THE EFFECTS OF BANDLIMITING
ON THE DETECTION OF ASK SIGNALS

A Thesis

Presented to

the Faculty of the Department of Electrical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science in Electrical Engineering

---

by

Soon Young Kwon

December, 1971

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ABSTRACT

The effects of bandlimiting on the detection of amplitude shift keying (ASK) signals disturbed by additive Gaussian noise have been analyzed using the average threshold method and the average probability density function method. A sample detector is used. For a small signal-to-noise (S/N) ratio, both methods give almost the same probability of bit error for noncoherent detection, but for a large S/N, the average probability density function method gives a smaller probability of error than does the average threshold method. For coherent detection only the average probability density function method has been used.

For the same energy of the signal, the results show that the probability of bit error using a sampler detector is optimum for BT = 0.6 for the noncoherent detection and for BT = 0.7 for the coherent detection of ASK signals. For $BT \leq 1.0$, the sampling time $t_s = 0.5T$ is optimum for the coherent and noncoherent detection. For $BT \geq 1.0$, the optimum sampling time t_s is variable as discussed in Chapter VI.

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CHAPTER I

INTRODUCTION

The binary communication system consists of the transmitter, the channel, and the receiver. The binary message is transmitted by the signals $s_0(t)$ (corresponding to information "m_0"), and $s_1(t)$ (corresponding to information "m_1") through the channel which may be a wire link or a radio link. In the passage through the channel, the transmitted signal is disturbed by unwanted, random signals known as noise. The receiver must decide which signal was transmitted on the basis of received signal, since the transmitted signal was disturbed by noise.

In Chapter II, the optimum decision rule, which maximizes the probability of making correct decisions at the receiver, will be defined for the binary amplitude shift keying (ASK) signal. Noncoherent detection of the ASK signal will be considered using an infinite bandwidth system with an additive Gaussian noise channel.

In practice, the restriction of the system bandwidth is inevitable because of transmitter and receiver filtering. The effects of bandwidth limiting on the detection of ASK signals will be considered in Chapter III.

In Chapter IV, the probability of bit error for the bandlimited system, using a sampler detector, will be calculated by two methods. Noncoherent detection using an envelope detector of the ASK signal will be assumed.

In Chpater V, the probability of bit error, using a sampler detector will be considered for the coherent detection of the bandlimited ASK signal, and the results will be compared with those in Chapter IV.

In Chapter VI, comparison of the results obtained in this thesis with those derived in reference [4] will be made.

CHAPTER II

OPTIMUM RECEIVER FOR THE BINARY COMMUNICATION CHANNEL IN THE PRESENCE OF ADDITIVE WHITE GAUSSIAN NOISE

2.1 Decision Rule of the Optimum Receiver

The communication system under consideration is shown in Figure 2.1. Here the set of messages $\{m_i\}$ is mapped onto the set of signals $\{s_i(t)\}$ by the transmitter. The signal is passed through the channel and is corrupted by additive white Gaussian noise process $n(t)$. Thus the received signal $r(t)$ is the sum of $s_i(t)$ and $n(t)$.

The receiver will map the output $r(t)$ onto the set of messages $\{m_i\}$. The receiver will be chosen such that it will maximize the probability of correct decision denoted by $P[C]$, or equivalently it will minimize the probability of error denoted by $P[E]$.

Since we are concerned with the binary ASK signal the decision rule which minimizes the probability of error is to map $r(t)$ onto m_0 if and only if

$$P[m_0/r(t)] \geq P[m_1/r(t)] \quad , \tag{1}$$

otherwise $r(t)$ is mapped onto m_1.

2.2 Amplitude Shift Keying Signal

One of the simplest digital modulation schemes is the

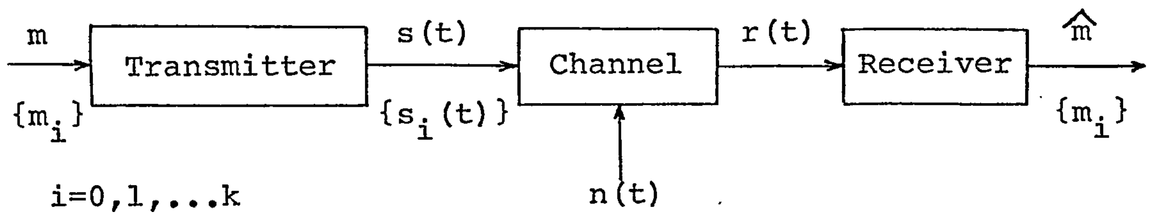

Figure 2.1. COMMUNICATION CHANNEL

amplitude shift keying (ASK) where the carrier wave amplitude is switched "on" and "off" as shown in Figure 2.2. The ASK signal can be generated as shown in Figure 2.3. Here the modulating signal is

$$s_i(t) = \begin{cases} A_i & nT \leq t \leq (n+1)T \\ 0 & \text{elsewhere} \end{cases}$$

where

$$A_i = \begin{cases} A & \text{for } m_1 \\ 0 & \text{for } m_0 \end{cases}$$

Thus the ASK signal is written as

$$X_c(t) = A_i \cos w_c t$$

2.3 Envelope Detection of the ASK Signal

The coherent detection of the ASK signal has been developed in several references [1, 2, 3]. Also the effects of bandlimiting on the coherent detection of these signals using an integrate-and-dump filter has been discussed in reference [4]. Since we are concerned with the effects of bandlimiting on the noncoherent detection of ASK signals, we will first develop the theory for the envelope detection for the infinite bandwidth case and then study the effects due to limiting the

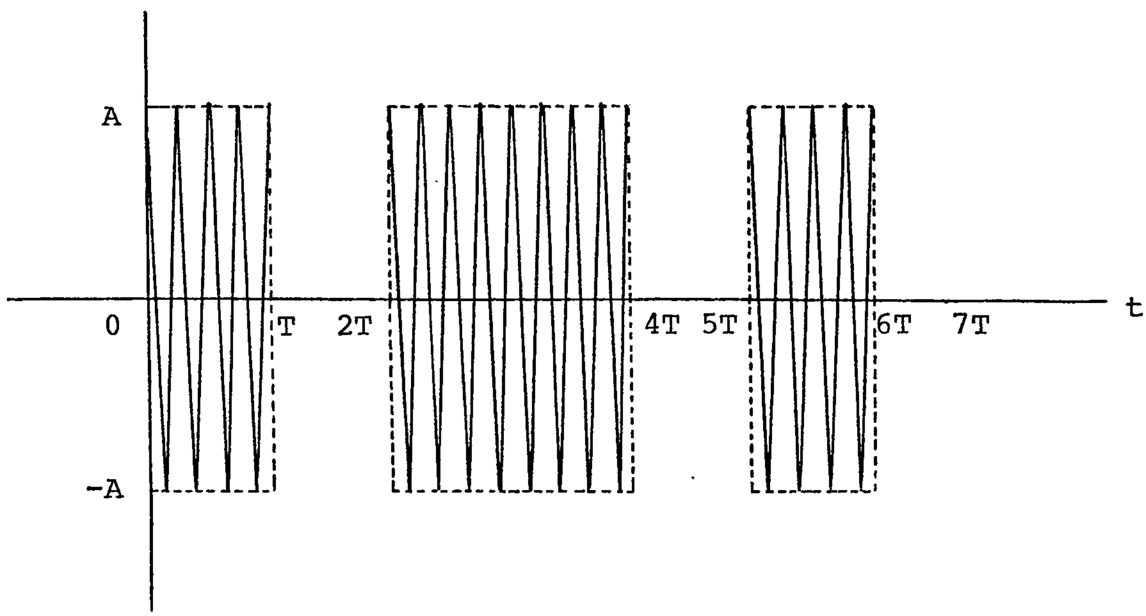

Figure 2.2. ASK Waveform for Message 1011010

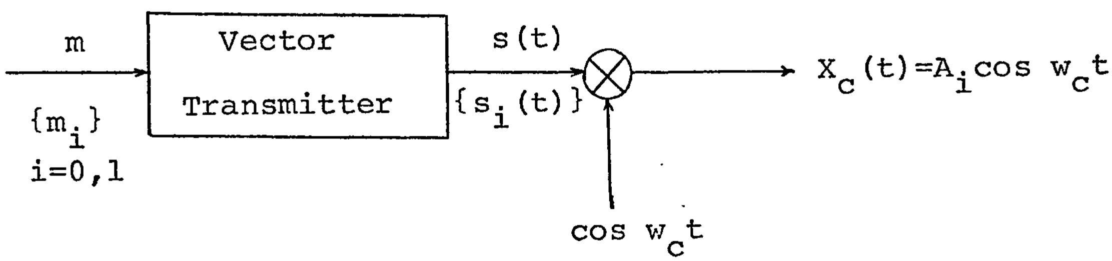

Figure 2.3. Block Diagram of ASK Signal Generator

bandwidth of the system.

The communication system using an envelope detector is shown in Figure 2.4. Here n(t) is an additive bandpass Gaussian noise. The additive bandpass Gaussian noise can be expressed [5] as

$$n(t) = n_c(t) \cos w_c t - n_s(t) \sin w_c t \tag{2}$$

where $n_c(t)$ and $n_s(t)$ are Gaussian distributed and statistically independent random variables with variance N and zero mean.

The received signal r(t) is

$$\begin{aligned} r(t) &= X_c(t) + n(t) \\ &= A_i \cos w_c t + n_c(t) \cos w_c t - n_s(t) \sin w_c t \\ &= [A_i + n_c(t)] \cos w_c t - n_s(t) \sin w_c t \\ &= R(t) \cos [w_c t + \emptyset(t)] \end{aligned} \tag{3}$$

where

$$R(t) = \sqrt{(A_i + n_c(t))^2 + n_s^2(t)}$$

and

$$\emptyset(t) = \tan^{-1} \frac{n_s(t)}{A_i + n_c(t)}$$

Let I(t) be equal to

$$I(t) = A_i + n_c(t).$$

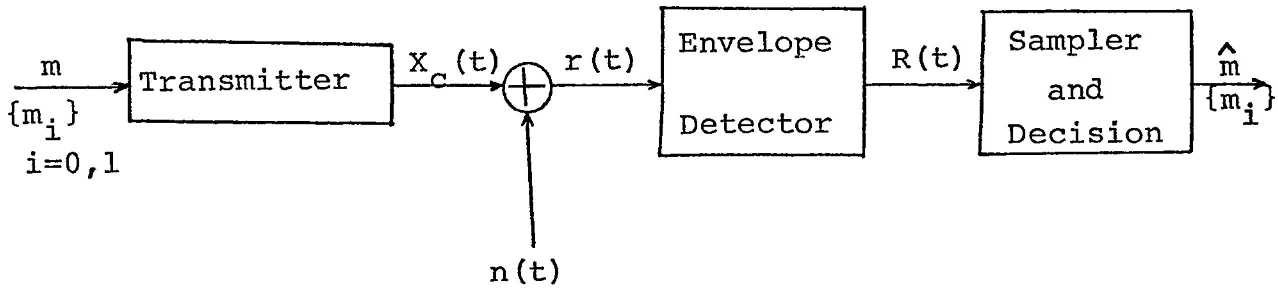

Figure 2.4. Envelope Detector Receiver

Then I(t) is Gaussian distributed with mean equal to A_i and variance equal to N. Since $n_c(t)$ and $n_s(t)$ are statistically independent, I(t) and $n_s(t)$ are also statistically independent. The joint probability density function of I(t) and $n_s(t)$ is

$$P_{I(t),n_s(t)}(\alpha,\beta) = P_{I(t)}(\alpha)\cdot P_{n_{s(t)}}(\beta)$$

$$= \frac{1}{2\pi N} \exp^{-\{(\alpha-A_i)^2 + \beta^2\}/2N} \qquad (4)$$

with the transformation

$$I(t) = R(t)\cos \phi(t) \qquad R(t) \geq 0$$

$$n_s(t) = R(t)\sin \phi(t) \qquad -\pi \leq \phi(t) \leq \pi$$

the joint density function of R(t) and $\phi(t)$ can be written as

$$P_{R(t),\phi(t)}(r,\theta) = \frac{r}{2\pi N}\exp^{-\frac{r^2-2A_i r\cos\theta+A_i^2}{2N}}$$

$$\text{for } r \geq 0$$

$$-\pi \leq \theta \leq \pi$$

The marginal density function of R(t) can be obtained as follows

$$P_{R(t)}(r) = \int_{-\pi}^{\pi} P_{R(t),\phi(t)}(r,\theta)\,d\theta$$

$$= \frac{r}{2\pi N} \exp^{-\frac{(A_i^2+r^2)}{2N}} \int_{-\pi}^{\pi} \exp^{(A_i r/N)\cos\theta} d\theta$$

$$= \frac{r}{N} \exp^{-\frac{A_i^2+r^2}{2N}} I_0(\frac{A_i r}{N}) \quad \text{for } r \geq 0 \tag{5}$$

where

$$I_0(X) = \frac{1}{2\pi} \int_0^{2\pi} e^{X\cos\theta} d\theta$$

is the modified Bessel function of the first kind and zero order. When m_0 is sent through the channel, $A_i = 0$ and the envelope of the received signal is the envelope of noise signal alone with the density function

$$P_{R(t)}(r) = \frac{r}{N} e^{-\frac{r^2}{2N}}, \quad r \geq 0 \tag{6}$$

which is a Rayleigh function. When m_1 is sent, $A_i = A$ and the envelope of the received signal has the density function

$$P_{R(t)}(r) = \frac{r}{N} e^{-\frac{A^2+r^2}{2N}} I_0(\frac{A\cdot r}{N}) \qquad r \geq 0 \tag{7}$$

which is called a non-central Rayleigh function.

When the signal is large compared to the noise, the distribution of the envelope is almost Gaussian, and when the signal is small compared to noise then the distribution of the envelope is almost Rayleigh distributed as shown in Figure 2.5.

Using a mixed Bayes' rule, the optimum decision rule of (1) maps R(t) onto "m_0" if and only if

$$P[m_0] \quad P_{R(t)} \quad (r/m_0) \geq P[m_1] \quad P_{R(t)}(r/m_1) \tag{8}$$

For the case $P[m_0] = P[m_1] = \frac{1}{2}$, Equation (8) becomes

$$P_{R(t)}(r/m_0) \geq P_{R(t)}(r/m_1), \tag{9}$$

which is the optimum decision rule for making a decision on the mapping R(t) into "m_0".

Under the previous assumptions, the probability of error will be represented by the shaded areas of Figure 2.5. The optimum threshold d is well approximated [6] by

$$d = \frac{A}{2}\sqrt{1 + 8N/A^2} \quad .$$

The exact derivation of the optimum threshold can be found by letting

$$P_{R(t)}(r/m_0) = P_{R(t)}(r/m_1) \tag{10}$$

Eq. (10) can be written as

$$\frac{r}{N} \; e^{-\frac{r^2}{2N}} = \frac{r}{N} \; e^{-\frac{A^2+r^2}{2N}} \quad I_0 \left(\frac{Ar}{N}\right)$$

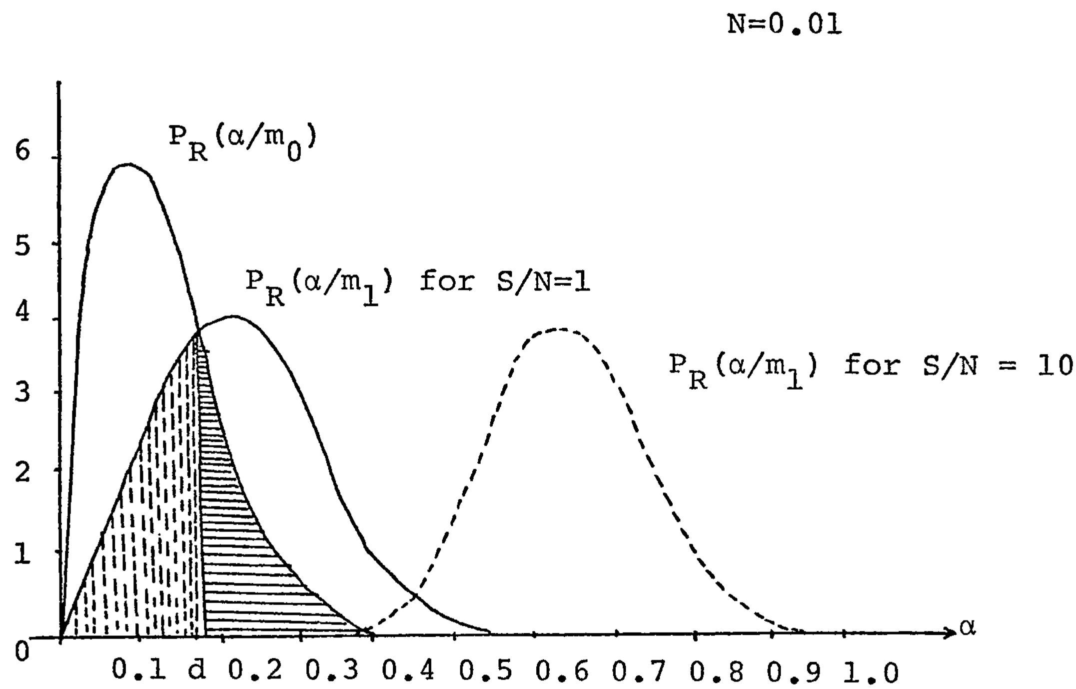

Figure 2.5. Probability Density Function of the Envelope

and when simplified, it becomes

$$I_0(\frac{Ar}{N}) = e^{\frac{A^2}{2N}} \quad . \tag{11}$$

The solution of (11) for the optimum threshold d, is a function of signal-to-noise ratio,

$$(S/N) = (A^2/4)/N = A^2/4N.$$

which is listed in Table (2.1).

Thus the probability of error when "m_0" is sent, denoted by P_{e0}, is

$$\begin{aligned} P_{e0} &= \int_d^{\infty} P_{R(t)}(r/m_0)\,dr \\ &= \int_d^{\infty} \frac{r}{N}\, e^{-\frac{r^2}{2N}}\, dr \\ &= e^{-\frac{d^2}{2N}} = e^{-(\frac{1}{\sqrt{2}} \cdot \frac{d}{\sqrt{N}})^2} \end{aligned} \tag{12}$$

and the probability of error when "m_1" is sent, deonted by P_{e1}, is

$$\begin{aligned} P_{e1} &= \int_o^{d} P_{R(t)}(r/m_1)\,dr \\ &= \int_o^{d} \frac{r}{N}\, e^{-\frac{A^2+r^2}{2N}}\, I_0(\frac{Ar}{N})\,dr \end{aligned}$$

Table 2.1

S/N ; $A^2/4N$

A ; amplitude of the c - w

N ; noise power

d ; optimum threshold

P [E] ; probability of error

S/N (db)	$d/\sqrt{N}$	$\log_{10} P$ [E]
-1.08	1.69	-0.5360
0.00	1.75	-0.5875
1.21	1.84	-0.6727
1.94	1.91	-0.7348
2.92	2.02	-0.8355
4.08	2.18	-0.9842
5.01	2.34	-1.1491
6.02	2.51	-1.3304
7.04	2.72	-1.5790
7.96	2.94	-1.8506
8.94	3.21	-2.2153
9.97	3.53	-2.6893
11.01	3.91	-3.2938
12.04	4.33	-4.0543
13.05	4.81	-4.9997
13.98	5.28	-6.0515
15.04	5.91	-7.5784

$$= \int_{0}^{d/\sqrt{N}} x\, e^{-\frac{\left(\sqrt{\frac{2S}{N}}\right)^2 + x^2}{2}} I_0\left(\sqrt{\frac{2S}{N}}\, x\right) dx$$

$$= 1 - Q\left(\sqrt{\frac{2S}{N}}\,,\ \frac{d}{\sqrt{N}}\right) \tag{13}$$

where

$$Q(\alpha,\beta) = \int_{\beta}^{\infty} x e^{-\frac{\alpha^2 + x^2}{2}} I_0(\alpha x)\, dx.$$

Thus the bit-error probability is

$$P[E] = P[m_0]\, P_{e0} + P[m_1]\, P_{e1}$$

$$= \frac{1}{2}\, [P_{e0} + P_{e1}] \tag{14}$$

P[E] is tabulated in Table (2.1) as a function of signal-to-noise ratio (S/N) = $A^2/4N$. The probability of bit error vs S/N is compared in Figure 2.6 for noncoherent detection of ASK and coherent detection of ASK using a sample detector. The values of optimum threshold vs S/N are plotted in Fig. 2.7.

