## MAGNETIC PERMEABILITY PROFILES IN ADSORPTION BEDS

A Thesis

Presented to the Faculty

of the Department of Chemical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree Master of Science in Chemical Engineering

by

Robert N. McGill

August, 1972

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#### ABSTRACT

The rate of adsorption of an adsorbate in a fluid flowing through a bed of adsorbent depends on kinetic and mass transfer effects. These phenomena cause a dispersion of the adsorbate species, both in the fluid and on the solid. The proper design of a system to remove efficiently and effectively these species requires a knowledge of the mechanism of the adsorption. With such information, dispersion may be minimized, yielding maximum utilization of the adsorbent.

Numerous mathmatical models have been proposed for the movement and dispersion of the adsorption wave. These models are either theoretical with adjustable parameters or empirical for a particular system. The parameters are usually obtained from effluent breakthrough curves. Multiple analyses with variable length beds are necessary to determine the dispersion with length.

In 1946 Thiele commented that "theory is ahead of experimental work in this field". The theory has been developed considerably since then, but experimental techniques are essentially unchanged. Recently, Richardson proposed a method for determining adsorption profiles for adsorbents that change magnetically upon adsorption. The voltage induced into a coil of wire in a changing magnetic field is proportional to the permeability and the amount of material inside the coil. By moving a bed of magnetic material into the coil, the change in the voltage induced into the coil with the position of the bed relative to the coil should be directly proportional to the change in the concentration of the material entering the coil. However, a real bed induces a voltage into the coil in a non-linear This thesis describes a technique for determining the "end effects" and accounting for them mathmatically to render adsorption data for realistic adsorbate profiles. Although considerably more complex than Richardson's "ideal coil" method, the technique developed will describe a two parameter equation relating dispersion of the concentration wave to its position in the bed. From magnetic measurements, the validity of models can be evaluated as the adsorbate wave moves down the bed, thus reducing the number of beds required by previous methods. An improvement of this technique is suggested for obtaining a third parameter which would provide sufficient data to reject a proposed model with measurements for a single profile.

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#### CHAPTER ONE: INTRODUCTION

Adsorption dynamics and in particular frontal chromatography data are necessary in the design of systems to remove economically undesired components by physical or chemical adsorption. Whether the system is a purification bed, a guard chamber, or a drying tube, the desired operation for maximum use of the adsorbent or catalyst requires conditions such that the "front" of the adsorption wave travels down the bed with a minimum dispersion.

The theory of frontal chromatography has received considerable attention. Most of the work has been devoted to the development of models for prediction of the movement of the adsorption wave down the bed. The methods presently available for verification of these models are either destructive and inaccurate or they are based on secondary measurements.

If a fluid containing one component that is strongly adsorbed is passed continuously through a packed adsorption bed, the material adsorbed will distribute on the adsorbent, forming a concentration wave in the direction of the flow. The dispersion of this wave is dependent on the chemical kinetic and mass transfer relations and on the equilibrium relation for the adsorption. The equations characterizing this process for an isothermal, constant gas density (low adsorbate concentration) case are:

$$\frac{\partial n}{\partial t} + \frac{\partial N}{\partial t} + U \frac{\partial n}{\partial x} = D^* \frac{\partial^2 n}{\partial x^2}$$
(1)

$$\frac{\partial N}{\partial t} = Q(n,N)$$
 (2)



The initial and boundary conditions generally used are:

$$t = 0, x \ge 0, n = 0$$
 N = 0 (3)

$$t > 0, x = 0, n = n_0, N = N^*$$
 (4)

 $t \ge 0, x = \omega, n = 0, N = 0$  (5)

A major problem is in determining the functional relation for the depositing of the adsorbate on the solid since Q can be dependent on mass transfer as well as chemical kinetics. Most investigators determine the best model and its associated parameters through analysis of the bed effluent adsorbate concentration or "breakthrough" curve. Rimpel, et. al. (21) describe two approaches that can be used to test various models.

One method of comparison is an integral approach. In this method, the proposed model for equation '2' is written in terms of a measurable quantity, the gas phase concentration at the bed exit, and the assumed or known equilibrium relation. Rimpel describes how equilibrium information may be obtained from the breakthrough curves. Equation '1' is then solved and theoretical curves generated for comparison with experimental results. The problem with this approach is the solution of equation '1', since solutions are available for particular cases only.

One of the first solutions of this equation was by Bohart and Adams (5). They assumed the following relation for the depletion of the activity of charcoal used to remove chlorine from air:

$$\frac{\partial N}{\partial t} = -k n (N^* - N)$$
(6)

This leads to the following time and position dependent concentration profile of the adsorbate on the adsorbent:

$$\frac{N}{N^*} = \frac{1 - EXP(-A t/t_s)}{1 + EXP[-A t/t_s] [EXP(A x/L) - 1]}$$
(7)

where A -- the number of transfer units -- is related to kinetic and mass transfer effects, and  $t_s$  is the stoichiometric time for complete depletion of a bed of length Le.

Thomas (25) used the following expression for an ion exchange reaction, assuming opposing second order rates:

$$\frac{\partial N}{\partial t} = k, n(N^* - N) - k_2 N(n_0 - n)$$
(8)

A solution for this model can be found in terms of integrals and Bessel functions. Thomas suggested that k, and k, could best be found by fitting experimental breakthrough curves.

Thiele (24) presented a review of work prior to 1946. In addition to the above mentioned expressions, he includes those of Wicke (29), accounting for axial dispersion which was neglected by Bohart and Adams and by Thomas. He derived an equation for the case of a linear adsorption isotherm and uniform initial conditions. His integrated expression follows:

$$\frac{N}{N^*} = \frac{1}{2} \left[ 2 - ERF \left( \frac{X + \alpha}{\beta} \right) - ERF \left( \frac{X - \alpha}{\beta} \right) \right]$$
(9)  
where:  $\alpha = \frac{Ut}{\phi + K - K\phi}, \quad \beta = 2 \left[ \frac{D^* \phi t}{\phi + K - K\phi} \right]^{\frac{1}{2}}$ 

Klotz (13) considered the special case where diffusion is the rate controlling step (no axial dispersion). For a linear isotherm, diffusion limitations predict:

$$\frac{N}{N^*} = EXP \left[ -\frac{K s x}{\phi U P_g} \right]$$
(10)

for times when the bed front is not at equilibrium concentration, and

$$\frac{N}{N^*} = EXP\left[-\frac{K}{\phi}\frac{s}{\rho_g}\left(\frac{x}{U}-\frac{n_o}{N^*}\right)-1\right]$$
(11)

when N equals N\* at x equal zero. Klotz also considered the case where adsorption is controlling, with the same results as obtained by Bohart and Adams. However, he found no satisfactory theoretical expression describing the removal of toxic gases with charcoal and resorted to an empirical approach.

Amundson (2,3) developed integral expressions for the case of general initial and boundary conditions and more general kinetics. His equations reduce to those of Bohart and Adams and Thomas when the associated assumptions are made.

More recently, Acrivos (1), Cooney and Lightfoot (9), and Rhee and Amundson (19) have presented asymptotic solutions which predict constant pattern concentration profiles, that is profiles whose shape is independent of time, providing the time is sufficient. Constant

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pattern profiles exist even for non-equilibrium, non-zero D\* as long as the adsorption isotherm is convex  $(j^2N/\partial n^2 < 0)$ .

Rachinski (18) reviewed the cases of convex, linear, and concave isotherms, with and without axial dispersion considerations. In all cases he predicts a constant pattern profile for convex isotherms. Concave isotherms cause progressive blurring of the concentration front proportional to time to at least the first power. Linear isotherms result in blurring with time to at least the one-half power.

Assuming negligible axial dispersion, linear adsorption isotherms, no radial gradients, and reversible first order kinetics, Masamune and Smith (15) developed a series of complex numerical solutions accounting for mass transfer effects. They considered the adsorption process as a three step mechanism:

- 1) diffusion of the adsorbate from the bulk of the fluid to the external surface of the adsorbent (external diffusion)
- 2) diffusion down the pores of the adsorbent (internal diffusion)
- 3) adsorption (or reaction) with the active surface of the pore.

A possible step omitted would be the diffusion through surface compounds or adsorbed species. These investigators developed breakthrough curves for all three steps, combinations of two steps, and a single step controlling.

If the system can be related to a chromatographic column, Glueckauk and Coates (12) have shown that equilibrium usually does not exist and is approached only at low velocities and for small particles, a situation generally not desirable for industrial applications. According to Clark (8), chemisorption always produces convex isotherms. However, other than chemical kinetic effects (e.g. pore or solid phase diffusion) may cause apparent non-convex isotherms. In addition, hydrodynamic effects such as variable radial velocity would cause a spreading of the front. Masamune and Smith (15) point out that axial dispersion is usually negligible except at conditions of low velocity or for large particles.

These models mentioned are only a few of the possible solutions of equation '2'. Some of these are quite complex and require numerical solutions. To avoid these complications and to test other models for which solutions are not readily available, a differential approach may be used. From experimental breakthrough curves for various length beds,  $(\partial n/\partial t)$ ,  $(\partial n/\partial x)$ , and  $(\partial^2 n/\partial x^2)$  can be evaluated. The rate of adsorption,  $(\partial N/\partial t)$ , could then be evaluated directly from equation '1'. Various models for the rate of adsorption could then be compared with that calculated from equation '1' to test the validity of the model.

Multiple runs at various bed lengths and experimental conditions would be needed to evaluate a proposed model. To fully describe the system, runs must be made at various temperatures (exponential dependence of rate indicates chemical kinetics control), flow velocities (dependence indicates external diffusion limitations), particle sizes (dependence shows pore diffusion limitations), and size of the active solid (dependence indicates solid diffusion limitations).

If  $(\partial N/\partial t)$  data could be obtained directly, the need for multiple bed length runs could be eliminated. Accuracy could also be improved since second derivitive data would not be necessary. With such data and an equilibrium relation, a model could be tested by equation '2'. The purpose of this study has been to develop a technique for obtaining such data.

