# Coalitional Games with Overlapping Coalitions for Interference Management in Small Cell Networks

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### Abstract

In this paper, we study the problem of cooperative interference management in an OFDMA two-tier small cell network. In particular, we propose a novel approach for allowing the small cells to cooperate, so as to optimize their sum-rate, while cooperatively satisfying their maximum transmit power constraints. Unlike existing work which assumes that only disjoint groups of cooperative small cells can emerge, we formulate the small cells' cooperation problem as a *coalition formation game with overlapping coalitions*. In this game, each small cell base station can choose to participate in one or more cooperative groups (or coalitions) simultaneously, so as to optimize the tradeoff between the benefits and costs associated with cooperation. We study the properties of the proposed overlapping coalition formation game and we show that it exhibits negative externalities due to interference. Then, we propose a novel decentralized algorithm that allows the small cell base stations to interact and self-organize into a stable overlapping coalitional structure. Simulation results show that the proposed algorithm results in a notable performance advantage in terms of the total system sum-rate, relative to the noncooperative case and the classical algorithms for coalitional games with non-overlapping coalitions.

### **Index Terms**

small cell networks, game theory, interference management, cooperative games

# I. INTRODUCTION

Small cell networks are seen as one of the most promising solutions for boosting the capacity and coverage of wireless networks. The basic idea of small cell networks is to deploy small cells, that are serviced by plug-and-play, low-cost, low-power small cell base stations (SBSs) able to connect to existing backhaul technologies (e.g., digital subscription line (DSL), cable modem, or a wireless backhaul) [1]. Types of small cells include operator-deployed picocells as well as femtocells that can be installed by end-users at home or at the office. Recently, small cell networks have received significant attention from a number of standardization bodies including 3GPP [1]–[7]. The deployment of SBSs is expected to deliver high capacity wireless access and enable new services for the mobile users while reducing the cost of deployment on the operators. Moreover, small cell networks are seen as a key enabler for offloading data traffic from the main, macro-cellular network [9].

The successful introduction of small cell networks is contingent on meeting several key technical challenges, particularly, in terms of efficient interference management and distributed resource allocation [8]-[16]. For instance, underlying SBSs over the existing macro-cellular networks leads to both cross-tier interference between the macrocell base stations and the SBSs and cotier interference between small cells. If not properly managed, this increased interference can consequently affect the overall capacity of the two-tier network. There are two types of spectrum allocation for the network operator to select. The first type is orthogonal spectrum allocation, in which the spectrum in the network is shared in an orthogonal way between the macrocell and the small cell tiers [12]. Although cross-tier interference can be totally eliminated using orthogonal spectrum allocation, the associated spectrum utilization is often inefficient [9]. The second type is co-channel assignment, in which both the macrocell and the small cell tiers share the same spectrum [13]. As the spectrum in the network is reused through co-channel assignment, the spectrum efficiency can be improved compared to the case of orthogonal spectrum allocation. However, both cross-tier interference and co-tier interference should be considered in this case. A lot of recent work has studied the problem of distributed resource allocation and interference management in small cells. These existing approaches include power control [17]–[20], fractional frequency reuse [21] [22], interference alignment [23], interference coordination [24], the use of cognitive base-stations [25] [26], and interference cancelation [27]–[30]. In [28], the authors use interference cancelation in poisson networks with arbitrary fading distribution to decode the k-th strongest user. The authors in [29] investigated the performance of successive interference cancelation for uplink cellular communications. The successive interference cancelation method is also adopted in [30] to calculate the aggregate throughput in random wireless networks. A distributed algorithm is proposed for minimizing the overall transmit power in two-tier networks in [31]. The problem of joint throughput optimization, spectrum allocation and access control in two-tier femtocell networks is investigated in [32]. In [33], the authors proposed an auction algorithm for solving the problem of subcarrier allocation between macrocell users and femtocell users. In [34] and [35], the authors develop two Stackelberg game-based formulations for studying the problem of optimizing the performance of both macrocells and femtocells while maintaining a maximum tolerable interference constraint at the macrocell tier. The potential of spatial multiplexing in noncooperative two-tier networks is studied in [36].

Most existing works have focused on distributed interference management schemes in which the SBSs act noncooperatively. In such a noncooperative case, each SBS accounts only for its own quality of service while ignoring the co-tier interference it generates at other SBSs. Here, the co-tier interference between small cells becomes a serious problem that can significantly reduce the system throughput, particularly in outdoor picocell deployments. To overcome this issue, we propose to enable cooperation between SBSs so as to perform cooperative interference management. The idea of cooperation in small cell networks has only been studied in a limited number of existing work [37]–[41]. In [37], the authors propose a cooperative resource allocation algorithm on intercell fairness in OFDMA femtocell networks. In [38], an opportunistic cooperation approach that allows femtocell users and macrocell users to cooperate is investigated. In [39], the authors introduce a game-theoretic approach to deal with the resource allocation problem of the femtocell users. In [40], a collaborative inter-site carrier aggregation mechanism is proposed to improve spectrum efficiency in a LTE-Advanced heterogeneous network with orthogonal spectrum allocation between the macrocell and the small cell tiers. The work in [41] propose a cooperative model for femtocell spectrum sharing using a cooperative game with transferable utility in partition form [42]. However, the authors assume that the formed coalitions are disjoint and not allowed to overlap, which implies that each SBS can only join one coalition at most. This restriction on the cooperative abilities of the SBSs limits the rate gains from cooperation that can be achieved by the SBSs. Moreover, the authors in [41] adopt the approach of orthogonal spectrum allocation that is inefficient on spectrum occupation for the two-tier small cell networks.

The main contribution of this paper is to develop a novel cooperative interference management model for small cell networks in which the SBSs are able to participate and cooperate with multiple coalitions depending on the associated benefit-cost tradeoff. We adopt the approach of co-channel assignment that improves the spectrum efficiency compared to the approach of orthogonal spectrum allocation used in [41]. We formulate the SBSs' cooperation problem as an overlapping coalitional game and we propose a distributed, self-organizing algorithm for performing overlapping coalition formation. Using the proposed algorithm, the SBSs can interact and individually decide on which coalitions to participate in and on how much resources to use for cooperation. We show that, as opposed to existing coalitional game models that assume disjoint coalitions, our proposed approach enables a higher flexibility in cooperation. We study the properties of the proposed algorithm, and we show that it enables the SBSs to cooperate and selforganizing into the most beneficial and stable coalitional structure with overlapping coalitions. To our best knowledge, this is the first work that studies overlapping coalition formation for small cell networks. Simulation results show that the proposed approach yields performance gains relative to both the noncooperative case and the classical case of coalitional games with non-overlapping coalitions such as in [41].

The rest of the paper is organized as follows: In Section II, we present and motivate the proposed system model. In Section III, the SBSs' cooperation problem is formulated as an overlapping coalition formation game and a distributed algorithm for overlapping coalition formation is introduced. Simulation results are presented and analyzed in Section IV. Consequently, conclusions are drawn in Section V.

