SOME PRACTICAL ASPECTS OF CAPITAL BUDGETING USING CHANCE CONSTRAINED PROGRAMMING

by

ROBERT STEPHEN HOYLE

THESIS

Presented to the Faculty of the Graduate School of

Engineering at the University of Houston

in Partial Fulfillment of the Requirements for

the Degree of

MASTER OF INDUSTRIAL ENGINEERING
(OPERATIONS RESEARCH OPTION)

THE UNIVERSITY OF HOUSTON

August 1976

ACKNOWLEDGEMENTS

Deep appreciation and gratitude are expressed to all of the members of this thesis committee for their counsel in the preparation and finalization of this writing. A special thanks is due particularly to Dr. Dawkins and Dr. Poage for their patience and cooperation in allowing me the opportunity to complete this somewhat lengthy work during several years of *in absentia* research. Most of all this thesis is dedicated to my wife, Betty, for all the hours of preliminary and final typing, plus the many years of nights, weekends and holidays she gladly sacrificed in the course of my studies.

Robert S. Hoyle
Manager, Management Sciences
Tesoro Petroleum Corporation
San Antonio, Texas
August 1976

SOME PRACTICAL ASPECTS OF CAPITAL BUDGETING USING CHANCE CONSTRAINED PROGRAMMING

by

ROBERT STEPHEN HOYLE

An Abstract of a Thesis

Presented to the Faculty of the Graduate School of

Engineering at the University of Houston

in Partial Fulfillment of the Requirements for

the Degree of

MASTER OF INDUSTRIAL ENGINEERING
(OPERATIONS RESEARCH OPTION)

THE UNIVERSITY OF HOUSTON

August 1976

ABSTRACT

The use of mathematical programming(MP) in capital budgeting(CB) industry practice appears to be rare even though the concept has been in the literature for a number of years and seems well-suited to the problem. The hypothesis is proposed in this thesis that perhaps the inclusion of uncertainty in the MP formulation, via chance constrained programming (C^2P), would enhance the industry utilization of MP in CB--since the uncertainty data would better fit the sophistication of a MP approach.

A review of some previous CB theoretical works and industries' response to these is presented in order to address some of the practical difficulties in the adoption of more sophisticated CB techniques by industry. From this survey, trends in CB practice appear to substantiate the claim that C^2P may indeed show some promise for increasing the use of MP in CB. It remains to investigate some of the details of actual industry implementation of such a concept.

An overview of C^2P and CB is discussed, and to clarify this discussion some details of a "fairly realistic" C^2P/CB formulation are presented. Data gathering aspects are included in the discussion, mostly from a standpoint of making uncertainty estimates. Computational problems are addressed, and linearization of the non-linear chance constraints leads into discussions of integer linear programming (ILP) and some heuristic ILP approaches. Finally, trends toward MP and CB are presented, and implementation factors are discussed such as 1) "educational" requirements between managers and management scientist and 2) political factors that can destroy all attempts to implement more sophisticated CB techniques.

TABLE OF CONTENTS

Chapter

LIST OF FIGURES

FIGURE

CHAPTER I

INTRODUCTION AND DISCUSSION OF BACKGROUND MATERIAL

Introduction

As used in this thesis, capital budgeting(CB) is a resource allocation problem usually encountered by higher management of large corporations when they are faced with deciding which capital projects, among a given set of capital projects, should be selected for implementation. Oftentimes this is also referred to as the investment portfolio selection problem of CB.

The essence of the problem is in defining project alternatives available, determining their cash flow, and finding some measure(s) of their economic profitability. The origination and detail of these projects are usually made at lower echelons of the company, but it remains for top management to ultimately decide which projects best contribute toward the overall, integrated objectives of the firm.

Frequently, project selection is complicated by political and organizational objectives which are not readily subject to quantitative analysis but must rely on management's intuition and "feel" for the problem. However, there are several decision making areas in the CB process which are, for the most part, quantifiable. For example, the project set selected (as a whole) should consider at least the following quantitative items: 1) one or more measures of economic profitability, 2) internal and external cash requirements for the firm, 3) financial reporting to the investment community, and 4) projects interacting with each other as competitors or complements. If management is to select the "best" project set from the large number of possible project set combinations, where "best" somehow includes all of the above quantitative requirements, a considerably complex evaluation could be required.

One of the methods theorists have proposed for assisting management in these quantifiable aspects of the CB selection process is mathematical programming(MP)[1]. This recommended approach has been in the literature for a number of years; however, its use in industry still appears to be relatively rare even though it seems well-suited for efficiently handling many of the quantifiable complexities mentioned above.

Apparently one obstacle is in overcoming some of the lack of education of management on this subject--but not all the blame can be placed here. A significant problem area seems to be in addressing the CB problems of management in such a way that the approach is realistic (but not overwhelmingly complicated) and at the same time computationally expedient. The trade-offs between a "simple but accurate and fast" CB model are not easy to define.

This thesis proposes to generally address the problem of why MP techniques have not found much acceptance in CB practice. Related to this, the thesis specifically discusses some of the prospects for industrial

---

[1]MP is the general descriptor for techniques such as linear programming, integer programming, non-linear programming, etc.

application of the more advanced mathematical programming/capital budgeting (MP/CB) technique, chance constrained programming(C^2P).

Chapter I presents a review of some previous theoretical works in CB, and this is followed by a discussion of the impact these theoretical works have had on industry. For the most part the "practice" side of the discussion is derived from surveys and descriptions in the literature on the subject, but some comes from the author's own experiences within his company and information gathered from similar companies.

From the review of earlier works and study of practices, the possibility of a chance constrained programming/capital budgeting(C^2P/CB) model appears promising. Even though the basic theoretical background and computational aspects are very involved, it is felt that the model can be introduced in a simplified form which intuitively appeals to management's sense of uncertainty in the CB problem. The remaining aspect is to keep the data gathering and computational difficulties to a minimum.

The approach taken in this thesis is to first present an overview of C^2P and its application to CB; this is done in Chapter II. General discussions of some pertinent aspects on objective functions, constraint sets, financing, and uncertainty considerations are presented.

Since some of the Chapter II subject matter may be a little abstract, an example problem is formulated in Chapter III to further clarify these points. However, the <u>main</u> purpose of the example problem is to construct the elements of a "fairly realistic" model and examine the difficulties encountered in formulation, data gathering and solution that might be encountered in an industry application.

Although the example problem is not actually solved, an analysis is conducted into what would be involved in data gathering and computer implementation. Also, a "fully realistic" example is beyond the scope of this thesis; so even though complete realism is not covered in the model, the formulation and analysis does at least provide <u>some</u> feel for the practicality of incorporating uncertainty in the CB process <u>via</u> C^2P.

Chapter IV discusses the data gathering problems, primarily in the area of uncertainty estimates. Chapter IV also discusses the computational aspects, mainly in the area of a heuristic, sub-optimal integer <u>linear</u> programming technique (which is made possible by some linear approximations to the non-linear chance constraints). The fast, flexible capability of such a heuristic computational procedure is discussed in the light of management's natural curiosity to "what if" the model's parameters.

Some implementation aspects are discussed in Chapter V. This includes the integration of the C^2P/CB model with existing CB procedures and other corporate models, as well as the problems of selling the idea to management. Finally, Chapter VI presents the summary and conclusions which can be drawn from the foregoing analysis and makes some recommendations for further study.

Background

There is a considerable amount of literature on capital investment and the related area of CB. One obvious reason for this is that the subject transcends such a wide range of disciplines like economics, finance, accounting and engineering. Each of these areas have made numerous and valuable contributions to the theory and practice of CB.

