STOCHASTIC-QUANTUM COMPARISON

THROUGH A FRICTIONLESS

STOCHASTIC EXPERIMENT

A Thesis

Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

By

Joe David Regester

August, 1972

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ABSTRACT

A stochastic single-slit experiment is used to exhibit a counterexample to the proposal by several investigators that quantum phenomena is equivalent to a frictionless stochastic process. The connection between Brownian motion and quantum mechanics is made by relating the diffusion coefficient and mean drift velocity of the Smoluchowski equation to $\hbar/2m$ and $h/m\lambda$ respectively. This is the connection usually made in relating quantum mechanics to Brownian motion. The omission of the damping term leads to an effective wavelength for the stochastic test problem which is changing in time and implies that is not the ideal stochastic test model to consider. The stochastic single-slit experiment is scaled to conform with a physical single-slit experiment which is known to agree with quantum calculations. An intensity distribution is developed by using Langevin's equation without damping to calculate (with a computer) the position of particles acted on by a random force. The intensity distribution is then compared to the diffraction pattern produced by the physical experiment and no similarity is noted.

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I. INTRODUCTION

This thesis is to test the hypothesis that Quantum phenomena can be accounted for as arising from stochastic processes. In particular it has been suggested by several investigators[1-4] that the motion of a particle, or ensemble of identical particles, described by the Schrödinger equation is equivalent to Brownian motion. This thesis presents a direct test of this idea through the numerical calculation of the motion of a beam of classical particles through a single-slit with the particles being subject to classical Brownian fluctuations. The crux of this test is to ascertain whether such a beam-slit experiment can exhibit the diffraction effects called for by Quantum Mechanics.

In Section II of this paper the basis for this stochastic connection to Quantum Theory will be reviewed. In Section III the single-slit beam diffraction experiment, which was used as the basis for the Stochastic-Quantum comparison, will be described. In Section IV the test problem will be described and the results and discussion given in Section V.

II. QUANTUM MECHANICS AND BROWNIAN MOTION

Of the several attempts[1-4] to exhibit the equivalence of the quantum mechanical motion of a particle to Brownian motion, that due to L. De La Peña-Auerbach and Leopoldo S. García-Colín[1] is perhaps the most direct.

They start with the Schrödinger equation

$$i\hbar \, {\partial}/{\partial t}\, \Psi = -{\hbar^2}/{2m}\, \nabla^2 \Psi + V \Psi \tag{1}$$

and write Ψ in the usual form

$$\Psi = e^{R+iS} \tag{2}$$

where R and S are real, dimensionless functions of the coordinates and the time. By the substitution of equation (2) into (1) and equating real and imaginary parts, there results the system of equations

$$\partial/\partial t\, R = -\frac{\hbar}{2m} \nabla^2 S - \frac{\hbar}{m} \nabla R \cdot \nabla S \tag{3}$$

$$-\partial/\partial t\, S = -\frac{\hbar}{2m} \nabla^2 R - \frac{\hbar}{2m}\left[(\nabla R)^2 - (\nabla S)^2\right] + \frac{1}{\hbar} V . \tag{4}$$

Equation (3), by using the integrating factor e^{2R} , may be written in the form

$$\partial/\partial t\, e^{2R} = -\frac{\hbar}{m} \nabla \cdot \left[e^{2R} \nabla S \right] . \tag{5}$$

Letting

$$\rho = e^{2R} = |\Psi|^2 \tag{6}$$

equation (5) becomes

$$\partial/\partial t\, \rho + \nabla\cdot\left[\hbar/m\, \rho\, \nabla S \right] = 0 \tag{7}$$

which is the continuity equation[5] of classical physics and is the basis for interpretating ρ as the probability density and Ψ as the probability amplitude. Here also $(\hbar/m)\nabla S$ must be interpreted as a "flux velocity vector".

Introducing a new function Q defined by

$$Q = R + S \tag{8}$$

equation (3) takes the form (in terms of R and Q)

$$\partial/\partial t\, R = \frac{-\hbar}{2m}\left[\nabla^2 Q - \nabla^2 R\right] - \frac{\hbar}{m}\nabla R\cdot(\nabla Q - \nabla R) \tag{9}$$

which, with the aid of equation (6), may be written as

$$\partial/\partial t\, \rho + \nabla\cdot\left[\left(\frac{\hbar}{m}\nabla Q\right)\rho - \frac{\hbar}{2m}\nabla\rho\right] = 0\,. \tag{10}$$

This equation has the form of a Smoluchowski equation[6] of a Brownian particle acted on by an external force K per unit mass given by

$$K = \hbar/m\, \beta\, \nabla Q\,. \tag{11}$$

This also corresponds to a diffusion coefficient D given by

$$D = \hbar/2m \tag{12}$$

and where β , in the stochastic sense, is a viscosity "damping" coefficient but in the quantum sense must be a parameter which is a measure of the interaction between the particle and the vacuum.

Schrödinger's equation is considered valid for all time intervals. On the other hand, Smoluchowski's equation which was derived directly from Schrödinger's equation is, in the theory of Brownian motion, valid only for time intervals such that $\beta\Delta t \gg 1$[6]. This leads to two possibilities: either Smoluchowski's equation is not equivalent to Schrödinger's equation because of the added restrictions not contained in Schrödinger's equation, or the same restrictions which apply to Smoluchowski's equation also apply to the Schrödinger equation.

Peña-Auerbach and García-Colín assume the second condition, $\beta\Delta t \gg 1$, as an oversimplified form of the time-energy uncertainty relation with the physical interpretation being, "as the time intervals used to measure the mean value of the energy of the particles grows, the dispersion in the measured value is reduced, because more and more fluctuations are taken into account".

Rewritting equation (4) and taking its expectation value to obtain

$$-\hbar\langle \partial/\partial t\, S\rangle = \frac{\hbar^2}{2m}\langle (\nabla S)^2-(\nabla R)^2-\nabla^2 R\rangle + \langle V\rangle \tag{13}$$

along with the relations

$$\hat{E} = i\hbar\, \partial/\partial t \tag{14}$$

$$\langle \hat{E}\rangle = -\hbar\langle \partial/\partial t\, S\rangle \tag{15}$$

$$\langle \hat{P}^2\rangle = -\hbar^2\langle \nabla^2 R + (\nabla R)^2 - (\nabla S)^2\rangle , \tag{16}$$

equation (13) may be written in the familiar form

$$\langle i\hbar\, \partial/\partial t\rangle = \langle \hat{E}\rangle = \langle \hat{P}^2/2m + V\rangle \,. \tag{17}$$

To remove the limitation that $\beta\Delta t \gg 1$, L. De La Peña-Auerbach and Leopoldo S. García-Colín[2] consider a quantum particle described by the classical Brownian motion in phase space of the Fokker-Planck equation[6] under the influence of a non-velocity-dependent force

$$\frac{D}{Dt}W = \nabla_u\cdot\left(\beta\hat{u}W + q\,\nabla_u W\right) \tag{18}$$

where $W(r,u,t)$ stands for the probability density evolving by a Markoff process[7], $\frac{D}{Dt}$ is the total macroscopic time derivative

$$\frac{D}{Dt} = \partial/\partial t + \hat{u}\cdot\nabla_r + \hat{K}\cdot\nabla_u \quad , \tag{19}$$

$\hat{K}$ is the external force per unit mass, β is a measure of the coupling between the system and its surroundings and q is the diffusivity of the system. The equation which will be derived will be valid for all times of the particle's motion.

Since W is a real positive function, let

$$W = e^{2R} \quad , \; R = f(\hat{r},\hat{u},t) \tag{20}$$

and define a vector $\hat{F}$ such that

$$\hat{F} = -\beta\hat{u} - (q/W)\,\nabla_u W \tag{21}$$

which has the meaning of a mean force per unit mass which is developed on the particle as it moves through its surroundings.

