SPIN-EXCHANGE CROSS SECTIONS OF ALKALI ATOMS

A Dissertation

Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

by

Chen-kun Chang

June 1967

ACKNOWLEDGMENTS

The author wishes to express his deep appreciation and thanks to Dr. R. H. Walker for his assistance and encouragement in the preparation of this work. Recognition is also given to the computer center of the Manned Space Craft Center of the National Aeronautics and Space Administration and to that of the University of Houston for their help towards the calculation of the results. The author also wishes to thank Dr. B. Oldham, who helped in setting up the calculations.

Furthermore, the author wishes to acknowledge the financial assistance rendered by the National Aeronautics and Space Administration during his last summer of residence, during which time the bulk of this thesis was written. In addition, the author wishes to extend his thanks to the physics department of the University of Houston for generous financial aid during the years of his graduate training.

SPIN-EXCHANGE CROSS SECTIONS OF ALKALI-ATOMS

An Abstract of a Dissertation

Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

Chen-kun Chang

June 1967

ABSTRACT

In the collision of atoms with one valence S electron outside closed shells (alkali-like atoms), an important feature of the process is the indistinguishability of electrons. This can give rise to an exchange of electrons between the colliding atoms. This process has come to be called spin-exchange scattering. These spin-exchange effects may be calculated using the usual scattering theories. This process is the subject of this paper.

To calculate numerically spin-exchange cross sections, the spin-dependent interaction characterizing the scattering is required. This interaction for two alkaline atoms is calculated by individually considering the interactions due to the valence S electrons and to the closed electronic shells of the two atoms. The two S electron interaction is generated phenomenonologically from the known hydrogen-hydrogen interatomic potential and the closed shell interaction from interatomic potentials of two inert atoms. Two undetermined parameters characterizing the scale of the interaction are introduced in this generation. These parameters are determined from experimentally known values of the equilibrium distance and the dissociation energy appropriate for the molecular state of the atomic system of interest. These theoretical potentials so obtained are in agreement with those constructed from band spectra. Unfortunately, these direct observations yield spin-dependent interaction potentials

only for the singlet state of the alkaline atoms. This technique is extended to obtain the interaction potentials for cases for which there is at the present time inadequate experimental information for their construction, and for those cases in which no stable molecule is formed. It is hoped that the good agreement of our phenomenonological calculation of potentials can be extrapolated to those cases for which no experimental check is possible. These interaction potentials are also used to calculate the second virial coefficients.

The present theories of spin-dependent scattering of alkali-like atoms are introduced, and our potentials are used to calculate various cross sections by semi-classical techniques and by partial wave analysis. The results of the semi-classical method compare fairly well with experimental ones. Six different scattering cross sections are calculated by partial wave analysis for each of the six alkaline atoms, H, Li, Na, K, Rb and Cs, for 124 different relative kinetic energies. These cross sections are finally averaged with Maxwellian distribution for temperature of range from $10^{o}k$ to $9500^{o}k$, and the partial results are shown in the following table.

Spin-Exchange Cross Sections (10^{-14} cm^2)

Systems	H^1-H^1	Li^3-Li^3	Na^{11}-Na^{11}	K^{19}-K^{19}	Rb^{85}-Rb^{85}	Cs^{133}-Cs^{133}
Temp.	325°K	405°K	405°K	405°K	504°K	504°K
Our cal.	0.130	1.018	1.207	2.374	2.565	2.706
Exp.	0.141	----	1-3	----	2.4±0.4	2.6±0.5
Ref.	1 - 2		3		4	4

1. K. M. Mazo, J. Chem. Phys. 34, 169 (1961).
2. A. F. Hildenbrandt, F. B. Booth, and C. A. Barth, J. Chem. Phys. 31, 273 (1959).
3. L. W. Anderson and A. T. Ramsey, Phys. Rev. 132, 712 (1963).
4. H. W. Moos And R. H. Sands, Phys. Rev. 135, A 591 (1964).

As shownin the table, the results obtained by partial wave analysis agree rather well with the experiments.

Comparing the spin-exchange cross sections calculated by the two methods shows that the cross sections calculated by semi-classical methods are too small by about 10% for the heavier atoms at the higher energies, and by about 50% for the light atoms at low energies.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF FIGURES

CHAPTER I

INTRODUCTION

An important relaxation mechanism in paramagnetic gases is the transfer of spin angular momentum during collisions between atoms. If the colliding atoms have unpaired electrons, the indistinguishability of the electrons allows the exchange of electrons between the colliding atoms with significant probability. This is the so-called spin-exchange scattering, the existence of which has supplied a tool for the measurement of several atomic parameters. In addition, many double resonance and radiofrequency spectroscopy experiments have been carried out in which spin-exchange scattering is important.[1-4] The same process is also of interest in radio astronomy.[5] The properties of collisions between hydrogen atoms are, of course, of astrophysical importance. Many experiments based upon optical pumping involving spin-exchange collisions have been reported. Some of these have exploited spin-exchange collisions to orient atoms and even free electrons, the latter being impossible to orient by a direct pumping process.[7,8,11,12] Additional interest in this problem arises from the information it can provide on the forces between atomic systems.

The utility of optical pumping as a tool and the importance of spin-exchange as a relaxation mechanism has prompted the experimental determination of many scattering cross sections, in particular

those of the alkaline atomic systems, such as hydrogen[15], sodium[16], rubidium[17,18], and cesium[17].

Origionally, a simple theory for spin-exchange between hydrogen atoms was developed by Purcell and Field[5] and by Wittke and Dicke[1]. The former pair of authors was concerned with the intensity of the 21-cm line in radio astronomy, the latter with line broadening in a precision measurement of the hyperfine structure of atomic hydrogen. The spin-exchange cross section for hydrogen collisions has been estimated by the former pair of authors using a strong collision approximation, by Dalgarno[19], and by Smirnov and Chibisov[24], using an impact parameter approximation and by Smith[6] using a semi-classical approximation. None of these approximations is valid at low energies where a quantal description must be employed. The Purcell and Field model, for example, predicts spin-exchange cross sections too small by about 35%, and a total cross section which has the wrong energy dependence and is typically incorrect by a factor of two. Dalgarno and Henry[20] have calculated the spin-exchange cross sections using a partial wave analysis for the atomic hydrogen without considering the identity of hydrogen atoms. Glassgold and Labedeff[21,22] also have computed the hydrogen spin-exchange cross sections using partial wave analysis and discussing exchange effects. But the hydrogen interaction potentials used by them are not accurate. Walker and Mays[23]

have extended the theory and calculated the spin-exchange cross sections of atomic hydrogen using an accurate interaction potential. Their results seem to agree well with experiments. The present author's work extends that of Walker and Mays to the other five alkaline atoms.

In our work we consider relative energies greater than the atomic hyperfine splittings, but small enough for the Born-Oppenheimer approximation to the valid and for the atoms to remain in ground states. For two hydrogen atoms, this restricts the relative energy to the range (measured in ^{o}K) from approximately 10 to 10^{5} ^{o}K. The restriction of the lowest relative energies for the other five alkaline atoms are roughly the same as that of hydrogen. Due to the larger masses of the other five alkaline atoms, the highest relative energies for the Born-Oppenheimer approximation to be valid are greater than 10^{5} ^{o}K. But the lowest atomic excitation energies of these five alkaline atoms are much smaller than that of hydrogen. This restricts the relative energies considered to the range from approximately 10 ^{o}K to 10^{4} ^{o}K for the five alkaline atoms. Neglecting all magnetic interactions, the above restrictions on the relative energy imply that the scattering can be treated as an elastic scattering problem. In this approximation, the atoms interact through a spin-dependent central potential. Associated with this potential are scattering phase shifts, which are obtained in the usual way by solving the Schrodinger equation.

The first goal of such an approach is the determination

of singlet and triplet spin-dependent interaction potentials of alkaline atoms. These potentials are not all available in the current literature, in fact only a few singlet potentials have been estimated from experiments. It is necessary, then, to first obtain these potentials. To this end we assume the interaction of the alkaline atoms to be a sum of two interaction potentials, one due to the two valence electrons, V_1, and another due to the cores of the two atoms, V_2. V_1 here includes the interactions between the two S electrons themselves and between the two S electrons and the effective charges of the two nuclei. The total interaction potential of two alkaline atoms, V, may then be expressed as $V = V_1 + V_2$. V_1 is generated from a scaling of the hydrogen-hydrogen interaction and V_2 from the interaction of the inert atoms which correspond to the cores. The interaction of the alkaline atoms so obtained are then compared to the potentials obtained experimentally from molecular spectra.[25] The results show surprisingly good agreement. These potentials are used to calculate the second virial coefficients which are given in Chapter III. These interaction potentials are also used for the numerical calculation of various cross sections.

In Chapter IV we use the WKB approximation to obtain semi-classically the spin-exchange cross sections for H-H, Li-Li, Na-Na, K-K, Rb-Rb and Cs-Cs systems. Spin-exchange cross sections for twenty-eight different relative kinetic energies are calculated for each of the six atomic systems. These results compare fairly well with the experimental ones.

Glassgold and Lebedeff's[21,22] and Walker and Mays'[23] work on the theory of spin-dependent scattering, results in formulae for calculating several kinds of spin-dependent cross sections including electronic spin-exchange, hyperfine transition and total polarized and unpolarized scattering cross section for both identical and non-identical atoms for systems of nuclear spin $\frac{1}{2}$. We have extended these relations for the calculation of cross sections of identical atoms of arbitrary nuclear spin. These results are used in conjunction with partial wave analysis to calculate numerically cross sections for alkali-alkali atomic collisions. We have employed a digital computer to calculate six types of scattering cross sections for 124 different relative kinetic energies for the six alkaline systems. Finally, each of the six types of cross sections is averaged over a Maxwellian distribution. The results again agree very well with experiments. Furthermore, comparing the two results by the semi-classical method and partial wave analysis suggests that the results by the semi-classical method are all too small by about 10% for high energy heavier atoms, and about 50% for low energy H-H and Li-Li systems.

CHAPTER II

GENERATION OF THE INTERACTION POTENTIALS BETWEEN ALKALI ATOMS

1. Introduction

As we have mentioned in Chapter I, the spin-dependent polarization phenomena in collision between atoms may be calculated using the usual scattering theories. Neglecting all magnetic interactions, previously mentioned restrictions on the relative kinetic energy imply that to a good approximation we may consider the processes as elastic. In this approximation, the atoms interact through a spin-dependent central potential. Associated with this potential are scattering phase shifts, which are obtained in the usual way by solving the Schrodinger equation. Therefore, the first step toward solving the problem is to obtain an accurate spin-dependent potential.

The simplest diatomic system is the H_2^+ ion. As shown originally by Burrau[57], the total energy E for the H_2^+ ion can be calculated exactly. Bates[58] has calculated this energy for nuclear separation distance up to seven atomic units. Cohen et al[59], using a variational method, calculated the energy up to 20 atomic units. Wind[26] has calculated the electronic energy for the ground state for nuclear separations up to 40 atomic units to seven significant figures. From

Wind's results it is not difficult to calculate the interaction potential between the neutral and ionic hydrogen atoms as a function of the nuclear separation including the energy arising from the repulsive Coulomb force of the two nuclei. These results are shown in Table 2.1.

The next simplest molecule is the neutral hydrogen molecule, H_2. This system has been investigated in very great detail. As discussed by Walker and Mays[23], the most accurate potentials may be obtained from the following authors: (all quantities are in atomic units)

(1) $r \leq 0.8$ molecular orbit calculations

(2) $0.8 \leq r < 4$ Kolos and Roothan[54]

(3) $4 \leq r \leq 10$ Dalgarno and Lynn[55]

(4) $r \geq 10$ Pauling and Beach[56]

for the singlet state and

(1) $r \leq 4$ Kolos and Roothan[54]

(2) $4 \leq r \leq 10$ Dalgarno and Lynn[55]

(3) $r \geq 10$ Pauling and Beach[56]

for the triplet state. Table 2.2 shows the potentials for the singlet and the triplet states.

Investigations concerning the more complicated atomic systems have been only partially successful because of the increasing complexity as we increase the number of electrons involved. The earliest attempt at the calculation of the interaction of two alkali-atoms apparently was that of Rosen and Ikehara[31]. They assumed that the valence S electrons of the alkaline atom has a hydrogen-like wave function scaled by parameters to be determined variationally. They used this

TABLE 2.1

Hydrogen-Hydrogen Ion Interaction Potential

r/a_o	V (Rydbergs)	r/a_o	V (Rydbergs)
0.05	0.370120×10^{2}	7.50	-0.758592×10^{-2}
0.30	0.393326×10	7.70	-0.649174×10^{-2}
0.50	0.153002×10	7.90	-0.555605×10^{-2}
0.70	0.634750	8.10	-0.475702×10^{-2}
0.90	0.219459	8.30	-0.407554×10^{-2}
1.10	0.717641×10^{-2}	8.50	-0.349428×10^{-2}
1.30	-0.105481	8.70	-0.299935×10^{-2}
1.50	-0.164646	8.90	-0.257769×10^{-2}
1.70	-0.193393	9.10	-0.221857×10^{-2}
1.90	-0.204212	9.30	-0.191283×10^{-2}
2.10	-0.204270	9.50	-0.165248×10^{-2}
2.30	-0.197862	9.70	-0.143044×10^{-2}
2.50	-0.187647	9.90	-0.124139×10^{-2}
2.70	-0.175315	10.10	-0.107999×10^{-2}
2.90	-0.161956	10.30	-0.942037×10^{-3}
3.10	-0.148285	10.50	-0.824407×10^{-3}
3.30	-0.134768	10.70	-0.723511×10^{-3}
3.50	-0.121711	10.90	-0.636965×10^{-3}
3.70	-0.109307	11.10	-0.562802×10^{-3}
3.90	-0.976747×10^{-1}	11.30	-0.498846×10^{-3}
4.10	-0.868789×10^{-1}	11.50	-0.443757×10^{-3}
4.30	-0.769471×10^{-1}	11.70	-0.396222×10^{-3}
4.50	-0.678802×10^{-1}	11.90	-0.355169×10^{-3}
4.70	-0.596591×10^{-1}	12.10	-0.319332×10^{-3}
4.90	-0.522519×10^{-1}	12.30	-0.288159×10^{-3}
5.10	-0.438431×10^{-1}	12.50	-0.261188×10^{-3}
5.30	-0.397043×10^{-1}	12.70	-0.237286×10^{-3}
5.50	-0.344630×10^{-1}	12.90	-0.216439×10^{-3}
5.70	-0.298384×10^{-1}	13.10	-0.198051×10^{-3}
5.90	-0.257755×10^{-1}	13.30	-0.181854×10^{-3}
6.10	-0.222210×10^{-1}	13.50	-0.167444×10^{-3}
6.30	-0.191225×10^{-1}	13.70	-0.154585×10^{-3}
6.50	-0.164309×10^{-1}	13.90	-0.143096×10^{-3}
6.70	-0.141001×10^{-1}	14.10	-0.132829×10^{-3}
6.90	-0.120873×10^{-1}	14.30	-0.123650×10^{-3}
7.10	-0.103538×10^{-1}	14.50	-0.115365×10^{-3}
7.30	-0.886380×10^{-2}	14.70	-0.107974×10^{-3}

TABLE 2-1 (Continued)

r/a_o	V (Rydbergs)
14.90	-0.101000×10^{-3}
15.10	-0.948608×10^{-4}
15.30	-0.892431×10^{-4}
15.50	-0.839382×10^{-4}
15.70	-0.792593×10^{-4}
15.90	-0.748336×10^{-4}
16.10	-0.707954×10^{-4}
16.30	-0.670105×10^{-4}
16.50	-0.634789×10^{-4}
16.70	-0.603199×10^{-4}
16.90	-0.571907×10^{-4}
17.10	-0.545383×10^{-4}
17.30	-0.518560×10^{-4}
17.50	-0.494719×10^{-4}
17.70	-0.470430×10^{-4}
17.90	-0.449568×10^{-4}
18.10	-0.429451×10^{-4}
18.30	-0.409633×10^{-4}
18.50	-0.392795×10^{-4}
18.70	-0.375211×10^{-4}
18.90	-0.358969×10^{-4}
19.10	-0.343472×10^{-4}
19.30	-0.330508×10^{-4}
19.50	-0.316948×10^{-4}

TABLE 2.2

Hydrogen-Hydrogen Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
.01	1.96200×10^{2}	1.97181×10^{2}
.02	9.62030×10	9.72139×10
.03	6.28736×10	6.39118×10
.04	4.62122×10	4.72758×10
.05	3.62187×10	3.73060×10
.06	2.95596×10	3.06689×10
.07	2.48062×10	2.59361×10
.08	2.12443×10	2.23932×10
.09	1.84769×10	1.96435×10
.10	1.62658×10	1.74488×10
.11	1.44502×10	1.56577×10
.12	1.29475×10	1.41692×10
.13	1.16786×10	1.29133×10
.14	1.05933×10	1.18401×10
.16	8.83645	1.01045×10
.18	7.47823	8.76404
.20	6.39931	7.69974
.225	5.33018	6.64499
.25	4.48483	5.81017
.275	3.80228	5.13482
.30	3.24183	4.57862
.325	2.77525	4.11371
.35	2.38233	3.72021
.375	2.04825	3.38357
.40	1.76186	3.09287
.455	1.26035	2.57677
.505	9.17543×10^{-1}	2.21580
.605	4.42370×10^{-1}	1.69450
.705	1.43860×10^{-1}	1.34153
.805	-4.81973×10^{-2}	1.09045
.905	-1.72576×10^{-1}	9.04887×10^{-1}
1.005	-2.52237×10^{-1}	7.63397×10^{-1}
1.205	-3.30917×10^{-1}	5.63900×10^{-1}
1.345	-3.47652×10^{-1}	4.65906×10^{-1}
1.405	-3.48879×10^{-1}	4.30918×10^{-1}
1.445	-3.48203×10^{-1}	4.09464×10^{-1}

TABLE 2.2 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
1.625	-3.34326×10^{-1}	3.27560×10^{-1}
1.825	-3.06003×10^{-1}	2.57291×10^{-1}
2.025	-2.71543×10^{-1}	2.02156×10^{-1}
2.265	-2.28116×10^{-1}	1.51181×10^{-1}
2.505	-1.92981×10^{-1}	1.18870×10^{-1}
2.745	-1.48029×10^{-1}	8.57006×10^{-2}
2.985	-1.14523×10^{-1}	6.53815×10^{-2}
3.545	-5.57458×10^{-2}	3.72075×10^{-2}
4.025	-2.39928×10^{-2}	7.34493×10^{-3}
4.505	-1.34454×10^{-2}	3.51838×10^{-3}
4.985	-8.02667×10^{-3}	9.97732×10^{-4}
5.545	-4.03660×10^{-3}	2.98300×10^{-4}
6.025	-1.73240×10^{-3}	-4.14194×10^{-5}
6.505	-7.55629×10^{-4}	-1.29861×10^{-4}
6.985	-4.40604×10^{-4}	-1.02750×10^{-4}
7.545	-1.77003×10^{-4}	-8.74368×10^{-5}
7.945	-1.23940×10^{-4}	-6.54480×10^{-5}
8.505	-8.17529×10^{-5}	-4.64852×10^{-5}
9.945	-2.01533×10^{-5}	-1.77925×10^{-5}
11.065	-6.61670×10^{-6}	-6.61670×10^{-6}
12.025	-4.90400×10^{-6}	-4.90400×10^{-6}
12.985	-3.03611×10^{-6}	-3.03611×10^{-6}
13.945	-1.94968×10^{-6}	-1.94968×10^{-6}
14.985	-1.24983×10^{-6}	-1.24983×10^{-6}

wave function to obtain the molecular energy neglecting the interaction due to the electrons in the closed shells. This treatment was not realistic since the closed shells play a very important role as had been pointed out earlier by James[32]. The calculations of scattering cross sections of alkaline atoms by Smirnov and Chibisov[24] employed potentials obtained in a manner similar to those of Rosen and Ikehara. Obviously, none of these authors has obtained potentials which agree with experimental results. The experimental determinations of the potentials yield only the singlet state. Our calculated results of the interaction potentials for the alkaline atoms agree well with those obtained from observation. Our work on the potentials is discussed in the following sections.

2. Theory

A comparison of the electronic configuration of the alkali and inert atoms indicates that the principal difference between these two groups is the number of electrons in the outer shell and the number of protons in the nucleus. That is, the alkali-atoms have one S electron in an incompletely filled subshell whereas the inert atoms have only completely filled subshells. Thus, the alkali-atoms can be considered to have an additional electron compared to the inert atoms. Therefore, since an alkali-atom has an added electron, then it must have an added proton in its nucleus because the atom is electrically neutral as a whole. But the sizes of the inner closed shells of two corresponding alkali-atoms and inert atoms are nearly the same,

especially for the heavier atoms. Furthermore,the experimental results suggest that the nuclear separations for minimum-interaction potentials of those heavier alkaline and inert diatomic systems are nearly the same. This fact will be utilized by our theory.

From the above discussion we then assume that the interaction of two alkaline atoms may be considered as a sum of two independent interactions, one due to the valence S electrons and another due to the closed shells of the two atoms. The interaction due to the valence S electrons here includes the interactions between the two S electrons themselves, and between the two S electrons and the effective charges of the two nuclei. We also assume that the interaction due to the closed shells is of the same form as the interaction of two inert atoms, and that the interaction due to the two S electrons has a hydrogen-like behavior, i.e. a modification of the hydrogen interaction. Then the interaction potential of two alkaline atoms, V, will be the sum of the interaction potentials due to the two S electrons, V_1, and the two inner closed shells V_2, i.e.

$$V = V_1 + V_2 \tag{2.1}$$

The general properties of a bonding diatomic interaction potential curve have been studied very extensively. Any potential is assumed to have the following properties

(1) It should be smooth and asymptotically approach zero as the nuclear separation becomes large.

(2) It should be a minimum at a certain nuclear separation, say $r = r_m$.

(3) It should become very large and positive at a certain nuclear

separation, say $r = r_c$, where $r_c < r_m$. Accordingly, each of the potential functions V, V_1, and V_2 should have the above mentioned properties. Let V, V_1, and V_2 only have minima at r_m, r_1 and r_2 respectively. Then the derivatives of the three functions, $\partial V/\partial r$, $\partial V_1/\partial r$ and $\partial V_2/\partial r$ should be zero only at r_m, r_1 and r_2 respectively, i.e.

$$(\partial V/\partial r)_{r=r_m} = (\partial V_1/\partial r)_{r=r_1} = (\partial V_2/\partial r)_{r=r_2} = 0 . \tag{2.2}$$

These conditions can be written

$$\partial V/\partial r = (r - r_m) f(r) , \tag{2.3}$$

$$\partial V_1/\partial r = (r - r_1) f_1(r) , \tag{2.4}$$

$$\partial V_2/\partial r = (r - r_2) f_2(r) , \tag{2.5}$$

where $f(r)$, $f_1(r)$ and $f_2(r)$ are unspecified functions, and $f(r) \neq 0$, $f_1(r) \neq 0$ and $f_2(r) \neq 0$ for any r in the range from $r = r_c$ to $r = \infty$.

The partial derivative of eq. (2.1) with respect to r is

$$\partial V/\partial r = \partial V_1/\partial r + \partial V_2/\partial r . \tag{2.6}$$

Substituting eqs. (2.3), (2.4) and (2.5) into eq. (2.6) we obtain

$$(r - r_m) f(r) = (r - r_1) f_1(r) + (r - r_2) f_2(r) , \qquad (2.7)$$

or

$$f(r)\, r_m - f_1(r) r_1 - f_2(r) r_2 = \left[f(r) - f_1(r) - f_2(r) \right] r . \qquad (2.8)$$

It is clear that this relation holds for any r in the range of $r_c \leq r < \infty$. There are an infinite number of different values of r in this range, and each of these values of r must satisfy eq. (2.8). Hence an infinite number of simultaneous linear equations arise from eq. (2.8). But there are only three unknowns, r_m, r_1 and r_2, to be determined from this set of equations. Therefore, in general, there is no unique solution unless some further conditions are imposed.

In particular if the following conditions are satisfied,

$$\begin{aligned} r_m &= r_1 = r_2 \\ f(r) &= f_1(r) + f_2(r) . \end{aligned} \qquad (2.9)$$

then a unique solution will be obtained. This condition implies that the three potentials V, V_1 and V_2 all have their minimum values at the same point. Thus placing $r_m = r_1 = r_2$ into eq. (2.8) we have

$$\left[f(r) - f_1(r) - f_2(r) \right] (r - r_m) = 0$$

This equation is automatically satisfied by the condition $f(r) = f_1(r) + f_2(r)$. The potential curve V so obtained conforms to the previously stated properties of a diatomic interaction.

As mentioned before, the size of the inner closed electronic shells of the heavier alkaline atoms are almost the same as those of the corresponding inert atoms. Then in the potential for the alkaline system, V_2 should fit the potential for the inert diatomic system, and V that for the corresponding alkaline system. Accordingly, V_2 and V should become a minimum at almost the same nuclear separation. This fact is verified by experimental investigations. The experimental values of the nuclear separations for minimum interaction potentials of alkaline and inert diatomic systems are shown in Table 2.3.

Table 2.3 [35-38]

Comparison of nuclear separations for minimum interaction potentials of alkaline and inert diatomic systems

Systems	r_m(Å)	D(ev)	Systems	r_2(Å)	$D_2(\times 10^{-3}$ ev)
Li-Li	2.6725	-1.14	He-He	3.135	-0.795
Na-Na	3.0786	-0.76	Ne-Ne	3.147	-3.29
K-K	3.923	-0.52	A-A	3.866	-10.7
Rb-Rb	4.127	-0.493	Kr-Kr	4.056	-13.7
Cs-Cs	4.465	-0.453	Xe-Xe	4.450	-20.0

Now let the minimum values of V, V_1 and V_2 be D, D_1 and D_2 respectively; then from eqs. (2.1) and (2.9) we know that

$$D = D_1 + D_2 \tag{2.10}$$

Here, we remember that V_1 is expected to be similar to the hydrogen-hydrogen interaction potential (Table 2.2). The nuclear separation for the minimum singlet hydrogen-hydrogen interaction potential can be obtained from Table 2.2, and is seen to be 0.7417 A. This is quite different from the corresponding values of alkaline-alkaline interaction as seen from Table 2.3. In order to satisfy the condition (2.9) we have to enlarge the scale of nuclear separation of Table 2.3. This may be done in a trival way by a transformation, namely

$$r \longrightarrow \frac{a}{a_o} r + b \quad . \tag{2.11}$$

We then require

$$r_m = \frac{a}{a_o} r_o + b \quad , \tag{2.12}$$

where r_m and r_o are the equilibrium distances of diatomic alkaline and hydrogen systems respectively, and a is the orbital radius of the S electron of the alkaline atom, and b is an undetermined constant.

On the other hand, the condition $D = D_1 + D_2$ states the minimum

of V should be equal to the sum of minima of V_1 and V_2. In order to satisfy these conditions we must reduce the energy scale of hydrogen-hydrogen interaction potential. Let V_o be the hydrogen-hydrogen interaction potential, including singlet and triplet states; then the energy scale may be reduced in a trivial way, namely

$$V_o \longrightarrow k V_o \quad , \quad (k < 1) . \tag{2.13}$$

Combining (2.11) and (2.13) we have

$$V_1 \left(\frac{a}{a_o} r + b \right) = k V_o(r) . \tag{2.14}$$

Now consider the three potentials V, V_1 and V_2 at nuclear separation $\frac{a}{a_o} r + b$. We have

$$V \left(\frac{a}{a_o} r + b \right) = V_1 \left(\frac{a}{a_o} r + b \right) + V_2 \left(\frac{a}{a_o} r + b \right) . \tag{2.15}$$

Substituting (2.14) into (2.15) yields

$$V \left(\frac{a}{a_o} r + b \right) = k V_o(r) + V_2 \left(\frac{a}{a_o} r + b \right) ,$$

or

$$V(r) = k V_o \left(\frac{a_o}{a} r - \frac{b}{a} \right) + V_2(r) , \tag{2.16}$$

which states that the interaction potential of alkaline atoms at

nuclear separation r is equal to k times the hydrogen interaction potential at nuclear separation $(\frac{a_0}{a} r - \frac{b}{a})$ plus the interaction potential of the corresponding inert atoms at nuclear separation r.

If D, D_0 and D_2 are the minimum values of the potentials for alkaline-alkaline, hydrogen-hydrogen and inert-inert atomic interaction potentials respectively; then from eqs. (2.10) and (2.13) we obtain

$$D_1 = k D_0$$

and

$$D = D_1 + D_2 = k D_0 + D_2 ,$$

or

$$k = (D - D_2) / D_0 . \tag{2.17}$$

From eq. (2.12) we find

$$b = r_m - \frac{a}{a_0} r_0 . \tag{2.18}$$

Combining these results with (2.16) gives

$$V(r) = V_0 \left(\frac{a_0}{a} r - \frac{r_m}{a} + \frac{r_0}{a_0}\right)(D - D_2)/D_0 +$$

$$+ V_2(r) . \tag{2.19}$$

In the above equation, the potential $V_0(r)$ and its parameters, r_0 and D_0 may be obtained from Table 2.2, and the parameters r_m ,

D and D_2 from Table 2.3; and a_0 and a from the literature[39,60]. The potential function $V_2(r)$ still has not been specified. The interaction potential of inert atoms may be adequately represented by one of the following three functions[27-29]:

$$\text{12:6 potential: } V_2(r) = -4D_2\left[\left(\frac{r_3}{r}\right)^{12} - \left(\frac{r_3}{r}\right)^{6}\right], \qquad (2.20)$$

$$\text{9:6 potential: } V_2(r) = -D_2\left[2\left(\frac{r_2}{r}\right)^{9} - \left(\frac{r_2}{r}\right)^{6}\right], \qquad (2.21)$$

$$\text{exp:6 potential: } V_2(r) = -\frac{D_2}{1-6/\alpha}\left[\frac{6}{\alpha}\exp\left(1-\frac{r}{r_2}\right) - \left(\frac{r_2}{r}\right)^{6}\right]. \qquad (2.22)$$

Here α , D_2 , r_2 and r_3 are all undetermined parameters, which may be determined from the experimental values of second virial coefficients, viscosity, thermal conductivity, diffusion coefficient, and crystal structures. With one of these functions for the one interaction, the interaction of alkaline atoms, $V(r)$, may be calculated.

3. Results

Using the three different forms of the interaction potentials of inert atoms, 12 : 6, 9 : 6 and exp : 6, we have calculated the interaction potentials of the five diatomic systems, Li-Li, Na-Na, K-K, Rb-Rb and Cs-Cs. The results calculated using the 9 : 6 potential fit best the experimental potentials constructed from band

spectra. In fact, they fit almost perfectly everywhere except in that portions from $r = 1.5$ rm to $r = 2$ rm, where rm is the equilibrium distances of the diatomic molecule. Our results are given in Tables 2.4-2.8 and plotted in Fig. 2-1.

