THE EFFECT OF INDUSTRY ORGANIZATION UPON PERFORMANCE:
A CASE STUDY
THE PRODUCTION OF PRIMARY ALUMINUM

A Dissertation

Presented to

the Faculty of the Department of Economics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

By

Harold A. Coward, Jr.

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DISSERTATION ABSTRACT

The familiar problem of social losses due to monopolization of an industry is addressed with the unfamiliar addition of a recognition that production costs may not be independent of industry structure. The study is specifically focused on the domestic primary aluminum industry. A linear-programming process model of aluminum production is used to analyze the production cost effects of certain sub-optimal institutional arrangements in the industry thought to reflect government intervention. Decontrol results in production cost savings, but any such savings are found to be vastly outweighed by usual monopoly losses unless output price is regulated.

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CHAPTER I

INTRODUCTION

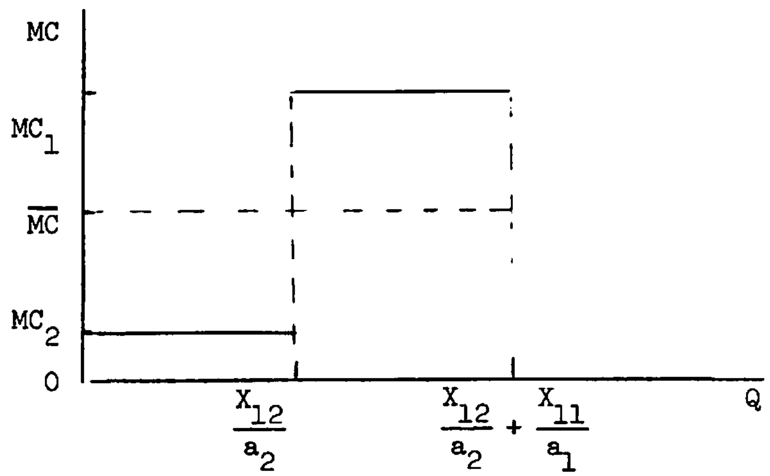
Imagine the following hypothetical situation. There exists an industry consisting of two firms, each of which owns one plant. Both plants exhibit fixed-proportions production in two productive inputs, X_1 and X_2, so that the production functions for the two plants are

$$Q_1 = \min \left\{ \frac{X_{11}}{a_1}, \frac{X_{21}}{b_1} \right\}$$

$$Q_2 = \min \left\{ \frac{X_{12}}{a_2}, \frac{X_{22}}{b_2} \right\}$$

where Q_1 and Q_2 are outputs of the respective plants, X_{11}, X_{12} and X_{21}, X_{22} are the amounts of X_1 and X_2 employed in the plants, and a_1, b_1 and a_2, b_2 are technological parameters. Further, fix $X_{11} = X_{12} = \bar{X}_1$, set $a_1 = a_2$ and let $b_1 > b_2$. If both firms buy X_2 in a competitive market so that they both pay the same price for X_2, a price invariant with the amount of X_2 employed, then the (constant) marginal cost of output in plant one will be higher than the (constant) marginal cost of output in plant two since b_1 and b_2 represent marginal X_2 per unit of Q requirements. Now let these two firms face a perfectly price inelastic demand curve for Q such that the amount of Q demanded in the market exactly equals $\frac{X_{11}}{a_1} + \frac{X_{12}}{a_2} = \frac{2\bar{X}_1}{a_1}$. This implies that in equilibrium (assuming blocked entry and a finite maximum price for the product) each firm will produce exactly half of total market output.

Now let demand decrease and a regulatory commission dictate that each firm continue to produce half of industry output; that is, let it fix market shares on the basis of proportion of ownership of the fixed productive input. Does marginal cost of output increase or decrease because the commission chooses a fixed market share rule rather than allowing a merger of the two firms? (The latter is the proposal made by the firms' lawyers.) The answer depends upon how one measures marginal cost and on how much demand decreases. The diagram below illustrates this.



The two solid horizontal lines represent the marginal cost schedule for the industry if the merger of the two firms is allowed. In the fixed market share case the heavy dashed horizontal line, which lies midway between the two solid lines would represent industry marginal cost since one-half of each marginal unit of Q would be produced in plant one and one-half would be produced in plant two. If the amount demanded is less than $\frac{X_{12}}{a_2}$, the regulatory commission's action increases marginal cost. If the amount demanded is greater than $\frac{X_{12}}{a_2}$ but less than or equal to $\frac{X_{12}}{a_2} + \frac{X_{11}}{a_1}$, the commission's decision decreases marginal cost. This conclusion, of course, involves some slight of hand. If one asks for marginal cost in the highest cost plant in operation, the answer for the fixed market share case is always MC_1, whereas in the merger case it is MC_2 if demand falls below $\frac{X_{12}}{a_2}$, the capacity of the low marginal cost plant. However, examination of industry average cost shows that the merger case results in lower average cost than the fixed share case unless demand is equal to $\frac{X_{12}}{a_2} + \frac{X_{11}}{a_1}$, where average cost in the two cases would be equal.

The situations previously described could have been presented in the form of two linear-programming problems. The merger case can be stated as

$$\min Z = p_2X_{21} + p_2X_{22}$$

s.t.

$$\begin{array}{lllll} b_1Q_1 & & -X_{21} & & \leq 0 \\ & b_2Q_2 & & -X_{22} & \leq 0 \\ a_1Q_1 & & & & \leq \bar{X}_1 \\ & a_2Q_2 & & & \leq \bar{X}_1 \\ -Q_1 & -Q_2 & & & \leq -Q \end{array}$$

where p_2 is the price of X_2, Q is total market demand, and all other variables and parameters are as previously defined. The addition of the following row to the constraint set represents the imposition of fixed (and equal) market shares.

$$-.5Q_1 + .5Q_2 = 0$$

It is an oft-referenced and easily demonstrated theorem that the value of a dual variable in the optimal solution to the dual of a linear-programming problem is equal to the rate of change of the value of the objective in the optimal primal solution with respect to the right hand side of the row in the primal problem to which the dual variable corresponds.[1] If the two linear-programming problems stated above are taken to be primal problems, then the dual variable values corresponding to the row $-Q_1-Q_2 \leq -Q$ represent industry marginal costs of output in the optimal solutions to the two problems. If those two problems were to be solved, the shadow price (dual value) of Q would be lower in the fixed market share case than in the merger case if Q were greater than the capacity of the low marginal cost plant and greater in the fixed market share case if Q were less than $\bar{X}_1$, plant capacity of the low cost plant. This is another way of stating the effects on industry marginal cost of the commission's decision concerning the rules by which the firms may operate. A slightly more complicated numerical example illustrating this point will be presented later after the structure of the model actually employed in simulations is presented.

An understanding of the previous simple example is necessary to the understanding of the results of simulations done with a much larger and more complicated process model formulated as a linear-programming problem of cost minimization. The production of milled

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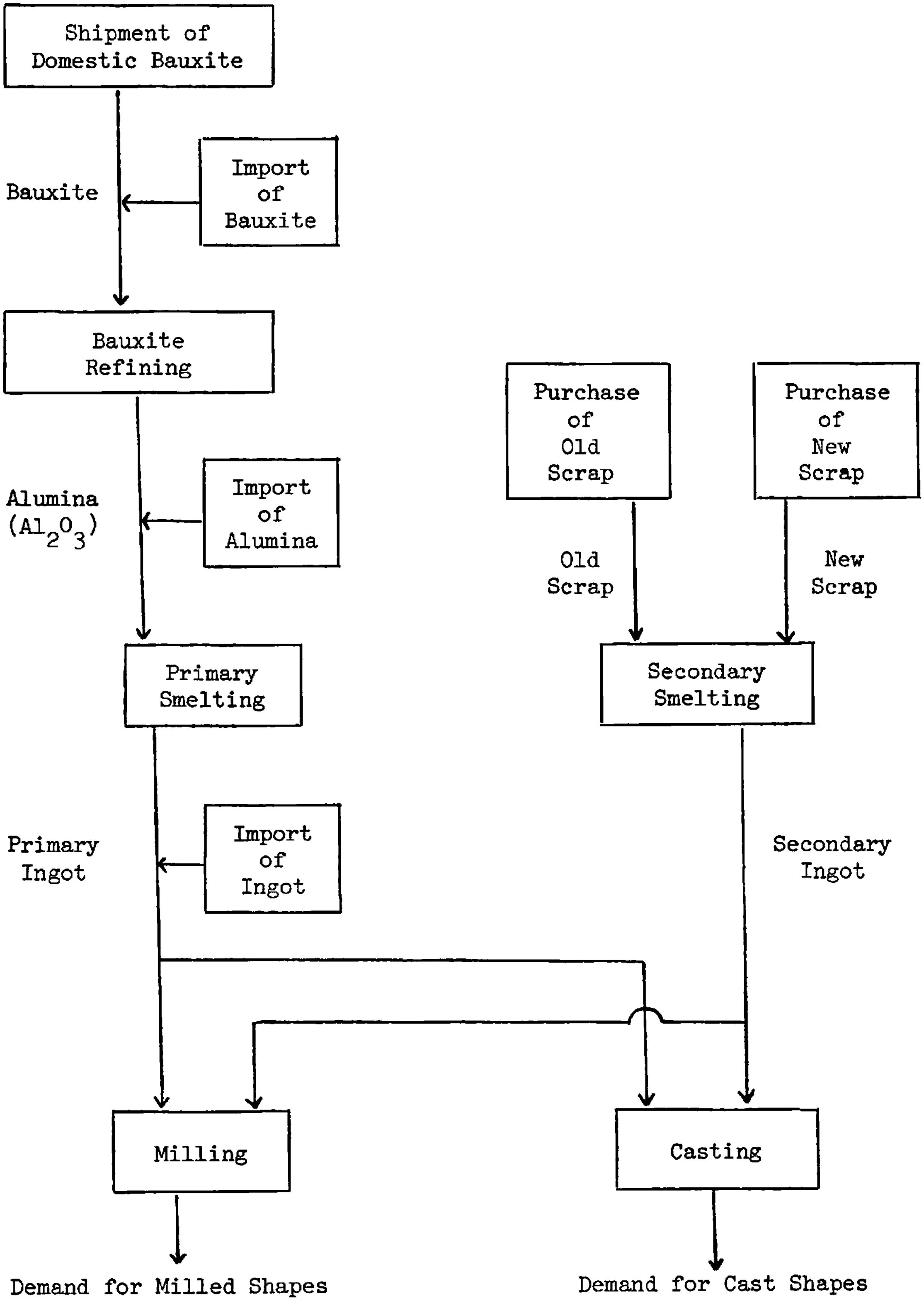
[1]See, for example, Intriligator, Michael D., Mathematical Optimization and Economic Theory, 1971, Prentice-Hall, Inc., Englewood Cliffs, N.J., pp. 86-88.

and cast aluminum shapes for consumption in the U.S., production of the primary and secondary aluminum inputs to domestic mills and foundries, and domestic production of some key inputs to the primary aluminum production process are linked by transportation networks to form a process model which moves from mined ore, in the case of primary production, and purchased scrap, in the case of secondary production, to production of milled and cast aluminum shapes. With one exception, that being imported Canadian primary aluminum, foreign production of inputs for domestic consumption are not modelled in any detail. However, importation of certain important inputs is allowed and occurs if domestic production is uneconomical or if domestic production capacity is insufficient to meet demands for the input at the next stage of production. Figure I-1 shows the flow of aluminum bearing materials from mines and scrap dealers to milles and foundries and indicates where imports can enter the process. The figure does not indicate all of the detail in the model. Energy inputs are given more explicit treatment. As well, production is disaggregated to the plant level in the case of alumina and aluminum production (allowance being made for plant ownership) and disaggregated to the regional level in the case of secondary production, milling, and casting. Non-zero unit costs are attached to the appropriate variables and the model becomes a static, one-period linear programming problem of the form

$$\text{minimize } Z = \sum_{i=1}^{n} p_i X_i$$

$$\text{subject to } AX \leq b$$

FIGURE I-1

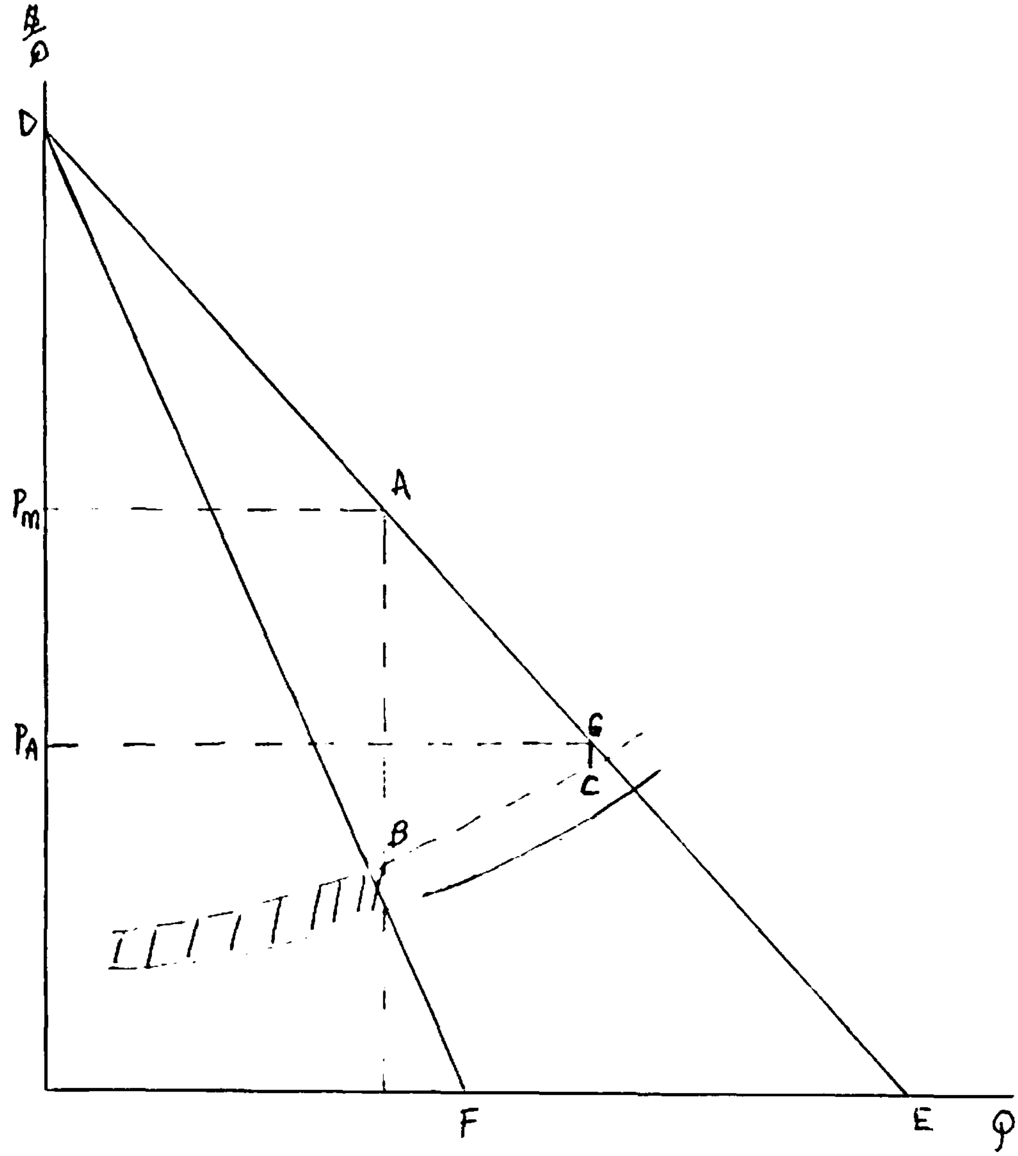

Shipment of Domestic Bauxite
Bauxite
Import of Bauxite
Bauxite Refining
Alumina (Al_2O_3)
Import of Alumina
Primary Smelting
Primary Ingot
Import of Ingot
Purchase of Old Scrap
Purchase of New Scrap
Old Scrap
New Scrap
Secondary Smelting
Secondary Ingot
Milling
Casting
Demand for Milled Shapes
Demand for Cast Shapes

where p_i is the unit cost of the i'th variable, A is a matrix of constants, X is a vector of the n choice variables, X_1, X_2, ..., X_n, and b is a vector of constants. A detailed discussion of the row constraints is found in the next chapter, while a description of data used to calculate objective and constraint coefficients and right hand sides is in Appendix A. It should be noted here that all data, with the exception of transport costs, are at 1974 levels. The results of simulations are presented in which the right hand sides of the final demand rows are varied from low to high levels in order to identify the industry marginal and average cost curves for domestically produced primary aluminum ingot.

There are restrictions in the constraint set reflecting institutional industry arrangements. Specifically, shares of primary shipments by U.S. firms for domestic consumption are constrained to be equal to a firm's ownership share of domestic capacity. As well, interfirm sales of intermediate products between firms vertically integrated to the production stage of the commodity in question are prohibited. For example, Alcoa may not sell Reynolds alumina since both own bauxite refineries. These institutional constraints are relaxed and the marginal and average cost curves of a domestic primary aluminum industry which is allowed to act as a monopolist are traced out by varying final demand. Comparison of the two sets of curves provides some measure of the effect on production cost of the existence of regulatory agencies. As hinted at previously, either a comparison of average cost or a careful definition of marginal cost is necessary before a sensible result emerges.

