COUNTING PROCEDURAL SKILL AND CONCEPTUAL KNOWLEDGE IN KINDERGARTEN AS PREDICTORS OF GRADE 1 MATH SKILLS

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Abstract

Though research has identified several possible factors that could be considered precursors of math difficulties in children, including cognitive, language, and number factors, there is not currently a consensus as to which are most critical. The present study focused on the role of two types of counting (procedural skill and conceptual knowledge) in kindergarten to predict math fluency, computation and applied reasoning performance in grade 1, which are direct antecedents of formal arithmetic. Their contribution was examined individually, and in the context of additional number (number identification and quantity discrimination), cognitive (working memory and phonological awareness) and behavior (behavioral inattention) factors. A step-by-step model building method showed that while both types of counting were predictive of each outcome, in the overall models the number factors accounted for variance over and above the counting predictors. Further, the number variables were the best predictors for each model, but secondary variables included verbal working memory and conceptual counting knowledge for fluency, phonological awareness and procedural counting for computation, and verbal and visuospaital working memory, phonological awareness, and procedural counting for the applied reasoning model. Therefore, counting procedural skill and conceptual knowledge should be considered when screening for early math difficulties, but their contributions should be considered along with other relevant number and cognitive factors for more robust prediction.

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Introduction

There is no current consensus regarding the most efficient predictors of later math performance, though many have been discussed. The focus of this study is on the role of the most proximal correlates, procedural counting skill and conceptual counting knowledge assessed in kindergarten, to math performance one year later, while also considering key relevant variables in cognitive domains. The overall goal is to improve clarity regarding the contributions of number and cognitive variables to math skill and their potential use in early identification. First, the prevalence of math difficulties and the importance of early identification will be discussed. Next, the language predictors of math performance will be examined. Then, other correlates of math performance including working memory and behavioral inattention will be described. Finally, number factors relevant to math development will be outlined, with a focus on two types of counting predictors, procedural skill and conceptual knowledge. This is a comprehensive review that includes factors related to math that go beyond the exact variables that are used in the current study, however the specific variables being utilized are explained in each section.

National status of mathematics

Mathematics difficulties are an important concern in the United States. In 2009, the National Assessment of Educational Progress (NAEP) conducted a study of mathematics skills in a sample of over 300,000 nationally representative fourth and eighth-graders (U.S. Department of Education, 2009). Students were assessed on five broad mathematics areas and placed into four achievement levels (advanced, proficient, basic, or below basic). Nationally, only 33% of fourth-grade and 25% of eighth grade students perform at the proficient level. There are eight states in which over 25% of fourth-graders perform below the basic level and 28 states with over

25% of eighth graders performing below the basic level. Rates of mathematical disabilities in children are similar to reading disability rates with approximately 6 to 7% of the school age population having a mathematical disability (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Geary, 1993). Further, 5 to 10% of students will have a learning difficulty in math before graduating from high school (Geary et al., 2009). The national deficit in mathematics skills is concerning since school-age math competence is correlated with ultimate academic achievement, as well as being necessary for everyday functioning later in life. For example, understanding spatial relations is crucial to reading maps, whole numbers and fractions are important for being able to manage money, and probabilities are required for proper financial planning (Mccloskey, 2007). Additionally, with the increase in technological advancements, more jobs require proficiency in mathematics (Clarke & Shinn, 2004).

This continuing concern has led to an increase in the number of research projects dedicated to learning more about both math development and math difficulties within the fields of psychology, education, and medicine. For example, Gersten, Clarke, and Mazzocco (2007) conducted a comparative literature search of reading disabilities and math learning disabilities (MLD) in the Pubmed and ERIC (Educational Resources Information Center) databases, and found that the amount of research on MLD is steadily increasing, even though research studies on RD still outpace those on MLD (a ratio of 14:1 from 1996 to 2005). Two issues are particularly prevalent in the research literature of MLD – a lack of universal agreement on identification or definition of MLD, and the search for what core cognitive processes and academic skills are involved in the early development of math. Even though the latter issue is more central to this study and will be considered in more detail, issues with identification are strongly linked to early cognitive and academic processes in math and will also be discussed.

Current issues in MLD identification

The concerns regarding identification and establishment of reliable precursors of academic difficulty are not unique to MLD; they have also arisen in the research on reading difficulties (RD). For example, one of the factors that led to a better understanding of the deficits as well as more successful interventions in this area was being able to identify, with consistency and at an early age, those who were going to struggle with acquiring reading skills (Hecht, Torgesen, Wagner, & Rashotte, 2001; Stanovich, 1988; Wagner & Torgesen, 1987). Early intervention in RD has been identified as one of the critical variables related to increasing student achievement in general (Clarke & Shinn, 2004). Additionally, specific marker variables and core cognitive processes, such as phonological awareness (PA) and rapid naming, have been successfully identified allowing for proper identification and intervention for RD (Catts, Gillispie, Leonard, Kail, & Miller, 2002; Fletcher, Lyon, Fuchs, & Barnes, 2007; Fletcher et al., 1994). Several developmental precursors for math difficulties have been identified; however, they are not yet as well established as are those for RD. Further, as in studies of RD subtypes, components of math difficulty (i.e. fluency, computation or problem solving) may have different predictors (Fuchs et al., 2008; Geary, 1993).

From studies of mathematics development, several factors are known to contribute to either an adequate or deficient performance. These factors include demographic variables, such as socio-economic status, gender (Jordan, Mulhern, & Wylie, 2009) and ethnic minority backgrounds (Strand, 1999); cognitive skills including working memory (Fuchs, 2006; reviewed in Raghubar, Barnes, & Hecht, 2010); and specific number precursors such as counting principles (Geary, 1999), magnitude comparison (Landerl, Bevan, & Butterworth, 2004) or subitizing (LeFevre et al., 2010). One model of relevant precursors that has been developed includes linguistic, spatial attention, and quantitative pathways (Lefevre et al., 2010) which can be related to neural circuits in the parietal lobe important for numerical processing (Dehaene, Piazza, Pinel, & Cohen, 2003). Consequently, relevant factors can be grouped into three areas: language processes traditionally related to reading; cognitive processes not specific to number; and specific numeric procedural and conceptual development, discussed in turn below.

Language factors in mathematical development and difficulty

Language skills have been associated with math skill development. This makes sense because numbers are stored verbally (Gelman & Butterworth, 2005) and language is required for formal math learning (e.g. to transition from counting to addition; Fletcher et al., 2007). Basic literacy skills have been shown to be predictive not only of reading skills, but also of later measures of math skills (Jordan et al., 2006). As demonstrated in the pathways model by LeFevre et al. (2010), linguistic measures (phonological awareness and vocabulary) contributed uniquely to several mathematical outcomes, and in some cases more so than the quantitative and spatial attention pathways. Further, children with MD who are good readers show greater progress in math development than do those that are poor readers (Jordan, Kaplan, & Hanich, 2002).

Phonological processes (PP) are considered one of the most significant language predictors of later math performance (Savage, Carless, & Ferraro, 2007). One model of PP consists of three parts: phonological memory, rate of access, and phonological awareness (Wagner & Torgesen, 1987). *Phonological memory* refers to the component of working memory called the phonological loop in the Baddeley-Hitch (1985) model (described later). *Rate of access* is the amount of time needed to retrieve phonological information (sound based representations) from long term memory storage. *Phonological awareness (PA)* is the awareness of and access to the sound structure of oral language. Tasks of phonological analysis (ability to identify sounds within words) and synthesis (ability to blend speech segments into syllables or words) are typically used to study PA. Hecht et al. (2001) found that all three of these phonological processes were strongly associated with the relationship of reading to calculations.

Geary (1993) suggested that phonological processes are important for math because computation requires the ability to create and maintain phonological representations. It has been suggested that learning the Arabic numerals and linking them to the appropriate labels is similar to developing lexical mappings when learning to read (LeFevre et al., 2010). Further, speech sound processes are necessary to solve mathematic problems. For example, in order to solve a computation problem, a child may translate the Arabic numbers into their verbal representations. This process is relevant to the retrieval of a phonologically based answer or using a counting based strategy to solve a simple arithmetic problem or more complex math that requires such retrieval as one step in a procedure (e.g. long division; Hecht et al., 2001). It is also possible that the memory representations required for math computations are partly supported by the same memory systems required for decoding and reading (Geary, 1993; 2003), though the exact mechanism is not yet completely understood. Empirically, phonological awareness at ages 4 to 6 has been shown to be related to differences in math computation skills at age 7 (Bryant, Maclean, Bradley, & Crossland, 1990). Savage et al. (2007) found that phonological awareness at age 5 predicted math outcomes at age 11 even after controlling for early literacy skills (word reading, decoding and letter sound knowledge). Wise et al. (2008) found that PA accounted for a significant amount of variance in some math outcome variables (addition and numeration subtests) when a stringent cutoff was used (15^{th} %ile).

Because phonological processes have been associated with math development, it would be expected that reading would be as well, given the well-established connection of phonological processes and reading skills. Further, because reading (RD) and math difficulties are highly comorbid, several studies have focused on comparing groups of children with both reading and math difficulties (MD/RD), to those with MD only or RD only (Fletcher, 2005; Fuchs, Fuchs, & Prentice, 2004; Geary, 1993; Gersten, Jordan, & Flojo, 2005; Jordan, Hanich, & Kaplan, 2003). Robinson, Menchetti and Torgessen (2002) discuss a two-factor theory of math fact learning that distinguishes children with MD/RD from those with MD only. In the MD/RD group, a deficit in phonological processing abilities leads to weak connections between number and number fact representations making retrieval difficult. Alternatively, children with MD only have weak number sense, resulting in not properly associating meaning to a number or to number fact connections, ultimately making retrieval difficult. Landerl et al. (2004) found no quantitative differences between the MD/RD and MD only groups in terms of their pattern of performance on numerical processing tasks with both groups performing poorly relative to controls (on measures of counting, number naming, number writing, and number comparison), though the MD/RD group usually had more errors indicative of more severe impairment.

While both PP and reading skills are related to math performance, they are also related to one another, and deficits of phonological processes are considered to be of greater theoretical importance to math difficulties. Of these processes, PA has been consistently predictive of later math outcomes (Bryant et al., 1990; Savage et al., 2007) and will therefore, be included in the present models.

Cognitive factors in mathematical development and difficulty

Working Memory. Several domains of cognition outside of language and number have also been identified as precursors to later math outcomes including working memory, executive functions (i.e. shifting, inhibition; Epsy et al., 2004), fluid intelligence, and attention (Fuchs et al., 2010); because they are relevant to many outcomes beyond math, these are often referred to as “domain general” predictors. Research has found associations between components of working memory (Rasmussen & Bisanz, 2005) and a wide variety of math skills (i.e. addition and subtraction facts or computations) even while controlling for IQ (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). The relationship of working memory to math development is a complicated one because there are multiple models of working memory which operationalize working memory differently making parallels among models difficult to make. Three models are explained below in an effort to describe these similarities and differences in models and how these are related to various math outcomes.

