EXCHANGE RATE DEVALUATION AND LABOR ABSORPTION IN A FOREIGN EXCHANGE CONSTRAINED ECONOMY: THE COLOMBIAN CASE

A Dissertation

Presented to

the Faculty of the Department of Economics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

Rolando F. Pelaez

August 1973

ACKNOWLEDGEMENTS

I would like to express my gratitude to Professors John Rowe and William Thomas for their advice and assistance. I am also indebted to Professors Sailors and Brosky who provided valuable suggestions.

EXCHANGE RATE DEVALUATION AND LABOR ABSORPTION IN A FOREIGN EXCHANGE CONSTRAINED ECONOMY: THE COLOMBIAN CASE

An Abstract of a Dissertation

Presented to

the Faculty of the Department of Economics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

Rolando F. Pelaez

August 1973

ABSTRACT

The paper revolves around the question of labor absorption in the process of economic development. The study deals specifically with Colombia, however it is felt that the principal findings may also prove valid for other less developed countries in similar situations. One section of the study investigates the effect of exchange rate devaluation on labor absorption in a foreign exchange constrained economy within the framework of a two-gap model. The model is specified in terms of a family of production functions that does not limit a priori the elasticity of substitution to a given value. Another section of the study investigates the causes of the employment lag in Colombia's manufacturing sector. Particular attention is given to the rate of growth of the real wage as an explanatory variable.

Chapter I presents an over-view of Colombia's unemployment and trade problems. The chapter also contains a brief review of explanations that have been advanced to account for the employment lag in the manufacturing sector of less developed countries.

Chapters II and III present a review of a variety of two-gap models. The policy implications of fixed coefficients Harrod-Domar type two-gap models are compared and contrasted with the policy implications of Nelson's neoclassical model.

Chapter IV contains a reformulation of Nelson's model with CES production functions and a reassessment of the policy alternatives in light of the more general formulation. The employment generating impact

of devaluation is shown to be uniquely dependent on the elasticity of substitution.

Chapter V investigates the relationship between the rate of growth of real wages and the rate of growth of employment in Colombia's manufacturing sector over the period 1959-1966. A labor demand relation is derived from a modified Fei-Ranis model of labor absorption, and fitted by the least squares method to a pooled cross section sample of 20 two-digit Colombian industries. The regression results lend support to the market imperfections hypothesis of the employment lag.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER ONE

INTRODUCTION

Unemployment in Colombia is a serious economic problem, and politically, a potentially explosive one. Colombia's chronic unemployment problem appears to be exacerbated by a foreign exchange bottleneck which imposes a limit on the rate of investment and on economic growth. The relationship between the availability of foreign exchange and economic advancement has recently been analyzed within the framework of two-gap models of development. This paper investigates the labor absorptive impact of devaluation in a foreign exchange constrained economy within the format of a neoclassical two-gap model. The paper deals specifically with the Colombian economy, however, it is felt that the major findings are applicable to other countries in similar situations.

The present chapter, descriptive in nature, is designed to prepare the ground for the discussion of gap models that follows in chapters two and three. More specifically, the purposes of this chapter are: (1) to present an overview of the unemployment problem in Colombia; (2) to review some of the explanations that have been offered to account for the employment lag in less developed countries; and (3) to trace the impact of the decline in Colombia's capacity to import after 1956 on economic activity.

The Unemployment Problem: An Overview of the Colombian Case

In 1967 the unemployment rate in the larger cities of Colombia was about 10-16 per cent, a figure four to six times greater than fifteen years earlier.[1] There are indications that unemployment has increased during the last two decades. First, the unemployment estimate of the 1964 census was three times greater than that of the census of 1951. Secondly, between March 1963 and March 1967 the unemployment rate in Bogota rose from 8.4 to 16.1 per cent.[2] Thirdly, the available data show that the unemployment rate in Bogota is lower than in other cities, but the increase in urban unemployment also occurred in other Colombian cities.[3]

However, between 1967 and 1970 urban unemployment appears to have decreased, apparently the result of an increase in the rate of growth of output over the period. The International Bank for Reconstruction and Development estimates that in 1970 Colombia's urban unemployment rate was about 10-12 per cent.[4]

In less developed countries the unemployment rate alone is not sufficient to give an accurate picture of the waste of human resources. In addition, two related problems must be considered: disguised

---

[1]Robert L. Slighton, Urban Unemployment in Colombia: Measurement, Characteristics, and Policy Problems, Research Memorandum 5393 (Santa Monica, California: The Rand Corporation, Jan. 1968), p. v.

[2]Ibid., p. 20.

[3]Ibid., p. 17.

[4]International Bank for Reconstruction and Development, Economic Growth of Colombia: Problems and Prospects (Baltimore, Md.: Johns Hopkins University Press, 1972), p. 8.

unemployment and underemployment. The International Labor Office study gives estimates of the extent of both of these problems.[5] The data presented in Table 1 show that in 1967 about 25 per cent of Colombia's urban labor force was either unemployed or underemployed.

The seriousness of Colombia's unemployment problem is made more evident if past trends of employment, output, and of labor force growth are projected into the future. During the last two decades, employment grew at an average rate of less than 5.0 per cent per year. Whatever may happen to the birth rate for the next ten to fifteen years, Colombia's labor force is expected to grow at an average annual rate of 3.5 per cent. Continuation of these trends means that by the year 1985 between thirty to forty per cent of Colombia's labor force will be unemployed.[6]

Unemployment in Colombia is a result of imbalance between the supply and the demand for labor as the economy expands. The rapid rate of growth of the labor force has been the consequence of a very rapid rate of demographic expansion. Between 1951 and 1964 the population grew at an average annual rate of 3.0 per cent.[7]

In urban areas, in addition to the natural increase in population, the picture is complicated by migration from rural areas. Between the censuses of 1951 and 1964, the population of the largest cities more

---

[5]International Labor Office, Towards Full Employment: A Programme for Colombia (Geneva, 1971).

[6]Ibid., p. 45.

[7]Ibid., p. 30.

TABLE 1

COLOMBIA: URBAN UNEMPLOYMENT AND UNDEREMPLOYMENT, 1967
(Percentage of Active Urban Labor Force)

	Total	Males	Females
Open unemployment[1]	14	12	19
Disguised unemployment[2]	7[6]	10	m.a.[5]
Open underemployment[3]	2	2	1
Disguised underemployment[4]	3	2	4
Total	26	26	24

[1]Persons without work and seeking it.

[2]Persons without work and who would probably seek it if unemployment were much lower.

[3]Persons working less than 32 hours per week and seeking to work longer.

[4]Persons working less than 32 hours per week, and who would probably seek longer hours if the opportunity were available.

[5]No estimate possible but probably substantial.

[6]The proportion of the labor force working less than 32 hours a week is larger than this figure which is obtained by expressing the number of hours of underemployment in units of 48 hours (i.e., in its full time equivalent) before the percentage is worked out.

Source:

International Labor Office, Towards Full Employment: A Programme for Colombia (Geneva, 1971), p. 18.

than doubled, growing at about 7 per cent a year.[8] The combined effect of population growth and migration produced a rate of growth of the urban labor force in excess of 4.0 per cent a year between 1951 and 1964.[9]

The Employment Lag in Manufacturing

The growth of employment in the non-agricultural sector has been inadequate to absorb the growing labor force. Particularly disappointing has been the relationship between the rates of growth of employment and output in the manufacturing sector.

Between 1950 and 1960 employment in Colombia's manufacturing sector grew at an average annual rate of only one-third of the rate of growth of manufacturing output (see Table 2). The Colombian situation, however, is not atypical. In sixty-eight per cent of the countries in Table 2 the rate of growth of employment is one-third or less than the rate of growth of output.

The employment lag is more pronounced for the Latin American countries than for the other less developed countries. Taking together all of the less developed countries in Table 2, a 1.0 per cent rate of growth of output is associated with a 0.62 per cent rate of growth of employment. For the Latin American countries the corresponding growth of employment is only 0.43 per cent.

In Latin America there are a number of extreme cases. For example, during the period 1950-60 Venezuela had an average annual rate

---

[8] Ibid., p. 31.

[9] Ibid., p. 31.

TABLE 2

AVERAGE ANNUAL RATE OF OUTPUT AND EMPLOYMENT GROWTH IN MANUFACTURING IN LESS DEVELOPED COUNTRIES

	Time[1] Period	Employment[1] Period	Output %	Employment %	Ratio E/O
All less developed countries	1955-65		7.1	4.4	.62
East and South East Asia	1955-65		8.1	5.0	.62
Latin America	1955-65		5.8	2.5	.43
Argentina	1950-60		4.4	-2.0	-.45
Brazil	1947-60	1949-59	9.8	2.6	.27
Chile	1950-60		5.4	1.7	.31
Colombia	1950-60		7.6	2.5	.33
Costa Rica	1957-63		6.3	1.7	.27
Mexico	1950-61		6.5	0.4	.06
Peru	1950-60		6.6	4.4	.67
Venezuela	1950-60		13.0	2.1	.16
Other					
India	1950-60		6.8	3.3	.49
Egypt	1956-60	1947-57	5.5	3.9	.71
Kenya	1954-64		7.6	-1.1	-.14
Zambia	1955-63		12.5	2.0	.16
Greece	1950-61		8.9	2.0	.22
Puerto Rico	1950-62		9.6	4.2	.44
Trinidad and Tobago	1951-60	1946-60	8.2	1.0	.12
Jamaica	1950-65		8.4	4.1	.49

[1]Unless otherwise noted in column 2, the time period for output and employment is the same.

Source:

Gene M. Tidrick, Wages Output, and the Employment Lag in Jamaica, Research Memorandum No. 40 (Williamstown, Mass.: Center for Development Economics, Williams College, December 1970).

of growth of manufacturing output of 13.0 per cent while employment in manufacturing only expanded at an average rate of 2.1 per cent. In Argentina, manufacturing output expanded at an average rate of 4.4 per cent for a decade, while employment actually decreased at an average rate of 2.0 per cent.

The position taken by most economists a few years back, that unemployment in the less developed countries would be remedied by industrialization, has, as Healey points out, become obsolete. As a consequence of the employment lag, or what is the same, because of productivity increases, industrialization has, in some cases, been associated with growing absolute unemployment.[10]

It is worth digressing to point out that preoccupation with the employment lag is not inconsistent with economic theory. Normally an increase in labor productivity is viewed as an achievement. However, in the presence of chronic and rising unemployment, an increase in the rate of growth of output per worker may produce difficulties. Slighton suggests that in high unemployment situations like Colombia's, output per worker is not a good criterion of economic performance.

> It is usually accepted that the objective of economic activity is the maximization of output per member of society under the assumption of freedom of choice with respect to supply of labor. Maximizing output per worker is consistent with maximizing output per capita only under the assumption that full employment is maintained. It should be abundantly clear by now that full employment is not a characteristic of the Colombian economy. It is thus quite possible that there could have been a distribution of investment in recent years that would have

---

[10]Derek T. Healey, "Development Policy: New Thinking About An Interpretation," Journal of Economic Literature (September 1972), p. 766.

yielded a lower rate of growth of output per worker but a higher rate of growth of total output.[11]

Slighton argues that part of the explanation for the employment lag is that firms have been induced to employ techniques which are more capital intensive than is desirable given Colombia's factor endowments.[12] There are differences of opinion concerning the causes of the employment lag. The following section examines some of the most usual explanations.

Explanations of the Employment Lag

Explanations of the employment lag can be arranged in two groups. One group emphasizes the role of market imperfections, the result of, for example, dysfunctional social legislation. By removing the causes of market imperfections, it is argued, the less developed countries can deal effectively with the employment lag. The other set of explanations argues that the employment lag is the natural result of forces over which the less developed countries have little or no control. The reasons given for the inevitability of the employment lag are varied. A list and brief discussion of some of these follows.

Labor saving bias in modern technology. The technology available to the less developed countries has evolved in the industrialized countries, where capital is the relatively plentiful factor, and hence cheap, and labor relatively scarce, and hence expensive. However, in the less developed countries relative factor supplies are just the

---

[11] Slighton, op. cit., pp. 48-49.

[12] Ibid., pp. 58-59.

opposite. Therefore, the labor saving bias in modern technology makes it ill-suited to the needs of underdeveloped countries. This is a controversial area and some difficult questions concerning the measurement of the bias in technological change have yet to be settled.[13] Nevertheless, it may be said that the prevailing view among economists is that historically technological progress has been dominated by a labor saving bias. The rationalization for that point of view has been presented by Hicks in The Theory of Wages. However, important dissenting opinions have been expressed. Notable among these is that of Marc Blaug.[14]

Learning by doing. The rate of growth of employment normally lags behind the rate of growth of output because of natural increases in labor productivity which are the result of learning by doing.

Increasing returns to scale. It is argued that as industrialization progresses, the size or scale of the average manufacturing establishment increases. Consequently, if the underlying production functions exhibit increasing returns to scale, employment growth lags behind output growth.

Intra-sectoral shifts. The employment lag is viewed as the inevitable result of changes in the output mix within the manufacturing

---

[13]The literature on the bias in technological change is extensive. For a recent effort see: Paul A. David and Th. Van de Klundert, "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960," The American Economic Review, Vol. LV (1965), pp. 357-394.

[14]For a fascinating discussion see: Marc Blaug, "A Survey of the Theory of Process Innovations," Economica, V. 30 (1963), pp. 13-32.

sector. Once import substitution in the "light" or relatively labor intensive industries has been completed, a more difficult phase begins. In this phase import substitution spreads to the more capital intensive industries which are characterized by lower incremental labor-output ratios.

Technological dualism. The manufacturing sector in less developed countries encompasses two sub-sectors which are technologically different from each other. There is a low productivity artisan sub-sector characterized by low wages and labor intensive methods of production, and a high productivity modern sub-sector characterized by high wages and capital intensive technology. The two sub-sectors compete with each other, however, over time the modern sub-sector gradually replaces artisan industry. Because the productivity of labor in modern industry is seven or eight times greater than in the artisan sub-sector, as employment expands in the former there is a greater than proportional decrease of employment in the latter.[15]

High level manpower bottleneck. Baer and Herve argue that some less developed countries suffer from a high level manpower bottleneck which retards the absorption of unskilled labor. The combination of a skilled labor bottleneck and very limited substitutability between skilled and unskilled labor makes unskilled labor a redundant input.[16]

---

[15]United Nations, El Proceso de Industrialización en América Latina (New York, 1965), pp. 46-48.