The sensible result does, in fact, emerge. At any particular output level the less restricted monopolistic industry experiences lower average cost and marginal cost and uses less energy (measured in terms of BTU's per ton of output) than the more restricted industry. One should not, therefore, advocate an unregulated monopoly in the domestic primary aluminum industry. An econometrically estimated demand curve for primary aluminum, described in Appendix B, is compared with the monopolist's marginal cost curve to determine the monopolist's equilibrium output and price. It is found that the production cost savings due to monopolization are outweighed by the deadweight loss caused by the unregulated monopolist's presence. Figure I-2 illustrates these results. The negatively sloped solid lines DE and DF represent the market demand and marginal revenue curves, respectively. The solid positively sloped line represents the monopoly marginal cost curve and P_m, the monopoly price. The broken positively sloped line represents the "actual" industry marginal cost curve. P_A is a representative actual price. The gain from monopolization of the industry is the shaded region. The gross social loss is the area of region ABCG. It is found that the area of ABCG is much larger than the shaded region.

FIGURE I-2



CHAPTER II
THE BASIC MODEL

An algebraic description of the linear-programming model employed in simulations is to be found in this chapter. Appendix A contains a data description.

The A matrix of the constraints contains five basic types of rows. These are accounting rows, materials balance rows, capacity rows, final demand rows, and rows in the market share constraints. The objective row will be discussed first, followed by descriptions of the constraint rows in the order listed above.

If Z is taken to be the value of the objective function, then for any given solution

$$Z = rCAP + \sum_{i=1}^{n} P_{ME_i} ME_i + \sum_{j=1}^{m} P_{TR_j} TR_j + \sum_{k=1}^{o} P_{CLN_k} CLN_k + \sum_{l=1}^{p} P_{DG_l} DG_l$$
$$+ \sum_{q=1}^{r} P_{TB_r} TB_r + \sum_{s=1}^{t} P_{ELT_t} ELT_t + \sum_{u=1}^{v} P_{I_v} I_v + \sum_{w=1}^{x} P_{SP_w} SP_w$$

The formulation above is simplified for clarity of presentation. The locational aspect of activities is ignored and subscripts portray only technological detail. The capital letter "P" represents the cost of the activity denoted by P's subscript and r is the rental rate on capital. CAP and the activity symbols are explained in Table II-1.

As Table II-1 implies, new capital equipment is measured in physical capacity units. The model is so constructed that firms are

TABLE II-1

INTERPRETATION OF OBJECTIVE FUNCTION SYMBOLS

Symbol	Interpretation
CAP	Total value of new capital equipment at purchase price per unit of physical capacity
ME_i	Physical amount of the i'th material on energy input
TR_j	Interplant or interregional shipment of the j'th aluminum-bearing input
CLN_k	Amount of calcined alumina produced by the k'th method
DG_l	Amount of alumina trihydrate produced from the l'th type of bauxite
TB_r	Activity level of the r'th turbine type
ELT_t	Amount of unblended molten primary aluminum produced by the t'th type of electrolysis
I_v	Amount of primary ingot imported by the v'th firm
SP_w	Amount of secondary ingot produced from the w'th type of scrap

assumed to rent any new capital so the cost of rental is the purchase price of a unit of an equipment type times the rental rate, assumed equal to the required rate of return of the renting firm. This underscores the static equilibrium nature of the model. With two exceptions to be noted later, taxes are ignored in cost calculations. Thus, P_{CAP} is a pre-income tax rate of return. The types of equipment available for renting are discussed in this chapter in the section on capacity rows. Some firms have initial endowments of certain types of equipment. These endowments are listed in Appendix A.

There are five types of plants contained in the model: bauxite refineries, primary smelters, secondary smelters, mills, and foundries. The major output of each plant type is shown in Figure I-1 in the introduction. Raw materials and energy sources available for purchase by refineries are: limestone, labor, natural gas, distillate fuel oil, coal, and electricity. The cost of bauxite, including any bauxite severence taxes in the case of imported bauxite, is included in the transport cost coefficient. Primary smelters may purchase petroleum coke, pitch binder, labor, coal, natural gas, distillate fuel oil, residual fuel oil, and electricity. Secondary smelters may purchase labor, electricity, five scrap types, and natural gas.

The five scrap types deserve some discussion at this point. Scrap explicitly accounted for in the model is of two basic types: old scrap, which is aluminum-bearing material which has been in the hands of consumers, and new scrap, which is aluminum-bearing material generated in production processes which use aluminum inputs. Examples of old scrap are used aluminum auto parts and used aluminum beverage containers. Examples of new scrap are clippings generated by fabricators or skimmings generated by primary smelters.[1] There are four types of old scrap and one type of new scrap available to the model's secondary smelters, the difference between the types of old scrap being price. The determination of the quantity of a scrap type available is described in Appendix B.

The transport activities, with the exception of imported bauxite and alumina, are per unit transportation costs for the type of transportation found typical to a particular shipping route. Alumina and bauxite imports include the foreign unit production cost (mining and/or refining) of the commodity being transported.

Not all purchased inputs used in a process are included explicitly in an activity vector of that process. This explains the presence of explicit cost coefficients for CLN, DG, TB, ELT, and SP.

Three major domestic firms - Alcoa, Reynolds, and Kaiser - own substantial primary smelting capacity located abroad.[2] These three firms are allowed to import primary ingot to mills and foundries at a unit cost equal to foreign production cost plus import duties plus transport cost to the receiving mill or foundry, the total unit cost symbolized previously as P_I.

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[1] See "Aluminum," Minerals Facts and Problems, 1976, Bureau of Mines, pp. 43-44.

[2] See "Aluminum," Minerals Yearbook, 1974, Bureau of Mines, pp.156-160.

The fourteen accounting rows (capital equipment purchases, two BTU-use rows, and eleven rows accounting for shipments of primary ingot by domestic firms) will be described next, beginning with the capital row.

That row has the form $\sum_{i=1}^{n} P_i X_i - CAP = 0$, where P_i is the purchase price of the i'th type of capital equipment and X_i is the number of units purchased. This row, due to the presence of CAP, ensures that the purchase price of a unit of capacity is multiplied by the rental rate, r in the objective function, before it enters the objective function.

The two BTU-use rows have the form

$$\sum_{i=1}^{n} BTU_i X_i - BTUUSE = 0$$

where BTU_i is the BTU equivalent of the i'th energy-bearing purchased raw input, X_i is the level of the i'th purchasing activity, and BTUUSE is an accounting activity.[3] There is one BTU-use row for domestic primary smelters and one for domestic bauxite refineries. These two rows are included because all inputs and outputs in the model, with the exception of recyclable process heat, are expressed in physical units, not BTU's, so the rows' presence makes energy use calculations simple.

The last type of accounting row provides a link between primary producer shipments and the market share constraints. Each

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[3] Conversion factors used for the BTU-use rows appear in Appendix A.

row appears as below

$$\sum_{i=1}^{n} X_{ij} - \bar{X}_j = 0$$

where X_{ij} is shipments by the j'th domestic firm from its i'th plant and $\bar{X}_j$ is an accounting activity representing total shipments by the j'th firm.

Materials and energy balance rows are written in the general form

$$\Sigma \text{ Uses} - \Sigma \text{ Sources} \leq 0$$

so that input rows in an activity have positive coefficients and output rows in an activity have negative coefficients. Except for the rows accounting for recyclable waste heat, all materials and energy balance rows are entered as equalities in the computer algorithm used since they would be satisfied as strict equalities in the optimal solution anyway due to the sense of the optimization.

The materials and energy balance rows appear as below if the material or energy is only purchased.

$$\sum_{k=1}^{n} a_{ijk} X_{ijk} - \bar{X}_{ij} = 0$$

where $\bar{X}_{ij}$ is the amount of the i'th input purchased at the j'th location (corresponding to ME_i, discussed as part of the objective function), X_{ijk} is the level of the k'th activity at the j'th location which uses the i'th input, and a_{ijk} is the amount of the i'th input used per unit of the k'th activity at the j'th location. Not all purchased inputs have location specific rows. Those which do are electricity at primary smelters and electricity at bauxite refineries. These two sets of rows are made location specific because in-house generation is modelled explicitly and because, in the case of electricity used at primary smelters, electricity prices are smelter specific.

These two sets of electricity rows are hybrids between rows for inputs which are only purchased and rows for inputs available only as outputs of a previous production stage. The hybrid rows appear as below

$$\sum_{k=1}^{n} a_{jk}X_{jk} - \bar{X}_j - \sum_{l=1}^{m} b_{jl}Y_{jl} = 0$$

where $\bar{X}_j$ is the amount of electricity purchased at the j'th location, a_{jk} is the amount of electricity used per unit of the k'th activity at the j'th location, X_{jk} is the level of that electricity using activity, b_{jl} is the output of the l'th generation method at the j'th location per unit of that generation method, and Y_{jl} is the level of that generating activity.

The row for an input which is the output of a previous stage of production could be of one of two types, those inputs transported and those which always remain within a plant. The latter type is as below

$$\sum_{k=1}^{n} a_{ijk}X_{ijk} - \sum_{l=1}^{m} b_{ijl}X_{ijl} = 0$$

where a_{ijk} is the amount of the i'th input used per unit of the k'th activity at the j'th location, X_{ijk} is the level of that activity, b_{ijl}

is the amount of the i'th input produced per unit of the l'th activity at the j'th location, and X_{ijl} is the level of that producing activity.

The rows for transported inputs are further subdivided into constraints which insure that use at a location does not exceed inputs shipped to a location (destination constraints) and constraints which insure that shipments from a location do not exceed production at that location (source constraints). If an input is shipped from a location, its row is as below

$$\sum_{k=1}^{n} TR_{ijk} - \sum_{l=1}^{m} a_{ijl}X_{ijl} = 0$$

where TR_{ijk} is total shipments of the i'th commodity from the j'th source to its k'th customer, a_{ijl} is the amount of the i'th commodity produced per unit of the l'th activity at the j'th location, and X_{ijl} is the level of that activity. The exception to this is bauxite, whose mining is not modelled explicitly though mine capacity is constrained.

If a commodity is shipped to a location, the constraint that ensures that use at that location does not exceed shipments to that location is

$$\sum_{l=1}^{n} a_{ikl}X_{ikl} - \sum_{j=1}^{m} TR_{ijk} = 0$$

where a_{ikl} is the amount of the i'th input used per unit of the l'th activity at the k'th location, X_{ikl} is the level of that activity, and TR_{ijk} is total shipments of the i'th input to the k'th location by the j'th supplier. Table II-2 classifies inputs by the type of materials and energy balance row that input possesses, with the exception of

TABLE II-2

INPUTS CLASSIFIED BY ROW TYPE

Input	Row Type*
Recyclable waste heat at refineries	1
High-pressure process steam at refineries and primary smelters	1
Low-pressure process steam at refineries	1
Alumina trihydrate at refineries	1
Anode coke at primary smelters	1
Anode paste at primary smelters	1
Pre-baked anodes at primary smelters	1
Molten primary aluminum	1
Calcined alumina at refineries	2
Primary ingot at primary smelters	2
Secondary ingot at secondary smelters	2
Calcined alumina at primary smelters	3
Primary ingot at mills and foundries	3
Secondary ingot at mills and foundries	3

* Row Type 1 - input remains in plant producing it
Row Type 2 - input shipped from plant
Row Type 3 - input shipped to plant

those inputs which are purchased only, or which can be either purchased or produced within the consuming plant or both, as these inputs have already been given explicit treatment in the text.

Capacity rows have the form

$$X_{ij} - X'_{ij} \leq \bar{X}_{ij}$$

where X_{ij} is the i'th activity possessing a capacity constraint at the j'th location and $\bar{X}_{ij}$ is that j'th location's original endowment. If investment in that type of capacity at that location is allowed, X'_{ij} is the level of such investment. There are two types of equipment in the model which can be improved through retrofit, which is most easily thought of as a transformation of one type of equipment into another type. Capacity rows for these equipment types then appear as below

$$X_{ij} + \sum_{k=1}^{n} X_{ikj} - X'_{ij} \leq \bar{X}_{ij}$$

$$X_{kj} - \sum_{i=1}^{m} X_{ikj} \leq \bar{X}_{kj}$$

where X_{ij}, X'_{ij}, and $\bar{X}_{ij}$ are as before, X_{kj} is the level of the activity using retrofitted capacity of the k'th type, X_{ikj} is the amount of the k'th type of retrofit fitted to the i'th original type of equipment, and $\bar{X}_{kj}$ is the initial endowment at the j'th location of the k'th type of retrofitted capacity. The first row is for an equipment type being transformed to a new type, and the second row is for the new type of equipment. Equipment or activities possessing capacity rows appear in Table II-3, where capacity which can be increased or retrofitted through investment is noted.

TABLE II-3

CLASSIFICATION OF CAPACITY TYPES

Activity or Equipment	Capacity Expansion Allowed	Retrofit Allowed
High-pressure boilers at refineries and primary smelters	yes	no
Generating turbines at refineries and primary smelters	yes	no
Bauxite mining	no	no
Bauxite digestion at refineries	yes	no
Rotary kiln calciners at refineries	yes	no
Fluidized bed calciners at refineries	yes	no
Capacity of foreign bauxite refineries	no	no
In-house hydroelectric generation at primary smelters	no	no
Importation from domestically owned foreign primary smelters	no	no
Existing Soderberg and Pre-Baked Anode electrolysis	no	yes
T_iB_2-plated cathode electrolysis (serves as retrofit for previous activity)	yes	no
New Hall electrolysis or Alcoa chlorination process	yes	no
Holding and casting furnace at primary smelters	yes	yes
Heat recuperator (retrofit for previous activity)	yes	no

Final demand rows take the form

$$-X_{ij} \leq -\bar{X}_{ij}$$

where $-X_{ij}$ is the level of demand for output of the i'th commodity in the j'th region and X_{ij} is the level of the activity producing the i'th commodity in the j'th region. There are two commodities feeding final demand, milled shapes and cast shapes, and five regions, two regions demanding no cast shapes.

Imports by Alcan (the Canadian-based primary producer) are constrained to be a percentage of primary shipments by domestic producers. Alcan's market share constraint is as below

$$\sum_{i=1}^{31} bX_i + \sum_{j=1}^{8} \sum_{k=1}^{3} bX_{jk} - X_1 = 0$$

where X_i is primary ingot production in the i'th domestic smelter, X_{jk} is importation of ingot from domestically owned foreign capacity by the k'th firm to the j'th destination, X_1 is Alcan's primary ingot production, and b is the percentage of domestic firm shipments Alcan's shipments are allowed to assume. Since the sense of optimization is minimization, production of primary ingot will equal shipments, so constraining production is equivalent to constraining shipments.

A typical row in the domestic firms' market share constraints is

$$(1 - M_i)X_i - M_i\left(\sum_{\substack{j=1 \\ j\neq 1}}^{n} X_j\right) = 0$$

where M_i is the allowed market share of the i'th firm, X_i is the accounting activity for primary ingot shipments for the i'th firm as previously described, and X_j is the accounting variable for the remaining n-1 j'th firms. An n x n matrix of coefficients so constructed would be singular, and so the market share constraints could never be in the optimal basis if each n'th firm had a market share constraint.[4]

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[4] I am indebted to Dr. Dave Pilati of Brookhaven National Labs for suggesting that the n x n matrix is singular. An explanation of the derivation of the market share rows and a proof of the singularity of the n x n matrix follows, for which the author must accept responsibility.

Retaining the notation of the text,

$$M_i = \frac{X_i}{\sum_{\substack{j=1 \\ j=i}}^{n} X_j} \qquad \forall_i$$

is the condition one wishes to impose. Any i'th condition can be rewritten as

$$M_i \left(\sum_{\substack{j=1 \\ j=i}}^{n} X_j \right) = X_i$$

or

$$(1 - M_i)X_i - M_i \sum_{\substack{j=1 \\ j\neq i}}^{n} X_j = 0 \qquad \forall_i$$

which is the form included in the model. Now consider the n x n matrix of market share constraint coefficients.

$$M = \begin{array}{cccc} (1 - M_1) & -M_1 & \cdots & -M_1 \\ -M_2 & (1 - M_2) & \cdots & -M_2 \\ \vdots & & & \\ -M_n & -M_n & \cdots & (1 - M_n) \end{array}$$

However, there are n rows and n + 1 columns in the market share submatrix. Due to the method of calculating market shares and the existence of repeating decimals, irrational numbers, and infeasible solutions,

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formed from all n constraint rows. Let M be denoted

$$M = [m_1, m_2, m_3, \ldots m_n]$$

where m_i is the i'th column of M. It will be shown that there exists at least one set of non-null scalars, $\{\alpha_i\}$, such that

$$\sum_{i=1}^{n} \alpha_i m_i = [0]$$

which establishes linear dependence among the columns of M and M's singularity. Let $\alpha_i = M_i$ to form

$$\sum_{i=1}^{n} M_i m_i = \begin{matrix} a_i \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{matrix}$$

and consider any row of the product in expanded form. Such a row appears as

$$(1 - M_j)M_j + \sum_{\substack{i=1 \\ i \neq j}}^{n} M_i(-M_j) = a_j$$

where j is the row under consideration. This reduces to

$$M_j(1 - \sum_{\substack{i=1 \\ i=j}}^{n} M_i) = a_j$$

But $\sum_{\substack{i=1 \\ i=j}}^{n} M_i = 1$, so $M_j(0) = a_j$

$$a_j = 0 \qquad \forall_j$$

which establishes M's singularity.

one row is dropped and one added. The row added is due to the addition of an activity which serves to take up the minute fraction of a percentage point of market share which is not allocated to any firm. This activity ships no ingot and only appears in the market share constraints. The resulting $n \times (n+1)$ matrix ensures the desired ratios between domestic firms' outputs, which, of course, is enough. The next chapter begins with a simple numerical example which captures the main features of the model and illustrates the unobvious effects of the market share equations on the optimal solution, as preface to presentation of simulation results.

