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Recycling Produced Water in Hydraulic Fracturing: A Comprehensive Analysis of its Impact on the Formations of the Appalachian Basin

A Thesis

Presented to

the Faculty of the Department of Chemical and Biomolecular Engineering
Petroleum Engineering Program

University of Houston

In Partial Fulfillment
of the requirements for the Degree
Master of Science
in Petroleum Engineering

by

Frederick B. Woodward

August 2015

Recycling Produced Water in Hydraulic Fracturing: A Comprehensive Analysis of its Impact on the Formations of the Appalachian Basin

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An Abstract

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Abstract:

Environmental interest, regulation changes, and costs have motivated the oil and gas industry to begin recycling produced water in high concentrations during new well stimulation. Accordingly, the potential impact of this practice on production should be investigated. In this thesis paper, tests were conducted to determine whether Marcellus produced water would cause incompatibilities in the Utica and Point Pleasant formations. A multivariate statistical analysis was then completed using a historical dataset of over 300 Marcellus wells to measure the effect of produced water used in stimulation had on well production. The results indicate that recycling produced water in high proportions, even from the Marcellus, should have no measurable impact on the productivity of Utica and Point Pleasant wells. This conclusion supports the use of recycled water not only to comply with regulations and address environmental concerns, but also as a method to reduce water management costs by at least 40 percent.

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I. Introduction

One of the most significant challenges the unconventional oil and gas industry currently faces is water management. Hydraulic fracturing can consume up to 12.6 million gallons of water per well and large portions of that water (10 – 50% depending on the formation) flows back over the life of the well; this water is known as produced water. Obtaining fresh water in the large quantities that are necessary to fracture a well and disposing of produced water can be costly and problematic. Recycling produced water in hydraulic fracturing can greatly decrease the total water management costs of a well. The objective of this study is to explore the re-use of produced water as a water management strategy and the potential impact of doing so on well productivity. Specifically, this research was prompted by the pending experimental re-use of water from the Marcellus shale to fracture wells in the Utica formation.

In order to investigate the recycling of produced water, it was necessary to first examine the interaction of the Marcellus produced water with the Utica formation. The primary concerns with the viability of recycling produced water are clay swelling, rock softening, and mineral scales, all of which are potential hindrances to well productivity. Roller oven shale stability, hardness reduction, and dynamic scale loop lab testing were performed to determine if using produced water would increase the likelihood or severity of these concerns. Additionally, the effectiveness of friction reducers in produced water was tested in the lab using a friction flow loop.

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¹ Historically, the term "flowback water" was used to refer to any fluids recovered from a treatment to a well, while the term "produced water" was used to describe only fluid that originated in the formation and was recovered. These definitions have often been confused, and now, "flowback water" is generally defined as fluids that are recovered within the first 60 days of the life of the well and "produced water" is any fluid recovered after that point. For simplicity, this paper will use the term "produced water" for any fluid recovered throughout the life of a well. For further discussion on the difference between flowback and produced water, see "U.S. Produced Water Volumes and Management Practices in 2012" by John Viel (2012).

A multivariate analysis of temporal data obtained from development of the Marcellus shale by Shell Exploration & Production Company (SEPCo) was then performed to evaluate any potential relationship between the reuse of produced water and well productivity. The outcome of the multivariate analysis confirms whether recycling high percentages of produced water has been detrimental to Marcellus well productivity. The results combined with the lab testing of the Utica cores, gives insight into whether this relationship is transferable to the Utica formation.

The next section of this thesis reviews existing research on recycling produced water and explains fundamental formation concepts. The methodology used in each of the tests and analyses is discussed in Section III. The final section presents the results of each part of the study, discusses the significance of the study and its weaknesses, and makes recommendations for additional research that should be performed.

II. Background

A. Past Research

Research on the topic of recycling produced water in fracturing operations began with enhanced oil recovery (EOR) and enhanced gas recovery studies (EGR). EOR research generally focuses on water-formation interactions and the fundamentals of chemistry. There are many papers that discuss the ideal salinity of injected brine for EOR and the need to prevent formation damage by ensuring that the injected water does not cause rock softening, scale, erosion or reduction in oil relative permeability due to high salinity brines. Romanuka et al. (2012) discuss the industry-accepted objective of achieving a "balance between improvement of oil recovery by low salinity brine injection and prevention of formation damage due to swelling and/or deflocculation of salinity-sensitive clays present in sandstone rocks." Morrow and

Carlisle (2012) present the widely accepted idea that improved recovery can be obtained by low salinity waterflooding for EOR applications, but acknowledge that "a consistent mechanistic explanation has not yet emerged" for the low salinity effect (LSE). Without being able to fully explain the mechanism, it would be difficult to apply a best practice for increasing oil well productivity to gas wells, especially those in shale formations. The field of EGR is far less robust than that of EOR but is more relevant for this thesis. Thomas (2005) discusses the mechanics and theory of EGR through waterflooding but fails to touch on water-gas-formation interaction and formation damage. Minimizing the ionic strength of injection brine for oil wells is necessary to increase relative oil permeability. For gas wells, increasing oil permeability is not applicable, so the focus in evaluating the ideal brine would be to control clay swelling, which would perhaps be achieved with higher salinity brines.

EOR and EGR research has provided the groundwork for water chemistry considerations with respect to recycling produced water and, as the topic has gained popularity in the industry, subsequent case studies have further investigated the specific case of using produced water in fracturing unconventional wells. Alongside service companies' work on the use of high TDS waters, operators have joined in publishing case studies on the successful use of 100% recycled water in fracturing new wells. In Eddy County, New Mexico, XTO performed cross-linked hydraulic fracture stimulations on seven Brushy Canyon sandstone wells using 100% produced water (Lebas et al., 2013). Their paper outlines the challenges and subsequent solutions they used to overcome the complexities of maintaining performance of the crosslink system in high salinities (285,000 TDS). Even with the difficulties encountered, the project averaged a savings of \$70,000-\$100,000 per well in water cost and greatly reduced the trucking requirements to support the operation by not having to travel large distances required to source fresh water.

Although the authors only mention the production of one of the wells, they report that the well productivity was aligned with that from offset wells during the first 14 days of production.

A recent study in Canada showed that using 52% produced water in the fracturing treatment increased initial well productivity. They also showed that when used in gelled hydraulic fracturing stimulation of vertical wells in the Milk River sandstone, it had no impact on long term production (Monroe et al., 2013). The study suggests that initial production increases may even be more prominent for horizontal wells using large volumes of produced water for stimulation. The authors attribute the increased production to the prevention of clay swelling. While the methods for analyzing the well productivities may lack thoroughness, the authors do present a 48 well, 10-year investigation into recycled water's effect on production.

Perhaps the most impressive case study that has explored recycled water and well productivity came from Statoil who released results in early 2015 of a four well project that looked into the feasibility of using produced water for a hybrid fracturing stimulation consisting of both slickwater and cross-linked gel (Schmidt et al., 2015). Two wells (one Middle Bakken, one Three Forks) were stimulated with base water that was 100% fresh water while two adjacent wells (drilled in the same formations) were stimulated with 100% produced water as base water. The initial results at the Three Forks well after five months of production show that the well stimulated with produced water as base water produced 32% more oil than the adjacent well that was stimulated with fresh water as base water. Similarly, the Middle Bakken well where produced water was used as base water produced 24% more than the Middle Bakken well where fresh water was used as base water. Unfortunately, the researchers did not directly discuss the geology of the wells or other possible reasons why these adjacent wells may have had different production rates. Due to regulations and water management plans that exist

in the region, the logistics of using 100% produced water was reported to be more expensive than fresh water by \$250,000 per well, but the authors argue that these costs can be greatly reduced as new technologies and best practices are created. A limited economic study shows this \$250,000 investment appears to be worth the additional increase they saw in production. In the first five months alone, the produced water stimulated wells combined to produce ~24,000 bbls more oil than then fresh stimulated offsets. In the current \$60/bbl oil market, the project easily covered the \$500,000 upfront investment.

Multivariate analysis is frequently used in the oil and gas industry to identify patterns, especially when there are many influencing factors. In evaluating the relationship between the use of produced water and well productivity, multivariate analysis enables the geology and completions geometry to be examined in more detail.² Cunningham et al. (2012) performed a case study using a linear regression model to determine key factors that influence well productivity in the Marcellus shale. According to the authors, their study provides value by incorporating additional information to assist in weighing the economics of key decisions regarding well geometry and completions design. Although they do not consider the impact of recycling produced water, the regression model they present establishes the structure for a model that could be used to evaluate the relationship between produced water and well productivity. The model presented in this paper will investigate this relationship after using a similar approach to define key factors that influence well production.

EOR and EGR have laid the foundation for studying the interaction between water and the formation, but the research is not directly applicable to water properties in hydraulic fracturing. Industry operators have begun to realize the financial, environmental, and

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² More information on regression analysis and multivariate analysis can be found in the methodology section of this paper and Appendix A.

reputational benefits that can be realized by large-scale campaigns to recycle produced water in hydraulic fracturing. Case studies in hydraulic fracturing have shown that the economics generally support the shift to using high proportions of produced water. Operators have conducted analysis reports indicating that produced water may be beneficial to the formation in both sandstones and shales with clear evidence to support their hypotheses. Most of these case studies, however, do not go far enough to provide a scientific basis for their results and fully evaluate the effect recycled water has on the formation and well productivity. This thesis presents a comprehensive study in which lab testing is performed on Utica / Point Pleasant core samples along with a multivariate analysis of a large dataset of wells to investigate the relationship between produced water usage and production in the Marcellus shale.

