, \quad (2.20)$$

$$\frac{dm_i}{dt} + \alpha_i\zeta_i + \frac{\zeta_i^2B_i^2}{2R_i} = 0, \quad (2.21)$$

$$m_i(T) = 0 \quad (2.22)$$

and

$$\alpha_i = C - \frac{\zeta_iB_i^2}{R_i}. \quad (2.23)$$

Finally,  $\xi_i$  satisfies the equation below,

$$\frac{d\xi_i}{dt} = -\frac{R_i\sigma^2u_i}{2}. \quad (2.24)$$

The optimal control variable can be obtained as follows:

$$u_i^* = -\frac{B_i}{R_i} \frac{\partial v_i}{\partial x} = -\frac{B_i(T_ix + \zeta_i)}{R_i}. \quad (2.25)$$

As it is shown, the optimal control function constitutes a feedback Nash Equilibrium to the stochastic differential game.

## 2.2 Overview of Satisfaction Games

In real life distributed systems, agents generally do not have knowledge of their opponents strategies. In this context, most game theoretic solution concepts are hardly applicable. Therefore, it is needed to define equilibrium concepts that do not require complete information and are achievable through learning, over repeated play. The satisfaction form is a game theoretical formulation which models systems where players are not interested in maximizing their own utility, rather in satisfying their own constraints [24].

Let us define the game as

$$\mathcal{G}' = (\mathcal{K}, \mathcal{A}^K, \{f_k\}_{k \in \mathcal{K}}), \quad (2.26)$$

where  $\mathcal{K}$  and  $\mathcal{A}^K$  follow the previous definitions and the correspondence  $f_k : \mathcal{A}^{(K-1)} \rightarrow \mathcal{A}$ , called *satisfaction* correspondence, is defined as

$$f_k(\mathbf{a}_{-k}) = \left( a_k \in \mathcal{A} : \sum_{\ell \in \mathcal{L}_k} \infty_{\{\xi_\ell(a_k, \mathbf{a}_{-k}) \geq \Gamma\}} = L_k \right). \quad (2.27)$$

Basically (2.27) is a correspondence which, given the action chosen by the other players, selects all the actions that satisfy the individual constraints. Here, a player can use any of its actions independently of all the other players. The dependence on the other players' actions plays a role only in determining whether a player is satisfied or not.

In this game formulation, the solution concept we adopt is the satisfaction equilibrium (SE) [24] defined as follows:

**Definition 2.2.** (*Satisfaction equilibrium*). A satisfaction equilibrium of game  $\mathcal{G}'$  is

an action profile  $\mathbf{a}' \in \mathcal{A}^K$  such that  $\forall k \in \mathcal{K}$ ,

$$a'_k \in f_k(\mathbf{a}'_{-k}). \quad (2.28)$$

The SE is an action profile where all players are simultaneously satisfying their constraints. In other words, if there exists at least one SE, then  $L^* = L$ , since all players can be satisfied. However, an SE does not always exist for a given game. For instance, if not all the communications can simultaneously take place with the minimum required QoS in the network modeled by the game  $G$ , an SE simply does not exist. An extensive discussion on the existence and multiplicity of an SE in finite games is provided in [25].

### 2.2.1 Efficient Satisfaction Equilibrium

Consider that player  $k$  assigns a cost to each of its actions  $a_k$ , which we denote by  $c_k(a_k)$ . For all  $k \in \mathcal{K}$ , the cost function  $c_k : \mathcal{A}_k \rightarrow [0, 1]$  satisfies the following condition:  $\forall (a_k, a'_k) \in \mathcal{A}_k^2$ , it holds that

$$c_k(a_k) < c_k(a'_k), \quad (2.29)$$

if and only if,  $a_k$  requires a lower effort than action  $a'_k$  when it is played by player  $k$ . In the QoS problem, the effort can be associated with the transmit power or the processing time required to implement a given transmit/receive configuration [26]. Thus, considering the effort or cost of individual actions, one SE which is particularly interesting in the QoS provisioning problem is the one that requires the lowest individual effort.

**Definition 2.3** (Efficient Satisfaction Equilibrium). An action profile  $\mathbf{a}^*$  is an ESE

for the game  $G$ , with cost functions  $\{c_k\}_{k \in \mathcal{K}}$ , if for all  $k \in \mathcal{K}$ ,

$$(i) \quad a_k^* \in f_k(\mathbf{a}_{-k}^*) \text{ and} \quad (2.30)$$

$$(ii) \quad \forall a_k \in f_k(\mathbf{a}_{-k}^*), c_k(a_k) \geq c_k(a_k^*). \quad (2.31)$$

The effort associated by each player with each of its actions does not depend on the choices made by other players. Thus, an ESE  $\mathbf{a}^* \in \mathcal{A}$ , if it exists, is one SE at which player  $k$  is satisfied by using the action  $a_k^*$  that requires the minimum effort among all the actions in  $f_k(\mathbf{a}_{-k})$ . Nonetheless, the existence of an SE does not imply the existence of an ESE.

## 2.2.2 Modeling Drop-ins and Drop-outs

Consider a game played only by a subset  $\mathcal{J} \subset \mathcal{K}$  of the players of the game  $G$  and denote it by

$$G(\mathcal{J}) = \left( \mathcal{J}, \{\mathcal{A}_k\}_{k \in \mathcal{J}}, \{f_k^{(\mathcal{J})}\}_{k \in \mathcal{J}} \right). \quad (2.32)$$

The function  $f_k^{(\mathcal{J})} : \mathcal{A}_{\mathcal{J}} \rightarrow 2^{\mathcal{A}_k}$  determines the set of actions that satisfy the individual constraints of player  $k$  given the actions adopted by the subset of players  $\mathcal{J}$ . In the game  $G(\mathcal{J})$ , players in  $\mathcal{K} \setminus \mathcal{J}$  do not play any role in the decisions adopted by the players in  $\mathcal{J}$ . More precisely, the game  $G(\mathcal{J})$  is obtained when the players in the set  $\mathcal{K} \setminus \mathcal{J}$  have decided to drop out of the original game  $G$  [27].

A player  $j$  drops out of the game  $G$  by playing the action corresponding to a standby state of the link which is denoted by  $A_j^{(0)}$ . In the game  $G$ , such an action  $A_j^{(0)}$  satisfies the following condition for all  $j \in \mathcal{J}$ :

$$f_k^{(\mathcal{J})}(\mathbf{a}_{\mathcal{J} \setminus \{j\}}) = f_k \left( (\mathbf{a}_{\mathcal{J} \setminus \{j\}}, \mathbf{A}_{\mathcal{K} \setminus \mathcal{J}}^{(0)}) \right), \quad (2.33)$$

where the action profile  $\mathbf{A}_{\mathcal{K} \setminus \mathcal{J}}^{(0)}$  represents an action profile in which all players  $k \in$

$\mathcal{K} \setminus \mathcal{J}$  use the action  $A_k^{(0)}$ . The equality in (2.33) shows that when a set of players  $\mathcal{K} \setminus \mathcal{J}$  choose to play their actions  $A_k^{(0)}$  in the game  $G$ , they do not play any role in the choice of the actions of the players in  $\mathcal{J}$ .

The relevance of a game  $G(\mathcal{J})$ , given a set  $\mathcal{J} \subseteq \mathcal{K}$ , stems from the fact that if the game  $G$  does not have an SE, the set  $\mathcal{J}$  can be chosen in order to allow the satisfaction of the largest population of players. That is,  $\mathcal{J}$  can be constructed such that the sub-game  $G(\mathcal{J})$  is the game with the largest population that possesses an SE. We refer to these action profiles as  $N$ -Person Satisfaction Points ( $N$ -PSPs) of the game  $G$ .

**Definition 2.4** ( $N$ -Person Satisfaction Point ( $N$ -PSP)). Assume the game in satisfaction form  $G$  does not possess an SE. Then, an action profile  $(\mathbf{a}_{\mathcal{J}}^*, \mathbf{A}_{\mathcal{K} \setminus \mathcal{J}}^{(0)})$  is said to be an  $N$ -PSP, if  $|\mathcal{J}| = N$  and  $G(\mathcal{J}) = \left( \mathcal{J}, \{\mathcal{A}_k\}_{k \in \mathcal{J}}, \{f_k^{(\mathcal{J})}\}_{k \in \mathcal{J}} \right)$  is the sub-game with the largest set of players that has an SE.

When a game  $G$  possesses at least one SE, any SE is a  $K$ -PSP. That is, when the simultaneous satisfaction of all individual constraints is feasible, SE and  $K$ -PSP are identical notions.

## Chapter 3

# Stochastic Dynamic Hydrothermal Scheduling in a Smart Grid network

### 3.1 Introduction

Recent efforts on smart grids [1, 2, 3, 13, 14, 15, 28] are motivated in part by the increasing demand for electric power, growing interest in finding pollution-free and sustainable energy supply sources, and inadequacy of the current transmission system. Energy storages can balance supply and demand of the electricity market, and mitigate supply side uncertainties. Among grid energy storages, pumped-storage plants generally have the largest available capacity. A pumped storage plant stores off-peak energy using water which is later used for generation during peak periods. Other types of energy storing devices and plug-in electrical vehicles have limited use in power systems due to their relatively small capacity and high costs.

Pumped-storage plants are usually operated within an overall system which contains thermal generation due to very high operating cost of thermal power plants compared to the operating cost of hydro power plant. The hydrothermal generation scheduling is concerned with both hydro plant scheduling and thermal plant dispatching. A variety of optimization methods have been proposed for planning the optimal operations of hydrothermal power systems. The scheduling problems considering deterministic and stochastic programming models have been studied for different time horizons. The planning horizons considered are long-medium term (1 to 3 years) [38, 39, 40], or short-term (weeks to a day) [41, 42]. For short-term models, the optimal operation scheduling of the available generating plants is defined for the following 24 hours. Specifically, authors in [43] introduced the stochastic programming models for the short-term hydro-thermal scheduling problem under uncertain demand.

Authors in [44] developed the stochastic scheduling models for the short-term hydropower production considering the uncertainty of natural inflows in reservoirs.

A widely used paradigm for modeling the hydrothermal power plants behavior in the oligopolistic electricity markets is so called the Nash-Cournot model, dealing with the analysis of the market equilibria [45]. The Nash-Cournot approach assumes that each strategic power plants decides its generation level supposing the energy outputs by the remaining strategic power plants are known. The market scheme is thus simulated through a game: the first strategic power plant chooses its profit-maximizing output under the assumption that the production of the other strategic power plants is known. This is repeated for each strategic power plant that decides its generation level based upon the most recent decisions of the others, until reaching a Nash equilibrium, where no power plant can profit from changing its output levels given the output of all other strategic power plants [46]. In [47], some theoretical results concerning the Cournot model applied to short-term electricity markets are presented. Authors in [48] address a short term hydrothermal scheduling problem using differential dynamic programming but not in a game fashion. The problem is decomposed into a thermal subproblem and a hydro subproblem that are solved in parallel through a constraint relaxed iterative algorithm not in a game fashion. A hydrothermal power exchange market that incorporates network constraints is proposed in [49], the Nash-Cournot equilibrium solution of the market is achieved using the Nikaido-Isoda function, which is derived from the profit maximization functions calculated by the generating companies. The reservoir dynamic is not incorporated in the system model.

In this chapter, we study the competitive interactions between an autonomous pumped-storage hydropower plant and a thermal-power plant in order to optimize power generation and storage. The instantaneous market price can be modeled as a Cournot duopoly game [29, 30]. Here, the dynamic comes from the water volume in

the reservoir, and the stochastic captures the natural inflow and loss to the reservoir. The hydro plant decides how much power to produce, and the thermal plant decides how much to sell to the market or sell to the hydro plant for pump-up storage. The major contributions of this section are:

- We propose a game-theoretical framework in which the thermal and pumped-storage power plants are networked and the thermal plant has the choice to sell the power to the pumped-storage plant.
- We solve the stochastic the Hamilton–Jacobi–Bellman (HJB) equation and obtain an optimal closed-form solutions for both thermal and hydro players.
- We analyze the outcome of interactions between two players and prove it constitutes a feedback Nash equilibrium solution.
- We demonstrate through simulations that the proposed framework can reduce the peak to average ratio and total energy generation of the thermal plant, which help plant operation and reduce CO2 emission with respect to the case that hydro and thermal power plants are working in isolation.

The rest of this chapter is organized as follows: In Section 3.2, the system model is given, and the game is constructed. In Section 3.3, we study the close-form solutions and properties of the proposed game. Simulation results are shown in Section 3.4. Finally, conclusions are drawn in Section 3.5. For better readability, important variables and parameters used in this section are listed in Table 3.1.

## 3.2 System Model and Game Formulation

We consider a smart grid network with one pumped-storage hydro power plant and one thermal power plant as two price makers. Each price maker power plant has autonomy to maximize its own profit by adjusting its generation volume. The

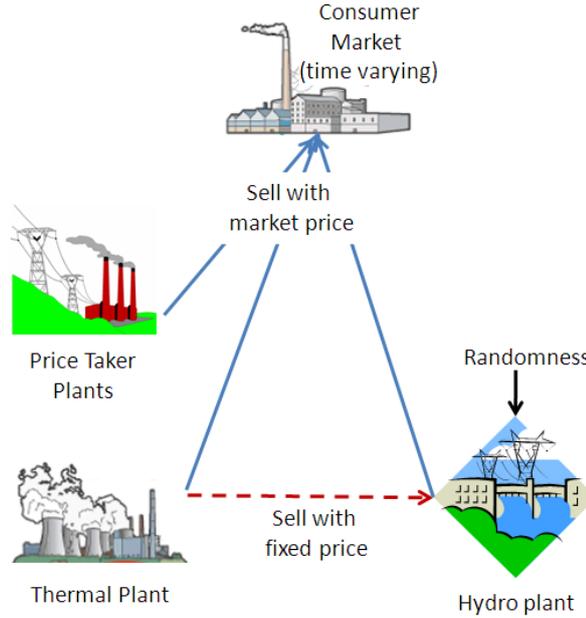
**Table 3.1:** Variables and parameters for system model.

Symbol	Description
$x$	reservoir volume
$r_H$	water discharge rate
$s_H$	water spillage rate
$p_T^g$	thermal plant output to sell to the market
$p_T^s$	thermal plant output to store
$p_H$	pumped-storage output
$w$	natural inflows to the storage
$D$	the total demand
$\beta$	storage leakage rate
$\eta_1$	turbine efficiency
$\eta_2$	generator efficiency
$g$	acceleration of gravity
$p$	market price
$K$	fixed price from thermal to hydro

power can be sold in the market, and the unit power price depends on the demand and supply, and is dynamic over different periods of time. Alternatively, the thermal power plant can sell its power at a fixed price to the pumped-storage hydro plant by storing the energy in the reservoir. The state of available hydroelectric energy depends on the amount of water stored in the reservoirs. The uncertainty of natural inflows and outflows to the reservoir is modeled as stochastic processes. The overall system model is illustrated in Figure 3.1.

Based on the system setup, a quantitative 2-player differential game is defined with the following components<sup>1</sup>: A *time interval*  $[0, T]$  is specified a priori and denotes the duration of the evolution of the game. In this section,  $[0, T]$  represents each hour in a one-day duration. An infinite set with some topological structure is defined for each power plant and is called the *action space*, whose elements are the control functions. For the pumped-storage hydro plant the action  $u_1(t) = r_H(t)$  is the discharged water from the dam; and for the thermal plant, the action  $u_2(t) = p_T^g(t)$  is how much power

<sup>1</sup>Here, we consider a two-player game, and multiple price maker player games will be studied in our future study.



**Figure 3.1:** System and game model.

to sell to the market from its output. Notice that we define  $u_1$  and  $u_2$  here since the definitions will make the analysis clear in the following sections. The actions of the power plant and thermal plant will affect the market price as well as the storage volume in the reservoir. The goal is to study the optimal strategies to control the actions, and analyze the interaction between the two plants.

In the rest of this section, we first discuss the dynamic model for the stored water volume of the reservoir in Section 3.2.1. Then we study how the market price is obtained in Section 3.2.2. Next, we formulate the controls for the hydro plant and thermal plant, in Section 3.2.3 and Section 3.2.4 respectively. Finally, we change the problem formulation in the standard form to simplify analysis in Section 3.2.5.