Substitution of Equations (20) and (21) back into Equation (19) gives

$$\frac{D}{Dt}R = -\frac{1}{2}\nabla_u \hat{F} - \hat{F}\cdot\nabla_u R\ . \tag{22}$$

Since $\hat{F}$ is velocity-dependent, we assume it may be derived from a velocity-dependent potential S such that

$$\hat{F} = 2q\nabla_u S\ . \tag{23}$$

Substitution of Equation (23) into (22) gives

$$\frac{D}{Dt}R = -q\nabla_u^2 S - 2q\nabla_u R\cdot\nabla_u S\ . \tag{24}$$

Defining a probability amplitude Ψ such that

$$\Psi = e^{R+iS} \tag{25}$$

and

$$W = \Psi^*\Psi \tag{26}$$

Equation (24) becomes

$$i\frac{D}{Dt}\Psi = -q\nabla_u^2\Psi + \Omega\Psi \tag{27}$$

where

$$\Omega = -\frac{D}{DS}S + q[\nabla_u^2 R + (\nabla_u R)^2 - (\nabla_u S)^2]. \tag{28}$$

This is the extended form of Schrödinger's equation defining the probability amplitude Ψ in phase space, where the potential energy is given by the function Ω.

This connection of the Fokker-Planck equation to

Schrödinger's equation is purely formal since on the left side of equation (27) we have D/Dt instead of d/dt and on the right, the Laplacian acts on the velocity coordinates and Ω is a space-velocity-dependent function.

Ordinary quantum mechanics is retrieved by taking an asymptotic limit which corresponds to times longer than the relaxation time of the particle. This corresponds to times for which the description of the system by a Smoluchowski equation is equivalent to the description of the system by the Fokker-Planck equation.

When discussing the kinematics of a quantum particle[7], one must assume that there is no friction involved in order to preserve Galilean covariance. This leads Edward Nelson[4] to adopt the kinematics of the Einstein-Smoluchowski[8] theory and the dynamics as in the Ornstein-Uhlenbeck theory[9] to study Brownian motion in a medium with zero friction.

As Nelson shows in his treatment of the kinematics of a Markoff process[7], there are two velocities to be considered which evolve in time according to the equations

$$d/dt\,\hat{u} = -\nu\nabla(\nabla\cdot\hat{u}) - \nabla(\hat{v}\cdot\hat{u}) \tag{29}$$

$$d/dt\,\hat{v} = \hat{a} - (\hat{v}\cdot\nabla)\hat{v} + (\hat{u}\cdot\nabla)\hat{u} + \nu\nabla^2\hat{u}\,, \tag{30}$$

where $\hat{u}$ and $\hat{v}$ are the osmotic and current velocities respectively. Since the hypothesis is that particles in empty space are subject to Brownian motion and macroscopic bodies do not appear to exhibit such behavior, he assumed that the diffusion coefficient ν is inversely proportional to the mass and set

$$\nu = \hbar / 2m . \tag{31}$$

Since the Ornstein-Uhlenbeck theory is used to describe the dynamics of the particle's motion, the acceleration term in equation (30) is given by $\hat{F}/m$. Thus the system of equations (29) and (30) may be rewritten, using equation (29), in the form

$$\frac{\partial}{\partial t}\hat{u} = -\frac{\hbar}{2m}\nabla(\nabla\cdot\hat{v}) - \nabla(\hat{v}\cdot\hat{u}) \tag{32}$$

$$\frac{\partial}{\partial t}\hat{v} = \frac{1}{m}\hat{F} - (\hat{v}\cdot\nabla)\hat{v} + (\hat{u}\cdot\nabla)\hat{u} + \frac{\hbar}{2m}\nabla^2\hat{u} . \tag{33}$$

In order to obtain the Time-Independent Schrödinger equation, it is necessary to assume the force to be deriveable from a potential such that

$$\hat{F} = -\nabla V \tag{34}$$

and assume that $\hat{v}$ is zero which means that the solutions are stationary in time. Under these conditions equation (32) becomes

$$\frac{\partial}{\partial t}\hat{u} = 0 \tag{35}$$

and equation (33) is

$$\hat{u}\cdot\nabla\hat{u} + \frac{\hbar}{2m}\nabla^2\hat{u} = \frac{1}{m}\nabla V . \tag{36}$$

In Nelson's treatment of the kinematics of a Markoff process, $\hat{u}$ is the gradient

$$\hat{u} = \nu\nabla \ln \rho , \tag{37}$$

so that

$$(\hat{u}\cdot\nabla)\hat{u} = \frac{1}{2}\nabla\hat{u}^2 \tag{38}$$

and

$$\nabla^2\hat{u} = \nabla(\nabla\cdot\hat{u}). \tag{39}$$

This means that equation (36) becomes

$$\nabla\left[\frac{1}{2}\hat{u}^2 + \frac{\hbar}{2m}\nabla\cdot\hat{u}\right] = \frac{1}{m}\nabla V \tag{40}$$

or

$$\frac{1}{2}\hat{u}^2 + \frac{\hbar}{2m}\nabla\cdot\hat{u} = \frac{1}{m}[V - E] \tag{41}$$

where E is a constant with dimensions of energy. Multiply by $m\rho$ and integrate equation (41) to obtain

$$E = \int \frac{1}{2} m\hat{u}^2 \rho\, d^3x + \int V\rho\, d^3x \ . \tag{42}$$

Thus E is the average value of $\frac{1}{2}m\hat{u}^2 + V$ and may be interpreted as the mean energy of the particle.

By the change of dependent variable

$$R = \frac{1}{2}\ln\rho \tag{43}$$

and letting

$$\psi = e^R \tag{44}$$

so that

$$\rho = \psi^2 , \tag{45}$$

equation (41) becomes equivalent to the time-independent

Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi = [E - V]\Psi . \tag{46}$$

To find the general time-dependent Schrödinger equation, assume that $\hat{V}$ is the gradient

$$\hat{V} = \hbar/m\, \nabla S . \tag{47}$$

Keeping R as before (equation 43), equation (44) becomes

$$\Psi = e^{R+iS} \tag{48}$$

which is shown to satisfy the Schrödinger equation

$$\frac{d}{dt}\Psi = i\hbar/2m\,\nabla^2\Psi - \frac{i}{\hbar}V\Psi + i\alpha(t)\Psi , \tag{49}$$

where the potential S can always be chosen such that

$$\alpha(t) = 0 . \tag{50}$$

It is well to note that by the substitution of equation (48) into (49) and equating real and imaginary parts, the system of equations which result are just those of L. De La Peña-Auerbach and Leopoldo S. García-Colín, i.e. equations (3) and (4). This would imply that since Peña-Auerbach and García-Colín have already shown that these equations are equivalent to a Smoluchowski equation of a Brownian particle acted on by an external force, then the equations derived by Edward Nelson must suffer from the same restriction which applies to the Smoluchowski equation; this restriction being that $\beta\Delta t \gg 1$.

III. SINGLE-SLIT DIFFRACTION OF A THERMAL POTASSIUM BEAM

To test the hypothesis that Brownian motion is equivalent to a quantum dynamics, we first needed the results of a physical experiment in the quantum domain to compare with the results of our Stochastic experiment. John A. Leavitt and Francis A. Bills[10] made experimental observations of the diffraction of de Broglie matter waves of a full, thermal, neutral, atomic Potassium beam (λ =0.175 Å) by a single slit. The experimental diffraction patterns were all reproducible and in good general agreement with the predictions of de Broglie and simple scalar Fresnel diffraction theory[11](Figure 3).

The experimental arrangement utilized by these investigators (Figure 1) employed a source to slit distance of 96 cm, slit to detector distance of 100 cm and an oven temperature of 533 K. The slit was 23.0 ($\pm$ 0.4)μ in width and the source slit was 2.5 ($\pm$ 1)μ in width. Their comparison of the experimental diffraction patterns with the theoretical predictions of quantum theory are reproduced in Figure 2.