The Appendix to this thesis and Swalm's 1967 bibliography(1) provide examples of extensive, but certainly not exhaustive, surveys of some of the literature in this field; and both illustrate the wide range of subject matter. Topics range from theory and practice on project economic profitability (such as present value, rate of return, the cost of capital, discount rate, etc) to elaborate, pure economic theories on capital consumption strategy, supply and demand in uncertain markets, and social wealth and welfare. Application literature includes plant and warehouse sizing and location in both public and private sector industries, replacement or maintenance of equipment, selection of R&D projects, stock and bond (securities) portfolio selection, etc.

The following discussion is intended to be an excursion into some of this extensive literature on CB. Its purpose is to 1) provide background materials for the reader, 2) help justify the rationale for selection of this thesis topic, and 3) gather information which can be used later in the thesis topic evaluation.

One of the primary motivating factors in selecting a topic was to find something in the CB (and hopefully, MP) field that could be used in a practical industrial situation and also contribute meaningfully to the academic literature.

By reviewing some of the works in CB, it was possible to get an indication of what topics were of interest to industry--at least as perceived by academic writers. Of equal importance, however, was to research the findings of various surveys on actual CB industry practice to see which of the subjects promoted by theorists were and were not being adopted.

Of those being implemented, some questions which arose were: how extensive was their use, why were they accepted, and did this acceptance by industry forecast any trends in future adoption of more sophisticated techniques? For those ideas not being used, the question of "why not" was asked--was it primarily because of time-lags in education of management, etc, or were the concepts just not "practical"?

Finally, for concepts which appeared promising from a practical standpoint, attempts were made to gather from the "practice" surveys more information on what kind of general changes would be needed to achieve implementation. From all of this, C^2P approaches to CB appeared to show promise and satisfy the desire by the author to do more study in the applied MP area.

Hopefully, the following review will convince the reader that indeed C^2P does show some potential; and perhaps he may more clearly see for himself (by reference to the cited surveys, etc) some other areas where research should and should not be concentrated to help solve some of industry's "real world" CB problems.

Some Earlier Works in Capital Budgeting:

Perhaps one of the earliest works related to modern-day CB is by that of a mathematical economist, Irving Fisher(2). In 1930, he published a book which was a summary and extension of an earlier (1906) book of his entitled <u>The Nature of Capital and Income</u>. In the 1930 work he dealt primarily with the subject of interest rates, the time value of money and "human impatience" to spend.

His ideas perhaps laid a large part of the foundation for current mathematical theories in capital expenditure evaluations, for the book is frequently referenced in later theoretical works. It appears that many of the subsequent publications of authors in the field of CB are amplifications of Fisher's writings on discounted cash flow rate of return, present value, opportunity costs, optimal investment decisions, and risk.

After Fisher's work it seems a large part of the economists and professional/academic (theorists) areas devoted most of their attention to other matters relating to public welfare and economics--a more macroscopic approach than is usually dealt with in CB. In 1951, however, Joel Dean(3) published a significant book, Capital Budgeting. In it he directly addressed many facets of the CB problem which corporate managers still face today.

Unlike Fisher's work, Dean did not elaborate on discounted cash flow theory and mathematics, but instead accepted this as a valid means of measuring project economic worth--and from there proceeded to develop an analytical framework for systematizing management's approach to the CB problem. He used ideas and problems he had gathered from working with actual companies (in his consulting practice) like Socony-Vacuum Oil Company, Gulf Oil, Shell Oil, Standard Oil, General Electric, Johns-Mansville and others.

Dean discussed the problems of generating good investment opportunities, screening these for economic worth, including uncertainty into the analysis, and classifying projects by type. Also, he addressed the problem of the length of the "planning period" and concluded that seldom does a company go beyond two years, in practice. Longer-range plans are generated but not really incorporated into the CB picture until the future is more certain.

The discussion on economic evaluation by Dean is straightforward. He talks about payout period and some of its deficiencies, leading into the preferable discounting techniques of present value and rate of return. For risk he suggests modifying required rate of return, adjusting the discount rate if using present value, changing the life expectancy of the project, and finally, actually making probability estimate of various outcomes.

The supply of money for capital expenditure from internal funding and outside sources is discussed. The "cost of capital" computations are presented as being determined from the sources gathered. Finally, the "capital rationing" (used in this sense as how much capital should be spent) problem is related to the intersection of the "cost of capital" and "demand for capital" curves. That is, projects should be accepted as long as their rate of return is above that of the cost of funds. This subject he presents in several ways--a fluctuating effective rate (supply and demand curves for cost of capital), a minimum rate (fixed cost of capital), and exception rates (to account for maintaining competitive position even if the rate of return is below the cost of funds). The essence of the above "capital rationing" system was presented in Fisher's work, but Dean more succinctly applied it to actual project evaluations.

Dean devotes entire chapters to specific projects such as replacement type investments, expansion projects, new product investments and other capital outlays for strategic investments and risk reduction (such as diversification). The book concludes by discussing the effect of timing decisions on investments--such as postponing versus obsolescence risks, implementing now versus excess capacity risks, etc.

All-in-all, Dean's work was and still is a significant contribution to the CB problem and has become, along with Fisher's book, one of the "classics" in the field.

Several years later (1955), Lorie and Savage(4) published an article which dealt more directly with the CB problem in a capital rationing situation. Capital rationing in this sense meant a self-imposed or externally imposed predetermined limit on how much capital was to be made available for the capital budget. Dean's "capital rationing" is used more to determine how much capital to allocate and is not a budget ceiling. Lorie and Savage (L-S) presented three problems they had encountered while consulting with a large industrial firm: 1) given the cost of capital, which group of investments should be selected from a fixed capital outlay (i.e. budget constraint); 2) given a fixed sum for capital investments in <u>two or more</u> budget periods, which group of investments should be undertaken and 3) what to do when the projects are not independent.

L-S pointed out defects in Dean's capital rationing approach under dependent (mutually exclusive) project assumptions, multi-period budget constraints, and multiple solutions for the rate of return criterion. Their approach to the single period capital rationing problem was to rank projects by their ratios of present value to current year cost (sometimes called the present value profitability index) and then to select projects from the top of the list until the budget is exhausted. This approach differs from Dean's primarily in the use of the present value profitability index, rather than Dean's discounted cash flow rate of return (DCF-ROR), and the limitation on spending (capital rationing) due to a budget ceiling. The DCF-ROR can be shown to be an incorrect ranking tool for dependent, mutually exclusive projects and budget ceiling problems due to several limitations--primarily an implicit re-investment assumption of cash flows from the project at the calculated rate of return and possibly multiple solutions for the rate of return when the cash flow stream of a project contains alternations in signs.[2]

The L-S procedure works well for divisible projects; however, for indivisible projects only a trial and error solution is proposed for determining the optimal budget set. Multiple period budgets require simultaneous, multiple trial and error solution, and this kind of difficulty later contributed to the use of the MP approach in CB.

L-S's work along with Dean's (and several others not mentioned here) promoted additional CB publications in journals and texts during the late 1950's and early 1960's. Among the most notable of the books were those of Solomon(5), Bierman and Smidt(6), and Grant and Ireson(7). Journal publications from this period cited frequently in the modern-day literature are Solomon(8,9), Bierman and Smidt(10), Modigliani and Miller(11), Hirshleifer(12), McLean(13), and Baldwin(14). Many other authors were

---

[2]A survey by Fremgen(23) in 1973 showed that multiple solutions to DCF-ROR do occur in industry (about 15% of the time), and thus the subject is not just of academic interest.

also publishing during this time, and all were attempting to educate (and persuade) industry in the "newly developed" approaches to CB.