### II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider the downlink transmission of an Orthogonal Frequency Division Multiple Access (OFDMA) small cell network composed of N SBSs and a macro-cellular network having a single macro base station (MBS). The access method of all small cells and the macrocell is closed access. Let  $\mathcal{N}=\{1,...,N\}$  denote the set of all SBSs in the network. The MBS serves W macrocell user equipments (MUEs), and each SBS  $i\in\mathcal{N}$  serves  $L_i$  small cell user equipments

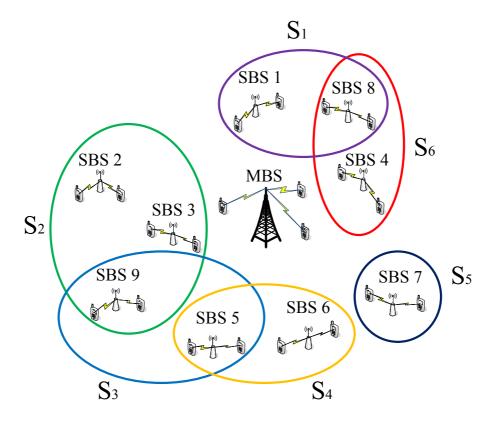


Fig. 1. An illustrative example of the proposed cooperative model in small cell networks.

(SUEs). Let  $\mathcal{L}_i = \{1, ..., L_i\}$  denote the set of SUEs served by an SBS  $i \in \mathcal{N}$ . Here, SBSs are connected with each other via a wireless backhaul. Each SBS  $i \in \mathcal{N}$  chooses a subchannel set  $\mathcal{T}_i$  containing  $|\mathcal{T}_i| = M$  orthogonal frequency subchannels from a total set of subchannels  $\mathcal{T}$  in a frequency division duplexing (FDD) access mode. The subchannel set  $\mathcal{T}_i$  serves as the initial frequency resource of SBS  $i \in \mathcal{N}$ . The MBS also transmits its signal on the subchannel set  $\mathcal{T}$ , thus causing cross-tier interference from MBS to the SUEs served by the SBSs. Moreover, the SBSs are deployed in hot spot indoor large areas such as enterprises where there are no walls not only between each SBS and its associated SUEs, but also between all the SBSs. Meanwhile, the MBS is located outdoor, so there exist walls between the MBS and the SBSs.

In the traditional noncooperative scenario, each SBS  $i \in \mathcal{N}$  transmits on its own subchannels. The set of the subchannels that SBS i owns is denoted as  $\mathcal{T}_i$ , where  $\mathcal{T}_i \subseteq \mathcal{T}$ . SBS i occupies the whole time duration of any subchannel  $k \in \mathcal{T}_i$ . Meanwhile, the MBS transmits its signal to the MUEs on several subchannels from  $\mathcal{T}$ , with each MUE occupying one subchannel at each time

slot. When the SBSs act noncooperatively, each SBS uses all the subchannels from  $\mathcal{T}_i$  to serve its SUEs  $\mathcal{L}_i$ . For each subchannel  $k \in \mathcal{T}_i$ , only one SUE  $u \in \mathcal{L}_i$  is served on subchannel k. SUE u has access to the full time duration of subchannel k. We denote the channel gain between transmitter j and the receiver u that owns subchannel k in SBS i by  $g_{j,i_u}^k$  and the downlink transmit power from transmitter j and the receiver u that occupies subchannel k in SBS i by  $P_{j,i_u}^k$ . The rate of SBS  $i \in \mathcal{N}$  in the noncooperative case is thus given by

$$\Upsilon_i = \sum_{k \in \mathcal{T}_i} \sum_{u \in \mathcal{L}_i} \log_2 \left( 1 + \frac{P_{i,i_u}^k g_{i,i_u}^k}{\sigma^2 + I_{MN} + I_{SN}} \right), \tag{1}$$

where  $\sigma^2$  represents the variance of the Gaussian noise,  $I_{MN} = P_{w,i_u}^k g_{w,i_u}^k$  is the cross-tier interference from the MBS w to a SUE served by SBS i on subchannel k, and  $I_{SN}$  denotes the overall co-tier interference suffered by SUE u that is served by SBS i on subchannel k, as

$$I_{SN} = \sum_{j \in \mathcal{N}, j \neq i} P_{j, i_u}^k g_{j, i_u}^k.$$
 (2)

We note that, in dense small cell deployments, the co-tier interference between small cells can be extremely severe which can significantly reduce the rates achieved by the SBSs [1]. Nevertheless, due to the wall loss and the long distance between MBS and SUEs, the downlink cross-tier interference is rather weak compared to the co-tier interference between small cells. Thus, in this work, we mainly deal with the downlink co-tier interference suffered by the SUEs from the neighboring SBSs. In order to deal with this interference problem, we propose a novel model in which the SBSs are allowed to cooperate with one another as illustrated in Fig. 1. In such a cooperative network, the SBSs can cooperate to improve their performance and reduce co-tier interference.

In particular, depending on the signal to noise and interference ratio (SINR) feedbacks from their SUEs, the SBSs can decide to form cooperative groups called *coalitions* so as to mitigate the co-tier interference between neighboring SBSs within a coalition. The SBSs can be modeled as the players in a coalitional game. Due to the possibility of having an SBS participating in multiple coalitions simultaneously as shown in Fig. 1, we state the following definition for a coalition [47]:

**Definition 1.** A partial coalition is given by a vector  $\mathcal{R} = (\mathbf{R}_1, ..., \mathbf{R}_N)$ , where  $\mathbf{R}_i$  is the subset of player i's resource set distributed to this coalition. The *support* of a partial coalition  $\mathcal{R}$  is defined as  $\text{Supp}(\mathcal{R}) = \{i \in \mathcal{N} | \mathbf{R}_i \neq \emptyset\}$ .

In what follows, we will omit the word "partial", and refer to partial coalitions as coalitions. The SBSs in the network act as players. After joining a coalition  $\mathcal{R}$ , SBS  $i \in \text{Supp}(\mathcal{R})$  allocates part of its frequency resource into this coalition  $\mathcal{R}$ . Within each coalition  $\mathcal{R}$ , the SBSs can jointly coordinate their transmission so as to avoid the collisions. The resource pool of coalition  $\mathcal{R}$  can be defined as follows:

$$\mathcal{T}_{\mathcal{R}} = \bigcup_{i \in \text{Supp}(\mathcal{R})} \mathbf{R}_i, \tag{3}$$

where  $\mathbf{R}_i$  denotes the subset of the frequency resources in terms of orthogonal frequency subchannels that SBS  $i \in \text{Supp}(\mathcal{R})$  dedicates to the resource pool of coalition  $\mathcal{R}$  and satisfies that  $\mathbf{R}_i \subseteq \mathcal{T}_i$ . Here, we assume that each SBS i will devote all its frequency resources to different coalitions that it decides to join in the network, i.e.,

$$\bigcup_{\{\mathcal{R}|i\in \text{Supp}(\mathcal{R})\}} \mathbf{R}_i = \mathcal{T}_i. \tag{4}$$

Note that given SBS i, for any two coalitions  $\mathcal{R}^t$  and  $\mathcal{R}^k$  in the network that satisfy  $i \in \text{Supp}(\mathcal{R}^t)$  and  $i \in \text{Supp}(\mathcal{R}^k)$ , we have  $\mathbf{R}_i^t \cap \mathbf{R}_i^k = \emptyset$ .

Without loss of generality, we consider that, whenever a coalition  $\mathcal{R}$  successfully forms, the transmissions inside  $\mathcal{R}$  will be managed by a local scheduler using the time division multiple access (TDMA) approach as in [44]. The subchannels in  $\mathcal{T}_{\mathcal{R}}$  are divided into several timeslots. Each SBS can access only a fraction of all the time-slots when transmitting on a specific subchannel. By doing so, the whole superframe duration of each subchannel can be shared by more than one SBS. Hence, the downlink transmissions from each SBS in the coalition to its SUEs are separated. Consequently, no more than one SBS will be using the same subchannel on the same time-slot within a coalition, thus efficiently mitigating the interference inside the coalition  $\mathcal{R}$ . However, as the resource pools of different coalitions may not be disjoint, the system can still suffer from inter-coalition interference. Here, we note that the proposed approach is still applicable under any other coalition-level interference mitigation scheme.