CHAPTER III
RESULTS AND CONCLUSIONS

The problem presented below is a very simple example of the industry model outlined in Chapter II.

$$\text{Minimize } Z = 40X_1 + 50X_2 + 10X_3 + 20X_4$$

subject to

$$
\begin{array}{lr}
X_1 & \leq 300 \\
X_2 & \leq 600 \\
X_3 & \leq 1000 \\
X_4 & \leq 400 \\
.66X_5 - .33X_6 - .33X_7 - .33X_8 & = 0 \\
-.33X_5 + .66X_6 - .33X_7 - .33X_8 & = 0 \\
-.01X_5 - .01X_6 - .01X_7 + .99X_8 & = 0 \\
 & \\
-X_1 - X_2 - X_3 - X_4 & \leq -1500 \\
X_1 - X_5 & = 0 \\
X_2 - X_6 & = 0 \\
X_4 - X_7 & = 0
\end{array}
$$

Variables X_1, X_2, and X_4 represent the output of three firms, each firm being constrained by rows five through seven of the constraint matrix to produce on-third of the sum of their outputs. Variables X_5, X_6, and X_7 are accounting variables linking the shipments by the three firms to the market share constraints, while X_8 is a variable taking up the slack undistributed percentage points of market share. X_8's presence is due to the irrationality of one-third. X_3 is the output of a fringe source whose share of shipments is constrained only by its capacity constraint, the fourth row, and optimality considerations. The eighth row is a final demand row.

If one is interested in the effect of the market share constraints on the optimal solution, one way to measure that effect is examination of the shadow price or dual variable value of the final demand row in two different problems, one containing the market share constraints (the problem as stated in the text) and one identical to the first problem except that the market share constraints are removed. Another way to measure the effect of explicitly constrained shares is comparison of the ojbective function values in the optimal solutions to the two problems.

If the two problems presented are solved, a comparison on the two bases mentioned yields conflicting results! The value of the objective function in the optimal solution to the problem containing the market share constraints is 28417.5 and the shadow price of the final demand row is 36.835. If the market share constraints are removed and the problem resolved, the value of the objective function is 22000.0, while the shadow price of the final demand row is 40.0.

The shadow price in the second case is the objective function coefficient of X_1, due to the facts that capacity of X_3 and X_4 are fully utilized and the value of X_2 is zero, which makes X_1 the high marginal cost source for final demand in the optimal solution.

In the first case (the problem as originally stated), X_1, X_2, X_3, and X_4 are all greater than zero in the optimal solution due to the presence of the market constraints and the fact that all of final demand cannot be satisfied by X_3. Suppose that one now asks which case uses the source with the highest marginal cost rather than comparing on the basis of lowest shadow price of the final demand row. The answer in the first case would be 50, the marginal cost of X_2, while in the second case it would be 40, the marginal cost of X_1, and the results of this comparison would not conflict with the comparison of objective function values.

The values of X_1, X_2, and X_4 in the optimal solution to the first case are 168.35, 168.35, and 163.29, respectively. This shows that shipments from the source whose row is omitted from the market share constraints are shorted by an amount equal to the value of the fictitious source, X_8, which is 5.06 in the optimal solution. The smaller the number of percentage points of market share allocated to the fictitious firm, the greater the quality of the market share constraints. See Appendix A for a discussion of the market share constraints in the model employed in simulations. The shares allocated X_1, X_2, and X_4 in the optimal solution to the first case are approximately 0.3367, 0.3367, and 0.3266, respectively. If the objective function coefficients of these three activities are averaged, with the allocated shares used as weights, the result if 36.835, the shadow price of the final demand row in the optimal solution to the first case. That is, the shadow price reported is a market share-weighted average of marginal cost of the three sources possessing market share constraints. It is this effect which causes the original marginal cost comparison to conflict with the objective function comparison. An understanding of this effect is necessary to an understanding of the results of simulations done with the model of Chapter II to be presented in this chapter.

Results from two sets of simulations will be presented. Both sets share the common factor of proportional variation of the absolute values of all final demand row right hand sides by decrements of five percent, with the actual 1974 levels as reported in Appendix A taken as the starting point. This variation is done in two situations. The first situation uses the version of the model referred to as the base case, as described in Chapter II and Appendix A. The second situation uses a version of the model in which the domestic market share constraints are non-binding and those capacity purchases and alumina and bauxite transfers disallowed in the base case are permitted. This second case will be referred to as the monopoly case, as the resulting industry structure in this case effectively allows domestic primary aluminum firms to collude in minimizing joint costs and maximizing total industry profits. Even in the second case, though, the share of total ingot shipments to mills and foundries going to domestic primary producers is limited. The marginal cost of secondary ingot and the share of shipments allocated to Alcan impose such limitations. One would expect, though, that the monopoly case would exhibit economically more efficient production at any output level than the base case.

This turns out to be true if the criteria for comparison are carefully selected. The columns of Table III-1 show the ambiguous results obtained from a carelessly selected criterion. This table contains the shadow price for the rows representing primary ingot at mills and foundries. The marginal cost reported is the highest dual variable value present in the solution. Due to differences between plant marginal costs and the fact that not all plants ship to all regions, this shadow price differs between regions. The rank order of regions never changes, though, and the comparison between the base case and monopoly case yields the same results as those shadow prices reported in Table III-1 for any region at any demand level. The highest shadow price was chosen for illustrative purposes. Figure III-1 presents these results graphically.

It is readily seen that the two schedules cross at a level of demand around .6 of the 1974 level. This intersection explains how average cost can behave the way it does, as shown in Table III-2, when marginal cost is higher in the monopoly case than in the base case at higher levels of demand.

The percentage decrements in primary production do not exactly match the percentage changes in the final demands as final demand is decreased. The levels of primary production shown in Table III-2 are standardized on 4874503.0, the level of domestic shipments by domestic primary producers resulting when the level of final demand equals one. Figure III-2 shows the contents of Table III-2 graphically. It can readily be seen from either the table or the figure that the monopoly

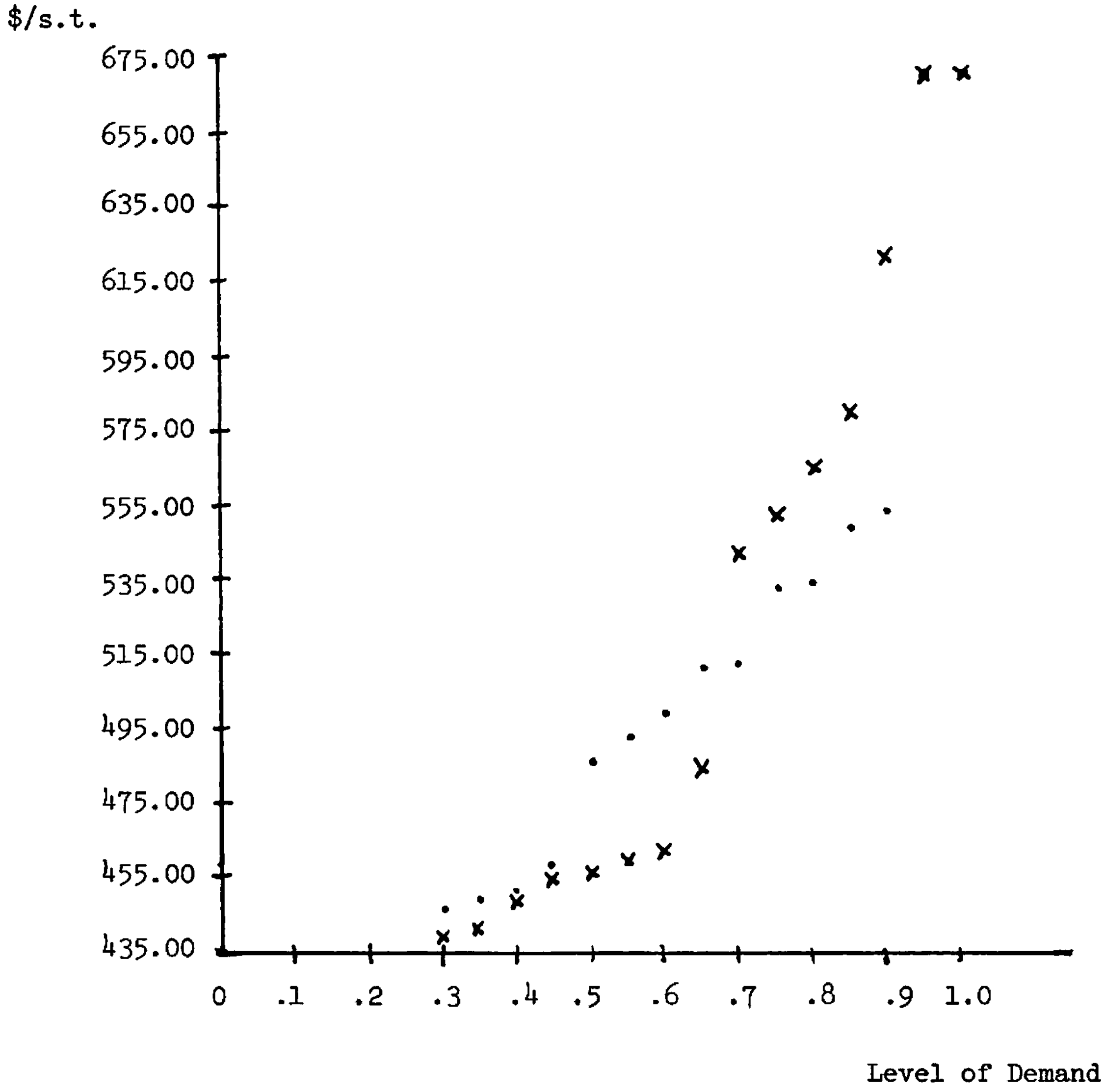
TABLE III-1

RESULTS OF SIMULATIONS, UNADJUSTED MARGINAL COSTS OF PRIMARY INGOT

Level of Demand	Raw MC Base Case ($/s.t.)	Raw MC Monopoly Case ($/s.t.)
1.00	670.33	670.33
.95	670.33	670.33
.90	552.64	621.03
.85	548.59	579.62
.80	533.14	564.57
.75	531.34	552.65
.70	512.88	542.92
.65	511.51	484.56
.60	499.12	460.42
.55	493.96	458.96
.50	485.40	455.07
.45	456.80	453.12
.40	450.48	447.62
.35	449.60	439.38
.30	446.35	438.04

FIGURE III-1

MARGINAL COST - UNADJUSTED



X - Monopoly Case

• - Base Case

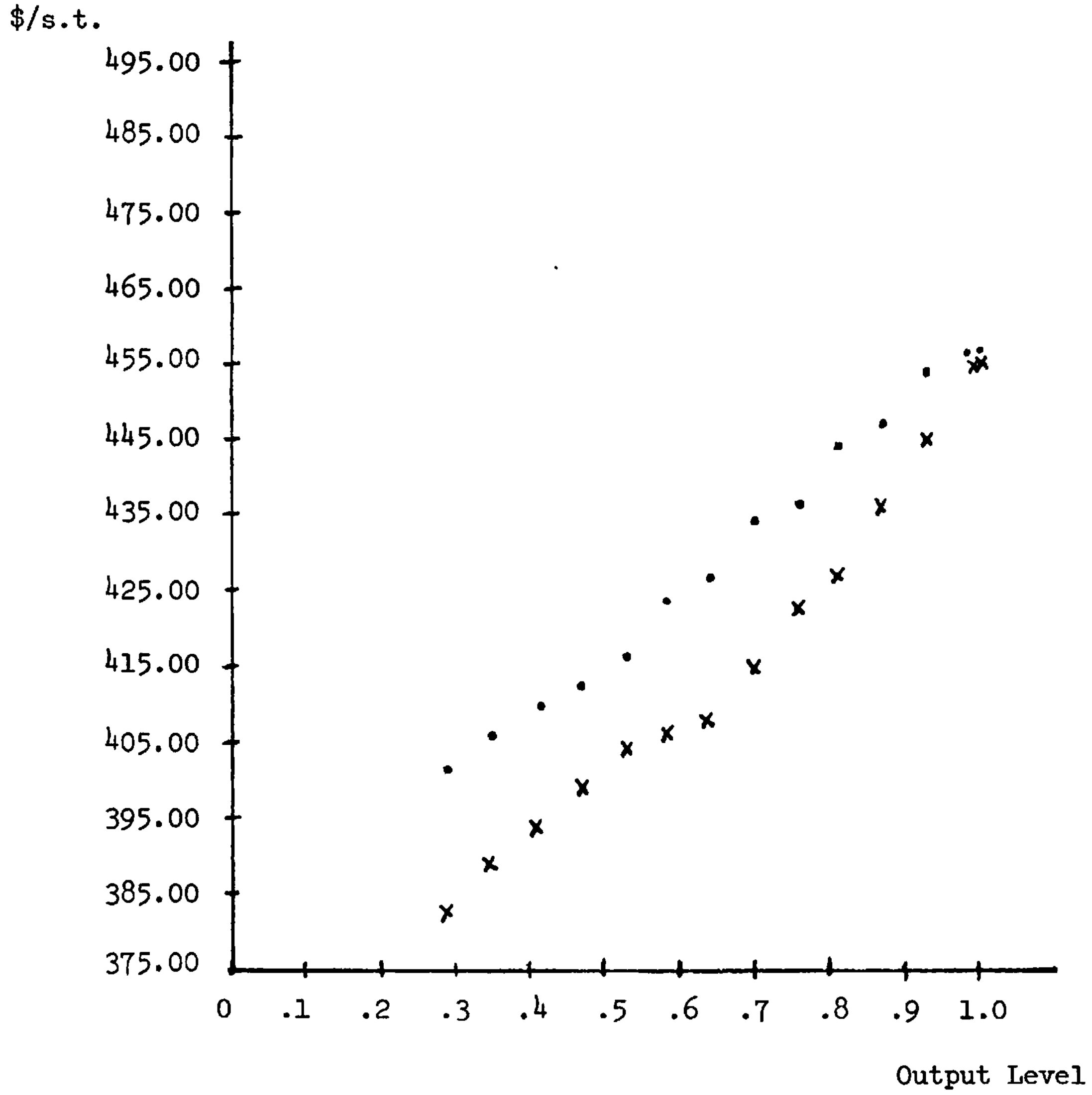
TABLE III-2

RESULTS OF SIMULATIONS
AVERAGE DELIVERED COST OF PRIMARY INGOT

Output Level	AC Base Case ($/s.t.)	AC Monopoly Case ($/s.t.)
1.00	458.85	455.64
.99	458.20	453.81
.93	453.00	444.60
.87	448.32	436.81
.81	443.19	428.65
.76	438.26	420.73
.70	433.36	414.27
.64	427.58	409.66
.58	422.33	406.38
.53	416.64	402.74
.47	412.18	398.59
.41	409.15	393.82
.35	405.93	388.58
.29	401.73	382.55

FIGURE III-2

AVERAGE COST OF PRIMARY INGOT



X - Monopoly Case

• - Base Case

case is always more efficient than the base case if a comparison is made on the basis of average cost. Average costs are not equal at a demand level of one, where all existing plants are operating at full capacity, because of the disallowance of certain transfers of bauxite and alumina in the base case. The difference in average cost shown at this demand level gives some indication of the importance at the industry level of this disallowance.

Only fourteen production levels are shown, though the two sets of simulations contain fifteen solutions to the model. This is because at final demand levels of one and .95 primary production is identical, all of the decrease occurring at the expense of secondary production. The average cost figures reported in Table III-2 for an output level of one are those resulting from a final demand level of .95. Average cost when final demand is at the 1974 level is slightly lower (a few cents) in both the monopoly case and the base case while the difference between the two is almost identical. It is lower at a final demand level of one because one higher cost transfer of primary ingot is not present in the optimal basis at this demand level.

