REALIZATION OF

OPTIMUM ORTHOGONAL MULTIPLEXING SYSTEMS

A Dissertation

Presented to

the Faculty of the Department of Electrical Engineering

University of Houston

In Partial Fulfillment

of the Requirements of the Degree

Doctor of Philosophy in Electrical Engineering

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by

Thomas Williams Jr.

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ABSTRACT

Several sets of orthonormal functions suitable for use in orthogonal multiplexing systems were derived. The functions were selected on the basis of ease of implementation since orthogonal multiplex systems perform equally well for all signal waveshapes in channels with additive white Gaussian noise.

The functions were analyzed and mathematical relationships wwere derived for message distortion due to time, frequency, and amplitude truncation. The orthomux system was also simulated on an IBM 7090 digital computer using the DSL-90 simulation language to evaluate the effects of realistic channel models. The computer programs used in this simulation are described.

The real exponential sets of orthogonal functions are proposed as the most easily realizable from the standpoint of equipment simplicity. The Gram-Schmidt procedure is used to derive these exponential sets whose interval of orthogonality is either finite or infinite. Another exponential set is generated such that it is orthogonal to any constant in order to implement it with A.C.-coupled amplifiers. System parameter equations are derived so that a system can be designed to give a desired level of performance. Time

truncation crosstalk is found to be less than five percent for a system with a time constant equal to four, whereas frequency truncation crosstalk of less than five percent is possible for a single pole filter with unit normalized bandwidth for synchronous reception. The bandwidth of the real exponential set is comparable with that required for frequency division multiplexing for a system with ten or fewer channels and one percent distortion. An amplitude limiting of the real exponential set to seventy percent of the magnitude of the peak value produces less than five percent distortion.

Functions based on powers of t are included because these can be readily generated using operational amplifiers. Digital orthogonal functions having a peak-to-average power ratio of unity are also discussed because of their desirability for use in a communication system which is part of a larger digital system.

System implementation and performance improvement techniques generally applicable to orthomux systems are presented. Additional investigation must be carried out to select functions which are optimum for more severe channel constraints.

To Iris and Tarsol

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CHAPTER I

INTRODUCTION

Effective communications was long ago recognized by mankind as one of the most important factors in determining the success or failure of a particular endeavor. In recent years people have become aware of the general applicability of information theory to all phases of communications, whether person to person, machine to machine, or machine to person. For instance, communication between individuals in such diverse environments as aviation and banking is being studied. Communication models are assumed, the flow-rate of information is measured, speech coding habits (as among jet pilots) are recorded, and so on. Attempts are then made to improve communications with the purpose of improving system performance. Digital communication between machines has been the subject of an intense concentration of investigative effort for twenty years. The transmission of television information in color is a remarkable recent development. The modern formulation of the mathematical foundation of information theory together with the development of statistical analysis techniques has answered many questions and offers hope for the solution of many other problems.

In the beginning, however, theories were not exotic, nor were the aims and needs of people complex. As with all things, primitive communication techniques were simple solutions developed to satisfy the simple requirements of the early ages. The story is a very interesting one, as is almost any history of the progress of mankind in a struggle to domesticate an otherwise hostile environment.

In early times acoustic signals were transmitted by voice, horns, bells, gongs, and drums. Men had perceived the usefulness of the speed with which sound travelled through the air. Probably visual signaling came into existence at about the same time as acoustics signalling. There are records of a visual system in regular use for message transmission as long ago as 300 B.C. This system used several light sources at fixed locations and letters were represented by a code which was implemented by igniting the appropriate light sources. Several semaphore systems were operated in Europe around the year 1800. Visual signals and their verifications were transmitted from station-to-station at such low rates as one signal per minute. This early closed loop system was a recognition of the need for accuracy in the handling of important information because the high cost of transmission of a message acted as a filter on trivia. The semaphor line from Paris to Toulon actually had 120 stations covering the 475 miles. Codes and signalling devices were improved, and transmission speeded up. Flag signals and optical shutters adopted by the military at early dates are still being used to this day.

Around 1747, Watson demonstrated that electricity could be transmitted over a two mile length of wire. Additional discoveries concerning the fundamental nature of electricity by such people as Faraday, Henry, Ohm, and others made possible the development of a practical communications system using electricity as the information bearer. Weber and others built telegraph systems which worked very well. Several of the early codes, including the Morse code, assigned the shortest codes to the most frequently used letters of the alphabet. Practical telegraph systems progressed from one way transmissions (simplex) to two way or bi-directional transmission (duplex) to the sending of a number of messages simultaneously over a single pair of wires (multiplex). The first multiplex systems used time division, with the machines at each terminal running in time synchronism.

Discovery of electromagnetic radiation and advanced mathematical techniques led to the development of wireless telegraphy. The invention of the vacuum tube then opened the door to the modern world of communications.

A second major type of multiplexing was discovered and made practical by the development of frequency domain theory and applications, notably in the design of filters which are capable of passing or rejecting selected frequencies. This is termed frequency division multiplexing.

Continued efforts to improve communication techniques led to the development of systems using orthogonal functions. These systems have used the sine and cosine functions until recently, when investigations into the suitability of other functions for use in multiplexing systems were undertaken. This effort is related to a more general area of investigation called signal design.

CHAPTER II

SYSTEM SELECTION AND OPTIMIZATION CRITERIA

Progress in communication theory has been matched by remarkable advances in the field of electronics components. Solid state circuits have made possible the manufacture of systems of vast complexity by increasing system reliability and performance while at the same time decreasing size, weight, and power consumption. Thus the system designer has a considerable body of theory to call on and the manufacturing capability to implement previously unrealizable designs. An example of a modern system employed on scientific satellites is a general purpose digital computer programmed by ground commands received by the on-board telemetry system to process the raw data from sensors so that only the important parameters are telemetered to earth. This saves bandwidth and allows the spacecraft to transmit more total information during its useful lifetime. This communication-computation package (Cliff, 1967) is expected to weigh less than one hundred pounds, consume less than twenty-five watts, and have a projected goal of 95 per cent probability of one year life in orbit. Despite these advances, and in some cases because of them, the design engineer must make crucially important decisions.

In the early stages of a communication system design an engineer should consider:

1) the system performance specification,
2) the characteristics of the input information and a description of the channel,
3) the freedom of choice allowed to arrive at an optimum system.

A system performance specification includes requirements on error rates, information handling rates, and reliability, etc. The designer may also be subjected to additional constraints in the form of funding and time limitations, and the type of hardware he could use. The optimization criteria must be clearly and definitively stated for mathematical formulation of system performance. Such optimization criteria may be one of these listed below:

1) $\overline{|M_1(t) - M_1^*(t)|^N}$, $N = 0,1,2,\ldots$ [2-1]

where

$M_1(t)$ = Input Message

$M_1^*(t)$ = Received Message

$N = 2$ is the Mean Square Criterion

2) $P[M_1^*(t) - M_1(t)]$ [2-2]

3) $P[|M_1(t) - M_1^*(t)| > \varepsilon]$ [2-3]

where ε is an arbitrary small positive constant.

The first criterion with N=2 is often used. The correlation detection process (or matched filter) is the optimum detection technique for a system with additive white Gaussian noise (Davenport, 1958), as shown in Appendix A, for any signal waveshape. This allows the signal or waveform used in a system to be so selected that its properties are optimally suited to the requirements of a specific application without sacrificing performance.

The remainder of this work is concerned with the realization of systems using correlation detection for a variety of orthogonal waveforms. All of these systems are optimum for signal detection in additive white Gaussian noise. The freedom of choice in selection of a waveform allows the investigation of sets of waveforms which appear to be optimum under additional criteria, such as ease of implementation, peak to average power ratio, and bandwidth. Implementation of the new types of multiplexing systems is discussed and digital computer programs are developed to allow the determination of orthogonal system distortion under various conditions such as time and frequency truncation and additive Gaussian noise.

CHAPTER III

ORTHOGONAL MULTIPLEXING

Introduction

Multiplexing is the combining of several messages in such a way that they can be transmitted over a channel on a single carrier, for example, and then be individually recovered at the receiver. The only multiplexing techniques which have been used to any great extent in the past are frequency division multiplexing and time division multiplexing. Since the recovery of the individual signals at the receiver is dependent on the orthogonality of the signals, and since there exist many types of orthogonal functions, then many other multiplexing schemes may be devised. Ballard (1962) analyzed a system based on the Legendre polynomials. The general orthogonal multiplexing technique was referred to as "orthomux." A search of the literature shows that very little has been done in the orthomux field to determine the relative advantages of the various possible systems with regard to implementation and performance.

The operation of an orthogonal multiplexing system is based on the integral

$$\int_0^T O_i(t)\ O_j(t)dt = \begin{cases} K, & \text{if } i=j \\ 0, & \text{if } i \neq j \end{cases} \qquad [3\text{-}1]$$

where $O_i(t)$ is the i^{th} channel transmitted signal and $O_j(t)$ is the j^{th} channel signal generated by the receiver, and usually the signals are orthonormal; that is, each signal is amplitude normalized to yield the mean square value of unity. This distributes the power equally among the signals in order to create an equal signal-to-noise ratio condition. The system operation is explained by Figures (3-1) and (3-2). The orthomux system is a pulsed system in that each block in the transmitter and receiver operates over a period of time T and then resets and starts another cycle. Identical orthogonal function generators are used in the transmitter and receiver to produce the set of orthogonal functions required for system operation. The transmitter and receiver operate in frequency synchronism, with receiver operation delayed in time enough to accomodate signal propagation time. A single message would travel through channel one, for instance, of the transmitter system in this manner:

(1) The message input of channel one of the transmitter $M_1(t)$ is sampled and the sampled value is held for one cycle of operation as the constant value $\overline{M_1(t)}$;

(2) The message sample is multiplied with the orthogonal signal $O_1(t)$ from channel one to form $\overline{M_1(t)}\ O_1(t)$; this product is then added to the signals of the other channels to form

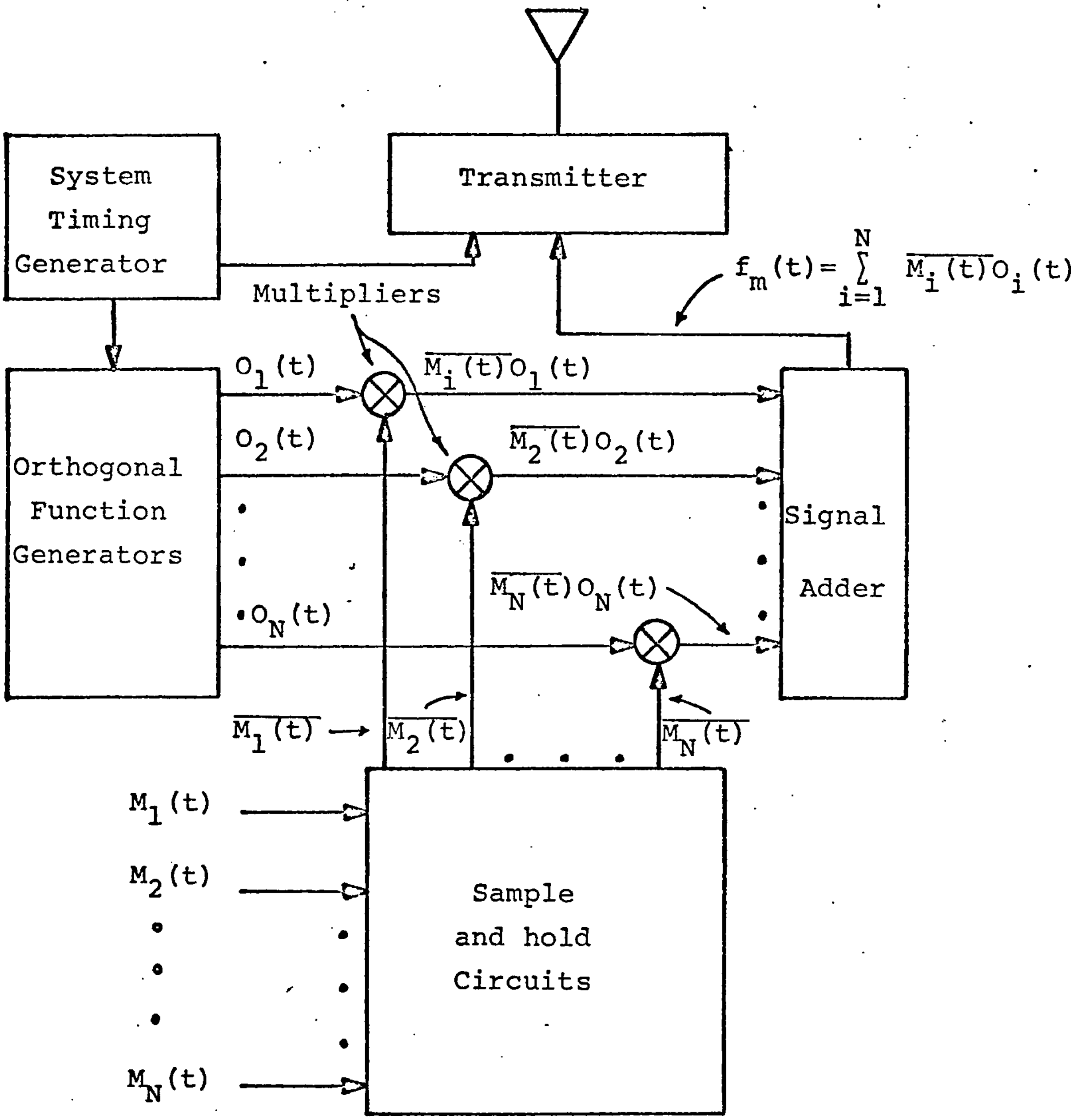

Figure 3-1. General orthogonal multiplex system transmitter

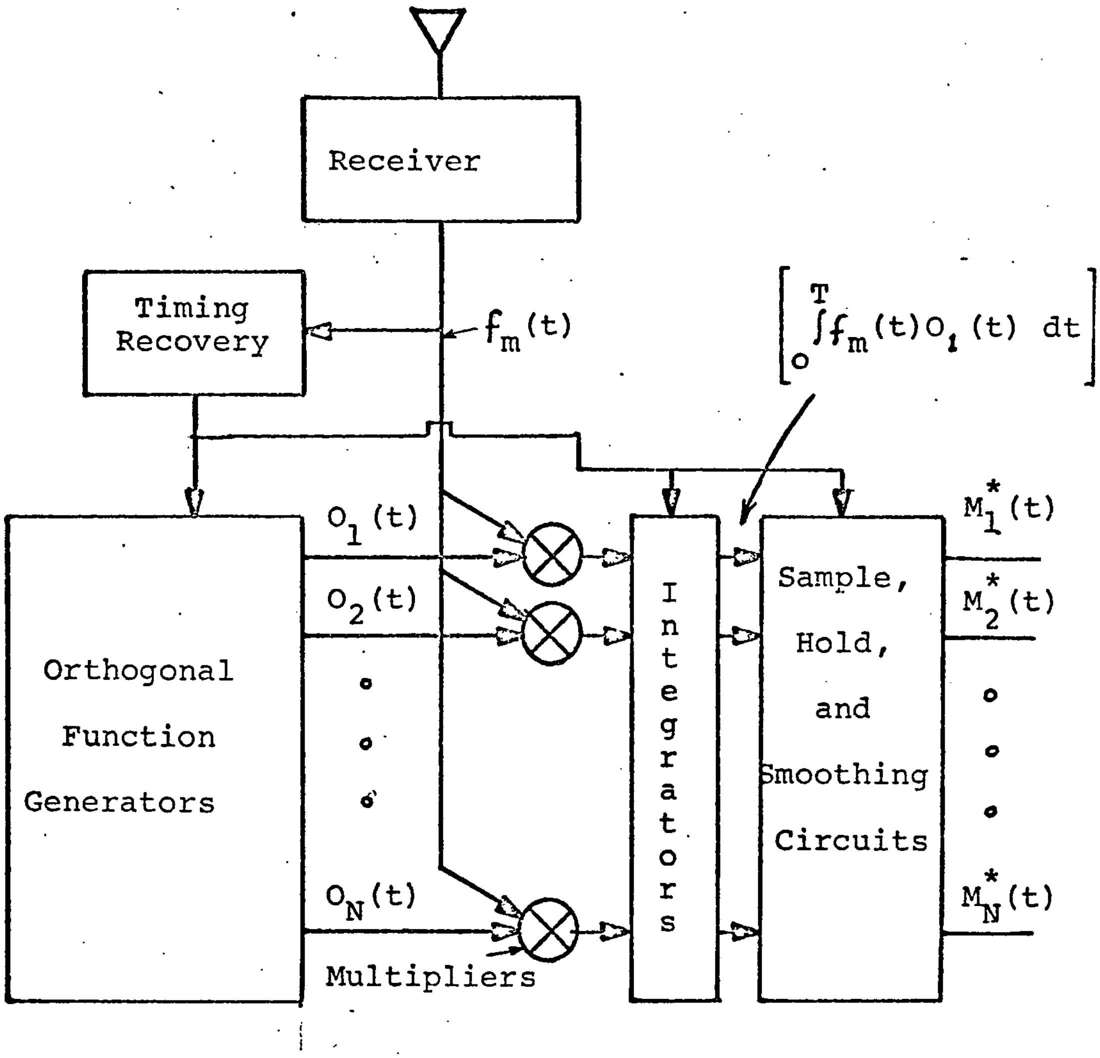

Figure 3-2. General orthogonal multiplex system receiver

$f_m(t)$. This composite signal is sent to the link transmitter for final processing before transmission over the path to the receiver. This might entail shifting the entire signal spectrum to a high frequency for transmission over a radio link, for example. The entire process described above is repeated continuously for each channel of the system. A timing signal generated by the transmitter controls the sequence of operations in the transmitter and the receiver. At the receiver, the received message is processed to recover the composite message signal $f_m(t)$, system timing information, and any other information necessary to recover the individual channel messages. The synchronized receiver generates a set of orthogonal functions identical to those in the receiver but delayed in time by propagation time, as shown in Figure (3-2). The message in channel one is recovered in the following way:

(a) The first orthogonal function is multiplied with the composite input message to form $O_1(t)f_m(t)$;

(b) This product is integrated to produce the expression

$$\int_0^T O_1(t)\,[f_m(t) + \eta]\,dt, \text{ and } \eta = \text{Noise} \qquad [3\text{-}2]$$

which upon expansion of $f_m(t)$ becomes

$$\int_0^T O_1(t)\,[\overline{M_1(t)}O_1(t) + \overline{M_2(t)}O_2(t) + \ldots +$$

$$\overline{M_N(t)}O_N(t)]\,dt + \int_0^T [O_1(t)\eta]\,dt \qquad [3\text{-}3]$$

(c) Finally, the receiver samples the integrator output at time t=T, and the sampled output is smoothed as one of a stream of levels of period T, by a smoothing network whose output is $M_1^*(t)$.

The difference between $M_1^*(t)$ and $M_1(t)$ is the channel error. The smoothing filter is not needed in the case of digital messages. Figures (3-3), (3-4), and (3-5) show functions suitable for use in an orthomux system.

As stated in Chapter II, the optimum process for the detection of a signal in the presence of additive white Gaussian noise is a correlation process, where the shape of the signal and time or arrival of the signal at the receiver are known, and the information is amplitude modulated. Specifically, the correlation process maximizes the signal-to-noise ratio or in the case of a digital signal, it minimizes the probability of error. This is true for the simple

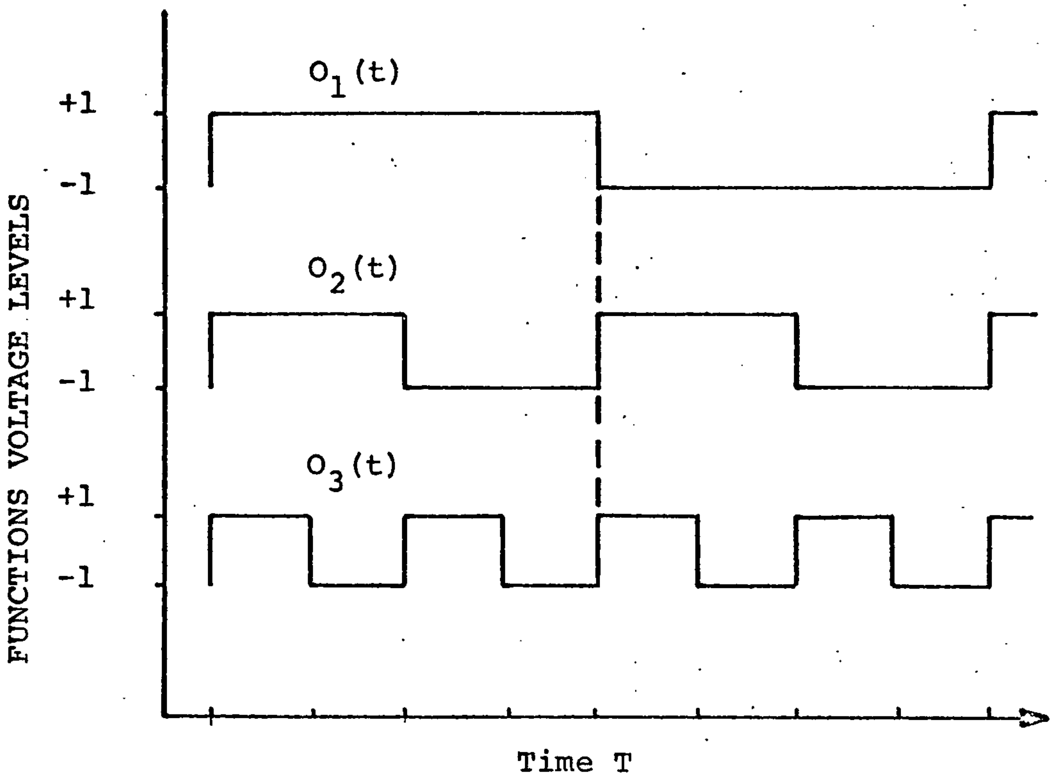

Figure 3-3. A set of binary orthogonal waveforms

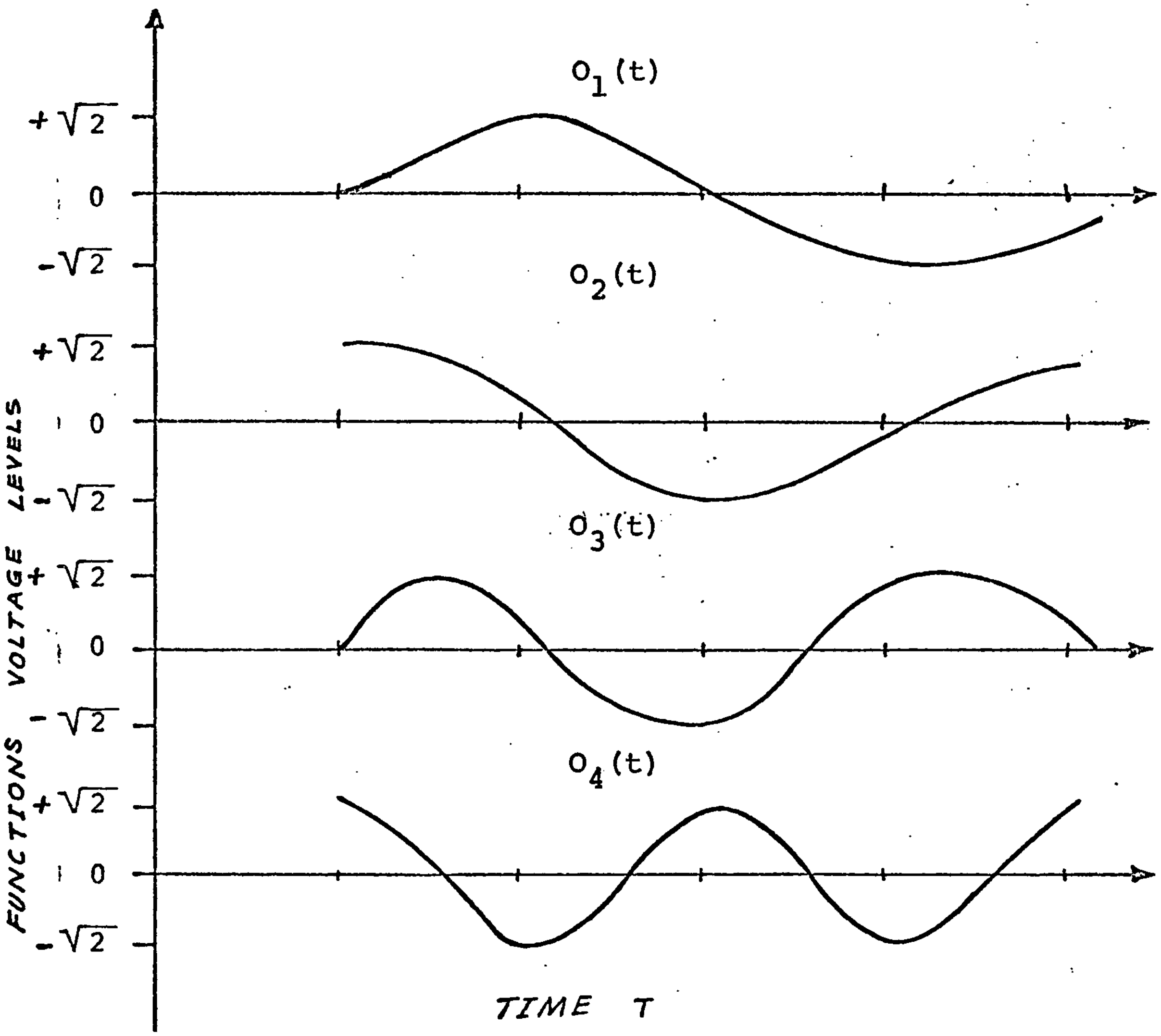

Figure 3-4. Sinusoidal orthogonal waveforms suitable for use in a frequency division multiplexing system

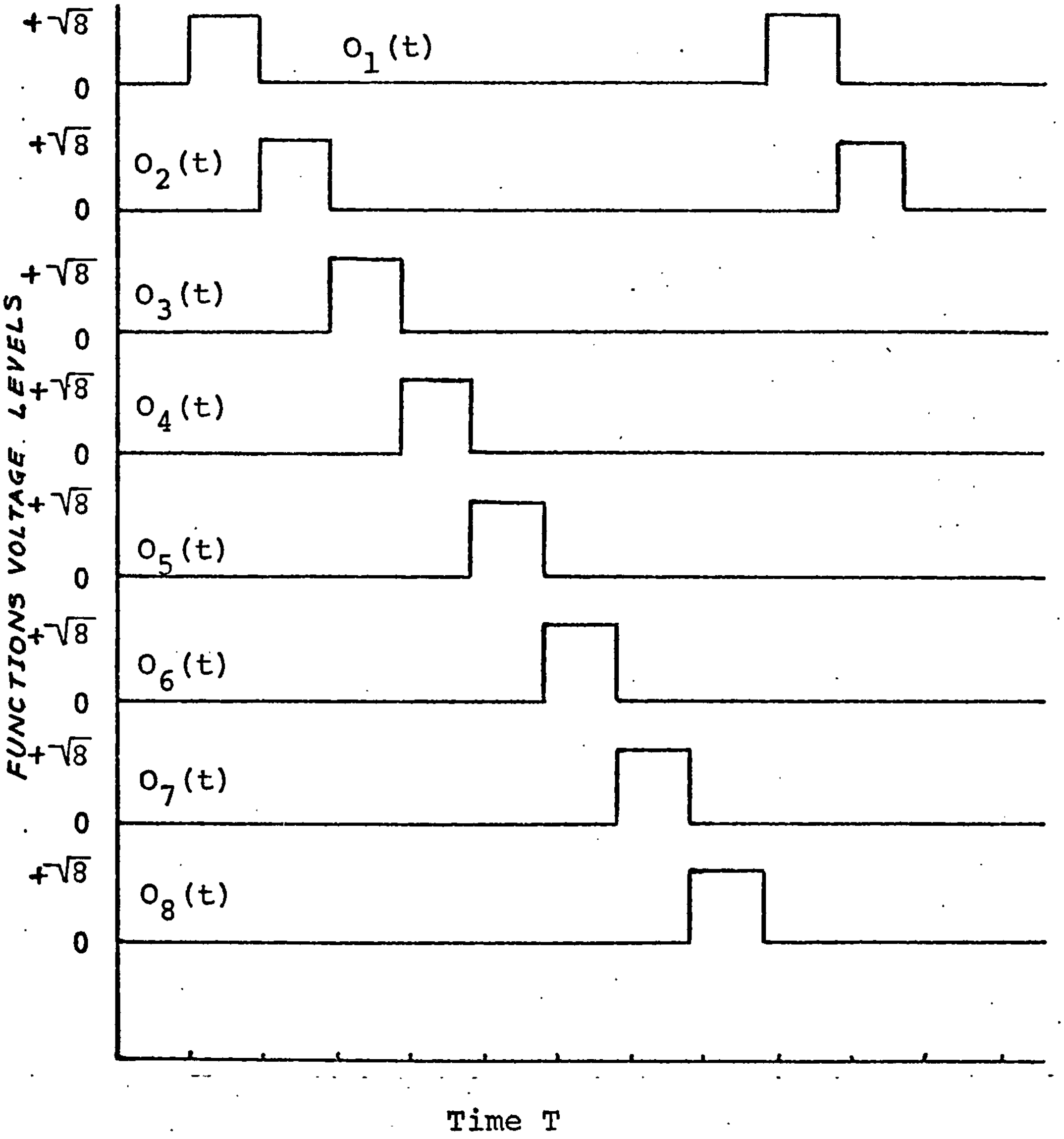

Figure 3-5. Orthogonal waveforms for time division multiplexing

baseband channel with additive Gaussian noise. However, if the channel differs from this simple channel with unrestricted bandwidth, then the optimum multiplexing system depends on the assumptions made about the channel itself. For the simple baseband channel with additive white Gaussian noise all the orthomux systems perform equally well (Davenport, 1958). Thus for the simple baseband channel the optimum orthomux system would be the system which is relatively simple to construct, and it turns out to be the system based on the real exponential set, since the exponentials are so very easy to generate. For a channel which band-limits the frequency spectrum of the transmitted signals, consideration must be given to the problem of the resultant distortion of the received message. In the case of a peak power-limited channel, the optimum multiplexing system uses binary (digital) waveforms. These waveforms have an optimum peak-to-average power ratio of one. The types of binary combination methods leading to the simplest implementation are also discussed.

Each of the types of signals which are optimum in terms of the considerations discussed above are investigated as to their implementation in the following chapters. In addition several other sets are considered because of one or more interesting properties. First a brief review of previous applicable work in the area of orthogonal multiplexing is given.

Review of Previous Investigations

Interest in orthogonal multiplexing has come about because of the following advantages:

(1) Orthogonality of the signals gives a theoretical minimum of zero crosstalk between channels.