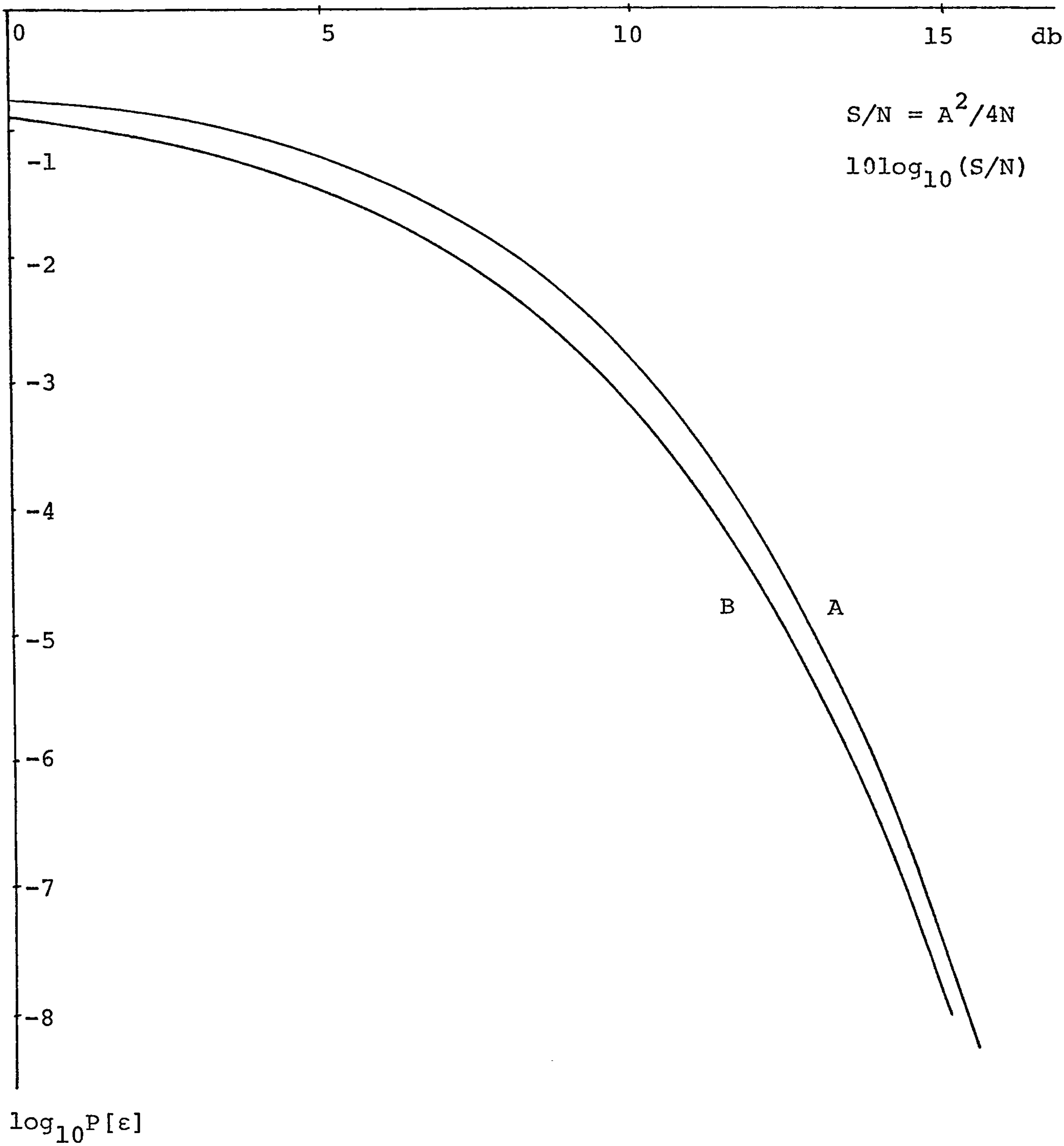

Figure 2.6. Probability of Bit-Error vs S/N

A: Noncoherent Detection of ASK
B: Coherent Detection of ASK

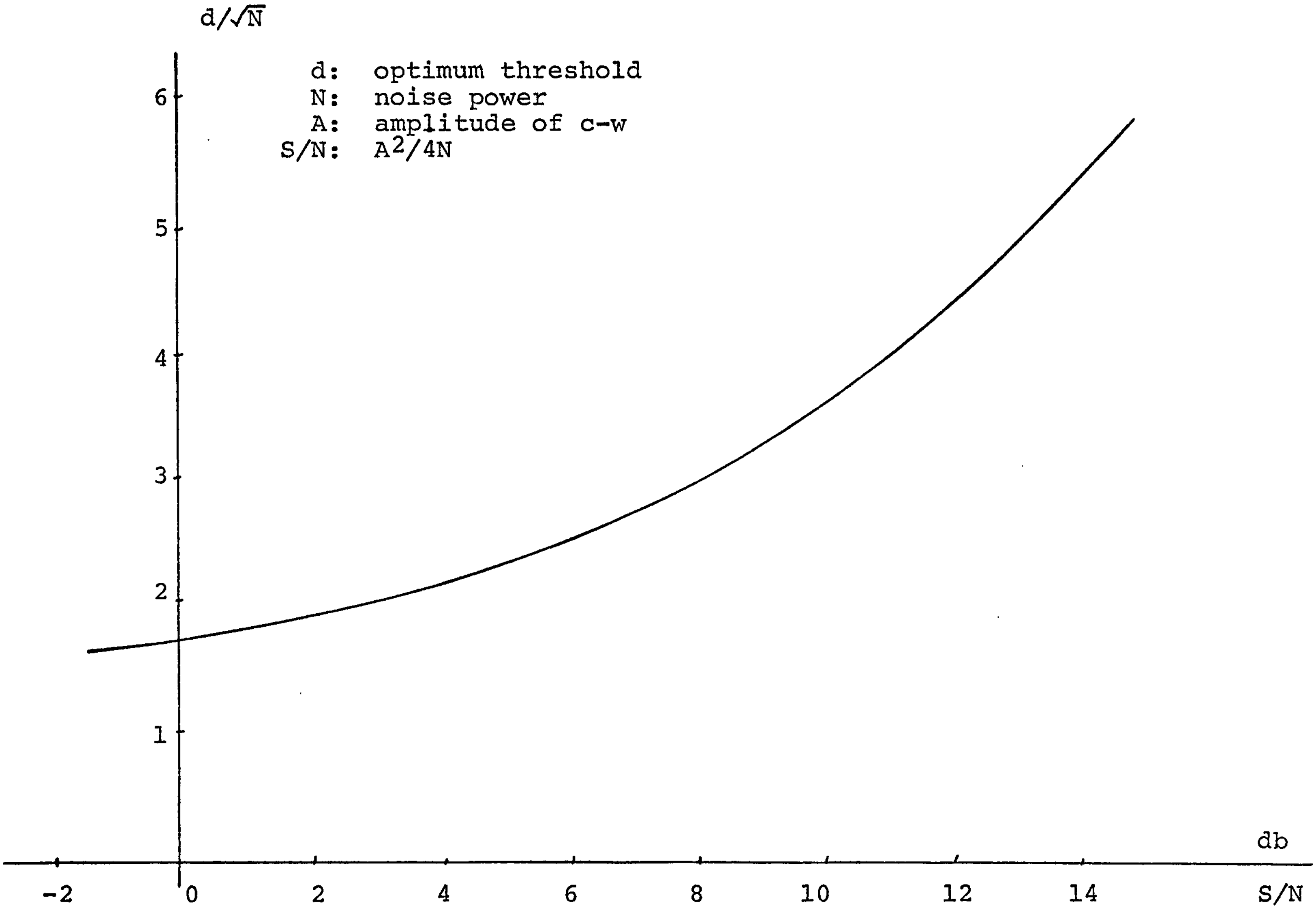

Figure 2.7. Optimum Threshold vs S/N

CHAPTER III

ENVELOPE DETECTION FOR THE FILTERED SIGNAL

In Chapter II we assumed that the signal component of the received waveform was unaffected by transmission except for the addition of noise. Actually the transmitted signal is disturbed not only by the addition of noise but also by the channel. The channel distortion cannot be neglected if the bandwidth of the channel is limited. Bandwidth limiting will cause energy loss of the desired signal, but more importantly it will introduce interference. This interference consists of intersymbol interference (signal waveform smearing in time) and intermodulation interference (aliasing effects). Aliasing effects can be neglected if carrier frequency is much larger than the bandwidth of the modulating signal, which will be assumed in this thesis.

In this chapter the performance of the envelope detector for the filtered signal in the presence of additive white Gaussian noise will be considered.

3.1 Filtered-Signal Channel

The filtered-signal channel with additive white Gaussian noise is pictured in Figure 3.1. The filtered signal is specified by

$$s_{fi}(t) = s_i(t)*h(t) = \int_{-\infty}^{\infty} s_i(\alpha)\ h(t - \alpha)\ d\alpha \qquad (15)$$

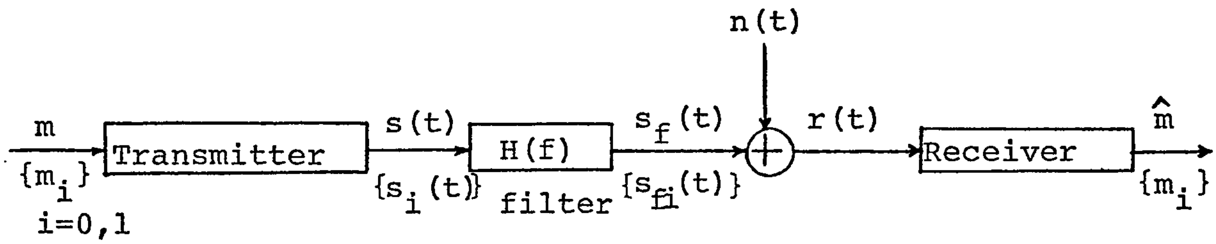

Figure 3.1. The Filtered Signal Channel with Additive White Gaussian Noise

where $h(t)$ is the impulse response of the filter $H(f)$.

The receiver for the additive white Gaussian noise with filtering is identical to the one in Chapter II for the channel without filtering, since the condition that m_i was transmitted uniquely specifies the output of the filter $s_{fi}(t)$. Thus the optimum receiver selects $m = m_o$ for $P[m_o] = P[m_1]$, if and only if

$$P_{r(t)}(\alpha/m_o) \geq P_{r(t)}(\alpha/m_1) \tag{16}$$

which is the same statement as

$$P_{r(t)}(\alpha/s_{f0(t)}) \geq P_{r(t)}(\alpha/s_{f1(t)}) \tag{17}$$

where $s_{f0(t)}$ and $s_{f1(t)}$ correspond to the output of the filter due to input m_0 and m_1, respectively.

3.2 Envelope Detection of the Filtered Signal

The receiver for the filtered signal using an envelope detector is pictured in Figure 3.2. Here $\{m_i\}$ is the set of binary input message, and $\{s_i(t)\}$ is the corresponding outputs of the vector transmitter where

$$s_i(t) = \begin{cases} A_i & nT \leq t \leq (n+1)T \\ 0 & \text{elsewhere} \end{cases}$$

The filter $H(f)$ is an ideal lowpass filter with bandwidth B depicted in Figure 3.3.

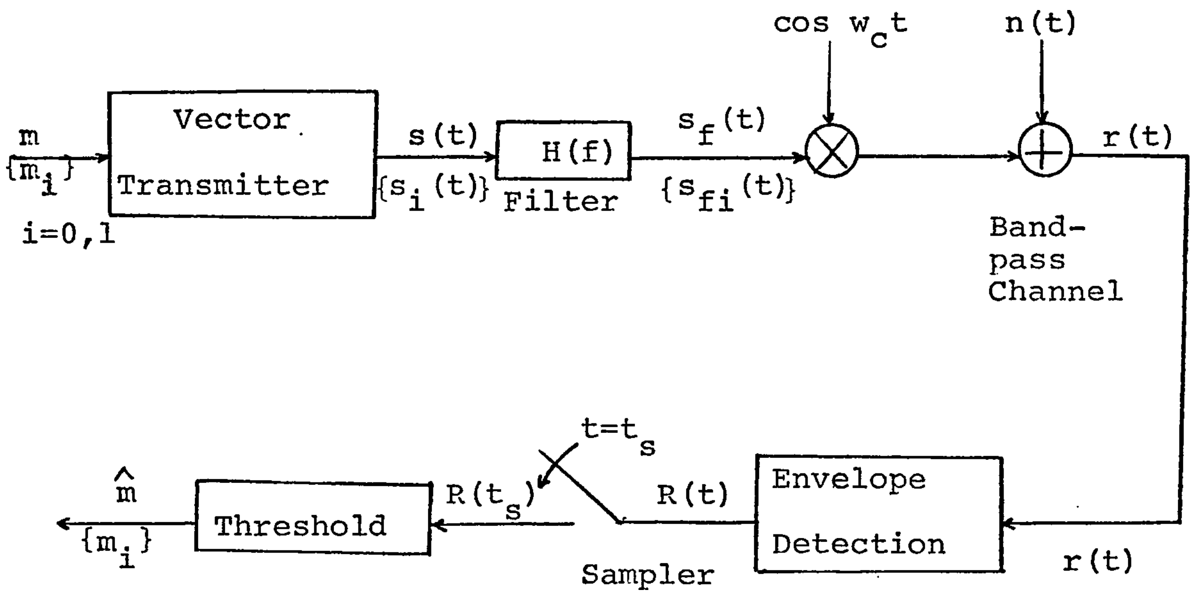

Figure 3.2. Ideal Bandpass Additive White Gaussian Noise Channel With Filtered Signal Using an Envelope Detector Receiver.

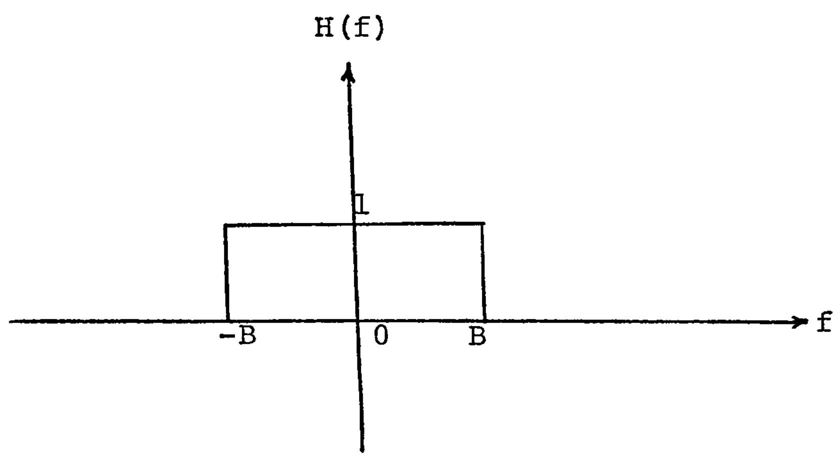

Figure 3.3. Ideal Lowpass Filter

The filtered signal $s_{fi}(t)$ can be written as

$$s_{fi}(t) = s_i(t)*h(t) = \int_{-\infty}^{\infty} s_i(\xi)\ h(t - \xi)\ d\xi$$

where $h(t)$ is the impulse response of the system, which is the inverse Fourier transform of $H(f)$. The transmitted signal $X_c(t)$ is then equal to

$$X_c(t) = s_{fi}(t) \cos w_c t$$

The envelope detector detects the envelope $R(t)$ of the signal $r(t)$ which is the sum of the bandpass noise $n(t)$ and the transmitted signal $X_c(t)$. The envelope output is sampled at $t = t_s$ and on the basis of the sampled signal $R(t_s)$, the decision is made as to which message was transmitted.

The optimum threshold which minimizes the probability of error can be obtained by letting

$$P_{R(t)}(r/m_0) = P_{R(t)}(r/m_1) .$$

Also the optimum decision rule can be stated as follows:

$R(t_s)$ is mapped onto "m_0", if and only if

$$P_{R(t_s)}(r/m_0) \geq P_{R(t_s)}(r/m_1) .$$

3.3 Intersymbol Interference by Limiting the Bandwidth of the Filter

The n-th bit of information can be represented by

$$s_n(t) = \begin{cases} A_n & nT \le t \le (n+1)T \\ 0 & \text{elsewhere} \end{cases} \tag{18}$$

where A_n is equal to A when "m_1" is transmitted and zero otherwise. The response of the lowpass filter due to the n-th bit is

$$s_{fn}(t) = s_n(t)*h(t) = F^{-1}[S_n(f)\ H(f)]$$

where

$$S_n(f) = F[s_n(t)]$$

$$= \int_{-\infty}^{\infty} s_n(t)\ e^{-j2\pi ft}\ dt$$

$$s_{fn}(t) = \int_{-\infty}^{\infty} S_n(f)\ H(f)\ e^{j2\pi ft}\ df$$

$$= \int_{-B}^{B} \left\{ \int_{nT}^{(n+1)T} A_n e^{-j2\pi ft}\ dt \right\} e^{j2\pi ft}\ df$$

$$= \int_{-B}^{B} A_n\ T\ \frac{\sin \pi fT}{fT}\ e^{-j2\pi f\{(2n+1)T-2t\}} df$$

After changing variables, the above can be written as

$$s_{fn}(t) = \int_{-\pi BT}^{\pi BT} \frac{1}{\pi} A_n \frac{\sin x}{x} e^{-j\{2n+1-2t/T\}X} dX$$

Since $\frac{\sin X}{X}$ is an even function, we have

$$s_{fn}(t) = \frac{2A_n}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(2n+1-2t/T)X \, dX \qquad (19)$$

Thus $s_{fn}(t)$ is a function of A_n , BT , t/T , and n and can be expressed as

$$s_{fn}(t) = A_n SI(BT, t/T, n)$$

where

$$SI(BT,t/T,n) = \frac{2}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(2n+1-2t/T)X \, dX$$

$$= \frac{2}{\pi} \int_0^{\pi BT} \frac{1}{X} \frac{1}{2} \{\sin(2n+1-2t/T+1)X + \sin(-2n-1 + 2t/T+1)X\} dx$$

$$= \frac{1}{\pi} \int_0^{\pi BT} \frac{1}{x} \{\sin(2n+2-2t/T)x - \sin(2n-2t/T)x\} dx$$

$$= \frac{1}{\pi} \left[\int_0^{(2n+2-2t/T)\pi BT} \frac{\sin Y}{Y} dY - \int_0^{(2n-2t/T)\pi BT} \frac{\sin Y}{Y} dY \right]$$

$$= \frac{1}{\pi} \int_{(2n-2t/T)\pi BT}^{(2n+2-2t/T)\pi BT} \frac{\sin X}{X} dX$$

Table 3.1 shows values of SI(BT, t/T, n) as a function of BT for different values of t/T and n .