The presence of diffraction is clearly demonstrated by the occurrence of the two well-defined fringes on the experimental patterns. The observed fringes are only 9.4% above central-beam intensity instead of the 14.1% above central-beam intensity of the theoretical fringes. These investigators attribute the discrepancy to a combination of the following factors:

(1) uncertainty in the measured width of the source slit,

(2) high-frequency vibrations of the apparatus,

(3) defects in the diffracting slit edges,

(4) angular misalignment of the beam elements.

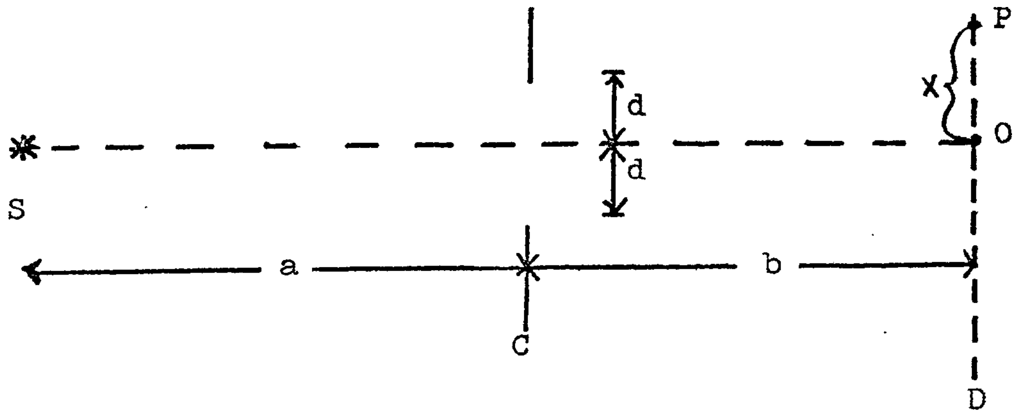
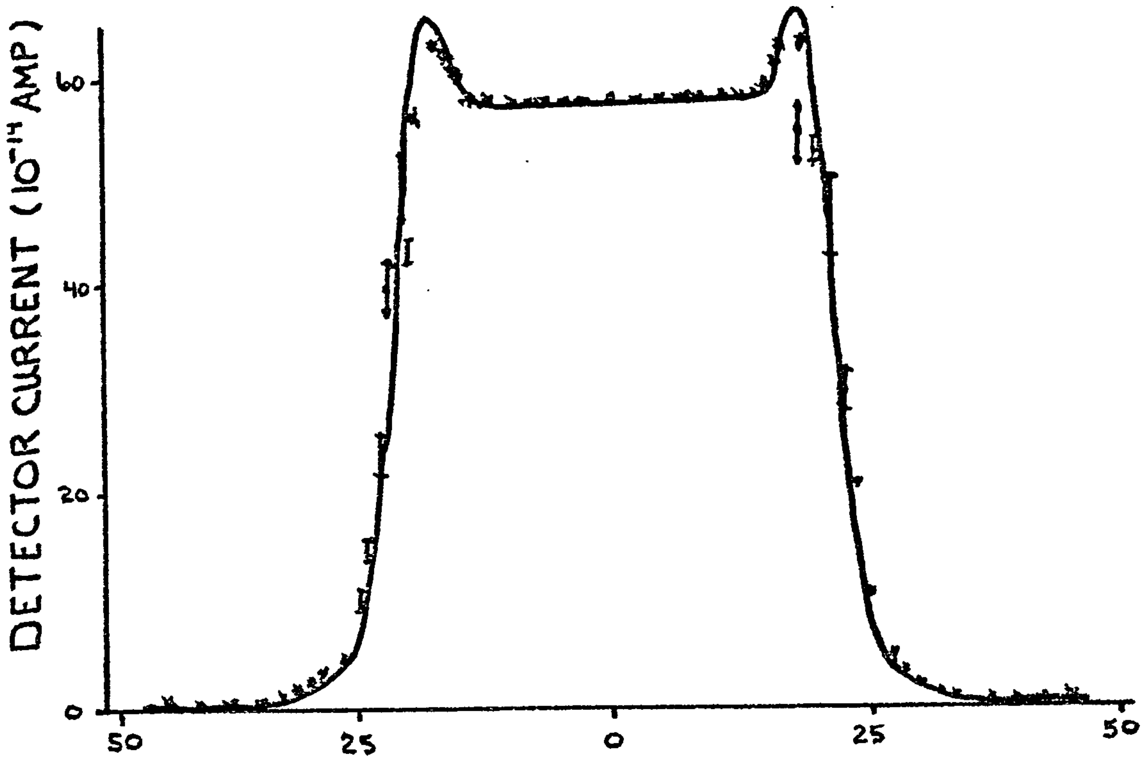


FIGURE 1

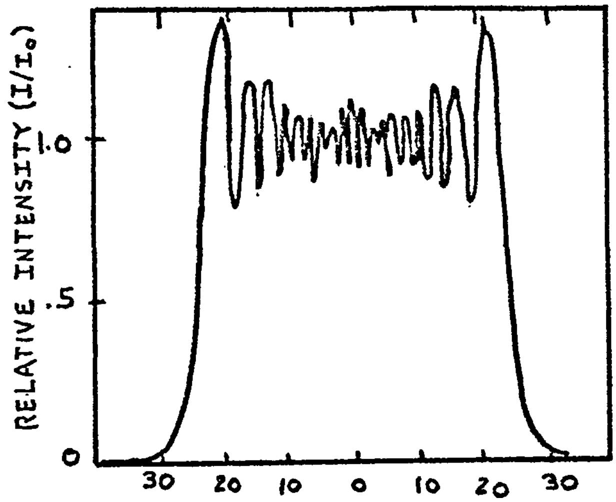
Arrangement of line source (S), slit (C), and detector (line P in observation plane D) for calculation of the diffraction pattern of a slit of width 2d. (Leavitt and Bills, Reference 10)



DETECTOR POSITION IN MICRONS

FIGURE 2

The single-slit diffraction pattern of a full thermal atomic K beam-direct comparison of theory and experiment. The solid curve is the prediction of de Broglie's hypothesis and simple scalar Fresnel diffraction theory, and the points are the experimental beam profiles. [Source slit width = 2.5($\pm$1)μ, collimating (diffracting) slit width = 23.0($\pm$0.4)μ, detector wire diameter = 3.0($\pm$0.1)μ, source-to-collimator distance= 96 cm, collimator-to-detector distance= 100 cm, oven-slit temperature = 533°K, and source-slit temperature = 400°K. The solid curve was calculated for a ^{39}K beam; the experimental points were obtained with a natural K beam(93% ^{39}K, 7% ^{41}K.] (Leavitt and Bills, Reference 10)



DETECTOR POSITION (X) IN MICRONS

FIGURE 3

Calculated velocity-selected atomic ^{39}K-beam diffraction pattern of a 25.4-μ slit(infinitesimal source and detector widths, a =96cm, b =100cm, λ =0.175 Å). (Leavitt and Bills, Reference 10)

IV. STOCHASTIC TEST PROBLEM

A stochastic experiment was performed to test the predictions of Brownian motion with those of quantum theory through the results of the diffraction experiment of Leavitt and Bills. This stochastic test problem consisted of the numerical calculation (with a computer) of the motion of a beam of particles subject to Brownian fluctuations through a single-slit. The crux of this stochastic test was to determine if the diffraction pattern, of the Leavitt and Bills experiment, could be reproduced through this stochastic model. The computer program with experimental data are given in Appendix I and II.