### 3.2.1 Dynamic Model

An infinite set  $\mathcal{X}$  with a certain topological structure is called the *trajectory space* of the game. Its elements are denoted as  $\{x(t), 0 \leq t \leq T\}$  and constitute the

permissible state trajectories of the game. In our case,  $x(t) \geq 0, \forall t \in [0, T]$  is the current volume of the pumped-storage plant's reservoir <sup>2</sup>. The reservoir dynamics can be characterized as a linear differential equation [48]:

$$\frac{dx(t)}{dt} = -\rho_0\beta x(t) + \rho_1 p_T^s(t) + \rho_2(w - \vartheta_H), \quad (3.1)$$

where  $x$  is the reservoir volume in ( $m^3$ ),  $\beta$  is reservoir leakage rate,  $\rho_0, \rho_1$  and  $\rho_2$  are the constant factors,  $w$  represents random fluctuations modeled as Gaussian noise with zero mean<sup>3</sup> and variance  $\sigma^2$ . In addition,  $p_T$  is the total power generated by the thermal plant and  $p_T^s$  represents the amount power that the thermal plant decides to store. We assume the initial state  $x_0$  is known. It is important to note that, this dynamic model of reservoir is applicable to the normal operation of the pumped-storage power plant and does not consider extreme cases, such as dead storage level or flooding condition. Alternatively boundary conditions can be coped by adding barrier functions [29]. However, there will be no closed-form solution as derived in the sequel and only numerical solutions can be obtained.

Since the generation of the thermal power plant has a slow response to load changes, for simplicity, we assume  $p_T = p_T^g(t) + p_T^s(t)$  to be constant in this section. If  $p_T$  changing slowly over time, a similar approach applies. Finally, the total water released at time  $t$  is shown by  $\vartheta_H(t)$  which can be obtained as:

$$\vartheta_H = r_H + s_H, \quad (3.2)$$

where  $r_H$  is the water discharge rate in ( $m^3/s$ ), and  $s_H$  is water spillage rate in ( $m^3/s$ ) assumed to be constant over time. <sup>4</sup> The pumped-storage power plant generation at

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<sup>2</sup>Here, we omit the maximal power storage constraint due to the difficulty of analysis. From the simulation results, we show the dynamic range of the storage, which is within practical ranges.

<sup>3</sup>non-zero mean case can be studied by adding a constant in (3.1).

<sup>4</sup>In practice, spillage rate is not a constant. However, we can model the randomness together with  $w$ .

time  $t$ ,  $p_H(t)$ , can be estimated as [51]:

$$p_H(t) = \eta(x(t))r_h(t), \quad (3.3)$$

where  $\eta$  is function of the net head or, equivalently, the volume of the stored water in the reservoir. Since  $x$  and  $r_H$  perturbations are small compared with the normal values of these parameters (operating point) for large reservoirs, the linear small disturbance approximation [52] can be used to write the hydro generation as:

$$p_H(t) = W_1 r_H(t) + W_2 x(t) = W_1 u_1(t) + W_2 x(t). \quad (3.4)$$

Let the operating point be  $(x^\dagger, r_h^\dagger)$ ,  $W_1$  and  $W_2$  can then be computed as follows [52]:

$$W_1 = g\eta_1\eta_2x^\dagger \quad (3.5)$$

and

$$W_2 = g\eta_1\eta_2r_h^\dagger. \quad (3.6)$$

Replacing equations (3.2) and (3.4) in equation (3.1), the storage dynamic equation can be rewritten as function of the system state and control variables as follows:

$$\begin{aligned} \frac{dx(t)}{dt} &= f[x(t), u_1(t), u_2(t)]dt + \rho_2 w \\ &= \{-\rho_0\beta x(t) + \rho_1[p_T - p_T^g(t)] - \rho_2\{r_H(t) + s_H\}\} + \rho_2 w. \end{aligned} \quad (3.7)$$

### 3.2.2 Market Price Model

Using the market price model introduced in [45], we assume the price takers have a quadratic operating cost

$$C(O) = \frac{O^2}{2\alpha}, \quad (3.8)$$

where  $O$  is total generation of price taker power plants and  $\alpha$  is a scalar parameter. Using this assumption, given a market spot price  $p$ , the price taker plant generation can be obtained by setting their marginal generation cost equal to the market price. The marginal generation cost can be obtained the derivative of with respect to  $O$ . Therefore, the price taker plants generation is a linear function of the spot price  $p$ , i.e.,  $O(p) = \alpha p$ .

Consider  $D$  as the total demand, and  $O(p)$  as the total generation of price takers as discussed above. The total output of price maker power plants,  $p_H(t) + p_T^g(t)$ , should satisfy the residual demand:

$$D_r = D - O(p) = D - \alpha p. \quad (3.9)$$

Therefore, the spot price can be obtained as:

$$p = \{D - [p_H(t) + p_T^g(t)]\}/\alpha. \quad (3.10)$$

Without loss of generality, we assume  $\alpha = 1$  in the rest of this section for simplicity.

### 3.2.3 Control for Hydro Plant

The revenue of the hydro plant, given the generation of other plants as given is:

$$g_1(t) = [D - p_H(t) - p_T^g(t)] p_H(t) - K p_T^s(t), \quad (3.11)$$

where the first term in (3.11) is the profit of selling power to the market, and the second term is the cost that the thermal player sells to the hydro player to store the power in the reservoir (by pumping water up). Here  $K$  represents the constant price (e.g. per a long-term contract) for selling from the thermal player to the hydro player.

We can rewrite (3.11) with state  $(x(t))$  and control  $(u_1(t))$  and  $u_2(t))$  as

$$g_1(t) = \{D - W_1 u_1(t) - W_2 x(t) - u_2(t)\} [W_1 u_1(t) + W_2 x(t)] - K[p_T - u_2(t)]. \quad (3.12)$$

The stochastic dynamic game of the pumped-storage hydro player is to control its discharged water  $r_H$  so as to maximize the following *utility* over the time interval

$$v_1(x) = \max_{u_1(t)=r_H(t)} L_1 = E_w \left\{ \int_0^T g_1(t) dt \right\}. \quad (3.13)$$

### 3.2.4 Control for Thermal Plant

Similarly, the stochastic dynamic game of the thermal player is to control its selling  $u_2(t) = p_T^g(t)$  and storing  $p_T^s$  so as to maximize its *utility*. Since we assume  $p_T$  to be a constant, the thermal plant's action can be uniquely determined by  $u_2(t)$ . The thermal plant tries to maximize the following:

$$g_2(t) = \{D - p_H(t) - p_T^g\} p_T^g(t) + K p_T^s(t) \{\epsilon_2 p_T^2 + \epsilon_1 p_T + \epsilon_0\} \quad (3.14)$$

and

$$v_2(x) = \max_{u_2(t)=p_T^g(t)} L_2 = E_w \left\{ \int_0^T g_2(t) dt \right\}. \quad (3.15)$$

In (3.14), the first term is the profit to sell in the market, the second term is the profit to sell to the hydro player, and the third term is the power generation cost [53]. Here  $\epsilon_0$ ,  $\epsilon_1$  and  $\epsilon_2$  are constants. We can rewrite (3.14) as

$$g_2(t) = \{D - W_1 u_1(t) - W_2 x(t) - u_2(t)\} u_2(t) + K [p_T - u_2(t)] - \{\epsilon_2 p_T^2 + \epsilon_1 p_T + \epsilon_0\}. \quad (3.16)$$

### 3.2.5 Standard Form Notation

**Table 3.2:** Change of variables table.

$X$	$\frac{3W_2x}{\sqrt{2}} + \frac{P[e]-u_T^g}{2\sqrt{3}}$
$U_1$	$W_1r_H + \frac{W_2x}{2} + \frac{P[e]-u_T^g}{2}$
$U_2$	$u_T^g - \frac{D-(W_1r_H^*+W_2x)-K}{2}$
$A$	$\frac{5\rho_0\beta-\rho_1W_2-3\rho_2W_2}{-4W_2(\frac{3}{\sqrt{2}}+\frac{1}{4\sqrt{3}})}$
$B_1$	$\frac{-\sqrt{2}\rho_0\beta-6\sqrt{3}\rho_1W_2+(\sqrt{2}-12\sqrt{3})\rho_2W_2}{-W_2(12\sqrt{3}+\sqrt{2})}$
$B_2$	$\frac{2\rho_0\beta+12\sqrt{3}\rho_1W_2+(\sqrt{2}+6\sqrt{3})\rho_2W_2}{-W_2(12\sqrt{3}+\sqrt{2})}$
$C$	$\rho_1u_T - \rho_2s_H + \frac{P[e](A+\sqrt{3}B_1-\sqrt{3}B_2)}{2\sqrt{3}} + \frac{B_2K}{2}$
$Q_H, R_H$	2
$Q_T, Q_H^f, Q_T^f$	0
$N_H$	$-Ku_T^s - \frac{(D-u_T^{g*})^2}{3}$
$R_T$	-1
$N_T$	$Ku_T - \left\{ \epsilon_2[u_T^g(t) + u_T^s(t)]^2 + \epsilon_1[u_T^g(t) + u_T^s(t)] + \epsilon_0 \right\} + \frac{(D-(W_1r_H^*+W_2x^*)-K)^2}{4}$

Notice that the utility functions are not in a standard form of the controls. So we change the variables as shown in Table 3.2. By changing the variable, the problem in (3.7) is an affine-quadratic differential game [33] with dynamics in the form of<sup>5</sup>

$$\frac{dX}{dt} = f(X, U_1, U_2)dt + \rho_2w = (AX + B_1U_1 + B_2U_2 + C) + \rho_2w. \quad (3.17)$$

From (3.12) and (3.14), the pumped-storage and thermal plants payoff functions are, respectively, in forms of

$$g_1 = \frac{1}{2}Q_HX^2 + R_HU_1^2 + N_H, \quad (3.18)$$

$$h_1 = \frac{1}{2}Q_H^fX^2, \quad (3.19)$$

<sup>5</sup>for notational simplicity time index  $t$  is omitted

$$g_2 = \frac{1}{2}Q_T X^2 + R_T U_2^2 + N_T, \quad (3.20)$$

and

$$h_2 = \frac{1}{2}Q_T^f X^2. \quad (3.21)$$

In summary, the pumped-storage plant and thermal plant control  $u_1(t) = r_H(t)$  and  $u_2(t) = p_T^g(t)$ , respectively. The dynamics in (3.17) is a linear function of state  $x(t)$  and those two controls, and the utility functions are in the linear quadratic forms of the controls.

### 3.3 Game Analysis and Performance

As an overview, differential games are the extension of the optimal control problems[33]. The Nash feedback equilibrium strategies are derived from a system of The Hamilton-Jacobi-Bellman (HJB) equations for the value functions of the players. The solution is obtained backwards in time using dynamic programming. That is, starting from all possible final states, the optimal action at each final time is selected, we then induce backward one step at time and determine the optimal action at each stage. This process is repeated until the initial time or stage is reached.

In this section we analyze the nonzero sum stochastic differential game of the thermal and pumped-storage power plants based on the models in the previous section. In the sequel, we will study the payoff maximization of pumped-storage plant and thermal plant given the other's strategy is fixed in Section 3.3.1 and Section 3.3.2, respectively. Finally, we discuss the outcome of the proposed games in Section 3.3.3.

#### 3.3.1 Pumped-Storage Player Payoff Maximization

First, given the thermal plant's strategy, the pumped-storage plant calculates its the best strategy through the stochastic HJB equation. This equation is considered

to be the first-order necessary and sufficient condition obeyed by the optimal value function and can be used to find the optimal time paths of the state, costate, and control variables. If the HJB equation is solvable (either analytically or numerically), an optimal feedback control can be obtained by taking the maximizer involved in the HJB equation [33]. To optimize the utility function in (3.13), the HJB equation for the pumped-storage power plant can be written as follows:

$$-\frac{\partial v_1}{\partial t} = \max_{U_1} \left\{ \frac{(\rho_2)^2 \sigma^2}{2} \frac{\partial^2 v_1}{\partial X^2} + \frac{\partial v_1}{\partial X} f(X, U_1, U_2^*) + g_1 \right\} \quad (3.22)$$

and

$$v_1[X(T)] = h_1[X(T)]. \quad (3.23)$$

In order to obtain the optimal action for each player of game, first the related value functions have to be derived, for the special case of affine-linear quadratic game, the value function has the unique solution [33] which should satisfy a set of first order differential equations. The closed form solution for the optimal action of hydro plant is presented through the following lemma.

**Lemma 3.1.** *Using the HJB equation, the optimal control policy for the pumped-storage hydro plant can be obtained as:*

$$U_1^* = -\frac{B_1}{R_H} \frac{\partial v_1}{\partial X} = -\frac{B_1(T_H X + \zeta_H)}{R_H}. \quad (3.24)$$

*Proof.* The value function of the hydro player can be written as:

$$v_1[t] = \frac{1}{2} T_H(t) X(t)^2 + X(t) \zeta_H(t) + \xi_H(t) + m_H(t), \quad (3.25)$$

where  $T_H(t)$  satisfies the following Riccati differential equations [33]

$$\frac{dT_H}{dt} + 2T_H F_H + Q_H + \frac{T_H^2 B_1^2}{R_H} = 0, \quad (3.26)$$

$$T_H(T) = Q_H^f, \quad (3.27)$$

and

$$F_H = A - \frac{T_H B_1^2}{R_H}. \quad (3.28)$$

$\zeta_H$  and  $m_H$  can be obtained from the following differential equations, respectively:

$$\frac{d\zeta_H}{dt} + \frac{F_H \zeta_H + T_H \zeta_H B_1^2}{R_H} + T_H B_1 = 0, \quad (3.29)$$

$$\zeta_H(T) = 0, \quad (3.30)$$

$$\frac{dm_H}{dt} + \alpha_H \zeta_H + \frac{\zeta_H^2 B_1^2}{2R_H} = 0, \quad (3.31)$$

$$m_H(T) = 0, \quad (3.32)$$

and

$$\alpha_H = C - \frac{\zeta_H B_1^2}{R_H}. \quad (3.33)$$

Finally,  $\xi_H$  satisfies the equation below

$$\frac{d\xi_H}{dt} = -\frac{R_H \sigma^2 U_1}{2}. \quad (3.34)$$

The optimal control variable can be obtained as follows:

$$U_1^* = -\frac{B_1}{R_H} \frac{\partial v_1}{\partial X} = -\frac{B_1(T_H X + \zeta_H)}{R_H}. \quad (3.35)$$

□

### 3.3.2 Thermal Player Payoff Maximization

Next, we determine the optimal strategy of the thermal plant given the optimal strategy of the hydro plant. To optimize the utility in (3.15), the stochastic HJB

equation of the proposed game for the thermal player can be represented by following equation

$$-\frac{\partial v_2}{\partial t} = \max_{U_2} \left\{ \frac{(\rho_2)^2 \sigma^2}{2} \frac{\partial^2 v_2}{\partial X^2} + \frac{\partial v_2}{\partial X} f(X, U_1^*, U_2) + g_2 \right\} \quad (3.36)$$

and

$$v_2[X(T)] = h_2[X(T)]. \quad (3.37)$$

The optimal action for the thermal plant can be obtained through the following lemma.