(2) A correlation detection process assures maximum rejection of noise and interference.

(3) Orthogonal multiplexing is optimum in several practical ways which depend on the channel characteristics. The different orthogonal sets allow the designer to select a set on the basis of ease of implementation or optimum in the sense of peak-to-average-power ratio, for instance.

Ballard (1962) pointed out that Legendre functions may be generated using cascaded solid state circuits. The first three Legendre functions are:

$$P_0(x) = 1 \qquad [3\text{-}4]$$

$$P_1(x) = X \qquad [3\text{-}5]$$

$$P_2(x) = \tfrac{1}{2}[3x^2-1] \qquad [3\text{-}6]$$

and so forth, where

$$x = \frac{2}{T}[t - \frac{T}{2}] \ , \text{ for } 0 \le t \le T \qquad [3\text{-}7]$$

Higher ordered polynomials are obtained using the Rodrigue's formula

$$P_N(x) = \frac{1}{2^N N!} \frac{d^N}{dx^N} [x^2 - 1]^N \ , \text{ for } n=0,1,2,. . . \qquad [3\text{-}8]$$

Note that all the functions have zero mean and a peak value of one except for $P_0(x)$. The first three functions are shown in Figure (3-6). Even order functions have even symmetry and odd order functions have odd symmetry, and their orthogonality interval extends from -1 to +1.

Equations for the frequency spectrum of the Legendre polynomials are tabulated in Ballard's paper and the effect of filtering or frequency truncation on crosstalk is mentioned in addition to the details of implementation.

Karp and Higuchi (1963) analyzed a system based on the modified Hermite polynomials which are said to have superior time-bandwidth compression properties. Their interval of orthogonality is from $t=-\infty$ to $t=+\infty$, but the functions can be truncated in time at the expense of introducing crosstalk into the system. This system is complex and therefore its implementation is not simple. The generating function for the modified Hermite polynomials is (Karp, 1963)

$$H_N(t) = \frac{(-1)^N a^{(N-\frac{1}{2})}}{\pi^{\frac{1}{4}}\sqrt{N!}\ 2^{N/2}}\, e^{t^2/2a^2}\left[\frac{d^N}{dt^N}(e^{-t^2/a^2})\right] \qquad [3\text{-}9]$$

Titsworth (1963) has described a Boolean-Function-Multiplexed system with the advantage that the average and peak powers are the same since it is a digital system.

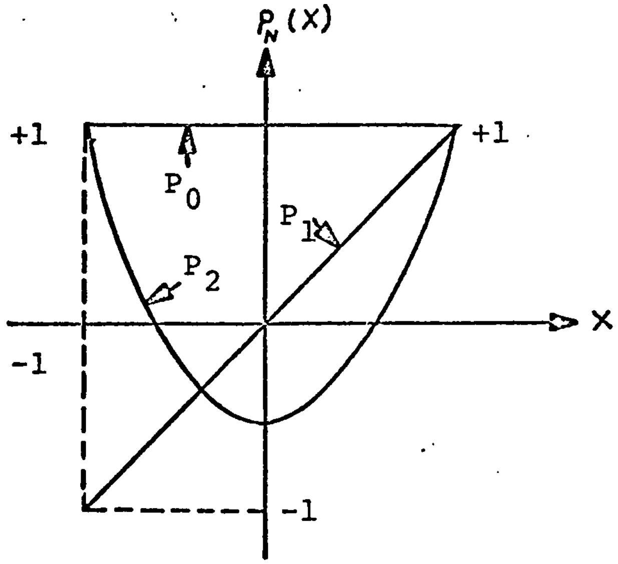

Figure 3-5. The first three Legendre polynomials

CHAPTER IV

EXPONENTIAL ORTHOGONAL FUNCTIONS

In this chapter exponential orthogonal functions are analyzed with a view to their application in pulsed multiplex systems. The interest in the exponential functions with real exponents is based on their being relatively easy to be generated. Methods of implementation and performance improvement techniques for this system are given later.

A sequence of functions $g_i(t)$, is given below:

$$\begin{aligned} g_1(t) &= e^{-pt} \\ g_2(t) &= e^{-2pt} \\ &\vdots \qquad\qquad \text{for } 0 \leq t \leq \infty \qquad [4\text{-}1] \\ g_N(t) &= e^{-Npt} \end{aligned}$$

with p as a real positive constant, may be used to obtain a set of orthogonal functions $O_i(t)$ over an interval given below:

$$O_1(t) = \sqrt{2p}\; e^{-pt} \qquad [4\text{-}2]$$

$$O_2(t) = 4\sqrt{p}\; e^{-pt} - 6\sqrt{p}\; e^{-2pt} \qquad [4\text{-}3]$$

$$O_3(t) = \sqrt{6p}(3e^{-pt} - 12e^{-2pt} + 10e^{-3pt}) \qquad [4\text{-}4]$$

or

$$O_N(t) = \sum_{k=1}^{N} C_{Nk}\; e^{-kpt} \quad \text{for } 0 \leq t \leq \infty \qquad [4\text{-}5]$$

The method by which this set of functions shown in Figure 4-1 is obtained is described in Appendix B.

Time Truncation Crosstalk

Since the exponentials decay rapidly, it is possible to truncate the signal at a finite time T and preserve most of their desirable characteristics. This truncation causes crosstalk which can be calculated using the basic orthogonality integral

$$\phi_{NM}(T) = \int_0^T O_N(t)\ O_M(t)\ dt \qquad [4\text{-}6]$$

The crosstalk is calculated by substituting the defining equations for the orthogonal signals in Equation [4-6]

$$\phi_{NM}(T) = \int_0^T \Big(\sum_{k=1}^{N} C_{Nk}\ e^{-kpt}\Big)\Big(\sum_{\lambda=1}^{M} C_{M\lambda}\ e^{-\lambda pt}\Big)dt \qquad [4\text{-}7]$$

$$\phi_{NM}(T) = \sum_{k=1}^{N}\ \sum_{\lambda=1}^{M} C_{Nk}\ C_{M\lambda} \int_0^T e^{-(k+\lambda)pt}\ dt \qquad [4\text{-}8]$$

$$\phi_{NM}(T) = \sum_{k=1}^{N}\ \sum_{\lambda=1}^{M} \frac{C_{Nk}\ C_{M\lambda}}{(k+\lambda)p} - \sum_{k=1}^{N}\ \sum_{l=1}^{M} \frac{C_{Nk}\ C_{M\lambda}}{(k+\lambda)p}\ e^{-[k+\lambda]pt}$$

[4-9]

or $$\phi_{NM}(T) = \delta_{NM} - E_{NM} \qquad [4\text{-}10]$$

and $$\delta_{NM} = \sum_{k=1}^{N}\ \sum_{\lambda=1}^{M} \frac{C_{Nk}\ C_{M\lambda}}{(k+\lambda)p} \qquad [4\text{-}11]$$

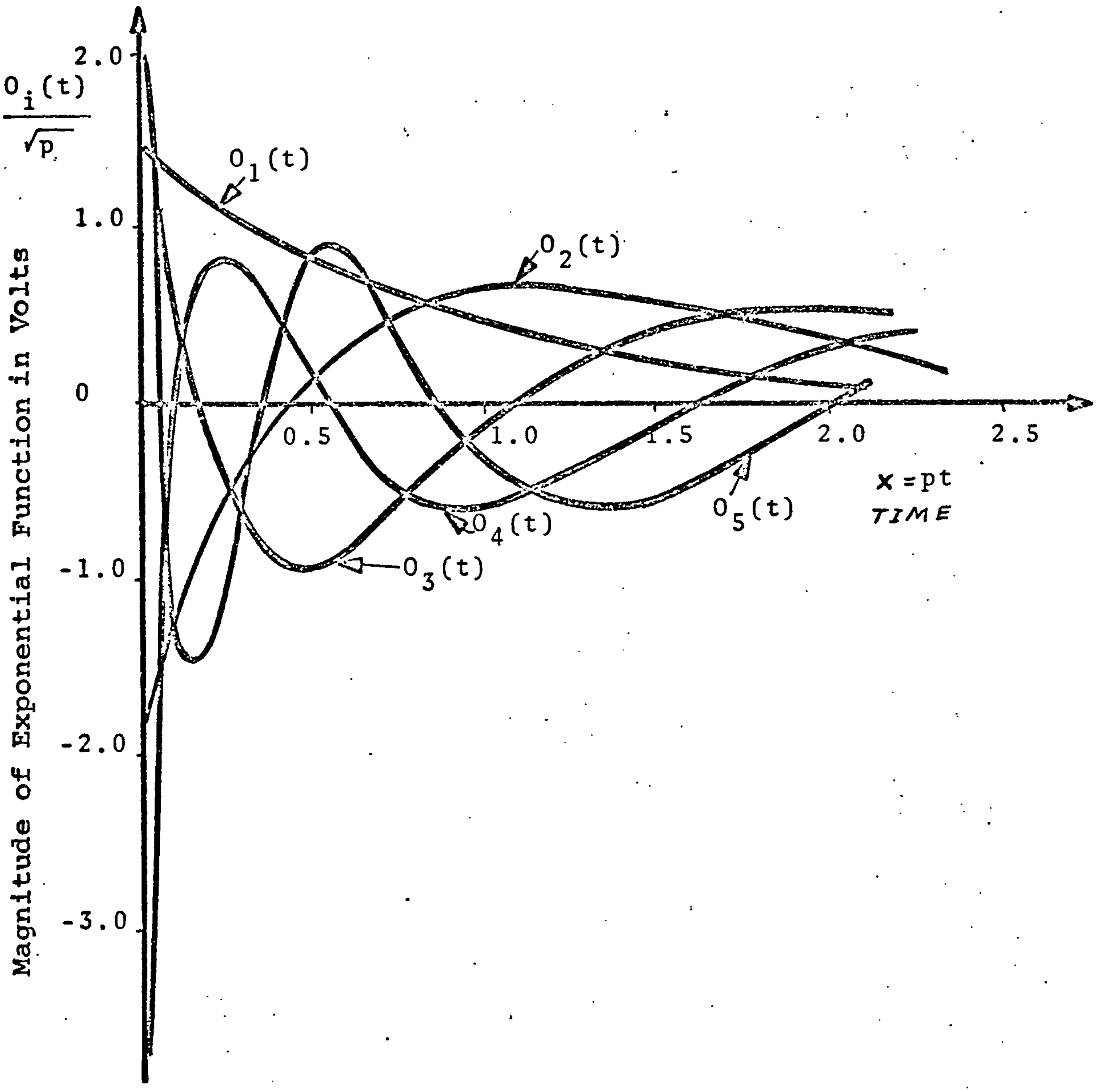

Figure 4-1. Normalized plot of real exponential set vs pT

where

$$\delta_{NM} = \begin{cases} 1, & N = M \\ 0, & N \neq M \end{cases} \qquad [4\text{-}12]$$

$E_{NM}(T)$ = crosstalk term in M^{th} receiver channel due to the N^{th} channel signal.

$\phi_{NM}(T)$ = total output of the M^{th} channel due to the N^{th} channel signal.

Figures (4-2) and (4-3) show $E_{NM}(T)$ (crosstalk) variation with pT.

Spectrum

The complex line spectrum of a periodic function with basic minimum frequency ω_1 is given by

$$C_\beta = \frac{1}{T}\int_0^T f(t)e^{-j\beta\omega_1 t}\,dt \qquad [4\text{-}13]$$

where $\beta = 0,1,2, \ldots$

T = Period of $f(t)$

The power spectrum of the N^{th} exponential function is found as follows:

$$C_\beta = \frac{1}{T}\int_0^T \sum_{k=1}^{N} C_{Nk}\, e^{-kpt}\, e^{-j\beta\omega_1 t}\,dt \qquad [4\text{-}14]$$

$$C_\beta = \frac{1}{T}\sum_{k=1}^{N} C_{Nk}\left[\frac{1 - e^{-T(kp+j\beta\omega_1)}}{kp + j\beta\omega_1}\right] \qquad [4\text{-}15]$$

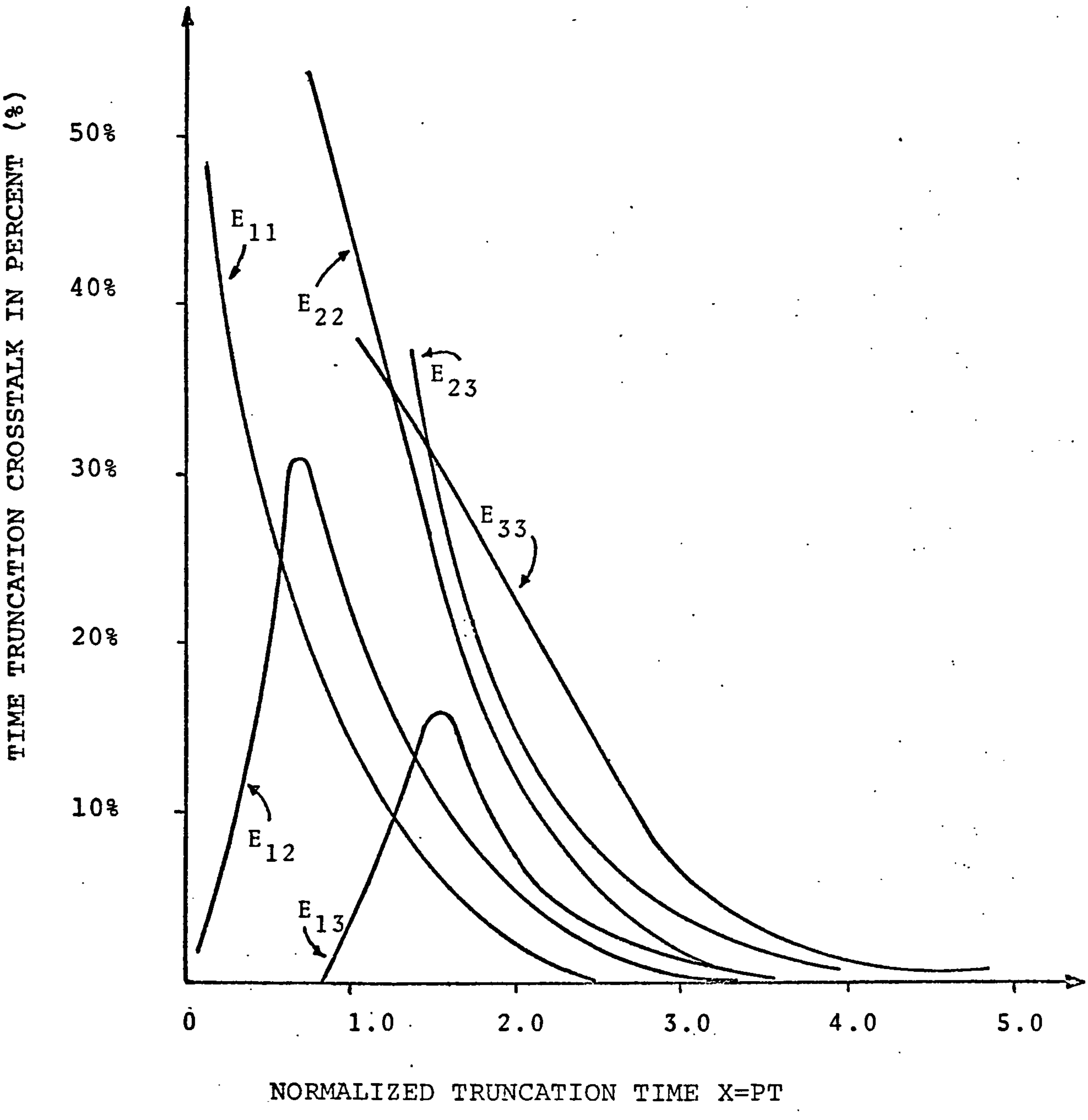

Figure 4-2. Crosstalk caused by time truncation of exponentially generated orthogonal functions

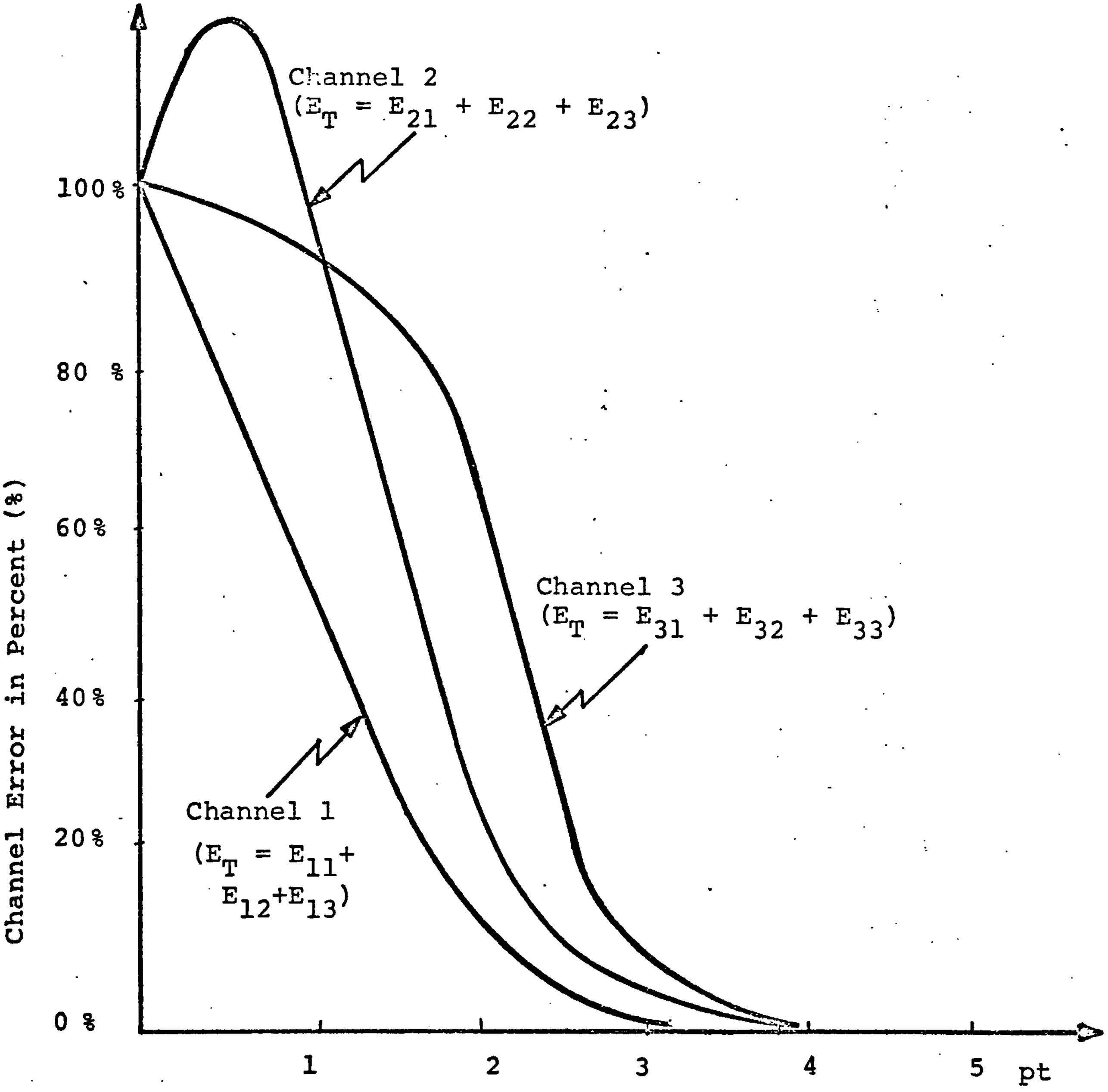

Figure 4-3. Total time truncation crosstalk in each channel for a three channel system using the real exponential set

As T becomes large enough to reduce the time truncation crosstalk to tolerable limits (less than 5 per cent for example), the above equation can be approximated by:

$$C_\beta = \frac{1}{T}\sum_{k=1}^{N} \frac{C_{Nk}}{(kp+j\beta\omega_1)} = \frac{1}{pT}\sum_{k=1}^{N} \frac{C_{Nk}}{(k+jX)} \qquad [4\text{-}16]$$

A substitution of the numerical values for the coefficients in Equation [4-16] results in an expression for the power spectrum as given in Equation [4-17].

$$|C_\beta|^2 = \frac{2}{pT^2}\left(\frac{N}{X^2+N^2}\right) \qquad [4\text{-}17]$$

where N is the index number of the orthogonal function. This equation is plotted in Figure (4-4). The highest order exponential of a particular function dominates the bandwidth expression because in the time domain the highest order exponent is most compressed, or has very short decay time.

<u>Frequency Truncation Crosstalk</u>

Crosstalk is introduced when the function under study is passed through a filter which causes amplitude or phase distortion. The effects of frequency truncation are studied by passing the exponential function through a one pole RC filter. The impulse response of the network is given as

$$h(t) = \frac{1}{RC}\, e^{-t|RC}, \quad \text{for } t \geq 0 \qquad [4\text{-}18]$$

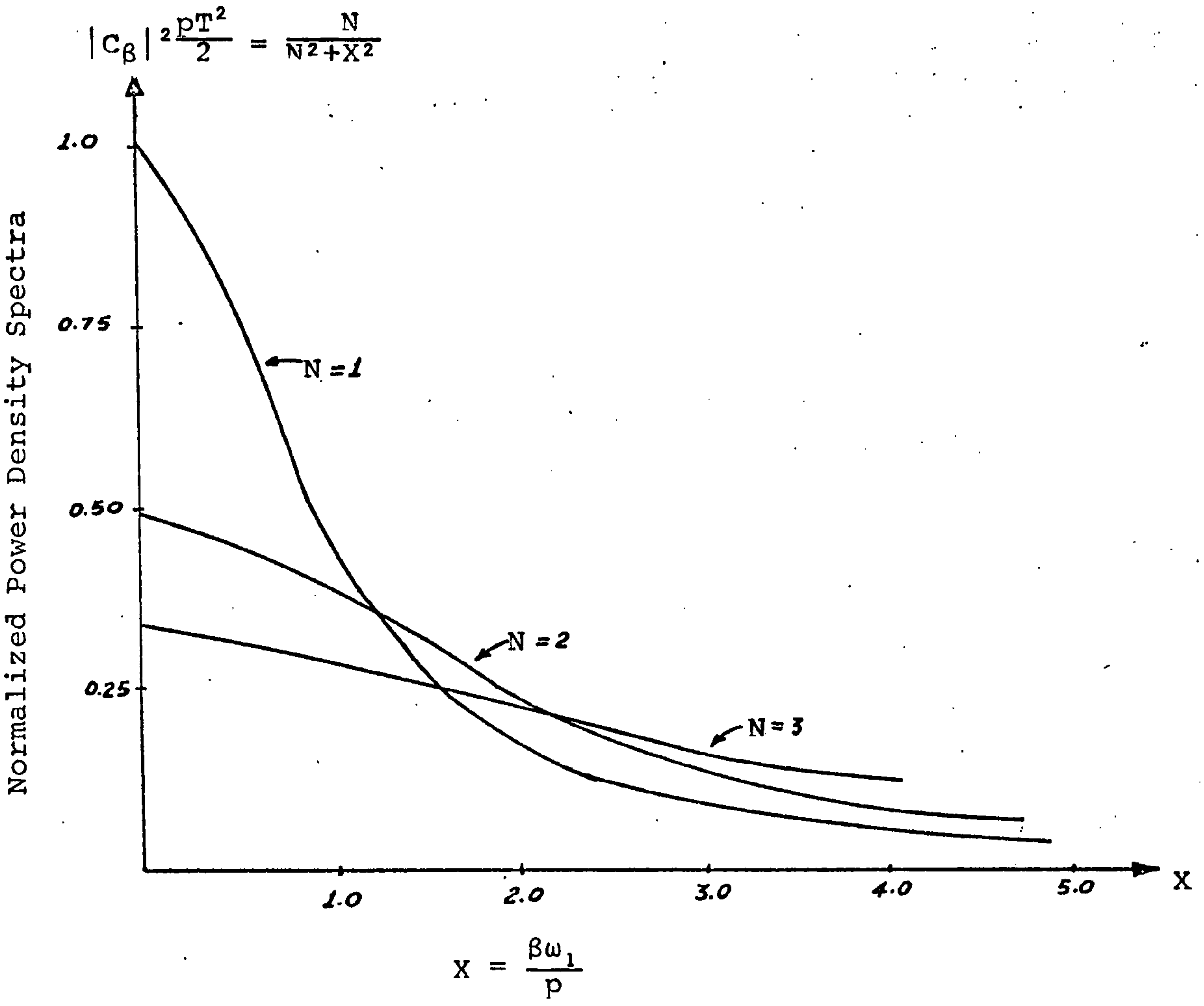

Figure 4-4. Envelope of the normalized power density spectra versus $X = \frac{\beta\omega_1}{p}$ for the real exponential set

The real exponential function inputs $e_i(t)$ are given by

$$e_i(t) = O_N(t) = \sum_{k=1}^{N} C_{Nk}\, e^{-kpt} \;, \quad 0 \le t \le T \qquad [4\text{-}19]$$

for which the output would be

$$e_{ON}(\tau) = \int_0^{\tau} \sum_{k=1}^{N} C_{Nk}\, e^{-kpt}\left[\frac{1}{RC} e^{-\frac{1}{RC}(\tau - t)}\right] dt \qquad [4\text{-}20]$$

$$e_{ON}(\tau) = \frac{e^{-\tau/RC}}{RC} \sum_{k=1}^{N} C_{Nk}\left[\frac{1 - e^{-(kp - 1/RC)\tau}}{kp - 1/RC}\right] \qquad [4\text{-}21]$$

Figure (4-5) shows the effect of a one pole filter on the input signal given by

$$fm(t) = O_1(t) + O_2(t) + O_3(t) \qquad [4\text{-}22]$$

where the $O_i(t)$ are the first three of the exponential signals. The graph shows the input signal and the output for two values of bandwidth. The crosstalk resulting from bandlimiting is obtained by substituting Equation [4-20] into Equation [4-6] as

$$\phi_{NM}(T) = \int_0^{T} \sum_{k=1}^{N} C_{Nk}\left[\frac{e^{-\tau/RC} - e^{-kp\tau}}{RCkp - 1}\right]\left(\sum_{\ell=1}^{M} C_{M\ell}\, e^{-\ell p\tau}\right) d\tau$$

[4-23]

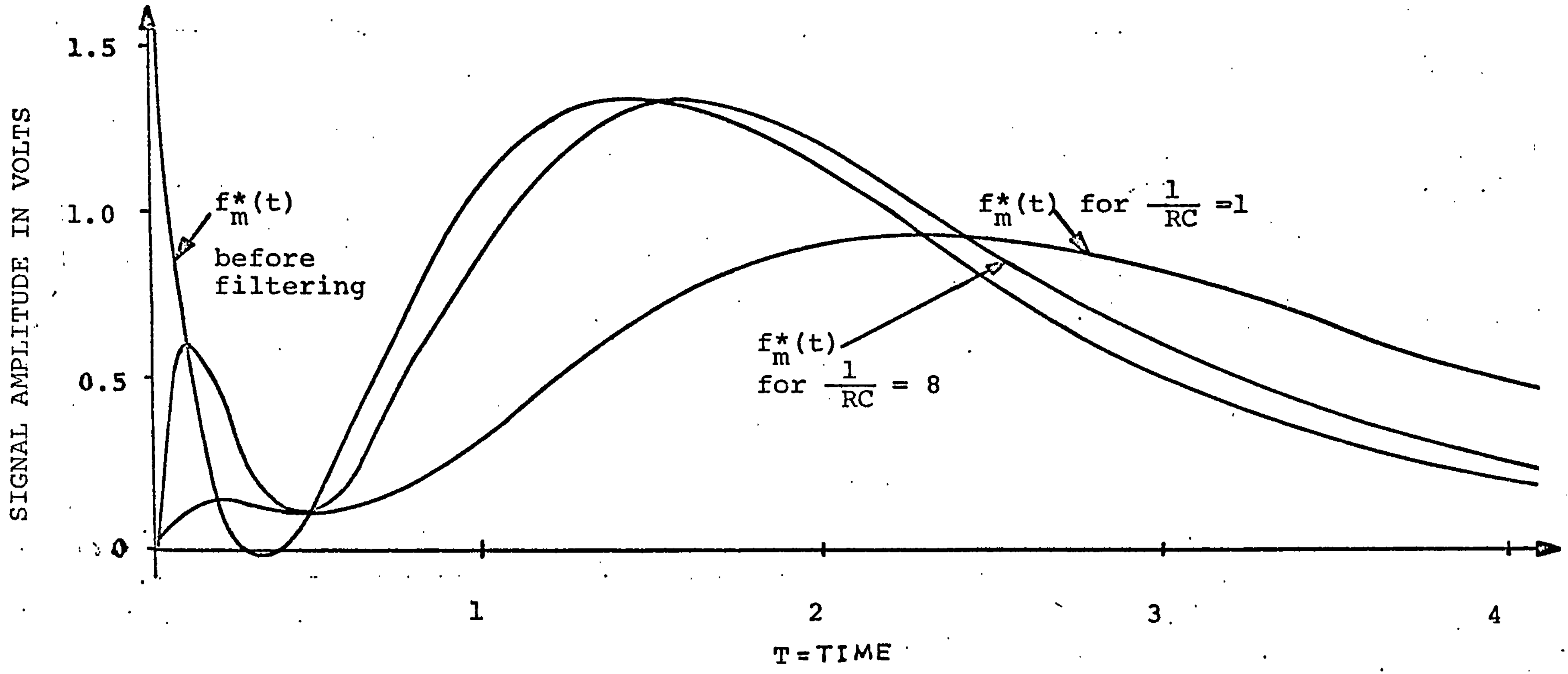

Figure 4-5. The effect of one pole filter on the composite transmitted waveform for several values of RC

$$\phi_{NM}(T) = \sum_{k=1}^{N} \sum_{\lambda=1}^{M} \frac{C_{Nk}\ C_{M\lambda}}{(RCkp-1)(\lambda p + 1/RC)}(1 - e^{-T(\lambda p+1/RC)}) -$$

$$\sum_{k=1}^{N} \sum_{\lambda=1}^{M} \frac{C_{Nk}\ C_{M\lambda}}{(RCkp-1)(\lambda p + 1/RC)}[1 - e^{-T(kp+\lambda p)}]$$

[4-24]

The error introduced in a message is shown in Figures (4-6) and (4-7) for two values of bandwidth.

One way to reduce the error due to the delay of the signal caused by the filter is to delay the operation of the receiver by an equal amount. The effect of this is shown in Figure (4-8). For the correct amount of delay in the receiver operation the error is greatly reduced. At the point of minimum error the receiver is said to be synchronized with the incoming signal.