While cooperation can lead to significant performance benefits, it is also often accompanied by inherent coordination costs. In particular, for the proposed SBS cooperation model, we capture the cost of forming coalitions via the amount of transmit power needed to exchange information. In each coalition  $\mathcal{R}$ , each SBS  $i \in \operatorname{Supp}(\mathcal{R})$  broadcasts its data to the other SBSs in the coalition in order to exchange information. Here, each SBS needs to transmit the information to the farthest SBS in the same coalition. We assume that, during information exchange, no transmission errors occur. So the power cost incurred for forming a coalition  $\mathcal{R}$  is given by:

$$P_{\mathcal{R}} = \sum_{i \in \text{Supp}(\mathcal{R})} P_{i,j^*},\tag{5}$$

where  $P_{i,j^*}$  is the power spent by SBS i to broadcast the information to the farthest SBS  $j^*$  in a coalition  $\mathcal{R}$ . Meanwhile, for every coalition  $\mathcal{R}$ , we define a maximum tolerable power cost  $P_{\text{lim}}$ .

### III. SMALL CELL COOPERATION AS AN OVERLAPPING COALITIONAL GAME

In this section, we will develop an overlapping coalition formation (OCF) game model to solve the problem of co-tier interference management in two-tier small cell networks. OCF games have been recently introduced in the game theory literature [45]–[48]. OCF games have been applied in cognitive radio networks and smartphone sensing [49] [50]. The goal is to leverage cooperation for maximizing the system performance in terms of sum-rate while taking into account the cooperation costs. A distributed OCF algorithm is proposed so as to solve the developed game. First, we will introduce some basic definitions of OCF games in order to provide the basis of our problem solving framework.

# A. SBS Overlapping Coalitional Game Formulation

We can model the cooperation problem in small cell network as an OCF game with transferable utility in which SBSs in  $\mathcal{N}$  act as *players*. To model the OCF game, we assume that each SBS treats the subchannels it possesses as the resources that it can distribute among the coalitions it joins. Here, we make the following definitions [47]:

**Definition 2.** A discrete OCF game  $G = (\mathcal{N}, v)$  with a transferable utility (TU) is defined by a set of players  $\mathcal{N}$  and a value function v assigning a real value to each coalition  $\mathcal{R}$ .

We also note that  $v(\emptyset) = 0$ . Each player  $i \in \mathcal{N}$  can join multiple coalitions. In the following, we will omit the word "discrete", and refer to discrete OCF games as OCF games.

**Definition 3.** An overlapping coalitional structure over  $\mathcal{N}$ , denoted as  $\mathcal{CS}$ , is defined as a set  $\mathcal{CS} = (\mathcal{R}^1, ..., \mathcal{R}^l)$  where l is the number of coalitions and  $Supp(\mathcal{R}^t) \subseteq \mathcal{N}$ ,  $t \in \{1, ..., l\}$ .

In the proposed OCF game  $G = (\mathcal{N}, v)$ , the players, i.e., the SBSs  $\mathcal{N} = \{1, ..., N\}$  can choose to cooperate by forming coalitions and sharing their frequency resources. Here, we consider, without loss of generality, that the resource pool of a coalition  $\mathcal{R}$  is divided among the SBSs in  $\mathcal{R}$  using a popular criterion named *proportional fairness*, i.e., each SBS  $i \in \text{Supp}(\mathcal{R})$  gets an share  $f_i \in [0,1]$  of the frequency resources from the coalition  $\mathcal{R}$  through the TDMA scheduling process of the proposed local scheduler, and the share satisfies that

$$\sum_{i \in \text{Supp}(\mathcal{R})} f_i = 1 \tag{6}$$

and

$$\frac{f_i}{f_j} = \frac{|\mathbf{R}_i|}{|\mathbf{R}_j|},\tag{7}$$

where  $|\mathbf{R}_i|$  denotes the number of subchannels in  $\mathbf{R}_i$ .

The proportional fairness criterion guarantees that the SBSs that dedicate more of its own frequency resources, i.e., subchannels to the coalition deserve more frequency resources back from the resource pool of the coalition. Furthermore, due to the TDMA process within each coalition, interference inside single coalition can be neglected, and, thus, we focus on mitigating the inter-coalition interference.

In our model, the inter-coalition interference leads to negative externalities, implying that the performance of the players in one coalition is affected by the other coalitions in the network. Therefore, the utility  $U(\mathcal{R}, \mathcal{CS})$  of any coalition  $\mathcal{R} \in \mathcal{CS}$ , which corresponds to the sum-rate achieved by  $\mathcal{R}$ , will be dependent on not only the members of  $\mathcal{R}$  but also the coalitional structure  $\mathcal{CS}$  due to inter-coalition interference as follows:

$$U(\mathcal{R}, \mathcal{CS}) = \sum_{i \in \text{Supp}(\mathcal{R})} \sum_{k \in \mathcal{T}_{\mathcal{R}}} \sum_{u \in \mathcal{L}_i} \gamma_{i, i_u}^k \log_2 \left( 1 + \frac{P_{i, i_u}^k g_{i, i_u}^k}{\sigma^2 + I_{MO} + I_{SO}} \right), \tag{8}$$

where  $\gamma_{i,i_u}^k$  denotes the fraction of the time duration during which SBS i transmits on channel k to serve SUE u,  $P_{i,i_u}^k$  indicates the transmit power from SBS i to its own SUE u on subchannel

 $k, g_{i,i_u}^k$  is the according channel gain,  $I_{MO} = P_{w,i_u}^k g_{w,i_u}^k$  denotes the cross-tier interference from the MBS w to SUE u served by SBS i on subchannel k and  $\sigma^2$  represents the noise power. Moreover, in (8), the term  $I_{SO}$  denotes the overall co-tier interference suffered by SUE u that is served by SBS i on subchannel k and is defined as follows:

$$I_{SO} = \sum_{\mathcal{R}' \in \mathcal{CS} \setminus \mathcal{R}} \sum_{j \in Supp(\mathcal{R}'), j \neq i} P_{j,i_u}^k g_{j,i_u}^k, \tag{9}$$

where  $P_{j,i_u}^k$  and  $g_{j,i_u}^k$  denote, respectively, the downlink transmit power and the channel gain from SBS  $j \in \text{Supp}(\mathcal{R}')$  to the considered SUE u served by SBS i on subchannel k.

Given the power cost of any coalition  $\mathcal{R} \in \mathcal{CS}$  defined in (5), the value of coalition  $\mathcal{R}$  can be defined as follows:

$$v(\mathcal{R}, \mathcal{CS}) = \begin{cases} U(\mathcal{R}, \mathcal{CS}), & \text{if } P_{\mathcal{R}} \leq P_{\lim}, \\ 0, & \text{otherwise.} \end{cases}$$
 (10)

As the utility in (10) represents a sum-rate, then the proposed OCF game has a transferable utility (TU), since the sum-rate can be appropriately apportioned between the coalition members (i.e., via a proper choice of a coding strategy).