The three theoretical perspectives described are the Baddeley-Hitch model of working memory (Baddeley & Hitch, 1974), Engle’s model of controlled attention (Engle, Kane, & Tuholski, 1999), and Miyake’s executive model (Miyake, Friedman, Emerson & Witzki, 2000). Several studies show that working memory relates to math skills in the context of models of both controlled attention (Colom, Escorial, Shih & Privado, 2007; Tuholski, Engle, & Baylis, 2001) and of executive function (Bull & Scerif, 2001; van der Sluis, de Jong, & van der Leij, 2007). However, most relations of working memory with math have been tested using the Baddeley model (Geary, 1993; McLean & Hitch, 1999; Meyer, Salimpoor, Wu, Geary, & Menon, 2009; Rasmussen & Bisanz, 2005). Therefore, after briefly reviewing the other models, this review will focus on the math relationship with Baddeley’s model.

The first model of working memory is that of controlled attention, which is associated with capacity (Engle, Tuholski, Laughlin, & Conway, 1999). It is described as a domain-free (general) attentional capacity to actively maintain or suppress representations in working memory (Engle et al., 1999). In this system, there are considered to be two components. First is *short term memory* which refers to memory representations of visuospatial and phonological information that achieve temporary maintenance of information when activated (Hambrick, Wilhelm, & Engle, 2001). The second component is *domain-free executive attention*, which is important for maintaining task relevant information while also suppressing interfering information that is irrelevant to the task at hand. This component, which is also called working memory capacity, has similarities with the central executive in the Baddeley-Hitch model in that it is a domain general resource (Hambrick, Wilhelm, & Engle, 2001).

In terms of its relation to math processes, Tuholski and colleagues (2001) found that those with shorter controlled attention spans had significantly larger reaction times than those with longer attention spans when counting was required in an enumeration task (i.e. counting *n* number of objects) but the groups had similar reaction times when the enumeration task was in the subitizing range (which is considered a "preattentive" process; Tuholski et al., 2001). Additionally, when two types of distracters (conjunctive – lines of a different color in the same direction as the target; disjunctive – lines of a different color in a different direction than the target) were added to the task, reaction times increased significantly only in the shorter span group, suggesting that controlled attention is a required process for counting.

The next model of working memory, one suggested by Miyake and colleagues (Miyake et al, 2000), is discussed in the context of executive function (EF). Past research has been done concerning the unity, or non-unity, of different constructs within EF including inhibition, shifting sets, and updating or monitoring working memory representations in typically developing preschoolers (Wiebe, Espy, & Charak, 2008) as well as in a variety of clinical populations (Duncan, Johnson, Swales, & Freer, 1997; Levin et al., 1991; Robbins et al., 1998; Welsh, Pennington, & Groisser, 1991). EF subfunctions in the model of Miyake et al. (2000) include inhibition, shifting, and updating. *Inhibition* refers to the ability to deliberately inhibit dominant or automatic responses when instructed or when necessary (Miyake et al., 2000). *Shifting* between mental sets or tasks, also called attention or task switching, has been explained as the disengaging of attention to an irrelevant task in order to actively engage in the relevant task (Miyake et al., 2000). However, another description of shifting suggests that it involves the ability to perform a new operation despite interference or negative priming (Allport & Wylie, 2000). *Updating* involves monitoring and coding new information relevant to the current task, which therefore involves active manipulation rather than passive storage (Lehto, Juujarvi, Kooistra, & Pulkkinen, 2003; Miyake et al., 2000).

Research on 6 to 8 year old children with math difficulties has shown impairments on tasks related to inhibiting a learned strategy and switching to a new strategy, as assessed with the Wisconsin Card Sorting Task (WCST; Bull & Scerif, 2001). However, after accounting for naming skill (i.e. a non-executive factor), this relationship between poor inhibition and switching with math difficulty (assessed with a math fluency measure) was no longer found (in children in grades 4 to 5; van der sluis et al., 2007). Children with lower mathematics abilities were also slower on an incongruent number version of a Stroop task (inhibition), which Bull and Scerif (2001) suggest may be a result of reduced attention for numerical symbols or decreased automaticity of number identification. In a longitudinal study from preschool to grade 3, shifting did predict later math and reading achievement at each grade level, however did not predict achievement over the course of the study (Bull, Espy, & Wiebe, 2008). Further, the shifting effect may change over time, or it may arise from outside factors (e.g., naming) or differ according to the type of outcome (Bull & Scerif, 2001). These types of results highlight the general need for clarification in terms of which predictors are considered as well as the target of prediction (i.e. type of mathematics outcome) to derive a more comprehensive picture of relevant factors.

As noted, the Baddeley WM model has been the most thoroughly explored with regard to mathematics. According to this model, there are three primary components of working memory: the central executive, phonological loop, and the visuospatial sketch pad, which all interact (Baddeley; Baddeley & Hitch, 1974). The *central executive* is an attentional controlling system which has several regulatory roles including coordination of access and retrieval from other systems (Baddeley & Hitch, 1974; Gathercole, Pickering, Knight, & Stegmann, 2004; Rasmussen & Bisanz, 2005), and also a supervisory role over the integration of information from the visuospatial sketchpad and phonological loop (Wu et al., 2008). The *phonological loop* is a temporary storage mechanism for maintaining and rehearsing verbal information that is subject to rapid decay; the *visuospatial sketch pad* is responsible in a similar way for storage of visuospatial material (Gathercole, Pickering, Ambridge, & Wearing, 2004).

The central executive is related to math problem solving ability (Rasmussen & Bisanz, 2005), though other components of this model also have demonstrated relationships with other types of math skill. For example, the phonological loop appears to be important for counting and holding information in complex calculations (Mclean & Hitch, 1999). There is also evidence of poor performance on phonological loop measures (digit span) being related to poor problem solving abilities (Rasmussen & Bisanz, 2005). These difficulties could be explained through the vulnerability of the phonological loop to rate of decay, which would be related to difficulties in holding problem associations (Fuchs et al., 2006). Finally, visuospatial working memory is related to numerical estimation (Khemani & Barnes, 2005), and poor performance on visuospatial working memory tasks has been associated with deficits in early (preschool) nonverbal math achievement (Rasmussen & Bisanz, 2005). In relation to one another, verbal working memory (phonological loop) has been shown to be more predictive of math computational skills than visuospatial working memory in some studies (Keeler & Swanson, 2001) and in a meta-analysis by Swanson and Jerman (2006), verbal working memory had the largest effect size when discriminating children with math difficulty from average achieving children. In contrast, Holmes and Adams (2006) found that both the central executive and visuospatial sketchpad uniquely predicted variance in curriculum based measures of math computations for children ages 7 and 8, while the phonological loop did not.

Studies have also shown that children may rely on different components of working memory at different stages of development, with a shift in reliance from visuospatial working memory to verbal working memory with age (Rasmussen & Bisanz, 2005). For example, in a study comparing 5 and 10 year old children, the younger children relied more heavily on visual working memory (visuospatial sketch pad) while the older children relied more on verbal working memory (Hitch, Halliday, Schaafstal, & Schraagen, 1988). In a study of 7 to 8 year olds, Holmes and Adams (2006) found that the central executive and the visuospatial sketchpad were more predictive of math outcomes than the phonological loop. Older children (9 to 10 years old) in this study were found to use phonological loop for easy problems while using the visuospatial sketchpad for more difficult problems. Similarly, working memory assessed by the counting span task was related to performance on simple and complex problems in grade 1, but not in grades 3 or 5 for children with and without MD (Geary, Hoard, Byrd-Craven, & DeSoto, 2004). However, there are exceptions as another study found that the central executive and phonological loop significantly predicted math reasoning scores in 2nd grade, while in 3rd grade the visuospatial sketchpad was predictive of mathematical reasoning and numerical operations (Meyer, Salimpoor, Wu, Geary, & Menon, 2009).

Each model reviewed has contributed to the understanding of how different aspects of working memory relate to math development. However, the wide variety of measures of working memory, as well as math outcomes, has made definitive conclusions about this relationship difficult. In general, children with math difficulties have deficits in working memory including difficulty holding material related to the phonological loop in memory in order to solve problems (Mclean & Hitch, 1999) and difficulty inhibiting and set shifting which can lead to deficits in solving math problems (skills associated with the central executive; Bull & Scerif, 2001; Rasmussen & Bisanz, 2005; Raghubar et al., 2010). Since the relation of working memory and math performance is complicated, it is important to examine multiple components of working memory and math outcomes as well as appropriately operationalize the measures and model used to do so. This is particularly important as many commonly used measures of working memory in the above studies involve numbers or counting (e.g., Digit Span, Counting Span; Raghubar et al., 2010). The present study will focus on measures of both number and non-number based storage and manipulation. Baddeley and Hitch would likely classify these as phonological and visuospatial working memory, respectively. More basic elements of number and language are also considered, given that working memory measures also involve these processes.

Attention. Attention is another important factor associated with mathematics difficulties. Although there are similarities in the theoretical frameworks of attention and working memory (i.e. Engle's model of controlled attention), their combined contributions to math development have not yet been adequately studied (Raghubar et al., 2010). Attentional resources have been suggested to be necessary for children to initiate and direct their processing of information, comprehend, and retrieve information for different tasks (Geary, Hoard, & Hamson, 1999).

Two rather different ways of assessing attention have been studied in children with MD: sustained attention or vigilance, usually assessed by continuous performance tests (Huckeba, Chapieski, Hiscock, & Glaze, 2008; Lindsay, Tomazic, Levine, & Accardo, 2001); and behavioral inattention, typically measured with parent and teacher rating scales (Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Fuchs et al., 2006; Raghubar et al., 2009). A study of children with Tourette's Syndrome (TS) examined the relations between sustained attention (using the Test of Variables of Attention, TOVA; Greenberg, 1990), visuospatial ability (Beery Visual Motor-Integration; Beery, Buktenica, & Beery, 2006) and arithmetic achievement (K-TEA computations and KeyMath calculations given in structured and unstructured settings; Huckeba et al., 2008). The results showed that while children with TS as a whole had attention and math difficulties, children with TS whose attention skills were within the average range did not differ from typically developing children on arithmetic performance. This suggests that inattention contributed to the arithmetic deficits beyond the effect of TS.

Another study (Lindsay et al., 2001) looked at the relation between a continuous performance task (Conner's Continuous Performance Test, (CPT; Conners, 1994) and math performance in two mutually exclusive groups defined on the basis of discrepancy of IQ and math performance: those with dyscalculia (a 15 point IQ-achievement discrepancy score *only*) and low functioning dyscalculia (15 point IQ-achievement discrepancy score *with* arithmetic score below 25th %ile). Both groups made more inattentive errors (omission errors) and were inconsistent (indexed by high standard error of response time) relative to controls. The low functioning group was more inconsistent and prone to errors than the dyscalculia group. Further, measures of inconsistency and impulsivity (commission errors) were the only variables to significantly predict arithmetic scores over and above IQ and reading scores, suggesting that these variables may be more related to math in this type of assessment than the inattentive (omission errors) variable.