TABLE 2.3

Lithium-Lithium Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
1.0819	0.12886×10^{4}	0.12887×10^{4}
1.1669	0.61815×10^{3}	0.61837×10^{3}
1.2378	0.36193×10^{3}	0.36217×10^{3}
1.3086	0.21881×10^{3}	0.21905×10^{3}
1.3937	0.12401×10^{3}	0.12427×10^{3}
1.4787	0.72770×10^{2}	0.73039×10^{2}
1.5637	0.44051×10^{2}	0.44326×10^{2}
1.6488	0.27426×10^{2}	0.27706×10^{2}
1.7338	0.17519×10^{2}	0.17803×10^{2}
1.8189	0.11458×10^{2}	0.11744×10^{2}
1.9039	0.76579×10^{1}	0.79456×10^{1}
1.9180	0.71748×10^{1}	0.74627×10^{1}
2.0173	0.46156×10^{1}	0.49039×10^{1}
2.1023	0.32249×10^{1}	0.35128×10^{1}
2.1732	0.24224×10^{1}	0.27094×10^{1}
2.2724	0.16511×10^{1}	0.19365×10^{1}
2.3432	0.12696×10^{1}	0.15534×10^{1}
2.4283	0.93588	0.12175×10^{1}
2.5275	0.66325	0.94193
2.6976	0.37386	0.64671
2.8676	0.21000	0.47626
3.0094	0.12525	0.38560
3.0944	0.88413×10^{-1}	0.34508
3.2645	0.35234×10^{-1}	0.28431
3.4346	0.21843×10^{-4}	0.24129
3.6046	-0.24010×10^{-1}	0.20933
3.7747	-0.40763×10^{-1}	0.18460
3.9731	-0.54205×10^{-1}	0.16187
4.3133	-0.67677×10^{-1}	0.13272
4.6534	-0.73826×10^{-1}	0.11128
4.9936	-0.75686×10^{-1}	0.94642×10^{-1}
5.3338	-0.74909×10^{-1}	0.81224×10^{-1}
6.0141	-0.68940×10^{-1}	0.60631×10^{-1}
6.6944	-0.60239×10^{-1}	0.45459×10^{-1}
7.3747	-0.50815×10^{-1}	0.34028×10^{-1}
8.0550	-0.41644×10^{-1}	0.25508×10^{-1}
9.3022	-0.26987×10^{-1}	0.15360×10^{-1}
10.662	-0.15067×10^{-1}	0.94624×10^{-2}

TABLE 2.3 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
12.023	-0.70983×10^{-2}	0.33777×10^{-2}
13.384	-0.34562×10^{-2}	0.99622×10^{-3}
14.744	-0.20252×10^{-2}	0.36180×10^{-3}
16.105	-0.12097×10^{-2}	0.11189×10^{-3}
17.465	-0.59180×10^{-3}	0.20700×10^{-4}
18.826	-0.24525×10^{-3}	-0.23375×10^{-4}
20.187	-0.11686×10^{-3}	-0.27394×10^{-4}
21.547	-0.64391×10^{-4}	-0.21489×10^{-4}
22.908	-0.31373×10^{-4}	-0.17309×10^{-4}
24.269	-0.20915×10^{-4}	-0.12197×10^{-4}
25.629	-0.16121×10^{-4}	-0.91789×10^{-5}
26.990	-0.11272×10^{-4}	-0.70007×10^{-5}
28.350	-0.67612×10^{-5}	-0.49872×10^{-5}
29.711	-0.33872×10^{-5}	-0.33918×10^{-5}
31.072	-0.22130×10^{-5}	-0.22130×10^{-5}
32.432	-0.11495×10^{-5}	-0.11495×10^{-5}
33.793	-0.11007×10^{-5}	-0.11007×10^{-5}
35.153	-0.10712×10^{-5}	-0.10712×10^{-5}
36.514	-0.83897×10^{-6}	-0.83897×10^{-6}
37.875	-0.66355×10^{-6}	-0.66355×10^{-6}
39.235	-0.52962×10^{-6}	-0.52962×10^{-6}
40.596	-0.42628×10^{-6}	-0.42628×10^{-6}
41.957	-0.34579×10^{-6}	-0.34579×10^{-6}
43.317	-0.28254×10^{-6}	-0.28254×10^{-6}

TABLE 2.4

Sodium-Sodium Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
1.7170	0.87900×10^{2}	0.88048×10^{2}
1.8195	0.23115×10^{2}	0.23278×10^{2}
1.9074	0.14213×10^{2}	0.14386×10^{2}
1.9952	0.93785×10^{1}	0.95597×10^{1}
2.0831	0.64200×10^{1}	0.66091×10^{1}
2.1710	0.45153×10^{1}	0.47094×10^{1}
2.2589	0.32469×10^{1}	0.34450×10^{1}
2.3467	0.23803×10^{1}	0.25812×10^{1}
2.4346	0.17749×10^{1}	0.19778×10^{1}
2.5225	0.13436×10^{1}	0.15478×10^{1}
2.6103	0.10307×10^{1}	0.12355×10^{1}
2.6982	0.79980	0.10047×10^{1}
2.7861	0.62661	0.83116
2.8740	0.49477	0.69856
2.9618	0.39296	0.59565
3.0497	0.31330	0.51460
3.1376	0.25017	0.44984
3.2694	0.17802	0.37486
3.4451	0.11055	0.30303
3.6209	0.64249×10^{-1}	0.25190
3.7966	0.31549×10^{-1}	0.21404
3.9724	0.79399×10^{-2}	0.18503
4.1481	-0.93759×10^{-2}	0.16217
4.3239	-0.22199×10^{-1}	0.14371
4.4996	-0.31730×10^{-1}	0.12850
5.0562	-0.48316×10^{-1}	0.94167×10^{-1}
5.4076	-0.52382×10^{-1}	0.79229×10^{-1}
5.7591	-0.53642×10^{-1}	0.67462×10^{-1}
6.1106	-0.53102×10^{-1}	0.57910×10^{-1}
6.8136	-0.48918×10^{-1}	0.43208×10^{-1}
7.5166	-0.42775×10^{-1}	0.32377×10^{-1}
8.2196	-0.36098×10^{-1}	0.24225×10^{-1}
8.9226	-0.29592×10^{-1}	0.18154×10^{-1}
10.211	-0.19181×10^{-1}	0.10928×10^{-1}
11.617	-0.10710×10^{-1}	0.67304×10^{-2}

TABLE 2.4 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
13.023	-0.50459×10^{-2}	0.24026×10^{-2}
14.429	-0.24569×10^{-2}	0.70880×10^{-3}
15.835	-0.14397×10^{-2}	0.25747×10^{-3}
17.241	-0.86000×10^{-3}	0.79671×10^{-4}
18.647	-0.42071×10^{-3}	0.14777×10^{-4}
20.053	-0.17434×10^{-3}	-0.16588×10^{-4}
21.459	-0.83072×10^{-4}	-0.19460×10^{-4}
22.865	-0.45773×10^{-4}	-0.15269×10^{-4}
24.271	-0.22301×10^{-4}	-0.12301×10^{-4}
25.677	-0.14867×10^{-4}	-0.86695×10^{-5}
27.083	-0.11460×10^{-4}	-0.65246×10^{-5}
28.489	-0.80142×10^{-5}	-0.49767×10^{-5}
29.895	-0.48069×10^{-5}	-0.35455×10^{-5}
31.300	-0.24082×10^{-5}	-0.24115×10^{-5}
32.706	-0.15735×10^{-5}	-0.15735×10^{-5}
34.112	-0.10307×10^{-5}	-0.10307×10^{-5}
35.518	-0.78279×10^{-6}	-0.78279×10^{-6}
36.924	-0.76185×10^{-6}	-0.76185×10^{-6}
38.330	-0.59667×10^{-6}	-0.59667×10^{-6}
39.736	-0.47194×10^{-6}	-0.47194×10^{-6}
41.142	-0.37670×10^{-6}	-0.37670×10^{-6}
42.548	-0.30322×10^{-6}	-0.30322×10^{-6}
43.954	-0.24598×10^{-6}	-0.24598×10^{-6}
45.360	-0.20099×10^{-6}	-0.20099×10^{-6}

TABLE 2.5

Potassium-Potassium Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
1.5930	0.16155×10^{4}	0.16156×10^{4}
1.7385	0.71819×10^{3}	0.71830×10^{3}
1.8632	0.38324×10^{3}	0.38336×10^{3}
1.9880	0.21305×10^{3}	0.21317×10^{3}
2.1127	0.12272×10^{3}	0.12285×10^{3}
2.2374	0.72946×10^{2}	0.73081×10^{2}
2.3621	0.44590×10^{2}	0.44728×10^{2}
2.4869	0.27952×10^{2}	0.28091×10^{2}
2.6116	0.17926×10^{2}	0.18066×10^{2}
2.7363	0.11737×10^{2}	0.11878×10^{2}
2.8610	0.78314×10^{1}	0.79733×10^{1}
2.9857	0.53167×10^{1}	0.54586×10^{1}
3.1105	0.36671×10^{1}	0.38088×10^{1}
3.2352	0.25663×10^{1}	0.27075×10^{1}
3.3599	0.18197×10^{1}	0.19601×10^{1}
3.4846	0.13057×10^{1}	0.14451×10^{1}
3.6094	0.94668	0.10849×10^{1}
3.7964	0.59398	0.73032
4.0459	0.32567	0.45901
4.2953	0.17950	0.30948
4.5448	0.96291×10^{-1}	0.22269
4.7942	0.46978×10^{-1}	0.16964
5.0437	0.16677×10^{-1}	0.13550
5.2931	-0.25352×10^{-2}	0.11238
5.5426	-0.15030×10^{-1}	0.95961×10^{-1}
5.8336	-0.24400×10^{-1}	0.82016×10^{-1}
6.3325	-0.33065×10^{-1}	0.65629×10^{-1}
6.8314	-0.36718×10^{-1}	0.54446×10^{-1}
7.3303	-0.37767×10^{-1}	0.46118×10^{-1}
7.8292	-0.37357×10^{-1}	0.39538×10^{-1}
8.8269	-0.34268×10^{-1}	0.29545×10^{-1}
9.8247	-0.29863×10^{-1}	0.22192×10^{-1}
10.822	-0.25146×10^{-1}	0.16638×10^{-1}
11.820	-0.20585×10^{-1}	0.12487×10^{-1}
13.649	-0.13325×10^{-1}	0.75309×10^{-2}
15.645	-0.74363×10^{-2}	0.46445×10^{-2}

TABLE 2.5 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
17.640	-0.35037×10^{-2}	0.16557×10^{-2}
19.636	-0.17063×10^{-2}	0.48648×10^{-3}
21.631	-0.99979×10^{-3}	0.17584×10^{-3}
23.627	-0.59716×10^{-3}	0.53718×10^{-4}
25.623	-0.29231×10^{-3}	0.93370×10^{-5}
27.618	-0.12133×10^{-3}	-0.12061×10^{-4}
29.614	-0.57916×10^{-4}	-0.13853×10^{-4}
31.609	-0.31958×10^{-4}	-0.10828×10^{-4}
33.605	-0.15621×10^{-4}	-0.86950×10^{-5}
35.600	-0.10421×10^{-4}	-0.61277×10^{-5}
37.596	-0.80265×10^{-5}	-0.46075×10^{-5}
39.592	-0.56156×10^{-5}	-0.35116×10^{-5}
41.587	-0.33774×10^{-5}	-0.25037×10^{-5}
43.583	-0.17041×10^{-5}	-0.17063×10^{-5}
45.578	-0.11173×10^{-5}	-0.11173×10^{-5}
47.574	-0.73515×10^{-6}	-0.73515×10^{-6}
49.569	-0.55873×10^{-6}	-0.55873×10^{-6}
51.565	-0.54071×10^{-6}	-0.54071×10^{-6}
53.561	-0.42363×10^{-6}	-0.42363×10^{-6}
55.556	-0.33518×10^{-6}	-0.33518×10^{-6}
57.552	-0.26762×10^{-6}	-0.26762×10^{-6}
59.547	-0.21547×10^{-6}	-0.21547×10^{-6}
61.543	-0.17484×10^{-6}	-0.17484×10^{-6}
63.538	-0.14289×10^{-6}	-0.14289×10^{-6}

TABLE 2.6. Rubidium-Rubidium Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
1.9785	0.48830×10^{3}	0.48840×10^{3}
2.1240	0.23994×10^{3}	0.24005×10^{3}
2.2487	0.14247×10^{3}	0.14258×10^{3}
2.3735	0.87185×10^{2}	0.87304×10^{2}
2.4982	0.54735×10^{2}	0.54860×10^{2}
2.6229	0.35155×10^{2}	0.35283×10^{2}
2.7476	0.23049×10^{2}	0.23179×10^{2}
2.8724	0.15397×10^{2}	0.15530×10^{2}
2.9971	0.10464×10^{2}	0.10597×10^{2}
3.1218	0.72239×10^{1}	0.73586×10^{1}
3.2465	0.50598×10^{1}	0.51950×10^{1}
3.3713	0.35915×10^{1}	0.37267×10^{1}
3.4960	0.25805×10^{1}	0.27154×10^{1}
3.6207	0.18748×10^{1}	0.20092×10^{1}
3.7454	0.13758×10^{1}	0.15095×10^{1}
3.8701	0.10186×10^{1}	0.11514×10^{1}
3.9949	0.75997	0.89169
4.1820	0.49531	0.62516
4.4314	0.28336	0.41034
4.6808	0.16166	0.28545
4.9303	0.89224×10^{-1}	0.20960
5.1797	0.44689×10^{-1}	0.16151
5.4292	0.16507×10^{-1}	0.12967
5.6786	-0.17690×10^{-2}	0.10767
5.9281	-0.13853×10^{-1}	0.91848×10^{-1}
6.2191	-0.23021×10^{-1}	0.78322×10^{-1}
6.7180	-0.31573×10^{-1}	0.62417×10^{-1}
7.2169	-0.35187×10^{-1}	0.51631×10^{-1}
7.7158	-0.36224×10^{-1}	0.43663×10^{-1}
8.2147	-0.35822×10^{-1}	0.37407×10^{-1}
9.2125	-0.32819×10^{-1}	0.27953×10^{-1}
10.210	-0.28565×10^{-1}	0.21009×10^{-1}
11.208	-0.24031×10^{-1}	0.15762×10^{-1}
12.205	-0.19659×10^{-1}	0.11837×10^{-1}
14.035	-0.12717×10^{-1}	0.71449×10^{-2}
16.030	-0.70951×10^{-2}	0.44099×10^{-2}
18.026	-0.33436×10^{-2}	0.15698×10^{-2}
20.021	-0.16288×10^{-2}	0.45945×10^{-3}
22.017	-0.95437×10^{-3}	0.16522×10^{-3}

TABLE 2.6 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
24.012	-0.57006×10^{-3}	0.49799×10^{-4}
26.008	-0.27924×10^{-3}	0.80362×10^{-5}
28.004	-0.11611×10^{-3}	-0.12043×10^{-4}
29.999	-0.55528×10^{-4}	-0.13566×10^{-4}
31.995	-0.30690×10^{-4}	-0.10568×10^{-4}
33.990	-0.15056×10^{-4}	-0.84597×10^{-5}
35.986	-0.10052×10^{-4}	-0.59638×10^{-5}
37.981	-0.77371×10^{-5}	-0.44811×10^{-5}
39.977	-0.54169×10^{-5}	-0.34132×10^{-5}
41.973	-0.32681×10^{-5}	-0.24360×10^{-5}
43.968	-0.16622×10^{-5}	-0.16643×10^{-5}
45.964	-0.10943×10^{-5}	-0.10943×10^{-5}
47.959	-0.72360×10^{-6}	-0.72360×10^{-6}
49.955	-0.55054×10^{-6}	-0.55054×10^{-6}
51.950	-0.52956×10^{-6}	-0.52956×10^{-6}
53.946	-0.41513×10^{-6}	-0.41513×10^{-6}
55.942	-0.32863×10^{-6}	-0.32863×10^{-6}
57.937	-0.26252×10^{-6}	-0.26252×10^{-6}
59.933	-0.21146×10^{-6}	-0.21146×10^{-6}
61.928	-0.17166×10^{-6}	-0.17166×10^{-6}
63.924	-0.14035×10^{-6}	-0.14035×10^{-6}

TABLE 2.7

Cesium-Cesium Interaction Potential

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
2.6173	0.14137×10^{3}	0.14146×10^{3}
2.7628	0.68341×10^{2}	0.68438×10^{2}
2.8875	0.45020×10^{2}	0.45124×10^{2}
3.0122	0.30454×10^{2}	0.30562×10^{2}
3.1369	0.20983×10^{2}	0.21095×10^{2}
3.2617	0.14684×10^{2}	0.14799×10^{2}
3.3864	0.10418×10^{2}	0.10536×10^{2}
3.5111	0.74857×10^{1}	0.76054×10^{1}
3.6358	0.54404×10^{1}	0.55614×10^{1}
3.7605	0.39960×10^{1}	0.41176×10^{1}
3.8853	0.29636×10^{1}	0.30856×10^{1}
4.0100	0.22175×10^{1}	0.23396×10^{1}
4.1347	0.16726×10^{1}	0.17945×10^{1}
4.2594	0.12708×10^{1}	0.13922×10^{1}
4.3842	0.97168	0.10924×10^{1}
4.5089	0.74700	0.86696
4.6336	0.57681	0.69580
4.8207	0.39363	0.51093
5.0701	0.23723	0.35194
5.3196	0.14136	0.25320
5.5690	0.81079×10^{-1}	0.18983
5.8185	0.42288×10^{-1}	0.14782
6.0679	0.16831×10^{-1}	0.11906
6.3174	-0.14801×10^{-3}	0.98724×10^{-1}
6.5668	-0.11607×10^{-1}	0.83882×10^{-1}
6.8578	-0.20432×10^{-1}	0.71121×10^{-1}
7.3567	-0.28750×10^{-1}	0.56160×10^{-1}
7.8556	-0.32275×10^{-1}	0.46155×10^{-1}
8.3545	-0.33284×10^{-1}	0.38885×10^{-1}
8.8534	-0.32901×10^{-1}	0.33254×10^{-1}
9.8512	-0.30062×10^{-1}	0.24838×10^{-1}
10.849	-0.26094×10^{-1}	0.18691×10^{-1}
11.846	-0.21906×10^{-1}	0.14042×10^{-1}
12.844	-0.17894×10^{-1}	0.10559×10^{-1}
14.673	-0.11556×10^{-1}	0.63866×10^{-2}
16.669	-0.64440×10^{-2}	0.39495×10^{-2}
18.665	-0.30389×10^{-2}	0.13998×10^{-2}
20.660	-0.14818×10^{-2}	0.40470×10^{-3}

TABLE 2.7 (Continued)

r/a_o	V_s (Rydbergs)	V_t (Rydbergs)
22.656	-0.86830×10^{-3}	0.14314×10^{-3}
24.651	-0.51875×10^{-3}	0.41222×10^{-4}
26.647	-0.25466×10^{-3}	0.48612×10^{-5}
28.642	-0.10646×10^{-3}	-0.12455×10^{-4}
30.638	-0.51226×10^{-4}	-0.13318×10^{-4}
32.634	-0.28459×10^{-4}	-0.10281×10^{-4}
34.629	-0.14119×10^{-4}	-0.81605×10^{-5}
36.625	-0.94538×10^{-5}	-0.57601×10^{-5}
38.620	-0.72621×10^{-5}	-0.43206×10^{-5}
40.616	-0.50958×10^{-5}	-0.32857×10^{-5}
42.611	-0.31047×10^{-5}	-0.23530×10^{-5}
44.607	-0.16178×10^{-5}	-0.16197×10^{-5}
46.602	-0.10782×10^{-5}	-0.10782×10^{-5}
48.598	-0.72362×10^{-6}	-0.72362×10^{-6}
50.594	-0.55243×10^{-6}	-0.55243×10^{-6}
52.589	-0.52217×10^{-6}	-0.52217×10^{-6}
54.585	-0.41011×10^{-6}	-0.41011×10^{-6}
56.580	-0.32522×10^{-6}	-0.32522×10^{-6}
58.576	-0.26022×10^{-6}	-0.26022×10^{-6}
60.571	-0.20993×10^{-6}	-0.20993×10^{-6}
62.567	-0.17065×10^{-6}	-0.17065×10^{-6}
64.563	-0.13972×10^{-6}	-0.13972×10^{-6}

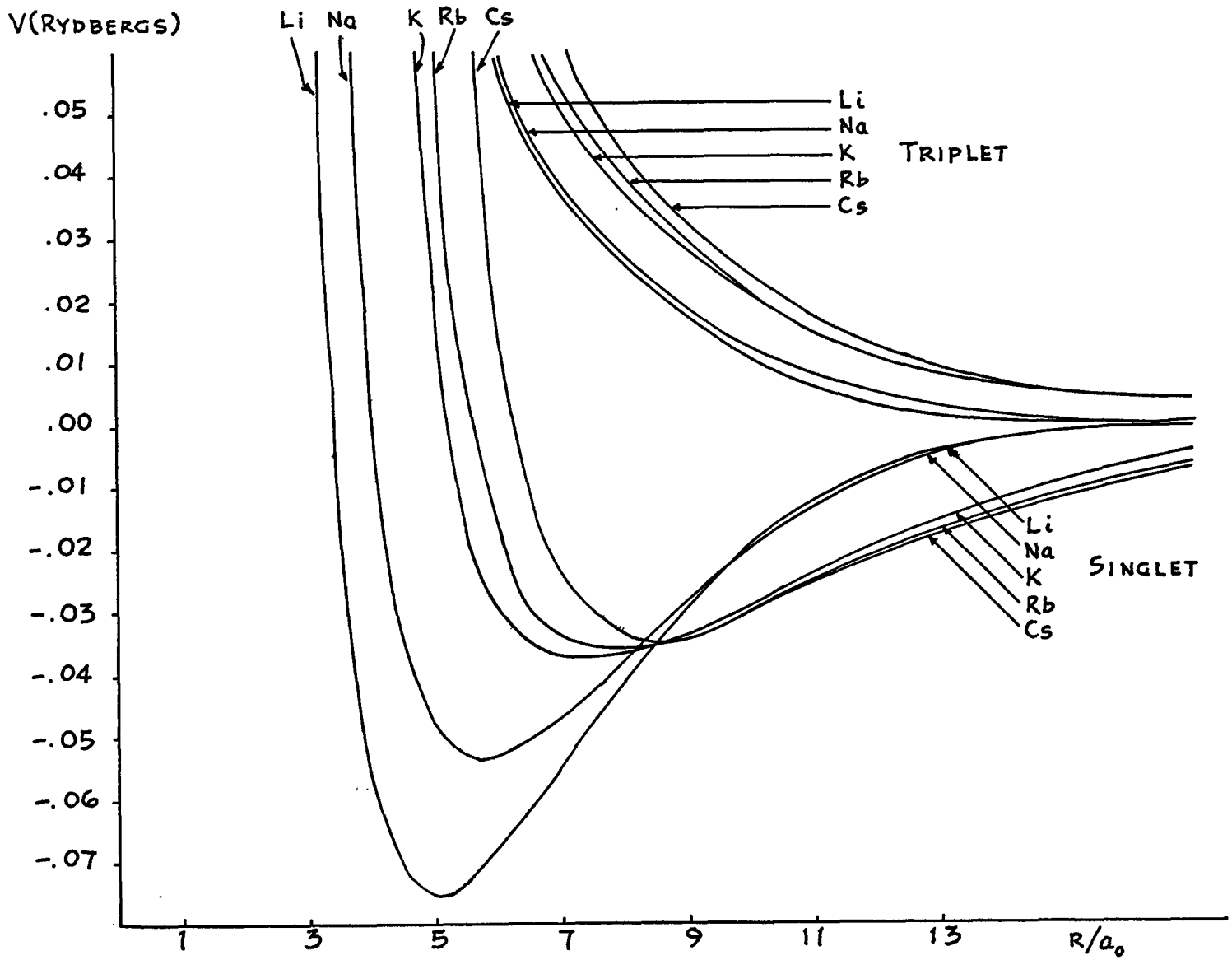

FIG. 2.1 INTERACTION POTENTIALS OF ALKALI-ATOMS

CHAPTER III

CALCULATION OF VIRIAL COEFFICIENT

In the preceding chapter, we have given a method to phenomenologically calculate the interaction of alkali-atoms. These interaction forces between atoms are intimately related to the equation of state and the transport properties of gases and liquids. The statistical mechanical theories which relate the bulk properties to the intermolecular forces are highly developed for dilute gases and developed to a lesser extent for dense gases and liquids. Once the law of force has been determined, it is possible to calculate the equation of state or transport properties. In this manner one reduces the description of a great number of equilibrium and non-equilibrium phenomena to a common basis and understands their interrelation from a molecular viewpoint.

In this chapter we concern ourselves with the virial equation of state, which is generally written in the form

$$p\tilde{V}/RT = 1 + B(T)/\tilde{V} + C(T)/\tilde{V}^2 + D(T)/\tilde{V}^3 + \cdots, \quad (3.1)$$

where $\tilde{V}$ is the volume of the system per mole. The coefficients B(T), C(T), D(T) . . . are called the second, third, fourth . . . virial coefficients. By means of statistical mechanics these virial

coefficients may be expressed in terms of the intermolecular potential functions. For example, if the potential V between two atoms in the gas is a function only of the interatomic separation r, the second coefficient is shown to be[33]

$$B(T) = -\frac{2\pi\tilde{N}}{3kT}\int_0^\infty r^3 \frac{dV}{dr} \exp(-V/kT)\,dr \,, \qquad (3.2)$$

where $\tilde{N}$ is the Avogadro's number and k is the Boltzmann constant.

Consider the low density monatomic molecular gases of alkali-atoms. Our potentials in Chapter II describe the interaction potentials between two single alkaline atoms. These potentials, singlet and triplet, are only functions of the nuclear separations, therefore eq. (3.2) may be applied. We have calculated individually the coefficients $B_s(T)$ for singlet potentials, and $B_t(T)$ for triplet potentials, and averaged with the probability ratio for occurrence of the two states, i.e.

$$\bar{B}(T) = \left[B_s(T) + 3\,B_t(T)\right]/4 \,. \qquad (3.3)$$

The temperature ranges for which we have carried out the calculations are from 100 ^{o}k to 10^4 ^{o}k. Our results are given in Table 3.1 and Figure 3-1. The second virial coefficients of these alkaline diatomic systems are not available in the earlier literatures. In fact, there is still no experimental determination of these coefficients.

TABLE 3.1

Second Virial Coefficients of Monatomic Alkali-Gases (In Mol. Volume)

T (^{o}k) sys	Hydrogen	Lithium	Sodium	Potassium	Rubidium	Cesium
300	-1.522×10^{35}	-2.912×10^{13}	-4.250×10^{8}	-2.582×10^{5}	-1.281×10^{5}	-3.265×10^{4}
310	-1.464×10^{35}	-8.153×10^{12}	-1.733×10^{8}	-1.382×10^{5}	-7.044×10^{4}	-1.887×10^{4}
320	-1.318×10^{35}	-2.473×10^{12}	-7.481×10^{7}	-7.701×10^{4}	-4.022×10^{4}	-1.130×10^{4}
330	-1.259×10^{35}	-8.070×10^{11}	-3.399×10^{7}	-4.449×10^{4}	-2.378×10^{4}	-6.979×10^{3}
340	-1.117×10^{35}	-2.814×10^{11}	-1.619×10^{7}	-2.656×10^{4}	-1.451×10^{4}	-4.440×10^{3}
350	-1.079×10^{35}	-1.043×10^{11}	-8.049×10^{6}	-1.634×10^{4}	-9.112×10^{3}	-2.900×10^{3}
360	-9.887×10^{34}	-4.083×10^{10}	-4.162×10^{6}	-1.033×10^{4}	-5.876×10^{3}	-1.941×10^{3}
370	-9.152×10^{34}	-1.683×10^{10}	-2.231×10^{6}	-6.704×10^{3}	-3.883×10^{3}	-1.328×10^{3}
380	-8.489×10^{34}	-7.272×10^{9}	-1.237×10^{6}	-4.452×10^{3}	-2.623×10^{3}	-9.278×10^{2}
390	-7.895×10^{34}	-3.282×10^{9}	-7.068×10^{5}	-3.020×10^{3}	-1.810×10^{3}	-6.605×10^{2}
400	-7.478×10^{34}	-1.542×10^{9}	-4.156×10^{5}	-2.091×10^{3}	-1.272×10^{3}	-4.786×10^{2}
410	-6.862×10^{34}	-7.516×10^{8}	-2.509×10^{5}	-1.474×10^{3}	-9.105×10^{2}	-3.524×10^{2}
420	-6.291×10^{34}	-3.794×10^{8}	-1.552×10^{5}	-1.057×10^{3}	-6.624×10^{2}	-2.635×10^{2}
430	-5.783×10^{34}	-1.977×10^{8}	-9.823×10^{4}	-7.703×10^{2}	-4.893×10^{2}	-1.997×10^{2}
440	-5.394×10^{34}	-1.062×10^{8}	-6.349×10^{4}	-5.697×10^{2}	-3.666×10^{2}	-1.534×10^{2}
450	-5.001×10^{34}	-5.865×10^{7}	-4.186×10^{4}	-4.272×10^{2}	-2.784×10^{2}	-1.193×10^{2}
460	-4.581×10^{34}	-3.325×10^{7}	-2.811×10^{4}	-3.246×10^{2}	-2.140×10^{2}	-9.380×10
470	-4.118×10^{34}	-1.932×10^{7}	-1.921×10^{4}	-2.496×10^{2}	-1.664×10^{2}	-7.455×10
480	-3.785×10^{34}	-1.148×10^{7}	-1.334×10^{4}	-1.941×10^{2}	-1.308×10^{2}	-5.985×10
490	-3.476×10^{34}	-6.973×10^{6}	-9.402×10^{3}	-1.526×10^{2}	-1.039×10^{2}	-4.849×10
500	-3.115×10^{34}	-4.322×10^{6}	-6.724×10^{3}	-1.211×10^{2}	-8.334×10	-3.964×10
510	-2.701×10^{34}	-2.730×10^{6}	-4.874×10^{3}	-9.707×10	-6.744×10	-3.267×10

TABLE 3.1 (Continued)

T_{sys} (^{o}k)	Hydrogen	Lithium	Sodium	Potassium	Rubidium	Cesium
520	-2.382×10^{34}	-1.755×10^{6}	-3.578×10^{3}	-7.848×10	-5.504×10	-2.714×10
530	-2.056×10^{34}	-1.148×10^{6}	-2.658×10^{3}	-6.399×10	-4.527×10	-2.271×10
540	-1.756×10^{34}	-7.629×10^{5}	-1.997×10^{3}	-5.259×10	-3.753×10	-1.914×10
550	-1.493×10^{34}	-5.147×10^{5}	-1.516×10^{3}	-4.354×10	-3.133×10	-1.623×10
560	-1.215×10^{34}	-3.522×10^{5}	-1.116×10^{3}	-3.630×10	-2.633×10	-1.385×10
570	-9.369×10^{33}	-2.443×10^{5}	-9.007×10^{2}	-3.047×10	-2.227×10	-1.189×10
580	-7.049×10^{33}	-1.717×10^{5}	-7.039×10^{2}	-2.574×10	-1.895×10	-1.026×10
590	-5.129×10^{33}	-1.221×10^{5}	-5.548×10^{2}	-2.187×10	-1.622×10	-8.902
600	-3.534×10^{33}	-8.785×10^{4}	-4.409×10^{2}	-1.869×10	-1.396×10	-7.762
610	-2.015×10^{33}	-6.390×10^{4}	-3.531×10^{2}	-1.606×10	-1.207×10	-6.800
620	-1.061×10^{33}	-4.697×10^{4}	-2.848×10^{2}	-1.387×10	-1.050×10	-5.984
630	-2.840×10^{32}	-3.488×10^{4}	-2.314×10^{2}	-1.204×10	-9.167	-5.288
640	-7.280×10^{31}	-2.614×10^{4}	-1.893×10^{2}	-1.050×10	-8.041	-4.693
650	-1.946×10^{31}	-1.977×10^{4}	-1.558×10^{2}	-9.195	-7.084	-4.180
660	-5.415×10^{30}	-1.508×10^{4}	-1.290×10^{2}	-8.087	-6.256	-3.737
670	-1.566×10^{30}	-1.160×10^{4}	-1.075×10^{2}	-7.142	-5.564	-3.353
680	-4.695×10^{29}	-8.994×10^{3}	-9.005×10	-6.331	-4.958	-3.018
690	-1.458×10^{29}	-7.026×10^{3}	-7.583×10	-5.633	-4.434	-2.726
700	-4.683×10^{28}	-5.528×10^{3}	-6.418×10	-5.030	-3.979	-2.469
750	-2.522×10^{26}	-1.839×10^{3}	-2.988×10	-2.993	-2.422	-1.568
800	-2.614×10^{24}	-7.040×10^{2}	-1.536×10	-1.906	-1.573	-1.056
850	-4.645×10^{22}	-3.027×10^{2}	-8.569	-1.283	-1.076	-7.456×10^{-1}
900	-1.294×10^{21}	-1.433×10^{2}	-5.114	-9.036×10^{-1}	-7.691×10^{-1}	-5.471×10^{-1}
950	-5.259×10^{19}	-7.362×10	-3.231	-6.607×10^{-1}	-5.694×10^{-1}	-4.144×10^{-1}
1000	-2.949×10^{18}	-4.051×10	-2.142	-4.986×10^{-1}	-4.343×10^{-1}	-3.224×10^{-1}

TABLE 3.1 (Continued)

P_{sys} T (^{o}k)	Hydrogen	Lithium	Sodium	Potassium	Rubidium	Cesium
1000	-2.949×10^{18}	-4.051×10	-2.142	-4.986×10^{-1}	-4.343×10^{-1}	-3.224×10^{-1}
1500	-3.658×10^{10}	-9.907×10^{-1}	-1.650×10^{-1}	-7.935×10^{-2}	-7.279×10^{-2}	-5.909×10^{-2}
2000	-4.284×10^{6}	-1.641×10^{-1}	-4.456×10^{-2}	-2.680×10^{-2}	-2.463×10^{-2}	-1.982×10^{-2}
2500	-1.938×10^{4}	-5.552×10^{-2}	-1.866×10^{-2}	-1.133×10^{-2}	-1.011×10^{-2}	-7.435×10^{-3}
3000	-5.429×10^{2}	-2.601×10^{-2}	-9.416×10^{-3}	-4.861×10^{-3}	-3.977×10^{-3}	-2.107×10^{-3}
3500	-4.303×10	-1.443×10^{-2}	-5.134×10^{-3}	-1.617×10^{-3}	-8.871×10^{-4}	$+5.911 \times 10^{-4}$
4000	-6.524	-8.795×10^{-3}	-2.825×10^{-3}	-1.961×10^{-4}	$+8.399 \times 10^{-4}$	$+2.095 \times 10^{-3}$
4500	-1.523	-5.656×10^{-3}	-1.451×10^{-3}	$+1.282 \times 10^{-3}$	$+1.870 \times 10^{-3}$	$+2.983 \times 10^{-3}$
5000	-4.808×10^{-1}	-3.738×10^{-3}	-5.762×10^{-4}	$+1.961 \times 10^{-3}$	$+2.510 \times 10^{-3}$	$+3.525 \times 10^{-3}$
5200	-3.235×10^{-1}	-3.176×10^{-3}	-3.155×10^{-4}	$+2.159 \times 10^{-3}$	$+2.695 \times 10^{-3}$	$+2.695 \times 10^{-3}$
5800	-1.169×10^{-1}	-1.934×10^{-3}	$+2.662 \times 10^{-4}$	$+2.583 \times 10^{-3}$	$+3.088 \times 10^{-3}$	$+3.088 \times 10^{-3}$
6400	-5.164×10^{-2}	-1.122×10^{-3}	$+6.470 \times 10^{-4}$	$+2.837 \times 10^{-3}$	$+3.318 \times 10^{-3}$	$+3.318 \times 10^{-3}$
7000	-2.643×10^{-2}	-5.668×10^{-4}	$+9.030 \times 10^{-4}$	$+2.987 \times 10^{-3}$	$+3.447 \times 10^{-3}$	$+3.447 \times 10^{-3}$
7600	-1.511×10^{-2}	-1.742×10^{-4}	$+1.079 \times 10^{-3}$	$+3.070 \times 10^{-3}$	$+3.514 \times 10^{-3}$	$+3.514 \times 10^{-3}$
8200	-9.407×10^{-3}	$+1.110 \times 10^{-4}$	$+1.201 \times 10^{-3}$	$+3.110 \times 10^{-3}$	$+3.540 \times 10^{-3}$	$+3.539 \times 10^{-3}$
8800	-6.258×10^{-3}	$+3.222 \times 10^{-4}$	$+1.287 \times 10^{-3}$	$+3.121 \times 10^{-3}$	$+3.538 \times 10^{-3}$	$+3.538 \times 10^{-3}$
9400	-4.388×10^{-3}	$+4.812 \times 10^{-4}$	$+1.345 \times 10^{-3}$	$+3.112 \times 10^{-3}$	$+3.519 \times 10^{-3}$	$+3.519 \times 10^{-3}$
10000	-3.209×10^{-3}	$+6.023 \times 10^{-4}$	$+1.385 \times 10^{-3}$	$+3.091 \times 10^{-3}$	$+3.487 \times 10^{-3}$	$+3.487 \times 10^{-3}$
10600	-2.428×10^{-3}	$+6.953 \times 10^{-4}$	$+1.411 \times 10^{-3}$	$+3.061 \times 10^{-3}$	$+3.448 \times 10^{-3}$	$+3.448 \times 10^{-3}$

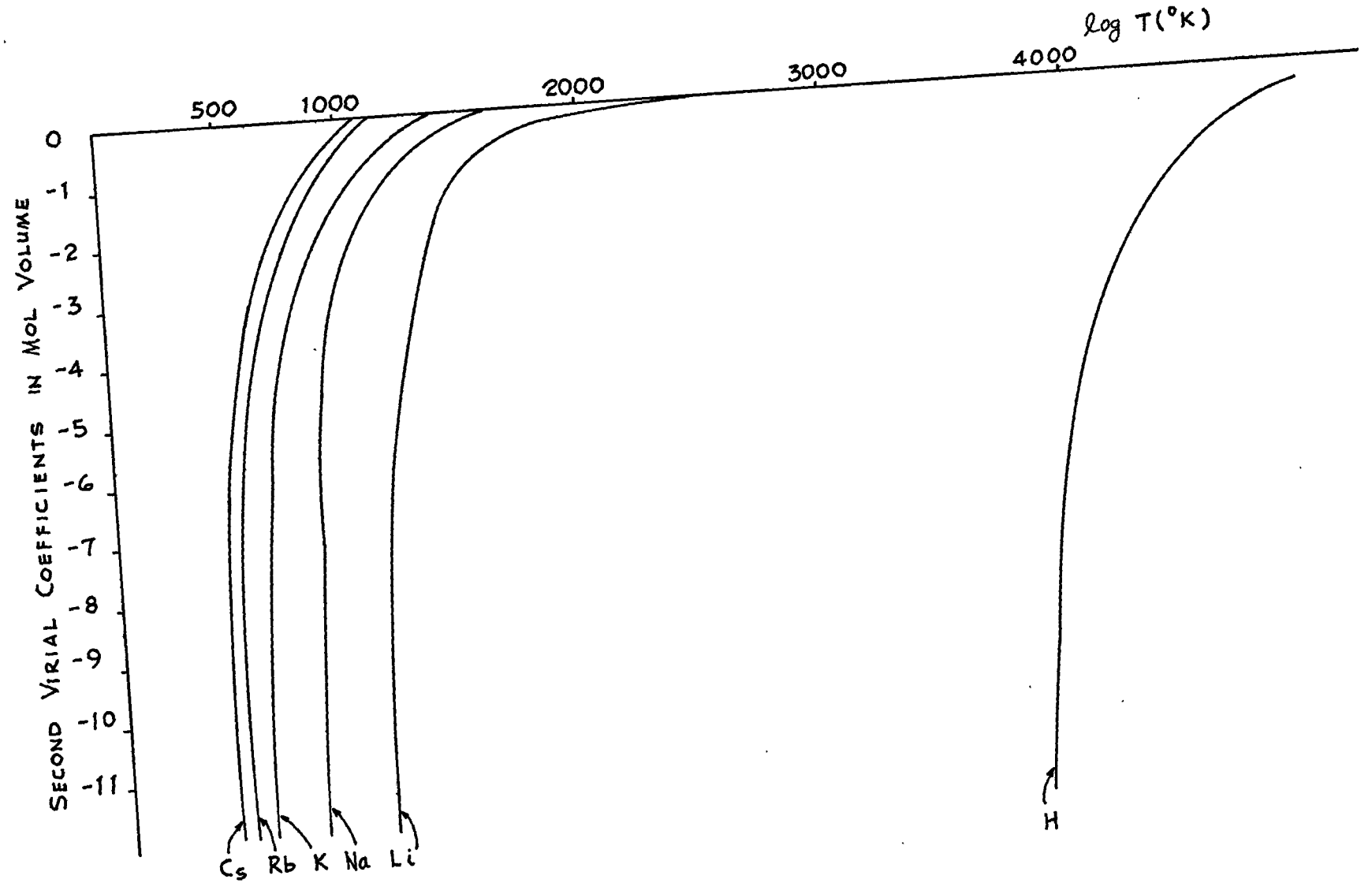

FIG. 3·1 SECOND VIRIAL COEFFICIENTS OF MONATOMIC ALKALI-GASES

CHAPTER IV

SEMI-CLASSICAL THEORY OF SPIN-DEPENDENT CROSS SECTIONS

In Chapter II we have investigated the interaction between alkali atoms, and have obtained their interaction potentials. We now use these potentials to calculate the spin-exchange cross sections by a semi-classical approach. The following section introduces the basic theory of spin-dependent scattering.