**Lemma 3.2.** *The optimal control policy for the thermal plant can be obtained as follows:*

$$U_2^* = -\frac{B_2}{R_T} \frac{\partial v_2}{\partial X} = -\frac{B_2(T_T X + \zeta_T)}{R_T}. \quad (3.38)$$

*Proof.* The value function of thermal player can be written as:

$$v_2[t] = \frac{1}{2} T_T(t) X(t)^2 + X(t) \zeta_T(t) + X(t) \xi_T(t) + m_T(t). \quad (3.39)$$

Here,  $T_T(t)$  satisfies the following Riccati differential equations [33]

$$\frac{dT_T}{dt} + 2T_T F_T + Q_T + \frac{T_T^2 B_2^2}{R_T} = 0, \quad (3.40)$$

$$T_T(T) = Q_T^f, \quad (3.41)$$

and

$$F_T = A - \frac{T_T B_2^2}{R_T}. \quad (3.42)$$

$\zeta_T$  and  $m_T$  can be obtained through similar steps in (3.29)-(3.34):

$$\frac{d\zeta_T}{dt} + \frac{F_T \zeta_T + T_T \zeta_T B_2^2}{R_T} + T_T B_2 = 0, \quad (3.43)$$

$$\zeta_T(T) = 0, \quad (3.44)$$

$$\frac{dm_T}{dt} + \alpha_T \zeta_T + \frac{\zeta_T^2 B_2^2}{2R_T} = 0, \quad (3.45)$$

$$m_T(T) = 0, \quad (3.46)$$

$$\alpha_T = C - \frac{\zeta_T B_2^2}{R_T}, \quad (3.47)$$

and  $\xi_T$  from the equation below

$$\frac{d\xi_T}{dt} = -\frac{R_T \sigma^2 U_2}{2}. \quad (3.48)$$

The optimal control variable can be obtained as follows:

$$U_2^* = -\frac{B_2}{R_T} \frac{\partial v_2}{\partial X} = -\frac{B_2(T_T X + \zeta_T)}{R_T}. \quad (3.49)$$

□

### 3.3.3 Nash Equilibrium

For a two-person differential game in the form discussed in the previous subsections, a two-tuple strategies constitutes a feedback Nash equilibrium solution [33] as defined below:

**Definition 3.3.** *A set of controls  $U_i^*(t, X), i \in 1, 2$  constitute a feedback Nash equilibrium of the game, if there exists functions  $v_i(X, t)$  satisfying the following relations  $\forall i$  :*

$$\begin{aligned} v_i(X, t) &= E_w \left[ \int_0^T g_i \{t, X^*(t), U_1^*[t, X(t)], U_2^*[t, X(t)]\} dt \right] \\ &\geq E_w \left[ \int_0^T g_i \{t, X^{[i]}(t), U_i[s, X(s)], U_{j \neq i}^*[t, X(t)]\} dt \right], \end{aligned} \quad (3.50)$$

where

$$\frac{dX^{[i]}(t)}{dt} = f\{t, X^{[i]}(t), U_i[t, X(t)], U_{j \neq i}^*[t, X(t)]\} \quad (3.51)$$

and

$$\frac{dX^*(s)}{dt} = f\{t, X^*(t), U_1^*[t, X(t)], U_2^*[t, X(t)]\}. \quad (3.52)$$

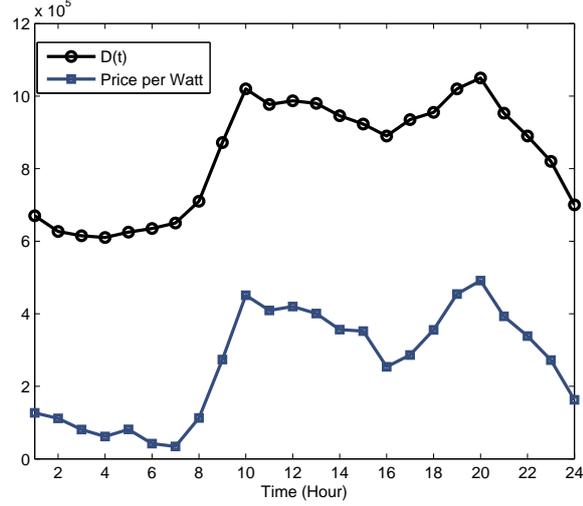
If there exists the value functions that are twice continuously differentiable, then the two partial differential feedback Nash equilibrium solutions in continuous time can be characterized using the stochastic HJB equations [33], which are necessary conditions of the candidate optimal control strategy. The feedback Nash equilibrium is defined as follows (Theorem 6.27[33]):

**Theorem 3.4.** *A set of feedback strategies  $U_i^*(t, X^*)$  lead to a feedback Nash equilibrium solution to the game, and  $X^*(t), 0 \leq t \leq T$  is the corresponding state trajectory, if there exist suitably smooth functions  $V_i$  satisfying the following rectilinear parabolic partial differential equations:*

$$\begin{aligned} -\frac{\partial v_i(X, t)}{\partial t} &= \max_{U_i(t, X)} \left\{ \frac{(\rho_2)^2 \sigma^2}{2} \frac{\partial^2 v_i(X, t)}{\partial X^2} \right. \\ &\left. + \frac{\partial v_i(X, t)}{\partial X} f[t, X, U_i, U_{j \neq i}^*] + g_i[t, X, U_i, U_{j \neq i}^*] \right\}. \end{aligned} \quad (3.53)$$

**Lemma 3.5.** *The feedback Nash equilibrium exists for the proposed game in (3.17), (3.18), (3.19), (3.20), and (3.21).*

*Proof.* Since the value functions in equations (3.25) and (3.39) are twice continuously differentiable, the derived optimal solution by HJB equations characterize the feedback Nash equilibrium solutions.  $\square$

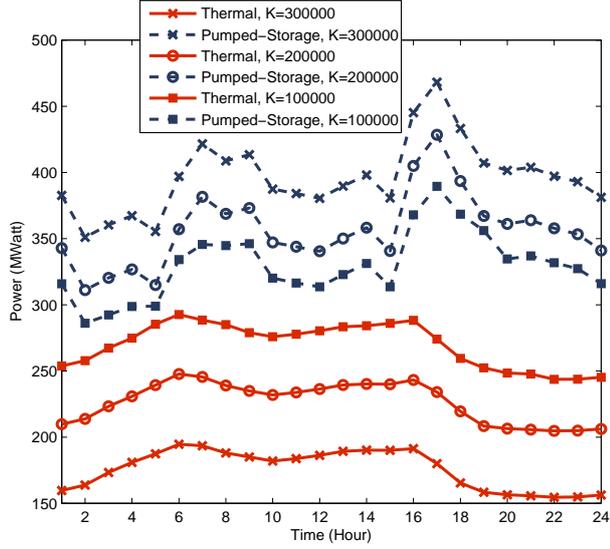


**Figure 3.2:** Daily demand  $D(t)$  versus market price based on data from [57].

### 3.4 Simulation Results

In this section, to clarify the game analysis results derived in Section 3.3, we investigate the performance of the proposed game numerically. The simulations are performed for each hour in a one-day duration. The thermal and pumped-storage plants have the maximum generation capacities of  $500MW$  and  $300MW$ , respectively. Other parameters are set as follows:  $\beta = 0.05$ ,  $\rho_0 = 3.4 * 10^{-4}$ ,  $\rho_1 = 3.83 * 10^{-5}$ ,  $\rho_2 = 0.001$ ,  $\sigma^2 = 0.5$ ,  $\eta_1 = 0.9$ ,  $\eta_2 = 0.96$ ,  $x^\dagger = 550Mm^3$ ,  $r_h^\dagger = 103 * 10^{-1} \frac{m^3}{s}$ ,  $x_{max} = 717Mm^3$  and  $\epsilon_2 = \epsilon_1 = \epsilon_0 = 0.5$ . The above characteristic values for the hydro plant's reservoir are selected based on [56]. The constant price  $K$  for thermal player to sell to the hydro player is assumed to be equal to 100000, unless we otherwise define. Figure 3.2 shows fluctuations of total demand  $D$  according to [57], and how the market price per Watt ( $\{D - p_H(t) - p_T^g(t)\}$ ) is affected accordingly. Notice that the market price can be affected by many other players in the market. Here we consider only the thermal and hydro players so that the interactions can be clearly demonstrated.

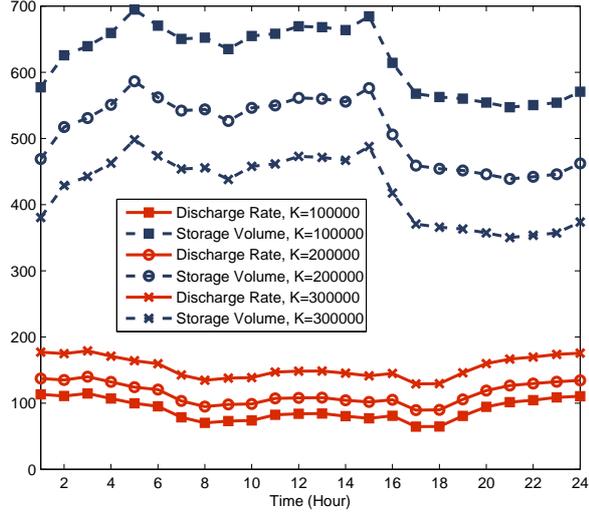
In Figure 3.3, the pumped-storage plant generation amount ( $p_H$ ) and thermal



**Figure 3.3:** Comparison of the output power versus  $K$ .

plant generation decision ( $p_T^g$ ) are compared over time. The amount of generation of the thermal plant and pumped-storage plant both follow the fluctuations of demand. However, compared with the demand in Figure 3.2, the thermal player has less fluctuation, and the hydro (pumped storage plant) player compensates for the fluctuation. This can help practical operations since the thermal player has a slow response to meet the demand's fluctuations. Also, we change the price per Watt of the electricity that the thermal plant sells power to the pumped-storage plant,  $K$  from 100000 to 300000. By increasing  $K$  the thermal plant finds selling its output power to the pumped-storage plant more beneficial than selling it to the market. Therefore, the amount of power that the thermal plant generates for selling to the market decreases and the generation of pumped-storage plant increases, respectively to satisfy the demand.

The reservoir volume and water discharge rate of the pumped-storage plant in each hour is shown in Figure 3.4. Comparing with Figure 3.3, the reservoir volume varies with the generation value of the pumped-storage plant. For instance when the plant decides to increase its output power, the discharge rate increases and the

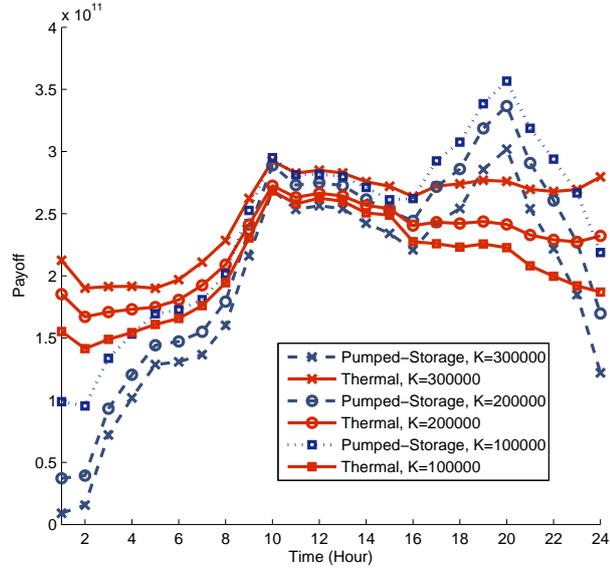


**Figure 3.4:** Reservoir volume and discharge rate.

reservoir level decays. When  $K$  increases, the volume drops and the discharge rate increases. Moreover, notice that the power generation is a function of both reservoir volume and water discharge rate.

In Figure 3.5, the payoffs of power plants from equations (3.11)-(3.14) are compared for different values of  $K$ . For all scenarios, the variations of the payoff functions are correlated to the changes in the plants outputs and the market values. Accordingly, by increasing  $K$  the thermal plant's payoff increases and that of the pumped-storage decreases.

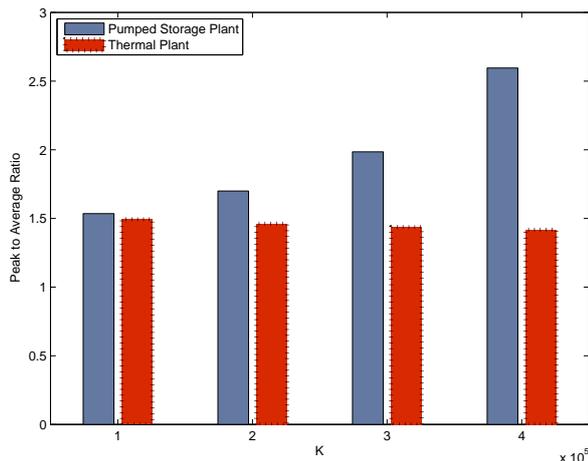
In Figure 3.6, the peak to average ratios of thermal and pumped-storage output power are depicted as  $K$  varies. As the price per Watt from the pumped-storage plant increases, the thermal plant prefers to sell its output to the pumped-storage plant rather than to sell it to the market price. Consequently, the peak to average ratio of the thermal plant decreases by increasing  $K$  from from 100000 to 400000. This also demonstrates that the proposed framework can reduce the peak to average ratio of the thermal plant by introducing the pricing mechanism between the thermal and hydro players.



**Figure 3.5:** Comparison of payoff versus  $K$ .

Figure 3.7 shows comparison of the output power of thermal and pumped-storage plants as the mean of stochastic inflow to reservoir varies. Increasing the mean of incoming water with constant variance increases the output of the pumped-storage plant. This provides a guidance for the system to operate during the dry and wet seasons.

The average output power of the thermal and pumped-storage plant in two scenarios is shown in Figure 3.8. In the first scenario, it is assumed that there is no interaction between two power plants, and they just sell their outputs to the market ( $K = 0$ ). In the second scenario, plants are networked with suggested price from pumped-storage plant ( $K = 200000$ ). It can be seen that the later case, increases the participation of pumped-storage plant in demand satisfaction and reduces the thermal's one, which yields a greener choice compared to the first scenario.



**Figure 3.6:** Comparison of output power peak to average ratio versus  $K$ .

### 3.5 Summary

In this chapter, we studied the problem of optimal generation and storage for two types of competitive power plants in smart grid networks. We have proposed a stochastic dynamic game approach to model their competition. The market price is based on the Cournot duopoly game model. The thermal power player can sell its power to the hydropower player at a fixed price or to the market at the market price. Based on the stochastic HJB equation, we derive the strategies for both players if the other's action is fixed. We showed that there exists the feedback Nash equilibrium strategies for the proposed game. Simulation results demonstrate the properties of the proposed game and suggest how the two types of power plants need to adjust their generating and storage decision variables to maximize their revenues. It is demonstrated that the proposed framework and games can reduce the peak to average ratio and total energy generation for the thermal plant, which helps power plant operation and reduces CO2 emission.

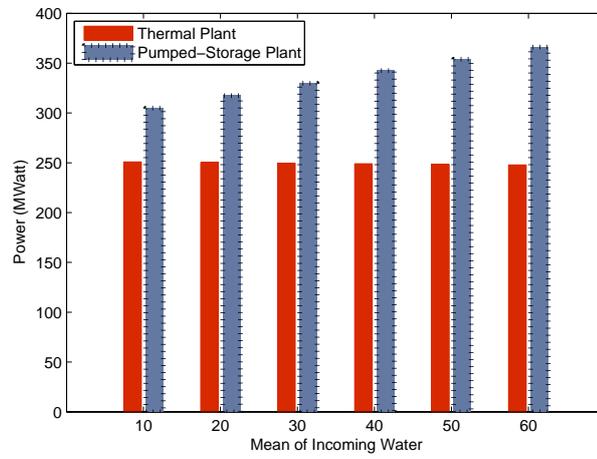


Figure 3.7: Power output versus mean of incoming water to the reservoir.

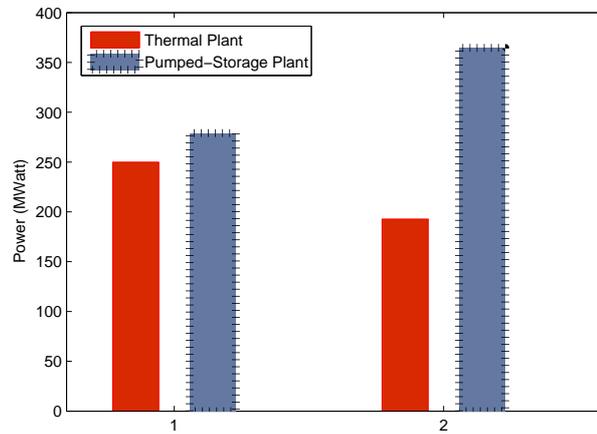


Figure 3.8: 1-Two plants are not networked, 2-Two plants are networked.

## Chapter 4

### Distributed Dynamic Control for Smart Buildings

#### 4.1 Introduction

Demand Response (DR) programs are implemented by utility companies to control the energy consumption at the customer side of the meter. Two popular DR approaches are direct load control (DLC) and smart pricing. In DLC [60, 61, 62, 63], an aggregator can remotely control the operations and energy consumption of certain consumer appliances. In contrast, in smart pricing, users are encouraged to individually and voluntarily manage their load, e.g., by reducing their consumption at peak price hours. This can be done using automated Energy Consumption Scheduling (ECS) devices [64]. For each user, the ECS finds the best load schedule to minimize the user's electricity cost while fulfilling the user energy needs. This can lead to autonomous demand response programs that burden a minimal control overhead on utilities.

A common analytical tool to study autonomous DR systems is game theory [65], that provides a framework to study rational interactions and outcome in a distributed manner. In [66], a stochastic game is developed to model an hourly energy auction in which generators and consumers participate as adaptive agents.