The stochastic equations of motion are taken to be

$$m\ddot{Y} = \sum_{i=1}^{\infty} I_i \, \delta(t - t_i) \tag{51}$$

$$m\ddot{X} = \sum_{i=1}^{\infty} I_i \, \delta(t - t_i) \tag{52}$$

for the test problem. The I_i's are the random impulses that the particle receives at random times, t_i, $i=1,2\cdots$. These equations correspond to Newton's law

$$F = ma \tag{53}$$

where the dynamics of the stochastic test problem is given by

$$F = \sum_{i=1}^{\infty} I_i \, \delta(t - t_i) \, . \tag{54}$$

We now integrate the equation in Y over the limits $t_n \leq t < t_{n+1}$, noting that the equation in X is treated in similiar manner except the initial velocity is taken to be zero, to obtain

$$m[\dot{Y}(t) - \dot{Y}(t_n)] = I_n \,. \tag{55}$$

The random impulse I_n is replaced by

$$\Delta V_n = \frac{1}{m} I_n \tag{56}$$

which says that the random impulse I_n will impart to the particle a random velocity increment ΔV_n . The ΔV_n's are stochastically selected using a random number generator[12] and the distribution function for the ΔV_n's is Gaussian with mean zero.

Letting

$$\dot{Y}(t_n) = V_n \tag{57}$$

along with Equation(56), Equation (55) takes the form

$$\dot{Y}(t) = V_n + \Delta V_n \,. \tag{58}$$

Integrating this equation over the same limits as before gives

$$Y(t) - Y_n = [V_n + \Delta V_n](t - t_n) \tag{59}$$

and by letting

$$t = t_{n+1} - \epsilon \quad , \epsilon \to 0^+ \tag{60}$$

$$\Delta t = t_{n+1} - t_n \tag{61}$$

$$\Delta Y_n = Y_{n+1} - Y_n \tag{62}$$

there results

$$\Delta Y_n = [V_n + \Delta V_n]\Delta t \,. \tag{63}$$

The additional assumption is now made that the Δt's are not stochastic which means that all Δt's are now equal.

The velocity at step n is given by

$$V_n - V_{n-1} = \Delta V_{n-1} \tag{64}$$

which is the system of equations

$$\begin{aligned} V_n &- V_{n-1} = \Delta V_{n-1} \\ V_{n-1} &- V_{n-2} = \Delta V_{n-2} \\ V_{n-2} &- V_{n-3} = \Delta V_{n-3} \\ &\vdots \\ V_1 &- V_0 = \Delta V_0 \,. \end{aligned} \tag{65}$$

Summing these equations gives

$$V_n - V_0 = \sum_{m=0}^{n-1} \Delta V_m \,, \tag{66}$$

which is then written in the more convenient form

$$V_n = V_0 + \sum_{m=0}^{n-1} \Delta V_m \,. \tag{67}$$

Using this in Equation (63) for ΔY yields

$$\Delta Y_n = [V_0 + \sum_{m=0}^{n-1} \Delta V_m + \Delta V_m]\Delta t \tag{68}$$

which becomes

$$\Delta Y_n = [V_0 + \sum_{m=0}^{n} \Delta V_m]\Delta t \,. \tag{69}$$

This equation determines the evolution of the ΔY_n's.

The expectation value for ΔY_n is

$$\langle \Delta Y_n \rangle = \langle [V_0 + \sum_{m=0}^{n} \Delta V_m] \Delta t \rangle \tag{70}$$

or

$$\langle \Delta Y_n \rangle = V_0 \Delta t + \sum_{m=0}^{n} \langle \Delta V_m \rangle \Delta t \tag{71}$$

which collapses to

$$\langle \Delta Y_n \rangle = V_0 \Delta t \tag{72}$$

since the ΔV_m's are Gaussian with mean zero. The dispersion, using Equations (69) and (72), is found to be

$$\langle (\Delta Y_n - \langle \Delta Y_n \rangle)^2 \rangle = \langle (\sum_{m=0}^{n} \Delta V_m)^2 (\Delta t)^2 \rangle \tag{73}$$

which, denoting the dispersion by $\langle (\delta Y_n)^2 \rangle$, becomes

$$\langle (\delta Y_n)^2 \rangle = (\Delta t)^2 \langle (\sum_{i=0}^{n} \Delta V_i)(\sum_{m=0}^{n} \Delta V_m) \rangle \; . \tag{74}$$

The velocity increments imparted to the particle at different times are uncorrelated, therefore

$$\langle \Delta V_i \Delta V_m \rangle = 0 \quad , i \neq m \tag{75}$$

and the dispersion Equation (74) becomes

$$\langle (\delta Y_n)^2 \rangle = (\Delta t)^2 \sum_{m=0}^{n} \langle (\Delta V_m)^2 \rangle \tag{76}$$

or, by using n as the number of steps that the particle has taken, the dispersion is

$$\langle (\delta Y_n)^2 \rangle = n \sigma_v (\Delta t)^2 \tag{77}$$

with

$$\sigma_v = \langle (\Delta V_m)^2 \rangle \quad . \tag{78}$$

The stochastic test problem evolves according to Equation (69), but in order for this to correspond to the experiment of Leavitt and Bills, specific conditions must be imposed on σ_v ; these are now examined.

In Brownian motion the diffusion coefficient is given by[6,13]

$$D = \frac{1}{2} \frac{\langle (\delta Y_n)^2 \rangle}{\Delta t} \tag{79}$$

where $\langle (\delta Y_n)^2 \rangle$ is calculated over the transition probability distribution. On the other hand, from the results of Section II the diffusion coefficient in the quantum domain is found to be given by

$$D = \hbar / 2m \ . \tag{80}$$

Relating the two expressions for D yields

$$\frac{\langle (\delta Y_n)^2 \rangle}{\Delta t} = \hbar / m \quad . \tag{81}$$

Similarly, the mean drift velocity for a Brownian particle[6] has been shown to be

$$V = \frac{\langle \Delta Y_n \rangle}{\Delta t} \tag{82}$$

while in the quantum domain the velocity is given by de Broglie as

$$V = h/m\lambda = \frac{2\pi}{\lambda}\frac{\hbar}{m} \quad . \tag{83}$$

Relating the stochastic velocity to the quantum velocity through Equation (82) and (83) gives

$$\frac{\langle \Delta Y_n \rangle}{\Delta t} = \frac{2\pi}{\lambda}\frac{\hbar}{m} \quad . \tag{84}$$

Similarly using the diffusion relation, Equation (79), for the term $\hbar/m$ in Equation (84) gives

$$\frac{\langle \Delta Y_n \rangle}{\Delta t} = \frac{2\pi}{\lambda}\frac{\langle (\delta Y_n)^2 \rangle}{\Delta t} \quad . \tag{85}$$

After replacing the terms $\langle \Delta Y_n \rangle$ and $\langle (\delta Y_n)^2 \rangle$ in Equation (85) by their equivalents through Equations (72) and (77) we have

$$\frac{V_0 \Delta t}{\Delta t} = \frac{2\pi}{\lambda}\frac{n \sigma_v (\Delta t)^2}{\Delta t} \tag{86}$$

and upon rearranging this becomes

$$\lambda = \frac{2\pi \sigma_v}{V_0} n \Delta t \quad . \tag{87}$$

Equation (87) expresses the sought after relationship between the parameters which condition the stochastic experiment and the effective wavelength. Notice should be taken that the effective wavelength grows in time in relation to the number of steps taken by the particle.

Denote the source to slit distance by S and the slit to detector distance by L. The average number of steps a particle will take from the source to the slit and from the slit to the detector are given respectively by

$$n_s \approx S / V_o \Delta t \tag{88}$$

$$n_L \approx L+S / V_o \Delta t \ . \tag{89}$$

We now show how Equation (87) for the wavelength can be used, together with the physical geometry of the slit experiment, to set up scaling conditions to match the stochastic scale to the experiment of Leavitt and Bills.

