GENERALIZED RAY TRACING IN
A MOST GENERAL OCEAN

A Thesis

Presented to

the Faculty of the Department of Electrical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

---

by

Malkiat S. Sohel

August 1969

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DEDICATED

To S. Surjan Singh Sohel, my beloved father, who left this world on 20 June 1968.

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ABSTRACT

The problem of ray tracing in a water mass moving with an exponentially decaying velocity in three-dimensional space is representative of, for instance, the Gulf Stream in the Gulf of Mexico and river deltas and most of all, many other restricted channels lying between large land masses around the world including tidal effects in many areas. A general expression for the curvature of the ray path in an ocean channel whose watermass is assumed to be moving with a three-dimensional mass velocity decreasing exponentially is derived under the assumption that the speed of sound decreases exponentially with depth.

The ray path data for the source located at various depths and for initial ray depression angles of $\theta_o = 20^\circ, 30^\circ, 45^\circ$ and 60°, respectively, for both still and moving watermass, both at constant velocity and spatially varying mass velocity. The percentage error in each case is calculated and presented.

The speed of sound variations data available off the Florida coast is analyzed for random variation of the speed of sound with depth superimposed on the usual exponential variation. Its statistical parameter, density function, mean, variance, auto-correlation and decorrelation distance are calculated and presented. The expression for the curvature of ray path in moving inhomogeneous medium, assuming the above model, is also presented.

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CHAPTER I

INTRODUCTION

For the last 25 years ocean acoustics have become a subject of great importance due to its various applications in war as well as in peace. The ocean is an inhomogeneous medium and may be characterized by its refractive index changing with location, temperature and salinity.

In saline water of the ocean, light as well as radio waves are attenuated to a much larger extent than the mechanical energy. The signal becomes distorted, weakened and delayed. Another important parameter which must be taken into account while discussing any ocean problem is the transmission loss in the medium. It is the combined effect of the spreading as well as the absorption by the medium. The general absorption is accompanied by the scattering, leakage out of the sound channels, air bubbles and biological effects.

The sound propagation in the ocean is governed by water pressure, temperature, and salinity. In deep water propagation, the effect of salinity is negligible. Salinity is the total amount of the solid material in grams contained in one Kg of salt water when all the bromine and iodine have been replaced by an equivalent amount of chlorine, all the carbonates converted into oxide, and all organic matter has been completely oxidized. S is the total measure of the solid salt disolved in the order of 35 parts per thousand.

Considerable experimental data (Wilson, 1960) has been used to determine the dependence of C the velocity of sound on these parameters:

$$C = 141000 + 421t - 3.7t^2 + 1.10S + .0189d \quad (1-1)$$

where C = speed of sound, ft/sec
t = temperature, $^{\circ}$C
P = pressure, dynes/cm^2, and
d = depth, feet

1.1 Graphical Representation of the Velocity Profile in the Ocean

The sound velocity profile in the ocean can be obtained from hydrographic observations of temperature, pressure and salinity, as shown in Fig. 1-1. The ocean body may be divided into different layers for purposes of further propagation analysis: The Surface Layer, Seasonal Thermocline, Main Thermocline, and Deep Isothermal Layer.

1.1.1 Surface Layer

This layer is just below the surface. It is effected due to daily changes near the surface; for example, the churning of the surface due to wind, heating, and cooling

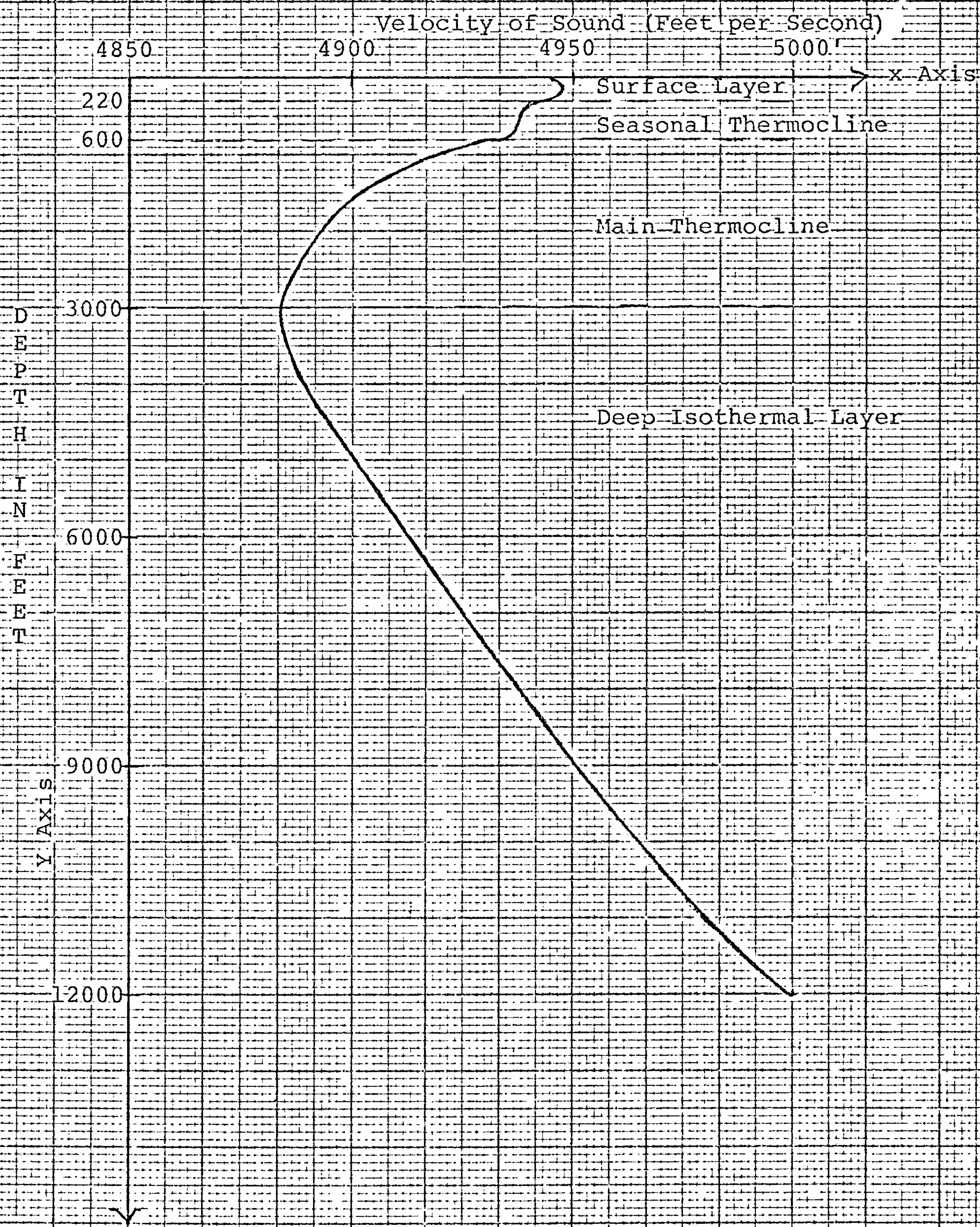


Fig. 1-1. Sound Velocity Profile in Layered Ocean

effects, etc. At certain times this layer may be replaced by the layer in which the temperature changes with the depth.

1.1.2 Seasonal Thermocline

The temperature decreases with depth in this layer.

1.1.3 Main Thermocline

There is no effect of the local changes in this layer but the temperature further decreases with the depth (refer to Fig. (1-2).

1.1.4 Deep Isothermal Layer

Pressure plays the most important part in this layer, and temperature variations are negligible. The speed of sound increases due to an increase in the water pressure.

The speed of sound changes with location such as in the North Atlantic Ocean. The sound minimums are not as close to the surface as in the case of the South Atlantic Ocean.

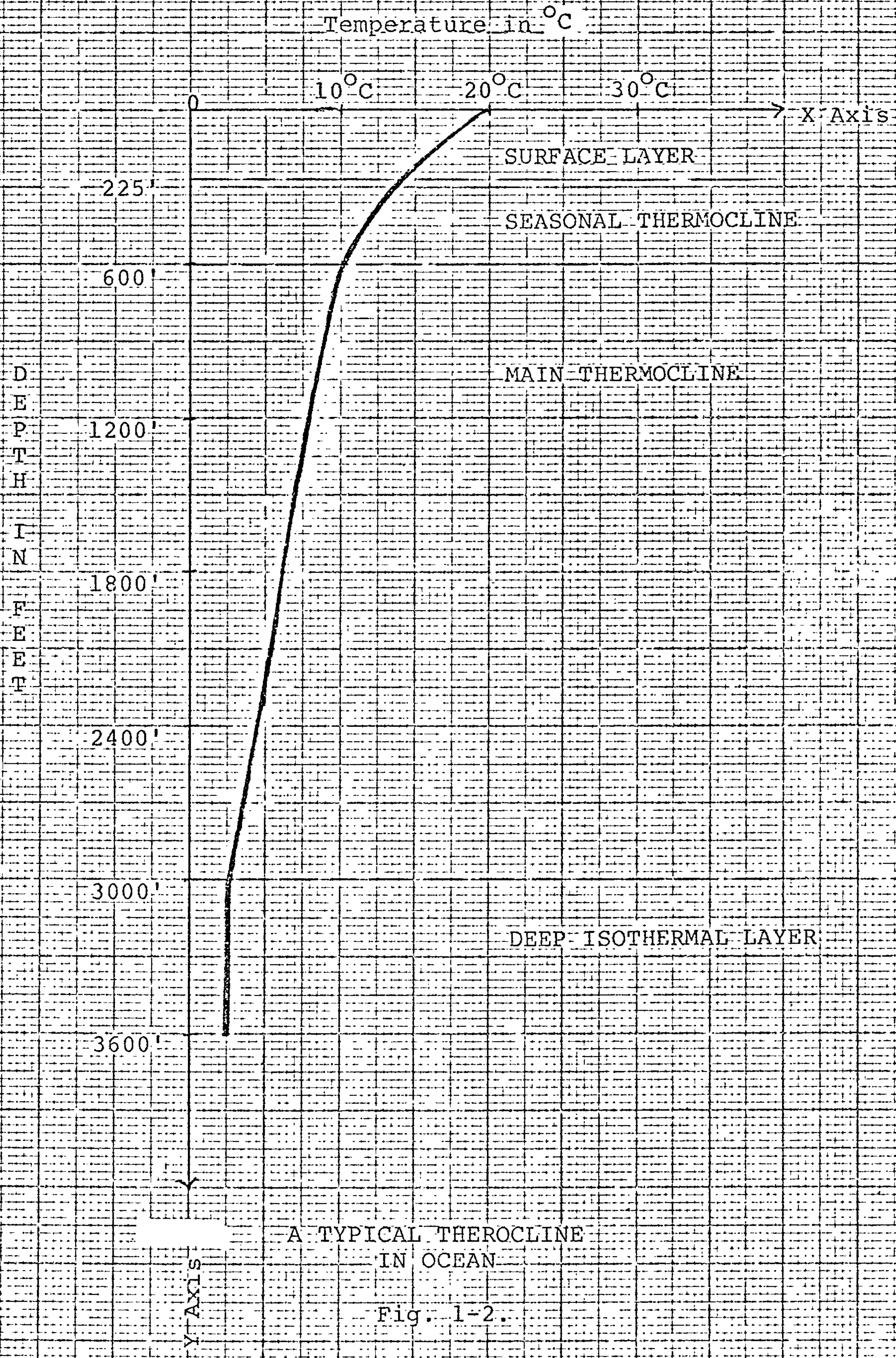
1.2 Theory Discussion

The wave propagation is, in general, governed by the following wave equation.

$$\nabla^2 \Phi = \frac{1}{C^2}\frac{\partial^2 \Phi}{\partial t^2} \tag{1-2}$$

where Φ = the particle velocity, pressure or velocity potential, and

C = sound velocity



A TYPICAL THEROCLINE
IN OCEAN

Fig. 1-2.

One can solve Eq. (1-2) in many ways. The most commonly used methods are the normal mode solution and the ray solution.

In high frequencies and short range propagation, ray theory is preferred due to its practical advantages and simplicity.

1.2.1 Ray Solutions

The wave theory can be transformed to ray optics and then an Eikonal equation, the solution of which is found in terms of wave fronts and rays. The transformation may be effected as below.