In [67], authors proposed a game theoretic demand response scheme to replace traditional centralized load prediction with a distributed load prediction system that involves user participation. Authors in [68] employed the Cournot game model to analyze the market effect of a demand response aggregator on both shifting and reducing deferrable loads. Authors in [69] developed a hybrid day-ahead and real-time consumption scheduling for a number of houses that participate in a demand side program based on game theory. The interaction between the service provider and the users is modeled as a Stackelberg game in [70] to derive the optimal real-time

electricity price and each user's optimal power consumption. In [71], a residential energy consumption scheduling framework is proposed, which attempts to achieve a desired trade-off between minimizing the electricity payment and minimizing the waiting time for the operation of household appliance in presence of real-time prices using price prediction. In [72], game theory is used for demand side management to reduce the peak-to-average-ratio in aggregate load demand. In [30], a tutorial is given for the game-theoretic methods on microgrid systems, demand-side management, and smart grid communications.

Different from the prior work in [66]-[30], in this paper, we focus on game-theoretic analysis of price-based DR programs where controllable load types are associated with *dynamic systems* and can be modeled using differential equations. Examples of such loads include heating, ventilation, and air conditioning (HVAC), water heating, refrigeration, and plug-in electric vehicles. In particular, we apply techniques from stochastic differential games [33]. To the best of our knowledge, this paper is the first work to study differential games in the context of price-based DR programs. The contributions in this chapter can be summarized as follows:

1. We study the strategic interactions between a Nash Cournot electricity market and multiple energy-smart buildings to construct a two-level stochastic differential game framework. At the upper level, the market offers a vector of hourly prices to end users. At the lower level, the energy-smart buildings as the lower level participate in demand response by managing controllable dynamic load in response to hourly prices set by the market.
2. We focus on smart buildings equipped with renewable resources generators, local energy storage and controllable HVAC units, in which users are able to respond to real-time grid conditions like electricity prices and weather conditions in order to minimize their cost.

3. We derive the optimal closed-form control strategies for each energy-smart building obtained by solving stochastic the Hamilton-Jacobi-Bellman (HJB) equation. We analyze the outcome of interactions between two levels and constitute a feedback Nash equilibrium solution.
4. The proposed technique comparing with the day-ahead pricing method, makes the load profile more flat and reduces the peak-to-average ratio (PAR) of aggregate load.
5. Using simulation results we show that by implementing our proposed stochastic differential DR game model, we can minimize the electricity cost of buildings.

The rest of this chapter is organized as follows. The system model is described in Section 4.2. The stochastic differential game is constructed and its solution to the proposed game is derived in Section 4.3. Simulation results are presented in Section 4.4. Conclusions are drawn in Section 4.5.

## 4.2 System Model

In this section, we explain the system model that incorporates the impact of demand response on both supply and demand sides when real-time pricing is used. As illustrated in Figure 4.1, we study a two-level design framework: at the upper level, at the beginning of each time interval, e.g., at each hour, the market decides on a price to pass on to the end-users in the lower level, based on the total demand data from the lower level during the last time interval. The ECS unit of each building minimizes the cost of electricity consumption. Since there are multiple buildings competing for the electricity resources, the system can be analyzed using game theory [65].

### 4.2.1 Smart Building Consumers

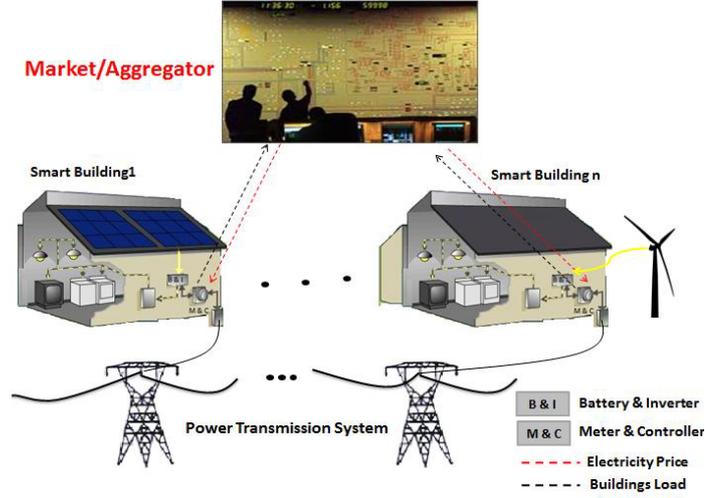
Consider a total of  $N$  energy-smart buildings that participate in demand response program. Each building has two specific controllable loads: an air conditioner with a controllable thermostat, and an always-connected battery. We also assume that a renewable source of energy, e.g., a residential wind turbine or a roof-top solar panel, is available in each building, with its generated output to be used to charge the battery. Uncontrollable appliances with a total and known consumption of  $l(t)$  constitute the rest of the building power consumption. Given the price that is a function of the optimal strategy of the upper level player, the decision variables available for consumers at each building  $i = 1, \dots, N$  are:

$$\begin{aligned} u_1^i &= \text{power draw from battery for home usage} \quad \text{and} \\ u_2^i &= \text{air conditioner usage of electricity.} \end{aligned}$$

And the dynamic *states* include:

$$\begin{aligned} x_1^i &= \text{the energy stored in the battery array} \quad \text{and} \\ x_2^i &= \text{the indoor temperature of home.} \end{aligned}$$

The output power of the renewable generator is indeed random. In our analysis, it is modeled as  $W + e^i$ , where  $W$  is the renewable output prediction that is obtained using a day-ahead forecasting method and  $e^i$  denotes the prediction error which is a Gaussian random variable with zero mean and variance  $\sigma^2$ . As an example, the amount of power generated by a wind turbine can be modeled as a function of wind speed. As for the outside temperature, we assume that its day-ahead predictions are used based on standard weather forecasting data. For each smart building, the



**Figure 4.1:** The interactions between the aggerator and individual buildings.

dynamics of the states can be modeled using the following differential equations:

$$\begin{bmatrix} \dot{x}_1^i \\ \dot{x}_2^i \end{bmatrix} = \begin{bmatrix} -\beta^i & 0 \\ 0 & (\epsilon^i - 1) \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -\gamma^i(1 - \epsilon^i)K^i \end{bmatrix} \begin{bmatrix} u_1^i \\ u_2^i \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} W + \begin{bmatrix} e^i \\ (1 - \epsilon^i)t_{OD}^i \end{bmatrix}. \quad (4.1)$$

The differential equation in the first row in (4.1) models the dynamic of the battery's state-of-charge. The differential equation in the second row models the variation in the building's indoor temperature. Here,  $\beta^i$  denotes the battery leakage rate. As the battery dynamic equation shows the output of renewable resource  $W + e^i$  acts as the input to the battery, and the amount of power that is discharged for usage in the building acts as the output of battery.

The thermal model in (4.1) is based on the a building thermal model in [73]. Here,  $\epsilon^i$  is the factor of inertia of the building which is a function of time constant of the building and overall thermal conductivity,  $\gamma^i$  is a factor capturing the efficiency of the air conditioning unit to cool the air inside the building,  $t_{OD}^i$  is the outside temperature

and  $K^I$  is a constant that depends on the performance of the air conditioning unit and the total thermal mass. The air conditioning unit uses power  $u_2^i$  to cool down the home's indoor temperature. Note that, in this model, our focus is only on the cooling scenario. The results for the heating scenario are similar and can be obtained by changing the sign of  $-\gamma i(1 - \epsilon^i)K^i$  from negative to positive. At the beginning of each time interval<sup>1</sup>, given the  $M \times 1$  demand vector  $\mathbf{U}_D$  from all  $N$  feeders that all buildings are connected to, a grid operator checks total available generation in the market and determines the price. Considering an estimated quadratic cost function for the oligopolistic electricity market [74], yields the electricity spot price as a linear function of aggregated building consumptions[75]. This model would help to study the impact of large-scale buildings' power generation and consumption on spot price as follows:

$$p = p_c + \frac{[\sum_{j=1}^N d^j - \sum_{j=1}^G g^j]}{\alpha}, \quad (4.2)$$

where  $p_c$  is a constant price factor decided by market,  $d^j$  and  $g^j$  are the electricity consumption and generation of building  $j$  respectively and  $\alpha$  is a scalar parameter. Increasing  $\alpha$  reduces the impact of buildings on spot price. From Section 4.2.1, for each smart building  $i$ , the total power consumption can be calculated as

$$d^i = l^i - u_1^i + u_2^i. \quad (4.3)$$

A price factor which can be defined as

$$\tau^i = \sum_{j \neq i}^N d^j - \sum_{j=1}^G g^j \quad (4.4)$$

is reported from the aggregator to the ECS device of building  $i$  hourly. Given the

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<sup>1</sup>Without loss of generality, we assume the time intervals is one hour in this paper. Other time interval can be implemented in a similar way

hourly price information from market including  $p_{const}$ ,  $\alpha$  and  $\tau^i$ , the management unit can estimate the price.

### 4.3 Differential Game Analysis

If a centralized control of all buildings is feasible, then one can formulate a stochastic dynamic optimization problem to control the operation of the battery storage and air conditioner units in all buildings so as to maximize the aggregate utility of all users. For each user, the utility function depends on both the cost of electricity and how comfortable the temperature feels like. An alternative approach is to use game theory to a distributed optimization framework to be implemented by each smart building using just local information and is able to address some of our key optimization challenges such as a) The heterogeneous nature of building ECS systems. b) The complexity of interactions among smart buildings. c) The non-linear the formulated optimization problems. Note that, centralized optimization across all buildings is not practical due to the need for collecting private information.

Next we explain our proposed dynamic game formulation and discuss some of its properties. In particular, we prove that the optimal solutions constitute a feedback Nash equilibrium for the formulated game.

#### 4.3.1 Game Formulation

For each time  $t$ , the stochastic dynamic game of each smart building  $i$  is to control the battery output used for building usage,  $u_1^i(t)$ , and the air conditioner electricity usage,  $u_2^i(t)$ , so as to minimize the cost. We model the cost at time  $t$  as a quadratic

function of the building power consumption:

$$\begin{aligned}
\mu^i[u_1^i(t), u_2^i(t)] = & \tag{4.5} \\
p(t)[l^i(t) - u_1^i(t) + u_2^i(t)] + \eta[x_2^1(t) - x_d]^2 = & \\
\frac{1}{\alpha}[\alpha p_c + \sum_{i=1}^N(l^i(t) - u_1^i(t) + u_2^i(t) - g^i(t))] & \\
[l^i(t) - u_1^i(t) + u_2^i(t)] + \eta^i[x_2^i(t) - x_d^i]^2, &
\end{aligned}$$

where the first term represents the cost of the building electricity consumption, and the second term models penalty of temperature differences from the desired value,  $t_d$ . By minimizing the objective function in (4.5), we achieve the optimal policies that can balance the trade-off between user comfort and electricity cost minimization by controlling the HVAC usage and local energy storage, given the current states of the system which follows the dynamics in (4.1).

Next, we introduce the expected utility function of each building over the random nature of renewable energy during a time period of interest, e.g., one day, as follows:

$$L^i = E_w \left\{ \int_0^T \mu^i(t) dt + h^i[x(T)] \right\}, \tag{4.6}$$

where  $h[x(T)]$  is the terminal condition for value function. The value function of  $u_1^1(t)$  and  $u_2^1(t)$  can be written as

$$v^i(x, t) = \min_{u_1^i(t), u_2^i(t)} L. \tag{4.7}$$

Without loss of generality, we assume that

$$h^i[x(T)] = v^i(x, T) = 0. \tag{4.8}$$

To convert our stochastic differential optimization problem into a linear quadratic

format, we use changes of variables

$$\mathbf{X} = [x_1^1, x_2^1, x_1^2, x_2^2, \dots, x_1^i, x_2^i - x_d, x_1^n, x_2^n]^T \quad (4.9)$$

and

$$\begin{aligned} \mathbf{U}^i = & \left[ u_1^1 - \frac{\sum_{j \neq i}^n l^j - \sum_{j=1}^n g^j + \alpha p_c}{2(n-1)}, -u_1^2 - \frac{\sum_{j \neq i}^n l^j - \sum_{j=1}^n g^j + \alpha p_c}{2(n-1)} \right. \\ & , \dots, u_1^i - \frac{l^i}{2}, -u_2^i - \frac{l^i}{2}, \dots, \\ & \left. u_1^n - \frac{\sum_{j \neq i}^n l^j - \sum_{j=1}^n g^j + \alpha p_c}{2(n-1)}, -u_2^n - \frac{\sum_{j \neq i}^n l^j - \sum_{j=1}^n g^j + \alpha p_c}{2(n-1)} \right]. \end{aligned} \quad (4.10)$$

As a result, the game dynamics can be written in matrix form as follows:

$$\begin{aligned} \dot{\mathbf{X}} &= f[\mathbf{X}(t), \mathbf{U}(t)] + \rho \mathbf{W} \\ &= \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t) + \mathbf{C} + \rho \mathbf{W}, \end{aligned} \quad (4.11)$$

where

$$\mathbf{A} = \begin{bmatrix} -\beta^1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & \epsilon^1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & -\beta^2 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \epsilon^2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & -\beta^n & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & \epsilon^n \end{bmatrix}_{2n \times 2n}, \quad (4.12)$$

$$\mathbf{B} = \begin{bmatrix} -1 & 0 & 0 & \dots & 0 \\ 0 & -\gamma^1 K^1 (1 - \epsilon^1) & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -1 & 0 \\ 0 & 0 & \dots & 0 & -\gamma^n K^n (1 - \epsilon^n) \end{bmatrix}_{2n \times 2n}, \quad (4.13)$$

$$\mathbf{C} = \begin{bmatrix} e^1 + \frac{l^1}{2} \\ (1 - \epsilon^1)(t_{OD}^1 + \frac{\gamma^1 K^1 (l^1)}{2}) + \epsilon^1 x_d^1 \\ \vdots \\ e^n + \frac{l^n}{2} \\ (1 - \epsilon^n)(t_{OD}^n + \frac{\gamma^n K^n (l^n)}{2}) + \epsilon^n x_d^n \end{bmatrix}_{2n \times 1}, \quad (4.14)$$

and

$$\rho = \begin{bmatrix} 1, 0, 1, \dots, 1, 0 \end{bmatrix}_{1 \times 2n}^T. \quad (4.15)$$

The cost function in (4.5) can be rewritten as follows:

$$\begin{aligned} \mu^i[\mathbf{X}(t), \mathbf{U}(t)] &= \\ &= \frac{1}{2}[\mathbf{X}(t)^T \mathbf{Q}_s^i \mathbf{X}(t) + \mathbf{U}(t)^T \mathbf{R}^i(t) \mathbf{U}(t)]. \end{aligned} \quad (4.16)$$

Finally, for the value function in (4.7), we have:

$$\begin{aligned} v^i(X, t) &= \min_{\mathbf{U}(t)} L \\ &= \min_{\mathbf{U}(t)} E_w \left\{ \int_0^T \mu^i[\mathbf{X}(t), \mathbf{U}(t)] dt + h^i[\mathbf{X}(T)] \right\}, \end{aligned} \quad (4.17)$$

where

$$h^i[\mathbf{X}(T)] = \frac{1}{2} \mathbf{X}^T \mathbf{Q}_{sf}^i \mathbf{X}, \quad (4.18)$$

$$\mathbf{R}^i(t) = \frac{1}{\alpha} \begin{bmatrix} 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \dots & 1 & 1 & -1 & 1 & 1 & \dots & 1 \\ 1 & \dots & 1 & -1 & 1 & 1 & 1 & \dots & 1 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{2n \times 2n}, \quad (4.19)$$

and

$$\mathbf{Q}_s^i(t) = \begin{bmatrix} 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & \dots & 0 & \eta & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{2n \times 2n}. \quad (4.20)$$

The stochastic differential optimization problem in (4.17)-(4.20) has an affine quadratic format. Next, we use dynamic programming to derive the optimal control solution for each building.

### 4.3.2 Solution Based on Dynamic Programming

Differential games are the extension of the basic optimal control problem and their analysis relies heavily on concepts and techniques in optimal control theory [33]. Equilibrium strategies in the feedback form are best studied by looking at a system of Hamilton-Jacobi-Bellman (HJB) equations for the value functions of various players. Using dynamic programming, the solution is obtained backwards in time. That is, we start at all possible final states with the corresponding final times. The optimal action at each final time is selected, we then proceed back one step in time and determine the optimal action at each stage. This process is repeated until the initial time or stage is reached. The core of dynamic programming when it is applied to continuous-time optimal control is the partial differential equation (PDE) of an HJB formulation.