Richardson (20) proposed a method of obtaining solid phase pro-

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files by a magnetic technique which must be restricted to adsorption that causes a measurable change in the magnetic permeability of the adsorbent. This technique utilizes the phenomena that the voltage induced in a coil of wire in the presence of a changing magnetic field is related to the magnetic permeability of the material inside the coil. Thus by moving a bed of magnetic material into the coil, the change in voltage induced in the coil should be related to the increased amount of material in the coil. The magnetic permeability profiles obtained could provide a means of obtaining concentration profiles of the adsorbed species for adsorbents that change magnetically upon adsorption. One such system of industrial importance that could be studied is NiO/Ni/NiS<sub>X</sub>, <u>i.e.</u> the reduction of a nickel oxide catalyst and the sulfur poisoning of the reduced nickel. Other ferromagnetic catalyst such as cobalt, iron, and iron oxides could also be investigated. If the method were made sensitive enough, studies could also be made of the chemisorption on paramagnetic materials.

The coil required for Richardson's method must be near ideal, that is, one with a step change in the field of mutual induction at the point of entrance to the coil, or at least a change much sharper than any zone being investigated. An alternate method proposed by this investigator has been to use non-ideal coil data to obtain sufficient parameters to describe the concentration wave. Both these methods have been investigated and are presented, along with their associated limitations, in the following sections.

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## CHAPTER TWO. THEORY OF PERMEAMETER MEASUREMENTS

Electromagnetic induction methods have been used for the measurement of magnetic susceptibilities for more than 100 years. Barnet (4) describes an apparatus similar to one used by W. Weber in 1852. His equipment, as shown below, consisted of two coils, c and c', connected opposingly with their axis parallel to a constant magnetic field, <u>H</u>.



Figure 1. Simple permeameter

If a sample S of susceptibility  $\chi$  is inserted into coil c, it will obtain a magnetic moment  $\underline{M} = \chi \underline{H}$ . The flux seen by the galvanometer G upon insertion of S, assuming negligible demagnetization factor, is  $\hat{\Phi} = \underline{gM} = \underline{g\chi \underline{H}}$ , where g is the coil constant. The effect can be doubled by movement of the sample S from c to c', producing a flux change of  $\hat{\Phi} = 2\hat{\Phi} = 2\underline{g\chi \underline{H}}$ . Barnet further suggested that instead of moving the sample, the field could be reversed.

The reversing of the field in a periodic manner is the basis of the alternating current induction method for measuring permeabilities as described by Broersma (6). With no sample in either coil, the voltage induced into each is the same. Since the coils are wired opposingly, the net voltage as seen by the galvanometer is zero. If a sample of susceptibility  $\mathcal{X}$  and filling factor f is inserted into coil c, the net voltage detected is proportional to the change of f and  $\mathcal{X}$  from that of air.

A number of investigators have used this phenomena to study the adsorption of gases on ferromagnetic catalysts, since adsorption often results in a change in the permeability of the sample. Selwood and co-workers (22,23,26) have used such a technique to study the adsorption of hydrogen, oxygen, and carbon monoxide on nickel catalyst for varying conditions of pretreatment, temperature, and pressure. Geus and co-workers (10,11) studied the adsorption of hydrogen, oxygen, nitrogen, and nitrous oxide on nickel catalyst with a permeameter. Umeda (27) also studied hydrogen adsorption on dispersed metal systems.

The net voltage of the secondary coils is proportional to the magnetization of the sample only if certain restrictions are satisfied. First, the ferromagnetic particles must be small enough so that the magnetization will reach a maximum value during an alternating field cycle. Calculation of the relaxation time for a particular system have been discussed by Néel (17) and by Weil (28). Most dispersed nickel catalysts satisfy this condition at room temperature and line frequency (22).

A second restriction is that the field of induction of the secondary rust be uniform throughout the sample. The above investigators assured this by using a coil much longer than their samples. If this is not the case, additional considerations must be made.

If a current I passes through a solenoid of length  $\Psi$  and turns  $\dot{N}$ , a magnetic field <u>H</u> is created inside the solenoid:

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$$\underline{\mathbf{H}} = \frac{\mathbf{N} \mathbf{I}}{\Psi}$$
(12)

At a constant applied voltage, the inductance of the solenoid is dependent on the magnetic properties of the material inside the core. Since the current is dependent upon the solenoid's inductance, <u>H</u> is influenced by a change of the amount of magnetic material inside. Once inside the solenoid, movement of any magnetic material has no effect on <u>H</u>. The magnetic induction <u>B</u> in the solenoid is dependent on <u>H</u> and on the magnetization M of the material in the solenoid.

$$\underline{\mathbf{B}} = \mathbf{u}_{\mathbf{o}} \left( \underline{\mathbf{H}} + \underline{\mathbf{M}} \right) \tag{13}$$

where  $u_o$  is the permeability of air. The magnetization term denotes the extra flux attributed to a magnetic material. At locations away from the magnetic sample, the magnetization is zero and <u>B</u> is proportional to <u>H</u>.

For many materials, it has been observed that  $\underline{M}$  is proportional to the magnetic field strength:

$$\underline{\mathbf{M}} = \chi \underline{\mathbf{H}} \tag{14}$$

The proportionality constant  $\mathcal X$  is called the magnetic susceptibility. Thus:

$$\underline{B} = u_o (\underline{H} + \underline{H}) = u\underline{H}$$
(15)

where u is defined as the permeability of the sample. For ferromagnetic materials, the magnetic susceptibility,  $\underline{M}/\underline{H}$ , is not a constant but depends on the field strength. Thus, the magnetic permeability is not independent of

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the field strength. Ferromagnetic permeabilities are also dependent on the sample temperature and the previous history of the material. They vary with field strength from zero, through a maximum, and decline to zero at very high fields.

The flux density associated with <u>B</u> near a magnetic material is shown below as depicted by Carter (7).



Figure 2. Magnetic induction near a ferromagnetic sample

The flux density at large distances from the sample is the same as if no sample were present. However, almost all the flux (dependent on the permeability of the sample) passes through the sample, yielding a higher value of <u>B</u> in the neighborhood of the sample. Figuratively speaking, when the flux comes near the sample, it bends -- taking advantage of the easier path through the sample. The voltage induced into the secondary is given by:

$$V_{2} = -\frac{d}{dt} (\tilde{N} \Phi) = -\frac{d}{dt} \int_{0}^{\tilde{N}} \int_{0}^{r} 2\pi r \underline{B} dr d\tilde{N}$$
(16)

From figure '2' it can be seen that <u>B</u> is varying over a region outside the sample, thus causing a voltage to be induced into the secondary before the bed begins to enter the secondary. This voltage will be referred to as "end effects". The component of <u>B</u> normal to a surface can not change discontiniously. If the flux of <u>B</u> passes from one material to another of differing permeability, as is the case at both ends of the sample, the angle of emergence  $\theta_0$  can be related to the angle of approach by:

$$\operatorname{Tan} \theta_{o} = \frac{u_{o}}{u} \operatorname{Tan} \theta \tag{17}$$

The relation between these angles is shown below:



## Figure 3. Magnetic Induction Change With Permeability

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Thus for ferromagnetic materials with large relative permeabilities  $(u/u_o)$ , the lines of <u>B</u> emerge practically normal to the surface, causing increased end effects.

If these end effects can be made small enough relative to a magnetic material concentration zone, the voltage induced will be dependent only on the material inside the secondary so that:

$$V_2 \propto \int u(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}$$
(18)

integrated over the length of bed inside the coil.

A function "g(x)" can be defined which accounts for the proportionality constant at a particular value of x and the constants required to convert f to the length of bed inside the coil. This function will be assumed independent of the bed. The axial permeability can be normalized relative to the average permeability of a known bed to yield C, a term representating a concentration of magnetic material. If a coil coordinate x, and a sample coordinate  $\ell$  are defined as shown below in figure "4",



Figure 4. Coil & Sample Coordinate Systems

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as the bed is moved into the coil, the secondary voltage will be represented by:

$$V_2(\ell) = \int_0^\ell g(x) C(\ell - x) dx$$
 (19)

Differentiating the voltage with respect to the bed movement gives:

$$\frac{\mathrm{d} V_2(\ell)}{\mathrm{d} \ell} = \int_0^\ell g(\mathbf{x}) \frac{\partial}{\partial \ell} \left[ C(\ell - \mathbf{x}) \right] \mathrm{d} \mathbf{x} + g(\mathbf{x}) C(\ell - \mathbf{x}) \frac{\mathrm{d} \mathbf{x}}{\mathrm{d} \ell} \bigg|_{\mathbf{x} = \ell} - g(\mathbf{x}) C(\ell - \mathbf{x}) \frac{\mathrm{d} \mathbf{x}}{\mathrm{d} \ell} \bigg|_{\mathbf{x} = 0}$$
(20)

If the rear of the bed is never past the start of the coil, the lower limit of integration is constant and the last term is zero. If in addition the concentration is constant, the integral term also becomes zero so that:

$$\frac{dV_2(\ell)}{d} = g(\mathbf{x}) C(\ell - \mathbf{x})$$
(21)

evaluated at  $x = \ell$ . Thus a method of experimentally determining g is available.

For an ideal coil (one with a step change in g, zero outside the coil and some non-zero constant value inside) equation '20' can be evaluated:

$$\frac{dV_2(l)}{dl} = g(x) \frac{d}{dl} \int_{0}^{l} C(l-x) dx = g(0) C(l)$$
(22)

Thus the derivitive of the voltage with position is directly proportional to the axial concentration. Normalizing with respect to a uniform bed of constant concentration,

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$$\frac{dV_{c}}{d\ell} = \frac{g(\mathbf{x}) C(\ell)}{g(\mathbf{x}) C_{o}(\ell)} = \frac{C(\ell)}{C_{o}(\ell)}$$
(23)

and the concentration profile is defined relative to some known profile.