[16]Werner Baer and Michael Herve, "Employment and Industrialization in Developing Countries," Quarterly Journal of Economics (February 1966), pp. 88-107.

It is worthwhile to point out that the advocates of such views do not accompany their arguments with sufficient supporting evidence.

Whether there is a high level manpower bottleneck is, of course, an empirical question. An investigation of the relationship between unemployment and educational attainment by Turnham and Jaeger does not lend support to the thesis. The authors report that, relative to the working population, the unemployed as a group tend to be better educated.[17] Unemployment rates by level of education and age are available for two Colombian cities and are presented in Table 3. It is interesting to observe that the unemployment rate for the group with one year or less of education is lower than the unemployment rate for the groups having up to eight years of education. That finding is not unique to Colombia. Turnham and Jaeger report that with the exception of Puerto Rico, the unemployment rate among the illiterate urban population is lower than the unemployment rate for the group of primary and secondary school leavers in a large sample of countries of Asia, Africa, and Latin America.[18]

Market Imperfections

As pointed out earlier, an alternative explanation of the employment lag argues that because of market imperfections, factor prices do not correctly reflect the true economic or intrinsic value of the factors to which they relate. More specifically, it is argued that the

---

[17]David Turnham and Ingelies Jaeger, The Employment Problem in Less Developed Countries (Paris: O.E.C.D., 1971), p. 50.

[18]Ibid., p. 50.

TABLE 3

UNEMPLOYMENT RATES BY LEVEL OF EDUCATION AND AGE
(per cent)

	Sample			
	Bogotá March 1966	Bogotá March 1965	Bogotá March 1964	Cali March 1965
	A. Unemployment Rates by Level of Education			
Level of Education (years)				
0-1	8	7	6	7
2-4	12	10	6	13
5	11	12	9	19
6-8	13	10	8	11
9-10	8	8	8	10
11	8	8	5	9
12-15	10	8	4	16
16+	2	3	4	--
	B. Unemployment Rates by Age			
Age				
14-19	20	19	14	27
20-24	17	16	9	17
25-29	9	4	6	11
30-39	6	5	4	8
40+	4	5	4	8

Source:

Robert L. Slighton, Urban Unemployment in Colombia: Measurement, Characteristics, and Policy Problems, RM 5393 (Santa Monica, Calif.: The Rand Corporation, 1968).

price of labor has been driven above, and that of capital below, its true or shadow price, inducing firms to operate at higher capital-labor ratios than is desirable from a social point of view. This thesis is lucidly presented by Tinbergen,[19] and further elaborated by Ranis.[20]

The following factors have propitiated the substitution of capital for labor: (1) increases in labor costs brought about by trade union pressure, and where unions are weak, by government legislation of minimum wage laws and other fringe benefits;[21] (2) domestic inflation combined with fixed exchange rates which give rise to a chronic over-valuation of the currencies of many less developed countries, resulting in the lowering of the price of imported inputs, i.e., capital, relative to the price of labor;[22] and (3) ceilings on interest rates and preferential treatment for capital goods imports, which lower the price of capital to artificial levels providing incentives to substitute capital for labor.[23]

Although several hypotheses have been offered to account for the employment lag, the fact remains that a great deal of work needs to be done in order to quantify the forces at work. The question of the causes of the employment lag is one of considerable importance to Colombia, and

---

[19]Jan Tinbergen, The Design of Development (Baltimore, Md.: Johns Hopkins Press, 1958), pp. 39-41.

[20]Gustav Ranis, "Production Functions, Market Imperfections and Economic Development," Economic Journal, Vol. 72 (June 1972), pp. 344-354.

[21]Ibid., p. 346.

[22]Slighton, op. cit., p. 59.

[23]Ibid., p. 58.

to other less developed countries, because of the widely different policy implications. For example, if increases in real wages have been mainly responsible for the employment lag, the problem has been self-inflicted and it is within the capabilities of less developed countries to design appropriate policies. Conversely, if the impact of wage behavior has been insignificant, the hypotheses which stress the inevitability of the employment lag would gain validity.

This concludes the descriptive overview of the unemployment problem and of the employment lag. In the final chapter of this dissertation an attempt is made to throw light on the causes of the employment lag in Colombia's manufacturing sector. Let us now take a look at the trade constraint.

The Trade Constraint

This section relates the poor growth performance of Colombia since the mid-1950's to the stagnation of the capacity to import and to the structure of the manufacturing sector. An attempt is made to underscore the relationship between the trade constraint and investment activity.

To a very large extent Colombia's manufacturing sector has evolved in response to a policy of import substitution. The initial impetus to industrialization was provided by the Depression of the 1930's, when, due to a sharp decline in the capacity to import, Colombia was forced to begin supplying her needs for non-durable consumer goods.[24] Import substitution received another push during World War II, and again

---

[24]United Nations, p. 19.

during the first half of the 1950's, when, aided by record high earnings from coffee exports, the Colombian economy registered a high rate of investment and rapid growth. Between 1950 and 1956 gross fixed investment averaged approximately 25 per cent of gross domestic product, value added in manufacturing rose at a rate of 7.0 per cent per year, and gross domestic product increased at a rate of 5.1 per cent per year.[25]

Sheahan points out that the high rate of investment during the 1950-56 period was made possible by ready access to imports of capital goods and intermediate materials, and by the high level of protection granted to domestic firms.[26] As shown in Table 4, between 1950 and 1956 imports of capital goods more than doubled. During this period the main factor behind Colombia's rising capacity to import was a very high world market price for the country's main export--coffee.

In order to gain a better picture of the importance of the coffee sector in Colombia's economy, it suffices to mention that in the early 1950's proceeds from coffee exports accounted for about eighty per cent of total exports. The rise in proceeds from coffee exports between 1949 and 1954 was a phenomenal 126 per cent. At the same time, Colombia's capacity to import more than doubled. (See Table 5.) However, beginning in 1954 coffee prices began to weaken, leading to a fall in exports which intensified after 1956. During the period 1958-63, Colombia's exports averaged two-thirds of the level reached during

---

[25]John Sheahan, "Imports, Investment, and Growth--Colombia," Development Policy--Theory and Practice, ed. G. F. Papanek (Cambridge, Mass.: Harvard University Press, 1968), p. 94.

[26]Ibid.

TABLE 4

COMPOSITION OF COLOMBIAN IMPORTS, 1950-68 (IN MILLIONS OF U.S. DOLLARS)

Year	Consumer Goods	Fuels	Construction Materials	Raw and Intermediate Materials	Capital Goods	Unclassified	Total
1950	76.1	10.2	25.0	130.5	121.0	1.8	364.7
51	74.1	16.6	26.9	160.1	136.5	2.2	416.4
52	74.3	19.3	27.8	141.3	150.4	2.2	415.4
53	104.9	25.6	43.9	158.8	210.3	3.2	546.7
54	144.1	29.8	52.5	193.9	246.9	4.5	671.8
55	114.8	24.7	51.9	210.2	263.4	4.3	669.3
56	80.6	23.2	56.6	229.6	263.3	3.9	657.2
57	53.3	19.1	40.0	213.5	152.0	4.7	482.6
58	32.8	9.8	32.9	191.4	128.8	4.2	399.9
59	30.3	8.6	20.4	204.2	149.3	2.8	415.6
60	34.6	10.3	23.6	229.7	216.4	4.0	518.6
61	48.1	12.5	24.7	238.2	228.3	5.3	557.1
62	36.0	13.5	25.7	243.9	208.1	12.8	540.3
63	31.4	9.3	24.2	234.0	195.7	11.4	506.0
64	37.1	6.4	27.3	268.0	236.5	11.0	586.3
65	17.3	3.2	3.6	195.9	232.7	0.7	453.4
66	43.8	2.0	2.7	349.3	275.8	0.7	674.3
67	17.4	3.5	8.4	211.7	254.9	1.0	496.9
68	33.4	1.2	8.4	266.7	332.6	1.0	643.3

Sources: (1950-1964): John Sheahan, "Imports, Investment and Growth--Colombia," Development Policy-Theory and Practice, ed. Gustav F. Papanek (Cambridge, Mass.: Harvard University Press, 1968), p. 96.

(1965-1968): United Nations, Statistical Bulletin for Latin America, Vol. VIII, No. 2, October 1971.

TABLE 5

STRUCTURE OF COLOMBIAN EXPORTS (MILLIONS OF U.S. DOLLARS)

Year	Coffee	Petroleum	Other Exports	Total
1948	225	45	37	307
49	242	58	35	335
50	306	65	23	394
51	359	73	52	484
52	380	71	32	483
53	492	76	37	605
54	550	76	43	669
55	487	61	49	597
56	413	71	68	552
57	390	72	49	511
58	355	65	41	461
59	363	73	37	473
60	333	80	52	465
61	308	68	59	435
62	332	61	70	463
63	303	77	67	447
64	394	75	79	548
65	344	88	107	539
66	328	71	109	508
67	321	61	128	510
68	351	36	171	558
69	344	57	206	607
70	467	59	206	732

Source:

International Monetary Fund, International Financial Statistics, 1972 Supplement. Washington, D.C.

1953-56. And, as shown in Table 5, in 1969 the total exports of Colombia still remained below the level reached fifteen years earlier.

As a result of the contraction in export earnings after 1956, imports had to be cut back. Since most capital goods and a substantial part of intermediate materials had to be imported, the result was a slow down of economic activity and a decrease in the investment ratio. Table 6 shows that between 1954-56 and 1957-59 fixed investment fell from 25 to 16 per cent of gross domestic product. It can be seen in Table 4 that during the same period there was a 75 per cent decrease in imports of capital goods, and only a 12 per cent decrease in imports of intermediate materials. The relatively small decrease in imports of intermediate materials was the result of the dependency of the economy on those imports for operation of existing capacity.

The composition of Colombian imports at the end of the 1960's serves to illustrate that country's predicament. In 1967-68 imports of consumer goods accounted for only 4.0 per cent of total imports. The above indicates the extent to which industrialization had been pursued through import substitution of consumer goods. On the other hand, imports of raw and intermediate materials, and of capital goods accounted for 42 per cent and 51 per cent respectively of total imports.

One of the results of the import substitution strategy has been the emergence of firms which depend on imports of intermediate materials for operation, and of capital goods for expansion. The sine qua non for the viability of these firms is a very high level of protection. It would be erroneous to assume that Colombia has succeeded in solving the problem of external dependence through import substitution, as might be

TABLE 6

COLOMBIAN INVESTMENT-GDP RATIOS AND GDP GROWTH RATES, 1950-65
(IN MILLIONS OF PESOS AT 1958 PRICES)

	(1) Average Gross Fixed Investment	(2) Fixed Investment as Percentage of GPD	(3) Rate of Growth of GDP	
1950-53	3,126	19.9	(1950-53)	5.2
1954-56	4,763	25.1	(1953-56)	5.0
1957-59	3,480	16.6	(1956-59)	3.9
1959-62	4,484	18.4	(1959-62)	4.7
1963-65	4,495	16.3	(1962-65)	4.3

Source:

John Sheahan, "Imports, Investment and Growth--Colombia," Development Policy-Theory and Practice, ed. Gustav F. Papanek (Cambridge, Mass.: Harvard University Press, 1968), p. 96.

implied by the low ratio of imports of consumer goods to total imports. On the contrary, because of the high imported input intensity of domestic industry, the capacity to import is a main determinant of the rate of growth of output and of employment in that sector.

Savings and the Trade Gap

Most students of the Colombian experience are in agreement that the slower rate of growth following 1956 was the result of the decrease in the investment rate. The conventional interpretation of a falling investment rate views it as the result of excessive consumption and of a decrease in the savings rate. In that event, the prescription commonly given is to curtail consumption in order to stimulate savings, investment, and the growth rate. However, if a poor savings and investment performance is not the result of excessive consumption, being instead a consequence of a foreign exchange constraint, the conventional prescription does not work.

It is interesting to observe that after 1956, when the investment rate fell, the demand for licenses to import capital goods exceeded the actual rate of licensing, indicating that with a greater supply of foreign exchange, investment would have been higher.[27]

Sheahan is not alone in suggesting that stagnation of export earnings is the main cause of Colombia's poor growth performance. A study of that country's economy by Jaroslav Vanek concludes that foreign

---

[27]Sheahan, op. cit., p. 98.

exchange, and not savings, is the effective constraint.[28] More recent studies of Colombia by Nelson,[29] and by Chenery and Eckstein[30] corroborate Vanek's conclusion of foreign exchange gap dominance. Although there is agreement among the above authors in the diagnosis of trade gap dominance, the policy recommendations emanating from these studies are quite dissimilar. The following two chapters take a closer look at a variety of two-gap models and at the policy recommendations.

---

[28]Jaroslav Vanek, Estimating Foreign Resource Needs for Economic Development (New York: McGraw-Hill Book Company, 1967), p. 141.

[29]Richard Nelson, A Study of Industrialization in Colombia: Part I. Analysis, RM 5412 (Santa Monica, Calif.: The Rand Corporation, 1967).

[30]Hollis B. Chenery and Peter Eckstein, "Development Alternatives for Latin America," Journal of Political Economy, 78 (July-August 1970), 966-1006.

CHAPTER TWO

TWO-GAP MODELS OF DEVELOPMENT

The relative importance of domestic savings and of foreign exchange as limits to growth has been the subject of much debate during the last two decades. It would be fair to say that, at least initially, much of the debate was lacking in rigor and followed mostly along qualitative lines. More recently, a number of two-gap models have appeared which attempt to provide a more rigorous framework. Two-gap models attempt to quantify the contribution to the development process of domestic savings, investment, and export performance. The purpose of this chapter is to review the two-gap literature. Throughout the chapter emphasis is placed on those studies which deal with Colombia.

The Basic Two-Gap Approach

The basic idea shared by all gap models is that accelerated development requires supplies of a number of resources, most of which may be available in sufficient quantities. However, the supply of one or a few resources (skills, savings, or foreign exchange) may be so limited that a bottleneck appears which frustrates growth and impedes full utilization of the other resources. Foreign assistance by supplying the critical resource can make possible a higher rate of growth and full employment of domestic resources.

Most gap models have a two-fold purpose. First, to ascertain

which constraint is binding in a specific situation, and secondly, to determine the inflow of foreign resources necessary for a target rate of growth of GNP to be achieved. The methodology employed by the various authors is essentially the same and is outlined below.