The second way of assessing attention, which is strongly related to math performance, is measures of behavioral inattention, which is a reduced ability to maintain the focus of attention (Fuchs et al., 2005; 2006). Research has shown that high ratings of behavioral inattention are related to lower academic achievement (Merrell & Tymms, 2001). Behavioral inattention ratings have been shown to be predictive of arithmetic skill (adding and subtracting single digit numbers), algorithmic computation and arithmetic word problems in first graders (Fuchs et al., 2006). Further, in a model relating numerical competencies (arithmetic, algorithmic computation, and arithmetic word problems) to various measures, Fuchs et al. (2006) found behavioral inattention to be the only factor to uniquely predict all three types of math competencies. Behavioral inattention was also a unique predictor of estimation skill in children in grade 3 (Seethaler & Fuchs, 2005). Raghubar et al. (2009) found that higher levels of ratings of inattention in children in third and fourth grade were related to higher multi-digit computation and math fact errors. As well, children in the math learning difficulty group (MLD) were rated as more inattentive than a low achievement group (LA) and a control group (Cirino et al., 2007). Similarly in a study by Duncan et al. (2007), teacher ratings of attention were found to be a modest, but consistent predictor of later academic achievement across several ages (age 4 – 5 to age 13 – 14). Children with short attention spans will have difficulty remember instructions and steps to solve problems. It has been suggested that a limited working memory capacity contributes to inattentive behaviors in children and these deficits will often co-occur (Gathercole et al., 2007). For example, children identified on the basis of having poor working memory skills exhibited similar behavior profiles involving primarily inattentive behaviors (i.e. difficulty attending in class, making careless errors, high levels of distractibility) rather than hyperactivity (Alloway, Gathercole, Kirkwood, & Elliot, 2009).

In line with behavior ratings, a common way that the influence of attention with regard to math is studied is in the context of attention deficit hyperactivity disorder (ADHD), because ADHD and math difficulties show strong overlap. For instance, the incidence of MD in children with ADHD is about five times that of the general population (31% vs. 6-7%; Zentall, 2007). Similarly, about 31% of students with MD in numeric operations also had ADHD (Mayes, Calhoun, & Crowell, 2000). The persistent inability or difficulty to automatize basic computation skills seen in children with ADHD may be due to their difficulty sustaining attention long enough to adequately learn the information (Zentall, 2007). Interestingly, among several learned skills, (number computations, low and high imagery words) only number computations was not automatized in children with ADHD as compared to age and IQ-matched peers (Ackerman, Anhalt, & Dykman, 1986). Children with ADHD respond more quickly to problems than their typically developing peers (Banaschewski et al., 2003). Inattention and disorganization have been shown to be related to difficulties with math computation (Marshall & Hynd, 1997). Research on the influence of medication within ADHD (specifically methylphenidate variations) found children on medication improved in number correct on math computation measures (Lopez, Silva, Pestreich, & Muniz, 2003 & Muniz, 2003). Comparing children with and without the hyperactivity component of ADHD, researchers have reported significantly lower scores on measures of mathematics computation skills for children with predominately inattentive problems relative to those with hyperactivity (Marshall, Schafer, O'donnell, Elliott, & Handwerk, 1999).

In sum, research involving children with ADHD and MD has contributed to a better understanding of the attention problems related to deficits in math performance, specifically difficulty with automatizing number facts. Both sustained attention and behavioral inattention have been shown to play a role in math development. However, only the two above referenced studies focused on sustained attention and math development. The Huckeba et al. (2008) study, however, utilized a specific population (e.g. TS), which limits the generalizability of the results, and the Lindsay et al. (2001) study did not find the inattention variable to be uniquely predictive of math outcomes. Because of these reasons as well as the larger representation of behavioral inattention in the literature, behavioral measures of inattention will be considered in this study.

Number factors in mathematical development and difficulty

So far the discussion has primarily focused on the relation of domain general cognitive variables and language with math performance. However, as predictors increase their similarity with the criterion (math performance), stronger predictive relationships might be expected. This appears to be the case in RD as PA and decoding skills have been suggested to be the best predictors of later reading skills (Wagner & Torgesen, 1987). Similarly, with respect to interventions, it is more beneficial to teach an academic skill rather than provide training in a general area or core cognitive process and expect that training improve the target academic skill (Fletcher et al., 2007). Therefore, the following section will focus on domain specific number variables as predictors of math performance.

Number sense, or number competence, has been suggested to be as important to mathematics learning as phonemic awareness is to reading development (Gersten & Chard, 1999). However, there is a lack of clarity as to how number sense is defined. According to Geary et al. (2009), number sense includes non-verbal and implicit understanding of both absolute and relative magnitude of sets of non-symbolic (i.e. objects or dots) or symbolic (i.e. Arabic numerals) factors. Another definition includes having the ability to add and subtract small quantities, compare magnitudes, and count, or more generally an understanding of numbers and their relationships (magnitudes, comparisons, calculations, etc.; Locuniak & Jordan, 2008). The development of the child's facility and flexibility with numbers is influenced by their environment, which includes informal teaching by parents or other adults prior to schooling (Robinson et al., 2002). However, studies have shown number sense, specifically quantity discrimination, to be evident in infants as young as 6 months with improvements in discrimination of smaller ratio sets between 6 and 9 months (Lipton & Spelke, 2003). Having a well-developed number sense is hypothesized to lead to a better ability to solve calculations (Gersten & Chard, 1999; Locuniak & Jordan, 2008). Several models of precursors have included number related or quantitative knowledge (Aunio et al., 2006; Jordan, Kaplan, Nabors Olah, & Locuniak, 2006; Lefevre et al., 2010) in predicting math outcomes. For example, LeFevre and colleagues (2010) found quantitative knowledge (non-symbolic estimation) to be related to both concepts of math procedures and computations. Number sense is related to conceptual understanding of math procedural knowledge (ability to use algorithms to solve problems; Chong & Siegel, 2008), which is why it has been suggested that early interventions that involve improving number sense should be implemented with children who demonstrate weak number sense (Gersten & Chard, 1999).

Using a number sense battery developed to screen kindergartners at risk for learning difficulties, Jordan and colleagues (Jordan et al., 2006; 2008; 2009) examined the contribution of number sense to mathematical development in a series of studies. An exploratory factor analysis of the battery resulted in two dimensions, basic number skills and conventional arithmetic (Jordan, Kaplan, Olah, & Locuniak, 2006). The basic number skills factor included measures of counting (count to 10 and identify correct/incorrect counts), number identification, number comparison, nonverbal calculation (addition and subtraction using chips), estimating dot set sizes, and patterns within number and color combinations. The conventional arithmetic factor included measures of story problems (same addition and subtraction problems as in nonverbal calculation presented orally) and number combinations (same addition and subtraction problems as in nonverbal calculation presented orally, “How much is 3 and 2?”). Their model is consistent with a previously described number sense dimension of higher (secondary skills including conventional educational activities) and lower functions (elementary knowledge; Dehaene, 2001).

Jordan and colleagues (2008) found that number sense in kindergarten predicted calculation fluency over and above age, reading, vocabulary, working memory and spatial reasoning, with number combinations (e.g. How much is 2 and 1?) and number knowledge (e.g. Which is bigger, 4 or 5?) being uniquely predictive. The authors used the number sense battery to identify kindergarten students at-risk for mathematical difficulties. Performance below the 25^{th}%ile on both of the significant predictors in their model (number knowledge and combinations) was used as the screening cut off score. The authors report a true negative rate of 84%, where children who were *not* identified by the screening measure were also *not* identified as having a difficulty in grade 2 on a calculation fluency measure. Further, they achieved a true positive rate of 52%, where those identified by screening measure continued to have a difficulty in grade 2 on a calculation fluency measure. Another study by Jordan and colleagues found that number competence level in kindergarten, as indicated by performance on the battery as a whole, was predictive of a composite math measure (calculation and math problem solving) at the end of grade 3 (Jordan, Kaplan, Ramineni, & Locuniak, 2009). Additionally, the rate of growth from kindergarten to grade 3 in number competence was a significant predictor of 3rd grade math achievement level.

Cirino (2011) explored the relation of number sense factors to math in kindergarten using a latent factor approach. Five latent number variables were found: symbolic comparison, non-symbolic comparison, symbolic labeling, rote counting, and counting knowledge. All factors, with the exception of non-symbolic comparison, were strongly related to small sums addition (single digit addends), with counting knowledge above .60 and symbolic labeling above .70. When linguistic ability (PA and rapid automatized naming) and spatial working memory were also included, these skills were predictive of small sums addition as well, but their effect was mediated by measures of symbolic quantity, including counting. However, this study focused only within kindergarten and a single mathematical outcome. The present study utilizes a subsample of this work, but is focused on more and later assessed mathematical outcomes.

Number sense is clearly important for math performance, but is assessed in numerous ways. An important distinction exists between symbolic and non-symbolic factors. While non-symbolic factors are emphasized in many conceptualizations of number sense (e.g., Butterworth, 2005), symbolic aspects of number predictors, such as counting, are more predictive of math outcomes than non-symbolic number based factors (Cirino, 2011). Further, research has shown that children with MLD are more impaired on symbolic tasks than non-symbolic tasks (De

Smedt & Gilmore, 2011). Therefore, it is these symbolic factors (number identification, quantity discrimination) that will be emphasized with regard to number in the current study. In doing so, the present study considers symbolic factors that do or do not involve counting per se separately from one another.

Counting in mathematical development and difficulty

The role of counting per se is particularly important for later math skill given its explicit role in the transition to formal arithmetic (Geary, 2004), and given that counting is suggested to be a fundamental skill in relation to future mathematical achievement (Carrasumada, Vendrell, Ribera, & Montserrat, 2006). Counting is also considered the first formal computational system (i.e. with a specific set of rules and language) a child acquires (Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989). Empirically, counting skill and knowledge have been identified as strongly predictive components of number sense to math skill (Geary, 1993; Stock, Desoete, & Roeyers, 2007), but there are few studies which examine its impact separate from other measures of numerosity (and other constructs). Counting can be further partitioned into separate but related components of procedural skill and conceptual knowledge. An immature understanding of the counting principles and the increase in the amount of procedural errors in counting made by children with MD are related to poor arithmetic skills in young children (Geary, 2004). Similarly, an understanding of counting principles contributes to the development of addition skills (Geary, 2004). These two aspects of counting are further discussed below.

Procedural counting. Procedural counting skill refers to the ability to correctly sequence numbers (Koponen, Aunola, Ahonen & Nurmi, 2007), and for present purposes is closely associated with numerical sequencing without reference to external stimuli. It is typically assessed with measures such as counting objects, successfully identifying the number of an object in an array, oral rote counting to a specific number, or counting backward (Lefevre et al., 2006). Geary (2004) suggests that procedural knowledge is supported by language systems. In a study exploring the relationship among number sense components (number identification, counting, quantity discrimination, and missing number – e.g., Which number comes next?), correlations of counting with the other number tasks were the lowest, suggesting it may contribute unique information about math development and performance (Clarke & Shinn, 2004), as opposed to composite number sense batteries. LeFevre and colleagues (2006) found procedural counting skill to increase linearly from kindergarten to grade 1 in terms of both accuracy and speed.