Richardson (20) employed this relation in a study of the sulfur poisoning of nickel catalysts, but did not verify the related assumptions. This work investigates the limitations associated with this approach and studies means of reducing these limitations. In addition, a technique for obtaining profiles for non-ideal coils is developed.

#### CHAPTER THREE. EXPERIMENTAL EQUIPMENT

The basic equipment used in this study was the same as that used by Richardson (20). However, extensive modifications and improvements have been made to increase the precision of the measurements and to give additional information.

An alternating current magnetic field was produced inside a solenoid consisting of a twelve inch cylindrical form -- two inches in diameter -- on which 10,000 turns of Alpha Heavy Formvar magnet wire were wound in 14 layers. The original cooling coils were removed since they contributed to undesirable magnetic fields when the sensitivity of the equipment was improved. Cooling was also found to be unnecessary for room temperature operation at line voltage.

The magnet was powered by stabilized line voltage from a Sola Type CVS-1 constant voltage transformer. The primary voltage and current were monitered with an A.C. voltmeter. The working volume of the primary was a cylinder, two inches in diameter and twelve inches in length. The primary coil was mounted inside a heavy iron cabinet for magnetic shielding. Several feet of working room above and below the magnet were provided for sample insertion. The primary sat on four leveling bolts attached to a wooden frame to avoid unsymmetrical fields. The leveling bolts also allowed for vertical adjustment of up to two inches.

The secondary coils consisted of two matched coils, wound opposingly on a non-ferrous tube. Glass, copper and paper tubes were all used. Being wound opposingly and matched, ideally no net output voltage should exist across the pair. The final matching of the coils in most cases was accomplished by adding extra turns to one of the secondaries so that the null voltage was a minimum. This was necessary because of slight geometrical differences in the coils.

Numerous secondary coils have been wound and evaluated. These coils were in all cases precision wound with Alpha Heavy Formwar #36 magnet wire. The most recently used coils consisted of two pairs of matched coils. One pair had individual coils 500 turns wide by ten layers; the other consisted of 300 turn coils, fifteen turns wide. Both pairs were wound on a 5/8 inch diameter Pyrex glass tube.

The output of the 5000 turn pair was adjusted to a minimum value with an external General Radio variable mutual inductor powered by the same current source as the primary coil. The output of the 300 turn pair required amplification before measurement. This was accomplished with a Tektronic Type RM122 Low-Level Preamplifier operated as a differential amplifier with an approximate gain of 100. The differential balance of the amplifier was also used in matching the signals from the two coils.

Since the system was subject to some drift (particularly if the amplifier was being used), after a minimum null was obtained by positioning of the secondaries, a small piece of iron was moved toward the measuring coil to create a small bias. This bias was in phase with the measured signal and served also to remove the small differences between the null point and measured signals (initial movement of a sample into the coil caused a slight phase shift with little change in induced voltage).

Unlike Richardson's system, the secondaries were fixed during measurement. Although the magnetic field was reasonably constant inside the solenoid, the differential signals magnified small changes. The small change that occured when the secondary coils were moved was utilized in

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obtaining a better null. An optimum position for a minimum difference in the two coils existed. For this reason, the secondaries were attached to a fine gear drive for precise axial positioning.

Since changes in the position of the secondaries produced voltage changes, even without a sample in the coils, measurements were made by movement of the bed instead of the coils. Moving the sample affects only the measuring coil, since slight changes caused by insertion of a sample into the primary were reflected in both coils and subtracted. The velocity of the bed was required to be constant since the distance parameter was obtained from the scan time. Also, no jerky motion of the bed could be tolerated if the signal was to be differentiated. A device was constructed that satisfied these requirements (see figure '5'). A reversible 72 rpm syncronous motor (Superior Electric Slo Syn Type SS150) turned a 3/4 inch diameter brass shaft, threaded right-handed at one end and lefthanded at the other with 16 threads per inch. The circular motion of the shaft was converted to a vertical motion of the bed support. The opposing threads allowed for future differential scanning techniques.

Delrin sleeve bearings at contact points reduced drag and minimized jerky motion. Vertical motion of the threaded rod was eliminated by thrust bearings at each end of the rod. End play in the bed support was minimized by two precision linear bearings for each support.

Since the relative position of the bed and the coil was important, a means of reproducing the starting point of a scan was a requisite. This was accomplished by a microswitch triggered by the position of the bed support. Closing of this switch starts the sweep of a Texas Instrument X-Y recorder (X axis was the time or position axis) used to record the secondary signal. Provisions were also available for scanning on the

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return.

As the bed of ferromagnetic or paramagnetic material approached a secondary coil, a voltage was induced into that coil dependent on the permeability and shape of the sample and on the effective shape of the secondary field of induction as was explained in Chapter Two. This increase in voltage was either first amplified or sent directly to a Hewlett Packard Model 400E A.C. Voltmeter where the signal could be read directly or could be amplified, rectified, and sent to the Y base of the X-Y recorder. In the cases where the derivative of the voltage was needed, a Cahn Time Derivative Computer, Mark II was inserted between the voltmeter D.C. output and the recorder.

Typical scans of the voltage and the derivative of the voltage with position of the bed relative to the secondary coil are shown in figures '6' and '7'. The first of these was obtained with the 5,000 turn, 10 layer coil (approximately 3 inches long) when a 1-1/8 inch length uniform bed traversed the coil. All beds studied were 3/8 inch diameter. The magnetic material used for the beds was made by reducing and then steaming a commercial fluid bed catalyst (Harshaw Iron Oxide, Fe-0303P, 20% FezO3 mounted on alumina). The reduction and steaming yielded a catalyst containing Fe, FezO3 and Fe3O.

Figure '7' illustrates the relations obtained when the bed is much longer than the coil. In this case, the same bed was moved into a 300 turn coil, 15 turns wide (<u>ca</u>. 0.1 inch long). The negative derivative signal shows the bed emerging from the backside of the coil.

A wiring schematic is given in figure '8', with a description of previously undefined components in Table 1.



Figure 5. Mechanical features of differential scanning permeameter



Figure 7. Typical Output Signals When the Coil is Shorter Than the Bed



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Figure 8. Wiring Schematic

# TABLE 1.

# UNDEFINED COMPONENTS OF CIRCUIT DIAGRAM (FIGURE 8.)

COMPONENT	FUNCTION	
SW-1	Main power switch (DPST).	
SW-2	Selector switch (DPDT) for either stabilized power input or external source (variable frequency supply.	
S₩ <b>-</b> 3	Selector switch (SPDT) for reading on the TVM either the primary voltage or the primary current (via. the voltage drop across R-1).	
SW-4	Selector switch (DPDT) for reading on the TVM either the voltage from SW-3 or SW-6.	
SW-5	Selector switch (DPDT) allowing signal to be ampli- fied if desired.	
sw-6	Selector switch (DP4T) for reading on the TVM either 1. individual secondary output 2. sum of long coil voltages 3. sum of short coil voltages 4. individual secondary output.	
R-1	10 ohm resistor for determining primary current.	
R-2	800 ohm resistor to reduce voltage to variable inductor.	

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### CHAPTER FOUR. DEVELOPMENT OF METHOD

Equations were developed in Chapter Two for the prediction of the induced voltages and the derivative of these voltages with bed movement. The general case was shown (equations '19' and '20'), as well as specific cases (equations '21' and '22'). The problem is in developing a technique for prediction of concentration profiles from experimentally measurable quantities.

An exact method of determining profiles exist when the magnetic field of induction (g) is a step function. If such a case existed, a point concentration would be defined by the derivative of the voltage at the step. Because of the simplicity of this method, considerable effort was devoted to the design of an ideal coil or one ideal enough to approximate realistic profiles.

The solution of equations '19' and '20' require an additional assumption of an equation describing the concentration profile. However, if an equation with enough adjustable parameters can be defined, this deficiency is a minor one. The mathematics involved are more complex, requiring numerical solution.

This chapter discusses both approaches.

## A. IDEAL COIL APPROXIMATION

If an ideal coil can be designed so that point axial concentrations can be related directly to the derivative of the induced voltage with position, exact profiles can be obtained with a minimum of effort. A uniformly wound coil does not meet the required criteria. Figure '9' shows the error in neglecting end effects for one inch beds of various profiles





Errors in Ideal Coil Assumption Using a Real Field of Induction (for simulated one inch beds)

---- Actual concentration and profile predicted for "ideal coil".

---- Profile obtained if end effects are considered.

described by the following relations:

$$C(\ell) = \ell \operatorname{Tan} \theta \qquad \qquad \ell \langle D_o \operatorname{Cot} \theta \\ = 1 \qquad \qquad \qquad \ell \rangle D_o \operatorname{Cot} \theta \qquad (24)$$

where D is the maximum bed diameter. The coil used was a three inch uniform coil. The dashed lines of figure '9' represent concentration profiles and the signal that would be obtained if the coil were ideal. The solid lines represent the real coil data relative to an initial two inch bed. As can be seen, the wider the gradient portion of the concentration profile, the better the ideal coil assumption becomes.

This is better illustrated by approximating g by a sinusoidal up to maximum and by a constant at points of greater x, <u>i.e.</u>:

and assuming linear bed gradients represented by:

Using these relations, figure '10' was constructed. The half cycle length for the rise of g required to approximate a given profile is illustrated. For a reasonably accurate approximation it is necessary that g must reach a maximum in less than one-tenth the width of the concentration gradient.

Numerous variables have been studied to obtain data for the design of a coil approximating the ideal case. Since improvements were small compared to that needed, only qualitative comparisons are given.







Coil Parameters:

A study of the effect of primary current was conducted using a 10 layer, 3000 turn coil. The current was varied from 50 to 200 milliamperes. The most "ideal" curves were obtained using the lowest current. This was expected since at low fields the permeability increases with field strength, causing (as discussed in Chapter Two) increased end effects. The differences were small and posibly not significant. Decreasing the current results in a loss of sensitivity and is not recommended.