Using Equation (88) for n , in Equation (87) and dividing both sides by the distance from the source to the slit, S , gives

$$\lambda_s / S \approx 2\pi \sigma_{v_s} / V_o^2 \tag{90}$$

Treating Equation (89) in similiar manner gives

$$\lambda_L / L+S \approx 2\pi \sigma_{v_L} / V_o^2 \tag{91}$$

From the experiment of Leavitt and Bills the ratios λ / S and $\lambda / L+S$ were easily obtained. Equating those ratios with Equations (90) and (91) gives

$$\lambda_e / S = \lambda_s / S \approx 2\pi \sigma_{v_s} / V_o^2 \tag{92}$$

and

$$\lambda_e / L+S = \lambda_L / L+S \approx 2\pi \sigma_{v_L} / V_o^2 \ . \tag{93}$$

From Equations (92) and (93), the ratio of the wavelength to the distance is the important quantity and not the individual element.

To assure that the wavelength in the slit for the stochastic test problem is the same as the wavelength used in the physical experiment, the appropriate value for σ_v must be used in Equation (90). To find σ_v we first need to determine the value for the initial velocity V_o .

Choose the number of steps, n , that the particle will make from the source to the slit to be 100. Equation (88) then becomes

$$V_o \Delta t \approx S/100 \approx 1 \tag{94}$$

where the distance, S, from the source to the slit is fixed by the geometry of the physical experiment, i.e. $S \approx 100$ cm.

Statistical mechanics[13] gives the average velocity of a particle in a dilute gas to be

$$V_o = \bar{V} = \sqrt{3KT/m} \tag{95}$$

where K is Boltzmann's constant and the temperature, T , is fixed by the physical experiment. The time interval, Δt, can now be determined from Equation (94) as

$$\Delta t \approx 1/V_o \approx \sqrt{m/3KT} \, . \tag{96}$$

Using Equation (95) in Equation (90) the expression for σ_v is

$$\sigma_v = \frac{\lambda_e V_o^2}{2\pi S} = \frac{3KT}{2m}\frac{\lambda_e}{\pi} \, , \tag{97}$$

where λ_S of the stochastic test problem is equal to λ_e, of the physical experiment.

From the physical experiment, S and L have nearly the same value, which means that in the stochastic test problem the wavelength in the region for the slit to the detector will be two times greater than the wavelength in the region from the source to the slit (Equations 92 and 93) for the value of σ_v . This should cause a blurring in the fringes of the intensity distribution pattern.

V. RESULTS AND DISCUSSION

In the stochastic test problem, the particles were emitted from a line source located above ($Y>0$) a single-line-slit. The position, Equation(63), of a particle at each step, Y_n, was conditioned by a random increment, ΔV_n, in the velocity, Equation (56). The random velocity increment was calculated through a Gaussian transition probability distribution with mean zero and a standard deviation, σ_V (Equation 78). The time parameter, Δt, was chosen to be nonstochastic, meaning all time intervals are equal. Each particle was released from the line source with an initial velocity in the negative Y-direction. The initial velocity of each particle in the X-direction was set equal to zero. This assured that the movement of the particles in the X-direction were not biased. The position of each particle was calculated until the particle went from positive Y to negative Y. When this condition occurred, the slope of the lines from the position (X_{n-1},Y_{n-1}) to (X_n,Y_n) and from (X_{n-1},Y_{n-1}) to edges of the slit, was calculated and compared, to determine if the slope of the line from (X_{n-1},Y_{n-1}) to (X_n,Y_n) lay between the slopes from (X_{n-1},Y_{n-1}) to the edges of the slit. If this condition was true, then the particle went through the slit. If this condition was false, the particle hit the wall and its motion was terminated. In this case another particle was released from the line source and the above procedure starts again.

If the stochastic particle went through the slit, its position was calculated continually until the particle traveled a "slit-to-detector" distance L. When the Y-position of the particle passed the point L, the equation of the line from (X_{n-1}, Y_{n-1}) to (X_n, Y_n) was constructed. Using Y was equal to L at the detector along with the equation of the line from (X_{n-1}, Y_{n-1}) to (X_n, Y_n), the X-intercept was found and the final position of the stochastic particle was known. The particle was then placed in the "detector box" corresponding to the position of (X,L).

The above procedure was repeated until a total of 2×10^5 particles had been emitted from the line source. An intensity distribution was formed by counting the number of particles in each detector box as a function of distance from the center of the slit.

The values for the above parameters along with the values for the parameters used in the physical experiment of Leavitt and Bills are given in Tables 1 and 2.

Before discussing the intensity distribution of random particles, let us look at a weak point in the design of the stochastic test problem. The damping term in the stochastic test problem was omitted to be in accord with the arguments of Nelson given in Section II. This omission of the damping term meant that the relaxation time of the particle was never reached. This led to a wavelength for the stochastic test problem which was changing in time, or with distance from the source (Equation 87). In the stochastic experiment, by using the parameters of the Leavitt and Bills experiment, the wavelength at the detector was twice the wavelength at the slit. By including damping, the relaxation time of the particle would be reached and an effective wavelength could be defined for all regions of the stochastic experiment.

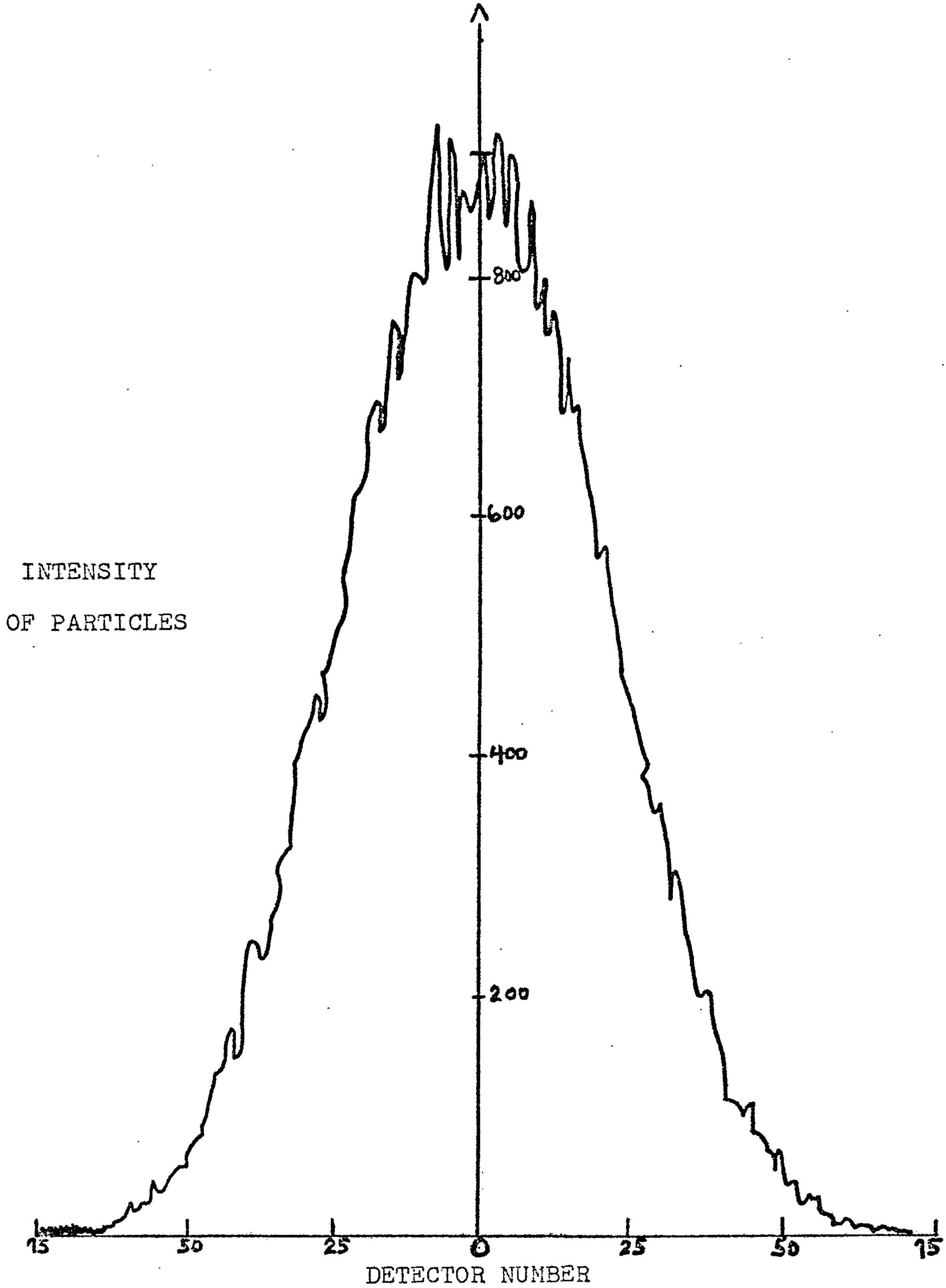
Obviously, using a time-dependent wavelength was not a desirable condition, and implies that either: this was not the proper stochastic test problem because the wavelength was changing in time or that the identification of stochastic quantities to quantum quantities made in Equation (80) and (84) was not correct.