Historical estimates of the marginal savings rate, the marginal capital-output ratio, the rate of growth of exports, and the marginal import rate are obtained and assumed to remain stable over time. Given the capital-output ratio, for a target rate of growth of GNP to be reached a specific savings rate is necessary. Similarly, given the historical relationship between imports and output, it is possible to determine the rate of growth of imports necessary for the target rate of growth of GNP to be reached. The necessary savings rate and the necessary rate of growth of imports are then compared with the historical estimates, and measures of the trade and savings gaps are obtained. The savings gap is the difference between the investment level required to achieve a given level of output and the savings which the economy is capable of generating.[1] The trade gap is the difference between import requirements at a given level of income and the value of exports generated by the economy. Given the structure of internal demand and the target rate of growth of GNP, foreign assistance requirements will be determined by the larger of the two gaps.[2]

A variety of gap models have evolved around the basic framework

---

[1]Hollis B. Chenery and Peter Eckstein, "Development Alternatives for Latin America," Journal of Political Economy, 78 (July-August 1970), p. 968.

[2]Ibid.

outlined above. In terms of the number of countries studied the paper by Chenery and Strout[3] is the most comprehensive and will be reviewed first.

The authors investigate the interaction between external assistance and internal performance that will make possible the transition to self-sustaining growth. For analytical purposes the development process is viewed in terms of stages or phases. In Phase I the savings constraint is dominant but external assistance allows investment to grow at a constant rate until it reaches a level adequate to sustain the target rate of growth. It is postulated that at the higher rate of growth made possible by foreign assistance the savings constraint gradually decreases in intensity so that the inflow of foreign resources may be reduced over time. In such an event the country enters Phase II. In this phase foreign assistance decreases but investment continues growing at the same constant rate because of a rise in the marginal savings rate. The decrease of the foreign capital inflow in Phase II also means that exports must rise faster than imports in order to reduce the pressure on the balance of payments. The empirical evidence presented by the authors suggest that many countries have been unable to bring about this required adjustment, and that the trade gap has become dominant.[4] When the trade gap is binding and is larger than the savings gap, actual savings fall short of potential savings, and Phase III begins. In

---

[3]Hollis B. Chenery and Alan M. Strout, "Foreign Assistance and Economic Development," American Economic Review, 56 (September 1966), p. 687

[4]Ibid., p. 689.

Phase III the country may suffer from import shortages, excess capacity, and a falling savings rate. Foreign aid requirements in this case, are determined by the trade gap, and exceed the level required by the savings gap.[5] It is clear from the above that the ability to increase exports plays a crucial role in the transition to self-sustaining growth. Finally, the phases described by Chenery and Strout need not follow one another, a country may be in Phase III without having passed through the other two.

The basic model described above was applied to Pakistan, a country which Chenery and Strout describe as being in Phase I. The authors estimate the amount of foreign assistance and the time that it would take for Pakistan to achieve a self-sustaining growth rate of 6 per cent under alternative assumptions about domestic performance.

Perhaps of greater interest is the author's attempt to identify dominance of gap for each country in a sample of 31 less developed countries. The countries are divided into four groups by comparing saving and trade performance over the period 1957-62 with what would have been needed to attain a 5 per cent rate of growth of GNP. Countries for which saving and trade performance were adequate to produce the growth target are classified as Group A countries. Group B includes those countries for which the trade gap impedes attainment of the growth target, and Group C is comprised of countries with savings gap dominance. Finally, Group D includes countries with unsatisfactory savings and trade performance.

---

[5] Ibid., p. 691.

About twenty per cent of the countries fall in Group B (trade gap dominance), and thirty-two per cent fall in Group D. Altogether sixty-one per cent of the countries fail to satisfy one or both criteria. The authors point out that one of the most revealing features of the grouping of countries is the predominant role played by exports. For example, eighty-three per cent of the Group A countries have export growth rates of six per cent or more and could reach the growth target even if the ratio of imports to GNP remained constant. The unsatisfactory performance of the Group D countries is attributed to the stagnation of export earnings which has led to falling saving rates and to increased requirement for external capital.[6]

The grouping of countries also shows certain differences between the Latin American countries and the other countries. The median marginal savings rate for the Latin American group is ten per cent or about half that of the other group. For the period 1957-62 the ratio of imports to GNP was declining in ten out of fourteen countries while it was increasing in a majority of the others.[7] Moreover, a large number of Latin American countries fall in Group D which corresponds to Phase III in the model.

Trade Gap Dominance and Savings

The question which arises at this point is whether the poor saving performance of the Latin American countries is the result of the stagnation of export earnings as the model implies. The idea, simply

---

[6]Ibid., p. 710.

[7]Chenery and Eckstein, op. cit., p. 967.

stated by Vanek,[8] is that domestic investment opportunities vary closely with the availability of foreign exchange and therefore domestic saving varies in response to those opportunities. But Chenery and Eckstein[9] depart from that interpretation, suggesting that foreign exchange obtained by exports will have a positive effect on saving whereas foreign capital inflows (aid and foreign investment) will have an adverse effect. They argue that inflows of external capital will serve as a substitute for external capital in financing investment.

Chenery and Eckstein test these relationships by fitting a regression equation in which gross domestic saving (S) is assumed to be a function of GNP (V), the inflow of foreign capital (F), and the share of exports in GNP (E/V). The results are as follows: "In twelve out of sixteen cases, the impact of additional foreign capital on saving was found to be negative, and in every case but two the impact of the export share was positive."

The regression results together with the fact that the foreign capital inflow increased for almost all countries and the export share decreased, the authors argue, helps to clarify the saving performance of Latin America over the fifteen-year period. And, since under the assumption of savings gap dominance it would be difficult to explain the negative impact of a foreign capital inflow on domestic savings, the hypothesis of trade gap dominance receives support.[10]

---

[8]Jaroslav Vanek, Estimating Foreign Resource Needs for Economic Development (New York: McGraw-Hill, 1967), p. 141.

[9]Chenery and Eckstein, op. cit., p. 975.

[10]Ibid., p. 976.

Chenery and Eckstein also fit an import regression equation in which imports (M) are assumed to be a function of consumption (C), investment (I), reserves of gold and foreign exchange (R) and export earnings (E).

Together, the trade and savings equations, conclude the authors, buttress the hypothesis of trade gap dominance. In the specific case of Colombia the equations are as follows:

$$\Delta S = 0.21\Delta V + 0.50\Delta E$$

$$\Delta M = 0.55\Delta I + 0.11\Delta C + 0.42\Delta E + 0.33\Delta R$$

The equations show that a one dollar decrease in exports would reduce potential saving and investment by fifty cents and would directly reduce imports by forty-two cents with a further decrease attributable to the decline in investment.[11]

An earlier study of the Colombian economy by Vanek also describes a dominant foreign exchange constraint.[12] Both Vanek and Chenery assume very limited substitutability between domestic and imported inputs in the short run.

> Given the overall rate of growth and the development structure postulated for the projection period (say 10 years), total investment and imports of capital and intermediate goods and services are inelastic or noncompressible.[13]

---

[11] Ibid., p. 980.

[12] Vanek, op. cit.

[13] Ibid., p. 107.

Policy Implications of the Basic Two-Gap Model

Vanek reaches pessimistic conclusions regarding Colombia's ability to improve her growth performance by policies designed to increase domestic savings, for example through taxation.

> Except in the most unlikely situation where some investment projects can be found not requiring foreign capital goods and/or intermediate products, that policy (increasing savings) cannot lead to any stimulation of economic growth.[14]

The point is that under conditions of trade gap dominance the main limit to the investment rate is not domestic savings but the capacity to import raw and intermediate materials, capital goods, and technical services. Under these conditions, if the government attempts to cut back on consumption in order to stimulate domestic savings and investment, the likely result will be deflation, unemployment, and excess capacity.[15] Unemployment results because, although all the imports released by cutting back consumption would be used in investment activities, the greater import intensity (lesser domestic input intensity) of investment means that not all the domestic resources released from consumption activities would be reemployed in investment. Therefore, in the absence of foreign aid in the standard two-gap model, growth and employment objectives, at least in the short run, are in conflict. Vanek[16] states clearly that the only policy that will succeed in raising the rate of growth in the short or intermediate run is the acquisition of additional foreign resources. But due to the nature of Colombia's

---

[14] Ibid., p. 112.

[15] Ibid., p. 112.

[16] Ibid., p. 112.

exports (coffee and other primary products) the possibility of drastically increasing export earnings in the short run is ruled out; therefore foreign aid alone, argues Vanek emerges as capable of doing the job.

According to Nelson[17] the pessimistic conclusions reached by the standard two-gap model regarding domestic development efforts are the result of taking relative factor prices as fixed, or what amounts to the same thing, fixed coefficients in production. A devaluation of the effective exchange rate, argues Nelson, would induce business firms to substitute domestic inputs for imported inputs and enable countries like Colombia to increase investment and employment simultaneously.

It may be said that the difference between Vanek and Chenery on the one hand, and Nelson on the other, is that the former authors view the development process along Harrod-Domar fixed production coefficient lines whereas the latter views it in a neo-classical input substitution spirit.

Nelson's Diagrammatic Presentation of the Basic Harrod-Domar Type Two-Gap Model

The model assumes that the economy has available to it four basic activities: domestic production of investment goods, imports of investment goods, domestic production of consumer goods, and imports of consumer goods. A unit level of an activity is defined as a dollar's worth, and it is assumed that all activities require imports. The

---

[17]Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, RM-5680 (Santa Monica, Calif.: The Rand Corporation, 1968).

economy's activity matrix appears as follows:

		I_p	I_m	C_p	C_m	
Output	I	1	1	0	0	
	C	0	0	1	1	
	M	a_1	1	a_2	1	Activity Matrix
Input	V	b_1	0	b_2	0	

The column headings are the four activities, and the subscripts p and m stand for production and imports respectively. Nelson also assumes that the production of investment goods is more import and less domestic input intensive than the production of consumer goods. In terms of the input coefficients: $a_1 > a_2 > 0$, and $b_2 > b_1 > 0$

If a given capacity to import and a given domestic production capacity are assumed the constraints on the activities can be written as follows:

$$M \geq a_1 I_p + a_2 C_p + I_m + C_m$$

$$V \geq b_1 I_p + b_2 C_p$$

The import and domestic capacity constraints circumscribe the economy's consumption, investment choice set to the frontier a - b - d in Figure 1. Along the a-b facet import capacity is fully utilized but there is unutilized domestic capacity. Along the b-d facet both domestic capacity and import capacity are fully utilized; imports are made up of intermediate materials to be used in the production of investment

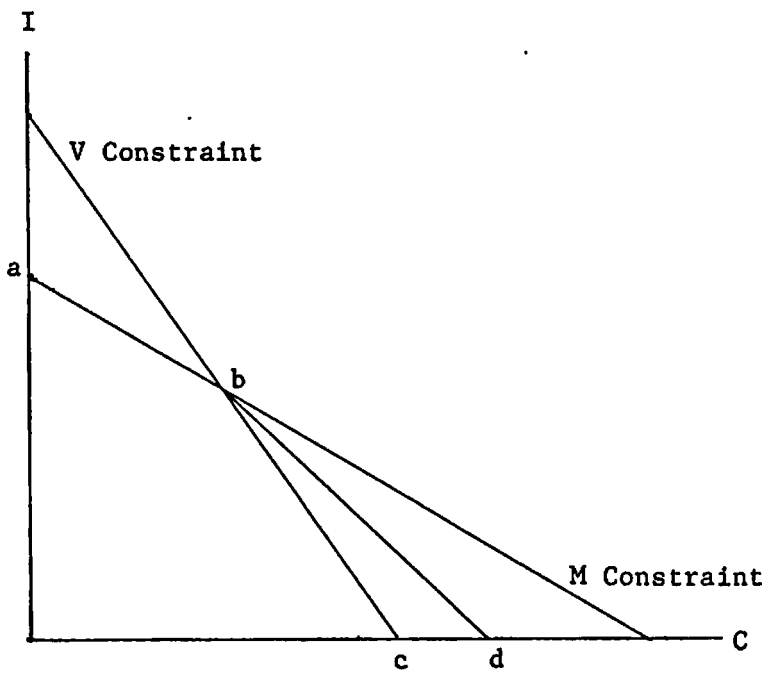

Figure 1.

Source: Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, R.M. 5680, The Rand Corporation, Santa Monica, California, 1968, p. 6.

and consumer goods, and in addition, some consumer and investment goods are directly imported.[18]

Harrod-Domar type gap models assume the capital-value added ratio is constant, and that labor is in excess supply, so that an increase in value added is proportional to the investment level. Then, as Nelson points out, with some changes in units the earlier figure shows the growth-consumption, opportunities available to the economy. In Figure 1.B the investment axis has been expressed in terms of changes in value added, and a savings constraint has been introduced. The savings constraint can be interpreted as a minimum level of consumption.[19]

Figure 1.B shows that the growth-consumption choice set open to the economy is limited to the frontier e - b - d. If now the horizontal axis refers to C/V and the vertical axis to $\Delta V/V$, the growth rate of the economy is determined by the savings constraint and the import constraint.[20] Figure 2 captures the essence of the models by Chenery and Vanek. Those authors, as we have already seen, investigate the effect on the growth rate of shifting one or the other of these constraints.

Given the savings constraint and the import constraint M_1 the highest attainable growth rate is g_1. If the target growth rate is g_2 an increase in import capacity from M_1 to M_2 allows the economy to reach

---

[18]Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, Vol. 78, No. 3 (May-June 1970), p. 548.

[19]Ibid., p. 549.

[20]Ibid., p. 549.

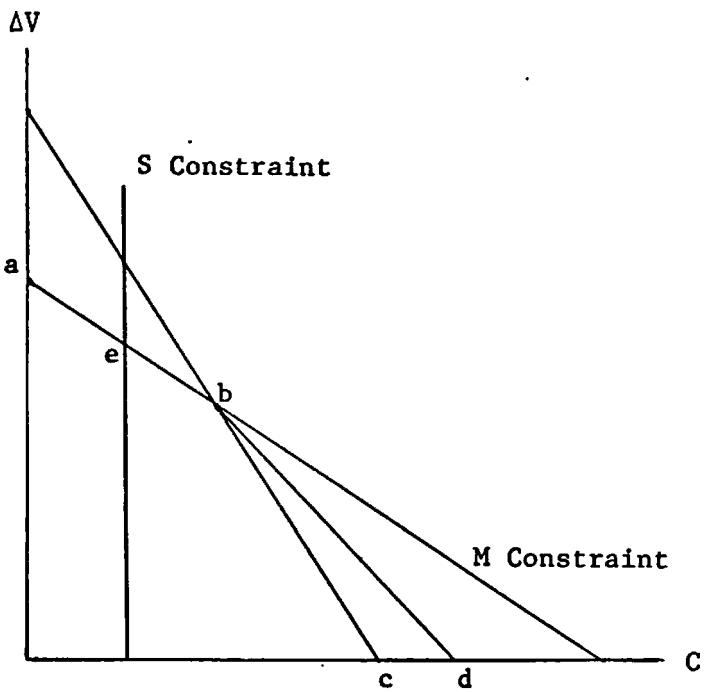

Figure 1B.