Furthermore, we can define the payoff of an SBS  $i \in \text{Supp}(\mathcal{R})$  as follows

$$x^{i}(\mathcal{R}, \mathcal{CS}) = f_{i}v(\mathcal{R}, \mathcal{CS}), \tag{11}$$

where  $f_i$  is the fraction of the frequency resource that SBS  $i \in \text{Supp}(\mathcal{R})$  gets from coalition  $\mathcal{R}$ . Note that, if SBS  $i \notin \text{Supp}(\mathcal{R})$ , then we have  $x^i(\mathcal{R}, \mathcal{CS}) = 0$ .

Suppose there are l coalitions in the coalitional structure  $\mathcal{CS}$ . Thus, we have  $\mathcal{CS} = (\mathcal{R}^1, ..., \mathcal{R}^l)$ . An imputation for  $\mathcal{CS}$  is defined as  $\mathbf{x} = (\mathbf{x}_1, ..., \mathbf{x}_l)$ , where  $\mathbf{x}_j = (x^1(\mathcal{R}^j, \mathcal{CS}), ..., x^N(\mathcal{R}^j, \mathcal{CS}))$ . Moreover, an outcome of the game is denoted as  $(\mathcal{CS}, \mathbf{x})$  with  $\mathcal{CS}$  as the coalitional structure and  $\mathbf{x}$  as the imputation.

Thus, the total payoff  $p^i(\mathcal{CS})$  received by SBS i from the coalitional structure  $\mathcal{CS}$  is calculated as the sum of the payoffs of SBS i from all the coalitions it is currently participating in, which is given by:

$$p^{i}(\mathcal{CS}) = \sum_{j=1}^{l} x^{i}(\mathcal{R}^{j}, \mathcal{CS}), \tag{12}$$

where  $CS = (R^1, ..., R^l)$ .

Consequently, in the considered OCF game, the value of the coalition structure  $\mathcal{CS} = (\mathcal{R}^1, ..., \mathcal{R}^l)$  can be defined as follows

$$v(\mathcal{CS}) = \sum_{j=1}^{l} v(\mathcal{R}^{j}, \mathcal{CS}). \tag{13}$$

Note that  $v(\mathcal{CS})$  is also the system payoff.

# B. Properties of the Proposed Small Cells' OCF Game

**Definition 4.** An OCF game  $(\mathcal{N}, v)$  with a transferable utility (TU) is said to be *superadditive* if for any two coalitions  $\mathcal{R}^1, \mathcal{R}^2 \in \mathcal{CS}$ ,  $v\left(\mathcal{R}^1 \cup \mathcal{R}^2, \mathcal{CS}'\right) \geq v\left(\mathcal{R}^1, \mathcal{CS}\right) + v\left(\mathcal{R}^2, \mathcal{CS}\right)$  with  $\mathcal{R}^1 \cup \mathcal{R}^2 \in \mathcal{CS}'$ .

**Theorem 1.** The proposed OCF game  $(\mathcal{N}, v)$  is non-superadditive.

*Proof:* Consider two coalitions  $\mathcal{R}^1 \in \mathcal{CS}$  and  $\mathcal{R}^2 \in \mathcal{CS}$  in the network with the players of  $\operatorname{Supp}(\mathcal{R}^1) \cup \operatorname{Supp}(\mathcal{R}^2)$  located far enough such that  $P_{\mathcal{R}^1 \cup \mathcal{R}^2} > P_{\lim}$ . We also suppose that  $\mathcal{R}^1 \cup \mathcal{R}^2 \in \mathcal{CS}'$ . Therefore, according to (10),  $v\left(\mathcal{R}^1 \cup \mathcal{R}^2, \mathcal{CS}'\right) = 0 < v\left(\mathcal{R}^1, \mathcal{CS}\right) + v\left(\mathcal{R}^2, \mathcal{CS}\right)$ . Thus, the proposed OCF game is not superadditive.

This result implies that the proposed game can be classified as a coalition formation game [42] [43]. One of the main features of the OCF game is that it allows different coalitions to overlap, i.e., an SBS i can simultaneously join more than one coalition. In order to capture this overlapping feature, we allow each SBS i to divide its frequency resource into several parts, each of which is dedicated to a distinct coalition. To better understand this model, consider an SBS  $i \in \mathcal{N}$  which is a player in the OCF game. The initial frequency resource of SBS i is the subchannel set  $\mathcal{T}_i$  which is measured in orthogonal subchannels. Here, we present the following definition:

**Definition 5.** An SBS unit  $\lambda_m^i$  is defined as the minimum indivisible resource (or part) of SBS i which has access to a single subchannel  $c(\lambda_m^i)$  from  $\mathcal{T}_i$ . The number of the SBS units that SBS i owns is M, i.e., the number of subchannels in  $\mathcal{T}_i$ . We also have  $\bigcup_{m=1}^M c(\lambda_m^i) = \mathcal{T}_i$ .

In the studied OCF game, if an SBS i is a member of a coalition  $\mathcal{R}$ , i.e.,  $i \in \text{Supp}(\mathcal{R})$ , then at least one SBS unit  $\lambda_m^i$  is dedicated to coalition  $\mathcal{R}$ . An SBS can decide to remove one unit from the current coalition and dedicate it to a new coalition when such a move leads to a preferred coalitional structure (i.e., a higher utility).

Next we discuss the stability of the solutions for the proposed OCF game. We mainly consider a stable solution concept for OCF games whenever only one of the players consider to reallocate a resource unit at a time. The proposed stability concept is defined as follows:

**Definition 6.** Given an OCF game  $G = (\mathcal{N}, v)$  with a transferable utility (TU) and a player  $i \in \mathcal{N}$ , let  $(\mathcal{CS}, \mathbf{x})$  and  $(\mathcal{CS}', \mathbf{y})$  be two outcomes of G such that  $(\mathcal{CS}', \mathbf{y})$  is the resulting outcome of one reallocation of an SBS unit of i from  $(\mathcal{CS}, \mathbf{x})$ . We say that  $(\mathcal{CS}', \mathbf{y})$  is a *profitable deviation* of i from  $(\mathcal{CS}, \mathbf{x})$  if the transformation from  $(\mathcal{CS}, \mathbf{x})$  to  $(\mathcal{CS}', \mathbf{y})$  is feasible.

Given the above definition, we can define a stable outcome as follows:

**Definition 7.** An outcome  $(\mathcal{CS}, \mathbf{x})$  is *stable* if no player  $i \in \mathcal{N}$  has a profitable deviation from it.

The corresponding coalitional structure  $\mathcal{CS}$  from a stable outcome  $(\mathcal{CS}, \mathbf{x})$  is a stable coalitional structure. When no SBS units can be switched from one coalition to another or be put alone, the coalitional structure is stable. In order to compare two coalitional structures, we introduce the following definition:

**Definition 8.** Given two coalitional structures  $\mathcal{CS}_P = (\mathcal{R}^1, ..., \mathcal{R}^p)$  and  $\mathcal{CS}_Q = (\mathcal{C}^1, ..., \mathcal{C}^q)$  which are both defined on the player set of  $\mathcal{N} = \{1, ..., N\}$ , an  $\mathit{order} \succeq_i$  is defined as a complete, reflexive and transitive binary relation over the set of all coalitional structures that can be possibly formed.  $\mathcal{CS}_P$  is preferred to  $\mathcal{CS}_Q$  for any player i when  $\mathcal{CS}_P \succeq_i \mathcal{CS}_Q$ .