Assume the solution of the differential Eq. (1-2) as

$$\Phi = A(X,Y,Z)\, e^{-jKS(X,Y,Z)}\, e^{jWt} \tag{1-3}$$

where $S(X,Y,Z)$ is an Eikonal or surface, $K = W/C_o$ for Eq. (1-3) to be the solution of Eq. (1-1). Substituting Eq. (1-3) into Eq. (1-1) and equating real and imaginary parts, yields:

$$\nabla^2 A - |\nabla S|^2 \left(A W^2/C_o^2\right) = \left(-W^2/C^2\right) A \tag{1-4}$$

$$2\left((\nabla S)\cdot(\nabla A)\right) + A \nabla^2 S = 0 \tag{1-5}$$

Further simplification of Eq. (1-4) yields

$$|\nabla S|^2 - n^2 - \frac{\lambda_0^2}{4\pi}\left(\frac{\nabla^2 A}{A}\right) = 0 \tag{1-6}$$

The following approximations are used to derive an Eikonal equation from the wave equation.

$$\frac{\nabla^2 A}{A} \ll 1$$

which implies that fractional change in the space rate of the amplitude should be very small.

$$\nabla^2 S \ll 1$$

or a change in the curvature of the wavefronts must be very small.

$$\frac{\lambda \delta c'}{c} \ll 1$$

a functional change in the velocity gradient over a wave length should be small as compared to 1.

Using the above approximations, Eqs. (1-5) and (1-6) yield

$$S_x^2 + S_y^2 + S_z^2 = n^2 \tag{1-7}$$

Equation (1-7) is called an Eikonal equation, a solution of which is not the solution of the wave equation but its solution, $S(x,y,z)$ = constant, represents a surface in three-dimensional space coordinates for a given value of S and at some given time. The amplitude may or may not be the same but all the points on the surface will be in phase. This surface is called the wave surface or wave front; at another time, a different surface will be generated as shown by Huygen's Principle in ray optics.

The normals to these wave surfaces give the direction of energy propagation called rays (refer to Fig. 1-3).

1.3 Three-Dimensional Ray Tracing

The ray equations may also be developed using Fermat's Principle, according to which a ray path between any two points is also a path of stationary time (refer to Fig. 1-4). For example, (Officer, 1958)

$$F = C_0 \int_{M_1}^{M_2} dt = C_0 \int_{M_1}^{M_2} \frac{ds}{C} = \int_{M_1}^{M_2} \eta \, ds \tag{1-8}$$

The extreme value, maximum or minimum, is given by

$$ds = \sqrt{\left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{dy}{d\sigma}\right)^2 + \left(\frac{dz}{d\sigma}\right)^2} \cdot d\sigma \tag{1-9}$$

where S = arc length

σ = parameter,

such that

$$X = X(\sigma), \quad Y = Y(\sigma), \quad Z = Z(\sigma)$$

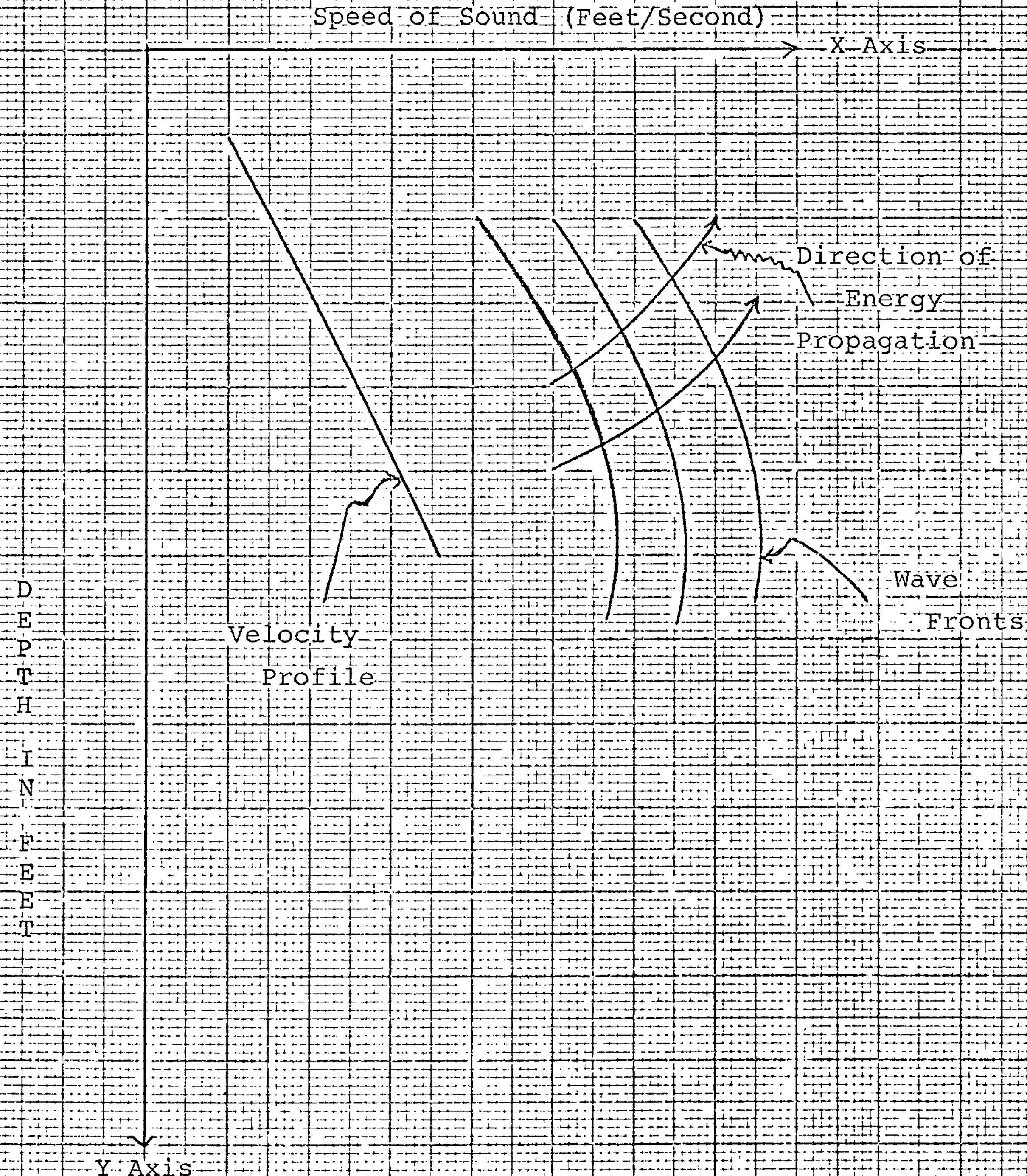


Fig. 1-3. Propagation of Wave Fronts
Depending on the Velocity Profile

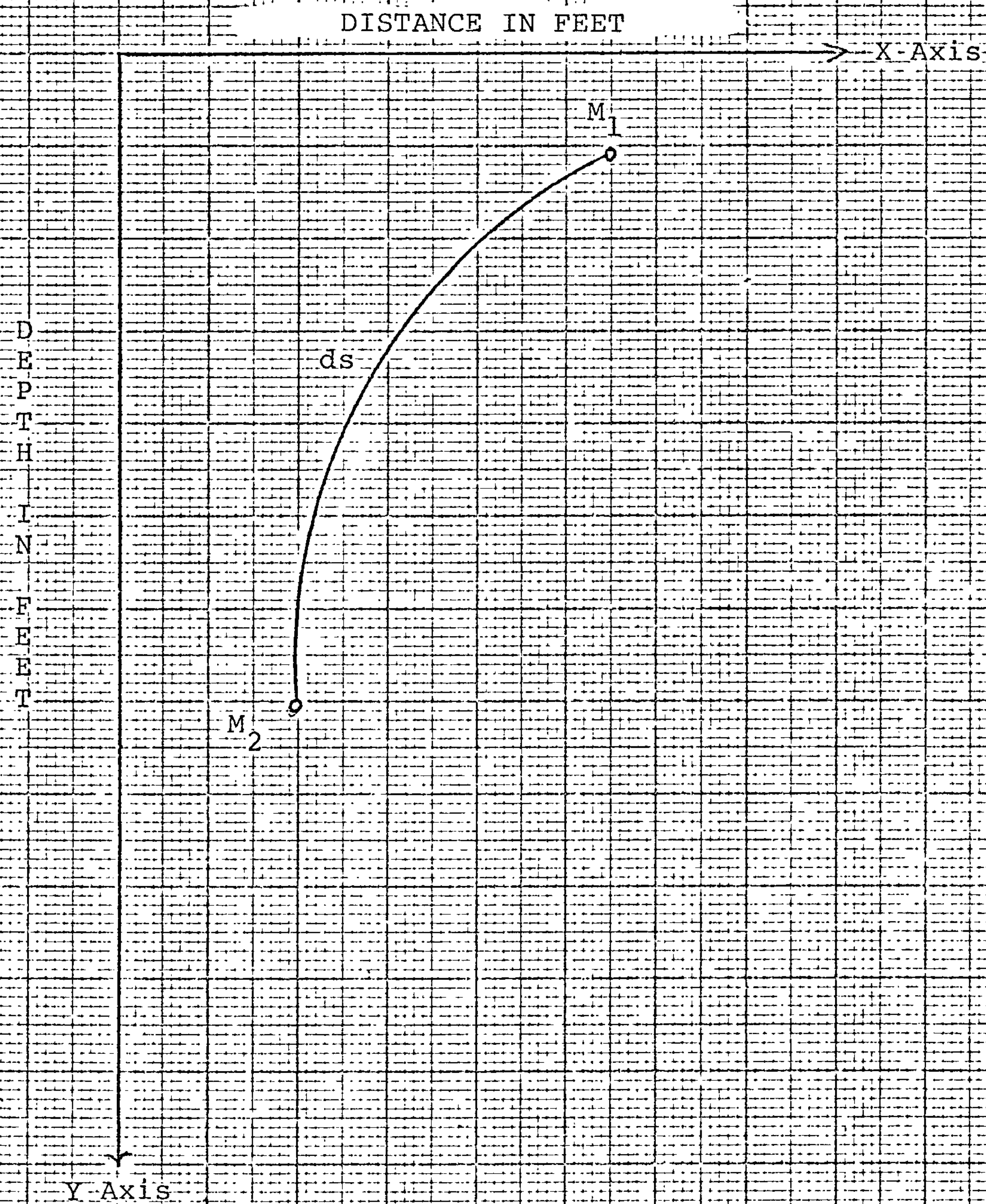


Fig. 1-4. Ray Path

A substitution of Eq. (1-9) in Eq. (1-8) yields

$$F = \int_{M_1}^{M_2} n\sqrt{x'^2+y'^2+z'^2}\, d\sigma = \int_{M_1}^{M_2} G(x,y,z,x',y',z')\, d\sigma \qquad (1\text{-}10)$$

This line integral gives the extremum value if Euler's equation is satisfied.

$$\frac{\partial G}{\partial x} - \frac{d}{d\sigma}\left(\frac{\partial G}{\partial x'}\right) = 0 \qquad (1\text{-}11)$$

$$\frac{\partial G}{\partial y} - \frac{d}{d\sigma}\left(\frac{\partial G}{\partial y'}\right) = 0 \qquad (1\text{-}12)$$

$$\frac{\partial G}{\partial z} - \frac{d}{d\sigma}\left(\frac{\partial G}{\partial z'}\right) = 0 \qquad (1\text{-}13)$$

which means:

$$\sqrt{x'^2+y'^2+z'^2}\,\frac{\partial n}{\partial x} - \frac{d}{d\sigma}\left(\frac{n x'}{\sqrt{x'^2+y'^2+z'^2}}\right) = 0 \qquad (1\text{-}14)$$

$$\sqrt{x'^2+y'^2+z'^2}\,\frac{\partial n}{\partial y} - \frac{d}{d\sigma}\left(\frac{n y'}{\sqrt{x'^2+y'^2+z'^2}}\right) = 0 \qquad (1\text{-}15)$$

$$\sqrt{x'^2+y'^2+z'^2}\,\frac{\partial n}{\partial z} - \frac{d}{d\sigma}\left(\frac{n z'}{\sqrt{x'^2+y'^2+z'^2}}\right) = 0 \qquad (1\text{-}16)$$

Using Eq. (1-9) in Eqs. (1-14), (1-15) and (1-16), one obtains

$$\frac{d}{ds}\left(n\frac{dx}{ds}\right) = \frac{\partial n}{\partial x} \tag{1-17}$$

$$\frac{d}{ds}\left(n\frac{dy}{ds}\right) = \frac{\partial n}{\partial y} \tag{1-18}$$

$$\frac{d}{ds}\left(n\frac{dz}{ds}\right) = \frac{\partial n}{\partial z} \tag{1-19}$$

These are the ray equations in vector form. They can be written as

$$\frac{d}{ds}(n\vec{r}') = \nabla n \tag{1-20}$$

where $\vec{r}' = \frac{d\vec{r}}{ds}$ = unit tangent vector, and

$\vec{r}$ = position vector

If the refractive index is given as a function of three space variables, Eq. (1-20) with $\vec{r}' = \frac{d\vec{r}}{ds}$ allows one to find the equation of trajectories $\vec{r} = \vec{r}(s)$ with given initial conditions.