To show the effect that wavelength has on the stochastic test problem, the intensity distribution shown in Figure 4 was calculated using a wavelength approximately two thousand times greater than the wavelength used for the distribution shown in Figure 5. In Figure 5, symmetry has been used to increase, in effect, the number of particles collected in each detector-box. This increase of particles is accomplished by adding together the number of particles in the detector-boxes which are at equal distances from the center of the slit on the right and the left. This means that the intensity distribution is forced to be completely symmetric as it would in fact be for an infinite number of particles. The area of interest, in the comparison of the intensity distribution of the stochastic experiment and the experiment of Leavitt and Bills, is the top portion of the distribution curve.

Figure 6 is an expanded view of the top portion of the symmatrized intensity distribution. Figure 7 is an expanded view of the top portion of the actual intensity distribution without the forced condition of symmetry. In both Figures 6 and 7 the fluctuations in the top portion of the curve are within the limits of normal statistical fluctuations, that is the fluctuations are of the order of $\sqrt{N_a}$, where N_a is the number of particles in a detector box. This much fluctuation can be expected by chance alone and hence these deviations are not statistically significant.

Comparing the intensity distribution from the stochastic test problem (Figures 5, 6, and 7) to the diffraction pattern of Leavitt and Bills (Figure 2), the conclusion must be that the stochastic experiment failed to produce an equivalent intensity distribution pattern. Thus, for this case, it would seem that a counterexample has been found for the results of Section II.

A definite conclusion should not be made until the stochastic experiment has been reformulated to include damping. With damping included, the wavelength would not be changing in time and a different intensity distribution might well be developed.



