An Electrical Circuit Model of Circle of Willis to Predict Stroke

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Background

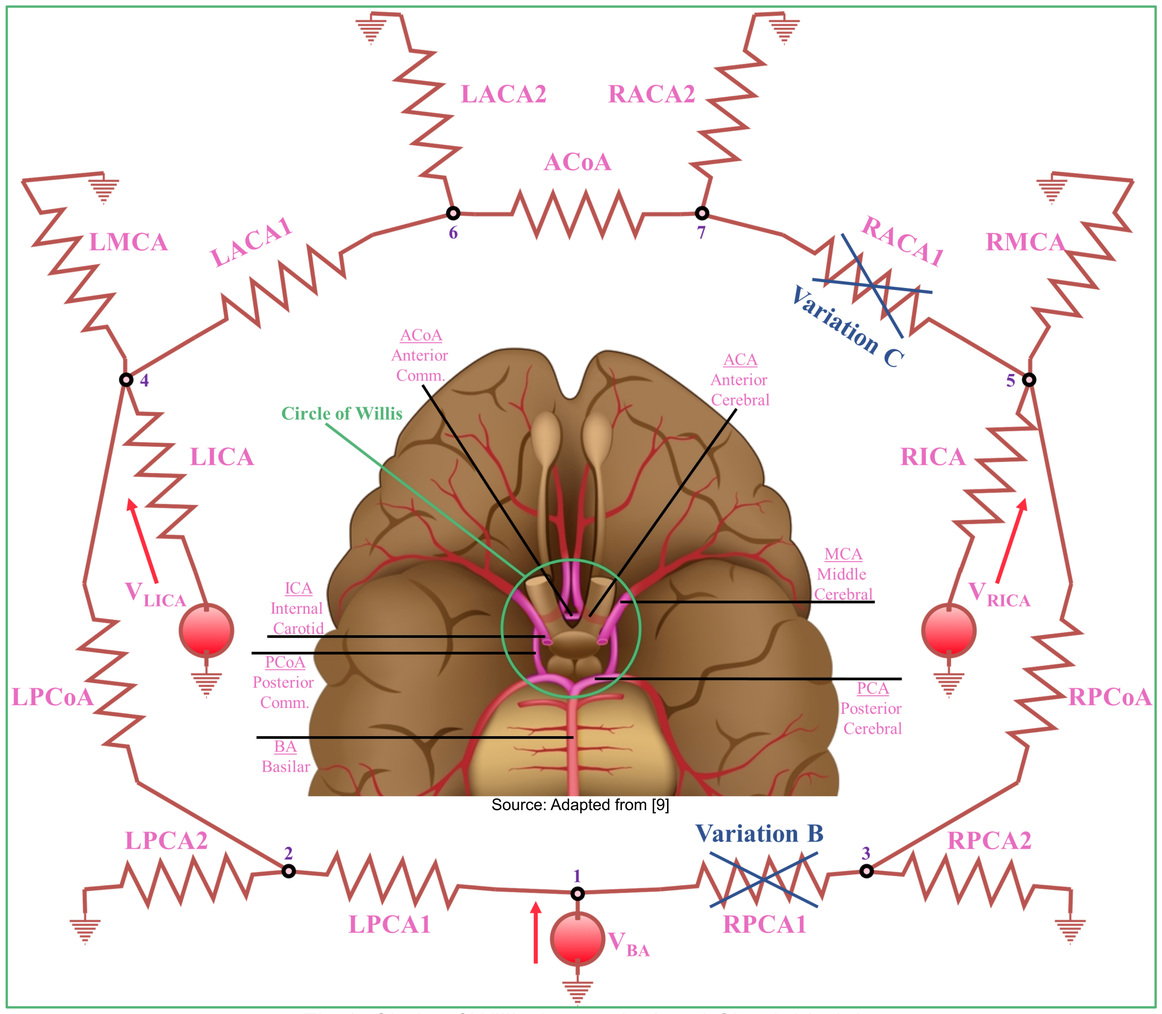
- Circle of Willis (CoW) distributes blood flow to the brain.
- Hypothesis: Structural incomplete variation (present in ~50% of humans) of CoW affects probability of stroke.
- Goal: Predict individuals who are more prone to stroke before they get one using structure of CoW.

Methodology

Modeling blood pressure difference using circuit analysis:

Pressure Difference - ΔP, Dynamic Viscosity - μ, Potential Difference - Δv, Length$_{Artery}$ - L, Electrical Resistance - r, Radius$_{Artery}$ - R, Electric Current - i, Volumetric Flow Rate - Q

- Poiseuille's Law: $\Delta P = \frac{8\mu L}{\pi R^4} * Q$.
- Ohm's Law: $\Delta v = r * i$.
- 5 sets of clinical data on artery dimensions[1,2,3,4,5] used to compute models.
- {min, mean, max}$^{8\ lengths+8\ radii}$=3^{16}=~43mil models: randomly generated dimensions.
- 5 sets of dimensions X ~43 mil combinations = ~215 mil models simulated.
- Utilized MATLAB(Parallel Computing & Symbolic Math Toolbox) & HPC(High Performance Clusters) at CACDS.
- Models validated with 3 sets of empirical clinical data on volumetric flow rates[6,7,8].
- Find models where flow rate$_{Model}$ = flow rate$_{Clinical}$ ∀ arteries in CoW.