Source: Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, RM-5680, The Rand Corporation, Santa Monica, California, 1968, p. 6.

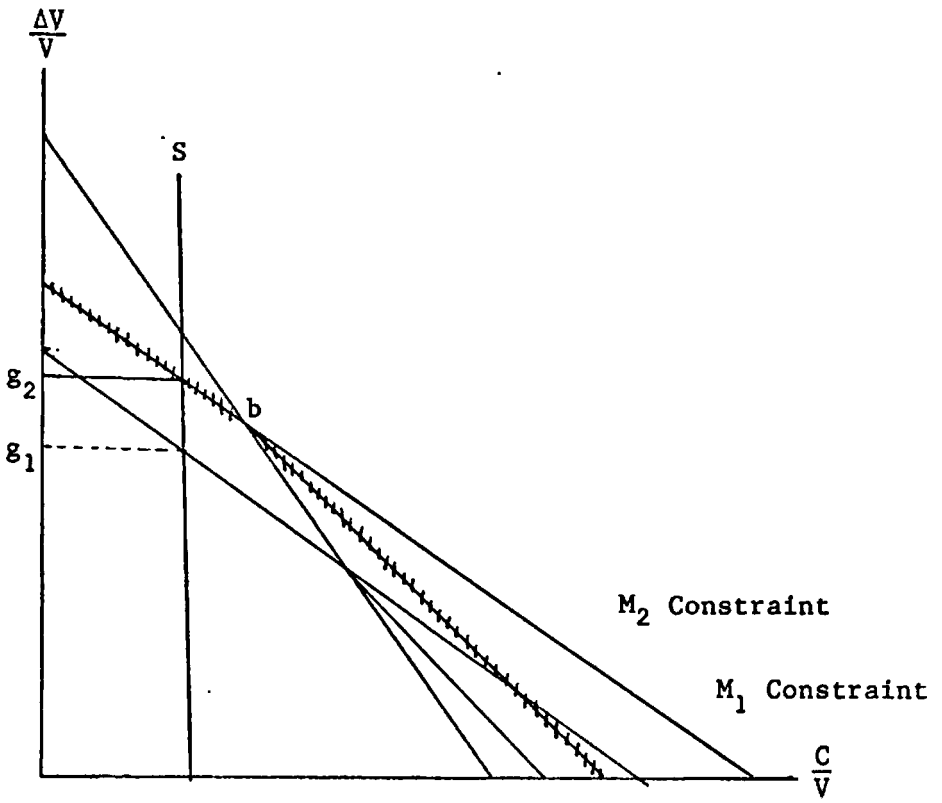

Figure 2.

Source: Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, Vol. 78, No. 3 (May-June 1970), p. 550.

the growth target. In the absence of foreign assistance, that is, given the import constraint M_1, a shift to the left of the savings constraint also allows the economy to reach the growth target g_1. But it takes a lot of reduced consumption to bring about a relatively small increase in the growth rate. However, if the savings constraint is to the right of b, an increase in savings (shifting the savings constraint to the left) produces a relatively large increase in the growth rate. On the other hand, if the savings constraint is to the left of b, increasing the growth rate by shifting the savings constraint to the left will have the undesirable effect of increasing unemployment.[21]

The above formulation differs slightly from that of Vanek in that an increase in savings just like an increase in import capacity always produces an increase in the growth rate. However, the growth increasing effect of shifting the savings constraint to the left is greater when the savings constraint is to the right of b. By the same token the growth increasing effect of an increase in import capacity is greater when the savings constraint is to the left of b.[22] In terms of this formulation, what Vanek is saying is that the savings constraint in Colombia's case is to the left of b so that more foreign aid is needed. The foreign exchange gap which he tries to estimate is the increase in import capacity given the savings constraint which would increase the economy's growth rate from g_1 to g_2.

As we have seen the standard two-gap formulation makes the

---

[21] Ibid., p. 549

[22] Ibid., p. 550.

outcome of the development effort of a foreign exchange constrained economy highly dependent on external assistance. Yet, as Nelson points out, if more aid is not forthcoming, this approach is of little value to policy makers when choosing among alternative courses of action. More precisely, the standard two-gap model conveys a sense of helplessness with regard to the effectiveness of domestic development efforts. Nelson shows that this is to be expected given the very restrictive assumption of fixed coefficients in production. However, the moment substitution possibilities are considered, the question of inducements through factor price changes comes to the fore. Nelson[23] concludes that taking into account substitution possibilities in production, there is a great deal that less developed countries can do to solve their unemployment and growth problems even in the absence of external assistance. The following chapter examines Nelson's neoclassical two-gap model.

---

[23]Nelson, op. cit. (1968), p. 44.

CHAPTER THREE

NELSON'S NEOCLASSICAL TWO-GAP MODEL

This chapter reviews Nelson's model and compares it to the standard two-gap model discussed in the previous chapter.

The most complete statement of the model appears in a research memorandum of the Rand Corporation.[1] In that publication the model is developed in terms of Cobb-Douglas production functions, the policy options are discussed in detail, and some illustrative calculations of the impact of devaluation on growth and employment are presented. The model also appears in a more compact form in the Journal of Political Economy.[2] There the model is formulated in terms of a production function restricted to linear homogeneity, but the presentation is only qualitative and diagrammatic. The presentation below covers only the essential points of the model and serves as a benchmark against which to compare the model reformulated in the next chapter with CES production functions.

The formulation of the model incorporates the following characteristics of Colombia's economy: (1) imports are mostly intermediate

---

[1]Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, RM-5680 (Santa Monica, Calif.: The Rand Corporation, 1968).

[2]Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, Vol 78, No. 3 (May-June 1970).

goods to be used in the production of consumer and investment goods, and direct purchases of capital goods; and (2) the import intensity of investment is greater than that of consumption. Nelson also makes the following simplifying assumptions: (1) the capital-labor ratio is the same in consumption and investment activities; and (2) full competitive equilibrium prevails in both factor and product markets in the short and the long run.[3]

The Model

The production of investment and consumer goods utilizes imports and a domestic value added input. The domestic value added input is produced by domestic capital and labor as follows:

$$V = V(L, K) \tag{1}$$

Consumer and investment goods are produced according to the following functions:

$$C = C(M_c, V_c) \tag{2}$$

$$I = I(M_i, V_i) \tag{3}$$

All the production functions are assumed to be homogeneous of degree one. It is also assumed that investors demand a certain rate of return on capital and that the capital stock is adjusted so as to achieve that rate of return.

Given the assumptions of the model, Nelson expresses the price

---

[3]Ibid., p. 552.

of consumer and investment goods in terms of the basic input prices: the money wage (W), the rate of return on capital (i), and the nominal exchange rate (E). This in turn allows him to express the input coefficients and hence the constraints on the economy's activities in terms of the basic factor prices. Nelson then investigates how the constraints on the economy's activities shift as relative factor prices change.

The activity matrix on page 31 yields the following input coefficients:

$$a_1 = \frac{M_I}{I}\ ,\ a_2 = \frac{M_C}{C}\ ,\ b_1 = \frac{V_I}{I}\ ,\ \text{and}\ b_2 = \frac{V_C}{C}$$

Assuming the same capital-labor ratio in the production of consumer and investment goods:

$$\frac{K}{V} = c_1\ ,\ \text{and}\ \frac{L}{V} = c_2$$

Ignoring the direct import activities the constraints can be written:

$$M \geq a_1 I + a_2 C$$

$$K \geq c_1 b_1 I + c_1 b_2 C$$

$$L \geq c_2 b_1 I + c_2 b_2 C$$

Notice that the V constraint can be interpreted as the more binding of of the K and L constraints.[4]

It is also assumed that the labor constraint never becomes

---

[4] Ibid., p. 554.

binding before the capital constraint so that:

$$L \geq \frac{c_2}{c_1} K^*$$

where K^* = capital actually employed.

The input coefficients: a_1, a_2, c_1b_1, c_1b_2, c_2b_1, and c_2b_2 are all a function of $\frac{W}{E}$ and i. Therefore for any $\frac{W}{E}$ and i the constraints are linear and have the relative slopes shown in Figure 3.

The K and L constraints have the same slope because the capital-labor ratio is by assumption the same in investment and in consumption. The slope of the M constraint is explained by the higher import intensity of investment, that is, $a_1 > a_2$.[5]

Figure 3 has been drawn assuming given factor prices, it always shows unemployment of at least one factor. At a high investment-consumption ratio, the import constraint becomes binding with the result that there is unemployed labor and underutilized capital. At a low investment-consumption ratio, the capital constraint becomes dominant and there is unemployed labor, while import capacity is slack. There is a ratio of investment to consumption (point b) which produces full utilization of import capacity and of the capital stock. However, in Figure 3 there is always unemployment of labor.

Figure 3 replicates Vanek's model. It can be seen that any attempt made to increase the investment-consumption ratio above that at point b has the undesirable result of increasing unemployment. But

---

[5]Ibid., p. 555.

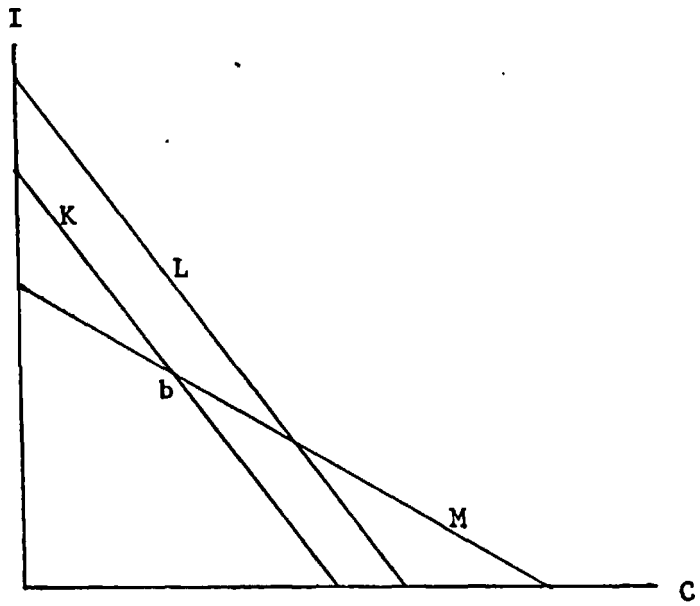

Figure 3.

Source: Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, Vol. 78, No. 3 (May-June 1970), p. 556.

notice that Figure 3 is drawn for a given set of factor prices; if factor prices change, the constraints must shift and the inevitability of unemployment disappears.

Let us now explore with Nelson[6] how the constraints will shift as a result of changes in factor prices. A decrease in $\frac{W}{E}$ or an increase in (i) decreases the amount of capital utilized to produce a given output and shifts out the capital constraint. A decrease in $\frac{W}{E}$ or an increase in (i) increases the quantity of labor employed to produce a given output and shifts in the labor constraint. Finally, a decrease in $\frac{W}{E}$ or (i) lowers the volume of imports needed to produce a given output and shifts out the foreign exchange constraint. All of this adds up to the fact that for any given ratio of investment to consumption there is an $\left(i, \frac{W}{E}\right)$ combination which makes all three constraints intersect at the same point. At that point there is full employment of all three factors.

It should be pointed out that full employment of all factors requires not only that factor prices be "right," but also a combination of fiscal and monetary policy so that aggregate demand also be "right."[7]

Policy Options

The significance of Nelson's contribution lies in showing that once the input coefficients have been made flexible by changes in factor prices, the redundant input phenomenon vanishes, and the economy is no longer condemned to unemployment while it passively awaits more foreign aid. Instead, a whole range of policy options now appear possible.

---

[6]Ibid., p. 556.

[7]Ibid., p. 558.

Taking point b in Figure 3 as the point of departure, Nelson mentions the following policies. First, it would be possible for the economy to expand employment so as to obtain full employment of labor, capital, and import capacity and to produce a balanced expansion of investment and consumption. Secondly, it would be possible to use the increase in employment to expand investment, holding consumption constant. Finally, between these two there are many possibilities, i.e., the increase in employment may be used to produce non-proportional expansions of investment and consumption.[8]

Contrary to the pessimism which permeates Vanek's fixed coefficients model, Nelson's neoclassical model emphasizes the effectiveness of domestic policy in enabling Colombia to increase her growth rate and to solve her unemployment problem. To illustrate let us follow Nelson along the first maneuver outlined above. Assume that the economy is initially at point b in Figure 3 where there is full utilization of import capacity and capital and unemployment of labor.

> A lower $\frac{W}{E}$ will shift the M constraint outward (by reducing the import intensity of both consumption and investment), shift in the labor constraint (by inducing a rise in the labor intensity of both activities), and pivot outwards the K constraint (by reducing capital intensity).[9]

As shown in Figure 3A, output increases as the result of a balanced expansion of investment and consumption along the I-C ray from point b to point c. The instrument used to implement the maneuver discussed is a devaluation of the import exchange rate. The question now arises:

---

[8]Ibid., p. 559.

[9]Nelson, op. cit. (1968), pp. 36-37.

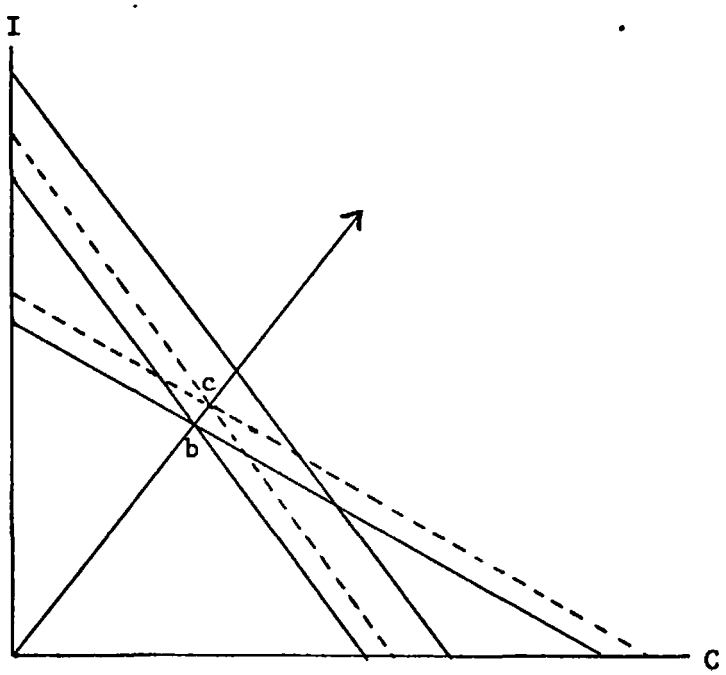

Figure 3A.