Primary frequencies from 30 Hz to 3000 Hz were investigated at a constant primary voltage. Indications were that higher frequencies were preferable. Again, this can be attributed to a lowering of the primary current. However, Selwood (22) points out that the relaxation time of the particles becomes a limiting factor at higher frequencies.

It was imposible to obtain a complete null for the secondaries. The best null obtained was .01 percent of the voltage of a single coil. Most beds yielded voltages of 1 to 10 percent of that for a single coil. The various nulling techniques did not significantly effect the sharpness of the derivative curve.

#### Coil Geometry:

A final attempt at producing an ideal coil was by variation of the secondary coil geometry. The winding of several thousand turns of wire in a desired manner was time consuming and tedious. Much greater changes were noted using this approach, however. Numerous coils were tried, a few of which will be discussed.

Coils with the geometries shown in figure 'll' were investigated. Of the first three of these, the convex coil geometry gave the most

-29-


# Figure 11. Coi

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Coil Geometries

-30-

ideal results. The geometry of induction of this coil meshes best with the extended field created by the sample as was shown in figure '2'. The reverse windings of the last case were added to try to improve on the convex coil geometry by subtracting turns at the end of the coil. The number of reverse turns was optimized for the best "g". The trend observed was as expected, but still not sufficient for application of the ideal coil analysis.

To reduce the number of coils needed to find an optimum, a variable, movable secondary was added to the system of coils. The basic pair of fixed coils for this investigation was a five layer, 2500 turn matched set. A movable pair consisting of the five layer arrangement shown in figure '12' was added. The movable pair was wound on a cylinder of diameter to allow them to fit over the fixed coils. Any combination of the layers of this movable pair could be connected in phase or opposing the fixed pair. Again, the results were not sufficient for ideal coil analysis. These investigations indicate that a coil can probably not be constructed that will describe a concentration gradient of less than several bed diameters by the simple derivative approach.



Figure 12. Movable Coil Geometry

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#### B. MATHMATICAL TREATMENT OF REAL COIL DATA

Further investigations were made to develop a means of determining concentration profiles by accounting for end effects. One approach was the solution of equation '19' for C using real coil data. Dividing the integral into small increments so that both g and C could be approximated by an average value in a single increment, equation '19' could be written as a set of equations describing the secondary voltage as l changed:

$$V_{2}(\ell_{i}) = g(\mathbf{x}) C(\ell - \mathbf{x}) d\mathbf{x}$$

$$= \mathbf{x} \left[ \underline{g}(\mathbf{x}_{i}) \underline{C}(\ell_{i}) + \underline{g}(\mathbf{x}_{i}) \underline{C}(\ell_{2}) + \dots + \underline{g}(\mathbf{x}_{n}) \underline{C}(\ell_{n}) \right]$$

$$V_{2}(\ell_{2}) = \mathbf{x} \left[ \underline{g}(\mathbf{x}_{i}) \underline{C}(\ell_{i}) + \underline{g}(\mathbf{x}_{3}) \underline{C}(\ell_{2}) + \dots + \underline{g}(\mathbf{x}_{n}) \underline{C}(\ell_{n}) \right]$$

$$\cdot$$

$$V_{2}(\ell_{n}) = \mathbf{x} \left[ \underline{g}(\mathbf{x}_{n}) \underline{C}(\ell_{i}) + \underline{g}(\mathbf{x}_{nn}) \underline{C}(\ell_{2}) + \dots + \underline{g}(\mathbf{x}_{nn}) \underline{C}(\ell_{n}) \right]$$

$$(27)$$

This method did not provide unique solutions because g was a smooth, increasing function. For small increments, successive equations were only slightly different since V was also an increasing function.

The same problem was encountered when C was approximated by a polynomial:

$$V_2(l) = g(x)[a + b(l-x) + c(l-x)^2 + ...] dx$$
 (28)

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The maximum number of coefficients that could be specified was two, and then only for zones several bed diameters wide.

The minimum number of parameters needed to describe a concentration profile is two, one defining the zone position and the other the zone width. If the bed is completely inside a long coil so that g is constant over the entire length of the bed, the zone position can be found as an effective length (<u>i.e.</u> a length equivalent to a uniform bed containing the same amount of ferromagnetic material). Normalizing with respect to an uniform bed of concentration  $C_o$  and length  $L_o$ , an effective length can be found:

$$\frac{\Psi_{2}(L)}{\Psi_{2}(L_{o})} = \frac{\int_{0}^{L} C(\ell - x) dx}{\int_{0}^{L} C_{o}(\ell - x) dx} = \frac{C_{o} L}{C_{o} L_{o}} = \frac{L}{L_{o}}$$
(29)

where L is the effective length.

It was previously shown that a single uniform coil will not provide a second parameter for sharp zones. To obtain another parameter, a coil of considerably different g would be required. One possibility would be a coil only a few turns in width. With the bed much longer than the coil, greater emphasis would be given to a narrower portion of the bed. Such a coil was studied to see if it would provide data for calculating a parameter related to the zone width.

An equation that can describe a number of profile shapes is the Bohart and Adams equation (5). Using this equation, the concentration of magnetic material is described by:

$$C(\ell/L_{o}) = C_{o}(\ell/L_{o}) \left[ \frac{EXP(A \ell/L_{o})}{B' + EXP(A \ell/L_{o})} \right]$$
(30)

where A is related to the zone width and B' is a function of the position of the zone. One relation between A and B' can be found by the integration described in equation '29', using the concentration relation given above, and a measured effective length:

$$\frac{L}{L_{o}} = 1 + \frac{1}{A} \ln \left[ \frac{B' EXP(-A) + 1}{B' + 1} \right]$$
(31)

The short coil can also be used to generate another relation between A and B'. Rewriting equation '19' for a particular value of  $\mathcal{L}$  so that the integration is in the direction of increasing concentration, and using equation '30' to describe the concentration:

$$V_{2}(\boldsymbol{\ell}) = \int_{B^{\prime}}^{\boldsymbol{\ell}} g(\boldsymbol{\ell}-\mathbf{x}) \left[ \frac{EXP(A \boldsymbol{\ell}/L)}{B^{\prime} + EXP(A \boldsymbol{\ell}/L)} \right]$$
(32)

An analog computer was used for the above integration. A diagram of the analog circuit is shown in figure '13'. g was approximated by:

$$g = EXP \left[ a + b(\ell - x) + (\ell - x)^{2} \right]$$
(33)

where the coefficients a, b, and c were determined by a least square fit.

The method was unsatisfactory for two reasons. One was that computer scaling was difficult due to the exponential nature of one parameter. Secondly, realistic profiles have values of A from 10 to 100 or even higher, and equations become non-unique for the g used when A is greater than five. The increased accuracy of digital computers extend the range to about ten.

Since these methods did not yield unique results, an experimental







-35-

comparison of known profile beds was made. Profiles were devised by machining regular shaped objects out of a non-magnetic material that would fit tightly inside a glass tube. Then a supported iron oxide catalyst was poured over the object. Two different profile types were studied. Cones were machined from Teflon, giving concentration gradients as described by figure '14'. "Inverted cone" beds were formed for cone angles of 90°, 45°, 30°, 15°, and 10°.

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Experimental voltage profiles were as predicted from equation '19' for these beds. The model predicted-- and it was experimentally observed -- that all profiles were identical. No significant differences could be seen in the long or short coil voltage outputs with position among any of the beds. Thus voltage -- position data can not be used to distinguish realistic profiles.

Differences in the voltages were not significant relative to the precision of the experimental equipment, but the model predicted that differences did exist. The computer program for the model was modified to predict the derivative of the voltage since this should magnify the differences. Differences were observed in predicted derivatives that indicated discrimination between profiles could be made. The predicted profiles were obtained by numerically integrating equation '20', using the inverted cone equations for C and experimentally determined values for g. For experimental determinations, duplicate runs were made of each profile. The ferromagnetic material was reweighed each time to account for possible inhomogeneity of the bed material. A comparison of predicted and experimental results is shown (table '2'). The average results of the two runs were normalized relative to the 45° front bed because of low results obtained for the 90° front, later determined to be due to

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 $C = 1 - (1 - \frac{2}{R \cdot \cot \theta})^2 \qquad 0 < 2 < R \cdot \cot \theta$ 

C = 1

R•Cot 0 < 2 < L

-37-

an axial ridge in the tube. With the exception of the 90° front bed, all experimental results are within 2 percent of the predicted values. The largest discrepancy was for the 10° front, which could be partially due to errors in machining a long cone of a flexible material as Teflon. Measurement of this cone indicated some concavity which would cause larger experimental values. Only peak values were used in the tabular comparison, since exact positioning was difficult. The complete profile as predicted by the model is shown in figure '15' for each bed.

#### TABLE 2

# MEASURED AND PREDICTED PEAK SHORT COIL DERIVATIVES FOR INVERTED CONE BEDS

Cone Angle 0	$(dV_2/d\ell)_{max} / (dV_2/d\ell)_{45^{\circ}max}$		
	Predicted	Measured	
90°	1.02	0,98	
45°	1.00	1.00	
30°	0.97	0.98	
15°	0.87	0,86	
10°	0.76	0.775	

-38-



Another more linear profile was obtained by slicing Teflon cylinders at different angles as shown in figure '16'. The experimental and predicted results of this study are shown in table'3'. Again an agreement within 2 percent was obtained and the greatest discrepancy was for the greatest angle bed.

The predicted results are in every case within two percent of experimental determinations. As can be seen, the method can differentiate zones of less than a bed radius (3/16 inches for the beds evaluated). Peak voltages obtained for all beds using the long coil were within two percent. Illustrations of this precision and actual experimental profiles are shown in the appendix.