1.4 Ray Equation for a Moving Ocean

In Section 1.3, the ray equation was developed for a motionless inhomogeneous ocean, but in practice one has to take into account the motion of the ocean especially near bays and harbors, in river deltas or in Gulf areas.

The general vector Eikonal equation for a moving inhomogeneous medium (Uginicius, 1965) is given as

$$(\eta\vec{r}')' + \vec{r}' \times (\nabla \times \vec{V}) = \nabla \eta \tag{1-21}$$

where $\eta(x,y,z)$ = refractive index of the medium

$\vec{r}' = \frac{dx}{ds}\hat{a}_1 + \frac{dy}{ds}\hat{a}_2 + \frac{dz}{ds}\hat{a}_3$ = unit tangent vector

$\vec{V}(x,y,z)$ = water mass velocity

Equation (1-21) can be solved further to find the ray path.

CHAPTER II

EXPONENTIAL OCEAN MODEL - GENERAL CASE

2.1 Ocean Model

The velocity of the water mass under consideration is assumed to decrease exponentially along X, Y and Z axes. Furthermore, the speed of sound is assumed to decrease exponentially with depth only.

$$\vec{V} = V_0 (\hat{a}_1 + b\hat{a}_3) e^{-\alpha x - \beta y - \gamma z} \tag{2-1}$$

$$C = C_0 e^{-AY} + C_1 \tag{2-2}$$

where α, β, γ, A = attenuation constants

b, C_0, V_0 = known constants

C_1 = random variable, and

$\hat{a}_1, \hat{a}_3$ = unit vectors

along X and Z directions, respectively. Such an ocean model represents a typical case of Gulf streams and other such locations where it is practically impossible to neglect the effects of tides and currents in sound transmission problems.

2.2 Most General Ray Path

One can derive an expression for the curvature of ray paths in a moving inhomogeneous medium from the general vector

Eikonal equation (Ugincius, 1965):

$$(n\vec{z}')' + \vec{z}' \times (\nabla \times \vec{V})\frac{n}{C}\left(1 - \frac{\vec{V}\cdot\vec{z}'}{C}\right) = \nabla n \tag{2-3}$$

The various terms used in this work are defined as:

$\vec{V}(X,Y,Z)$ = water mass velocity

$n = \frac{C_0}{C(X,Y,Z)}$ = refractive index of the medium

$\vec{z}' = \frac{dx}{ds}\hat{a}_1 + \frac{dy}{ds}\hat{a}_2 + \frac{dz}{ds}\hat{a}_3$ = unit tangent vector

$n' = \frac{dn}{ds}$

S = arc length

An expansion of the first term on the left hand side of Eq. (2-3) yields:

$$n'\vec{z}' + n\vec{z}'' + \vec{z}' \times (\nabla \times \vec{V})\frac{n}{C}\left(1 - \frac{\vec{V}\cdot\vec{z}'}{C}\right) \tag{2-4}$$

An expansion of the term $\vec{z}' \times (\nabla \times \vec{V})$, using Eqs. (2-1) and (2-2) yields:

$$\begin{aligned}\vec{z}' \times (\nabla \times \vec{V}) = {} & \hat{a}_1\left(-y'\frac{\partial V_x}{\partial y} - z'\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\right) \\ & + \hat{a}_2\left(x'\frac{\partial V_x}{\partial z} + z'\frac{\partial V_z}{\partial y}\right) \\ & + \hat{a}_3\left(x'\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) - y'\frac{\partial V_z}{\partial y}\right)\end{aligned} \tag{2-5}$$

A further simplification is obtained by substituting Eq. (2-1) and Eq. (2-2) in Eq. (2-5):

$$\begin{aligned}\vec{z}' \times (\nabla \times \vec{V}) = {} & \hat{a}_1\,(y'\beta V + z'\gamma V - z'b\alpha V) - \\ & \hat{a}_2\,(x'\beta V + z'b\beta V) - \\ & \hat{a}_3\,(x'\gamma V - x'b\alpha V - y'b\beta V)\end{aligned} \tag{2-6}$$

and Eq. (2-3) reduces to

$$\frac{dn}{ds}\left(\frac{dx}{ds}\hat{a}_1+\frac{dy}{ds}\hat{a}_2+\frac{dz}{ds}\hat{a}_3\right)+\left(n\ \frac{d^2x}{ds^2}\hat{a}_1+\frac{d^2y}{ds^2}\hat{a}_2+\frac{d^2z}{ds^2}\hat{a}_3\right)$$

$$+\Big(\hat{a}_1\ (y'\beta V+z'\gamma V-z'b\alpha V)-$$

$$\hat{a}_2\ (x'\beta V+z'b\beta V)\ -$$

$$\hat{a}_3\ (x'\gamma V-x'b\alpha V-y'b\beta V)\Big)\left(\frac{n}{c}\left(1-\frac{\vec{V}}{c}\cdot(x'\hat{a}_1+z'\hat{a}_3)\right)\right) \quad (2\text{-}7)$$

$$=\nabla n$$

where

$$n=\frac{c_o}{c(y)}\ ,\quad \frac{dn}{ds}=-\frac{c_o}{c^2}\frac{dc}{ds}\ ,=-\frac{c_o}{c^2}c'$$

$$\frac{\partial n}{\partial y}=-\frac{c_o}{c^2}\frac{\partial c}{\partial y}=\frac{Ac_o}{c}$$

Substituting the above relations in Eq. (2-7) and separating $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$ components, yields:

$$x''=\frac{c'}{c}x'-(y'\beta+z'\gamma-z'b\alpha)\frac{V}{c}\left(1-\frac{\vec{V}}{c}\cdot(x'\hat{a}_1+z'\hat{a}_3)\right) \quad (2\text{-}8)$$

$$y''=\frac{c'}{c}y'+A+(x'\beta+z'b\beta)\frac{V}{c}\left(1-\frac{\vec{V}}{c}\cdot(x'\hat{a}_1+z'\hat{a}_3)\right) \quad (2\text{-}9)$$

$$z''=\frac{c'}{c}z'+(\gamma x'-x'b\alpha-y'b\beta)\frac{V}{c}\left(1-\frac{\vec{V}}{c}\cdot(x'\hat{a}_1+z'\hat{a}_3)\right) \quad (2\text{-}10)$$

The curvature K is given by

$$K^2=x''^2+y''^2+z''^2 \quad (2\text{-}11)$$

Equations (2-8), (2-9), (2-10) and (2-11) yield the following result when $\left(\frac{V}{c}\right)^2$ is assumed negligible as compared to unity

$$K^2 = A^2(1-y'^2) + 2A\,(x'\beta + z'b\beta)\frac{V}{C}\left(1-\frac{V}{C}(x'+z')\right) \tag{2-12}$$

This expression can now be evaluated, but for the purpose of this project, one can have the following practical assumptions for simplification.

2.3 Special Case

One can neglect $\left(\frac{V}{C}\right)^2$ terms as compared to unity and also $V_z \ll V_x$, $b=0$ and attenuation constant $\gamma = 0$.

Equation (2-12) can be rewritten as:

$$K^2 = A^2(1-y'^2) + 2\frac{V}{C}A\beta x' \tag{2-13}$$

Since the arc length S is a function of x , y , and z axes, it is assumed the variations of S along x and z axes are negligible for the first order approximations of $V \ll C$ and C is a function of y only. One can see from the application of Snell's law that ray path, in general, lies in a plane normal to the xz plane and, without loss of generality, the ray path lies in the xy plane.

For $\frac{dx}{ds} \approx 1$ (which is true even for a very shallow case), Eq. (2-13) can be rewritten as

$$K^2 \approx \left(\frac{d^2y}{ds^2}\right)^2 = A^2\left(1-\left(\frac{dy}{ds}\right)^2\right) + 2\frac{V}{C}A\beta \tag{2-14}$$

In Eq. (2-14), S is just a parameter. It has already been confirmed that the ray path lies in the xy plane, so the parameter S can be changed to x without any loss of generality in ray tracing. Thus Eq. (2-14) yields:

$$\left(\frac{d^2y}{dx^2}\right)^2 = A^2\left(1-\left(\frac{dy}{dx}\right)^2\right) + 2\frac{V}{C}A\beta \tag{2-15}$$

2.4 Ray Path in a Medium with Space Variable Mass Velocity

In the First Case,

$$\vec{V} = V_0\, e^{-\alpha x - \beta y}\,\hat{a}_1$$

$$C = C_0\, e^{-Ay} \quad \text{for} \quad b = C_1 = \gamma = 0$$

Equation (2-15) gives

$$y''^2 = A^2(1-y'^2) + 2A\beta\frac{V_0}{C_0}\, e^{-\alpha x - \beta y + Ay} \tag{2-16}$$

In order to see the influence of various parameters on the ray path calculations, the following approximate set of values for the various constants are selected for the assumed model ocean case.

$$\begin{aligned} C_0 &= 5000 \text{ ft/sec} \\ V_0 &= 3.5 \text{ ft/sec} \\ \alpha &= 0.001 \text{ ft}^{-1} \\ \beta &= 0.1756 \text{ ft}^{-1} \\ A &= 1.325 \times 10^{-5} \text{ ft}^{-1} \end{aligned} \tag{2-17}$$

The previous set of values suggests that one may neglect the term $A^2(1-y'^2)$ in Eq. (2-16) as A^2 is negligible as compared to unity. Equation (2-16) yields

$$y''^2 = 2A\beta \frac{V_0}{C_0} e^{-\alpha x - \beta y + Ay} \tag{2-18}$$

Let $2A\beta \frac{V_0}{C_0} = K'$

$\beta - A = K_2$

where K_1' and K_2 are constants, Eq. (2-18) becomes

$$y'' = \sqrt{K'}\ e^{-\frac{\alpha}{2}x - \frac{K_2}{2}y} \tag{2-19}$$

which is further simplified using Taylor's series and neglecting the higher order terms. Equation (2-19) yields

$$y'' = \sqrt{K'}\left(1 - \frac{\alpha}{2}x - \frac{K_2}{2}y\right) \tag{2-20}$$

or

$$y'' + Ly = \sqrt{K'}\left(1 - \frac{\alpha}{2}x\right) \tag{2-21}$$

where $L = \frac{\sqrt{K'}K_2}{2}$. Equation (2-21) is a linear differential equation. The complete solution can be given as

$$y = \frac{\sqrt{K'}}{L}\left(1 - \frac{\alpha}{2}x\right) + M_1 \cos\sqrt{L}\,x + M_2 \sin\sqrt{L}\,x \tag{2-22}$$

The constants M_1 and M_2 can be evaluated from initial conditions. The following are cases considered for detailed study.

2.5 Initial Conditions

The follwoing set of values are chosen for the initial conditions:

Surface Level $Y = 0 \; , \quad X = 0$

Source Positions

$$X = 0 \; , \quad Y = (n-1) \times 10^2 \, ft$$
$$n = 1, 2, \ldots \ldots$$

and

$$X = 0 \; , \quad \frac{dY}{dx} = \tan \theta_0$$

where $\theta_0 = 20^\circ, 30^\circ, 45^\circ, 60^\circ$

2.5.1 Source at Surface Level

When the source is located at just below the water surface or $X = 0 \; , Y = 0$, then by substituting the above in Eq. (2-22), one can find the constants of integration after which the equation turns out to be complete as under

$$Y = \frac{\sqrt{K_1'}}{L}\left(1 - \frac{\alpha}{2}\right)x - \frac{\sqrt{K_1'}}{L} \cos\sqrt{L}\, X + \frac{1}{\sqrt{2}}\left(\tan\theta_0 + \frac{\sqrt{K_1'}\alpha}{2}\right)\sin\sqrt{L}\, X \qquad (2\text{-}23)$$

where $\sqrt{K_1'}$, α, θ_0 and L are known constants. The ray paths are plotted in Fig. 2-1 for $\theta_0 = 20^\circ, 30^\circ, 45^\circ, 60^\circ$.