Now consider the stochastic control problem in the stochastic control problem in (4.17)-(4.20) can be derived using the stochastic HJB equation [33]:

$$-\frac{\partial v^i}{\partial t} = \min_{\mathbf{U}^i(t)} \left\{ \frac{\sigma^2}{2} \frac{\partial^2 v^i(\mathbf{X}, t)}{\partial X^2} + \nabla_X v^i(\mathbf{X}, t) f^i[\mathbf{X}(t), \mathbf{U}(t)] + \mu^i[\mathbf{X}(t), \mathbf{U}(t)] \right\}.$$

The HJB equation above is a sufficient condition to obtain the optimal solutions. If our value function solves this partial differential equation, then the optimal controls that achieve the minimum cost can be readily obtained from the value function. In general, the HJB equation does not have a classical (smooth) solution. Although some efforts have been made in the past, e.g., to obtain the viscosity solution [76], or the minimax solution [77]. However, for the special case of a affine-linear quadratic game, where the system dynamics are described by a set of linear differential equations and the cost function is quadratic, the value function has the unique solution which should satisfy a set of first order differential equations. Therefore, a closed form solution for the optimal action can be obtained for this special case.

According to [33], the value function for an affine-linear quadratic problem has

the following solution for  $v(t)$ :

$$v^i(t) = \frac{1}{2} \mathbf{X}(t)^T \mathbf{Z}^i(t) \mathbf{X}(t) + \mathbf{X}(t)^T \zeta^i(t) + \xi^i(t) + \mathbf{m}^i(t), \quad (4.21)$$

where  $\mathbf{Z}^i$  satisfies the following Riccati differential equations:

$$\dot{\mathbf{Z}}^i + \mathbf{Z}^i \check{\mathbf{F}}^i + \check{\mathbf{F}}^{i,T} \mathbf{Z}^i + \mathbf{Z}^i \mathbf{B} \mathbf{R}^{i-1} \mathbf{B}^T \mathbf{Z}^i + \mathbf{Q}_s^i = 0, \quad (4.22)$$

$$\mathbf{Z}^i(T) = 0_{2n \times 2n}, \quad (4.23)$$

where

$$\check{\mathbf{F}}^i = \mathbf{A} - \mathbf{B} \mathbf{R}^{i-1} \mathbf{B}^T \mathbf{Z}^i. \quad (4.24)$$

$\zeta^i$  and  $\mathbf{m}^i$  can be obtained from the following differential equations, respectively:

$$\dot{\zeta}^i + \check{\mathbf{F}}^i \zeta^i + \mathbf{Z}^i \mathbf{B} \mathbf{R}^{i-1} \mathbf{B}^T \zeta^i + \mathbf{Z}^i \mathbf{B}^i = 0, \quad (4.25)$$

$$\zeta^i(T) = 0_{2n \times 2n}, \quad (4.26)$$

$\mathbf{m}^i(T) = 0$ , and  $\mathbf{m}^i$  is obtained from:

$$\dot{\mathbf{m}}^i + \gamma^{i,T} \zeta^i + \frac{1}{2} \zeta^i \mathbf{B} \mathbf{R}^{i-1} \mathbf{B}^T \zeta^i = 0 \quad (4.27)$$

and

$$\gamma^i = \mathbf{c}' - \mathbf{B} \mathbf{R}^{i-1} \mathbf{B}^T \zeta^i. \quad (4.28)$$

Finally, the optimal control strategy can be obtained as

$$\begin{aligned}\mathbf{U}^{i*}(t, X) &= -\alpha \mathbf{R}^i(t)^{-1} \mathbf{B}(t)^T \nabla_{\mathbf{x}} v^i(t, \mathbf{X})^T \\ &= -\alpha \mathbf{R}^i(t)^{-1} \mathbf{B}(t)^T \left\{ \frac{\mathbf{X}(t)^T [(\mathbf{Z}^i)^T + \mathbf{Z}^i]}{2} + \zeta^{iT} \right\}.\end{aligned}\quad (4.29)$$

For the proposed game in (4.11)-(4.17),  $\mathbf{R}^i, i = 1, \dots, N$ , matrices in (4.19) are singular block matrices (therefore non-invertible). Using the Singular Value Decomposition (SVD) factorization method, we have  $\mathbf{R}^i = \mathbf{E}^i \mathbf{\Sigma}^i \mathbf{T}^{i*}$ , where  $\mathbf{E}^i$  is a real unitary matrix,  $\mathbf{\Sigma}^i$  is a rectangular diagonal matrix with singular values of  $\mathbf{R}^i$  on the diagonal, and  $\mathbf{T}^{i*}$  (the conjugate transpose of  $\mathbf{T}^i$ ) is a real unitary matrix. Using SVD factorization, a Moore–pseudoinverse of matrix  $\mathbf{R}^i$  can be calculated as  $\mathbf{R}^{i+} = \mathbf{T}^i \mathbf{\Sigma}^{i+} \mathbf{E}^{i*}$ .

Using Moore–Penrose pseudoinverse of matrix  $\mathbf{R}^i$  in (4.29), the following Lemma shows that player  $i$  does not need knowledge of other players' states.

**Lemma 4.1.** *For each player  $i$ , the associated columns of the other players in Moore–Penrose pseudoinverse matrix of  $\mathbf{R}^i$  are zeros.*

*Proof.* For each player  $i$ , matrix  $\mathbf{R}^i$  is written as follows:

$$\mathbf{R}^i(t) = \begin{bmatrix} 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \dots & 1 & 1 & -1 & 1 & 1 & \dots & 1 \\ 1 & \dots & 1 & -1 & 1 & 1 & 1 & \dots & 1 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{2n \times 2n}, \quad (4.30)$$

where all rows are zeros except row  $i$  and  $i + 1$ . According to Theorem 3 in [82], if matrix  $\mathbf{A}$  is partitioned row-wise as  $\mathbf{A} = (\mathbf{A}_1 \ :: \ \mathbf{A}_2)^T$ , the matrix  $\mathbf{M}$  of form

$\mathbf{M} = (\mathbf{A}_1^+ \mathbf{A}_2^+) = \mathbf{A}^+$  if the following relationships are satisfied:

$$\mathbf{A}_1^+ \mathbf{A}_2 = 0, \quad \mathbf{A}_2^+ \mathbf{A}_1 = 0, \quad (4.31)$$

$$(\mathbf{A}_1 \mathbf{A}_1^+)^* (\mathbf{A}_2 \mathbf{A}_2^+)^* = 0, \quad (4.32)$$

and

$$\mathbf{A}_1 \mathbf{A}_1^+ \mathbf{A}_2 \mathbf{A}_2^+ = 0. \quad (4.33)$$

Matrix  $\mathbf{R}^i$  can have 3 case of row-wise decomposition : 1) non zeros columns are in matrix  $\mathbf{R}_1^i$  and matrix  $\mathbf{R}_2^i$  is a zero  $n \times n$  matrix; 2) non zeros columns are in matrix  $\mathbf{R}_2^i$  and matrix  $\mathbf{R}_1^i$  is a zero  $n \times n$  matrix; 3) both  $\mathbf{R}_1^i$  and  $\mathbf{R}_2^i$  matrices have just one non-zero rows. For all these 3 cases, the necessary conditions in (4.31-4.33) are satisfied. Therefore  $\mathbf{R}^{i+} = (\mathbf{R}_1^{i+} \mathbf{R}_2^{i+})$ . For all 3 cases, zero rows in  $\mathbf{R}_1^i$  and  $\mathbf{R}_2^i$  are associated with zero columns in  $\mathbf{R}_1^{i+}$  and  $\mathbf{R}_2^{i+}$ . Therefore, all columns in  $\mathbf{R}^{i+}$  except column  $i$  and  $i + 1$  are zeros and cancel out the state information of the other players but player  $i$ .  $\square$

After calculation of optimal actions from (4.29), to obtain the original optimal control decisions  $\mathbf{u}^*$ , the following change of variables should be used:

$$\begin{aligned} \mathbf{u}^* = & [U_1^1 + \frac{\sum_{j \neq i}^n l^i - \sum_{j=1}^n g^i + \alpha p_c}{2(n-1)}, \\ & -(U_1^2 + \frac{\sum_{j \neq i}^n l^i - \sum_{j=1}^n g^i + \alpha p_c}{2(n-1)}), \dots, U_1^i + \frac{l^i}{2}, -(U_2^i + \frac{l^i}{2}) \\ & \dots, U_1^n + \frac{\sum_{j \neq i}^n l^i - \sum_{j=1}^n g^i + \alpha p_c}{2(n-1)}, \\ & -(U_2^n + \frac{\sum_{j \neq i}^n l^i - \sum_{j=1}^n g^i + \alpha p_c}{2(n-1)})]. \end{aligned} \quad (4.34)$$

**Table 4.1:** Building consumption scheduling algorithm

---

For each hour $t=1:24$
Update the market price.
Compute $\zeta$ using (4.25).
Compute vector of optimal decisions, $U^*$ , according to (4.29).
Use change of variables in (4.34) to transform $U^*$ to $u^*$ .
Update the total hourly demand according $q^i$ .
Send back the total hourly demand to the market.
End

---

Next, each building reports its total consumptions to the upper level, and the market makes its decisions based on the total bus load vector  $\mathbf{U}_D = [U_{D1}, \dots, U_{DM}]^T$ , where  $U_{Dj} = \sum_{i=1}^{n_j} q^i$ . Here,  $M$  and  $n_j$  are the total number of feeders and the total number of buildings connected to bus  $j$ , respectively. In summary, the daily building load control algorithm is shown in Table 4.1.

### 4.3.3 Properties and Discussion

In this section we show that the optimal control solution constitutes a feedback Nash Equilibrium to the stochastic differential game.

The  $N$ -person differential game discussed in Section 4.3.2 can be rewritten in the following form for each player  $i$ , where  $i = 1, \dots, N$ :

$$\begin{aligned} \dot{\mathbf{X}}^i &= f[\mathbf{X}^i(t), \mathbf{U}^i(t)] + \rho \mathbf{W} \\ &= \mathbf{A}\mathbf{X}^i(t) + \mathbf{B}\mathbf{U}^i(t) + \mathbf{C} + \rho \mathbf{W} \end{aligned} \quad (4.35)$$

and

$$\begin{aligned} v^i(X^i, t) &= \min_{\mathbf{U}(t)} L \\ &= \min_{\mathbf{U}(t)} E_w \left\{ \int_0^T \mu^i[\mathbf{X}^i(t), \mathbf{U}(t)] dt + h[\mathbf{X}^i(T)] \right\}, \end{aligned} \quad (4.36)$$

where

$$\mathbf{U}^i(t) = [U_1^i, U_2^i]^T \quad (4.37)$$

and

$$\mathbf{U}(t) = [U_1^1(t), U_2^1(t), \dots, U_1^i(t), U_2^i(t), \dots, U_1^N(t), U_2^N(t)]^T. \quad (4.38)$$

For this game, the  $N$ -tuple strategies that are defined below constitute a feedback Nash equilibrium solution [33].

**Definition 4.2.** *For an  $n$ -person game as defined in (4.35)-(4.38), a set of controls  $U^*(t, \mathbf{X}), \forall i = 1, \dots, N$ , constitutes a feedback Nash equilibrium of the formulated dynamic game if there exists functions  $v^i(\mathbf{X}, t), \forall i = 1, \dots, N$ , that satisfy the following relations:*

$$\begin{aligned} v_i(\mathbf{X}, t) &= E_w \left[ \int_0^T \mu^i \{t, \mathbf{X}^*(t), \mathbf{U}^*\} dt \right] \\ &\geq E_w \left[ \int_0^T \mu^i \{t, \mathbf{X}(t), \mathbf{U}\} dt \right], \end{aligned}$$

where

$$\mathbf{U} = [U_1^{1*}, U_2^{1*}, \dots, U_1^i, U_2^i, \dots, U_1^{N*}, U_2^{N*}] \quad (4.39)$$

and

$$\mathbf{U}^* = [U_1^{1*}, U_2^{1*}, \dots, U_1^{i*}, U_2^{i*}, \dots, U_1^{N*}, U_2^{N*}]. \quad (4.40)$$

If there exists a function  $v^i(\mathbf{X}, t)$  that is twice continuously differentiable, then the two partial differential feedback Nash equilibrium solutions in continuous time can be characterized using the stochastic HJB equations, which are necessary conditions of

**Table 4.2:** Number of buildings connected to each Bus.

bus 1	bus 2	bus 3	bus 4	bus 5	bus 6	bus 7
0	2100	9400	4800	760	1120	0
bus 8	bus 9	bus 10	bus 11	bus 12	bus 13	bus 14
0	2950	900	350	6100	1350	1490

the candidate optimal control strategy. This is summarized in the following theorem [33].

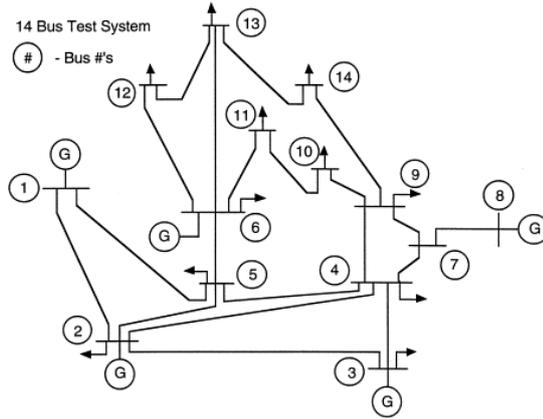
**Theorem 4.3.** *A set of feedback strategies  $U^*(\mathbf{X}, t)$  leads to a feedback Nash equilibrium for stochastic differential game in (4.35)-(4.38), and  $X^*(t), 0 \leq t \leq T$  is the corresponding state trajectory, if there exist suitably smooth functions  $v^i(t)$ , for  $i = 1, \dots, N$  satisfying the following rectilinear parabolic partial differential equations:*

$$-\frac{\partial v^i(\mathbf{X}, t)}{\partial t} = \min_{\mathbf{U}^i(t)} \left\{ \frac{(\rho_2)^2 \sigma^2}{2} \frac{\partial^2 v^i(\mathbf{X}, t)}{\partial \mathbf{X}^2} + \frac{\partial v^i(\mathbf{X}, t)}{\partial \mathbf{X}} f[t, \mathbf{X}, \mathbf{U}^{[i]}(t)] + \mu^i[t, \mathbf{X}, \mathbf{U}^*(t)] \right\}. \quad (4.41)$$

Next, to prove that our proposed game has the feedback Nash equilibrium, we note that the differential game in (4.11)-(4.17) is in the affine linear form. Therefore, the value function in (4.21) is twice continuously differentiable and the derived optimal solutions by the HJB equations can indeed characterize the feedback Nash equilibrium.

## 4.4 Simulation Results

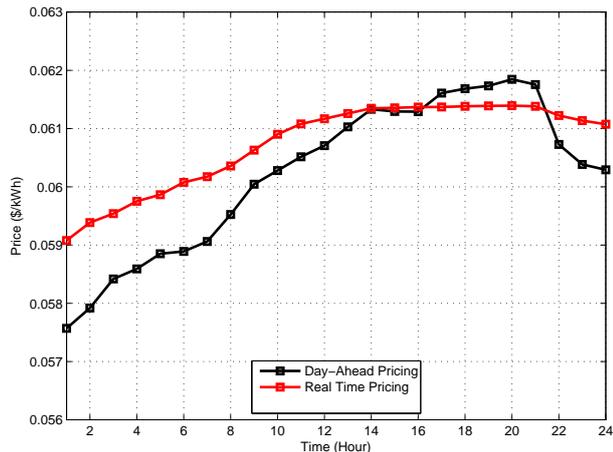
In this section, we numerically investigate the performance of the proposed stochastic differential game to confirm and complement the results presented in the previous sections. Consider the IEEE 14-bus test system in Figure 4.2, where the load at each bus is the summation of several homogeneous smart building load at that bus. The



**Figure 4.2:** The fourteen bus system studied in simulations.

number of buildings that are connected to each bus is shown in Table 4.2. The bus and the line parameters are set according to the model in [78]. According to [79], the lead-acid batteries which are suitable for energy-smart buildings are generally 85-95 % efficient. Therefore, for simulation purposes the value for the leakage rate of the batteries is considered to be  $\beta = 0.05$ . The value for  $\epsilon$  as the factor of inertia of the building is a function of the time constant of the building which can be defined as the energy stored per unit area in the construction per unit change in heat flux. Finally, the overall thermal conductivity is calculated based on the real data for a typical building in Texas provided by [80] as 0.5. The price factors in equation (4.2) are set as  $p_c = 0.055\$KWh$  and  $\alpha = 1$ .