Yet another basis for comparison is the marginal cost of the highest marginal cost plant in operation in the two cases. This basis also yields the reasonable result of greater efficiency in the monopoly case, as can be seen from Table III-3 and Figure III-3. It will be noticed that marginal cost at a demand level of one is not identical with that reported in Table III-1. This is because the figure reported by the computer and recorded in Table III-1 for this demand level is

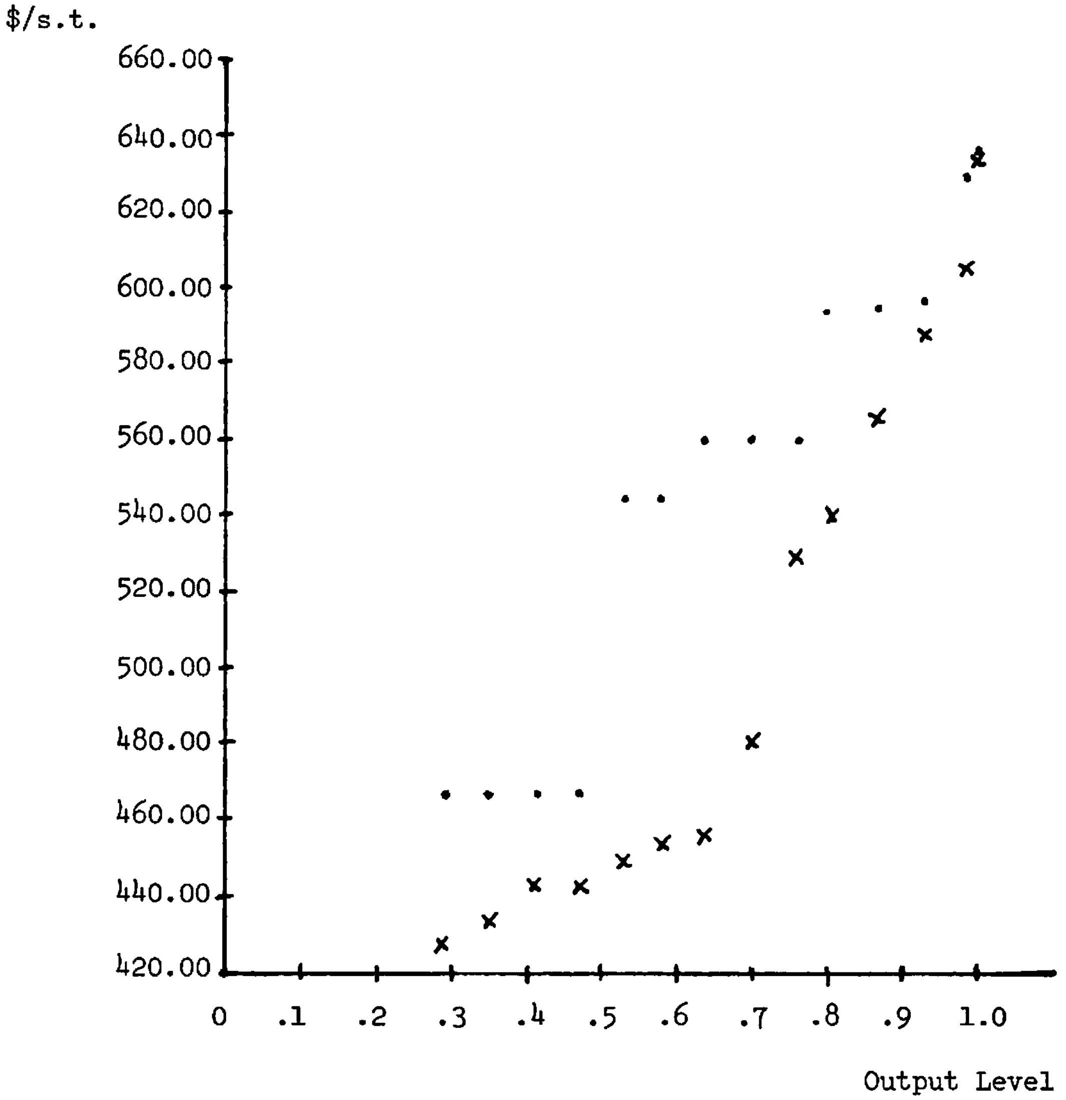
TABLE III-3

RESULTS OF SIMULATIONS
MARGINAL DELIVERED COST OF THE HIGHEST
MARGINAL COST PLANT

Output Level	MC Base Case ($/s.t.)	MC Monopoly Case ($/s.t.)
1.00	632.63	632.38
.99	627.57	603.11
.93	594.99	586.23
.87	592.63	562.84
.81	592.63	538.98
.76	558.18	528.73
.70	558.18	478.29
.64	558.18	454.72
.58	543.69	453.13
.53	543.69	448.83
.47	465.79	440.59
.41	465.79	440.59
.35	465.79	431.52
.29	465.79	425.65

FIGURE III-3

MARGINAL COST - ADJUSTED



X - Monopoly Case

• - Base Case

the unit cost of secondary ingot produced from new scrap, the high marginal cost source for mills and foundries at this demand level.

Table III-3 and Figure III-3 indicate that relatively higher marginal cost plants lose market share to the lower marginal cost plants between the base case simulations and the monopoly case simulations. In primary aluminum production, a relatively energy intensive process, high energy users tend to be high cost producers. Thus, one would expect energy use in primary production to be lower in the monopoly case than in the base case. Table III-4 shows that this expectation is correct. Energy use is identical in this table at an output level of one as this level represents full capacity utilization in the domestic industry and thus the output share of each plant is identical in the two cases. At less than full capacity utilization though, the monopoly case uses less energy than the base case. The figures for the higher output levels in the base case, which represent industry averages, compare favorably with the results of other studies of energy consumption in primary aluminum production, after compensation for differences in assumed energy conversion factors.[1] A schedule for energy consumption in the Bayer process is not presented, as energy consumption per ton of alumina does not vary much between the two cases or as demand varies for either case. Average consumption at bauxite

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[1] For a summary of such studies, see "Energy Consumption Data Base," Energy and Environmental Analysis, Inc., March 3, 1977, Vol. III, Chapter 4, Part 5, Table 3-11.

TABLE III-4

RESULTS OF SIMULATIONS
DOMESTIC BTU CONSUMPTION PER SHORT TON
OF PRIMARY ALUMINUM

Output Level	Base Case (10^6 BTU/s.t.)	Monopoly Case (10^6 BTU/s.t.)
1.00	194.30	194.30
.99	193.89	193.81
.93	192.01	190.93
.87	189.93	188.21
.81	187.61	185.02
.76	185.35	181.14
.70	183.03	177.99
.64	180.29	175.70
.58	177.51	173.51
.53	174.16	170.84
.47	170.23	165.63
.41	166.79	159.55
.35	162.42	154.69
.29	157.29	149.36

refineries is about $12.0X10^6$ BTU's per short ton of alumina, which is slightly lower than other published estimates. The figures resulting from the model are lower due to the greater utilization of co-generation and recycled waste heat in the optimal solutions of the simulations than in the actual industry.[2]

If a monopolistic primary aluminum industry is more efficient on the bases of production cost and energy use than the existing base case industry, the natural question to ask would be, "Assuming they would wish to, why not allow the domestic firms to merge into one firm?" The answer is that monopolization of the aluminum industry is fine as long as the monopolist is coerced to charge a price equal to his marginal production cost. If the monopoli is unregulated, it seems probable that a net social loss results. True, there is a gross social gain in that any level of output is obtained at a lower total cost. But comparison of the monopolist's marginal cost curve with a demand curve for primary aluminum, whose econometric details are contained in Appendix B, and its attendant marginal revenue curve suggest that the gross social loss due to lower output at a higher price in the monopoly case far outweighs any production cost savings. Since investment is allowed in the simulations, the long-run version of the demand curve is used for comparison. Figure III-4 shows the demand curve used, along with that region of the monopoly case marginal cost curve which cuts the marginal revenue schedule, the actual price and output in the industry in 1974,

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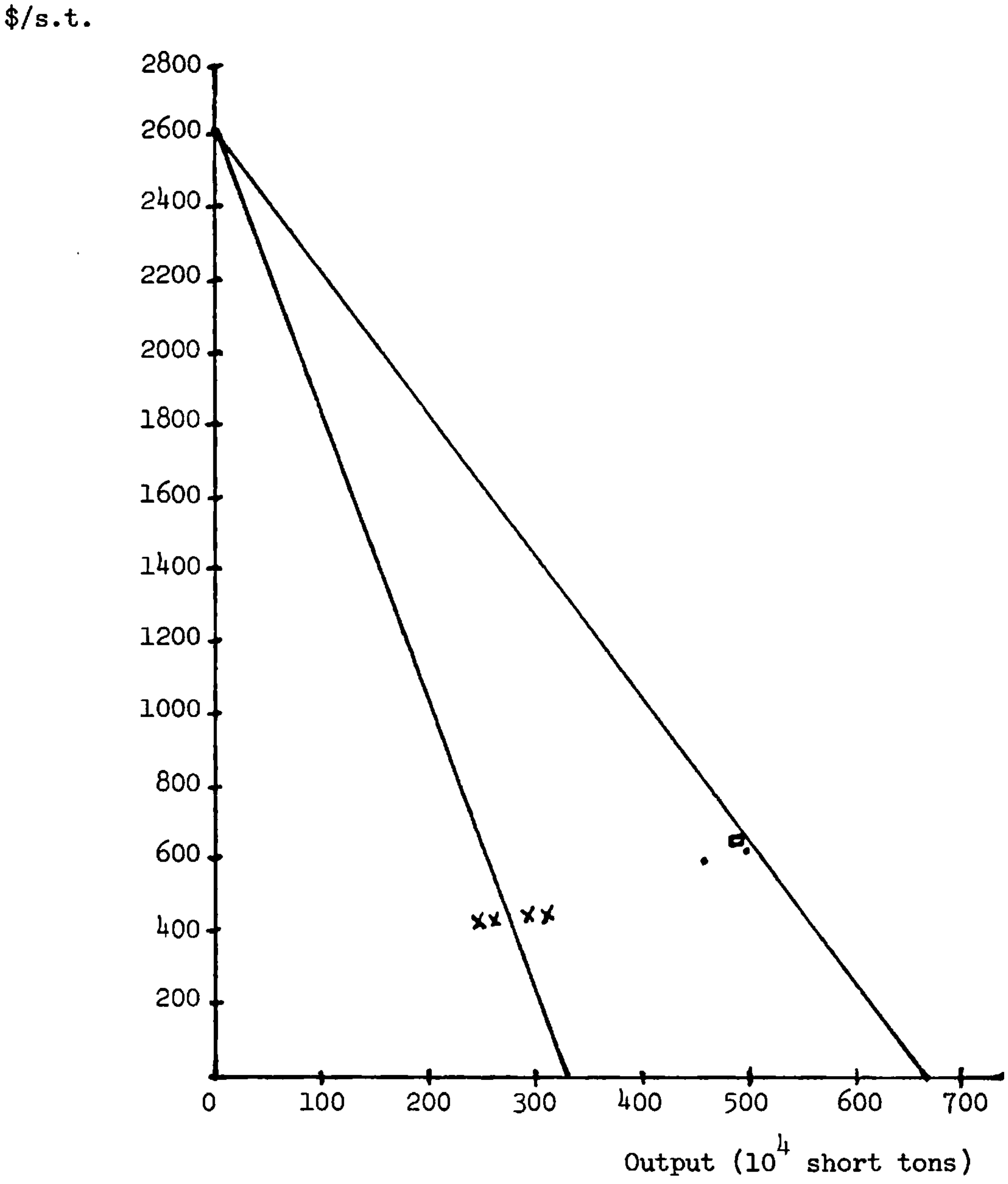
[2]Ibid, Table 3-10.

FIGURE III-4

ILLUSTRATION OF MONOPOLIST'S OUTPUT



X - Monopoly Case

• - Base Case

◻ - Actual

and the region of the base case marginal cost curve near the actual price and output. It can be seen that marginal cost is less than price in the base case, a symptom of imperfect competition, but that actual output is much greater than that level which would maximize total industry profits, a result not uncommon in models of non-cooperative oligopoly.[3]

The points on the marginal cost curves shown in Figure III-4 are those for output levels in the original sets of simulations discussed previously. It can be seen that the decrements of 5% of 1974 levels don't make a fine enough grid to pinpoint the monopolist's output and price. Simulations using finer grids (smaller decrements) in the region where the marginal cost and marginal revenue curve appear to cross reveal that the marginal revenue curve passes through a discontinuity in the marginal cost curve. Unfortunately, although the exact location of this discontinuity can be determined, it is extremely difficult to construct a solution such that it yields the primary output where the discontinuity occurs. This is because the final demands are varied proportionately and primary output is determined within the model. A level very close to the discontinuity but slightly below it is employed in the calculations to follow.[4] The difference between the two is only three and one-half tenths of one percent. Given the relative magnitudes

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[3] See Intriligator, M. D., Mathematical Optimization and Economic Theory, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971, Chapter 8, or for that matter, any good principles textbook.

[4] Output at the discontinuity is 2721917.1 short tons. The level of primary output used for welfare comparisons is 2712499.1 short tons.

of gross social loss and cost savings, this source of error is unimportant.

Production cost savings at the monopolist's output level are calculated as 4.06×10^7 dollars, the difference between total costs in the monopoly case at the monopolist's output (1.0989×10^9 dollars) and total costs in the base case at the monopoly output (1.1395×10^9 dollars). The gross social loss is calculated as the sum of lost consumer's surplus and lost producer's surplus. Lost consumer's surplus is $1/2\,\Delta P\,\Delta Q$, where ΔP is the difference between the monopolist's price and actual price and ΔQ is the difference between actual output and monopolist's output.[5] This consumer loss is calculated to be $1/2(856.4)(2149588.9) = 9.2045 \times 10^8$ dollars. The price and quantity for the monopolist is $1,537.60 per short ton and 2712499.1 short tons while the actual price is $681.20 per short ton and quantity demanded at this price is 4862088.0 short tons, approximately eight-tenths of one percent below the actual level for 1974.[6] Lost producer's surplus is the actual price times the difference between actual output and monopoly output, minus the difference between base case total variable costs at the actual output and base case total variable costs at the monopolist's output. Producer's surplus lost is then equal to $\$681.2(4862088 - 2712499.1) - \$1.09 \times 10^9) = \$3.7429 \times 10^8$. Gross social loss is then $\$1.2947 \times 10^9$. Net social loss due to monopoliza-

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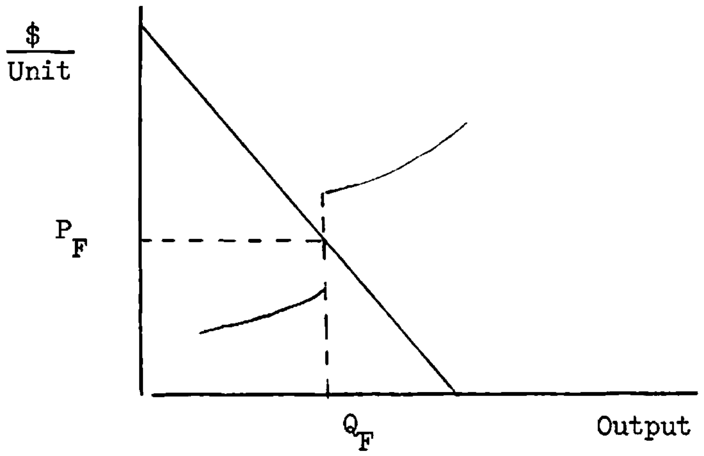
[5] The economics of this calculation may be found in any good principles text or an introductory text on public finance.

[6] See Aluminum Statistical Yearbook - 1975, Aluminum Association, p. 3.

tion, the difference between gross social loss and production cost savings is then $1,2541 x 10^9, or about thirty-eight percent of the gross value of primary output in 1974 and not an insubstantial amount.

The natural question now concerns the gains possible were regulators to set a price such that the monopolist maximizes profits by producing an output such that price is equated to marginal cost. Simulations in which the level of final demand is increased beyond the 1974 level reveal a substantial discontinuity in the marginal cost curves at that output level corresponding to full capacity utilization in primary production. The discontinuity results from the divergence between domestic and foreign production costs, as importation from domestically owned foreign capacity occurs before investment in additional domestic capacity. For those who don't feel that the marginal cost of output produced abroad is relevant for these calculations, let it be noted that an increase in production coming from new domestic capacity has the same effect on marginal production cost. The situation facing the policy maker is illustrated in the diagram below. Output level "Q_F" corresponds to full domestic capacity utilization, while price "P_F" is a price set such that the market clears, no output is produced such that marginal cost exceeds price, but all units such that price exceeds marginal cost are produced. The negatively sloped line represents the demand curve. The discontinuous curve represents the monopolist's marginal cost curve.



At price P_F, price is only slightly lower and output slightly higher than actual. Consumer surplus gain ($1/2\,\Delta P\,\Delta Q$) is only \$3.08 x 10^4, while the producer surplus gain, calculated as before, is \$1.08 x 10^6. Cost savings are a little larger as the difference between total variable costs in the base case and total variable costs in the monopoly case at the actual output level is \$2.13 x 10^7. Total gain is then \$2.24 x 10^7.

A final word is in order concerning the sensitivity of the model and problems which it does not address. A sensitivity analysis of the objective coefficients in the base case run reveals that two sets of coefficients need vary very little for the optimal basis to change.[7]

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[7] See Hadley, G., Linear Programming, Addison-Wesley Publishing Company, Menlo Park, 1963, Chapter 11.

One set is the objective function coefficients for those activities transporting aluminum-bearing materials. Most of them can vary only five to ten percent above their current level without causing a basis change. Only rail transport rates show a clear upward trend from 1974 to 1977 and only primary ingot is shipped primarily by rail. If the rail rates for primary ingot shipments are reduced by twenty-five percent and the base case simulations reproduced for demand levels of one and one-half, the level of primary shipments does not change for either demand level. Therefore, it appears that changes in single rates change only the regional pattern but not the level of primary shipments. This is not too surprising since the twenty-five percent change in rail rates translates into about a two and one-half percent (or less) change in the marginal cost of primary ingot.

The other set has only one element, the rental rate on capital equipment. It can almost double before causing a basis change, but can only decrease from its assumed value of .3 to a value of .291 before causing a basis change. It is encouraging that almost no significant investment occurs in the simulations, as the general feeling in the industry is that further investment in domestic capacity is unwarranted. Perhaps the fact that the apparent markup on average variable cost on primary ingot in 1974 was a little greater than 30 percent gives some room for error on the low side for this coefficient.

The only investment present in simulation solutions for either case is in co-generation, waste heat recycling, and electricity generating capacity. That is, the release of proprietary smelting technology in the monopoly case does not induce investment in new smelting capacity. Only those smelters already endowed with generating capacity choose to invest in further capacity. However, the investment allowed in the simulations is limited in that only capacity of existing plants may be expanded. No plants may be built at other than existing locations. This factor may cause understatement of the amount of investment that would be undertaken.

As well, the cost-benefit calculations of this chapter reflect external costs and external benefits only to the degree that potential externalities have already been internalized in market prices. Since the calculations involve a comparison of the base case and monopoly case, it might be argued that the difference in externalities in the two cases is minor. If that is not the case, one can place less confidence in the welfare conclusions being drawn.

Finally, all welfare calculations contained herein are the result of a very narrow partial comparative static equilibrium analysis and do not address the issues which arise in discussions of "second-best" policy prescriptions.

APPENDIX A
DATA DESCRIPTION

This appendix is for those unfamiliar with the aluminum production process and those curious about the values of coefficients used in simulations. The presentation is organized around Figure I-1, a section of this appendix being devoted to a description of the activities in the model which represent each major production stage. Primary and then secondary production is described from bauxite and scrap to smelter shapes. Milling and casting also have a section. A description of the production process being modelled introduces each section. Table A-1 contains a list of non-aluminum-bearing input prices and BTU-conversion factors, except for smelter-purchased electricity.