Procedural counting skill is associated with MLD regardless of IQ or reading difficulties (Geary, 2007). It has been suggested that counting contributes to success in mathematics development for at least two reasons. First, counting allows for the automatic use of math related information which would permit other cognitive resources to be devoted to more complex tasks, such as problem solving (Gersten & Chard, 1999; Resnick, 1989). For example, as counting is used to solve addition problems early in development, the correct solution becomes associated with a specific problem (i.e. that 2 plus 3 is always 5). Also, counting functions as a back-up strategy to retrieval in the learning of new arithmetic knowledge (Jordan et al., 2006).

Conceptual counting. Conceptual knowledge refers to the child's *understanding* of counting procedures; that is, knowledge of principles that govern how and why counting works. Gelman and Galistel (1978) described five counting principles which have been divided into essential and non-essential principles based on whether or not mastery of the principle is required for correct counting (Briars & Siegler, 1984; Kamawar et al., 2010; Laupa & Becker, 2004). The essential principles (or the how-to-count principles) include one-to-one correspondence, stable-order, and cardinality; the non-essential principles are abstraction and order-irrelevance. The developmental trajectory of these types of counting knowledge has not been fully determined. Some research has shown conceptual knowledge develops prior to procedural knowledge (Gelman & Gallistel, 1978; Gelman & Meck, 1983), while others propose it is procedural knowledge that develops first (Briars & Sigler, 1984; Frye, 1989). It is also possible that the developmental timing of these counting components depends on the type of task, or that their development may be iterative (Rittle-Johnson, Siegler, & Alibali, 2001).

The one-to-one correspondence principle involves understanding that one must tick off items in an array using one and only one tick for each individual item. Gelman and Galistel (1978) state that in order to follow this principle, a child has to coordinate *partitioning* (step by step maintenance of items that have to be counted and those that have already been counted) and *tagging* (the summoning up of distinct tags one at a time) such that these two processes both begin and end together. An example of a strategy that would ensure this coordination is pointing to the items while counting. Violations of this principle include counting an item more than once or skipping an item (error in the partitioning process), using the same tag twice (error in the tagging process), and a failure to coordinate partitioning and tagging. There is evidence of the ability to partition appropriately as young as 3 years old (Potter & Levy, 1968; Sophian, 1988). Wynn (1992) found evidence of comprehension of the one-to-one principle in children ages 2 to 3, however, a good understanding of the principle may not be established until age 5 (Briars & Siegler, 1984).

The stable order principle requires the tags used to correspond to items in an array must be chosen in a repeatable (or stable) order (Gelman & Galistel, 1978). This requires a stable list that is as long as the number of items in the array. Further, the extent to which children are able to adhere to this principle is related to set size (i.e. the larger the set size, the more difficult it is for children to abide by the principle). There is evidence children have established this principle in kindergarten (LeFevre, et al., 2006) with most children in grade 1 mastering an understanding at an adult level (Stock, Desoete, & Roeyers, 2009).

The cardinality principle reflects the understanding that the number tag applied to the final item in the set represents the total number of items in the set, giving the final tag a special significance (Gelman & Galistel, 1978). In other words, the child must be able to indicate the last number assigned represents the numerosity of the array. This principle has a developmental relationship to the one-to-one correspondence and stable order principles such that it incorporates the two and should develop later. There is debate surrounding when children are able to master this principle. Gelman and Meck (1983) suggest children master the principle by age 3 while other suggest it is only beginning to be understood at age 3 and half (Wynn, 1992) or even not until 5 years of age (Freeman, Antonucci, & Lewis, 2000). A lack of understanding of this principle is usually evidenced when after counting an array of objects, the last number counted does not equal the answer to the question "How many all together?"

The abstraction principle involves the understanding that the essential principles can be applied to any array or collection of units (i.e. blocks, animals, objects; Gelman & Galistel, 1978). The order-irrelevance principle states that the order of enumeration is irrelevant, or in other words, the order in which items are tagged does not matter (Gelman & Galistel, 1978). Adults understand that the order in which items are partitioned and tagged (the processes required for one-to-one correspondence) is not important. This principle requires the understanding and incorporation of the previous principles in that an item is a *thing* not a one or a two (abstraction), the verbal tags are arbitrary (stable order) and the same cardinal number results regardless of the order of counting (cardinality). Therefore this last principle is also concerned with the understanding of some of the properties of numbers, not just the ability to count.

Comparing procedural and conceptual counting knowledge in math development. Studies using combined measures of procedural skill and conceptual knowledge show a clear relation with math performance. Aunola and colleagues (2004) examined growth trajectories of overall math competence (using a measure which includes basic arithmetic, knowledge of numbers, and word problems) in preschool to grade 2 and found counting ability to predict both initial math performance as well as growth. Procedural skill and conceptual knowledge may also predict math skills differentially. For example, counting ability (procedural skill) has been shown to predict single digit calculation fluency (Koponen et al, 2007). In a study using a combination of the essential counting principles (stable order, one-to-one, and cardinality) in kindergarten to predict grade 1 arithmetic and numerical facility (math facts), Stock and colleagues (2009) found conceptual knowledge to account for 14% of the variance in math computations and 5% of the variance in number fact knowledge. Duncan and colleagues (2007) found that cardinality as a measure of school readiness was one of the most predictive factors of later school achievement.

Mathematical development progresses in a hierarchical manner such that basic skills are learned first in order for more complex skills to develop and allow for redistribution of attention (Aunola et al., 2004). In typical development of arithmetic skills (i.e. addition or subtraction computation), children use either finger counting strategies or verbal counting strategies. Regardless of the type of strategy utilized, common counting procedures have been identified, “counting on” (min or max) or “counting all” (or sum; Geary, 2004). Counting on involves stating the larger value addend and then counting a number of times equal to the value of the smaller addend. For example, to add 6 + 4, the child would count 6, 7, 8, 9, 10. Counting on also includes stating the lower value and then counting the appropriate number equal to the larger value to achieve the answer. Therefore the same problem would be solved by counting 4, 5, 6, 7, 8, 9, 10. The counting all procedure involves counting both addends starting from 1. The shift from more frequently using the counting all procedure to the counting on procedure suggests an improvement in a child's conceptual understanding of counting (Geary, 2004). However, children with MLD appear to be delayed in their ability to use counting strategies to solve addition problems. For example, they rely on their fingers for counting longer (rather than developing other strategies), are delayed in adopting the counting on procedure, and have more errors in counting (Geary, 2004; Geary et al., 2009).

In kindergarten, children have already begun to demonstrate an understanding of the essential counting principles (Geary, 1993), with the one-to-one principle possibly being the first to be mastered by most children (Stock et al., 2009). However, children in kindergarten also may believe that the non-essential counting principles are essential demonstrating a rigid and immature understanding of the principles that is most likely a result of observing counting procedures (Geary et al., 2009). Further, LeFevre et al. (2006) found that children with lower math skills performed better than average and high skilled children in kindergarten and grade 1 on identifying unusual but correct counts while still performing more poorly on identifying incorrect counts. This suggests that the ability to separate the essential and non-essential principles may be a more complex task that takes more time to develop. Additionally, the way good versus poor math performers accept unusual counts is not the same. The authors suggested that the non-essential principles are used in initial development of counting ability but are eventually viewed as less critical, beginning around grade 2. LeFevre and colleagues (2008) found that conceptual knowledge (specifically order irrelevance) was related to basic number concepts (KeyMath Test- Revised Numeration subtest which includes counting, sequencing numbers, number identification, and rounding; Connolly, 2000), though only in kindergarten and not in grades 2 or 4. Geary et al. (1999) found that children with MD in grade 1 were less likely to identify incorrect counts and to accept unusual counts as correct, which could suggest an incomplete conceptual understanding.

In summary, two studies found that a combined measure of counting procedural skill and conceptual knowledge was related to math at the same time point (Jordan et al., 2007; Stock et al., 2009) and three studies showed relations across time points (Aunola et al., 2006; LeFevre et al., 2006; Kamawar et al., 2010). For procedural counting skill, one study found a correlation with math outcomes at one time point (Cirino, 2011) and four studies across time points (Geary, 1999; Koponen et al., 2007; LeFevre et al., 2006; 2008). Four studies found conceptual counting knowledge to be related to math performance both at one time point as well as across time points (Geary, 2004; 2007; LeFevre et al., 2006; 2008).

Individual differences in general number sense (e.g., counting, number identification) are more easily detected in older children (Methe, Hintze, & Floyd, 2008; 3rd grade or above) which has led to more studies with older children as they have already acquired some basic knowledge of math (Aunola et al., 2004) and counting skills have been established (Geary, 2004; with the exception of LeFevre, 2006; Aunola, 2004; Geary, 2007 with preschool or kindergarten age children). Geary's work shows that typically developing children in grade 1 are still using finger counting strategies on addition and subtraction problems on 64% of problems, suggesting that counting skills are an essential factor in math development at this age. Therefore, this study focuses on children in kindergarten to grade 1.

Present study

The primary goal of this study is to evaluate counting procedural skill and conceptual knowledge in kindergarten as precursors to three types of mathematics achievement in grade 1: fluency, computation, and applied reasoning. Other number variables (number identification and quantity discrimination), are included to determine what contribution counting procedural skill and conceptual knowledge have over and above other domain-specific predictors. Additionally, previously discussed cognitive (spatial working memory and PA) and behavioral (behavior ratings of attention) predictors are included.

To our knowledge, there are thirteen studies in the literature that involve the relation of counting skills to mathematics performance in children, and so it is important to situate the current study in this context. These studies are summarized in Table 1. While these studies clearly add to our cumulative knowledge, there remain several gaps in the literature that require further clarity.

Of the existing studies, some assessed only conceptual counting knowledge (Geary et al., 2004; Geary et al., 2007; Kamawar et al., 2010), and others include only one outcome measure, such as math reasoning (Geary et al., 2004) or computations (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Aunola, Ahonen, & Nurmi, 2007). Additionally, many studies focused on number (domain specific) factors only and did not test domain general factors (Jordan & Locuniak, 2008; Kamawar et al., 2010; LeFevre et al., 2006; Stock et al., 2009). Some studies use a cross-sectional, rather than longitudinal design (Kamawar et al., 2010; Lefevre et al., 2006), or only one time point (Cirino, 2011). Jordan and colleagues' series of studies (2006; 2008; 2009) include a number sense battery which measures both procedural skill and conceptual counting knowledge; however, they did not distinguish between the effects of procedural skill and conceptual knowledge in predicting level of performance and growth in mathematical skills over time, and looked at the impacts across different math outcomes in separate studies rather than together.

Therefore the contribution of the current study is that it is longitudinal in nature, interested in the relative contribution of multiple number, cognitive and behavioral predictors, as well as multiple mathematical outcomes (fluency, computation, and applied reasoning). Identifying specific skills early in development that contribute to later math difficulties (similar to PA and rapid naming in RD) will allow for earlier and more explicit interventions. For example, the field of RD has specific interventions geared toward the type of RD the child has (i.e. word recognition, fluency, or comprehension), which has been shown to play a large part in the increasing success of reading interventions (Fletcher et al., 2007). Analogously, procedural skill and conceptual knowledge may be associated with different kinds of math outcomes or one may hold more promise for identifying math difficulty later in development.