To study how the proposed demand response method affects electricity scheduling at the buildings level, we compare the performance of our algorithm with a more realistic day-ahead pricing scenario. In the day ahead pricing scenario, the 24-hour price profile is assumed to be known to the ECS from day before and taken from real data [80] of one day consumption as shown in Figure 4.3. We also show the real-time pricing in the same figure. For simplicity, we focus on one building at bus number 2 as an example. The outdoor temperature, the mean of wind turbine output, and the uncontrollable load are depicted in Figure 4.4(a), Figure 4.4(b) and Figure 4.4(c),



**Figure 4.3:** Daily price for two pricing scenarios.

respectively [80].

In Figures 4.5(a) and 4.5(b), the daily states of the considered building for the mentioned three methods are depicted. For all three scenarios, as price tends to increase, the battery tends to discharge in order to cover a portion of the building power consumption. Furthermore, the indoor temperature tends to increase due to lowering the air conditioner’s load during peak hours. We can also see that, for all scenarios, the variations of both battery level and the indoor temperature are correlated to the changes in price values. Here, the average usage from the battery reduces by around 10% for real-time pricing in peak hours (18-20) compared to the day-ahead pricing case.

Next, we compare our proposed joint real-time pricing at upper level and demand response at lower level, with the two other design scenarios. The corresponding daily load profiles are shown in 4.6. For both day-ahead and real-time pricing techniques, the peak load is reduced at around 8:00 PM. However, the use of the proposed real-time pricing technique yields a more flat load shape compared to other methods.

Figure 4.7 studies how the PAR in aggregate load varies as the mean of the daily outdoor temperature increases for three pricing scenarios. As it is shown in Figure

**Table 4.3:** Average daily market profit vs. building’s cost

	Real-time Pricing	Day-ahead Pricing
Market Profit (\$)	48609	48741
Average Building’s Cost (\$)	229	241

4.6 compared to the other technique, the proposed real pricing technique makes the load curve more flat. Therefore, the PAR of associated load curve is also have the least value. Due to the fact under high temperature, users tend to let AC unit stay on for longer time, e.g., even at night when the load is low. Consequently, the PAR decreases as the temperature increases, roughly in form of a linear function.

The mean daily cost of the considered building versus the mean daily outdoor temperature are shown in Figure 4.8. We can see that the real-time pricing combined with the proposed stochastic differential game can reduce the daily cost. The price value under the real-time pricing has higher gradient since all building users increase their power consumption in the case of a high temperature, which results in a higher price. Moreover, when the temperature is high, the HVAC units are needed to stay functioning in order to keep the indoor temperature in the comfort range of users. Therefore, there is no major difference among different algorithms in this case as Figure 4.8 suggests.

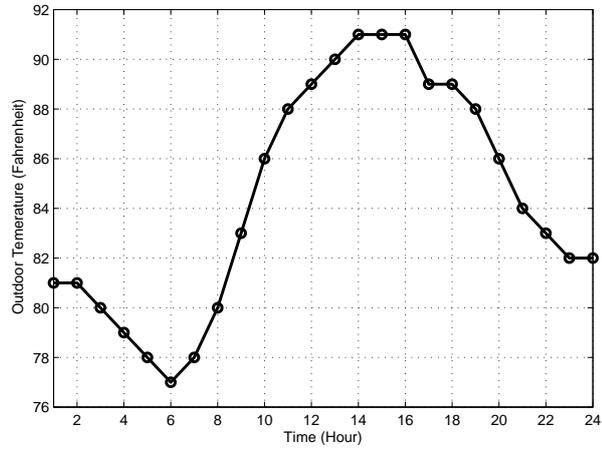
The average daily market profit versus the building’s costs are compared in Table 4.3. The profit is calculated as the total market revenue reduced from the total generation cost. The generation cost per hour is calculated according to [81] as  $0.00128D^2 + 6.48D + 459$ , where D is the total demand. As the table suggests, by deploying the real-time pricing technique compared to the day-ahead pricing, the buildings can reduce their cost approximately 5% while the market will face 0.27 % reduction in the daily profit, which is quite insignificant. This can be explained by the fact that the real-time pricing only lowers the consumption in high price hours, which does not hurt the supply side due to their savings of expensive fuel during peak

hours.

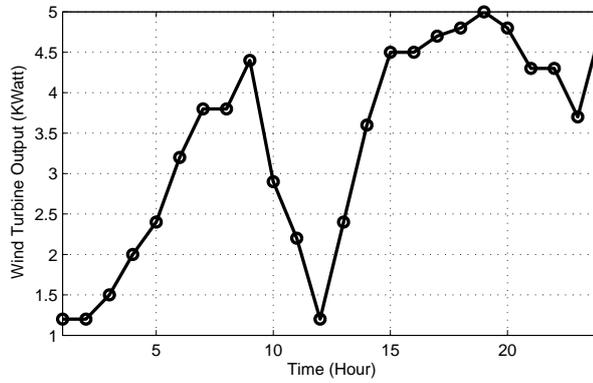
Finally the impact of buildings on price is studied in Figure 4.9 by comparing the average daily consumption of a sample building at Bus no. 2 for different value of  $\alpha$  in equation (4.2). Increasing  $\alpha$  reduces the impact of buildings on spot price and yields an almost constant price. However, since HVAC units are considered as the only controllable load in system model, the consumption would not increase beyond certain range for  $\alpha \geq 4$ .

## 4.5 Summary

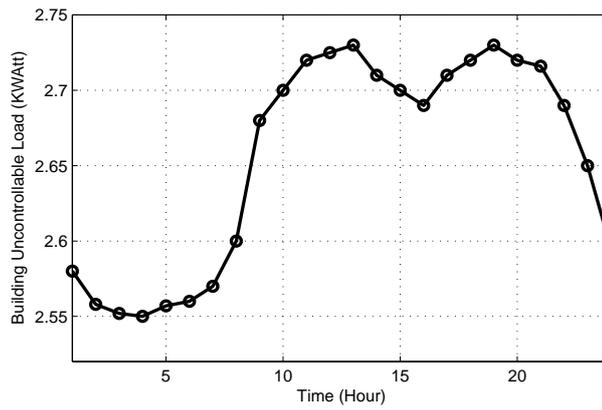
In this chapter, we developed a stochastic differential game model for autonomous demand response when the price of electricity varies during the day. The model explains how end-users can decrease their electricity bill when having dynamic load, where the load dynamics formulated as differential equations. Real-time pricing also gives the aggregator the opportunity to influence end users load profile through pricing of power. We studied the interaction between the market and buildings using a two-level differential game model. To gain insights, two dynamic states are particularly investigated: the battery's state-of-charge and the room temperature. The HJB equation is used to study the solution of the formulated game among different buildings. As simulation results over two different pricing scenarios show, the proposed method reduces the overall power consumption of all users, by storing the energy when the price is low and by later discharging it when the price is high. The peak-to-average ratio in aggregate load demand as well as overall energy cost are also greatly reduced.



(a) Outdoor Temperature

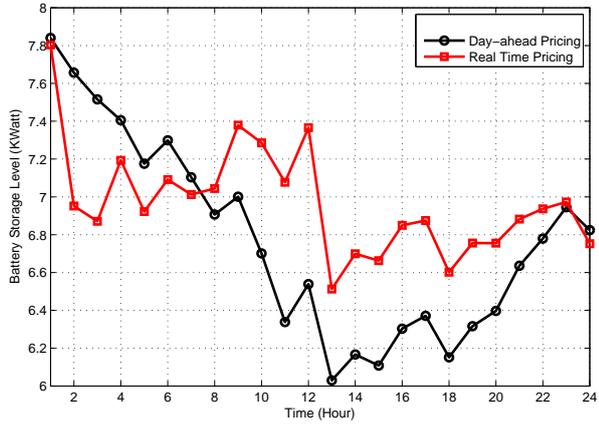


(b) Predicted Output of Wind Turbine

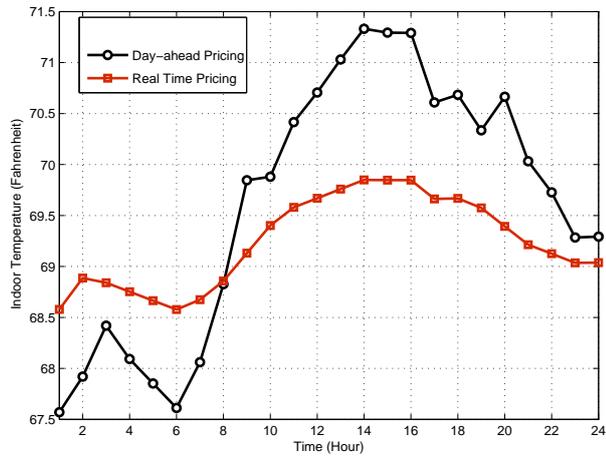


(c) Uncontrollable Load

**Figure 4.4:** Region characteristics at Bus 2.



(a) Battery Storage Level



(b) Indoor Temperature

Figure 4.5: System states for a sample building at Bus 2.

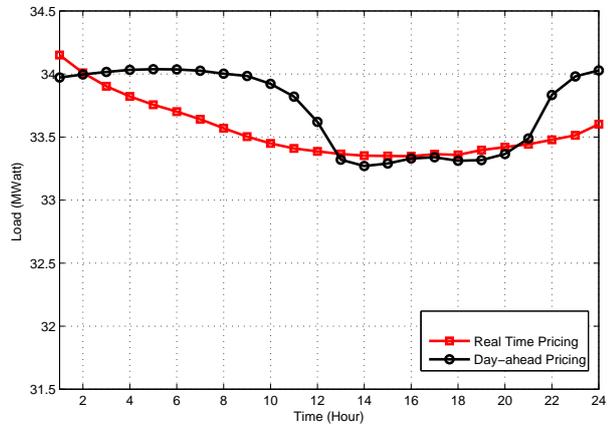


Figure 4.6: Total daily load at Bus 2.

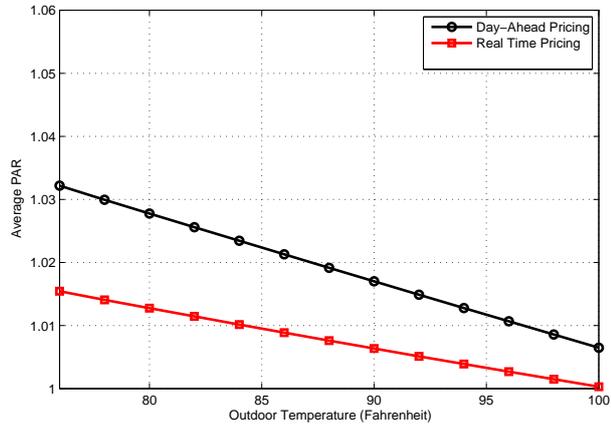


Figure 4.7: Comparison of load PAR for two pricing scenarios.

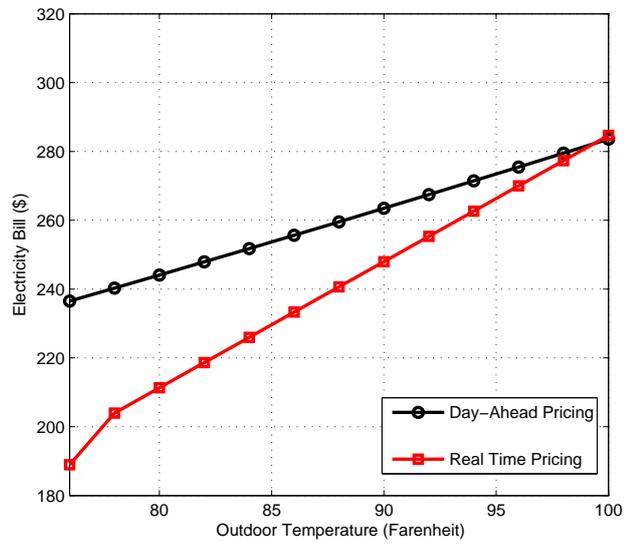


Figure 4.8: Daily cost vs. outdoor temperature for two pricing scenarios.

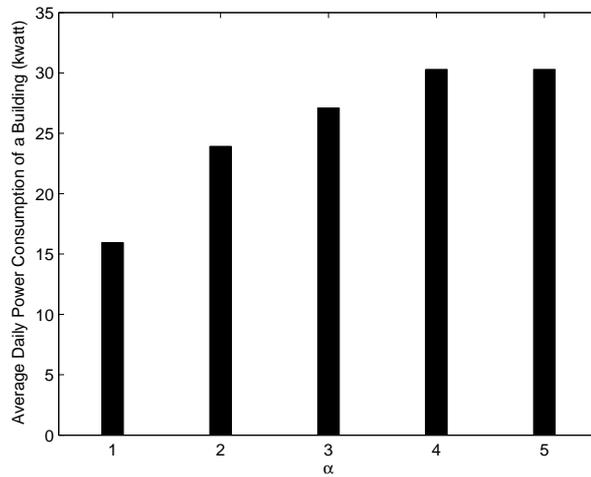


Figure 4.9: Impact of buildings power consumption on price.

## Chapter 5

### Distributed Control of HVAC Systems in Smart Buildings

#### 5.1 Introduction

Buildings consume almost 70% of the total electricity generated in the US [83, 84]. One of the major energy-consuming systems of a building is the heating, ventilation and air conditioning (HVAC) system. More precisely, an HVAC system might exhaust more than 65% of the total electrical energy consumed by a building [85, 86, 87]. The high energy consumption of HVAC systems raises energy costs as well as environmental concerns. Therefore, a desirable capability of future “smart buildings” is energy reduction via fine-grained control of HVAC systems. For instance, an HVAC system can be conceived as an autonomous system that adjusts the indoor temperature of different locations in the building based on the occupancy [88].

A growing body of research suggests that efficient control of HVAC systems might significantly increase the energy efficiency of future smart-buildings. For instance, in [89], the authors explored the problem of computing optimal control strategies for time-scheduled operation taking into consideration building operation schedules, e.g., night-setback, start-up and occupied modes, as well as a predicted weather profile. In [90], dynamic and real-time simulation models are developed to study the thermal, hydraulic, mechanical, environmental and energy performance of smart buildings. In [91], the authors presented a simulation tool, “QUICK Control”, to predict the effect of changing control strategies on indoor comfort and energy consumption.

For occupancy-based control of HVAC systems, a fundamental requirement is a sensor network to capture the occupancy changes in real time. In [92], the authors proposed several control strategies based on real time occupancy monitoring and occupancy predictions.

In this paper, tools from game theory and multi-agent learning are used to design

a cost-efficient distributed control for HVAC systems. This work takes into consideration the electricity cost and predetermined ranges of desirable temperatures during certain periods for all locations in the building that are subject to temperature control. The main game-theoretic tool of this analysis is the notion of a satisfaction equilibrium (SE) [24]. Basically, an SE is an equilibrium concept in which players do not seek any benefit maximization but only the satisfaction of their own individual constraints. This equilibrium is thoroughly studied in the context of the distributed control of HVAC systems. More importantly, a simple algorithm based on the notion of trial-and-error learning [94] that is capable of achieving an SE is also presented.

The rest of this paper is organized as follows. Section 5.2 introduces the system model. Section 5.3 describes two games, one in the satisfaction form and another one in the normal form, to model the problem of distributed control of HVAC systems. Therein, a fully distributed learning algorithm is also provided to achieve the equilibria of the corresponding games. Section 5.5 presents some simulation results, and Section 5.6 concludes this work.

## 5.2 System Model

Consider a smart-building with  $n$  zones subject to temperature control. Each zone is managed by an HVAC system that is fully independent of all the other HVAC units in the building. The goal is to reduce the electricity cost by dynamically adjusting the operation of the HVAC units such that two main constraints are satisfied: (i) The temperature of a given room  $i$ , with  $i \in \{1, \dots, n\}$  must be kept within a desirable range for certain predetermined periods; and (ii) The total power load dedicated for heating, ventilation and air conditioning must not exceed a predetermined threshold.