Source: Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, RM-5680, The Rand Corporation, Santa Monica, California, 1968, p. 38.

what is the employment generating impact of a given devaluation? With the Cobb-Douglas specification, Nelson arrives at the conclusion that the above maneuver permits employment to expand in exactly the same percentage as the devaluation. Of course, the increase in employment will not be immediate, according to Nelson something like a five-year adjustment period might be involved.[10]

Turning again to the unemployment problem in Colombia, Nelson raises the question: by how much should the import exchange rate be raised in order to produce full employment? Considering that the urban unemployment rate is in excess of 10 per cent, and that an increase in urban employment is likely to induce additional migration to the cities, Nelson estimates that an increase in employment of 15 per cent is a reasonable objective. Nelson suggests that a devaluation of 15 per cent "might do the trick."[11]

Since specification of the model along Cobb-Douglas lines restricts the elasticity of substitution to a value of unity, the question arises: to what extent is the relationship between employment and the exchange rate derived by Nelson dependent on the Cobb-Douglas assumption? Nelson does not throw light on this question. He simply states that he does not know the extent to which the implications of the model are sensitive to the Cobb-Douglas specification. He suspects that the quantitative results are affected, however, states that most of the

---

[10] Ibid., p. 50.

[11] Ibid., p. 53.

qualitative results would hold under "more realistic assumptions."[12]

With respect to the elasticity of substitution, Nelson recognizes that: "one would be tempted to assume a less than unitary elasticity of substitution between domestic inputs and imports in investment."[13] However, he feels that the Cobb-Douglas specification is a necessary simplification. "The principal reason is that the structure of import demand . . . leads to a webb of multiplicative relationships that are tractable under Cobb-Douglas assumptions and next to impossible under other forms."[14]

In the next chapter the present author shows that Nelson's model is tractable in terms of a family of production functions that does not limit a priori the value of the elasticity of substitution to unity. The next chapter also contains a reassessment of the policy options in the light of the more general formulation.

---

[12] Ibid., pp. 22-23.

[13] Ibid., pp. 15-16.

[14] Ibid., p. 15.

CHAPTER FOUR

NEOCLASSICAL TWO-GAP MODEL WITH CES PRODUCTION FUNCTIONS

The purpose of this chapter is to demonstrate the tractability of Nelson's neoclassical model with CES production functions. The model is developed mathematically, and the policy implications are contrasted with those reached by Nelson with the Cobb-Douglas specification.

The Model

For simplicity's sake it is assumed that the capital-labor ratio is the same in the production of investment and consumer goods. It is also assumed that there are three basic inputs: capital, labor, and imports, which are employed in the production of investment and consumer goods. Domestic capital and labor are viewed as producing a domestic value added input as follows:

$$V = [K^{-\alpha} + \gamma_1 L^{-\alpha}]^{-\frac{1}{\alpha}} \qquad (1)$$

where L and K stand for labor and capital respectively, and V is value added in real terms. Capital goods are produced from domestic inputs of capital and labor, and imports as follows:

$$I = [M_i^{-\alpha} + \gamma_2 V_i^{-\alpha}]^{-\frac{1}{\alpha}} \qquad (2)$$

where I and M are capital goods and imports, and V was defined above.

Consumer goods are produced according to the following production function:

$$C = [M_c^{-\alpha} + \gamma_3 V_c^{-\alpha}]^{-\frac{1}{\alpha}} \qquad (3)$$

Equations (2) and (3) can also be written in terms of the basic factors of production.

$$I = \{M_i^{-\alpha} + \gamma_2[K^{-\alpha}+\gamma_1 L^{-\alpha}]\}^{-\frac{1}{\alpha}} \qquad (2')$$

$$C = \{M_c^{-\alpha} + \gamma_3[K^{-\alpha}+\gamma_1 L^{-\alpha}]\}^{-\frac{1}{\alpha}} \qquad (3')$$

In all of the above production functions γ is a factor intensity parameter, and (α) is related to the elasticity of substitution (σ) by: $\sigma = \frac{1}{1+\alpha}$. The elasticity of substitution between any pair of inputs is the same as that between every other pair. All the production functions exhibit constant returns to scale.

We will follow Nelson with the assumption that business firms hire factors of production up to the point where the value of the marginal product of each factor equals its price. Let W be the money wage rate, i the equilibrium rate of return on capital, P_i the price of capital goods, $r = P_i \cdot i$ the rental rate of a unit of capital, and P the price of domestic value added. In equilibrium we have the following relationships between inputs, outputs, and their prices.

$$\frac{L}{V} = \left(\frac{P}{W}\right)^{\sigma} \gamma_1^{\sigma} \qquad (4)$$

$$\frac{K}{V} = \left(\frac{P}{r}\right)^{\sigma} \tag{5}$$

From the consumer goods production function:

$$\frac{M_c}{C} = \left(\frac{P_c}{E}\right)^{\sigma} \tag{6}$$

$$\frac{V_c}{C} = \left(\frac{P_c}{P}\right)^{\sigma} \gamma_3^{\sigma} \tag{7}$$

where P_c is the price of consumer goods and E is the nominal exchange rate.

From the investment goods production function:

$$\frac{M_i}{I} = \left(\frac{P_i}{E}\right)^{\sigma} \tag{8}$$

$$\frac{V_i}{I} = \left(\frac{P_i}{E}\right)^{\sigma} \gamma_2^{\sigma} \tag{9}$$

The prices of the various outputs can now be expressed as a function of the prices of the various inputs.

$$P = \{r^{\alpha\sigma} + \gamma_1^{\sigma} W^{\alpha\sigma}\}^{\frac{1}{\alpha\sigma}} \tag{10}$$

$$P_c = \{E^{\alpha\sigma} + \gamma_3^{\sigma} P^{\alpha\sigma}\}^{\frac{1}{\alpha\sigma}} \tag{11}$$

$$P_i = \{E^{\alpha\sigma} + \gamma_2^{\sigma} P^{\alpha\sigma}\}^{\frac{1}{\alpha\sigma}} \tag{12}$$

For derivation of equations (10-12) see appendix.

Substituting $r = P_1 i$ in equations (10-12), these equations can be expressed in terms of the basic input prices--the wage rate, the rate of return on capital, and the exchange rate.

$$P = \left\{\frac{(Ei)^{\alpha\sigma} + \gamma_1^{\sigma} W^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha\sigma}} \tag{13}$$

$$P_c = \left\{E^{\alpha\sigma} + \gamma_3^{\sigma}\left[\frac{(Ei)^{\alpha\sigma} + \gamma_1^{\sigma} W^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right]\right\}^{\frac{1}{\alpha\sigma}} \tag{14}$$

$$P_1 = \left\{\frac{E^{\alpha\sigma} + (\gamma_1\gamma_2)^{\sigma} W^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha\sigma}} \tag{15}$$

and,

$$r = \left\{\frac{E^{\alpha\sigma} + (\gamma_1\gamma_2)^{\sigma} W^{\alpha\sigma}}{i^{-\alpha\sigma} - \gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha\sigma}} \tag{16}$$

The real wage rate--the money wage divided by the price of consumer goods--appears below.

$$\frac{W}{P_c} = \left\{\left(\frac{E}{W}\right)^{\alpha\sigma} + \gamma_3^{\sigma}\left[\frac{(E/W)^{\alpha\sigma} i^{\alpha\sigma} + \gamma_1^{\sigma}}{(1 - i^{\alpha\sigma}\gamma_2^{\sigma})}\right]\right\}^{-\frac{1}{\alpha\sigma}} \tag{17}$$

In Chapter 2 the input coefficients were derived from the activity matrix and used to write the constraints on the economy's activities. Ignoring the direct import activities the constraints can

be written as follows:

$$M \geq a_1 I + a_2 C$$

$$K \geq c_1 b_1 I + c_1 b_2 C$$

$$L \geq c_2 b_1 I + c_2 b_2 C$$

Equations (4-9) can now be used to convert the constant input coefficients of the constraints into variable coefficients. Equations (10-16) enable us to write these coefficients in terms of the basic input prices W, i and E. The explicit derivation of the constraints is shown in the appendix. The constraints are as follows:

$$M \geq \left\{\frac{1 + (\gamma_1\gamma_2)^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}} I$$

$$+ \left\{\frac{1 + i^{\alpha\sigma}(\gamma_3^{\sigma} - \gamma_2^{\sigma}) + (\gamma_1\gamma_3)^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}} C \qquad (18)$$

$$K \geq \gamma_2^{\sigma} i^{-\sigma} I$$

$$+ \left\{\frac{i^{-\alpha\sigma} - \gamma_2^{\sigma} + \gamma_3^{\sigma} + (\gamma_1\gamma_3)^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma} i^{-\alpha\sigma}}{1 + (\gamma_1\gamma_2)^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma}}\right\}^{\frac{1}{\alpha}} \gamma_3^{\sigma} C \qquad (19)$$

$$L \geq \left\{\frac{\left(\frac{E}{W}\right)^{\alpha\sigma} + (\gamma_1\gamma_2)^{\sigma}}{i^{-\alpha\sigma} - \gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}} \gamma_1^{\sigma} K^* \qquad (20)$$

For any $\frac{W}{E}$ and i the constraints are linear and can be shown diagrammatically as in Figure 3 in the previous chapter. But as the basic factor prices change the constraints shift. We can now explore shifts in the constraints produced by changes in W, E, and i.

Beginning with the import constraint we notice that: $\frac{\partial M}{\partial E} < 0$, therefore an increase in E lowers the import intensity of a given output and shifts out the M constraint in Figure 3. Also, $\frac{\partial M}{\partial W} > 0$, and $\frac{\partial M}{\partial i} > 0$, therefore, increases in W and i result in an increase in the import intensity of a given output and shift in the M constraint. Turning to the capital constraint: $\frac{\partial K}{\partial W} > 0$, thus an increase in W shifts in the capital constraint, and $\frac{\partial K}{\partial i} < 0$, $\frac{\partial K}{\partial E} < 0$, thus increases in i and E shift out the capital constraint. Finally, the labor constraint yields the following: $\frac{\partial L}{\partial E} > 0$, $\frac{\partial L}{\partial i} > 0$, and $\frac{\partial L}{\partial W} < 0$ that is an increase in E and i and a decrease in W shift in the labor constraint.

At this juncture we are back with Nelson, and it is not difficult to demonstrate that given a ratio of investment to consumption there exists a combination of i and $\frac{W}{E}$ for which all three constraints can be made to go through the same point.

Ignoring the import constraint and assuming a target ratio of

I-C we can plot all the combinations of i and $\frac{W}{E}$ that induce business firms to use capital and labor in the same ratio as the country's endowments. Let that ratio be denoted $\left(\frac{K}{L}\right)^*$.

Rearranging equation (20)

$$\left(\frac{K}{L}\right)^* = \left\{\frac{\left(\frac{W}{E}\right)^{-\alpha\sigma} + (\gamma_1\gamma_2)^{\sigma}}{i^{-\alpha\sigma} - \gamma_2^{\sigma}}\right\}^{-\frac{1}{\alpha}} \gamma_1^{-\sigma} \qquad (21)$$

If we plot the $\left(\frac{K}{L}\right)^*$ schedule in i and $\frac{W}{E}$ space it must slope upwards from left to right because $\frac{\partial(K/L)}{\partial(W/E)} > 0$, and $\frac{\partial(K/L)}{\partial i} < 0$. Therefore for the K-L ratio to remain unchanged, i and $\frac{W}{E}$ must move in the same direction.

Ignoring this time the labor constraint and assuming the same target ratio of I-C we can plot all the combinations of i and $\frac{W}{E}$ that induce business firms to employ M and K in the same ratio as the country's endowments. Let that ratio be denoted $\left(\frac{M}{K}\right)^*$. From equations (5), (6), (7), and (16) one can derive the equation for the M-K ratio in the production of consumer goods.

$$\left(\frac{M}{K}\right)^* = \left\{\frac{1 + (\gamma_1\gamma_2)^{\sigma}\left(\frac{W}{E}\right)^{\alpha\sigma}}{i^{-\alpha\sigma} - \gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}} \gamma_3^{-\sigma} \qquad (22)$$

Since $\frac{\partial(M/K)}{\partial(W/E)} > 0$, and $\frac{\partial(M/K)}{\partial i} > 0$, $\frac{W}{E}$ and i must move in opposite

directions for the (M/K) ratio to remain constant. Therefore the (M/K) schedule slopes downwards and to the right. The (M/K) ratio in investment is of the same form.

Recounting, the $(K/L)^*$ schedule shows all the combinations of i and $\frac{W}{E}$ which produce full employment of capital and labor. The $(M/K)^*$ schedule shows all the combinations of i and $\frac{W}{E}$ for which there is full utilization of import capacity and of the capital stock. Therefore full employment of all three inputs is only possible where the schedules intersect in Figure 4.

The moment the intersection of the schedules is discussed, two questions beg for attention. First, is intersection always possible regardless of the value of the elasticity of substitution? Secondly, what is the trend of the point of intersection of the schedules as the elasticity of substitution assumes different values? In both the Rand Research memorandum and the J.P.E. article Nelson leaves these questions partly unanswered.