The response of the lowpass filter due to the infinite pulse train can be expressed as

$$W(t) = \sum_{n=-\infty}^{\infty} s_{fn}(t) = \sum_{n=-\infty}^{\infty} A_n SI(BT,\ t/T,\ n)$$

$$= \frac{2A_0}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos\ (1 - 2t/T)XdX$$

$$+ \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A_n}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos\ (2n+1 - 2t/T)XdX \qquad (20)$$

The first term is the desired signal and the second term is the intersymbol interference due to limiting the bandwidth of the filter. As $B \to \infty$, the first term becomes $s_0(t)$ and the second term becomes zero, for $0 \le t \le T$, which was the case discussed in Chapter II. Mathematically,

$$\lim_{B\to\infty} \frac{2A_0}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(1 - 2t/T)XdX$$

$$= \lim_{B\to\infty} \int_{-\pi BT}^{\pi BT} \frac{A_0 \sin X}{\pi X} \cos(1 - 2t/T)XdX$$

$$= \lim_{B\to\infty} \int_{-B}^{B} \{ \int_0^T A_0\ e^{-j2\pi ft}\ dt\}\ e^{j2\pi ft}\ df$$

$$= s_0(t)$$

TABLE 3.1

SI(BT, t/T, n)

BT = 0.5

t/T	n = -2	n = -1	n = 0	n = 1	n = 2
0.50	-0.167×10^{-1}	0.756×10^{-1}	0.873	0.756×10^{-1}	-0.167×10^{-1}
0.55	-0.522×10^{-1}	0.350×10^{-1}	0.869	0.120	-0.339×10
0.60	0.158×10^{-1}	-0.171×10^{-2}	0.860	0.167	-0.511×10
0.65	0.304×10^{-1}	-0.343×10^{-1}	0.844	0.217	-0.680×10
0.70	0.436×10^{-1}	-0.625×10^{-1}	0.823	0.269	-0.840×10
0.75	0.550×10^{-1}	-0.862×10^{-1}	0.795	0.322	-0.988×10
0.80	0.646×10^{-1}	-0.105×10^{0}	0.763	0.377	-0.112
0.85	0.721×10^{-1}	-0.120	0.725	0.431	-0.123
0.90	0.774×10^{-1}	-0.130	0.684	0.485	-0.131
0.95	0.806×10^{-1}	-0.136	0.638	0.538	-0.136

BT = 0.7

t/T	n = -2	n = -1	n = 0	n = 1	n = 2
0.50	0.478×10^{-1}	-0.833×10^{-1}	0.107×10^{1}	-0.833×10^{-1}	0.478×10^{-1}
0.55	0.382×10^{-1}	-0.101	0.107×10^{1}	-0.563×10^{-1}	0.573×10^{-1}
0.60	0.263×10^{-1}	-0.111	0.105×10^{1}	-0.204×10^{-1}	0.633×10^{-1}
0.65	0.137×10^{-1}	-0.114	0.102×10^{1}	0.246×10^{-1}	0.663×10^{-1}
0.70	0.105×10^{-2}	-0.109	0.971	0.784×10^{-1}	0.659×10^{-1}
0.75	-0.110×10^{-1}	-0.995×10^{-1}	0.916	0.140	0.619×10^{-1}
0.80	-0.220×10^{-1}	-0.851×10^{-1}	0.851	0.209	0.543×10^{-1}
0.85	-0.314×10^{-1}	-0.676×10^{-1}	0.779	0.285	0.432×10^{-1}
0.90	-0.388×10^{-1}	-0.480×10^{-1}	0.701	0.365	0.289×10^{-1}
0.95	-0.439×10^{-1}	-0.275×10^{-1}	0.618	0.449	0.119×10^{-1}

TABLE 3.1 (CONT'D)

BT = 1.0

t/T	n = -2	n = -1	n = 0	n = 1	n = 2
0.50	-0.130×10^{-1}	-0.564×10^{-1}	0.118×10^{1}	-0.546×10^{-1}	-0.130×10^{-1}
0.55	-0.123×10^{-1}	-0.534×10^{-1}	0.117×10^{1}	-0.528×10^{-1}	-0.123×10^{-1}
0.60	-0.106×10^{-1}	-0.455×10^{-1}	0.114×10^{1}	-0.408×10^{-1}	-0.102×10^{-1}
0.65	-0.795×10^{-1}	-0.344×10^{-1}	0.109×10^{1}	-0.189×10^{-1}	-0.677×10^{-2}
0.70	-0.480×10^{-2}	-0.220×10^{-1}	0.103×10^{1}	0.141×10^{-1}	-0.219×10^{-2}
0.75	-0.147×10^{-2}	-0.944×10^{-2}	0.948	0.589×10^{-1}	0.321×10^{-2}
0.80	0.171×10^{-2}	0.195×10^{-2}	0.858	0.116	0.896×10^{-2}
0.85	0.445×10^{-2}	0.113×10^{-1}	0.760	0.185	0.145×10^{-1}
0.90	0.653×10^{-2}	0.182×10^{-1}	0.657	0.265	0.192×10^{-1}
0.95	0.782×10^{-2}	0.223×10^{-1}	0.553	0.354	0.224×10^{-1}

BT = 2.0

t/T	n = -2	n = -1	n = 0	n = 1	n = 2
0.50	0.668×10^{-2}	0.318×10^{-1}	0.903	0.318×10^{-1}	0.668×10^{-2}
0.55	0.544×10^{-2}	0.259×10^{-1}	0.922	0.247×10^{-1}	0.535×10^{-2}
0.60	0.232×10^{-2}	0.119×10^{-1}	0.974	0.366×10^{-2}	0.167×10^{-2}
0.65	-0.135×10^{-2}	-0.349×10^{-2}	0.104×10^{1}	-0.268×10^{-1}	-0.312×10^{-2}
0.70	-0.418×10^{-2}	-0.147×10^{-1}	0.110×10^{1}	-0.560×10^{-1}	-0.723×10^{-2}
0.75	-0.522×10^{-2}	-0.187×10^{-1}	0.112×10^{1}	-0.694×10^{-1}	-0.888×10^{-2}
0.80	-0.425×10^{-2}	-0.152×10^{-1}	0.109×10^{1}	-0.510×10^{-1}	-0.708×10^{-2}
0.85	-0.179×10^{-2}	-0.682×10^{-2}	0.101×10^{1}	0.119×10^{-1}	-0.211×10^{-2}
0.90	0.111×10^{-2}	0.272×10^{-2}	0.860	0.125	0.444×10^{-2}
0.95	0.337×10^{-2}	0.986×10^{-2}	0.676	0.284	0.101×10^{-1}

where $s_0(t)$ is equal to A for $0 \le t \le T$ and equal to zero elsewhere. The second term is equal to

$$\lim_{B\to\infty} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A_n}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(2n + 1 - 2t/T)X dX$$

$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \lim_{B\to\infty} \int_{-B}^{B} \{ \int_{nT}^{(n+1)T} A_n e^{-j2\pi ft} dt\} \quad e^{j2\pi ft} df$$

$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} s_n(t)$$

$$= 0 \qquad\qquad \text{for} \quad 0 \le t \le T$$

The transmitted signal due to the infinite pulse train is

$$X_c(t) = w(t) \cos w_c t$$

$$= [\frac{2A_0}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(1 - 2t/T)X dX] \quad \cos w_c t$$

$$+ [\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A_n}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(2n + 1 - 2t/T)X dX] \quad \cos w_c t$$

The noise power N can be represented as a function of the parameters of the channel. Since the white Gaussian noise power spectrum is

$$s_w(f) = N_0/2 \;, \qquad -\infty < f < \infty$$

then the power in the bandpass nosie is

$$E\,\{n^2(t)\} = \overline{n^2(t)} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df$$

where $H(f)$ is the transfer function of the channel filter which is shown in Figure 3.4. For the bandpass channel in Figure 3.4, the noise power is

$$E\,\{n^2(t)\} = \frac{N_0}{2}[\int_{-f_c-B}^{-f_c+B} |1|^2\, df + \int_{f_c-B}^{f_c+B} |1|^2\, df]$$

$$= \frac{N_o}{2}\cdot 2\cdot 2\cdot B = 2BN_0 = N$$

If we limit intersymbol interference to the n preceding and subsequent bits, we then have 2^{2n} different patterns to consider. Denote $P_i(\varepsilon)$ as the probability that the center bit is detected in error given that i-th pattern is transmitted. Since each pattern will occur with the same probability, the average bit-error probability $P[\varepsilon]$ can be expressed as

$$P[\varepsilon] = \frac{1}{2^{2n}} \sum_{i=1}^{2^{2n}} P_i(\varepsilon) \tag{21}$$

where $P_i(\varepsilon)$ is the probability of error for the i-th pattern, i.e.,

$$P_i(\varepsilon) = \frac{1}{2}\,[P_i(\varepsilon/m_o) + P_i(\varepsilon/m_1)]$$

Denote by $f_0(t)$ the output of the lowpass filter when "m_0"

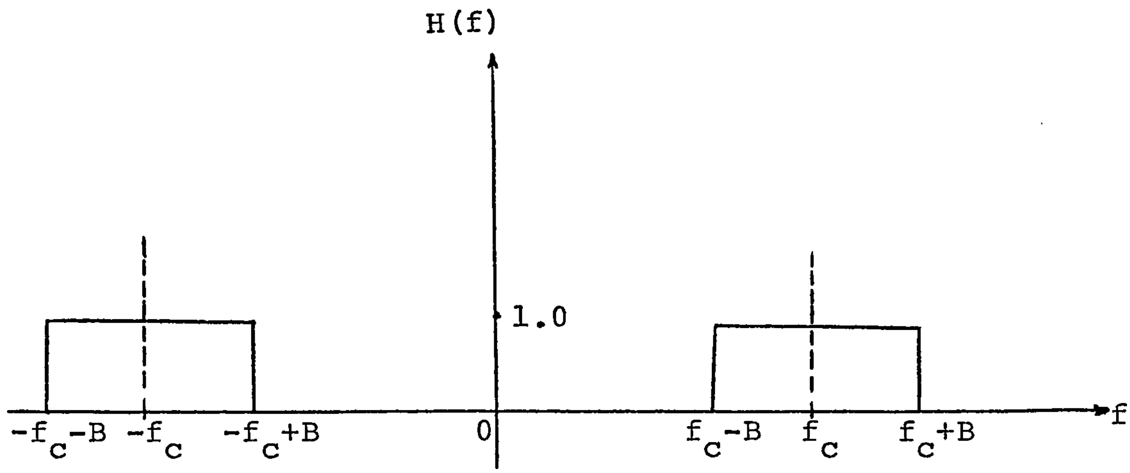

Figure 3.4. Bandpass Channel Characteristic

is transmitted. This will be expressed as

$$\{f_0(t)\} = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2A_n}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(2n+1 - 2t/T)X \, dX$$

$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} A_n \, SI(BT, t/T, n)$$

$$= A \, SI_0(BT, t/T) \tag{22}$$

where

$$SI_0(BT, t/T) = \frac{1}{A} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} A_n \, SI(BT, t/T, n)$$

Also denote by $f_1(t)$ the output of the lowpass filter when "m_1" is transmitted. This will be expressed as

$$\{f_1(t)\} = \{f_0(t)\} + \frac{2A}{\pi} \int_0^{\pi BT} \frac{\sin X}{X} \cos(1 - 2t/T)X \, dX$$

$$= \{f_0(t)\} + A \, SI(BT, t/T, 0)$$

$$= A \, SI_1(BT, t/T) \tag{23}$$

If we could neglect the intersymbol interference for $n \geq M$ there will be 2^{2M} different bit patterns around the bit under detection. These patterns will be shown in Figure 3.5. The lowpass filter output $w(t)$, when the i-th pattern is transmitted, will be denoted by $f_{0,i}(t)$ for $A_0 = 0$ and $f_{1,i}(t)$

$$A_{-M}\ \underbrace{\cdots}_{M\ \text{bits}}\ A_{-1}\ \ A_0\ \ A_1 \cdots A_M$$

bit under detection

Adjacent i-th pattern	A_{-M}	$A_{-(M-1)}$	...	A_{-1}	A_0	A_1	...	A_{M-1}	A_M
1	0	0	...	0	A_0	0	...	0	0
2	0	0	...	0	A_0	0	...	0	A
.	.	.		.	.	.		.	.
.	.	.		.	.	.		.	.
.	.	.		.	.	.		.	.
2^{2M}	A	A	...	A	A_0	A	...	A	A

Figure 3.5. List of the Adjacent Patterns

for $A_0 = A$. As an example consider the second adjacent pattern

$$0\ 0\ .\ .\ .\ 0\ A_0\ 0\ .\ .\ .\ 0\ A\ .$$

The lowpass filter output $w(t)$, when $A_0 = 0$, will be denoted by $f_{0,2}(t)$. Then

$$f_{0,2}(t) = \frac{2A}{\pi}\int_0^{\pi BT} \frac{\sin X}{X} \cos(-2M + 1 - 2t/T)X\,dX$$

$$= A\ SI(BT,\ t/T,\ -2M)$$

$$= A\ SI_{0,2}(BT,\ t/T) \tag{24}$$

and $w(t)$, when $A_0 = A$, will be denoted by $f_{1,2}(t)$, where

$$f_{1,2}(t) = f_{0,2}(t) + \frac{2A}{\pi}\int_0^{\pi BT} \frac{\sin X}{X} \cos(1 - 2t/T)X\,dX$$

$$= A\ SI_{1,2}(BT,\ t/T) \tag{25}$$

The effects of bandlimiting on the detection of ASK signals discussed in this chapter will be considered in the calculation of bit-error probability in Chapter IV.

CHAPTER IV

PROBABILITY OF ERROR CALCULATION

In this chapter the probability of bit error is calculated using two different methods, the average threshold method and the average probability density function method.

4.1 The Average Threshold Method

The output of the filter for the i-th pattern can be written as

$$f_{0,i}(t) = \sum_{n=-M, n\neq 0}^{M} \frac{2A_n}{\pi} \int_{0}^{\pi BT} \frac{\sin X}{X} \cos(2n+1 - 2t/T)X dX$$
$$= A\ SI_{0,i}(BT, t/T)$$

for no pulse in the moddle of the pulse train, and

$$f_{1,i}(t) = f_{0,i}(t) + \frac{2A}{\pi} \int_{0}^{\pi BT} \frac{\sin X}{X} \cos(1 - 2t/T)X dX$$
$$= A\ SI_{1,i}(BT,\ t/T)$$

for a pulse in the middle of the pulse train.

The conditional probability density functions for the envelopes are

$$P_R(\alpha/m_0) = P_R(\alpha/f_{0,i})$$
$$= \frac{\alpha}{N} e^{-\frac{f_{0,i}^2 + \alpha^2}{2N}} I_0\left(\frac{\alpha\ f_{0,i}}{N}\right)$$

$$P_R(\alpha/m_1) = P_R(\alpha/f_{1,i})$$

$$= \frac{\alpha}{N} e^{-\frac{f_{1,i}^2+\alpha^2}{2N}} \quad I_0\left(\frac{\alpha f_{1,i}}{N}\right) \tag{26}$$

where the first density function is for the zero input and the second density function is for the nonzero input. The conditional density functions for the envelope of the i-th pattern are shown in Figure 4.1.

The optimum threshold for the given i-th pattern will be denoted "d_i" and the bit-error probability $P_i[\varepsilon]$ can be calculated as

$$P_i[\varepsilon] = \frac{1}{2}\left[\int_{d_i}^{\infty} P_R(\alpha/f_{0,i})d\alpha + \int_{o}^{d_i} P_R(\alpha/f_{1,i})d\alpha\right]$$

$$= \frac{1}{2}\left[Q\left(\sqrt{\frac{2S}{N} \quad SI_{0,i}^2(BT,t/T)}\ ,\ \frac{d_i}{\sqrt{N}}\right)\right.$$

$$\left. + 1 - Q\left(\sqrt{\frac{2S}{N} \quad SI_{1,i}^2(BT,t/T)},\ \frac{d_i}{\sqrt{N}}\right)\right]$$

For another pattern, the bit-error probability will be,

$$P_j[\varepsilon] = \frac{1}{2}\left[\int_{d_j}^{\infty} P_R(\alpha/f_{0,j})d\alpha + \int_{o}^{d_j} P_R(\alpha/f_{1,j})d\alpha\right]$$

$$= \frac{1}{2}\left[Q\left(\sqrt{\frac{2S}{N} SI_{0,j}^2 \ (BT,t/T)},\ \frac{d_j}{\sqrt{N}}\right)\right.$$

$$\left. + 1 - Q\left(\sqrt{2S/N \cdot SI_{1,j}^2(BT,t/T)},\frac{d_j}{\sqrt{N}}\right)\right]$$

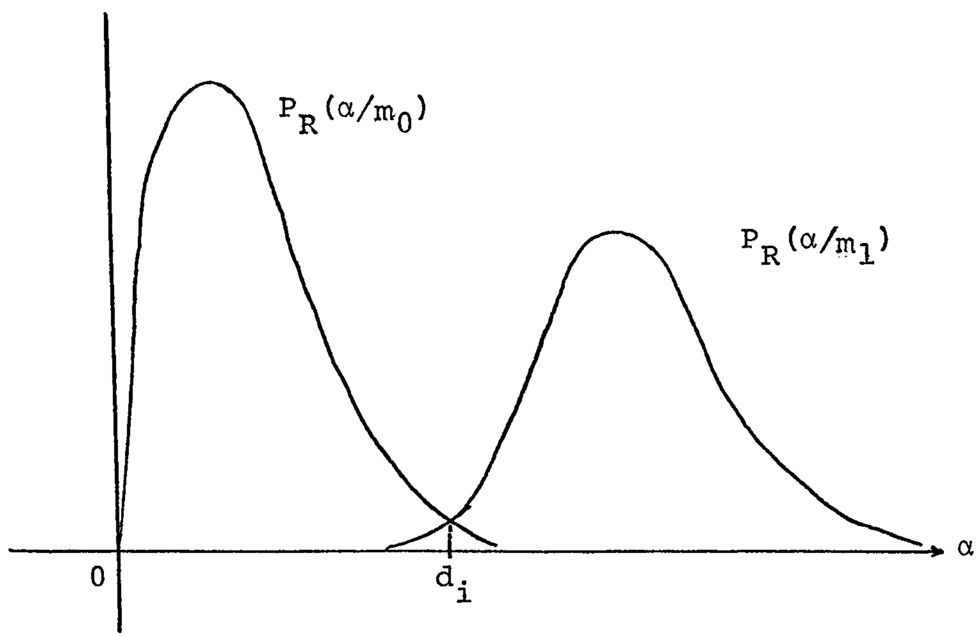

Figure 4.1. Conditional Probability Density Functions for the i-th Pattern

For M=2, the different sequences, conditional probability density functions, and the probabilities of error are