A Lower Bound on pT

The crosstalk effects of time truncation crosstalk on the real exponential set may be studied as given in Equation [4-9].

$$E_{NM}(T) = \sum_{k=1}^{N} \sum_{\lambda=1}^{M} \frac{C_{Nk}\ C_{M\lambda}\ e^{-(k+\lambda)pT}}{(k + \lambda)p} \qquad [4\text{-}25]$$

For large values of pT, only the first term of Equation [4-25] is significant as shown in Figure (4-2) and may be rewritten as

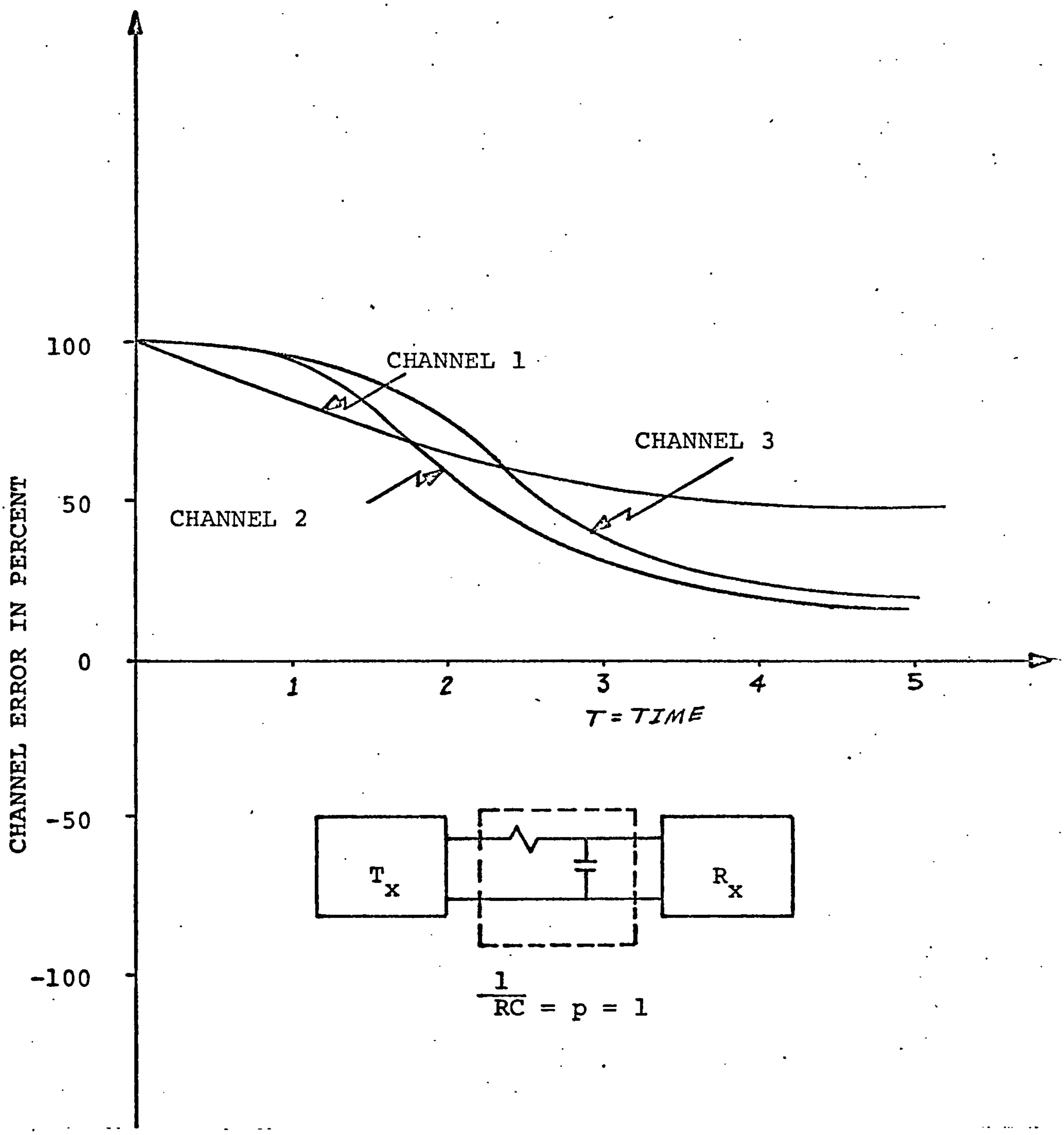

Figure 4-6. System error for one pole filter channel model with 1/RC equal to one

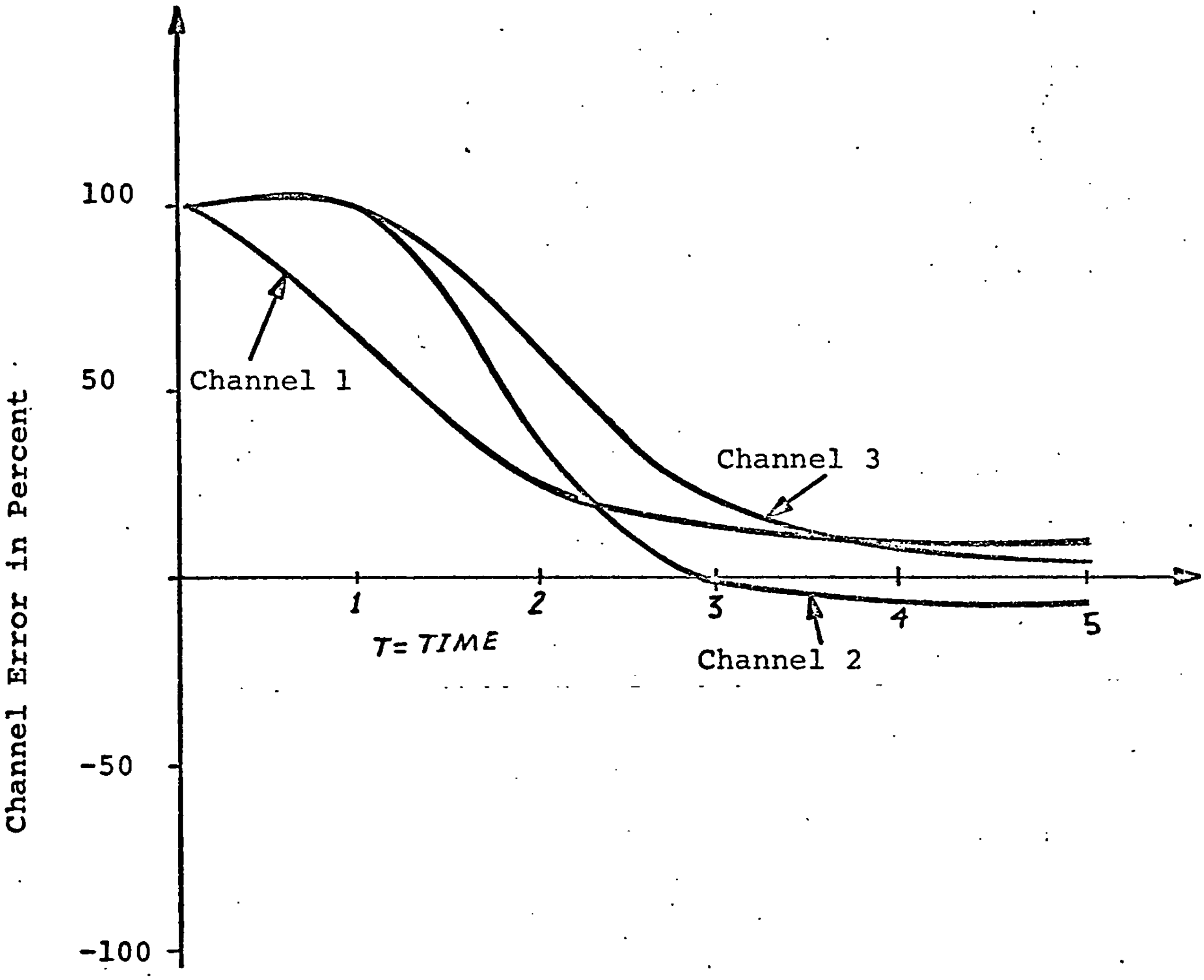

Figure 4-7. System error for a one pole channel model with 1/RC equal to eight

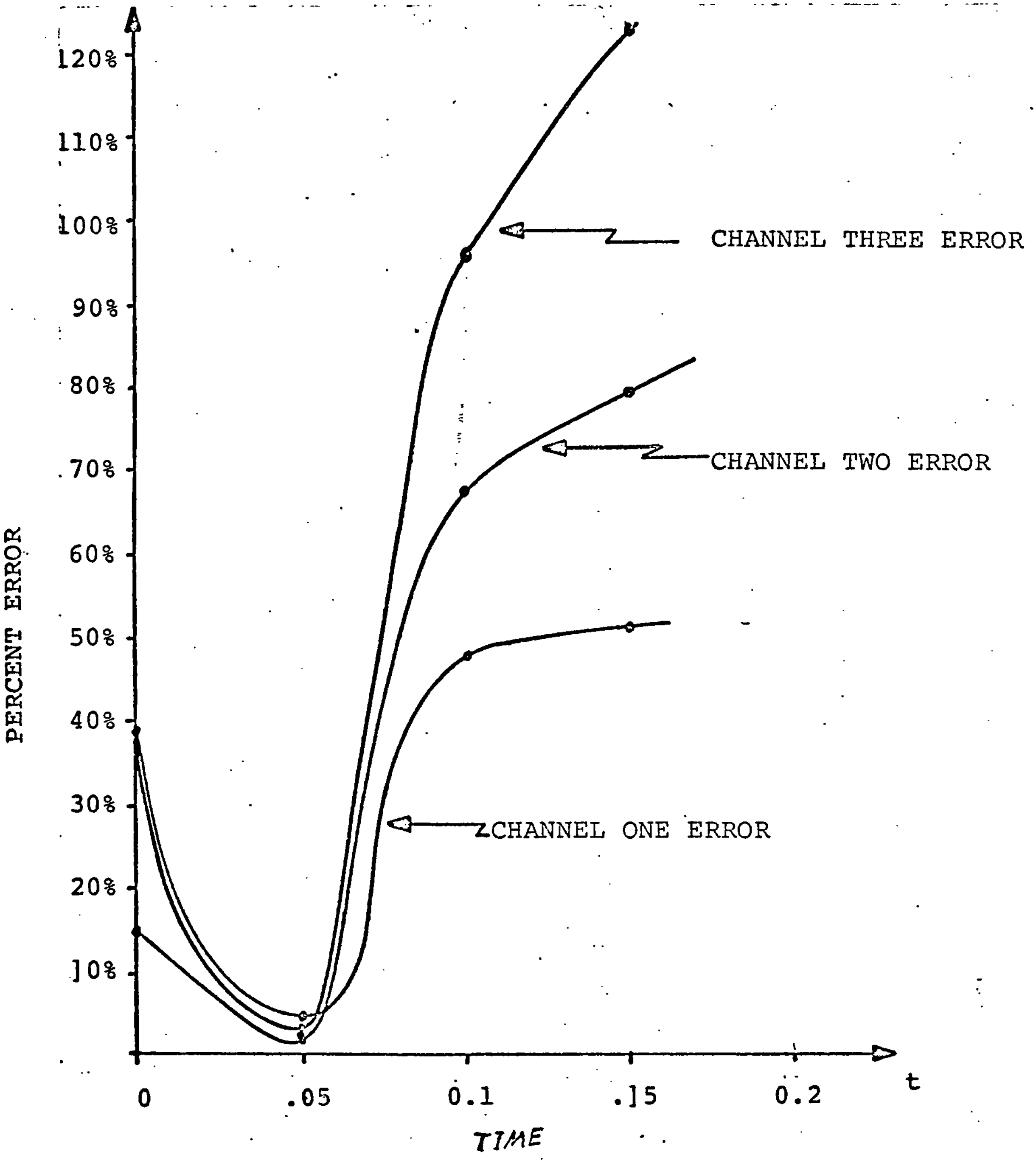

Figure 4-8. Error due to the delay caused by the channel

$$E_{NM}(T) = \varepsilon_{NM}(T) > \frac{C_{N1}\ C_{M1}\ e^{-2pT}}{2p} \qquad [4\text{-}26]$$

or

$$e^{2pT} > \frac{C_{N1}\ C_{M1}}{2p\varepsilon_{NM}(T)} \qquad [4\text{-}27]$$

or

$$T > \frac{1}{2p}[\mathrm{Ln.}(\frac{C_{N1}\ C_{M1}}{p}) - \mathrm{Ln.}(2\varepsilon_{NM}(T)] \qquad [4\text{-}28]$$

The coefficients of the initial terms in Equations [4-2] through [4-4] are given by the sequence $C_{11} = \sqrt{2p}$, $C_{21} = 4\sqrt{p}$, $C_{31} = 3\sqrt{6p}$, and the general expression for the N^{th} channel is therefore given by

$$C_{N1} = N\sqrt{2Np} \qquad [4\text{-}29]$$

and for the M^{th} channel the expression is

$$C_{M1} = M\sqrt{2Mp} \qquad [4\text{-}30]$$

The worst case of crosstalk occurs for M = N channels so that Equation [4-28] becomes

$$T > \frac{1}{2p}\{\mathrm{Ln.}[2N^3] - \mathrm{Ln.}[2\varepsilon_{NN}(T)]\} \qquad [4\text{-}31]$$

$$T > \frac{1}{2p}\{\mathrm{Ln.}[2] + 3\mathrm{Ln.}[N] - \mathrm{Ln.}[2] - \mathrm{Ln.}[\varepsilon_{NN}(T)]\} \qquad [4\text{-}32]$$

$$T > \frac{1}{2p}\{3\mathrm{Ln.}[N] - \mathrm{Ln.}[\varepsilon_{NN}(T)]\} \qquad [4\text{-}33]$$

or

$$pT > \frac{1}{2}[3\mathrm{Ln.}(N) - \mathrm{Ln.}[\varepsilon_{NN}(T)]] \qquad [4\text{-}34]$$

This equation is plotted in Figure (4-9) for several values of $\varepsilon_{NN}(T)$, and the lower bound on pT necessary for a required amount of crosstalk due to time truncation can be readily selected from this figure.

The bandwidth of a system based on such an exponential set is now compared with that of common multiplex systems. The Nyquist sampling theorem states

$$T \leq \frac{1}{2B_m} \qquad [4\text{-}35]$$

where B_m is the message bandwidth and T is the sampling period. Equation [4-17] yields an expression for system bandwidth given by

$$B_s = \frac{Np}{2\pi} \qquad [4\text{-}36]$$

or

$$p = \frac{2\pi B_s}{N}$$

A minimum sampling rate given by Equation [4-35] is used for purposes of comparing the various systems. A substitution of Equations [4-35] and [4-37] in Equation [4-34] yields

$$\left(\frac{2\pi B_s}{N}\right)\left(\frac{1}{2B_m}\right) > \frac{1}{2}[3Ln.(N) - Ln.(\varepsilon_{NN}(T))] \qquad [4\text{-}38]$$

or

$$B_s = \frac{NB_m}{2\pi}[3Ln.(N) - Ln.(\varepsilon_{NN}(T))] \qquad [4\text{-}39]$$

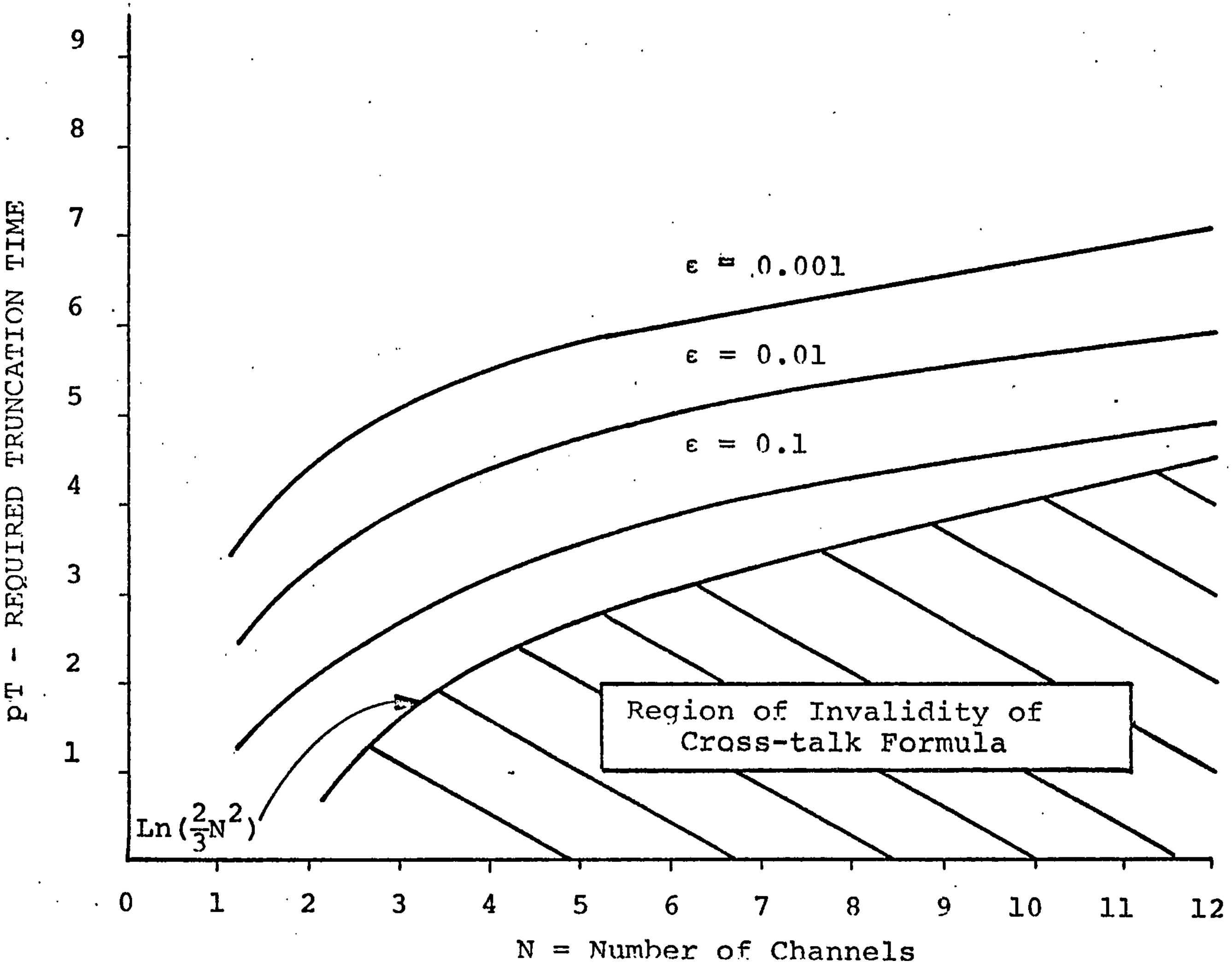

Figure 4-9. Truncation time required for specified crosstalk level $pT > \frac{1}{2}[3 Ln N - Ln \epsilon]$ where $\epsilon = MAX. E_{NM}$.

Equivalent expressions for other systems are given below

$$B_s = 2NB_m \text{ , for ordinary frequency division multiplexing.} \qquad [4\text{-}40]$$

$$B_s = 4NB_m \text{ , for time division multiplexing.} \qquad [4\text{-}41]$$

The results in the form of bandwidth versus number of channels are shown in Figure (4-10).

Peak Limiting Distortion

Figure (4-11) illustrates peak amplitude limiting of the composite waveform given by

$$f_m(t) = \sum_{N=1}^{R} \overline{m_N(t)} O_N(t) \qquad [4\text{-}42]$$

$\overline{m_N(t)}$ = sample value of $m_N(t)$, the N^{th} channel message

$O_N(t)$ = N^{th} orthogonal function

R = number of channels

Clipping results in a system when the peak amplitude capability of the system is exceeded, and it may occur during one or more intervals of each period of operation of the repetitive system. The peak signal amplitude of the real exponential set occurs at the t = 0 and increases as the number of functions increase. The peak amplitude of the individual members of the real exponential set at t = 0 is shown by Figure (4-12). Amplitude limiting of $f_m(t)$ causes distortion, and the equation for the receiver output in the case of

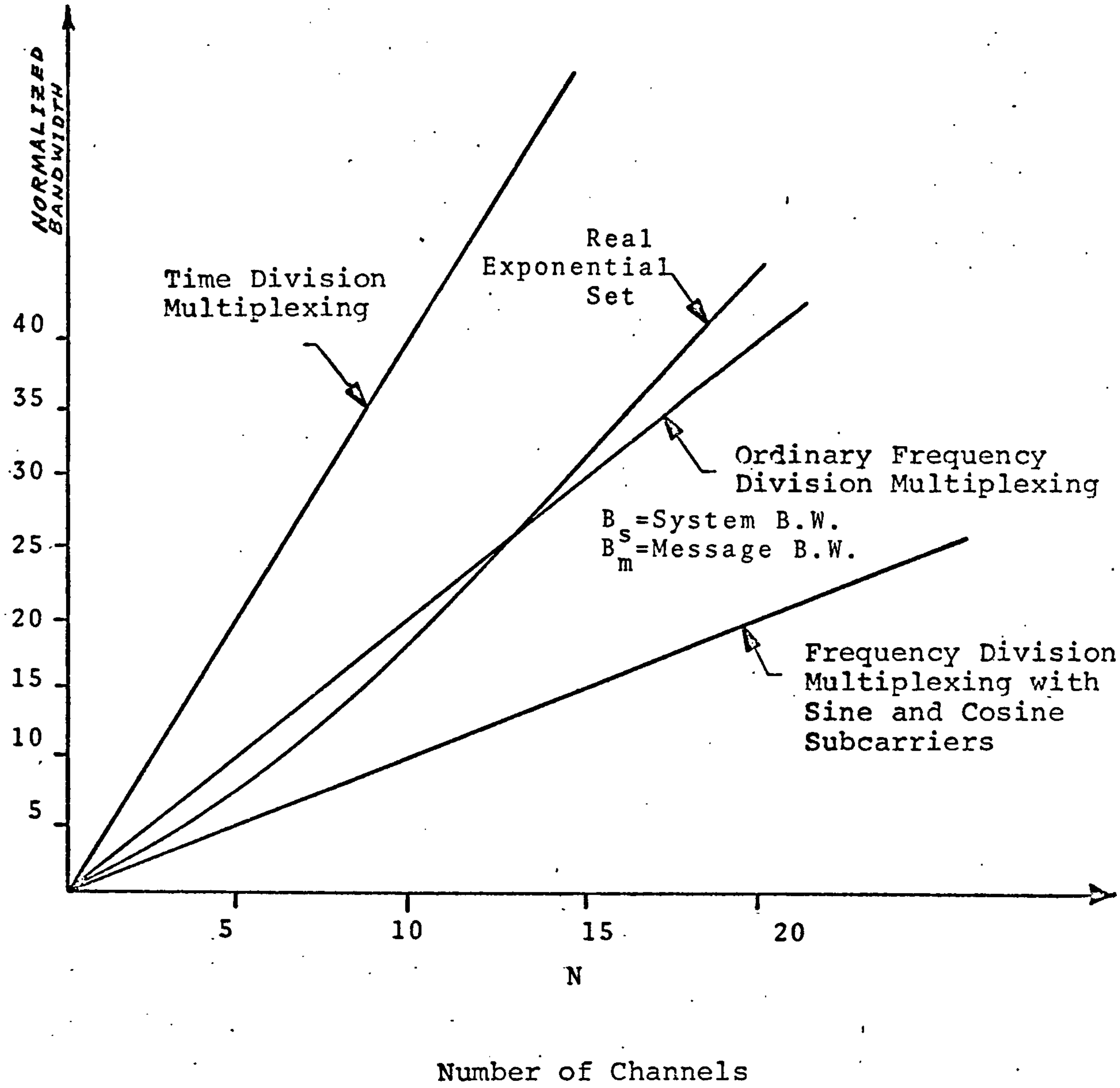

Figure 4-10. Normalized bandwidth versus number of channels

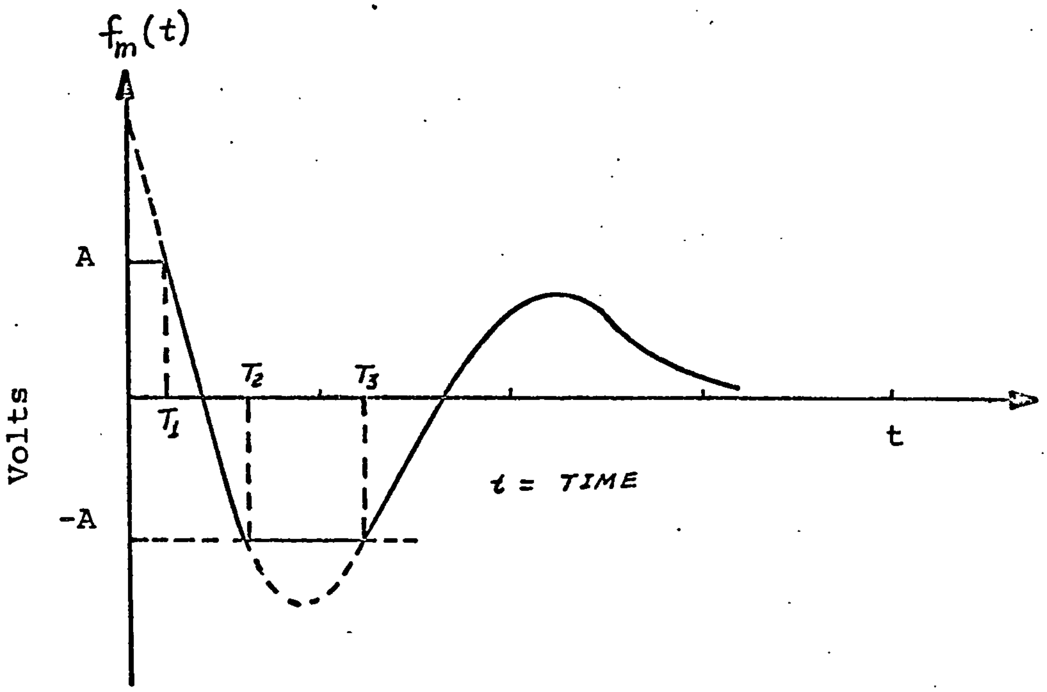

Figure 4-11. The function is shown limited at a peak amplitude of $f_m(t) = |A|$ for $0 \le t \le T_1$ and $T_2 \le t \le T_3$.

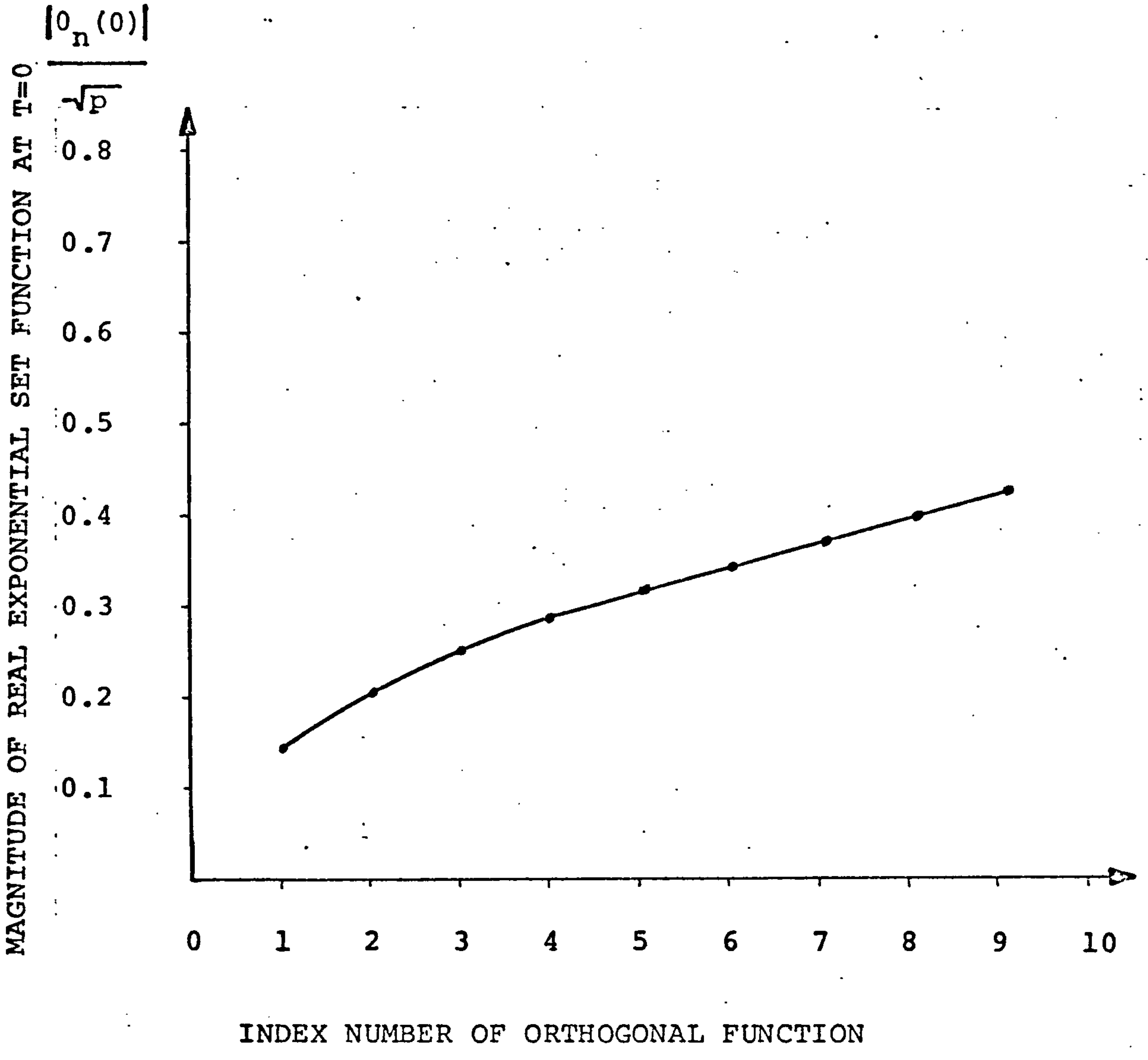

Figure 4-12. The magnitude of real exponential function at T=0 versus the index number

amplitude limiting to the value of

$$f_m(t) = \pm A$$

where A = constant in the interval $0 \leq t \leq T$, is

$$m_J^*(T) = \int_0^{T_1} AO_J(t)\,dt + \int_{T_1}^{T} f_m(t)O_J(t)\,dt \qquad [4\text{-}43]$$

where T_1 = time at which limiting ceases

T = pulse period

$m_J^*(T)$ = output of J^{th} channel at receiver.