> There are some difficult questions regarding the conditions under which the curves will intersect, and under which they will intersect only once. The assumption that marginal products are positive within the range of relevant factor-proportion variation would appear sufficient to assure intersection. This implies, for example, that the nation's capital-labor ratio is not so low that the marginal productivity of labor at full employment is zero (as will happen at very low K/L if the elasticity of substitution is less than one). It also implies that imports are not so scarce that domestic inputs have zero marginal productivity in capital goods production (as could happen at high V/M ratios if the elasticity of substitution is less than one). Conditions for the intersection, if there is one, to be unique, are less clear to me.[1]

---

[1]Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange-Constrained Economy," Journal of Political Economy, Vol. 78, No. 3 (May-June 1970), p. 558

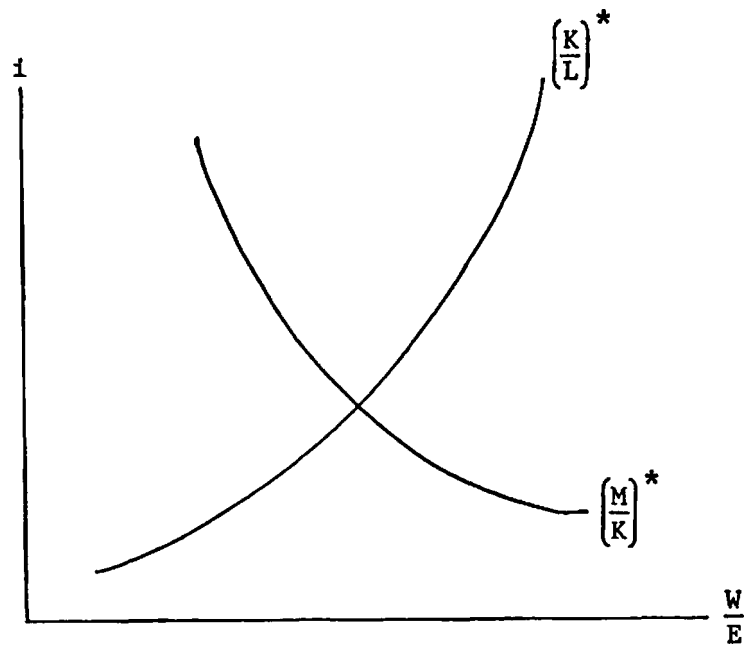

Figure 4.

Source: Richard R. Nelson, "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, Vol. 78, No. 3 (May-June 1970), p. 557.

Let us now investigate the question of the intersection of the schedules. Equations (21) and (22) define the $(K/L)^*$ and $(M/K)^*$ schedules respectively. From equation (21)

$$[(\frac{W}{E})^{-\alpha\sigma} + (\gamma_1\gamma_2)^{\sigma}]\gamma_1^{\alpha\sigma} (\frac{K}{L})^{\alpha} + \gamma_2^{\sigma} = i^{-\alpha\sigma} \tag{23}$$

and from equation (22)

$$1 + (\gamma_1\gamma_2)^{\sigma} (\frac{W}{E})^{\alpha\sigma} (\frac{M}{K})^{-\alpha} \gamma_3^{-\alpha\sigma} + \gamma_2^{\sigma} = i^{-\alpha\sigma} \tag{24}$$

Substituting (23) into (24) we "get rid" of the i, and rearranging and multiplying through by $\left(\frac{W}{E}\right)^{\alpha\sigma}$ we obtain

$$(\gamma_1\gamma_2)^{\sigma} [(\frac{W}{E})^{\alpha\sigma}]^2 + [1 - (\frac{M}{L})^{\alpha} (\gamma_1\gamma_3)^{\alpha\sigma} (\gamma_1\gamma_2)^{\sigma}](\frac{W}{E})^{\alpha\sigma} - (\frac{M}{L})^{\alpha} (\gamma_1\gamma_3)^{\alpha\sigma} = 0 \tag{25}$$

Equation (25) is a quadratic, and its solution yields two roots. The schedules intersect at the points defined by the roots below.

The roots are: $\begin{cases} \frac{W}{E} = \left(\frac{M}{L}\right)^{\frac{1}{\sigma}} \gamma_1\gamma_3 \\ \frac{W}{E} = -\frac{1}{(\gamma_1\gamma_2)^{\sigma}} \end{cases}$

We discard the negative root. Then we have two cases only one of which has economic significance, and that is Case 1.

Case 1

$$0 < \frac{M}{L} < 1$$

Case 2

$$\frac{M}{L} \geq 1$$

In Case 1 we have:

$$\frac{W}{E} = \lim_{\sigma \to 0} \left(\frac{M}{L}\right)^{\frac{1}{\sigma}} \gamma_1 \gamma_3 = 0$$

and,

$$\frac{W}{E} = \lim_{\sigma \to \infty} \left(\frac{M}{L}\right)^{\frac{1}{\sigma}} \gamma_1 \gamma_3 = \gamma_1 \gamma_3$$

That is, as σ increases from zero the point of intersection of $(K/L)^*$ and $(M/K)^*$ moves to the right in Figure 4. The economic interpretation of this finding is straightforward. For relatively small values of the elasticity of substitution a relatively high effective exchange rate (E/W), is needed for full utilization of all inputs.

The Policy Options Re-examined

We are now ready to re-examine the policy maneuver in which devaluation gives rise to an increase in employment and to a balanced expansion in investment and consumption. Our point of departure again is point b in Figure 3, where there is full utilization of import capacity and the capital stock, and an excess supply of labor. It would

be interesting to compare the impact of devaluation on employment, the model specified along CES lines, with the Cobb-Douglas specification used by Nelson. Before, however, a clarification concerning devaluation. Nelson separates the import and export exchange rates for policy purposes, in the maneuver under discussion only the import rate is devalued.[2] Naturally, with a given export exchange rate import capacity is constant. If the export exchange rate is allowed to vary the capacity to import also changes, the magnitude of the change depending on the exchange rate elasticity of exports. The effects of devaluation on exports have been discussed extensively in the literature but the effects on factor substitution have received little attention. Nelson's model focuses on the latter.

As shown in the previous chapter (holding W constant), there is an increase in E and i that shifts out the M and K constraints and shifts in the L constraint. The result is that all three constraints can be made to intersect to the right of point b along the same I-C ray as illustrated by Figure 3A. At the new equilibrium point there is greater employment, more consumption and investment, the same level of imports, and the same capital stock.

Demonstration

Equation (20) yields the change in employment induced by changes in E and i holding the capital stock constant. The proportionate change in employment is:

---

[2]Richard R. Nelson, The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy, RM 5680 (Santa Monica, Calif.: The Rand Corporation, 1968), p. 49.

$$\frac{dL}{L} = \sigma\left\{\frac{1}{1 + (\gamma_1\gamma_2)^{\sigma}\left(\frac{W}{E}\right)^{\alpha\sigma}}\ \frac{dE}{E} + \frac{1}{1 - \gamma_2^{\sigma}\, i^{\alpha\sigma}}\ \frac{di}{i}\right\} \qquad (26)$$

From equation (1),

$$\frac{dV}{V} = \frac{\partial V}{\partial L}\ \frac{dL}{V} = \gamma_1\left(\frac{V}{L}\right)^{\alpha}\frac{dL}{L} \qquad (27)$$

and substituting equation (4) in equation (27)

$$\frac{dV}{V} = \gamma_1^{(1-\alpha\sigma)}\left(\frac{W}{P}\right)^{\alpha\sigma}\frac{dL}{L} \qquad (28)$$

Since consumption and investment are expanding proportionately, it can be said that labor and domestic capacity (V) are also expanding proportionately in the production of consumption and investment goods.[3] Thus,

$$\frac{dV}{V} = \frac{dV_i}{V_i} = \frac{dV_c}{V_c} \qquad (29)$$

Total demand for imports can be written:

$$M = \left\{\left(\frac{M}{V}\right)_c\frac{V_c}{V} + \left(\frac{M}{V}\right)_i\frac{V_i}{V}\right\} V \qquad (30)$$

But from equations (6) - (9) and (13) we can derive:

$$\left(\frac{M}{V}\right)_c = \left\{\frac{i^{\alpha\sigma} + \gamma_1^{\sigma}\left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma}\,\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}}\gamma_3^{-\sigma} \qquad (31)$$

---

[3] Ibid., p. 50.

and

$$\left(\frac{M}{V}\right)_1 = \left\{\frac{i^{\alpha\sigma} + \gamma_1^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma} \gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}} \gamma_2^{-\sigma} \tag{32}$$

Substituting (31) and (32) in (30)

$$M = \left\{\left[\frac{i^{\alpha\sigma} + \gamma_1^{\sigma} \left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma} \gamma_2^{\sigma}}\right]^{\frac{1}{\alpha}} \left[\gamma_3^{-\sigma} \frac{V_c}{V} + \gamma_2^{-\sigma} \frac{V_1}{V}\right]\right\} V \tag{33}$$

In this equation the term $\left(\gamma_3^{-\sigma} \frac{V_c}{V} + \gamma_2^{-\sigma} \frac{V_1}{V}\right)$ is constant.

Equation (33) defines the demand for foreign exchange and since in this maneuver we are considering only a devaluation of the import rate, the supply of foreign exchange has not changed, that is in equation (33) M is constant.

Differentiating equation (33) and dividing by M

$$\frac{dM}{M} = \frac{\partial M}{\partial E} \frac{dE}{M} + \frac{\partial M}{\partial i} \frac{di}{M} + \frac{\partial M}{\partial V} \frac{dV}{M}$$

and remembering that $\frac{dM}{M} = 0$, and evaluating the partials we have:

$$0 = -\frac{\sigma\,\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}}{E[i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}]}\,dE + \frac{\sigma}{i^{\sigma}\,[i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}]}\,di$$

$$+\frac{\sigma}{i^{\sigma}\,\gamma_2^{-\sigma}\,(1-i^{\alpha\sigma}\,\gamma_2^{\sigma})}\,di + \left(\frac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\frac{dL}{L} \qquad (34)$$

Substituting (26) in (34)

$$\left[\frac{\sigma\,\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}}{i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}} - \left(\frac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\,\frac{\sigma}{1+(\gamma_1\gamma_2)^{\sigma}\,(W/E)^{\alpha\sigma}}\right]\frac{dE}{E}$$

$$= \left[\frac{\sigma}{i^{-\alpha\sigma}[i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}]} + \frac{\sigma\,i^{\alpha\sigma}\,\gamma_2^{\sigma}}{1-i^{\alpha\sigma}\,\gamma_2^{\sigma}} + \left(\frac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\,\frac{\sigma}{1-\gamma_2^{\sigma}\,i^{\alpha\sigma}}\right]\frac{di}{i}$$

(35)

Solving for $\frac{di}{i}$

$$\frac{di}{i} = \left\{\frac{\dfrac{\sigma\,\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}}{i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}} - \left(\dfrac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\,\dfrac{\sigma}{1+(\gamma_1\gamma_2)^{\sigma}\,(W/E)^{\alpha\sigma}}}{\dfrac{\sigma}{i^{-\alpha\sigma}[i^{\alpha\sigma}+\gamma_1^{\sigma}\,(W/E)^{\alpha\sigma}]} + \dfrac{\sigma\,i^{\alpha\sigma}\,\gamma_2^{\sigma}}{1-i^{\alpha\sigma}\,\gamma_2^{\sigma}} + \left(\dfrac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\dfrac{\sigma}{1-\gamma_2^{\sigma}\,i^{\alpha\sigma}}}\right\}\frac{dE}{E}$$

(36)

Substituting (36) back in (26)

$$\frac{dL}{L} = \frac{\sigma}{1 + (\gamma_1\gamma_2)^{\sigma}\,(W/E)^{\alpha\sigma}}\,\frac{dE}{E}$$

$$+\frac{\sigma}{1-\gamma_2^{\sigma}\,i^{\alpha\sigma}}\left\{\frac{\dfrac{\sigma\,\gamma_1^{\sigma}(W/E)^{\alpha\sigma}}{i^{\alpha\sigma}+\gamma_1^{\sigma}(W/E)^{\alpha\sigma}} - \left(\frac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}\,\dfrac{\sigma}{1+(\gamma_1\gamma_2)^{\sigma}(W/E)^{\alpha\sigma}}}{\dfrac{\sigma}{i^{-\alpha\sigma}[i^{\alpha\sigma}+\gamma_1^{\sigma}(W/E)^{\alpha\sigma}]} + \dfrac{\sigma\,i^{\alpha\sigma}\gamma_2^{\sigma}}{1-i^{\alpha\sigma}\gamma_2^{\sigma}} + \dfrac{\sigma}{1-\gamma_2^{\sigma}i^{\alpha\sigma}}\left(\frac{W}{P}\right)^{\alpha\sigma}\gamma_1^{(1-\alpha\sigma)}}\right\}\frac{dE}{E}$$

(37)

Substituting for P as in equation (13), rearranging and simplifying, equation (37) yields:

$$\frac{dL}{L} = \sigma\frac{dE}{E} \tag{38}$$

The labor absorptive impact of devaluation depends on the elasticity of substitution. That result is not surprising. It can now be seen that the result obtained by Nelson with the Cobb-Douglas specification is a special case of the more general CES case. Only when the elasticity of substitution equals one is the increase in employment proportional to the devaluation. There is, of course, no <u>a priori</u> reason why the elasticity of substitution must be equal to one.

Let us now investigate the effect of devaluation on investment and consumption. Earlier we saw that a change in relative factor prices produced by devaluation can be made to increase both investment and

consumption along a given C-I ray. It can be shown that the increase in both consumption and investment occurs because employment increases in both activities. Equation (2') shows that the production of investment goods is a function of the quantities of imported inputs (M), domestic labor (L), and domestic capital (K) employed. In the present maneuver both M and K are held constant, therefore, differentiating equation (2') with respect to L and dividing by I yields an expression for the percentage increase in investment.

$$\frac{dI}{I} = \left(\frac{I}{L}\right)^{\alpha} \gamma_1\gamma_2 \frac{dL}{L} \tag{39}$$

Substituting (38) in (39) gives the percentage increase in investment associated with a given devaluation.

$$\frac{dI}{I} = \sigma\left(\frac{I}{L}\right)^{\alpha} \gamma_1\gamma_2 \frac{dE}{E} \tag{40}$$

Similarly, using equation (3') it can be shown that the percentage increase in consumption is:

$$\frac{dC}{C} = \sigma\left(\frac{C}{L}\right)^{\alpha} \gamma_1\gamma_3 \frac{dE}{E} \tag{41}$$

Thus, equations (38), (40), and (41) show that the option is open to obtain both more employment and a higher rate of growth without external assistance. The conflict between growth and employment objectives, a characteristic of rigid-coefficients type gap models like Vanek's here disappears.

Having demonstrated the tractability of Nelson's model under CES conditions, let us examine two questions that beg for attention the

moment devaluation is considered as a tool of labor absorption.