#### TABLE 3.

# MEASURED AND PREDICTED PEAK SHORT COIL DERIVITIVES FOR SLICED CYLINDER BEDS

ANGLE 0 (avj ax) max	$(dv/dx)_{max} / (dv/dx)_{90^{\circ}max}$		
Predicted	Measured		
90° 1.00	1.00		
60° 0.975	0.985		
45° 0.94	0.94		
30° 0.86	0.865		
15° 0.62	0.63		

-40-



### SLICED CYCLINDER CONCENTRATION GRADIENTS



where  $A = 1 - \frac{\cancel{R} \cdot \text{Tan } \Theta}{R}$ 

 $C = 1 \qquad \qquad \frac{2R}{Tan \theta} \left\langle \mathbf{z} \right\rangle L$ 

-41-





-42-

To use this method to determine parameters associated with a particular adsorption model, calibration of g for the short coil must first be accomplished with a bed of known concentration and length. This could be done with the initial bed or some standard bed. Uniform concentration beds would make mathmatical interpretation of g simpler. Beds to be tested must be shorter than the long coil. Voltage data from the long coil, obtained with the bed completely inside the coil, would give directly the effective length relative to a known or initial bed, From the effective length, and a knowledge of g, a zone parameter could be varied until the model for the short coil output predicts the correct peak value for the short coil derivative with position. Alternately, curves could be constructed from the model relating the effective length and maximum derivative to a zone width parameter. An illustration of such a graphical interpretation is shown in figure '18' for assumed sliced cylinder profiles. The variance of the parameter as the zone progresses down the bed would provide initial screening of models predicting the adsorption. Repeated analysis at varying conditions ( temperature, flow rate, concentration, particle size, and crystallite size) would be necessary for a complete study of the validity of a model.

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## FIGURE 18.

RELATION BETWEEN ZONE WIDTH AND MEASURABLE QUANTITIES

FOR "SLICED CYLINDER" BEDS



#### CHAPTER FIVE. LIMITATIONS AND RECOMMENDATIONS

The method discussed in the previous chapter will define two independent parameters in an assumed model describing the ferromagnetic concentration zone of a bed. Parameters can be determined for a particular model that are related to the zone position and spread. Most theoretically developed equations for adsorption waves use only two adjustable parameters, so this is not a severe limitation.

For all constant volume beds studied, the peak long coil voltage was observed to be within two percent of the same value. Using two percent as a maximum error associated with derivative peak measurements, a maximum propagated error was determined for the zone width of a sliced cylinder bed. For a bed with an effective length of three bed diameters and a zone of one diameter, a maximum error in the zone width of 0.2 bed diameters was predicted. For the details of this error analysis, see the appendix.

Some discrimination between models for the concentration gradient can be made. A comparison of two equal volume beds, one a 30° sliced cylinder and the other a 15° inverted cone is shown in figure '19'. Both beds gave near identical maximum derivative signals. However, the faster rise of the differentiated signal for the inverted cone was a result of the faster concentration rise. These results were obtained theoretically for the short coil, and it is doubtful that the differences could be detected experimentally since the experimental system was constructed to give reproducable position data for a single bed, Changing beds resulted in an error in the estimated starting point. More work is advisable in optimizing the short coil geometry, since a FIGURE 19. DISTINQUISH

DISTINQUISHING PROFILE EQUATIONS



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coil with a shorter effective length would probably be more discrimatory. The previous studies on coil geometry suggest ways of reducing this effective length (i.e. reverse windings).

Using just maximum signal data, the technique developed in this thesis can be used to describe the kinetics of adsorption with fewer experimental beds than required by techniques presently available. After assuming a model representing the movement of the adsorption zone, the model parameters can be determined from experimental measurements. The adsorption wave can be studied as it moves down the bed, thus eliminating the multiple bed analyses required by previous methods. If the technique suggested for obtaining additional data from the shape of the derivative curve can be perfected, even fewer measurements would be required. Based on an evaluation of the experimental results and theoretical predictions as presented in this thesis, the following conclusions were made:

1.) A secondary coil can probably not be designed that will yield concentration data directly from voltage or derivative of the voltage data for profiles less than several diameters in width.

2.) A single, long uniform coil can not provide any data other than the effective length of a bed for realistic adsorption profiles.

3.) A two coil system (one a long uniform coil with a uniform field greater than the bed length from which the effective length can be obtained and the other a short coil from which the zone width can be found using the differentiated voltage output) will define two parameters in a concentration profile equation.

4.) Information can be obtained from the shape of the derivative of the short coil voltage with relative bed and coil position that will discriminate between assumed concentration models.

5.) This technique has the advantage over other methods of determining concentration profiles for adsorption waves in that it is nondestructive, and yields direct data.

6.) The information obtainable by this method, along with breakthrough curve analysis, will be a valuable tool for determining kinetic data for adsorption systems that change magnetically upon adsorption.

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APPENDIX

### COMPUTER PROGRAM FOR PREDICTING VOLTAGE AND DERIVITIVE PROFILES

The program used to test a particular model was specific to the model. Small changes were made in the main program when converting from inverted cone profiles to sliced cylinder profiles. Using experimentally determined values of g and effective length, the parameter associated with zone width was varied until agreement with experiment was obtained.

The values of g were obtained as suggested by equation 21. Measurements were made with an uniform bed of length much greater than any bed to be tested and containing the same material and axial density as the beds that were to be evaluated. Either the voltage was determined and differentiated numerically, or the derivitive was recorded directly, using the Cahn derivitive computer. Due to slight irregularities in the bed movement, some filtering of the signal was required. This resulted in a lag in the responce and an attenuation of the peak signal. Because of this, the calibration was generally also measured with the differentiator at the same filter setting. A single equation representing g over the full length of induction could not be found. For this reason, g was divided into small increments where a linear or nearly linear representation could be used. Generally the relation used was:

$$g(x) = EXP(a + bx + cx^2)$$

The coefficients were determined by either a least square computer fit or from selected points. Other forms of g may also be used, as indicated in the program. The form used is specified by the value of the input parameter 'ID'. The effective length of a particular bed was determined from long coil voltages using equation '30'. Since the profiles of the beds tested were known, these lengths were verified.

The present program requires successive estimations of the correct zone width by requiring that the angle of the cone or cylinder be specified. This could easily be modified to require the computer to determine the "best" values corresponding to measurements of the derivitive peak.

The integration of equations '19' and '20' were performed sequentially as the bed moved into the coil. Numerical integration was performed using Gauss's Quadrature Method with the number of quadrature points 'NQ' and values 'R' and 'U' specified by the programmer. (This method can be found in most books on numerical methods. One such source is J. R. Scarborough's <u>Numerical Mathmatical Analysis</u>, published by the John Hopkins Press.) The integration is performed in segments of specified length 'DL'. This length is chosen so that the maximum is easily distinguished and so that g is a single function in the segment.

Several recommendations for the improvement of the program should be made. One would be to use external subroutines for the model for concentration, thus minimizing the changes in the main program required. Another suggested change would be to incorporate an optimizing routine to determine the best fit of a particular equation to experimental values. A final deficiency that should be remedied is to correct the length parameter for integration so that values are determined relative to the back of the bed instead of to the changing front. Presently the results must be shifted by an ammount equal to the effective length of the bed.

A computer listing along with a sample output follows.

-52-

```
•53-
    PROGRAM FOR PREDICTING V-X PROFILES FOR CULICAL SHAPED BEDS
С
С
    OL = INCREMENTAL POSITION CHANGE & INTEGRATION RANGE
С
    NSET = MUNBER OF DATA SETS 4, B, &C -
С
    NO = NUMBER OF QUADRATURE POINTS
    TANT = TANGENT OF COME ANGLE
С
    NICR1 = INCREMENTS IN CHANSING CONCENTRATION PORTION
С
С
    VICR2 = INCREMENTS IN CONSTANT CONCENTRATION PART
С
    FL1 = LENGTH OF CHANGING CONCENTRATION SECTION
    NICR = TOTAL NUMBER INCREMENTS OF INTEGRATION
С
С
    NPTS = NUMBER OF POINTS IN V-X OUTPUT
С
    DG = INITIAL VALUE OF D
    FLL = BED LENGTH
C
C
    FLO = MACHINING EPROR
С
    RAD = BED RADIUS
С
    NPTSC = COUNTER FOR NPTS
С
    SLI &SL2 = INEEGRATIUN RANSE
    SL = MCDIFIED INTEGRATION LIMITS FOR GAUSS METHOD
С
С
    NUMERICAL INTEGRATION BY GAUSS! CUADRATURE METHOD
С
    ID = 1, G = A+B*X+C*X**2)
C
    iD = 2, G = B + 2 * C * X
С
    ID = 3, G = EXP(A+B*X+C*X**2)
С
    ID = 4, G = (B+2*C*X)*EXP(A+B*X+C*X**2)
    DV = DERIVITIVE OF VOLTAGE WITH POSITIC!
С
    NUPV = NURBER OF POINTS PER VALUE OF A, B,C
С
      DIMENSION U(15),R(15),Y(15),A(200),B(200),C(200),X(200),V(200),
     1XF(200),VT(200),TITL(200),DV(200),DVT(200),DY(200)
  100 FORMAT (415, F5.1)
  101 FURMAT (SF10.5)
102 FURMAT (16F5.1)
  103 FORMAT (20X,G12.4,10X,G12.4, 1CX,G12.4)
  104 FORMAT (2044)
 200 FOPMAT (//,(6G12.4))
 203 FORMAT (1H1,//,10X, ' INCREMENTAL VALUES OF VOLTAGE')
 204 FORMAT (1H1,//,30X, ' DISTANCE VOLTAGE PROFILES',//,20X,20A4,////,
     120X, ' LENGTH ',13X,' VOLTAGE',10X,' DERIVATIVE',//)
      READ (5,100) NSET, NQ, ID, NUPV, DL
      READ (5,101) (U(I),I=1,NQ)
      READ (5, 101) (R(I), I=1, NQ)
      READ (5,102) (A(I),I=1,NSET,NUPV)
      READ (5,102) (B([),I=1,NSET,NUPV)
      READ (5,102) (C(I),I=1,NSET,NUPV)
      MNSET = NSET-NUPY
      DC 17 I=1, MNSET, HUPV
     DO 17 J=1,NUPV
      A(I+J-1) = A(I)
     B(I+J-1) = B(I)
  1^{7} C(I+J-1) = C(I)
      WRITE (6,100) WSFT, NO, ID, NUPV, DL
     WRITE (6,200) (U(I),I=1,40)
     WRITE (5,200) (R(I), I=1, 10)
     WRITE (6,200) (A(I), I=1, NSET)
     WRITE (0,200) (B(I), I=1, NSET)
     WRITE (6,200) (C(I), I=1, MSET)
9999 READ (5,104) (TITL(I), I=1,20)
```