An expansion of Equation [4-43] yields

$$m_J^*(T) = A \int_0^{T_1} \sum_{k=1}^{J} G_{Jk}e^{-kpt}\,dt +$$

$$\sum_{N=1}^{R} \sum_{\lambda=1}^{N} \sum_{k=1}^{J} m_N(t)C_{N1}C_{Jk} \int_{T_1}^{T} e^{-(\lambda+k)pt}dt \qquad [4\text{-}44]$$

and on evaluation of this integral one obtains

$$m_J^*(T) = A \sum_{k=1}^{J} \left[\frac{1 - e^{-kpT_1}}{kp}\right] +$$

$$\sum_{N=1}^{R} \sum_{\lambda=1}^{N} \sum_{k=1}^{J} \overline{m_N(t)}C_{N1}C_{Jk}\left[\frac{e^{-(\lambda+k)pT_1} - e^{-(\lambda+k)pT}}{(\lambda + k)p}\right] [4\text{-}45]$$

Figure (4-13) shows the effect of peak amplitude limiting on channel response in the form of percent error. For a three-channel system, the error is less than 5 per cent if the amplitude is allowed to reach 70 per cent of its peak value.

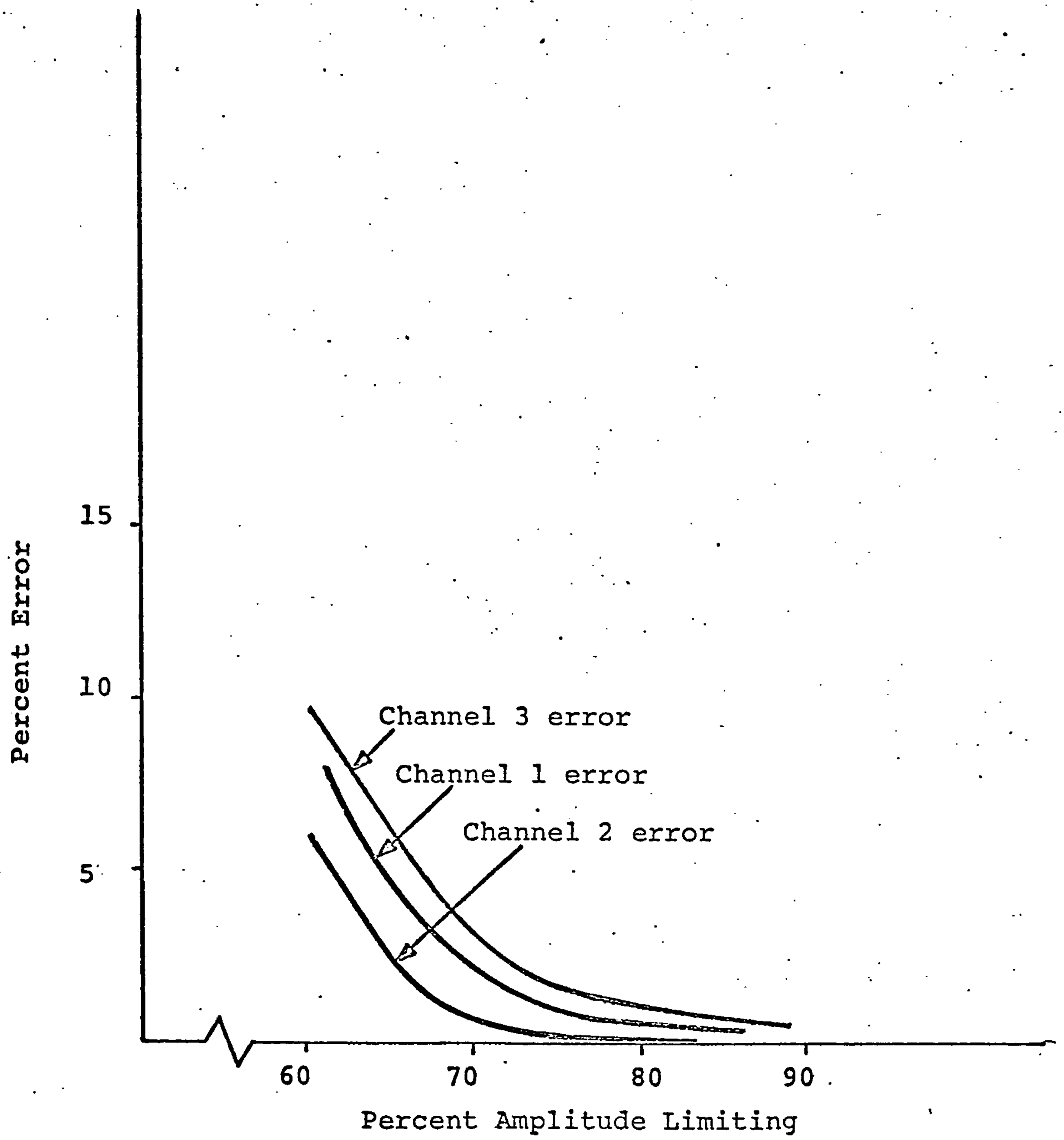

Figure 4-13. The effect of amplitude limiting on channel response. The amplitude is limited to a percentage of the peak value in both the positive and negative swings.

CHAPTER V

THE ORTHONORMAL POLYNOMIAL SET

The circuitry necessary to generate polynomial sets is relatively simple because its elements are linearly independent functions like

$$\{t^0, t^1, t^2, \ldots, t^n, \ldots\}. \qquad [5\text{-}1]$$

This set can be generated using analog integrators and amplifiers. A general polynomial would be of the form

$$\phi_N(t) = \sum_{k=0}^{N} \alpha_{Nk} t^k \qquad [5\text{-}2]$$

An example of such a set are the Legendre polynomials which form the basis for an Orthomux system described by Ballard (1962). The first few Legendre polynomials are

$$O_0(x) = 1 \qquad [5\text{-}3]$$

$$O_1(x) = x \qquad [5\text{-}4]$$

$$O_2(x) = \frac{1}{2}[3x^2 - 1] \qquad [5\text{-}5]$$

with the orthogonality interval of $-1 \leq x \leq 1$. If the following substitution is made for x in the above equations;

$$x = (t - \frac{T}{2}) \qquad [5\text{-}6]$$

the orthogonality interval becomes

$$0 \leq t \leq T$$

and the polynomials take on the form

$$O_0(t) = 1 \qquad [5\text{-}7]$$

$$O_1(t) = 2t/T - 1 \qquad [5\text{-}8]$$

$$O_2(t) = 6t^2/T^2 - 6t/T + 1 \qquad [5\text{-}9]$$

The system should have equal power in each channel, so it is necessary to normalize these functions, and it is done by evaluating the integrals

$$K_N^2 \int_0^T O_N^2(t)\,dt = 1 \qquad [5\text{-}10]$$

to obtain K_N as

$$K_0 = \sqrt{1/T}$$

$$K_1 = \sqrt{3/T}$$

$$\vdots$$

$$K_N = \sqrt{(2N+1)/T} \qquad [5\text{-}11]$$

Therefore these functions O_1, O_2, . . . , O_N, . . . do not have the same value at $t = T$. The orthonormal Legendre polynomials become

$$O_0(t) = \sqrt{1/T} \qquad [5\text{-}12]$$

$$O_1(t) = \sqrt{3/T}\left(\frac{2t}{T} - 1\right) \qquad [5\text{-}13]$$

$$O_2(t) = \sqrt{5/T}\ (6t^2/T^2 - 6t/T + 1) \qquad [5\text{-}14]$$

and this set would have equal power distribution in each channel. It is necessary to produce stable positive and negative voltages to set the initial values of the ordinary Legendre polynomials. A different voltage must be derived from the reference voltages for each of the polynomials in the modified set because the initial values are all different.

Zero Initial Value Set

In this section another polynomial set has been constructed using the Gram-Schmidt procedure, where each function has an initial value of zero as shown in Appendix B. The first three polynomials are

$$O_1(t) = \sqrt{3}t \quad [5\text{-}15]$$

$$O_2(t) = \sqrt{5}(4t^2 - 3t) \quad [5\text{-}16]$$

$$O_3(t) = \sqrt{7}(15t^3 - 20t^2 + 6t) \quad [5\text{-}17]$$

where T is normalized to unity. The base set used here is $(t^1, t^2, \ldots, t^n, \ldots)$ and Figure (5-1) shows the first three functions of the zero initial value set.

System Bandwidth Requirements

An expression for the power density spectrum is for the polynomials functions discussed above. In general,

$$O_N(t) = \sum_{k=1}^{N} \alpha_{Nk} t^k \quad [5\text{-}18]$$

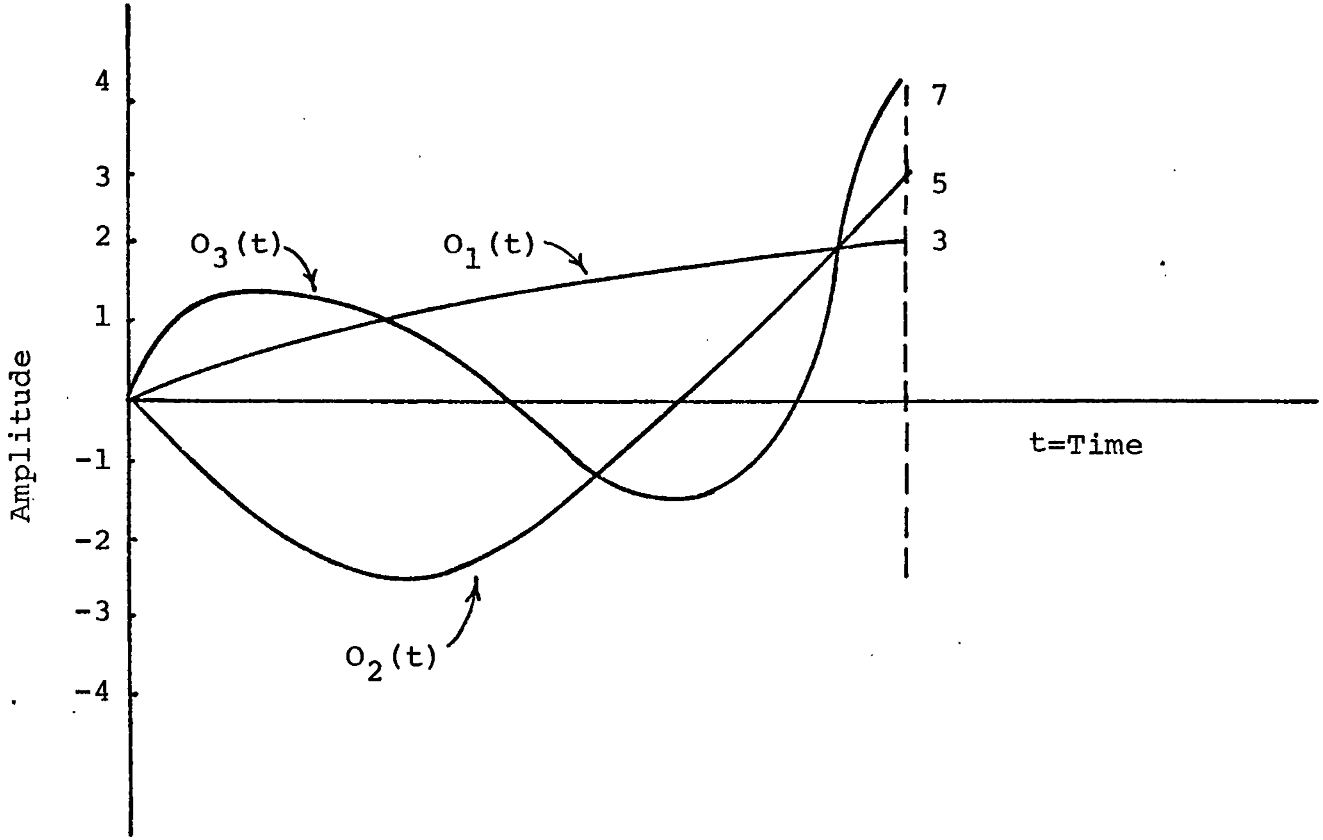

Figure 5-1. The First Three Polynomials of the Zero Initial Value Set

and the Fourier coefficients α_{Nk} are given below

$$a_\beta = 2\int_0^1 t^k \cos(\beta\omega_1 t)\,dt \quad , \quad \beta = 0,1,2,... \qquad [5\text{-}19]$$

$$b_\beta = 2\int_0^1 t^k \sin(\beta\omega_1 t)\,dt \quad , \quad \beta = 1,2,... \qquad [5\text{-}20]$$

$$c_\beta^2 = a_\beta^2 + b_\beta^2 \qquad [5\text{-}21]$$

Integration by parts yields the following:

$$a_\beta = 2\{\frac{1}{\beta\omega_1}t^k \sin(\beta\omega_1 t)\Big|_0^1 - \frac{k}{\beta\omega_1}\int_0^i t^{k-1} \sin(\beta\omega_1 t)\,dt\} \qquad [5\text{-}22]$$

$$b_\beta = 2\{-\frac{1}{\beta\omega_1}t^k \cos(\beta\omega_1 t)\Big|_0^1 + \frac{k}{\beta\omega_1}\int_0^1 t^{k-1} \cos(\beta\omega_1 t)\,dt\} \qquad [5\text{-}23]$$

which can be easily calculated by a computer program. The first two calculations give the following results for $O_1(t)$

$$|c_\beta|^2 = 3/(\beta\pi)^2 \qquad [5\text{-}24]$$

and for $O_2(t)$,

$$|c_\beta|^2 = [5/(\beta\pi)^2][1 + 16/(\beta\pi)^2] \qquad [5\text{-}25]$$

The dominant term in the above equations for large values of β is given by

$$|c_\beta|^2 \approx \frac{2N + 1}{(\beta\pi)^2} \qquad [5\text{-}26]$$

where N is the index number of the function and $\beta = 0,1,2,$ .... An evaluation of the crosstalk in a system due to frequency truncation is easily done by a computer program for each filter. The bandwidth of the simulated filter should be varied from a minimum value given by $f_0 = 1/T$, which is the system repetition rate, to an upper limit of approximately ten times the minimum value or until the distortion reaches tolerable values. There is no crosstalk due to time truncation for a system based on the zero initial value set of polynomials since the interval of orthogonality will be the same as the system period. Thus crosstalk would occur due only to amplitude and frequency truncation and interference.

Peak Voltage

The zero initial value set has unit energy in each channel for the orthogonality interval, and the peak value for $O_n(t)$ occurs at t equal to unity and is given by

$$O_N(t=1) = \sqrt{2N + 1} \qquad [5\text{-}27]$$

This makes it evident that the problem of peak voltages poses a very serious limitation, and it becomes more serious when the composite signal is formed by summing the various individual polynomials.

Implementation

The orthomux system based on polynomials can be built using analog integrating circuits. Figure (5-2) shows a block diagram of such a system based on such polynomials.

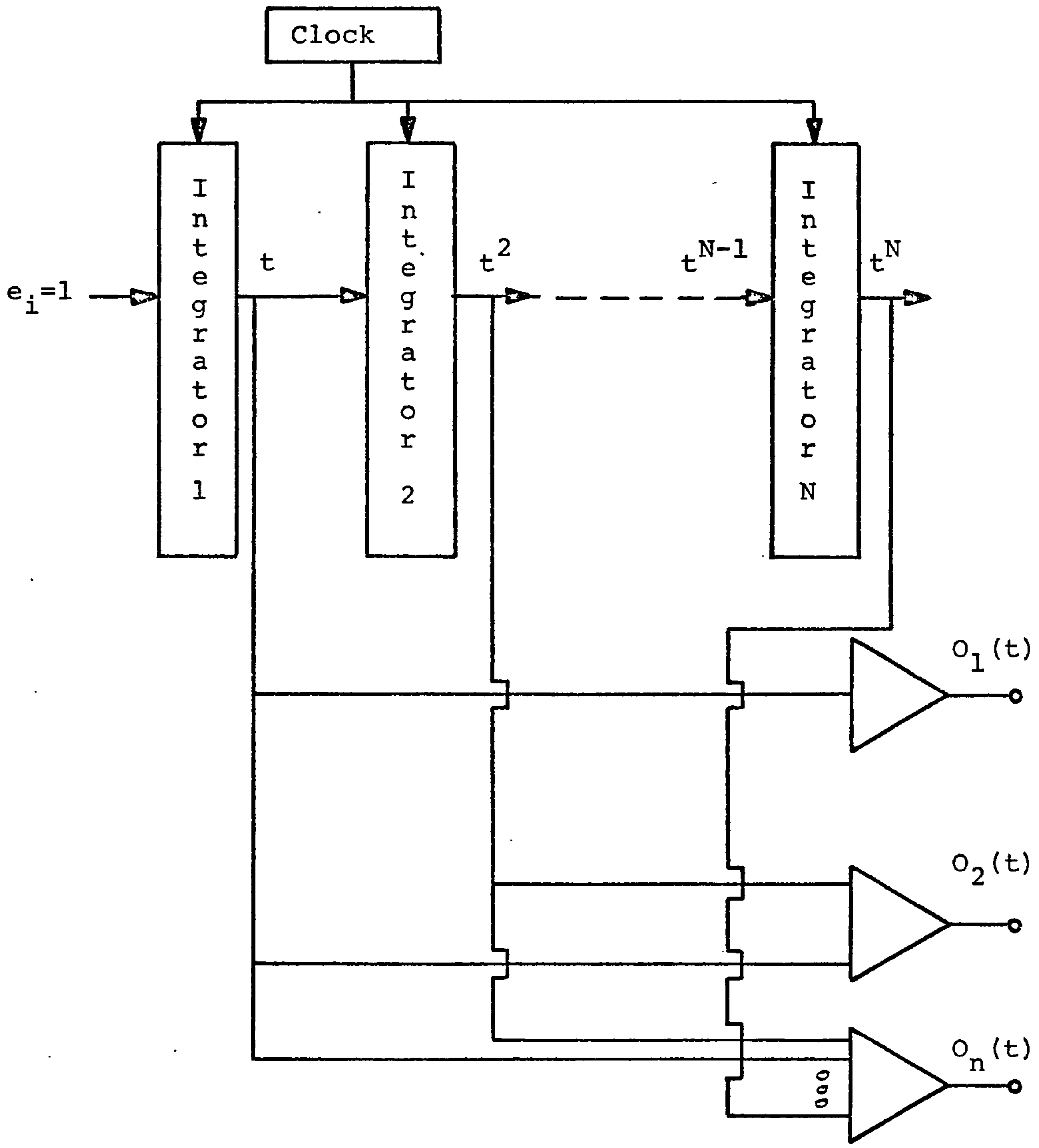

Figure 5-2

An implementation of the polynomial set

CHAPTER VI

AN ORTHOGONAL DIGITAL SYSTEM

The preceding chapters dealt with analog signal sets suitable for use in an orthogonal multiplexing system, and now a possible orthomux system using orthogonal or orthonormal digital waveforms is discussed in this chapter including the formation of sets of orthogonal digital functions suitable for such a system.

A digital orthomux system is designed in the same manner as an analog system, in that the code words used in such a system are orthogonal to each other. An example of an orthonormal set suitable for use as a basis for an orthomux system is:

$$R = [R_0, R_1, \ldots] \qquad [6\text{-}1]$$

$$R_N = \mathrm{Sgn}\ [\mathrm{Sin}(2^N \pi t)] \qquad [6\text{-}2]$$

This is the family of Radamacher functions, and their region of orthogonality is $0 \leq t \leq 1$. The Sgn function is defined as

$$\begin{aligned} \mathrm{Sgn}(a) &= 1 \quad (a > 0) \\ \mathrm{Sgn}(a) &= 1 \quad (a < 0) \\ \mathrm{Sgn}(a) &= 0 \quad (a = 0) \end{aligned} \qquad [6\text{-}3]$$

The first three Radamacher functions are shown in Figure 6-1. A simple method of forming an orthogonal binary set is by employing the Hadamard matrix. A Hadamard matrix is a square matrix whose elements are restricted to take on values of plus and minus one, and has the property that the row vectors as well as the column vectors are mutually orthogonal. Formation of Hadamard matrices of the order 2^k where k is an integer equal to or greater than zero is possible using the relationship

$$H_{2^{K+1}} = \begin{vmatrix} H_{2^K} & H_{2^K} \\ H_{2^K} & \overline{H_{2^K}} \end{vmatrix} \qquad [6-4]$$

For example, H_1 can be immediately written as $H_1 = [1]$ (or $H_1 = [0]$). If $H_1 = [1]$ is selected, the matrix H_2 takes the form

$$H_2 = \begin{vmatrix} H_1 & H_1 \\ H_1 & \overline{H_1} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \qquad [6-5]$$

and the matrix H_4 is given by

$$H_4 = \left|\begin{array}{cc:cc} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ \hdashline 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right| = \begin{vmatrix} x_{1i} \\ x_{2i} \\ x_{3i} \\ x_{4i} \end{vmatrix} \qquad [6-6]$$

and so on. A useful definition of correlation for binary waveforms is

$$P = A - B \qquad [6-7]$$

where

A = number of like terms, and

B = number of unlike terms.

Application of this definition to Equation [6-6] shows that either the row vectors or column vectors are a suitable set of functions for an orthogonal multiplexing system. Another set of orthogonal waveforms is given by

$$\begin{aligned} F_1 &= [\ 1, \quad 1, \quad 1, \quad -1] \\ F_2 &= [\ 1, \quad 1, \quad -1, \quad 1] \\ F_3 &= [\ 1, \quad -1, \quad 1, \quad 1] \\ F_4 &= [-1, \quad 1, \quad 1, \quad 1] \end{aligned} \qquad [6\text{-}8]$$

If the value zero is substituted for the positive ones in Equation [6-8], the resulting orthogonal set is recognized as that used in time division multiplexing. In all these orthogonal digital codes, n information bits are transmitted in 2^n symbols. Time division multiplexing has the optimum peak-to-average power ratio of unity and this also allows the use of hard limiting at the receiver which limits the noise. However, it requires more accurate synchronization of the receiver in order to prevent excess message distortion, since the energy of each message is concentrated in a narrow time span. Systems using Equation [6-6] as a basis have the advantage of having the impulse noise distributed evenly among the channels and thus the synchronization problem becomes less critical.

Implementation of the digital system is accomplished by using cascaded bistable multivabrator circuits which give the outputs of Equation [6-2] as shown in Figure (6-1). The additional wave forms which must be formed to complete the orthogonal set are formed by combinational logic blocks. An eight-channel code is given by

$$\begin{aligned}
DC &= [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \\
A &= [1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0] \\
B &= [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0] \\
C &= [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0] \\
D &= [1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1] \\
E &= [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1] \\
F &= [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1] \\
G &= [1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0]
\end{aligned} \qquad [6\text{-}9]$$

where DC is a level, and A, B, and C are formed by cascading an astable multivibrator and two bistable multivibrators. D, E, F, and G are derived from A, B, and C, using and-or logic circuits which satisfy the Boolean functions given by

$$D = \overline{A}\ \overline{B} + AB \qquad [6\text{-}10]$$

$$E = \overline{A}\ \overline{C} + AC \qquad [6\text{-}11]$$

$$F = \overline{B}\ \overline{C} + BC \qquad [6\text{-}12]$$

$$G = \overline{A}\ (B\overline{C} + \overline{B}C) + A\ F \qquad [6\text{-}13]$$

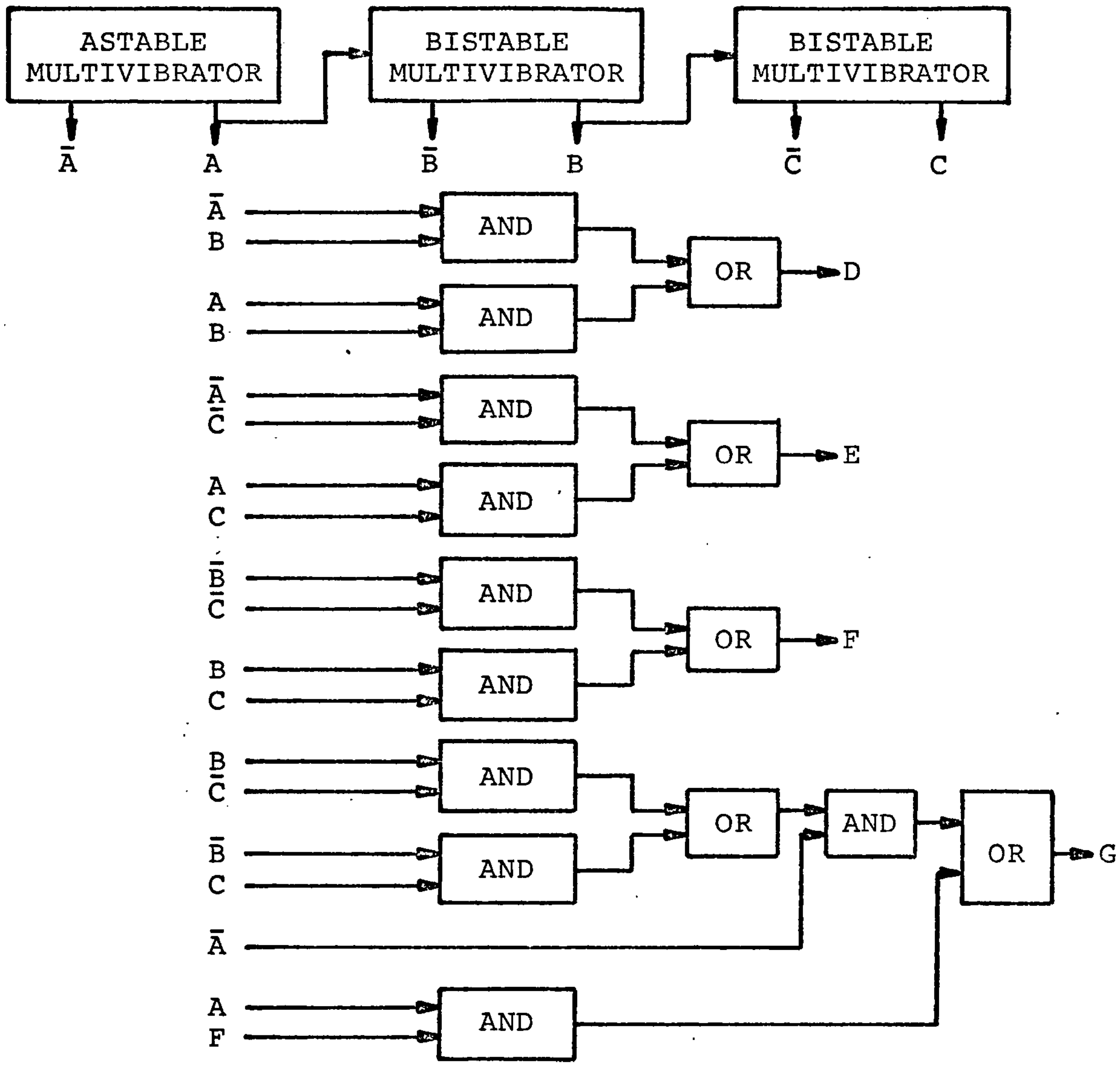

Figure 6-1. Logic circuitry for realization of an orthogonal digital set

A message can be impressed on an orthogonal function by transmitting either the function or its inverse, which is formed by passing the function through a simple inverting amplifier or by ordinary analog methods. A digital system does not require analog multipliers in either the transmitter or receiver, since multiplication by plus or minus one can be accomplished by a selective inverting amplifier controlled by the appropriate orthogonal waveform. In the case of a digital system it is possible to devise a method of combining the orthogonal functions by means of a Boolean logic transformation. Titsworth (1963) has proposed a system employing a majority logic in the transmitter to combine the channel waveforms and correlation detection in the receiver. The advantage of such a system is that the peak-to-average power ratio is unity as is the case with time division multiplexing, and hard limiting at the receiver is possible. This is not an orthomux system and, according to Titsworth, performance is about two decibels poorer than the systems previously discussed.

CHAPTER VII

TECHNIQUES TO IMPROVE SYSTEM PERFORMANCE

The performance of orthomux communication systems may be limited because of the following factors:

1) the finite time truncation of a signal set with an infinite orthogonality interval,

2) large peak-to-average signal voltage ratio, and

3) excessive bandwidth requirement of the signal set.

These factors are often responsible for distortion of the received messages, since crosstalk is the inevitable result of time, amplitude, or frequency truncation of the transmitted signal. If the resultant distortion becomes intolerable, it is necessary to take steps to improve the system performance, and the remainder of this chapter is a discussion of methods to achieve this objective.

Finite Time Truncation

It is sometimes desirable to use a signal set which is orthogonal over an infinite interval. The reason for this is that the system parameters can be selected so that the crosstalk is tolerable at the minimum truncation time, and therefore will be less for longer truncation times. Thus a basic system may be operated at any repetition rate below some maximum value. In some cases it may be desirable to completely eliminate the crosstalk due to time truncation, and this can often be accomplished by applying the Gram-Schmidt procedure to the linearly independent set of functions which are used to construct an orthonormal set of functions under consideration. For instance, consider the real exponential set given by Equation [4-5] which has an orthogonality interval of $0 \leq t \leq \infty$. The linearly independent set is given by $e^{-pt}, e^{-2pt}, \ldots, e^{-npt}$. The first two functions of the real exponential set orthonormal over the interval $0 \leq t \leq T$ were calculated using the Gram-Schmidt procedure outlined in Appendix B, and are given by:

$$\phi_1 = (2p/B)^{\frac{1}{2}} e^{-pt} \qquad [7\text{-}1]$$

$$\phi_2 = \frac{(2C/3B)e^{-pt} - e^{-\ pt}}{[(D/4p) - (2C^2/9pB)]^{\frac{1}{2}}} \qquad [7\text{-}2]$$

where

$B = 1 - e^{-2pt}$

$C = 1 - e^{-3pt}$

$D = 1 - e^{-4pt}$

T = Orthogonality Interval, and

p = System Parameters

These functions may be expressed in the following form:

$$\phi_1 = C\ e^{-pt} \qquad [7\text{-}3]$$

$$\phi_2 = C\ e^{-pt} + C_3 e^{-2pt} \qquad [7\text{-}4]$$

where

$$C_1 = (2p/B)^{1/2}$$

$$C_2 = \frac{2C/3B}{[(D/4p) - (2C^2/9pB)]}$$

and

$$C_3 = \frac{1}{[(D/4p) - (2C^2/9pB)]}$$

It is evident that reducing the orthogonality interval from an infinite to a finite value does not increase the complexity of the circuitry required to generate this set of functions. The crosstalk due to time truncation is completely removed and the system performance improved by a corresponding amount, however. This general result for ϕ_i may be verified by allowing the interval T to increase without bound until in the limit B = C = D = 1, and the orthogonal functions become

$$\phi_1 = \sqrt{2p}e^{-pt} \qquad [7\text{-}5]$$

$$\phi_2 = 4\sqrt{p}e^{-pt} - 6\sqrt{p}e^{-2pt} \qquad [7\text{-}6]$$

Peak-to-Average Ratio

A practical communication system has a finite allowable peak-to-average power ratio and if operation is attempted outside this limit, the transmitted signal is subject to amplitude limiting resulting in message distortion. A certain amount of amplitude limiting is tolerable in order to assure the maximum utilization of the transmitter capability. A knowledge of the amplitude distribution of the message signals is very useful in this connection and can sometimes be obtained. The simulation program given in Appendix C is used to determine the exact message distortion of each channel as a function of amplitude limiting. Typical values of peak signal levels are given below for several orthonormal sets with N representing the number of channels and T the magnitude of the orthogonality interval:

1) for time division multiplexing V (peak), the composite signal peak is given by

$$V(\text{peak}) = \sqrt{N/T} \qquad [7\text{-}7]$$

2) for frequency division multiplexing,

$$V(\text{peak}) \leq N\sqrt{N/T} \qquad [7\text{-}8]$$

3) for the real exponential set which is orthonormal over the interval $0 \leq t \leq \infty$, the peak value ϕ_k of the k^{th} signal of the set, is given by

$$\phi_k(\text{peak}) = \sqrt{2pk} \qquad [7\text{-}9]$$

and the composite signal peak is given by

$$V(\text{peak}) = \sum_{k=1}^{N} \phi_k = \sqrt{2p}\sum_{k=1}^{N} \sqrt{k} \qquad [7\text{-}10]$$

4) for the polynomial set given by Equations [7-14], [7-15], and [7-16] the composite peak value is

$$V(\text{peak}) = \sum_{k=1}^{N} \frac{\sqrt{2k+1}}{\sqrt{T^{2k+1}}} \qquad [7\text{-}11]$$

There are several methods which may be capable of lowering the peak signal value of an orthomux system. Each particular orthomux system needs to be examined, since all the techniques are not suitable for all systems. In the case of frequency division multiplexing, it is clearly desirable to use sine functions rather than cosine functions as the orthogonal set, for the peaks of the sine functions occur at different points in the orthogonality interval and thus the peak value of the sum of the sine function subcarriers is less than the cosine functions. The peak value for the cosine set is

$$V(\text{peak}) = \sum_{k=1}^{N} A \cos(k\omega t) = NA \qquad [7\text{-}12]$$

where for the sine set it is

$$V(\text{peak}) = \sum_{k=1}^{N} A \text{ sine } (k\omega t) < NA \qquad [7\text{-}13]$$

where

N = total number of channels, and

A = peak value of the individual channels.