Consequently, for any player  $i \in \mathcal{N}$ , given two coalitional structures  $\mathcal{CS}_P$  and  $\mathcal{CS}_Q$ ,  $\mathcal{CS}_P \succeq_i \mathcal{CS}_Q$  means that player i prefers to allocate its frequency resources in the way that  $\mathcal{CS}_P$  forms over the way that  $\mathcal{CS}_Q$  forms, or at least, player i prefers  $\mathcal{CS}_P$  and  $\mathcal{CS}_Q$  indifferently. Moreover, if we use the asymmetric counterpart of  $\succeq_i$ , denoted as  $\succ_i$ , then  $\mathcal{CS}_P \succ_i \mathcal{CS}_Q$  indicates that player i strictly prefers to allocate its frequency resources in the way that  $\mathcal{CS}_P$  forms over the way that  $\mathcal{CS}_Q$  forms.

Different types of orders can be applied to compare two coalition structures. This includes two major categories: individual payoff orders and coalition payoff orders. For individual payoff orders, each player's individual payoff in the game is mainly used to compare two coalitional structures. In contrast, for a coalition payoff order, the payoff of the coalitions in the game is mainly used to compare two coalitional structures. In our OCF game case, we aim at increasing

the total payoff of the coalitional structure in a distributed way. When two coalitional structures are compared, both the individual payoff and the coalition payoff will be considered.

When an SBS i decides to change the allocation of its unit  $\lambda_m^i$  from the current coalition, it may either allocate it to another existing coalition or make it alone, i.e., allocate it to another completely new and independent coalition consisting of only SBS i. Accordingly, we propose the following two orders to compare two coalitional structures:

**Definition 9.** Consider a coalitional structure  $\mathcal{CS}_P = \{\mathcal{R}^1, ..., \mathcal{R}^l\}$  and an SBS unit  $\lambda_m^i$  satisfying  $\lambda_m^i \in \mathbf{R}_i^t \in \mathcal{R}^t$ , where  $t \in \{1, ..., l\}$ . For a coalition  $\mathcal{R}^k$  with  $k \in \{1, ..., l\}$  and  $k \neq t$ , a new coalitional structure is defined as  $\mathcal{CS}_Q = \{\mathcal{CS}_P \setminus \{\mathcal{R}^t, \mathcal{R}^k\}\} \cup \{\mathcal{R}^t \setminus \{\{\lambda_m^i\}\}, \mathcal{R}^k \cup \{\{\lambda_m^i\}\}\}$ . In order to transform  $\mathcal{CS}_P$  into  $\mathcal{CS}_Q$ ,  $\lambda_m^i$  must be switched from the current coalition  $\mathcal{R}^t$  to another coalition  $\mathcal{R}^k$ . Then, the *switching order*  $\triangleright_S$  is defined as

$$\mathcal{CS}_{Q} \triangleright_{S} \mathcal{CS}_{P} \Leftrightarrow
\begin{cases}
p^{i}(\mathcal{CS}_{Q}) > p^{i}(\mathcal{CS}_{P}), \\
v(\mathcal{CS}_{Q}) > v(\mathcal{CS}_{P}), \\
\sum_{\lambda_{m}^{j} \in \mathbf{R}_{j}^{k} \in \text{Supp}(\mathcal{R}^{k})} p^{j}(\mathcal{CS}_{Q}) \geq \sum_{\lambda_{m}^{j} \in \mathbf{R}_{j}^{k} \in \text{Supp}(\mathcal{R}^{k})} p^{j}(\mathcal{CS}_{P}).
\end{cases} (14)$$

The switching order  $\triangleright_S$  indicates that three conditions are needed when an SBS switches one of its units from one coalition to another. These conditions are: (i) The individual payoff of SBS i is increased, (ii) the total payoff of the coalitional structure is increased, and (iii) the payoff of the newly formed coalition  $\mathcal{R}^k$  is not decreased.

**Definition 10.** Consider a coalitional structure  $\mathcal{CS}_P = \{\mathcal{R}^1, ..., \mathcal{R}^l\}$  and an SBS unit  $\lambda_m^i$  satisfying  $\lambda_m^i \in \mathbf{R}_i^t \in \mathcal{R}^t$ , where  $t \in \{1, ..., l\}$ . A new coalitional structure is defined as  $\mathcal{CS}_E = \{\mathcal{CS}_P \setminus \{\mathcal{R}^t\}\} \cup \{\mathcal{R}^t \setminus \{\{\lambda_m^i\}\}\} \cup \{\{\lambda_m^i\}\}\}$ . In order to transition from  $\mathcal{CS}_P$  to  $\mathcal{CS}_E$ ,  $\lambda_m^i$  needs to be removed from the current coalition  $\mathcal{R}^t$  and put in an independent coalition  $\{\{\lambda_m^i\}\}$ . Then, the *independent order*  $\triangleright_I$  is defined as

$$CS_E \triangleright_I CS_P \Leftrightarrow \begin{cases} p^i(CS_E) > p^i(CS_P), \\ v(CS_E) > v(CS_P). \end{cases}$$
(15)

The independent order  $\triangleright_I$  implies that two conditions are needed when an SBS unit is removed from its current coalition and made independent: (i) The individual payoff of SBS i is increased and (ii) the total payoff of the coalitional structure is increased.

Moreover, we denote  $h(\lambda_m^i)$  as the history set of the SBS unit  $\lambda_m^i$ .  $h(\lambda_m^i)$  is a set that contains all the coalitions that  $\lambda_m^i$  was allocated to in the past. The rationale behind the history set  $h(\lambda_m^i)$  lies in that an SBS i is prevented from allocating one of its SBS units  $\lambda_m^i$  to the same coalition twice.

Using the two orders above, each SBS can make a distributed decision to change the allocation of its units, and thus, the coalitional structure. Moreover, the individual payoff, the coalition payoff, and the total payoff are considered when a reallocation is performed. For every coalitional structure  $\mathcal{CS}$ , the switching order and the independent order provide a mechanism by which the players, i.e., the SBSs can either reallocate its SBS units from one coalition to another coalition or make its SBS units act independently. Here, no global scheduler is required for performing the comparisons between pairs of coalitional structures. Furthermore, when one of the two orders is satisfied, in order to change the coalitional structure, we need to compare the new coalition including  $\lambda_m^i$  to the coalitions in the history set  $h(\lambda_m^i)$ . If the new coalition including  $\lambda_m^i$  is the same with one of the previous coalition members from  $h(\lambda_m^i)$ , then the reallocation of  $\lambda_m^i$  cannot be done and the coalitional structure remains unchanged. Otherwise, if the new coalition including  $\lambda_m^i$  is different from any of the previous coalition members from  $h(\lambda_m^i)$ , then we update  $h(\lambda_m^i)$  by adding the new coalition into it. Finally, to solve the proposed game, we propose a distributed algorithm that leads to a stable coalitional structure while significantly improving the overall network performance, as described next.