800
600
400
200
INTENSITY
OF PARTICLES
75
50
25
0
25
50
75
DETECTOR NUMBER

FIGURE 4

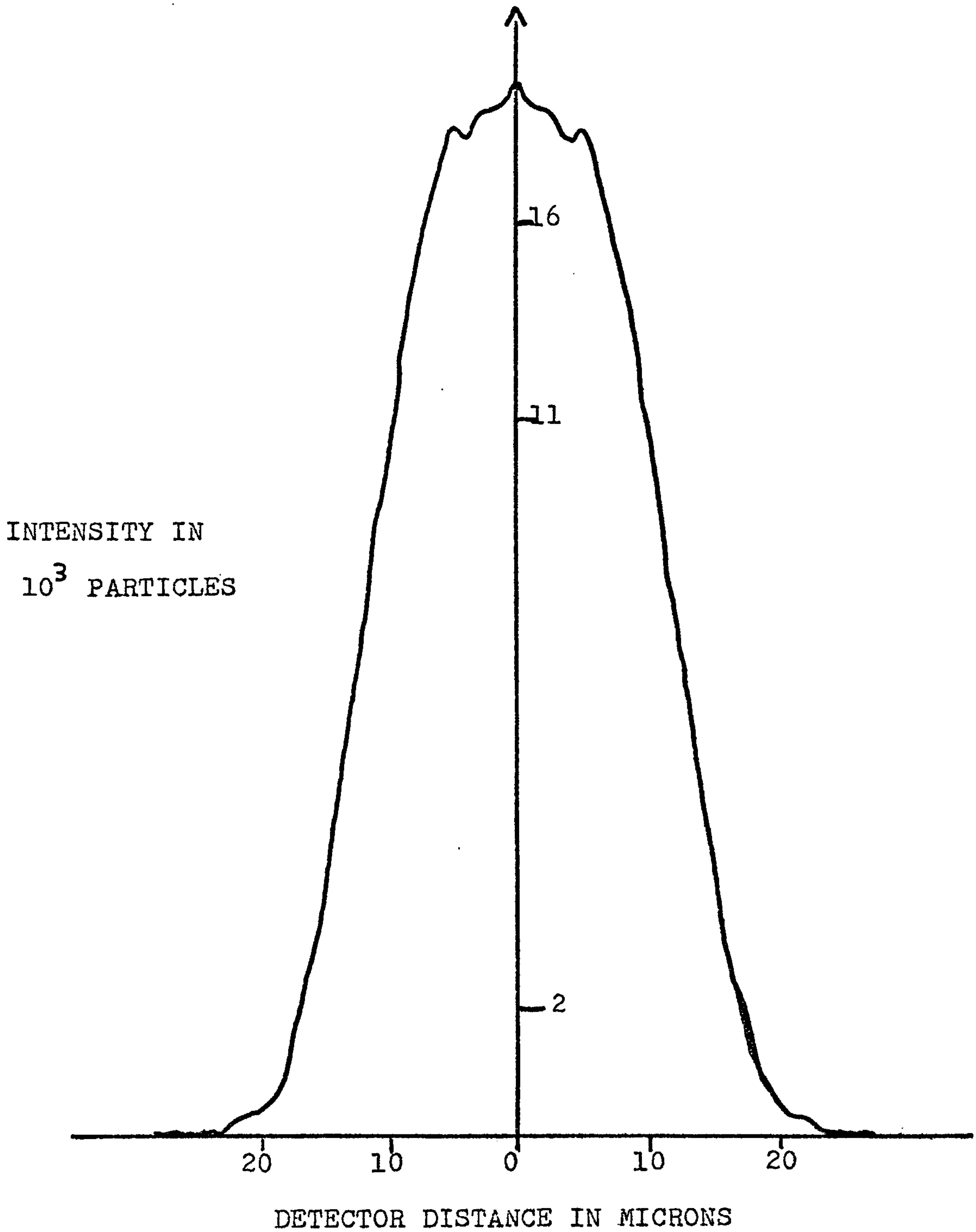

16
11
2
INTENSITY IN
10^3 PARTICLES
20
10
0
10
20
DETECTOR DISTANCE IN MICRONS

FIGURE 5

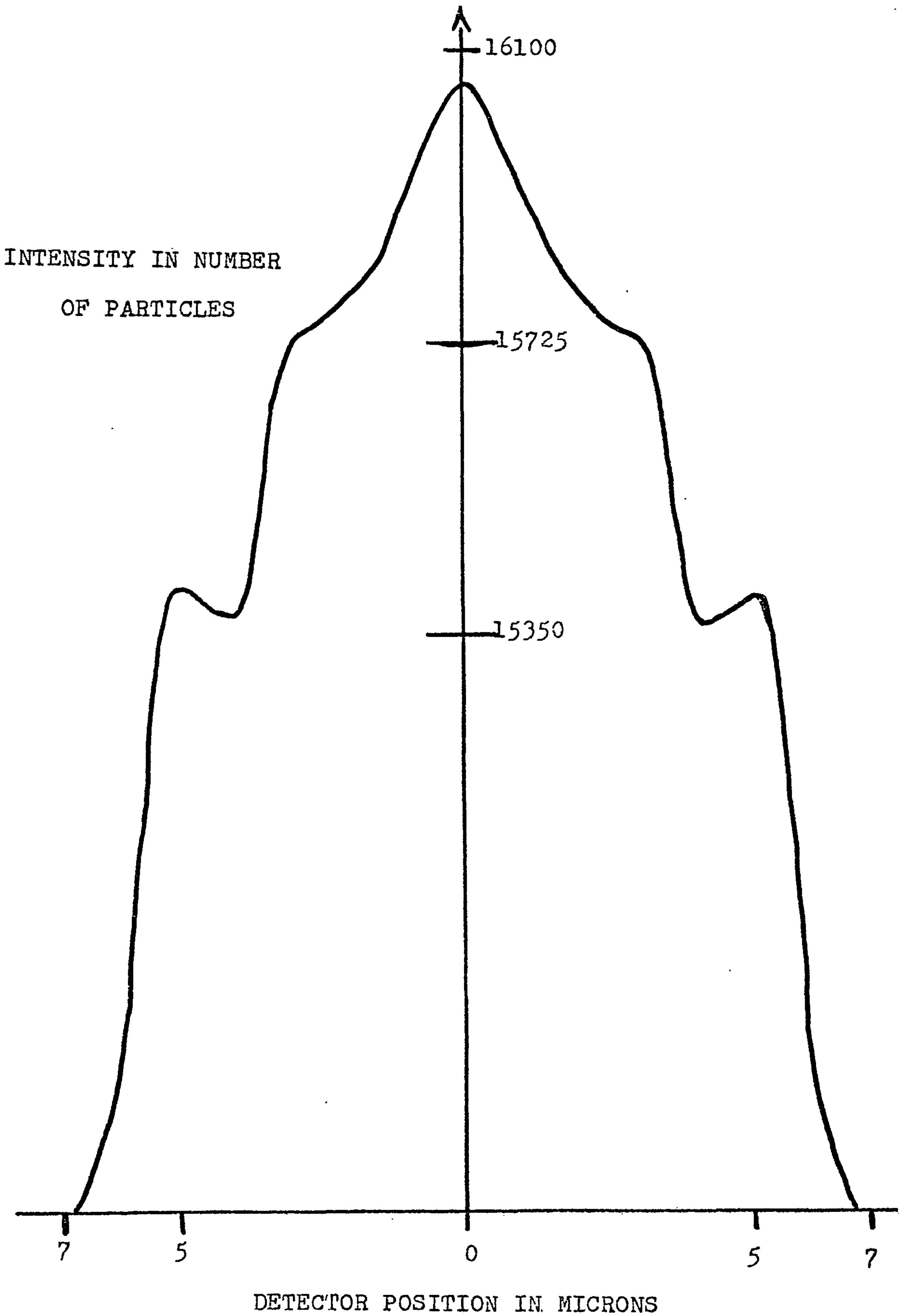

16100
15725
15350
INTENSITY IN NUMBER OF PARTICLES
7
5
0
5
7
DETECTOR POSITION IN MICRONS

FIGURE 6

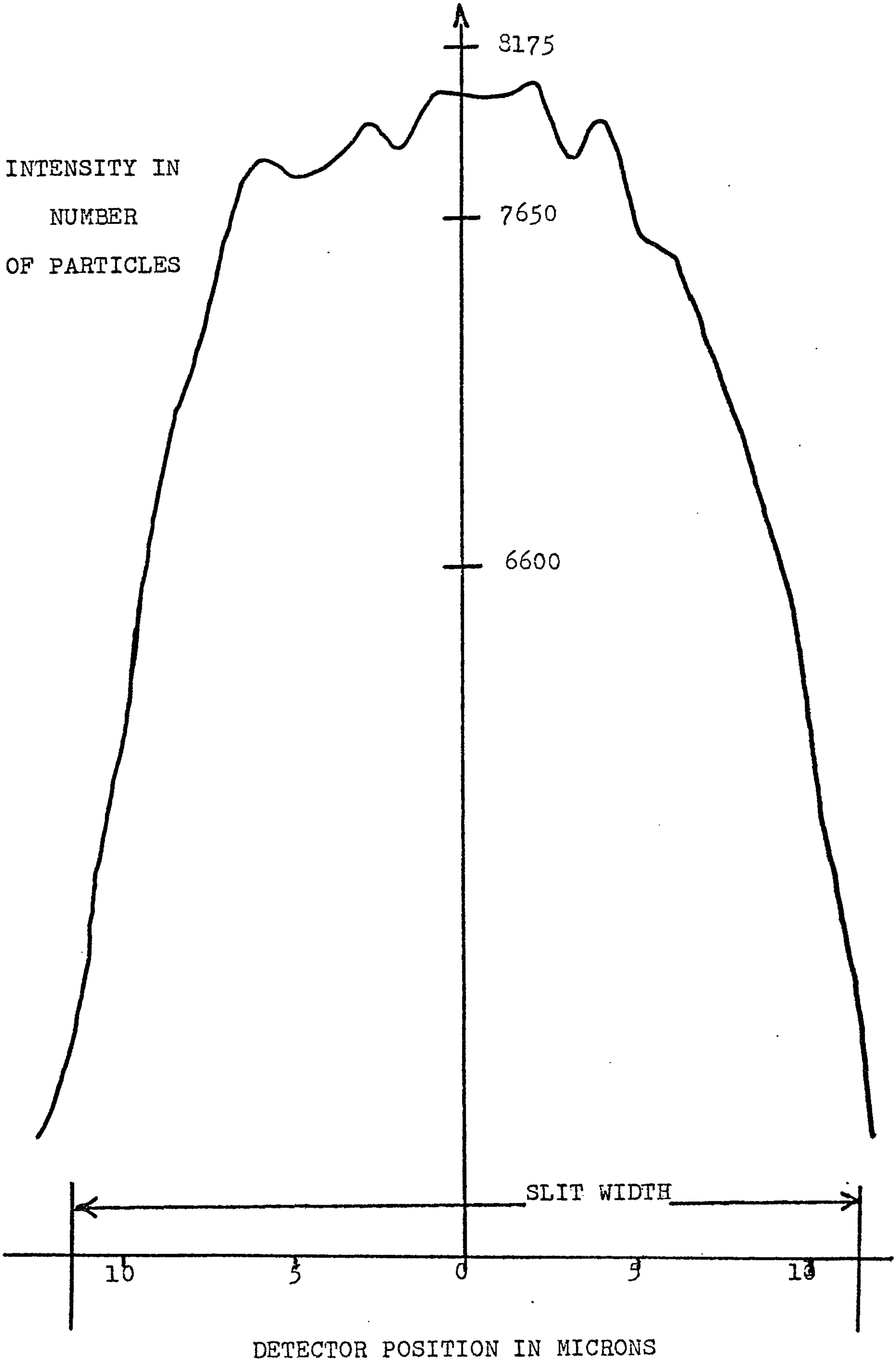

8175
INTENSITY IN NUMBER OF PARTICLES
7650
6600
SLIT WIDTH
10
5
0
5
10
DETECTOR POSITION IN MICRONS

FIGURE 7

PHYSICAL EXPERIMENT

PARAMETER	SYMBOL	VALUE
WAVELENGTH	λ	0.175×10^{-8} cm
SLIT WIDTH	D	23×10^{-4} cm
DETECTOR WIDTH	C	3×10^{-4} cm
SOURCE WIDTH	P	2.5×10^{-4} cm
DISTANCE FROM SOURCE TO SLIT	S	96 cm
DISTANCE FROM SLIT TO DETECTOR	L	100 cm
RATIO OF WAVELENGTH TO SLIT WIDTH	λ/D	0.76×10^{-6}
RATIO OF SLIT WIDTH TO DETECTOR WIDTH	D/P	0.7×10^{-5}
TEMPERATURE	T	533 °K