The first question relates to the real wage cost of a devaluation-induced increase in employment. It can be seen from equation (17) that the real wage (W/P_c) is inversely related to the effective exchange rate (E/W). More precisely, the elasticity of the real wage with respect to the effective exchange rate is equal to minus one. Assuming a value of the elasticity of substitution, say $\sigma = 0.5$, it can be seen that in order to produce a 15 per cent increase in employment, a 30 per cent devaluation is needed. Such a devaluation implies a 30 per cent decrease in the real wage. Clearly, to the extent that the model corresponds with reality and that workers are aware of the real wage cost of devaluation, implementation of the policy in question may be very difficult.

The second question relates to the value of the elasticity of substitution and to the specification of the model presented in this chapter. Since the labor absorptive impact of devaluation depends uniquely on the value of the elasticity of substitution, it would be convenient to know the probable range of that parameter. In recent years the literature has been deluged with estimates of the elasticity of substitution. A review of that literature is, however, clearly beyond the scope of this paper. Nevertheless, suffice it to say that according to Streissler "reputable estimations" have not produced elasticities of substitution above unity for the whole economy although these do exist for certain industries. For the whole economy, estimates

range from one-third to close to unity.[4] These figures refer only to the elasticity of substitution between capital and labor. In the present model there are three inputs, and therefore three elasticities of substitution, which have been assumed to be equal. The only justification for that assumption is, of course, analytical convenience. It may be argued that the elasticity of substitution between imported and domestic inputs is smaller than the conventional elasticity of substitution between capital and labor. A more realistic model would then be specified in terms of a three-input production function with different elasticities of substitution between every pair of inputs. Preliminary work by the present author indicates that the model is not tractable under that alternative specification.

In conclusion, because of the reasons discussed above, it appears doubtful that devaluation can accomplish as much as is suggested by Nelson's Cobb-Douglas model. This is particularly so in situations where drastic decreases in real wages are needed to produce small increases in employment. However, if the growth of employment has been retarded by rapid increases in real wages, the result of, for example, market imperfections, devaluation may be an ideal tool to moderate the growth of wages and thus to facilitate labor absorption.

The next chapter is an effort to throw light on the relationship between wage behavior and employment growth in Colombia's manufacturing sector for the period 1959-66.

---

[4]Erich Streissler, "Long Term Structural Changes in the Distribution of Income," Zeitschrift fur Nationalokonomie 29 (1969), p. 54.

CHAPTER FIVE

THE COLOMBIAN CASE: SOME EMPIRICAL RESULTS

As pointed out in Chapter 1 the unemployment picture in less developed countries has been complicated by the employment lag in the manufacturing sector. The employment lag is according to some authors, a result of market imperfections which distort the relationship between relative factor prices and factor endowments. Thus in a country with a chronic and rising unemployment problem the price of labor may be pushed up relative to the price of capital, providing firms with incentives to substitute capital for labor. As we have seen, market imperfections may take the form of overvalued exchange rates. In that case, exchange rate devaluation may be necessary in order to bring factor prices in line with factor endowments. Alternatively, the employment lag may have nothing to do with market imperfections, being instead the product of technological change, learning by doing, etc. In short, the product of forces over which less developed countries have little or no control.

In this chapter the Fei-Ranis[1] model of labor absorption is used to shed light on the causes of the employment lag in Colombia's manufacturing sector. The Fei-Ranis model is designed to measure the contribution of capital accumulation and of technological change to labor

---

[1]John C. H. Fei and Gustav Ranis, "Innovation, Capital Accumulation and Economic Development," American Economic Review (June 1963), pp. 283-313.

absorption under the assumption that real wages are constant. According to the model the growth of industrial employment is a direct function of the rate of capital accumulation and of the intensity of neutral technological change. In addition, innovations which are biased in the labor using direction may make significant contributions to labor absorption.[2]

Notably absent from the model is the rate of growth of the real wage in the industrial sector. The authors justify the omission of the wage variable on the grounds that ". . . to the extent that there exists a reserve army of the disguised unemployed in the agricultural sector, it is unlikely that significant upward pressures on industrial real wages will persist."[3] The proposition that in a dualistic economy the real wage tends to remain constant is an important part of Lewis' model of development with unlimited supplies of labor, and was at one time assumed to have widespread validity. More recently, however, it has been discredited and even Lewis has abandoned it.[5] In the present case of Colombia, the real wage in the manufacturing sector has been far from constant, increasing over the period 1959-66 at an average annual rate

---

[2]Ibid., p. 285.

[3]Ibid., p. 284.

[4]W. Arthur Lewis, "Economic Development with Unlimited Supplies of Labor," The Manchester School (May 1954), 22, pp. 139-92.

[5]W. Arthur Lewis, "A Review of Economic Development," American Economic Review Papers and Proceedings (May 1965), pp. 1-16.

of 4.6 per cent.[6] However, the Fei-Ranis model can be modified so as to yield a labor demand relation with the growth of wages as a variable. To an examination of that formulation we presently turn.

The Model

Production in the manufacturing sector is assumed to take place along a CES production function represented by:

$$Q = F(K,L,t) \tag{1}$$

where Q, K, L, and t stand for output, capital, labor, and time respectively. The value of the elasticity of substitution is not specified a priori. The only restriction placed by Fei-Ranis on the production function is that it be homogeneous of degree one in its first two arguments.

Making the usual assumption about competitive imputations it can be shown that:[7]

$$\eta_L = \eta_k - \frac{\sigma}{\phi_k}\eta_w + \frac{\sigma}{\phi_k}J - \sigma B \tag{2}$$

Equation (2) shows that the rate of growth of employment (η_L) varies directly with the rate of growth of the capital stock (η_k), with the innovational intensity (J), and with the innovational bias (B) if the

---

[6]Growth rates computed by the author from Dane, Anvario General de Estadistica, Bogotá, Colombia, 1962, pp. 756-759; and Dane, Ibid., 1966-67, pp. 62-65.

[7]For derivation see appendix to this chapter.

innovation is labor using.[8] The rate of growth of employment varies inversely with the rate of growth of wages (η_w), and with the innovational bias if the innovation is capital using. The elasticity of substitution between capital and labor (σ), and capital's share in value added (ϕ_k) also appear in equation (2). The elasticity of substitution exerts a contra-puntal effect on the rate of growth of employment. First of all it can be seen that the greater the elasticity of substitution, the greater the damping impact on employment of an increase in the real wage. On the other hand, the greater the elasticity of substitution the greater the labor absorptive impact of a neutral innovation. The reason for this should be obvious. The elasticity of substitution is a measure of the rapidity with which diminishing returns to a factor of production set in. A neutral innovation shifts the MPP_L schedule to the right, therefore, the greater the elasticity of substitution, the greater the amount of labor that can be admitted before the marginal product of labor falls to the level of the wage rate.

Although equation (2) may be used to investigate the effects of capital accumulation and technological change on the growth of employment it is not suited to the study of the causes of the employment lag. What is needed is a labor demand relation with output as one of the variables. As shown in the appendix to this chapter that equation is easily derived, and is shown below.

---

[8]The bias in innovation in the Hicksian sense is defined as follows. Let H_L and H_K be the time rate of increase of the marginal product of labor and capital respectively. Let the bias be equal to: $B = H_K - H_L$. If $H_L \gtreqless H_K$ the bias is: labor using, neutral, and capital using.

$$\eta_L = \eta_Q - \sigma\,\eta_w - J(1-\sigma) - \phi_k\,\sigma B \qquad (3)$$

where η_Q is the rate of growth of value added, the other variables have already been defined. It can be seen that the last three terms on the right hand side of equation (3) may cause the rate of growth of employment to lag behind the rate of growth of output. An increase in the wage rate exerts an unambiguously damping impact on employment. On the other hand, if innovations are biased in the labor using direction ($B < 0$), their contribution to employment is positive.

In the absence of technological progress, and the wage rate held constant, equation (3) reduces to:

$$\eta_L = \eta_Q \qquad (4)$$

that is, the employment lag disappears as output and employment expand pari-passu.

Let us now examine the role of the elasticity of substitution. If innovations are Hicks neutral and the elasticity of substitution is unitary equation (3) reduces to:

$$\eta_L = \eta_Q - \eta_w \qquad (5)$$

In this case the rate of growth of wages is the sole reason for the employment lag. Equation (3) also shows that if the elasticity of substitution is less than unitary, Hicks neutral innovations with wages held constant result in employment growing less rapidly than output. Conversely, if the elasticity of substitution is greater than unitary Hicks neutral technological change with wages held constant results in

employment growing more rapidly than output. However, if wage increases occur the elasticity of substitution exerts a contra-puntal effect on employment. If the rate of growth of wages is positive, other things equal, the greater the elasticity of substitution the slower the rate of growth of employment.

The Employment Lag in Colombia's Manufacturing Sector

Let us now take a closer look at the causes of the employment lag in Colombia's manufacturing sector. Data on employment, wages, and value-added are available for 20 two-digit Colombian industries for the period 1959-1966. Since both (J) and (B) are unknown, equation (3) cannot be estimated directly. However, the last two terms on the right hand side of equation (3) can be collapsed to yield:

$$\eta_L = a - \sigma \eta_w + \eta_Q \qquad (6)$$

where $a = -[J(1-\sigma) + \phi_k \sigma B]$.

The coefficient of the rate of growth of value added is unitary under constant returns to scale, less than unitary under increasing returns, and greater than unitary under decreasing returns to scale. In the most plausible case, that is, with the elasticity of substitution less than unitary and technological change biased in the labor saving direction the first term on the right hand side of equation (6) is negative.

Equation (6) was estimated by the least squares method for the pooled cross section sample of 20 Colombian industries. There are advantages and disadvantages associated with that approach. The

disadvantage is that a very restrictive assumption must be made, namely, that every industry in the sample operates along the same production function. However, according to Tidrick it is not clear how this is more restrictive than the assumption underlying frequent attempts to estimate an aggregate manufacturing wage and output coefficient.[9] In defense of the present approach Eriksson points out that it mitigates two problems encountered in least-squares regression analysis of time series, namely, multicollinearity and identification.[10] Eriksson opines that given statistically significant regression and determination coefficients the approach here adopted should show the relative contribution of the independent variables to the explanation of employment growth.[11] Lastly, in view of the paucity of data the pooled cross section approach is almost forced and a number of authors have adopted it.[12]

Regression Results

The estimation of equation (6) yielded:

---

[9]Gene M. Tidrick, Wages, Output and the Employment Lag in Jamaica, RM-40 (Williamstown, Mass.: Center for Development Economics, Williams College, 1970), p. 26.

[10]John R. Eriksson, Wage Change and Employment Growth in Latin American Industry, RM-36 (Williamstown, Mass.: Center for Development Economics, Williams College, 1970), p. 13.

[11]Ibid., p. 14.

[12]Henry H. Bruton, Employment, Productivity and Import Substitution, RM-44 (Williamstown, Mass.: Center for Development Economics, Williams College, 1972); John R. Eriksson, op. cit.; Lloyd G. Reynolds and Peter Gregory, Wages, Productivity and Industrialization in Puerto Rico (Irwin, Ill.: 1965); and Gene M. Tidrick, op. cit.

$$\eta_L = -0.12 - 0.32\,\eta_w + 0.61\,\eta_Q \qquad R^2 = 0.70$$

$$(0.96)\quad (0.18)\quad (0.09)$$

First of all, it is worth mentioning that the signs are as expected, this can be considered an accomplishment when working with less developed country data. Secondly, the estimate of the elasticity of substitution is well within the range of estimates obtained by other authors for the manufacturing sector of other countries. Thirdly, the coefficient of the rate of growth of output is very significant at the 95 per cent level, while the coefficient of the rate of growth of wages is barely significant at the same level. Fourthly, the coefficient of the rate of growth of value added is less than unitary, indicating the existence of economies of scale.

The question may now be asked: what would have been the rate of growth of employment had wages behaved differently? The estimated equation shows that had real wages remained constant, other things equal, employment would have expanded at an average annual rate of 4.08 per cent. This is to be contrasted with the actual average annual growth rate of employment, 2.7 per cent shown in Table 7. However, even with constant wages employment would have lagged behind output, the latter registering an average annual growth rate of 6.9 per cent.

The main factors behind the employment lag appear to be the rate of growth of wages, and economies of scale. Had real wages remained constant and had the coefficient of the rate of growth of output being unitary, employment would have expanded at an average annual rate of 5.3 per cent, almost double the actual rate of growth. The

TABLE 7

GROWTH RATES OF RELEVANT VARIABLES, COLOMBIA'S MANUFACTURING SECTOR 1959-1966

	Average Annual Growth Rates	Percentage Growth Rates
Value added (real)	6.9	59
Employment	2.7	20
Real wages	4.6	37
Capital	5.5	45
Labor productivity	4.2	33

Source:

Growth rates computed by the author from: Dane, Anuario General de Estadistica (Bogotá, Colombia: 1962), pp. 756-759; and Dane, Anuario General de Estadistica (1966-67), pp. 62-65.

coefficient of the rate of growth of output indicates very high economies of scale. A possible explanation for that phenomenon is that over the period in question the rate of growth of large establishments was greater than that of artisan shops with the result that the size of the average establishment increased. If the degree of returns to scale of modern firms exceeds that of firms in the artisan sub-sector, different rates of growth of output for the two sub-sectors will give the appearance of economies of scale. However, not having data for value added, employment, and wages by firm size, the above interpretation must remain as conjectural.

Equation (6) was also estimated in a time series format using average annual rates of growth for the aggregate manufacturing sector.

$$\eta_L = 1.60 - 0.38\,\eta_w + 0.40\,\eta_Q \qquad R^2 = 0.26$$

$$(0.70) \quad (0.44) \quad (0.35)$$

As is to be expected with only seven observations, the statistical results do not look very good. Nevertheless, it is interesting to observe that assuming constant wages and the same rate of growth of output, the average rate of growth of employment predicted by the time series equation is 4.36 per annum. This figure is very close to the one obtained with the pooled cross section sample.

The regression results lend support to the argument that the rate of growth of the real wage rate has been a cause of the employment lag in Colombia's manufacturing sector. In a country with 10 to 16 per cent of the urban labor force unemployed and where the unemployment rate

is four or five times higher than 20 years earlier it is difficult to accept the wisdom of policies which by raising the price of labor induce business firms to economize on their use of labor. On the other hand there are advantages associated with a policy of reducing market imperfections so as to allow the price of labor to more accurately reflect the economic value of that factor of production. First, in a less developed economy where the edges of social conflict are sharpened by extreme inequality in the distribution of income it is desirable from a humanitarian as well as from a practical point of view to improve the standard of living of the poor. In the absence of an efficient progressive tax structure one of the most efficient ways to alleviate poverty is through increasing employment. It is difficult to justify on economic or on moral grounds a policy which produces a small, elite industrial labor force but which condemns large segments of the population to live in poverty. Secondly, reducing the rate of growth of the real wage rate in the manufacturing sector will probably reduce Colombia's external dependence. The point here is that the import content of mass goods consumed by the proletariat is bound to be lower than that of commodities consumed by a better paid elite work force.