Central issues espoused in most of these works were the use of discounted cash flow (DCF) techniques such as rate of return (DCF-ROR), present value (PV), or profitability index (PI = total present value/present value of capital outlay). These subjects were generally advocated to be considerably superior to the accounting rate of return (ROI) or payback period (PP) so widely adopted by industry as project evaluation and ranking criteria.

Spin-offs from these DCF topics were debates on the relative merits of the individual measures. The controversy between DCF-ROR and PV is still not completely decided, although most agree PV techniques are generally superior*. Many thousands of words have been written on the theoretical deficiencies of the DCF-ROR; such as the assumption on re-investment of generated funds at the rate of return calculated, and multiple solutions (or none) to the DCF-ROR. However, despite its deficiencies, DCF-ROR still continues to find wide support.

The PV measure also suffers from the limitation of having to pre-determine an appropriate discount rate. Since a project's PV will appear economically different at various discount rates, the use of the appropriate discount rate has become a major controversial issue usually involving subjects such as the "hurdle rate", "aspiration rate", or "cost of capital". The "cost of capital" and discount rate controversy is still being discussed today in the literature.

---

*This continuing theoretical controversy over the "right" economic criteria has no doubt slowed the acceptance of DCF methods by industry. Firms may be thinking "the theorists cannot even agree among themselves, so we'll just wait until the issue is resolved before adopting any changes".

PV also fails to give an efficiency measure (like a rate of return), so its derivative profitability index (PI) was developed to provide an efficiency criterion. The only problem is that the numbers derived are "abstract" to management. Where a DCF-ROR of 15% can be compared with a bank or bond interest rate, to what can one compare a PI of 1.35? These and many other related topics were the central theme(s) of the above literature.

The Response of Industry:

Although there was a large effort by theorists and academia to gain industry's attention on the more appropriate means for economic evaluations and CB, the acceptance of these techniques was still slow.

One of the earliest attempts to measure how much of the current CB theory was being applied in practice was Istvan's 1961 monograph(15). His DBA dissertation was based on a survey of then current (1958) CB practices at 48 large diverse corporations in the U.S. In this he personally interviewed 147 financial executives, ranging from controllers to CB specialists (the preponderance were controllers, tax advisors or treasurers). Analyses were conducted on the forms and administration procedures but of most interest were the results on techniques for economic evaluations. His conclusions were that there <u>had not</u> been extensive adoption, among the firms studied, of the theoretically superior techniques of capital expenditure analysis. Furthermore he concluded that no basic agreement was found among the management of the companies regarding how much concern and effort should be expended to develop such a CB decision process.

As a measure of economic acceptability Istvan found that many of the firms used the PP as their primary economic indicator (13 of 48); a majority were using ROI (24 of 48); and only a small portion were even using DCF techniques (5 of 48 using the DCF-ROR and PV). Istvan's conclusions regarding the poor showing of the theoretically-correct measures of economic worth basically fall into the position of 1) management saying "we've been too busy making money to worry about setting up a better CB system"; or 2) a lack of education and understanding on management's part (which hopefully would be resolved over time) of the benefits of a better system.

Although Istvan's survey was not very encouraging, later surveys evidence the increasing tendency towards the DCF concepts and more away from the PP and ROI criteria for project selection. A survey in 1966 by Christy(16) still showed a heavy reliance on ROI and PP; however, Christy's results indicated a slight increase of DCF methods to 14% (compared to Istvan's 10%). Most of the firms employing DCF evaluations were chemical (and although not explicitly stated, probably petro-chemical) companies, simply because they are very capital intensive industries which tie up large amounts of funds for very long periods of time. DCF methods are of the most advantage to investments of this nature.

A 1966 report(17) by Kempster of the National Association of Accountants (NAA) and a 1963 Conference Board report by Pflomm(18) both hedge somewhat on the subject. The NAA report states the sample size was too small to conclude anything (28 firms surveyed, half using no discounting and half using discounting). The Conference Board report simply says there is much controversy over the various criteria and most management of companies they surveyed indicated they thought as long as the criteria was applied consistently in the company it did not matter too much which was used. The fallacy of this argument appears to be that a company can be doing everything completely consistent internally, but be "losing its shirt" to the competition who is also being consistent, but at the same time more "accurate" with DCF methods.

Another survey, by Williams in 1970(19), showed a marked increase to 69 of 100 firms using DCF methods--specifically 55% using DCF-ROR; 14% using PV. However, 51% were still using PP as a supplemental criteria. The reasons cited in the survey for not using DCF methods was similar to Istvan's findings--a lack of understanding at some management levels of DCF techniques; also a preference for PP benchmarks was listed as a strong reason for not using DCF criteria. Similarly, a study by Klammer(20) in 1972 showed a 57% figure (of 184 surveyed) for firms using DCF methods, and PP as a primary criterion had dropped to 12%. Another survey by Neuhauser and Viscione in 1973(21) showed wide acceptance of DCF measures (usually with larger budgets), but most companies maintained their older methods (such as PP,ROI, etc) since it provided a link with the past, served as a transition tool, and kept management "more comfortable". This seems like a very pragmatic approach in which the new DCF techniques are benchmarked against the older ones to lend credence to the new measures.

Finally, a survey by Petty et al(22) in 1975 showed 58% of the 109 corporations queried, used DCF methods for evaluating new product lines, and 50% used this for existing product lines. The PP had dropped to 11% and 12%, respectively. Interestingly, this survey is one of the first to directly question the use of the Profitability Index (PI)--the criterion proposed by Lorie and Savage(4) in their capital rationing ranking problem. The PI percentage was low, 1% to 2%, but this is not surprising since DCF rate of return is still preferred over PI as a ranking tool primarily because (as stated in the NAA report) 1) management is accustomed to using rate of return, 2) DCF-ROR can be directly compared to bank and bond interest rates, etc and 3) PI gives an abstract number to which management cannot relate. However, the mere appearance of PI in a survey such as this shows that industry is at least "listening" to some of the theoreticians, and perhaps a better ranking device may evolve.

From the above discussions, it appears evident that DCF techniques are becoming more accepted by industry in general, although use is still far from being universal. For those using DCF methods, the DCF-ROR is significantly favored over PV or its derivative PI. The majority of DCF-ROR users follow Dean's simplified, constant cost of of capital approach in selecting projects (i.e. any above the cost of capital are acceptable). There is little to indicate that Dean's more sophisticated approach of having the spending limit be determined by the variable cost of capital schedule (variable with amount spent) is being used.

Unfortunately Christy's study(16) keeps bringing up a point in contradiction to the "scientific" DCF approach to CB. In his survey one of his major findings was that there was no relation between the CB practices of firms and their <u>earnings performance</u>. He concludes that DCF project selection techniques proposed as "scientific" were of no greater benefit than other forms of project evaluation; and that current practices of industry in this regard, although unscientific, appeared adequate.

Christy's data may indicate there is little true value for DCF methods. However, certain biases could be present causing these conclusions. One is that the number of companies using DCF techniques in his survey was small (about 14%). The technique was probably new to them, or they may have been using them incorrectly thus offsetting their advantage. Another explanation is that since the adoption of DCF techniques was still relatively new at the time of Christy's survey, the benefits of these techniques had not completely shown up in changes to earnings. Also, as Istvan pointed out in his survey, during times of prosperity scientific analysis is not always necessary--practically any approach will give reasonably profitable results.