### C. Proposed Algorithm for SBSs' OCF

We propose a new distributed OCF algorithm based on the switching order and the independent order as shown in Table I. This algorithm is mainly composed of three phases: environment sensing, overlapping coalition formation and intra-coalition cooperative transmission. First of all, the network is partitioned by  $|\mathcal{N}|$  single coalitions, each of which contains a noncooperative SBS with all its SBS units. Thus, the SBSs act noncooperatively in the beginning. Then, through environment sensing, the SBSs can generate a list of existing coalitions in the network [51]–[53]. Successively, for each of the SBS units, the corresponding SBS decides whether to reallocate this SBS unit based on the switching order and the independent order. The history set of the SBS

# TABLE I THE OVERLAPPING COALITION FORMATION ALGORITHM

### \* Initial State:

The network consists of noncooperative SBSs, and the initial coalitional structrure is denoted as  $\mathcal{CS} = \{\{\mathcal{T}_1\}, ..., \{\mathcal{T}_N\}\}.$ 

# \* Each Round of the Algorithm:

- \* Phase 1 Environment Sensing: For each SBS unit  $\lambda_m^i$ , SBS i discovers the existing coalitions in  $\mathcal{N}$ .
- \* Phase 2 Overlapping Coalition Formation:

#### Repeat

- a) For each SBS unit  $\lambda_m^i$ , SBS i lists the potential coalitions that  $\lambda_m^i$  may join. Suppose the current coalitional structure is  $\mathcal{CS}_P = \{\mathcal{R}^1,...,\mathcal{R}^l\}$ . Then there exists l possible coalitional structures  $\mathcal{CS}_Q^1,...,\mathcal{CS}_Q^{l-1},\mathcal{CS}_E$ .
- b) For each SBS unit  $\lambda_m^i$ , SBS i decides whether to let  $\lambda_m^i$  switch to another coalition in the current coalitional structure based on the switching order and the history set  $h(\lambda_m^i)$ . The switching order stands when  $\mathcal{CS}_Q^k \triangleright_S \mathcal{CS}_P, k \in [1, l-1]$ .
- c) For each SBS unit  $\lambda_m^i$ , SBS i decides whether to let  $\lambda_m^i$  become independent from the current coalition based on the independent order and the history set  $h(\lambda_m^i)$ . The independent order stands when  $\mathcal{CS}_E \triangleright_I \mathcal{CS}_P$ .

**Until** convergence to a stable coalition structure  $CS^*$ .

 $\star$  Phase 3 - Inner-coalition Cooperative Transmission: Scheduling information is gathered by each SBS  $i \in \operatorname{Supp}(\mathcal{R})$  from its coalition members, and transmitted within the coalition  $\mathcal{R}$  afterwards.

unit should also be taken into account when the reallocation process is performed by the SBSs. Every time a reallocation is completed, the system payoff will be improved respectively. Note that the system payoff throughout the paper refers to  $v(\mathcal{CS})$ . When no reallocation is possible, the second phase of overlapping coalition formation terminates and a stable overlapping coalitional structure is formed. Consequently, in the third phase of cooperative transmission within each coalition, the scheduling information is broadcasted from each SBS to the other SBSs within the same coalition. In summary, the proposed overlapping coalition formation algorithm enables the SBSs in the network to increase their own payoff as well as the system payoff without hurting the other members of the newly formed coalition in each iteration and self-organize into a stable overlapping coalition structure in a distributed way.

While the OCF game is constructed with the SBSs as the players, every change of the coalitional structure is caused by the reallocation of the SBS units. As the SBS units are assigned in a distributed way by the SBSs, they seek to improve their individual payoffs without causing the payoff of the newly formed coalition to decrease. Meanwhile, as the goal of this work is to maximize the system payoff in terms of sum-rate, the coalition formation process must also consider the improvement of the total payoff of the network. Thus, the proposed algorithm will also ensure that the system payoff will be increased every time the coalitional structure changes due to the reallocation of an SBS unit.

**Theorem 2.** Starting from the initial network coalitional structure, the convergence of the overlapping coalition formation algorithm is guaranteed.

*Proof:* Given the number of the SBSs and the number of the subchannels that each SBS initially possesses, the total number of possible coalitional structures with overlapping coalitions is finite. As each reallocation of the SBSs' units causes a new coalitional structure with a higher system payoff than all the old ones, the proposed algorithm prevents the SBSs from ordering its units to form a coalitional structure that has previously appeared. Consequently, each reallocation of the SBSs' units will lead to a new coalitional structure, and given the finite number of these structures, our algorithm is guaranteed to reach a final coalitional structure with overlapping coalitions.

Next, we prove that the final coalitional structure is a stable coalitional structure.

**Proposition 1:** Given the switching order and the independent order, the final coalitional structure  $\mathcal{CS}^*$  resulting from the overlapping coalition formation algorithm is stable.

*Proof:* If the final coalitional structure  $\mathcal{CS}^*$  is not stable, then there exists a reallocation of one of the resource units of i that can change the current coalitional structure  $\mathcal{CS}^*$  into a new coalitional structure  $\mathcal{CS}'$ . Hence, SBS i can perform an operation on one of its SBS units based either on the switching order or the independent order, which contradicts with the fact that  $\mathcal{CS}^*$  is the final coalitional structure resulted from the convergence of the proposed OCF algorithm. Thus, the final coalitional structure is stable.

# D. Distributed Implementation of the OCF Algorithm

The proposed algorithm can be implemented distributedly, since, as explained above, the reallocation process of the SBS units can be performed by the SBSs independently of any centralized entity. First, for neighbor discovery, each SBS can rely on information from the control channels which provides the needed information including location, frequency, number of users and so on for assisting the SBSs to cooperate and form overlapping coalitions [51]–[53]. After neighbor discovery, the SBSs seek to engage in pairwise negotiations with the neighboring members in other coalitions. In this phase, all the players in the network investigate the possibility of performing a reallocation of its SBS units using the switching order and the independent order. Finally, in the last phase, the scheduling process is executed within each formed coalition.

Next, we investigate the complexity of the overlapping coalition formation phase. Given a present coalitional structure  $\mathcal{CS}$ , for each SBS unit from an SBS, the computational complexity of finding its next coalition, i.e., being reallocated by its corresponding SBS, is easily computed to be  $O(N \times M)$  in the worse case, where N is the number of the SBSs and M is the number of the SBS units that each SBS possesses. The worst case occurs when all the SBS units are allocated in a noncooperative way. As coalitions begin to form, the complexity of performing a reallocation of an SBS unit becomes smaller. This is due to the fact that when an SBS attempts to move one of the SBS units from one coalition to another, the complexity is dependent on the number of coalitions within the coalitional structure. Thus, the complexity is reduced when the number of coalitions is smaller than  $N \times M$ . Furthermore, finding all feasible possible coalitions seems to be complex at first glance. But due to the cost of the coalition formation, the SBS networks mainly deals with small coalitions rather than large coalitions. Moreover, both the system payoff and the individual payoff are considered when each reallocation of an SBS unit is performed. Thus, the constraints in Definitions 9 and 10 reduce the number of iterations needed for finding the final stable outcome. Consequently, for each SBS that is willing to reallocate its SBS units, the complexity of finding the feasible coalitions to cooperate will be reasonable.