Hypotheses. The overall hypothesis of this study is that both procedural counting skill and conceptual counting knowledge are important longitudinal predictors of several math outcomes (fluency, computation, and applied reasoning), even in the context of other known relevant predictors. This general hypothesis is built up over five sub-hypotheses in order to assess these relations in a systematic and comprehensive manner.

First, the relation of procedural counting skill and conceptual counting knowledge to each of the three math outcome measures, fluency, computation, and applied reasoning, will be evaluated; each of the counting variables are expected to demonstrate a significant relation with each of the math outcomes, based on the literature reviewed above.

Second, the relation of the number, cognitive and behavioral variables to fluency, computation, and applied reasoning will also be investigated, with each again expected to show significant relations to all three math outcomes.

The third hypothesis involves evaluating the unique contributions of procedural counting skill and conceptual counting knowledge in predicting fluency, computation, and applied reasoning. There is relatively little literature to guide how procedural skill and conceptual knowledge will predict the specific types of math outcomes; however, we expect that conceptual knowledge will account for variance in the math reasoning model over and above procedural counting skill since reasoning and problem solving involve computations as well as identifying what type of computation to perform. In contrast, procedural skill is expected to account for variance in the math fluency and computation models over above that of the conceptual knowledge variables, as Koponen et al. (2007) found.

The fourth hypothesis evaluates the contribution of the number, cognitive and behavioral predictors for fluency, computation, and applied reasoning. In this regard, the number variables are expected to account for variance over and above the cognitive predictors. While the cognitive and behavioral variables discussed above (PA, reading, working memory, and behavioral inattention) contribute to components of math development, some (Jordan et al., 2009; Locuniak & Jordan, 2008) have suggested that numeric variables, such as number identification, are most central to future math performance as they provide a foundational base for future learning.

Finally in a model that includes all the relevant predictors (counting, number, cognitive, and behavioral), it is hypothesized that the counting variables will account for variance in fluency, computation, and applied reasoning over and above the number, cognitive and behavioral predictors, as previous work has demonstrated symbolic variables to be more predictive of math performance than non-symbolic ones (Koponen et al., 2007; Krajewski & Schneider, 2009; Lefevre et al., 2010).

Method

Participants

Participants were from a larger study of math skills in kindergarten students (Cirino, 2011). Students (N = 194; 48.45% female) from a single large urban district were evaluated in kindergarten (mean age = 6.16, SD = 0.32) and then again in grade 1 (N = 193; mean age = 7.16, SD = 0.32). Children who had data at both time points were included in this study. Students were from eight schools and 37 classrooms where English was the language of instruction. Participant characteristics comparing those who continued the study and those who did not are summarized in Table 2.

Procedures

Students were assessed in two 30-minute sessions in their schools mostly on consecutive days by trained examiners. Students were first assessed in Spring of their Kindergarten year and again in Spring of first grade.

Measures

Kindergarten predictors.

Procedural Counting. *Oral Counting* was adapted from AIMsweb (Clarke & Shinn, 2004) and involves asking a student to count aloud from "1" until told to stop. The total of correctly identified numbers in one minute and errors are recorded, which were converted to a numbers-per-second metric. Test-retest reliabilities range from .78 - .80 (Clarke & Shinn, 2004). A *Counting Down* measure was also used. In this measure, children counted down from 10 and 20 as quickly as possible.

Conceptual Counting. In the *Count out objects* measure, children see pictures of boxes and cars randomly arrayed on a page and are told to "count out loud, all of the *things* on this page" and then immediately asked, "how many are there altogether?" The task consisted of five items: 9 (5 boxes, 4 cars), 13 (6 boxes, 7 cars), 15 (8 boxes, 7 cars), 14 (7 boxes, 7 cars), and 8 (4 boxes, 4 cars). Errors in counting principles were recorded (Gelman & Gallistel, 1978). An abstraction error was when only one type of object (e.g., cars) was counted. A one-to-one correspondence error was when an object was double counted. Stable order error was when the child counted out of sequence. A cardinal count error was when a child responded to the "how many" question with a number other than the last number counted. Internal consistency from this sample was $\alpha = .68$. Sums of the error scores were created from each trial.

The second conceptual counting measure was a 10-item version of *Puppet Counting* that follows Geary's procedure (Geary, Brown, & Samaranayake, 1991; Geary et. 1999; Geary et al., 2000). The puppet counts an array of alternating red and green dots (a) correctly in typical left-to right fashion (3 trials); (b) correctly though by counting all the red dots and then all the green dots (psuedoerrors, 4 trials); or (c) incorrectly by double counting the first dot (3 trials). Double count error trials, which assess the one to one counting principle, were included in the analyses. Sample internal consistency for the error count trials was $\alpha = .80$.

Number tasks. *Number identification* asks children to identify 15 numbers (4, 8, 3, 7, 6, 84, 17, 25, 33, 12, 79, 100, 150, 264, 333). Number correct will be used in this study. This has been used in previous research since it is often included in curriculum (Jordan et al., 2006). Sample internal consistency was $\alpha = .86$. *Quantity discrimination* is an AIMSweb measure (Clarke & Shinn, 2004) that consists of 28 sets of Arabic numbers where children are asked to circle the greater number. Alternate form and test-retest reliability were good, range from .85 to .93 (Clarke & Shinn, 2004; Lembke & Foegen, 2009). Number correct was used.

Cognitive. Visuospatial working memory was assessed with *Spatial Working Memory* (Cirino, 2011). A series of nameable shapes, or a star, are presented one at a time in one of four quadrants of a page. The child is asked to first identify whether the shape is a star for each stimulus shown, then to recall the position of all the shapes in a series, in sequential order. There were two practice trials and three trials with systematically increasing series lengths (blocks) of 2, 3, 4, and 5. The measure was discontinued if students incorrectly recalled the order of all three series within a block. The total raw score was used where a point is awarded for each correct sequence recalled. Raw scores across blocks for Trial 1, 2 and 3 had a sample internal consistency of α = .76.

Digit Span (DS) from the Test of Memory and Learning (TOMAL; Reynolds & Bigler, 1994) was included as a measure of phonological working memory. The backwards subtest was used where students were read a string of numbers and asked to repeat them in reverse order back to the examiner. Test-retest reliability for this age group was .61 and internal consistency was α = .92. Number correct was used.

Phonological Awareness was assessed using the *Phoneme Elision* subtest from the *Comprehensive Test of Phonological Processing* (CTOPP; Wagner, Torgesen, & Rashotte, 1999). The *Phoneme Elision* task involves hearing a whole word, and then being asked to remove a sound from the beginning, middle, or end of the word, and state the result, which is always a new word. Sample internal consistency was α = .88.

Behavioral. *The Strengths and Weaknesses of ADHD and Normal Behavior (SWAN-IV; Swanson et al., 2005)*. SWAN-IV is an 18-item teacher rating scale of inattention and hyperactivity/ impulsivity rated on a 7-point Likert scale that ranges from -3 to +3. Each behavior corresponds to specific ADHD criteria identified in the *Diagnostic and Statistical Manual of Mental Disorders-Fourth Edition, Text* Revision (American Psychiatric Association, 2004) which factors into two scales, inattention and hyperactivity/impulsivity. The inattention scale was used in this study.

Grade 1 Outcome Measures.

In an effort to include a comprehensive array of outcomes, two measures were utilized for each outcome type. However, in order to be more efficient (i.e. reduce the number of models examined) measures were combined to create composite scores. While it is possible that doing so would decrease the differential predictions per measure, since both tasks measure the same construct, it is not expected that there would be much differentiation. Correlations were examined to determine if it was appropriate to create composite scores.

Computation. This is a composite measure of *Woodcock-Johnson-Third Edition (WJ-III) Calculation,* and *Wide Range of Achievement Test-Third Edition (WRAT-3) Arithmetic.* The WJ-III calculation subtest consists of addition and subtraction of single and multi digit problems for this age range. Test-retest reliability in this age range for this task is 0.96 (McGrew, Schrank, & Woodcock, 2007). The WRAT-4 arithmetic subtest involves number identification, counting, number comparisons, and other tasks for very young children; at school age, the task is primarily of computations that increase in difficulty. Test-retest reliability in this age range for this task is 0.87 (Wilkinson, 1993).

Fluency. *Small Sums Addition and Subtraction* combined score was used. Addition items include 55 single-digit problems that sum to 10 or less. Subtraction items include 55 single-digit problems. Problems were arranged in vertical format with eight rows of five problems per sheet, over two sheets. Students were asked to complete as many problems as they could in two minutes. A composite sum of the number correct minus the number incorrect on the addition and subtraction tests was used.

Math Fluency Woodcock-Johnson - III Tests of Academic Achievement (WJ-III;Woodcock, Mcgrew, & Mather, 2001). Participants solve as many single digit problems (addition, subtraction, and multiplication) as possible in 3 minutes. Standard scores were used in analyses for both measures. Test-retest reliability in this age range for this task is 0.90 (McGrew et al., 2007).

Math Reasoning. *Woodcock-Johnson - III Tests of Academic Achievement (WJ-III;Woodcock et al., 2001)*. The *Applied Problems* subtest consists of math word problems that are presented to the subject and read out loud by the examiner. The items range in difficulty and the use of pencil and paper is allowed. Test-retest reliability in this age range for this task is 0.88 (McGrew et al., 2007). *Single Digit Story Problems* was first developed by Riley, Greeno, and Heller (1983) and has since been adapted (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2003; Riley & Greeno, 1998). The measure includes items of addition and subtraction read out loud by the examiner that includes three categories of problems: change, combine and compare. A composite score was created for the Applied Problems and Single Digit Story problems measures.

Analyses

The first two hypotheses evaluating the relationship between the predictor and outcome variables were tested using bivariate Pearson's correlation. The data being tested is interval (i.e. continuous variable where equal intervals on the score represent equal differences; Field & Miles, 2010). For the sampling distribution to be considered normal, the variables being included in the correlations must be normally distributed. Therefore, each variable was tested for appropriate skewness and kurtosis. The correlation coefficients were used to determine the direction and size of the relation between predictor variables.

Hypotheses 3, 4, and 5, which involved using the predictors to determine the amount of variance that can be explained for each outcome, were tested with multiple regression analyses. Assumptions of regression (homoscedasticity, independence and normality of residuals, and no multicolliearity) were tested using the regression diagnostics. A plot of the residuals was used to show the distribution of the variables and to test that the variance of the residuals does not differ by level, meaning the assumption of homoscedasticity is not violated. Assessing the level of skewness and kurtosis was used to test the normality of the residuals. Correlations and specific tests (e.g., tolerance) were used to test the multicollinearity of the predictors.