But perhaps the most probable of all biases is that the use of specific criteria is only one part of the CB process. A large portion of the earnings of capital investments rests on managements' shoulders in other quantifiable and non-quantifiable areas. Fremgen's survey in 1973(23) showed that the most critical phase of CB to management was the project definition and cash flow estimates. Even the best criteria procedure would be neutralized if these basics were not done correctly.

Comparing earnings performance and CB project selection criteria of a company is a difficult task that appears unresolved. Istvan purposefully avoided this issue by stating "unfortunately it is not possible to prove that profitability is greater with this measure, but all the executives of the firms now using time-adjusted rate of return are of the opinion that it has increased".

The best approach would seem to be a comparison of a specific company's (or industry's) average earnings over a period of years before and after DCF techniques were employed. The problem here is removing the effect of external factors such as economic recessions or inflations as time changes.

In any case, the surveyed industry practices clearly show that DCF techniques are being widely adopted. The majority of managers are far too pragmatic to adopt these techniques just because they are fashionable or to give the appearance of sophistication (although, granted this does happen). It seems more likely that someome in industry must truly be convinced that these "scientific" approaches are more profitable to the firm--although measurement of this is difficult, if not impossible.[3]

Another aspect of interest usually addressed in the surveys was that of the use of capital rationing (budget ceilings) in CB. Although 34 of the 48 companies studies in Istvan's survey were using a rationing budget (i.e. no financing), his survey did not deal significantly with the capital rationing aspects of how competing projects are selected (such as in the L-S problem), except to state that they were normally selected by the ranking of the primary indicator. Several (10) did not use budget

---

[3]The benefit/cost measurement problem of any management science technique is a common theme running through all types of applications, and it is only when industry is favorably convinced either by "gut feel" or tangible results, that the techniques are accepted. This point will come up again in later discussions of this thesis; in fact, it is probably the central issue--is there a favorable benefit/cost ratio for using more sophisticated CB techniques (i.e. C^2P) in industry?

rationing. Of those that used DCF techniques in capital rationing (two firms), they employed DCF-ROR as a ranking tool.

The other studies on industrial practices in CB also discuss the applicability of the capital rationing problem, again through in little detail as to ranking mechanism. The 1963 Conference Board report by Pflomm(18) cites evidence that uses of the capital budget involve "selecting and assigning priorities to projects, the sum total of which frequently far exceeds the funds available". The NAA report by Kempster (17) somewhat supports the contention that a fixed capital budget size is used; however, they found that 20 of the 28 firms employed a more flexible budget form where external financing would be tapped if more favorable projects became available.

The 1966 study by Christy(16) on CB practices indicated that for the majority (90%) of the 108 companies sampled, shortage of cash (either from internal generation or selling stocks or bonds) was <u>not</u> of significance. Their limitations were more on the lack of attractive investments or a physical inability to digest projects as fast as desired.

Christy's survey results in this area seems to be in general agreement with the above cited NAA report; however, the NAA report also states, "Care must be taken not to attach too much significance to these samples of company practice. Changes in circumstances may force a company to change from one approach to the other"--probably a true statement, since changes in the economic climate and money market with time would change a manager's perspective on adopting capital rationing.

Therefore, the timing of the surveys may account for some of the difference in emphasis shown toward capital rationing. For example, a more recent (1973) survey by Fremgen(23) found that 73% of the 177 companies queried experienced capital rationing either from limitations on borrowing dictated by management or the money market. 64% of these firms said they faced capital rationing every year, while the remainder experienced this less frequently. Petty's survey in 1975(22) also showed capital rationing to be of significance; some 58% of the survey respondents stated a self-imposed arbitrary expenditure ceiling existed for CB. Petty points out that this does not really resolve the question as to what extent this rationing impedes investment.

Capital rationing in multiple periods (as posed by Lorie and Savage) does not appear to be of practical interest--probably because most firms only deal with a one-year budget. However, Istvan indicated that a small portion of firms use a two-year budget, and the 1970 study by Williams(19) showed 57% of the 110 companies surveyed made capital expenditure projections at least four years into the future. The definition of whether this falls into the current-year CB process was not made clear in Williams' questionnaire, however, and other research indicates the multi-period aspects of CB seldom go past two years. If they do, these plans fall more into the category of long-range planning (which is indirectly included in budgeting but not in the form of budget limits).

The conclusion to be drawn here is that firms probably do, from time-to-time, experience capital rationing (budget ceilings), and the single period approach of Lorie and Savage would probably be appropriate in some instances. However, as pointed out earlier, their approach suffers some difficulties in application, and the previously cited surveys find little evidence that the Lorie-Savage model is being used at all. When companies do face budget ceilings, it appears that most simply use their established primary economic indicator (usually DCF-ROR) to rank the investments until the budget is exhausted. The problems with DCF-ROR multiple solutions and reinvestment assumptions (which can lead to project allocations with less total project <u>set</u> present value than by using PI as a ranking criteria) tend to <u>not</u> be generally recognized by industry.

Uncertainty and Capital Budgeting:

As indicated in the preceding section, the adoption of more scientific approaches to CB (namely DCF techniques) has slowly but steadily increased; however, acceptance is far from being complete in industry. Perhaps one of the reasons is due to the inherent uncertainties associated with project evaluations; that is, the vagueness of forecasting cash flows for projects--which are themselves only indefinite proposals--preclude the adoption by many managers of more scientific methods. The claim that "fine grain analysis is not justified with such coarse data" is understandable--"it's like putting a $100 saddle on a $10 horse", as one manager put it.

This is basically the argument in Christy's survey(16) for the poor showing of DCF methods in industry application. He points out that, of the various aspects in project evaluation and CB, the estimation of cash flows is of primary importance to accurate evaluation of project economics (ranking just behind the importance of project conception); and that advocates of DCF techniques, etc can justifiably be accused of a "misplaced emphasis" in CB theory. Several references he cites support his contention.

Furthermore, Christy states that "in the absence of reliability in cash flow forecasts, the manager is forced back upon the use of judgment; i.e. qualitative criteria and simple quantitative considerations. The simpler quantitative methods eliminate spurious accuracy and involve fewer hidden assumptions; and as a means of ranking projects for profitability, average ROI or PP is as likely to yield an accurate set of rankings as DCF methods using unreliable cash flow estimates. He claims "the businessman's clearest opportunity to improve his CB effort lies in improving his estimates of future cash flows, and only in this manner can the way be paved for the adoption of more rigorous methods of project ranking and selection".

Fremgen's survey(23) specifically questioned management on Christy's points and came to the same conclusion on the "misemphasis". Fremgen found that 51% of those responding claimed project definition and cash flow estimation were the most critical; 44% claimed these aspects were the most difficult in CB. 27% said that project analysis and ranking was the most critical, while only 12% judged this phase of CB to be the most difficult. He concludes that the subject of CB receiving the most attention in the literature (financial analysis and ranking) is neither the most critical or the most difficult, and that perhaps theoreticians should devote more of their time in the "proper direction".

Istvan(15) also found that managers believed the cash flow estimation was the most difficult and important part of the CB process. However, at least one of those he questioned, viewed DCF techniques as having an advantage of reducing the margin or error in estimates of cash flows in the future. In this sense it provides a measure of risk evaluation by de-emphasizing the more distant uncertain cash flows.