1. Basic Theory

Let the origin of co-ordinates be located at the center of mass of the colliding atoms. The Schrodinger equation for the complete system in atomic units is

$$\left(H - \frac{1}{2M}\nabla_r^2\right) X(\vec{x}, \vec{r}) = E\, X(\vec{x}, \vec{r}), \qquad (4.1)$$

where $\vec{x}$ represents the position vectors of the electrons, $\vec{r}$ is the vector joining the nuclei, H is the Hamiltonian for the system when the nuclei are held fixed, M is the reduced mass, and E is the total energy. If the velocities of the nuclei are negligible in comparison with the velocities of the electrons, then the Born-Oppenheimer[9] approximation may be applied. This approximation implies

$$H\psi_n(\vec{x},\vec{r}) = V_n(r)\psi_n(\vec{x},\vec{r}), \qquad (4.2)$$

where ψ_n are the eigenfunctions and $V_n(r)$ the eigenvalues of H. At the same time, the motion of the nuclei may be described by the wave equation

$$\left[-\frac{\nabla_r^2}{2M} + V_n(r)\right] U_n(\vec{r}) = E\,U_n(\vec{r}), \qquad (4.3)$$

where $V_n(r)$ is called the interaction potential of the two atoms, and E is the total energy. In the process under consideration, the two atoms may approach each other adiabatically along more than one possible interaction curve.

For slow collisions the refinements discussed by Bates and McCarroll[61] eventually reduce to the assumption that eq. (4.1) may be solved by a substitution of the form

$$\chi(\vec{x},\vec{r}) = F_s(\vec{r})\psi_s(\vec{x},\vec{r}) + F_t(\vec{r})\psi_t(\vec{x},\vec{r}), \qquad (4.4)$$

where ψ_s corresponds to the singlet state of the molecule and ψ_t to the triplet state, coupling to other states being ignored. Using eq. (4.2) it follows that

$$\left(\frac{\nabla^2}{2M}+E-V_s(r)\right)F_s(\vec{r})+\frac{1}{M}\int\psi_t^*\nabla\psi_s\,d\vec{x}\cdot\nabla F_s+$$

$$\frac{1}{2M}\int\psi_t^*\nabla^2\psi_s\,d\vec{x}\,F_s+\frac{1}{M}\int\psi_t^*\nabla\psi_t\,d\vec{x}\cdot\nabla F_t+\int\frac{1}{M}\psi_t^*\nabla^2\psi_t\,d\vec{x}F_t=0. \quad (4.5)$$

A similar equation in which s and t are interchanged holds for $F_t(\vec{r})$. The coupling terms in eq. (4.5) will usually be small[62,63], so that to a good approximation Fs satisfies

$$\left(\frac{\nabla^2}{2M}+E-V_s(r)\right)F_s(\vec{r})=0\ , \quad (4.6)$$

and F_t satisfies

$$\left(\frac{\nabla^2}{2M}+E-V_t(r)\right)F_t(\vec{r})=0\ . \quad (4.7)$$

We are ignoring the small change in relative velocity that occurs in a spin-exchange process so that we require solutions of (4.6) and (4.7) which behave asymptotically as

$$F_{s,t}(\vec{r})\sim e^{ikz}+e^{ikr}r^{-1}f_{s,t}(\theta,\phi)\ , \quad (4.8)$$

where (r,θ,ϕ) are the spherical polar co-ordinates of $\vec{r}$. Equations (4.6) and (4.7) may be solved in the conventional way[64] giving the amplitudes $f_{s,t}$, as

$$f_{s,t}(\theta,\phi)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left\{\exp(2i\delta_l^{s,t})-1\right\}P_l(\cos\theta)\ , \quad (4.9)$$

where $\delta_\ell^{s,t}$ are phase shifts and $k=\sqrt{2ME}$. The phase shifts may be obtained by solving the following radial wave equation

$$\frac{d^2R_\ell^{s,t}}{dr^2}+\left\{k^2-2MV_{s,t}-\frac{\ell(\ell+1)}{r^2}\right\}R_\ell^{s,t}=0 \quad , \qquad (4.10)$$

The solution of eq. (4.10) behaves asymptotically as

$$R_\ell^{s,t}\sim k^{-1}\sin\left(kr-\tfrac{1}{2}\ell\pi+\delta_\ell^{s,t}\right) . \qquad 4.11)$$

All scattering cross sections may be expressed in terms of phase shifts $\delta_\ell^{s,t}$. Therefore the first step towards calculating the spin-exchange cross sections is solving the wave equation (4.10).

2. Semi-classical Approach of Cross Sections

It is well known that we still lack a method to obtain the exact solution of the wave equation (4.10) with an arbitrary potential V. Some reasonable approximation methods must be employed. Using the WKB approximation, one may obtain the solution of eq. (4.10) as[42]

$$R_\ell^{s,t}\simeq\frac{1}{P^{1/2}}\cos\left[\int_{r_0}^{r}P\,dr-\frac{\pi}{4}\right] , \qquad (4.12)$$

where

$$P^2=k^2-2M\bar{V}_{s,t}-(\ell+\tfrac{1}{2})^2/r^2>0 \quad , \qquad (4.13)$$

and r_o is the distance of "closest approach" or the classical "turning point of the particle, i.e. the value of r such that

$$P(r_o) = 0 \tag{4.14}$$

Substituting eq. (4.13) into (4.12) and putting $V_{s,t} = 0$, we obtained the solution for a free particle system as

$$R_{\ell}^{s,t}_{\text{(free Partical)}} = \frac{1}{p^{1/2}} \cos\left[\int_{r_o}^{r} \sqrt{k^2-(\ell+\tfrac{1}{2})^2/r^2}\, dr - \frac{\pi}{4}\right]. \tag{4.15}$$

Then the phase shifts may be obtained by matching the difference between the two phase angles of $R_{\ell}^{s,t}{}_{r\to\infty}$ and $R_{\ell}^{s,t}$ (free particle); i.e.

$$\delta_{\ell}^{s,t} = \lim_{r\to\infty}\left[\int_{r_o}^{r} P\,dr - \int_{r_o}^{r} \sqrt{k^2-(\ell+\tfrac{1}{2})^2/r^2}\, dr\right]. \tag{4.16}$$

Substituting eq. (4.13) into (4.16) yields

$$\delta_{\ell}^{s,t} = \lim_{r\to\infty}\left\{\int_{r_o}^{r}\left[k^2-(\ell+\tfrac{1}{2})^2/r^2\right]^{1/2}\left[1-2MV_{s,t}/(k^2-(\ell+\tfrac{1}{2})^2/r^2)\right]^{1/2} dr\right.$$

$$\left. -\int_{r_o}^{r}\left[k^2-(\ell+\tfrac{1}{2})^2/r^2\right]^{1/2} dr\right\} \tag{4.17}$$

$$= \lim_{r\to\infty} \left\{ \int_{r_0}^{r} \left[k^2 - (l+\tfrac{1}{2})^2/r^2\right]^{1/2} \left[1 - MV_{s,t}/(k^2-(l+\tfrac{1}{2})^2/r^2) + \right.\right.$$

$$\left. + \tfrac{1}{2}(MV_{s,t})^2/(k^2-(l+\tfrac{1}{2})^2/r^2)^2 - \cdots\cdots\right] dr$$

$$\left. - \int_{r_0}^{r} \left[k^2 - (l+\tfrac{1}{2})^2/r^2\right]^{1/2} dr \right\}. \qquad (4.18)$$

If we assume that k and l are large, so that

$$\left| 2MV_{s,t} \right| / \left| \sqrt{k^2-(l+\tfrac{1}{2})^2/r^2} \right|^2 \ll 1 \ , \qquad (4.19)$$

for any r except in the neighborhood of r_0, then we may neglect all the terms which have a large power of $\left| MV_{s,t} \right| / \left| \sqrt{k^2-(l+\tfrac{1}{2})^2/r^2} \right|^2$, and get

$$\delta_l^{s,t} = -\int_{r_0}^{\infty} \left[MV_{s,t} / \sqrt{k^2-(l+\tfrac{1}{2})^2/r^2} \right] dr$$

$$= -\frac{M}{k} \int_{r_0}^{\infty} \left[V_{s,t} / \sqrt{r^2-(l+\tfrac{1}{2})^2/k^2} \right] r\,dr \ . \qquad (4.20)$$

Putting $r = r_0$ into eq. (4.13), we obtain, from eq. (4.14),

$$P^2(r_0) = k^2 - 2M V_{s,t} - (l+\tfrac{1}{2})^2/r_0^2 = 0 \qquad (4.21)$$

Using the condition (4.19) we may neglect $2MV_{s,t}$ in eq. (4.21) which then yields

$$(l+\tfrac{1}{2}) = k\, r_0 \qquad (4.22)$$

Substituting eq. (4.22) into eq. (4.20) gives the phase shift in the WKB approximation, namely

$$\delta_l^{s,t} = -\frac{M}{k}\int_{r_0}^{\infty}\left[V_{s,t}/\sqrt{1 - r_0^2/r^2}\right] dr. \qquad (4.23)$$

Using density metric techniques, the spin-exchange cross section σ_{NX} for nonidentical atoms may be expressed in terms of phase shifts[22,23] as

$$\sigma_{NX} = \frac{\pi}{2k^2}\sum_{l}^{\infty}(2l+1)\sin^2(\delta_{l,t} - \delta_{l,s}), \qquad (4.24)$$

where $\delta_{l,t}$ and $\delta_{l,s}$ are the phase shifts of singlet and triplet states respectively and which may be obtained from eq. (4.23). Substituting (4.23) into (4.24) yields

$$\sigma_{NX} = \frac{\pi}{2k^2}\sum_{l}(2l+1)\sin^2\left[\frac{M}{k}\int_{r_0}^{\infty}(V_t - V_s)/\sqrt{1 - r_0^2/r^2}\,dr\right]. \qquad (4.25)$$

For the semi-classical approach, l and k are assumed to be large and can be treated as continuous. Then a summation of infinite terms approaches an integration. From eq. (4.22) we obtain

$$\sum_{l}^{\infty} (2l+1) \longrightarrow \int_{0}^{\infty} (2 k r_0) d(k r_0) \quad . \tag{4.26}$$

Substituting eq. (4.26) into eq. (4.25) yields

$$\sigma_{NX} = \pi \int_{0}^{\infty} r_0 \sin^2 \left[\frac{M}{k} \int_{r_0}^{\infty} (V_t - V_s) / \sqrt{1 - r_0^2/r^2} \, dr \right] dr_0 \quad . \tag{4.27}$$

The above equation is a semi-classical approximation of the spin-exchange cross section for nonidentical atoms.

The singlet and triplet interaction potentials, $V_{s,t}$ of the diatomic alkaline systems may be obtained from Tables 2.2, and 2.4-2.8. The spin-exchange cross sections obtained from eq. (4.27) are shown in Table 4.1. The spin-exchange cross sections are calculated for twenty-eight different relative kinetic energies for each system. The results compare fairly well with the experimental ones[15-18] except for the H-H system, which suggests that the semi-classical method is not valid for this system.

TABLE 4.1. Semi-Classical Spin-Exchange Cross Sections of Alkali-Atoms (πa_o^2)

sys E (^oK)	Hydrogen	Lithium	Sodium	Potassium	Rubidium	Cesium
10	14.69	144.1	171.2	391.9	382.7	426.8
20	14.63	140.3	176.0	320.2	381.0	410.1
30	13.62	139.6	144.1	327.8	370.2	394.2
40	14.38	136.6	137.0	329.8	330.9	370.5
50	13.96	128.2	140.7	300.3	347.6	403.8
60	14.05	135.6	152.5	315.9	327.3	336.8
70	13.49	130.5	149.4	287.7	322.0	345.8
80	12.66	122.2	151.7	270.0	308.8	342.6
90	13.04	134.5	145.8	315.7	325.8	327.4
100	12.32	116.2	138.3	296.5	299.6	291.4
200	12.29	121.9	138.6	272.9	270.8	312.5
300	11.08	114.3	138.8	276.3	282.9	334.3
400	11.84	108.2	121.6	279.1	299.3	302.3
500	10.85	101.0	128.8	258.7	278.5	264.7
600	10.58	94.32	118.8	274.1	272.2	271.3
700	10.11	107.8	119.0	245.5	237.7	297.0
800	9.836	97.34	116.5	240.2	270.3	287.6
900	10.13	96.36	120.0	232.3	256.9	271.3
1000	10.83	101.8	118.3	242.8	251.3	293.8
2000	9.452	95.54	108.7	213.3	236.8	239.0
3000	9.173	87.13	105.8	190.8	218.5	230.3
4000	9.682	85.98	106.5	208.8	212.8	240.2
5000	8.560	91.81	106.4	201.4	224.8	226.1
6000	8.543	88.82	97.98	203.8	203.4	231.8

TABLE 4.1 (Continued)

sys E (^{o}k)	Hydrogen	Lithium	Sodium	Potassium	Rubidium	Cesium
7000	8.600	82.72	106.5	194.5	216.2	224.7
8000	8.735	81.02	90.32	199.3	211.8	206.9
9000	8.418	87.52	103.1	191.9	213.5	229.1
10000	8.391	79.12	96.76	192.9	200.0	230.4

CHAPTER V

PARTIAL WAVE ANALYSIS OF SPIN-DEPENDENT CROSS SECTIONS

In the previous chapter we have calculated the spin-exchange cross sections using a semi-classical approximation method. One of the central approximations of this method is replacing a summation by integration. This is perhaps valid for the heavier atoms at higher relative kinetic energies. But the main disadvantage of this method for heavier atoms is the difficulty of performing the integration. Further, the relevant interaction potentials in the region of small nuclear separations, $r < r_c$, are unknown. As shown in Tables 2.4-2.8, the smallest values for which the potentials are known are about one atomic unit. Therefore, to start the integration of eq. (4.27) from r_o smaller than this value is not feasible. However, eq. (4.27) may be rewritten as

$$\sigma_{NX} = \int_0^{r_c} [\qquad] dr_o + \int_{r_c}^{\infty} [\qquad] dr_o \,. \tag{5.1}$$

Hence, due to the lack of knowledge of the potential in the region $0 < r \leq r_c$ the integration of the first term can not be performed. Now let us change the first term back to its original form, i.e.

$$\int_0^{r_o} [\qquad] dr_o \longrightarrow \sum_{l=0}^{l_c} [\qquad] \equiv \sigma_L \,, \tag{5.2}$$

where

$$\ell_c = k r_c + \frac{1}{2} \quad .$$

The value of ℓ_c can be calculated and corresponds to a large value for large k. For example, the K-K system at relative energy corresponding to the temperature of 10^4 ^{o}k we have $\ell_c \simeq 130$. Therefore, the cross section, σ_L, due to the partial waves of $\ell = 0$ to $\ell = \ell_c = 130$ cannot be obtained. This is the disadvantage of the semi-classical method at high relative energies depicting heavy atoms. Furthermore, at low relative energies for light atoms, i.e. small values of k, the disadvantage of the semi-classical method is more acute than that of heavy atoms at high relative energies, because the deviation of r_o values from continuity is larger in this case. Obviously, therefore, the semi-classical method is not entirely justified, and consequently a partial wave analysis involving numerical integration method will be employed.

1. Numerical Integration

The equation to be solved now is

$$\frac{d^2 R_\ell}{d r^2} = \left[\frac{\ell(\ell+1)}{r^2} - k^2 + 2M V \right] R_\ell \quad . \tag{5.3}$$

Let

$$y = \frac{dR}{dr} \quad , \tag{5.4}$$

and this leaves

$$\frac{dy}{dr} = \left[\frac{l(l+1)}{r^2} - k^2 + 2MV\right]R. \qquad (5.5)$$

In this form we have two simultaneous equations of first order. These equations may be solved by the so called Runge-Kutta technique[51], provided initial conditions are known. The initial conditions chosen as $r_0 = r_c$, $R_0 = 0$, and y_0 = arbitrary value, where r_c is the closest nuclear separation at which the potential is known. This corresponds to replacing the unknown portion of the potential by a square cell of infinite height. Then the phase shifts may be obtained by matching the phase difference at a point for which the potential V approaches zero. These calculations were carried out on a digit computer. Once the phase shifts are obtained, several different kinds of spin-dependent cross sections may be calculated from the formulae predicted by the theory of spin-dependent scattering. With the exception of hyperfine transitions for identical atoms, these formulae are available in the current literature. In the appendix we discuss these various cross sections, we merely state the results here.

(A) Non-idential atoms:

(1) Total unpolarized cross section[21-23],

$$\sigma_{NU} = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\left[\frac{1}{4}\sin^2\delta_l^{s} + \frac{3}{4}\sin\delta_l^{t}\right], \qquad (5\text{-}6)$$

(2) Spin-exchange cross section,[21-23]

$$\sigma_{NX}=\frac{\pi}{2k^2}\sum_{\ell}^{\infty}(2\ell+1)\sin^2(\delta_\ell^{\,t}-\delta_\ell^{\,s}) \quad , \tag{5.7}$$

(3) Hyperfine spin-transition cross section[21-23] ($f=i+\frac{1}{2}\rightarrow f'=i-\frac{1}{2}$),

$$\sigma_{Ny}=\frac{2i}{2i+1}\sigma_{NX} \quad , \tag{5.8}$$

(B) Identical atoms:

(1) Total cross section,[21-23]

$$\sigma_{IU}=\frac{4\pi}{k^2}\sum_{\ell}^{\infty}(2\ell+1)\left\{\left[2-\frac{2}{2i+1}(-1)^{2i+1+\ell}\right]\left(\frac{1}{4}\right)\sin^2\delta_\ell^{\,s}+\right.$$

$$\left.+\left[2+\frac{2}{2i+1}(-1)^{2i+1+\ell}\right]\left(\frac{3}{4}\right)\sin^2\delta_\ell^{\,t}\right\}, \tag{5.9}$$

(2) Spin-exchange cross section,[21-23]

$$\sigma_{Ix}=\frac{\pi}{2k^2}\sum_{\ell}^{\infty}(2\ell+1)\left\{\left[2-\frac{2}{2i+1}(-1)^{2i+1+\ell}\right]\sin^2\delta_\ell^{\,s}+\right.$$

$$\left.+\left[2+\frac{2}{2i+1}(-1)^{2i+1+\ell}\right]\sin^2\delta_\ell^{\,t}\right\}, \tag{5.10}$$

(3) Hyperfine spin-transition cross section ($f = i+\frac{1}{2} \rightarrow f' = i-\frac{1}{2}$)

(See Appendix)

$$\sigma_{Iy} = \frac{4i}{2i+1}\left\{\frac{1}{2}\sigma_{NX} + \left[\left(\frac{1}{2i+1}\right)^2 - \frac{1}{2(2i+1)}\right]\sigma_T + \frac{\sigma_{NU}}{2(2i+1)} + \right.$$

$$\left. + \frac{\pi}{2k^2(2i+1)}\sum_{\ell}^{\infty}(2\ell+1)\left[(-1)^{\ell}\sin^2\delta_{\ell}^{t} - (-1)^{\ell}\sin^2\delta_{\ell}^{s}\right]\right\}, \quad (5.11)$$

where

$$C_T = -\frac{1}{2}\sigma_{NX} + \frac{\pi}{2k^2}\sum_{\ell}^{\infty}(2\ell+1)\left(\sin^2\delta_{\ell}^{s} + \sin^2\delta_{\ell}^{t}\right). \quad (5.12)$$

We have calculated these six scattering cross sections for 124 different relative energies for the alkaline systems. The results are listed in Tables 5.1-5.6 and are plotted in Figures 5.1-5.6. Comparing the cross sections calculated by semi-classical method, Table 4.1, and by partial wave analysis, Tables 5.1-5.6, shows that all of the results obtained by semi-classical arguments are too small by about 10% for the heavy atoms and by as much as 50% for the lighter ones.

2. Averaged Spin-dependent Cross Sections

In the preceding section we have calculated the spin-dependent cross sections as functions of relative kinetic energy E corresponding to which we define a temperature T by E = 3/2 kT.

Now we consider the energy distribution of the atoms in a system of temperature T_o as Maxwellian distribution, i.e.

$$dN_E = \frac{2N}{\sqrt{\pi}}(kT_o)^{-3/2} E^{1/2} \exp(-E/kT_o)\, dE, \quad (5.13)$$

where N is the total number of atoms in the system. Then the averaged spin-dependent cross section will be

$$\bar{\sigma}(T_0) = \left(\int_0^\infty \sigma \, dN_E\right) \Big/ \left(\int_0^\infty dN_E\right)$$

$$= \left(\int_0^\infty \sigma T^{1/2} \exp\left(-\frac{3T}{2T_0}\right) dT\right) \Big/ \left(\int_0^\infty T^{1/2} \exp\left(-\frac{3T}{2T_0}\right) dT\right) \quad (5.14)$$

The exact value of the last integration on the right side of eq. (5.14) may be found from mathematical tables. It is

$$\int_0^\infty T^{1/2} \exp\left(-\frac{3T}{2T_0}\right) dT = \left(2T_0^3 \pi / 3\right)^{1/2} \quad (5.15)$$

This exact value may be used to check the accuracy of $\bar{\sigma}(T_0)$ computed by a digit computer. Let

$$A = \left[\left(2T_0^3 \pi / 3\right)^{1/2} - \int_0^{\infty\ (\text{computer})} T^{1/2} \exp\left(-\frac{3T}{2T_0}\right) dT\right] \Big/ \left(2T^3 \pi / 3\right)^{1/2},$$

the smallness of which gives an indication of the accuracy of the numerical integration. Averaged cross sections are shownin Tables 5.7-5.12. For the temperature range from 50°K to 2000°K, the values are small. We then expect the averaged cross sections to be most accurate in this region. The results are given in Tables 5.7-5.12 and plotted in Figures 5.7-5.12. A comparison of these results with experimental results is shown in Table 5.13. We see that quite good qualitative agreement has been obtained. Here, the experimental value for the hydrogen system was calculated by Mazo[15] using the EPR linewidth data of Hildebrandt, Booth, and Barth[4]. The experimental errors are not shown here. The spin-flip cross section defined by Mazo is a factor of two larger than the spin-exchange cross section

used in our calculation. The experimental value of the Sodium system was calculated by Anderson and Ramsey[16] using the measured line shapes of the various radio-frequency Zeeman transitions. The experimental values for the Rubilium and Cesium systems were measured by Moos and Sands[17] using paramagnetic resonance techniques to examine the Zeeman transitions. Unfortunately,we still lack knowledge of the experimental values for Lithium and Potassium systems. Our calculated values in Table 5.13 were obtained from the second column of Tables 5.7-5.12, corresponding to the spin-exchange cross sections of nonidential alkali atoms. The values in the fifth column of Tables 5.7-5.12 are the corresponding values for identical atoms, which seem to be too large in comparrision with the experimental values. This indicats that our treatment of the effects due to identical atoms is inadequate. In fact, the interaction between the two nuclear spins of the two colliding atoms is not strong enough for the exchange effect to be taken into account. Ho wever, if the effects due to identical atoms are to be considered, the exchange effect due to the inner closed electronic shells must also be considered. It would appear that this leads to a cancellation.

TABLE 5.13

Spin-Exchange Cross Sections (10^{-14} cm^2)

Systems	H^1-H^1	Li^3-Li^3	Na^{11}-Na^{11}	K^{19}-K^{19}	Rb^{85}-Rb^{85}	Cs^{133}-Cs^{133}
Temp.	325°K	405°K	405°K	540°K	540°K	540°K
Our Cal.	0.130	1.018	1.207	2.374	2.565	2.706
Exp.	0.141	----	1 - 3	~~2.2±4~~	2.4±0.4	2.6±0.5
Ref.	15		16		17	17

TABLE 5.1*

Spin-Dependent Cross Sections of Hydrogen Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f=1 \rightarrow f'=0$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{IY} Hyperfine transition cross sections of identical atom from $f=1 \rightarrow f'=0$

E(^{o}k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
10	54.80	14.24	7.120	134.0	36.48	22.50
11	55.88	14.54	7.270	138.8	38.28	23.41
12	56.97	14.87	7.435	142.1	39.78	24.17
13	57.08	14.52	7.263	144.3	41.10	24.81
14	59.56	15.70	7.851	145.7	42.28	25.36
15	61.37	16.31	8.156	146.9	43.52	25.90
16	63.95	17.22	8.610	148.2	44.96	26.50
17	67.89	18.70	9.352	150.5	46.99	27.34
18	74.16	21.24	10.620	155.0	50.09	28.65
19	83.60	25.38	12.690	162.5	54.71	30.73
20	95.46	31.06	15.530	172.3	60.50	33.55
21	105.18	36.32	18.160	180.7	65.17	36.14
22	108.44	38.84	19.420	181.3	66.59	37.34
23	106.30	38.64	19.320	177.4	65.31	37.18
24	102.26	37.20	18.600	170.4	62.25	36.12
25	98.34	35.56	17.780	166.1	60.87	35.49
26	95.17	34.10	17.050	161.2	59.08	34.69
27	92.76	32.90	16.450	157.8	57.68	34.03
28	90.94	31.92	15.960	154.9	56.58	33.47
29	89.59	31.12	15.560	152.5	55.74	33.02
30	88.59	30.46	15.230	150.8	55.11	32.65
32	87.30	29.42	14.710	148.5	54.28	32.09
34	86.62	28.62	14.310	147.4	53.90	31.72

* This table is calculated using Walker and Mays'[23] results of scattering amplitudes.