1

2

3

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0

1

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9

0

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3

4

5

5

7

8

```
READ(5,102) FLL, TANT, FLO, RAD, XO
    XI = XC
    PI = 3.14159
    DO 12 I=1,NSET
    X(I) = XI + DL
12 \times I = \times (I)
  CALCULATE INCREMENTS IN EACH SECTION
    FL1 = PAD/TANT
    IF (FL1 - FLL) 15,16,16
16 FL1 = FLL
 15 CONTINUE
    NICR1 = FL1/DL
    IF(FL1-NICR1*DL-7.3001)10,20,20
-20 NICR1 = -3ICR1 + 1
 10 CONTINUE
    IFLLD = I \times T(FLL/DL)
    FLLD = FLOAT(IFLLD)
    IF (FLL/0L - FLLD-0.0001) 22,23,23
 22 \text{ MICR} = \text{IFLLD}
    GO TC 24
 23 NICR = IFLLD + 1
 24 CONTINUE
    NPTS = NSET - NICR +1
    D = CL * NICR
    DO 30 NPISC = 1, NPIS
    JJ = NPTSC + NICP-1
    SL1 = -.3
    IF (T_{11}T_{-50}) 13,14,14
 14 \text{ NICR1} = 1.
    FL1 = 0.0
    OV(1) = A(JJ) + B(JJ) * (D+XO) + C(JJ) * (D+XO) * 2
    IF (ID.EQ.3) DV(1) = EXP(DV(1))
    V(1) = 0.0
    GD TC 52
 13 CONTINUE
    DU 40 I=1,NICR1
    IF (I-NIC31)41,42,42
 41 \, \text{SL}^2 = \, \text{SL}^1 + \, \text{DL}^2
    GO TC 43
 42 \, SL2 = FL1
.43 CONTINUE
    V(I) = 0.0
    DV(I) = 0.0
    DO 50 J=1,NO
    SL = (SL2-SL1) \neq U(J) + 0.5 \neq (SL1+SL2)
    T23T = 4(JJ+1-I) + B(JJ+1-I) * (D-SL+X0) + C(JJ+1-I) * (D-SL+X0) * *2
    TESF1 = B(JJ+1-I) + 2 \cdot C(JJ+1-I) \cdot (D-SL+XO)
     IF (ID.EW.1) Y(J) = TEST
    1F(10.E0.2) Y(J) = TEST1
    IF (ID.EQ.3) Y(J) = EXP(TEST)
     IF (ID.EQ.4) Y(J) = TEST1 * EXP(TEST)
    CST = RAD-SL*TANT
    H = SL * TANT
    DY(J) = Y(J)*2.*(TANT/RAD-SL*(TANT/RAD)**2)
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47 Y(J) = Y(J) \neq (2 \cdot \pi SL + TANT/RAD - (SL + TANT/RAD) + 2)
      DV(I) = DV(I) + (DY(J)*R(J))*(SL2-SL1)
   50 V(I) = V(I) + (Y(J) * R(J)) * (SL2 - SL1)
   40 \, \text{SL1} = \text{SL2}
    CALCULATION IF END OF BED IS IN VARIABLE SECTION & ID = 3 SC FRONT
С
      DY(NICR1) = EXP(A(JJ+1-NICR1)+B(JJ+1-NICR1)*(C-FL1+XO)+C(JJ+1-NICR)
     1)*(D-FL1+\chi_{0})**2)
      H = FL1 \neq FANT
      TERM = ABS(2.*H*RAD-H*H)
      DY(NICR1) = DY(NICR1)*(2.*FL1*TANT/RAP-(FL1*TANT/RAD)**2)
      DV(NICR1) = DV(NICR1) - DY(NICR1)
       IF (FLL-FL1-0.0001) 53,53,54
   54 DV(MICR1) = DV(MICR1) + DY(MICR1)
С
    CONSTANT CONCENTRATION PART
   52 DU 65 I=NICRI;NICR
       IP1 = I+1
      OV(IP1) = 0.0
      V(IP1) = 0.0
       IF (I-MICR1) 61,61,62
   51 SL1 = FL1
      SL2 = NICR1 * DL
      GD TO 65
   62 IF(I-NICR) 53,64,64
   64 \text{ SL1} = \text{DL}*(\text{NICR}-1)
      SL2 = FLL
      GO TC 65
   63 \text{ SL1} = \text{SL2}
       SL2 = SL2 + DL
   65 CONTINUE
      00 70 J=1,NQ
       SL = (SL2-SL1)*U(J)+0.5*(SL1+SL2)
       TEST = A(JJ+1-I) + B(JJ+1-I)*(D-SL+XO)+C(JJ+1-I)*(D-SL+XO)**2
       TEST1 = B(JJ+1-I) + 2.*C(JJ+1-I)*(D-SL+X0)
       IF (ID.EQ.1) Y(J) = TEST
       IF (ID.EQ.2) Y(J) = TEST1
       IF (ID.EQ.3) Y(J) = EXP(TEST)
       IF (ID.EQ.4) Y(J) = TEST1 \times EXP(TEST)
   70 V(I) = V(I) + (Y(J) + (SL2 - SL1))
   60 CONTINUE
      DVL = A(JJ+1-NICR) + B(JJ+1-NICR) * (D+XO-FLL) + C(JJ+1-NICR) * (D+XO-FL)
     1)**2
       IF(IC.EO.3) DVL = EXP(DVL)
       DV(NICR) = DV(NICR) - DVL
   53 \text{ VT(NPTSC)} = 0.0
       DVT(NPTSC) = 0.0
       DO 80 K=1, NICR
       DVT(NPTSC) = DVT(NPTSC) + DV(K)
   80 \text{ VT(NPTSC)} = \text{VT(NPTSC)} + \text{V(K)}
       XF(JPTSC) = X(JJ)
       XERMF = X(JJ)
       D = C + DL
   30 CENTINUE
       WRITE (6,204) (TITL(I), I=1,20)
       DO 90 I=1, NPTSC
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-0.4746 -0.3708 -0.2029 0.0 0.2029 0.3708 0.4746

0.6474E-01 0.1399 0.1909 0.2090 0.1909 0.1399 0.6474E-01

i	-3.910	-3.910	-3.910	-3.910	-3.910	-3.910
	-3.910	-3.910	-3.910	-3.910	-3.910	-3.910
	-3.910	-3:910	-4.640	-4.640	-4.640	-4.640
	-4.640	-4.540	-4.640	-4.640	-4.640	-4.640
	-4.640	-4.640	-4.640	-4.640	-4.640	-5.060
	-5.060	-5.000	-5.060	-5.060	-5.060	-5.060
	-5.060	-6.430	-6.430	-6.430	-6.430	-6.430
	-6.430	-6.430	-6.430	-6.430	-6.430	-6.430
	-6.430	-4.340	-2.050	-2.050	-2.050	-0.3500
	1.591	2.367	4.2°2	4.282	4.282	5.900
,	5.900	7.130	7.130	7.130	8.100	8.100
(	8.100	0.100	0.100	C.100 C.200	8.100	0 • 1 U) 8 • 2 0
	8-820	3.329	8.820	3.520	8 820	8.820
1	9.100	9,100	9.100	9,150	9.100	9,100
	9.100	9.100	9.100	9.100	9.100	9.100
	9.100	9.100	9.100	9.100	9.100	9.100
	9.100	9.100				
		0.0000	0 (020	0 ( 0 0	0 ( 0 0 0	<u> </u>
	0.6930	6.6930	0.6930	0.6930	0.6930	0.6930
	0.6930	0.6930	0.6930	0.6930	0.6930	0.6930
	0.6930	0.6930	1.056	1.056	1.956	1.056
	1.056	1.056	1.056	1.056	1.055	1:056
	1.056	1.056	1.056	1.056	1.056	1.177
•	1.177	1.177	1.177	1.177	1.177	1.177
	1.177	1.496	1.496	1.496	1.496	1.496
	1.496	1.496	1.496	1.496	1.496	1.496
•	1.496	1.115	0.7070	0.7070	0.7070	0.4165
	0.930CE-0	01 -0.1160	-0.3440	-0.3440	-0.3440	-0.5940
	-0.5940	-0.7780	-0.7780	-0.7780	-0.9160	-0.9160
	-0.9160	-0.9160	-0.9160 •	-0.9150	-0.9150	-0.9160
	-1.010	-1.010	+1.010	-1.010		-1.010
•	-1.610	-1.040	-1.040	-1.040	-1.040	-1.040
	-1.040	-1.040	-1.040	-1.040	-1.040	-1.040
	-1.040	-1.040	-1.040	-1.040	-1.040	-1.040
	-1.040	-1.040				
1	0.0		0 0	<b>C</b> 0	0.0	0 0
•	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0
	Q•C	0.0	0.0	0.0	0.0	C.O
	C • O	0.0	C.O	0.0	0.0	0.0
	0.0	0.0	0.0	C.O	C.C	0.0
	0.0	0.0	0.0	0.0	0.0	G • O
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
		0.0				0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	*0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	C.C	<b>0.</b> 0	6.0	0.0
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### DISTANCE VULTAGE PROFILES

2 GRAM 90 DEG INVERTED CONE, L = 3.00 (1.124 IN.)

VOLTAGE DERIVATIVE LENGTH 3.000 C.2094 0.2234 C.2335 3.100 0.2506 3.200 0.2752 0.2504 3.30% 0.3027 0.2903 0.3334 9.3236 3.401 0.3607 3.500 0.3676 3.600 0.4061 C.4028 C.4615 0.4495 3.700 0.4936 0.5209 3.300 C.5878 0.5540 3.900 4.000 0.6165 C.6632 4.100 5.7422 0.6859 0.7664 6.8439 4:200 4.300 0.8561 C.9518 1.112 4.400 0.9592 4.500 1.079 1.296 4.630 1.219 1.510 4.700 1.759 1.382 1.573 2.049 4.800 2.386 4.900 1.794 2.051 2.778 5.000 2.351 3.230 5.100 3.756 5.200 2.700 3.105 4.357 5.300 3.577 5.077 5.400 4.124 5.902 5.500 6.562 4.745 5.600 5.429 7.075 5.700 6.161 7.587 5.300 6.947 8.135 5.900 8.347 6.000 7.765 8.401 5.100 9.603 9.439 8.233 6.200 7.973 6.300 10.25 7.658 11.03 6.400 6.500 11.78 7.347 12.49 6.801 6.600 13.14 6.328 6.700 5.301 13.75 5.738 5.198 6.900 14.29 14.79 4.634 7.000 4.144 15.23 7.100 3.613 15.62 7.200 .7.300 15.95 3.106 2.585 16.24 7.400 2.069 16.47 7.500 1,551 7.500 16.65

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7.7.0	16.78	1.024
7.300	16.85	C.4807
7,900	15.87	-C.8872E-01
8.000	16.84	-C.6934
8.100	16.73	-1.424
8,200	16.55	-2.142
8.300	15.30	-2.923
8.400	15.96	-3.799
3.500	15.53	-4.739