Williams' 1970 study(19) actually questioned managers on why they were not using DCF methods; most replied because they did not understand it or preferred PP standards (very similar results to Istvan's)--only 9% listed the forecasting of cash flows as the problem. The argument of Christy against DCF, mainly due to cash flow uncertainties, is possibly not "all the story"--especially considering the DCF future cash flow de-emphasizing aspect discussed above. Also, the "troublesome" aspect of DCF techniques is possibly not well-founded since Istvan maintains that most DCF-oriented managers feel that once the cash flow estimates are made, DCF calculations are not much more troublesome than any of the other criteria but provided considerably more reliable data.

In any case, whether cash flow uncertainty is the main hindrance to adoption of more scientific methods or not, there seems a strong possibility that judiciously selected uncertainty analysis on cash flows and financial indicators would improve the feelings management has toward the more sophisticated CB techniques, and at the same time, improve the quality of their decision making.

The theoreticians were already anticipating the uncertainty analysis problem in CB and were publishing works at an early date such as Dean's(3), Carter's(24), Schlaiefer's(25), Grayson's(26), Grant and Ireson's(7), and Farrar's(27)--all covering at least some aspects of uncertainty analysis in CB.

Perhaps the earliest forms for incorporating uncertainty into a PV analysis was the use of a "risk adjusted" discount rate. That is, instead of using the "risk free" discount rate where the returns from a project were known with certainty, the discount rate should be increased to allow for the fact that the returns may be unfavorable or lower than expected. This would cause the riskier projects to bear the burden of having their cash flows discounted more to make up for the uncertainty of the project. The concept is simple: for a riskier project one would require a higher return for acceptance or rejection than for a less risky project. (An equivalent method using DCF-ROR is to have rate of return greater than the risk adjusted discount rate). However, the principal difficulty with this approach is in determining how much the discount rate should be "adjusted" to reflect the uncertainty of the project. Most advocates agree the "risk class" of the project should be used, and usually they resort to the securities market for an assessment of this.

However, the risk class and security market correlation is still an unresolved subject. Many authors have proposed the risk class concept dating back to Fisher's early works, continuing with Hershliefer's 1961 paper(28) and even extending up to Lewellyn and Long's 1972 paper(29) which produced several counters and replies(30,31,32). These later articles show the concept is not "dead", but most authorities in the field feel it is one of the cruder manners with which to deal with uncertainty.

Another conceptually similar approach is the certainty equivalent method in which the uncertain cash flows are adjusted by a factor (between 0.0 and 1.0) to transform them to a certain cash flow. Again, the difficulty of specifying the adjustment factors precludes this technique from being accepted by the majority of theorists.

An approach similar in methodology to the certainty-equivalent technique but more theoretically correct in handling uncertainty is to apply probability factors (also between 0.0 and 1.0) to the various outcomes of a cash flow stream and derive its expected value. The discounted PV resulting from this is the expected value since it is composed of a linear sum of discounted expected value cash flows. DCF-ROR in this manner is not truly mathematical expected value for <u>all</u> cases, but it is a consistently applied uncertainty weighting technique which may be viewed for the most part as "expected value".

Grayson's book(26) is an example of the expected value approach. His work was very practically oriented, and directed primarily at oil and gas exploration project economic evaluations. In his Chapter 9 on "Statistical Decision", he illustrates how the use of specific probabilities could be subjectively assigned to various outcomes affecting cash flows. Weighting these cash flow outcomes by their probabilities, Grayson showed how the mathematical expected value for PV of the project could be calculated. Thus, he had included the various uncertain outcomes of the project in calculating the DCF economics and concisely expressed them as a single term, expected value. Of course, expected value was certainly not a new concept, and Grayson's book was not the first to apply this to project evaluations. However, he and others used this to incorporate some of the aspects of uncertainty into DCF measures, thus improving their overall information content and acceptability by management.

Although expected value was a theoretically correct and a more convenient way for expressing some aspects of uncertainty, it does not fully convey the uncertainty of the project. It was still a single figure and gave no information as to what other specific outcomes might occur. The standard deviation was one measure of the project dispersion but also it could not completely describe other outcomes or their probabilities without some assumptions on the form of the resulting probability distribution.

Frederick Hillier's work in 1963(33,34) was one of the first attempts to provide this "other outcome" additional information in a practical manner. His approach was to derive the expected value and variance of several economic indicators (such as PV and DCF-ROR) by analytical means. By proper assumptions concerning the form of the resulting probability distrubition, statements about the probability of the outcome being greater (or smaller) than a specified figure could be made. Information such as this would allow management to assess the trade-offs between various projects exhibiting different expected values, variances, and probabilities of losses (or gains).

Hillier developed basically two cases for his model to derive the variance of the economic indicator, PV. He first assumed independence of net cash flows for each period, and required estimates of the variance of each period's net cash flow (which he likens to refinements in doing a sensitivity study). The second case assumes the net cash flow of each period is formed from two components--one independent from all other periods, the other perfectly correlated (correlation coefficient = 1) with all periods. That is, circumstances causing one period's cash flow to deviate W amount would also cause all other period's cash flow to deviate W amount.

The assumptions concerning the form of the probability distribution are discussed mostly from a normality standpoint, showing how the Central Limit Theorem will make PV approximately normal even if the individual period cash flows are not normal.

Although Hillier's work was a significant step toward trying to make the information content of DCF techniques more consistent with their degree of sophistication (i.e. making the output as refined as the computational techniques), it still left several areas open. For one, his assumptions on perfect correlation of cash flows and normality of the PV function were not always valid. In some cases the estimation of "central tendencies" and "dispersions" of cash flows, which he assumed equivalent to the mean and standard deviation respectively, could be difficult to estimate.

For example, the estimation of means and standard deviations for cash flows of a complicated economic project involving many processes to generate revenue and incur costs, each with their associated uncertainties, could tax even the most seasoned manager or financial analyst.

Hertz(35) was one of the first to popularize this problem by proposing what is known as Monte Carlo simulation. This technique, which was not new and had been applied earlier in engineering studies[4], was

---

[4]See also in Grayson's book(26).

adapted to generating probability distributions (along with their mean and variance) of capital investment economic indicators. The procedure basically assigns subjective probability distributions to key variables which generate the revenue and costs for a period's cash flow. These distributions are each randomly sampled and a calculation of the appropriate indicator is performed. The random sampling process is repeated a large number of times and the resulting PV's,etc are pooled together to form a probability distribution which can be used much like that proposed by Hillier earlier for augmenting management's decision making.

The main advantage of the Monte Carlo technique is that very complicated processes can be handled,and in some cases this is the only way to generate an expected value for the economics, regardless of whether the probability distribution is used or not. The disadvantage is the large amount of additional data and analysis required, and even on a computer, calculations (number of random trials) can absorb hours.

Both Hillier and Hertz's approach to generating economic indicator probability distributions and their use was deficient in at least one respect. The manager still had to look at the probability distributions of various projects and decide which ones would be selected. This was largely an intuitive process in which the decision maker would decide that a project with an expected value of, say, $100,000 (but a 30% chance of losing $10,000) was preferrable to another project whose expected value was $50,000 (with only a 5% chance of losing $5,000).

This type of information was almost "overkill". Management had earlier expressed concern that DCF techniques were much more detailed than the cash flow estimates going into them; now, they were overwhelmed with economics which spanned <u>all</u> of the possible uncertain outcomes of the cash flows used to calculate them. There was simply too much information to be digested in a practical manner, even by a relatively sophisticated manager.

For handling this much information, a way must be devised to succinctly express the outcomes of a probability distribution as a single number--like expected value does. However, this single measure must include the <u>preferences</u> that management might have for various outcomes; that is, include reasons why one decision maker would reject a project if his probability of loss being greater than $10,000 is 30% or more, even if the expected value is very large.