CHAPTER SIX

CONCLUSIONS

The study has shown that Nelson's neoclassical two-gap model is tractable with CES production functions. The results obtained by Nelson with the Cobb-Douglas specification are not qualitatively different from the results obtained with the CES specification, but certainly they are quantitatively different. It is shown that the CES specification produces a more realistic model, at the same time achieving a higher degree of generality.

The conflict between growth and employment objectives which is so much a part of fixed coefficients two-gap models vanishes the moment substitution possibilities in production are brought into the picture. Just as in Nelson's model the option is open to obtain both an increase in employment and in economic growth through changes in relative factor prices brought about by devaluation. However, the increase in employment is equi-proportional to the devaluation only in the Cobb-Douglas case.

There are a number of reasons why it is thought that Nelson's appraisal of the efficiency of devaluation as a tool of labor absorption is too optimistic. First, it is unlikely that the elasticity of substitution for the whole economy is unitary. In the most likely case ($\sigma<1$), it may take a very large devaluation to produce a small increase in employment. It is significant that the estimate of the elasticity of

substitution between capital and labor in Colombia's manufacturing sector obtained in Chapter Five (σ=0.32), is well below unity. Second, the elasticity of substitution between imported and domestic inputs is probably less than that between domestic inputs. Finally, due to the real wage cost of devaluation there may be situations in which it is politically impossible to produce full employment through devaluation.

To recognize the difficulties associated with devaluation is not to negate that there may be situations in which devaluation may be called for in order to produce an increase in employment. This is particularly true in labor surplus economies in which the employment lag is a result of market imperfections. In those cases devaluation may be an ideal way to moderate, or even to bring about a decrease in real wages.

Unemployment is considered to be one of the most important socio-economic problems of Colombia. The regression results presented in Chapter Five indicate that increases in real wages have been an important determinant of the employment lag in the manufacturing sector. Increase in real wages in manufacturing aggravate the urban unemployment problem by: (1) inducing firms to substitute capital for labor; and (2) inducing migration from rural to urban areas. Over the period 1959-1966 real wages in the manufacturing sector increased by 37 per cent, exceeding the rate of growth of productivity. It is important that policy makers in labor surplus economies be made aware of the fact that increases in real wages exact a price in terms of foregone employment.

BIBLIOGRAPHY

Baer, Werner and Herve, Michael. "Employment and Industrialization in Developing Countries," Quarterly Journal of Economics, 80 (February 1966), 88-107.

Blaug, Marc. "A Survey of the Theory of Process Innovations," Economica, 30 (1963), 13-32.

Bruton, Henry H. Employment, Productivity and Import Substitution. RM-44. Williamstown, Massachusetts: Center for Development Economics, Williams College, 1972.

Chenery, Hollis B. and Eckstein, Peter. "Development Alternatives for Latin America," Journal of Political Economy, 78 (July-August 1970), 966-1006.

Chenery, Hollis B. and Strout, Alan M. "Foreign Assistance and Economic Development," American Economic Review, 56 (September 1966), 679-733.

David, Paul A. and Van de Klundert, Th. "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960," American Economic Review, 55 (1965), 357-394.

Eriksson, John R. Wage Change and Employment Growth in Latin American Industry. RM-36. Williamstown, Massachusetts: Center for Development Economics, Williams College, 1970.

Fei, John C. H., and Ranis, Gustav. "Innovation, Capital Accumulation and Economic Development," American Economic Review, 53 (June 1963), 283-313.

Healey, Derek T. "Development Policy: New Thinking About an Interpretation," Journal of Economic Literature, 10 (1972), 757-797.

International Bank for Reconstruction and Development. Economic Growth of Colombia: Problems and Prospects. Baltimore: Johns Hopkins University Press, 1972.

International Labor Office. Towards Full Employment: A Programme for Colombia. Geneva, 1971.

Lewis, W. Arthur. "Economic Development with Unlimited Supplies of Labor," The Manchester School, 22 (May 1954), 139-192.

Lewis, W. Arthur. "A Review of Economic Development," American Economic Review Papers and Proceedings, 55 (May 1965), 1-16.

Nelson, Richard. A Study of Industrialization in Colombia: Part I. Analysis. RM-5412. Santa Monica, California: The Rand Corporation, 1967.

Nelson, Richard R. The Effective Exchange Rate, Employment, and Growth in a Foreign Exchange Constrained Economy. RM-5680. Santa Monica, California: The Rand Corporation, 1968.

Nelson, Richard R. "The Effective Exchange Rate: Employment and Growth in a Foreign Exchange Constrained Economy," Journal of Political Economy, 78 (May-June 1970), 546-564.

Ranis, Gustav. "Production Functions, Market Imperfections and Economic Development," Economic Journal, 72 (June 1972), 344-354.

Reynolds, Lloyd G., and Gregory, Peter. Wages, Productivity and Industrialization in Puerto Rico. Illinois: Irwin, 1965.

Sheahan, John. "Imports, Investment and Growth--Colombia," in Development Policy--Theory and Practice, edited by Gustav F. Papanek. Cambridge, Massachusetts: Harvard University Press, 1968.

Slighton, Robert L. Urban Unemployment in Colombia: Measurement, Characteristics, and Policy Problems. RM-5393. Santa Monica, California: The Rand Corporation, 1968.

Streissler, Erich. "Long Term Structural Changes in the Distribution of Income," Zeitschrift fur Nationalokonomie, 29 (1969), 39-110.

Tidrick, Gene M. Wages, Output, and the Employment Lag in Jamaica. RM-40. Williamstown, Massachusetts: Center for Development Economics, Williams College, 1970.

Tinbergen, Jan. The Design of Development. Baltimore: Johns Hopkins Press, 1958.

Turnham, David and Jaeger, Ingelies. The Employment Problem in Less Developed Countries. Paris: O.E.C.D., 1971.

United Nations. El Proceso de Industrializacion en America Latina. New York, 1965.

Vanek, Jaroslav. Estimating Foreign Resource Needs for Economic Development. New York: McGraw Hill, 1967.

APPENDIX TO CHAPTER FOUR

Equations (10-12) in the text are derived as follows:

From the main text we have:

$$\frac{L}{V} = \left(\frac{P}{W}\right)^{\sigma} \gamma_1^{\sigma} \qquad (4)$$

and

$$\frac{K}{V} = \left(\frac{P}{r}\right)^{\sigma} \qquad (5)$$

From equations (4) and (5):

$$P = \frac{\left\{\frac{L}{V} W^{\sigma} + \frac{K}{V} r^{\sigma}\right\}^{\frac{1}{\sigma}}}{1 + \gamma_1^{-\sigma}} \qquad (I)$$

The price of domestic value added (P) must now be expressed in terms of the wage rate (W) and the rental per unit of capital (r). From equation (1) in the text:

$$\frac{L}{V} = \left\{\left(\frac{K}{L}\right)^{-\alpha} + \gamma_1\right\}^{-\frac{1}{\alpha}} \qquad (II)$$

but

$$\frac{F_k}{F_L} = \frac{r}{W}, \text{ therefore } \frac{K}{L} = \left(\frac{r}{W}\right)^{-\sigma} \gamma_1^{-\sigma} \qquad (III)$$

and substituting (III) into (II)

$$\frac{L}{V} = \{\left(\frac{r}{W}\right)^{\alpha\sigma} \gamma_1^{\alpha\sigma} + \gamma_1\}^{\frac{1}{\alpha}} \qquad \text{(IV)}$$

Similarly it can be shown that:

$$\frac{K}{V} = \{1 + \gamma_1[\left(\frac{r}{W}\right)^{-\alpha\sigma} \gamma_1^{-\alpha\sigma}]\}^{\frac{1}{\alpha}} \qquad \text{(V)}$$

Substituting (IV) and (V) into (I), and rearranging one obtains equation (10).

$$P = \{r^{\alpha\sigma} + \gamma_1^{\sigma} W^{\alpha\sigma}\}^{\frac{1}{\alpha\sigma}} \qquad (10)$$

Equations (11) and (12) are derived in the same manner.

Let us now take a look at the constraints, equations (18-20). The derivation of the coefficients of the constraints is straightforward. We show the first one.

The import constraint in the text is reproduced below.

$$M \geq a_1 I + a_2 C$$

From the activity matrix:

$$a_1 = \frac{M_1}{I} \text{ and } a_2 = \frac{M_C}{C}$$

but equation (8) in text shows that:

$$\frac{M_1}{I} = \left(\frac{P_1}{E}\right)^{\sigma} \qquad (8)$$

If we now substitute equation (15) in the text in (8):

$$a_1 = \left\{\frac{1 + (\gamma_1\gamma_2)^{\sigma}\left(\frac{W}{E}\right)^{\alpha\sigma}}{1 - i^{\alpha\sigma}\gamma_2^{\sigma}}\right\}^{\frac{1}{\alpha}}$$

Similarly, a_2 can be obtained by substituting equation (14) in equation (6) in the text.

APPENDIX TO CHAPTER FIVE

Production Concepts

(1) $Q = F(K,L,t)$

1_a $F_L = \frac{\partial F}{\partial L}$ (marginal product of labor)

1_b $F_K = \frac{\partial F}{\partial K}$ (marginal product of labor)

1_c $F_{LL} = \frac{\partial^2 F}{\partial L^2}$ (rate of change of MPP_L with respect to K)

1_d $F_{KK} = \frac{\partial^2 F}{\partial K^2}$ (rate of change of MPP_K with respect to K)

1_e $F_{KL} = F_{LK} = \frac{\partial^2 F}{\partial K \partial L} = \frac{\partial^2 F}{\partial L \partial K}$ (rate of change of MPP_K[MPP_L] with respect to L[K])

1_f $\phi_k = F_K \frac{K}{Q} > 0$ (elasticity of output with respect to K)

1_g $\phi_L = F_L \frac{L}{Q} > 0$ (elasticity of output with respect to L)

1_h $\epsilon_{LL} = -\frac{F_{LL}}{F_L} L > 0$ (elasticity of the MPP_L with respect to L)

1_i $\epsilon_{LK} = \frac{F_{LK}}{F_L} K > 0$ (elasticity of the MPP_L with respect to K)

1_j $\epsilon_{KK} = \frac{F_{KK}}{F_K} K > 0$ (elasticity of the MPP_K with respect to K)

1_k $\epsilon_{KL} = \frac{F_{KL}}{F_K}$ $K > 0$ (elasticity of the MPP_K with respect to L)

The assumption of constant returns to scale implies:

(2) $Q = F_K K + F_L L$ (Euler's Theorem)

and

(2a) $1 = \phi_k + \phi_L$

Differentiating (2) partially with respect to K and then with respect to L, we obtain

(3) $KF_{KK} + LF_{LK} = KF_{KL} + LF_{LL} = 0$

from (3),

(3a) $\frac{K}{L} = -\frac{F_{LK}}{F_{KK}} = -\frac{F_{LL}}{F_{KL}}$

From (3) and (3a), we obtain

(4) $\epsilon_{LK} = \epsilon_{LL}$; $\epsilon_{KL} = \epsilon_{KK}$

(5) $\Gamma = \frac{\phi_L}{\phi_K} = \frac{\epsilon_{KK}}{\epsilon_{LL}} > 0$

Fei-Ranis[1] define the elasticity of substitution between capital and labor

---

[1]John C. H. Fei and Gustav Ranis, Development of the Labor Surplus Economy (Irwin, Ill.: 1964), p. 76.

$$\sigma = \frac{1}{\epsilon}$$

and they go on to show that:[2]

(6) $$\epsilon = \epsilon_{LL} + \epsilon_{KK}$$

Derivation of Labor Demand Relation

We can write the MPP_L,

(7) $$F_L = F_L(K,L,t)$$

Differentiating (7) with respect to time and dividing through by F_L, one obtains:

$$\frac{1}{F_L}\frac{dF_L}{dt} = \frac{F_{LK}}{F_L}\frac{dK}{dt} + \frac{F_{LL}}{F_L}\frac{dL}{dt} + \frac{F_{Lt}}{F_L}$$

Making the usual competitive imputations, $w = F_L$ and

$$\eta_w = \frac{1}{F_L}\frac{dF_L}{dt} \qquad (\eta_w = \text{rate of growth of the wage rate})$$

and making use of (4)

$$\eta_w = \epsilon_{LL}\,\eta_K - \epsilon_{LL}\,\eta_L + H_L$$

where $H_L = \frac{F_{Lt}}{F_L}$ (rate of increase of MPP_L due to the passage of time)

---

[2] Ibid., p. 106.

Solving for η_L (rate of growth of employment)

$$(8) \qquad \eta_L = \eta_K - \frac{\eta_w}{\epsilon_{LL}} + \frac{H_L}{\epsilon_{LL}}$$

Combining (5) and (6), one obtains

$$(9) \qquad \epsilon_{LL} = \frac{\phi_k}{\sigma}$$

Substituting (9) into (8)

$$(10) \qquad \eta_L = \eta_K \frac{\sigma}{\phi_k} \eta_w + \frac{\sigma}{\phi_k} H_L$$

From the production function, we have the familiar result,

$$(11) \qquad \eta Q = \phi_k \eta_K + \phi_L \eta_L + J$$

where $J = \frac{F_t}{Q} > 0$ (the intensity of innovation)

it can be shown that

$$(12) \qquad J = \phi_L H_L + \phi_K H_K$$

(H_K = rate of increase of MPP_K due to the passage of time)

We can now define the bias in innovation

$$(13) \qquad B = H_K - H_L$$

if $B > 0$ innovations are capital using in the sense of Hicks

Combining (10), (12) and (13), one obtains

$$(14) \qquad \eta_L = \eta_K - \frac{\sigma}{\phi_k}\eta_w + \frac{\sigma}{\phi_k}J - \sigma B$$

Finally solving for η_K in equation (11) and substituting into equation (12), one obtains

$$(15) \qquad \eta_L = \eta_Q - \sigma\eta_w - J(1-\sigma) - \phi_k \sigma B$$