For Hypothesis 3, which assessed the amount of variance the counting variables can explain in each outcome, three models were tested including all of the counting procedural skill and conceptual knowledge variables to predict the three types of outcomes: fluency, computation, and applied reasoning. In order to test the amount of variance each counting type predicts over and above the other, hierarchical regression was used beginning with the procedural counting variables and then adding the conceptual counting variables for the fluency and computation models. For the applied reasoning model, the conceptual variables were added first, followed by the procedural variables. Then the R^2 change for the models were evaluated by calculating an observed F value based on the sum of squares of the error term for the models and comparing it to a critical F value with the degrees of freedom associated with the full model. For Hypothesis 4, three models were tested using the number, cognitive and behavioral variables to predict each outcome. For hypothesis 5, a model including all counting, number, cognitive and behavioral variables was tested predicting each outcome. To assess the amount of variance accounted for by the counting variables over and above the other number, cognitive and behavioral variables, model comparisons were used.

Several covariates were considered such as demographic variables since these have been previously documented as having specific relevance for math (Cirino, 2011; Jordan et al., 2006). Since lower socioeconomic status (SES) is a risk factor for learning difficulties (Duncan et al, 2007), the effect of socio-economic status (whether the student receives free lunch; RFL) was included in the analyses. Additionally, gender differences were tested; however significant effects were not expected as gender differences do not become apparent until later in academic development (Rosselli, Ardila, Matute, & Inozemtseva, 2009). Age at baseline was also considered, though the age variance is relatively small so an effect was not expected (Swanson & Jerman, 2006). First, correlations between RFL, gender, race and age to the math outcomes were tested. When significant relations were found between a demographic variable and the math outcome measure, the variable was added to the appropriate regression models.

Results

Preliminary results. Descriptive statistics for the predictor and outcome variables are reported in Table 3. There were two procedural counting skill variables (oral counting and speeded counting down), five conceptual counting knowledge variables (one to one, stable order, abstraction, cardinal and double count errors), two number variables (number identification and quantity discrimination), three cognitive variables (spatial working memory, digit span and phonological awareness), and one behavioral factor (behavioral inattention). The procedural skill variables had a normal distribution as evidenced by skewness and kurtosis that are less than 1, and there was no evidence of outliers or heterogeneity of variance (no trends in residuals). For the conceptual knowledge variables, one to one and double count error were normally distributed with skewness and kurtosis around 1. Even though the cardinal error variable had a skewness and kurtosis greater than one, there was still enough variance in order to interpret the results. However, the stable order and abstraction error variables were very skewed as well as leptokurtotic. Therefore, these two variables were not included in the final analyses. None of the number, cognitive or behavioral variables were skewed or kurtotic as evidenced by statistics less than 1. Further, residual plots showed no evidence of heterogeneity of variance for the variables, with the exception of number identification.

Composites were used to create the outcome variables. Fluency was a summed variable of small sums addition and subtraction. The correlation for the addition and subtraction variables was $r = .50$, $p < .0001$. This composite was significantly correlated with the WJ-III Math Fluency subtest ($r = .63$). The computation variable was created using a composite of the raw score from the WJ Calculation subtest and the written arithmetic items from the WRAT-3 Arithmetic subtest. Because the early items on the WRAT-3 involved mostly counting, it was expected that these items would inflate the relation of the counting variables to the computation outcome score so these items were taken out of the composite. The correlation of the complete WRAT-3 Arithmetic subtest with the WJ-III Calculation subtest was $r = .82$, $p < .0001$, which was similar to the correlation without the early items $r = .80$, $p < .0001$. This computation outcome was significantly correlated with standardized test scores (Stanford Procedures test; $r = .71$) and Total Math test; $r = .73$). The applied reasoning outcome was a composite of WJ Applied Problems and Story Problems (which was first standardized with a mean of 100 and standard deviation of 15). This variable was significantly correlated with the Stanford Problem Solving test ($r = .78$) and the Total Math test ($r = .80$), which had an intercorrelation of $r = .94$. All of the outcome variables were normally distributed with skewness and kurtosis less than 1 and had no evidence of outliers or heterogeneity of variance.

Hypothesis 1: The relation of procedural counting skill and conceptual counting knowledge to Fluency, Computation, and Applied Reasoning. Correlations between all the predictor and outcome variables are presented in Table 4. All measures of procedural counting skill (oral counting and counting down) and conceptual counting knowledge (one to one, cardinal and double count error) were significantly related to each math outcome (fluency, computation, and applied reasoning). The procedural counting skill variables correlations with the outcomes ranged from .33 to .52. The conceptual counting knowledge variables correlations ranged from -.21 to -.48. The double count error variable had a positive correlation with the outcomes because the higher scores corresponded to fewer errors; the correlations ranged from .32 - .36.

Hypothesis 2: The relation of the number, cognitive and behavioral variables to Fluency, Computation, and Applied Reasoning. All the number variables (number identification and quantity discrimination), cognitive (SWM, DS and PA), and behavioral (behavioral inattention) variables were significantly related to each math outcome. The number variables were highly related to each outcome, all with correlations of .58 or greater at $p < .0001$. The other cognitive and behavioral predictors were also significantly related to each outcome.

Hypothesis 3: Variance explained by procedural skill and conceptual knowledge counting variables for Fluency, Computation, and Applied Reasoning. The results of these regression models are summarized in the top portions of Table 5. For each outcome, model comparisons were used to test the amount of variance attributable to each type of counting predicting over and above the other. Even though specific hypotheses were made for each outcome, comparisons were tested both ways (i.e. procedural over conceptual and conceptual over procedural) since the information on this in the literature is sparse. In the fluency model, procedural skill and conceptual knowledge accounted for similar amounts of variance, 25.08% and 30.38%, respectively. A model including both types of counting variables was significant [$F(5,179) = 23.61$; $p < .0001$, $R^2 = .40$] with counting down, cardinal and double count error being uniquely predictive. Using model comparisons, the procedural variables did add a significant amount of explained variance over and above the conceptual variables, as hypothesized [$F(4,179) = 6.34$; $p = .01$, R^2 change = .08]. Additionally, the conceptual knowledge variables added a significant amount of variance over and above the procedural counting skills [$F(5,179) = 8.95$; $p = .01$, R^2 change = .15].

In the computation model, conceptual knowledge accounted for 25.84% of the variance and procedural skill accounted for 17.40%. A model including all counting variables was also significant for computation [$F(5,181) = 16.21$; $p < .0001$, $R^2 = .31$] with counting down, cardinal and double count error being uniquely predictive. In comparing these two models, both types of counting added significant variance over and above the other type.

Procedural counting skill accounted for 28.74% of the variance in the applied reasoning model and conceptual counting knowledge accounted for 21.93%. A model including all counting variables was significant for applied reasoning [$F(5,181) = 20.03$; $p < .0001$, $R^2 = .36$] with counting down, cardinal and double count error contributing unique amounts of variance. In comparing these two models, both types of counting added significant variance over and above the other type.

Hypothesis 4: The contribution of the number, cognitive, and behavioral predictors for Fluency, Computation, and Applied Reasoning. The number variables accounted for a significant amount of the variance in the fluency, computation, and applied reasoning models

(54.50%, 49.08% and, 51.71%, respectively), with both variables being unique contributors. Similarly, the cognitive and behavioral variables accounted for significant amount of variance in the models (44.25%, 37.64% and 53.17%), with each variable contributing uniquely (with the exception of digit span for the computation model). When the number, cognitive, and behavioral variables were combined into the same models, the significance of the individual variables was affected. In the fluency model, the number and working memory variables were significant [F (6,171) = 43.62; p <.0001, R^2 = .60]. In the computation model, the number variables, spatial working memory, and PA were uniquely predictive [F (6,173) = 31.87; p <.0001, R^2 = .53]. In the applied reasoning model, the number variables, verbal and spatial working memory, and PA were uniquely predictive [F (6,173) = 50.42; p <.0001, R^2 = .64].

Hypothesis 5: Both procedural skill and conceptual knowledge are important longitudinal predictors of a variety of math outcomes, even in the context of several other relevant predictors. The results of the regression models are summarized in the lower portion of Table 5. First, a correlation between the covariates (age, gender, free lunch status, and race) and the outcome variables was conducted. Free lunch status was significant for all models. Additionally, age was significantly related to fluency and race was significantly related to applied reasoning. Therefore the appropriate covariates were added to the models. However, none of the covariates were significant in the overall models, so they are were removed from the final models and not reported in Table 5.

Some of the procedural skill and conceptual knowledge variables remained predictive in the context of other relevant predictors for each model. In the fluency model, cardinal and double count error remained unique predictors (β = -0.187, p = .005; β = 0.106, p = .045, respectively). The number variables were strongly uniquely predictive. Of the cognitive variables, only digit span was a unique predictor. The overall model accounted for 64.93% of the variance in fluency. Comparing the full model to the number, cognitive, and behavioral model showed that counting added a significant amount of variance over and above the other factors [F (9,162) = 1.93; p =.05, R^2 change = .04]. For computation, the overall model accounted for 56.01% of the variance. Oral counting down was the only uniquely predictive counting variable (β = -0.184, p = .010). The number variables again were uniquely predictive. PA was the only uniquely predictive cognitive or behavioral variable. Comparing the full model to the counting variables only model showed that counting did not add a significant amount of variance over and above the other factors. However, when adding only the unique counting predictors to the model, the amount of variance added was significant over and above the other factors [F (2,171) = 3.04; p =.05, R^2 change = .03]. For applied reasoning, the overall model accounted for 65.77% of the variance. The procedural skill counting variables (oral counting and counting down) significantly contributed to the model (β = -0.119, p = .058; β = 0.149, p = .023; respectively). The number variables were unique contributors, as well as both measures of working memory and PA. Comparing the full model to the number, cognitive and behavioral model showed that counting did not add a significant amount of variance over and above the other factors, even when only the significant variables were added.

Follow up analyses. Not all of the counting variables uniquely predicted the outcomes in each model as expected, and so some follow-up analyses were conducted. Given the strong relationship of number and counting, the degree of shared variance among these variables was evaluated with multiple regressions to test if the counting variables were predictive of the two number variables. For number identification, the counting variables together accounted for 40.46% of the variance. Both procedural skill variables and cardinal errors were unique predictors. For quantity discrimination, 34.93% of the variance was accounted for with oral counting, cardinal and double count error contributing significantly. The impact of these shared relations was further evaluated with regression analyses predicting the outcomes but including only the counting and number variables. For fluency, 59.07% of the variance was accounted for by the counting and number variables as compared to 54.51% with only the number variables, with the contribution of counting over and above the number variables being significant [$F(9,162) = 1.93$; $p = .05$, R^2 change = .05]. The cardinal and double count error variables were unique contributors (β = -0.158, $p = .019$; β = 0.122, $p = .024$; respectively). In the computation model, 54.88% of the variance was accounted for by the counting and number variables as compared to 52.26% by the number variables alone. Therefore, the R^2 change for the models was 2.62%, which was not significant. Oral counting and cardinal error contributed significantly to the model (β = -0.167, $p = .017$; β = -0.142, $p = .045$; respectively). In the applied reasoning model, 55.92% of the variance was accounted for by the counting and number variables as compared to 51.71% by the number variables alone. Counting variables did show an R^2 increase of 4.21%, which was significant ($p = .05$). Counting down was a unique predictor (β = 0.239, p = .0006).