TABLE 5.1 (Continued)

E(°k)	σ_{NU}	σ_{Nx}	σ_{Ny}	σ_{Iu}	σ_{Ix}	σ_{Iy}
36	86.33	27.98	13.990	147.3	53.85	31.50
38	86.31	27.46	13.730	148.2	54.13	31.42
40	86.44	27.00	13.500	149.7	54.66	31.46
42	86.71	26.62	13.310	151.9	55.43	31.62
44	87.10	26.32	13.160	154.6	56.41	31.89
46	87.53	26.08	13.040	157.7	57.52	32.25
48	87.94	25.88	12.940	160.9	58.70	32.66
50	88.29	25.72	12.860	164.1	59.82	33.07
52	88.54	25.58	12.790	167.0	60.82	33.46
54	88.58	25.40	12.700	169.6	61.63	33.78
56	88.47	25.22	12.610	171.6	62.18	34.01
58	88.16	25.00	12.500	173.1	62.51	34.16
60	87.66	24.74	12.370	174.1	62.58	34.20
62	86.99	24.44	12.220	174.6	62.40	34.15
64	86.20	24.12	12.060	174.6	62.02	34.01
66	85.29	23.78	11.890	174.2	61.49	33.81
68	84.27	23.40	11.700	173.5	60.79	33.54
70	83.19	23.02	11.510	172.5	59.97	33.22
75	80.27	22.02	11.010	169.0	57.56	32.26
80	77.21	21.02	10.510	164.3	54.79	31.15
85	74.18	20.06	10.030	159.0	51.85	29.97
90	71.30	19.16	9.582	153.2	48.89	28.77
95	68.84	18.43	9.216	147.4	46.06	27.61
100	67.34	18.08	9.040	142.2	43.65	26.61
110	73.73	22.16	11.080	140.7	43.58	27.03
120	63.70	19.66	9.830	123.0	35.56	24.27
130	55.08	15.93	7.965	107.7	28.72	21.15
140	57.35	15.20	7.601	117.1	27.96	21.89
150	49.16	13.08	6.543	93.2	22.71	18.20
160	48.47	12.38	6.192	91.2	22.03	17.68
170	48.89	12.00	6.001	92.3	22.74	17.74
180	50.28	11.92	5.962	96.5	24.75	18.35
190	52.41	12.11	6.055	102.9	27.75	19.41
200	54.87	12.44	6.223	110.3	31.21	20.68
220	60.00	13.08	6.543	125.7	38.13	23.17
240	71.44	16.96	8.481	159.3	54.22	30.28
260	65.78	13.14	6.570	140.9	44.51	24.92
280	66.53	11.95	5.976	141.0	44.26	24.00
300	67.63	11.02	5.512	141.5	44.52	23.38
320	68.73	10.68	5.340	141.5	44.75	23.03
340	69.59	10.65	5.329	140.3	44.49	22.66
360	70.38	11.02	5.514	138.3	43.94	22.35
380	70.59	11.58	5.790	135.3	42.89	22.03
400	69.58	11.84	5.924	131.2	41.23	21.58

TABLE 5.1 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
420	67.49	11.71	5.858	126.3	39.07	20.93
440	65.41	11.57	5.785	121.8	37.01	20.34
460	64.59	12.02	6.010	119.1	35.74	20.11
480	64.96	13.31	6.657	118.2	35.26	20.41
500	60.78	12.44	6.220	113.3	32.69	19.72
550	53.69	10.20	5.101	106.4	28.78	18.37
600	54.81	10.98	5.494	110.3	30.21	19.16
650	53.10	10.19	5.097	112.8	31.23	19.66
700	55.28	10.29	5.148	118.4	34.13	20.57
750	57.88	10.36	5.180	122.7	36.68	21.19
800	62.16	11.20	5.601	129.4	40.51	22.41
850	62.87	11.09	5.547	132.1	42.28	23.02
900	62.42	10.25	5.127	129.1	41.10	22.08
950	61.89	9.73	4.866	126.2	39.77	21.25
1000	62.21	10.00	5.004	126.6	39.93	21.35
1100	63.24	12.12	6.061	124.6	38.61	21.40
1200	54.42	9.19	4.597	108.3	30.21	17.87
1300	54.25	10.25	5.125	107.3	29.78	18.15
1400	54.42	11.12	5.562	104.0	28.50	17.84
1500	50.84	9.14	4.574	96.5	25.28	16.08
1600	54.07	9.98	4.992	102.3	28.51	17.25
1700	57.32	10.85	5.426	110.7	32.82	18.98
1800	57.66	10.52	5.260	112.0	33.35	18.94
1900	56.69	9.33	4.668	111.8	33.06	18.40
2000	57.40	9.62	4.814	115.1	34.54	19.07
2200	55.96	9.85	4.925	116.0	35.21	19.80
2400	52.39	9.31	4.657	106.2	30.89	18.21
2600	52.65	10.35	5.175	108.9	32.50	19.41
2800	50.64	9.33	4.669	104.1	29.95	18.08
3000	52.34	9.74	4.871	108.1	31.74	18.70
3500	54.80	9.61	4.807	108.4	32.47	18.28
4500	50.31	9.50	4.752	99.3	28.04	16.89
5000	50.49	9.68	4.840	101.1	29.44	17.52
5500	50.68	8.71	4.357	104.8	30.87	17.84
6000	52.05	9.24	4.621	105.9	32.36	18.14
6500	51.58	9.51	4.755	103.5	31.21	17.71
7000	48.54	8.73	4.369	95.1	27.15	15.95
7500	48.26	9.29	4.645	94.5	27.12	16.18
8000	46.65	8.75	4.379	92.8	26.57	15.95
8500	47.82	8.95	4.475	97.0	28.92	16.83
9000	48.69	9.37	4.688	98.96	30.04	17.30
9500	48.39	8.67	4.337	96.86	29.10	16.47
10000	48.85	9.12	4.560	96.69	29.12	16.49

TABLE 5.2

Spin-Dependent Cross Sections of Lithium Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f=2 \rightarrow f'=1$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{IY} Hyperfine transition cross sections of identical atom from $f=2 \rightarrow f'=1$

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
10	1950.4	251.5	188.6	3899.7	873.8	522.1
11	1964.8	227.2	170.4	3922.3	900.3	505.8
12	1965.6	214.7	161.0	3941.7	925.0	502.5
13	1983.9	193.3	145.0	3961.1	946.7	477.7
14	2017.5	181.1	135.8	4029.6	997.1	471.6
15	1988.2	167.8	125.8	3986.0	996.5	464.0
16	1987.8	169.2	126.9	3975.1	1011.5	462.0
17	1920.6	162.3	121.7	3825.7	953.5	439.4
18	1877.1	168.5	126.3	3760.4	936.3	443.4
19	1808.2	163.2	122.4	3637.5	892.2	433.9
21	1702.9	157.6	118.2	3384.7	804.0	400.2
22	1682.2	158.2	118.6	3351.3	803.7	400.1
23	1643.9	160.8	120.6	3302.8	793.6	405.2
24	1646.4	145.4	109.1	3314.7	813.3	393.9
25	1651.1	150.3	112.7	3311.7	827.1	392.7
26	1636.1	142.1	106.6	3257.1	815.6	375.3
27	1640.4	141.1	105.8	3262.9	835.0	374.6
28	1609.7	148.2	111.2	3220.1	827.7	382.8
29	1598.9	138.6	103.9	3202.0	832.0	373.7
30	1569.0	145.7	109.3	3144.0	815.0	374.1
31	1504.6	140.3	105.2	3001.7	755.8	354.2
32	1468.0	142.3	106.7	2934.4	736.6	352.0
33	1413.7	148.4	111.3	2834.3	699.8	352.5

TABLE 5.2 (Continued)

E(°k)	σ_{NU}	σ_{Nx}	σ_{Ny}	σ_{IU}	σ_{Ix}	σ_{Iy}
34	1367.6	146.1	109.6	2744.9	668.2	345.8
35	1317.1	141.5	106.1	2643.7	632.2	334.9
36	1299.5	145.9	109.4	2594.9	618.3	330.2
37	1277.1	148.6	111.4	2544.8	605.6	325.8
38	1257.8	149.9	112.4	2498.4	590.7	320.0
39	1254.2	142.9	107.1	2498.2	602.5	315.2
40	1266.4	145.3	109.0	2535.0	629.1	323.2
42	1258.2	136.5	102.4	2531.5	652.8	320.1
44	1255.7	131.7	98.8	2518.7	669.4	312.7
45	1244.3	130.4	97.8	2493.6	666.6	308.3
46	1226.4	129.2	96.9	2457.2	657.3	303.5
48	1177.6	127.1	95.3	2360.0	625.3	293.4
50	1126.4	126.0	94.5	2255.5	589.4	284.4
52	1088.1	127.2	95.4	2174.2	563.6	278.9
54	1070.3	130.4	97.8	2136.7	555.7	277.8
55	1068.9	132.6	99.4	2134.4	558.3	279.0
56	1053.2	130.4	97.8	2105.4	555.9	274.7
58	1067.4	135.3	101.4	2134.4	575.6	279.9
60	1067.6	135.2	101.4	2132.9	588.8	279.1
62	1077.5	135.6	101.7	2149.2	599.8	280.6
64	1064.1	132.6	99.5	2122.1	595.7	276.2
65	1044.7	131.2	98.4	2083.5	584.4	271.8
66	1042.4	129.9	97.4	2081.8	581.7	271.2
68	1015.7	128.7	96.5	2031.7	560.7	266.8
70	987.4	129.6	97.2	1974.6	535.4	263.1
75	947.2	129.3	96.9	1890.5	497.9	257.0
80	965.1	129.8	97.3	1926.5	520.6	258.2
85	995.3	130.2	97.7	1987.4	559.9	262.7
90	981.6	130.0	97.5	1955.6	554.6	259.1
95	918.9	129.9	97.4	1842.4	512.3	252.5
100	883.0	129.8	97.3	1759.6	479.1	244.4
110	873.1	128.4	96.3	1764.1	503.7	247.8
120	874.2	120.2	90.1	1743.6	500.9	236.2
130	831.9	123.2	92.4	1676.3	469.5	236.6
140	843.0	122.1	91.6	1693.4	486.3	234.2
150	811.9	111.6	83.7	1610.3	460.3	216.0
160	831.9	120.8	90.6	1641.8	465.5	225.1
170	795.5	121.7	91.2	1608.5	456.8	226.3
180	776.7	114.6	86.0	1546.2	450.3	216.7
190	764.7	118.0	88.5	1533.4	444.3	218.1
210	749.4	119.4	89.5	1506.4	428.2	214.4
230	776.3	115.7	86.8	1548.9	448.2	214.8

TABLE 5.2 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
230	776.3	115.7	86.8	1548.9	448.2	214.8
250	746.3	119.7	89.8	1492.2	432.2	214.4
270	718.8	118.7	89.0	1414.4	410.8	203.9
290	738.8	116.1	87.1	1471.0	430.7	210.4
310	701.7	114.6	85.9	1380.8	405.3	198.8
325	685.4	104.6	78.4	1400.6	415.2	198.4
350	705.8	111.3	83.4	1422.9	419.8	203.3
370	664.1	114.7	86.0	1334.5	395.3	194.3
390	686.8	105.2	78.9	1380.4	394.0	194.9
405	681.4	117.3	88.0	1363.6	399.5	201.8
410	675.3	111.4	83.6	1335.4	387.8	195.3
430	716.6	112.3	84.2	1413.6	403.5	200.3
450	697.6	114.1	85.6	1384.0	404.9	198.5
470	687.9	116.5	87.4	1366.1	391.1	203.7
490	644.8	112.3	84.2	1295.7	381.1	193.5
510	675.6	102.6	76.9	1337.0	383.7	185.9
540	620.3	110.6	83.0	1218.9	371.1	184.7
600	621.6	98.8	74.1	1252.8	361.2	179.0
650	666.3	110.4	82.8	1320.5	380.1	192.8
700	679.2	110.8	83.1	1362.7	393.0	199.3
750	619.3	106.0	79.5	1248.4	359.8	183.5
800	563.1	100.5	75.4	1131.6	336.4	171.3
850	629.4	106.1	79.6	1284.8	379.6	190.7
900	648.0	102.7	77.0	1300.1	373.8	185.5
950	606.2	102.4	76.8	1198.6	344.4	176.1
1000	577.0	108.9	81.6	1157.9	342.5	180.0
1100	637.7	105.3	78.9	1267.3	370.2	185.1
1200	569.4	99.3	74.5	1130.3	335.5	168.8
1300	606.8	98.7	74.0	1217.7	345.3	174.9
1400	598.7	96.5	72.3	1190.9	346.9	173.9
1500	546.0	98.0	73.5	1079.7	319.4	164.7
1600	591.3	101.4	76.0	1195.1	345.9	178.2
1700	621.8	90.22	67.6	1239.9	357.7	171.1
1800	550.3	92.5	69.4	1095.8	324.7	160.2
1900	576.2	91.38	68.5	1138.6	327.9	161.9
2000	604.5	100.0	75.0	1212.3	340.9	178.1
2250	553.9	94.1	70.6	1089.9	319.9	161.9
2500	572.8	94.6	70.9	1159.6	330.3	167.4
2750	558.0	97.2	72.9	1113.7	320.7	166.9
3000	568.5	92.7	69.5	1152.5	326.4	165.5
3250	546.0	87.6	65.7	1113.5	318.0	158.9

TABLE 5.2 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{Ny}	σ_{IU}	σ_{Ix}	σ_{Iy}
3500	552.6	97.7	73.3	1123.9	313.8	168.8
3750	536.5	102.6	76.9	1066.3	311.7	167.6
4000	555.3	91.5	68.6	1101.5	319.2	162.0
4250	549.7	86.4	64.8	1080.2	306.4	156.4
4500	538.7	91.0	68.3	1082.0	308.9	159.8
4750	549.5	94.4	70.8	1084.0	317.3	162.0
5000	538.7	95.6	71.7	1074.9	305.4	164.6
5500	543.0	84.9	63.7	1092.9	312.1	154.7
6000	533.0	93.4	70.0	1046.5	299.6	159.0
6500	525.4	88.5	66.4	1042.2	298.5	152.3
7000	520.5	87.3	65.4	1031.0	297.4	152.1
7500	517.7	92.7	69.5	1051.5	295.9	158.3
8000	521.5	85.3	63.9	1046.7	302.7	154.2
8500	502.4	82.0	61.5	1024.8	290.7	148.5
9000	497.6	92.4	69.3	989.9	287.1	152.5
9500	513.1	91.2	68.4	1014.3	290.8	151.4
10000	504.2	83.7	62.8	1004.6	289.8	148.3

TABLE 5.3

Spin-Dependent Cross Sections of Sodium Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f = 2 \rightarrow f' = 1$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{IY} Hyperfine transition cross sections of identical atom from $f = 2 \rightarrow f' = 1$

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
10	1800.2	190.1	142.6	3604.6	965.2	446.1
11	1881.8	198.4	148.8	3755.6	1028.9	461.8
12	1880.8	197.3	147.9	3747.1	971.7	461.9
13	1825.1	187.3	140.5	3625.6	884.6	445.1
14	1932.1	177.0	132.8	3857.9	943.4	459.2
15	2045.2	159.4	119.5	4070.7	1023.0	458.7
16	2090.4	154.8	116.1	4180.0	1058.7	466.3
17	2043.6	145.2	108.9	4082.7	1020.4	449.7
18	1997.8	146.7	110.0	3987.9	977.3	444.0
19	1961.3	149.3	112.0	3923.9	956.7	442.2
20	1964.3	153.9	115.4	3924.2	964.4	445.1
21	1978.3	161.6	121.2	3961.8	989.3	456.4
22	1972.5	163.2	122.4	3958.1	997.6	457.2
23	1937.7	159.7	119.7	3865.0	959.5	442.1
24	1933.8	159.2	119.4	3853.1	943.5	442.5
25	1916.6	163.6	122.7	3835.3	935.8	446.8
26	1898.6	170.1	127.5	3805.2	938.5	448.7
27	1897.3	171.6	128.7	3798.1	956.4	451.5
28	1866.7	177.0	132.7	3734.7	937.0	445.3
29	1837.9	169.2	126.9	3667.5	915.4	435.0
30	1814.9	159.1	119.3	3621.8	895.0	421.8
31	1809.8	163.5	122.6	3620.7	894.4	431.1
32	1788.1	159.3	119.5	3584.9	884.7	422.1
33	1793.1	161.1	120.8	3583.7	899.3	420.1

TABLE 5.3 (Continued)

E(°k)	σ_{Nu}	σ_{Nx}	σ_{Ny}	σ_{Iu}	σ_{Ix}	σ_{Iy}
34	1782.3	166.3	124.7	3554.3	894.4	422.7
35	1763.4	171.9	128.9	3528.0	881.8	425.1
36	1756.0	169.1	126.8	3506.5	886.3	423.5
37	1718.0	157.9	118.4	3438.3	870.7	411.9
38	1671.9	163.6	122.7	3354.5	832.8	405.2
39	1683.7	155.1	116.3	3341.2	844.2	397.1
40	1664.2	144.1	108.1	3349.6	839.3	388.0
42	1637.5	144.7	108.5	3269.3	829.5	383.3
44	1663.9	158.4	118.8	3353.7	862.0	404.9
45	1643.5	156.9	117.6	3270.7	825.5	392.1
46	1602.3	166.5	124.8	3188.6	805.5	391.9
48	1589.3	156.5	117.3	3204.5	810.8	390.5
50	1554.8	141.4	106.0	3094.0	784.0	361.8
52	1579.1	149.3	112.0	3154.7	818.6	375.5
54	1465.1	156.3	117.2	2932.9	766.9	361.7
55	1459.2	151.6	113.7	2903.4	757.0	356.4
56	1508.3	155.7	116.8	2995.0	772.7	368.8
58	1455.5	161.5	121.1	2934.1	752.6	372.7
60	1486.7	161.0	120.7	2957.9	765.5	367.8
62	1474.7	149.3	112.0	2928.5	752.8	357.2
64	1453.2	163.0	122.2	2934.7	765.5	374.3
65	1471.1	146.9	110.2	2961.5	773.3	361.4
66	1409.9	156.9	117.7	2773.1	734.0	344.7
68	1362.0	150.6	112.9	2712.7	703.3	340.0
70	1363.6	158.7	119.0	2730.4	709.8	350.5
75	1332.8	153.7	115.3	2648.5	706.5	336.5
80	1361.9	154.8	116.1	2757.4	717.0	354.5
85	1313.6	140.0	105.0	2656.9	692.3	330.7
90	1262.7	153.8	115.4	2566.8	696.2	334.0
95	1256.2	150.8	113.1	2505.4	681.6	319.7
100	1242.8	144.0	108.0	2491.8	660.9	316.5
110	1221.3	153.5	115.1	2462.2	663.8	321.7
120	1185.9	132.3	99.2	2359.5	640.4	300.1
130	1179.2	151.0	113.2	2363.7	654.9	314.0
140	1125.4	150.2	112.6	2250.9	615.6	304.2
150	1102.9	139.2	104.4	2202.6	614.0	287.6
160	1035.4	148.3	111.2	2091.8	588.8	284.0
170	1088.2	143.8	107.9	2179.0	599.6	291.1
180	1058.9	139.6	104.7	2126.4	592.0	284.2
190	1052.3	144.4	108.3	2077.4	588.9	277.3
200	967.5	141.2	105.9	1936.5	544.3	265.8
210	1029.0	153.3	115.0	2047.9	580.4	287.8
230	934.9	141.2	105.9	1823.8	512.4	254.4

TABLE 5.3 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
250	917.7	140.1	105.0	1840.0	538.7	260.2
270	937.7	138.9	104.1	1877.4	519.3	260.8
290	941.7	138.5	103.8	1907.0	538.8	265.3
310	887.8	143.1	107.3	1753.3	511.0	254.8
325	901.9	127.3	95.4	1802.5	516.5	247.2
350	866.5	134.8	101.1	1746.8	499.7	250.2
370	859.2	151.0	113.2	1731.1	516.1	260.4
390	844.2	127.1	95.3	1682.3	476.1	239.6
405	846.2	128.7	96.5	1672.1	476.8	232.9
410	837.2	127.3	95.5	1673.8	493.2	237.6
430	934.7	136.6	102.5	1861.5	531.0	257.2
450	839.7	128.7	96.5	1673.4	478.2	235.1
470	870.0	128.5	96.3	1736.4	495.7	241.9
490	880.8	123.1	92.3	1793.8	502.7	247.1
510	835.0	125.6	94.2	1672.6	480.2	234.8
540	836.1	138.5	103.8	1680.3	484.3	243.9
600	809.3	124.3	93.2	1565.0	459.0	218.9
650	812.7	130.9	98.2	1618.4	480.6	232.4
700	760.4	123.7	92.8	1533.7	439.3	218.8
750	783.6	134.1	100.6	1565.0	458.9	230.3
800	832.2	121.0	90.7	1685.8	476.7	234.5
850	808.9	121.9	91.4	1609.0	461.6	222.7
900	761.1	124.2	93.1	1551.7	449.8	226.6
950	770.6	116.7	87.5	1535.5	437.3	212.9
1000	732.9	119.4	89.5	1445.4	430.2	209.0
1100	710.9	109.7	82.3	1421.7	419.3	201.8
1200	735.0	116.9	87.7	1462.7	418.1	210.0
1300	800.4	122.6	91.9	1586.6	453.0	222.6
1400	700.0	117.0	87.7	1386.4	404.2	205.4
1500	648.2	115.9	86.9	1291.3	381.8	195.4
1600	733.7	119.3	89.4	1449.8	422.3	208.9
1700	733.1	114.4	85.8	1471.4	422.1	208.4
1800	677.4	119.3	89.4	1326.6	381.0	198.2
1900	696.2	109.9	82.4	1371.9	400.1	194.1
2000	703.9	111.9	83.9	1409.5	410.5	204.2
2250	672.0	115.0	86.3	1358.2	394.0	201.6
2500	721.8	105.6	79.2	1431.3	415.5	199.0
2750	659.2	114.4	85.8	1322.6	381.5	196.1
3000	690.8	113.1	84.89	1400.0	399.7	205.0
3250	659.8	114.3	85.77	1307.5	381.2	193.6
3500	652.9	109.6	82.21	1319.4	379.0	196.2
3750	692.0	109.2	81.93	1382.1	391.1	198.8

TABLE 5.3 (Continued)

E(^{o}k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
4000	635.2	112.2	84.17	1248.7	366.7	186.8
4250	631.3	103.7	77.84	1250.0	360.1	183.9
4500	672.7	106.1	79.60	1316.4	367.5	189.6
4750	638.4	104.5	78.44	1268.5	372.0	187.2
5000	642.9	111.3	83.52	1282.0	376.0	191.1
5500	635.0	106.5	79.90	1294.1	357.9	189.5
6000	635.8	103.4	77.57	1246.9	374.3	181.8
6500	625.1	109.4	82.07	1228.7	344.0	185.8
7000	635.5	111.9	83.97	1259.2	364.5	190.6
7500	631.2	101.2	75.91	1270.3	358.3	181.8
8000	626.1	94.5	70.93	1235.2	348.7	175.1
8500	606.4	106.0	79.50	1210.6	349.6	181.9
9000	630.0	109.1	81.85	1275.1	361.4	188.3
9500	616.8	104.2	78.21	1232.2	344.4	184.2
10000	602.7	101.7	76.33	1216.6	345.9	178.6

TABLE 5.4

Spin-Dependent Cross Sections of Potassium Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f=2 \rightarrow f'=1$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{IY} Hyperfine transition cross sections of identical atom from $f=2 \rightarrow f'=1$

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
10	3964.9	428.8	321.6	7962.0	2016.0	992.0
11	3869.5	319.9	239.9	7723.8	1939.1	881.4
12	3950.7	307.7	230.8	7922.7	1997.1	901.8
13	3874.4	322.5	241.8	7697.7	1981.7	881.6
14	3896.3	334.9	251.2	7777.4	1973.2	898.6
15	3685.7	363.7	272.8	7380.3	1882.2	899.7
16	4000.4	371.9	278.9	8026.2	2031.4	950.7
17	3877.2	395.6	296.7	7740.1	1944.0	951.0
18	3970.7	399.1	299.3	8014.6	1990.1	983.5
19	4020.0	427.7	320.7	7998.3	1965.0	991.7
20	4031.8	350.0	262.5	8088.2	2035.5	944.8
21	3938.7	352.7	264.5	7873.9	1985.4	930.9
22	4004.6	340.2	255.1	8084.6	2001.5	937.6
23	3835.5	364.0	273.0	7628.8	1876.7	916.8
24	3998.2	353.4	265.0	8028.6	2000.3	946.0
25	3941.6	346.6	259.9	7865.1	1941.8	923.6
26	3988.5	339.6	254.7	7952.1	1950.2	923.9
27	4076.0	321.6	241.2	8192.1	2015.1	935.9
28	4058.2	283.7	212.8	8070.6	1990.4	881.4
29	3979.9	292.6	219.5	7975.2	1984.1	899.8
30	3920.9	322.4	241.8	7850.1	1918.7	900.9
31	3857.0	299.5	224.6	7731.9	1913.2	875.1
32	3803.1	318.2	238.6	7577.3	1882.9	871.3
33	3853.7	328.3	246.2	7740.6	1920.7	898.0

TABLE 5.4 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{Ny}	σ_{Iu}	σ_{IX}	σ_{Iy}
34	3986.5	310.0	232.5	7968.7	1989.4	899.4
35	3850.3	305.1	228.8	7634.1	1893.8	863.7
36	3879.3	306.8	230.1	7829.2	1943.3	896.1
37	3774.0	317.5	238.1	7472.3	1851.7	856.9
38	3787.7	288.7	216.5	7589.2	1872.8	850.5
39	3784.8	291.9	218.9	7516.3	1829.1	846.9
40	3810.4	324.0	243.0	7679.5	1870.5	896.0
42	3672.0	304.0	228.0	7269.4	1789.1	834.5
44	3680.5	315.6	236.7	7361.9	1850.9	847.3
45	3561.6	316.2	237.1	7086.5	1792.5	832.5
46	3568.9	328.1	246.0	7186.6	1797.0	855.0
48	3613.1	280.0	210.0	7236.3	1819.3	820.2
50	3494.2	305.5	229.1	6959.4	1709.3	808.6
52	3563.0	301.0	225.7	7130.3	1776.8	821.8
54	3591.4	329.7	247.3	7158.3	1791.7	846.9
55	3463.4	319.9	239.9	6946.6	1747.2	825.4
56	3466.3	331.4	248.6	6899.9	1737.9	825.5
58	3413.1	312.1	234.1	6777.1	1720.5	808.5
60	3286.7	324.0	243.0	6508.3	1630.8	791.4
62	3276.0	299.4	224.5	6627.7	1673.8	785.8
64	3203.4	283.6	212.7	6378.8	1624.8	752.6
65	3288.1	288.7	216.5	6572.7	1652.5	763.5
66	3206.1	267.2	200.4	6420.5	1624.3	739.3
68	3354.4	289.2	216.9	6783.6	1720.5	787.9
70	3262.9	282.2	211.6	6506.2	1653.0	748.7
75	3189.9	320.2	240.1	6472.9	1642.8	793.9
80	3098.5	272.9	204.7	6222.7	1580.3	722.4
85	2912.1	290.9	218.2	5820.9	1508.9	706.3
90	2969.9	321.8	241.3	5885.6	1500.5	735.9
95	2886.6	288.8	216.6	5708.2	1455.4	691.1
100	2982.2	306.0	229.5	5912.0	1504.6	722.5
110	2707.8	295.1	221.3	5480.6	1407.0	689.8
120	2662.0	281.5	211.1	5288.9	1401.8	651.4
130	2521.5	292.4	219.3	4968.5	1292.8	633.7
140	2473.9	303.9	227.9	5024.8	1335.0	656.1
150	2420.0	280.2	210.1	4824.5	1300.0	607.5
160	2391.2	274.5	205.9	4794.0	1281.6	604.1
170	2358.3	285.3	214.0	4757.9	1283.7	618.2
180	2392.5	260.0	195.0	4858.3	1305.4	606.8
190	2311.3	254.6	191.0	4598.8	1225.9	570.1
200	2300.4	289.2	216.9	4578.2	1240.7	602.7
210	2270.7	278.3	208.7	4547.9	1222.8	583.1
230	2010.4	262.2	196.6	4015.4	1086.1	534.2

TABLE 5.4 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
250	2036.7	284.6	213.5	4069.4	1152.1	560.3
280	2057.7	277.1	207.8	4112.2	1111.4	554.4
290	1966.7	254.6	191.0	3921.6	1111.9	516.6
310	1899.1	272.7	204.5	3786.9	1053.1	528.4
325	1987.0	278.4	208.8	3957.3	1101.4	549.3
350	1894.2	246.2	184.7	3739.3	1075.4	493.5
380	1838.9	268.3	201.2	3713.4	1065.8	521.6
390	1829.5	275.9	206.9	3684.6	1043.7	519.8
405	1840.6	277.1	207.8	3687.5	1047.5	516.7
410	1855.9	266.3	199.7	3704.2	1070.4	510.0
430	1907.3	236.5	177.4	3818.4	1084.0	501.0
450	1832.1	267.8	200.8	3596.4	990.3	495.0
480	1737.4	277.9	208.4	3498.4	1023.5	507.3
490	1724.3	276.4	207.3	3424.2	972.9	495.9
510	1770.8	252.2	189.1	3555.4	1008.4	486.3
540	1676.3	250.5	187.8	3335.4	975.1	462.1
600	1703.3	275.9	206.9	3351.2	971.4	487.1
650	1655.7	226.7	170.0	3327.1	953.5	446.7
700	1585.1	252.5	189.4	3188.4	922.9	461.3
750	1716.8	240.6	180.4	3454.4	964.9	472.6
800	1572.2	242.9	182.1	3168.2	911.0	452.3
850	1566.4	246.4	184.8	3130.0	924.1	447.9
900	1531.5	241.0	180.7	3001.4	875.7	425.1
950	1455.2	234.9	176.1	2865.8	866.0	408.8
1000	1495.5	250.7	188.0	2976.5	857.7	438.4
1100	1511.8	234.1	175.6	3022.1	851.3	423.9
1200	1499.0	232.2	174.2	2983.5	854.9	424.3
1300	1479.6	243.7	182.8	2963.2	858.2	427.6
1400	1400.9	232.0	174.0	2763.3	810.6	402.3
1500	1409.8	226.3	169.7	2858.9	839.6	413.8
1600	1431.0	237.3	177.9	2870.4	860.4	417.9
1700	1311.4	224.1	168.0	2622.8	784.6	388.2
1800	1422.6	223.5	167.6	2853.1	829.7	410.9
1900	1446.4	233.0	174.7	2871.8	824.4	414.6
2000	1468.5	214.6	160.9	2940.6	853.2	407.6
2250	1196.2	218.8	164.1	2391.5	727.6	362.7
2500	1435.6	226.4	169.8	2926.4	837.4	421.4
2750	1296.1	216.7	162.5	2560.2	743.6	376.2
3000	1303.9	199.2	149.4	2594.2	762.2	362.6
3250	1373.7	227.5	170.6	2767.6	799.4	403.8
3500	1242.8	220.3	165.2	2479.8	728.3	371.7

TABLE 5.4 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
3750	1298.1	219.4	164.5	2601.9	734.2	379.7
4000	1276.9	213.7	160.2	2583.7	747.9	378.3
4250	1210.5	213.1	159.8	2400.9	722.4	364.3
4500	1242.1	231.0	173.2	2417.6	686.9	369.9
4750	1280.9	209.1	156.8	2591.6	732.6	378.9
5000	1215.1	209.8	157.3	2461.7	729.6	365.5
5500	1228.2	198.0	148.5	2476.3	715.4	360.2
6000	1208.5	211.0	158.2	2444.7	726.3	366.6
6500	1181.9	205.4	154.1	2399.2	694.4	357.0
7000	1186.4	201.7	151.3	2411.4	681.8	356.5
7500	1199.2	207.7	155.8	2408.6	693.8	354.6
8000	1237.9	206.5	154.9	2484.7	699.8	365.2
8500	1179.5	208.0	156.0	2400.0	670.2	360.8
9000	1132.5	199.3	149.4	2303.1	659.9	341.4
9500	1224.5	192.2	144.2	2394.7	691.7	345.7
10000	1184.7	200.0	150.0	2410.4	676.1	353.0

TABLE 5.5

Spin-Dependent Cross Sections of Rubidium Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f = 3 \rightarrow f' = 2$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{IY} Hyperfine transition cross sections of identical atom from $f = 3 \rightarrow f' = 2$

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
10	4949.9	435.8	363.1	9860.4	2411.0	945.8
11	4783.4	477.4	397.8	9569.1	2337.4	976.2
12	4760.3	377.2	314.3	9511.0	2313.7	887.5
13	4694.3	383.3	319.4	9398.7	2330.4	882.1
14	4572.9	385.0	320.8	9160.1	2259.6	873.9
15	4482.7	417.0	347.5	8960.6	2243.1	886.3
16	4234.2	406.5	338.8	8471.4	2159.4	843.4
17	4119.6	396.9	330.7	8255.0	2031.3	829.1
18	4144.7	407.7	339.8	8254.9	2046.7	830.0
19	3952.4	409.2	341.0	7892.8	2004.8	817.0
20	4147.5	413.7	344.8	8352.8	2096.9	850.5
21	4210.8	398.9	332.4	8409.3	2142.9	833.4
22	4002.2	390.5	325.4	8006.6	1997.8	811.1
23	3999.2	409.5	341.2	7967.4	2021.3	811.6
24	4075.7	376.4	313.6	8116.3	2080.3	799.4
25	3936.7	405.9	338.2	7916.6	2016.4	818.9
26	3928.7	402.5	335.4	7829.3	1987.2	800.0
27	3960.6	407.1	339.2	7949.5	2035.5	815.7
28	3911.1	375.7	313.1	7838.5	1984.6	786.1
29	3881.6	360.0	300.0	7768.1	1956.9	771.1
30	3922.3	381.9	318.2	7859.2	1991.4	791.6
31	4004.8	352.9	294.1	8012.3	2005.0	777.1
32	4084.7	337.0	280.8	8155.9	2076.1	767.1

TABLE 5.5 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
33	4084.7	375.4	312.8	8164.1	2060.1	796.4
34	3973.8	377.3	314.4	7931.6	2017.9	787.1
35	4055.2	381.5	317.9	8177.8	2054.6	816.6
36	4068.4	356.8	297.3	8101.0	2023.0	775.7
37	4026.8	378.4	315.3	8077.1	2010.6	804.5
38	4079.4	382.2	318.5	8136.7	2000.2	801.5
39	3984.2	324.4	270.4	7974.8	1969.0	743.9
40	4075.5	341.4	284.5	8142.3	2066.5	770.5
42	4025.5	334.4	278.7	8057.7	2049.2	761.6
44	4102.0	348.3	290.2	8164.7	2069.5	776.7
45	4127.6	330.2	275.1	8268.3	2056.8	771.3
46	3920.0	342.6	285.5	7852.4	1954.1	758.4
48	4091.5	368.3	306.9	8244.7	2041.0	814.2
50	4069.8	349.6	291.3	8137.0	1989.2	780.3
52	4075.3	378.1	315.0	8137.5	2025.0	801.7
54	4208.0	352.9	294.1	8412.4	2074.0	795.1
55	4151.1	307.8	256.5	8328.8	2053.6	757.2
56	4096.7	318.5	265.4	8192.9	2003.5	757.1
58	4012.0	331.7	276.4	7994.6	1992.9	753.9
60	4023.8	330.6	275.5	8032.5	1973.5	760.0
62	3887.6	337.5	281.3	7796.4	1933.0	750.7
64	3930.1	350.8	292.4	7899.0	1956.6	775.9
65	3881.0	366.3	305.3	7741.1	1921.7	767.9
66	3878.1	356.8	297.3	7755.4	1887.6	763.1
68	4076.0	334.7	278.9	8162.8	2015.2	769.5
70	3946.6	336.2	280.2	7831.9	1944.0	741.2
75	4011.3	329.6	274.6	8013.2	1971.6	750.7
80	4026.8	321.1	267.5	8026.8	1970.3	743.9
85	3820.2	330.9	275.7	7666.9	1884.6	741.0
90	3696.4	311.6	276.4	7385.3	1881.5	721.5
95	3673.5	303.2	252.7	7328.9	1805.3	686.4
100	3673.3	307.2	256.0	7374.8	1825.1	704.3
110	3577.6	322.1	268.4	7176.5	1785.7	701.5
120	3449.5	351.0	292.5	6950.9	1695.0	714.4
130	3382.7	301.2	251.0	6773.2	1687.1	659.4
140	3389.5	306.6	255.5	6793.9	1737.1	666.1
150	3308.5	342.9	285.8	6583.7	1636.6	675.6
160	3136.7	286.4	238.7	6261.3	1572.5	607.1
170	3057.0	307.0	255.8	6101.5	1570.9	622.3
180	3026.2	316.2	263.5	6027.1	1545.4	623.8
190	3040.5	315.9	263.2	6031.8	1512.9	619.9
200	3049.3	277.8	231.5	6089.6	1578.8	592.1

TABLE 5.5 (Continued)