B. Water Formation Compatibility

When looking at the interaction between water and the formation, the three primary mechanisms that can lead to poor well performance are clay swelling, rock softening, and scale precipitation.

Clay Swelling

Clay swelling is a type of formation damage that can clog pore throats and restrict hydrocarbon flow. It is caused by an ion imbalance between clays in the formation and fluids introduced to the system. If the fluids entering the formation contain an insufficient concentration of positively charged cations, a negative charge imbalance in the clays will attract water molecules to fill the gaps between the clay crystals. When the repulsion of the counterion clouds in water-clay structures exceed the forces holding the clay structures together, the clay swells. Swelling clays reduce permeability and can impair production of hydrocarbons.

A common practice to prevent clay swelling is the use of high salinity brines, such as potassium chloride, as completion and workover fluids. Waters with high salinity have high concentrations of cations, which replace the water molecules in the clay structure and inhibit the counter-ion forces that cause swelling. Produced water generally has high salinity and has been shown to perform better to prevent clay swelling than a fresh water alternative (Gdanski, 1999).

Rock Softening

Rock softening can cause formation rocks to fail and close in both propped and unpropped fractures. Un-propped fractures can fall apart or become more complex thereby restricting flow. Propped fractures can succumb to proppant embedment in softer rocks, making the fracture narrower and less permeable. With time, under formation temperatures, all fluids can cause rock softening. The degree of softening can be controlled by choosing the best completion fluid. In theory, the more similar the chemistry of the completion fluid is to the natural formation waters, the less softening would be expected since the formation would be in equilibrium. Therefore, introduction of a Marcellus-produced water, which is chemically more similar to Utica produced water, should cause less additional rock softening than fresh water. To confirm this, lab testing of core samples from the selected well sites was conducted.

Mineral Scales

Mineral scales have been a topic of concern in oil and gas production since the early days in the industry. Scales are inorganic solids that can precipitate from water and deposit in formations, wellbore tubulars, and surface facilities and they lead to production restrictions, workover costs, and safety concerns. Most fluids native to the formation are in equilibrium with the reservoir rock, and a change must occur in order to cause scale precipitation and

subsequent deposition. Examples of such changes are mixing of two incompatible waters and variances in temperature or pressure.

There are four main types of scale: carbonates, sulfates, sulfides, and salts. Carbonate scales are the most common and are very quick to form. Calcium carbonate or calcite CaCO₃ formation in oil and gas wells is governed by the following equation:

$$Ca^{2+} + 2 HCO_3 \rightarrow CaCO_3(s) + CO_2 + H_2O.$$
 (1)

Calcium carbonate can precipitate as shown in Eq. (1) if CO_2 is removed from the system, which can occur by a decrease in pressure. Also, and specifically with hydraulic fracturing applications, calcite can form if calcite-based brines or produced water are injected into the formation where they begin to equilibrate with CO_2 gas that is already present in the formation. Calcite scale may be the easiest to mitigate, but it can form quickly and cause well integrity issues in a matter of days (Bellarby, 2009).

Sulfate scales contain Group II metals (known as the alkali earth metals), which have a +2 valence from loss of their outermost two electrons in their s-orbitals. Calcium, strontium, barium and radium are the most reactive of the Group II metals. The formation of sulfate scales requires the presence of these metal ions and sulfates. Generally, sulfate levels are very low in formation waters but exist in fresh water and can be found in very high levels (2000+ ppm) in seawater. Concern for sulfate scales is, therefore, raised when formation waters are mixed with fresh water. (The use of seawater is not applicable to on-shore gas wells.) Barium sulfate, or barite, is the most common sulfate scale and is the first to form, following the equation

$$Ba^{2+} + SO_4^{2-} \rightarrow BaSO_4(s)$$
. (2)

It is also the most problematic of the scales due to difficulties that arise in its removal process; barium sulfate is insoluble in traditional acids. The removal of barium sulfate usually requires both mechanical and chemical remediation techniques (Bellarby, 2009). The presence of radium sulfate in the crystal lattice structures of barium sulfate can create additional health, safety and environment (HSE) exposures, since radium sulfate exhibits radioactive properties and can be harmful to humans if ingested.³ The best course of action for addressing barium scale is prevention rather than mitigation after formation. The only preventative measure, which has become common practice when SO₄ levels are below 100 ppm, is the use of an inhibitor pumped with the stimulation treatment (Frigo, 1999).

Sulfide scales are the product of a sulfide ion and a metal ion. In general, they are less common than other scales. Lead, zinc and iron sulfides have all been reported; but lead and zinc are most often associated with high pressure, high temperature wells. Therefore, they are not as much of a concern in Marcellus, Utica, and Point Pleasant formations. Iron sulfide (FeS) scale is usually the result of iron formed by corrosion reacting with sulfide ions, although it can form in more than one crystalline structure. Iron sulfide scale is acid soluble, but acid remediation treatments can create additional HSE concerns through the formation of toxic hydrogen sulfide gas (Bellarby, 2009).

Salt deposition is the final type of scale encountered in the oil and gas industry. It is uncommon and is the result of oversaturated sodium chloride (Halite) usually found in very high salinity brines or very small water quantities. Halite scale can precipitate if there is a temperature drop, or more commonly, when "a large amount of water evaporates" (Frigo, 2011). The injection of produced water from the Marcellus formation, which has very high

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³ Barium sulfate and radium sulfate are both considered Normally Occurring Radioactive Material (NORM).

salinities, into the Utica formation presents a scenario in which this tendency may increase. The Utica formation has higher temperatures and consequently higher temperature drops during production. Similarly, Utica wells also experience greater pressure drops which could lead to a higher percentage of water evaporation. As a result, TDS levels are being managed in the Utica shale until further information can be attained on the matter.

Using Marcellus-produced water should cause less clay swelling and rock softening than fresh water would due to Marcellus-produced water's high salinity and more similar composition to the formation. The effect on rock softening can be confirmed by testing core samples of the particular formation. The Marcellus-produced water's effect on scale formation is a more complex issue and requires in-depth chemical analysis of the water. The results section of this thesis discusses in detail the testing that was performed on the Utica formation and the Marcellus-produced water to evaluate the interaction between the water and the formation.

III. Methodology

A. Economics

The current oil and gas market highlights the need to reduce costs in drilling and completing new wells. Profitability relies on being able to deliver the most productive wells safely and at the lowest cost. There are many factors used to determine the lowest cost option for water management including water acquisition, trucking, treatment, storage, and disposal. These economic considerations for water management are greatly dependent on the geographic region being considered due to varying regulatory requirements and water availability across regions. This study is mostly limited to the discussion of options for water management within the state of Pennsylvania.

When compared to other states, Pennsylvania is unique for a number a reasons. Water disposal in deep injection wells is essentially non-existent due to the geology of the region (Rassenfoss, 2011). As of 2013, the state had only 8 disposal wells that accepted water from oil and gas producers with a total capacity of 8667 BWPD (McCurdy, 2013). That capacity represents less than 10% of the 93,400 barrels of water generated daily by oil and gas production in Pennsylvania (Veil, 2015). The nearest disposal well options involve trucking the water to Ohio which can cost \$8 to \$15 per barrel from Northeastern Pennsylvania. After disposal fees are added, the cost to dispose of this water can be as much as \$20 to \$25 per barrel.

Treatment and discharge options in Pennsylvania have been significantly limited by regulators. In 2011, the Pennsylvania Department of Environmental Protection asked 15 publically owned treatment facilities to stop handling produced water from the Marcellus shale. The push from regulators has encouraged operators to begin developing new methods to dispose of their produced water and has led to an environment where water recycling from hydraulic fracturing has begun to flourish.

Generally, as the two case studies conducted by XTO and Statoil revealed, the best option for stimulating a new well is to do so in a way that allows recycling all the produced water from the nearby wells in the field. To further investigate this conclusion, this study created an economic model that scoped three different scenarios for produced water management. The model considered the yearly costs with a drilling schedule of 14 wells for 2015. Using temporal data from SEPCo's operation in Pennsylvania, the three models were all built to have the same infrastructure, drilling, and base operating costs. They also had the same

royalties, taxes, and gas sales rates. However, the three scenarios were adjusted to reflect the cost implications of managing water in three different ways.

- Scenario 1 of the proposed model represents a plan in which all produced water is sent to treatment or disposal and all water that was used in hydraulic fracturing treatments is fresh water.
- Scenario 2 is very similar to the current plan that is employed by SEPCo in which most
 water produced in the field is re-used in fracturing new wells. The produced to fresh
 ratio for the fracturing treatments is one to one and only about 4% of total fieldproduced water is sent to treatment facilities.
- Scenario 3 exemplifies a plan where all water that is used to fracture new wells is
 produced water. The difference in total water needed to fracture new wells and field
 production is made up by using water from other operators and treated water from
 local treatment facilities known as credit water⁴.

B. Lab Testing

The laboratory testing examined the impact of using Marcellus-produced water to hydraulically fracture new Utica wells⁵. The testing was divided into two categories: chemical compatibility and reservoir compatibility. The chemical compatibility testing ensured that the chemicals added during the hydraulic fracturing jobs would be compatible with the various Marcellus-produced waters would be recycled. Chemical compatibility tests are performed frequently and are routine, but the testing performed in this study represents the first time they were performed by SEPCo at 100% produced water loading, which was the proposed plan for

⁵ This lab testing is directed specifically at the Utica formation and not the Marcellus. The Marcellus was not previously formally tested for this compatibility largely due to a belief accepted industry-wide that reintroducing Marcellus-produced water into the Marcellus formation was not damaging to the formation.