The peak value for the normalized sine function set is shown in Table 7-1.

The system based on both the sine and cosine functions utilizes bandwidth two times as effectively as the sine-only system and also has a lower peak signal value for an equal number of channels. The peak signal value for the sine and cosine subcarrier system for eight channels was calculated to be

$$V(\text{peak}) = 7.59$$

which is less than the value listed for the eight-channel system in Table 7-1. The peak of the sine and cosine system contains less energy than that of the sine-only system, and this is responsible for further reducing the distortion due to amplitude limiting. Table 7-2 shows the effect of amplitude limiting in terms of message distortion for the sine-and-cosine system. It is therefore preferable from bandwidth and peak-to-average power considerations to use both the sine and cosine functions for frequency division multiplexing systems.

The peak signal value of orthogonal systems based on functions such as the real exponential set and the polynomial set can be reduced by shifting the level of the signal so that the constant or D.C. value is removed. This makes the set orthogonal to any constant as well. An example of such an orthonormal polynomial set is given by

Table 7-1. The peak signal value of the orthogonal system based on sine functions only.

Number of Channels N	V(Peak)	θ(Radians)
1	1.41	1.57
2	2.48	.93
3	3.5	.67
4	4.51	.52
5	5.53	.43
6	6.55	.36
7	7.57	.31
8	8.57	.27
9	9.6	.25
10	10.6	.23

Table 7-2. Message distortion as a function of amplitude limiting for an eight-channel system. The odd numbered channels are sine channels, the even numbered channels are cosine channels.

Channel Number	Percent Message Distortion for Limiting at .8 V(Peak)	Percent Message Distortion for Limiting at .6 V(Peak)
1	2.7	8.5
2	10.0	28.0
3	5.2	15.6
4	8.8	24.9
5	7.1	20.3
6	7.0	19.7
7	8.3	22.2
8	4.7	13.1

$$\phi_1 = 1/\sqrt{T} \quad [7\text{-}14]$$

$$\phi_2 = \sqrt{3/T^3}[2t - T] \quad [7\text{-}15]$$

$$\phi_3 = \sqrt{5/T^5}[6t^2 - 6Tt + T^2] \quad [7\text{-}16]$$

where $0 \leq t \leq T$ is the orthogonality interval, ϕ_i may be a constant, and the other ϕ_is will still be orthogonal. It is of course possible to shift this set by a constant amount in the transmitter so that the positive and negative peaks are equal, but the equipment necessary to remove the messages in the receiver would be very complex. Another way to improve the condition is to multiply the composite signal $f_m(t)$

$$f_m(t) = m_1(t)O_1(t) + m_2(t)O_2(t) + \ldots \quad [7\text{-}17]$$

where

$m_i(t)$ = message in the i^{th} channel, and

$O_i(t)$ = i^{th} channel orthogonal function

by another function $P(t)$, which has a small or zero value at the point in time where the composite signal peaks. This is particularly effective for the real exponential set where the peak value of each function occurs at $t = 0$. It is necessary to generate the reciprocal of $P(t)$ in the receiver in order to recover the messages. The transmitted signal is thus

$$Tx = f_m(t)P(t) \quad [7\text{-}18]$$

and the functions generated in the receiver are given by

$$(Rx)_j = P^{-1}(t)O_j(t) \qquad [7\text{-}19]$$

where

$P^{-1}(t) = 1/P(t)$

$j = j^{th}$ channel

The j^{th} channel detection operation takes the form given by

$$\int_0^T (f_m(t)P(t))(P^{-1}(t)O_j(t))\,dt \qquad [7\text{-}20]$$

which becomes

$$\int_0^T f_m(t)O_j(t)\,dt = m_j(t) \qquad [7\text{-}21]$$

and thus the message is recovered correctly. Figure (7-1) depicts such an operation. The improvement is dependent on the function selected for P(t).

System Bandwidth Considerations

The bandwidth requirements of a system often determine the suitability of a given set. A comparison of the bandwidth requirements of orthogonal multiplexing systems based on practical functions, such as sines, cosines, and real exponentials shows that the system based on both sines and cosines (sometimes called super frequency division

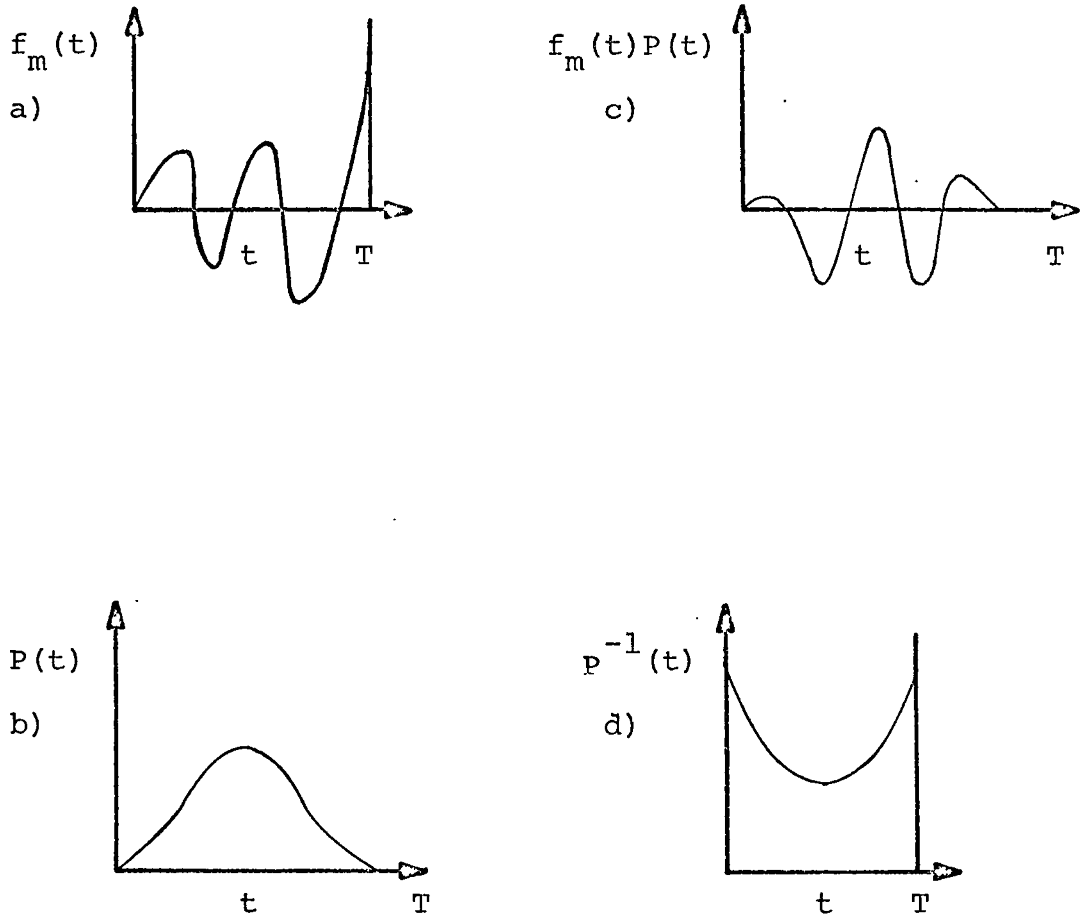

Figure 7-1

Waveforms formed in the transmitter and receiver multiplexing) is considerably better in this respect (see Figure (4-10)). It is possible to compress the spectrum of a signal by smoothing it in the time domain. One way this can be accomplished is by inverting the time domain waveforms of odd functions such as the odd ordered Legendre polynomials.

CHAPTER VIII

SYSTEM IMPLEMENTATION

The orthogonal multiplexing system contains several functions whose complexity and expense of realization in the transmitter and receiver is considered next. Specific functional operations considered here are function generation, multiplication, filtering, and signal detection.

Function Generation

Suitable functions for use in orthogonal multiplexing are the exponentials, sines, polynomials of time, and digital signals, or their products. The real exponential linearly independent set uses these functions

$$\{e^{-pt}, e^{-2pt}, \ldots, e^{-kpt}\} \qquad [8\text{-}1]$$

These functions are the response of an RC network to an impulse function input (Figure (8-1)). These may also be produced by a variation of this circuit which is shown in Figure (8-2). The capacitor is charged rapidly by a short duration pulse from a low impedance source. The diode conducts when the pulse is positive and as the pulse voltage discharges through the resistor to produce

$$e_0 = Ee^{-t/RC}, \qquad t \geq 0 \qquad [8\text{-}2]$$

The values of RC are selected to be equal to $1/np$ for $n = 1, 2, \ldots$ and thus all the basic functions are generated for the real exponential set. In combining the basic exponentials to form a function such as

$$O_2 = C_{21}e^{-pt} + C_{22}e^{-2pt} \qquad [8\text{-}3]$$

it is necessary to use operational amplifiers with the gains adjusted to give the coefficients the correct value. It is simpler to build these amplifiers for "A.C. coupled" or "capacitively coupled" rather than direct coupled service. It is possible to use capacitively coupled amplifiers provided the orthogonal set has no d.c. component in any member function, which implies that these functions must be orthogonal to any constant term. A set of orthonormal functions has been formed which are based on the real exponential functions given by

$$e^{-0pt},\ e^{-pt},\ e^{-2pt},\ \ldots,\ e^{-kpt} \qquad [8\text{-}4]$$

These orthonormal functions also have a finite interval of orthogonality, and the first three are given by

$$\phi_1 = 1/\sqrt{T} \qquad 8\text{-}5$$

$$\phi_2 = \frac{e^{-pt} - (A/pT)}{[(B/2p) - (A^2/p^2T]} \qquad [8\text{-}6]$$

$$\phi_3 = \frac{e^{-2pt} - K_1 e^{-pt} + K_2}{\{g_3,\ g_3\}^{\frac{1}{2}}} \qquad [8\text{-}7]$$

where

$$A = 1 - e^{-pt}$$

$$B = 1 - e^{-2pt}$$

$$K_1 = \frac{2pTC - 3AB}{3BpT - 6A^2}$$

$$K_2 = \frac{A}{3pT}\left(\frac{2pTC - 3AB}{BpT - 2A^2}\right) - \frac{B}{2pT}$$

$$\{g_3, g_3\}^{\frac{1}{2}} = \left[\frac{-24K_1K_2A+6(K_1^2+K_2)B - 8K_1C + 3D + 12K_2^2Tp}{12p}\right]^{\frac{1}{2}}$$

This set has a lower peak-to-average ratio than the real exponential set discussed in Chapter IV by the amount the elimination of the constant or d.c. term removes.

Polynomials in powers of t are easily formed, as shown in Figure (8-3). Functions of the form

$$y = Ke^{-a^2t^2} \qquad [8\text{-}8]$$

are generated by the circuit of Figure (8-4) with electronic multipliers in order to achieve a high speed operation in a communication system. Accurate electronic multipliers require non-linear components the cost of which may be prohibitive.

An alternate solution is to approximate the function with a series expansion. If it is desired to use the normal pulse from $0 \leq t \leq T$ to lower the peak power requirements of the zero initial value set, then the $e^{-a^2t^2}$ signal can be formed as a sum of even powers of t from already available

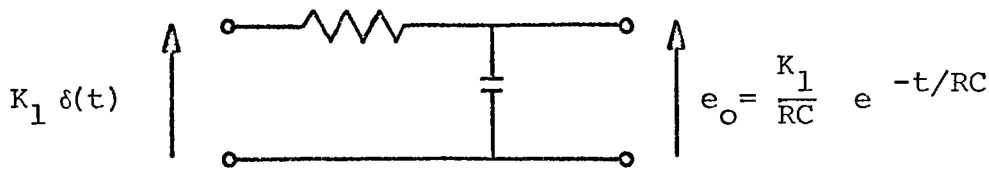

Figure 8-1. Exponential functions generating RC network

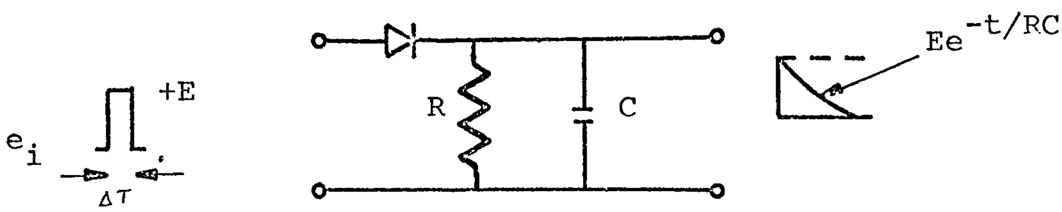

Figure 8-2. Exponential functions generating RC network with diode isolation

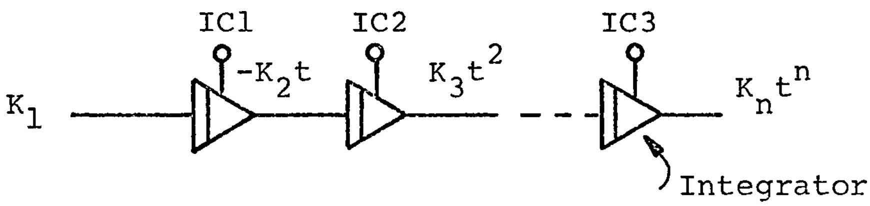

Figure 8-3. Polynomial generating system

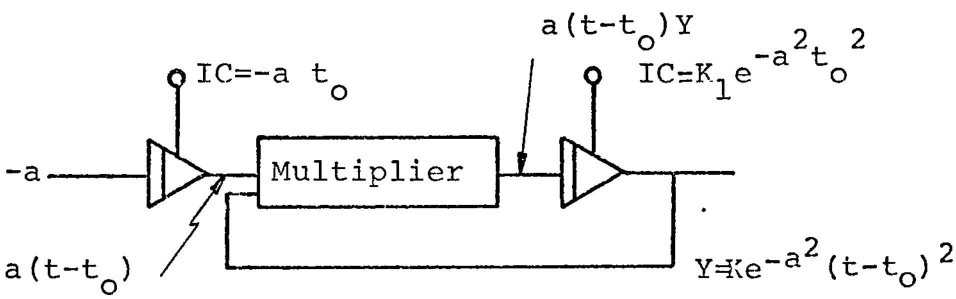

Figure 8-4. Function generation circuit

functions. The set of waveforms commonly used in analog multiplexing systems is the sine-cosine set. This has resulted in the development of sinusoidal signal sources which are stable and are capable of operating over a wide range of frequencies. One particular form of stable sinusoidal sources is used widely in the frequency synthesizer. An example of a system for generating orthogonal digital signals is given in Chapter VI.

Multiplication

Circuits used to satisfactorily perform four quadrant multiplication are expensive, and therefore the use of these devices should be avoided wherever possible. Multiplication is also necessary if the correlation detection process is used in the receiver.

The need for multiplication in the transmitter of the orthomux system based on the real exponential set can be avoided by using the sampled value of the message waveform as the input. This is shown diagrammatically in Figure (8-5) and the waveform for

$$\overline{m_1(t)}O_1(t) = \overline{\sin(\omega t)}e^{-t/RC} \qquad [8\text{-}9]$$

where the bar denotes the sampled value. This technique is shown in the photograph of Figure (8-6).

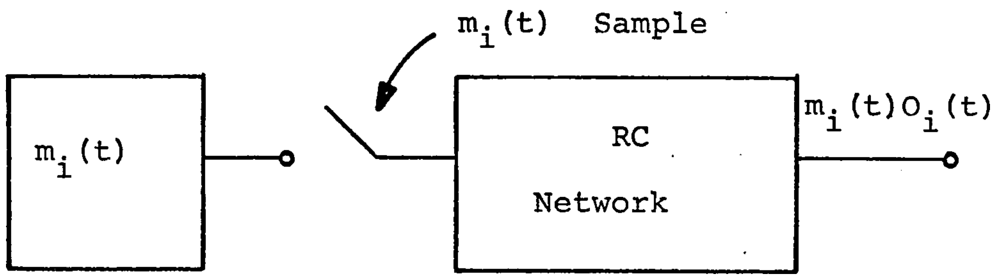

Figure 8-5 . Function generation without multiplication

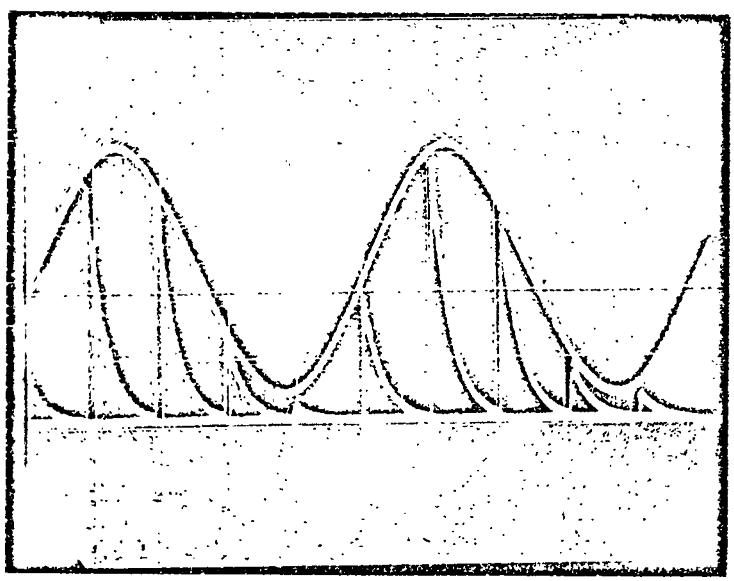

Figure 8-6 . Formation of the first exponential for the case of a sinusoidal message

Filtering

Distortionless transmission and reception of a waveform is desirable in a communication system and is achieved by the use of synchronous correlation detection. Amplitude distortion or non-linear phase shift causes crosstalk or interference in the receiving channels. It is often necessary to limit the signal energy level outside a bandpass in order to avoid interference with other systems. The design of a filter to minimize distortion is the next requirement. For distortionless transmission, a bandpass filter must have (Panter, 1965)

$$H(j\omega) = K, \ \omega_1 \leq \omega \leq \omega_2 \qquad [8\text{-}10]$$

$$H(j\omega) = 0 \quad \text{otherwise} \qquad [8\text{-}11]$$

$$\theta(\omega) = \omega t_0 \pm n\pi \qquad [8\text{-}12]$$

The output of this ideal filter is a replica of the input waveform but delayed in time. The operation of the receiver of the orthogonal multiplex system can be delayed to compensate for this and thus the undistorted messages are recovered. Such an ideal filter cannot be realized but modern filter theory does allow the realization of filters which give maximally flat amplitude or phase response. The Butterworth filter has a maximally flat amplitude response characteristic, and the transfer function is given by

$$T(S) = \left[\frac{T_0}{S^N + b_{n-1}S^{N-1} + \ . \ . \ . + b_1 S + b_0}\right] \qquad [8\text{-}13]$$

The solutions to the denominator polynomial in Equation [8-13] maximally flat amplitude case lead to the Butterworth polynomials. Equation [8-13] also represents the general form of the Bessel, or maximally flat delay filter. The denominator in this case yields the Bessel polynomial. These filters are tabulated in Weinberg (1962). A set of functions that have characteristics between the Butterworth and Bessel filters are the transitional Butterworth-Thompson filters (Peless and Murakami, 1957). The crosstalk resulting from filtering can be formulated as in Equation [4-23] and is easily calculated, using a computer program such as the one in Appendix C, "Polynomial Functions and Channel Filter."

Signal Detection

The correlation process of signal detection is used in the orthomux receiver. Actually only one value is calculated, at τ equal to zero, thus implying that the functions are occurring in synchronism. This is the ideal case when the receiver is perfectly synchronized. The response of a correlation detector in the case of the real exponential set is discussed here. A message input of unity in the first channel yields an output shown in Figure (8-7), where crosstalk terms are ignored:

$$\int_0^T (\sqrt{2p}e^{-pt})(\sqrt{2p}e^{-pt})dt = (1-e^{-2pT}) \qquad [8\text{-}14]$$

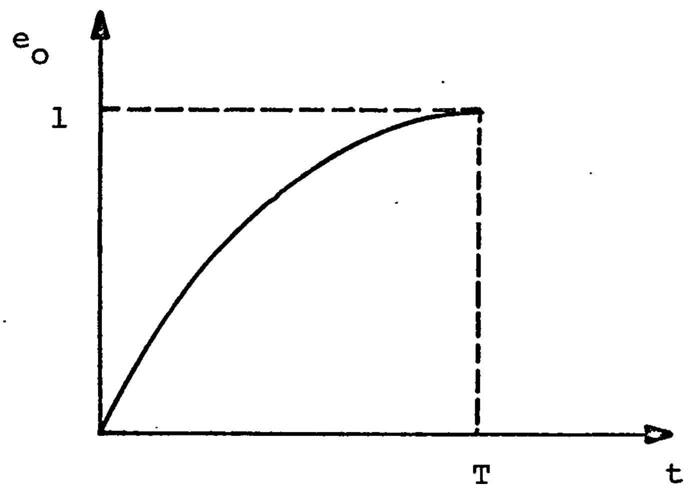

Figure 8-7. Correlation detection output

The detector output approaches the correct value as an asymptote. The correlation detector requires a multiplier and integrator with inputs as shown. At the end of the integration period the output of the integrator is sampled and the integrator is reset. The sampled outputs are held and passed through a smoothing filter to recover the analog input message.

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APPENDIX A

PERFORMANCE IN THE PPESENCE OF NOISE

In many communications applications the noise is Gaussian and has a power spectrum that is almost flat up to frequencies much higher than the significant signal frequencies. This type noise is called white Gaussian and is defined as the stationary, zero-mean Gaussian process with a power spectrum of

$$G(f) = \frac{n_0}{2} \text{ watts/Hz.}, \ -\infty \leq f \leq \infty \qquad \text{(A-1)}$$

The filter which maximizes the ratio of the peak value of the signal amplitude to the rms noise is called a matched filter, since the impulse response of the filter is matched to the signal pulse shape. This is demonstrated in connection with the freedom of selection of signal waveshape. Finally, the results of a digital computer simulation of an orthogonal multiplexing system operating with additive noise, which has a Gaussian amplitude distribution is presented at the end of the appendix. Assume an input signal $f(t)$ to a linear filter with a transfer function $H(\omega)$. The output of the filter is given by

$$g(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}F(\omega)H(\omega)d\omega \qquad \text{(A-2)}$$

If the filter input is white Gaussian noise, the output is

$$N = \frac{n_0}{2} \int_{-\infty}^{\infty} |H(\omega)|^2 df = \frac{n_0}{2} \int_{-\infty}^{\infty} h^2(\tau) d\tau \tag{A-3}$$

The ratio of the signal squared to the mean noise power squared at time t_0 is

$$\frac{g^2(t_0)}{N} = \frac{S}{N} = \frac{\left[\int_{-\infty}^{\infty} f(\tau) h(t_0 - \tau) d\tau \right]^2}{\frac{n_0}{2} \int_{-\infty}^{\infty} h^2(\tau) d\tau} \tag{A-4}$$

For the case of the orthogonal multiplexing system, the signal f(t) is known and the energy E can be considered a known constant

$$E = \int_{-\infty}^{\infty} f^2(t) dt = \int_{-\infty}^{\infty} |F(\omega)|^2 df \tag{A-5}$$

If S/N is divided by E, it gives the following expression which must be maximized.

$$\frac{\left[\int_{\infty}^{\infty} f(\tau) h(t_0 - \tau) d\tau \right]^2}{\int_{-\infty}^{\infty} f^2(t) dt \int_{-\infty}^{\infty} h^2(\tau) d\tau} \tag{A-6}$$

Schwarz's inequality states that

$$\left[\int_{-\infty}^{\infty} f(\tau) h(t_0 - \tau) d\tau \right]^2 \leq \int_{-\infty}^{\infty} f^2(t) dt \int_{-\infty}^{\infty} h^2(\tau) d\tau \tag{A-7}$$

Using the equality the expression for S/N may be maximized to yield

$$f(\tau) = h(t_0-\tau) \quad \text{or} \quad h(t) = f(t_0-t) \tag{A-8}$$

and in the frequency domain $H(\omega) = F^*(\omega)e^{-j\omega t_o}$ where $F^*(j\omega)$ is the complex conjugate of $F(j\omega)$. This is the impulse response of the matched filter, or the filter which has an impulse response which is a replica of the signal but inverted, as shown in Figure A-1.

The signal-to-noise ratio of the system using the matched filter is found by substituting Equation (A-8) into Equation (A-4).

$$\frac{S}{N} = \frac{\int_{-\infty}^{\infty} f^2(\tau)\,d\tau}{n_0/2} = \frac{2E}{n_0} \tag{A-9}$$

Thus the maximum value of the peak signal-to-noise ratio is dependent only on the signal energy and the noise spectral density and is independent of the pulse shape. This allows great freedom in the selection of signal waveshapes for orthogonal multiplexing systems.

A digital computer program written in the Digital Simulation Language is included to show how well the orthomux system performs in the presence of noise with Gaussian amplitude distribution. The results for several ratios of signal-to-noise are included. The results for each run are identified by the identical heading for P1. The computer program follows the orthomux system layout, as shown in

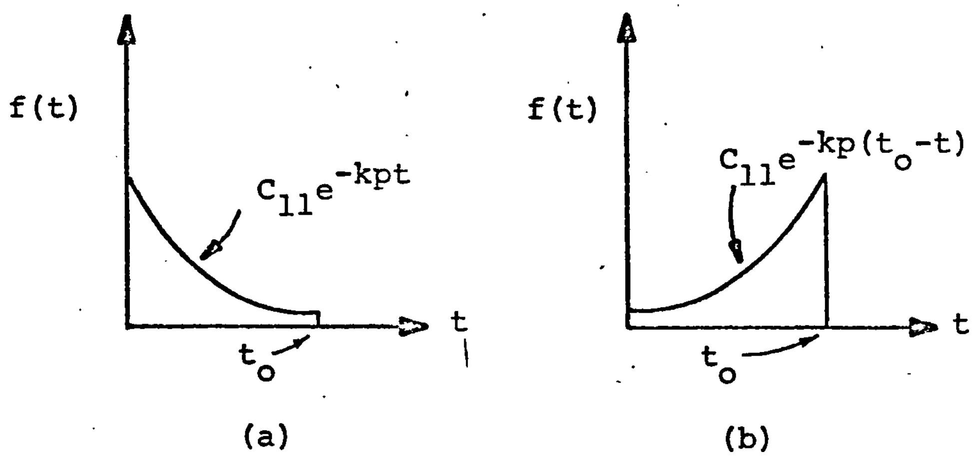

Figure A-1. Exponential waveforms referred to in the text

Figures A-2, A-3, and A-4. Figures A-2 and A-3 are the conventional block diagrams of an orthomux system with the notation used throughout the thesis, while Figure A-4 gives the notation used in the computer program. The notation is almost identical and the flow of the computer routine can be easily identified. The digital simulation language is non-procedural and consists essentially of a group of subroutines which can be closely related to analog computer blocks. In this case, the computer printout shows the error does not increase significantly as the noise power is increased from S/N=10 to S/N=1/5. The value of Y, the normally distributed amplitude function, is also printed to show the magnitude and random variation of this quantity.