Adjacent Pattern	Adjacent Pattern Sequence	$f_{0,i}$	$f_{1,i}$	$P_R(\alpha/f_{0,i})$	$P_R(\alpha/f_{1,i})$	d_i	$P_i[\varepsilon]$	d_i	$P_i[E]$
1	0 0 A_0 0 0	$f_{0,1}$	$f_{1,1}$	$P_R(\alpha/f_{0,1})$	$P_R(\alpha/f_{1,1})$	d_1	$P_1[\varepsilon]$	d	$P_1[E]$
2	0 0 A_0 0 0	$f_{0,2}$	$f_{1,2}$	$P_R(\alpha/f_{0,2})$	$P_R(\alpha/f_{1,2})$	d_2	$P_2[\varepsilon]$	d	$P_2[E]$
.	.	.		.	.	.	.	.	.
.	.	.		.	.	.	.	.	.
.	.	.		.	.	.	.	.	.
16	A A A_0 A A	$f_{0,16}$	$f_{1,16}$	$P_R(\alpha/f_{0,16})$	$P_R(\alpha/f_{1,16})$	d_{16}	$P_{16}[\varepsilon]$	d	$P_{16}[E]$
						d	$P[\varepsilon]$	d	P [E]

Thus the probability of bit-error $P[\varepsilon]$ is given by

$$P[\varepsilon] = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_i[\varepsilon]$$

$$= \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{2}\left[\int_{d_i}^{\infty} P_R(\alpha/f_{0,i})d\alpha + \int_{o}^{d_i} P_R(\alpha/f_{1,i})d\alpha \right]$$

From a practical point of view it is desirable to fix the threshold. One way to do is to consider d as the average of all thresholds

$$d = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} d_i$$

and the corresponding probability of error is, for M=2

$$P[E] = \frac{1}{16}\Bigg[\frac{1}{2}\int_d^\infty P_R(\alpha/f_{0,1})d\alpha + \frac{1}{2}\int_o^d P_R(\alpha/f_{1,1})d\alpha$$

$$+ \frac{1}{2}\int_d^\infty P_R(\alpha/f_{0,2})d\alpha + \frac{1}{2}\int_o^d P_R(\alpha/f_{1,2})d\alpha$$

$$+ \ldots$$

$$+ \frac{1}{2}\int_d^\infty P_R(\alpha/f_{0,16})d\alpha + \frac{1}{2}\int_o^d P_R(\alpha/f_{1,16})d\alpha\Bigg]$$

$$= \frac{1}{32}\sum_{i=1}^{16}\Bigg[Q\left(\sqrt{2S/N\ SI^2_{0,i}(BT,t/T)},\ d/\sqrt{N}\right)$$

$$+ 1 - Q\left(\sqrt{2S/N\ SI^2_{1,i}(BT,t/T)},\ d/\sqrt{N}\right)\Bigg] \qquad (27)$$

which in general can be written as

$$P[E] = \frac{1}{2^{2M}}\sum_{i=1}^{2^{2M}} \frac{1}{2}\left[\int_d^\infty P_R(\alpha/f_{0,i})d\alpha + \int_o^d P_R(\alpha/f_{1,i})d\alpha\right]$$

$$= \frac{1}{2^{2M+1}}\sum_{i=1}^{2^{2M}}\Bigg[Q\left(\sqrt{2S/N\ SI^2_{0,i}\ (BT,t/T)},d/\sqrt{N}\right)$$

$$+ 1 - Q\left(\sqrt{2S/N\ SI^2_{1,i}\ (BT,t/T)},\ d/\sqrt{N}\right)\Bigg] \qquad (28)$$

4.2 Average Probability Density Function Method

Another way of finding the probability of bit-error is to calculate the average of the conditional probability densities for a zero pulse and the conditional probability density for the nonzero pulse. These averages are

$$P_R(\alpha/m_0) = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_R(\alpha/f_{0,i}), \text{ for } m_0$$

and

$$P_R(\alpha/m_1) = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_R(\alpha/f_{1,i}), \text{ for } m_1$$

Thus the probability of bit-error is

$$P[E] = \frac{1}{2}\left[\int_{\beta}^{\infty} P_R(\alpha/m_0)\,d\alpha + \int_{o}^{\beta} P_R(\alpha/m_1)\,d\alpha\right] \tag{29}$$

where β is the optimum threshold, which is the intersection of the average probability density functions $P_R(\alpha/m_0)$ and $P_R(\alpha/m_1)$.

The probability of error can be written as

$$P[E] = \frac{1}{2}\left[\int_{\beta}^{\infty} \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_R(\alpha/f_{0,i})\,d\alpha + \int_{o}^{\beta} \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_R(\alpha/f_{1,i})\,d\alpha\right]$$

$$= \frac{1}{2^{2M+1}} \sum_{i=1}^{2^{2M}} \left[Q\left(\sqrt{2S/N\ SI_{1,i}^2(BT,t/T)},\ \beta/\sqrt{N}\right) + 1 - Q\left(\sqrt{2S/N\ SI_{1,i}^2(BT,t/T)},\ \beta/\sqrt{N}\right)\right] \tag{30}$$

where β is a solution of the equation

$$\sum_{i=1}^{2^{2M}} \left\{ P_R(\beta/f_{0,i}) - P_R(\beta/f_{1,i}) \right\} = 0$$

or

$$\sum_{i=1}^{2^{2M}} \left\{ \frac{\beta}{N} e^{-\frac{f_{0,i}^2+\beta^2}{2N}} I_0\left(\frac{\beta f_{0,i}}{N}\right) - \frac{\beta}{N} e^{-\frac{f_{1,i}^2+\beta^2}{2N}} I_0\left(\frac{\beta f_{1,i}}{N}\right) \right\} = 0$$

or

$$\sum_{i=1}^{2^{2M}} \left\{ e^{-\frac{f_{0,i}^2}{2N}} I_0\left(\frac{\beta f_{0,i}}{N}\right) - e^{-\frac{f_{1,i}^2}{2N}} I_0\left(\frac{\beta f_{1,i}}{N}\right) \right\} = 0 \quad (31)$$

Table 4.1 shows the probability of bit-error as a function of signal-to-noise ratio for $BT \leq 1.0$ using the two different methods for M=2. The sampling time t_s is set at 0.5T where the signal value is maximum [4]. However for $BT \geq 1.0$ the optimum sampling time t_s is variable and can be determined from Table 4.3 and Figure 4.3. Tables 4.1 and 4.2 are plotted in Figures 4.2 and 4.3, respectively. For $BT \geq 1.0$ the probability of error is tabulated in Table 4.3 for differet sampling time t_s using the method in 4.2. Table 4.3 is plotted in Figure 4.4.

Table 4.1

Probability of Error by Methods (1) and (2)

(1) : Average threshold method.

(2) : Average probability density function method.

$S/N=A^2/4N$

A : Amplitude of the rectangular pulse.

d : Average threshold.

β : Optimum threshold.

t_s : 0.5T (sampling time).

M : 2.

BT=0.5

S/N db	Threshold method (1) $d/\sqrt{N}$	Threshold method (2) $\beta/\sqrt{N}$	Average probability of Error by (1)	Average probability of Error by (2)
-6.02	1.50	1.50	0.4285×10^{0}	0.4285×10^{0}
0		1.72		0.2812×10^{0}
3.52	2.05	2.04	0.1506×10^{0}	0.1506×10^{0}
6.02		2.42		0.6820×10^{0}
7.96	2.85	2.83	0.2691×10^{-1}	0.2680×10^{-1}
9.54		3.26		0.9403×10^{-2}
10.88	3.75	3.70	0.2995×10^{-2}	0.2957×10^{-2}
12.04		4.15		0.8596×10^{-3}
13.06	4.70	4.62	0.2448×10^{-3}	0.2344×10^{-3}
13.98		5.09		0.6112×10^{-4}

BT = 0.6

S/N db	Threshold Method (1) $d/\sqrt{N}$:	Threshold Method (2) $\beta/\sqrt{N}$:	Average probability of error by (1)	Average probability of error by (2)
-6.02	1.51	1.51	0.4147	0.4147
0		1.78		0.2502
3.53	2.15	2.15	0.1182	0.1182
6.02		2.58		0.4528×10^{-1}
7.96	3.05	3.02	0.1460×10^{-1}	0.1456×10^{-1}
9.54		3.49		0.3939×10^{-2}
10.88	4.04	3.97	0.9590×10^{-3}	0.9321×10^{-3}
12.04		4.46		0.1996×10^{-3}
13.06	5.07	4.96	0.4481×10^{-4}	0.4105×10^{-4}
13.98		5.47		0.9953×10^{-5}

BT = 0.7

S/N db	Threshold Method (1) $d/\sqrt{N}$:	Threshold Method (2) $\beta/\sqrt{N}$:	Average probability of error by (1)	Average probability of error by (2)
-6.02	1.51	1.52	0.4129	0.4129
0		1.79		0.2467
3.52	2.17	2.17	0.1157	0.1157
6.02		2.60		0.4437×10^{-1}
7.96	3.09	3.05	0.1463×10^{-1}	0.1453×10^{-1}
9.54		3.52		0.4112×10^{-2}
10.88	4.10	4.00	0.1107×10^{-2}	0.1049×10^{-2}
12.04		4.49		0.2496×10^{-3}
13.06	5.15	4.99	0.6760×10^{-4}	0.5761×10^{-4}
13.98		5.51		0.1445×10^{-4}

BT = 0.8

S/N db	Threshold Method (1) $d/\sqrt{N}$	Threshold Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	Average probability of error by (2)
-6.02	1.52	1.52	0.4131	0.4131
0		1.79		0.2478
3.52	2.18	2.18	0.1176	0.1176
6.02		2.63		0.4631×10^{-1}
7.96	3.13	3.13	0.1590×10^{-1}	0.1587×10^{-1}
9.54		3.62		0.4773×10^{-2}
10.88	4.19	4.15	0.1319×10^{-2}	0.1305×10^{-2}
12.04		4.69		0.3286×10^{-3}
13.06	5.28	5.23	0.7826×10^{-4}	0.7691×10^{-4}
13.98		5.79		0.1779×10^{-4}

BT = 0.9

S/N db	Threshold Method (1) $d/\sqrt{N}$	Threshold Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	Average probability of error by (2)
-6.02	1.52	1.52	0.4069	0.4069
0		1.82		0.2344
3.52	2.23	2.23	0.1041	0.1041
6.02		2.70		0.3719×10^{-1}
7.96	3.21	3.20	0.1113×10^{-1}	0.1112×10^{-1}
9.54		3.73		0.2798×10^{-2}
10.88	4.30	4.27	0.6200×10^{-3}	0.6158×10^{-3}
12.04		4.83		0.1219×10^{-3}
13.06	5.42	5.39	0.2356×10^{-4}	0.2335×10^{-4}
13.98		5.96		0.5861×10^{-5}

BT = 1.0

S/N db	Threshold Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	by (2)
-6.02	1.53	1.53	0.4025	0.4025
0		1.83		0.2251
3.52	2.26	2.25	0.9503×10^{-1}	0.9501×10^{-1}
6.02		2.73		0.3134×10^{-1}
7.96	3.25	3.24	0.8344×10^{-2}	0.8337×10^{-2}
9.54		3.77		0.1971×10^{-2}
10.88	4.35	4.32	0.3271×10^{-3}	0.3246×10^{-3}
12.04		4.88		0.5239×10^{-3}
13.06	5.48	5.44	0.9753×10^{-5}	0.9655×10^{-5}
13.98		6.02		0.3831×10^{-5}

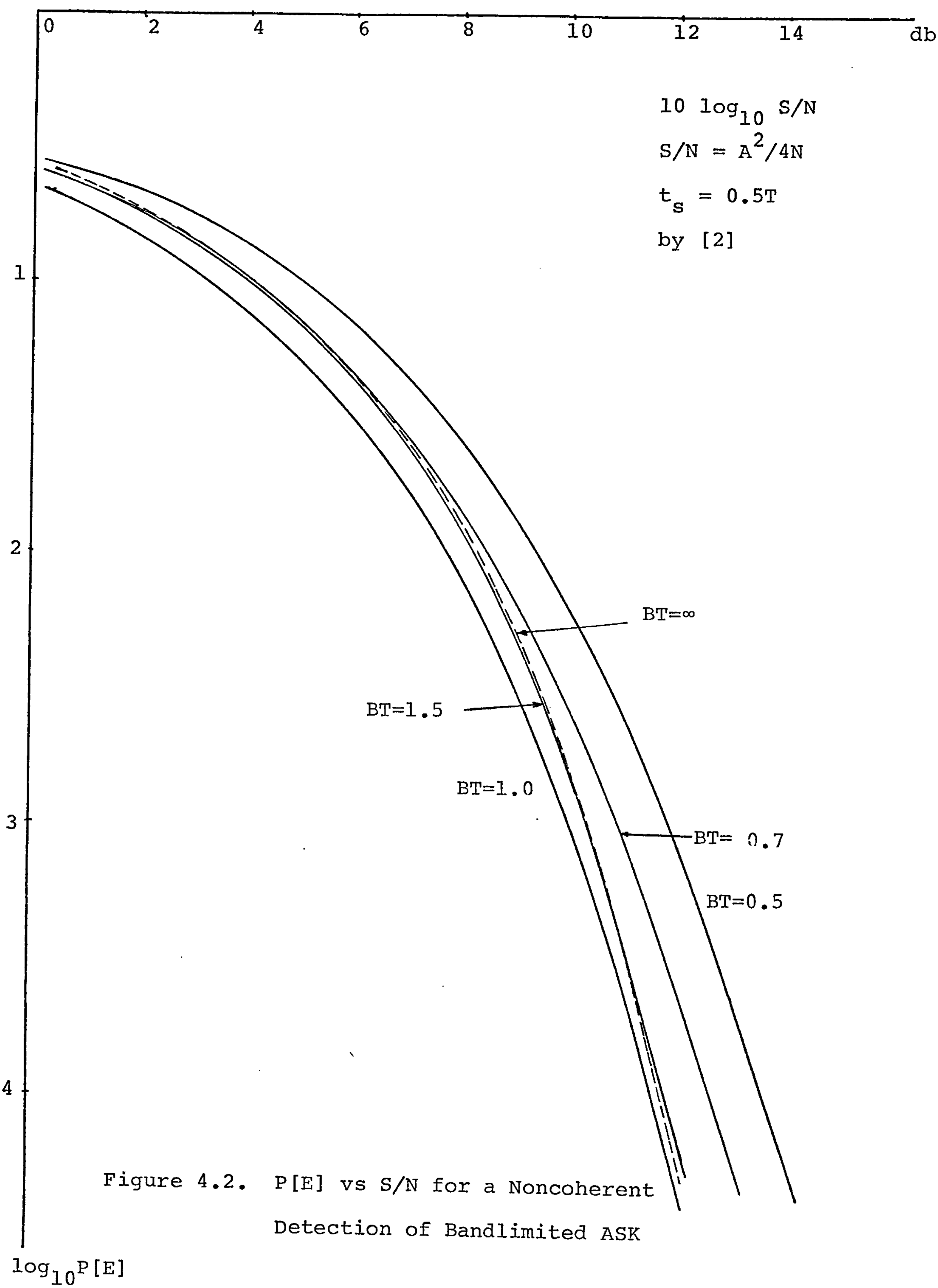

Figure 4.2. P[E] vs S/N for a Noncoherent Detection of Bandlimited ASK

Table 4.2

Probability of error by methods (1) and (2)

(1) ; Average threshold method

(2) ; Average probability density function method

E ; Signal energy

$\frac{E}{N_O} = \frac{S}{N} \cdot 2BT$

d ; Average threshold

β ; Optimum threshold

t_s ; Sampling time (0.5T)

intersymbol interference ; 2 proceeding & 2 following pulse.

BT = 0.5

S/N_O	Threshold		Average probability of error	
db	Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	by (1)	by (2)
-6.05	1.50	1.50	0.4285	0.4285
0.0		1.72		0.2812
3.52	2.05	2.04	0.1506	0.1506
6.02		2.42		0.6820×10^{-1}
7.96	2.85	2.83	0.2691×10^{-1}	0.2686×10^{-1}
9.54		3.26		0.9403×10^{-2}
10.88	3.75	3.70	0.2995×10^{-2}	0.2957×10^{-2}
12.04		4.15		0.8596×10^{-3}
13.06	4.70	4.62	0.2448×10^{-3}	0.2344×10^{-3}
13.98		5.09		0.6112×10^{-4}

BT = 0.6

E/N_0 db	Threshold Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	by (2)
-5.26	1.51	1.51	0.4147	0.4147
0.79		1.78		0.2502
4.31	2.15	2.15	0.1182	0.1182
6.81		2.58		0.4528×10^{-1}
8.75	3.05	3.02	0.1460×10^{-1}	0.1456×10^{-1}
10.33		3.49		0.3939×10^{-2}
11.67	4.04	3.97	0.9590×10^{-3}	0.9321×10^{-3}
12.83		4.46		0.1996×10^{-3}
13.85	5.07	4.96	0.4481×10^{-4}	0.4105×10^{-4}
14.77		5.47		0.9953×10^{-5}

BT = 0.6

E/N_0 db	Threshold Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	by (2)
-4.59	1.51	1.52	0.4129	0.4129
1.46		1.79		0.2467
4.98	2.17	2.17	0.1157	0.1157
7.48		2.60		0.4437×10^{-1}
9.42	3.09	3.05	0.1463×10^{-1}	0.1453×10^{-1}
11.00		3.52		0.4112×10^{-2}
12.34	4.10	4.00	0.1107×10^{-2}	0.1049×10^{-2}
13.50		4.49		0.2496×10^{-3}
14.52	5.15	4.99	0.6760×10^{-4}	0.5761×10^{-4}
15.44		5.51		0.1445×10^{-4}

BT = 0.8

E/N_0 db	Threshold Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	by (2)
-4.01	1.52	1.52	0.4131	0.4131
2.04		1.79		0.2478
5.56	2.18	2.18	0.1176	0.1176
8.06		2.63		0.4631×10^{-1}
10.00	3.13	3.11	0.1590×10^{-1}	0.1587×10^{-1}
11.58		3.62		0.4773×10^{-2}
12.92	4.19	4.15	0.1319×10^{-2}	0.1305×10^{-2}
14.08		4.69		0.3286×10^{-3}
15.10	5.28	5.23	0.7826×10^{-4}	0.7691×10^{-4}
15.02		5.79		0.1779×10^{-4}

BT = 0.9

E/N_0 db	Threshold Method (1) $d/\sqrt{N}$	Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	by (2)
-3.50	1.52	1.52	0.4069	0.4069
2.55		1.82		0.2344
6.07	2.23	2.23	0.1041	0.1041
8.57		2.70		0.3719×10^{-1}
10.50	3.21	3.20	0.1113×10^{-1}	0.1112×10^{-1}
12.09		3.73		0.2798×10^{-2}
13.43	4.30	4.27	0.6200×10^{-3}	0.6158×10^{-3}
14.59		4.83		0.1219×10^{-3}
15.61	5.42	5.39	0.2356×10^{-4}	0.2335×10^{-4}
16.53		5.96		0.5861×10^{-5}