 $^{^{4}}$ An example of credit water composition can be seen in Treated Sample 1 in Figure 1.

future Marcellus wells. Reservoir compatibility testing ensured that the Marcellus-produced water did not lead to increased scale formation, clay swelling, erosion, or rock softening in the Utica formation.

Water Sample Set

The water sample set used for the lab testing was designed to model the range of water qualities for base water that could be used during a fracturing treatment. Over time, total dissolved solids (TDS) and ion levels in produced waters from a given well tend to increase. For the sample set to be representative of the field, produced water from both old and new wells was needed. Flowback water is defined as water from wells flowing less than 60 days, while produced water is from wells flowing more than 60 days. (See Table 1 below for complete ion analysis of source waters used in this study.) Eight total samples were taken, from two different formations—the Utica and Marcellus— that represented both flowback stage and produced water⁶.

- Marcellus Sample 1 and 3 are from wells over two years old and come from an area that produces the highest TDS levels in the field.
- Utica Sample 2 is the oldest Utica well in the area.
- Marcellus Sample 2 and Utica Sample 1 represent wells in flowback stage.
- Treated Sample 1 is oil and gas produced water from a nearby water treatment facility.
 The facility is paid to take produced water from oil and gas operators and treat it to reduce iron content and adjust the pH.
- Fresh Sample 1 and 2 were taken from fresh water impoundments in the area.

⁶ Wells in Appalachia with laterals drilled in either the Utica shale or underlying Point Pleasant shale are both commonly referred to as Utica wells (as in this case). When necessary, the distinction between the two will be made in this paper.

Table 1: Source Water Sample Set. Chemical analysis of water samples used for the fracturing chemistry, core testing, and scale testing.

Sample Name	Fresh Sample 1	Fresh Sample 2	Treated Sample 1	Utica Sample 1	Utica Sample 2	Marcellus Sample 1	Marcellus Sample 2	Marcellus Sample 3
Total Barium (Ba) mg/L	< 0.35	1	9,663	726	2,593	3,529	14,090	3,349
Total Calcium (Ca) mg/L	16	34	19,800	5,350	17,080	35,740	23,440	39,170
Total Iron (Fe) mg/L	<0.7	<0.7	<0.7	112	88	<0.7	118	95
Total Potassium (K) mg/l	<1.75	<1.75	285	1,000	2,497	442	254	498
Total Lithium (Li) mg/L	<0.7	<0.7	211	<0.7	84	211	194	231
Total Magnesium (Mg) mg/L	3	5	1,499	543	1,248	3,328	1,512	3,632
Total Manganese (Mn) mg/L	< 0.35	< 0.35	< 0.35	< 0.35	< 0.35	< 0.35	< 0.35	< 0.35
Total Sodium (Na) mg/L	9	50	44,060	22,020	62,270	63,810	49,330	60,900
Total Strontium (Sr) mg/L	<0.7	1	5,078	2,760	6,851	4,406	6,182	4,361
Chloride (Cl ⁻) mg/L	4	105	122,500	47,320	129,300	155,500	134,700	165,000
Sulfates (SO ₄) mg/L	10	13	<1.25	<1.25	<1.25	<1.25	<1.25	<1.25
Total Alkalinity (CaCO ₃) mg/L	53	90	107	224	86	56	62	71
Bicarbonates (HCO ₃) mg/L	65	110	131	273	104	69	76	87
Carbonates (CO ₃) mg/L	0	0	0	0	0	0	0	0
Hydroxides (OHT) mg/L	0	0	0	0	0	0	0	0
Total Hardness (CaCO ₃) mg/L	53	106	55,610	15,600	47,790	103,000	64,760	112,800
TDS Calculated mg/L	73	261	188,200	76,360	212,400	258,800	209,200	269,200
Specific Gravity	1.00	1.00	1.15	1.06	1.15	1.18	1.17	1.19
pH	7.7	6.8	6.0	6.0	6.9	5.5	5.0	5.3
Temperature (ºF)	69	69	69	69	69	69	69	69
lon Balance (%)	3	-8	-2	1	4	7	0	5

Fracturing Chemistry

Chemical performance is one of the most important factors in effectively placing sand in a formation during a hydraulic fracturing treatment. Fracturing chemicals depend on water quality, chemical composition, and temperature for them to perform properly. High rate 'slickwater' fracturing was SEPCo's only planned method to stimulate the subject wells. Therefore, the primary concern for this study was friction reducer (FR) performance in the range of water salinities that could be used in the field.

There are two main types of FRs used in high rate slickwater fracturing. Anionic FRs have traditionally been the favorite FR in hydraulic fracturing due to their low cost relative to cationic FRs. Anionic FRs perform very well in the low TDS waters that have historically been used for hydraulic fracturing. However, TDS levels have risen above the traditional limit of

50,000 ppm due to the increased use of produced water, and cationic FRs have become the more economic option due to their superior performance at higher TDS (Fichter, 2010).

From an operational perspective, the goal is to place sand in formation in the most cost-effective way. Minimizing FR loading while still achieving the desired pressures that prevent equipment damage and pressure up-charges from service companies is necessary. The logistics of water delivery require the ability to provide mix water that may range from very low (e.g., fresh water) to very high TDS (e.g., produced water), while maintaining the most cost-effective chemical formulation. Thus, extensive lab testing of the performance of different FRs using a range of source waters is imperative in order to achieve the goal.

The FR lab testing was conducted at the Calfrac Technology Center in Louisville,

Colorado. First, the samples listed in Table 1 were shipped to the lab in 260 gallon totes. The

constituents of the samples were analyzed using inductively coupled plasma (ICP) and standard

titration techniques. Based on these results, synthetic brines were prepared to match the water

quality of the samples. The synthetic brines were then circulated through the friction flow loop

system. The friction flow loop consists of several hundred feet of stainless steel tubing in an oval

configuration. There are five individual test sections that make up the overall length of tubing

and each is equipped with a pressure transducer. The water injection tub utilizes a variable

frequency drive to control the pump rate. During the test, flow rate, pressure differential, and

temperature are recorded for the base case (brine alone) and treated case (brine with FR). The

flow loop operates on-the-fly and is a one pass system to best simulate field conditions. The

base pressure across the loop is measured for the brine of interest and allowed to stabilize prior

to FR injection. Once a stable flow rate and pressure drop is achieved, the chemical pumps are

activated at the desired FR loadings. Friction properties are calculated based upon the base

pressure and treated pressure drop for each test section. The friction reducer effectiveness, or percent friction reduction (%FR), is calculated from flow loop data by

$$\%FR = \frac{(\Delta P_b - \Delta P_t)}{\Delta P_b} \times 100,\tag{3}$$

where ΔP_b is the base pressure, and ΔP_t is the treated pressure differential after FR addition.

The use of synthetic brines for the initial testing allows many runs to be performed to determine the chemical composition (FR and base water proportions) that performs best. An average of 50% friction reduction is targeted across the loop. If the 50% reduction cannot be achieved by addition of cationic or anionic FRs alone, non-emulsifying surfactants can be added at low loadings (0.5-1.0 gpt) to help augment the performance of the friction reducer. Once the slickwater systems are optimized using synthetic brines, the results are verified using the actual source waters.

Usually, the performance of the FR using the synthetic versus actual brine is similar. However, this is not always the case. Chemicals added in a water treatment process, which are not detectable by analysis with Inductively Coupled Plasma (ICP) and standard titration techniques, can adversely affect the friction reducer performance. One such water source (Treated Sample 1) was identified in this study. The presence of these chemicals and the subsequent resolution needed are described in the results section but this example serves as a confirmation of the importance of lab testing with actual source waters rather than only with synthetic brines.

Core testing

The cores for the roller oven shale stability and hardness reduction testing were collected during drilling operations of two wells across the Point Pleasant and Utica formations.

The first test performed on these samples was roller oven shale stability testing, which simulates the circulation and contact of the fracturing fluid through the formation and measures the effect of erosion the fracturing fluid may have on the formation. The core samples are ground and sieved so that the particles being tested are between 0.425 mm and 2mm (10-40 mesh). The particles are then split equally into 10 gram samples using a spinning Riffler. Each sample is submerged in 50 ml of the fracturing test fluids and placed in a roller oven for 24 hours at the average Utica formation temperature of 175 degrees. After the samples are pulled from the roller oven, they are sieved through a 70 mesh screen (0.269 mm) before rinsed with fresh water and dried. The roller oven (RO) value is calculated as the percentage of the original 10 gram sample that passed through the 70 mesh screen. According to the modified API RP 13i procedure, a RO value of 0.5 represents a sample with no sensitivity to the fluid, while a RO value of 10 represents extreme sensitivity.

Hardness reduction testing compares the hardness of each core sample before and after submersion in test fluids and yields a percentage change in rock hardness. Each core sample is first trimmed to be of equal size and thickness then polished to have smooth surfaces. Samples were measured six times on one surface and six times on the opposite surface with an Equotip 3 calibrated for Leeb's reference hardness scale. The samples were then placed in individual containers and submerged in the test fluids for 24 hours. Next, the samples were removed and left to drain for one hour after which the same six hardness measurements per side were taken. The mean Leeb's hardness was calculated for each sample and converted to a Brinell hardness reference and unconfined compressive strength (UCS).

Scale Testing

Before laboratory testing for scale formation, modeling is conducted to forecast the conditions when scaling will be most severe. The French Creek Downhole Sat™ software is used to determine scale onset. As described in the Fracturing Chemistry section, a routine water analysis and extended metals analyses are performed on the subject water samples using (ICP) and standard titration techniques and the results are entered into the French Creek Downhole SAT™ scale modeling software. The software is capable of calculating saturation indexes of many common and uncommon scales, as well as modeling scale potentials at varying conditions such as temperature, pressure, and pH. SEPCo OLI software was also used to predict scaling and produced results similar to the French Creek model. The lab analyst based the Dynamic Tube Block testing on the worst-case scenario according to the model results.