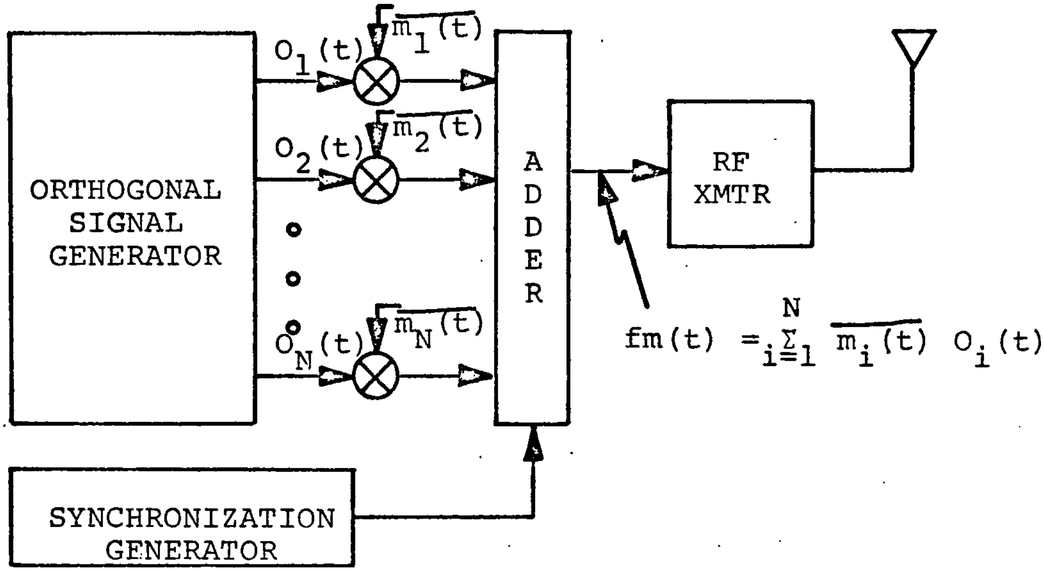

Figure A-2 Transmitter block diagram

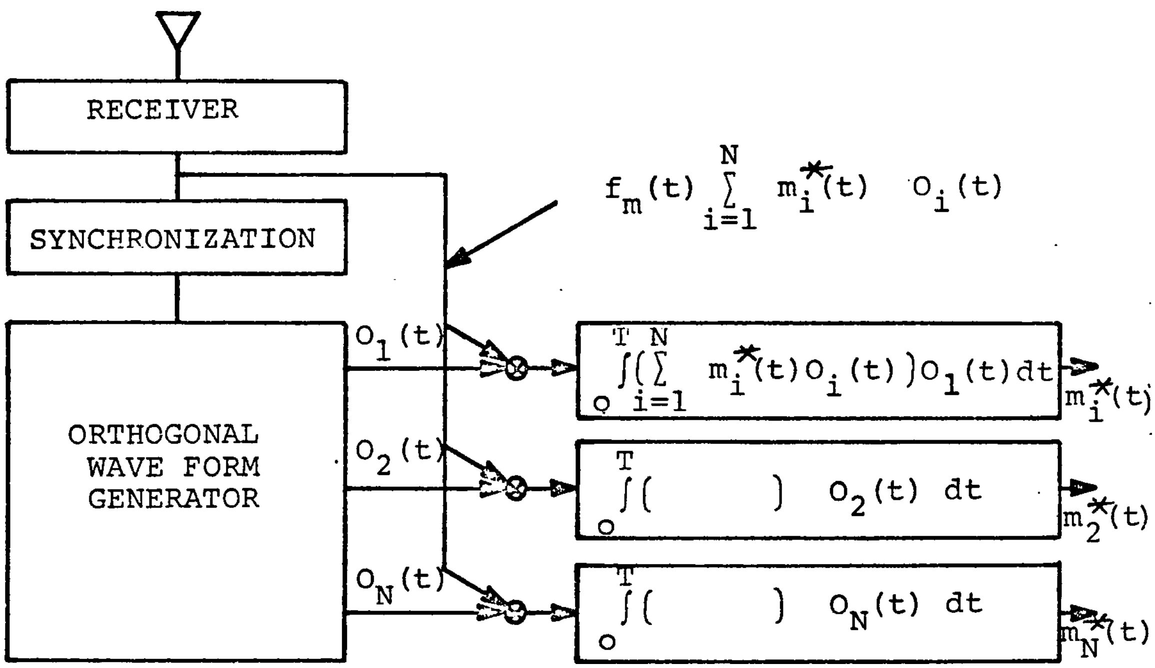

Figure A-3 Receiver block diagram

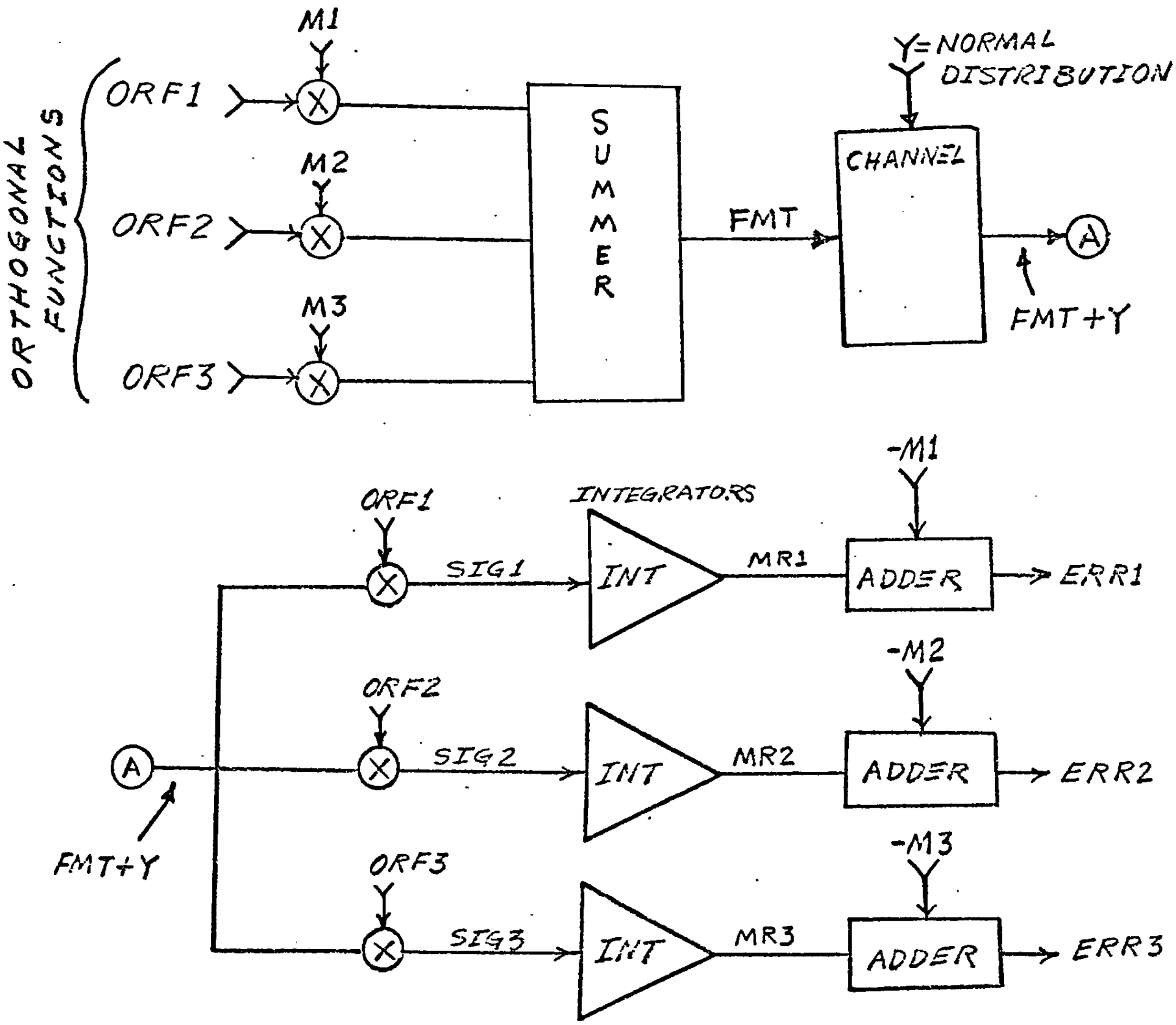

Figure A-4 Computer Program Notation

INPUT FØR DSL/90 TRANSLATØR

```
$IEDIT          SYSLB3,SRCH.
$IBLDR MAIN
$IBLDR CENTRL
$IEDIT
TITLE DSL/90 ADDITIVE GAUSSIAN NOISE SIMULATION 3/12/67
      ØRF1=C11*EXP(-TIME)
      ØRF2=C21*EXP(-TIME)+C22*EXP(-2.*TIME)
      ØRF3=C31*EXP(-TIME)+C32*EXP(-2.*TIME)+C33*EXP(-3.*TIME)
      FMT=ØRF1*M1+ØRF2*M2+ØRF3*M3+Y
      Y=NØRMAL(P1,P2,P3)
      SIG1=FMT*ØRF1
      MR1=INTGRL(0.,SIG1)
      SIG2=FMT*ØRF2
      MR2=INTGRL(0.,SIG2)
      SIG3=FMT*ØRF3
      MR3=INTGRL(0.,SIG3)
      ERR1=MR1-M1
      ERR2=MR2-M2
      ERR3=MR3-M3
CØNTRL FINTIM=5.,DELT=.01
PARAM M1=1.,M2=1.,M3=1.,P1=1,P2=0.,P3=.245
CØNST C11=1.4142,C21=4.,C22=-6.,C31=7.3485,C32=-29.394,C33=24.495
RANGE ØRF1,ØRF2,ØRF3,FMT,Y
INTEG MILNE
PRINT .1,Y,FMT,MR1,MR2,MR3,ERR1,ERR2,ERR3
END
TITLE SAME SYSTEM P1=3
PARAM P1=3
END
TITLE SAME SYSTEM P1=5
PARAM P1=5
END
TITLE SAME SYSTEM P1=7
PARAM P1=7
END
TITLE SAME SYSTEM P1=9,P3=.347,S/N=5
PARAM P1=9,P3=.347
END
TITLE SAME SYSTEM P1=11,P3=.548,S/N=2
PARAM P1=11,P3=.548
END
TITLE SAME SYSTEM P1=13,P3=.778,S/N=1
PARAM P1=13,P3=.778
END
TITLE SAME SYSTEM8 P1=15,P3=1.095,S/N=1/2
PARAM P1=15,P3=1.095
END
TITLE SAME SYSTEM,P1=17,P3=1.73,S/N=1/5
PARAM P1=17,P3=1.73
END
TITLE SAME SYSTEM,P1=19,P3=2.45,S/N=1/10
PARAM P1=19,P3=2.45
END
STOP
```

SAME SYSTEM P1=5, S/N = 10

TIME	Y	FM1	MR1	MR2
0.	2.8823E-01	2.1519E 00	0.	0.
1.000E-01	3.3551E-01	1.0519E 00	1.6695E-01	-2.0775E-01
2.000E-01	2.4399E-01	4.1103E-01	2.1466E-01	-2.4933E-01
3.000E-01	1.7847E-01	1.6759E-01	2.2261E-01	-2.5358E-01
4.000E-01	2.9505E-01	3.2434E-01	2.2159E-01	-2.5340E-01
5.000E-01	-2.0401E-01	-1.8195E-02	2.3135E-01	-2.5219E-01
6.000E-01	-3.1899E-01	7.3861E-02	2.5517E-01	-2.4297E-01
7.000E-01	-2.2702E-01	3.8227E-01	2.9034E-01	-2.2108E-01
8.000E-01	-2.9881E-01	5.1205E-01	3.3890E-01	-1.8102E-01
9.000E-01	-3.0625E-01	6.7829E-01	3.9255E-01	-1.2647E-01
1.000E 00	-2.9362E-01	8.3099E-01	4.5076E-01	-5.7354E-02
1.100E 00	1.5061E-01	1.3806E 00	5.0900E-01	2.0858E-02
1.200E 00	3.6726E-01	1.6697E 00	5.6623E-01	1.0582E-01
1.300E 00	-1.9706E-01	1.1482E 00	6.1928E-01	1.9129E-01
1.400E 00	2.3942E-01	1.6017E 00	6.6850E-01	2.7637E-01
1.500E 00	-4.9559E-01	8.6211E-01	7.1315E-01	3.5819E-01
1.600E 00	6.6434E-02	1.4020E 00	7.5309E-01	4.3513E-01
1.700E 00	-3.2989E-01	9.6977E-01	7.8943E-01	5.0829E-01
1.800E 00	3.3375E-01	1.5869E 00	8.2097E-01	5.7421E-01
1.900E 00	-5.5341E-01	6.4566E-01	8.4823E-01	6.3312E-01
2.000E 00	-2.9794E-01	8.4176E-01	8.7191E-01	6.8576E-01
2.100E 00	-3.3803E-03	1.0737E 00	8.9229E-01	7.3226E-01
2.200E 00	2.3385E-03	1.0153E 00	9.0989E-01	7.7334E-01
2.300E 00	-3.6710E-01	5.8139E-01	9.2503E-01	8.0937E-01
2.400E 00	-1.2883E-01	7.5598E-01	9.3790E-01	8.4057E-01
2.500E 00	1.9275E-01	1.0154E 00	9.4841E-01	8.6645E-01
2.600E 00	-3.0659E-01	4.5613E-01	9.5750E-01	8.8915E-01
2.700E 00	2.5072E-01	9.5602E-01	9.6486E-01	9.0775E-01
2.800E 00	3.4032E-01	9.9105E-01	9.7131E-01	9.2423E-01
2.900E 00	-3.1443E-01	2.8474E-01	9.7625E-01	9.3699E-01
3.000E 00	3.8525E-02	5.8923E-01	9.8026E-01	9.4745E-01
3.100E 00	8.8271E-02	5.9363E-01	9.8400E-01	9.5727E-01
3.200E 00	-1.5705E-01	3.0604E-01	9.8673E-01	9.6450E-01
3.300E 00	-3.6930E-01	5.4506E-02	9.8895E-01	9.7042E-01
3.400E 00	-2.7669E-01	1.1074E-01	9.9080E-01	9.7537E-01
3.500E 00	-1.5205E-01	2.0175E-01	9.9278E-01	9.8071E-01
3.600E 00	3.6479E-02	3.5928E-01	9.9418E-01	9.8451E-01
3.700E 00	-1.0415E-01	1.9013E-01	9.9512E-01	9.8706E-01
3.800E 00	3.2140E-01	5.8947E-01	9.9591E-01	9.8921E-01
3.900E 00	-1.7308E-01	7.0965E-02	9.9641E-01	9.9057E-01
4.000E 00	1.9838E-02	2.4187E-01	9.9692E-01	9.9199E-01
4.100E 00	-2.5003E-01	-4.8125E-02	9.9752E-01	9.9363E-01
4.200E 00	5.7196E-01	7.5546E-01	9.9790E-01	9.9468E-01
4.300E 00	-1.0834E-01	5.8373E-02	9.9821E-01	9.9554E-01
4.400E 00	-1.6342E-02	1.3506E-01	9.9843E-01	9.9615E-01
4.500E 00	-2.0395E-02	1.1656E-01	9.9878E-01	9.9712E-01
4.600E 00	3.2605E-01	4.5079E-01	9.9882E-01	9.9723E-01
4.700E 00	-1.5744E-01	-4.4266E-02	9.9895E-01	9.9761E-01
4.800E 00	4.5295E-02	1.4795E-01	9.9909E-01	9.9799E-01
4.900E 00	-1.3179E-01	-3.8689E-02	9.9923E-01	9.9838E-01
5.000E 00	1.1365E-01	1.9804E-01	9.9933E-01	9.9866E-01

VARIABLE	MINIMUM	MAXIMUM
ORF1	9.5288E-03	1.4142E 00
ORF2	-2.0000E 00	6.6667E-01

P1=5 , S/N =10

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
2.0324E-01	-8.3305E-01	-1.2077E 00	-7.9676E-01
2.1526E-01	-7.8534E-01	-1.2493E 00	-7.8474E-01
2.1218E-01	-7.7739E-01	-1.2536E 00	-7.8782E-01
2.1304E-01	-7.7841E-01	-1.2534E 00	-7.8696E-01
2.0319E-01	-7.6865E-01	-1.2522E 00	-7.9681E-01
1.7879E-01	-7.4483E-01	-1.2430E 00	-8.2121E-01
1.4627E-01	-7.0966E-01	-1.2211E 00	-8.5373E-01
1.0970E-01	-6.6110E-01	-1.1810E 00	-8.9030E-01
8.1882E-02	-6.0745E-01	-1.1265E 00	-9.1812E-01
6.7335E-02	-5.4924E-01	-1.0574E 00	-9.3266E-01
6.9885E-02	-4.9100E-01	-9.7914E-01	-9.3012E-01
9.0025E-02	-4.3377E-01	-8.9418E-01	-9.0997E-01
1.2502E-01	-3.8072E-01	-8.0871E-01	-8.7498E-01
1.7289E-01	-3.3150E-01	-7.2363E-01	-8.2711E-01
2.2968E-01	-2.8685E-01	-6.4181E-01	-7.7032E-01
2.9204E-01	-2.4691E-01	-5.6487E-01	-7.0796E-01
3.5896E-01	-2.1057E-01	-4.9171E-01	-6.4104E-01
4.2530E-01	-1.7903E-01	-4.2579E-01	-5.7470E-01
4.8949E-01	-1.5177E-01	-3.6688E-01	-5.1051E-01
5.5071E-01	-1.2809E-01	-3.1424E-01	-4.4929E-01
6.0788E-01	-1.0771E-01	-2.6774E-01	-3.9212E-01
6.6080E-01	-9.0107E-02	-2.2666E-01	-3.3920E-01
7.0916E-01	-7.4973E-02	-1.9063E-01	-2.9084E-01
7.5253E-01	-6.2099E-02	-1.5943E-01	-2.4747E-01
7.8964E-01	-5.1586E-02	-1.3355E-01	-2.1036E-01
8.2307E-01	-4.2495E-02	-1.1085E-01	-1.7693E-01
8.5111E-01	-3.5138E-02	-9.2251E-02	-1.4889E-01
8.7648E-01	-2.8691E-02	-7.5768E-02	-1.2352E-01
8.9650E-01	-2.3752E-02	-6.3009E-02	-1.0350E-01
9.1317E-01	-1.9737E-02	-5.2545E-02	-8.6827E-02
9.2905E-01	-1.6002E-02	-4.2731E-02	-7.0954E-02
9.4091E-01	-1.3269E-02	-3.5496E-02	-5.9092E-02
9.5072E-01	-1.1047E-02	-2.9576E-02	-4.9278E-02
9.5901E-01	-9.1997E-03	-2.4626E-02	-4.0992E-02
9.6803E-01	-7.2195E-03	-1.9291E-02	-3.1973E-02
9.7450E-01	-5.8160E-03	-1.5492E-02	-2.5497E-02
9.7889E-01	-4.8769E-03	-1.2938E-02	-2.1113E-02
9.8260E-01	-4.0905E-03	-1.0792E-02	-1.7404E-02
9.8497E-01	-3.5918E-03	-9.4261E-03	-1.5028E-02
9.8744E-01	-3.0782E-03	-8.0145E-03	-1.2559E-02
9.9033E-01	-2.4814E-03	-6.3705E-03	-9.6719E-03
9.9219E-01	-2.1001E-03	-5.3164E-03	-7.8106E-03
9.9371E-01	-1.7894E-03	-4.4557E-03	-6.2850E-03
9.9480E-01	-1.5699E-03	-3.8464E-03	-5.2013E-03
9.9652E-01	-1.2231E-03	-2.8826E-03	-3.4834E-03
9.9672E-01	-1.1815E-03	-2.7663E-03	-3.2751E-03
9.9741E-01	-1.0450E-03	-2.3855E-03	-2.5921E-03
9.9808E-01	-9.1077E-04	-2.0103E-03	-1.9175E-03
9.9878E-01	-7.7209E-04	-1.6223E-03	-1.2188E-03
9.9929E-01	-6.7160E-04	-1.3408E-03	-7.1069E-04

SAME SYSTEM P1=9,P3=.347,S/N=5

TIME	Y	FMT	MR1	MR2
0.	4.0790E-01	2.2716E 00	0.	0.
1.000E-01	3.3153E-02	7.4951E-01	1.6592E-01	-2.0631E-01
2.000E-01	-1.4577E-02	1.5247E-01	2.1370E-01	-2.4780E-01
3.000E-01	9.6192E-02	8.5318E-02	2.2160E-01	-2.5206E-01
4.000E-01	3.6119E-01	3.9049E-01	2.2375E-01	-2.5252E-01
5.000E-01	5.4911E-01	7.3493E-01	2.2863E-01	-2.5177E-01
6.000E-01	2.2614E-02	4.1546E-01	2.5336E-01	-2.4220E-01
7.000E-01	1.3534E-01	7.4463E-01	2.8912E-01	-2.2009E-01
8.000E-01	4.1573E-01	1.2266E 00	3.3883E-01	-1.7893E-01
9.000E-01	8.3177E-02	1.0677E 00	3.9641E-01	-1.2042E-01
1.000E 00	-4.5575E-01	6.6886E-01	4.5390E-01	-5.2149E-02
1.100E 00	2.1836E-01	1.4484E 00	5.1292E-01	2.7132E-02
1.200E 00	4.7981E-01	1.7823E 00	5.7049E-01	1.1258E-01
1.300E 00	2.4564E-01	1.5909E 00	6.2478E-01	2.0003E-01
1.400E 00	4.4616E-01	1.8084E 00	6.7500E-01	2.8683E-01
1.500E 00	1.7114E-01	1.5288E 00	7.2060E-01	3.7035E-01
1.600E 00	6.1677E-01	1.9524E 00	7.6164E-01	4.4941E-01
1.700E 00	5.1382E-01	1.8135E 00	7.9795E-01	5.2248E-01
1.800E 00	-4.2894E-01	8.2426E-01	8.3050E-01	5.9049E-01
1.900E 00	2.5912E-02	1.2250E 00	8.5847E-01	6.5094E-01
2.000E 00	-6.2149E-01	5.1821E-01	8.8188E-01	7.0298E-01
2.100E 00	-7.6138E-01	3.1572E-01	9.0194E-01	7.4877E-01
2.200E 00	-1.3662E-01	8.7630E-01	9.1919E-01	7.8904E-01
2.300E 00	-1.0329E-02	9.3816E-01	9.3385E-01	8.2393E-01
2.400E 00	1.4586E-01	1.0307E 00	9.4600E-01	8.5338E-01
2.500E 00	-2.9571E-02	7.9312E-01	9.5699E-01	8.8041E-01
2.600E 00	-5.0131E-01	2.6140E-01	9.6560E-01	9.0191E-01
2.700E 00	-4.9492E-01	2.1038E-01	9.7264E-01	9.1972E-01
2.800E 00	-3.9259E-01	2.5814E-01	9.7926E-01	9.3664E-01
2.900E 00	-1.1882E-01	4.8034E-01	9.8438E-01	9.4788E-01
3.000E 00	-4.0034E-01	1.5037E-01	9.8865E-01	9.6100E-01
3.100E 00	8.9478E-02	5.9484E-01	9.9227E-01	9.7050E-01
3.200E 00	-8.0015E-02	3.8307E-01	9.9533E-01	9.7861E-01
3.300E 00	2.8344E-01	7.0725E-01	9.9795E-01	9.8560E-01
3.400E 00	-2.1893E-01	1.6850E-01	9.9997E-01	9.9101E-01
3.500E 00	-3.0591E-01	4.7890E-02	1.0016E 00	9.9545E-01
3.600E 00	2.3259E-01	5.5539E-01	1.0030E 00	9.9917E-01
3.700E 00	-1.3933E-01	1.5494E-01	1.0044E 00	1.0030E 00
3.800E 00	-9.0142E-02	1.7793E-01	1.0054E 00	1.0058E 00
3.900E 00	-5.8120E-01	-3.3716E-01	1.0064E 00	1.0085E 00
4.000E 00	4.4470E-01	6.6673E-01	1.0072E 00	1.0107E 00
4.100E 00	2.5172E-01	4.5362E-01	1.0080E 00	1.0127E 00
4.200E 00	-4.4496E-02	1.3901E-01	1.0085E 00	1.0141E 00
4.300E 00	1.0924E-01	2.7596E-01	1.0090E 00	1.0156E 00
4.400E 00	5.7440E-01	7.2581E-01	1.0093E 00	1.0165E 00
4.500E 00	5.1550E-02	1.6900E-01	1.0096E 00	1.0174E 00
4.600E 00	2.8727E-01	4.1200E-01	1.0097E 00	1.0175E 00
4.700E 00	4.7440E-01	5.8757E-01	1.0098E 00	1.0180E 00
4.800E 00	-2.3268E-01	-1.3003E-01	1.0101E 00	1.0187E 00
4.900E 00	-1.2259E-01	-2.9509E-02	1.0102E 00	1.0189E 00
5.000E 00	3.4965E-01	4.3404E-01	1.0103E 00	1.0192E 00

VARIABLE	MINIMUM	MAXIMUM
ØRF1	9.5288E-03	1.4142E 00
ØRF2	-2.0000E 00	6.6667E-01

P1 = 9, S/N = 5

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
2.0156E-01	-8.3408E-01	-1.2063E 00	-7.9844E-01
2.1330E-01	-7.8630E-01	-1.2478E 00	-7.8670E-01
2.1029E-01	-7.7840E-01	-1.2521E 00	-7.8971E-01
2.0856E-01	-7.7625E-01	-1.2525E 00	-7.9144E-01
2.0362E-01	-7.7137E-01	-1.2518E 00	-7.9638E-01
1.7829E-01	-7.4664E-01	-1.2422E 00	-8.2171E-01
1.4512E-01	-7.1088E-01	-1.2201E 00	-8.5488E-01
1.0784E-01	-6.6117E-01	-1.1789E 00	-8.9216E-01
7.7927E-02	-6.0359E-01	-1.1204E 00	-9.2207E-01
6.3602E-02	-5.4610E-01	-1.0521E 00	-9.3640E-01
6.6208E-02	-4.8708E-01	-9.7287E-01	-9.3379E-01
8.6431E-02	-4.2951E-01	-8.8742E-01	-9.1357E-01
1.2218E-01	-3.7522E-01	-7.9997E-01	-8.7782E-01
1.7099E-01	-3.2500E-01	-7.1317E-01	-8.2901E-01
2.2888E-01	-2.7940E-01	-6.2965E-01	-7.7112E-01
2.9296E-01	-2.3836E-01	-5.5059E-01	-7.0704E-01
3.5975E-01	-2.0205E-01	-4.7752E-01	-6.4025E-01
4.2814E-01	-1.6950E-01	-4.0951E-01	-5.7186E-01
4.9401E-01	-1.4153E-01	-3.4906E-01	-5.0599E-01
5.5453E-01	-1.1812E-01	-2.9702E-01	-4.4547E-01
6.1084E-01	-9.8058E-02	-2.5123E-01	-3.8916E-01
6.6275E-01	-8.0807E-02	-2.1096E-01	-3.3725E-01
7.0955E-01	-6.6147E-02	-1.7607E-01	-2.9045E-01
7.5051E-01	-5.3997E-02	-1.4662E-01	-2.4949E-01
7.8925E-01	-4.3013E-02	-1.1959E-01	-2.1075E-01
8.2091E-01	-3.4403E-02	-9.8088E-02	-1.7909E-01
8.4776E-01	-2.7359E-02	-8.0278E-02	-1.5224E-01
8.7380E-01	-2.0744E-02	-6.3363E-02	-1.2620E-01
8.9458E-01	-1.5616E-02	-5.0116E-02	-1.0542E-01
9.1229E-01	-1.1351E-02	-3.9001E-02	-8.7710E-02
9.2766E-01	-7.7346E-03	-2.9499E-02	-7.2340E-02
9.4094E-01	-4.6720E-03	-2.1393E-02	-5.9058E-02
9.5253E-01	-2.0477E-03	-1.4402E-02	-4.7473E-02
9.6160E-01	-2.7537E-05	-8.9866E-03	-3.8401E-02
9.6910E-01	1.6204E-03	-4.5465E-03	-3.0899E-02
9.7543E-01	2.9918E-03	-8.3230E-04	-2.4567E-02
9.8196E-01	4.3895E-03	2.9685E-03	-1.8041E-02
9.8679E-01	5.4124E-03	5.7609E-03	-1.3214E-02
9.9160E-01	6.4234E-03	8.5299E-03	-8.4003E-03
9.9540E-01	7.2144E-03	1.0704E-02	-4.5982E-03
9.9898E-01	7.9534E-03	1.2741E-02	-1.0206E-03
1.0014E 00	8.4582E-03	1.4136E-02	1.4428E-03
1.0040E 00	8.9786E-03	1.5577E-02	3.9958E-03
1.0056E 00	9.2944E-03	1.6454E-02	5.5565E-03
1.0072E 00	9.6193E-03	1.7358E-02	7.1686E-03
1.0073E 00	9.6541E-03	1.7455E-02	7.3422E-03
1.0082E 00	9.8322E-03	1.7952E-02	8.2333E-03
1.0097E 00	1.0117E-02	1.8747E-02	9.6607E-03
1.0100E 00	1.0185E-02	1.8938E-02	1.0005E-02
1.0104E 00	1.0261E-02	1.9152E-02	1.0392E-02

SAME SYSTEM P1=11,P3=.548,S/N=2

TIME	Y	FMT	MR1	MR2
0.	6.4391E-01	2.5076E 00	0.	0.
1.000E-01	3.1674E-01	1.0331E 00	1.7173E-01	-2.1341E-01
2.000E-01	7.2904E-02	2.3995E-01	2.2278E-01	-2.5733E-01
3.000E-01	1.0978E 00	1.0869E 00	2.2760E-01	-2.6001E-01
4.000E-01	-1.6483E 00	-1.6190E 00	2.2537E-01	-2.5943E-01
5.000E-01	-1.3363E-01	5.2189E-02	2.3266E-01	-2.5807E-01
6.000E-01	5.0365E-01	8.9649E-01	2.5106E-01	-2.5084E-01
7.000E-01	4.6960E-01	1.0789E 00	2.8854E-01	-2.2748E-01
8.000E-01	-5.1276E-01	2.9810E-01	3.3722E-01	-1.8727E-01
9.000E-01	3.0634E-01	1.2909E 00	3.9052E-01	-1.3309E-01
1.000E 00	-1.9603E-01	9.2858E-01	4.4807E-01	-6.4841E-02
1.100E 00	6.7746E-01	1.9075E 00	5.0453E-01	1.0904E-02
1.200E 00	7.4659E-01	2.0491E 00	5.6097E-01	9.4693E-02
1.300E 00	-3.0548E-01	1.0397E 00	6.1623E-01	1.8373E-01
1.400E 00	7.5789E-01	2.1201E 00	6.6559E-01	2.6905E-01
1.500E 00	1.7529E-02	1.3752E 00	7.0854E-01	3.4772E-01
1.600E 00	-4.9909E-01	8.3650E-01	7.4795E-01	4.2369E-01
1.700E 00	7.4431E-01	2.0440E 00	7.8303E-01	4.9430E-01
1.800E 00	9.5243E-02	1.3484E 00	8.1564E-01	5.6243E-01
1.900E 00	-1.9052E-01	1.0086E 00	8.4375E-01	6.2314E-01
2.000E 00	-2.8155E-01	8.5814E-01	8.6679E-01	6.7438E-01
2.100E 00	4.7473E-01	1.5518E 00	8.8693E-01	7.2033E-01
2.200E 00	-2.9292E-01	7.2001E-01	9.0400E-01	7.6017E-01
2.300E 00	7.2304E-01	1.6715E 00	9.1871E-01	7.9520E-01
2.400E 00	-1.2746E-02	8.7206E-01	9.3096E-01	8.2489E-01
2.500E 00	1.8518E-01	1.0079E 00	9.4154E-01	8.5093E-01
2.600E 00	-9.3003E-02	6.6971E-01	9.5051E-01	8.7333E-01
2.700E 00	8.8702E-01	1.5923E 00	9.5776E-01	8.9165E-01
2.800E 00	-1.3289E 00	-6.7820E-01	9.6434E-01	9.0847E-01
2.900E 00	-6.0594E-01	-6.7689E-03	9.6938E-01	9.2149E-01
3.000E 00	-6.6423E-01	-1.1352E-01	9.7328E-01	9.3168E-01
3.100E 00	-9.6823E-01	-4.6237E-01	9.7735E-01	9.4236E-01
3.200E 00	-2.6624E-01	1.9665E-01	9.8025E-01	9.5002E-01
3.300E 00	4.0322E-01	8.2703E-01	9.8244E-01	9.5587E-01
3.400E 00	-1.5981E-01	2.2761E-01	9.8447E-01	9.6131E-01
3.500E 00	7.2426E-01	1.0781E 00	9.8622E-01	9.6602E-01
3.600E 00	-6.6477E-01	-3.4197E-01	9.8754E-01	9.6960E-01
3.700E 00	-6.4023E-02	2.3025E-01	9.8872E-01	9.7281E-01
3.800E 00	-4.5606E-01	-1.8798E-01	9.8983E-01	9.7585E-01
3.900E 00	2.8000E-01	5.2404E-01	9.9075E-01	9.7837E-01
4.000E 00	-8.4936E-02	1.3710E-01	9.9149E-01	9.8040E-01
4.100E 00	2.4325E-01	4.4515E-01	9.9223E-01	9.8245E-01
4.200E 00	1.9607E-01	3.7958E-01	9.9295E-01	9.8443E-01
4.300E 00	-9.4564E-01	-7.7893E-01	9.9326E-01	9.8530E-01
4.400E 00	3.5999E-01	5.1139E-01	9.9382E-01	9.8683E-01
4.500E 00	3.3567E-01	4.7312E-01	9.9405E-01	9.8749E-01
4.600E 00	7.1075E-02	1.9581E-01	9.9412E-01	9.8767E-01
4.700E 00	-5.0653E-01	-3.9346E-01	9.9431E-01	9.8822E-01
4.800E 00	9.8279E-03	1.1246E-01	9.9439E-01	9.8843E-01
4.900E 00	5.1518E-01	6.0827E-01	9.9456E-01	9.8891E-01
5.000E 00	-1.6688E-01	-8.2482E-02	9.9474E-01	9.8941E-01