Discussion

The goal of the present study was to determine the contribution of procedural counting skill and conceptual counting knowledge to three different math outcomes in the context of other relevant variables. Since many studies consider each type of predictor separately (i.e. focusing separately on number based, cognitive, or behavioral predictors), this study was unique in combining these variables into one model to better understand the relation to several types of math outcomes, and doing so in a fairly large sample and also over time.

What is the role of counting variables to math outcomes?

We found that as expected, procedural skills significantly accounted for variance over and above the conceptual knowledge variables for fluency and computation. Also, conceptual counting knowledge accounted for significant unique variance over and above the procedural skill model for applied reasoning. These results are consistent with previous literature indicating the relevance of these counting skills independently (Koponen et al., 2007). However, further analyses showed that each type of counting explained a significant amount of variance over and above the other type for all outcomes (fluency, computation, and applied reasoning). Therefore, distinguishing between these types of counting may not significantly add to the ability to differentially predict in which area of math a child might struggle. However, this also suggests that assessing *both* types of counting is relevant, and would contribute significantly to predict math performance, and therefore which to emphasize in an instructional context. For example, ensuring children know the correct counting sequence as well as using explicit instruction of the counting principles (i.e. focusing on both the oral sequences to solidify the number-word sequences, as well as the means by which those number words are associated with objects) will give children a strong foundation to learn more complex math concepts and computation. Whether or not students specifically at-risk for mathematics difficulty would benefit differentially from exposure to varying amounts of these types of instruction could benefit from future research.

What is the role of number, cognitive and behavioral variables to math outcomes?

As expected, the number variables alone were significant predictors in each model accounting for over half the variance in each of the math outcomes. Further, with the exception of digit span in the computation model, *each* number, cognitive, and behavioral predictor contributed uniquely to *each* outcome, when considered separately. These models are consistent with previous literature showing a role for number (Cirino, 2011; Gersten & Chard, 1999; Geary, 2009; Jordan et al., 2006; 2009; Locuniak & Jordan, 2008), working memory (Mclean & Hitch, 1999; Raghubar et al., 2010; Rasmussen & Bisanz, 2005; Swanson & Jerman, 2006), and behavioral inattention (Fuchs, Fuchs & Compton et al., 2006; Merrell & Tymms, 2001; Seethaler & Fuchs, 2005) to be significant predictors of complex math outcomes.

In the models with number, cognitive and behavior variables together, both number measures remained unique predictors in each model, but the contribution of the other predictors was more variable. In the fluency model, both of the working memory variables were unique additional predictors. Verbal working memory (DS) may be important because in order to quickly and accurately answer addition and subtraction problems, it is necessary to know the correct sequence without counting. Children using counting on their fingers as a strategy for computing addition and subtraction problems will be slower and possibly less accurate than the children who have committed these to memory (Geary, 2004). Also of note is that PA was not a unique predictor of fluency. It is possible this is related to our measure of fluency. While this was a timed measure, it was a pencil and paper task rather than and aurally presented one which would rely more heavily on retrieval.

In the computation model, number variables, spatial working memory and phonological awareness were unique predictors. This is consistent with the literature that has found younger children tend to use visual working memory in math computation (Hitch et al., 1988; Holmes & Adams, 2006; Rasmussen & Bisanz, 2005). It is possible children at this age have a mental number line which enables them to assess magnitudes and distance between numbers in a visual manner to help them successfully compute problems. Language systems have been shown to support math computation as the first step in solving a problem is having the ability to read the number. Further, math facts are commonly taught by oral repetition in order to memorize them, which can be considered learning the oral phonological representations of the number words (Robinson et al., 2002). It is suggested that children with math difficulties struggle with tasks that involve making connections between phonological representations of numbers, rather than a general memory deficit (Robinson et al., 2002).

For the applied problems model, both types of working memory and PA remained unique predictors. This is consistent with the literature that linguistic factors are related to performance on math problem solving (Hecht et al., 2001; LeFevre et al., 2010; Savage et al., 2007) as well as the ability to hold and manipulate information in order to solve the problem. In general, applied problems had predictors from all domains except behavioral inattention, and more of the variables had unique contributions here than in the fluency and computational model, which is consistent with the array of skills and concepts assessed with such a measure.

Surprisingly, behavioral inattention did not remain uniquely significant in any of the above models. This may be because much of the literature that supports the relation of behavioral inattention and math skills has been with older children (Duncan et al., 2007; Fuchs et al., 2006; Raghubar et al., 2009; Seethaler & Fuchs, 2005). In the age range of this study (6 – 7 years old), the math problems on these measures are not as susceptible to inattentive errors. For example, the problems involve only one type of operation at a time rather than having to switch between four operations and do not require properly aligning numbers or decimal points to solve the problems. Another possibility is that past research has not taken into account other relevant factors to the same extent as the current study has longitudinally. For example, the meta-analysis by Duncan et al., (2007) included early math and reading achievement, hyperactivity and social skills ratings and verbal measures, but did not include PA or working memory. It has also been suggested that difficulties with attention and working memory overlap in children with math difficulties (Alloway et al, 2009; Gathercole et al., 2007). However, in this study, behavioral inattention was a unique predictor in the presence of the working memory variables, but not when the number and counting variables were included which suggests that a relationship exists between behavioral inattention and the domain specific variables.

Why were the number variables more important than the counting variables?

When the above models were expanded to also include procedural and conceptual counting, both number variables remained relevant for all three outcomes, whereas the contribution of the counting variables were more variable. There are several potential reasons why number variables were consistently unique predictors in the final models, more so than the counting factors. First, it is possible that the number tasks measured skills that are antecedent to the counting skills necessary to solve a math problem. For example, in order for a child to compute 1 + 3, they must first recognize the number (number identification), and then, depending on the counting strategy used (i.e. max vs. min), determine which is greater (quantity discrimination). While it is not necessary to use quantity discrimination, it would be more efficient. Only after they have figured this out can they then use counting strategies to determine the answer. Therefore even though counting is the more proximal skill to the outcome, the task cannot be completed without the more elemental number skills. Another possibility is that the counting measures used were aurally presented as opposed to the visually presented number measures and math outcomes which both involve Arabic numerals, therefore making the counting measures less proximal to the outcome. It would be interesting to assess the math outcomes with an aural task instead of a visual task and see if the results are similar.

Current findings in the context of previous models

Even though the overall goal of this study was to determine the role that different counting types have in the prediction of several math outcomes, the use of a step-by-step design in the analyses helped us evaluate in detail the role of relevant factors for math. While some individual predictors overlapped for each outcome (i.e. number variables significant for each model), we did have distinct trends of predictors. Additionally, as might be expected, more predictors were significant as the outcomes became more broad and complex.

The findings from this study can be discussed in terms of previously proposed models. Cirino (2011) found five latent factors in kindergarten (symbolic and non-symbolic comparison, symbolic labeling, rote counting, and counting knowledge). While these distinctions are important for theories of math development and understanding, when these variables are used practically to predict later outcomes, differentiating between them may be less important. However, Koponen et al. (2007) did find that procedural counting skill was predictive of fluency (single digit calculation) in grade 4, while conceptual counting knowledge was predictive of more complex computation. Further, in that study, procedural counting skill mediated the effect conceptual counting had on fluency. This was not the case in our fluency or computation models, as it was actually the conceptual variables which were significant and not the procedural counting variables for fluency and just the opposite for computation. Also, there is the possibility that the distinctions in counting types may manifest themselves at later time points, particularly if more robust counting variables were utilized.

LeFevre and colleagues (2010) suggest three separate pathways exist when predicting math performance: quantitative, linguistic, and spatial attention. The quantitative factors in the present study (counting and number variables) did appear to represent a separate pathway to all three types of math outcomes. While the cognitive tasks differed in this study, the SWM, behavioral inattention, and PA factors did not appear to represent separate pathways to math outcomes, but rather were possibly mediated by the counting and number variables. This is similar to Krajewski and colleagues' (2009) suggestion that PA and visual-spatial WM directly influence more basic number skills (described as number words not linked with quantities - similar to the number variables) that then indirectly affect later more complex math skills and competency. Further, in our follow up regressions of counting variables predicting the number variables, the results showed that PA and SWM were significant predictors of number identification and quantity discrimination. However, our results found PA was predictive of computation and applied reasoning and SWM was directly predictive of applied reasoning, which differs from their work (Krajewski & Schneider, 2009) which found neither PA nor SWM to be directly related to math competency in school. The authors refer to PA as a "necessary but not sufficient" prerequisite for understanding math concepts due its indirect impact on later math achievement via early skills. Our findings are partially consistent with that conclusion as PA and SWM demonstrated indirect effects on fluency and were predictive of each number variable, however direct effects were also found.

The findings in this study support the importance of specific number factors, including both counting types, in predicting math outcomes, which is similar to results from Jordan's studies (Jordan et al., 2002; Jordan et al., 2006; Jordan et al., 2009; Jordan et al., 2010; Locuniak & Jordan, 2008). The contributions of the cognitive factors are less straightforward. It is possible the counting variables partially mediate the effect of working memory on fluency and computation since working memory is required to monitor place while counting (Swanson & Jerman, 2006). However, verbal and visual working memory had direct effects on the fluency and applied reasoning outcomes. This has been found in research using cognitive factors to predict placement in low achieving or math learning difficulty groups (Geary et al., 2009). PA, or other linguistic factors, may represent a separate pathway as suggested by LeFevre et al. (2010).

Given their strength in prediction across the range of ability as demonstrated here, the results of this study suggest that exposure to both counting procedures and knowledge of counting principles, in conjunction with Arabic numerals would be useful in helping to identify children who are likely to struggle with math skills. Curricula vary in the extent that they emphasize conceptual relationships such as manipulables, versus emphasize number recognition, sequencing, and number combinations, though the present results suggest that the latter may be more related to the kinds of paper and pencil math outcomes assessed here, though clearly both play a role. For students who struggle in academic areas, an explicit and systematic focus on core skills that are most closely related to the desired outcome (i.e., words in reading, Arabic numerals in math) may be even more important. The contribution of the cognitive variables were found to be relevant, though the present results highlight the need to further understand their direct versus indirect impact, particularly to the extent that they are implicated in the performance of the more proximal numeric and counting variables examined here. In this way, such skills (or behaviors) can still exert influence on children's classroom performance, particularly as math skills become more differentiated and involve more problem solving or extended algorithmic procedures. Therefore, assessment of these factors (working memory, PA and behavioral inattention) can still be beneficial in guiding interventions by identifying skills that may compound or otherwise interfere with the more direct elements of the intervention.