SECTION I
BAUXITE MINING AND REFINING

Bauxite, the only ore of aluminum now used in non-experimental plants, is found in temperate and tropic zones all over the world.[1] The major form of mining is open-pit.[2] The majority of primary aluminum

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[1] Hill, V. G., "Bauxite and the Aluminum Industry - Reserves and Technological Alternatives," <u>Materials and Society</u>, 1977, Pergamon Press, Great Britain, pp. 3-6.

[2] Hibbard, W. R., Jr., "U.S. Aluminum Industry - Mineral to Wire," pp. 1-2.

TABLE A-1

NON-ALUMINUM BEARING INPUT PRICES
(OTHER THAN SMELTER-PURCHASED ELECTRICITY)
AND BTU-CONVERSION FACTORS

Input	Unit	Price per Unit (dollars)	BTU Equivalent (10^6 BTU)
Limestone	short ton	2.00[2]	-
Labor	hour	6.00[2]	-
Distillate Oil	gallon	.29[1]	.1387[5]
Residual Oil	gallon	.28[1]	.1497[5]
Natural Gas	10^3 cubic ft.	.698[1]	1.035[5]
Coal	short ton	22.24[1]	25.8[5]
Pitch	gallon	.24[1]	.16[6]
Petroleum Coke	short ton	58.75[3]	30.0[6]
Electricity	10^3 kwh	20.2[4]	10.5[5]

SOURCES:

1 "Energy Consumption Data Base, Vol. III, Chapter 4, Part 5, Primary Metals Industry, SIC33," Energy and Environmental Analysis, Inc., March 3, 1977, p. 227.

2 "Policy Implications of Producer Country Supply Restrictions: The World Aluminum Bauxite Market," Charles River Associates, Cambridge, Mass., 1976, p. 34.

3 Ibid, p. 39.

4 Statistical Yearbook of the Electric Utility Industry, Edison Electric Institute.

5 Elliot-Jones, M.F., "Aluminum - SIC3324 and 3352," in Intensive Studies of Energy Use in Manufacturing Industries, p. 572.

6 "Energy Use Patterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flowsheets, High Priority Commodities), June 7, 1975, p. 10.

smelted in the U.S. is produced from ores originating in foreign bauxite mines.[3] Imported bauxite is usually transported on chartered freighters.[4] Imported bauxite, along with any mined domestically, is refined to alumina (Al_2O_3) in plants located along the Gulf Coast and near the domestic mines in Arkansas. Domestic ores are of lower grade than most foreign ores and require a special refining process before they are economical.[5]

The process for refining bauxite to alumina included in the model is the American Bayer process. In that process, bauxite is crushed and screened and mixed with a caustic soda solution. Steam is applied to this solution, which is then filtered. The filtrate, called green liquor, is then pumped to large tanks and seeded with crystals of aluminum trihydrate ($Al_2O_3 \cdot 3H_2O$). The sodium aluminate in the green liquor is then transformed to sodium hydroxide and alumina trihydrate. The trihydrate crystals are then washed and the spent liquor is recycled after ingredient makeup additions.

The presence of silica in the ore causes a proportionate loss of alumina and soda in the waste material known as red mud, which results from filtration of the steam-treated solution. It is high silica content which causes domestic ores to require a special refining process.

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[3] "Bauxite," Minerals Yearbook, 1974, Bureau of Mines, pp. 201-202.

[4] Phone conversation with industry representatives.

[5] "Aluminum," Minerals Facts and Problems, 1976, Bureau of Mines, pp. 38, 46.

After washing, the crystals are calcined to remove water of hydration in calciners of two types, rotary kiln and fluidized bed.[6] Use of fluidized bed calciners reduces energy requirements of the calcination stage but is currently employed only in Alcoa's domestic and foreign refineries.[7]

The calcined alumina is then shipped by rail or barge to primary smelters where, along with any imported alumina, it is electrolytically reduced to aluminum. This reduction is described in the next section.

There are eight domestic and three foreign sources of alumina and one domestic and three foreign sources of bauxite in the model. Foreign mining and refining capacities of domestic firms were lumped together so that, for instance, Alcoa and Revere transport alumina from the same Jamaican refinery in the model while they don't in the real world. All domestic capacity is modelled at the plant level. The location and capacities of foreign and domestic bauxite mines and refineries, along with ownership where appropriate, are found in Tables A-2 and A-3.

The objective function coefficients for those activities transporting bauxite from mines to refineries appear in Table A-4. Reference codes for Table A-4 may be found in Table A-2 and Table A-3.

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6 "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Battelle Columbus Laboratories, pp. 148-153.

7 "Alcoa Saves Energy on the Way to Aluminum with Fluid Flash Calciners," <u>Engineering and Mining Journal</u>, April, 1974, p. 23.

TABLE A-2

BAUXITE SOURCES AND CAPACITIES
ON AN ANNUAL BASIS

Code	Source	Capacity (short tons)
M1	Arkansas	1,080,000.0
M2	Jamaica	13,800,000.0
M3	Surinam	3,800,000.0
M4	Guinea	4,500,000.0

SOURCE: Hill, V. G., "Bauxite and the Aluminum Industry - Reserves and Technological Alternative," Materials and Society, 1977, Pergamon Press, Great Britain, p. 3.

TABLE A-3

ALUMINA SOURCES, OWNERSHIP, AND CAPACITIES
ON AN ANNUAL BASIS

Code	Source	Ownership	Capacity (short tons)
R1	Australia		4,705,000.0
R2	Jamaica		3,110,000.0
R3	Surinam		1,500,000.0
R4	Mobile, AL	Alcoa	1,025,000.0
R5	Point Comfort, TX	Alcoa	1,350,000.0
R6	St. Croix, VI	Martin Marietta	370,000.0
R7	Baton Rouge, LA	Kaiser	1,025,000.0
R8	Gramercy, LA	Kaiser	800,000.0
R9	Burnside, LA	Ormet (Conalco)	600,000.0
R10	Hurricane Creek, AR	Reynolds	840,000.0
R11	Corpus Christi, TX	Reynolds	1,385,000.0

SOURCE: "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, pp. 201, 208-210.

TABLE A-4

DELIVERED UNIT COSTS OF BAUXITE BY SOURCE AND DESTINATION IN DOLLARS PER SHORT TON

Destination	Source			
	M1	M2	M3	M4
R1	-[1]	-	-	-
R2	-	-	-	-
R3	-	-	-	-
R4	13.18	22.67	21.06	33.12
R5	14.15	22.67	21.06	33.12
R6	14.28	21.56	19.96	33.12
R7	11.28	22.67	21.06	33.12
R8	11.53	22.67	21.06	33.12
R9	11.46	22.67	21.06	33.12
R10	9.83	24.51	22.91	34.97
R11	14.31	22.67	21.06	33.12

[1] Dash denotes not applicable.

As stated in Chapter II, these coefficients are the sum of mining costs, transport costs, and, when applicable, bauxite export taxes. Table A-5 contains mining cost estimates employed.

The three countries exporting bauxite in the model are all members of the International Bauxite Association, an international cartel of bauxite exporting countries.[8] All three levy a bauxite severence tax on exports according to the following formula:

$$T = a + p(P_{AL})(Alcon)$$

where T is the tax-per-ton on export bauxite, a is a fixed charge, p is a stated percentage, P_{AL} is the price of U.S. primary ingot, and Alcon is the aluminum content of the bauxite being taxed. Table A-6 contains the assumed values of p, a, and Alcon used. The price of ingot assumed is $681.20.[9]

Bauxite and alumina ocean freight rates are almost identical,[10] and were used interchangeably in the calculation of coefficients. All ocean freight rates are charter rates obtained from Shipping Statistics and Economics for late 1977 and early 1978. As stated in the introduction, transport costs are all at the late 1977 to early 1978 level.

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[8] For more on the potential effects of this cartel, see Alan, Mary M., "Bauxite Aluminum Industry: U.S. Technology Transfer to Resource - Rich Developing Countries," National Science Foundation, Washington, D.C., 1975.

[9] This is the average annual price per ton of virgin ingot listed in Metals Statistics, 1977, The American Metal Market Co., New York, 1977, p. 26.

[10] Industry source.

TABLE A-5

UNIT BAUXITE MINING COSTS BY SOURCE
IN DOLLARS PER SHORT TON

	Source			
	M1	M2	M3	M4
Cost	9.83[1]	7.63[2]	7.61[2]	7.61[2]

SOURCES:

[1] "Buaxite," Minerals Yearbook, Bureau of Mines, 1974, p. 203.

[2] "Policy Implications of Producer Country Supply Restrictions: The World Aluminum Bauxite Market," Charles River Associates, Cambridge, Mass., 1976, p. 21.

TABLE A-6

BAUXITE TAX SCHEDULE VALUES BY COUNTRY

Country	a	p	Alcon
Jamaica	.55[1]	.075[1]	.2326[1]
Surinam	.50[1]	.06[1]	.2532[2]
Guinea	.50[3]	.074[3]	.2326[2]

SOURCES:

1 "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, pp. 214-215.

2 Estimated (see Tables A-8 and A-9 and text).

3 Estimated from difference between average value reported in Minerals Yearbook and production and transport cost estimates in source of note 2 to Table A-5.

This is true due to the difficulty of obtaining transport cost data. Current figures were difficult enough to obtain, much less figures for 1974. The only way to substitute away from transportation in the model is through secondary production, as all secondary smelting occurs in the same region to which a secondary smelter ships its output and so incurs no transport cost. Since transportation is ten percent or less of the value of commodities transported in the model, it is hoped any distortion is small. At least relative transport costs within the primary production block are probably measured accurately. No straightforward indexing method for these costs was apparent so no deflation was attempted.

In Table A-4, shipments from M1 and shipments to R10 contain a barge rate as part of transport costs. Bauxite and alumina are bulk exempt commodities and barge shipping rates on them are not regulated nor are companies carrying such commodities required to report rates actually charged.[11] Barge rates in the model are estimated through the following procedure. A company representative provided an estimate for the rate charged a particular aluminum company for a particular haul. Estimates for the rates charged for other hauls were obtained by assuming that the rate charged is proportional to mileage, with the exception of Intracoastal Waterway shipments. Computation of mileage between shipping point and destination was accomplished with the aid of

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[11] Shipping industry source.

navigational tables.[12] The rate per ton-mile obtained from the industry representative's estimate was then multiplied times mileage between other points to obtain barge shipping rates for bauxite and alumina with a one-dollar fixed charge added to Intracoastal Waterway shipments. The Light List referenced in the notes to the text, along with conversations with port authorities and aluminum company personnel allowed determination of which shipments are by barge and which are by rail.

Interfirm transfers of bauxite between primary aluminum producers vertically integrated to the mining stage are prohibited in the base case run of Chapter III. Table A-7 shows which transfers are allowed in the base case. Once again, the reference code used in Table A-7 may be found in Tables A-2 and A-3. The exact source for R9 was not known so it is allowed to buy least-cost bauxite, except for domestically produced bauxite which requires special processing equipment. All other entries in A-7 are based on ownership of a mine in the producing country.[13] Ownership might be whole or in partnership with other firms.

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[12] Mileages were computed using the following sources:
Light List, Volume V, Mississippi River System, Dept. of Transportation, United States Coast Guard, 1976
"Table of Distances, Gulf Intracoastal Waterway from Harvery Lock, La., to Carrabelle, Fla.," Dept. of the Army, U.S. Army Corps of Engineers
"Table of Distances, Gulf Intracoastal Waterway New Orleans District," Dept. of the Army, U.S. Army Corps of Engineers
"Mileages - Gulf Intracoastal Waterway Main Channel," Dept. of the Army, U.S. Army Corps of Engineers

[13] See Hill, V. G., "Bauxite and the Aluminum Industry - Reserves and Technological Alternatives," Materials and Society, Pergamon Press, Great Britain, 1976, p. 3., and "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, p. 212.

TABLE A-7

BAUXITE TRANSPORT NETWORK FOR BASE CASE
BY SOURCE AND DESTINATION

Destination	Source			
	M1	M2	M3	M4
R1	-[1]	-	-	-
R2	-	-	-	-
R3	-	-	-	-
R4		X[2]	X	X
R5		X	X	X
R6				X
R7		X		
R8		X		
R9		X	X	X
R10	X	X		
R11	X	X		

[1] Dash denotes not applicable

[2] "X" denotes that the transfer implied by a column and row intersection is allowed

The characteristics of bauxite refining are shown in Tables A-8 and A-9 by ore grade. Bauxite from a producing country was assumed to be of constant grade but, as is evident from Table A-8, grades differ between countries. In the base case run, both optimality considerations and the constraints placed on transfer of bauxite determine which refineries process a particular bauxite type.

Two ancillary processes to bauxite refining, steam generation, and lime production are explicitly modelled. The possibility of co-generation of steam and electricity at refineries enters by way of activities which furnish plants with co-generation capacity. The characteristics of steam generation, lime production, and co-generation are described in Tables A-10, A-11, and A-12, respectively.

The capacities of domestic refineries may be expanded through investment in digestion capacity, calcination capacity, and, if justified, co-generation facilities or waste-heat using boilers. Calcination capacity endowments of domestic refineries are in Table A-2. Bauxite digestion capacity is obtained by multiplying calcination capacity by 1.7345, the number of units of alumina trihydrate per unit of calcined alumina. Initial endowments in all waste-heat using boilers and co-generation facilities are set at zero. Per unit purchase prices of these types of capacity appear below.

The reader is here reminded that all capacity purchase prices are multiplied by a rental rate before entering the objective function. The rental rate assumed for the simulations is three-tenths as thirty percent was taken to be a representative average pre-tax required rate

TABLE A-8

CHARACTERISTICS OF ACTIVITIES PRODUCING ALUMINA-TRIHYDRATE FROM BAUXITE BY SOURCE OF BAUXITE

Input	Unit[1]	Source			
		M1	M2	M3	M4
Bauxite	short ton	1.6015[2]	1.2533[2]	1.1766[2]	1.2843[6]
Lime	short ton	.04[2]	.04[2]	.04[2]	.04[2]
Low-pressure steam	short ton	1.90[3]	1.90[3]	1.738[3]	1.738[3]
Cost	dollars	10.48[4]	8.82[4]	10.00[4]	8.82[4]
Electricity	10^3 kwh	.061[5]	.061[5]	.061[5]	.061[5]

SOURCES:

[1] Matrix coefficients listed in this and subsequent tables are in units of input per unit of output. The "Unit" column gives units for the input's row. Outputs, unless otherwise noted, are in short tons.

[2] "Evaluation of the Theoretical Potential for Energy Conservation In Seven Basic Industries," Batelle Laboratories, pp. 147-150.

[3] Steam requirements vary with ore type. The coefficients used are calculated to account for this and are drawn from the source listed in not 2 and from the following two sources:
"Revised and Updated Cost Estimates for Producing Alumina from Domestic Raw Materials," Bureau of Mines, 1975.
"Potential for Energy Conservation in Nine Selected Industries," Gordian Associates, 1974.

[4] "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, p. 203 and "Policy Implications of Producer Country Supply Restrictions: The World Aluminum Bauxite Market," Charles River Associates, Cambridge, Mass., 1976, p. 34. The differences in objective function coefficients between ore grades is due to differences in makeup soda requirements caused by differences in silica content of the ores (see text).

[5] "Energy Use Patterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flowsheets, High Priority Commodities)," June 7, 1975, pp. 9-10.

TABLE A-8
SOURCES

[6] Alan, Mary M., "Bauxite Aluminum Industry: U.S. Technology Transfer to Resource-Rich Developing Countries," National Science Foundation, Washington, D.C., 1975, p. A-5.

TABLE A-9

CHARACTERISTICS OF ACTIVITIES PRODUCING CALCINED ALUMINA
BY TYPE OF CALCINER

		Type	
Input	Unit	Rotary Kiln	Fluidized Bed
Alumina trihydrate	short ton	1.7345[1]	1.7345[1]
Natural Gas	10^3cubic ft.	4.0[2]	2.8[5]
Labor	hour	1.1[3]	1.1[3]
Cost	dollars	1.27[4]	1.27[4]
Non-Al-bearing output			
Recyclable waste heat	10^6 BUT	.2[5]	.2[5]

SOURCES:

1 "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle Laboratories, p. 150.

2 "Energy Use Patterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flowsheets, High Priority Commodities)," June 7, 1975, p. 10.

3 "Revised and Updated Cost Estimates for Producing Alumina from Domestic Raw Materials," Bureau of Mines, 1975, p. 51.

4 "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, p. 203.

5 "Alcoa Saves Energy on the Way to Aluminum with Fluid Flash Calciners," Engineering and Mining Journal, April, 1974, p. 23.

TABLE A-10

CHARACTERISTICS OF STEAM GENERATION ACTIVITIES
BY TYPE OF STEAM AND ENERGY SOURCE[1]

		Steam Type/Energy Source						
		High-Pressure			Low-Pressure			
Input	Unit	Oil	Gas	Coal	Oil	Gas	Coal	Waste Heat
Distillate Oil	gallon	21.62[2]			18.02[2]			
Natural Gas	10^3 cubic ft.		2.9[2]			2.415[2]		
Coal	short ton			.12[2]			.097[2]	
Recyclable Waste Heat	10^6 BTU							2.875[3]

SOURCES:

[1] The output of one unit of a high-pressure boiler is one ton of high-pressure steam. The output of one unit of a low-pressure boiler is one unit of low-pressure steam.

[2] "Major Fuel Burning Installation Pricing Information for Boilers and Combustors," Stewert Associates, Inc., November, 1976, figures 1 and 2.

[3] Assumed 80% efficient.

TABLE A-11

CHARACTERISTICS OF LIME-PRODUCING ACTIVITY

Input	Unit	Level in Activity
Distillate Oil	gallon	.936[1]
Natural Gas	10^3 cubic feet	1.681[1]
Coal	Short ton	.1006[1]
Limestone	Short ton	1.896[2]

SOURCES:

[1] "Potential For Energy Conservation in Nine Selected Industries," Gordian Associates, April, 1974, p. 162.