VARIABLE	MINIMUM	MAXIMUM
ORF1	9.5288E-03	1.4142E 00
ORF2	-2.0000E 00	6.6667E-01

P1=11, S/N=2

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
2.0823E-01	-8.2827E-01	-1.2134E 00	-7.9177E-01
2.1988E-01	-7.7722E-01	-1.2573E 00	-7.8012E-01
2.1817E-01	-7.7240E-01	-1.2600E 00	-7.8183E-01
2.1985E-01	-7.7463E-01	-1.2594E 00	-7.8015E-01
2.1232E-01	-7.6732E-01	-1.2581E 00	-7.8768E-01
1.9353E-01	-7.4894E-01	-1.2508E 00	-8.0647E-01
1.5890E-01	-7.1146E-01	-1.2275E 00	-8.4110E-01
1.2226E-01	-6.6278E-01	-1.1873E 00	-8.7774E-01
9.4605E-02	-6.0948E-01	-1.1331E 00	-9.0540E-01
8.0081E-02	-5.5193E-01	-1.0648E 00	-9.1992E-01
8.2409E-02	-4.9547E-01	-9.8910E-01	-9.1759E-01
1.0224E-01	-4.3903E-01	-9.0531E-01	-8.9776E-01
1.3870E-01	-3.8377E-01	-8.1627E-01	-8.6130E-01
1.8669E-01	-3.3441E-01	-7.3095E-01	-8.1331E-01
2.4121E-01	-2.9146E-01	-6.5228E-01	-7.5879E-01
3.0290E-01	-2.5205E-01	-5.7631E-01	-6.9710E-01
3.6745E-01	-2.1697E-01	-5.0570E-01	-6.3255E-01
4.3596E-01	-1.8436E-01	-4.3757E-01	-5.6404E-01
5.0204E-01	-1.5625E-01	-3.7686E-01	-4.9796E-01
5.6161E-01	-1.3321E-01	-3.2562E-01	-4.3839E-01
6.1809E-01	-1.1307E-01	-2.7967E-01	-3.8191E-01
6.6946E-01	-9.5999E-02	-2.3983E-01	-3.3054E-01
7.1648E-01	-8.1286E-02	-2.0480E-01	-2.8352E-01
7.5777E-01	-6.9037E-02	-1.7511E-01	-2.4223E-01
7.9510E-01	-5.8460E-02	-1.4907E-01	-2.0490E-01
8.2807E-01	-4.9489E-02	-1.2667E-01	-1.7193E-01
8.5570E-01	-4.2236E-02	-1.0834E-01	-1.4430E-01
8.8158E-01	-3.5662E-02	-9.1528E-02	-1.1842E-01
9.0200E-01	-3.0621E-02	-7.8505E-02	-9.7996E-02
9.1823E-01	-2.6715E-02	-6.8325E-02	-8.1772E-02
9.3551E-01	-2.2648E-02	-5.7637E-02	-6.4488E-02
9.4807E-01	-1.9755E-02	-4.9976E-02	-5.1933E-02
9.5776E-01	-1.7560E-02	-4.4129E-02	-4.2239E-02
9.6688E-01	-1.5530E-02	-3.8685E-02	-3.3117E-02
9.7483E-01	-1.3785E-02	-3.3981E-02	-2.5166E-02
9.8093E-01	-1.2463E-02	-3.0403E-02	-1.9068E-02
9.8645E-01	-1.1281E-02	-2.7189E-02	-1.3549E-02
9.9171E-01	-1.0167E-02	-2.4146E-02	-8.2900E-03
9.9609E-01	-9.2469E-03	-2.1626E-02	-3.9052E-03
9.9964E-01	-8.5099E-03	-1.9600E-02	-3.6313E-04
1.0032E 00	-7.7654E-03	-1.7548E-02	3.2420E-03
1.0067E 00	-7.0509E-03	-1.5574E-02	6.7252E-03
1.0083E 00	-6.7356E-03	-1.4699E-02	8.2760E-03
1.0110E 00	-6.1849E-03	-1.3171E-02	1.0994E-02
1.0122E 00	-5.9458E-03	-1.2506E-02	1.2180E-02
1.0125E 00	-5.8817E-03	-1.2326E-02	1.2500E-02
1.0135E 00	-5.6854E-03	-1.1778E-02	1.3484E-02
1.0139E 00	-5.6113E-03	-1.1570E-02	1.3857E-02
1.0147E 00	-5.4398E-03	-1.1090E-02	1.4722E-02
1.0156E 00	-5.2625E-03	-1.0594E-02	1.5618E-02

SAME SYSTEM P1=13,P3=.778,S/N=1

TIME	Y	FMT	MR1	MR2
0.	9.1374E-01	2.7774E 00	0.	0.
1.000E-01	-4.9892E-01	2.1743E-01	1.6429E-01	-2.0435E-01
2.000E-01	-1.2128E-02	1.5492E-01	2.1497E-01	-2.4805E-01
3.000E-01	8.5239E-01	8.4152E-01	2.1589E-01	-2.4886E-01
4.000E-01	1.0389E 00	1.0682E 00	2.2299E-01	-2.4953E-01
5.000E-01	7.7780E-01	9.6362E-01	2.3012E-01	-2.4848E-01
6.000E-01	-1.2036E-01	2.7249E-01	2.5620E-01	-2.3840E-01
7.000E-01	-1.8319E-01	4.2610E-01	2.9163E-01	-2.1654E-01
8.000E-01	2.7377E-01	1.0846E 00	3.3778E-01	-1.7823E-01
9.000E-01	7.7682E-01	1.7614E 00	3.9485E-01	-1.2029E-01
1.000E 00	5.2716E-02	1.1773E 00	4.5100E-01	-5.3456E-02
1.100E 00	3.3378E-01	1.5638E 00	5.0871E-01	2.4140E-02
1.200E 00	-3.9008E-01	9.1239E-01	5.6626E-01	1.0955E-01
1.300E 00	-1.5772E 00	-2.3196E-01	6.2004E-01	1.9620E-01
1.400E 00	-5.7037E-01	7.9188E-01	6.7064E-01	2.8363E-01
1.500E 00	4.2111E-01	1.7788E 00	7.1517E-01	3.6517E-01
1.600E 00	-6.4484E-02	1.2711E 00	7.5572E-01	4.4333E-01
1.700E 00	-4.9536E-01	8.0430E-01	7.9145E-01	5.1521E-01
1.800E 00	7.9020E-01	2.0434E 00	8.2046E-01	5.7589E-01
1.900E 00	7.3622E-01	1.9353E 00	8.4907E-01	6.3770E-01
2.000E 00	-1.0929E 00	4.6770E-02	8.7301E-01	6.9091E-01
2.100E 00	-3.4600E-01	7.3110E-01	8.9332E-01	7.3725E-01
2.200E 00	2.4562E-01	1.2585E 00	9.0980E-01	7.7573E-01
2.300E 00	-6.5488E-02	8.8300E-01	9.2429E-01	8.1024E-01
2.400E 00	-1.0353E 00	-1.5050E-01	9.3700E-01	8.4104E-01
2.500E 00	1.3671E 00	2.1898E 00	9.4739E-01	8.6662E-01
2.600E 00	-1.2241E-01	6.4030E-01	9.5599E-01	8.8808E-01
2.700E 00	1.1580E-01	8.2110E-01	9.6369E-01	9.0757E-01
2.800E 00	8.5475E-01	1.5055E 00	9.7108E-01	9.2646E-01
2.900E 00	1.0180E 00	1.6171E 00	9.7640E-01	9.4022E-01
3.000E 00	-2.8777E-01	2.6294E-01	9.8043E-01	9.5073E-01
3.100E 00	-2.8914E-01	2.1622E-01	9.8316E-01	9.5789E-01
3.200E 00	-1.7833E-01	2.8475E-01	9.8607E-01	9.6558E-01
3.300E 00	4.7075E-01	8.9456E-01	9.8804E-01	9.7086E-01
3.400E 00	1.0695E-01	4.9437E-01	9.8959E-01	9.7501E-01
3.500E 00	1.8777E 00	2.2315E 00	9.9088E-01	9.7849E-01
3.600E 00	-7.0606E-01	-3.8326E-01	9.9294E-01	9.8405E-01
3.700E 00	1.5553E-01	4.4981E-01	9.9401E-01	9.8697E-01
3.800E 00	7.8780E-01	1.0559E 00	9.9465E-01	9.8873E-01
3.900E 00	-2.4530E-01	-1.2563E-03	9.9555E-01	9.9120E-01
4.000E 00	5.8674E-01	8.0877E-01	9.9589E-01	9.9212E-01
4.100E 00	-8.1558E-01	-6.1368E-01	9.9643E-01	9.9362E-01
4.200E 00	8.1300E-01	9.9651E-01	9.9718E-01	9.9568E-01
4.300E 00	1.0786E 00	1.2454E 00	9.9773E-01	9.9721E-01
4.400E 00	3.3165E-01	4.8305E-01	9.9803E-01	9.9806E-01
4.500E 00	-4.3339E-01	-2.9594E-01	9.9804E-01	9.9807E-01
4.600E 00	1.0432E-01	2.2906E-01	9.9818E-01	9.9845E-01
4.700E 00	-7.8214E-01	-6.6897E-01	9.9826E-01	9.9870E-01
4.800E 00	1.6312E-01	2.6577E-01	9.9841E-01	9.9910E-01
4.900E 00	-8.8400E-01	-7.9091E-01	9.9866E-01	9.9981E-01
5.000E 00	1.8974E-01	2.7413E-01	9.9873E-01	1.0000E 00

VARIABLE	MINIMUM	MAXIMUM
ØRF1	9.5288E-03	1.4142E 00
ØRF2	-2.0000E 00	6.6667E-01

P1=13, S/N=1

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
1.9970E-01	-8.3571E-01	-1.2043E 00	-8.0030E-01
2.1148E-01	-7.8503E-01	-1.2480E 00	-7.8852E-01
2.1158E-01	-7.8411E-01	-1.2489E 00	-7.8842E-01
2.0519E-01	-7.7701E-01	-1.2495E 00	-7.9481E-01
1.9795E-01	-7.6988E-01	-1.2485E 00	-8.0205E-01
1.7124E-01	-7.4380E-01	-1.2384E 00	-8.2876E-01
1.3838E-01	-7.0837E-01	-1.2165E 00	-8.6162E-01
1.0386E-01	-6.6222E-01	-1.1782E 00	-8.9614E-01
7.4140E-02	-6.0515E-01	-1.1203E 00	-9.2586E-01
6.0402E-02	-5.4900E-01	-1.0535E 00	-9.3960E-01
6.3111E-02	-4.9129E-01	-9.7586E-01	-9.3689E-01
8.3271E-02	-4.3374E-01	-8.9045E-01	-9.1673E-01
1.1877E-01	-3.7996E-01	-8.0380E-01	-8.8123E-01
1.6786E-01	-3.2936E-01	-7.1637E-01	-8.3214E-01
2.2434E-01	-2.8483E-01	-6.3483E-01	-7.7566E-01
2.8771E-01	-2.4428E-01	-5.5667E-01	-7.1229E-01
3.5339E-01	-2.0855E-01	-4.8479E-01	-6.4661E-01
4.1454E-01	-1.7954E-01	-4.2411E-01	-5.8546E-01
4.8185E-01	-1.5093E-01	-3.6230E-01	-5.1815E-01
5.4369E-01	-1.2699E-01	-3.0909E-01	-4.5631E-01
6.0067E-01	-1.0668E-01	-2.6275E-01	-3.9933E-01
6.5030E-01	-9.0203E-02	-2.2427E-01	-3.4970E-01
6.9661E-01	-7.5707E-02	-1.8976E-01	-3.0339E-01
7.3940E-01	-6.2996E-02	-1.5896E-01	-2.6060E-01
7.7607E-01	-5.2608E-02	-1.3338E-01	-2.2393E-01
8.0768E-01	-4.4013E-02	-1.1192E-01	-1.9232E-01
8.3707E-01	-3.6306E-02	-9.2429E-02	-1.6293E-01
8.6615E-01	-2.8921E-02	-7.3541E-02	-1.3385E-01
8.8775E-01	-2.3597E-02	-5.9780E-02	-1.1225E-01
9.0450E-01	-1.9567E-02	-4.9274E-02	-9.5505E-02
9.1607E-01	-1.6845E-02	-4.2118E-02	-8.3927E-02
9.2869E-01	-1.3935E-02	-3.4416E-02	-7.1315E-02
9.3743E-01	-1.1956E-02	-2.9141E-02	-6.2571E-02
9.4439E-01	-1.0408E-02	-2.4990E-02	-5.5613E-02
9.5026E-01	-9.1193E-03	-2.1515E-02	-4.9740E-02
9.5976E-01	-7.0628E-03	-1.5945E-02	-4.0244E-02
9.6476E-01	-5.9909E-03	-1.3031E-02	-3.5244E-02
9.6780E-01	-5.3457E-03	-1.1268E-02	-3.2196E-02
9.7209E-01	-4.4457E-03	-8.8016E-03	-2.7906E-02
9.7372E-01	-4.1095E-03	-7.8755E-03	-2.6284E-02
9.7635E-01	-3.5664E-03	-6.3771E-03	-2.3650E-02
9.7998E-01	-2.8226E-03	-4.3215E-03	-2.0023E-02
9.8269E-01	-2.2703E-03	-2.7912E-03	-1.7312E-02
9.8419E-01	-1.9659E-03	-1.9450E-03	-1.5806E-02
9.8421E-01	-1.9628E-03	-1.9344E-03	-1.5785E-02
9.8491E-01	-1.8234E-03	-1.5455E-03	-1.5090E-02
9.8535E-01	-1.7363E-03	-1.3017E-03	-1.4653E-02
9.8607E-01	-1.5928E-03	-8.9990E-04	-1.3929E-02
9.8735E-01	-1.3392E-03	-1.9004E-04	-1.2650E-02
9.8770E-01	-1.2694E-03	5.7220E-06	-1.2297E-02

SAME SYSTEM P1=15,P3=1.095,S/N=1/2

TIME	Y	FMT	MR1	MR2
0.	1.2854E 00	3.1491E 00	0.	0.
1.000E-01	-6.3669E-01	7.9668E-02	1.6408E-01	-2.0370E-01
2.000E-01	5.6079E-01	7.2784E-01	2.0898E-01	-2.4289E-01
3.000E-01	-2.7088E-01	-2.8175E-01	2.2005E-01	-2.4832E-01
4.000E-01	-1.0273E 00	-9.9805E-01	2.1455E-01	-2.4732E-01
5.000E-01	7.8073E-01	9.6655E-01	2.2983E-01	-2.4540E-01
6.000E-01	-2.2278E-01	1.7007E-01	2.4685E-01	-2.3911E-01
7.000E-01	-7.0295E-01	-9.3663E-02	2.8061E-01	-2.1790E-01
8.000E-01	8.6686E-01	1.6777E 00	3.2577E-01	-1.8070E-01
9.000E-01	-3.7580E-01	6.0874E-01	3.7625E-01	-1.2922E-01
1.000E 00	2.7256E 00	3.8502E 00	4.3402E-01	-6.0772E-02
1.100E 00	-1.6565E 00	-4.2648E-01	4.9762E-01	2.4581E-02
1.200E 00	-7.1738E-01	5.8510E-01	5.5598E-01	1.1122E-01
1.300E 00	-1.1489E 00	1.9629E-01	6.0823E-01	1.9547E-01
1.400E 00	2.8661E-01	1.6489E 00	6.5632E-01	2.7862E-01
1.500E 00	8.3526E-01	2.1930E 00	7.0147E-01	3.6130E-01
1.600E 00	-1.3021E 00	3.3466E-02	7.4047E-01	4.3644E-01
1.700E 00	-1.0216E 00	2.7806E-01	7.7701E-01	5.0996E-01
1.800E 00	-1.6721E-01	1.0860E 00	8.0970E-01	5.7832E-01
1.900E 00	-6.6167E-02	1.1329E 00	8.3589E-01	6.3491E-01
2.000E 00	3.2698E-01	1.4667E 00	8.6049E-01	6.8962E-01
2.100E 00	1.2029E 00	2.2800E 00	8.8221E-01	7.3918E-01
2.200E 00	4.2293E-01	1.4359E 00	9.0052E-01	7.8190E-01
2.300E 00	4.7558E-01	1.4241E 00	9.1515E-01	8.1673E-01
2.400E 00	-1.2956E 00	-4.1079E-01	9.2789E-01	8.4761E-01
2.500E 00	4.3893E-01	1.2616E 00	9.3898E-01	8.7492E-01
2.600E 00	-1.9192E 00	-1.1565E 00	9.4867E-01	8.9914E-01
2.700E 00	-7.3677E-01	-3.1467E-02	9.5553E-01	9.1648E-01
2.800E 00	-3.6645E-02	6.1408E-01	9.6169E-01	9.3223E-01
2.900E 00	-5.9415E-01	5.0166E-03	9.6643E-01	9.4450E-01
3.000E 00	1.2598E 00	1.8105E 00	9.7107E-01	9.5660E-01
3.100E 00	-1.6199E 00	-1.1145E 00	9.7517E-01	9.6738E-01
3.200E 00	-4.2469E-01	3.8396E-02	9.7882E-01	9.7703E-01
3.300E 00	-2.0741E 00	-1.6503E 00	9.8105E-01	9.8299E-01
3.400E 00	-9.2060E-01	-5.3317E-01	9.8318E-01	9.8870E-01
3.500E 00	-5.4092E-01	-1.8712E-01	9.8444E-01	9.9208E-01
3.600E 00	3.4550E-01	6.6830E-01	9.8635E-01	9.9726E-01
3.700E 00	5.5136E-02	3.4941E-01	9.8713E-01	9.9938E-01
3.800E 00	2.1478E 00	2.4159E 00	9.8816E-01	1.0022E 00
3.900E 00	-1.7496E 00	-1.5056E 00	9.8898E-01	1.0044E 00
4.000E 00	-9.1035E-01	-6.8831E-01	9.8911E-01	1.0048E 00
4.100E 00	1.1514E-01	3.1705E-01	9.8966E-01	1.0063E 00
4.200E 00	-3.7928E-02	1.4558E-01	9.8967E-01	1.0064E 00
4.300E 00	6.8394E-01	8.5066E-01	9.9033E-01	1.0082E 00
4.400E 00	-2.1828E 00	-2.0314E 00	9.9053E-01	1.0087E 00
4.500E 00	2.1834E 00	2.3209E 00	9.9062E-01	1.0090E 00
4.600E 00	4.2120E-01	5.4594E-01	9.9052E-01	1.0087E 00
4.700E 00	2.2553E-01	3.3870E-01	9.9052E-01	1.0087E 00
4.800E 00	-3.7002E-01	-2.6737E-01	9.9071E-01	1.0093E 00
4.900E 00	-1.2549E-02	8.0537E-02	9.9093E-01	1.0099E 00
5.000E 00	5.8809E-01	6.7248E-01	9.9080E-01	1.0095E 00

VARIABLE	MINIMUM	MAXIMUM
ØRF1	9.5288E-03	1.4142E 00
ØRF2	-2.0000E 00	6.6667E-01

P1=15, S/N=1/2

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
1.9848E-01	-8.3592E-01	-1.2037E 00	-8.0152E-01
2.0988E-01	-7.9102E-01	-1.2429E 00	-7.9012E-01
2.0491E-01	-7.7995E-01	-1.2483E 00	-7.9509E-01
2.0944E-01	-7.8545E-01	-1.2473E 00	-7.9056E-01
1.9401E-01	-7.7017E-01	-1.2454E 00	-8.0599E-01
1.7653E-01	-7.5315E-01	-1.2391E 00	-8.2347E-01
1.4544E-01	-7.1939E-01	-1.2179E 00	-8.5456E-01
1.1134E-01	-6.7423E-01	-1.1807E 00	-8.8866E-01
8.5371E-02	-6.2375E-01	-1.1292E 00	-9.1463E-01
7.0683E-02	-5.6598E-01	-1.0608E 00	-9.2932E-01
7.3339E-02	-5.0238E-01	-9.7542E-01	-9.2666E-01
9.3852E-02	-4.4402E-01	-8.8878E-01	-9.0615E-01
1.2848E-01	-3.9177E-01	-8.0453E-01	-8.7152E-01
1.7528E-01	-3.4368E-01	-7.2138E-01	-8.2472E-01
2.3254E-01	-2.9853E-01	-6.3870E-01	-7.6746E-01
2.9340E-01	-2.5953E-01	-5.6356E-01	-7.0660E-01
3.6059E-01	-2.2299E-01	-4.9004E-01	-6.3941E-01
4.2945E-01	-1.9030E-01	-4.2168E-01	-5.7055E-01
4.9110E-01	-1.6411E-01	-3.6509E-01	-5.0890E-01
5.5470E-01	-1.3951E-01	-3.1038E-01	-4.4530E-01
6.1565E-01	-1.1779E-01	-2.6082E-01	-3.8435E-01
6.7064E-01	-9.9477E-02	-2.1810E-01	-3.2936E-01
7.1743E-01	-8.4854E-02	-1.8327E-01	-2.8257E-01
7.6036E-01	-7.2112E-02	-1.5239E-01	-2.3964E-01
7.9952E-01	-6.1023E-02	-1.2508E-01	-2.0048E-01
8.3517E-01	-5.1326E-02	-1.0086E-01	-1.6483E-01
8.6131E-01	-4.4466E-02	-8.3516E-02	-1.3869E-01
8.8558E-01	-3.8312E-02	-6.7770E-02	-1.1442E-01
9.0481E-01	-3.3565E-02	-5.5503E-02	-9.5187E-02
9.2413E-01	-2.8925E-02	-4.3400E-02	-7.5874E-02
9.4159E-01	-2.4826E-02	-3.2617E-02	-5.8412E-02
9.5739E-01	-2.1181E-02	-2.2968E-02	-4.2614E-02
9.6728E-01	-1.8946E-02	-1.7007E-02	-3.2723E-02
9.7684E-01	-1.6818E-02	-1.1298E-02	-2.3155E-02
9.8256E-01	-1.5564E-02	-7.9170E-03	-1.7442E-02
9.9138E-01	-1.3654E-02	-2.7440E-03	-8.6218E-03
9.9503E-01	-1.2873E-02	-6.1684E-04	-4.9695E-03
9.9993E-01	-1.1837E-02	2.2135E-03	-7.4811E-05
1.0038E 00	-1.1025E-02	4.4390E-03	3.7982E-03
1.0044E 00	-1.0892E-02	4.8050E-03	4.4410E-03
1.0071E 00	-1.0335E-02	6.3411E-03	7.1410E-03
1.0072E 00	-1.0328E-02	6.3638E-03	7.1835E-03
1.0104E 00	-9.6717E-03	8.1810E-03	1.0404E-02
1.0114E 00	-9.4706E-03	8.7404E-03	1.1399E-02
1.0119E 00	-9.3754E-03	9.0064E-03	1.1876E-02
1.0114E 00	-9.4757E-03	8.7287E-03	1.1381E-02
1.0114E 00	-9.4770E-03	8.7262E-03	1.1378E-02
1.0123E 00	-9.2887E-03	9.2531E-03	1.2327E-02
1.0134E 00	-9.0698E-03	9.8661E-03	1.3432E-02
1.0128E 00	-9.1974E-03	9.5097E-03	1.2789E-02

SAME SYSTEM,P1=17,P3=1.73,S/N=1/5

TIME	Y	FMT	MR1	MR2
0.	2.0298E 00	3.8935E 00	0.	0.
1.000E-01	3.0420E 00	3.7584E 00	1.6543E-01	-2.0480E-01
2.000E-01	1.1794E 00	1.3465E 00	2.1347E-01	-2.4696E-01
3.000E-01	3.5107E-01	3.4019E-01	2.2422E-01	-2.5340E-01
4.000E-01	-1.4000E-01	-1.1070E-01	2.2264E-01	-2.5286E-01
5.000E-01	1.1779E 00	1.3637E 00	2.2837E-01	-2.5174E-01
6.000E-01	-1.5589E 00	-1.1660E 00	2.5169E-01	-2.4243E-01
7.000E-01	-8.1703E-01	-2.0774E-01	2.8827E-01	-2.1993E-01
8.000E-01	-1.4086E 00	-5.9769E-01	3.3252E-01	-1.8311E-01
9.000E-01	-3.5572E 00	-2.5726E 00	3.8740E-01	-1.2765E-01
1.000E 00	1.0483E 00	2.1729E 00	4.4615E-01	-5.8174E-02
1.100E 00	7.6696E-01	1.9970E 00	5.0705E-01	2.3488E-02
1.200E 00	7.2821E-01	2.0307E 00	5.6715E-01	1.1268E-01
1.300E 00	-2.0826E-01	1.1370E 00	6.1985E-01	1.9762E-01
1.400E 00	-1.5512E 00	-1.8892E-01	6.7004E-01	2.8436E-01
1.500E 00	2.6865E 00	4.0442E 00	7.1505E-01	3.6679E-01
1.600E 00	1.6225E 00	2.9581E 00	7.5509E-01	4.4402E-01
1.700E 00	3.0016E 00	4.3012E 00	7.9041E-01	5.1512E-01
1.800E 00	-5.8859E-01	6.6461E-01	8.2394E-01	5.8527E-01
1.900E 00	-3.9665E 00	-2.7675E 00	8.5118E-01	6.4412E-01
2.000E 00	9.3937E-01	2.0791E 00	8.7446E-01	6.9589E-01
2.100E 00	-8.0953E-01	2.6757E-01	8.9584E-01	7.4467E-01
2.200E 00	1.8828E 00	2.8957E 00	9.1295E-01	7.8459E-01
2.300E 00	1.5763E 00	2.5247E 00	9.2705E-01	8.1818E-01
2.400E 00	1.5356E 00	2.4204E 00	9.3915E-01	8.4751E-01
2.500E 00	-1.2164E-01	7.0105E-01	9.4861E-01	8.7083E-01
2.600E 00	4.0556E-01	1.1683E 00	9.5674E-01	8.9113E-01
2.700E 00	1.3385E 00	2.0438E 00	9.6320E-01	9.0748E-01
2.800E 00	-1.0985E 00	-4.4780E-01	9.6888E-01	9.2199E-01
2.900E 00	2.4014E 00	3.0006E 00	9.7449E-01	9.3648E-01
3.000E 00	-1.3341E 00	-7.8343E-01	9.7825E-01	9.4631E-01
3.100E 00	-8.8528E-01	-3.7992E-01	9.8227E-01	9.5687E-01
3.200E 00	-7.7593E-01	-3.1285E-01	9.8540E-01	9.6517E-01
3.300E 00	8.7143E-01	1.2952E 00	9.8815E-01	9.7248E-01
3.400E 00	4.4988E 00	4.8862E 00	9.8994E-01	9.7730E-01
3.500E 00	5.8306E-01	9.3686E-01	9.9182E-01	9.8237E-01
3.600E 00	1.6475E 00	1.9703E 00	9.9360E-01	9.8719E-01
3.700E 00	-2.7221E-01	2.2066E-02	9.9577E-01	9.9310E-01
3.800E 00	-8.7325E-01	-6.0517E-01	9.9580E-01	9.9319E-01
3.900E 00	-1.6189E 00	-1.3748E 00	9.9660E-01	9.9538E-01
4.000E 00	-1.3979E 00	-1.1759E 00	9.9773E-01	9.9849E-01
4.100E 00	-3.1827E 00	-2.9808E 00	9.9817E-01	9.9972E-01
4.200E 00	-2.0811E 00	-1.8976E 00	9.9827E-01	1.0000E 00
4.300E 00	-1.0074E 00	-8.4067E-01	9.9871E-01	1.0012E 00
4.400E 00	-1.4364E 00	-1.2850E 00	9.9868E-01	1.0011E 00
4.500E 00	1.4386E 00	1.5761E 00	9.9874E-01	1.0013E 00
4.600E 00	2.4447E 00	2.5694E 00	9.9890E-01	1.0018E 00
4.700E 00	4.0602E-03	1.1723E-01	9.9941E-01	1.0032E 00
4.800E 00	-1.5212E 00	-1.4185E 00	9.9966E-01	1.0039E 00
4.900E 00	2.8182E 00	2.9113E 00	9.9960E-01	1.0037E 00
5.000E 00	-1.4929E 00	-1.4085E 00	9.9976E-01	1.0041E 00

VARIABLE	MINIMUM	MAXIMUM
ØRF1	9.5288E-03	1.4142E 00
ØRF2	-2.0000E 00	6.6667E-01

PI=17, S/N =1/5

MR3	ERR1	ERR2	ERR3
0.	-1.0000E 00	-1.0000E 00	-1.0000E 00
1.9840E-01	-8.3456E-01	-1.2048E 00	-8.0160E-01
2.1118E-01	-7.8653E-01	-1.2470E 00	-7.8882E-01
2.0798E-01	-7.7578E-01	-1.2534E 00	-7.9202E-01
2.0909E-01	-7.7736E-01	-1.2529E 00	-7.9091E-01
2.0317E-01	-7.7163E-01	-1.2517E 00	-7.9683E-01
1.7930E-01	-7.4831E-01	-1.2424E 00	-8.2070E-01
1.4531E-01	-7.1173E-01	-1.2199E 00	-8.5469E-01
1.1229E-01	-6.6748E-01	-1.1831E 00	-8.8771E-01
8.3359E-02	-6.1260E-01	-1.1277E 00	-9.1664E-01
6.8169E-02	-5.5385E-01	-1.0582E 00	-9.3183E-01
7.0592E-02	-4.9295E-01	-9.7651E-01	-9.2941E-01
9.1634E-02	-4.3285E-01	-8.8732E-01	-9.0837E-01
1.2640E-01	-3.8015E-01	-8.0238E-01	-8.7360E-01
1.7516E-01	-3.2996E-01	-7.1564E-01	-8.2484E-01
2.3228E-01	-2.8495E-01	-6.3321E-01	-7.6772E-01
2.9502E-01	-2.4491E-01	-5.5598E-01	-7.0498E-01
3.6003E-01	-2.0959E-01	-4.8488E-01	-6.3997E-01
4.3080E-01	-1.7606E-01	-4.1473E-01	-5.6920E-01
4.9488E-01	-1.4882E-01	-3.5588E-01	-5.0512E-01
5.5507E-01	-1.2554E-01	-3.0411E-01	-4.4493E-01
6.1496E-01	-1.0416E-01	-2.5533E-01	-3.8504E-01
6.6638E-01	-8.7049E-02	-2.1541E-01	-3.3362E-01
7.1148E-01	-7.2948E-02	-1.8182E-01	-2.8852E-01
7.5227E-01	-6.0853E-02	-1.5249E-01	-2.4773E-01
7.8572E-01	-5.1386E-02	-1.2917E-01	-2.1428E-01
8.1562E-01	-4.3263E-02	-1.0887E-01	-1.8438E-01
8.4025E-01	-3.6795E-02	-9.2521E-02	-1.5975E-01
8.6258E-01	-3.1120E-02	-7.8007E-02	-1.3742E-01
8.8531E-01	-2.5514E-02	-6.3516E-02	-1.1469E-01
9.0097E-01	-2.1748E-02	-5.3692E-02	-9.9026E-02
9.1805E-01	-1.7733E-02	-4.3135E-02	-8.1946E-02
9.3164E-01	-1.4598E-02	-3.4834E-02	-6.8357E-02
9.4378E-01	-1.1855E-02	-2.7517E-02	-5.6222E-02
9.5186E-01	-1.0059E-02	-2.2696E-02	-4.8140E-02
9.6042E-01	-8.1805E-03	-1.7631E-02	-3.9580E-02
9.6864E-01	-6.4021E-03	-1.2810E-02	-3.1356E-02
9.7879E-01	-4.2320E-03	-6.9041E-03	-2.1212E-02
9.7896E-01	-4.1996E-03	-6.8095E-03	-2.1044E-02
9.8277E-01	-3.3998E-03	-4.6151E-03	-1.7229E-02
9.8819E-01	-2.2726E-03	-1.5147E-03	-1.1807E-02
9.9037E-01	-1.8255E-03	-2.7872E-04	-9.6335E-03
9.9085E-01	-1.7270E-03	-3.1218E-06	-9.1452E-03
9.9299E-01	-1.2909E-03	1.2051E-03	-7.0054E-03
9.9287E-01	-1.3193E-03	1.1296E-03	-7.1339E-03
9.9317E-01	-1.2593E-03	1.2983E-03	-6.8309E-03
9.9399E-01	-1.0957E-03	1.7565E-03	-6.0070E-03
9.9652E-01	-5.9211E-04	3.1630E-03	-3.4809E-03
9.9778E-01	-3.4172E-04	3.8637E-03	-2.2184E-03
9.9750E-01	-3.9815E-04	3.7075E-03	-2.4976E-03
9.9829E-01	-2.4255E-04	4.1443E-03	-1.7069E-03

APPENDIX B

OBTAINING THE ORTHOGONAL SET

There exist several procedures for obtaining an orthogonal or orthonormal set. The Gram-Schmidt procedure is described and used to construct a polynomial set which is orthonormal over the interval $0 \leq t \leq 1$.