E(^{o}k)	σ_{NU}	σ_{Nx}	σ_{Ny}	σ_{IU}	σ_{Ix}	σ_{Iy}
200	3049.3	277.8	231.5	6089.6	1578.8	592.1
210	2905.0	301.8	251.5	5837.8	1543.1	608.0
230	2876.0	322.1	268.4	5757.4	1500.9	612.4
250	2799.5	292.3	243.5	5526.6	1451.3	561.4
270	2760.3	307.5	256.2	5547.6	1411.2	594.6
290	2632.1	291.3	242.7	5285.7	1402.6	558.9
310	2499.3	279.3	232.7	4990.2	1315.0	527.8
325	2513.9	284.4	237.0	5009.9	1337.1	530.4
350	2417.5	324.6	270.5	4834.8	1317.0	559.1
370	2348.1	296.9	247.4	4737.4	1248.0	536.4
390	2253.7	274.4	228.6	4494.6	1225.9	501.5
405	2392.2	294.0	245.0	4769.3	1306.5	529.0
410	2269.0	290.7	242.3	4539.4	1238.6	507.5
430	2294.5	294.7	245.6	4592.0	1245.1	517.9
450	2121.5	300.4	250.3	4232.2	1170.4	499.5
470	2219.6	272.6	227.2	4461.3	1193.3	491.6
490	2197.9	285.7	238.0	4364.0	1233.2	497.7
510	2219.8	290.6	242.2	4437.3	1212.8	502.1
540	2180.8	282.9	235.8	4361.4	1190.6	495.9
600	2141.9	280.0	233.3	4298.7	1211.7	486.7
650	1984.7	269.8	224.8	3978.0	1085.1	458.8
700	1969.8	246.6	205.5	3922.0	1117.0	435.3
750	1914.8	287.4	239.5	3827.8	1077.9	463.2
800	1905.4	280.0	233.3	3788.4	1070.7	457.7
850	1902.4	279.4	232.8	3808.9	1079.2	463.9
900	1888.8	265.9	221.6	3805.4	1048.0	452.8
950	1793.5	288.8	240.6	3625.4	1056.8	466.3
1000	1719.1	260.8	217.3	3449.4	978.8	421.4
1100	1732.2	261.6	218.0	3432.9	990.4	417.7
1200	1812.9	268.9	224.1	3662.4	1049.8	449.1
1300	1666.5	255.0	212.5	3335.7	952.9	407.6
1400	1643.0	269.7	224.7	3282.4	958.5	419.7
1500	1776.0	257.5	214.6	3559.3	1002.8	428.4
1600	1625.0	248.0	206.6	3220.1	932.6	392.6
1700	1668.4	257.0	214.1	3334.0	974.9	411.5
1800	1552.3	267.8	223.2	3138.5	907.1	416.4
1900	1511.5	238.1	198.4	3008.5	890.5	373.3
2000	1552.7	245.3	204.4	3098.3	895.9	388.6
2250	1546.2	241.0	200.8	3093.5	883.7	383.5
2500	1587.9	236.4	197.0	3207.4	916.8	394.5
2750	1463.2	252.9	210.7	2910.1	847.3	386.8
3000	1441.7	225.1	187.6	2850.2	831.3	349.9

TABLE 5.5 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
3250	1447.0	240.3	200.3	2902.7	877.7	374.6
3500	1451.9	218.8	182.3	2887.2	837.5	353.4
3750	1453.4	228.4	190.3	2868.9	816.6	354.6
4000	1554.7	240.8	200.7	3100.5	904.1	379.9
4250	1418.1	259.8	216.5	2850.8	850.6	384.8
4500	1312.8	258.8	215.6	2625.4	801.3	370.5
4750	1362.9	247.9	206.6	2761.0	817.8	377.7
5000	1467.7	253.0	210.8	2985.8	893.9	398.6
5500	1417.5	248.9	207.4	2837.7	844.0	373.9
6000	1395.1	231.9	193.3	2775.9	818.0	352.4
6500	1460.5	254.3	211.9	2919.5	869.6	389.9
7000	1287.1	244.8	204.0	2584.9	777.4	363.3
7500	1357.9	248.5	207.1	2721.2	800.7	377.4
8000	1393.3	240.4	200.3	2773.4	829.2	364.9
8500	1284.1	245.7	204.7	2561.0	785.8	360.3
9000	1300.1	241.8	201.5	2566.8	745.5	349.6
9500	1382.1	238.8	199.0	2734.9	794.2	354.6
10000	1316.5	228.2	190.1	2607.8	788.4	339.8

TABLE 5.6

Spin-Dependent Cross Sections of Cesium Atom (πa_o^2)

σ_{NU} Total scattering of nonidentical atom

σ_{NX} Spin-exchange cross sections of nonidentical atom

σ_{NY} Hyperfine transition cross sections of nonidentical atom from $f=4 \rightarrow f'=3$

σ_{IU} Total scattering of identical atom

σ_{IX} Spin-exchange cross sections of identical atom

σ_{Iy} Hyperfine transition cross sections of identical atom from $f=4 \rightarrow f'=3$

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{Iy}
10	4991.1	457.9	400.6	10009.2	2524.1	869.8
11	5036.6	417.7	365.5	10068.0	2514.6	832.2
12	4875.7	417.3	365.2	9737.6	2412.9	816.8
13	4928.2	417.5	365.3	9841.0	2458.6	822.1
14	5146.4	426.4	373.1	10317.7	2520.8	859.6
15	5046.2	426.0	372.7	10111.7	2492.8	843.7
16	4887.7	412.0	360.5	9747.9	2361.6	810.1
17	5042.0	399.8	349.8	10135.0	2464.1	824.7
18	4737.2	403.6	353.1	9462.9	2328.1	787.5
19	4884.5	417.4	365.2	9768.8	2385.9	824.3
20	4917.7	428.4	374.8	9862.3	2428.0	837.2
21	4732.0	411.1	359.7	9474.8	2323.8	797.2
22	4704.9	417.0	364.9	9440.9	2326.0	806.3
23	4666.1	421.1	368.5	9347.4	2306.2	799.8
24	4600.0	427.8	374.3	9174.1	2237.5	793.3
25	4549.4	396.4	346.9	9103.6	2300.7	768.5
26	4351.5	438.8	384.0	8717.3	2177.4	788.8
27	4332.5	435.4	381.0	8631.4	2174.1	775.4
28	4411.9	389.6	340.9	8824.4	2246.8	750.2
29	4290.5	390.7	341.9	8594.0	2133.4	742.2
30	4298.0	409.0	357.9	8608.1	2156.7	756.1
31	4138.7	373.8	327.0	8263.5	2094.3	702.9

TABLE 5.6 (Continued)

E(°k)	σ_{Nu}	σ_{Nx}	σ_{Ny}	σ_{Iu}	σ_{Ix}	σ_{Iy}
32	4119.9	387.4	339.0	8264.0	2130.0	725.1
33	4133.9	413.4	361.7	8272.5	2095.6	744.0
34	4079.1	397.5	347.8	8193.7	2098.7	727.7
35	4277.0	402.0	351.8	8570.5	2202.7	749.1
36	4043.2	372.3	325.8	8095.0	2084.1	698.5
37	3955.9	384.7	336.6	7911.5	1988.5	704.1
38	4156.0	356.6	312.1	8314.7	2129.5	696.4
39	4157.0	406.6	355.8	8346.1	2115.6	750.6
40	4038.0	382.9	335.0	8092.6	2074.2	712.5
42	4126.4	329.1	287.9	8281.0	2093.6	669.8
44	4193.7	384.6	336.5	8316.6	2130.0	707.4
45	4150.2	359.2	314.3	8318.2	2119.8	701.4
46	4007.5	379.5	332.1	8024.3	2022.2	702.6
48	4072.9	371.5	325.0	8144.9	2077.8	700.8
50	4002.7	417.7	365.5	8021.0	2006.7	745.5
52	3974.3	344.5	301.5	7952.9	2024.7	666.5
54	4089.0	321.3	281.1	8151.8	2082.6	651.7
55	4077.8	347.7	304.3	8147.1	2025.6	681.1
56	4090.5	378.4	331.1	8207.6	2076.6	715.7
58	4155.1	337.2	295.0	8335.3	2128.7	678.9
60	4228.0	353.3	309.1	8489.7	2126.5	705.7
62	4144.4	386.6	338.3	8274.8	2026.7	721.0
64	4226.2	373.1	326.5	8486.7	2106.8	730.6
65	4283.0	381.2	333.5	8567.0	2123.9	732.1
66	4021.8	365.8	320.0	8046.2	1996.9	693.7
68	4214.7	345.0	301.9	8447.7	2076.7	695.8
70	4251.5	360.7	315.6	8463.1	2078.8	706.3
75	4076.0	346.5	303.2	8176.1	2035.2	683.5
80	4143.6	349.3	305.7	8288.1	2023.1	687.3
85	4246.5	367.2	321.3	8467.5	2087.7	711.2
90	4090.4	334.5	292.7	8180.1	2008.6	668.2
95	4118.5	322.0	281.8	8234.1	2043.6	664.7
100	4206.3	301.3	263.7	8399.0	2066.0	646.8
110	3926.7	336.2	294.2	7854.5	1924.2	658.0
120	4100.7	354.4	310.1	8218.0	2039.8	696.3
130	3978.0	340.4	297.9	7953.2	1967.6	667.6
140	3813.1	314.7	275.3	7594.2	1877.2	625.2
150	3803.7	330.0	288.8	7597.9	1904.9	640.9
160	3746.1	309.4	270.7	7471.1	1831.7	609.8
170	3728.3	307.0	268.6	7465.0	1832.8	613.3
180	3592.7	342.8	299.9	7205.2	1791.6	635.8
190	3631.9	333.4	291.7	7282.3	1821.1	629.1

TABLE 5.6 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
200	3430.6	324.0	283.5	6838.7	1692.1	602.9
210	3346.7	296.4	259.3	6704.5	1652.9	566.7
230	3377.9	310.5	271.7	6752.0	1711.1	585.6
250	3187.7	326.6	285.8	6385.3	1589.9	587.8
270	3123.2	303.1	265.2	6243.2	1602.3	553.2
290	3133.4	300.9	263.2	6262.6	1583.5	549.9
310	3034.5	320.0	280.0	6127.8	1561.0	572.2
325	3066.4	306.1	267.9	6188.8	1613.6	561.0
350	2959.2	297.1	259.9	5952.3	1524.7	540.4
370	2842.8	328.4	287.3	5681.5	1481.7	553.3
390	2868.5	297.9	260.7	5746.1	1509.0	528.6
405	2752.1	312.3	273.3	5480.4	1463.8	520.0
410	2680.9	292.4	255.8	5376.1	1396.1	509.3
430	2580.9	302.6	264.8	5130.6	1355.3	496.7
450	2665.9	318.1	278.3	5322.1	1408.9	520.3
470	2699.3	304.5	266.4	5360.9	1412.4	511.3
490	2543.5	307.0	268.6	5096.3	1357.0	504.2
510	2524.4	273.2	239.0	5044.4	1343.8	471.0
540	2540.8	307.0	268.6	5083.6	1366.4	502.1
600	2422.5	276.8	242.2	4880.0	1312.3	473.7
650	2364.5	300.4	262.8	4731.1	1304.2	476.2
700	2277.9	306.9	268.6	4568.2	1225.0	488.0
750	2184.6	292.1	255.6	4374.7	1228.3	458.7
800	2115.5	295.8	258.8	4234.0	1184.6	451.5
850	2181.1	283.8	248.4	4363.4	1186.5	452.7
900	2170.9	279.9	244.9	4346.0	1201.5	445.7
950	2053.5	280.2	245.2	4105.8	1158.4	434.1
1000	2015.0	303.8	265.8	4030.5	1132.6	449.7
1100	2135.4	270.4	236.6	4267.9	1187.7	430.4
1200	1847.4	275.5	241.0	3710.4	1049.6	415.0
1300	1953.3	287.9	251.9	3918.9	1116.9	435.4
1400	1818.6	270.7	236.9	3653.5	1050.3	411.0
1500	1884.5	296.0	259.0	3758.8	1084.1	430.5
1600	1894.2	311.9	272.9	3781.5	1121.4	446.6
1700	1874.7	281.7	246.5	3749.9	1092.5	420.9
1800	1827.3	295.0	258.1	3668.1	1082.6	432.7
1900	1818.0	289.8	253.6	3623.9	1034.6	414.1
2000	1709.1	269.4	235.7	3411.3	1003.4	392.9
2250	1713.4	289.8	253.6	3445.6	1055.7	412.2
2500	1603.5	272.8	238.7	3204.6	951.4	382.7
2750	1690.9	272.7	238.6	3422.0	1017.2	402.4

TABLE 5.6 (Continued)

E(°k)	σ_{NU}	σ_{NX}	σ_{NY}	σ_{IU}	σ_{IX}	σ_{IY}
3000	1676.4	258.4	226.1	3339.0	966.1	378.1
3250	1669.0	279.5	244.5	3357.2	968.0	402.7
3500	1663.3	276.7	242.1	3324.2	990.8	398.6
3750	1512.3	270.3	236.5	3038.4	902.9	378.8
4000	1500.2	269.6	235.9	2962.8	889.2	362.3
4250	1526.7	278.4	243.6	3045.8	910.5	380.7
4500	1517.1	255.7	223.8	3033.7	908.9	365.5
4750	1545.8	268.4	234.8	3106.0	911.2	383.6
5000	1584.5	254.7	222.9	3199.7	932.7	380.6
5500	1468.5	275.6	241.1	2954.4	906.4	378.9
6000	1358.9	261.2	228.5	2718.1	841.5	354.1
6500	1517.2	264.5	231.5	3039.3	896.5	369.5
7000	1485.4	253.2	221.5	2971.4	891.3	356.2
7500	1394.3	264.6	231.5	2774.7	825.0	356.3
8000	1425.1	234.4	205.1	2844.9	845.2	335.2
8500	1474.8	265.9	232.7	2934.0	862.6	367.9
9000	1400.7	257.5	225.3	2786.2	841.7	350.2
9500	1341.6	254.7	222.8	2663.9	804.2	343.5
10000	1463.2	258.6	226.3	2927.7	849.1	368.9

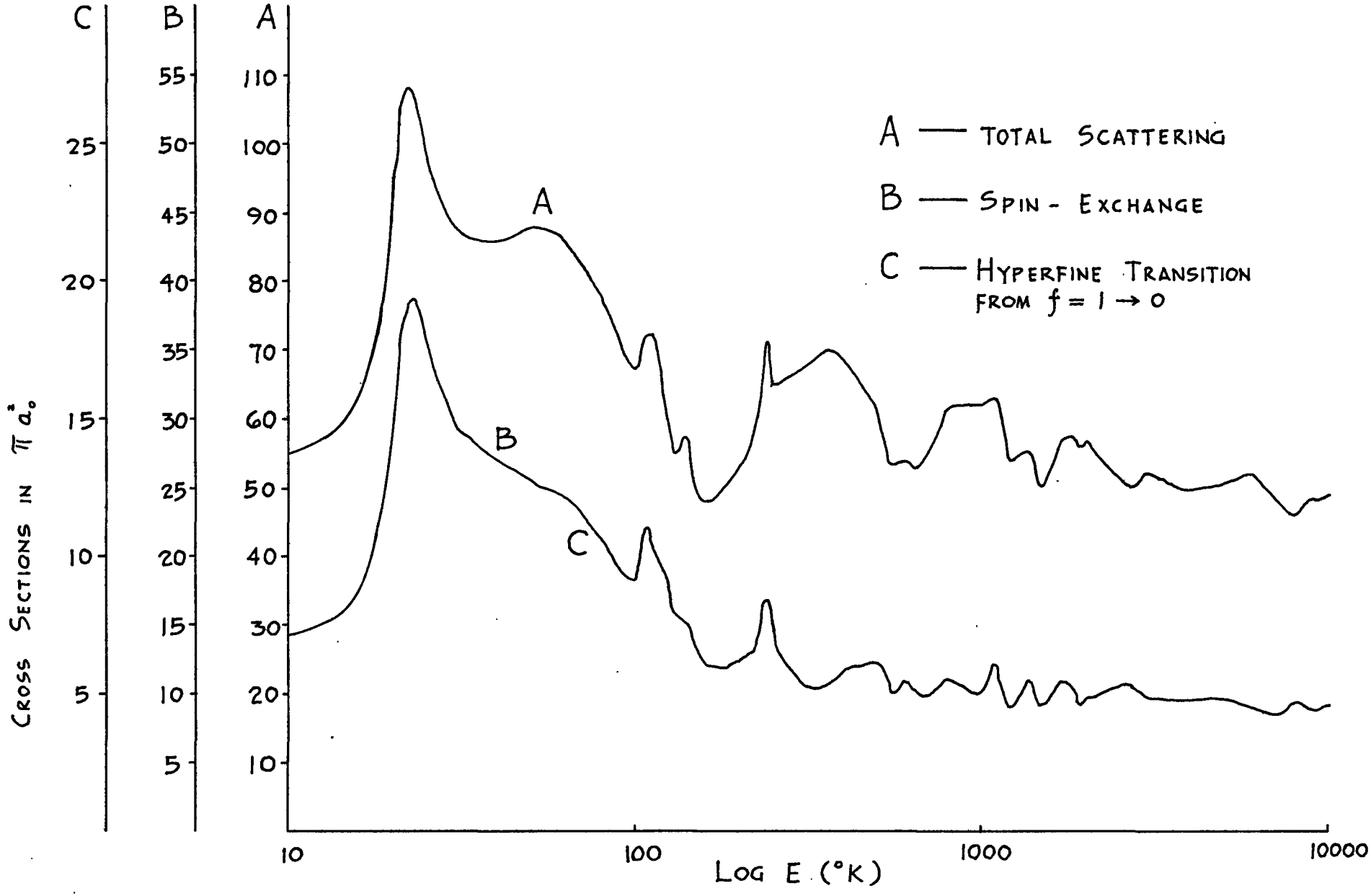

FIG. 5.1 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL HYDROGEN ATOM AS A FUNCTION OF RELATIVE ENERGY E

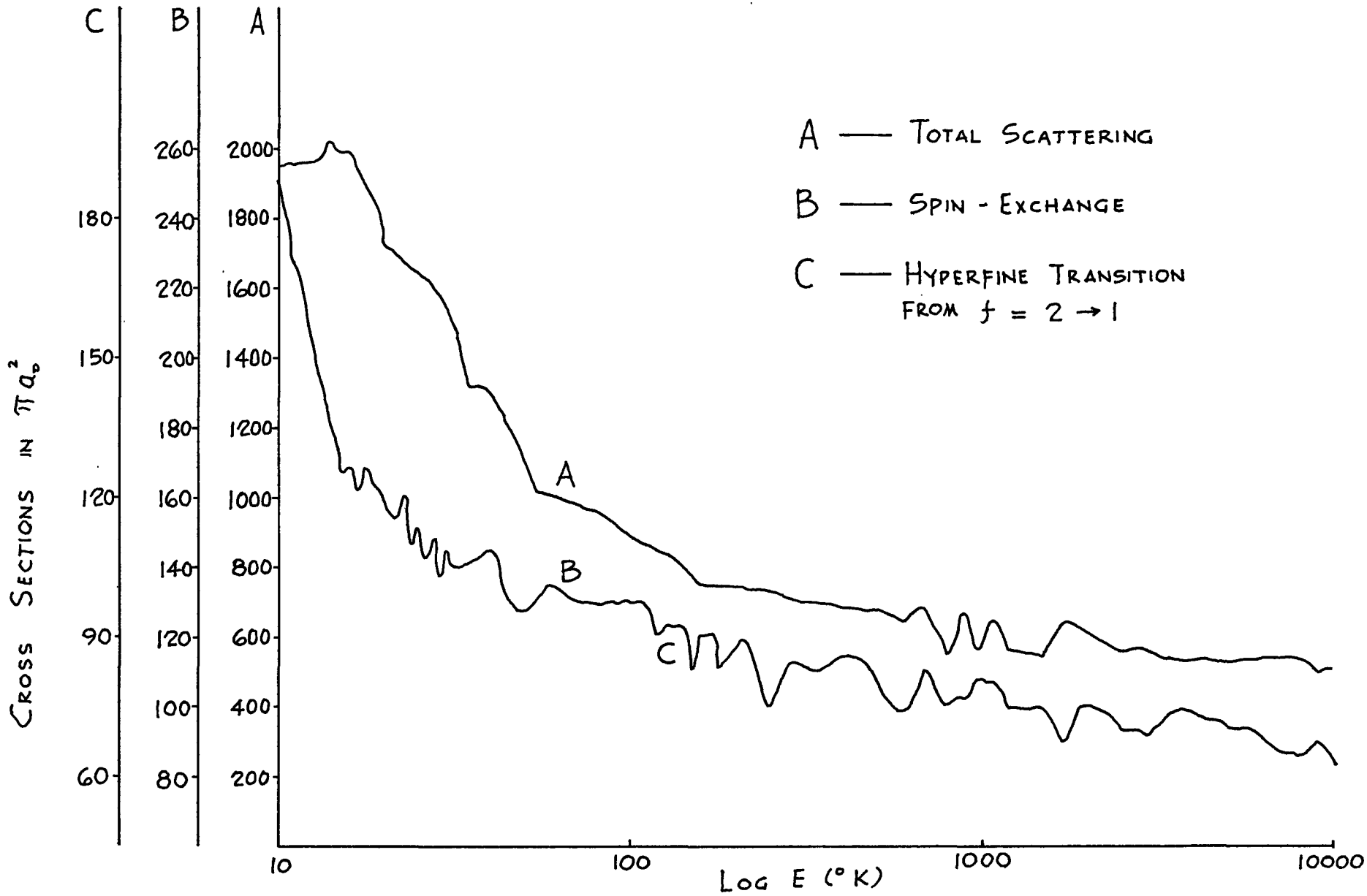

FIG. 5.2 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL LITHIUM ATOM AS A FUNCTION OF RELATIVE ENERGY E

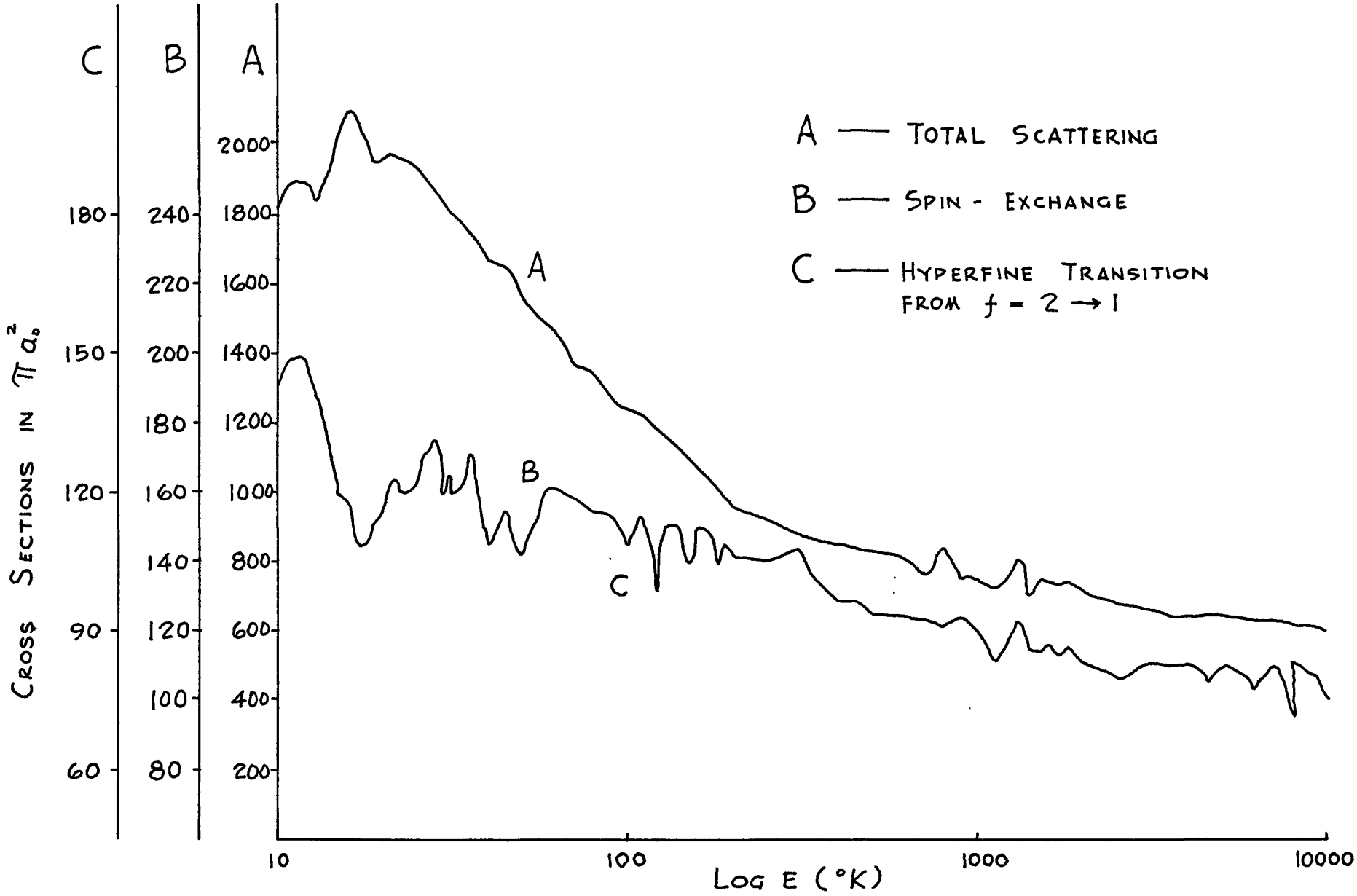

FIG. 5.3 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL SODIUM ATOM AS A FUNCTION OF RELATIVE ENERGY E

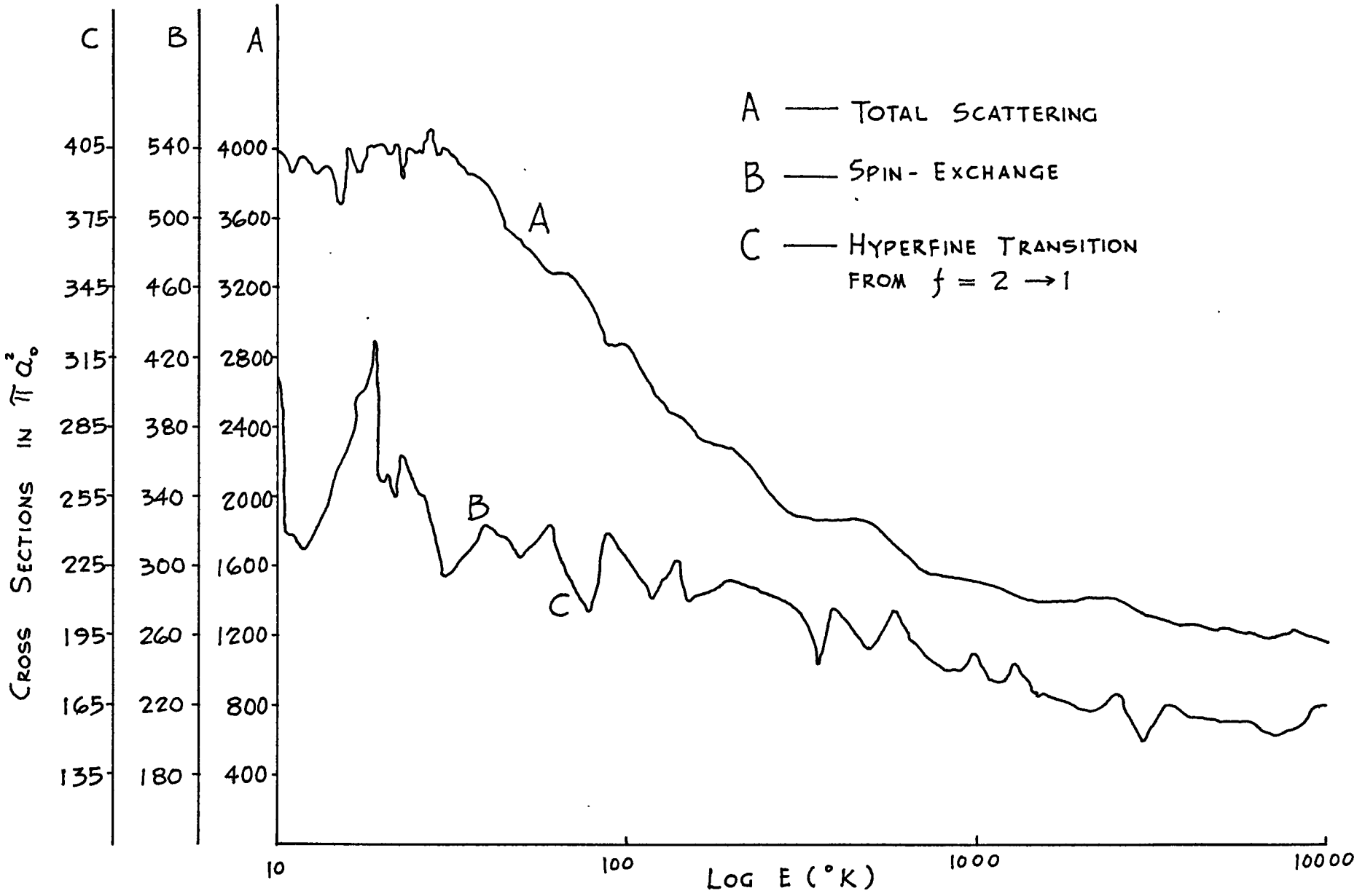

FIG. 5.4 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL POTASSIUM ATOM AS A FUNCTION OF RELATIVE ENERGY E

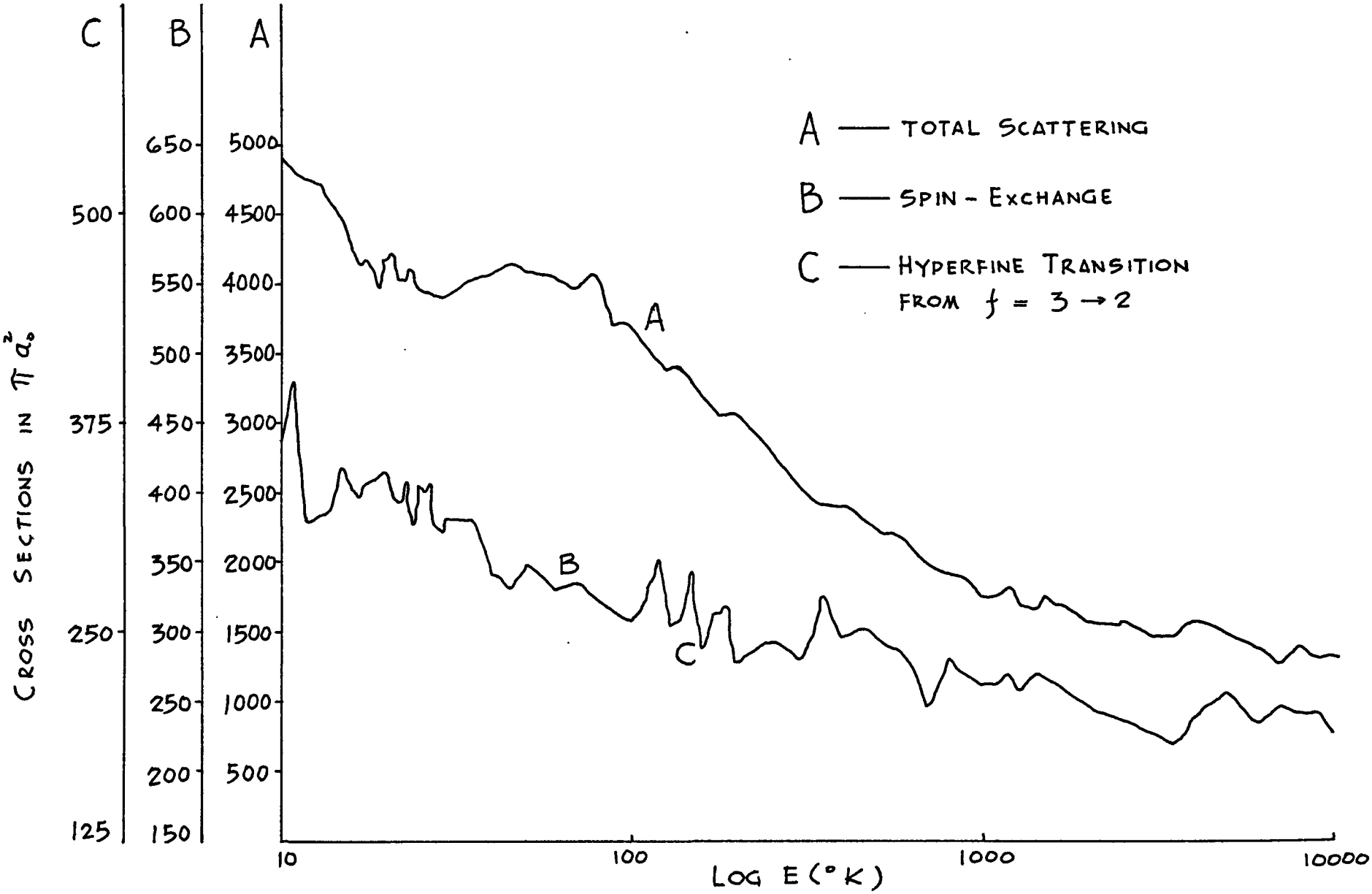

FIG. 5·5 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL RUBIDIUM ATOM AS A FUNCTION OF RELATIVE ENERGY E

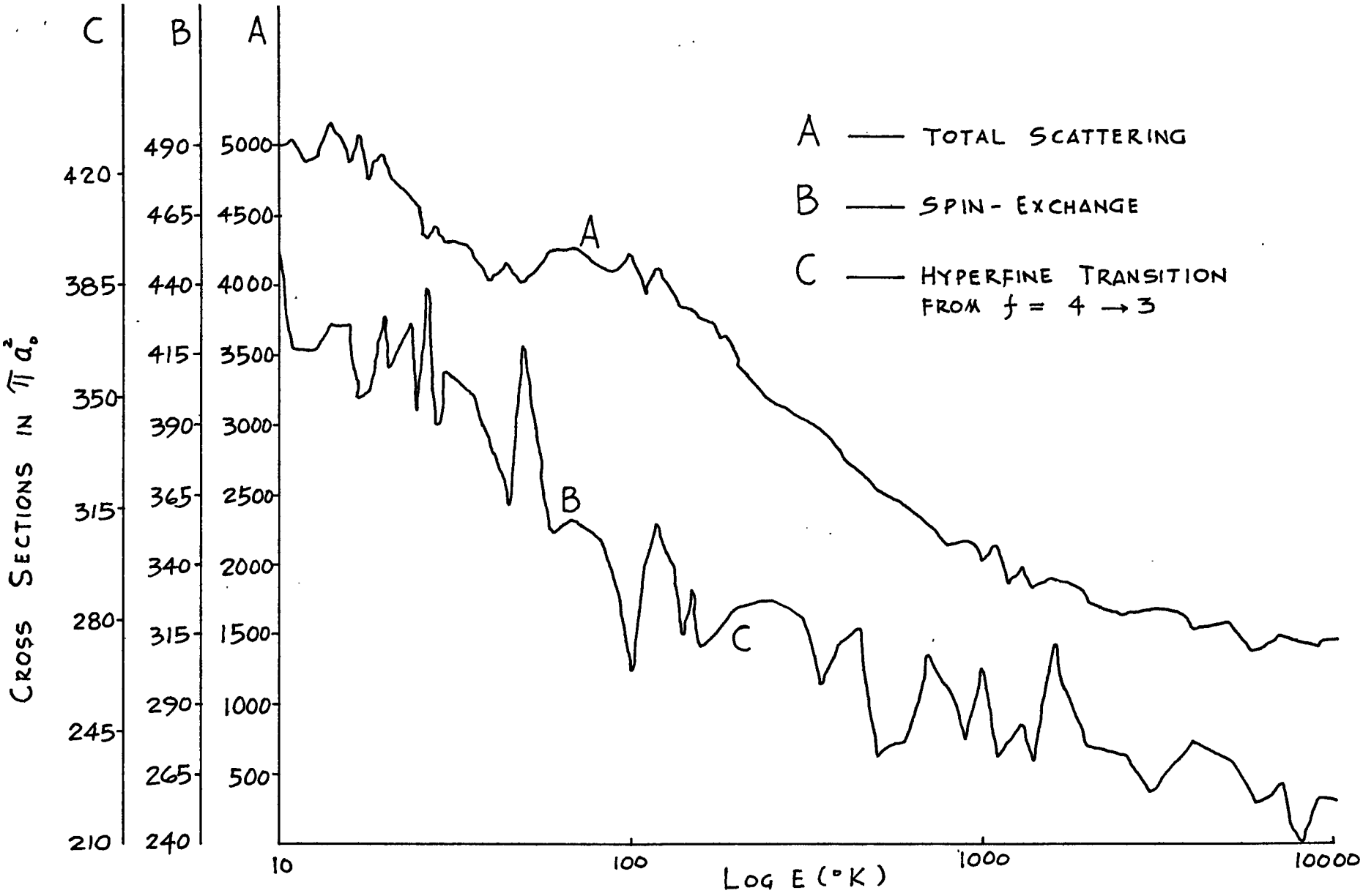

FIG. 5.6 SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL CESIUM ATOM AS A FUNCTION OF RELATIVE ENERGY E

TABLE 5.7

Averaged Spin-Dependent Cross Sections

of Hydrogen Atom ($\pi\ a_o^2$)