The Dynamic Tube Block – scale loop apparatus was constructed in-house at Calfrac Well Services Corp. The apparatus is simple in that pressure and temperature can be matched to surface, bottom hole, or any condition in between. The pressure differential is measured across a test section of microbore tubing. The test section tubing has a small ID, so that the onset of scale precipitation and plugging will be detectable as minute changes in differential pressure. In order to properly evaluate scale formation, two synthetic brines are formulated, one containing incompatible anions and the other containing incompatible cations. Scale formation is isolated to the test section by injecting the two brines separately through individual heat exchange coils and combined just ahead of the test section. A temperature reading is taken at this point, and differential pressure is measured across the test section.

C. Statistical Analysis

Model Form

Statistical modeling is a powerful tool capable of providing insight into complex relationships. Alongside modern advancements in computing technologies, it has developed into a robust field with a vast array of tools for analyzing datasets. Selecting the best method to evaluate a particular dataset can be as difficult as carrying out the actual analysis. The goal is to find a useful way to interpret and predict relationships between variables by assigning the best-fitting formulae to these relationships. There are many challenges in determining the best model to analyze a particular dataset or what data is relevant to the model. The size of the data set, relationships between independent variables, outliers in the data, and the motivation for the study itself should all be considered. Although it is possible to achieve a very close-fitting and informative model, it is important to keep in mind the limits of each method of statistical analysis. In the words of renowned statistician George E.P. Box's: "Essentially, all models are wrong, but some are useful."

The approach used in this study was that recommended by Jablonowski and MacEachern (2009): "Unless the analyst has definitive knowledge or strong hypotheses regarding a specific functional form, it is most common to specify a simple linear relationship as a starting point." In the end, it was determined that an adequate model for this study was a linear model due to the desire to obtain results that were highly interpretable and the high level of noise in the data. Flexibility and interpretability are terms that are used often to describe the complexity of a model. Simple models are considered less flexible but easy to interpret. With complexity comes the ability to create a closer-fitting model, but the results can often be very difficult to relate back to the input variables. The primary focus of the study was to determine the relationship between the predictor variable water composition on the response variable

productivity. The study did not need to accurately predict well productivity of future wells. For this reason, a simple and highly interpretable model was better suited. Additionally, since the dataset was influenced by many unknown and random variables causing it to have high levels of noise, more complex functions would have over-fit the data and led to high variance. Small differences or trends in the dataset may be missed or overlooked by using a less-complex, linear model. However, such small differences are likely insignificant and caused by random error inherent in the data. In statistics, this is called the bias-variance tradeoff, which is explained further in Appendix A.

Dataset and Production Constraints

This study specifically investigated wells in the Marcellus shale operated by SEPCo. The dataset contained approximately 330 horizontal wells in SEPCo's Tioga County asset. Over the course of time in Appalachia, SEPCo has learned many lessons regarding both geology and fracturing geometry of the Marcellus formation in Tioga County, allowing the asset to be maximized with respect to the Estimated Ultimate Recovery (EUR) for gas production by focusing on well completion and geology. Wells that were poor performers were investigated to determine what geologic attributes may have led to their constrained production. The following four characteristics were identified:

Formation Thickness - For most of Tioga County, there is a fairly dependable linear
relationship between Marcellus thickness and EUR; however, this linear relationship
does not apply to the Texas Creek area. The reasoning for the underproduction in Texas
Creek is not fully understood but the hypotheses is that the subsurface surrounding the
Marcellus formation in Texas Creek suffers from high complexity.

- Proximity to Strike-Slip Faults Strike-slip faulting leaves a wide path of structural complexity, which has been proven to impair production. If the fault is close enough to the wellbore that the induced hydraulic fracture reaches it, the fault will "...act as a fluid migration pathway, thus hindering production in the area." (Roberts and Gao, 2013). As a result, screen-outs become frequent and it is difficult to place the designed proppant volumes into the formation.
- <u>Large Deformation Zones</u> These zones contain increased structural complexity leading
 to lower EUR. Additionally, it is more difficult to stay in the target zone when drilling
 through deformation zones, so that the wells contain sections that are not within the
 targeted lower Marcellus formation.
- Well Proximity to Salt Domes Salt domes have hindered wells in two ways. First, they
 can cause the dip angle of the Marcellus formation around them to be large. Second,
 wells near salt domes have experienced salt precipitation issues during production that
 have led them to salt-up the casing and tubing.

Wells that fell into the above categories were labeled as "constrained" and were all dropped from the dataset. The inclusion of these wells in the statistical study would bias the results towards factors that influence them, which the remaining wells in the dataset were not subject to.

There were a number of trial wells that have been conducted in Tioga County in an attempt to better understand the area. Stage length trials and proppant loading trials were dropped from the dataset. These trial wells are outliers and they skew the density distribution of the dataset. Examples of these outliers are highlighted in Figure 1.

After the above wells were eliminated from the dataset, data for 116 wells remained. Of the 116 wells, 87 wells recorded data on the amount of produced water that was recycled; wells drilled before June of 2011 did not require the same level of reporting to the state, and therefore water usage numbers were not recorded. Although this dataset is small, according to James et al. (2013), a dataset with 30 or more observations (wells) is considered to be statistically significant.

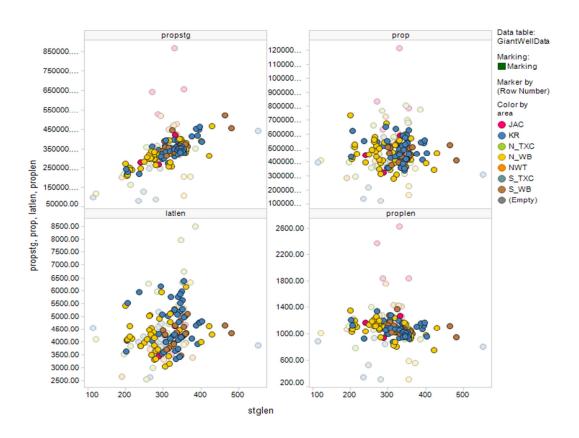


Figure 1: Identification of Outliers. Well data that were included in the analysis are shown in bold while outliers are shown muted. All parameters are graphed vs. stage length (stglen).

Proppant/stage (propstg), Total proppant (prop), Lateral length (latlen), Proppant/ft (proplen).

IV. Results & Discussion

A. Economics

Scenario 1 has significantly higher costs than other scenarios primarily due to the large distances associated with trucking miles and the higher premiums paid for treatment and disposal. Most treatment facilities and disposal wells can only accept a given volume of produced water per day. Operators may exceed these limits by paying a premium price per barrel. Also, Scenario 1 does not take advantage of the sunk cost of trucking produced water. The water will need to be trucked in other scenarios as well, however, if it is trucked to a fracturing site, it essentially equates to free trucking for the capital expenditures (capex) budget. Scenarios 2 and 3 both highly benefit from this free trucking.

Scenario 2 re-uses almost all water that is produced from the field, reducing the water operating expenditures (opex) by \$16 million per year. Nearly all of this opex cost reduction is attributable to a lower price per barrel for treatment, by only sending 4% of field produced water to treatment facilities. Capex costs are also reduced by \$300,000 per well by replacing fresh water with produced water. This scenario represents a 59% reduction in total water management costs compared with Scenario 1.

Scenario 3 is the most cost-effective water management option. All water produced by the field is re-used. Additional water is acquired from other operators and nearby treatment facilities at a credit of \$1.00 per barrel after trucking costs are paid, and no fresh water is needed. The \$9.5 million represents the minimum opex needed to store, truck, and coordinate the water management plan with no water being sent outside the field. Scenario 3 represents a 40% overall reduction in water management costs compared to Scenario 2.

⁷ Capex are all costs associated with drilling and completing new wells. Opex are all costs associated with wells while in production.

Obviously, the impact on well production plays a very important role in determining the best water management system. If produced water were determined to have a negative impact on production, the economics can quickly shift to make the scenario 1 the best option. To understand how the three scenarios for produced water management affect profitability, the "per well production for equal NPV" variable was included. Due to sensitivity of the data, the actual EURs and NPVs in the model will not be discussed; instead, both have been normalized. Scenario 2 is used as the base case, and well productivity was set to 1.000 BCF per well. This productivity level in Scenario 2 created an NPV of \$10 million. In order to have the same NPV, Scenario 1 would have to produce 1.064 BCF wells, and Scenario 3 would have to create 0.978 BCF wells.

In other words, if the re-use of produced water at 50% loading (Scenario 2) had a negative impact on production that was greater than a 6.4% decrease in well productivity, it would no longer be economical to re-use any produced water and Scenario 1 would be the best choice. Likewise, if Scenario 3 resulted in wells that were 2.2% less productive than Scenario 2, Scenario 2 would be more economical than Scenario 3.

Table 2: Cost Overview of the Three Water Management Scenarios.

	Scenario 1	Scenario 2	Scenario 3
Use of produced water %	0%	50%	100%
Total water management cost (\$MM)	\$34.372	\$14.186	\$8.571
Per well production for equal NPV (BCF)	1.064	1.000	0.978

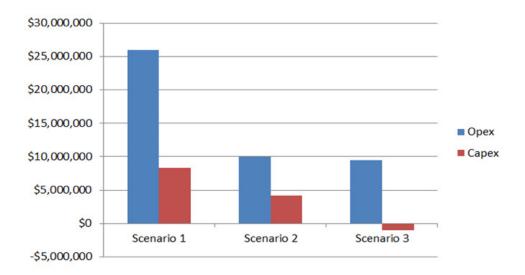


Figure 2: Total Yearly Capex and Opex Water Costs for the Three Produced Water Management Scenarios.