If a finite or infinite linearly independent set is given as

$$(e_1, e_2, \ldots e_k, \ldots) \tag{B-1}$$

it is possible to construct an orthonormal set

$$(\phi_1, \phi_2, \ldots \phi_k, \ldots) \; . \tag{B-2}$$

This method yields ϕ_k as a linear combination of only the first k elements, or

$$\phi_k = \sum_{j=1}^{k} a_{kj} e_j \tag{B-3}$$

First ϕ_1 is formed as

$$\phi_1 = e_1/\{e_1, e_1\}^{\frac{1}{2}} \tag{B-4}$$

where

$$\{e_1, e_1\} = \int_a^b |g_1(t)|^2 dt \tag{B-5}$$

and $a \leq t \leq b$ is the orthogonality interval.

Then one forms g_2 as

$$g_2 = e_2 - \{e_2, \phi_1\}\phi_1 \tag{B-6}$$

$$\phi_2 = g_2/\{g_2, g_2\}^{\frac{1}{2}} \tag{B-7}$$

The general form is then

$$g_k = e_k - \sum_{i=1}^{k-1} \{e_k, \phi_i\}\phi_i \tag{B-8}$$

$$\phi_k = g_k/\{g_k, g_k\}^{1/2} \tag{B-9}$$

The orthonormal polynomial set is then constructed. If a set

$$\{t, t^2, \ldots, t^n, \ldots\} \tag{B-10}$$

is given, and it is linearly independent because

$$a_1 t + a_2 t^2 + \ldots + a_n t^n = 0$$

for all t if, and only if, $a_1 = a_2 = \ldots = a_n = 0$, then ϕ_1 is given by

$$\phi_1 = \frac{e}{\{e_1, e_1\}^{1/2}} = \frac{t}{[\int_0^1 t^2 dt]^{1/2}} = \sqrt{3}\ t \tag{B-12}$$

and g_2 is given by

$$g_2 = e_2 - \{e_2, \phi_1\}\phi_1 \tag{B-13}$$

$$\{e_2, \phi_1\} = \int_0^1 t^2(\sqrt{3}\ t)dt = \frac{\sqrt{3}}{4} \tag{B-14}$$

$$g_2 = t^2 - \frac{3}{4}t \tag{B-15}$$

so that

$$\phi_2 = \frac{t^2 - \frac{3}{4}t}{[\int_0^1 (t^4 - \frac{3}{2}t^3 + \frac{9}{16}t^2)dt]^{1/2}} = \sqrt{5}(4t^2 - 3t) \tag{B-16}$$

The scaling value which makes the set orthonormal is given by

$$K_n \sqrt{2n + 1} \tag{B-17}$$

and the sum of the coefficients of each polynomial equals one. Then one can formulate ϕ_4 and g_4 as

$$\phi_4 = K_4 \sqrt{9}\, g_4 \tag{B-18}$$

$$g_4 = e_4 - \sum_{i=1}^{3} \{e_4, \phi_i\}\phi_i \tag{B-19}$$

or

$$\phi_4 = \frac{K_4\sqrt{9}}{56} (56t^4 - 105t^3 + 60t^2 - 10t) \tag{B-20}$$

so that $K_4 = 56$ and

$$\phi_4 = \sqrt{9}\, (56t^4 - 105t^3 + 60t^2 - 10t) \tag{B-21}$$

which is the correct result. Thus the resulting four polynomials are

$$\phi_1 = \sqrt{3}\, t \tag{B-22}$$

$$\phi_2 = \sqrt{5}\, (4t^2 - 3t) \tag{B-23}$$

$$\phi_3 = \sqrt{7}\, (15t^3 - 20t^2 + 6t) \tag{B-24}$$

$$\phi_4 = \sqrt{9}\, (56t^4 - 105t^3 + 60t^2 - 10t) \tag{B-25}$$

APPENDIX C

Computer Programs

This appendix contains several computer programs which greatly assist the users in evaluating the worth of any set of orthogonal functions for use as the signal set for an orthogonal multiplexing system. The programs are written in DSL-90, a digital simulation language which is a part of the IBM share library. Instruction manuals are available from the IBM Corporation. DSL-90 is a non-procedural language which is composed of subroutines which can be closely related to analog computer blocks, and thus is ideal for use in simulating many physical systems. The DSL-90 language is based on Fortran IV and is very easy to use. The notation throughout most of the programs follows that shown in Figure C-1 and therefore it is possible to follow the flow of the programs without being familiar with the language. To use these programs to assist in the evaluation of a signal set it is only necessary to change the orthogonal functions in the program to the desired set and change the system parameters. In this collection each program will be preceded by a description of the purpose of the program and how the program can be adapted to another set of functions. The printout on page 107 is the DSL-90 deck which must go before the main program deck.

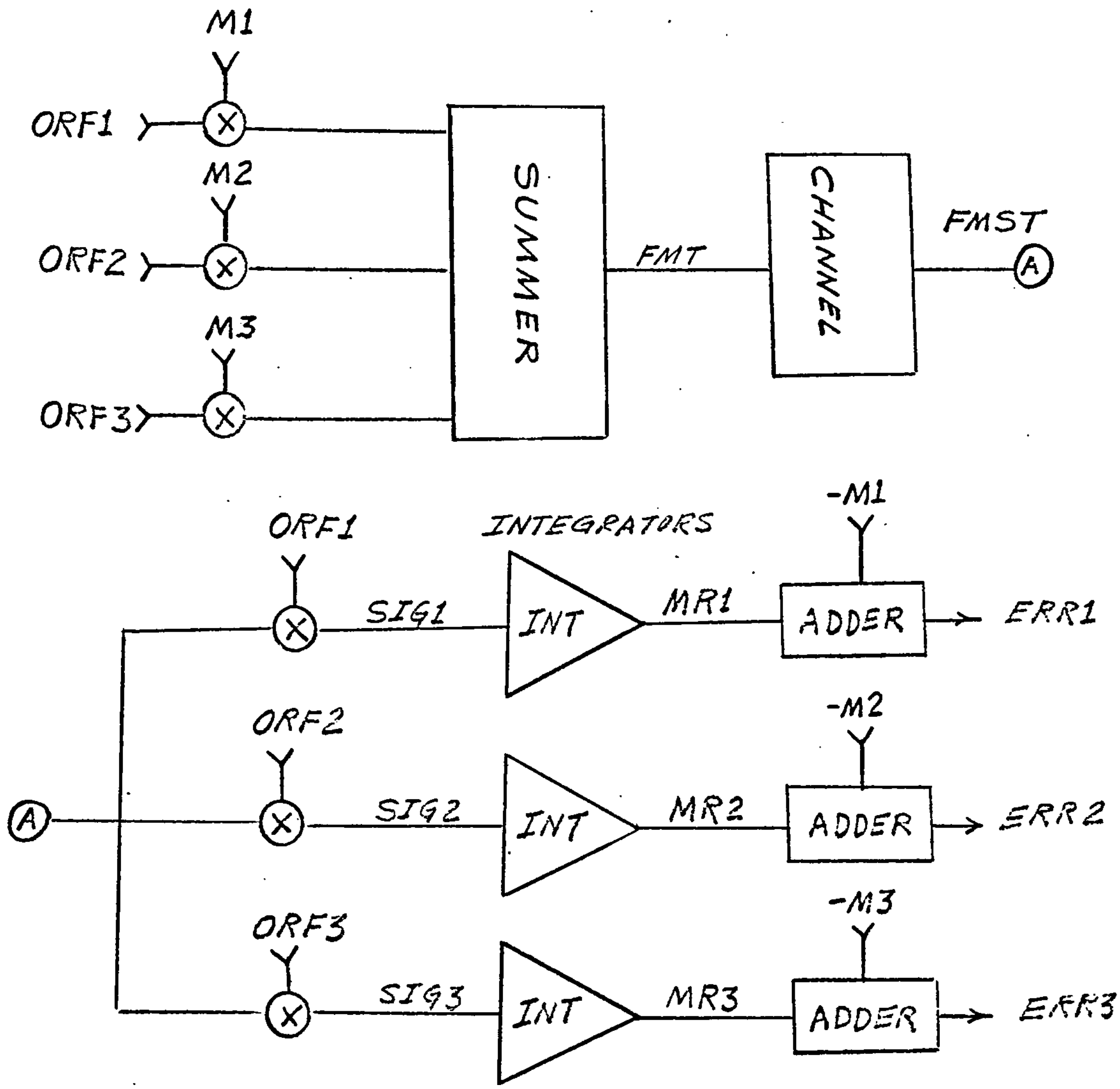

Figure C-1   Digital Computer Notation

DSL-90 DECK


```
$ID                                          010  05000  S031   TOM WILLIAMS
$REWIND         SYSCK1
$REWIND         SYSLB3
$EXECUTE        IBJOB
$IBJOB          MAP,FIOCS
$IEDIT          SYSLB3,SRCH
$IBLDR CKSTOP
$IBLDR CONTIN
$IBLDR FINISH
$IBLDR INTEG
$IBLDR JIGSAW
$IBLDR NAME
$IBLDR OUTIN
$IBLDR OUTPUT
$IBLDR PDWRMX
$IBLDR SCAN
$IBLDR SORT
$IBLDR STORE
$IBLDR TRANSL
$IEDIT
$DATA
$IEDIT          SYSLB3,SRCH
$IBLDR MAIN
$IBLDR CENTRL
$IEDIT
```

000025

Program 1

Title: Periodic Function Frequency Distribution

The Real Exponential Set

This program is used to calculate the Fourier coefficients of each orthogonal function and the composite waveform. The program notation is readily identified because of the similarity to the notation used in many texts.

$$A_0 = \frac{1}{T}\int_0^T f(t)\,dt \tag{C-1}$$

$$A_n = \frac{2}{T}\int_0^T f(t)\ \cos(n\omega_0 t)\,dt \tag{C-2}$$

$$B_n = \frac{2}{T}\int_0^T f(t)\ \sin(n\omega_0 t)\,dt \tag{C-3}$$

$$C_n = \sqrt{A_n^2 + B_n^2} \tag{C-4}$$

Thus for example:

AN1 = A_n of ORF1

AN2 = A_n of ORF2

ANT = A_n of FMT=ORF1 + ORF2 + ORF3

This program yields the information necessary to determine signal bandwidth in order to compare the bandwidth with that of other signals or to use to design a channel filter. In order to use this program to calculate the bandwidth of another signal set it is necessary to change ORF1, ORF2, ORF3, and change the coefficient values and the period T on the param card.

```
TITLE PERIODIC FUNCTION FREQUENCY DISTRIBUTION
TITLE THE REAL EXPONENTIAL SET
      ORF1=C11*EXP(-TIME)
      ORF2=C21*EXP(-TIME)+C22*EXP(-2.*TIME)
      ORF3=C31*EXP(-TIME)+C32*EXP(-2.*TIME)+C33*EXP(-3.*TIME)
CONST C11=1.4142,C21=4.,C22=-6.,C31=7.3485,C32=-29.39,C33=24.5,T=5.
      FMT=ORF1+ORF2+ORF3
      SIG1=ORF1*(SIN(N*6.28*TIME/T))
      SIG2=ORF2*(SIN(N*6.28*TIME/T))
      SIG3=ORF3*(SIN(N*6.28*TIME/T))
      SFMT=FMT*(SIN(N*6.28*TIME/T))
      CIG1=ORF1*(COS(N*6.28*TIME/T))
      CIG2=ORF2*(COS(N*6.28*TIME/T))
      CIG3=ORF3*(COS(N*6.28*TIME/T))
      CFMT=FMT*(COS(N*6.28*TIME/T))
      INTS1=INTGRL(0.,SIG1)
      BN1=(2.*INTS1)/T
      INTS2=INTGRL(0.,SIG2)
      BN2=(2.*INTS2)/T
      INTS3=INTGRL(0.,SIG3)
      BN3=(2.*INTS3)/T
      INTS4=INTGRL(0.,SFMT)
      BNT=(2.*INTS4)/T
      INTC1=INTGRL(0.,CIG1)
      AN1=(2.*INTC1)/T
      INTC2=INTGRL(0.,CIG2)
      AN2=(2.*INTC2)/T
      INTC3=INTGRL(0.,CIG3)
      AN3=(2.*INTC3)/T
      INTC4=INTGRL(0.,CFMT)
      ANT=(2.*INTC4)/T
      A01=(INTGRL(0.,ORF1))/T
      A02=(INTGRL(0.,ORF2))/T
      A03=(INTGRL(0.,ORF3))/T
      A0T=(INTGRL(0.,FMT))/T
      CN1=SQRT(AN1**2.+BN1**2.)
      CN2=SQRT(AN2**2.+BN2**2.)
      CN3=SQRT(AN3**2.+BN3**2.)
      CNT=SQRT(ANT**2.+BNT**2.)
CONTRL FINTIM=5.,DELT=.01
PARAM N=1.
PRINT 1.,AN1,BN1,CN1,AN2,BN2,CN2,AN3,BN3,CN3,...
      ANT,BNT,CNT,A01,A02,A03,A0T
INTEG MILNE
END
TITLE N=2
PARAM N=2.
END
TITLE N=3
PARAM N=3.
END
TITLE N=4
PARAM N=4.
END
TITLE N=5
PARAM N=5.
```

```
END
TITLE N=6
PARAM N=6.
END
TITLE N=7
PARAM N=7.
END
TITLE N=8
PARAM N=8.
END
TITLE N=9
PARAM N=9.
END
TITLE N=10
PARAM N=10.
END
TITLE N=11
PARAM N=11.
END
TITLE N=12
PARAM N=12.
END
TITLE N=13
PARAM N=13.
END
TITLE N=14
PARAM N=14.
END
TITLE N=15
PARAM N=15.
END
STOP
1        END OF FILE

         000089
```

Program 2

Title: Periodic Function Power Spectrum Zero Initial Value Polynomials and Gaussian Product

This program is similar to the previous one in that the power spectral density is calculated for an orthogonal set. Both C and C squared are printed for several values of N. This program is written to also evaluate the effect of multiplying the orthogonal set by another function which is in this case the Gaussian function.

$$\int_0^T [\underbrace{(FMT)P(t)}_{\text{PRODUCT FUNCTION EVALUATED}}][(ORF1)P^{-1}(t)]dt \qquad (C-5)$$

The advantages are reduced peak-to-average power requirements and reduced bandwidth. The value of the Fourier coefficients for the product function are CM and CSQM, or C modified and C^2 modified. This program provides a print-out of the values of FMT and FMTM, the product or modified function. In order to use this program to evaluate other functions it is necessary to substitute the desired orthogonal functions and the improvement factor function $p(t) = y_1$ for the present set, and change the values on the param card.

```
TITLE PERIODIC FUNCTION POWER SPECTRUM
TITLE ZIV POLYNOMIALS AND GAUSSIAN PRODUCT
      ORF1=(SQRT(3.))*(TIME-K)
      ORF2=(SQRT(5.))*((4.*((TIME-K)**2.))-(3.*(TIME-K)))
      ORF3=(SQRT(7.))*((15.*((TIME-K)**3.))-(20.*...
      ((TIME-K)**2.))+(6.*(TIME-K)))
      FMT=(ORF1+ORF2+ORF3)
      FMTM=FMT*Y1
       Y1=EXP(K1*(TIME**2.))
      FUNA=FMT*(COS(6.28*N*TIME/T))
      INTA=INTGRL(0.,FUNA)
      FUNB=FMT*(SIN(6.28*N*TIME/T))
      INTB=INTGRL(0.,FUNB)
      A0=(INTGRL(0.,FMT))/T
      AN=(2.*INTA)/T
      BN=(2.*INTA)/T
      CSQRD=((AN)**2.)+((BN)**2.)
      C=SQRT(CSQRD)
      FUNAM=FMTM*(COS(6.28*N*TIME/T))
      FUNBM=FMTM*(SIN(6.28*N*TIME/T))
      A0M=(INTGRL(0.,FMTM))/T
      INTAM=INTGRL(0.,FUNAM)
      INTBM=INTGRL(0.,FUNBM)
      ANM=(2.*INTAM)/T
      BNM=(2.*INTBM)/T
      CSQM=((ANM)**2.)+((BNM)**2.)
      CM=SQRT(CSQM)
INTEG MILNE
PRINT .05,AN,C,CSQRD,ANM,CM,CSQM,FMT,FMTM,ORF3
PARAM N=1.,K=0.,K1=-1.,T=1.
CONTRLFINTIM=1.,DELT=.001
END
TITLE N=2
PARAM N=2.
END
TITLE N=3
PARAM N=3.
END
TITLE N=4
PARAM N=4.
END
TITLE N=5
PARAM N=5.
END
TITLE N=6
PARAM N=6.
END
TITLE N=7
PARAM N=7.
END
TITLE N=8
PARAM N=8.
END
STOP
```

000054

Program 3

Steady State

Polynomial Signals and Channel Filter

The purpose of this program is to determine the distortion caused by a filter inserted in the channel to restrict signal energy to a certain pass-band. The program goes through several cycles and more can easily be added using the Title, Contin, and Param cards as examples. A more complex filter can be used (for example, an n stage Butterworth, etc.) to test any design. DSL-90 contains complex functions commands which allow the evaluation of any filter design. The notation for this program is standard and it is necessary to modify the "param", orthogonal functions, and possibly the "contrl" cards to use another set of functions.

```
 $IEDIT          SYSLB3,SRCH
 $IBLDR MAIN
 $IBLDR CENTRL
 $IEDIT
 TITLE STEADY STATE
 TITLE PØLYNØMIAL SIGNALS AND CHANNEL FILTER
 TITLE RC=.05
       ØRF1=(SQRT(3.))*(TIME-K)
       ØRF2=(SQRT(5.))*((4.*((TIME-K)**2.))-(3.*(TIME-K)))
       ØRF3=(SQRT(7.))*((15.*((TIME-K)**3.))-(20.*...
       ((TIME-K)**2.))+(6.*(TIME-K)))
       FMT=M1*ØRF1+M2*ØRF2+M3*ØRF3
       FMST=REALPL(0.,RC,FMT)
       SIG1=FMST*ØRF1
       SIG2=FMST*ØRF2
       SIG3=FMST*ØRF3
       INT1=INTGRL(0.,SIG1)
       INT2=INTGRL(0.,SIG2)
       INT3=INTGRL(0.,SIG3)
       FMT2=C1*FMT
       FMST2=REALPL(0.,RC,FMT2)
       SIG12=FMST2*ØRF1
       SIG22=FMST2*ØRF2
       SIG32=FMST2*ØRF3
       INT12=INTGRL(0.,SIG12)
       INT22=INTGRL(0.,SIG22)
       INT32=INTGRL(0.,SIG32)
       MR1=INT1-INT12
       MR2=INT2-INT22
       MR3=INT3-INT32
       ERR1=MR1-M1
       ERR2=MR2-M2
       ERR3=MR3-M3
 CØNST M1=1.,M2=1.,M3=1.
 PARAM K=0.,C1=0.,RC=.05
 CØNTRL FINTIM=1.,DELT=.001
 INTEG MILNE
 PRINT .02,ERR1,ERR2,ERR3,FMT,FMST,INT3,INT32,INT2,INT22
 END
 TITLE CYCLE TWØ
 CØNTIN
 CØNTRL FINTIM=2.,DELT=.001
 PARAM K=1.,C1=1.
 END
 TITLE CYCLE THREE
 CØNTRL FINTIM=3.,DELT=.001
 CØNTIN
 PARAM K=2.,C1=1.
 END
 TITLE RC=.005, CYCLE ØNE
 PARAM K=0.,C1=0.,RC=.005
 CØNTRL FINTIM=1.,DELT=.001
 END
 TITLE CYCLE TWØ
 CØNTRL FINTIM=2.,DELT=.001
```

```
CØNTIN
PARAM K=1.,C1=1.
END
TITLE CYCLE THREE
CØNTIN
CØNTRL FINTIM=3.,DELT=.001
PARAM K=2.,C1=1.
END
TITLE CYCLE FØUR
CØNTIN
CØNTRL FINTIM=4.,DELT=.001
PARAM K=3.,C1=1.
END
STØP
```

Program 4

Polynomial Functions/Synch Error

This program is used to determine the distortion caused by synchronization error in the receiver of the orthomux system. Time delay (TD) is varied from 0 to .3 seconds of a 1 second normalized period. The polynomial functions and system parameters can be changed to evaluate any other set, it will also be necessary to change the bandwidth for the new set and possibly the "control" time. For the particular set under consideration the print out runs for 1.5 seconds. The receiver period is from t=TD to t=TD+1. for each cycle, so the receiver output for each cycle should be read from the print out at t=TD+1.

```
TITLE POLYNOMIAL FUNCTIONS / SYNCH ERROR / BW=1
TITLE TIME DELAY EQUAL TO ZERO
      ORF1=(SQRT(3.))*(TIME-K)
      ORF2=(SQRT(5.))*((4.*((TIME-K)**2.))-(3.*(TIME-K)))
      ORF3=(SQRT(7.))*((15.*((TIME-K)**3.))-(20.*...
      ((TIME-K)**2.))+(6.*(TIME-K)))
      ORF12=(SQRT(3.))*(TIME-TD)
      ORF22=(SQRT(5.))*((4.*((TIME-TD)**2.))-(3.*(TIME-TD)))
      ORF32=(SQRT(7.))*((15.*((TIME-TD)**3.))-(20.*...
      ((TIME-TD)**2.))+(6.*(TIME-TD)))
      FMT=M1*ORF1+M2*ORF2+M3*ORF3
      FMST=REALPL(0.,BW,FMT)
      ST1=STEP(TD)
      DEL1=ORF12*ST1
      DEL2=ORF22*ST1
      DEL3=ORF32*ST1
      SIG1=FMST*DEL1
      SIG2=FMST*DEL2
      SIG3=FMST*DEL3
      INT1=INTGRL(0.,SIG1)
      INT2=INTGRL(0.,SIG2)
      INT3=INTGRL(0.,SIG3)
      ERR1=M1-INT1
      ERR2=M2-INT2
      ERR3=M3-INT3
INTEG MILNE
CONTRLFINTIM=1.5,DELT=.001
PARAM BW=.053,K=0.,TD=0.,M1=1.,M2=1.,M3=1.
PRINT .01,ERR1,ERR2,ERR3,ORF1,DEL1,ORF2,DEL2,ORF3,DEL3
END
TITLE TD=.05
PARAM TD=.05
END
TITLE TD=.1
PARAM TD=.1
END
TITLE TD=.15
PARAM TD=.15
END
TITLE TD=.2
PARAM TD=.2
END
TITLE TD=.25
PARAM TD=.25
END
TITLE TD=.3
PARAM TD=.3
END
STOP
'     END OF FILE
```

Program 5

Title: Distortion Due to Amplitude Limiting

This program is used to calculate the distortion caused by limiting the maximum swing of the composite function FMT (often called clipping) to specific values. In order to use this program with other functions it is necessary to change the orthogonal functions (ORF1, etc.) and the "contrl" and "Param" cards.

```
TITLE DISTORTION DUE TO AMPLITUDE LIMITING
TITLE NO LIMITING
          ORF1=(SQRT(3.))*(TIME-K)
          ORF2=(SQRT(5.))*((4.*((TIME-K)**2.))-(3.*(TIME-K)))
          ORF3=(SQRT(7.))*((15.*((TIME-K)**3.))-(20.*...
          ((TIME-K)**2.))+(6.*(TIME-K)))
          FMT=ORF1*M1+ORF2*M2+ORF3*M3
          FMST=LIMIT(P1,P2,FMT)
          SIG1=FMST*ORF1
          SIG2=FMST*ORF2
          SIG3=FMST*ORF3
          MR1=INTGRL(0.,SIG1)
          MR2=INTGRL(0.,SIG2)
          MR3=INTGRL(0.,SIG3)
          ERR1=MR1-M1
          ERR2=MR2-M2
          ERR3=MR3-M3
INTEG MILNE
CONTRL FINTIM=1.,DELT=.001
PRINT .02,FMST,ERR1,ERR2,ERR3,ORF1,ORF2,ORF3,FMT
PARAM M1=1.,M2=1.,M3=1.,P1=-100.,P2=100.,K=0.
RANGE ORF1,ORF2,ORF3,FMT
END
TITLE LIMITING AT .9 OF PEAK VALUE
PARAM P1=-5.95,P2=5.95
END
TITLE LIMITING AT .8 OF PEAK VALUE
PARAM P1=-5.3,P2=5.3
END
TITLE LIMITING AT .7 OF PEAK VALUE
PARAM P1=-4.66,P2=4.66
END
TITLE LIMITING AT .6 OF PEAK VALUE
PARAM P1=-3.99,P2=3.99
END
TITLE LIMITING AT .5 OF PEAK VALUE
PARAM P1=-3.33,P2=3.33
END
STOP
*         END OF FILE
```

000040

Program 6

Othomux System with Additive Gaussian Noise

This program is written in order to insert random numbers into the channel with a Gaussian amplitude distribution and these values are added to the signal value. As would be expected, the orthomux system is not affected much by this type interference, but it is an interesting use of the digital computer. As before, the program is changed by modifying the "param," "contrl," "const" and orthogonal function cards.

```
TITLE ORTHOMUX SYSTEM WITH ADDITIVE GUASSIAN NOISE
TITLE POLYNOMIAL SET
      ORF1=(SQRT(3.))*(TIME-K)
      ORF2=(SQRT(5.))*((4.*((TIME-K)**2.))-(3.*(TIME-K)))
      ORF3=(SQRT(7.))*((15.*((TIME-K)**3.))-(20.*...
      ((TIME-K)**2.))+(6.*(TIME-K)))
      FMT=M1*ORF1+M2*ORF2+M3*ORF3
      EMTY=EMT+Y
      Y=NORMAL(P1,P2,P3)
      SIG1=EMTY*ORF1
      MR1=INTGRL(0.,SIG1)
      SIG2=EMTY*ORF2
      MR2=INTGRL(0.,SIG2)
      SIG3=EMTY*ORF3
      MR3=INTGRL(0.,SIG3)
      ERR1=MR1-M1
      ERR2=MR2-M2
      ERR3=MR3-M3
CONTRL FINTIM=1.,DELT=.001
CONST K=0.
PARAM M1=1.,M2=1.,M3=1.,P1=1.,P2=0.,P3=.245
RANGE Y,EMT,EMTY
INTEG MILNE
PRINT .01,ERR1,ERR2,ERR3,Y,EMT,EMTY
END
TITLE S/N=1
PARAM P1=3.,P2=0.,P3=1.73
END
TITLE S/N=1/5
PARAM P1=5.,P2=0,P3=3.87
END
TITLE S/N=1/10
PARAM P1=7.,P2=0,P3=5.48
END
STOP
1      END OF FILE
```

000036