$\bar{\sigma}_{NU}$ Averaged total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{NY}$ Averaged hyperfine transition cross sections of nonidentical atom from $f=1 \rightarrow f'=0$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{IX}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{IY}$ Averaged hyperfine transition cross sections of identical atom from $f=1 \rightarrow f'=0$

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
10	72.31	21.67	10.83	151.48	48.03	28.21
11	73.69	22.30	11.15	152.16	48.75	28.58
12	74.89	22.84	11.42	152.74	49.38	28.90
13	75.94	23.31	11.65	153.24	49.95	29.19
14	76.86	23.71	11.85	153.67	50.45	29.44
15	77.66	24.05	12.02	154.06	50.89	29.66
16	78.37	24.33	12.16	154.41	51.29	29.86
17	78.98	24.58	12.29	154.72	51.65	30.03
18	79.52	24.78	12.39	155.01	51.97	30.19
19	79.99	24.95	12.47	155.27	52.26	30.33
21	80.75	25.21	12.60	155.73	52.76	30.56
22	81.05	25.30	12.65	155.92	52.96	30.66
23	81.31	25.37	12.68	156.10	53.14	30.74
24	81.53	25.42	12.71	156.25	53.30	30.81
25	81.71	25.46	12.73	156.39	53.44	30.87
26	81.86	25.49	12.74	156.50	53.56	30.93
27	81.98	25.50	12.75	156.60	53.65	30.97
28	82.08	25.50	12.75	156.67	53.73	31.01
29	82.14	25.50	12.75	156.72	53.80	31.03

TABLE 5.7 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
30	82.19	25.48	12.74	156.76	53.84	31.05
32	82.22	25.43	12.71	156.77	53.89	31.08
34	82.18	25.35	12.67	156.71	53.89	31.08
36	82.08	25.26	12.63	156.59	53.84	31.05
38	81.94	25.14	12.57	156.40	53.75	31.02
40	81.75	25.02	12.51	156.16	53.62	30.96
42	81.53	24.89	12.44	155.88	53.47	30.90
44	81.29	24.75	12.37	155.55	53.29	30.82
46	81.02	24.60	12.30	155.18	53.09	30.73
48	80.73	24.45	12.22	154.79	52.87	30.64
50	80.44	24.30	12.15	154.37	52.64	30.54
52	80.13	24.14	12.07	153.93	52.39	30.43
54	79.81	23.98	11.99	153.48	52.14	30.33
56	79.49	23.83	11.91	153.01	51.88	30.21
58	79.17	23.67	11.83	152.54	51.62	30.10
60	78.84	23.51	11.75	152.06	51.36	29.99
62	78.52	23.35	11.67	151.58	51.10	29.87
64	78.20	23.20	11.60	151.10	50.84	29.75
66	77.88	23.04	11.52	150.62	50.58	29.64
68	77.56	22.89	11.44	150.15	50.32	29.52
70	77.25	22.74	11.37	149.68	50.06	29.41
75	76.50	22.37	11.18	148.53	49.44	29.13
80	75.79	22.01	11.00	147.44	48.86	28.86
85	75.12	21.67	10.83	146.41	48.31	28.60
90	74.49	21.34	10.67	145.44	47.79	28.35
95	73.90	21.02	10.51	144.54	47.31	28.11
100	73.36	20.72	10.36	143.69	46.86	27.89
110	72.38	20.16	10.08	142.17	46.06	27.47
120	71.54	19.65	9.82	140.85	45.37	27.10
130	70.82	19.18	9.59	139.70	44.78	26.76
140	70.18	18.75	9.37	138.68	44.25	26.45
150	69.63	18.35	9.17	137.78	43.80	26.17
160	69.14	17.99	8.99	136.98	43.39	25.91
170	68.70	17.66	8.83	136.25	43.02	25.68
180	68.30	17.35	8.67	135.59	42.69	25.46
190	67.95	17.07	8.53	134.99	42.39	25.25
200	67.62	16.80	8.40	134.43	42.12	25.07
220	67.05	16.33	8.16	133.43	41.64	24.73
240	66.55	15.91	7.95	132.56	41.22	24.43
260	66.12	15.54	7.77	131.78	40.86	24.16
280	65.73	15.22	7.61	131.09	40.53	23.92

TABLE 5.7 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
300	65.37	14.93	7.46	130.46	40.24	23.71
320	65.05	14.67	7.33	129.88	39.98	23.52
340	64.76	14.43	7.21	129.34	39.74	23.34
360	64.48	14.21	7.10	128.84	39.52	23.18
380	64.23	14.02	7.01	128.37	39.32	23.03
400	63.99	13.84	6.92	127.93	39.12	22.89
420	63.76	13.68	6.84	127.50	38.94	22.76
440	63.55	13.52	6.76	127.10	38.77	22.64
460	63.34	13.38	6.69	126.72	38.61	22.52
480	63.15	13.25	6.62	126.35	38.46	22.42
500	62.96	13.13	6.56	126.00	38.31	22.31
550	62.53	12.86	6.43	125.16	37.97	22.08
600	62.14	12.62	6.31	124.39	37.66	21.87
650	61.77	12.42	6.21	123.68	37.37	21.68
700	61.43	12.24	6.12	123.01	37.11	21.51
750	61.12	12.09	6.04	122.38	36.86	21.35
800	60.82	11.95	5.97	121.79	36.63	21.21
850	60.54	11.82	5.91	121.23	36.42	21.08
900	60.28	11.71	5.85	120.71	36.22	20.96
950	60.03	11.60	5.80	120.21	36.03	20.84
1000	59.80	11.51	5.75	119.74	35.86	20.74
1100	59.36	11.34	5.67	118.88	35.54	20.55
1200	58.97	11.20	5.60	118.10	35.26	20.38
1300	58.61	11.07	5.53	117.39	35.00	20.23
1400	58.28	10.96	5.48	116.75	34.77	20.10
1500	57.98	10.87	5.43	116.16	34.57	19.98
1600	57.70	10.78	5.39	115.61	34.38	19.87
1700	57.44	10.71	5.35	115.11	34.20	19.77
1800	57.20	10.64	5.32	114.64	34.04	19.68
1900	56.98	10.58	5.29	114.20	33.89	19.59
2000	56.76	10.52	5.26	113.79	33.75	19.51
2200	56.38	10.42	5.21	113.04	33.50	19.37
2400	56.04	10.34	5.17	112.37	33.28	19.25
2600	55.73	10.27	5.13	111.77	33.09	19.14
2800	55.46	10.20	5.10	111.23	32.91	19.04
3000	55.21	10.15	5.07	110.74	32.75	18.95
3500	54.68	10.03	5.01	109.71	32.42	18.77
4500	53.93	9.88	4.94	108.21	31.95	18.50
5000	53.65	9.83	4.91	107.66	31.78	18.41
5500	53.42	9.78	4.89	107.20	31.64	18.33
6000	53.23	9.75	4.87	106.82	31.52	18.26
6500	53.06	9.72	4.86	106.49	31.42	18.21

TABLE 5.7 (Continued)

T (^{o}k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
7000	52.92	9.69	4.84	106.20	31.33	18.16
7500	52.80	9.67	4.83	105.96	31.26	18.12
8000	52.69	9.65	4.82	105.74	31.19	18.08
8500	52.59	9.63	4.81	105.55	31.13	18.05
9000	52.51	9.62	4.81	105.38	31.08	18.02
9500	52.43	9.61	4.80	105.23	31.04	18.00
10000	52.36	9.60	4.80	105.09	30.99	17.97

TABLE 5.8

Averaged Spin-Dependent Cross Sections

of Lithium Atom (πa_o^2)

$\bar{\sigma}_{NU}$ Averaged total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{NY}$ Averaged hyperfine transition cross sections of nonidentical atom from $f=2 \rightarrow f=1$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{IX}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{IY}$ Averaged hyperfine transition cross sections of identical atom from $f=2 \rightarrow f=1$

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
10	1845.1	181.2	135.9	3690.0	894.3	448.4
11	1823.7	178.5	133.9	3647.4	885.9	442.7
12	1802.5	176.1	132.1	3604.9	877.4	437.2
13	1781.5	173.9	130.4	3562.9	868.8	431.9
14	1760.8	171.9	128.9	3521.6	860.3	426.8
15	1740.5	170.1	127.6	3481.2	851.8	422.0
16	1720.7	168.4	126.3	3441.6	843.6	417.3
17	1701.4	166.8	125.1	3403.0	835.5	412.8
18	1682.7	165.4	124.0	3365.5	827.6	408.5
19	1664.4	164.0	123.0	3329.0	820.0	404.3
20	1646.6	162.7	122.0	3293.5	812.5	400.3
21	1629.4	161.5	121.1	3259.1	805.3	396.5
22	1612.7	160.4	120.3	3225.7	798.3	392.8
23	1596.5	159.3	119.5	3193.3	791.5	389.2
24	1580.9	158.3	118.7	3161.9	784.9	385.8
25	1565.6	157.4	118.0	3131.5	778.5	382.5
26	1550.9	156.5	117.3	3102.0	772.3	379.3
27	1536.6	155.6	116.7	3073.4	766.4	376.2
28	1522.8	154.8	116.1	3045.7	760.6	373.2
29	1509.4	154.0	115.5	3018.9	755.0	370.4

TABLE 5.8 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{Iu}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
30	1496.4	153.3	115.0	2992.9	749.5	367.6
31	1483.8	152.6	114.4	2967.7	744.3	364.9
32	1471.5	151.9	113.9	2943.2	739.2	362.3
33	1459.7	151.3	113.4	2919.5	734.2	359.8
34	1448.2	150.6	113.0	2896.4	729.4	357.4
35	1437.0	150.0	112.5	2874.1	724.8	355.0
36	1426.1	149.5	112.1	2852.4	720.2	352.7
37	1415.6	148.9	111.7	2831.3	715.8	350.5
38	1405.3	148.4	111.3	2810.8	711.6	348.4
39	1395.4	147.9	110.9	2790.9	707.4	346.3
40	1385.7	147.4	110.5	2771.5	703.4	344.3
42	1367.1	146.5	109.8	2734.3	695.7	340.4
44	1349.5	145.6	109.2	2699.1	688.4	336.7
45	1341.1	145.2	108.9	2682.2	684.9	335.0
46	1332.8	144.8	108.6	2665.7	681.4	333.3
48	1316.9	144.0	108.0	2633.9	674.9	330.0
50	1301.8	143.3	107.4	2603.7	668.6	326.9
52	1287.5	142.6	106.9	2574.9	662.6	323.9
54	1273.7	141.9	106.4	2547.5	656.9	321.1
55	1267.1	141.6	106.2	2534.2	654.2	319.7
56	1260.6	141.3	106.0	2521.2	651.5	318.4
58	1248.1	140.7	105.5	2496.2	646.3	315.8
60	1236.1	140.1	105.1	2472.1	641.3	313.3
62	1224.6	139.6	104.7	2449.1	636.5	311.0
64	1213.6	139.1	104.3	2427.1	631.1	308.7
65	1208.2	138.8	104.1	2416.4	629.7	307.6
66	1203.0	138.6	103.9	2405.9	627.5	306.5
68	1192.8	138.1	103.6	2385.5	623.2	304.4
70	1183.0	137.7	103.2	2365.9	619.2	302.4
75	1160.1	136.6	102.4	2320.1	609.6	297.7
80	1139.2	135.6	101.7	2278.1	600.8	293.4
85	1119.9	134.7	101.0	2239.6	592.7	289.4
90	1102.2	133.9	100.4	2204.1	585.3	285.8
95	1085.7	133.1	99.8	2171.2	578.4	282.4
100	1070.5	132.4	99.3	2140.6	571.9	279.2
110	1043.0	131.1	98.3	2085.4	560.2	273.5
120	1018.8	129.9	97.4	2036.9	549.9	268.5
130	997.3	128.9	96.6	1993.9	540.6	264.0
140	978.1	127.9	95.9	1955.4	532.3	260.0
150	960.7	127.0	95.3	1920.6	524.8	256.4
160	945.0	126.2	94.6	1889.1	517.9	253.1
170	930.7	125.5	94.1	1860.3	511.5	250.1

TABLE 5.8 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
180	917.5	124.8	93.6	1833.9	505.7	247.3
190	905.4	124.1	93.1	1809.6	500.3	244.7
200	894.2	123.5	92.6	1787.0	495.3	242.3
210	883.8	122.9	92.2	1766.2	490.6	240.1
230	865.0	121.9	91.4	1728.5	482.0	236.1
250	848.5	120.9	90.7	1695.5	474.5	232.6
270	833.9	120.1	90.0	1666.2	467.7	229.5
290	820.9	119.3	89.4	1640.1	461.6	226.7
310	809.2	118.5	88.9	1616.5	456.1	224.2
325	801.1	118.0	88.5	1600.4	452.2	222.4
350	788.8	117.2	87.9	1575.8	446.4	219.8
370	779.9	116.6	87.5	1558.0	442.1	217.8
390	771.8	116.1	87.0	1541.7	438.1	216.0
405	766.0	115.7	86.7	1530.2	435.3	214.8
410	764.2	115.5	86.6	1526.6	434.4	214.4
430	757.2	115.1	86.3	1512.6	431.0	212.8
450	750.7	114.6	85.9	1499.5	427.8	211.4
470	744.6	114.1	85.6	1487.4	424.8	210.0
490	738.9	113.7	85.3	1476.0	421.9	208.8
510	733.6	113.3	85.0	1465.4	419.2	207.6
540	726.2	112.7	84.5	1450.5	415.5	205.9
600	713.1	111.6	83.7	1424.5	408.8	203.0
650	703.7	110.8	83.1	1405.7	403.9	200.8
700	695.4	110.1	82.5	1389.1	399.6	198.9
750	688.0	109.4	82.0	1374.2	395.6	197.1
800	681.3	108.7	81.5	1360.9	392.1	195.5
850	675.3	108.1	81.1	1348.9	388.8	194.1
900	669.7	107.6	80.7	1337.9	385.8	192.8
950	664.7	107.1	80.3	1327.8	383.1	191.6
1000	660.1	106.6	79.9	1318.6	380.5	190.4
1100	651.8	105.7	79.2	1302.2	375.9	188.4
1200	644.6	104.9	78.6	1287.9	371.9	186.6
1300	638.3	104.2	78.1	1275.4	368.3	185.1
1400	632.7	103.5	77.6	1264.3	365.1	183.7
1500	627.7	102.9	77.2	1254.3	362.2	182.4
1600	623.1	102.4	76.8	1245.3	359.5	181.3
1700	619.0	101.9	76.4	1237.1	357.1	180.2
1800	615.2	101.4	76.0	1229.6	354.9	179.3
1900	611.8	101.0	75.7	1222.8	352.8	178.4
2000	608.6	100.6	75.4	1216.4	350.9	177.6

TABLE 5.8 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
2250	601.6	99.7	74.8	1202.5	346.8	175.8
2500	595.8	99.0	74.2	1190.8	343.2	174.3
2750	590.8	98.3	73.7	1180.9	340.2	173.0
3000	586.5	97.8	73.3	1172.4	337.7	171.9
3250	582.9	97.3	73.0	1165.1	335.4	171.0
3500	579.7	96.9	72.6	1158.7	333.5	170.1
3750	576.9	96.5	72.4	1153.1	331.8	169.4
4000	574.4	96.2	72.1	1148.2	330.3	168.8
4250	572.2	95.9	71.9	1143.8	329.0	168.2
4500	570.3	95.6	71.7	1139.9	327.8	167.7
4750	568.5	95.4	71.6	1136.4	326.7	167.2
5000	566.9	95.2	71.4	1133.2	325.8	166.8
5500	564.2	94.9	71.1	1127.8	324.1	166.1
6000	562.0	94.6	70.9	1123.3	322.7	165.5
6500	560.1	94.3	70.7	1119.5	321.6	165.0
7000	558.4	94.1	70.6	1116.2	320.6	164.6
7500	557.0	93.9	70.4	1113.5	319.8	164.2
8000	555.8	93.8	70.3	1111.0	319.1	163.9
8500	554.8	93.6	70.2	1108.9	318.4	163.6
9000	553.8	93.5	70.1	1107.1	317.9	163.4
9500	553.0	93.4	70.0	1105.4	317.4	163.2

TABLE 5.9

Averaged Spin-Dependent Cross Sections

of Sodium Atom (πa_o^2)

$\bar{\sigma}_{NU}$ Averaged total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{Ny}$ Averaged hyperfine transition cross sections of nonidentical atom from $f = 2 \rightarrow f = 1$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{IX}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{Iy}$ Averaged hyperfine transition cross sections of identical atom from $f = 2 \rightarrow f = 1$

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
10	1916.9	171.4	128.6	3827.1	967.8	449.8
11	1912.4	170.4	127.8	3818.6	964.2	448.4
12	1906.8	169.5	127.1	3807.7	960.5	446.8
13	1900.3	168.7	126.5	3795.0	956.3	445.2
14	1893.1	168.0	126.0	3780.9	952.4	443.5
15	1885.4	167.4	125.5	3765.7	948.1	441.8
16	1877.2	166.8	125.1	3749.5	943.8	440.1
17	1868.7	166.3	124.7	3732.8	939.5	438.3
18	1859.9	165.8	124.3	3715.5	935.1	436.5
19	1851.0	165.3	124.0	3697.8	930.7	434.7
20	1841.9	164.9	123.7	3679.9	926.3	432.9
21	1832.8	164.5	123.4	3661.8	921.9	431.1
22	1823.6	164.1	123.1	3643.7	917.6	429.3
23	1814.5	163.8	122.8	3625.5	913.3	427.5
24	1805.3	163.4	122.5	3607.4	909.1	425.7
25	1796.2	163.1	122.3	3589.4	904.9	424.0
26	1787.2	162.8	122.1	3571.5	900.8	422.3
27	1778.3	162.5	121.8	3553.7	896.7	420.6
28	1769.4	162.2	121.6	3536.2	892.7	418.9
29	1760.6	161.9	121.4	3518.8	888.8	417.2

TABLE 5.9 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
30	1752.0	161.6	121.2	3501.6	884.9	415.6
31	1743.5	161.3	121.0	3484.7	881.1	414.0
32	1735.0	161.1	120.8	3468.0	877.4	412.4
33	1726.7	160.8	120.6	3451.5	873.7	410.8
34	1718.5	160.6	120.4	3435.3	870.1	409.3
35	1710.5	160.3	120.2	3419.3	866.6	407.8
36	1702.6	160.1	120.1	3403.6	863.1	406.3
37	1694.7	159.9	119.9	3388.1	859.7	404.8
38	1687.1	159.7	119.7	3372.8	856.3	403.4
39	1679.5	159.5	119.6	3357.8	853.0	402.0
40	1672.1	159.2	119.4	3343.0	849.8	400.6
42	1657.5	158.8	119.1	3314.2	843.5	397.9
44	1643.5	158.5	118.8	3286.2	837.5	395.3
45	1636.6	158.3	118.7	3272.6	834.5	394.0
46	1629.8	158.1	118.6	3259.1	831.6	392.7
48	1616.6	157.7	118.3	3232.9	825.9	390.3
50	1603.9	157.4	118.0	3207.5	820.4	387.9
52	1591.5	157.1	117.8	3182.9	815.1	385.6
54	1579.5	156.8	117.6	3159.0	810.0	383.3
55	1573.6	156.6	117.4	3147.3	807.5	382.2
56	1567.8	156.5	117.3	3135.8	805.0	381.1
58	1556.5	156.2	117.1	3113.3	800.2	379.0
60	1545.5	155.9	116.9	3091.5	795.6	377.0
62	1534.9	155.6	116.7	3070.2	791.0	375.0
64	1524.5	155.4	116.5	3049.6	786.6	373.1
65	1519.5	155.2	116.4	3039.5	784.5	372.1
66	1514.5	155.1	116.3	3029.5	782.3	371.2
68	1504.7	154.8	116.1	3010.0	778.2	369.3
70	1495.1	154.6	115.9	2991.0	774.1	367.6
75	1472.4	154.0	115.5	2945.6	764.4	363.3
80	1451.1	153.5	115.1	2903.0	755.4	359.3
85	1431.1	153.0	114.7	2863.0	746.8	355.5
90	1412.2	152.5	114.4	2825.2	738.8	352.0
95	1394.4	152.0	114.0	2789.6	731.2	348.6
100	1377.6	151.6	113.7	2755.9	724.0	345.4
110	1346.5	150.8	113.1	2693.7	710.7	339.6
120	1318.5	150.0	112.5	2637.5	698.6	334.2
130	1293.0	149.3	112.0	2586.4	687.6	329.4
140	1269.7	148.6	111.4	2539.8	677.5	324.9
150	1248.4	148.0	111.0	2497.0	668.2	320.8
160	1228.7	147.4	110.5	2457.5	659.7	317.0
170	1210.5	146.8	110.1	2421.1	651.7	313.5

TABLE 5.9 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
180	1193.7	146.2	109.6	2387.3	644.3	310.2
190	1178.0	145.7	109.2	2355.8	637.4	307.1
200	1163.3	145.1	108.8	2326.4	630.9	304.2
210	1149.6	144.6	108.5	2299.0	624.9	301.5
230	1124.7	143.7	107.7	2249.0	613.8	296.6
250	1102.6	142.8	107.1	2204.6	603.8	292.1
270	1082.8	141.9	106.4	2164.8	594.9	288.1
290	1064.9	141.1	105.8	2129.0	586.8	284.5
310	1048.7	140.4	105.3	2096.5	579.4	281.1
325	1037.5	139.8	104.8	2074.0	574.3	278.8
350	1020.4	138.9	104.2	2039.7	566.4	275.2
370	1007.9	138.3	103.7	2014.6	560.6	272.6
390	996.4	137.6	103.2	1991.4	555.2	270.1
405	988.3	137.2	102.9	1975.1	551.3	268.4
410	985.7	137.0	102.8	1969.9	550.1	267.8
430	975.7	136.4	102.3	1949.8	545.4	265.7
450	966.3	135.9	101.9	1931.0	541.0	263.6
470	957.6	135.3	101.5	1913.4	536.8	261.7
490	949.3	134.8	101.1	1896.9	532.9	259.9
510	941.6	134.3	100.7	1881.3	529.1	258.2
540	930.8	133.6	100.2	1859.5	523.9	255.8
600	911.5	132.3	99.2	1820.8	514.5	251.6
650	897.6	131.3	98.5	1792.7	507.6	248.4
700	885.1	130.4	97.8	1767.6	501.4	245.6
750	873.9	129.5	97.1	1745.0	495.8	243.1
800	863.8	128.7	96.5	1724.7	490.7	240.8
850	854.6	128.0	96.0	1706.1	486.0	238.7
900	846.1	127.3	95.5	1689.2	481.7	236.7
950	838.4	126.7	95.0	1673.6	477.8	234.9
1000	831.2	126.1	94.5	1659.2	474.1	233.3
1100	181.4	125.0	93.7	1633.6	467.4	230.3
1200	807.3	124.0	93.0	1611.2	461.6	227.8
1300	797.5	123.1	92.3	1591.5	456.4	225.5
1400	788.7	122.3	91.7	1574.0	451.7	223.4
1500	780.9	121.6	91.2	1558.3	447.5	221.6
1600	773.8	121.0	90.7	1544.2	443.7	220.0
1700	767.4	120.4	90.3	1531.3	440.2	218.4
1800	761.6	119.8	89.8	1519.5	437.0	217.1
1900	756.2	119.3	89.4	1508.7	434.0	215.8
2000	751.2	118.8	89.1	1498.8	431.2	214.6
2250	740.4	117.7	88.3	1477.2	425.2	212.1

TABLE 5.9 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
2500	731.5	116.8	87.66	1459.2	420.1	209.9
2750	723.9	116.1	87.0	1444.0	415.8	208.1
3000	717.5	115.4	86.5	1431.1	412.1	206.6
3250	712.1	114.8	86.1	1420.1	408.9	205.3
3500	707.3	114.3	85.7	1410.5	406.2	204.1
3750	703.2	113.9	85.4	1402.2	403.7	203.1
4000	699.6	113.5	85.1	1395.0	401.6	202.3
4250	696.4	113.2	84.9	1388.6	399.7	201.5
4500	693.6	112.9	84.7	1382.9	398.1	200.8
4750	691.1	112.6	84.4	1377.9	396.6	200.2
5000	688.8	112.4	84.3	1373.4	395.2	199.7
5500	685.0	112.0	84.0	1365.6	392.9	198.7
6000	681.8	111.6	83.7	1359.3	391.0	197.9
6500	679.2	111.3	83.5	1354.0	389.4	197.3
7000	677.0	111.1	83.3	1349.5	388.1	196.7
7500	675.1	110.9	83.1	1345.7	386.9	196.3
8000	673.4	110.7	83.0	1342.4	385.9	195.9
8500	672.0	110.5	82.9	1339.5	385.0	195.5
9000	670.7	110.4	82.8	1337.0	384.3	195.2
9500	669.6	110.2	82.7	1334.8	383.6	194.9

TABLE 5.10

Averaged Spin-Dependent Cross Sections

of Potassium Atom (πa_o^2)

$\bar{\sigma}_{NU}$ Averaged total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{Ny}$ Averaged hyperfine transition cross sections of nonidentical atom from $f = 2 \rightarrow f' = 1$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{Ix}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{Iy}$ Averaged hyperfine transition cross sections of identical atom from $f = 2 \rightarrow f' = 1$

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
10	3911.2	349.1	261.8	7824.2	1965.4	919.6
11	3909.0	348.2	261.1	7819.7	1962.6	918.6
12	3905.6	347.1	260.3	7812.8	1959.4	917.3
13	3901.0	345.9	259.4	7803.7	1955.7	915.7
14	3895.5	344.7	258.5	7792.5	1951.8	913.9
15	3889.0	343.5	257.6	7779.4	1947.5	911.9
16	3881.6	342.3	256.7	7764.6	1943.0	909.8
17	3873.6	341.1	255.8	7748.4	1938.3	907.5
18	3864.8	339.9	254.9	7730.8	1933.3	905.1
19	3855.5	338.7	254.0	7712.2	1928.2	902.7
20	3845.7	337.6	253.2	7692.5	1923.0	900.2
21	3835.5	336.5	252.4	7671.9	1917.6	897.6
22	3824.9	335.5	251.6	7650.7	1912.1	895.0
23	3814.0	334.4	250.8	7628.7	1906.6	892.4
24	3802.8	333.5	250.1	7606.3	1901.0	889.7
25	3791.4	332.5	249.4	7583.4	1895.3	887.1
26	3779.8	331.6	248.7	7560.1	1889.6	884.4
27	3768.1	330.7	248.0	7536.6	1883.9	881.7
28	3756.2	329.8	247.3	7512.8	1878.1	879.1
29	3744.3	329.0	246.7	7488.8	1872.4	876.4

TABLE 5.10 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{Iu}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
30	3732.2	328.2	246.1	7464.6	1866.6	873.7
31	3720.2	327.4	245.5	7440.4	1860.9	871.1
32	3708.0	326.6	245.0	7416.1	1855.1	868.5
33	3695.9	325.9	244.4	7391.8	1849.4	865.8
34	3683.8	325.2	243.9	7367.5	1843.7	863.2
35	3671.7	324.5	243.4	7343.2	1838.1	860.7
36	3659.6	323.8	242.9	7318.9	1832.4	858.1
37	3647.5	323.2	242.4	7294.8	1826.9	855.6
38	3635.5	322.6	241.9	7270.7	1821.3	853.1
39	3623.5	322.0	241.5	7246.7	1815.8	850.6
40	3611.6	321.4	241.0	7222.9	1810.3	848.1
42	3588.0	320.2	240.2	7175.5	1799.5	843.2
44	3564.7	319.1	239.3	7128.8	1788.9	838.5
45	3553.1	318.6	239.0	7105.7	1783.6	836.1
46	3541.7	318.1	238.6	7082.7	1778.4	833.8
48	3519.0	317.1	237.8	7037.3	1768.2	829.2
50	3496.7	316.2	237.1	6992.7	1758.1	824.7
52	3474.7	315.3	236.5	6948.8	1748.3	820.3
54	3453.2	314.4	235.8	6905.6	1738.6	816.0
55	3442.5	314.0	235.5	6884.3	1733.9	813.9
56	3432.0	313.6	235.2	6863.2	1729.1	811.8
58	3411.2	312.8	234.6	6821.5	1719.9	807.7
60	3390.7	312.0	234.0	6780.6	1710.8	803.6
62	3370.7	311.3	233.5	6740.5	1701.9	799.7
64	3351.0	310.6	232.9	6701.1	1693.2	795.8
65	3341.3	310.2	232.7	6681.7	1688.9	793.9
66	3331.6	309.9	232.4	6662.5	1684.6	792.0
68	3312.7	309.2	231.9	6624.6	1676.3	788.3
70	3294.1	308.6	231.4	6587.4	1668.1	784.7
75	3249.1	307.0	230.2	6497.5	1648.3	775.9
80	3206.2	305.6	229.2	6411.7	1629.5	767.6
85	3165.3	304.2	228.1	6329.9	1611.6	759.7
90	3126.2	302.9	227.2	6251.8	1594.6	752.1
95	3088.9	301.7	226.3	6177.2	1578.3	745.0
100	3053.2	300.6	225.4	6105.8	1562.8	738.1
110	2986.4	298.5	223.8	5972.2	1533.8	725.3
120	2925.0	296.6	222.4	5849.3	1507.3	713.5
130	2868.4	294.8	221.1	5736.0	1482.8	702.7
140	2816.0	293.2	219.9	5631.2	1460.3	692.6
150	2767.4	291.7	218.7	5533.8	1439.3	683.3
160	2722.2	290.2	217.7	5443.2	1419.8	674.7
170	2680.0	288.9	216.7	5358.6	1401.6	666.6

TABLE 5.10 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{ru}$	$\bar{\sigma}_{rx}$	$\bar{\sigma}_{ry}$
180	2640.5	287.7	215.7	5279.5	1384.6	659.0
190	2603.4	286.5	214.9	5205.2	1368.6	651.9
200	2568.6	285.0	214.0	5135.4	1353.6	645.2
210	2535.8	284.3	213.2	5069.6	1339.4	638.9
230	2475.5	282.3	211.7	4948.8	1313.3	627.2
250	2421.4	280.5	210.4	4840.3	1289.8	616.8
270	2372.6	278.8	209.1	4742.3	1268.6	607.3
290	2328.2	277.2	207.9	4653.2	2149.1	598.6
310	2287.6	275.7	206.8	4571.8	1231.3	590.6
325	2259.4	274.7	206.0	4515.3	1218.9	585.1
350	2216.1	273.0	204.8	4428.3	1199.8	576.5
370	2184.3	271.8	203.8	4364.7	1185.7	570.2
390	2154.8	270.6	202.9	4305.5	1172.5	564.3
405	2134.1	269.8	202.3	4263.9	1163.2	560.1
410	2127.4	269.5	202.1	4250.4	1160.2	558.8
430	2101.7	268.4	201.3	4199.0	1148.7	553.6
450	2077.7	267.4	200.5	4150.7	1137.9	548.8
470	2055.1	266.4	199.8	4105.5	1127.7	544.2
490	2033.8	265.4	199.1	4062.8	1118.0	539.9
510	2013.7	264.5	198.4	4022.6	1108.9	535.8
540	1985.6	263.2	197.4	3966.3	1096.1	530.1
600	1935.7	260.8	195.6	3866.3	1073.2	519.8
650	1899.3	258.9	194.2	3793.6	1056.4	512.2
700	1866.8	257.2	192.9	3728.7	1041.3	505.5
750	1837.6	255.6	191.7	3670.3	1027.7	499.3
800	1811.2	254.1	190.6	3617.5	1015.3	493.8
850	1787.1	252.7	189.5	3569.5	1003.9	488.7
900	1765.1	251.4	188.5	3525.6	993.5	484.0
950	1744.8	250.1	187.6	3485.3	983.8	479.7
1000	1726.1	249.0	186.7	3448.0	974.9	475.6
1100	1692.7	246.8	185.1	3381.4	958.8	468.4
1200	1663.5	244.9	183.6	3323.5	944.6	462.1
1300	1637.8	243.1	182.3	3272.5	932.1	456.5
1400	1615.0	241.5	181.1	3227.2	920.8	451.5
1500	1594.5	240.0	180.0	3186.6	910.6	447.0
1600	1576.0	238.7	179.0	3150.0	901.3	443.0
1700	1559.2	237.4	178.1	3116.7	892.9	439.3
1800	1543.8	236.3	177.2	3086.4	885.1	435.9
1900	1529.7	235.2	176.4	3058.6	877.9	432.8
2000	1516.8	234.2	175.7	3033.1	871.3	429.9
2250	1488.5	232.0	174.0	2977.6	856.7	423.6