8.5.9	15.02	-5.604
8.700	14.43	-6.208
3.300	13.73	-6.832
8.900	13.06	-7.382
9.000	12.30	-7.813
9.100	11.53	-7.753
9.200	10.74	-7.952
9.300	9.954	-7.724
9.400	9.193	-7.499
9.500	8.454	-7.225
9.500	7.752	-6.827
2.700	7.088	-6.431
9.800	6.469	-5.959
9.900	5.895	-5.521
10.00	5.364	-5.136
10.10	4.873	-4.690
10.20	4.425	-4.282
10.30	4.015	-3.910
10.40	3.642	-3.570
10.50	3.300	-3.259
10.60	2.989	-2.976
10.70	2.704	-2.717
10.82	2.445	-2.481
10.90	2.208	-2.265
11.00	1.991	-2.000

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## 2 GRAM 45 DEG INVERTED CONE, L = 3.10 (1.163 IN.)

LENGTH	VOLTACE	DERIVATIVE
3.200	0.2373	0.2190
3.300	<b>0</b> •2604	0.2442
3.400	0.2862	0.2723
3.500	0.3149	C.3035
3.600	0.3471	C•3398
3.700	0.3831	C.3819
3.800	0.4236	0.4302
3.900	0.4694	0.4354
4.000	0.5210	C.5477
4.100	0.5792	0.6181
4.200	0.6449	0.6973
4.300	0.7190	0.7866
4.436	0.8028	0.8943
4.500	0.8987	1.028
4.600	1.009	1.191
4.700	1.138	1.386
4.800	1.288	1.614
4.900	1.462	1.880
5.000	1.665	2.189
5.100	1.902	2.549
5.200	2.111	2.966
5.300	2.497	3.449
5.400 5.200	2.009	4 • UIU
5.600	2 902	4.003
5 700		5.501
5.800	5.010	6 662
5.900	5.705	7,233
6.000	6.452	7-698
6.100	7.241	8-044
6.200	8.056	8-233
6.300	8.881	8.241
6.400	9.699	8.092
6.500	10.50	7.848
6.600	11.26	7.504
6.700	12.00	7.091
6.900	12.68	6.617
6.900	13.32	6.101
7.000	13.90	5.570
7.100	14.43	5.042
7.200	14.91	4.507
7.300	15.33	3.981
7.400	15.71	3.467
7.500	16.03	2.956
7.600	16.30	2.435
7.700	16.51	. 1.917
7.300	16.68	1.396

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7.900	16.79	C.8629	
8.0C0	16.85	C.3103	
001.5	15.35	-C.2977	
3.200	15.73	-0.9421	
8.300	16.65	-1.632	
8.4UU	16.46	-2.379	
8.500	16.13	-3.195	•
2.600	15.82	-4.099	
8.700	15.36	-4.997	
2.800	14.32	-5.793	
8.900	14.21	-6.406	
9.000	13.54	-7.045	
9.100	12.31	-7.433	
9.200	12.05	-7.792	
9.300	11.26	-7.712	
9.405	10.47	-7.845	
9.500	9.696	-7.636	
9.600	8.943	-7.422	
9.700	8.215	-7.088	
9.300	7.526	-6.697	
9.900	6.876	-6.209	
10.00	6.273	-5.908	
10.10	5.713	-5.381	
10.20	5.195	-4.983	
10.30	4.719	-4.549	
10.40	4.284	-4.154	
10.50	3.887	-3.793	
16.60	3.524	-3.463	
10.70	3.193	-3.162	
10.80	2.891	-2.887	
10.90	2.615	-2.536	
11.00	2.363	-2.406	

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## DISTANCE VOLTAGE PROFILES

2 GRAM 30 DEG INVERTED CONE, L = 3.18 (1.191 IN.)

LENGTH	VOLTAGE	DERIVATIVE
3.300	0.2357	C.2172
3.400	0.2586	0.2422
3.500	C.2342	C.2700
3.600	0.3128	C.3019
3.700	0.3447	0.3385
3.800	0.3806	0.3804
3.900	0.4210	C.4281
4.000	0.4665	6.4924
4.100	0.5177	0.5439
4.200	0.5755	C.6135
4.300	0.6407	0.6921
4.400	0.7144	0.7349
4.500	0.7984	C.8977
4.600	0.8947	1.033
4.700	1.006	1.194
4.800	1.135	1.385
4.900	1.284	1.610
5.000	1.458	1.873
5.100	1.660	2.181
5.200	1.896	2.539
5.300	2.170	2.955
5.400	2.489	3.436
5.500	2.860	3.995
5.600	3.290	4.610
5.700	3.782	5.243
5.800	4.338	5.860
5.960	4.954	6.464
5.000	5.628	7.007
6.100	6.353	7.464
6.200	7.117	7.796
6.300	7.907	7.982
6.400	8.708	8.025
6.500	9.508	7.954
6.600	10.29	7.763
6.700	11.06	7.474
6.800	11.79	7.099
6.900	12.48	6.657
7.000	13.12	6.178
7.100	13.71	5.679
7.200	14.25	5.161
7.300	14.74	4.637
7.400	15.18	4.117
7.500	15.57	3.608
7.600	15.90	3.105
7.700	16.19	2.583
7.800	16.42	. 2.062
7.900	16.60	1.537

<b>8.000</b>	16.73	1.002
8.100	16.80	C.4331
8.200	16.81	-C.1628
3.300	16.75	-0.7945
8.400	16.65	-1.473
8.500	16.47	-2.208
3.600	16.21	-3.015
8.700	15.36	-3.908
3.800	15.42	-4.834
8.900	14.90	-5.669
9.000	14.30	-6.280
9.100	13.54	-6.866
9.200	12.93	-7.382
9.300	12.17	-7.778
9.400	11.38	-7.919
9.500	10.59	-7.874
9.600	9.813	-7.653
9.700	9.058	-7.436
9.800	8.326	-7.140
9.900	7.632	-6.752
10.00	6.975	-1.343
10.10	6.365	-5.877
10.20	5.799	-5.445
10.30	5.275	-5.054
10.40	4.792	-4.515
10.50	4.351	-4.214
10.50	3.948	-3.948
10.70	3.580	-3.513
10.80	3.244	-3.208
10.90	2.938	-2.927
11.00	2.658	-2.674

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## DISTANCE VULTAGE PROFILES

## 2 GRAM 15 DEG INVERTED CONE, L = 3.38 (1.268 IN.)

LENGTH	VOLTAGE	DERIVATIVE
2 7.00		
2 000	0.2020	
3.000	0.2970	0 2210
2 · 3 · 10	0.3616	0.3606
4.UC.	5 2004	
4.100	0 × 2930	0 6 5 4 2
4.200	0 4011	0.4000 C 51.27
4.200	0.5457	
4.400 7.500	0.5437	0 4401
4.300	6 4722	0.5501
4.500		0 9454
4 800	0.8520	0 9965
4.000 4.000	0.9531	1 150
5 000	1 093	1 221
5 100	1 226	1 540
5 200	1 302	1 789
5 300	1 595	2 076
5.400	1 200	2 • 0 / 0
5 500	2 070	2 804
5 600	2.372	2 2 2 4 3
5 700	2 719	3 713
5.800	3,115	4,198
5,900	3,559	4.699
5.000	4 . 054	E. 194
6.100	4.597	5,666
6.200	5.186	6,092
6.300	5.814	6.454
6.400	6.474	6.742
6.500	7.160	6.962
6.600	7.854	7.100
6.700	8.577	7.160
5.800	9.293	7.138
6.900	10.00	7.040
7.000	10.70	6.874
7.100	11.38	6.646
7.200	12.03	6.356
7.300	12.54	6.016
7.400	13.23	5.638
7.500	13.77	5.229
7.600	14.27	4.799
7.700	14.73	4.353
7.300	15.14	3.898
7.900	15.51	3.438
3.000	15.83	2.951
8.100	16.10	2.440
8.200	16.32	1.917
3.300	16.43	1.379

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0.405	1/ 50	C 0211	
8.491	16.54		
8.500	10.65	L.2382	
2.600	16.64	-0.3776	
8.700	16.57	-1.936	
3.300	16.43	-1.747	
3.900	16.22	2.523	
1.000	15.92	-3.380	
9.100	15.54	-4.308	
9.200	15.06	-5.166	
7.300	14.51	-5.890	
9.400	13.89	-6.524	
9.500	13.20	-7.186	
9.600	12.46	-7.533	
9.700	11.69	-7.832	
9.800	10.90	-7.873	
9.900	10.12	-7.721	
10.00	9.359	-7.501	
15.10	8.620	-7.283	
10.20	7.911	-6+890	
10.30	7.241	-0.512	
10.40	6.612	-6.053	
10.50	6.029	-5.610	
10-60	5.489	-5.199	
10.70	4.989	-4.789	
10.80	4.531	-4.374	
10-90	4.113	-3,995	
11.00	3,731	-3,648	
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## CISTANCE VOLTAGE PROFILES

## 2'GRAM 10 DEG INVERTED CONE, L = 3.58 (1.343 IN.)

LENGTH	VOLTAGE	DERIVATIVE
4.000	0.2958	C.2867
4.100	0.3262	0.3219
4.200	0.3603	0.3616
4.300	0.3987	C.4065
4.439	0.4418	C.4585
4.500	C.4907	0.5201
4.600	0.5462	C+5925
4.700	0.6096	0.6775
4.800	0.6822	0.1115
4.900	0.7657	0.8944
5.000	0.8618	1.031
5.100	0.9727	1.191
5.200	1.101	1.378
5.300	1.249	1.595
<b>5.4</b> 00	1.421	1.850
5.500	1.621	2.146
5.666	1.852	2.480
5.700	2.118	2.543
5.800	2.421	3.223
5.900	2.153	3.024
6.000	3.146	4.021
6.100	3.768	4.423
6.200	4.029	4.191
6.300		2.130 ·
		J • 4 3 2 5 4 0 2
	2.012	2+092 . 5 004
6 700	0 • 1 72 6 701	5.904
6 900	7 404	6 176
	8 025	6 236
7 000	0 • UZJ 8   KAQ	6 249
7 100	0 273	6 217
7.200	9.891	6.141
7.300	10-50	6.024
7.400	11.09	5.869
7.500	11.67	5.678
7 600	12.23	5.453
7.700	12.76	5,199
7.800	13.27	4-918
7,900	13.74	4,612.
8.000	14.19	4,294
8.100	14.60	3.930
8.200	14.97	3,556
8,300	15.31	3,151
8.400	15.60	2.704
8,500	15.85	2,232
8.600	16.05	1.734

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8.700	16.20	1.205
5.800	16.29	0.6437
8.900	16.32	0.4352E-01
9.000	16.30	-C.6020
9.100	16.20	-1.285
9.200	16.04	-2.050
9.300	15.79	-2.892
9.400	15.45	-3.827
9.500	15.02	-4.741