2.5.2 Source at a Depth of 100 Feet

In case of the source location at $x = 0, y = 100 ft$, one similarly obtains the ray paths as was done in the previous case, and the resulting data are shown in Fig. 2-2 for $\theta_0 = 20^\circ, 30^\circ, 45^\circ$, and 60°. The same data is tabulated in Tables I, II, and III.

The most significant results of this study is the error in case of the medium moving with constant velocity; i.e., $\alpha = 0$, $\beta = 0$, as compared to the case of medium with no horizontal variations of water mass; i.e., $\alpha = 0$.

2.6 Ray Path in a Medium with Constant Velocity

Consider Eq. (2.16):

$$y''^2 = A^2 (1 - y'^2) + 2A\beta \frac{V_0}{C_0} e^{-\alpha x - \beta y + Ay}$$

for $\alpha = 0$, $\beta = 0$.

This reduces to

$$y''^2 = A^2 (1 - y'^2) \qquad (2-23)$$

$$y = A (1 - y'^2)^{\frac{1}{2}} \qquad (2-24)$$

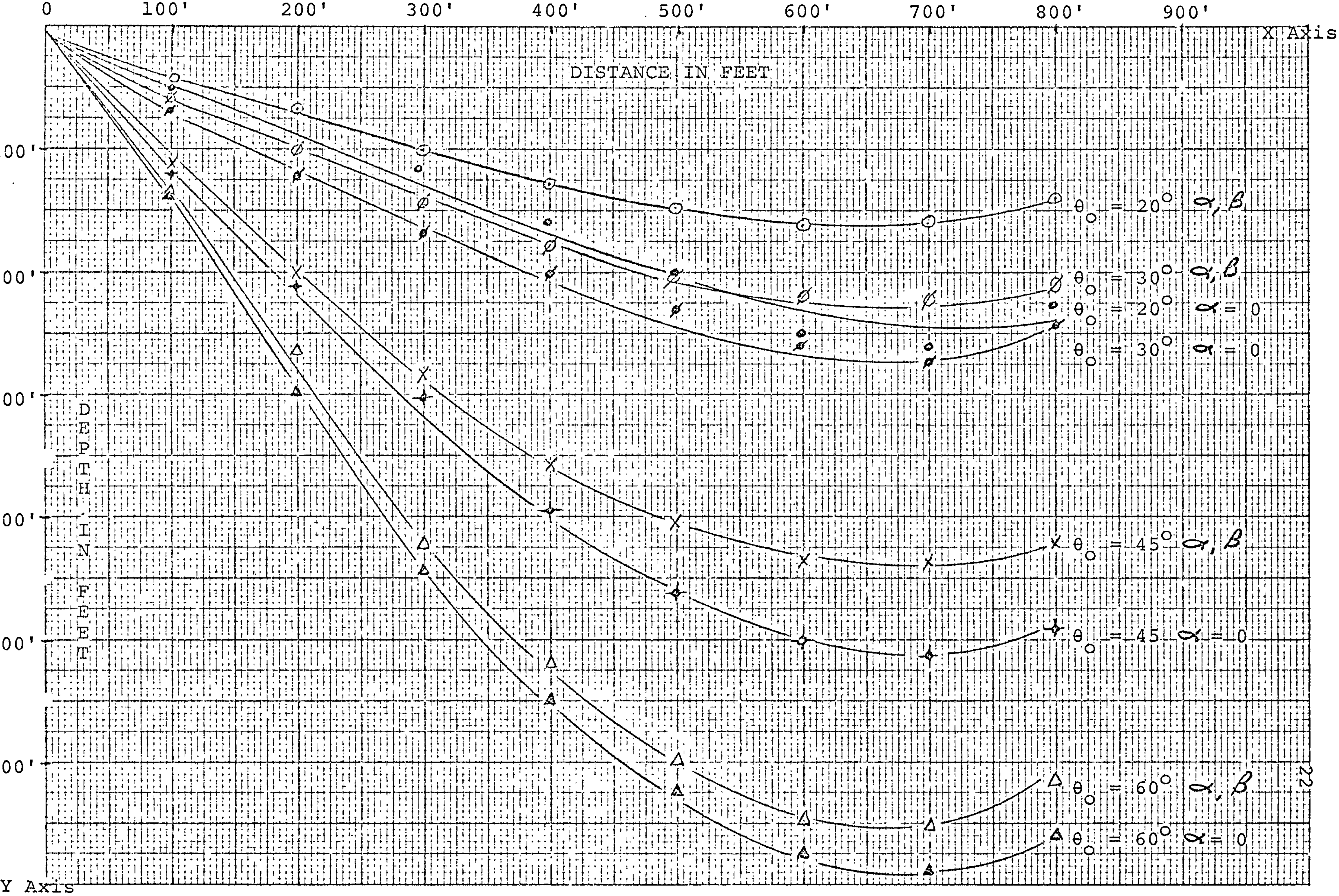


Fig. 2-1. Ray Path in Ocean Source at Surface Level

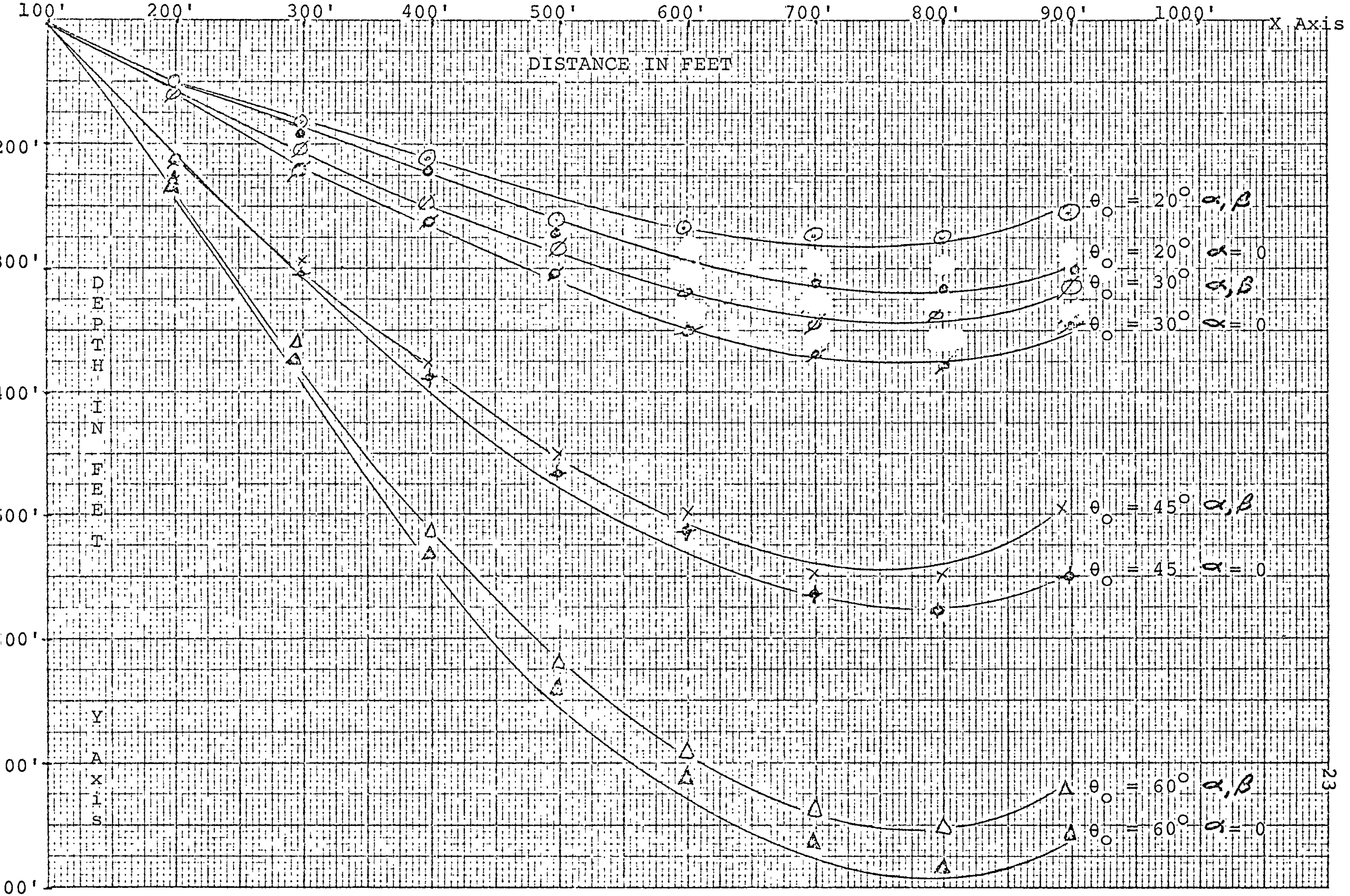


Fig. 2-2. Ray Path in Ocean Source at a Depth of 100 Feet

TABLE I

RAY PATH COORDINATES AT VARIOUS DEPTHS

Source at Surface Level

Horizontal Distance in Feet	Depth in Feet $\theta_o = 20^\circ$		Depth in Feet $\theta_o = 30^\circ$		Depth in Feet $\theta_o = 45^\circ$		Depth in Feet $\theta_o = 60^\circ$	
X	Y $\alpha,\beta \neq 0$	Y $\alpha = 0$, $\beta \neq 0$	Y $\alpha,\beta \neq 0$	Y $\alpha = 0$, $\beta \neq 0$	Y $\alpha,\beta \neq 0$	Y $\alpha = 0$, $\beta \neq 0$	Y $\alpha,\beta \neq 0$	Y $\alpha = 0$, $\beta \neq 0$
100	40.9	40.9	51.9	57.27	98.9	98.9	129.8	135.2
200	64.0	64.0	109.6	121.76	192.8	200.0	261.7	271.7
300	99.9	125.0	145.76	156.76	275.7	300.0	421.0	437.2
400	120.1	160.0	181.44	203.1	350.2	398.0	525.0	547.0
500	142.0	202.0	204.1	230.8	398.2	465.0	591.1	618.1
600	156.0	247.0	220.5	252.9	420.0	508.0	641.92	674.32
700	157.9	260.0	222.7	260.5	425.0	525.0	652.3	690.1
800	145.2	210.0	210.0	236.5	410.0	498.0	622.61	665.8

TABLE II

RAY PATH COORDINATES AT VARIOUS DEPTHS

Source 100' Deep

Horizontal Distance in Feet X	Depth in Feet $\theta_0 = 20^\circ$ Y α,β≠0	Depth in Feet $\theta_0 = 20^\circ$ Y α=0 β≠0	Depth in Feet $\theta_0 = 30^\circ$ Y α,β≠0	Depth in Feet $\theta_0 = 30^\circ$ Y α=0 β≠0	Depth in Feet $\theta_0 = 45^\circ$ Y α,β≠0	Depth in Feet $\theta_0 = 45^\circ$ Y α=0 β≠0	Depth in Feet $\theta_0 = 60^\circ$ Y α,β≠0	Depth in Feet $\theta_0 = 60^\circ$ Y α=0 β≠0
100	150.9	156.2	151.9	156.20	220.0	225.7	229.8	234.0
200	182.0	173.8	209.6	220.76	300.0	311.7	361.7	370.7
300	205.0	205.0	245.76	255.56	380.0	391.0	521.0	537.2
400	260.0	274.0	281.44	303.7	456.0	470.0	625.4	648.0
500	262.0	276.0	304.1	330.8	500.0	504.8	691.1	718.0
600	270.0	280.0	320.5	352.0	550.0	582.0	742.0	773.32
700	272.0	284.0	322.7	360.5	547.0	585.0	752.3	790.1
800	252.0	278.0	310.0	336.0	498.0	556.0	722.6	764.0

TABLE III

RAY PATH COORDINATES AT VARIOUS DEPTHS

Source at Surface (Medium with Constant Velocity)

Horizontal Distance in Feet X $\alpha, \beta = 0$	Depth in Feet $\theta_0 = 20^\circ$ Y $\alpha, \beta = 0$	Depth in Feet $\theta_0 = 30^\circ$ Y $\alpha, \beta = 0$	Depth in Feet $\theta_0 = 45^\circ$ Y $\alpha, \beta = 0$	Depth in Feet $\theta_0 = 60^\circ$ Y $\alpha, \beta = 0$
4000	1900	2500	4000	4000
8000	3500	5000	8000	8000
12000	5000	8000	12000	13000
16000	8000	12000	18000	18000
20000	9500	15500	22000	
24000	10000	19000		