TABLE 5.10 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
2500	1465.1	230.1	172.6	2931.5	844.4	418.4
2750	1445.4	228.5	171.4	2892.9	834.0	414.0
3000	1428.6	227.1	170.3	2860.2	825.1	410.3
3250	1414.3	225.9	169.4	2832.2	817.5	407.1
3500	1401.9	224.8	168.6	2808.1	810.8	404.3
3750	1391.1	223.9	167.9	2787.2	805.0	401.9
4000	1381.7	223.1	167.3	2768.9	799.8	399.8
4250	1373.4	222.4	166.8	2752.8	795.3	398.0
4500	1366.1	221.7	166.3	2738.6	791.3	396.3
4750	1359.6	221.1	165.8	2725.9	787.7	394.8
5000	1353.7	220.6	165.4	2714.6	784.4	393.5
5500	1343.7	219.7	164.7	2695.3	778.9	391.3
6000	1335.5	218.9	164.2	2679.4	774.3	389.4
6500	1328.7	218.3	163.7	2666.2	770.4	387.9
7000	1322.9	217.8	163.3	2655.0	767.1	386.6
7500	1318.0	217.3	163.0	2645.5	764.3	385.4
8000	1313.7	216.9	162.7	2637.3	761.9	384.5
8500	1310.0	216.5	162.4	2630.2	759.8	383.6
9000	1306.7	216.2	162.2	2623.9	757.9	382.9
9500	1303.9	215.9	161.9	2618.3	756.3	382.2

TABLE 5.11

Averaged Spin-Dependent Cross Sections

Rubidium Atom (πa_o^2)

$\bar{\sigma}_{NU}$ Averated total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{Ny}$ Averaged hyperfine transition cross sections of nonidentical atom from $f = 3 \rightarrow f' = 2$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{Ix}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{Iy}$ Averaged hyperfine transition cross sections of identical atom from $f = 3 \rightarrow f' = 2$

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
10	4384.6	403.8	336.5	8768.3	2181.2	862.9
11	4356.7	401.9	334.9	8712.6	2169.5	857.9
12	4332.0	400.1	333.4	8663.5	2159.0	853.4
13	4310.2	398.3	331.9	8620.0	2149.7	849.3
14	4290.7	396.5	330.4	8581.2	2141.2	845.5
15	4273.2	394.8	329.0	8546.3	2133.4	842.0
16	4257.4	393.2	327.6	8514.8	2126.3	838.7
17	4243.0	391.5	326.3	8486.1	2119.7	835.6
18	4229.8	389.9	324.9	8459.8	2113.6	832.6
19	4217.6	388.4	323.7	8435.5	2107.8	829.9
20	4206.3	386.9	322.4	8412.9	2102.4	827.3
21	4195.7	385.5	321.2	8391.7	2097.2	824.8
22	4185.7	384.1	320.0	8371.7	2092.3	822.4
23	4176.2	382.7	318.9	8352.8	2087.5	820.1
24	4167.2	381.4	317.8	8334.7	2083.0	817.9
25	4158.5	380.1	316.7	8317.4	2078.6	815.8
26	4150.2	378.8	315.7	8300.7	2074.3	813.7
27	4142.1	377.6	314.7	8284.5	2070.1	809.8
28	4134.2	376.4	313.7	8268.8	2066.0	809.8
29	4126.5	375.3	312.7	8253.5	2062.0	808.0
30	4119.0	374.2	311.8	8238.4	2058.1	806.1

TABLE 5.11 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
31	4111.6	373.1	310.9	8223.7	2054.2	804.3
32	4104.4	372.1	310.1	8209.1	2050.4	802.6
33	4097.2	371.1	309.2	8194.8	2046.6	800.9
34	4090.1	370.1	308.4	8180.6	2042.9	799.2
35	4083.1	369.1	307.6	8166.6	2039.2	797.6
36	4076.1	368.2	306.8	8152.6	2035.6	795.9
37	4069.2	367.3	306.0	8138.8	2031.9	794.3
38	4062.3	366.4	305.3	8125.0	2028.3	792.8
39	4055.4	365.5	304.6	8111.3	2024.8	791.2
40	4048.6	364.7	303.9	8097.7	2021.2	789.7
42	4035.0	363.1	302.5	8070.6	2014.2	786.8
44	4021.5	361.5	301.3	8043.6	2007.2	783.9
45	4014.8	360.8	300.6	8030.1	2003.8	782.4
46	4008.0	360.1	300.0	8016.7	2000.4	781.0
48	3994.6	358.7	298.9	7989.8	1993.5	778.2
50	3981.2	357.3	297.8	7965.1	1986.8	775.5
52	3967.9	356.1	296.7	7936.4	1980.1	772.9
54	3954.6	354.8	295.7	7909.8	1973.5	770.2
55	3947.9	354.2	295.2	7896.5	1970.2	769.0
56	3941.3	353.7	294.7	7883.3	1966.9	767.7
58	3928.1	352.5	293.8	7856.8	1960.4	765.1
60	3914.9	351.5	292.9	7830.5	1954.0	762.7
62	3901.8	350.4	292.0	7804.2	1947.6	760.2
64	3888.7	349.4	291.2	7778.1	1941.3	757.8
65	3882.2	348.9	290.8	7765.1	1938.1	756.6
66	3875.7	348.4	290.4	7752.1	1935.0	755.4
68	3862.7	347.5	289.6	7726.2	1928.8	753.1
70	3849.9	346.6	288.8	7700.4	1922.7	750.8
75	3818.0	344.5	287.1	7636.7	1907.6	745.2
80	3786.7	342.5	285.4	7574.0	1892.9	739.8
85	3755.9	340.7	283.9	7512.4	1878.6	734.5
90	3725.7	339.0	282.5	7451.9	1864.7	729.4
95	3696.1	337.4	281.2	7392.6	1851.1	724.5
100	3667.1	335.9	279.9	7334.5	1837.9	719.7
110	3610.9	333.2	277.6	7222.0	1812.5	710.6
120	3557.2	330.7	275.6	7114.3	1788.5	702.0
130	3505.7	328.5	273.7	7011.3	1765.6	693.9
140	3456.6	326.4	272.0	6912.8	1743.9	686.2
150	3409.6	324.5	270.4	6818.7	1723.2	678.9
160	3364.7	322.8	269.0	6728.8	1703.5	672.0
170	3321.7	321.2	267.6	6642.9	1684.8	665.4

TABLE 5.11 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
180	3280.6	319.6	266.4	6560.7	1666.9	659.1
190	3241.3	318.2	265.2	6482.0	1649.8	653.1
200	3203.7	316.9	264.0	6406.6	1653.5	647.4
210	3167.6	315.6	263.0	6334.5	1617.9	642.0
230	3099.8	313.2	261.0	6198.9	1588.6	631.7
250	3037.3	311.0	259.2	6073.8	1561.6	622.3
270	2979.5	309.0	257.5	5958.2	1536.7	613.6
290	2925.8	307.2	256.0	5851.0	1513.6	605.6
310	2875.9	305.5	254.6	5751.3	1492.2	598.1
325	2840.7	304.3	253.6	5681.0	1477.0	592.8
350	2785.9	302.4	252.0	5571.4	1453.4	584.6
370	2745.1	301.0	250.8	5489.9	1435.8	578.5
390	2706.8	299.7	249.7	5413.3	1419.3	572.8
405	2679.5	298.7	248.9	5358.9	1407.5	568.7
410	2670.7	298.4	248.7	5341.2	1403.7	567.4
430	2636.7	297.2	247.7	5273.2	1389.0	562.3
450	2604.5	296.1	246.7	5209.0	1375.1	557.4
470	2574.0	295.0	245.8	5148.1	1361.9	552.9
490	2545.1	293.9	244.9	5090.4	1349.4	548.5
510	2517.6	292.9	244.1	5035.5	1337.4	544.4
540	2478.9	291.5	242.9	4958.2	1320.6	538.5
600	2409.2	288.9	240.7	4819.0	1290.2	528.0
650	2357.8	286.9	239.0	47.6.3	1267.6	520.1
700	2311.5	285.0	237.5	4623.7	1247.2	513.0
750	2269.5	283.2	236.0	4539.8	1228.6	506.5
800	2231.3	281.6	234.7	4463.3	1211.6	600.6
850	2196.2	280.1	233.4	4393.3	1196.0	495.1
900	2164.0	278.7	232.2	4328.8	1181.6	490.1
950	2134.3	277.3	231.1	4269.4	1168.3	485.4
1000	2106.7	276.0	230.0	4214.3	1155.9	481.0
1100	2057.3	273.7	228.1	4115.3	1133.5	473.2
1200	2014.1	271.6	226.3	4029.0	1113.9	466.2
1300	1976.0	269.7	224.7	3952.8	1096.6	460.1
1400	1942.2	268.0	223.3	3885.0	1081.1	454.7
1500	1911.9	266.5	222.0	3824.4	1067.2	449.8
1600	1884.6	265.1	220.9	3769.8	1054.7	445.4
1700	1859.9	263.8	219.8	3720.3	1043.4	441.4
1800	1837.4	262.7	218.9	3675.3	1033.0	437.7
1900	1816.8	261.7	218.0	3634.3	1023.6	434.4
2000	1798.0	260.7	217.3	3596.6	1014.9	431.4
2250	1757.3	258.7	215.6	3515.2	996.1	424.9

TABLE 5.11 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{Iu}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
2500	1723.8	257.1	214.2	3438.1	980.7	419.6
2750	1695.8	255.8	213.2	3392.2	967.7	415.2
3000	1672.3	254.7	212.3	3345.1	956.8	411.6
3250	1652.2	253.8	211.5	3305.0	947.5	408.5
3500	1635.0	253.1	210.9	3270.6	939.6	405.8
3750	1620.1	252.5	210.4	3240.8	932.6	403.6
4000	1607.1	251.9	209.9	3214.8	926.6	401.6
4250	1595.7	251.5	209.6	3192.0	921.3	399.9
4500	1585.7	251.1	209.2	3171.8	916.6	398.4
4750	1576.8	250.7	208.9	3153.9	912.4	397.0
5000	1568.8	250.4	208.7	3137.9	908.7	395.9
5500	1555.2	249.9	208.2	3110.7	902.3	393.8
6000	1544.1	249.5	207.9	3088.3	897.0	392.2
6500	1534.8	249.2	207.6	3069.7	892.7	390.8
7000	1527.0	248.9	207.4	3054.0	889.0	389.7
7500	1520.4	248.7	207.2	3040.6	885.8	388.7
8000	1514.6	248.5	207.0	3029.0	883.1	387.9
8500	1509.6	248.3	206.9	3018.9	880.7	387.1
9000	1505.3	248.1	206.8	3010.1	878.6	386.5
9500	1501.4	248.0	206.7	3002.3	876.7	385.9

TABLE 5.12

Averaged Spin-Dependent Cross Sections

of Cesium Atom (πa_o^2)

$\bar{\sigma}_{NU}$ Averaged total scattering of nonidentical atom

$\bar{\sigma}_{NX}$ Averaged spin-exchange cross sections of nonidentical atom

$\bar{\sigma}_{Ny}$ Averaged hyperfine transition cross sections of nonidentical atom from $f = 4 \rightarrow f' = 3$

$\bar{\sigma}_{IU}$ Averaged total scattering of identical atom

$\bar{\sigma}_{Ix}$ Averaged spin-exchange cross sections of identical atom

$\bar{\sigma}_{Iy}$ Averaged hyperfine transition cross sections of identical atom from $f = 4 \rightarrow f' = 3$

T(^{o}k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
10	4823.5	416.6	364.6	9652.1	2390.5	813.3
11	4795.2	415.4	363.5	9595.6	2377.6	809.5
12	4767.5	414.2	362.4	9540.4	2365.2	805.8
13	4740.7	413.0	361.3	9486.9	2353.2	802.1
14	4715.0	411.7	360.2	9435.7	2341.8	798.5
15	4690.4	410.4	359.1	9386.7	2331.0	795.1
16	4667.1	409.2	358.0	9340.1	2320.7	791.8
17	4644.9	407.9	356.9	9295.9	2310.8	788.5
18	4624.0	406.6	355.8	9254.0	2301.5	785.4
19	4604.1	405.4	354.7	9214.3	2292.6	782.5
20	4585.4	404.2	353.6	9176.8	2284.2	779.6
21	4567.6	403.0	352.6	9141.4	2276.2	776.9
22	4550.9	401.8	351.5	9107.8	2268.6	774.2
23	4535.0	400.6	350.5	9076.1	2261.3	771.7
24	4520.1	399.4	349.5	9046.0	2254.4	769.2
25	4505.7	398.3	348.5	9017.4	2247.7	766.9
26	4492.2	397.2	347.5	8990.3	2241.4	764.6
27	4479.3	396.1	346.5	8964.5	2235.3	762.4
28	4467.1	395.0	345.6	8940.0	2229.5	760.3
29	4455.4	393.9	344.7	8916.6	2223.9	758.3

TABLE 5.12 (Continued)

T(°k)	$\bar{\sigma}_{NU}$	$\bar{\sigma}_{NX}$	$\bar{\sigma}_{NY}$	$\bar{\sigma}_{IU}$	$\bar{\sigma}_{IX}$	$\bar{\sigma}_{IY}$
30	4444.3	392.9	343.8	8894.2	2218.6	756.3
31	4433.6	391.9	342.9	8872.8	2213.4	754.4
32	4423.4	390.2	342.0	8852.3	2208.4	752.5
33	4413.6	389.9	341.2	8832.6	2203.6	750.8
34	4404.2	388.9	340.3	8813.6	2198.9	749.0
35	4395.1	388.0	339.5	8795.4	2194.4	747.3
36	4386.3	387.1	338.7	8777.8	2190.0	745.7
37	4377.9	386.2	337.9	8760.7	2185.7	744.1
38	4369.7	385.3	337.1	8744.2	2181.6	742.5
39	4361.7	384.4	336.4	8728.2	2177.6	741.0
40	4354.0	383.6	335.6	8712.7	2173.6	739.5
42	4339.1	381.9	334.2	8682.8	2166.0	736.7
44	4325.0	380.4	332.8	8654.3	2158.7	733.9
45	4318.1	379.6	332.1	8640.5	2155.2	732.6
46	4311.4	378.8	331.5	8627.0	2151.7	731.3
48	4298.4	377.4	330.2	8600.8	2144.9	728.8
50	4285.8	376.0	329.0	8575.4	2138.4	726.4
52	4273.6	374.6	327.8	8550.8	2132.0	724.0
54	4261.7	373.3	326.6	8526.8	2125.7	721.7
55	4255.8	372.7	326.1	8515.0	2122.7	720.6
56	4250.0	372.0	325.5	8503.3	2119.7	719.5
58	4238.6	370.8	324.5	8480.4	2113.7	717.4
60	4227.4	369.6	323.4	8457.8	2107.9	715.3
62	4216.4	368.5	322.4	8435.6	2102.1	713.2
64	4205.5	367.4	321.5	8413.8	2096.5	711.2
65	4200.1	366.9	321.0	8402.9	2093.7	710.3
66	4194.7	366.3	320.5	8392.2	2090.9	709.3
68	4184.1	365.3	319.6	8370.8	2085.4	707.4
70	4173.6	364.3	318.7	8349.7	2080.0	705.5
75	4147.8	361.9	316.7	8297.9	2066.8	701.0
80	4122.4	359.7	314.7	8247.1	2054.0	696.7
85	4097.5	357.7	312.9	8197.1	2041.5	692.6
90	4073.0	355.7	311.3	8148.0	2029.3	688.7
95	4048.8	353.9	309.7	8099.7	2017.4	684.9
100	4025.0	352.2	308.2	8052.0	2005.7	681.2
110	3978.3	349.1	305.5	7958.6	1983.2	674.2
120	3932.8	346.4	303.1	7867.8	1961.6	667.6
130	3888.6	343.9	300.9	7779.5	1940.8	661.3
140	3845.7	341.6	298.9	7693.7	1920.9	655.4
150	3804.0	339.5	297.1	7610.5	1901.7	649.7
160	3763.5	337.6	295.4	7529.7	1883.3	644.4
170	3724.2	335.9	293.9	7451.4	1865.5	639.2

TABLE 5.12 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{Iu}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
180	3686.2	334.2	292.4	7375.4	1848.5	634.3
190	3649.3	332.7	291.1	7301.9	1832.0	629.6
200	3613.6	331.3	289.9	7230.6	1816.1	625.0
210	3579.0	329.9	288.7	7161.5	1800.8	620.7
230	3513.0	327.5	286.5	7029.7	1771.8	612.5
250	3450.9	325.3	284.6	6905.8	1744.8	604.8
270	3392.6	323.3	282.9	6789.4	1719.5	597.7
290	3337.7	321.5	281.3	6679.8	1695.8	591.1
310	3286.0	319.9	279.9	6576.5	1673.5	584.9
325	3249.1	318.8	278.9	6502.8	1657.7	580.4
350	3191.1	317.0	277.4	6386.7	1632.9	573.5
370	3147.4	315.7	276.3	6299.5	1614.2	568.3
390	3106.0	314.5	275.2	6216.7	1596.6	563.5
405	3076.3	313.7	274.5	6157.5	1584.0	560.0
410	3066.7	313.4	274.2	6138.2	1579.9	558.8
430	3029.4	312.4	273.3	6063.6	1564.0	554.4
450	2993.8	311.4	272.4	5992.5	1549.0	550.3
470	2959.9	310.4	271.6	5924.8	1534.7	546.3
490	2927.6	309.6	270.9	5860.1	1521.0	542.5
510	2896.7	308.7	270.1	5798.4	1508.0	538.9
540	2852.9	307.6	269.1	5710.8	1489.5	533.9
600	2773.3	305.5	267.3	5551.5	1456.0	524.6
650	2713.9	304.0	266.0	5432.8	1431.1	517.8
700	2660.0	302.6	264.8	5324.8	1408.4	511.5
750	2610.7	301.4	263.7	5226.3	1387.8	505.9
800	2565.5	300.2	262.7	5135.9	1368.8	500.7
850	2523.9	299.2	261.8	5052.6	1351.4	495.9
900	2485.4	298.2	260.9	4975.7	1335.3	491.4
950	2449.7	297.3	260.1	4904.3	1320.3	487.3
1000	2416.5	296.5	259.4	4837.9	1306.4	483.5
1100	2356.6	294.9	258.0	4718.0	1281.2	476.6
1200	2303.8	293.5	256.8	4612.4	1258.9	470.4
1300	2257.0	292.3	255.7	4518.7	1239.0	465.0
1400	2215.0	291.1	254.7	4434.9	1221.1	460.1
1500	2177.2	290.0	253.8	4359.3	1205.0	455.6
1600	2143.0	289.0	252.9	4290.8	1190.2	451.6
1700	2111.8	288.1	252.1	4228.4	1176.7	447.9
1800	2083.3	287.2	251.3	4171.4	1164.3	444.4
1900	2057.1	286.4	250.6	4119.0	1152.9	441.3
2000	2033.0	285.6	249.9	4070.8	1142.3	438.4
2250	1980.4	283.9	248.4	3965.6	1119.1	432.0

TABLE 5.12 (Continued)

T(°k)	$\bar{\sigma}_{Nu}$	$\bar{\sigma}_{Nx}$	$\bar{\sigma}_{Ny}$	$\bar{\sigma}_{Iu}$	$\bar{\sigma}_{Ix}$	$\bar{\sigma}_{Iy}$
2500	1936.7	282.3	247.0	3878.1	1099.6	426.6
2750	1900.0	281.0	245.9	3804.6	1083.1	422.0
3000	1868.8	279.8	244.8	3742.2	1068.9	418.1
3250	1842.1	278.8	243.9	3688.8	1056.8	414.7
3500	1819.2	277.8	243.1	3642.7	1046.2	411.7
3750	1799.2	277.0	242.4	3602.6	1037.0	409.2
4000	1781.7	276.3	241.7	3567.6	1028.8	406.9
4250	1766.4	275.6	241.2	3536.7	1021.7	404.9
4500	1752.8	275.0	240.7	3509.5	1015.3	403.1
4750	1740.7	274.5	240.2	3485.2	1009.6	401.5
5000	1729.9	274.0	239.8	3463.4	1004.5	400.0
5500	1711.4	273.2	239.0	3426.3	995.8	397.5
6000	1696.3	272.5	238.4	3395.8	988.5	395.5
6500	1683.7	271.9	237.9	3370.3	982.5	393.7
7000	1673.0	271.4	237.5	3348.9	977.4	392.3
7500	1663.9	270.9	237.1	3330.5	973.0	391.0
8000	1656.1	270.6	236.7	3314.7	969.2	389.9
8500	1649.2	270.2	236.4	3300.8	965.9	389.0
9000	1654.2	269.9	236.2	8288.7	962.9	388.1
9500	1637.9	269.7	235.9	3278.0	960.4	387.4

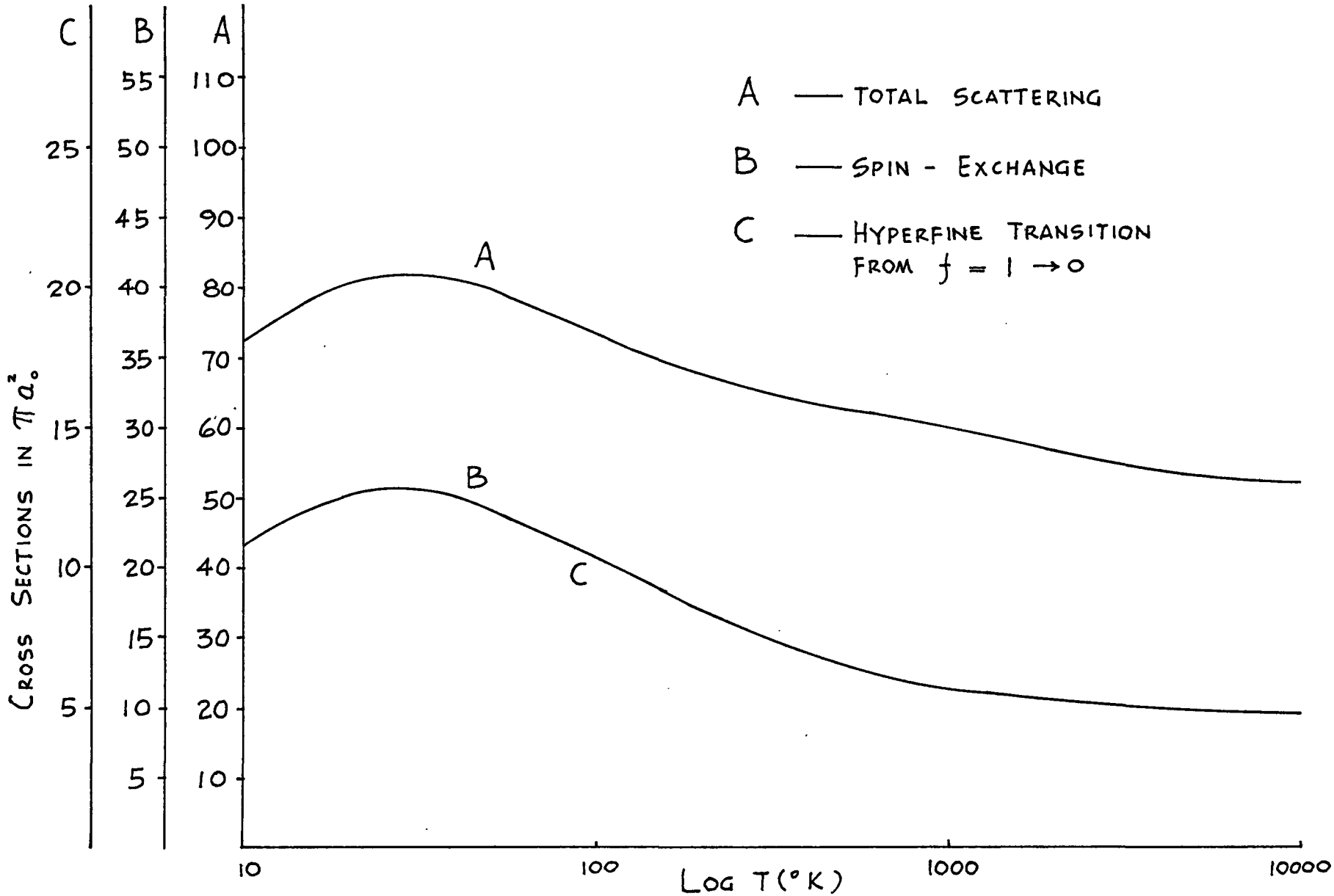

FIG. 5·7 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL HYDROGEN ATOM AS A FUNCTION OF TEMPERATURE T

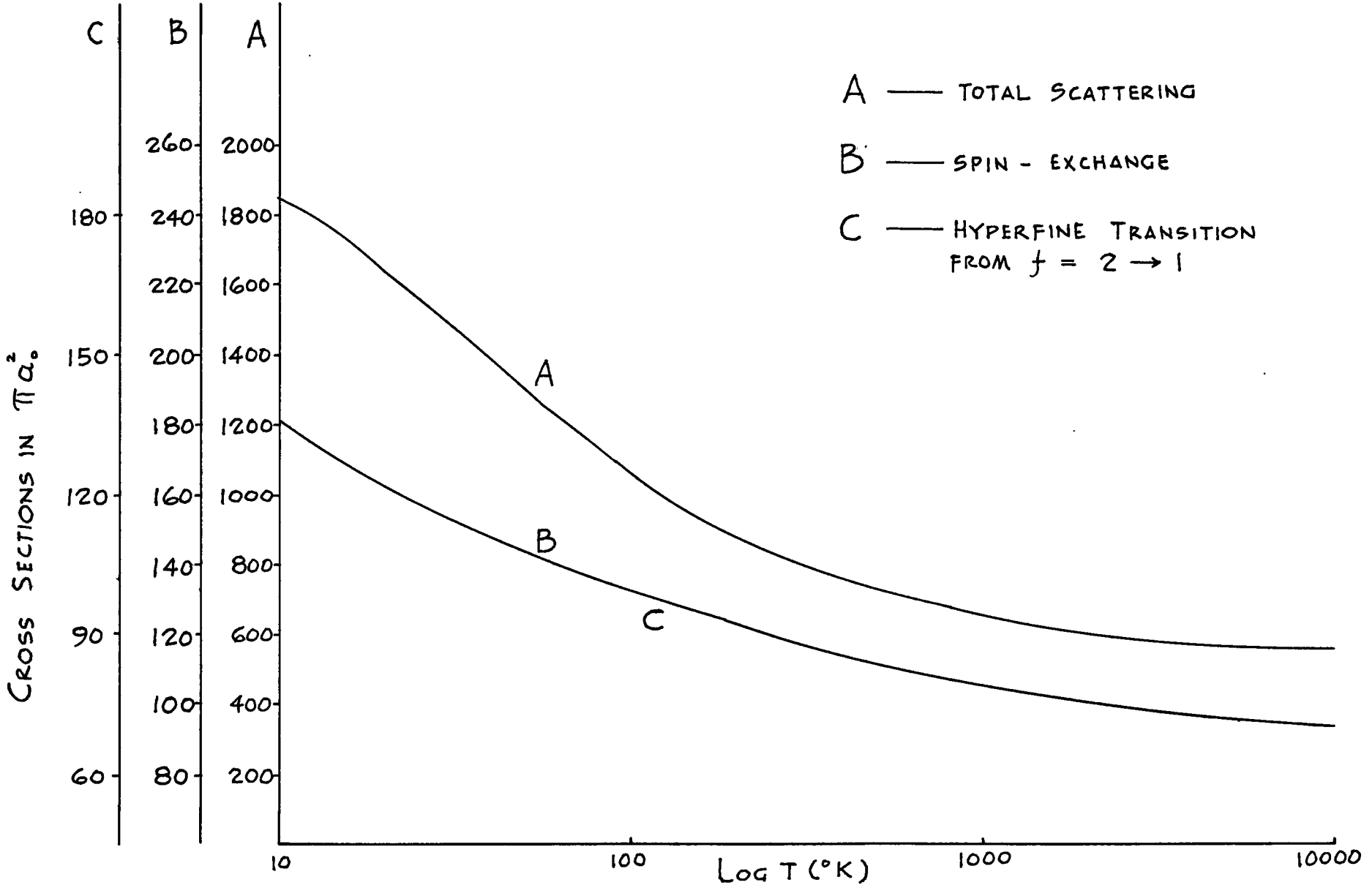

FIG. 5·8 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL LITHIUM ATOM AS A FUNCTION OF TEMPERATURE T.

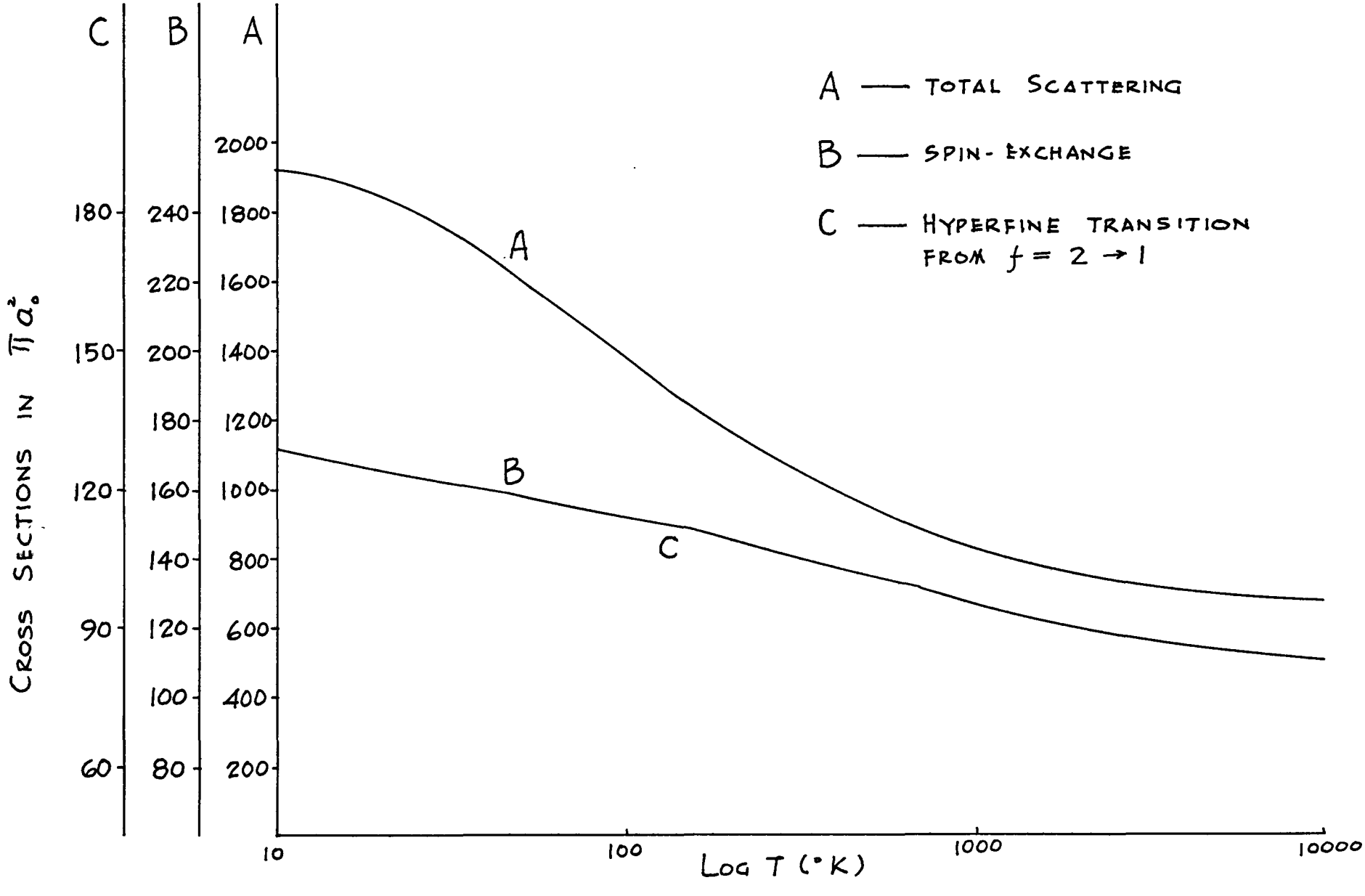

FIG. 5·9 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL SODIUM ATOM AS A FUNCTION OF TEMPERATURE T.