Limitations

Even though the number variables were more predictive, counting measures still explained about 30 – 40% of the variance in each model. This is still a significant amount considering how quickly you can assess these variables and how much they are able to explain complex math. However, there are some limitations. Since the test battery was time limited, it made it difficult to include more measures of counting. It may have been beneficial to include more trials of the conceptual counting tasks as well as possibly including another procedural counting measure such as starting to count from a given number (i.e. count to 20 beginning at number 5). It is possible that the nature of the counting measures that were used limited their impact. For example, the one to one error variable ranges from 0 to 5 with almost half of the children making no errors, and the stable order and abstraction variables could not be utilized. While a composite measure across error types might have improved the technical qualities of this predictor, for this study, keeping separate counting variables in the model made it possible to observe more specific relationships with the outcomes.

Additionally, the number variables were both symbolic. Since symbolic and non-symbolic tasks are both included in number sense, using a non-symbolic measure may have added some more information to the models. Including both would likely help to differentiate between a magnitude effect and a symbolic effect. However, several recent studies have found symbolic factors to be more significant than non-symbolic factors (De Smedt & Gilmore, 2011). In future studies it may be beneficial to include measures in the subitizing versus non-subitizing range. In regards to the working memory variables, it may also have been interesting to use an additional verbal working memory task that did not involve numbers (digit span) such as word span.

Since the focus of this study was on kindergarten precursors, identification and classification of children as having a difficulty or disability in math was beyond the scope of the present study. As mentioned in the Introduction, there is no consensus of the best way to identify children with MLD. Further, using cut off scores will arbitrarily demarcate a continuous distribution and there is a lack of evidence that differential prediction is possible along a continuum. There would also be the concern of restriction of range as it is likely there would be few children at the extremes of the distribution. Nonetheless, identification of risk status is an important goal, particularly from a practical perspective, and is an area that needs further study.

Conclusion

In sum, this study found that: (1) while counting procedural skill and conceptual knowledge are both strong predictors of several types of math outcomes, patterns of counting do not convincingly distinguish between different math outcomes, but both are important predictors; (2) number variables account for much of the variance in math outcomes over and above counting types, and (3) the influence of counting variables on math outcomes is effected by cognitive factors. Further, each outcome did have distinct predictors such that the more comprehensive outcome (applied reasoning) had a larger range of predictors. The counting variables did account for about 30 – 40% of the variance in each model, which is more than has been reported previously (Stock et al., 2009), and should be considered when screening for early math difficulties and when considering the most appropriate interventions. However, their contributions should be considered along with other number factors for more robust prediction.

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Table 1. Summary of Studies with Counting Variables

Authors	Age group	Time Points	Predictors	Math Outcomes
PRESENT STUDY	K – G1	2 – longitudinal	1. **Counting**: procedural & conceptual 2. **Number**: # ID & Quant Comp 3. **Cognitive**: SWM, Beh Inattn, PA **Other**: Demo (SES, gender)	1. **Fluency** (MF & small sums +/-) 2. **Computation** (Calc & arithmetic) 3. **Applied Reasoning** (Applied Probs & Single-digit story prob)
Cirino (in press)	K	One time point	1.Both 2.symbolic/nonsym comp; # ID, missing # 3.VSWM, PA, RAN	1.small sums 2. 3. Note: all at Time 1
Geary, Hoard, Byrd-Craven, DeSoto (2004)	G1, 3, 5	cross sectional & longitudinal (each G tested in fall – exp. tasks & spring –IQ & achievement)	1. Conceptual only (puppet) 2. 3. WM (counting span) Other:	1. 2. Simple/complex math (strategy) 3. Math reasoning
Geary, Hoard, Byrd-Craven, Nugent, & Numtee (2007)	K-G1	longitudinal	1. Conceptual only (puppet) 2. 3. WMTB-C (Baddeley) Other: PS	1. 2. Simple/complex math (strategy) 3.
Jordan, Kaplan, Nabors Olah, & Locuniak (2006)	K-G2	longitudinal	1. Within battery 2. Number sense battery 3. Other: Gender, SES, K start age	1. 2. 3. Note: Trajectories

Jordan & Locuniak, 2008	K-G2	longitudinal – 4 times for predictors	1. Within battery 2. Number sense battery 3. WM (DS), MR Other:	1. Math fact fluency 2. 3.
Jordan, Kaplan, Ramieni, & Locuniak, 2009	K – G3	longitudinal – 4 times for predictors, 5 times for outcomes	1. Within battery 2. Number sense battery 3. Other: Demo	1. 2. WJ Calc 3. Applied Problems
Aunola, Leskinen, Lerkkanen, & Nurmi (2006)	Preschool – G2	6 - longitudinal	1. Procedural & conceptual Combined into Counting Ability (1 to 1, stable, cardinality) T1 2. Number ID 3. Metacognitive knowledge – T1 Other: Visual Attn – T1	1. 2. Basic arithmetic 3. Word problems Note: Growth curve model with Math competence outcome combined
Koponen, Aunola, Ahonen, Nurmi (2007)	K– G4	2 - longitudinal	1. Conceptual (stable, cardinality) 2. 3. PA, gen cog ability Other: SES	1. 2. Single digit & multi digit addition & multiplication 3.
LeFevre et al. (2006); Kamawar, LeFevre et al. (2010) (also reported in 2008 conference proceedings paper)	K-2; 5-11 (K, 2, 3, 4, 5)	cross sectional	1. Procedural (count objects) & conceptual (puppet) 2. 3.	1. 2.Skill groups – numeration, addition/subtraction 3. Note: ANOVA (count type X grade X skill group)
Stock et al (2009)	K-G1	longitudinal	1. Conceptual (stable, 1 to 1, cardinality	1. Fluency 2. Mental arithmetic &

			2. 3.	number knowledge 3.

Table 2. Demographic Characteristics Comparing Included and Dropped Samples

Variable	Category/Scale	Included (*n* = 194)	Dropped (*n* = 93)
Age in Kinder	Years Mean (SD)	6.16 (0.32)	6.06 (0.33)
Gender	Female (%)	48.45%	48.39%
Ethnicity	African American	43.30%	63.44%
	Caucasian	24.23%	10.75%
	Hispanic	22.68%	18.28%
	Other	9.8%	7.53%
SES	Free/Reduced Lunch (%)	58.25%	66.67 %
K-BIT Verbal	Standard Score	98.80 (12.63)	
Small Sums Addition		4.73 (13.77)	2.93 (14.79)

Note. SES = Socioeconomic status measured by receiving a free or reduced lunch; K-BIT Verbal = Verbal IQ index from Kaufman Brief Intelligence Test given in grade 1

Table 3. Descriptive Statistics for Predictor and Outcome Variables

Variable	Category/Scale	N	Mean	Std Dev	Kurtosis	Skewness
Procedural Counting						
Oral Counting	#s per second	191	1.24	0.36	0.19	-0.04
Counting Down	seconds	190	2.42	1.59	0.91	0.91
Conceptual Counting						
One to One Error	0 – 5	193	0.82	0.99	1.05	1.18
Stable Order Error	0 – 3	193	0.06	0.31	52.12	6.72
Abstraction Error	0 – 5	193	0.21	0.87	22.84	4.76
Cardinal Error	0 – 9	192	1.52	1.93	2.72	1.68
Double Count Error	0 – 3	191	1.98	1.20	-1.15	-0.68
Number						
Number Identification	1 – 15	193	11.30	3.02	0.01	-0.73
Quantity Discrimination	-8 – 42	192	18.59	11.18	-0.60	-048
Cognitive and Behavioral						
Digit Span	3 – 19	189	9.98	3.47	-0.21	-0.21
Spatial Working Memory	0 – 10	192	2.96	2.32	0.32	0.86
Behavioral Inattention	-27 – 27	193	6.51	11.98	-0.47	-0.14
Phonological Awareness	4 – 19	185	10.04	2.95	-0.00	0.05
Outcomes						
Fluency	-47 – 74	189	17.88	23.85	-0.14	-0.25
Computation	0 – 25	193	14.55	4.72	0.26	-0.59
Applied Reasoning	74 – 137	193	104.83	13.38	-0.84	-0.04

Note. N = number of participants; Std Dev = standard deviation.

Table 4. Correlations between Kindergarten Predictors and Grade 1 Math Outcomes

	Counting Predictors					**Number and Cognitive Predictors**					
	OC	CD	OOE	CE	DCE	NI	QD	BI	SWM	DS	PA
Outcomes											
Fluency	0.407	0.481	-0.310	-0.481	0.362	0.654	0.597	0.471	0.414	0.550	0.493
Computation	0.330	0.404	-0.280	-0.439	0.323	0.630	0.600	0.465	0.384	0.445	0.512
Applied Reasoning	0.401	0.522	-0.214	-0.365	0.350	0.617	0.606	0.513	0.456	0.554	0.638

Note. OC = Oral Counting; CD = Counting Down; OOE = One to One Error; CE = Cardinal Error; DCE = Double Count Error; NI = Number Identification; QD = Quantity Discrimination; BI = Behavioral Inattention; SWM = Spatial Working Memory; DS = Digit Span; PA = Phonological Awareness; All correlations *p* <.0001.

Table 5. Regression Statistics for Each Math Outcome Model

Predictor	Fluency	Computation	Applied Reasoning
Hypothesis 3			
Oral Counting	0.071	0.026	0.065
Counting Down	0.259**	0.218**	0.359**
R^2 Procedural Model	*0.251***	*0.174***	*0.282***
One to One Error	0.004	0.002	0.043
Cardinal Error	-0.332**	-0.337**	-0.228**
Double Count Error	0.227**	0.197**	0.196**
R^2 Conceptual Model	*0.315***	*0.258***	*0.219***
R^2 Change Proc > Conc	*0.082***	*0.051***	*0.137***
R^2 Change Conc > Proc	*0.146***	*0.135***	*0.074***
Total Model R^2	**0.397****	**0.309****	**0.356****
Hypothesis 4			
Number Identification	0.377**	0.325**	0.177**
Quantity Discrimination	0.324**	0.337**	0.271**
Digit Span	0.228**	0.064	0.158**
Spatial Working Memory	0.111*	0.112*	0.175**
Behavioral Inattention	-0.036	-0.037	0.0349
Phonological Awareness	0.034	0.160**	0.293**
Total Model R^2	**0.605****	**0.525****	**0.636****
Hypothesis 5			
Number Identification	0.335**	0.339**	0.159*
Quantity Discrimination	0.285**	0.345**	0.289**
Digit Span	0.225**	0.072	0.150**
Spatial Working Memory	0.044	0.077	0.141**
Behavioral Inattention	-0.064	-0.065	0.027
Phonological Awareness	0.035	0.190**	0.280**
Oral Counting	-0.109	-0.184**	-0.119*
Counting Down	0.090	0.013	0.149**
One to One Error	-0.001	-0.030	0.023
Cardinal Error	-0.187**	-0.128	-0.025
Double Count Error	0.106*	0.038	0.043
R^2 Change Counting	*0.044**	*0.035*	*0.022*
Total Model R^2	**0.649****	**0.560****	**0.658****

Note. Numbers reported are standardized beta estimates. Hypothesis 3 estimates reported are from the total counting model. ** = <.01 * = <.05; Proc = Procedural counting skills; Conc = Conceptual Counting Knowledge