The subject of cardinal utility theory was brought to bear on such a problem of handling these preferences for various outcomes. Bernoulli was the first to analyze the fundamentals of this theory in 1732(36). His work was later proved with modern-day mathematics by Von Neumann and Morgenstern in 1947(37); and other authors have since then contributed greatly to the practical application of the theory--Friedman and Savage(38) and Schlaifer(25) to name just a few.

Bernoulli's concept was that the preference for a set of various outcomes could be expressed as a single number called <u>expected utility</u>. This was exactly like expected value, except, instead of summing the dollar value of the outcome times the probability of the outcome as in PV expected value, the probability of the outcome was multiplied by the "utility" of the outcome. Borch(39) presents a lucid description of this approach in the first four chapters of his book.

The concept hinges on the fact that a preference ranking of outcomes can be made for the outcomes and expressed as a measurable quantity on a scale. For instance, the utility of an investor for \$x from a project can be expressed as

utility = f(\$x)

where f is the function describing the relation between utility and dollars.

The key is in defining the utility function f(\$x) and many authors have written about how to do this. They fall basically into the class of descriptive and prescriptive (or normative) utility functions. Swalm(40) published a clear and interesting (but certainly not unique) paper on descriptive utility functions in which he showed how a utility function could be determined by questioning various individuals on their preferences for money under risk. This concept is in the form of a lottery in which you ask the participants how much money they would want with <u>certainty</u> in place of a game giving them R dollars with probability p and S dollars with probability (1 - p).

Several questions such as this are posed to the participants (with some questions to check for consistency) and a curve of utility versus money can be constructed. Grayson's book(26) shows an actual case in which he constructed a utility function in this way for an executive of the Beard Oil Company.

Other proponents of utility theory advocate the prescriptive technique in which a mathematical form of the decision makers utility function is hypothesized (such as hyperbolic, or exponential, linear, or combinations) and then coefficients to define the function are determined by asking the participant a limited number of questions to define just a few of his risk attitudes. Smith(41) presents a detailed examination of this approach relating to the petroleum industry.

Response of Industry:

Some of the previously cited surveys on CB practices in industry provide information as to what extent the writings of theorists dealing with CB and uncertainty have gotten across.

The earlier studies by Istvan(15), Pflomm(18), and Christy(16) did not address the uncertainty analysis aspect in their surveys although Christy felt that the lack of uncertainty analysis (i.e. uncertain cash flow forecasts) was a definite hindrance to the adoption of more sophisticated DCF methods. The NAA 1967 report by Kempster(17) discussed uncertainty analysis in the chapter on "More Advance Techniques" and indicated that sensitivity studies (an uncertainty analysis form where the sensitivity of the economic indicator is investigated as selected inputs are changed one at a time), "risk analysis" (i.e. expected value), and simulation (Monte Carlo types among others) were being tested on an <u>experimental</u> basis in several <u>large</u> companies.

Williams' 1970 survey(19) showed 8% of his respondents were using probabilities in conjunction with DCF methods, although he does not elaborate on how the probabilities are being used. Klammer(20) indicated that 21% handled risk by adjusting the discount rate (or raising the required rate of return), 10% shortened the PP, 13% determined probability distributions, and 3% considered covariance between the projects. He noted that the petroleum, refining, and chemical industries accounted for a large percentage of those using probability techniques--again probably because of their large capital investments and high degree of uncertainty.

Fremgen's(23) questions on risk and uncertainty showed 54% adjusting the primary economic indicator, 40% shortening the PP, 32% used probability estimates on cash flows (probably expected value), 29% used subjective judgment, and 8% were using Monte Carlo, sensitivity studies, etc.

The 1974 Converence Board report by Davey(42) dealt extensively with the techniques of risk analysis. They found 15/84 companies use modification of their hurdle rates or calculate economics on several sets of input data (sensitivity analysis). Those using the hurdle rate change said they encountered problems with this because of not knowing how much to adjust the hurdle rate. 19 of 84 perform sensitivity studies, 40 of 84 said they were doing risk analysis on projects, although the study did not break down as to what type (it implied these were expected value and Monte Carlo simulations being done).

Finally, Petty's 1975 survey(22) questioned management on how they defined "risk". This is an important point overlooked by many authors when writing on uncertainty analysis. Risk is more appropriately used to mean the probabilities of loss, either absolute, or loss from expected gain--in other words, the "downside" portion of the probability distribution. Uncertainty analysis covers both loss and gain; i.e. variations both above and below the expected values. Risk (loss) in the former sense is sometimes called semi-variance, while uncertainty is just variance.

Petty's survey showed that most management (40%) view risk as the concept defined above: Loss or semi-variance. However, 30% indicated that risk to them meant variations in either direction; i.e., uncertainty or variance. Mao's early work(43) with only 8 companies indicated semi-variance was the main concept, and although this still holds, variance appears to be coming into its own as a concept of "risk".

Petty also found that 61% of the firms still use PP to determine the risk of a project, 65% are using risk adjusted discount rates, 30% are measuring the variations of returns (sensitivity studies) or use Monte Carlo simulation. Several were using combinationsof the above methods on a more or less frequent basis (accounting for the greater than 100%).

Clearly, the above surveys show that uncertainty analysis is gaining ground in applications within industry. The one exception is utility functions. No mention of this was made in any of the surveys although considerable theoretical work has been done in the area. To this writer's knowledge he knows of no company using or contemplating using such a concept.

It appears the major problem with utility theory is that of determining what is the utility function form (by either prescriptive or descriptive techniques), who decides whose "utility" is to be used, and how one can keep it up to date once it has been defined. Unfortunately, the place one would expect utility theory to be employed is exactly where it is not--in the large companies. This is mainly because they are so large that their assets and resources make their utility function simply linear. For this case expected value and expected utility are identical (i.e. the investor is risk indifferent). Even medium-sized firms avoid the non-linear portion of their utility curve, where the losses become really damaging, and stay on the linear portion. Thus, they too have little use for utility theory.

All of these above surveys and statements tend to support in general the experiences and information gathered by the author of this thesis in his work experience. Proprietary documents on capital budgeting and project evaluation procedures for several large petroleum companies (44,45,46,47,48) show many of the previously discussed DCF and ranking techniques, as well as those uncertainty analyses (see Smith, ref. 49) outlined above are being adapted to current company operations.

Mathematical Programming and Capital Budgeting:

Although the previous discussions dealt with literature and industry advances in some areas of CB techniques, they mainly were concerned with analysis of individual projects. Many theorists feel that an equally important aspect of CB is the selection of an optimal set of projects.

Truly, this is what management does; they try to select the best set of projects submitted to them at budget time. The question is, can this process be improved by some more sophisticated quantitative technique than just ranking the individual projects (possibly adjusted for uncertainty) by their primary economic indicators? Are there other considerations that management must deal with when selecting the project set--such as budget constraints in different periods (either self-imposed or externally-imposed), interactions of different projects with each other and time, additional criteria to be met besides those considered by the economic indicators, financing aspects, etc?

Mathematical programming (MP) is the type of tool designed to solve problems of the nature as conjectured above, and there have been a large number of academic writings in this area. Again, the Appendix of this thesis shows some of these works from about 1964 to 1972 and the Bibliography by Larson(50) covers some 140 items from about 1955 to 1970.