TABLE 1

STOCHASTIC EXPERIMENT

PARAMETER	SYMBOL	VALUE
WAVELENGTH	λ	0.175×10^{-8} cm
SLIT WIDTH	D	23×10^{-4} cm
SOURCE WIDTH	P	2.5×10^{-4} cm
DETECTOR WIDTH	C	4.48×10^{-4} cm
DISTANCE FROM SOURCE TO SLIT	S	96 cm
DISTANCE FROM SLIT TO DETECTOR	L	100 cm
TIME INTERVAL	Δt	2.54×10^{-5} sec
INITIAL VELOCITY	V_o	3.93×10^{4} cm/sec
STANDARD DEVIATION	σ_v	4.48×10^{-3}
NUMBER OF STEPS FROM SOURCE TO SLIT	n	100
RATIO OF WAVELENGTH TO DISTANCE FROM SOURCE TO SLIT	λ/S	1.82×10^{-11}
RATIO OF WAVELENGTH TO DISTANCE FROM SLIT TO DETECTOR	$\lambda/L+S$	0.892×10^{-11}

TABLE 2

APPENDIX I

COMPUTER PROGRAM FOR STOCHASTIC EXPERIMENT

```
      INTEGER P,S,A,W,M,Z,N(60),TN,PP

      REAL B,XO,CO,D,L,YO,MT,MV,SDT,SDV,VELX,VELY,XF,X(2),
     C    Y(2),IV,V(5000)

  105 FORMAT(' CPUTIME=',I25,'MICROSECONDS'/)

  106 FORMAT(' CPU TIME/PARTICLE='I20,'MICROSECONDS'/)

  107 FORMAT(' NUMBER OF COLLISIONS',E15.8/)

    1 FORMAT( 10I10/)

   20 FORMAT(I10)

  100 FORMAT('  XO=',E15.8,5X,'  YO=',E15.8,5X,'  B=',E15.8,
     C        5X,'  D=',E15.8/      '  L=',E15.8,5X,'  CO=',
     C        E15.8,5X,'  MV=',E15.8,5X,'  MT=',E15.8/  ' SDT='
     C        ,E15.8,5X,' SDV=',E15.8,5X,'  IV=',E15.8,
     C        5X,'  A=',I3 /)

  300 FORMAT(' P=',I8/)

C     GEOMETRY DATA:

      DATA YO,D,L,A/96.,23E-4,100.,30/

C     DYNAMICAL DATA:

      DATA MT,MV,SDT,SDV,M,NNN/2.54E-5,0.,0.,4.48E-3,2000,50/

C     COUNTERS:

      DATA Z,S,P,TN,KK,K/0,0,0,0,1,2/

      DO 16 PP=1,60

   16 N(PP) = 0

      XX = 5.

      IV = -3.93E4

      XX =  XX + 1.

      CALL CPUTIM(IJ)
```

```
      DO 35 JJ = 1,5000
   35 V(JJ) = MV+(SDV/2)*GAUSS(XX)
      CALL CPUTIM(IJJ)
      IIJJ = (IJJ-IJ)*200
      WRITE(6,105)  IIJJ
      XO = -1.25E-4
      B = 1.25E-6
      CO = 1.E5
      WRITE(6,100) XO,YO,B,D,L,CO,MV,MT,SDT,IV,A
      CALL CPUTIM(III)
      AA = L/(-IV*MT)
      WRITE(6,107) AA
    9 S = S+1
      IF ( S .GE. M ) GO TO 2
    3 X(1) = XO + 2.*P*B
      Y(1) = YO
      VELX = 0
      VELY = 0
      GO TO 4
    2 P = P + 1
      S = 0
      IF ( P .LE. 2*nnn )  GO TO 3
      CALL CPUTIM(II)
      IIII = ( II-III )*200 / (M*NNN*2)
      WRITE(6,106) IIII
   25 WRITE(6,1)    ( N(Z),Z=1,60 )
      WRITE(6,300)  P
```

```
      DO 6  Z=1,60
    6 TN = TN + N(Z)
      WRITE(6,20)  TN
      S  =  2*M*NNN
      WRITE(6,20)  S
      GO TO 7
    4 VELX = VELX + V(KK) + V(K)
      VELY = VELY + V(KK+1) + V(K+1)
      KK = KK + 2
      K = K + 3
      IF ( KK .GE. 4998 )  KK=1
      IF ( K .GE. 4996 ) K = 1
      T = MT
      X(2) = X(1) + VELX*T
      Y(2) = Y(1) + ( VELY + IV )*T
      IF ( Y(2) .LT. 0. ) GO TO 8
      X(1) = X(2)
      Y(1) = Y(2)
      GO TO 4
    8 XOX = X(1)+((-Y(1))/(Y(2)-Y(1)))*(X(2)-X(1))
      IF ( XOX .LE. -D/2 )  GO TO 9
      IF ( XOX .GT. D/2 )  GO TO 9
   10 IF ( Y(2) .LT. L )  GO TO 11
      X(1) = X(2)
      VELX = VELX + V(KK) + V(K)
      VELY = VELY + V(KK+1) + V(K+1)
```

```
      KK = KK + 2
      K = K + 3
      IF ( K .GE. 4996 )  K=1
      IF ( KK .GE. 4998 )  KK=1
      T = MT
      X(2) = X(1) + VELX*T
      Y(2) = Y(1) + ( VELY + IV )*T
      GO TO 10
   11 XF = X(1)+((L-Y(1))/(Y(2)-Y(1)))*(X(2)-X(1))
      W = XF * CO
      N(W+30) = N(W+30) + 1
      GO TO 9
    7 END


      FUNCTION GAUSS(X)
      Y=0.
      DO 10 I=1,12
   10 Y = Y+RANDM(X)
      GAUSS = Y-6.
      RETURN
      END
```

```
      FUNCTION RANDM
      DATA I/11111/
      IF(ABS(X).GT.1.) GO TO 10
      I=11111
   10 RANDM= UDRNRT(I)
      RETURN
      END

      FUNCTION UDRNRT(I)
      I = IDRNRT(I)
      UDRNRT = FLOAT(I)*(2.**-35)
      UDRNRT = ABS (UDRNRT)
      RETURN
      END
```

APPENDIX II

INTENSITY OF PARTICLES IN EACH COLLECTOR BOX OF 1 MICRON WIDTH

DETECTOR BOX NUMBER TO RIGHT OF CENTER OF THE SLIT	NUMBER OF PARTICLES IN DETECTOR BOX	DETECTOR BOX NUMBER TO LEFT OF CENTER OF THE SLIT	NUMBER OF PARTICLES IN DETECTOR BOX
1	8033	1	8033
2	8054	2	7870
3	7842	3	7944
4	7937	4	7802
5	7598	5	7779
6	7556	6	7842
7	7222	7	7611
8	6953	8	7221
9	6581	9	6853
10	5929	10	6140
11	5405	11	5528
12	4669	12	4984
13	3860	13	4169
14	3273	14	3467
15	2545	15	2868
16	1874	16	2266
17	1348	17	1636
18	934	18	1178
19	564	19	790
20	325	20	474
21	177	21	294
22	88	22	190

DETECTOR BOX NUMBER TO RIGHT OF CENTER OF THE SLIT	NUMBER OF PARTICLES IN DETECTOR BOX	DETECTOR BOX NUMBER TO LEFT OF CENTER OF THE SLIT	NUMBER OF PARTICLES IN DETECTOR BOX
23	37	23	107
24	20	24	58
25	7	25	35
26	6	26	14
27	2	27	5
28	0	28	1
29	0	29	1
30	0	30	0

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