In order to solve Eq. (2-24), assume $y' = t$, $y'' = t'$, and substitute in Eq. (2-24), to obtain

$$\frac{dt}{dx} = A\left(1-t^2\right)^{\frac{1}{2}} \qquad (2\text{-}25)$$

$$\int \frac{dt}{(1-t^2)^{\frac{1}{2}}} = A\int dx + R_1 \qquad (2\text{-}26)$$

where R_1 is just a constant of integration. Equation (2-26) yields

$$t = \sin(AX+R_1) \quad , \quad t = \frac{dy}{dx} \qquad (2\text{-}27)$$

$$\int dy = \int \sin(Ax+R_1)\,dx + R_2$$

$$Y = -\frac{\cos(AX+R_1)}{A} + R_2 \qquad (2\text{-}28)$$

where R_1 and R_2 are constants of integration which can be evaluated from the initial conditions. One can consider the two boundary conditions or $Y=0$, for $X=0$ from Eq. (2-28); therefore

$$R_2 = \frac{\cos R_1}{A} \qquad (2\text{-}29)$$

Similarly, for $X=0$, $\frac{dy}{dx} = \tan\theta_0$ and Eq. (2-28) yields

$$R_1 = \sin^{-1}(\tan\theta_0) \qquad (2\text{-}30)$$

Substituting Eqs. (2-29) and (2-30) in Eq. (2-28), the complete solution becomes

$$y = -\frac{\cos(Ax + \sin^{-1}(\tan\theta_0))}{A} + \frac{\cos(\sin^{-1}(\tan\theta_0))}{A} \qquad (2\text{-}31)$$

The ray paths are plotted for $\theta_0 = 20^\circ, 30^\circ, 45^\circ$ and 60°, respectively, in Fig. 2-3.

The tables of values are also presented in this case for complete specifications (refer to Tables I, II and III).

2.7 No Horizontal Variations of Water Mass Velocity

Referring to Eq. (2-16)

$$y''^2 = A^2(1-y'^2) + 2A\beta\frac{V_0}{C_0}\, e^{-\alpha x - \beta y + Ay}$$

for $\alpha = 0$. This reduces to

$$y''^2 = A^2(1-y'^2) + 2A\beta\frac{V_0}{C_0}\, e^{-\beta y + Ay} \qquad (2\text{-}32)$$

A^2 is small as compared to unity. Then Eq. (2-32) yields

(2-33)

Let $2A\beta\frac{V_0}{C_0} = K_1'$ and $\beta - A = K_2$

$$y'' = \sqrt{K_1'}\, e^{-\frac{K_2}{2}y}$$

expanding by Taylor's series and neglecting second order terms compared to unity. This yields

$$y'' + Ly = \sqrt{K'} \qquad (2\text{-}34)$$

where $$L = \frac{\sqrt{K_1'}\, K_2}{2}$$

Equation (2-34) is a linear differential equation. One can obtain the complete solution as given below (Hayre and Sohel, 1969)

$$Y = \frac{\sqrt{K_1'}}{L} + M_1 \cos\sqrt{L}\,X + M_2 \sin\sqrt{L}\,X \qquad (2\text{-}35)$$

where M_1 and M_2 are constants of integration which can be evaluated from initial conditions. For a detailed study, one can donsider the following cases

$X = 0$, $Y = 0$ AND FOR $X = 0$, $\frac{dY}{dx} = \tan\theta_0$

substituting in Eq. (2-35)

$$M_1 = \frac{-\sqrt{K_1'}}{L}, \quad M_2 = \frac{\tan\theta_0}{\sqrt{L}} \qquad (2\text{-}36)$$

Substituting in Eq. (2-36), which yields

$$Y = \frac{\sqrt{K_1'}}{L} - \frac{\sqrt{K_1'}}{L}\cos\sqrt{L}\,X + \frac{\tan\theta_0}{\sqrt{L}}\sin\sqrt{L}\,X \qquad (2\text{-}37)$$

The constants K_1', L and θ_0 are known. The ray path is plotted for $\theta_0 = 20^\circ, 30^\circ, 45^\circ$ and 60° in Fig. 2-1.

The ray paths are plotted when the source is 100 feet deep and the bean is transmitted at angles of $\theta_0 = 20^\circ, 30^\circ, 45^\circ$ and 60° in Fig. 2-2. These are tabulated below (refer to Tables, I, II, and III).

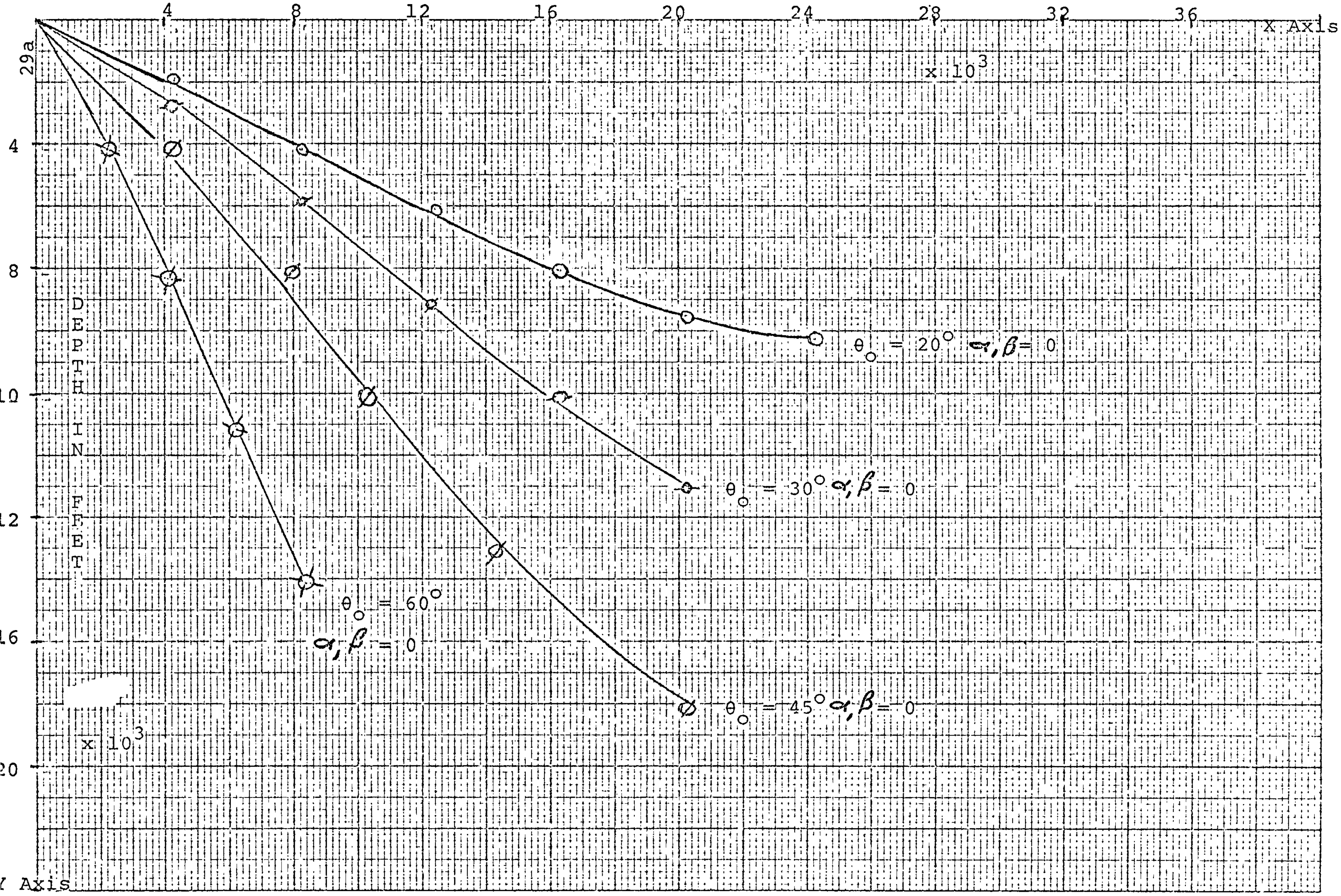


Fig. 2-3. Ray Path in a Media Moving with Constant Velocity

CHAPTER III

RANDOM VARIATIONS OF SOUND VELOCITY

The data on the variations of the speed of sound in the Atlantic Ocean off the Florida coast is analyzed by removing the usually expected exponential depth variation and then statistically examining the random component of the sound velocity. The probability density function of the random variable is plotted and other parameters such as the mean, variance and auto-correlation are also calculated. Decorrelation depth is calculated for each data run and an attempt is made to completely specify the statistical characteristics of the random variations in the speed of sound for such a case.

The speed of sound varies with salinity, depth, pressure, and temperature, the latter being the dominating factor, and has the overall exponential decay with depth. Extensive experimental data shows that the speed of sound varies randomly around its gross exponential decrease with depth. In this study some experimental data taken during a given day off the Florida coast at differnt locations is investigated to obtain the statistical characteristics. These are expected to vary considerably from one location to another. Furthermore, any such variations also effect the ray path propagation in the ocean.

3.1 Analysis of Data and Calculation of Attenuation Constants

The available data of the sound velocity varying with depth was plotted for each run. The scatter diagram (refer to Figs. 3-1, 3-2 and 3-3) clearly shows that the sound velocity is varying randomly with depth although a smoothed curve approximates the usual exponential for variation. Then the data on sound velocity was plotted on semilog paper to remove the exponential variation. Thus the exponential variations of the sound velocity in the ocean has the random component superimposed on it and it may be expressed as:

$$C = C_o e^{-AY} + C_1(Y) \tag{3-1}$$

where C_o = initial speed of sound = 1,554 meter/second

A = attenuation constant to be evaluated, and

$C_1(Y)$ = random component of the sound velocity

The attenuation constant A was obtained from semilog plots of C versus Y, and these were fitted to straight lines because of the straight line plots of all exponential variations, as shown below:

$$C = C_o e^{-AY} + C_1(Y)$$

$$Log\, C = Log\, C_o - AY + Log\, C_1(Y)$$

$$X_{11} = Log\, C_{11} = Log\, C_o - AY_{11} + Log\, C_1(Y) \tag{3-2}$$

Fig. 3-1. SCATTER DIAGRAM

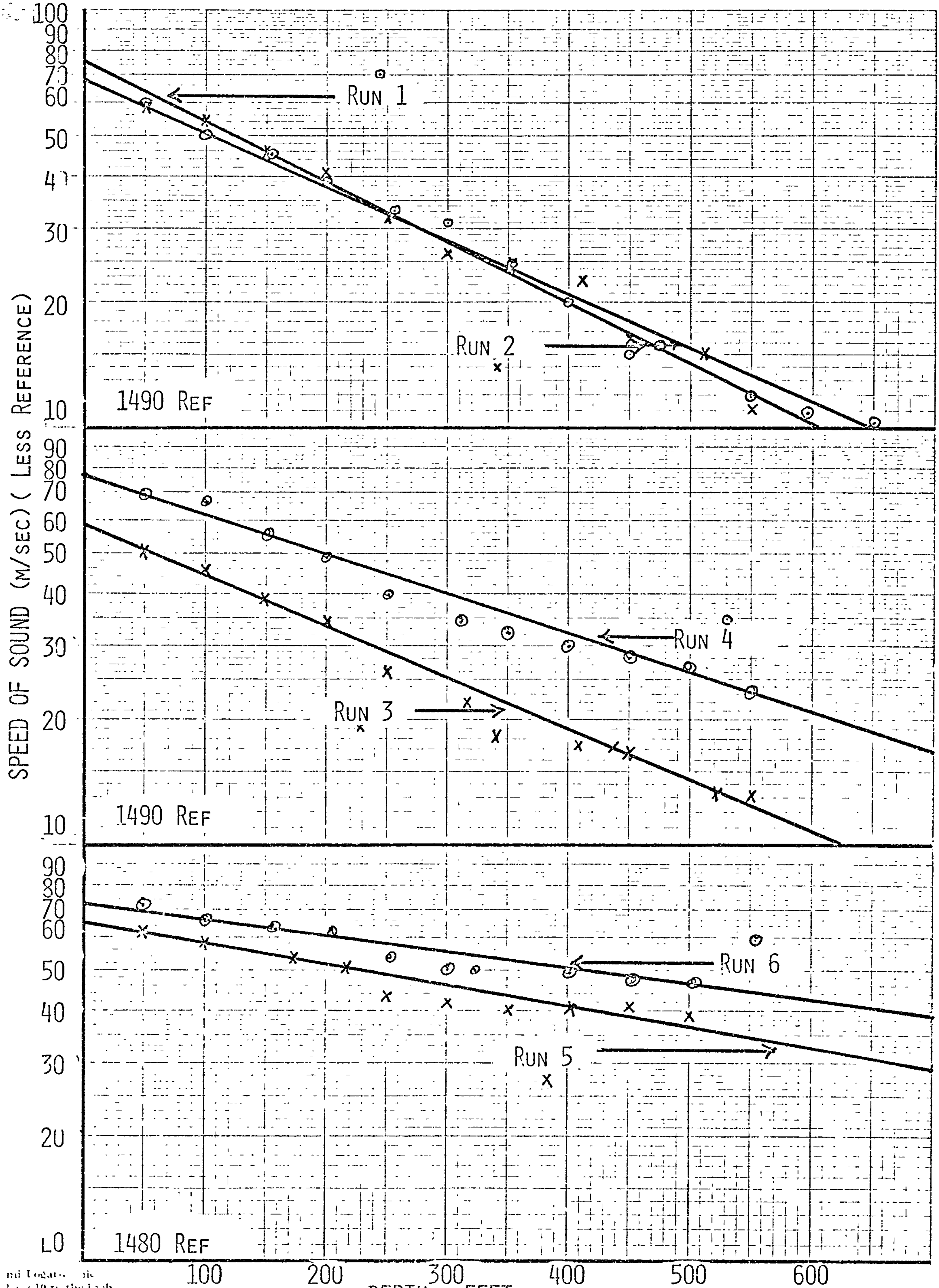
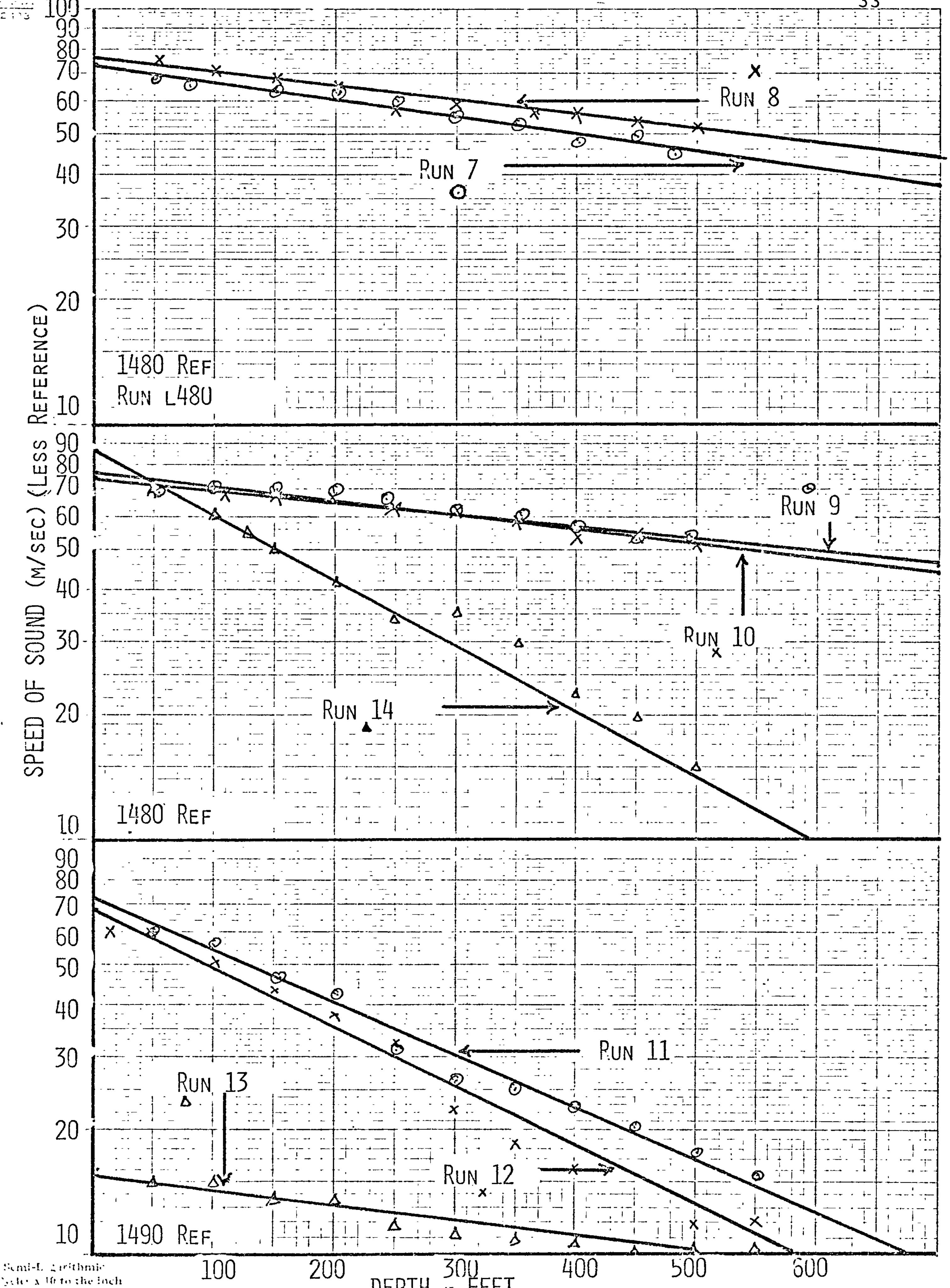


Fig. 3-2. SCATTER DIAGRAM



$$X_{22} = \log c_{22} = \log c_0 - A y_{22} + \log c_1 (y) \tag{3-3}$$

Thus Eqs. (3-2) and (3-3) yield:

$$\frac{X_{11} - X_{22}}{y_{11} - y_{22}} = -A \tag{3-4}$$

which is the slope of the line

$$X_{11} - X_{22} = 6.7 \text{ cm}$$
$$y_{11} - y_{22} = 7.2 \text{ cm}$$

or for Eq. (3-4), because

$$-\frac{6.7}{7.2} = A \tag{3-5}$$

or

$$A = -.93 \tag{3-6}$$

3.2 Statistical Characteristics of Random Part

In order to completely specify a random variable, one needs to find its probability density function, mean, and variance. As discussed in the previous section, the random part of the sound velocity varies above and below the approximate average exponential variation. The data thus obtained is sampled at a rate of one m/sec in each run and the histograms are plotted for each run as shown in Figs. 3-3, 3-4, 3-5 and 3-6.