Economics Discussion

The economics of the three scenarios demonstrate the importance of this study. When discussing profitability, one must understand how production will be impacted by the three water management strategies. Well productivity is the single most important factor when determining the feasibility of recycling produced water. A 2.2% decrease in production is small when considering the significant effects that formation damage can have on a well's productivity.

Economic models in the oil and gas industry can change drastically when applied to different basins or price markets. This model was set up to study the current situation in the Appalachian basin and would need to be restructured to reflect the economics of operating in other basins. The Statoil Bakken example (see background section) reveals that there is a negative cost relationship in recycling produced water for use in fracturing new wells. This is likely due to the need for a more complicated and expensive chemical formulation in the Bakken formation combined with the relatively cheap disposal costs for produced water in North

Dakota. The price of natural gas also affects the "per well production for equal NPV" variable. If the average price per MMBtu is increased by \$1.00 over the life of the wells, the 6.4% difference between Scenario 1 and 2 noted above becomes 4.6%, and the 2.2% difference between Scenario 2 and 3 becomes 1.3%. Thus, while this economic analysis is helpful for the current market in the Appalachian basin, it would need to be restructured to apply to other areas or price markets.

B. Lab Testing

Fracturing Chemistry

An anionic, polyacrylamide-based friction reducer was used with the two source waters that were lowest in salinity, Fresh Samples 1 and 2. Table 3 and Figure 3 summarize the results of the friction flow loop testing with Fresh Sample 2, synthetic brine and source water. The results represent the optimum loadings for each friction reducer tested.

Table 3: Anionic Friction Reducer Performance for Fresh Sample 2. Both synthetic and actual field water friction reduction is shown at FR loadings in gallons per thousand gallons (gpt).

Chemical Loading (gpt)		Friction Reduction (%)					
		3 sec	16 sec	29 sec	42 sec	55 sec	Overall Friction
Friction Reducer	Base Fluid Type						Reduction
Anionic FR @ 1.0 gpt	Synthetic	50	63	64	64	62	55.6
Anionic FR @ 1.0 gpt	Field Water	51	63	64	65	62	56.1

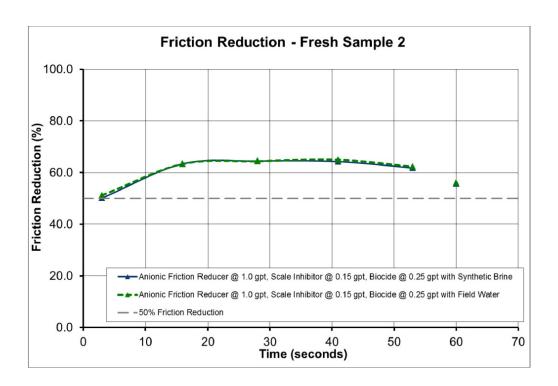


Figure 3: Anionic Friction Reducer Performance for Fresh Sample 2. Both synthetic and actual field water friction reduction is shown with chemical loadings in gallons per thousand gallons (gpt).

A cationic, polyacrylamide-based friction reducer was used with the higher salinity source waters. A non-emulsifying surfactant was added to the formulation containing Utica Sample 2 source water in order to reduce the emulsion stability of the friction reducer and allow quicker inversion of the polymers. Table 4 and Figure 4 show the results from the Utica Sample 2 compatibility testing.

The friction reduction measured for the synthesized Treated Sample 1 brine did not match that of the actual brine. It was hypothesized that an additive may have been added to the actual brine by the water processing facility during the treatment process. Upon further investigation, it was discovered that the surface tension of the actual source water was 30 dynes/cm, which is well below the 72 dynes/cm interfacial tension of pure water and air (Adamson & Gast, 1997). In order to perform further testing on the actual source water, a

Table 4: Cationic Friction Reducer Performance with Utica Sample 2. Both synthetic and actual field water friction reduction is shown at FR loadings in gallons per thousand gallons (gpt).

Chemical Loading (gpt)		Friction Reduction (%)						
Friction Reducer	Surfactant	Base Fluid Type	3 sec	16 sec	29 sec	42 sec	55 sec	Overall Friction Reduction
Cationic FR @	NA	Synthetic	0	53	64	67	65	50.6
Cationic FR @	Non-Emulsifier @ 1.0 gpt	Synthetic	3	70	71	70	67	56.9
Cationic FR @	NA	Field Water	10	46	53	53	52	47.3
Cationic FR @	Non-Emulsifier @ 1.0 gpt	Field Water	14	70	70	70	67	57.5

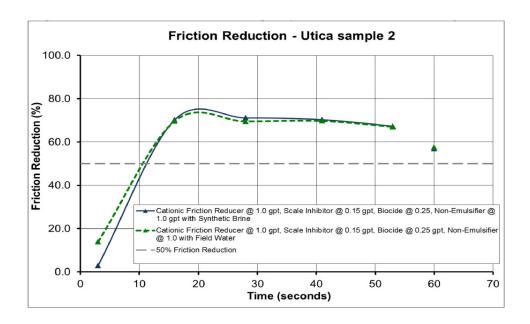


Figure 4: Cationic Friction Reducer Performance with Utica Sample 2. Both synthetic and actual field water friction reduction is shown with chemical loadings in gallons per thousand gallons (gpt).

second sample of the Treated Sample 1 source water was sent to the lab. The ICP and standard titration analysis on the second sample from the treatment facility established that the two waters were nearly identical, including the surface tension. Since a representative synthetic brine could not be formulated in the lab, the second sample from the treatment facility was used to test the system with the addition of the same non-emulsifying surfactant used on the Utica Sample 2. Table 5 and Figure 5 summarize the results of testing from this sample.

Addition of the non-emulsifying surfactant resulted in acceptable friction reduction of over 50%.

Table 5: Cationic Friction Reducer Performance with Treated Sample 1. Both synthetic and actual field water friction reduction is shown at FR loadings in gallons per thousand gallons (gpt).

Chemical Loading			Friction Reduction (%)					
Friction Reducer	Surfactant	Base Fluid Type	3 sec	16 sec	29 sec	42 sec	55 sec	Overall Friction Reduction
Cationic FR @ 1.0 gpt	NA	Synthetic	5	49	66	70	68	53.4
Cationic FR @ 1.0 gpt	NA	Field Water	6	25	32	34	37	29.2
Cationic FR @	NA	Re-Sample Field Water	2	11	11	12	15	10.6
Cationic FR @	Non-Emulsifier @ 1.0 gpt	Re-Sample Field Water	2	52	73	72	70	50.2

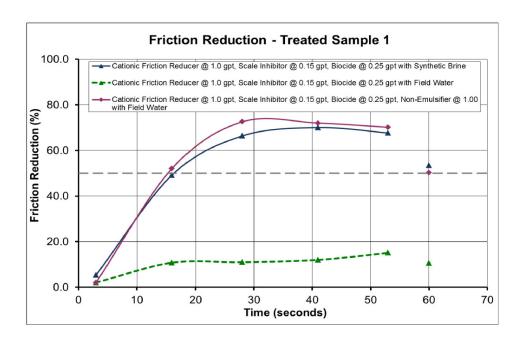


Figure 5: Cationic Friction Reducer Performance with Treated Sample 1. Both synthetic and actual field water friction reduction is shown with chemical loadings in gallons per thousand gallons (gpt).

Flow loop testing with a typical SEPCo slickwater system design yielded satisfactory results for most field waters provided. Where the typical design did not yield acceptable friction reduction (>50%), an enhanced design was developed by adding a non-emulsifying surfactant. The friction flow loop measurements using synthetic brines as base fluid duplicated (within 5%) those obtained using the actual source waters submitted as base fluid, with the exception of the Treated Sample 1 source water. Even though acceptable friction reduction was achieved with the addition of non-emulsifying surfactant, caution is recommended when and if water from a treatment facility is used.

Core Testing

The sample set used for roller oven shale stability and hardness reduction testing consisted of 26 core samples. The core samples were taken from two different wells drilled with

names are not used and instead are referred to as "Utica Wells A and B." Table 6 (below) lists the depths and formations of each core sample. The four water samples used for the core testing were Marcellus Samples 1 and 2, Treated Sample 1, and Fresh Sample 1. The four water samples were actual field waters and are the same as those that were used in the chemical compatibility testing (shown in Table 1). No Utica water samples were used as the Utica water samples should not create incompatibilities when re-introduced to the Utica formation. Diesel was added as a non-reactive control fluid for the roller oven shale stability testing. If the shale core samples display high erosion in diesel, it is an indication that the core samples may have poor strength and that percentage of fragmentation should be expected with any fluid regardless of the salinity.

Table 6: **Core Samples Used for Lab Testing.** The formation and sampling depth of core samples.