### IV. SIMULATION RESULTS AND ANALYSIS

For simulations, we consider an MBS that is located at the chosen coordinate of (1 km, 1 km). The radius of the coverage area of the MBS is 0.75 km. The number of MUEs is  $10 \cdot N$  SBSs

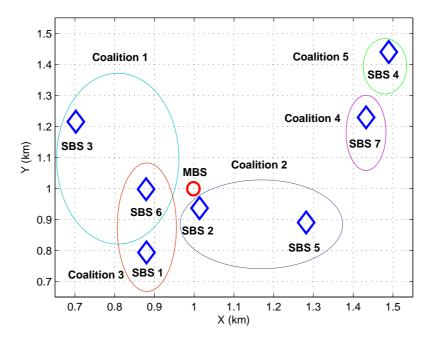


Fig. 2. A snapshot of an overlapping coalitional structure resulting from the proposed approach in a small cell network.

are deployed randomly and uniformly within a circular area around the MBS with a radius of 0.1 km. There is a wall loss attenuation of 20 dB between the MBS and the SUEs, and no wall loss between the SBSs and the SUEs. Each SBS has a circular coverage area with a radius of 20 m. Each SBS has 4 subchannels to use and serves 4 users as is typical for small cells [1]. The total number of subchannels in the considered OFDMA small cell network is 20. The bandwidth of each subchannel is 180 kHz [1]. The total number of time-slots in each transmission in TDMA mode is 4. The transmit power of each SBS is set at 20 dBm, while the transmit power of the MBS is 35 dBm. The maximum tolerable power to form a coalition  $P_{\rm lim}$  =100 dBm. The noise variance is set to -104 dBm.

In Fig. 2, we present a snapshot of an OFDMA small cell network resulting from the proposed algorithm with N=7 SBSs. The radius of the distribution area of SBSs is 0.7 km. The cooperative network shown in this figure is a stable coalitional structure  $\mathcal{CS}^*$ . Initially, all the SBSs schedule their transmissions noncooperatively. After using the proposed OCF algorithm, they self-organize into the structure in Fig. 2. This coalitional structure consists of 5 overlapping coalitions named Coalition 1, Coalition 2, Coalition 3, Coalition 4, and Coalition 5. The support

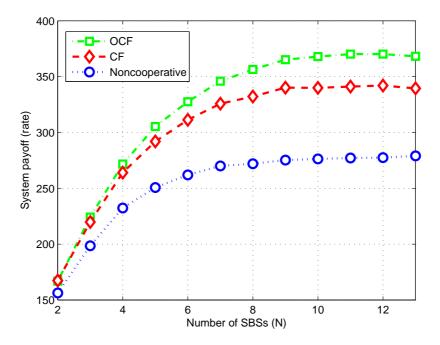


Fig. 3. Performance evaluation in terms of the overall system payoff as the number of SBSs N varies.

of Coalition 1 consists of SBS 3 and SBS 6. The support of Coalition 2 includes SBS 2 and SBS 5. The support of Coalition 3 includes SBS 1 and SBS 6. The support of Coalition 4 includes SBS 7. The support of Coalition 5 includes SBS 4. SBS 4 and SBS 7 have no incentive to cooperate with other SBSs as their spectral occupation is orthogonal to all nearby coalitions. Meanwhile, SBS 6 is an overlapping player because its resource units are divided into two parts assigned to different coalitions. The interference is significantly reduced in  $\mathcal{CS}^*$  as compared to that in the noncooperative case, as the interference between the members of the same coalition is eliminated using proper scheduling. Clearly, Fig. 2 shows that by adopting the proposed algorithm, the SBSs can self-organize to reach the final network structure.

Fig. 3 shows the overall system utility in terms of the total rate achieved by our proposed OCF algorithm as a function of the number of SBSs N compared with two other cases: the non-overlapping coalition formation (CF) algorithm in [41] and the noncooperative case. Fig. 3 shows that for small networks (N < 4), due to the limited choice for cooperation, the proposed OCF algorithm and the CF algorithm have a performance that is only slightly better than that of the noncooperative case. This indicates that the SBSs have no incentive to cooperate in a small-sized

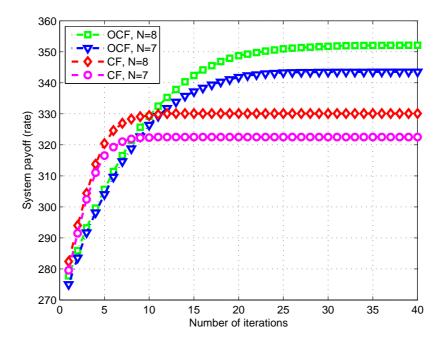


Fig. 4. System payoff vs. number of iterations.

network as the co-tier interference remains tolerable and the cooperation possibilities are small. As the number of SBS N increases, the possibility of cooperation for mitigating interference increases. Fig. 3 shows that, as N increases, the proposed OCF algorithm exhibits improved system performances compared to both the traditional coalition formation game and that of the noncooperative case. The performance advantage reaches up to 32% and 9% at N=10 SBSs relative to the noncooperative case and the classical CF case, respectively.

Fig. 4 shows the convergence process under different scenarios using the proposed OCF algorithm and the CF algorithm. We observe that, although the OCF algorithm requires a few additional iterations to reach the convergence as opposed to the CF case when both N=7 and N=8, this number of iterations for OCF remains reasonable. Moreover, Fig. 4 shows that the OCF algorithm clearly yields a higher system payoff than the CF case, with only little extra overhead, in terms of the number of iterations. Hence, the simulation results in Fig. 4 clearly corroborate our earlier analysis.

Fig. 5 shows the cumulative density function (CDF) of the individual SBS payoff resulting from the proposed OCF algorithm and the CF algorithm when the number of SBSs is set to

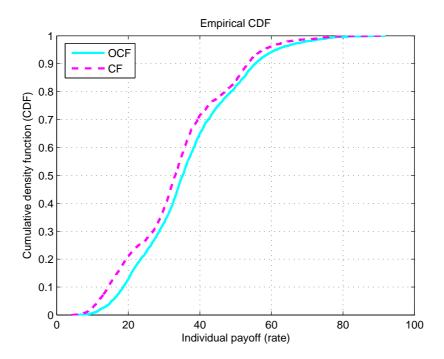


Fig. 5. Cumulative density function of the individual payoff for a network with N=10 SBSs.

N=10. From Fig. 5, we can clearly see that the proposed OCF algorithm performs better than the CF algorithm in terms of the individual payoff per SBS. For example, the expected value of the individual payoff for a network formed from the OCF algorithm is 36, while for a network formed from the CF algorithm the expected value is 33. This is due to that our proposed algorithm allows more flexibility for the SBSs to cooperate and form coalitions. Each SBS is able to join multiple coalitions in a distributed way by adopting our OCF algorithm, while it can only join one coalition at most in the CF case. Moreover, during each reallocation, the SBSs improve their own payoff without being detrimental to the other SBSs in the new coalition. This also contribute to a growth of the individual payoff of each SBS. In a nutshell, Fig. 5 shows that our proposed OCF algorithm yields an advantage on individual payoff per SBS over the CF algorithm.

Fig. 6 shows the growth of the system payoff of the network as the number of SBSs increases, under different maximum tolerable power costs of a coalition  $P_{\text{lim}}$ . Both the OCF algorithm and the CF case are considered in Fig. 6. The power cost incurred for forming each coalition is found from (5). From Fig. 6, we observe that, as the number of SBSs increases, the system payoff

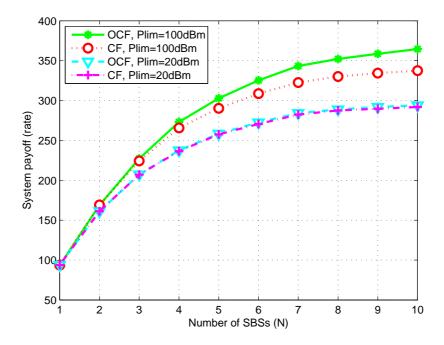


Fig. 6. System payoff as a function of number of SBSs N, for different maximum tolerable power costs.

under two conditions both grows. Moreover, the proposed OCF algorithm has a small advantage on the system payoff compared to the CF case when  $P_{\rm lim}=20$  dBm, while the advantage of the OCF algorithm over the CF case is more significant when  $P_{\rm lim}=100$  dBm. This is due to the fact that when  $P_{\rm lim}$  is low, the SBSs can hardly cooperate with other neighboring SBSs. Most SBSs choose to stay alone as the power cost of possible coalitions exceeds the maximum tolerable power cost. Thus, the system payoff of the OCF algorithm and of the CF algorithm are close. Furthermore, when  $P_{\rm lim}$  is high, each SBS is able to reallocate its SBS units to join neighboring coalitions and improve both the system payoff and its own payoff using the OCF algorithm. Meanwhile, the cooperation possibility of the SBSs under the CF case is also increased when  $P_{\rm lim}$  increases. Consequently, Fig. 6 shows that the OCF algorithm incurs a higher probability for the SBSs to cooperate than the CF case, especially when the maximum tolerable power cost of forming a coalition is high. Thus, our OCF algorithm achieves better system performances in terms of sum rate than the CF algorithm.