[2] "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle-Columbus Laboratories, p. 150.

TABLE A-12

CHARACTERISTICS OF CONGENERATION FACILITIES
BY ENERGY SOURCE AND TYPE OF TURBINE

Input	Unit	Turbine Type		
		Steam	Oil-fired Gas	Gas-fired Gas
Distillate oil	Gallon		32.78	
Natural gas	10^3 cubic ft.			4.39
High-pressure steam	Short Ton	1.0		
Cost	Dollars	.22	1.85	1.85

Output	Unit	Turbine Type		
		Steam	Oil-fired Gas	Gas-fired Gas
Low-pressure steam	Short Ton	1.0	1.0	1.0
Electricity	10^3 kwh	.0726	.462	.462

Source: "A Study of Inplant Electric Power Generation in the Chemical, Petroleum Refining and Paper and Pulp Industries," Thermo-Electron Corporation, pp. 3-1 to 3-26.

of return in U.S. industry.

Bauxite digestion capacity costs $214.78.[14] Bed calcination capacity is assumed to cost $27.46.[15] Both of these types of capacity are measured in short tons of output. Only Alcoa is allowed to purchase bed calcination capacity in the base case run of Chapter III. Oil-fired gas turbines and gas-fired gas turbines cost $12.09 and $10.36 per unit of capacity, respectively.[16] Capacity of turbines is measured as a pool of kilowatt-hours, not as kilowatt potential. Thus, one unit of activity, gas-fired gas turbine, uses .462 units of capacity (see Table A-12). The same holds for steam turbines, which cost $3.41 per unit of capacity.[17] High-pressure boiler prices vary by energy source as well. Oil-fired high pressure boilers are $2.25 per unit, gas-fired boilers are $2.21 per unit, while coal-fired boilers cost $3.37 per unit, capacity being measured in short tons of high-pressure steam.[18] Waste

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[14] "Environmental Considerations of Selected Energy Conserving Manufacturing Process Options: Vol. VIII. Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge, Mass., 1976.

[15] "Revised and Updated Cost Estimates for Producing Alumina from Domestic Raw Materials," Bureau of Mines, p. 22.

[16] "A Study of Inplant Electric Power Generation in the Chemical, Petroleum Refining and Paper and Pulp Industries," Thermo-Electron Corporation, pp. 3-24 and 3-26.

[17] Ibid.

[18] "Major Fuel Burning Installation Pricing Information for Boilers and Combustors," Stewert Associates, Inc., November, 1976, figures 1 and 2.

heat boiler capacity is measured in units of short tons of low-pressure steam and costs $2.88.[19] Where appropriate, capacity costs have been deflated to the 1974 level using the Wholesale Price Index for Machinery and Equipment.

Information on alumina transport costs is contained in the next section, with the exception of foreign production cost of alumina. This figure is included in the objective function coefficient of the activity which transports alumina produced at foreign refineries to primary smelters in North America. The costs assumed are $76.76 for Jamaican alumina, $62.34 for Surinam alumina, and $70.10 for Australian alumina.[20] All costs are costs per short ton of calcined alumina.

SECTION II
PRIMARY ALUMINUM SMELTING

Except for experimental plants, all of the primary aluminum production in the U.S. results from the electrolytic reduction of alumina by the Hall-Heroult process. In this process alumina is dissolved in a molten salt bath mainly consisting of cryolite. The anode and cathode of the mechanism are carbon, the anode sometimes resting no

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[19] "A Study of Inplant Electric Power Generation in the Chemical, Petroleum Refining and Paper and Pulp Industries," Thermo-Electron Corporation, p. 3-26.

[20] "Bauxite," Minerals Yearbook, Bureau of Mines, 1974, p. 206.

more than two inches above the cathode through the electrolytic bath may be fourteen inches deep. Molten aluminum is formed at the cathode while oxygen combines with carbon in the anode to form carbon dioxide so that the anode is consumed in the process.[21]

The two variants of the Hall process differ in the method anode replacement. The Soderberg technique employs a continuously replaced anode which is baked by process heat. An anode paste of calcined coke and pitch binder is fed into the top of a steel shell which extends into the bath. As the baked section of the anode near the bottom is consumed, paste flows down and is hardened by the heat of the bath and the reduction process. Current is fed into the anode through metal pins inserted into the anode either horizontally or vertically.[22]

The other variant employs fixed anodes which are pre-baked in gas-fired ovens and then suspended in the bath. This method requires periodic replacement of the anodes but is technically more efficient than the Soderberg technique in that it uses less electricity and carbon. Approximately two-thirds of U.S. primary capacity employs pre-baked anodes.[23]

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[21] "Aluminum," Minerals Facts and Problems, Bureau of Mines, 1976, pp. 49-50.)This is the best source for a summary introduction to market structure and technology in aluminum production.)

[22] Ibid.

[23] Elliot-Jones, M. F., "Aluminum - SIC 3324 and 3352," in Intensive Studies of Energy Use in Manufacturing Industries, pp. 532-534.

Molten aluminum is periodically removed from an electrolytic cell, or "pot", and blended with the output of other "pots" from the "potlines" to assure quality consistency. It is held in gas-fired holding furnaces until cast into ingots of various types which are held until shipment or further processing.[24] Some deliveries of molten metal are made but these are not significant.[25]

Primary smelter location is based on availability of large pools of cheap energy, smelters not necessarily locating near their customers.[26] Indeed, transcontinental shipments from smelters located in the Pacific Northwest to Eastern-based customers are substantial.[27] The locations and capacities of the thirty-one domestic smelters and one foreign smelter explicitly modelled appear in Table A-13.

The domestic primary aluminum industry was split into eleven firms for the purpose of market share constraint construction. The split was not based upon nominal control of capacity, a basis which would have resulted in twelve firms, but upon majority ownership of a nominal firm's capacity. Thus, Ormet Corporation does not appear but

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[24] "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle Columbus Laboratories, p. 156.

[25] Industry source.

[26] "Environmental Considerations of Selected Energy Conserving Manufacturing Process Options: Vol. VIII. Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge, Mass., 1976, p. V.

[27] Industry source.

TABLE A-13

LOCATION AND CAPACITIES OF DOMESTIC AND FOREIGN SMELTERS EXPLICITLY MODELLED BY TYPE OF ANODES

Reference Code	Location	Capacity (short tons) Soderberg	Pre-Baked
S1	Alcoa, TN	43200.0	230200.0
S2	Badin, NC		120200.0
S3	Evansville, IN		280000.0
S4	Massena, NY		134500.0
S5	Point Comfort, TX	185000.0	
S6	Rockdale, TX		284400.0
S7	Vancouver, WA		114700.0
S8	Wenatchee, WA		180000.0
S9	Columbia Falls, MT	180000.0	
S10	Sebree, KY		120000.0
S11	Lake Charles, LA		36000.0
S12	New Johnsonville, TN		141000.0
S13	Frederick, MD		88000.0
S14	Ferndale, WA		260000.0
S15	Chalmette, LA	260000.0	
S16	Mead, WA		220000.0
S17	Revenswood, WV		163000.0
S18	Tacoma, WA	81000.0	
S19	Goldendale, WA	115000.0	
S20	The Dalles, OR	90000.0	
S21	Hawesville, KY		180000.0
S22	New Madrid, MO		70000.0
S23	Hannibal, OH		250300.0
S24	Scottsboro, AL		112000.0
S25	Arkadelphia, AR	50500.0	17500.0
S26	Corpus Christi, TX	114000.0	
S27	Jones Mills, AR		125000.0
S28	Listerhill, AL	202000.0	
S29	Longview, WA	210000.0	
S30	Massena, NY	126000.0	
S31	Troutdale, OR		130000.0
S32	Southwestern Quebec	999999.0	

SOURCE: "Primary Aluminum Plants, Worldwide, Part One - Detail," Bureau of Mines, 1977, except for the smelter located in Quebec. This smelter represents Alcan's capacity. Utilization of this capacity is determined by Alcan's market share constraint. Alcan is provided an amount of primary capacity that is less than its actual total but that is sufficient to ensure model feasibility without requiring investment by the Canadian firm.

is instead included as part of Conalco's capacity. Conalco in turn is controlled by Alusuisse, as one finds from Moody's Index. The remaining allocations come from Aluminum Statistical Review, 1975 of the Aluminum Association, with the exception of the Eastalco and Intalco plants. Half of each of these plants is owned by Howmet, a subsidiary of Pechiney Ugine Kuhlmann, the French firm and half is owned by ALUMAX, a consortium of American and Japanese interests.[28] The plants remain separate to approximate this fact. Table A-14 lists assumed plant ownership and control by reference to the codes found in Table A-13.

Using the codes in Tables A-13 and A-3, Table A-15 lists objective function coefficient values for those activities transferring alumina from bauxite refineries to primary smelters. The reader is reminded that the coefficients for foreign refineries contain foreign alumina production costs as listed at the end of Section I of this appendix.

The base case run of Chapter III prohibits the transfer of alumina between firms owning bauxite refineries, though existing joint ventures, of course, have to be allowed. The one exception to this rule is Martin Marietta, whose smelters are allowed to use Australian alumina as Martin Marietta's bauxite refinery has insufficient capacity to supply both of Martin Marietta's smelters were they to operate at full capacity. Australia is chosen as the source as it is listed as a

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[28] See Aluminum Statistical Review - 1975 and "Pechinery Ugine Kuhlmann, 1976," their corporate report.

TABLE A-14

OWNERSHIP OF DOMESTIC PRIMARY CAPACITY

Company	Plants Owned
Alcoa	S1, S2, S3, S4, S5, S6, S7, S8
Anaconda	S9, S10
Conalco	S11, S12, S23
Eastalco	S13
Intalco	S14
Kaiser	S15, S16, S17, S18
Martin Marietta	S19, S20
National-Southwire	S21
Noranda	S22
Revere	S24
Reynolds	S25, S26, S27, S28, S29, S30, S31

TABLE A-15

COSTS FOR ALUMINA TRANSPORT ACTIVITIES BY SOURCE AND DESTINATION

(An explanation of the one- to four-figure suffix is found following the table.)

	Source			
Destination	R1	R2	R3	R4
S1	95.24JS1	93.50JS1	79.08JS1	14.14R1
S2	90.16JS3	88.42JS3	74.00JS3	11.66JS3
S3	84.27JW	82.53JW	68.11JW	4.70B
S4	93.57JS3	91.83JS3	77.41JS3	15.07JS3
S5	81.10S	79.36S	64.94S	2.95B
S6	92.35JS4	90.58JS4	76.19JS4	19.86R5
S7	77.90S	81.94S	67.52S	5.18S
S8	77.90S	81.94S	67.52S	5.18S
S9	77.90S	81.94S	67.52S	5.18S
S10	84.84JW	85.72JW	71.30JW	5.24B
S11	83.12JW	81.38JW	66.96JW	2.31B
S12	83.88JW	82.14JW	67.72JW	4.50B
S13	77.90S	80.66S	66.24S	3.90S
S14	81.10S	81.94S	67.52S	5.18S
S15	81.10S	79.36S	64.94S	1.50B
S16	77.90S	81.94S	67.52S	5.18S
S17	85.96JW	84.22JW	69.80JW	6.36B
S18	77.90S	81.94S	67.52S	5.18S
S19	77.90S	81.94S	67.52S	5.18S
S20	77.90S	81.94S	67.52S	5.18S
S21	84.44JW	82.70JW	68.28JW	4.85B
S22	83.28JW	81.76JW	67.34JW	3.90B
S23	86.25JW	84.51JW	70.09JW	6.65B
S24	84.96JW	83.22JW	68.80JW	5.36B
S25	85.48JW	83.74JW	69.32JW	4.88B
S26	81.10S	79.36S	64.94S	3.06B
S27	84.76JW	83.02JW	68.60JW	4.88B
S28	94.76JS1	93.02JS1	78.60JS1	13.66R1
S29	77.90S	81.94S	67.52S	5.18S
S30	92.27JS3	108.20JS6	93.75JS6	28.81R6
S31	77.90S	81.94S	67.52S	5.18S
S32	92.27JS3	108.20JS6	93.75JS6	28.81R6

TABLE A-15

	Source			
Destination	R5	R6	R7	R8
S1	19.44R2	16.74JS1	17.14R8	13.81R1
S2	11.66JS3	11.66JS3	22.22R1	21.82R1
S3	5.62B	5.77JW	2.77B	3.03B
S4	15.07JS3	15.07JS3	32.60R6	32.66R6
S5	0	2.60S	17.62R11	18.51R11
S6	11.25R4	13.85JS4	17.62R11	18.51R11
S7	5.18S	5.18S	5.18S	5.18S
S8	5.18S	5.18S	5.18S	5.18S
S9	5.18S	5.18S	5.18S	5.18S
S10	6.19B	8.96JW	3.34B	3.59B
S11	1.66B	4.62JW	2.19B	1.94B
S12	5.48B	5.38JW	2.60B	2.85B
S13	3.90S	3.90S	3.90S	3.90S
S14	5.18S	5.18S	5.18S	5.18S
S15	2.47B	2.60S	0.40B	0.15B
S16	5.18S	5.18S	5.18S	5.18S
S17	7.30B	7.46JW	4.46B	4.71B
S18	5.18S	5.18S	5.18S	5.18S
S19	5.18S	5.18S	5.18S	5.18S
S20	5.18S	5.18S	5.18S	5.18S
S21	5.79B	5.94JW	2.95B	3.20B
S22	4.85B	5.00JW	2.00B	2.25B
S23	7.60B	7.75JW	4.75B	5.00B
S24	6.31B	6.46JW	3.46B	3.71B
S25	5.82B	6.98JW	3.98B	4.23B
S26	1.16B	2.60S	3.02B	2.62B
S27	5.82B	6.26JW	3.98B	4.23B
S28	18.19R2	16.26JS1	14.79R8	17.19R1
S29	5.18S	5.18S	5.18S	5.18S
S30	34.85R7	31.41JS6	32.60R6	32.66R6
S31	5.18S	5.18S	5.18S	5.18S
S32	34.85R7	31.41JS6	32.60R6	32.66R6

TABLE A-15

Destination	Source R9	Source R10	Source R11
S1	18.31R1	16.49R9	19.44R2
S2	21.82R1	24.01R10	22.10R2
S3	2.95B	2.00B	5.79B
S4	32.66R6	27.38R10	34.85R7
S5	18.51R11	11.06R11	11.25R4
S6	18.51R11	11.06R11	11.25R4
S7	5.18S	9.56JW	5.18S
S8	5.18S	9.56JW	5.18S
S9	5.18S	9.56JW	5.18S
S10	3.52B	2.57B	6.35B
S11	2.01B	3.64B	1.82B
S12	2.78B	1.84B	5.61B
S13	3.90S	3.90S	3.90S
S14	5.18S	9.56JW	5.18S
S15	0.22B	1.85B	2.63B
S16	5.18S	9.56JW	5.18S
S17	4.64B	3.68B	7.47B
S18	5.18S	9.56JW	5.18S
S19	5.18S	9.56JW	5.18S
S20	5.18S	9.56JW	5.18S
S21	3.12B	2.17B	4.96B
S22	2.18B	1.22B	5.01B
S23	4.93B	3.97B	7.76B
S24	3.64B	2.68B	6.47B
S25	4.16B	2.53B	5.99B
S26	2.62B	4.47B	0
S27	4.16B	2.53B	5.99B
S28	17.19R1	11.68R10	18.19R2
S29	5.18S	9.56JW	5.18S
S30	32.66R6	27.38R10	34.85R7
S31	5.18S	9.56JW	5.18S
S32	32.66R6	27.38R10	34.85R7

TABLE A-15

NOTES:

Explanation of rate suffixes in Table A-14 -

The one- or two-letter parts of the suffixes denote the mode of transportation, as follows:

R - railroad
B - barge
S - steamship
JS - joint steamship and rail
JW - joint steamship and barge

The numeric parts of the suffixes refer to the railroad freight tariffs employed in figuring rail freight rates. They are as follows:

1) SFTB 2011-P
2) SWFB 357-C, Supplement 99
3) The rail rate components of these rates use Norfolk, VA, as the rail shipping point. Rail rates from Norfolk to Badin and Messena were provided by a railroad company official.
4) SWFB 60-K, Supplement 102
5) SWFB 2007-H, Supplement 153
6) SFTB 819-G, Supplement 52
7) SWFB 2005-J, Supplement 180
8) SFTB 817-G, Supplement 50
9) SWFB 2008-L
10) SWFB 34-A, Supplement 98
11) SWFB 2004-J, Supplement 105

An explanation of the sources for barge and railroads is found in Section I of this appendix. Shipments between the Gulf Coast or Caribbean and the Pacific Northwest have an estimated rate of the sum of a Panama Canal toll of $1.28/s.t. and the rate from the north coast of South America to the east coast of the U.S.

source of alumina for those two smelters in "Aluminum Plants, Worldwide, Part One - Detail" as published by the Bureau of Mines. It is known that alumina moving between aluminum companies is bought under long-term contracts. The previously mentioned BOM source lists alumina sources for individual smelters, where known. This listing is used to determine who had contracted with whom for alumina, unless the BOM information conflicts with the prohibition on interfirm transfers between firms integrated to the bauxite refining level or unless the countless phone calls made to company representatives yielded more specific information. The results of all this detective work appear in Table A-16, which shows which transfers of alumina are allowed in the base case run.

Once the alumina arrives at a smelter, it enters the "pot" for reduction to molten aluminum. The characteristics of existing smelting capacity appear in Tables A-17, A-18, and A-19. Table A-17 shows entries for the smelting activities' material balance rows, while Table A-18 lists the electricity requirement per ton of aluminum produced for each smelter. Table A-19 contains information on the ancillary anode-producing activities which accompany electrolysis.