Fig. 3-3. Histograms - Run 1 through 4

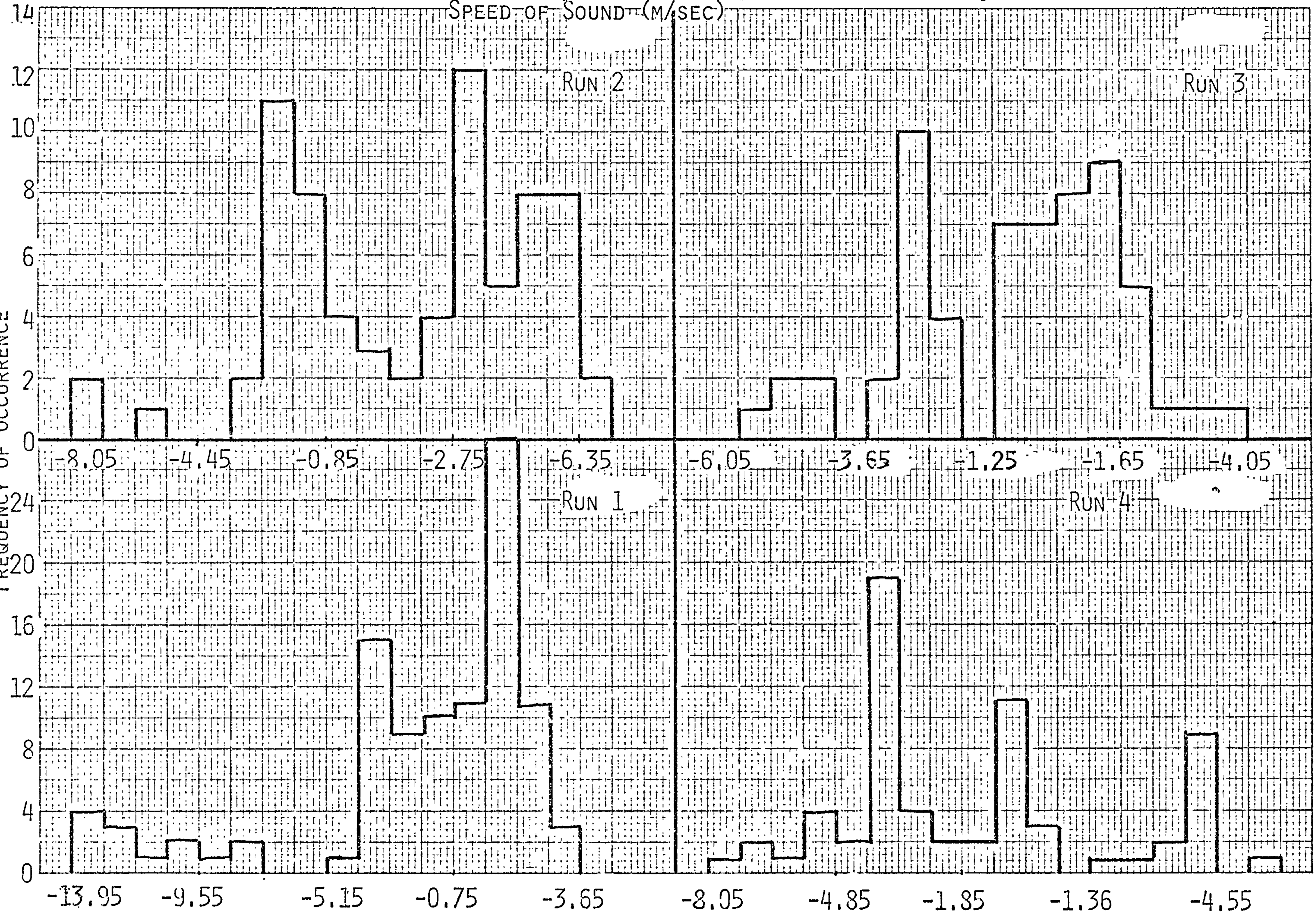


Fig. 3-4. Histograms - Run 5 through 8

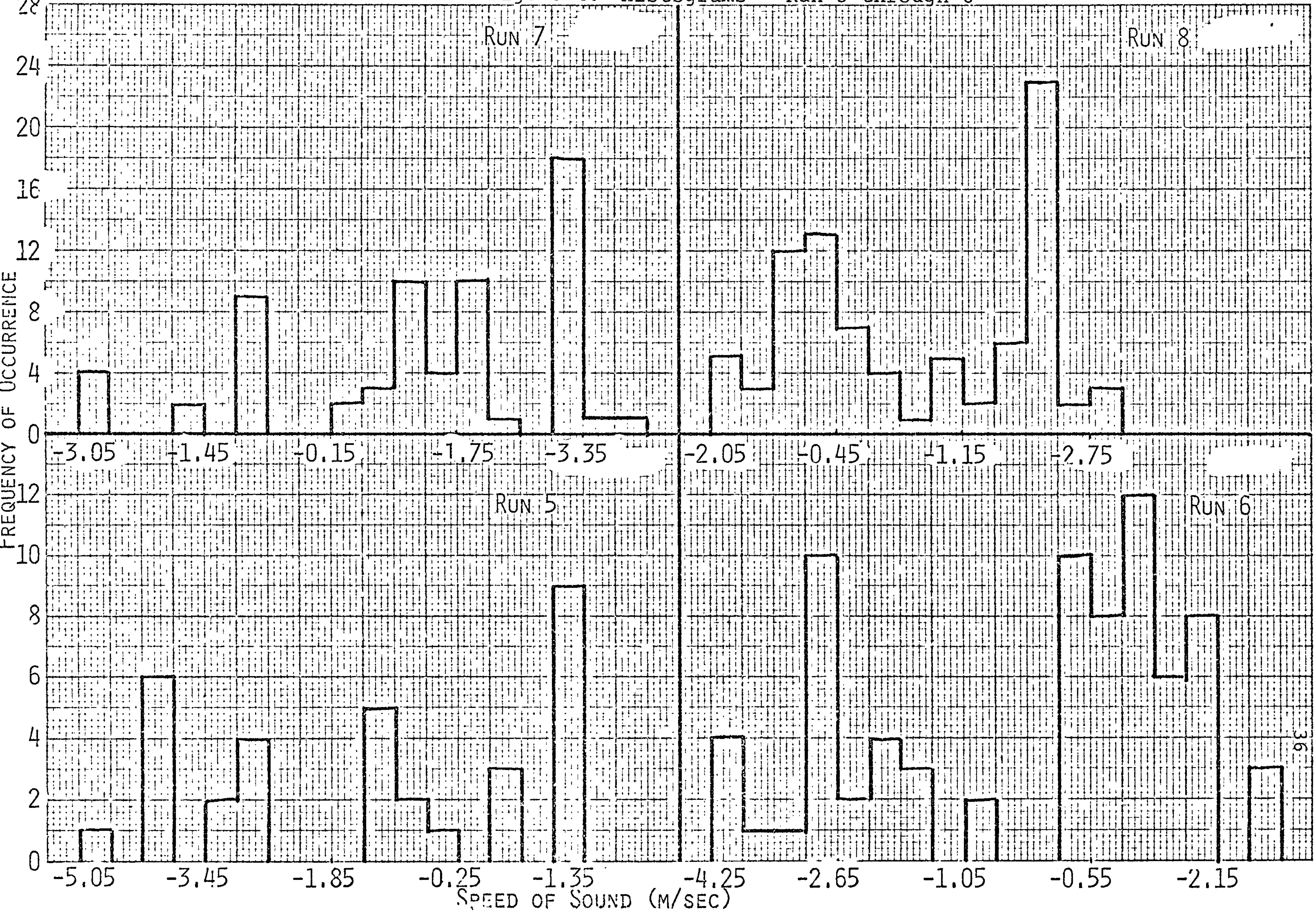


Fig. 3-5. Histograms - Run 9 through 12

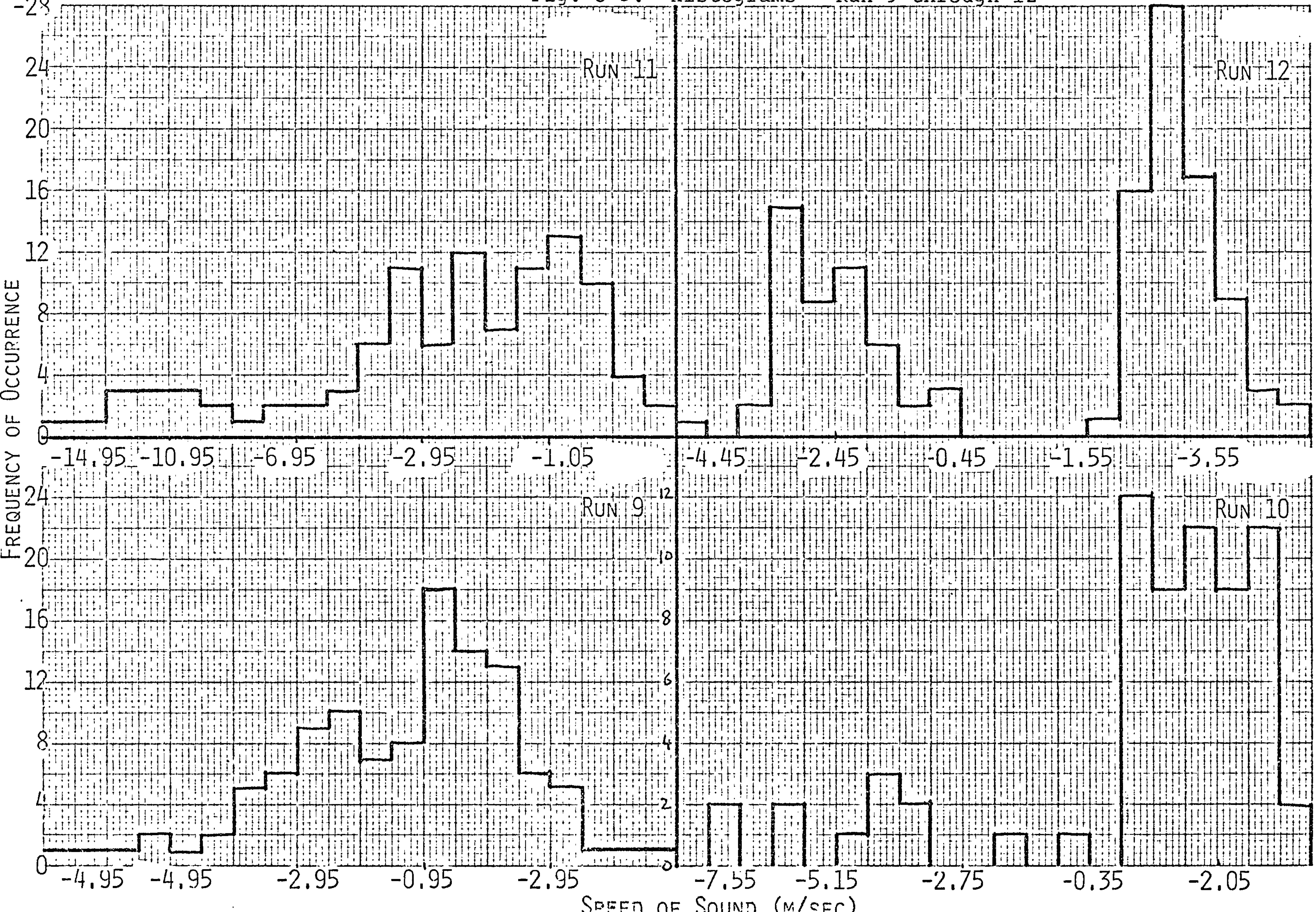
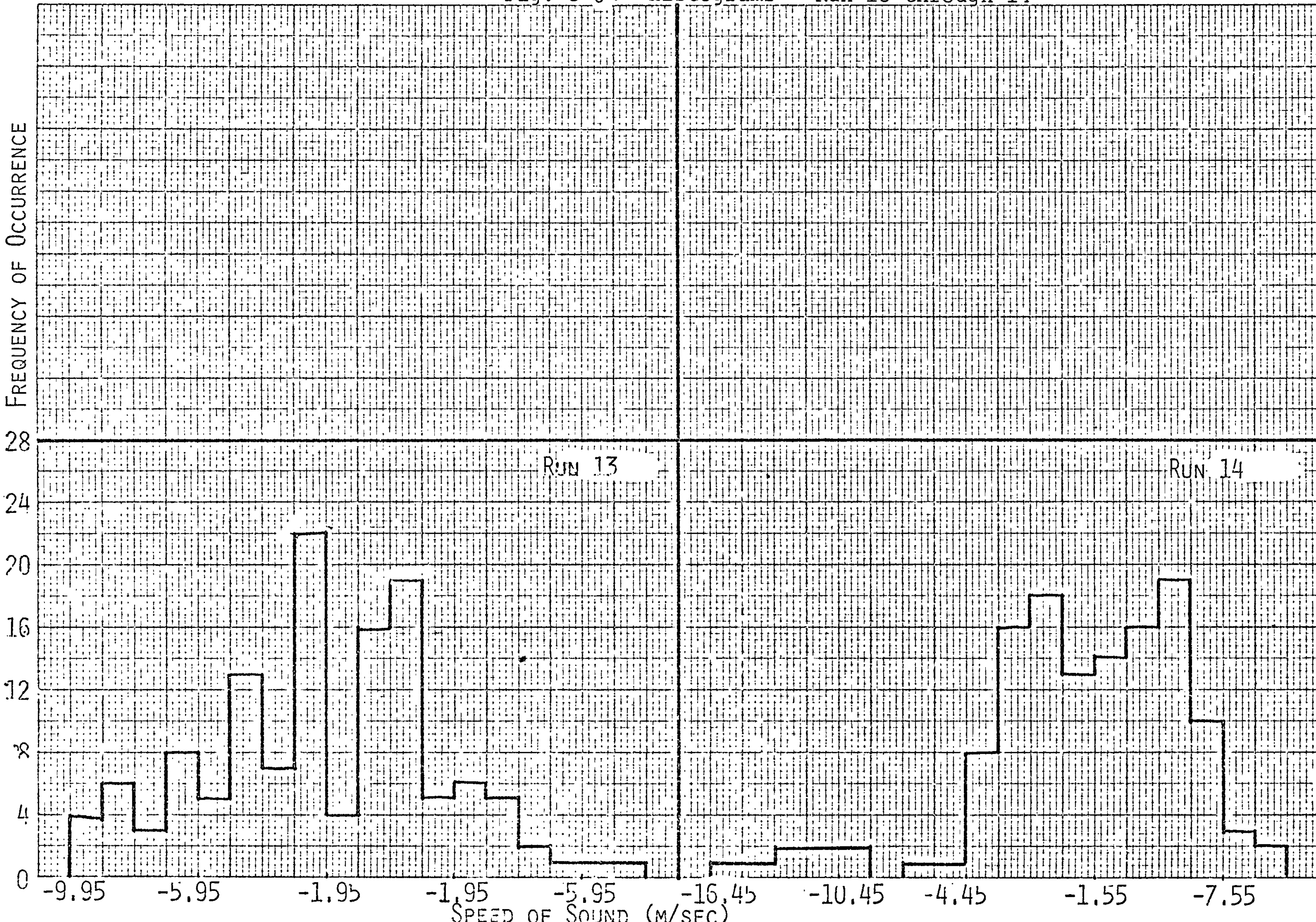


Fig. 3-6. Histograms - Run 13 through 14



3.3 Calculation of the Mean Value

The mean of this random variable can be calculated by the following relationship (Meyer, 1965)

$$\bar{C}_1 = A + \frac{\sum_{j=1}^{K} f_j d_j}{\sum_{j=1}^{K} f_j} \tag{3-7}$$

where f_j = frequency of occurrence of jth number

A = assumed mean

$d_j = C_{1j} - A$ = deviation of any class mark number from the assumed mean

3.4 Calculation of Standard Deviation

Further, one needs to find the variance of random variable because this parameter gives the degree of spreading of the data about the mean value. One can find the variance by the following relationship (Papoulis, 1967)

$$\sigma = \pm \sqrt{\frac{\sum_{j=1}^{N} \left(C_{1j} - \bar{C}_1\right)}{N}} \tag{3-8}$$

The percentage of the point included between $\bar{C}_1 \pm \sigma$ and $\bar{C}_1 \pm 2\sigma$ are 69.27% and 95.45% respectively for normally distributed data, but standard deviation is a valid measure of the spread of any data sample. The data available off the Forida coast was analyzed to obtain the mean, variance, and probability density functions. The density function in some cases seems to be Gaussian and in other cases either bimodal or multimodal, but later on in this chapter the distribution is assumed to be normal.

3.5 Decorrelation Distance

A complete specification of a random variable by density functions also requires its spatial auto-correlation function. In theory, two types of auto-correlation functions are assumed (Urick, 1967)

$$\rho(d) = e^{-|d/a|} \tag{3-9}$$

and

$$\rho(d) = e^{-(d/a)^2} \tag{3-10}$$

where d = distance

a = auto-correlation distance within which the speed of sound variations maintain coherence

In other words, within this distance data points are correlated with each other and at $d = a$ one obtains

$$\rho(d) = e^{-1} = .372 \tag{3-11}$$

and the variations are no longer in coherence with each other beyond this decorrelation distance. Either a linear combination of exponential forms or the Gaussian form is physically reliable at the origin should be zero for a stationary process or for continuously varying speed of sound.

In this study the correlation function is calculated as follows:

$$\rho_{c_1c_1}(y) = \frac{\sum_{K=0}^{N-K} C_{IN} - C_{IN-K}}{N-K} \tag{3-12}$$

where Y is the ocean depth coordinate. The auto-correlation functions and decorrelation depth are calculated for each run as shown in Table IV.