Well	Sample number	Depth (ft)	Formation
Utica Well A	1	10,974	Utica
Utica Well A	2	11,013	Utica
Utica Well A	3	11,053	Utica
Utica Well A	4	11,093	Utica
Utica Well A	5	11,113	Utica
Utica Well B	6, 7, 8, 9	11,705-11,747	Utica
Utica Well A	10	11,133	Point Pleasant
Utica Well A	11	11,163	Point Pleasant
Utica Well A	12	11,173	Point Pleasant
Utica Well A	13	11,215	Point Pleasant
Utica Well A	14	11,255	Point Pleasant
Utica Well A	15	11,295	Point Pleasant
Utica Well B	16, 17	11,761-11,768	Point Pleasant
Utica Well B	18, 19, 20	11,782-11,796	Point Pleasant
Utica Well B	21, 22, 23	11,810-11,824	Point Pleasant
Utica Well B	24, 25	11,832-11,838	Point Pleasant
Utica Well A	26	11,315	Trenton

Roller Oven Shale Stability Results

The core samples from the Utica formation demonstrated moderate to high erosion potential for all water types tested (Figure 6). For five of the six Utica formation cores, the RO values were highest using fresh water. For the Point Pleasant formation cores, core sample 10 was the only core that displayed high erosion tendency; all other cores displayed moderate to low erosion tendency. More importantly, all fluids performed similarly on all the Point Pleasant formation cores (Figure 7). Overall, the testing shows that the use of produced water should not cause more formation erosion when compared with fresh water, and in the case of the Utica formation, all produced water samples slightly outperformed fresh water.

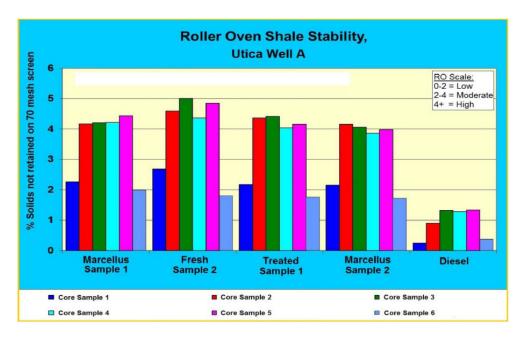


Figure 6: Roller Oven Shale Stability Results for the Utica Core Samples.

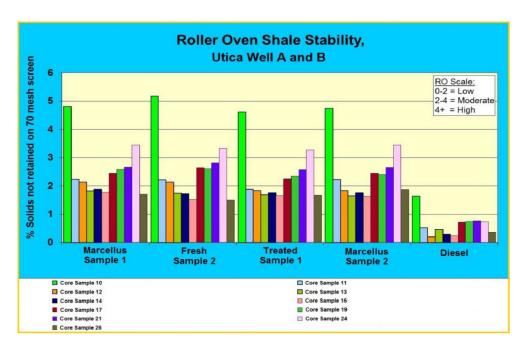


Figure 7: Roller Oven Shale Stability Results for the Point Pleasant Core Samples.

Hardness Reduction Testing Results

The hardness reduction testing results show that Utica Well B (Figure 8) demonstrated less softening after immersion in the test fluids compared to Utica Well A core samples (Figure 9) for cores obtained from both Utica and Point Pleasant formations. Utica Well B cores all displayed less than 10% reduction in Brinell hardness, while Utica Well A cores displayed Brinell hardness reduction of over 10%, and in one case, over 50%. Core samples with values not shown in the figures represent core samples that broke and became unsuitable for testing. The principal conclusion is that rock softening is sporadic and unpredictable. Some cores responded better to Fresh Sample 2, while others responded better to the produced water samples, but there is no clear trend indicating one water source was better overall. The conclusion is that fresh water did not prove to create less rock softening than produced waters.

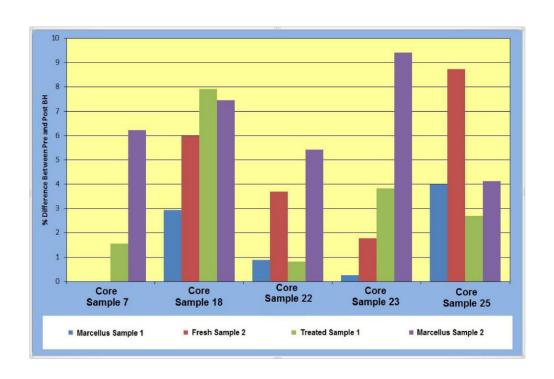


Figure 8: Hardness Reduction Testing Results for Utica Well B. Values displayed as a % difference between pre and post Brinell hardness (BH).

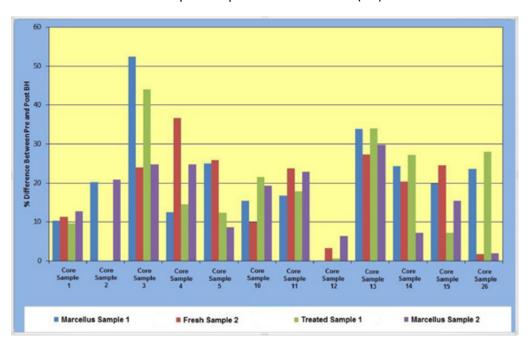


Figure 9: Hardness Reduction Testing Results for Utica Well A. Values displayed as a % difference between pre and post Brinell hardness (BH).

Core Testing Discussion

Core sampling is a very cost intensive operation and is not usually included in development well programs. The cost to add core testing to a drilling program varies, but can be as much as \$2 million per well when the additional cost to drill a pilot hole is considered. It is difficult to justify this cost solely for water-formation compatibility testing alone. However, core sampling is typically included in an *exploratory* well drilling program, as was the case for the wells that were a part of this research. The incremental cost to perform formation-water compatibility testing with pre-existing cores was only \$20,000. When considering the insight that these tests provide, it seems well worth the cost.

This study is a very specific case of introducing Marcellus water to the Utica formation. The results indicate that produced water from the Marcellus performs slightly better than fresh water as a base fluid for stimulation projects in the Utica formation. The trends observed for the core testing program of the two Utica -Point Pleasant wells should be transferable to all Utica – Point Pleasant wells in Tioga Country and even probably the greater Appalachian basin. Further testing is required to determine if these results are transferrable to similar scenarios in other shale basins.

Scale Loop

There are three scenarios that were modelled numerically for potential scale formation:

- 1. a mixture of produced water from the Utica and Marcellus formations;
- a mixture of produced water from the Utica formation with minimally treated source water, and;
- 3. a mixture of produced water from the Utica formation with fresh water.

When simulated, scenarios 1 and 2 did not exhibit any potential for scale formation. Scenario 3 demonstrated a slight incompatibility with respect to barite scale with a worst-case occurring with a 90% fresh, 10% Utica produced mixture. The sulfates levels in the fresh water are the reason that this scenario exhibited a potential for barite precipitation when simulated. However, when tested in the scale loop, no scale formation was detected. This result is further supported by years of field experience where Marcellus produced water has been mixed with fresh water with no barite scale detected. Marcellus produced waters have higher barium levels than Utica produced waters and would, therefore, be more prone to barite scaling.

The effect of using a scale inhibitor was demonstrated using the scale loop and two completely hypothetical, synthetic water sources. In Figure 10, a potential for scale formation was measured for a synthetic water source at both surface and downhole conditions, and compared to the same source water after the addition of a scale inhibitor which mitigated the onset of scale precipitation. These two water sources were not based on the composition of any of the waters in this study; they were formulated in the lab to create a barium sulfate scale when untreated. The results of the lab testing support the practice of depending on inhibition as the only means to prevent barium sulfate scales with waters containing SO_4 levels below 100 ppm.

Scale Testing Discussion

It should be noted that bi-carbonate scales are the toughest to model or predict. It was assumed that, due to the model's lack of robustness in predicting bi-carbonate scale potential, bi-carbonate scale could pose a problem during production. After investigating the Marcellus, Utica and Point Pleasant water chemistries, several items were noted. First, the Utica/Point Pleasant formation waters have bi-carbonate (HCO_3) concentrations in the 200-300 ppm range

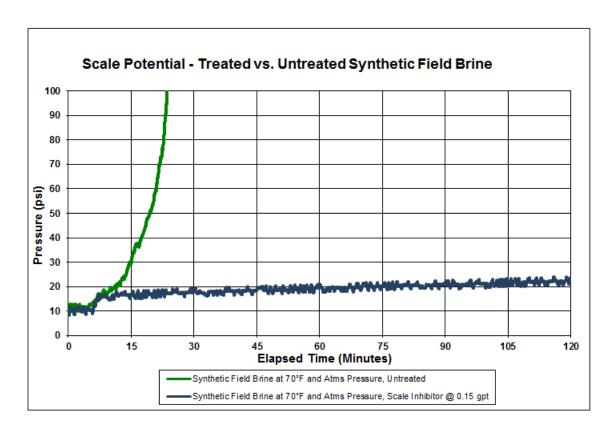


Figure 10: Scale Inhibitor Performance for Barium Sulfate Scale.

which is nearly three times the levels found in Marcellus waters, and also have calcium concentrations around 10,000-15,000 ppm, which is half of the levels found in the Marcellus waters. The combination of these two characteristics could lead to a slight incompatibility and the formation of $CaCO_3$ scale (calcite). However, the potential appears to be low, since the bicarbonate concentrations are below 1,000 ppm and usually scaling in the formation does not occur at these levels (Frigo, 1999). If scale were to form, it would most likely be in the tubulars and surface equipment. Therefore, it should be easily detectable. Additionally, mitigation of calcite is inexpensive and straight-forward, so that once detected, more formal prevention measures could be initiated. For SEPCo in Tioga County, PA, there have been no evidence of calcite scaling on the first five Utica / Point Pleasant wells that used Marcellus produced water for stimulation fluid. Future wells are planned and monitoring will continue.

Scale is one of the most difficult parameters to model and test for in the lab. The parameters that lead to scale precipitation evolve during the life of a producing well; water variability over time can add to these difficulties. This study attempted to test a representative sample for the range in water quality that exists in the field, but as new wells come on-line and new water sources are added to the system, scale tendencies can change. The results of this lab testing serve to increase the confidence scale will not occur but does not rule it out entirely. Constant monitoring of the quality from new sources is needed, and trial and error may still exist due to the complex nature of scale formation.