9.600	14.51	-5.546
9.700	13.92	-6.185
9.800	13.27	-6.849
9.900	12.56	-7.327
10.00	11.80	-7.669
10.10	11.03	-7.776
10.20	10.25	-7.692
10.30	9.495	-7.480
10.40	8.757	-7.271
10.50	8.044	-6.936
10.60	7.370	-6.558
10.70	6.734	-6.133
10.80	6.143	-5.684
10.90	5.596	-5.268
11.00	5.088	-4.875

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The data collected and values computed in this thesis were primarily given to illustrate a technique for measuring magnetic concentration profiles. Not enough data is currently available for a comprehensive error analysis. Since a new experimental technique should have some prediction of the validity of the determinations, a rough analysis of the maximum error has been made from available results.

As developed previously, the zone width Zw for the concentration to change from zero to a maximum is a function of the peak voltage obtained from the long coil (L) and the maximum in the short coil derivative signal Dm. That is:

$$Zw = Zw(L,Dm)$$
(35)

By the method of propagation of errors, the error in the zone width determination  $\Delta(2w)$  is a function of the errors in effective length and in maximum short coil derivative,  $\Delta L$  and  $\Delta(Dm)$  respectively, and the relations between these functions and the zone width. Considering only first order errors:

$$\Delta(Zw) = \frac{\partial Zw}{\partial L} \Delta L + \frac{\partial Zw}{\partial Dm} \Delta Dm \qquad (36)$$

Choosing a particular set of values as shown below for a sliced cylinder bed:

the partial derivatives were determined. Since complete relations were not available, only an estimate of the maximum was made. Using figure '18', the following were determined:

$$\frac{Z_{W}}{L} \approx \frac{1}{.3}, \qquad \frac{Z_{W}}{L} \approx \frac{1}{.05}$$

An analysis of the values for L and Dm showed that each could probably be obtained within two percent of the correct value. Again from figure '18',

 $\Delta$  L  $\approx$  (.02) (3)  $\approx$  .06 diameters

 $4 \text{ Dm} \approx (.02) (.8) \approx .016$ 

Thus, for this zone width and effective length sliced cylinder bed, the error can be determined from equation '36':

$$\Delta Zw = (\frac{1}{.3})(.06) + (\frac{1}{.05})(.016) \approx .4$$
 diameters

The difficulty in obtaining the partial derivatives illustrates the need for more results. The error above should be a maximum and actual errors may be much smaller. Most experimental values used in this thesis are given in Chapter Four. In order to illustrate that experimental curves are similar in magnitude and shape to predicted ones, actual data is shown on the following pages. Shown are a few comparisons when a particular study was made at constant recorder and voltmeter settings. Most data was attenuated for maximum signal and is not readily comparible.

Figure '20' illustrates the independence of the long coil peak voltage on the bed front for the inverted cone beds of constant volume. Similar deviations were observed for sliced cylinder beds.

Figure '21' shows the voltage derivative profiles for inverted cone beds. Recall that the tube for the 90° front was found to have radial imperfections. The distance between curves was for better illustration and has no real meaning. The final zero position should be approximately at the same point for all beds. Due to the "noise" associated with differentiating, generally some filtering of the signal was used. The curves given in figure '22' are more representative of the actual data used. The repeatibility of a single measurement is illustrated in figure '23'. ER SECKMAN PARE NO 101293

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Since concentration profiles were simulated by radial material variations, axial g values require that the variation of g for a particular axial position must be small. To verify that this was the case, radial g profiles were determined by measuring the derivative signal for a 1/16 inch i.d. bed at various radial positions. Measurements were made of g at the nine locations shown below:





Averaging radial effects at a particular radius, the g function was mapped both axially and radially. The results are shown in figure '25'. The "axial magnetic center" was assigned to the location of maximum g. The positive axial locations are positions toward the center of the primary coil. Beds enter from the negative side.

Applying corrections for radial variations resulted in changes in the derivative curves of magnitude much smaller than the detection limits for the experimental equipment.



Figure 25. Radial Values of g

### NOMENCLATURE

SYMBOL	QUANTITY	DIMENSIONS	UNITS USED IN TEXT
	Latin Symbols		
В	Magnetic induction	MT <sup>'</sup> Q <sup>-'</sup>	
d	Diameter	L	inches
D	Molecular diffusivity	L <sup>2</sup> T <sup>-'</sup>	
D*	Effective coefficient of longitud- anal dispersion	L <sup>2</sup> T <sup>-'</sup>	
H	Magnetic field strength	'L'T'Q	
I·	Primary coil current	T <sup>´</sup> Q	amperes
k,kį	Rate constants (eq'ns 628)	ML <sup>3</sup> T	
K	Mass transfer coefficient	ML <sup>-2</sup> T-'	
	Length of bed inside coil	L	inches
L	Effective bed length	L	inches
Lo	Initial bed length	L	inches
М	Magnetization of sample	LTQ	
n	Gas phase concentration of adsor- bate per volume of adsorbent	ML <sup>-3</sup>	
no	Initial value of n	ML <sup>3</sup>	
N	Solid phase concentration of adsor- bate per volume of adsorbent	ML <sup>3</sup>	
N*	Equilibrium value of N relative to n	МГ <sup>3</sup>	
Q	Rate of adsorption	ML <sup>3</sup> T <sup>-/</sup>	
r	Radius	L	inches
S	Surface area per unit volume	L <sup>-/</sup>	

t	Time since introduction of adsorbate	Т	
ts	Stoichiometric time for equilibrium adsorption	T	
u	Magnetic permeability	MLQ <sup>3</sup>	
uo	Reference magnetic permeability	MLQ	
U	Linear velocity of adsorbate in bed	LT <sup>-'</sup>	
V	Volume of bed	L <sup>3</sup>	cu. in.
V,	Primary applied voltage	ML <sup>2</sup> T <sup>-2</sup> Q <sup>-'</sup>	Volts
V2	Secondary induced voltage	ML <sup>2</sup> T <sup>-2</sup> Q <sup>-/</sup>	Volts
x	Position coordinate	L	inches

# Greek Symbols

S.	Gradient concentration length	L	
Pg	Gas phase density	ML <sup>3</sup>	
θ	Angle	a	degrees
$\tilde{arPsi}$	Halfwave length of simusiodal	L	Π radians
$ar{\Phi}$	Magnetic flux	ML <sup>2</sup> T <sup>'</sup> Q <sup>'</sup>	
$\Psi$	Solenoid length	L	

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#### Dimensionless Quantities

- A Number of transfer units
- B' Time dependent variable in Bohart and Adams equation
- C Concentration of magnetic material relative to some standard
- C<sub>o</sub> Initial value of C
- f Filling factor, fraction of volume filled
- k' Equilibrium constant (N/n)
- N Number of turns
- χ Magnetic susceptibility
- ø Bed porosity