Fig. 7 shows the relationship between the number of coalitions that each SBS joins and the number of SBSs under the proposed OCF case and the CF case. As the number of SBSs

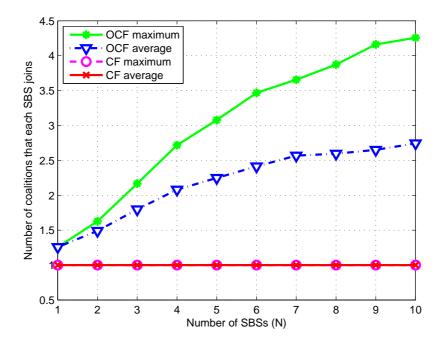


Fig. 7. Number of coalitions per SBS as a function of number of SBSs N.

increases, both the maximum and the average number of coalitions that each SBS joins also grows under the OCF case. While in the CF case, each SBS is only allowed to join one coalition at most no matter how the number of SBSs changes, thus causing the maximum number and the average number of coalitions that each SBS joins to remain the same when the number of SBSs increases. Fig. 7 shows that the incentive towards cooperation for the SBSs is more significant for the proposed OCF algorithm than for the CF case. Thus, The cooperative gain can be achieved more efficiently by using our OCF algorithm than the CF case when the SBSs are densely deployed in the network. The cooperative probability of the OCF algorithm represented by the maximum number of coalitions that each SBS joins is 325.75% larger than that of the CF case when N=10 SBSs are deployed in the network.

In Fig. 8, we show the system payoff in terms of sum-rate as the radius of the distribution area of SBSs varies. The number of SBSs in the network is set to N=10. We compare the system payoff of the proposed OCF algorithm, CF case and noncooperative case. Fig. 8 shows that as the radius of the distribution area of SBSs increases, the system payoff also increases. This is because both the co-tier interference and the cross-tier interference are mitigated when the SBSs

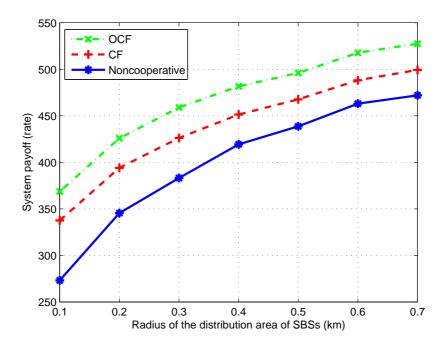


Fig. 8. System payoff vs. radius of the distribution area of SBSs for a network with N=10 SBSs.

are deployed in a larger area. Thus, the system payoff is improved for the OCF algorithm, the CF case as well as the noncooperative case. From Fig. 8, we can also observe that as the radius of the distribution area of SBSs varies, our OCF algorithm yields a higher system payoff than the CF case and the noncooperative case.

In Fig. 9, we continue to compare our OCF approach to the CF case and the noncooperative case in terms of system payoff as the total number of the available subchannels in the network changes. Here, N=10 SBSs are deployed in the network. Note that, we adopt the approach of co-channel assignment, i.e., the SBSs reuse the spectrum allocated to the macrocell. Fig. 9 shows that the system payoff of the proposed OCF algorithm, the CF case, and the noncooperative case are improved when the total number of available subchannels increases. This is due to the fact that when the number of available subchannels increases, the probability of conflicts on subchannels is greatly decreased. Thus, the interference in the two-tier small cell network is mitigated, causing the improvement of the system payoff in terms of sum-rate. Moreover, Fig. 9 shows that our proposed OCF algorithm outperforms the CF case and the noncooperative case in terms of system payoff when the total number of available subchannels increases.

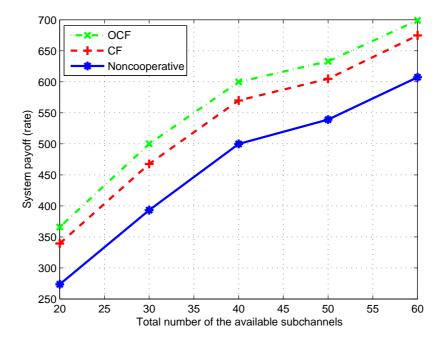


Fig. 9. System payoff vs. total number of subchannels for a network with  $N=10~\mathrm{SBSs}$ .

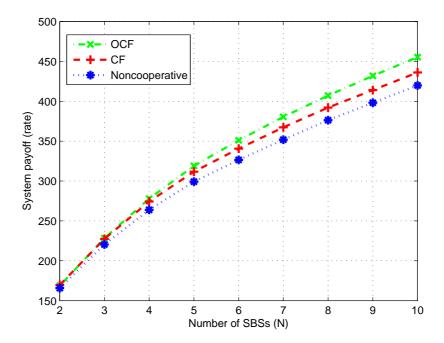


Fig. 10. Performance evaluation in terms of the overall system payoff with wall loss in the small cell tier as the number of SBSs N varies.

In Fig. 10, we modify the scenario by considering the wall loss between the MBS and the SUEs and the wall loss between the SBSs and the SUEs, both of which are set at 20 dB. In this scenario, the downlink cross-tier interference has a much greater impact on system performance than in the scenario where no wall exists between the SBSs and the SUEs such as in Fig. 3. As shown in Fig. 3 and Fig. 10, the advantage on system payoff of our OCF algorithm over the CF algorithm and the noncooperative case when no wall loss is considered between the SBSs and the SUEs is more significant than that when wall loss is assumed between the SBSs and the SUEs.

### V. CONCLUSIONS

In this paper, we have investigated the problem of cooperative interference management in small cell networks. We have formulated this problem as an overlapping coalition formation game between the small cell base stations. Then, we have shown that the proposed game has a transferable utility and exhibits negative externalities due to the co-tier interference between small cell base stations. To solve this game, we have proposed a distributed overlapping coalition formation algorithm that allows the small cell base stations to interact and individually decide on their cooperative decisions. By adopting the proposed algorithm, each small cell base station can decide on the number of coalitions that it wishes to join as well as on the resources that it allocates to each such coalition, while optimizing the tradeoff between its overall rate and the associated cooperative costs. We have shown that the proposed algorithm is guaranteed to converge to a stable coalition structure in which no small cell base station has an incentive to reallocate its cooperative resources. Simulation results have shown that the proposed overlapping coalitional game approach allows the small cell base stations to self-organize into cooperative coalitional structures while yielding notable rate gains relative to both the noncooperative case and the classical coalition formation algorithm with non-overlapping coalitions.

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