The molten primary aluminum drawn from the pots is taken to a holding and casting furnace for fluxing and blending. The characteristics of existing furnaces in the model are in Table A-20.

Electricity prices are specific to particular smelters in the model. This reflects the use of knowledge available concerning

TABLE A-16

TRANSFERS OF ALUMINA ALLOWED IN THE BASE CASE
RUN BY SOURCE AND DESTINATION

("X" denotes that the transfer implied by a row and column intersection is allowed)

Destination	Source										
	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11
S1		X	X								
S2		X	X								
S3	X	X	X	X	X						
S4		X	X								
S5					X						
S6				X	X						
S7	X										
S8	X										
S9		X									
S10		X									
S11			X								
S12	X	X									
S13	X										
S14	X										
S15							X	X			
S16	X										
S17							X	X			
S18	X										
S19	X					X					
S20	X					X					
S21					X			X			
S22		X					X	X			
S23									X		

TABLE A-16

Destination	Source										
	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11
S24		X									
S25		X								X	
S26											X
S27		X								X	
S28											X
S29		X									X
S30										X	X
S31		X									X
S32		X									

TABLE A-17

CHARACTERISTICS OF EXISTING ELECTROLYSIS METHODS
BY TYPES OF ANODE

Input	Unit	Anode Type: Soderberg	Anode Type: Pre-Baked
Pre-baked anode	Short Ton		.45[1]
Anode paste	Short Ton	.58[2]	
Alumina	Short Ton		1.93[1]
Cost	Dollars	81.38[3]	81.38[3]

SOURCES:

1 "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Bastelle Columbus Laboratories, , p. 154.

2 Elliot-Jones, M. F., "Aluminum - SIC 3324 and 3352," in <u>Intensive Studies of Energy Use in Manufacturing Industries</u>, pp. 533, 570.

3 "Environmental Considerations of Selected Energy Conserving Manufacturing Process Options: Vol. VIII Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge Mass., 1976, p. 54.

TABLE A-18

ELECTROCITY REQUIREMENTS OF ELECTROLYSIS BY SMELTER

Smelter	Requirement(10^3 kwh/s.t.)	Smelter	Requirement(10^3 kwh/s.t.)
S1	17.0S[1,2] 16.0P[3]	S17	16.0[3]
S2	16.0[3]	S18	18.0[4]
S3	16.0[3]	S19	13.0[4]
S4	16.0[3]	S20	17.0[2]
S5	17.0[2]	S21	16.0[3]
S6	16.0[3]	S22	16.0[3]
S7	14.0[4]	S23	16.0[3]
S8	13.0[4]	S24	16.0[3]
S9	16.0[4]	S25	17.0S[2] 16.0P[3]
S10	16.0[3]	S26	17.0[2]
S11	16.0[3]	S27	16.0[3]
S12	16.0[3]	S28	17.0[2]
S13	12.0[5]	S29	18.0[4]
S14	12.0[5]	S30	17.0[2]
S15	17.0[2]	S31	14.0[4]
S16	14.0[4]	S32	17.0[2]

TABLE A-18
NOTES AND SOURCES

NOTES AND SOURCES:

1 The two smelters with two entries have both Soderberg and pre-baked anode capacity. Soderberg electricity requirements are denoted by "S" and pre-baked by "P" for these two plants.

2 Elliot-Jones, M. F., "Aluminum - SIC 3324 and 3352," in Intensive Studies of Energy Use in Manufacturing Industries, pp. 532-533.

3 "Energy Use Paterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flowsheets, High Priority Commodities)," Batelle Columbus Laboratories, p. 11.

4 Hibbard, W. R., "U.S. Aluminum Industry - Decentralized Energy Sources and Captive Energy Sources," mimeo.

5 "Alumax-Turning Aluminum Capacity Upside Down," Business Week, March 6, 1978, p. 74.

TABLE A-19

CHARACTERISTICS OF ACTIVITIES PRODUCING
ANODES OR ANODE INGREDIENTS

		Activity		
Input	Unit	Anode Coke Production	Anode Baking	Anode Paste Production
Distillate Oil	gallon	1.154[1]		
Natural Gas	10^3 cubic ft.	1.1304[1]	4.65[3]	
Electricity	10^3 kwh	.0113[1]	.011[3]	
Pitch	gallon		63.2[3]	56.89[2]
Anode Coke	short ton		.889[2]	.8[2]
Raw Petroleum Coke	short ton	1.1108[2]		

SOURCES:

[1] "The Potential for Energy Conservation in Nine Selected Industries," Gordian Associates, New York, 1974, p. 103.

[2] "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle Columbus Laboratories, , p. 157.

[3] "Energy Use Patterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flowsheets, High Priority Commodities)," Batelle Columbus Laboratories, p. 10.

TABLE A-20

CHARACTERISTICS OF EXISTING HOLDING FURNACES

Input	Unit	Level per Unit of Activity
Natural Gas	10^3 cubic feet	4.83[1]
Molten Primary Aluminum	short ton	1.008[2]
Labor	hour	8.0[3]

SOURCES:

[1] Stephens, W. E., "Improved Methods and Equipment for Energy Savings in the Aluminum Industry," in _Aluminum Industry Conservation Workshop Papers_, Aluminum Association, p. 6.

[2] Ibid, p. 2.

[3] "Environmental Considerations of Selected Energy Conserving Manufacturing Process Options: Vol. VIII. Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge, Mass., 1976, p. 54.

the long-term contracts under which smelters purchase electricity. If specific contract terms were unknown but it was known that a smelter had some form of in-house electricity generation facilities, that smelter paid the average industrial rate for 1974. Table A-21 contains a list of electricity prices for smelters employed in the model.

Any in-house generation by smelters was modelled explicitly. Smelters known to possess such facilities were given starting capacities for generation. All others could, if optimal, purchase such capacity. The boilers generating steam for input to turbines have the same characteristics, including new capacity price, as the high pressure boilers appearing in Table A-10 in Section I of this appendix. Smelters were allowed to use only steam turbines as it was learned from conversations with industry personnel that only steam turbines are employed for thermal generation of electricity at smelters. The energy source for endowed high-pressure boiler capacity came from the same sort of source. The characteristics of the steam turbine employed at smelters appears in Table A-22, while Table A-23 contains endowed boiler capacity and endowed steam turbine capacity of smelters. Steam turbine capacity is expressed in kilowatt-hours while boiler capacity is in short tons of steam.

Two plants have their own hydroelectric generation facilities, Aloca's plants at Alcoa and Badin (S1 and S2). These plants were given initial endowments of hydroelectric generation capability, calculated by

TABLE A-21

ELECTRICITY PRICES PAID BY SMELTERS

Smelter	Price ($/$10^3$ kwh)	Smelter	Price ($/$10^3$ kwh)
S1	6.01[1]	S17	7.00[4]
S2	20.20[2]	S18	3.00[3]
S3	20.20[2]	S19	3.00[3]
S4	3.04[1]	S20	3.00[3]
S5	20.20[2]	S21	3.06[1]
S6	20.20[2]	S22	7.57[1]
S7	3.00[3]	S23	7.00[4]
S8	3.00[3]	S24	6.32[1]
S9	3.00[3]	S25	6.70[1]
S10	3.06[1]	S26	20.20[2]
S11	20.20[2]	S27	20.20[2]
S12	6.32[1]	S28	6.31[1]
S13	7.00[4]	S29	3.00[3]
S14	3.00[3]	S30	3.04[1]
S15	20.20[1]	S31	3.00[3]
S16	3.00[3]	S32	7.00[4]

SOURCES:

[1] "Energy-Economy Relationship," prepared for Bonneville Power Administration, Ernst and Ernst, June, 1976, pp. V-21 to V-25.

[2] Industrial user average price as reported in Statistical Yearbook of the Electric Utility Industry, Edison Electric Institute.

[3] "Alumax-Turning Aluminum Capacity Upside Down," Business Week, March 6, 1978, p. 74. All of these plants are located in the Northwest and pay what is considered the average BPA price for electricity.

[4] Considered an average price paid by plants not generating own electricity.

TABLE A-22

CHARACTERISTICS OF STEAM TURBINE USED BY SMELTERS

Input	Unit	Level per Unit of Activity
High-pressure Steam	short ton	1.0
Cost	dollars	.66[1]
Output Electricity	10^3 kwh	.243[2]

SOURCES:

[1] "A Study of Inplant Electric Power Generation in the Chemical, Petroleum Refining, and Paper and Pulp Industries," Thermo-Electron Corporation, pp. 3-23.

[2] "Energy Use Patterns in Metallurgical and Nonmetallic Mineral Processing (Phase 4 - Energy Data and Flow-sheets, High Priority Commodities)," Batelle Columbus Laboratories, p. 14, is used as a source for evaluating the efficiency of thermal electricity generation as a whole at smelters. Using the efficiency of the high-pressure boilers in Table A-10, steam turbine efficiency may be calculated.

TABLE A-23

INITIAL CAPACITIES OF BOILERS (BY TYPE) AND STEAM TURBINES AT SMELTERS

Smelter	Coal-Fired Boiler	Gas-Fired Boiler	Steam Turbine
S3	12298086.0[2]		2988435.0[1]
S5	8631213.9[2]		2097385.0[1]
S6		12491341.0[2]	3035396.0[1]
S11		2371773.6[2]	576341.0[1]
S15		18195534.0[2]	4421515.0[1]
S26	7978041.1[2]		1938664.0[1]
S27	3294131.6[2]		800474.0[1]

NOTE: Smelters not listed in this table receive initial endowments of zero of boiler and turbine capacity.

SOURCES:

1 "Energy Consumption Data Base, Vol. III, Chapter 4, Part 5, Primary Metals Industry, SIC33," Energy and Environmental Analysis, Inc., March 3, 1977, p. 220 lists sources of electricity used in primary smelters. Together with data from conversations with industry representatives, is is possible to calculate the capacities in the table. For instance, it was determined that two-thirds of the electricity consumed in S3 comes from steam turbines fed by coal-fired boilers. Turbine capacity for S3 is then two-thirds of the electricity S3 would use were it to operate at full capacity.

2 Boiler capacity is then that capacity necessary for S3 to use all of its turbine capacity as calculated by the procedure outlined in note 1.

the procedure outlined in the notes to Table A-23. S1 receives a pool of 2,946,688,000.0 kilowatt-hours while S2 receives 1,282,893,000.0 kilowatt-hours. The price of self-generated hydroelectricity is zero, as this is felt to reflect marginal cost.

Capacity of three energy-conserving smelting technologies can be purchased by primary smelters. One of these is a retrofit of existing capacity. In addition, the holding furnaces may be retrofitted, at a cost of $5.70 times the rental rate per unit of capacity, with a heat recuperator which preheats combustion air and lowers the natural gas requirement of the holding furnace to 3,960.6 cubic feet per ton.[29]

Two of the new smelting technologies are variations of the basic Hall-Heroult process. One, called the "New Hall" process in Table A-24, utilizes computer control of the electrolytic bath to increase energy efficiency. The other, called "TiB_2 cathodes" in Table A-24, replaces the conventional carbon cathodes with cathodes of titanium-diboride. Such a replacement reduces electricity requirements for a constant output from a "pot" two ways. First, the voltage drop at the interface between the cathode and the molten aluminum is reduced as carbon cathodes cause the formation of sludge and aluminum carbide at the interface, increasing all resistance. Second, it would be possible to leave only a thin wetting film of molten aluminum on the metal

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[29] "Using Recuperated Heat to Preheat Combustion Air," Aluminum Industry Energy Conservation Workshop Papers, Aluminum Association.

TABLE A-24

Input	Unit	Process: New Hall	Process: T_iB_2 Cathodes	Process: Alcoa
Alumina	short ton	1.93[1]	1.93[1]	1.93[1]
Electricity	10^3 kwh	12.0[1]	5	10.5[2]
Residual Fuel Oil	gallon			166.0[2]
Pre-baked Anode	short ton	.45[4]	5	
Cost	dollars	81.40[1]	96.48[3]	84.70[2]

SOURCES:

[1] "Environmental Considerations of Selected Energy Conserving Manufacturing Process Options: Vol. VIII. Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge, Mass., 1976, p. 55.

[2] Ibid, p. 69.

[3] Ibid, p. 76.

[4] "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle Columbus Laboratories, p. 154.

[5] See text.

cathode and decrease the distance between anode and cathode, thereby reducing all losses due to resistance in the electrolyte.[30]

The third new smelting technology, called "Alcoa" in Table A-24, produces aluminum through the electrolysis of aluminum chloride in an electrolytic bath of chlorine salts. In this process alumina is transformed into aluminum chloride by a two-step procedure, the first of which is carbon-coating of the alumina in a fluid-bed coking system and the second of which is actual chlorination. Electrolysis is accomplished with inert anodes, which affords an added advantage to this system in addition to its lower electricity requirement due to lower deomposition voltage and bath resistance than the conventional Hall-Heroult process.[31]

The operating characteristics of these three new smelting technologies as entered in the model appear in Table A-24. Except for electricity requirements and the objective function coefficient, "TiB_2 cathodes" is identical to existing Hall technology in any particular smelter. The new cost coefficient appears in Table A-24. The new electricity requirement coefficient is determined by multiplying the Hall electrolysis coefficient of original plant capacity (Table A-18) by .8.[32]

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[30] "Environmental Considerations of Selected Energy-Conserving Process Options: Vol. VIII. Alumina/Aluminum Industry Report," A. D. Little, Inc., Cambridge, Mass., 1976, pp. 70-71.

[31] Ibid, pp. 52-64.

[32] Ibid, pp. 71.

Initial endowments of the three new smelting technologies are set at zero, as is initial heat recuperator capacity. Capacity of the TiB_2 cathode retrofit option has a purchase price of $170.06[33] per short ton of capacity, while "new Hall" and "Alcoa" capacity is $1,473.85[34] per short ton, each, of course, multiplied by the rental rate before entering the objective function. Steam turbine capacity is available to smelters at a price of @.74 times the rental rate.

The three domestic firms allowed to import primary aluminum from foreign capacity - Alcoa, Reynolds, and Kaiser - may do so at a cost per short ton indicated in Table A-25. The destinations listed in that table are the locations of the regional mills and foundries and will be justified in Section III of this appendix. The costs shown in Table A-25 are total delivered costs of primary ingot, including production and transportation costs and the tariff levied on primary aluminum imports of $20.00 per short ton. It is assumed that production occurs in Europe or Africa, port of entry is Philadelphia, except for Charlotte, whose port of entry is assumed to be Mobile, and any further necessary transportation is by rail. Production costs are derived from average domestic production cost[34] and an assumed difference in domestic and foreign production costs of from 4 to 12 cents per pound higher abroad.[35] The average of 7 cents per pound was chosen for

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[33] Ibid, p. 76.

[34] Ibid, p. 54 with adjustments.

[35] "Aluminum Prices 1974-75," U.S. Executive Office of the President, Council on Wage and Price Stability, Washington, D.C., 1976, p. 44.

TABLE A-25

DELIVERED UNIT COST OF IMPORTED PRIMARY INGOT
BY DESTINATION

Destination	Cost ($/short ton)
Philadelphia, PA	792.63
Charlotte, NC	814.33
Birmingham, AL	808.13
Cleveland, OH	814.33
Los Angeles, CA	877.93

SOURCE: See text.

the model. Steamship transportation costs are from Shipping Statistics and Economics, March, 1978. Rail costs may be found in Table A-26. Foreign capacity endowments of Alcoa, Reynolds, and Kaiser, which they are not allowed to enlarge, appear in Table A-27. No importation other than that previously mentioned is allowed as historical figures from Minerals Yearbook indicate that other sources are unimportant. Transportation costs for the output of primary smelters are found in the next section, which concerns itself with milling and casting.

The final topic of this section is the market share constraints described in Chapter II. These constraints may seem to be a rather artificial behavioral structure. They are present in the model as a result, once again, of conversations with industry experts. It is found that when output of the big three firms falls, imports from their foreign-based smelters rise.[36] It is as if they import to maintain a share of domestic primary shipments consistent with their ownership share of domestic primary capacity. Price-cutting is the tool the minor firms are suspected of employing.[37] The market shares allocated to the eleven domestic firms (the "Mi's" of Chapter II) are in Table A-28. Alcan is allowed to import an amount of primary ingot equal to

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[36] Ibid, and "Aluminum," Minerals Yearbook, Bureau of Mines, 1974, pp. 156-160 and 154.

[37] "Aluminum Prices 1974-75," U.S. Executive Office of the President, Council on Wage and Price Stability, Washington, D.C., 1976 reports price-cutting by the minors in this period, while "Alumax-Turning Aluminum Capacity Upside Down," Business Week, March 6, 1978, p. 73 reports price-cutting by the minors in late '77 when demand slumped again.

TABLE A-26

RAIL TRANSPORT COSTS OF IMPORTED PRIMARY INGOT BY DESTINATION

Destination	Cost ($/short ton)
Philadelphia, PA	0[1]
Charlotte, NC	21.70[2]
Birmingham, AL	15.50[2]
Cleveland, OH	21.70[2]
Los Angeles, CA	85.30[3]

SOURCES:

[1] See text.

[2] Personal estimate derived from cost per ton for hauls of various lengths provided by railroad company official and estimate of railroad mileage between source and destination.

[3] TCFB 1-Z, Supplement 57.

TABLE A-27

FOREIGN PRIMARY PRODUCTION CAPACITIES OF ALCOA, REYNOLDS, AND KAISER

Company	Capacity (short tons)
Alcoa	237500.0
Reynolds	480500.0
Kaiser	334500.0

SOURCE: "Aluminum," Minerals Yearbook, Bureau of Mines, 1974, pp. 156-160.