Consider that temperature is controlled with a granularity of 1 hour. At the beginning of the day, HVAC unit  $i$  chooses its own daily power consumption vector, denoted by,  $\mathbf{L}_i = (l_i(1), \dots, l_i(t), \dots, l_i(24))$ , where  $l_i(t)$  denotes the power consumed

by HVAC unit  $i$  at hour  $t$ . Each HVAC unit chooses its vector  $\mathbf{L}_i$  from a finite set of vectors denoted by  $\mathcal{L}_i$ . The daily power consumption vector is selected in order to minimize the daily electricity cost

$$\Phi(\mathbf{L}, \mathbf{P}) = \sum_{i=1}^n \sum_{t=1}^{24} \mu_i(l_i(t), p(t)), \quad (5.1)$$

where  $\mathbf{L} = (\mathbf{L}_1, \dots, \mathbf{L}_n) \in \mathcal{L}_1 \times \dots \times \mathcal{L}_n$ ,  $\mathbf{P} = (p(1), \dots, p(24)) \in \mathcal{R}^{24}$ , with  $p(t)$  the hourly market price per energy unit. Note that the vector  $\mathbf{P}$  is assumed to be known by each HVAC unit at the beginning of the day. The function  $\mu_i : \mathcal{L}_i \times \mathcal{R} \rightarrow \mathcal{R}$  models the operation cost of HVAC  $i$  with power load  $l_i(t)$  and price  $p_i(t)$ . One example of the function  $\mu_i$  is presented in [95]. Therein, the function  $\mu_i$  is defined as follows:

$$\mu_i(l_i(t), p(t)) = c_1 p(t) l_i(t)^2 + c_2 p(t) l_i(t) + c_3, \quad (5.2)$$

where  $c_1$ ,  $c_2$  and  $c_3$  are constant parameters that are determined based on experimental data. One of the advantages of a quadratic model for the operation cost of an HVAC unit is that it is more realistic and flexible than other models, e.g., the translog cost model [96].

The total power load allocated for the HVAC system satisfies the following constraint at each 1-hour period  $t$  of the day:

$$\forall t \in \{1, \dots, 24\}, \quad \sum_{i=1}^n l_i(t) \leq L_r, \quad (5.3)$$

where the threshold value  $L_r$  is imposed by the characteristics of the power distribution feeder/bus of the building. This threshold is assumed to remain the same at each period  $t$ .

The temperature  $T_i(t)$  of zone  $i$  during period  $t$  must fall within the interval  $[\underline{T}_i^t, \bar{T}_i^t]$ . Note that this control is enforced only during certain periods. Hence, the

set  $\mathcal{I}_i$  denotes the periods over which this rule is enforced, i.e.,

$$\forall t \in \mathcal{I}_i, \quad \underline{T}_i^t \leq T_i(t) \leq \bar{T}_i^t. \quad (5.4)$$

For all the other periods  $t \in \{1, \dots, 24\} \setminus \mathcal{I}_i$  no temperature control is performed. The indoor temperature is predicted up to 24 hours in advance using a simple exponential decay model proposed by [73]. That is,

$$T_i(t+1) = \epsilon T_i(t) + (1 - \epsilon)(T_{OD}(t) - \gamma K l_i(t)), \quad (5.5)$$

where  $\epsilon$  is the thermal time constant of the building;  $\gamma$  is a factor capturing the efficiency of the air conditioning unit to cool the air inside the building ;  $T_{OD}$  is the outside temperature which is predicted a day ahead; and  $K$  is a conversion factor that is proportional to the performance coefficient of the HVAC unit divided by the total thermal mass. The HVAC unit uses power  $l_i(t)$  to control the temperature of zone  $i$  during period  $t$ . In this model, our focus is only on the cooling scenario. The results for a heating scenario can also be easily obtained by changing the sign of  $-\gamma K l_i(t)$ .

Equation (5.5) explains how the constraint (5.4) relates to the power consumed by the HVAC unit. The HVAC control units should select their consumption amounts in a way that the indoor temperature obtained from (5.5) falls within the comfort interval (5.4) and minimize the total cost in (5.1). That is, the vector  $\mathbf{L} = (\mathbf{L}_1, \dots, \mathbf{L}_n)$  must be chosen such that

$$\begin{aligned} \min_{\mathbf{L} \in \mathcal{L}_1 \times \dots \times \mathcal{L}_n} & \sum_{i=1}^n \sum_{t=1}^{24} \mu_i(l_i(t), p(t)), \\ \text{s.t.} & \sum_{i=1}^n l_i(t) \leq L_r, \quad \forall t \in \{1, \dots, 24\}, \quad \text{and} \\ & \underline{T}_i^t \leq T_i(t) \leq \bar{T}_i^t, \quad \forall t \in \mathcal{I}_i. \end{aligned} \quad (5.6)$$

The above problem can be solved at the beginning of the day such that the vector

$\mathbf{L}_i = (l_i(1), \dots, l_i(t), \dots, l_i(24))$  is entirely calculated by HVAC  $i$  only once per day; or at each period  $t$ , the HVAC determines its individual load  $l_i(t+1)$ . Both alternatives are studied in Section 5.4.

### 5.3 Game Formulation

The distributed control problem of the HVAC system described above can be modeled via two games formulations: a game in normal form [97] and a game in satisfaction form [24]. The related equilibrium solution are defined accordingly.

#### 5.3.1 Game in Normal Form

Consider the game in normal form

$$G = (\mathcal{N}, \mathcal{L}, \{\xi_i\}_{i \in \mathcal{N}}). \quad (5.7)$$

The set  $\mathcal{N} = \{1, \dots, n\}$  represents set of players. HVAC  $i$  is represented by player  $i$ . The set of actions of player  $i$  is the set of daily power consumption vectors  $\mathcal{L}_i$ . Hence, the set of actions of the game is  $\mathcal{L} = \mathcal{L}_1 \times \mathcal{L}_2 \times \dots \times \mathcal{L}_n$ . The payoff function  $\xi_i : \mathcal{L} \times \mathcal{R}^{24} \rightarrow \mathcal{R}$  of player  $i$  is defined by

$$\xi_i(\mathbf{L}, \mathbf{P}) = \frac{1}{\beta + 1} \left( 1 - \frac{\sum_{t=1}^{24} \mu_i(l_i(t), p(t))}{24\mu_{max}} + \beta \mathbf{1}_{\{\mathbf{L}_i \in f_i(\mathbf{L}_{-i})\}} \right), \quad (5.8)$$

where  $\beta$  is a design parameter,  $\mu_i$  is defined in (5.2) and  $\mu_{max}$  is the maximum cost user  $i$  can experience. The correspondence  $f_i : \mathcal{L}_1 \times \dots \times \mathcal{L}_{i-1} \times \mathcal{L}_{i+1} \times \dots \times \mathcal{L}_n \rightarrow 2^{\mathcal{L}_i}$  is defined as follows:

$$f_i(\mathbf{L}_{-i}) = \left\{ \mathbf{L}_i \in \mathcal{L}_i : \forall t \in \{1, \dots, 24\} \right. \\ \left. \sum_{i=1}^n l_i(t) \leq L_r \text{ and } \underline{T}_i^t \leq T_i(t) \leq \bar{T}_i^t \right\}. \quad (5.9)$$

The payoff function in (5.8) captures the tradeoff between satisfying the individual constraints of player  $i$  and minimizing the individual consumption cost. Note that increasing the value of  $\beta$  leads to the player focusing on in the satisfaction of its individual constraints. Alternatively, reducing the value of  $\beta$  leads the player to focus more on the reduction of the individual operating cost. This utility function was first proposed in [26] for the case of decentralized radio resource allocation in wireless networks.

An interesting outcome of the game  $G$  is a Nash Equilibrium (NE), which is defined as follows:

**Definition 1.** *An action profile  $\mathbf{L}^* \in \mathcal{L}$  of the game (5.7) is an NE if  $\forall i \in \mathcal{N}$  and  $\forall \mathbf{L}'_i \in \mathcal{L}_i$ ,*

$$\xi_i(\mathbf{L}^*_i, \mathbf{L}^*_{-i}, \mathbf{P}) \geq \xi_i(\mathbf{L}'_i, \mathbf{L}^*_{-i}, \mathbf{P}). \quad (5.10)$$

The interest in the NE stems from the fact that at such a state, none of the players can improve its payoff by unilaterally changing its action.

### 5.3.2 Game in Satisfaction Form

Consider the game in satisfaction form  $\hat{G} = (\mathcal{N}, \mathcal{L}, \{f_i\}_{i \in \mathcal{N}})$ . In the game  $\hat{G}$ , a player is said to be satisfied if it plays an action that satisfies its individual constraints. More importantly, once a player satisfies its individual constraints, it has no interest in changing its action, and thus, an equilibrium is observed when all players are simultaneously satisfied. This equilibrium is often referred to as the satisfaction equilibrium [24].

**Definition 2.** *An action profile  $\mathbf{L}^+ \in \mathcal{L}$  is a satisfaction equilibrium for the game*

$\hat{G} = (\mathcal{N}, \mathcal{L}, \{f_i\}_{i \in \mathcal{N}})$  if

$$\forall i \in \mathcal{N}, \quad \mathbf{L}_i^+ \in f_i(\mathbf{L}_{-i}^+). \quad (5.11)$$

The interest in the SE stems from the fact that at such a state, all players satisfy their individual constraints. However, no optimality can be claimed on the choices of the players with respect to the cost in (5.1).

## 5.4 The distributed learning algorithm

In this section, a learning algorithm based on a trial-and-error dynamics is proposed to distributively achieve an SE and/or NE.

### 5.4.1 Trial and Error Learning Algorithm

Player  $i$  locally implements a state machine. At iteration  $s$ , the state of player  $i$  is defined by the triplet

$$Z_i(s) = \{m_i(s), \bar{\mathbf{L}}_i(s), \bar{\xi}_i(s)\}, \quad (5.12)$$

where  $m_i(s)$  represents the ‘‘mood’’ of player  $i$ , that is, the way it reacts to the observation of the instantaneous observation  $\tilde{\xi}_i(s)$  of  $\xi_i(\mathbf{L}(s), \mathbf{P})$ , with  $\xi_i$  defined in (5.8) and  $\mathbf{L}(s)$  is the action played at iteration  $s$ .  $\bar{\mathbf{L}}_i(s) \in \mathcal{L}$  and  $\bar{\xi}_i(s) \in [0; 1]$  represent a benchmark action and a benchmark payoff, respectively. There are two possible moods: content ( $C$ ) and discontent ( $D$ ); and thus,  $m_i(s) \in \{C, D\}$ .

If at iteration  $s$  player  $i$  is content, it chooses action  $\mathbf{L}_i$  following the probability distribution

$$\pi_{i, \mathbf{L}_i} = \begin{cases} \frac{\epsilon^c}{|\mathcal{L}_i| - 1}, & \text{if } \bar{\mathbf{L}}_i \geq \mathbf{L}_i \quad \text{and} \\ 1 - \epsilon^c, & \text{if } \bar{\mathbf{L}}_i = \mathbf{L}_i, \end{cases}$$

where,  $\pi_{i,\mathbf{L}_i} = \Pr(\mathbf{L}_i(s) = \mathbf{L}_i)$ ,  $c > n$  is a constant and  $\epsilon > 0$  is an experimentation rate. In the following, we use the notation  $X \Leftarrow Y$  to indicate that variable  $X$  takes the value of variable  $Y$ . If player  $i$  uses its bench-marked action at iteration  $s$ , i.e.,  $\mathbf{L}_i(s) = \bar{\mathbf{L}}_i$ , and  $\tilde{\xi}_i(s+1) = \bar{\xi}_i(s)$  then the state  $Z_i(s)$  remains the same. Otherwise, it adopts a new bench-marked action and a new benchmark payoff:  $\bar{\mathbf{L}}_i(s+1) \Leftarrow \tilde{\mathbf{L}}_i(s)$ ,  $\bar{\xi}_i(s+1) \Leftarrow \tilde{\xi}_i(s)$ . The mood of player  $i$  is updated as follows: with probability  $\epsilon^{(1-\tilde{\xi}_i(s))}$  it sets its mood to content  $m_i(s+1) \Leftarrow C$ , and with probability  $1 - \epsilon^{(1-\tilde{\xi}_i(s))}$ , it sets it to discontent  $m_i(s+1) \Leftarrow D$ .

### 5.4.2 Properties

An essential condition for the mentioned learning algorithm to achieve a stochastically stable state is the interdependence property defined as follows [94].

**Definition 3.** (*Interdependent game*) An  $n$ -person game  $G$  on the finite action set  $\mathcal{L}$  is interdependent if for every non-empty subset  $\mathcal{J} \subset \mathcal{N}$  and every action profile  $\mathbf{L} = (\mathbf{L}_{\mathcal{J}}, \mathbf{L}_{-\mathcal{J}}) \in \mathcal{L}$  such that  $\mathbf{L}_{\mathcal{J}}$  is the action profile of all users in  $\mathcal{J}$ , there exists a player  $i \notin \mathcal{N}$  such that

$$\exists \mathbf{L}'_{\mathcal{J}} \neq \mathbf{L}_{\mathcal{J}} : \xi_i(\mathbf{L}'_{\mathcal{J}}, \mathbf{L}_{-\mathcal{J}}) \neq \xi_i(\mathbf{L}_{\mathcal{J}}, \mathbf{L}_{-\mathcal{J}}). \quad (5.13)$$

In other words, the interdependence condition states that it is not possible to divide the players into two separate subsets that do not interact with each other.

In the following, we assume that game  $G$  is interdependent. This is a reasonable assumption, since the power consumption choices of all players affects the set of conditions in (5.3) that all other players should satisfy. The following theorem states that the players' actions at the stochastically stable state of the learning algorithm maximize the social welfare of all players[94].

**Theorem 5.1.** *Under the dynamics defined by the mentioned learning algorithm, a*

state  $Z = (m, \mathbf{L}, \xi)$ , is stochastically stable if and only if the following conditions are satisfied:

- The action profile  $\mathbf{L}$  maximizes the social welfare function  $W : \mathcal{L} \times \mathcal{R}^{24} \rightarrow \mathcal{R}$ , defined as follows:

$$W(\mathbf{L}, \mathbf{P}) = \sum_{i \in \mathcal{N}} \xi_i(\mathbf{L}, \mathbf{P}). \quad (5.14)$$

- The mood of each player is content.

The next theorem states that by selecting  $\beta > n$ , the stochastically stable state of the dynamics described above is such that the largest set of players is satisfied.

**Theorem 5.2.** Let each player in the game  $G$  implement the learning algorithm described above with utility function  $\xi_i$  and  $\beta > n$ . Then, the action profile with the highest social welfare satisfies the highest number of players.

*Proof.* It is sufficient to show that if  $\mathbf{L}^*$  is an action profile that satisfies  $k^*$  players and  $\mathbf{L}'$  an action profile that satisfies  $k'$  players, and  $k^* > k'$ , then  $W(\mathbf{L}^*) > W(\mathbf{L}')$ .

We have  $W(\mathbf{L})$  as follows:

$$W(\mathbf{L}) \sum_{i \in \mathcal{N}} \frac{1}{\beta + 1} \left( 1 - \frac{\sum_{t=1}^{24} \mu_i(l_i(t), p(t))}{24\mu_{max}} + \beta \mathbf{1}_{\{\mathbf{L}_i \in f_i(\mathbf{L}_{-i})\}} \right), \quad (5.15)$$

and we also have:

$$0 \leq \sum_{i \in \mathcal{N}} \left( 1 - \frac{\sum_{t=1}^{24} \mu_i(l_i(t), p(t))}{24\mu_{max}} \right) \leq n. \quad (5.16)$$

Using the inequality (5.16), and the assumption that  $\mathbf{L}^*$  is an action profile which satisfies  $k^*$  players, we can write  $\frac{\beta k^*}{1+\beta} \leq W(\mathbf{L}^*) \leq \frac{n+\beta k^*}{1+\beta}$ . Similarly using (5.16) and the assumption that  $\mathbf{L}'$  is an action profile which satisfies  $k'$  players, we can write  $\frac{\beta k'}{1+\beta} \leq W(\mathbf{L}') \leq \frac{n+\beta k'}{1+\beta}$ . Since  $k^*, k' \in \mathbb{N}$ , we can write the assumption  $k' < k^*$  as

$k' \leq k^* - 1$ , which implies  $W(\mathbf{L}') \leq \frac{n+\beta k' - \beta}{1+\beta}$ . Using the assumption that  $\beta > n$ , we can write  $\frac{n+\beta k' - \beta}{1+\beta} < \frac{\beta k'}{1+\beta}$ . Following the set of inequalities, we can state  $W(\mathbf{L}') < \frac{\beta k'}{1+\beta} \leq W(\mathbf{L}^*)$ , which proves  $W(\mathbf{L}^*) > W(\mathbf{L}')$ .  $\square$

### 5.4.3 Online Scheduling

The day-ahead scheduling method as it is described in Section 5.2 can achieve a satisfaction equilibrium using the distributed learning method in the convergence as is proven in Theorem 2. Due to the constant change of the outdoor temperature and market electricity price, it is more practical that scheduling of HVAC units be performed on a hourly basis instead of a day-ahead one. In this section we explain the details of the hourly scheduling method.