C. Statistical Analysis

Regression Model and Variables

All regression analysis for this study was performed using *R*, a software package for statistical computing and graphics. The approach to this study was to first set-up a regression model without including produced water data as an independent variable in order to identify the other independent variables that may have a statistically significant impact on well productivity. The initial independent variables that were considered in the model were based on previous studies conducted by the asset to determine factors that influence production. T-statistics, which are further discussed in Appendix A, were used to help make this determination. For a model with more than 30 observations, a t-statistic above approximately two represents a 95% chance that the coefficient estimate does not include zero (James et al., 2013). Essentially, there is a >95% chance that the true value of the coefficient is non-zero; therefore, the independent variable is considered to be statically significant. Following this guideline, the following five independent variables were used in the base model:

- Marcellus thickness the thickness in feet from the Stafford limestone to the Onondaga
 limestone that includes the Oatka Creek shale and the Union Springs shale.
- Cherry Valley thickness the thickness in feet of the Cherry Valley limestone that separates the Oatka Creek shale and the Union springs shale. It is believed to be a slight barrier to hydraulic fracturing and acts to impede production with increasing thickness.
- Well dip measured in degrees and is defined here as the average vertical inclination from 90 degrees of the horizontal section of the well. Up-dip and down-dip wells have both proven to impair production in previous multivariate analysis studies.
- Lateral length the length in feet of the lateral section of the well measured from the landing point to plugged back total depth (PBTD).
- Stage length the average length of each stage of the fracturing job measured form
 plug to plug (or for stage 1 from PBTD to plug.) A recent trend towards shorter stage
 lengths has increased well productivity.⁸

The dependent variable was the EUR of each well in BCF. All independent variables were then checked for collinearity to ensure there was no relationship between them.

Collinearity was identified with total proppant and lateral length, and as a result, one of the variables was removed from the model.

Table 7 (below) shows the output from the base model before adding produced water.

In the regression table, one can find the standard error value, which represents the amount of

⁹ Wells in SEPCo Appalachian asset are designed based on proppant per lateral foot loading, and with few exceptions, this value is held constant, developing a very consistent relationship between the total proppant and lateral length variables. When both are included in the model, they register within the 95% confidence level but act to increase the standard error of each other. Lateral length was selected for the model but substituting total proppant produces results that are very similar.

⁸ Although decreasing stage length has shown a direct increase in well production, there must be a minimum length beyond which productivity declines again. The current value for this optimal length is believed to be 250'. Since the stage lengths that were included ranged from 200-400 foot stage spacing, it is believed that a linear relationship is still best for the model.

uncertainty in the coefficient measurement, and the t-statistics that can be compared to the independent variables used to create the base model. Following a procedure described by Jablonowski and MacEachern (2009): "To protect the proprietary nature of the data supplied by the operator, some of the independent variables were multiplied by a random factor. This changes the value of the coefficient estimate for that specific variable, but does not affect other coefficient estimates, nor does it affect tests for statistical significance."

Table 7: Regression Output from First Run of Model before Produced Water Was Added.

Regression Statistics				
R_2	0.52			
Adjusted R ₂	0.50			
Residual Standard Error	1.523			
Observations	116			
	Coefficients	Standard Error	t stat	p value
Intercept	-0.677	0.456	-1.484	0.1407
Marcellus thickness	1.637	0.285	5.736	8.63e-08
Cherry valley thickness	-1.825	0.806	-2.265	0.0254
Well dip	-0.015	0.007	-2.033	0.0444
Lateral length	0.208	0.032	6.474	2.75e-09
Stage length	-5.380	1.561	-3.447	0.0008

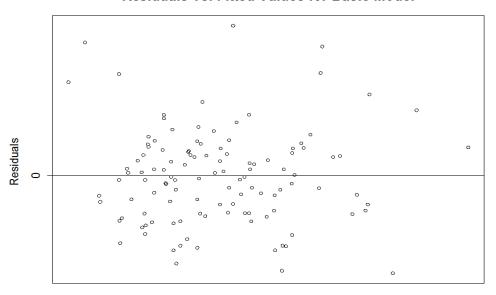
Model Authentication

R² measures the fit of a model to the data. By definition, it is the proportion of variance in the response variable that can be explained by the predictor variables (James et al., 2013). In this case, the model without the produced water percentage variable accounts for 52% (R² of 0.52) of the variance seen in well production. Using past statistical studies of well productivity prediction in shale formations as a reference, an R² value of 0.52 represents a sufficiently accurate model.

¹⁰ Of the 116 wells that were used in setting up the model, only 87 had data for percentage of produced water used to stimulate the well.

Residuals can also be used to assess the fit of a model. Residuals represent the difference between model-predicted EURs and the actual EUR. The residuals were checked and confirmed to be approximately normally distributed. In the residuals vs. fitted plot below (Figure 11)¹¹, the residuals are noted to have constant variance, confirming that the logic of the model is sound.

Residuals vs. Fitted Values for Basic Model



Fitted Values

Figure 11: Residuals Plotted Versus Fitted Values (Model Predicted Values).

Addition of the Produced Water Percentage Variable

Once a satisfactory model was created, the produced water variable was added as an independent variable. At first, a "fine-tuning" of the model was performed to address any issues with regard to model fit and accuracy when compared to the initial model that did not contain the produced water data. The estimate of the relationship between the percentage of produced water used and well productivity is the value of produced water variable coefficient

¹¹ The values and units on the axis were left out in order to protect proprietary data.

estimate. The model output in Table 8 shows the regression output with produced water added.

Table 8: Regression Output of Model with Produced Water Percentage Added.

Regression Statistics				
R_2	0.69			
Adjusted R_2	0.67			
Residual Standard Error	1.315			
Observations	87			
	Coefficients	Standard Error	t stat	p value
Intercept	-1.112	0.504	-2.205	0.0303
Marcellus thickness	2.044	0.362	5.651	2.37e-07
Cherry valley thickness	-2.732	0.910	-3.003	0.0036
Well dip	-0.028	0.008	-3.457	8.77e-04
Lateral length	0.239	0.033	7.300	1.87e-10
Stage length	-5.778	1.619	-3.569	6.10e-04
Produced water used, %	0.016	0.007	2.185	0.031

The initial review of the model including data for the percentage of produced water used to stimulate the well shows that the model fit is better, with an R² value of 0.69. In this case, the large increase in R² is likely due to the 29 wells that were dropped from the dataset because they lacked data on amount of produced water used for stimulation.¹² Produced water appears to have a positive impact on production, while meeting the t-statistic value for the 95% confidence interval.

Additional Outlier Wells

When the residuals are plotted vs. produced water (figure 12), there are a couple of outliers that become apparent. The model under-predicted the production of three wells that used 100% produced water and are some of the highest producing Marcellus wells in the asset.

All three wells were from the same pad, and they have a large impact on the positive correlation

¹² By definition, R² values will always increase when additional independent variables are added to the model.

between the produced water and well productivity. There is no known mechanism that would cause these wells to exceed the model's expected production. More likely, it is a combination of many unknown factors. These wells act as outliers disproportionately swaying the results to show a positive correlation with produced water. To understand the sensitivity of the results to these three wells, they were dropped from the dataset, and the model was re-run. If the iuse of produced water for well stimulation does enhance production, it should still hold with a dataset of 84 wells without these possible outliers. Table 9 shows the results of the statistical analysis after dropping these three wells from the dataset.

This model output has the same R² value for model fit but now shows a slightly negative estimate of the correlation between produced water and production. The three outlier wells clearly had strong influence on the results, so leaving them out is justified. However, the statistical analysis does not give reason to believe that the use of produced water is detrimental for well production. The t-statistic value for produced water is -0.28 indicating that the results very likely are due to random chance.

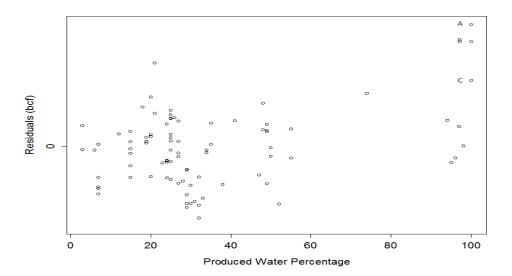


Figure 12: Residuals plotted vs Produced water percentage. Wells that were outliers that heavily influenced the positive correlation between produced water and well productivity are labeled with the letters A, B, and C.

Table 9: Regression Output with Three Outlier Wells Removed.

Regression Statistics				
R_2	0.69			
Adjusted R_2	0.67			
Residual Standard Error	1.187			
Observations	84			
	Coefficients	Standard Error	t stat	p value
Intercept	-0.740	0.466	-1.587	0.1165
Marcellus thickness	1.886	0.328	5.747	1.73e-07
Cherry valley thickness	-3.250	0.829	-3.992	1.89e-04
Well dip	-0.020	0.008	-2.671	0.0092
Lateral length	0.236	0.031	7.690	4.02e-11
Stage length	-6.154	1.463	-4.205	6.98e-05
Produced water used, %	-0.002	0.008	-0.28	0.7802

Other Dominant Variables

After additional study of the temporal data and its trends, additional insight is gained.

Figure 13 shows the Marcellus thickness, lateral length, total proppant, and produced water variables plotted versus time. Actual well productivity values cannot be shown for confidentiality, however trends can still be observed. During time period 1, there is a downward trend in well productivity, as well as in Marcellus thickness, lateral length, and total proppant.