Source: Adapted from [9]

Fig 1. Circle of Willis Anatomical and Circuit Model
(Var A:Complete CoW [~51% of pop], Var B:Missing PCA1 [~18% of pop], Var C:Missing ACA1 [~5% of pop])

Circuit equations: **(1)** $V_{BA} = V_1$. **(2)** $(V_2-V_1)/R_{LPCA1} + V_2/R_{LPCA2} + (V_2-V_4)/R_{LPCoA}=0$. **(3)** $(V_3-V_1)/R_{RPCA1} + V_3/R_{RPCA2} + (V_3-V_5)/R_{RPCoA}=0$. **(4)** $(V_4-V_2)/R_{LPCoA}+(V_4-V_{LICA})/R_{LICA}+V_4/R_{LMCA}+(V_4-V_6)/R_{LACA1}=0$. **(5)** $(V_5-V_3)/R_{RPCoA}+(V_5-V_{RICA})/R_{RICA}+V_5/R_{RMCA}+(V_5-V_7)/R_{RACA1}=0$. **(6)** $(V_6-V_4)/R_{LACA1} + (V_6-V_7)/R_{ACoA} + V_6/R_{LACA2} = 0$. **(7)** $(V_7-V_5)/R_{RACA1} + (V_7-V_6)/R_{ACoA} + V_7/R_{RACA2} = 0$.

Results

Table 1. Number of Models with 0 Error from each of the 5 sets of 3^{16} Simulated Models

CoW Flow Rate Papers	CoW Artery Diameter & Length Dimension Papers				
	Shatri[1]	Iqbal[2]	Hillen[3]	Maaly[4]	Karatas[5]
Zhao[7]	5,579,679	5,704,713	4,504,379	10,074,014	4,388,238
Hanjani[8]	3,834,810	4,308,819	3,461,063	7,851,375	3,128,537
Zarrinkoob[6]: Variation: A	803,543	551,452	350,674	1,636,613	636,790
Zarrinkoob[6]: Variation: A, B, C	0	**2,015**	**1,640**	0	0

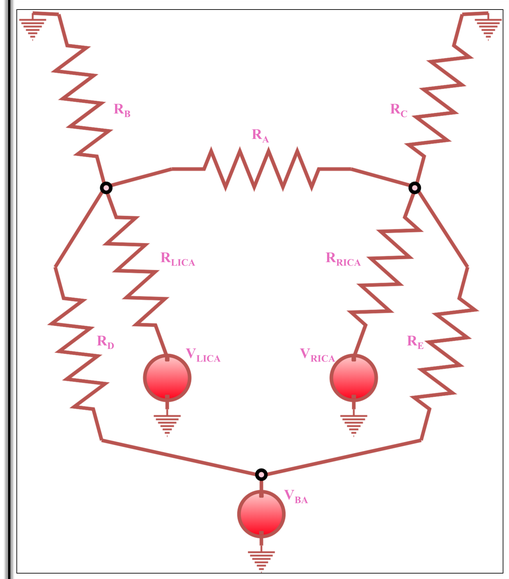


Fig 2. Circuit analysis yields simpler model of CoW with no loss of accuracy

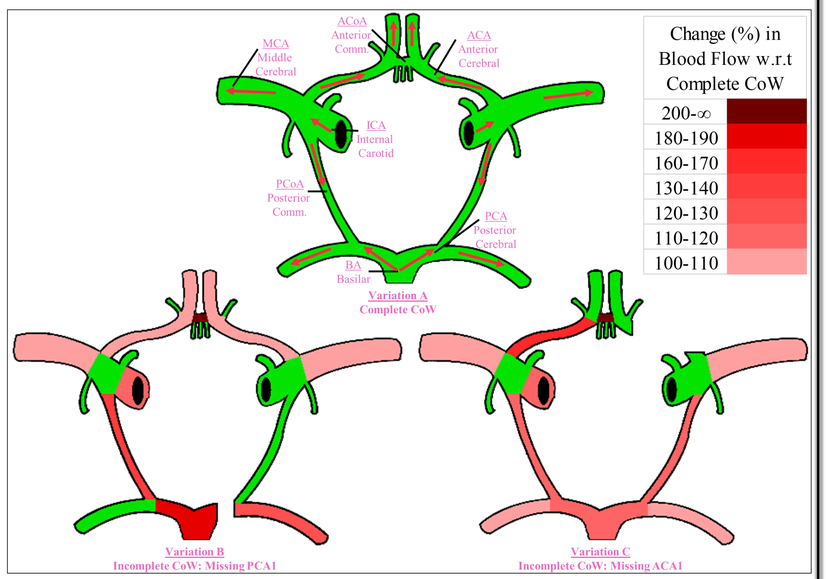


Fig 3. Incomplete CoW (Var. B and C) show larger than normal flow in some arteries - a potential concern

- 2,015 & 1,640 models from Iqbal[2] & Hillen[3] matched Zarrinkoob's[6] clinical flow rate for CoW variations A, B, & C.
- ~30mil & ~22mil models matched Zhao[7] & Hanjani's[8] clinical flow rate.

Summary

- **3,500+ circuit models obey anatomical measures and yield realistic, empirically verified flow rates.**
- ~215mil potential solutions obtained and compared with clinical data.
- Simplified CoW circuit and produced ranges for simplified resistances.
- Analyzed change in blood flow in CoW variations w.r.t complete CoW.
- Payoff: Solution models will be extended to conditions (incomplete CoW & blockages) with no clinical data to analyze probability of stroke.

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