One of the earliest (and often cited) articles in this area was in 1959 by Charnes, Cooper and Miller(51) in which they constructed a linear programming model of a firm's combined physical and financial operations. Total profit (not discounted) was maximized over a specified planning horizon subject to capacity constraints (on a warehouse) and constraints on cash position with time. Trade credit and borrowing-lending opportunities (short-term) were incorporated to include some of the aspects of an imperfect capital market.

The selection of specific projects was not considered explicitly, but rather the fundamental cost and revenue relations of the firm were modeled to determine the net cash flows by selecting to what extent a given project should be adopted. The dual variables of the cash constraints with time were examined to determine the contribution to profit of having an additional dollar to spend--thus the elusive "cost of capital".

Perhaps the most notable and widely acclaimed work in this area was Weingartner's 1963 Dissertation which was later (1967) reprinted in book form(52). His subject was basically the Lorie-Savage (L-S) problem(4) cast in a linear programming (LP) formulation. He first examines the L-S single period budget model and points out some of its weaknesses--such as not being able to handle project indivisibilities in an efficient manner since the problem is to examine all possible sets of projects and pick the best. However, the L-S multi-period budget case with non-independent projects is where Weingartner devotes most of his attention, since L-S's approach is very tedious (perhaps impossible for more than two periods) and directly amenable to LP solution.

This LP formulation tried to solve the problem of indivisibility by examining combinations of projects, rather than looking at just one project at a time. Although fractional projects are allowed in the model, Weingartner develops proofs that the number of fractional projects cannot exceed the number of periods for which budget constraints are imposed. So a good sub-optimal solution can usually be achieved with the LP model even when only integer (0,1) solutions are desirable.

Dependent projects are also taken into account by including mutually exclusive and contingency relationships among the decision variables. Again though, these hold strictly only for integer programming solutions. Fractional solutions of mutually exclusive projects, however, can often give valuable information that perhaps the mutually exclusive constraint should not be binding and that the projects should each be accepted at a lower level (assuming constant returns to scale). Some other types of non-financial constraints such as manpower, raw material or other scarce resources related to the specific projects are also considered.

The dual variables of the problem are examined and interpreted for evaluating whether adjustments should be made in the budget perhaps by going to external markets or by shifting budget funds from one period to another.

Integer programming (Gomory's cutting plane method) is adopted where integer (0,1) requirements are explicit, and Weingartner develops interpretations of the combined dual variables of the cutting plane constraints and original constraints (this subject, however, is very difficult and beyond the scope of this thesis "Background" discussion).

Finally, Weingartner develops (in Chapter 8) some alternate formulations of his basic model which he says "hopefully prove to be somewhat more directly applicable to real-world situations". This model is a compromise between that of the previously discussed Charnes, Cooper, and Miller(51) paper and that of the L-S problem in which he maximizes terminal wealth (non-discounted) subject to constraints on funds available for spending from internally generated sources and investment cash throw-offs. Short-term lending and borrowing are introduced and a later model places constraints on the amount which can be borrowed in a given period (under both perfect and imperfect capital markets).

Without a doubt, Weingartner's work remains as one of the most comprehensive treatments of MP and CB; and its impact on the works of future authors in this field (including the writer of this thesis) is very great indeed.

Although Weingartner's MP treatments have been widely acclaimed, they still contain some theoretically controversial areas, primarily concerned with the discount rate used for calculating PV.

Hershleifer, in 1958(12) was one of the first to address the problem of the Lorie-Savage CB model and the discount rate used to calculate the PV of the projects. He contended that an externally defined discount rate was inappropriate since the "cost of capital" was not generally known until the optimal investment set had been selected (due to the cost of capital being equal to the rate of return of the best project rejected). Thus in order to get the answer to the L-S model, where budgets were constrained per period and borrowing is not allowed, the answer must be known to start with. This has come to be known as the Hershleifer Paradox.

Later, in 1965, Baumol and Quandt(53) proposed a solution to the Hershleifer Paradox by including an externally defined utility curve as the objective function which shows the decision maker's preference for withdrawal of funds over time. Using duality theory they conclude that the correct discount rate can be determined by proper interpretation of the dual variables. The problem of integer requirements on the variables and the resulting dual interpretation difficulty is not discussed.

Lustzig and Schwab(54) recommended internal determination of the discount rate by starting with an arbitrary discount rate, solving the problem for an optimal solution, examining the internal rate of return (DCF-ROR) of the next most attractive portfolio, comparing this to the previous discount rate and repeating these steps until they are the same. Their approach appears simple, straightforward and based on the apparent logic that the discount rate should be the rate of return of the best foregone portfolio.

In 1971, Bernhard(55) countered Lustzig and Schwab's argument by showing how rate of return selection and iteration as they proposed would fail to select the "best" projects in certain instances (he used several examples to illustrate this). His contention is that Baumol and Quandt's argument is still correct and that externally defined discount rates are not possible with budget constraints; the only way to define them is through the utility function approach originally proposed by Baumol and Quandt.

All of these articles simply indicate that the discount rate, cost of capital, or whatever it is called is probably the most complex and controversial subject in CB theory. Lorie and Savage ignored it, Weingartner specifically stated he was going to ignore it, and many other authors [such as the more recent (1972) articles] ignore it.[5*]

Perhaps Baumol and Quandt do have a valid point, but the fact that they can determine discount rates by the use of a utility function does not appear to help any. It is not clear how they intend to use this information because their objective does not contain PV and thus needs no discount factor. And the determination or use of a utility function still remains to be accepted by industry for a number of reasons discussed earlier.

In any case, it has been shown that industry has accepted DCF techniques and it is becoming firmly entrenched into their CB procedures

---

[5]For example, see Bussey's(56) statement on page 3 in which he states, "Assuming a discount rate may be hazardous but this approach is justified on the grounds of an approximate solution is probably obtained plus theoretical considerations prevent any other approach, except to assume it is known".

*This continuing controversy over the discount rate may be causing some problems in acceptance of the MP/CB approach by industry. The same type of situation may be occuring here as mentioned in the footnote on page 11; i.e. "The theorists cannot even make up their own mind, so we'll just wait until it's resolved before adopting an approach".

more each year. To have a company switch from something that is considerably better than what they were previously using (non-DCF methods), and something they are beginning to understand (DCF techniques) to an abstract concept such as utility functions would seem to be a bit too much. Perhaps in time this approach (Baumol and Quandt's) might be advocated, but for the near-term future, the best one can do is to try and improve the system of CB within its existing, basic framework--changing the entire "framework" is unlikely to succeed.

Industry Response:

Although Weingartner's extensive work in the area of MP and CB first came out in 1963 as his doctoral dissertation and later in 1967 as a well-publicized book, the adoption by industry of this approach has been less than rapid--although there have been some applications.

The collection of papers from the symposium on "Corporate Simulation Models" which was presented in Schrieber's publication(57), illustrate a few of these applications. Dickson _et al_ published the results of a study on twenty operating, computer-assisted planning models. Their conclusion was that optimization models, _per se_, (i.e. LP, etc) were not employed by any of the firms. However, their analysis indicated that work on such optimization is beginning--particularly in the banking industry.

Gershefski's monograph(58) also indicates this trend. He was one of the first (and most widely publicized) principles to develop a fully-integrated model of an entire corporation while at Sun Oil Company in the late 1960's; and although the basic approach was deterministic simulation, his comments concerning the future were "a linear program will be developed so the model can be used for optimization studies".

Schrieber's book contains several other papers on LP CB applications. Chervany, et al reported on a LP formulation of the Northwestern National Bank of Minneapolis' resource allocation problem (loans and investments). Similarly, Cohen and Hammer(59) reported on another approach used by the Bankers Trust Company. Apparently Dickson's earlier conclusion concerning banks taking to the LP approach for CB has some validity.