3.6 Derivation of Curvature of Ray Path in a Moving Medium With Random Variations Superimposed on Speed of Sound

An analysis of the available data off the Florida coast suggests the following model for the speed of sound in a moving inhomogeneous water mass:

$$C = C_o e^{-\beta y} + C_1(Y) \tag{3-13}$$

where $C_1(Y)$ is the random part of the sound velocity and the water mass velocity is assumed to decrease exponentially along the X, Y, and Z axis.

$$\vec{V} = V_o(\hat{a}_1 + b\hat{a}_3) e^{-\alpha x - \beta y - \gamma z} \tag{3-14}$$

The general vector Eikonal equation for a moving inhomogeneous medium is given as (Uginicius, 1965)

$$(n\vec{r}')' + \vec{r}' \times (\nabla \times \vec{V})\frac{n}{c}\left(1 - \frac{\vec{V}}{c}\cdot\vec{r}'\right) = \nabla n \tag{3-15}$$

TABLE IV

STATISTICAL PARAMETERS

S. No.	Run No.	Mean (m/sec)	Variance $(m/sec)^2$	Standard Deviation	Decorrelation Distance (feet)
1	I	- .8362	4.35	2.313	210
2	II	1.51	10.23	3.19	190
3	III	- .732	6.89	2.62	209
4	IV	1.533	11.08	3.32	210
5	V	-1.218	4.721	2.17	205
6	VI	- .135	3.9881	1.997	189
7	VII	1.005	4.505	2.12	210
8	VIII	.553	2.52	1.58	180
9	IX	.80	4.00	2.00	156
10	X	.95	7.56	2.74	190
11	XI	2.04	5.85	2.41	185
12	XII	1.03	8.47	2.9	89
13	XIII	1.92	10.06	3.17	195
14	XIV	1.136	9.23	3.03	210

where $n = \frac{c_0}{c(x,y,z)}$ refractive index of medium

$\vec{V}(x,y,z)$ = water mass velocity

$\vec{\xi}'$ = unit tangent vector = $\frac{dx}{ds}\hat{a}_1 + \frac{dy}{ds}\hat{a}_2 + \frac{dz}{ds}\hat{a}_3$

S = arc length

$(\xi')'$ = derivative with respect to S

A substitution of Eqs. (3-13) and (3-14) in Eq. (3-15) and separating $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ components after some simplifications yields (see Chapter II for details)

$$x'' = \frac{c'}{c}x' - (y'\beta + z'\gamma - z'b\alpha)\frac{V}{c}\left(1 - \frac{V}{c}(x'+z')\right) \tag{3-16}$$

$$y'' = \frac{c'}{c}y' - \frac{1}{c}\frac{dc}{dy} + (x'\beta + z'b\beta)\frac{V}{c}\left(1 - \frac{V}{c}(x'+z')\right) \tag{3-17}$$

$$z'' = \frac{c'}{c}z' + (x'\gamma - x'b\alpha - y'b\beta)\frac{V}{c}\left(1 - \frac{V}{c}(x'+z')\right) \tag{3-18}$$

and for $V_z \ll V_x$, $\gamma = 0$, $z' = 0$, $\left(\frac{V}{c}\right)^2 \ll 1$

$$x'' = \left(\frac{c'}{c}\right)x' - (y'\beta)\frac{V}{c} \tag{3-19}$$

$$y'' = \left(\frac{c'}{c}\right)y' - \frac{1}{c}\left(\frac{dc}{dy}\right) + x'\beta\frac{V}{c} \tag{3-20}$$

$$z'' = \left(\frac{c'}{c}\right) z' \tag{3-21}$$

The curvature K is given by

$$k^2 = x''^2 + y''^2 + z''^2 \tag{3-22}$$

A substitution of Eq. (3-19), (3-20) and (3-21) into Eq. (3-22) yields

$$K^2 = \frac{1}{c^2}\left((1-y'^2)\left(\frac{dc}{dy}\right)^2 - 2\frac{dc}{dy}x'\beta V\right) \tag{3-23}$$

for $\left(\frac{V}{c}\right)^2 \ll 1$ and from Eq. (3-13) one gets

$$K^2 = \frac{1}{C^2}\left((1-y'^2)\left(c_1' - AC_0 e^{-Ay}\right)^2 - 2x'\beta V\left(c_1' - AC_0 e^{-Ay}\right)\right) \tag{3-24}$$

At this point it is desirable to average K^2 in order to obtain its

$$\langle K^2\rangle = \left\langle \frac{1}{c^2}\left((1-y'^2)(c_1' - AC_0 e^{-Ay})^2 - 2\beta x' V(c_1' - AC_0 e^{-Ay})\right)\right\rangle \tag{3-25}$$

where $\langle\ \rangle$ = statistical average over C_1

or

$$\langle K^2\rangle = \left\langle \frac{1}{c^2}\left(D(c_1' - E)^2 - F(c_1' - E)\right)\right\rangle$$

$$= \left\langle \frac{c_1'^2}{c^2}\right\rangle D + \left\langle \frac{c_1'}{c^2}\right\rangle (2DE + F) + \left\langle \frac{1}{c^2}\right\rangle \left(E^2/D + EF\right)$$

where

$$\begin{aligned} D &= (1-y'^2) \\ E &= A\, C_0\, e^{-Ay} \\ F &= 2\beta V x' \end{aligned} \tag{3-26}$$

and these are not random variables, Eq. (3-26) can be rewritten as

$$\langle K^2 \rangle = \left\langle \frac{C_1'}{C^2} \right\rangle D + (E^2 D - EF)\left\langle \frac{1}{C^2} \right\rangle + (2ED + F)\left\langle \frac{C_1'}{C^2} \right\rangle \tag{3-27}$$

The average of each of these terms is calculated below:

$$\left\langle \frac{1}{C^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{(c_1 + C_0 e^{-AY})^2} e^{-c_1^2/2\sigma^2} dc_1 \tag{3-28}$$

Let $C_0 e^{-AY} = T$

$$\left\langle \frac{1}{C^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{(c_1 + T)^2} e^{-c_1^2/2\sigma^2} dc_1 \tag{3-29}$$

Integrating by parts, Eq. (3-29) yields

$$\left\langle \frac{1}{C^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}} \left(\underbrace{\cancel{\left(- \frac{e^{-c_1^2/2\sigma^2}}{(T + c_1)} \right)_{-\infty}^{\infty}}}_{=0} - \frac{1}{\sigma^2} \int_{-\infty}^{\infty} \frac{c_1}{(T + c_1)} e^{-c_1^2/2\sigma^2} dc_1 \right) \tag{3-30}$$

$$\left\langle \frac{1}{C^2} \right\rangle = - \frac{1}{\sigma^3\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-c_1^2/2\sigma^2} dc_1 + \frac{T}{\sigma^3\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{(T + c_1)} e^{-c_1^2/2\sigma^2} dc_1 \tag{3-31}$$

On further simplification of Eq. (3-31) yields

$$\left\langle \frac{1}{C^2} \right\rangle = - \frac{1}{\sigma^2} + \frac{\cancel{T}}{\sigma^3\sqrt{2\pi}\,\cancel{T}} \int_{-\infty}^{\infty} \left(1 - \frac{c_1}{T} + \frac{c_1^2}{T^2}\right) e^{-c_1^2/2\sigma^2} dc_1$$

or

$$\left\langle \frac{1}{c^2} \right\rangle = -\frac{1}{\sigma^2} + \frac{1}{\sigma^2} - \frac{1}{\sigma^3\sqrt{2\pi}\,T}\int_{-\infty}^{\infty} c_1\, e^{-c_1^2/2\sigma^2} dc_1$$

$$+ \frac{1}{\sigma^3\sqrt{2\pi}\,T^2}\int_{-\infty}^{\infty} c_1^2\, e^{-c_1^2/2\sigma^2} dc_1 \tag{3-32}$$

$$\left\langle \frac{1}{c^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}\,T}\left(\underbrace{\cancel{e^{-c_1^2/2\sigma^2}}}_{=0} \right)_{-\infty}^{\infty} + \frac{1}{\sigma^3\sqrt{2\pi}\,T^2}\int_{-\infty}^{\infty} c_1^2\, e^{-c_1^2/2\sigma^2} dc_1$$

$$\left\langle \frac{1}{c^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}\,T^2}\left(\underbrace{\cancel{-c_1\, e^{-c_1^2/2\sigma^2}}}_{=0} \right)_{-\infty}^{\infty} + \frac{1}{\sigma\sqrt{2\pi}\,T^2}\int_{-\infty}^{\infty} e^{-c_1^2/2\sigma^2} dc_1$$

$$\left\langle \frac{1}{c^2} \right\rangle = \frac{1}{T^2}$$

$$\left\langle \frac{1}{c^2} \right\rangle = \frac{1}{T^2} \tag{3-33}$$

In case of $T \gg C_1$, one can neglect the first and second order terms of $\frac{c_1}{T}$ and $\left(\frac{c_1}{T}\right)^2$ $\therefore$ $\left\langle \frac{1}{c^2} \right\rangle = 0$

Also $$\left\langle \frac{c_1'}{c^2} \right\rangle = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{c_1'}{(T+c_1)^2}\, e^{-c_1^2/2\sigma^2} dc_1 \tag{3-34}$$

$$\left\langle \frac{c_1'}{c^2} \right\rangle = -\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{d}{d\gamma}\, \frac{1}{(T+c_1)}\, e^{-c_1^2/2\sigma^2} dc_1 \tag{3-35}$$

$$\left\langle \frac{c_1'}{c^2} \right\rangle = \frac{d}{d\gamma}\left(\frac{-1}{T\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-c_1^2/2\sigma^2} dc_1 \right) \tag{3-36}$$

for $T \gg C_1$

$$\left\langle \frac{C_1'}{C^2} \right\rangle = -\frac{d}{dY}\left(\frac{1}{T}\right)$$

Hence

$$\left\langle \frac{C_1'}{C^2} \right\rangle = -\frac{d}{dY}\left(\frac{1}{T}\right) = -\frac{d}{dY}\left(\frac{1}{C_0} e^{AY}\right) \tag{3-37}$$

$$\left\langle \frac{C_1'}{C^2} \right\rangle = -\frac{1}{C_0} A \, e^{AY} \tag{3-38}$$

Solving further

$$\left\langle \frac{C_1'^2}{(T+C_1)^2} \right\rangle = \int_{-\infty}^{\infty} C_1'^2 \, (T+C_1)^{-2} \, e^{-c_1^2/2\sigma^2} \, dC_1 \tag{3-40}$$

$$\because \; \frac{d}{dY}\left(C_1' (T-C_1)^{-1}\right) = C_1'' (T+C_1)^{-1} - (T+C_1)^{-2} C_1'^2$$

$$\cong -C_1'^2 \, (T+C_1)^{-2} \tag{3-41}$$

Equation (3-40) reduces to

$$\left\langle \frac{C_1'^2}{(T+C_1)^2} \right\rangle = -\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{d}{dY} \frac{C_1'}{(T+C_1)} \, e^{-c_1^2/2\sigma^2} \, dC_1 \tag{3-42}$$

which from Eq. (3-37) yields

$$\left\langle \frac{C_1'^2}{(T+C_1)^2} \right\rangle = \frac{A}{C_0} e^{AY} \tag{3-43}$$

A substitution of Eq. (3-32), (3-38), (3-43) and (3-26) yields

$$\langle K^2 \rangle = \frac{DA}{C_0} e^{AY} + \frac{1}{C^2} e^{2AY} + (2ED+F)\frac{1}{C_0} e^{AY} \tag{3-44}$$

Numerical values of all constants may now be substituted to obtain the mean square curvature $\langle K^2 \rangle$.

Thus the mean square value of the radius of curvature for random variation of the speed of sound superimposed on its usual exponential form is obtained.

CHAPTER IV

CONCLUSION

A theoretical analysis of ray path in a general type of ocean with or without mass velocity as well as with the random variation of the speed of sound superimposed on its usual variation was carried out in detail.

A numerical calculation of ray paths for both constant and exponentially (spatially) decaying water mass velocities shows that horizontal location errors, for a source at the surface level, due to the motion of the medium may be as large as 27%, 19%, 14%, 6.7%. For the source at 100' depth these may range over 12.69%, 6.9%, 5%, 4% for initial angles of the ray with the horizontal of 20°, 30°, 45°, and 60°, respectively. Similarly, maximum vertical location errors of the order of 25%, 19%, 14%, 6.7% for a source at surface level for transmissions of initial ray at angles of 20°, 30°, 45°, and 60°, respectively with the horizontal. The average radius of curvature for the ray path in a general ocean was also calculated to show the significance of the medium mass velocity in addition to sound velocity variations in ray tracing problems.

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