BT = 1.0

E/N_0 db	Threshold Method (1) $d/\sqrt{N}$	Threshold Method (2) $\beta/\sqrt{N}$	Average probability of error by (1)	Average probability of error by (2)
-3.04	1.53	1.53	0.4025	0.4025
3.01		1.83		0.2251
6.53	2.26	2.25	0.9503×10^{-1}	0.9501×10^{-1}
9.03		2.73		0.3134×10^{-1}
10.97	3.25	3.24	0.8344×10^{-2}	0.8337×10^{-2}
12.55		3.77		0.1971×10^{-2}
13.89	4.35	4.32	0.3271×10^{-3}	0.3246×10^{-3}
15.05		4.88		0.5239×10^{-4}
16.07	5.48	5.44	0.9752×10^{-5}	0.9655×10^{-5}
16.99		6.02		0.3831×10^{-5}

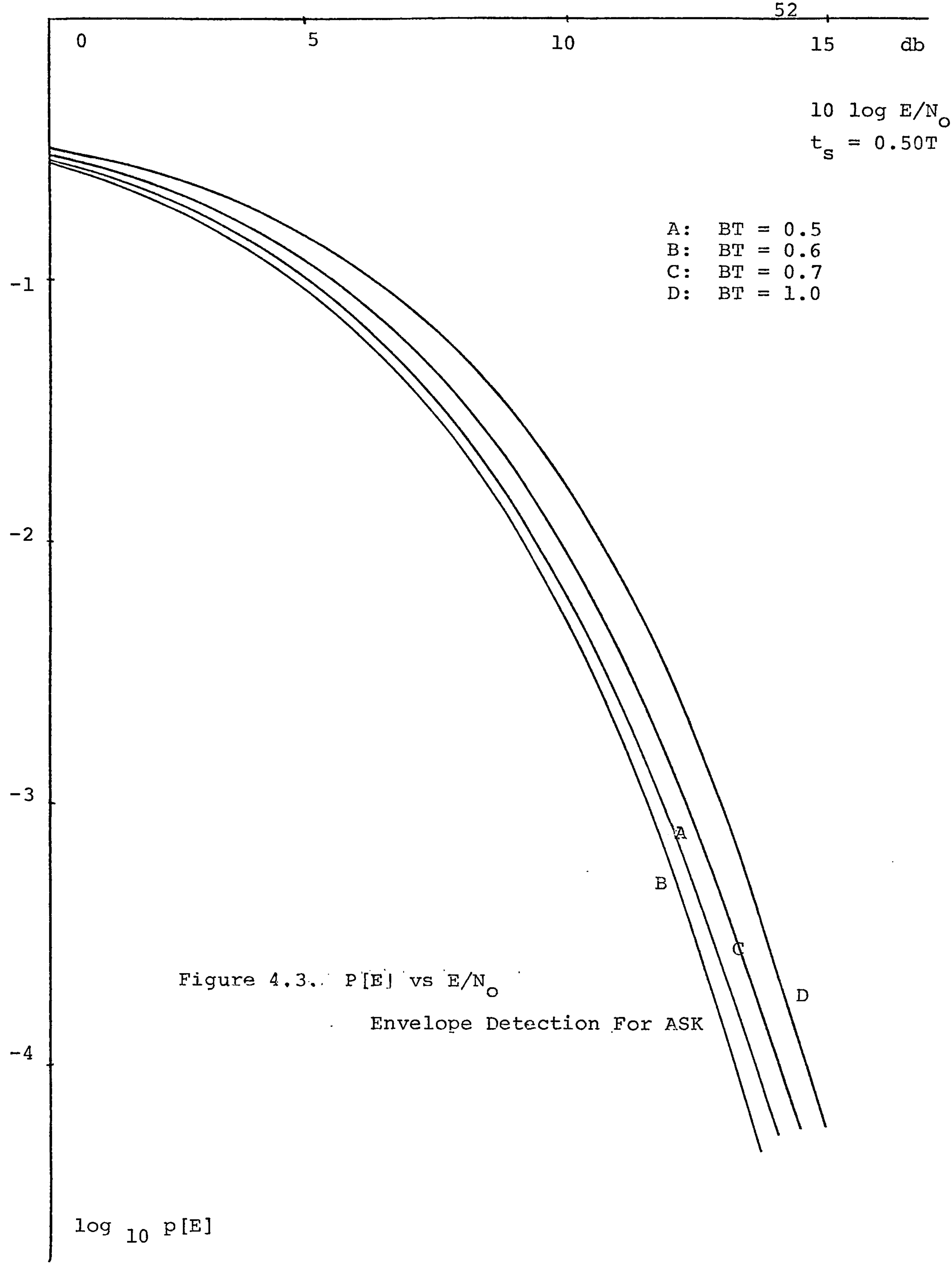

Figure 4.3. P[E] vs E/N$_o$

Envelope Detection For ASK

Table 4.3

Probability of Error by the Method (2) for BT $\geq$ 1

BT = 1.5

S/N	$t_s = 0.50T$		$t_s = 0.65T$		$t_s = 0.75T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.51	0.4165	1.51	0.4155	1.51	0.4170
0.00	1.76	0.2528	1.78	0.2524	1.77	0.2545
3.52	2.13	0.1188	2.14	0.1212	2.13	0.1217
6.02	2.54	0.4390×10^{-1}	2.56	0.4793×10^{-1}	2.54	0.4680×10^{-1}
7.96	2.98	0.1292×10^{-1}	3.00	0.1625×10^{-1}	2.97	0.1486×10^{-1}
9.54	3.44	0.2952×10^{-2}	3.45	0.4758×10^{-2}	3.43	0.3873×10^{-2}
10.88	3.91	0.5319×10^{-3}	3.92	0.1241×10^{-3}	3.89	0.8499×10^{-3}
12.04	4.39	0.7843×10^{-4}	4.39	0.2951×10^{-3}	4.35	0.1620×10^{-3}
13.06	4.87	0.1235×10^{-4}	4.87	0.6618×10^{-4}	4.82	0.2938×10^{-4}
13.98	5.36	0.4829×10^{-5}	5.36	0.1564×10^{-4}	5.30	0.7344×10^{-5}

S/N	$t_s = 0.80T$		$t_s = 0.85T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.51	0.4201	1.50	0.4256
0.00	1.75	0.2607	1.73	0.2742
3.52	2.10	0.1267	2.06	0.1427
6.02	2.50	0.4903×10^{-1}	2.45	0.6192×10^{-1}
7.96	2.92	0.1531×10^{-1}	2.86	0.2302×10^{-1}
9.54	3.37	0.3774×10^{-2}	3.29	0.7402×10^{-1}
10.88	3.38	0.7435×10^{-3}	3.74	0.2077×10^{-2}
12.04	4.30	0.1203×10^{-3}	4.20	0.5182×10^{-3}
13.06	4.76	0.1871×10^{-4}	4.66	0.1166×10^{-3}
13.98	5.24	0.5565×10^{-5}	5.14	0.2525×10^{-4}

BT = 2.0

S/N	$t_s = 0.50T$		$t_s = 0.60T$		$t_s = 0.75T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.50	0.4286	1.50	0.4206	1.52	0.4081
0.00	1.72	0.2763	1.75	0.2618	1.81	0.2358
3.52	2.05	0.1434	2.09	0.1277	2.20	0.1039
6.02	2.43	0.6099×10^{-1}	2.49	0.4963×10^{-1}	2.66	0.3591×10^{-1}
7.96	2.83	0.2156×10^{-1}	3.01	0.1211×10^{-1}	3.13	0.9991×10^{-2}
9.54	3.26	0.6311×10^{-2}	3.36	0.3828×10^{-2}	3.64	0.2218×10^{-2}
10.88	3.70	0.1537×10^{-2}	3.82	0.7500×10^{-3}	4.15	0.4068×10^{-3}
12.04	4.16	0.3184×10^{-3}	4.28	0.1197×10^{-3}	4.68	0.6474×10^{-4}
13.06	4.61	0.5865×10^{-4}	4.75	0.1835×10^{-4}	5.21	0.1130×10^{-4}
13.98	5.08	0.1197×10^{-4}	5.22	0.5517×10^{-5}	5.75	0.4287×10^{-5}

S/N	$t_s = 0.85T$		$t_s = 0.90T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.54	0.4175	1.50	0.4293
0.00	1.76	0.2549	1.72	0.2835
3.52	2.12	0.1208	2.04	0.1539
6.02	2.53	0.4154×10^{-1}	2.42	0.7141×10^{-1}
7.96	2.96	0.1345×10^{-1}	2.82	0.2920×10^{-1}
9.54	3.42	0.3114×10^{-2}	3.25	0.1079×10^{-1}
10.88	3.89	0.5683×10^{-3}	3.70	0.3592×10^{-2}
12.04	4.36	0.8446×10^{-4}	4.16	0.1094×10^{-2}
13.06	4.84	0.1312×10^{-4}	4.63	0.3040×10^{-3}
13.98	5.32	0.4931×10^{-5}	5.10	0.7750×10^{-4}

BT = 2.5

S/N db	$t_s = 0.50T$ $\beta/\sqrt{N}$	$t_s = 0.50T$ P [E]	$t_s = 0.60T$ $\beta/\sqrt{N}$	$t_s = 0.60T$ P [E]	$t_s = 0.75T$ $\beta/\sqrt{N}$	$t_s = 0.75T$ P [E]
-6.02	1.51	0.4190	1.51	0.4196	1.51	0.4149
0.00	1.75	0.2582	1.75	0.2600	1.77	0.2500
3.52	2.11	0.1241	2.11	0.1269	2.14	0.1171
6.02	2.51	0.4724×10^{-1}	2.51	0.5000×10^{-1}	2.57	0.4374×10^{-1}
7.96	2.94	0.1440×10^{-1}	2.93	0.1626×10^{-1}	3.01	0.1334×10^{-1}
9.54	3.39	0.3430×10^{-2}	3.38	0.4331×10^{-2}	3.47	0.3288×10^{-2}
10.88	3.85	0.6454×10^{-3}	3.83	0.9649×10^{-3}	3.95	0.6731×10^{-3}
12.04	4.32	0.9874×10^{-4}	4.29	0.1851×10^{-3}	4.43	0.1181×10^{-3}
13.06	4.79	0.1513×10^{-4}	4.75	0.3315×10^{-4}	4.91	0.2026×10^{-4}
13.98	5.27	0.5152×10^{-5}	5.22	0.7906×10^{-5}	5.41	0.5753×10^{-5}

S/N db	$t_s = 0.85T$ $\beta/\sqrt{N}$	$t_s = 0.85T$ P [E]	$t_s = 0.90T$ $\beta/\sqrt{N}$	$t_s = 0.90T$ P [E]
-6.02	1.51	0.4157	1.50	0.4232
0.00	1.77	0.2515	1.74	0.2680
3.52	2.14	0.1183	2.08	0.1349
6.02	2.55	0.4426×10^{-1}	2.47	0.5519×10^{-1}
7.96	2.99	0.1346×10^{-1}	2.89	0.1872×10^{-1}
9.54	3.46	0.3285×10^{-2}	3.33	0.5220×10^{-3}
10.88	3.93	0.6588×10^{-3}	3.78	0.1213×10^{-2}
12.04	4.40	0.1122×10^{-3}	4.24	0.2403×10^{-3}
13.06	4.88	0.1888×10^{-4}	4.70	0.4302×10^{-4}
13.98	5.37	0.5582×10^{-5}	5.17	0.9289×10^{-5}

BT 3.0

S/N	$t_s = 0.50T$		$t_s = 0.60T$		$t_s = 0.75T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.51	0.4125	1.50	0.4203	1.51	0.4176
0.00	1.78	0.2444	1.75	0.2612	1.76	0.2550
3.52	2.16	0.1110	2.10	0.1271	2.12	0.1209
6.02	2.59	0.3930×10^{-1}	2.50	0.4922×10^{-1}	2.53	0.4521×10^{-1}
7.96	3.05	0.1101×10^{-1}	2.92	0.1534×10^{-1}	2.96	0.1348×10^{-1}
9.54	3.52	0.2385×10^{-2}	3.37	0.3762×10^{-2}	3.42	0.3124×10^{-2}
10.88	4.01	0.4089×10^{-3}	3.83	0.7333×10^{-3}	3.89	0.5704×10^{-3}
12.04	4.51	0.5847×10^{-4}	4.29	0.1164×10^{-4}	4.36	0.8478×10^{-4}
13.06	5.01	0.9815×10^{-5}	4.76	0.1786×10^{-4}	4.84	0.1316×10^{-4}
13.98	5.52	0.4464×10^{-5}	5.23	0.5458×10^{-5}	5.32	0.4933×10^{-5}

S/N	$t_s = 0.85T$		$t_s = 0.90T$	
db	$\beta/\sqrt{N}$	P [E]	$\beta/\sqrt{N}$	P [E]
-6.02	1.52	0.4100	1.51	0.4180
0.00	1.80	0.2397	1.76	0.2560
3.52	2.19	0.1074	2.11	0.1218
6.02	2.63	0.3783×10^{-1}	2.52	0.4580×10^{-1}
7.96	3.10	0.1075×10^{-1}	2.96	0.1375×10^{-1}
9.54	3.59	0.2436×10^{-2}	3.41	0.3210×10^{-2}
10.88	4.10	0.4553×10^{-3}	3.88	0.5914×10^{-3}
12.04	4.61	0.7313×10^{-4}	4.35	0.8863×10^{-4}
13.06	5.13	0.1254×10^{-4}	4.82	0.1369×10^{-4}
13.98	5.66	0.4511×10^{-5}	5.30	0.4933×10^{-5}

BT = 3.5

S/N	$t_s = 0.50T$		$t_s = 0.70T$		$t_s = 0.80T$	
db	$\beta/\sqrt{\overline{N}}$	P [E]	$\beta/\sqrt{\overline{N}}$	P [E]	$\beta/\sqrt{\overline{N}}$	P [E]
-6.02	1.51	0.4179	1.51	0.4204	1.51	0.4162
0.00	1.76	0.2558	1.75	0.2617	1.77	0.2522
3.52	2.11	0.1217	2.10	0.1282	2.13	0.1184
6.02	2.52	0.4568×10^{-1}	2.50	0.5056×10^{-1}	2.55	0.4383×10^{-1}
7.96	2.96	0.1369×10^{-1}	2.92	0.1633×10^{-1}	2.99	0.1298×10^{-1}
9.54	3.41	0.3187×10^{-2}	3.37	0.4272×10^{-2}	3.45	0.3007×10^{-2}
10.88	3.88	0.5845×10^{-3}	3.82	0.9197×10^{-3}	3.92	0.5550×10^{-3}
12.04	4.35	0.8714×10^{-4}	4.28	0.1674×10^{-3}	4.40	0.8472×10^{-4}
13.06	4.83	0.1345×10^{-4}	4.74	0.2840×10^{-4}	4.88	0.1350×10^{-4}
13.98	5.31	0.4969×10^{-5}	5.21	0.6997×10^{-5}	5.37	0.4927×10^{-5}

S/N	$t_s = 0.90T$		$t_s = 0.95T$	
db	$\beta/\sqrt{\overline{N}}$	P [E]	$\beta/\sqrt{\overline{N}}$	P [E]
-6.02	1.51	0.4163	1.50	0.4312
0.00	1.77	0.2526	1.72	0.2895
3.52	2.13	0.1189	2.04	0.1636
6.02	2.55	0.4422×10^{-1}	2.42	0.8211×10^{-1}
7.96	2.98	0.1321×10^{-1}	2.82	0.3808×10^{-1}
9.54	3.44	0.3105×10^{-2}	3.25	0.1684×10^{-1}
10.88	3.91	0.5859×10^{-3}	3.69	0.6969×10^{-2}
12.04	4.39	0.9198×10^{-4}	4.16	0.2712×10^{-2}
13.06	4.87	0.1478×10^{-4}	4.64	0.9809×10^{-3}
13.98	5.35	0.5072×10^{-5}	5.12	0.3265×10^{-3}