For the hourly scheduling method, each HVAC unit chooses its hourly consumption  $l_i$  from a finite set denoted by  $\mathcal{L}'_i$ . The constrained electricity cost optimization of each zone as defined in (5.6) can be rewritten as follows at time interval  $t$ :

$$\begin{aligned} \min_{\hat{L} \in \mathcal{L}'_1 \times \dots \times \mathcal{L}'_n} \sum_{i=1}^n \mu_i(l_i(t), p(t)), \quad (5.17) \\ \text{s.t.} \quad \sum_{i=1}^n l_i(t) \leq L_r, \quad \text{and} \\ \underline{T}_i^t \leq T_i(t) \leq \bar{T}_i^t, \end{aligned}$$

where  $\hat{L}(t) = [l_1(t), \dots, l_n(t)]$ . The game definition for the normal and satisfaction forms change accordingly as follows.

#### 5.4.3.1 Game in Normal Form

The game in normal form is represented by the triplet,  $G = (\mathcal{N}, \mathcal{L}', \{\xi_i\}_{i \in \mathcal{N}})$ . Here,  $\mathcal{N} = \{1, \dots, n\}$  represents the set of players that are HVAC control units of  $n$  zones. The action of player  $i$  is its hourly power consumption,  $l_i(t)$ , and each player  $i$  has a finite set of actions,  $\mathcal{L}'_i$ . The set of actions of the game is the joint set of

	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21
Zone 1		■	■	□	■	■	□	■	■	■	■	□
Zone 2		□	■	■	□	■	■	□	□	■	■	■
Zone 3		■	■	■	■	□	□	□	■	■	□	□

**Figure 5.1:** The occupancy schedule of building zones.

players actions,  $\mathcal{L}' = \mathcal{L}'_1 \times \mathcal{L}'_2 \times \dots \times \mathcal{L}'_n$ . We introduce the payoff function of player  $i$ ,  $\xi'_i : \mathcal{L}' \rightarrow \mathcal{R}$  defined by

$$\xi'_i(l_i(t), \mathbf{L}_{-i}(t)) = \frac{1}{\beta + 1} \left( 1 - \frac{\mu_i(l_i(t), p(t))}{\mu_{max}} + \beta \mathbf{1}_{\{l_i(t) \in f'_i(\mathbf{L}_{-i}(t))\}} \right), \quad (5.18)$$

where we define  $f'_i : \mathcal{L}_1 \times \dots \times \mathcal{L}_{i-1} \times \mathcal{L}_{i+1} \times \dots \times \mathcal{L}_n \rightarrow 2^{\mathcal{L}_i}$  as follows:

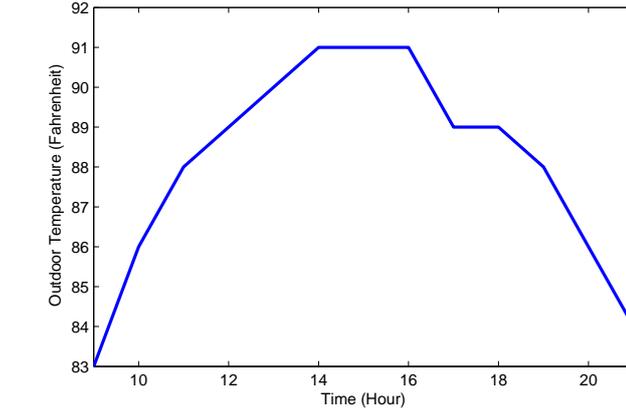
$$f'_i(\mathbf{L}_{-i}) = \left\{ l_i \in \mathcal{L}'_i : \sum_{i=1}^n l_i \leq L_r, \underline{T}_i^t \leq T_i(t) \leq \bar{T}_i^t \right\}.$$

#### 5.4.3.2 Game in Satisfaction Form

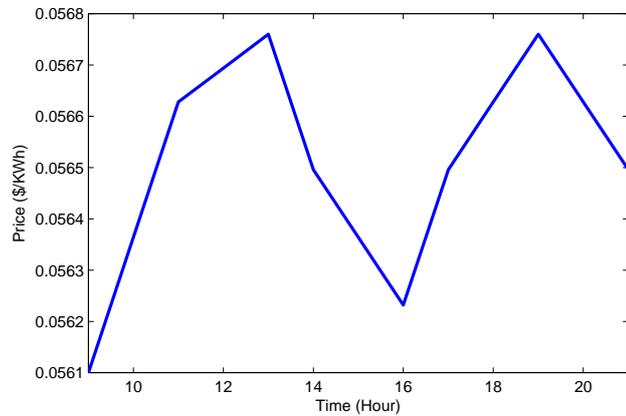
The game in the satisfaction form is defined by the triple  $\hat{G} = (\mathcal{N}, \mathcal{L}', \{f'_i\}_{i \in \mathcal{N}})$ . Similar to the properties of the distributed learning algorithm for 24-hour scheduling case discussed in Section 5.4.2, the distributed learning algorithm for the online scheduling case achieves the solution with the largest number of satisfied players.

**Theorem 5.3.** *Let each player in the game  $G$  implement the learning algorithm described above with utility function  $\xi_i$  and  $\beta > n$ . Then, the action profile with the highest social welfare is the solution with the largest number of satisfied users.*

The proof is similar to the proof of Theorem 2. In following, we present numerical results to demonstrate the performance of the proposed game.



(a) Outdoor Temperature

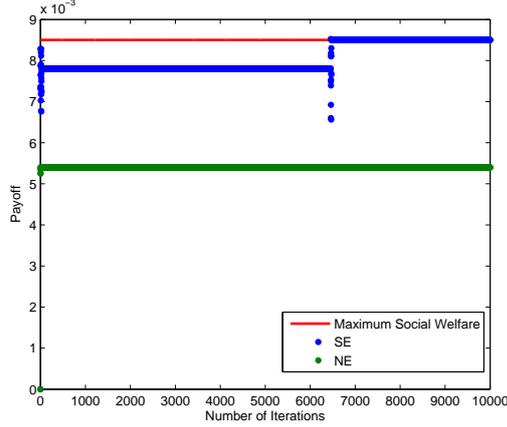


(b) Electricity Price

**Figure 5.2:** Characteristics of the building.

## 5.5 Simulation Results

In this section, we numerically investigate the performance of the proposed satisfaction game to confirm and complement the results presented in the previous sections. Consider a building divided into three independent zones for the task of temperature control based on occupancy. The occupancy schedule of the building for a 12 hour period is shown in Figure 5.1. In this figure, shaded blocks indicate the time slots during which the corresponding zones in the building are occupied and needed to be conditioned. For the occupied time slots, the lower and upper bound for indoor temperature for all zones are taken to be  $67F$  and  $79F$ , respectively. The outdoor

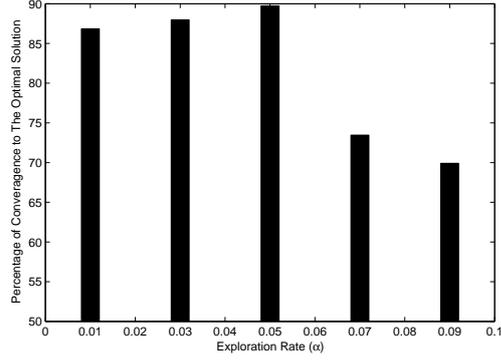


**Figure 5.3:** Payoff versus the number of iterations.

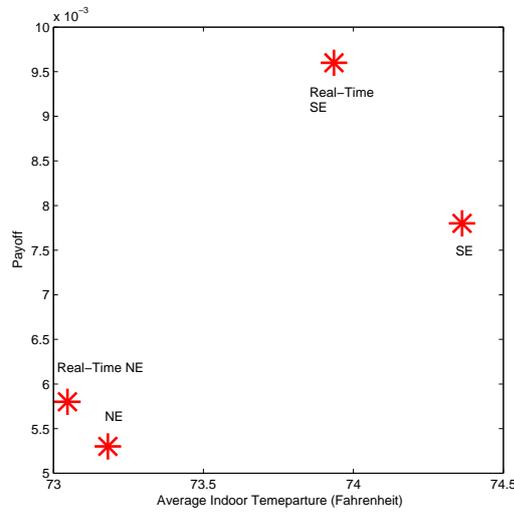
temperature and market price are depicted in Figure 5.2(a) and Figure 5.2(b), respectively according to data in [80].

Other simulation parameters are set as  $\epsilon = 0.7$ ,  $\lambda = 0.9$ ,  $K = 15$ ,  $c_1 = c_2 = c_3 = 1$ ,  $B = 100$  and  $L_r = 1.5 \text{ KW}$ . The energy consumption of the HVAC units is assumed to be chosen from the set  $\{0, 0.2, 0.4\}$  for both games. First, we compare the payoff of our proposed game in the satisfaction form with its payoff in the normal form and the payoff of the maximum social welfare solution as shown in Figure 5.3. The learning algorithm parameter is set at  $\alpha = 0.05$  and the number of iteration is 10000. The proposed learning algorithm is able to meet the optimal solution for most of the time (stochastically stable state). For the game in normal form, players can achieve NE after a few iterations, and the payoff of players is considerably lower than the maximum spacial welfare.

Figure 5.4 studies the effect of exploration rate ( $\alpha$ ) on the convergence of the game's payoff in satisfaction form to the maximum social welfare. As the figure shows, by decreasing the exploration rate from 0.05 to 0.01, the players tend to stick to their choices. Therefore, the learning algorithm might temporary stabilize at a non-optimal state. On the the other hand by increasing the exploration rate from 0.05 to 0.09, the payoff decreases again. This can be explained since for higher exploration



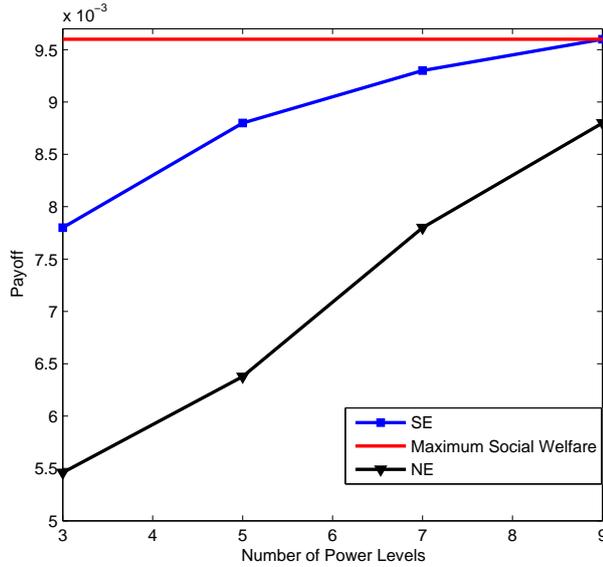
**Figure 5.4:** Satisfaction game convergence versus the exploration rate.



**Figure 5.5:** The payoff versus the average indoor temperature.

rates, players choose their actions more dynamically, and consequently the algorithm might not be stable even if it has converged to the maximum social welfare.

Next we compare the average conditioned indoor temperature during the scheduled time slots for zone 1 versus the achieved payoff for the four cases of day-ahead SE, day-ahead NE, real-time SE and real-time NE in Figure 5.5. Note that the payoff function for 12-hour cases are different from that for the real-time cases. However, the payoff achieved by the game in satisfaction form is higher than that of the game in normal form at the cost of higher average indoor temperature.



**Figure 5.6:** Payoff versus the number of power levels.

Finally, Figure 5.6 shows by increasing the number of granularity levels of the power consumed by HVAC units, the payoff increases and finally archives the maximum social welfare solution with the continuous action set. Having 9 power levels, the SE has a similar payoff to the maximum social welfare solution, and significantly better payoff compared to the NE payoff.

## 5.6 Summary

The proposed learning algorithm is able to control the power consumption levels of HVAC units in order to guarantee that the largest number of zones have stochastically stable indoor temperatures which are falling within the predetermined comfort range of users, while incurring the minimum electricity cost. To implement the proposed game in the satisfaction form and the proposed learning algorithm, the HVAC unit of each zone only requires little information from other zones. Simulation results demonstrate the properties of the proposed game and show how HVAC control units can reduce the building electricity cost effectively while satisfying the constraints.

## Chapter 6

### Conclusion and Future Work

#### 6.1 Conclusion

The smart grid is envisioned to be a large-scale cyber-physical system composed of intelligent nodes that can operate, communicate, and interact autonomously in order to efficiently deliver power to their consumers. The heterogeneous and decentralized nature of the smart grid motivates adoption of game theory for overcoming various design, control, and implementation challenges. In this dissertation, we proposed game theoretical frameworks for distributed dynamic control application in smart grid systems. We summarize the research that has been completed for each of these frameworks in this chapter.

First we studied the problem of optimal distributed generation and storage for thermal and pumped-storage power plants in a smart grid network. We proposed a stochastic differential game approach to model their competition and derived the optimal strategies for both players if the other's action is fixed using HJB equations. We showed that there exists the feedback Nash equilibrium strategies for the proposed game. Simulation results demonstrate the properties of the proposed game and suggest how the two types of power plants need to adjust their generating and storage decision variables to maximize their revenues. It is demonstrated that the proposed framework and games can reduce the peak to average ratio and total energy generation for the thermal plant, which helps power plant operation and reduces CO<sub>2</sub> emission.

Next, we considered smart buildings equipped with renewable resources generators, local energy storage and controllable HVAC units, in which users are able to respond to real-time grid conditions like electricity prices and weather conditions in order to minimize their cost. We modeled the strategic interactions between

such energy-smart buildings and a Nash-Cournot electricity market using a two-level stochastic differential game framework. At the upper level, the market offers a vector of hourly prices to end users. At the lower level, the energy-smart buildings as the lower level participate in demand response by managing controllable dynamic load in response to hourly prices set by the market. Specifically, two dynamic states in energy-smart buildings are investigated: the battery's state-of-charge and the room temperature. The solution of the formulated game among different buildings is derived by HJB equations. As simulation results show, the proposed method reduces the overall power consumption of all users, by storing the energy when the price is low and by later discharging it when the price is high. The peak-to-average ratio of aggregate load demand as well as overall energy cost is also greatly reduced.

Finally, we focused distributed control of HVAC systems as major consumption units in buildings. The problem of electricity cost efficient scheduling of HVAC units while satisfying predetermined ranges of desirable temperatures during certain periods for multiple zones in the building that are subject to temperature control is taken into consideration. The game-theoretic tool used to analysis this problem is satisfaction equilibrium, in which players do not seek any benefit maximization but only the satisfaction of their own individual constraints. The notion of satisfaction game enables players to make stochastically stable decisions with limited information from the rest of players. To achieve satisfaction equilibrium, a trial-and-error learning is proposed. We showed that this algorithm reaches stochastically stable control decisions that are equilibria and maximizers of the global welfare of the corresponding game. The propertied of proposed game and learning algorithms are studied for both day-ahead and real-time cases through simulation results.

## 6.2 Future Work

This section includes the overview of future research and possible progressions in the studied problems in this dissertation on distributed dynamic control schemes for smart grid networks. We introduce the following topics that can be chosen for further advanced research.

- Design of multi-stage game-theoretic frameworks for integration of distributed generation units to smart grids' electricity markets in order to achieve optimal trading prices among competing distributed generation entities as well as optimal generation or storage decisions.
- Incorporation of mean field games [98, 99] for distributed control applications with extremely large number of players such as large number of buildings, PHEV vehicles, and so on in order to avoid the high volume overhead of feedback information which enforce timing delays and data storage costs to the grid. In contrast to N-player games, where each player follows the evolution of the state of the game and the actions taken by all other players in order to maximize a given individual benefit, in the mean field game formulation, every players action is driven by the collective (or mean) behavior of all players and not by the individual actions of each other player. As a main consequence, it is possible in these games to follow the state trajectory of all players at once and to capture the behavior of the players depending only on their initial state and the joint distribution.
- Sensor-driven analysis to obtain full picture of almost every aspect of the building's states and dynamics for more efficient buildings energy management systems. For example, using infrared sensors to determine whether a space is occupied or not, how much daylight there is, whether the lights are on and what level of lighting they provide would enable design of more efficient lighting control

systems. Integrated data from sensors in electricity and water meters, lighting, HVAC systems, geothermal pumps [100] and other subsystems will provide immediate insight into the overall state of the building's dynamic systems, as well as design of energy efficient distributed controls.

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