TABLE A-28

MARKET SHARES OF DOMESTIC FIRMS

Firm	M_i (% of total shipments by domestic firms)
Alcoa	.3199755
Reynolds	.1984328
Kaiser	.1473491
Eastalco	.0179098
Intalco	.0529154
Conalco	.0869644
Noranda	.0142464
National-Southwire	.0366337
Revere	.0227943
Anaconda	.0610562
Martin Marietta	.0417217
Fictitious	.0000007

SOURCE: "Aluminum Plants, Worldwide, Part One - Detail," Bureau of Mines, 1977 provides plant capacities. After ownership is determined, market shares are calculated as the ratio of total domestic primary capacity owned by a firm to total primary domestic capacity.

ten percent of shipments by domestic firms. This is estimated from data on imports of Canadian primary aluminum found in Minerals Yearbook.

SECTION III
MILLING AND CASTING

Milled aluminum products are as diverse as one could want. They range from rolled sheet and plate to forgings to powder, the latter two representing a minor part of mill output. Other major milled product categories are extrusions, shapes formed by pressing heated aluminum through a mold and drawn shapes which are formed by puloing the stock through a mold (wire, for example). Rolled products (sheet, plate, and foil) are formed by rolling preheated ingots between large heavy rollers repeatedly until the desired thickness is attained. Rolled sheet, plate, and foil is the major category of milled products.[38]

Cast shapes, the output of foundries, fall into three major categories: permanent mold or investment castings, die castings, and sand castings. The difference between the three is quality of the cast shape's finish, each successive category possessing a rougher finish.[39]

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[38] See Aluminum Statistical Review - 1975, and The Story of Aluminum, both available from the Aluminum Association.

[39] See Aluminum Statistical Review - 1975, and call a foundry.

The right hand sides of the final demand rows in the model are determined from two sources, The Aluminum Statistical Review - 1975 and U.S. Department of Commerce studies as noted in a final report of the Institute for Energy Analysis.[40] The former provides total industry shipments of milled products and cast shapes. The latter provides the percentage of total shipments originating in each of the nine SIC regions for 1972. It is assumed that the 1972 percentages are a good approximation to the 1974 regional percentages. From there the calculation of final demand row right hand sides is a simple matter of multiplication but for one thing. There are only five regions at the milling and casting stage in the model. This is because transportation cost data generation imposes both a psychic and a time cost on the author/researcher. Regions producing what was considered to be an insignificant proportion of total milled products or cast shapes have their percentages added to the percentage of the closest significant region in order to simplify the transportation network connecting smelters with mills and foundries. An insignificant percentage is seven percent of the total or less. The results of

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[40] "An Evaluation of Certain Key Parts of the Natural Resources Defense Council Proposal to Satisfy the Future Elective Power Supply Requirements of the Pacific Northwest," Institute for Energy Analysis, September 30, 1977.

The Department of Commerce Studies quoted are Business and Defense Services Administration, Aluminum Factbook, U.S. Dept. of Commerce, Washington, updated, p. 58 and Bureau of Census, 1972 Census of Manufacturers: Consumption of Selected Metal Mill Shapes and Forms, Special Report Series, U.S. Dept. of Commerce, Washington, June, 1977.

all this aggregation and multiplication appear in Table A-29. It can be seen that aggregation resulted in five regional mills and three regional foundries, two regions having no foundry. The point sources for each region are determined by the author's judgment and his observation of maps portraying concentration of milling and casting activity found in The Aluminum Statistical Review - 1975. There are no known published figures on capacity of mills and foundries. Indeed, milling capacity in the sense capacity is used in this study may not make much sense. Therefore, mills and foundries have no capacity constraints.

The categoreis of milled products and cast shapes, as Table A-29 implies, are lumped into two categories for modelling purposes - milled products and cast shapes - so the activities producing these two products represent industry average materials and energy consumption. Table A-30 contains the characteristics of the milling and casting activities, which are identical across regions. Table A-31 shows the objective function coefficients for activities transferring primary ingot to mills and foundries from primary smelters. These coefficients represent transportation cost only.

SECTION IV
SECONDARY SMELTING

Secondary ingot produced in the U.S. results from more than a simple remelting of scrap aluminum. Scrap recycled for consumption may be classified into three categories: runaround or house or home

TABLE A-29

SHIPMENTS OF MILLED AND CAST ALUMINUM IN 1974
BY LOCATION AS ENTERED IN MODEL

Location	SIC Region	Cast Shapes (ton)	Milled Production (ton)
Philadelphia	Middle Atlantic	108337.5	1098415.5
Charlotte	South Atlantic		836888.0
Birmingham	East South Central	108337.5	679971.5
Cleveland	East North Central	650025.0	1673776.0
Los Angeles	Pacific		941499.0

SOURCE: See text.

TABLE A-30

CHARACTERISTICS OF ACTIVITIES PRODUCING
CAST SHAPES AND MILLED PRODUCTS

Input	Unit	Milling	Casting
Primary or Secondary Ingot	Short ton	1.0165[1]	1.0068[2]
Residual Fuel Oil	Gallon	.39[3]	2.12[3]
Distillate Fuel Oil	Gallon	3.9[3]	2.68[3]
Natural Gas	10^3 cubic feet	14.8[3]	19.02[3]
Coal	Short ton	.03[3]	.009[3]
Electricity	10^3 kwh	1.19[3]	1.28[3]

SOURCES:

[1] 33% of gross input assumed to result in runaround or house scrap, 95% of which is assumed recovered, Minerals Facts and Problems, BOM, 1976

[2] 13.6% of gross input assumed to result in runaround or house scrap, 95% of which is assumed recovered, Stephens, W. E., "Improved Methods and Equipment for Energy Savings in the Aluminum Industry," in Aluminum Industry Conservation Workshop Papers, Aluminum Association.

[3] Bureau of Census, 1972 Census of Manufacturers, Dept. of Commerce, Washington, D.C. and Aluminum Statistical Review - 1975, Aluminum Association.

TABLE A-31

TRANSPORT COSTS OF PRIMARY ALUMINUM BY SOURCE AND DESTINATION IN DOLLARS PER SHORT TON

Source	Destination				
	Phil.	Char.	L.A.	Cleve.	Birm.
S1	28.60[1]	18.20[1]	57.15[2]	26.00[1]	15.40[1]
S2	21.60[1]	8.00[1]	64.68[2]	26.00[1]	18.20[1]
S3	35.00[1]	26.00[1]	70.04[2]	21.60[1]	18.20[1]
S4	21.60[1]	38.20[1]	85.20[2]	26.00[1]	50.00[1]
S5	61.23[3]	35.32[3]	49.58[2]	52.28[3]	23.02[3]
S6	61.23[3]	35.32[3]	49.58[2]	52.28[3]	23.62[3]
S7	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S8	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S9	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S10	31.80[1]	26.00[1]	53.94[2]	18.20[1]	18.20[1]
S11	45.43[3]	30.27[3]	49.11[2]	40.59[3]	22.24[3]
S12	35.00[1]	21.60[1]	53.94[2]	28.60[1]	15.40[1]
S13	12.00[1]	18.20[1]	85.20[2]	18.20[1]	31.80[1]
S14	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S15	51.83[6]	26.20[7]	41.73[8]	47.20[6]	14.20[7]
S16	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S17	18.20[1]	21.60[1]	85.20[2]	12.00[1]	25.60[1]
S18	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S19	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S20	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S21	31.80[1]	26.00[1]	53.94[2]	21.60[1]	18.20[1]
S22	46.09[3]	26.60[3]	41.73[8]	34.40[3]	16.05[2]
S23	18.20[1]	21.60[1]	74.17[2]	8.00[1]	31.80[1]
S24	35.99[9]	16.00[7]	53.94[2]	25.98[9]	9.20[7]
S25	52.74[3]	28.67[3]	49.58[2]	41.28[3]	17.43[3]
S26	61.23[3]	35.32[3]	49.58[2]	52.28[3]	23.62[3]
S27	52.74[3]	28.67[3]	49.58[2]	41.28[3]	17.43[3]
S28	35.59[9]	19.80[7]	53.94[2]	25.98[9]	8.40[7]
S29	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S30	21.60[1]	38.20[1]	85.20[2]	26.00[1]	50.00[1]
S31	68.21[4]	50.80[4]	27.80[5]	68.21[4]	51.09[4]
S32	41.60[10]	58.20[10]	105.20[10]	46.00[10]	70.00[10]

TABLE A-31

SOURCES:

[1] Estimated from rates for hauls of various distances provided by railroad company official and author's estimate of rail mileage between points.

[2] TCFB 1-Z, Supplement 57.

[3] SWFB 310-G.

[4] TCFB 2-O, Supplement 10.

[5] TSFB 1-V (all rates for shipments between smelters located in the Pacific Northwest and Los Angeles use Portland, Oregon, as the shipping point rate basis).

[6] SFTB 859-F, Supplement 116.

[7] SFTB 869-I, Supplement 10

[8] TCFB 1-Z, Supplement 22.

[9] SFTB 859-F, Supplement 117

[10] The Alcan smelter uses Mossena, NY, for the shipping point rate basis but incurs the $20.00 per ton tariff on primary ingot as part of shipping costs in the model. Of course, this is only an approximation.

scrap, purchased new scrap, and old scrap. Home scrap is recycled by the same company or plant that generates it, this type of scrap resulting from a production process which uses aluminum as an input. Purchased new scrap is aluminum-bearing material produced as waste in a process which uses aluminum inputs which is sold for recycling by a scrap dealer or by the company generating it with the proviso that the aluminum-bearing articles sold as scrap have never entered the possession of consumers. Old scrap is recycled aluminum-bearing material which has been part of an article used by final consumers.[41]

Scrap purchased from dealers or customers, unless already sorted, must be segregated. Obvious impurities such as insulation or iron may be separated either manually or by mechanical crushers and magnetic separators.

If the composition of the segregated scrap is still uncertain, tests must be made of the melt and alloying additions may be necessary. Further, the melt must be fluxed and chlorinated to remove any remaining impurities. Flourination is an alternative to chlorination but is uncommon (unless engaged in on the sly) due to environmental standards concerning flourine gas emissions.[42]

The location of secondary smelters in the same region as

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[41] "Aluminum," <u>Minerals Facts and Problems</u>, Bureau of Mines, 1976, pp. 43-44.

[42] "Secondary Smelting of Aluminum Alloys," mimeo provided by Paul Smith, Gulf Metals Industries, Inc., Houston, 1977.

that to which they ship, or imposition of a prohibition on interregional shipments of secondary ingot, resulted from conversations with industry personnel. It was learned that it is common for secondary smelter I in Region A to trade secondary smelter II in Region B delivery of ingot by smelter II to smelter I's customer in Region B in return for scrap generated by smelter II's customer in Region A. Hence, no significant scrap transportation occurs. Therefore, the activities transferring secondary ingot to mills and foundries do so with no penalty in the objective function.

Recycling of runaround scrap at mills and foundries is not explicitly modelled. The energy use figures from which the matrix coefficients were obtained were total energy use figures and therefore included energy used for recycling of home scrap. Thus, the materials balance entry for aluminum ingot into milling and casting processes is stated in net rather than gross terms.

Table A-32 contains the characteristics of those activities producing secondary ingot from new and old scrap. These activities involve some aggregation as both new scrap and old scrap are very heterogeneous categories. The energy use figures shown are for what is considered to be a representative mix of the components of each category. Appendix B contains an explanation of the scrap availability levels imposed on the model.

TABLE A-32

CHARACTERISTICS OF ACTIVITIES PRODUCING SECONDARY INGOT BY TYPE OF SCRAP INPUT

Input	Unit	Scrap Type	
		Old	New
Old scrap	Short ton	1.0[1]	
New scrap	Short ton		1.0[1]
Natural gas	10^3 cubic feet	8.56[2]	6.64[2]
Electricity	10^3 kwh	.497[2]	.12[2]
Labor	Hours	8.9[3]	8.9[3]
Cost	Dollars	286.27[3]	286.27[3]

SOURCES:

1 Scrap availabilities used in the model are on a recovered metal basis (see Appendix B).

2 "Evaluation of the Theoretical Potential for Energy Conservation in Seven Basic Industries," Batelle Columbus Laboratories, , p. 166.

3 Bureau of Census, 1972 Census of Manufacturers, Dept. of Commerce, Washington, D.C. The labor figure is wages paid divided by the cost of labor assumed for the model. The objective function coefficient is cost of materials less the 1971 scrap price and energy costs explicitly accounted for divided by output of secondary smelters in 1971, estimated as gross returns divided by the 1971 price of secondary ingot. This quotient is then corrected by the percentage change in the WPI for metals and metal products.

APPENDIX B
DESCRIPTION OF REGRESSIONS EMPLOYED

SECTION I
SCRAP SUPPLIES

New scrap availability is determined by the following OLS regression.

$$QSN_t = -0.2009 + 0.2177\ QP_t \qquad \bar{R}^2 = .90 \qquad D.W. - 1.46$$
$$(0.0307) \quad (11.5080)$$

where QSN = annual recovery of new scrap on a recovered metal basis in 10^4 s.t.

QP = annual primary aluminum production in 10^4 s.t.

$\bar{R}^2$ is the multiple condition coefficient adjusted for degrees of freedom and D.W. is the Durbin-Watson statistic. Student's t-statistics appear in parenthesis under the regression coefficients. The source for both is Aluminum Statistical Review - 1975, Aluminum Association. The sample period is 1961 - 1975. The quantity of new scrap available on a recovered metal basis and its price appear in Table B-1.

Old scrap availability and prices also appear in Table B-1. These figures were calculated from the following OLS regression.

$$QSO = 0.3753 + 0.0202RPO - 0.0158POL + 0.0158NSHL + 0.7167QOL$$
$$(0.0846) \quad (1.6386) \quad (-1.3527) \quad (1.6755) \quad (2.6036)$$
$$\bar{R}^2 = .89 \qquad D.W. - 2.25$$

where QSO = old scrap recovered on a recovered metal basis in 10^4 short tons.

TABLE B-1

Scrap Type	Price ($/short ton)[1]	Quantity Available (s.t.)
Old Scrap 1	63.74	278900.0
Old Scrap 2	191.33[2]	15000.0
Old Scrap 3	319.04	15000.0
Old Scrap 4	446.75	15000.0
New Scrap	323.60[3]	1065420.0[4]

Notes and Sources:

[1]All prices are nominal prices in 1974 dollars.

[2]The price of each type of scrap is the average supply price in the interval on the supply schedule implied by the column entry, "Quantity Available."

[3]Price of aluminum clippings taken from Metals Statistics - 1976.

[4]Computed from regression explained in text.

RPO = price of scrap aluminum castings as reported in Metals Statistics in dollars per short ton divided by the WPI for Metals and Metal Products

POL = RPO_{t-1}

NSHL = net aluminum shipments (gross shipments minus secondary recovery) lagged one period in 10^4 short tons

QOL = QSO_{t-1}

$\bar{R}^2$ is the adjusted simple R^2, D.W. is the Durbin-Watson Statistic, and student's t-statistics for the regression coefficients appear in parentheses. The sample period is 1954-1975.

The data series for QSO is adjusted from 1954 to 1960 for the fact that previous to 1961 sweated pig is included in new scrap recovery. The change in the ratio of new scrap to old scrap between old scrap recovery figures for 1954-1960 are adjusted upward by the difference in percentages. The regression listed was run with a dummy variable taking the value of one in 1954-1960 included. The added dummy was found to be insignificant so it was concluded that the adjustment did not alter the character of QSO.

The equation estimated is derived from the following structural model.

$$QSO_t = aRPO_t + b \sum_{i=0}^{\infty} \lambda^i (1 - \lambda) NSHL$$

A Koyck transformation is performed on the above equation to obtain the equation estimated, which is[1]

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[1] Johnston, J., "Econometric Methods," 2nd edition, McGraw-Hill, New York, 1972, pp. 298-300.

$$QSO_t = aRPO_t - \lambda aRPO_{t-1} + b(1 - \lambda)NSHL + \lambda QSO_{t-1}$$

The own price elasticity of old scrap supply at the mean for this equation is about -.22. This compares favorably with another estimate obtained by Charles Rivers Associates.[2]

SECTION II
PRIMARY ALUMINUM DEMAND

The demand curve for primary ingot used in Chapter III is derived from the OLS regression below.

$$QP_t = \sum_{i=0}^{3} b_i P_{t-i} + \underset{(8.3993)}{.5599\,QT} + \underset{(2.4335)}{315.8068} \qquad \bar{R}^2 = .9558 \qquad D.W. = 1.48$$

$\bar{R}^2$ is the simple R^2 adjusted for degrees of freedom, D.W. is the Durbin-Watson Statistic, and student's t-statistics appear in parentheses below the regression coefficients. The b_i are estimated through the Almon procedure[3] and are as follows:

$$b_0 = -0.0863$$
$$b_1 = -0.1295$$
$$b_2 = -0.1295$$
$$b_3 = -0.0863$$

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[2] "Policy Implications of Producer Country Supply Restrictions: The World Aluminum Bauxite Market," Charles Rivers Associates, Cambridge, Mass., 1976.

[3] Johnston, J., "Econometric Methods," 2nd edition, McGraw-Hill, New York, 1972, pp. 292-298, contains an explanation of this procedure.

As may be apparent from the coefficient values, the b_i belong to a distribution of weights constrained to equal zero at $t+1$ and $t-4$. They, therefore, all share the same t-statistic of -2.0256. The variable mnemonica appear below.

QP = annual production of primary aluminum as taken from Aluminum Statistical Review - 1975 in 10^4 short tons

P_{t-i} = the price of virgin primary ingot as reported in Metals Statistics - 1977 in dollars per short ton divided by the WPI for Metals and Metal Products

QT = the sum of total shipments of milled products and cast shapes on an annual basis as taken from Aluminum Statistical Review - 1975 in 10^4 short tons

The sample period is 1958-1975. The short run own-price elasticity of demand evaluated at the mean is -.137 while the long run elasticity, once again evaluated at the mean, is -.69.

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