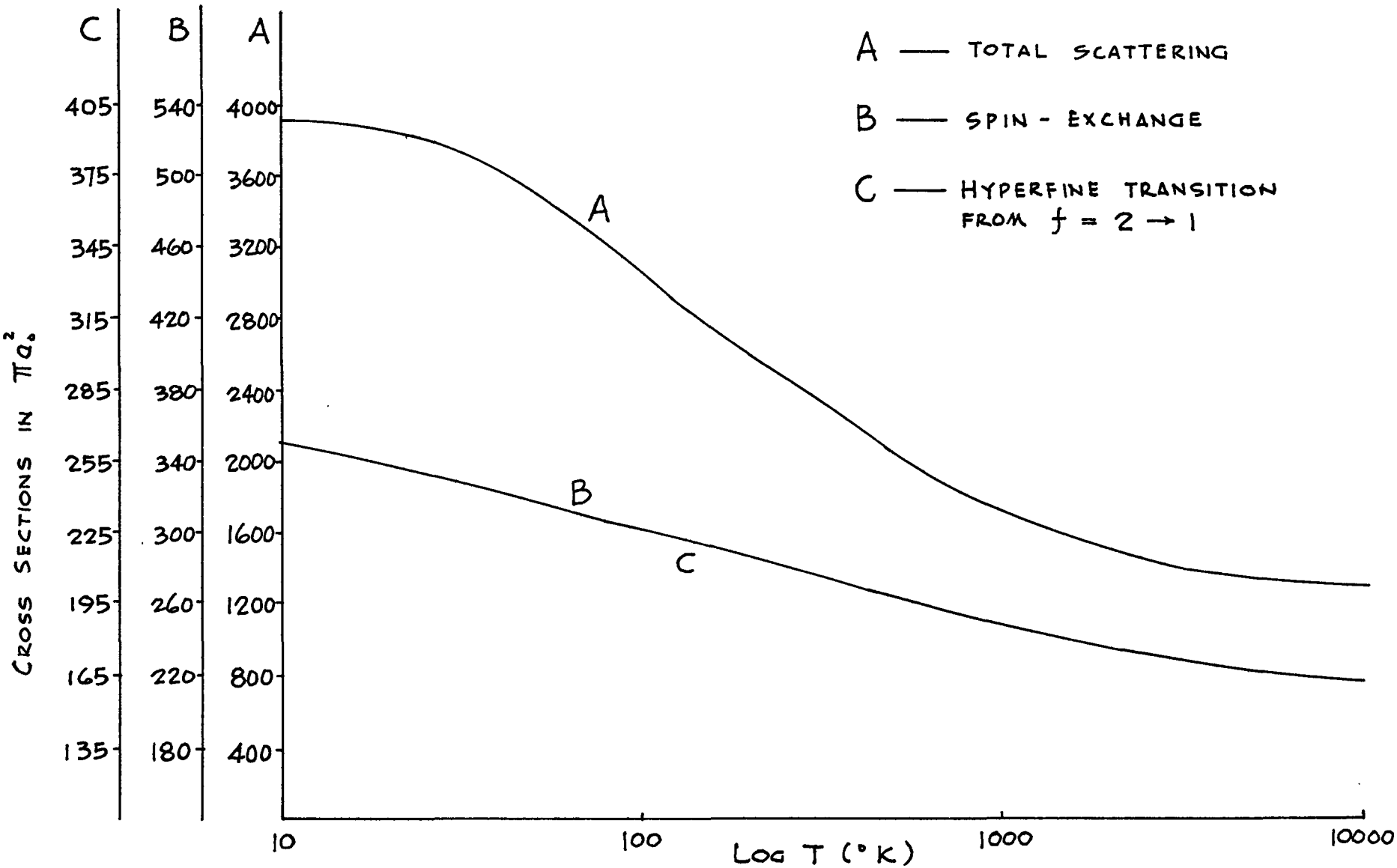

FIG. 5·10 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL POTASSIUM ATOM AS A FUNCTION OF TEMPERATURE T.

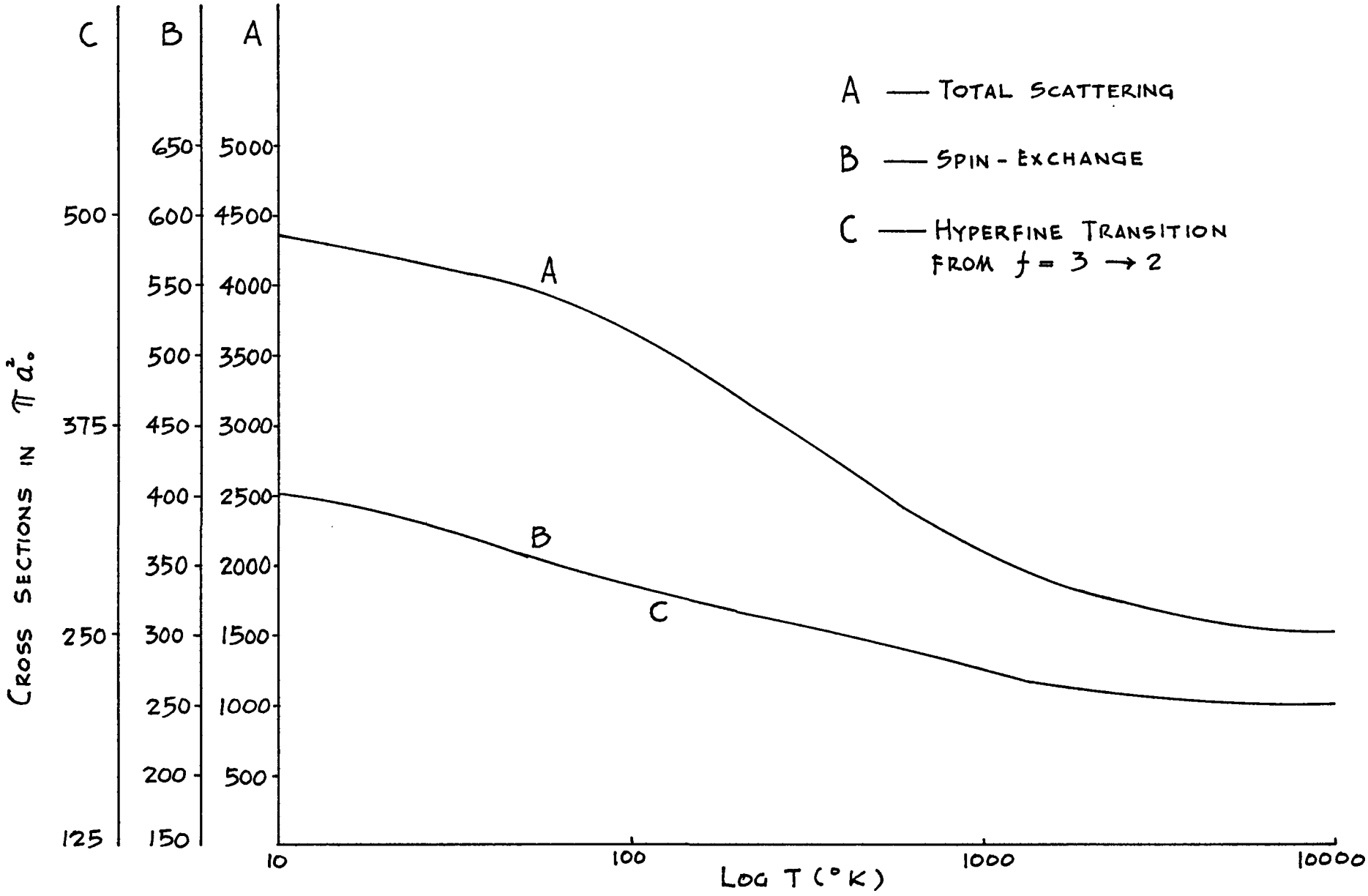

FIG. 5·11 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL RUBIDIUM ATOM AS A FUNCTION OF TEMPERATURE T

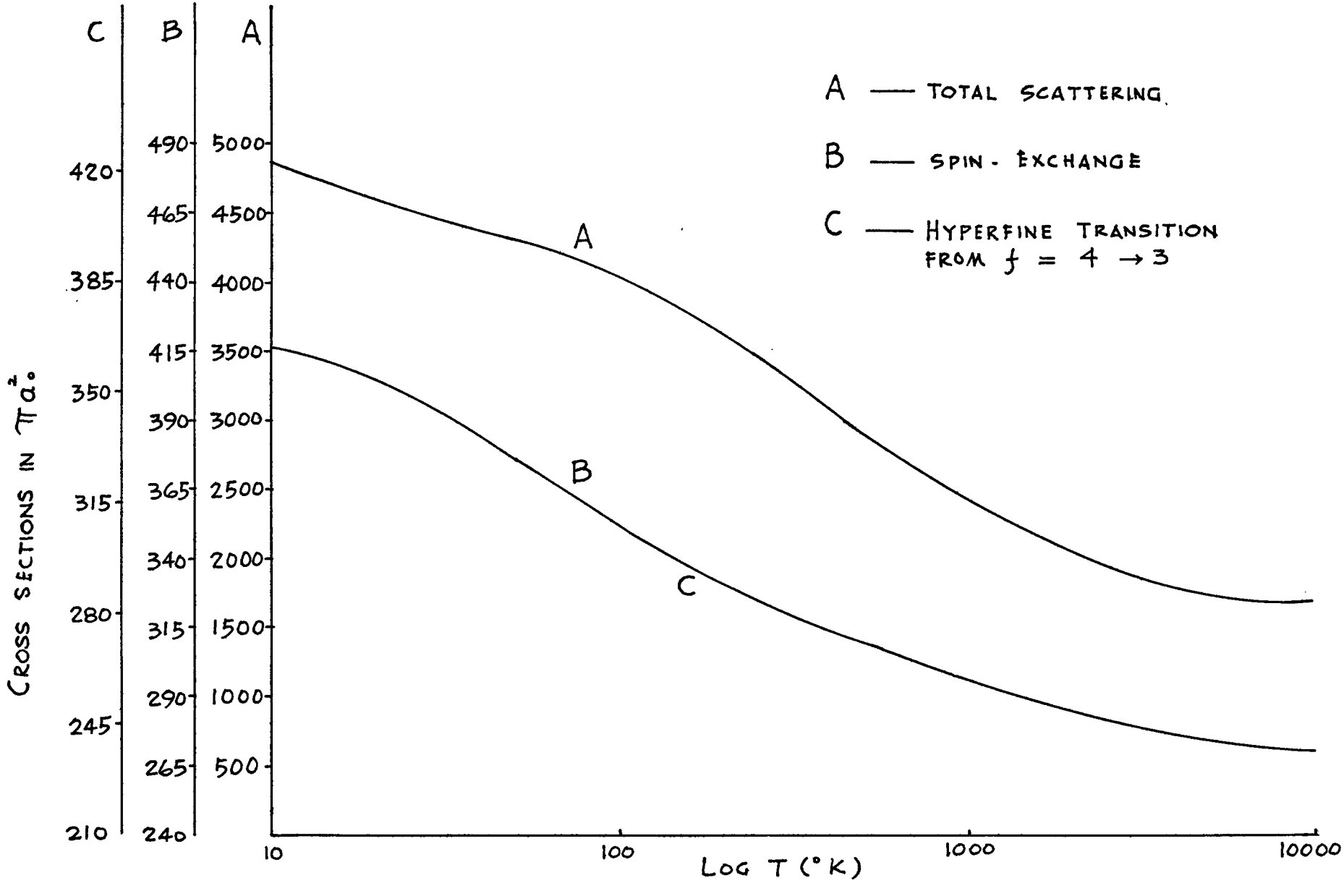

FIG. 5·12 AVERAGED SPIN-DEPENDENT CROSS SECTIONS OF NONIDENTICAL CESIUM ATOM AS A FUNCTION OF TEMPERATURE T.

CHAPTER VI

CONCLUSION

From the discussion in the preceding chapters it is seen that the acquisition of suitable interaction potential between atoms as a function of the nuclear separation is a difficult problem. Only a few simple diatomic systems have been solved, namely the hydrogen ion molecule and the neutral hydrogen molecule.

The present treatment concerns the interaction potential between two atoms with one S valence electron. The theory predicts that the interaction between these two atoms may be considered to be composed of the interaction due to the S electron and those electrons enclosed by it. The interaction potential between the two atoms with S electrons may then be generated from the interaction potential of the hydrogen molecule and the corresponding inert diatomic system. The results obtained are in good agreement with experimental results. It is also seen that the potentials obtained from direct observation apply only to the singlet state. Yet the present treatment yields potential curves including both the singlet and the triplet states. The technique of this theory may be extended to other diatomic or ionic systems. Also these potentials between alkaline atoms may be used to calculate their equation of state or their transport properties. Hence,

a great number of equilibrium and non-equilibrium phenomena of alkaline atomic systems may be studied.

These potentials are subsequently employed in the calculations of several kinds of spin-dependent cross sections including electronic spin-exchange, hyperfine transition and total unpolarized scattering cross sections for both identical and nonidentical six alkaline systems. In comparing the calculated electronic spin-exchange cross sections of both identical and nonidentical atoms with experimental results, it is found that the calculated cross sections of nonidential atoms agree very well with experiment and those for identical atoms are about a factor of two larger than the experimental ones, This indicats that consideration of atomic identity is not necessary in this case. In fact, the interaction between the two nuclear spins of the two colliding atoms is not strong enough for exchange effects to occur. However, atomic identity is to be considered, exchange effects due to the inner closed electronic shells must also be taken into account. Our results indicate that these effects tend to cancel.

In connection with spin-dependent cross sections and the interaction potentials of alkali atoms, there are three more interesting problems which can be investigated.

1. The scattering of alkaline atoms and alkaline ions. Here, we also have six different colliding systems, i.e. H^+-H, Li^+-Li, Na^+-Na, K^+-K, Rb^+-Rb, and Cs^+-Cs. The alkaline ion has spin 0

and alkaline atom $\frac{1}{2}$, so this problem really is a spin $\frac{1}{2}$ and spin 0 collision. The cause of spin-exchange in this case is also due to the transition of the valence S electron of the alkali atom from the atom to the ion. The required interaction potential between the ion and the atom also can be generated by the techinique we employed in the generation of the interaction potential between two alkali atoms. In this case we have to use the known interaction potential of hydrogen molecule ion (H^+-H) instead of the H-H interaction.

2. The scattering of unlike alkali atoms..In this case we can have 15 different diatomic systems formed from the 6 different alkali atoms. The same formulars used in the calculations of spin-dependent cross sections of like atoms can also be used in the calculation of spin-dependent cross sections of unlike diatomic alkali systems.

The interaction potential for the unlike atomic systems also can be calculated in a similar way to that used in the like atomic systems by only rescaling the parameters. Many of the spin-exchange cross sections of the unlike alkali atoms have been measured. It is hoped that calculated value would agree with the experiments.

3. The scattering of alkali atoms and inert atoms. Here we consider a spin $\frac{1}{2}$ and spin 0 collision. There are 30 different kinds of such diatomic systems formed from 6 alkali and 5 inert atoms. These problems are very important in the study of optical pumping techniques. The spin-flip cross sections for this type of scattering have been measured, but a quantitative theoretical description is still

lacking. In fact, the cause of the spin-flip is still debated. The most likely candidate for this interaction is the spin-orbit coupling of the valence S electron of the alkali atom to the electric field of the polarized inert atom. However, the interaction potential between the alkali and the inert atoms can be calculated by extending our techniques and calculations carried out to test this hypathesis.

APPENDIX

HYPERFINE SPIN TRANSITION CROSS SECTIONS OF IDENTICAL ATOMS WITH ARBITRARY SPIN

Density matrix techniques have been employed by earlier authors[21,23] to obtain the hyperfine transition cross sections for colliding identical atoms with nuclear spin $\frac{1}{2}$. This appendix is an extension of this techniques for identical atoms with arbitrary nuclear spin i .

Using density matrix techniques[21,23] the cross section for the hyperfine transition from fm to f'm' of an incident identical atom with nuclear spin i may be expressed as

$$\sigma_I(fm,f'm') = \frac{1}{g_2} T_r\left[(J_d + Q'_n K_d) P(fm)(J_d^* + Q'_n K_d^*) \times P(f'm')\right] + \frac{1}{4}\sum_{i=1}^{3} T_r\left[(J_x + Q'_n K_x)\hat{\delta}_i P_1(fm) \times (J_x^* + Q'_n K_x^*)\hat{\delta}_i P(f'm')\right], \tag{A.1}$$

where these symbols express the following relations:

p(fm)=projection operators for the hyperfine states of the incident atom,

$$g_2 = 2(2i+1), \tag{A.2}$$

$$Q'_n = \frac{1}{2i+1}(\hat{\tau}_1 \cdot \hat{\tau}_2), \tag{A.3}$$

$$J_d(\theta) = F_d(\theta) + \left[(-1)^{2i+1}/(2i+1)\right] F_d'(\pi-\theta), \tag{A.4}$$

$$K_d(\theta) = (-1)^{2i+1} F_d'(\pi-\theta) \tag{A.5}$$

$$J_x(\theta) = F_x(\theta) + \left[(-1)^{2i+1}/2i+1\right] F_x'(\pi-\theta), \tag{A.6}$$

$$K_x(\theta) = (-1)^{2i+1} F_x'(\pi-\theta), \tag{A.7}$$

$$F_d(\theta) = \frac{1}{4} f^s(\theta) + \frac{3}{4} f^t(\theta), \tag{A.8}$$

$$F_x(\theta) = \frac{1}{4}\left[f^t(\theta) - f^s(\theta)\right], \tag{A.9}$$

$$F_d'(\theta) = -\frac{1}{4} f^s(\theta) + \frac{3}{4} f^t(\theta), \tag{A.10}$$

$$F_x'(\theta) = \frac{1}{4} f^t(\theta) + \frac{1}{4} f^s(\theta), \tag{A.11}$$

$\hat{\tau}_1$, $\hat{\tau}_2$ and $\hat{\delta}_i$ = Pauli spin operators,

and $f^s(\theta)$ and $f^t(\theta)$ = scattering amplitudes of singlet and triplet states.

The special properties of the operator Q_n' cited above lead to the vanishing of all traces linear in Q_n', $T_r[Q_n'] = 0$, and thus to the expression

$$\sigma_I(fm, f'm') = T_r\left[|J_d|^2 P(fm) P(f'm') + |J_x|^2 \sum_i^3 \hat{\delta}_i P(fm) \hat{\delta}_i P(fm)\right]$$
$$+ |K_d|^2 \frac{1}{g_2} \sum_i^3 T_r\left[\hat{\tau}_i P(fm) \hat{\tau}_i P(f'm')\right] +$$
$$+ |K_x|^2 \frac{1}{g_2} \sum_i^3 \sum_j^3 \left[\hat{\tau}_i \hat{\delta}_j P(fm) \hat{\tau}_i \hat{\delta}_j P(f'm')\right]. \tag{A.12}$$

For the average cross sections, we have

$$\sigma_I(f,f') = \frac{1}{2f+1}\sum_{mm'}\sigma_I(fm,f'm') \tag{A.13}$$

$$= T_r\left[|J_d|^2 P(f)P(f') + |J_x|^2 \sum_i^3 \hat{\delta}_i P(f)\hat{\delta}_i P(f')\right] +$$

$$+ |K_d|^2 \frac{1}{q_2}\sum_i^3 T_r\left[\hat{\tau}_i P(f)\hat{\tau}_i P(f')\right] +$$

$$+ |K_x|^2 \frac{1}{q_2}\sum_i^3\sum_j^3 T_r\left[\hat{\tau}_i \hat{\delta}_j P(f)\hat{\tau}_i \hat{\delta}_j P(f')\right].$$

(A.14)

Both $\hat{\delta}_i$ and $\hat{\tau}_i$ are the Pauli operators, so let

$$\Delta(f,f') = \sum_i^3 T_r\left[\hat{\tau}_i P(f)\hat{\tau}_i P(f')\right]$$

$$= \sum_i^3 T_r\left[\hat{\delta}_i P(f)\hat{\delta}_i P(f')\right]. \tag{A.15}$$

The following table gives the values of this parameter,

Table A.1. The values of $\Delta(f,f')$

f' \ f	$i+\frac{1}{2}$	$i-\frac{1}{2}$
$i+\frac{1}{2}$	$\frac{(2i+2)(2i+3)}{2i+1}$	$\frac{8i(i+1)}{2i+1}$
$i-\frac{1}{2}$	$\frac{8i(i+1)}{2i+1}$	$\frac{(2i-1)(2i)}{(2i+1)}$

For the average projection operators we have

$$P(f) = \sum_{m=-j}^{j} P(fm) \quad , \tag{A.16}$$

and for the two hyperfine states $f = i \pm \frac{1}{2}$ these operators may be written as

$$P(i+\tfrac{1}{2}) = (2i+1)^{-1}(i + 1 + \hat{\delta} \cdot \hat{I}) \quad , \tag{A.17}$$

$$P(i-\tfrac{1}{2}) = (2i+1)^{-1}(i - \hat{\delta} \cdot \hat{I}) \quad , \tag{A.18}$$

or

$$P(f) = (2i+1)^{-1}\left[(f+\tfrac{1}{2}) + (-1)^{f-i-\frac{1}{2}}\, \hat{\delta} \cdot \hat{I}\right] , \tag{A.19}$$

where $\hat{I}$ is the unit operator.

Putting (A.19) into (A.15), we obtain

$$\Delta(f,f') = \left(\frac{2}{2i+1}\right)\left[3(f+\tfrac{1}{2})(f'+\tfrac{1}{2}) - (-1)^{f-f'} i(i+1)\right] , \tag{A.20}$$

where the $\Delta(f,f')$ are tabulated in Table A.1.

Now define

$$\Gamma(f,f') = \sum_{i}^{3} \sum_{j}^{3} T_n \left[\hat{\tau}_i \hat{\delta}_j P(f) \hat{\tau}_i \hat{\delta}_j P(f')\right] , \tag{A.21}$$

then from similar procedures we may obtain

$$\Gamma(f,f') = \left(\frac{2}{2i+1}\right)\left[3^2\left(f+\tfrac{1}{2}\right)\left(f'+\tfrac{1}{2}\right) + (-1)^{f+f'} i\,(i+1)\right] \qquad \text{(A.22)}$$

For convenience the $\Gamma(f,f')$ are tabulated in Table A.2.

Table A.2. The values of $\Gamma(f,f')$

f' \ f	$i+\frac{1}{2}$	$i-\frac{1}{2}$
$i+\frac{1}{2}$	$\left(\frac{2}{2i+1}\right)\left[9(i+1)^2+(-1)^{2i+1}\,i(i+1)\right]$	$\left(\frac{2}{2i+1}\right)\left[9i(i+1)+(-1)^{2i}\,i\,(i+1)\right]$
$i-\frac{1}{2}$	$\left(\frac{2}{2i+1}\right)\left[9(i+1)(i)+(-1)^{2i}\,i(i+1)\right]$	$\left(\frac{2}{2i+1}\right)\left[9(i)^2+(-1)^{2i-1}\,i(i+1)\right]$

Using (A.15) and (A.21), (A.14) may be reduced to

$$\sigma_I(f,f') = \delta_{ff'}|J_d|^2 + (2f+1)^{-1}\Delta(f,f')\left[|J_x|^2 + \frac{1}{g_2}|K_d|^2\right] + (2f+1)^{-1}\Gamma(f,f')\frac{1}{g_2}|K_x|^2, \qquad \text{(A.23)}$$

From Tables A.1 and A.2, and eq. (A.23) the cross section $\sigma_I(f, f')$ for nuclear spin i have been tabuleted in Table A.3.

Table A.3. The cross sections $\sigma_I(f,f')$

f' \ f	$i+\frac{1}{2}$	$i-\frac{1}{2}$
$i+\frac{1}{2}$	$\lvert J_d\rvert^2+\frac{2i+3}{2i+1}\left[\lvert J_x\rvert^2+\frac{1}{2(2i+1)}\lvert K_d\rvert^2\right]$ $+\frac{1}{2(2i+1)^2}\lvert K_x\rvert^2\left[q(i+1)+(-1)^{2i+1}i\right]$	$\frac{4(i+1)}{2i+1}\left[\lvert J_x\rvert^2+\frac{1}{2(2i+1)}\lvert K_d\rvert^2\right]$ $+\frac{\lvert K_x\rvert^2}{2(2i+1)^2}\left[q(i+1)+(-1)^{2i}(i+1)\right]$
$i-\frac{1}{2}$	$\frac{4i}{2i+1}\left[\lvert J_x\rvert^2+\frac{1}{2(2i+1)}\lvert K_d\rvert^2\right]$ $+\frac{\lvert K_x\rvert^2}{2(2i+1)^2}\left[qi+(-1)^{2i}i\right]$	$\lvert J_d\rvert^2+\frac{2i-1}{2i+1}\left[\lvert J_x\rvert^2+\frac{\lvert K_d\rvert^2}{2(2i+1)}\right]$ $+\frac{\lvert K_x\rvert^2}{2(2i+1)}\left[qi+(-1)^{2i-1}(i+1)\right]$

We integrate the six kinds of scattering amplitudes defined in eqs. (A.4-A.11) over sodid angle, using the relation between the phase shifts and the scattering amplitudes,

$$f_\ell^{s,t}(\theta)=\frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)\left(e^{2i\delta_\ell^{s,t}}-1\right)P_\ell(\cos\theta)\ , \qquad \text{(A.25)}$$

and obtain the following integrated scattering amplitudes:

$$\underline{F_d}=\frac{\pi}{4k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\lambda_{\ell,s}^2+9\lambda_{\ell,t}^2+6C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right];$$

$$\underline{F_x}=\frac{\pi}{4k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\lambda_{\ell,s}^2+\lambda_{\ell,t}^2-2C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right];$$

$$\underline{J_d}=\frac{\pi}{16k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[(5-4(-1)^\ell)\lambda_{\ell,s}^2+9(5-4(-1)^\ell)\lambda_{\ell,t}^2+18C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right];$$

$$\underline{J_x}=\frac{\pi}{16k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[5-4(-1)^\ell\lambda_{\ell,s}^2+(5+4(-1)^\ell\lambda_{\ell,t}^2-6C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right]; \qquad \text{(A.26)}$$

$$\underline{K_d}=\frac{\pi}{4k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\lambda_{\ell,s}^2+9\lambda_{\ell,t}^2-6C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right];$$

$$\underline{K_x}=\frac{\pi}{4k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\lambda_{\ell,s}^2+\lambda_{\ell,t}^2+2C_\ell\lambda_{\ell,s}\lambda_{\ell,t}\right],$$

where

$$\lambda_{\ell,s} = \sin \delta_\ell^{\,s} \;;$$

$$\lambda_{\ell,t} = \sin \delta_\ell^{\,t} \;;$$

$$C_\ell = \cos\left(\delta_\ell^{\,t} - \delta_\ell^{\,s}\right) . \qquad \text{(A.27)}$$

From Table A.3 and eqs. (A.26) and (A.27), we obtain the cross section for hyperfine transitions $f = i + \frac{1}{2} \longrightarrow f' = i - \frac{1}{2}$, for identical atoms each with nuclear spin i, namely

$$\sigma_{Iy} = \frac{4i}{2i+1}\left\{\frac{1}{2}\sigma_{NX} + \left[\left(\frac{1}{2i+1}\right)^2 - \frac{1}{2(2i+1)}\right]A + \frac{\sigma_{NU}}{2(2i+1)} + \right.$$

$$\left. + \frac{\pi}{2k(2i+1)}\sum_\ell^\infty (2\ell+1)\left[(-1)^\ell \sin\delta_\ell^{\,t} - (-1)^\ell \sin^2\delta_\ell^{\,s}\right]\right\}, \qquad \text{(A.28)}$$

where

$$\sigma_{NX} = \frac{\pi}{2k^2}\sum_\ell^\infty (2\ell+1)\sin^2\left(\delta_\ell^{\,t} - \delta_\ell^{\,s}\right) ,$$

$$A = -\frac{1}{2}\sigma_{NX} + \frac{\pi}{2k^2}\sum_\ell^\infty (2\ell+1)\left(\sin^2\delta_\ell^{\,s} + \sin^2\delta_\ell^{\,t}\right),$$

and

$$\sigma_{NU} = \frac{4\pi}{k^2}\sum_\ell^\infty (2\ell+1)\left[\frac{1}{4}\sin^2\delta_\ell^{\,s} + \frac{3}{4}\sin^2\delta_\ell^{\,t}\right].$$

BIBLIOGRAPHY

1. J. P. Wittke and R. H. Dicke, Phys. Rev. 103, 620 (1956).

2. R. Novick and H. E. Peters, Phys. Rev. Letters 1, 54 (1958).

3. P. Franken, R. Sands, and J. Hobart, Phys. Rev. Letters 1, 118 (1958).

4. A. F. Hildenbrandt, F. B. Booth, and C. A. Barth, J. Chem. Phys. 31, 273 (1959).

5. E. M. Purcell and G. B. Field, Astrophys. J. 124, 542 (1956).

6. F. J. Smith, Planet. Space Sci. 11, 126 (1963).

7. M. Ardit and T. R. Carver, Phys. Rev. 109, 1012 (1958).

8. W. W. Holloway, E. Luscher, and R. Novick, Phys. Rev. 126 2109 (1962).

9. M. Born and J. R. Oppenheimer, Ann. Physik 84, 457 (1927).

10. H. G. Dehmelt, Phys. Rev. 109, 381 (1958).

11. P. L. Bender, Phys. Rev. 132, 2154 (1963).

12. P. Franken, R. Sands, and J. Hobart, Phys. Rev. Letters 1, 52 (1958).

13. R. H. Lambert and F. M. Pipkin, Phys. Rev. 128, 198 (1962).

14. G. S. Hayne and H. C. Robinson, Bull. Am. Phys. Soc. 5, 411 (1960).

15. R. M. Mazo, J. Chem. Phys. 34, 169 (1961).

16. L. W. Anderson and A. T. Ramsey, Phys. Rev. 132, 712 (1963).

17. H. W. Moos and R. H. Sands, Phys. Rev. 135, A591 (1964).

18. S. M. Jarrett, Phys. Rev. 133, A111 (1964).

19. A. Dalgarno, Proc. Roy. Soc. A 262, 132 (1961).

20. A. Dalgarno and R. J. W. Henry, Proc. Phys. Soc. 83, 157 (1964).

21. A. E. Glassgold and S. A. Lebedeff, Annals of Phys. 28, 181 (1964).

22. A. E. Glassgold, Phys. Rev. 132, 2144 (1963).

23. B. W. Mays, Thesis (M.S.), University of Houston (1965).

24. B. M. Smirnov and M. I. Chibisov, Soviet Phys. JETP (USA) V. 21, 624 (Sept. 1965).

25. R. S. Mulliken, Rev. Modern Phys. 4, 16 (1932).

26. H. Wind, J. Chem. Phys. 42, 2371 (1965).

27. E. A. Mason and W. E. Rice, J. Chem. Phys. 22, 843 (1954).

28. E. A. Mason, J. Chem. Phys. 23, 49 (1955).

29. E. Whalley and W. G. Schneider, J. Chem. Phys. 23, 1644 (1955).

30. Y. P. Varshni, Rev. Mod. Phys. 29, 664 (1957).

31. N. Rosen and S. Ikehara, Phys. Rev. 43, 5 (1933).

32. H. James, Phys. Rev. 43, 589 (1933).

33. Hirschfelder, Curtiss, and Bird, Molecular Theory of Gases and Liquids (John Wiley and Sons, Inc., New York, 1954).

34. E. A. Guggenheim, Australian Rev. Pure Appl. Chem. 3, 1 (1953).

35. Tables of Interatomic Distances and Configuration in Molecules and Ions (The Chemical Society, London, 1958).

36. F. W. Loomis and R. E. Nusbaum, Phys. Rev. 39, 89 (1932).

37. W. H. Evans, R. Jacofson, T. R. Munson, and D. D. Wagman, J. Res. Natl. Bur. Std. (U.S.) 55, 83 (1955).

38. G. Herzberg, Molecular Spectra and Molecular Structure, V. 1, p. 373 (D. Van Nostrand Company, New Yor,, 1947).

39. J. K. Knipp, Phys. Rev. 53, 734 (1938).

40. Handbook of Chemistry and Physics, 39th edition (Chemical Rulfer, Inc., 1958).

41. G. Herzberg, Atomic Spectra and Atomic Structure, p. 219 (Dover Publications, Inc., New York).

42. Wu and Ohmura, Quantum Theory of Scattering, p. 39 (Prentice-Hall, Inc., New York, 1962).

43. W. W. Holloway, Jr. and E. Luscher, Nuovo Cimento 18, 1926 (1960).

44. L. W. Anderson, F. M. Pipkin, and J. C. Baird, Jr., Phys. Rev. 120, 1279 (1960).

45. F. M. Pipkin and R. H. Lambert, Phys. Rev. 127, 787 (1962).

46. L. W. Anderson, F. M. Pipkin, and J. C. Baird, Jr., Phys. Rev. 116, 87 (1959).

47. R. H. Lambert and F. M. Pipkin, Phys. Rev. 129, 1233 (1963).

48. L. W. Anderson, F. M. Pipkin, and J. C. Baird, Jr., Phys. Rev. 121, 1864 (1961).

49. H. G. Dehmelt, J. Phys. Radium 19, 866 (1958).

50. G. Stephenson, Proc. Phys. Soc. (London) A 64, 458 (1951).

51. J. B. Scarborough, Numerical Mathematical Analysis (The Johns Hopkins Press, Baltimore, 1930).

52. Tables of Spherical Bessel Function (Columbia University Press, New York, 1947).

53. D. M. S. Bagguley and J. H. E. Griffiths, Proc. Roy. Soc. (London) A 201, 366 (1950).

54. W. Kolos and C. C. J. Roothan, Rev. Mod. Phys. 32, 219 (1960).

55. A. Dalgarno and N. Lynn, Proc. Phys. Soc. A69, 821 (1956).

56. L. Pauling and J. Y. Beach, Phys. Rev. 47, 686 (1935).

57. K. D. Burrau, Videnskab Selaskab Mat. Fys. Medd. 14 (1927).

58. D. R. Bates, K. Ledsham, and A. L. Stewart, Phil. Trans. Roy. Soc. (London) A246, 215 (1954).

59. S. Cohen, D. L. Judd, and R. J. Riddell, University of California, Lawrence Radiation Laboratory, Repoet No. UCRL 8802.

60. J. C. Slater, Quantum Theory of Molecules and Solids V.2, p.103 (McGraw Hill Book Company, Inc., New York, 1965).

61. D. R. Bates and R. McCarroll, Proc. Roy. Soc. A, 245, 175 (1958).

62. A. Dalgarno and R. McCarroll, Proc. Roy. Soc. A, 237, 383 (1956).

63. D. W. Jepsen and J. O. Hirschfelder, J. Chem. Phys. 32, 1323 (1960).

64. N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions (Clarendon Press, Oxford, 1949).