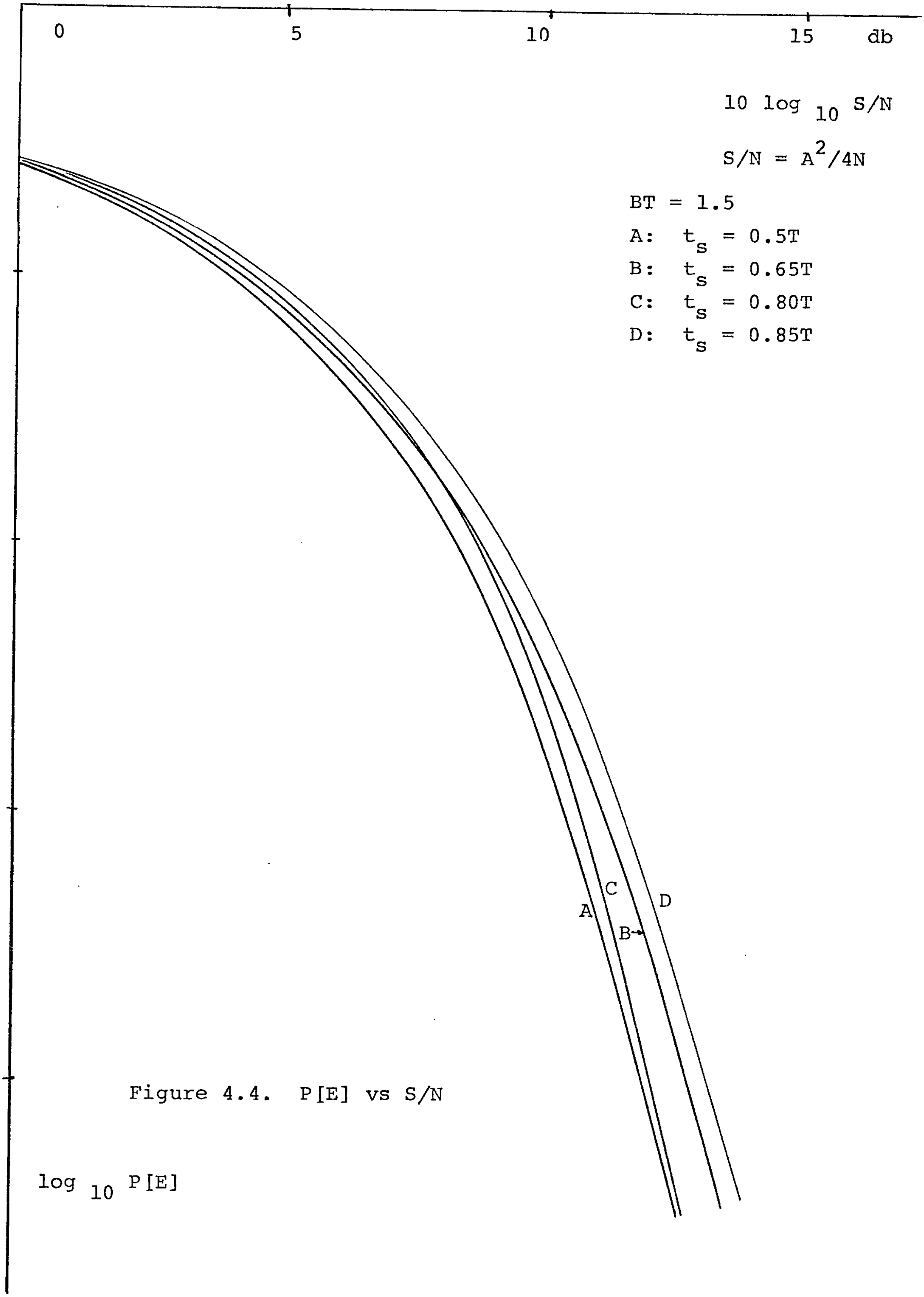

Figure 4.4. P[E] vs S/N

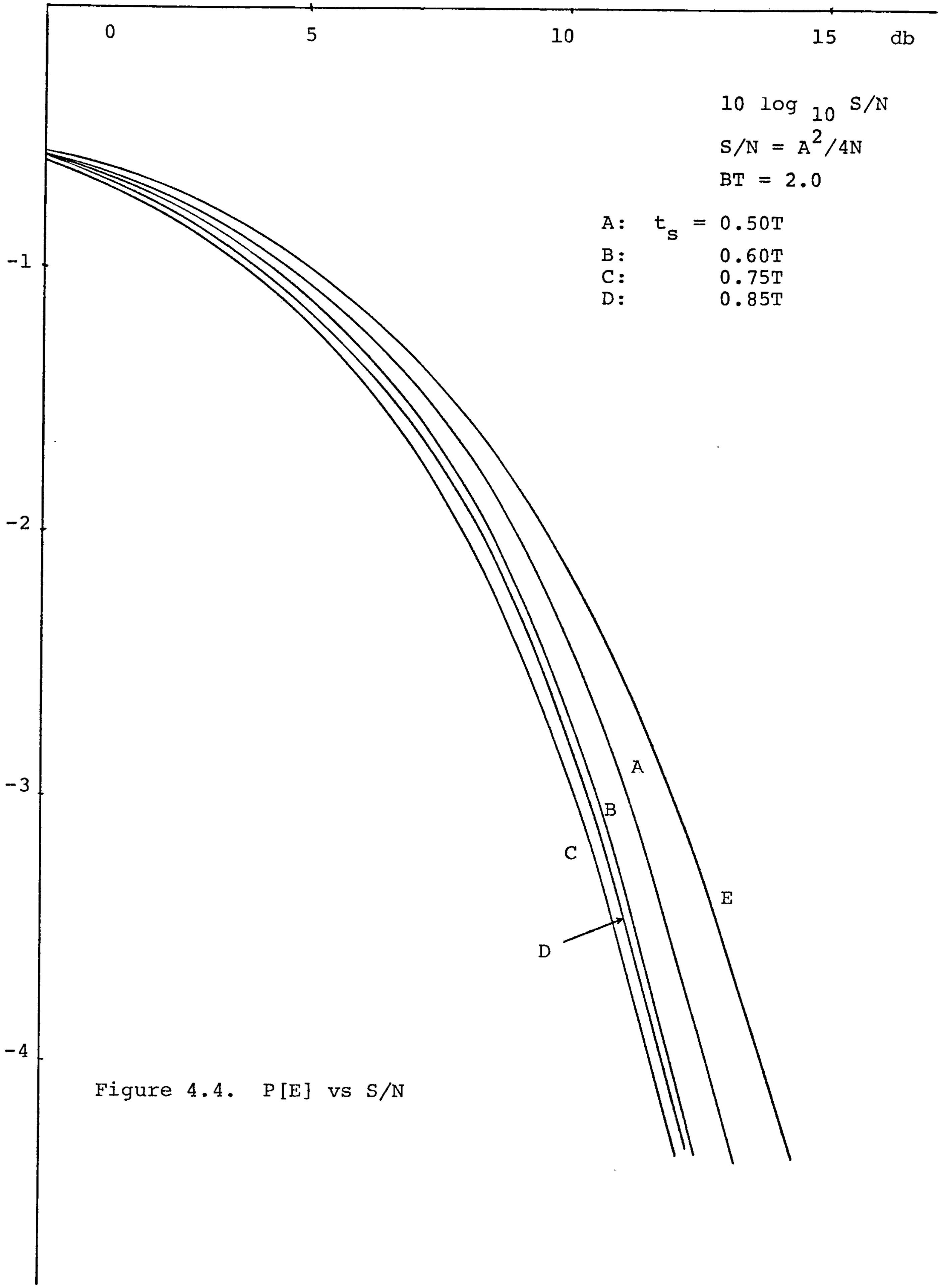

Figure 4.4. P[E] vs S/N

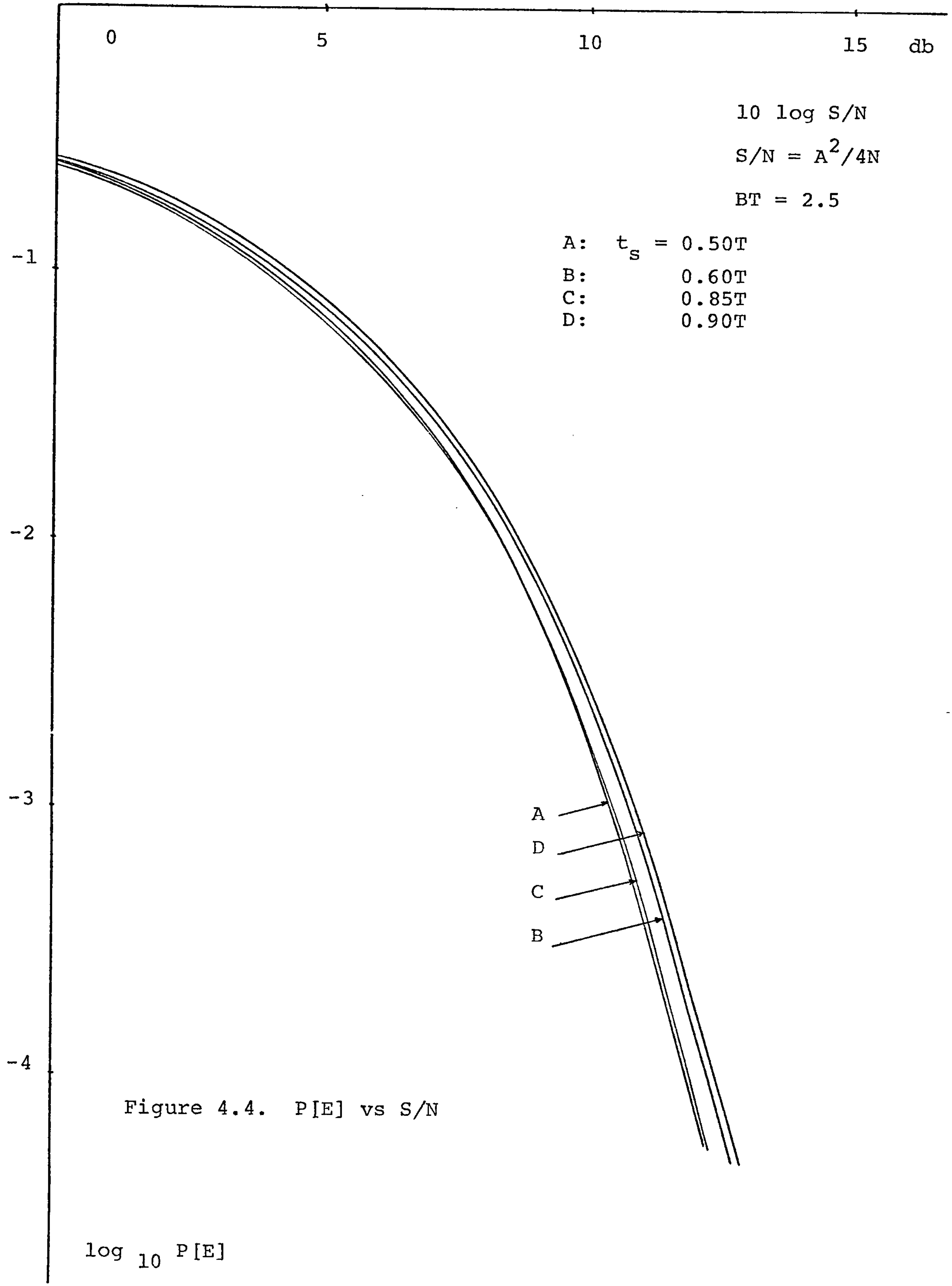

Figure 4.4. P[E] vs S/N

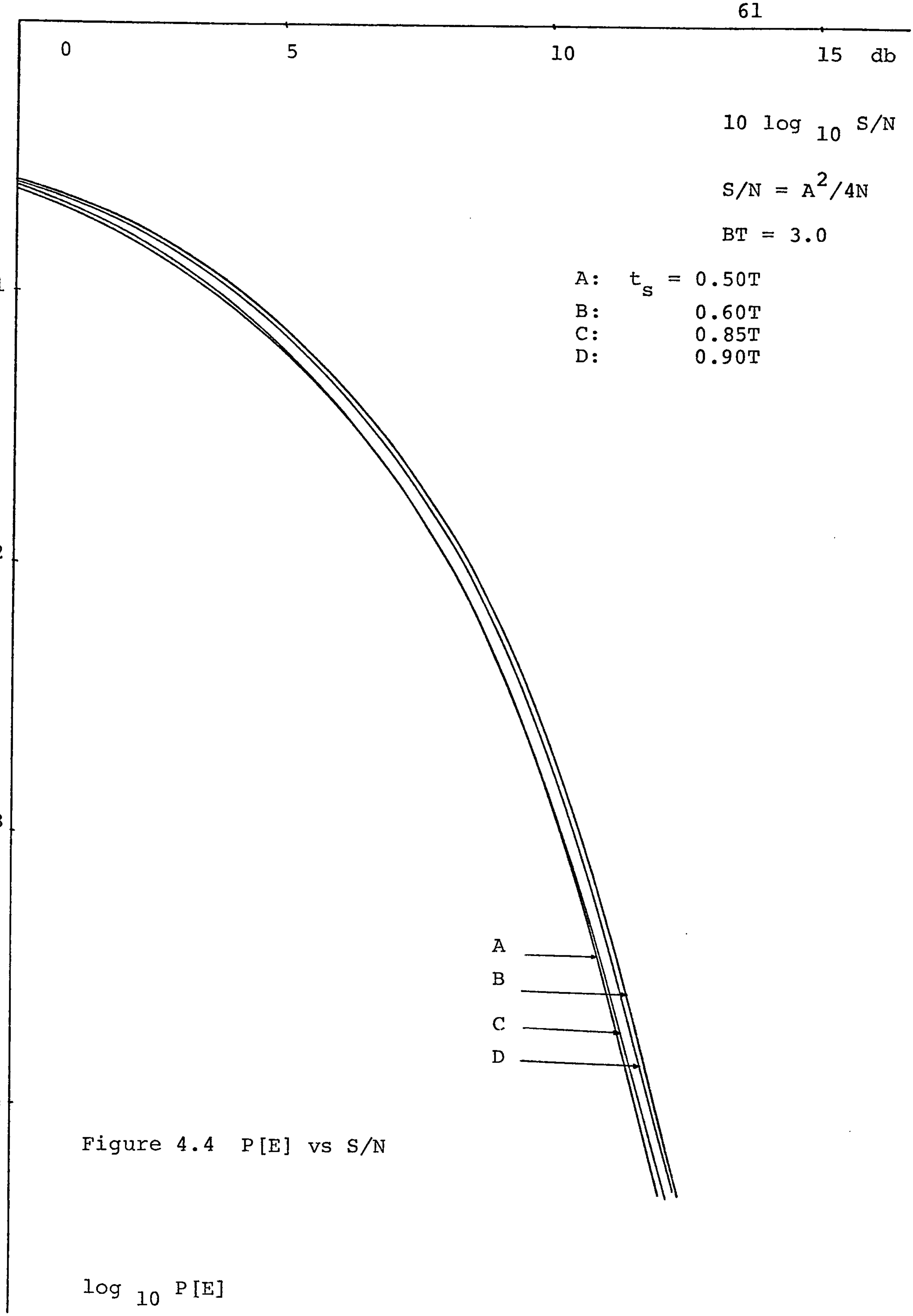

Figure 4.4 P[E] vs S/N

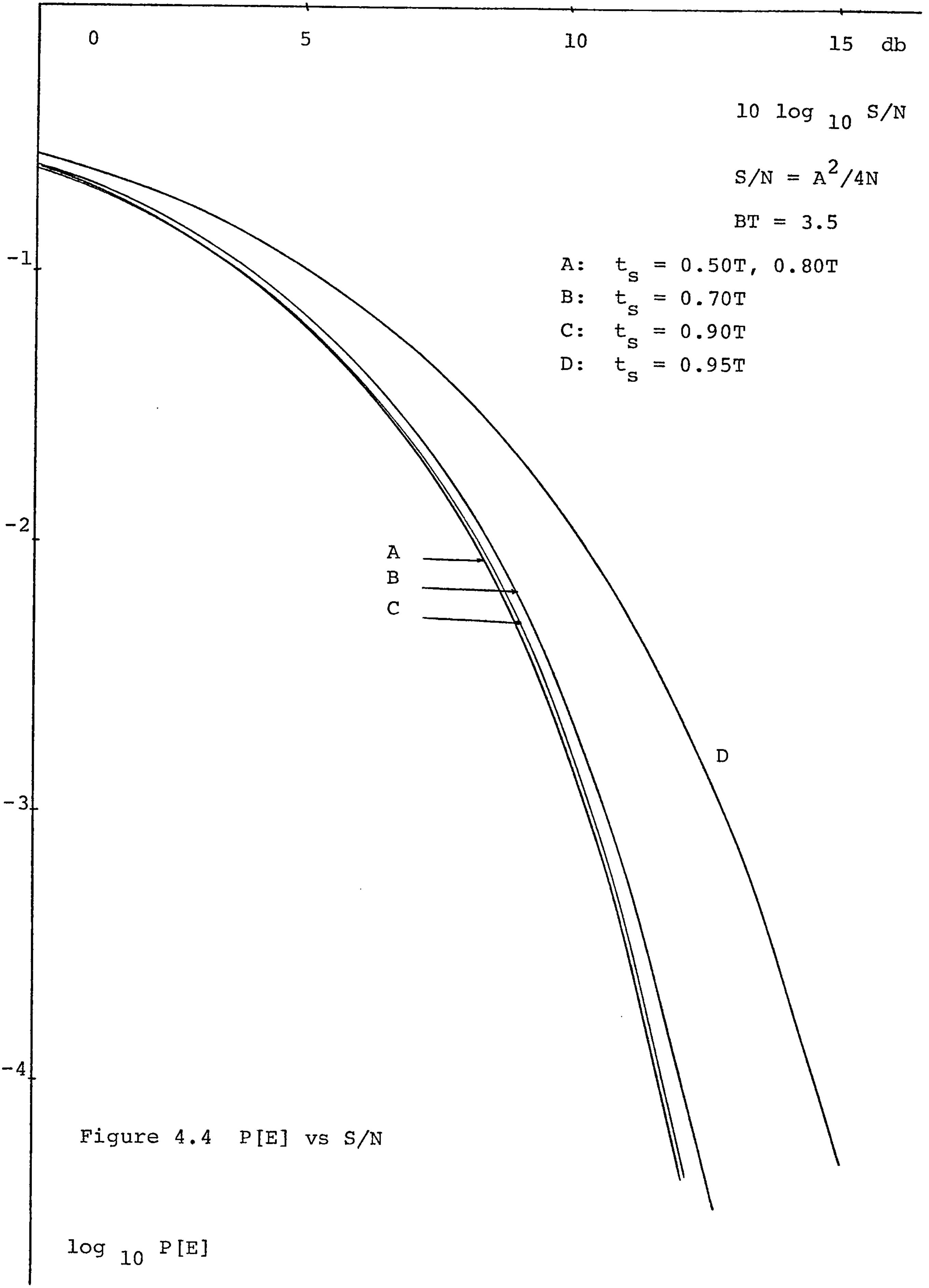

Figure 4.4 P[E] vs S/N

CHAPTER V

DETECTION OF ASK SIGNALS USING A SAMPLER

So far we have considered envelope detection of the ASK signal. Also in [4], the detection of ASK signal using an integrate-and-dump filter was considered. In this chapter, the detection of ASK signal using a sample detector is to be considered.

The system for the detection of the ASK signal using a sampler is shown in Figure 5.1. The output of the filter in the receiver $s_d(t)$ can be found to be

$$s_d(t) = \frac{1}{2}\,[w(t) + n_c(t)] \tag{32}$$

where $w(t)$ and $n_c(t)$ are as shown in (20) and (2), respectively.

The conditional probability density functions given that the i-th pattern is transmitted with no pulse in the middle is

$$P_{s_d}(\alpha/m_0) = P_{s_d}(\alpha/f_{0,i}) = \frac{1}{\sqrt{2\pi N}}\exp\left(-\frac{(\alpha-f_{0,i})^2}{2N}\right) \tag{33}$$

Similarly, for the case for a pulse in the middle of the pattern is

$$P_{s_d}(\alpha/m_1) = P_{s_d}(\alpha/f_{1,i}) = \frac{1}{\sqrt{2\pi N}}\exp\left(-\frac{(\alpha-f_{1,i})^2}{2N}\right) \tag{34}$$

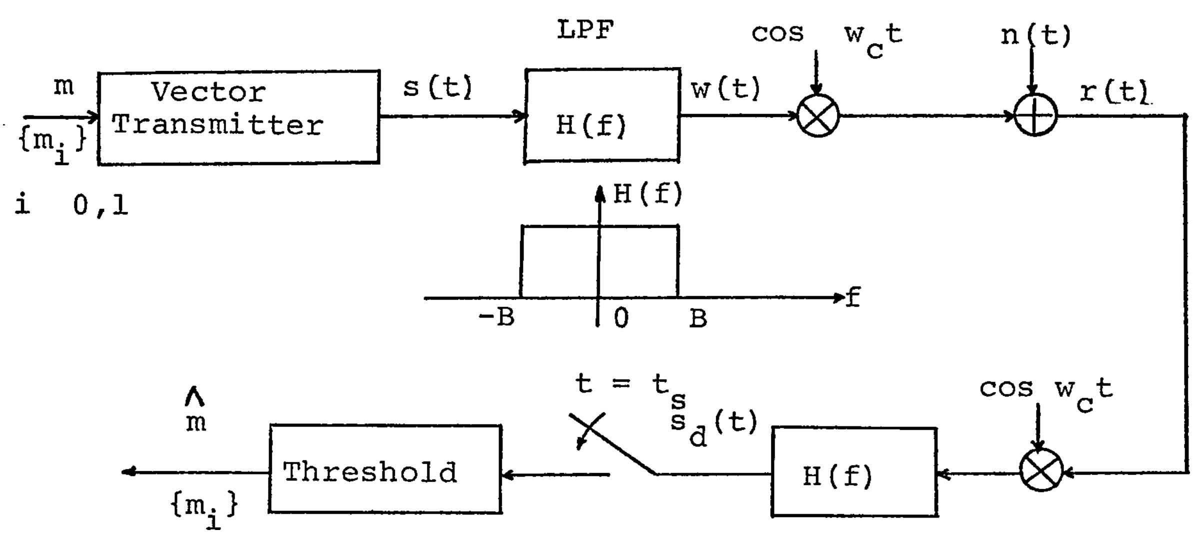

Figure 5.1. Sample Detector

Using the average probability density function method, the probability of bit-error is

$$P[E] = \frac{1}{2}\left[\int_{\beta}^{\infty} \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{0,i})\,d\alpha + \int_{-\infty}^{\beta} \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{1,i})\,d\alpha\right] \tag{35}$$

where β is an intersection of $\frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{0,i})$ and $\frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{1,i})$.