During this time period, use of produced water for stimulation (as a percentage) was rising. A model run using this time period only would likely show a negative correlation between produced water use and production because produced water percentage is confounded by the more dominant variables. Similarly, if you look at time period 2, where there is an increasing trend in well productivity, there is also an increasing trend in all four independent variables. A re-run of the model with this data alone would most likely give a positive correlation between all four variables and well productivity. The confounding effect is removed when all time periods are considered in the model. Produced water is not likely the cause of these small

trends; more likely, it is caused by the changes in the more dominant independent variables—
Marcellus thickness, total proppant, and lateral length. This represents a good example where an incorrect conclusion could have been made based on happenstance data rather than a controlled trial.

The number of wells operated by SEPCo in the Marcellus shale, or the observations, provided enough data to conduct the modeling in this study. Observations that did not represent future designs were identified as outliers and removed from the dataset. Collinearity was accounted for in selection of the variables, and only variables with statistical significance were used. The base model was checked for, and corrected against, all discrepancies that were identified using accepted methods of model authentication. The model was found to be acceptable and the produced water percentage variable was added to the equation. The new model was tested again and revealed that there was a positive correlation between well production and percentage of produced water used. Additional outlying wells were removed from the dataset, and the subsequent results showed that the percentage of produced water used has no statistically significant impact on well production. However, other factors, such as Marcellus thickness and total proppant are more dominant variables and have a larger influence well production.

Statistical Analysis Discussion

The statistical analysis revealed that there is no measurable statistical correlation between the use of produced water as base water for stimulation and well production, but the standard error of the coefficient estimate is 7.1 % of the mean well EUR. This is greater than the 6.4% difference (from the economic model results) needed to justify Scenario 2 (50% produced water) over Scenario 1 (100% fresh). Therefore, the true value of the coefficient for produced water could be greater than 6.4% and still lie within one standard error of the model predicted

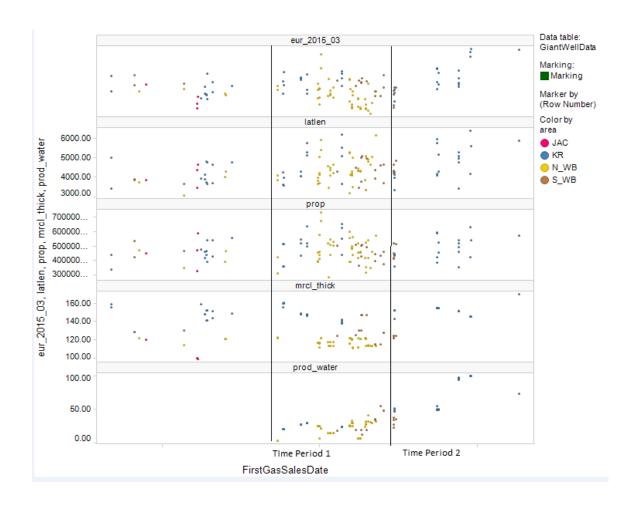


Figure 13: Time Trends in EUR, Location, and Design. Well EUR (eur_2015_03), lateral length (latlen), total proppant, Marcellus thickness (mrcl_thick), and Produced water percentage plotted vs. time to show trends in time period 1 and time period 2.

value. However, according to statistics, it is more likely that the true impact of produced water is substantially better than -6.4% and therefore the decision to recycle produced water is still justified by the results.

The model could be improved in several ways; the two most effective improvements would be to increase size of the dataset and to improve the knowledge around variables that effect well productivity in the region. The size of this dataset, while larger than other case studies on the subject and above the 30 observation statistical recommendation, is still quite

small. A larger dataset could reduce the standard error of the results. However, in order to increase the number of observations in the near future, it would require data sharing between operators and this may be a complicated task. Another way the model would increase accuracy would be a better understanding of the geologic features that affect well production. The geology of the Marcellus is studied continually by SEPCo in an effort to improve the data for this model as well other models attempting to define what variables affect well productivity. As progress is made in this research, future revisions of the model should improve.

V. Conclusion

This thesis used lab testing and temporal data to scientifically evaluate the production impacts resulting from recycling produced water in higher mix ratios than traditionally applied in the oil and gas industry. The conclusions of the thesis are as follows:

- The economic model revealed that total water management costs can be reduced by 59% through water recycling and using at least 50% percent produced water in fracturing new wells.
- Through the roller oven shale stability and hardness reduction testing, it was
 determined that using produced water from the Marcellus formation should not
 increase erosion or rock softening if used in stimulation of wells located within Utica
 formation.
- Modern fracturing treatment chemicals were shown to display acceptable performance
 in a wide range of water salinities (300 -270,000 mg/L TDS). FR testing identified a
 source water from a treatment facility that needed special treatment before use to
 achieve successful friction reduction.

- Scale testing revealed no significant incompatibilities that would prevent recycling large
 concentrations of various source waters in the Utica shale; however, scale testing was
 very difficult to fully evaluate in the lab and continuous monitoring is recommended.
- The conclusion of the multivariate analysis study is that there is no statistically significant correlation between the reuse of produced water for well stimulation and well production in the Marcellus shale.

The results of this thesis are significant to SEPCo as well as the greater oil and gas industry. This research presented lab testing and statistical analysis to support the decision to recycle high concentrations of produced water in hydraulic fracturing. Other operators in the greater Appalachian basin should be motivated by the water management cost reductions that water recycling with increasing produced water concentrations will not cause production impairment in the Marcellus and Utica formations. Beyond Appalachia, this study acts as an model to conduct a comprehensive analysis on the impact of recycling water. The methods presented in the statistical analysis outline a proven approach to identify if produced water has any correlation with production so long as adequate empirical data is available.

The age of research in recycling high concentrations of produced water for use in hydraulic fracturing is still immature. The error in this model can be improved with future studies and improved localized knowledge of Marcellus geology and the addition of more wells to the study. As more wells come on production in the Utica shale, the data will become available to run improved statistical analyses to investigate whether there is a correlation between the use of produced water and well productivity. Continued research outside of Appalachia would allow for results to be compiled and generalized as best practices to be applied globally. In order to make board conclusions and recommendations on recycling

produced water, the industry will need to gather and analyze additional data, and more importantly, increase the knowledge sharing between operators so the full benefits of the research is realized.

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Appendix A: Statistical Methods

Simple and Multiple Linear Regression Modeling

Linear regression is the simplest and earliest method of statistical analysis. It deploys a least mean squares best fit line through a set of data points. Simple linear regression is a two-dimensional plot with a quantitative response Y on the basis of a single predictor X along with an error term ϵ which is independent of X and is expressed as

$$Y = \beta_0 + \beta_1 X_1 + \varepsilon. \tag{4}$$

The least mean squares line minimizes the sum of all the squared Y-value differences between each point and the line by estimating the co-efficients β_0 and β_1 . Multiple linear regression adds more predictors to the model, making the equation:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 \dots + \varepsilon.$$
 (5)

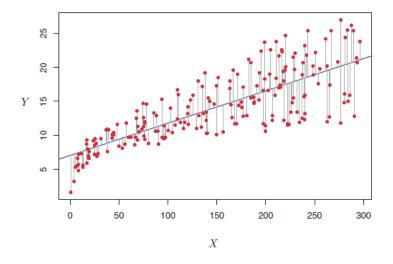


Figure 14: Example of simple linear regression. A data set (red points) with a least mean squares line (blue line) through the data (James et al., 2013).

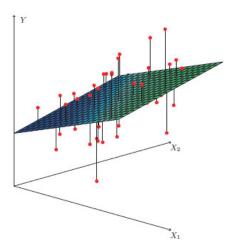


Figure 15: An Example of Multiple Linear Regression. There are two predictor variables X_1 and X_2 , and one response variable Y. The black vertical lines represent least mean square differences (James et al., 2013).

Bias variance

The Bias-Variance tradeoff is an important consideration when determining the best method to use for statistical analysis. *Variance* is the responsiveness of the model to changing datasets. For example, if a dataset were split in half and each half was used to create a best fit curve for the data, variance is a measure of how different these curves would be. Simple linear models usually have very low variance, while very complex models can have high variance as they create narrowly tailored curves to the two completely different datasets. *Bias* refers to the error that is introduced by using a simple model to represent a real-life problem, which may be much more complicated. In general, a more flexible model will have lower bias. The end goal of any statistical modeling method would be to create the model that has the lowest combination of bias and variance.

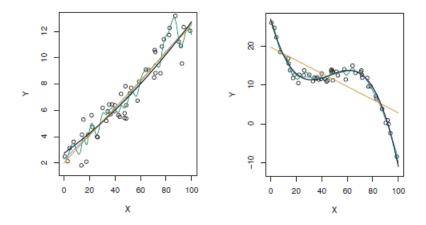


Figure 16: An Example of Bias and Variance. Left: The yellow linear regression line represents a model with low variance. Right: More flexible curves have less bias (green curve) and create a better fit for the data (James et al., 2013).

T-statistics

The t-statistic for the coefficient estimates is calculated using

$$t = \frac{\widehat{\beta}_i - \beta_i}{SE(\widehat{\beta}_i)} \tag{6}$$

where $\hat{\beta}_i$ is the coefficient estimate being tested, β_i is the true value for the coefficient, and $SE(\hat{\beta}_i)$ is the standard error for the coefficient estimate $\hat{\beta}_i$. By setting $\beta_i=0$, we are testing the confidence that our estimate $\hat{\beta}_i$ could actually equal zero in the true model (James et al., 2013).

 R^2

 R^2 values depend heavily on the nature of the data. In physics applications, where the analyst has a very good knowledge of the mathematical relationships between the variables, an R^2 value extremely close to 1.0 would be expected. However, in applications of biology, marketing, or psychology, R^2 values close to 0.1 can be considered a good fit.