Since $P_{s_d}(\alpha/f_{0,i})$ and $P_{s_d}(\alpha/f_{1,i})$ are Gaussian distributed, $\frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{0,i})$ and $\frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} P_{s_d}(\alpha/f_{1,i})$ are also Gaussian distributed with means

$$\overline{f_{0,i}} = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} f_{0,i} \tag{36}$$

and

$$\overline{f_{1,i}} = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} f_{1,i} \tag{37}$$

and the same variance σ^2. Thus

$$P_{s_d}(\alpha/m_0) = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{(\alpha-f_{0,i})^2}{2N}\right)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\alpha-\overline{f_{0,i}})^2}{2\sigma^2}\right) \tag{38}$$

and

$$P_{s_d}(\alpha/m_1) = \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{(\alpha - f_{1,i})^2}{2N}\right)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\alpha - \overline{f_{1,i}})^2}{2\sigma^2}\right) \tag{39}$$

The optimum threshold will be the intersection of $P_{s_d}(\alpha/m_0)$ and $P_{s_d}(\alpha/m_1)$, which is $\frac{\overline{f_{0,i}} + \overline{f_{1,i}}}{2}$.

From (30), then the probability of error is

$$P[E] = \frac{1}{2}\left[\frac{1}{2^{2M}} \int_{\beta}^{\infty} \sum_{i=1}^{2^{2M}} \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{(\alpha - f_{0,i})^2}{2N}\right) d\alpha\right.$$

$$\left. + \frac{1}{2^{2M}} \int_{-\infty}^{\beta} \sum_{i=1}^{2^{2M}} \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{(\alpha - f_{1,i})^2}{2N}\right) d\alpha\right]$$

$$= \frac{1}{2^{2M}} \int_{-\infty}^{\beta} \sum_{i=1}^{2^{2M}} \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{(\alpha - f_{1,i})^2}{2N}\right) d\alpha$$

$$= \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \int_{-\infty}^{\frac{(\beta - f_{1,i})}{\sqrt{N}}} \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)\, dx$$

$$= \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{2}\left[1 - \frac{2}{\sqrt{\pi}} \int_{0}^{\frac{(f_{1,i} - \beta)}{\sqrt{2N}}} \exp(-y^2) dy\right]$$

$$= \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{2}[1 - \mathrm{erf}(\frac{f_{1,i} - \beta}{\sqrt{2N}})]$$

$$= \frac{1}{2^{2M}} \sum_{i=1}^{2^{2M}} \frac{1}{2}[1 - \mathrm{erf}(\frac{1}{\sqrt{2N}} (f_{1,i} - \frac{\overline{f_{1,i}} + \overline{f_{0,i}}}{2}))]$$

$$= \frac{1}{2} \sum_{i=1}^{2^{2M}} \frac{1}{2^{2M}} [1 - \mathrm{erf}\{ \sqrt{\frac{2S}{N}} (SI_{1,i}(BT,t/T) - \frac{\overline{SI_{1,i}(BT,t/T)}}{2} - \frac{\overline{SI_{0,i}(BT,t/T)}}{2})\}] \tag{40}$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2)dy$$

is the error function.

The probability of bit error is calculated using (40). For BT $\leq$ 1.0, the sampling time t_s is set at 0.5T which was discussed in Chapter IV. The probability of bit error vs E/N_0, using a sampler detector for several BT, is compared with noncoherent detection of ASK, coherent detection of ASK, and coherent detection of phase shift keying (PSK) [4] as shown in Figure 5.2a. The probability of bit error vs S/N using a sample detector for several BT is compared with noncoherent detection of ASK and coherent detection of ASK as shown in Figure 5.2b.

For BT $\geq$ 1.0, the optimum sampling time t_s is variable but can be determined from Figures 5.2c, 5.2d, 5.2e, 5.2f, and 5.2g. The probability of bit error is also compared in Figure 5.2c for BT = 1.5, Figure 5.2d for BT = 2.0, Figure 5.2e for BT = 2.5, Figure 5.2f for BT = 3.0, and Figure 5.2g for BT = 3.5.

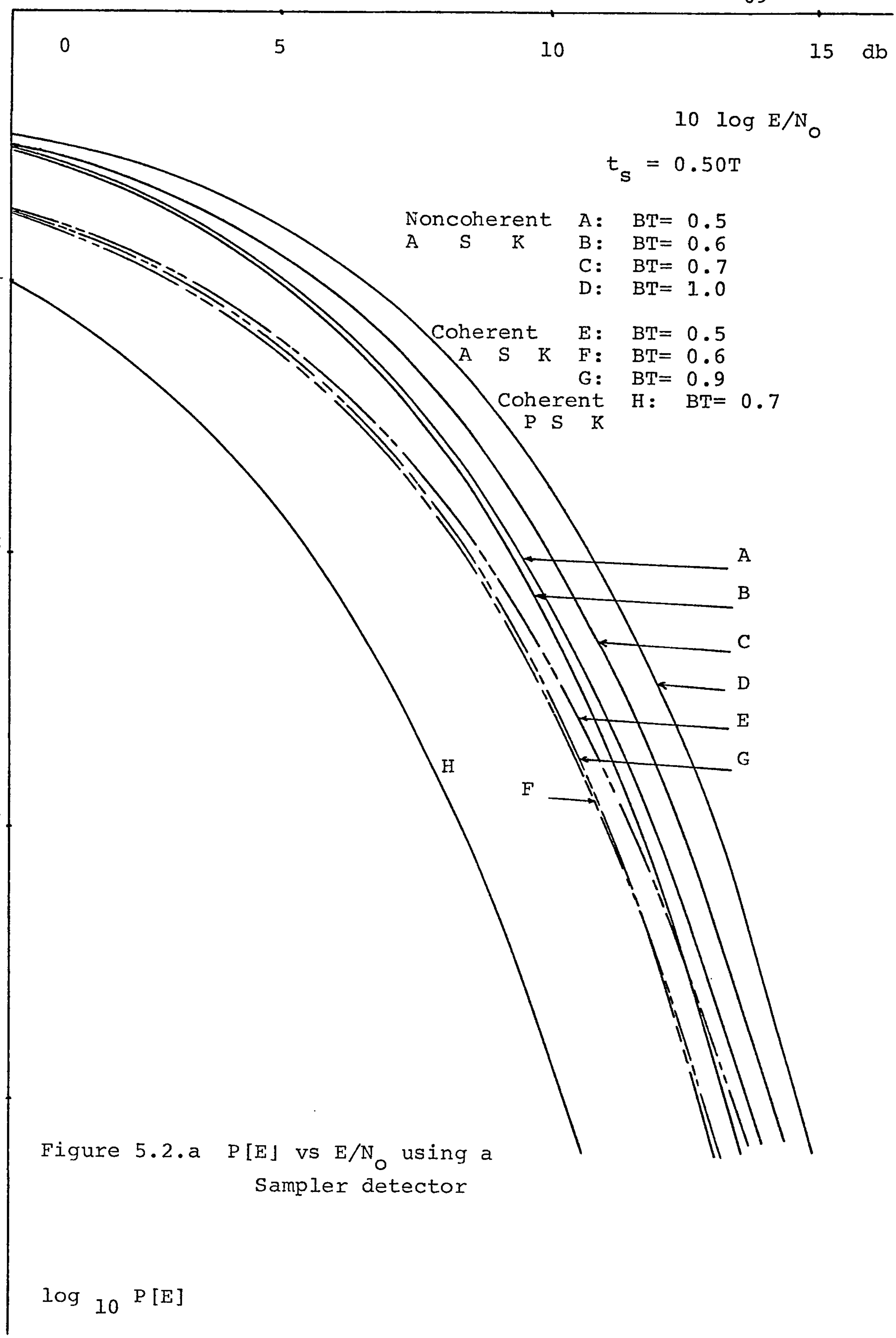

Figure 5.2.a P[E] vs E/N_o using a Sampler detector

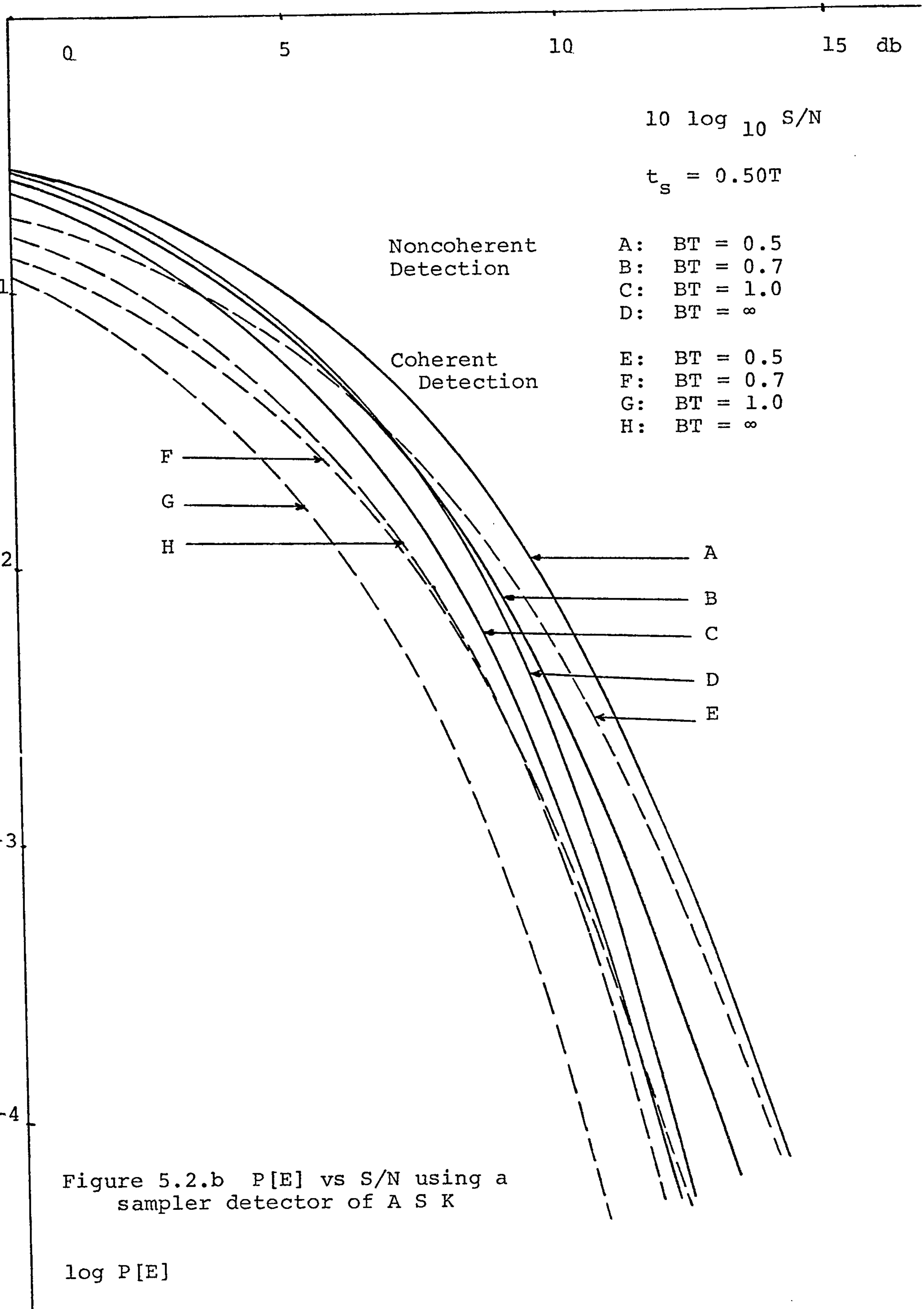

Figure 5.2.b P[E] vs S/N using a sampler detector of A S K

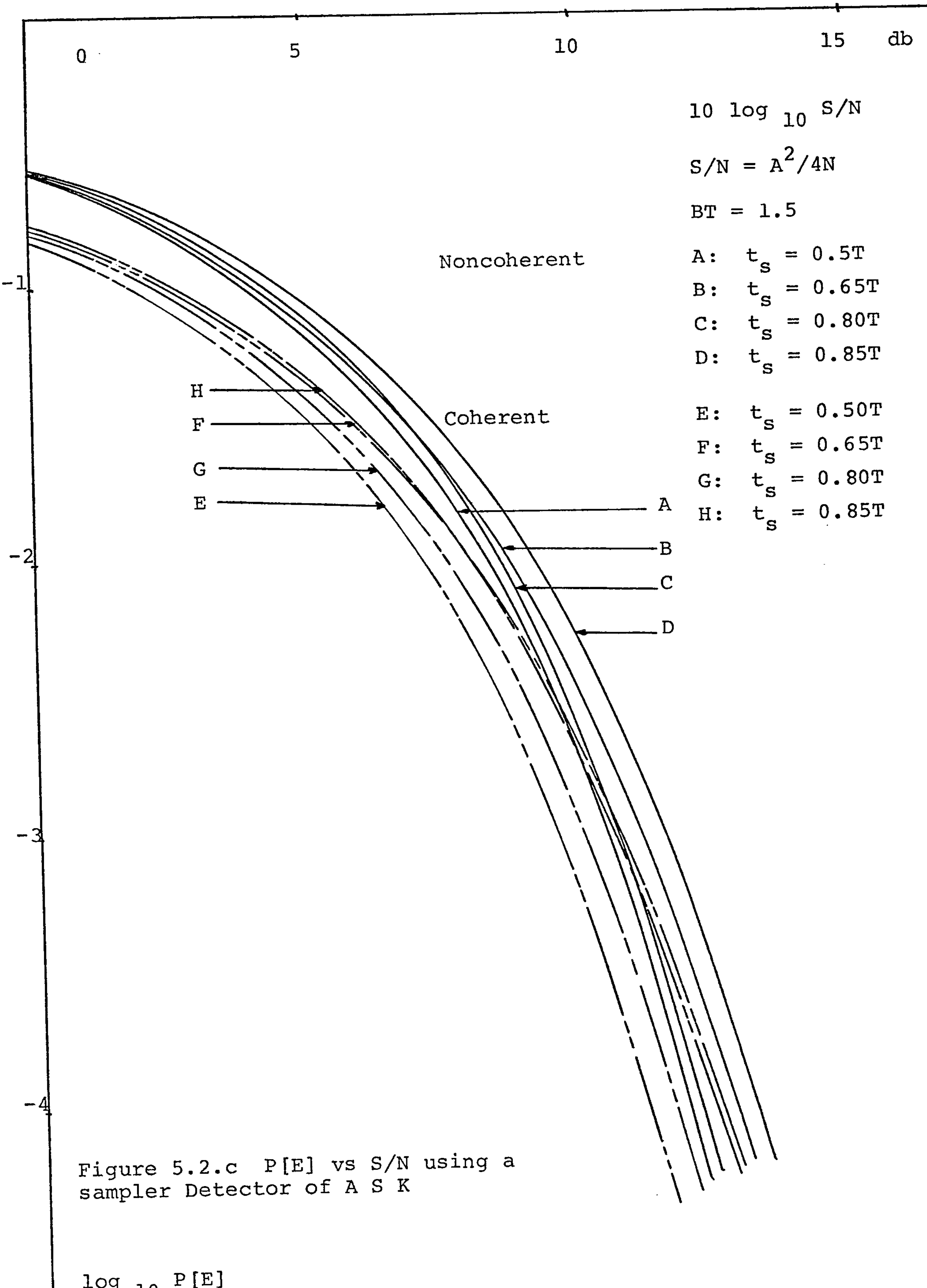

Figure 5.2.c P[E] vs S/N using a sampler Detector of A S K

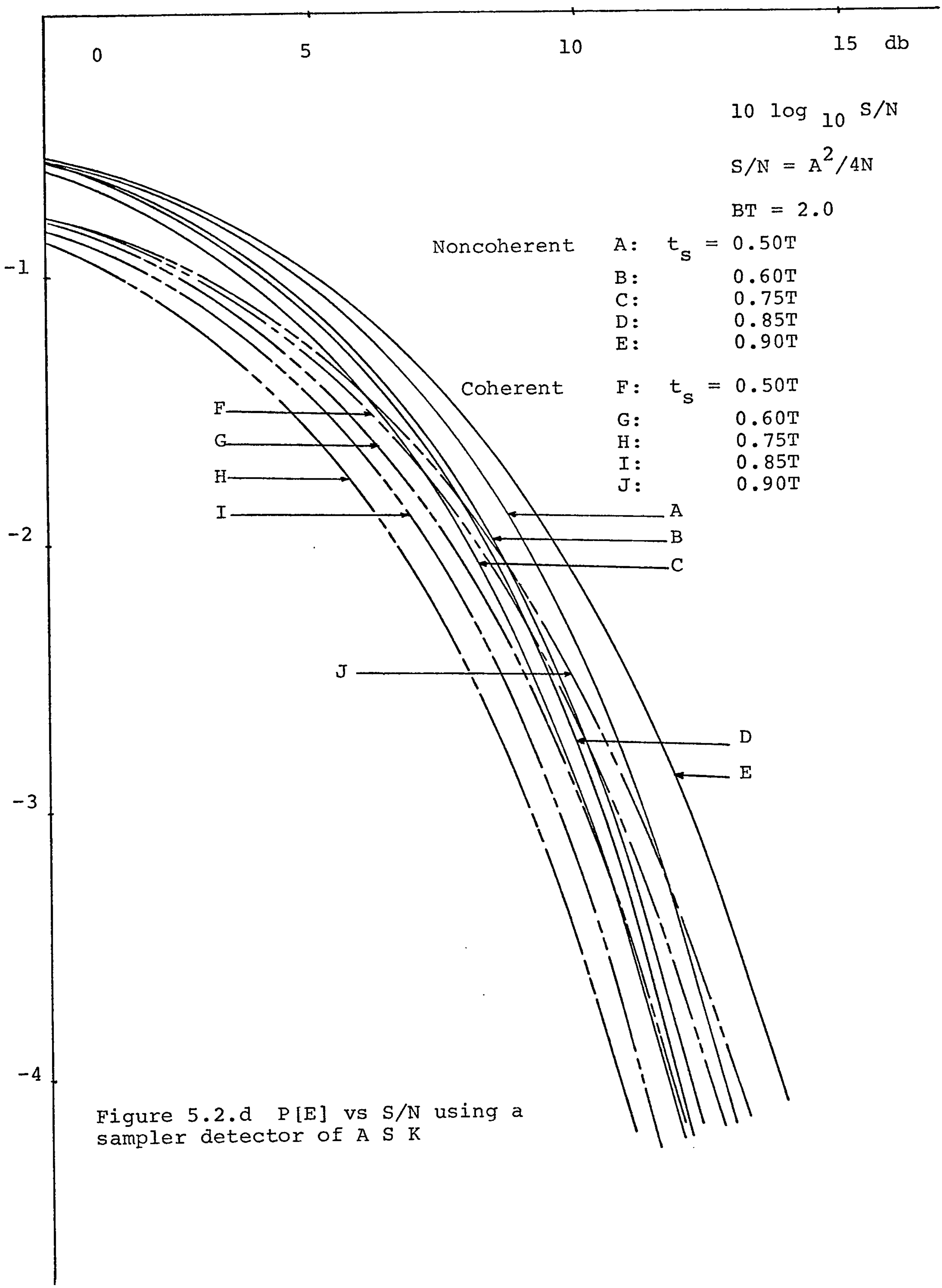

Figure 5.2.d P[E] vs S/N using a sampler detector of A S K

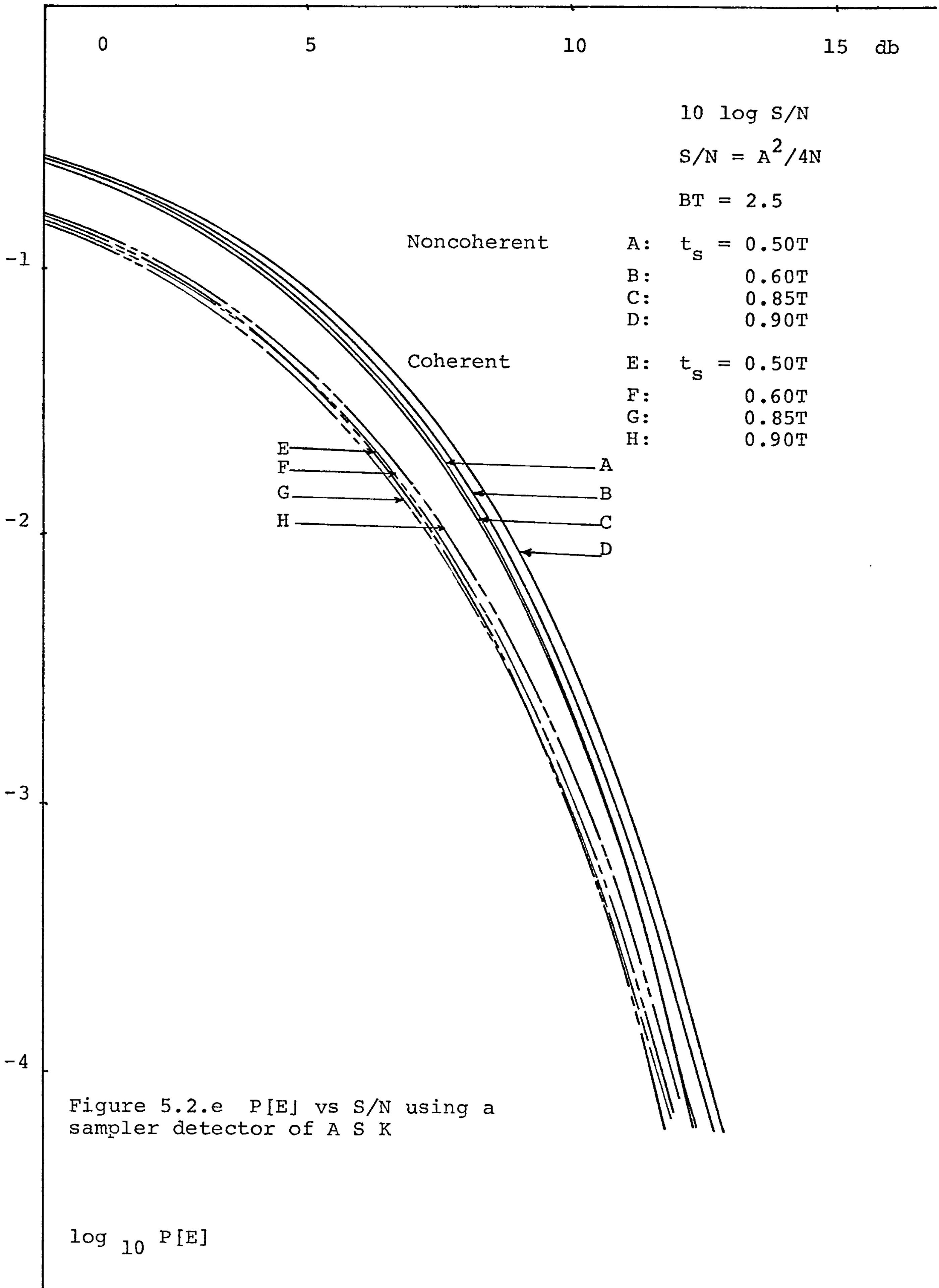

Figure 5.2.e P[E] vs S/N using a sampler detector of A S K

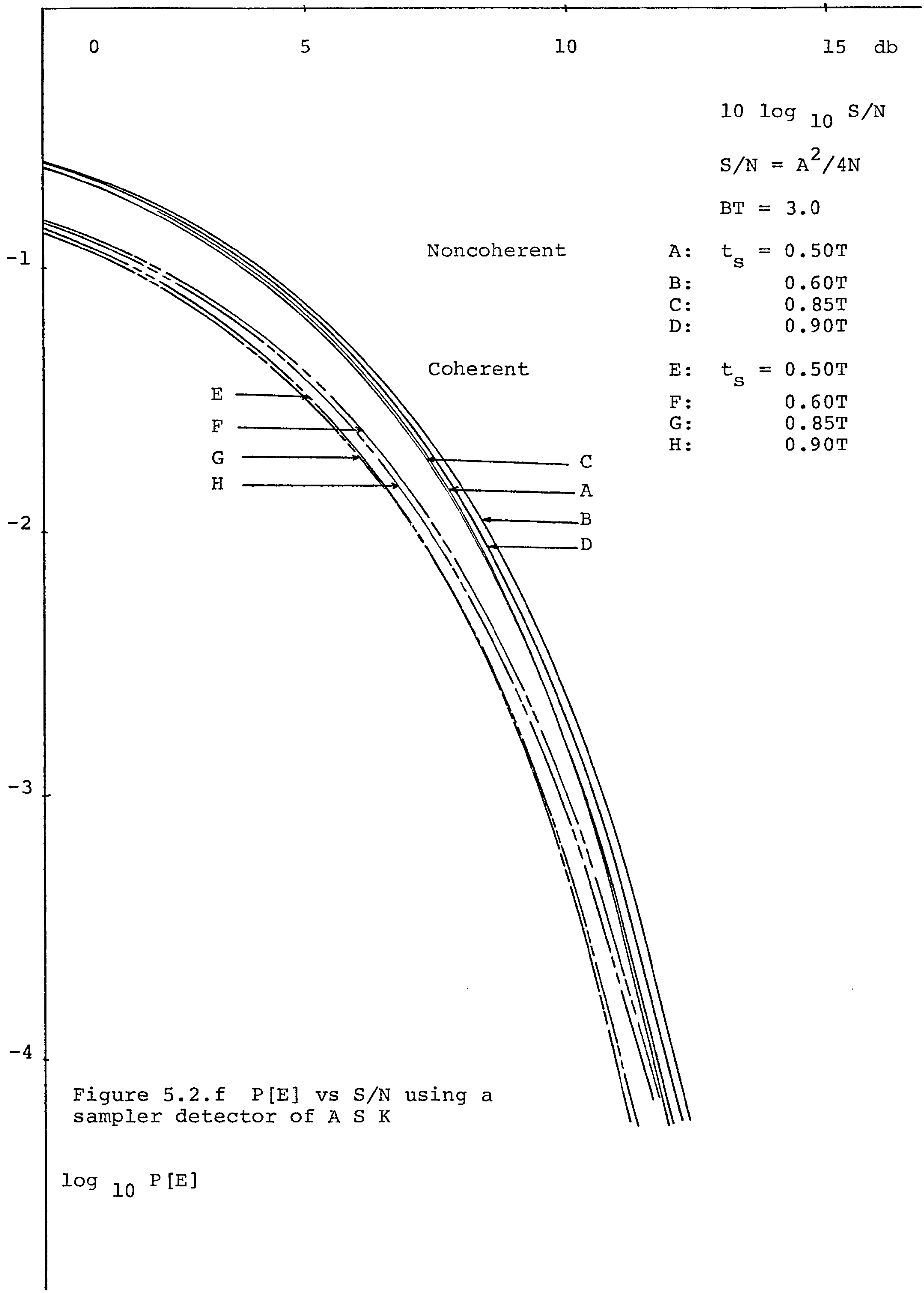

Figure 5.2.f P[E] vs S/N using a sampler detector of A S K

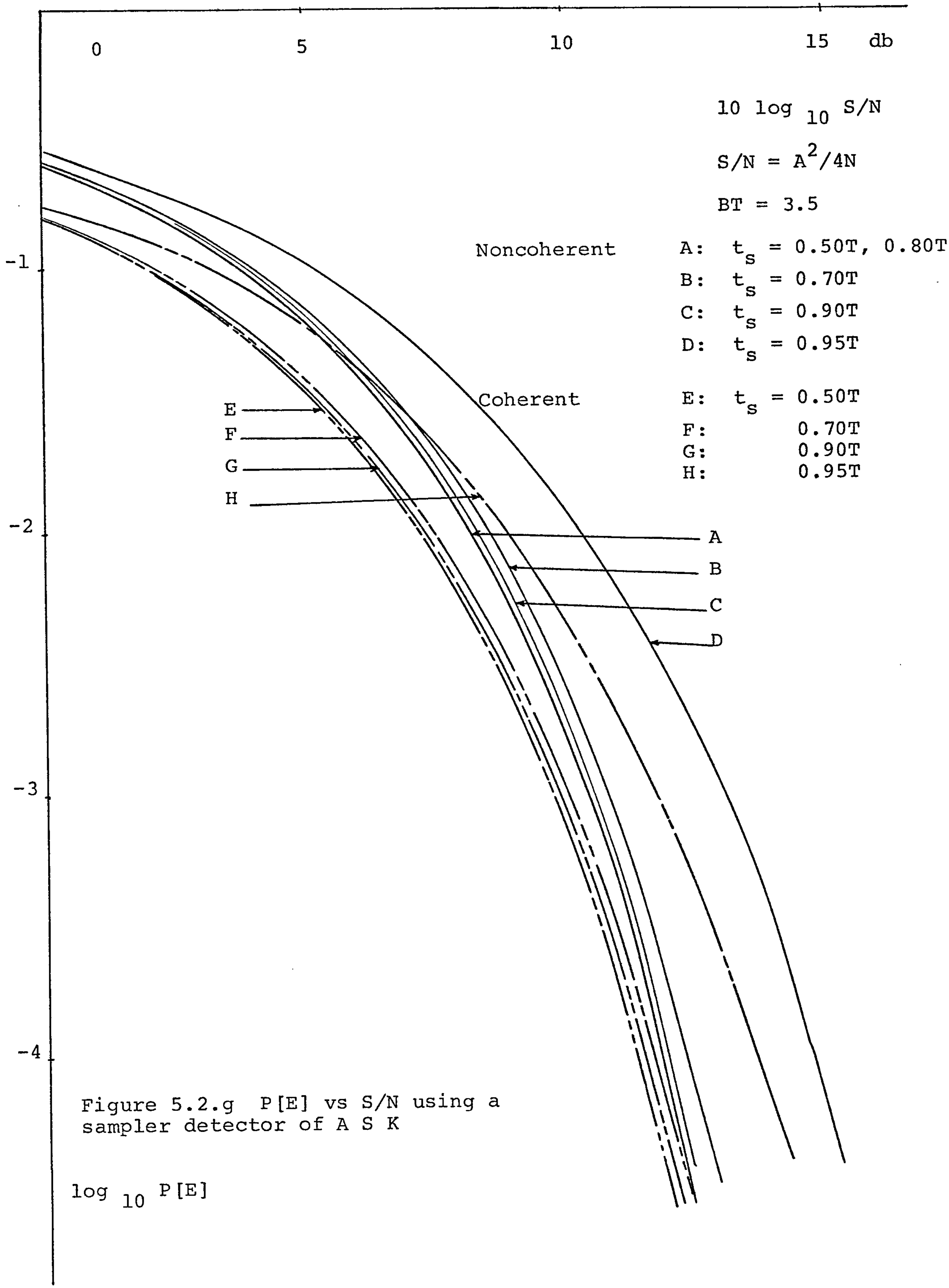

Figure 5.2.g P[E] vs S/N using a sampler detector of A S K

CHAPTER VI

CONCLUSIONS

The effects of bandlimiting on the performance of noncoherent detection of ASK signal distrubed by additive Gaussian noise have been analyzed by two methods, average threshold method and average probability density function method.

For a small S/N both methods have the almost same probability of bit error, but for a large S/N the average probability density function method gives smaller probability of bit error than the average threshold method does.

The intersymbol interference becomes more serious for a large S/N and for a smaller BT.

For the same energy of the signal, probability of bit error is minimum around BT = 0.6 for a noncoherent detection using a sample detector and around BT = 0.7 for a coherent detection.

For BT $\leq$ 1.0 sampling time t_s = 0.5T is optimum for a coherent or noncoherent detection of bandlimited ASK signal. For BT $\geq$ 1.0, optimum sampling time t_s is a variable. For example for noncoherent detection t_s = 0.5T for BT = 1.5, 0.75T for BT = 2.0, between 0.70T and 0.5T for BT = 2.5, between 0.85T and 0.5T for BT = 3.0 and between 0.8T and 0.5T for BT = 3.5.

For the coherent detection the optimum sampling time t_s is 0.5T for BT = 1.5, 0.75T for BT = 2.0, between 0.70T and 0.5T for BT = 2.5, 0.5T for BT = 3.0, and 0.5T for BT = 3.5. As the value of BT becomes greater and for a large S/N the optimum sampling time t_s becomes 0.5T.

The probability of bit error is compared in Figure 6.1 for the noncoherent detection using a sample detector, the coherent detection using a sample detector, and the coherent detection using an integrate-and-dump filter. The detection using an integrate-and-dump filter performs worse than the coherent and noncoherent detection using a sample detector for BT = 0.5.

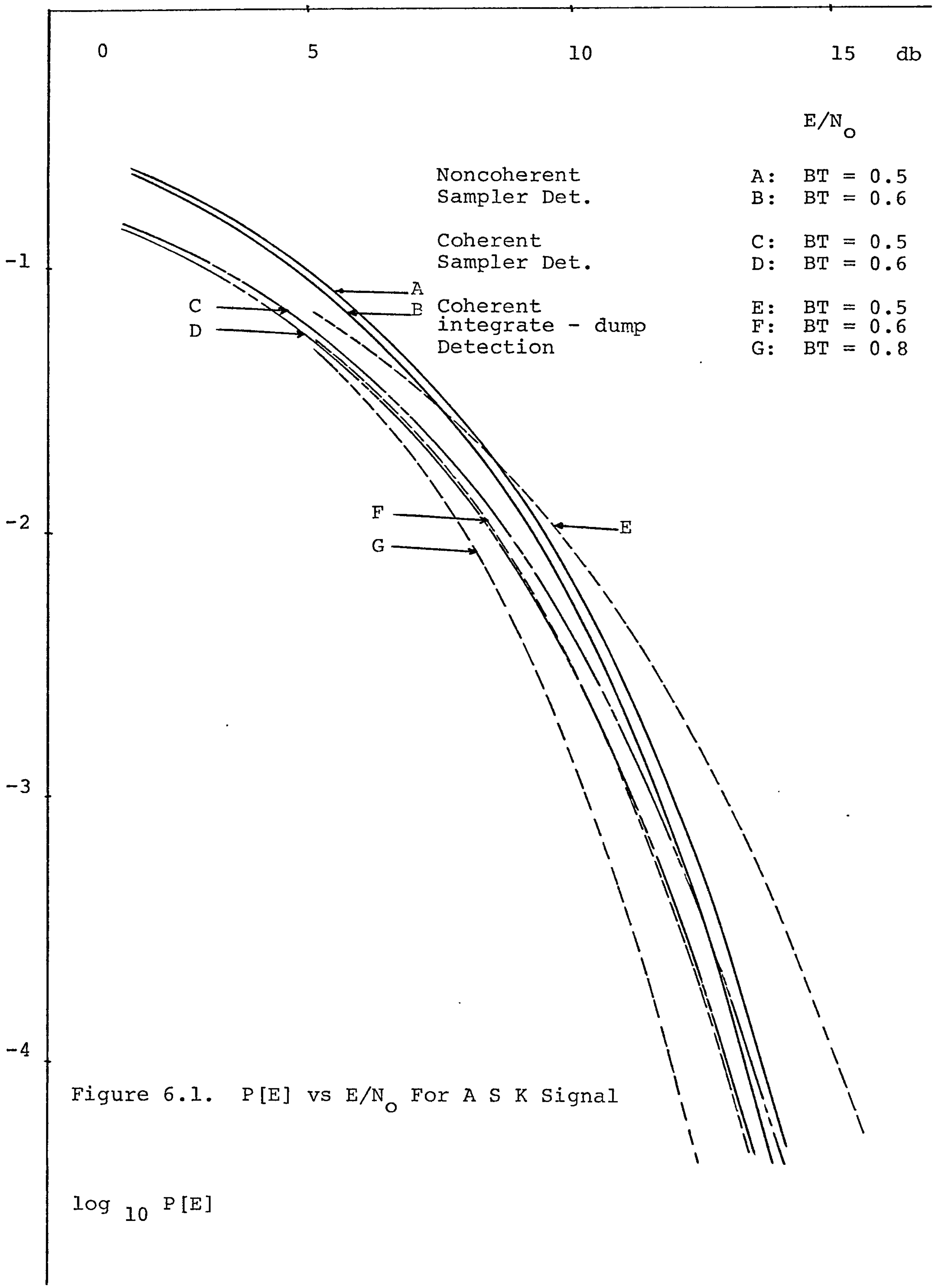

Figure 6.1. P[E] vs E/N$_o$ For A S K Signal

REFERENCES

1. Bennett, William R. and James R. Davey, "Data Transmission", McGraw-Hill, New York, 1965.

2. Helstrom, Carl W., "Statistical Theory of Signal Detection", Pergamon Press Inc., New York, 1968.

3. Vantrees, Harry L., "Detection, Estimation, and Modulation Theory", Wiley, New York, 1968.

4. Tu, Kwei, "The Effects of Bandlimiting on the Performance of Digital Communication Systems", Ph.D. Dissertation, Department of Electrical Engineering, University of Houston, Houston, Texas, 1970.

5. Schwartz, Mischa, "Communication Systems and Techniques", McGraw-Hill, New York, 1966.

6. Bruce, Carlson A., "Communication Systems; An Introduction to Signals and Noise in Electrical Communication", McGraw-Hill, New York, 1968.

7. Wozencraft, J. M. and Irwin M. Jacobs, "Principles of Communication Engineering", John Wiley and Sons, Inc., New York, 1967.

8. Watson, G. N., "A Treatise on the Theory of Bessel Functions", Cambridge at the University Press, 1962.

9. Bowman, Frank, "Introduction to Bessel Functions", Dover Publications Inc., New York, 1958.