Another work presented in Schrieber's book illustrates a non-banking application. Dickens, of the Boise Cascade Corporation, reported on the successful application of an LP model both for long-range and current CB needs.

As for the industry surveys cited earlier (Istvan, Christy, etc), few of these reported any direct application of LP to CB. Fremgen(23) reported 19% had used LP techniques (17% regularly) although he did not elaborate on the types of applications. Neuhauser(21) reported that 47% of the "larger" firms in this survey used LP techniques in capital budgeting to allocate resources subject to various forms of constraints. How many firms this actually involved is not revealed.

Two other CB applications using mathematical programming are presented by Woolsey(60,61) in a humorous discussion of some of the problems encountered by the unnamed firms. One of the most informative and interesting discussions of an actual application in an "unnamed multi-nation firm" (perhaps the International Utilities Corporation) is that of Hamilton and Moses(62,63). Their discussion presents a very readable description of the adoption of a mixed integer programming model for corporate decision making in financial planning and covers the spectrum from organizational problems in implementation to detailed descriptions of the objective function and constraint set.

From these brief cases presented above, it can be reasonably assumed that the MP approach to CB is not "taking industry by storm". But then again it is a relatively new tool; look how long it took DCF project evaluations to be widely accepted. Perhaps in that length of time, mathematical optimization techniques will be just as pervasive.

However, for this to happen one of the most important changes to be made is that industry must adopt a different approach in their philosophy of project evaluation at CB time. One of the fundamental pre-requisites for using MP techniques in selecting an optimal budget set is that the detailed economics of each project must be defined at budget time.

However, the 1963 Conference Board report(18) found that most companies do not require a great amount of "supporting detail" on their prospective projects at budget time (such as DCF indicators, etc) preferring to examine the detailed aspects as the projects are funded. The NAA research report(17) also discusses this aspect and concludes that most companies use a list of "partly analyzed" projects for submission at CB time rather than a list of "fully analyzed" ones. The advantage to this approach is that it does list the specific items of machinery, etc and it does permit a qualitative evaluation of the purposes involved; it also frees management from the requirement of calculating, detailed economic factors which are in many cases applied to only "vaguely conceived" projects.

The main disadvantage, however, is that is does not provide for any precise quantitative capital allocations to be used in planning purposes. Projects selected from the "partly analyzed" list may indeed show to be profitable as funding is due, and it is allocated its portion of the budget. Other projects may not be, and thus the company has perhaps made tentative arrangements for external financing on a project which on closer examination is not to be funded. Even for all of the projects selected on a partly analyzed basis and also later funded, the total impact on the corporation for all projects is not really known in advance and is just a gradually "unfolding" process. Such a process is not very conducive to accurate future planning in a firm.

Vandel's article(64) presents a very practical and mostly valid analysis of some of the limitations on CB theory. In this paper he discusses several "erroneous assumptions" made by theorists and one of them is that "students of CB conjure up a picture of top management sitting amid a bountiful array of capital expenditures opportunities, selecting those projects that will be of greatest strategic benefit". He goes on to say that this is just not the way it is.

However, both the NAA report by Kempster(17) and Williams' survey(19) show that detailed project evaluation is becoming more and more recognized by firms as being an important part of their CB procedures.

Mathematical Programming, Uncertainty, and Capital Budgeting:

As indicated in the previous discussion, MP applications are making some headway in industry CB practice, although only on a very limited scale.

The aspect of time and the educational process for managers to understand and use this tool is undoubtedly causing a portion of this slow progress. However, another factor that enters into the picture is uncertainty.

Weingartner discussed the uncertainty situation in the introduction to his book. He recognized that this aspect was important but also very complicating, and his response was that the problem analyzed under certainty was the first thing to do. He states that his work was not intended to be directly applicable to "real world" problems, but rather to present a framework on which to build. There were more than enough problems to be encountered and solved just using the deterministic model, without worrying about uncertainty, too.

His comments are well taken and meaningful, but nonetheless, before "real world" industry applications are attempted, some measure of the uncertainties involved in the management decision process should probably be included. This aspect is somewhat analogous to the findings of Christy's survey(16) in which he implied that DCF techniques were not being widely adopted possibly because the coarseness (uncertainty) of the data did not justify (in the manager's mind) the fine-grained DCF analysis. Indeed, the preceding discussions have indicated that DCF technique usage has risen considerably--as has the use of uncertainty analysis in conjunction with these methods.

Therefore, the incorporation of uncertainty into the MP model would seem to enhance its possibilities for adoption by industry. The only question is, can the uncertainty aspects (which are inherently an extremely complicated subject) be simplified to an accurate, realistic model that management would understand and use.

Again, the theorists and academic writers were already recognizing at an early date the uncertainty aspect of the MP/CB problem. One of the earliest and most notable works in this area is that of Harry Markowitz in 1952(65). This article was just after Joel Dean's book(3) and several years before the Lorie-Savage problem on capital budgeting(4); however, it dealt with the aspect of investment decisions <u>under risk</u> in what he termed the "portfolio selection problem".

Markowitz showed that the maximization of the expected value of an investor's quadratic utility function (actually expected value minus variance of the portfolio) for returns lead to a consideration of only "efficient portfolios" for which the variance of return is minimum at each available level of expected return. The set of efficient portfolios and the associated return-variance combinations form an "efficiency frontier" when plotted as return versus variance. The choice of a particular portfolio from the efficiency frontier is dependent upon the level of risk aversion which is implicit in the investor's personal utility function (a "risk aversion" coefficient associated with the variance).

This work was advanced for its time, and includes aspects of multi-project portfolio selection with uncertainty and utility functions--a concept which had been around many years in economic theory, but was not often applied to actual problems, especially CB. Eventually, Markowitz's paper was extended into a comprehensive treatment in a book published in 1959 (mostly dealing with selection of <u>securities</u> rather than projects), and his ideas have become the basis for many subsequent works on uncertainty and CB in a MP framework.

Another early work in the uncertainty area was by Joel Cord(67) in 1964. His approach was to use dynamic programming(DP) in sequentially accepting projects to maximize the total project set's interest rate of return (DCF-ROR) subject to a constraint on the budget in the first period and an additional constraint on the average variance of the portfolio.

The inclusion of the additional constraint on variance is handled by the Generalized Lagrange Multiplier Technique. However, Weingartner(52) showed that the existence of these multipliers was not always guaranteed, and even when they did exist, the numbers generated were often not the true optimal. In addition to this drawback, usually only a limited number of projects can be handled in this fashion due to the DP algorithm since it considers a very large number of possible combinations in searching for an optimal.

Also, Dantzig regards more than one kind of constraint as computationally untractable (even though Lagrange multipliers can be used, perhaps erroneously); so most CB problems of practical interest would probably fall out of scope of dynamic programming applications.

Another approach to uncertainty is using what is popularly known as "Stochastic Linear Programming". Salazar and Sen(68) published an interesting article in this area dealing with selecting capital investment portfolios. The approach is basically to combine Monte Carlo simulation techniques with 0-1 integer linear programming. Uncertain outcomes for various exogenous variables to the firm (such as Gross National Product, market competition, etc) are randomly selected and then given this, the set of uncertain cash flows for each period and each project is sampled. Using these data values the LP model is solved for the optimal (highest PV) project set. The whole process is repeated a large number of times to generate the whole range of outcomes for the uncertain situations.

Based on these samplings a heuristic procedure is defined for project ranking and selection. If the project appeared frequently in the optimal set of projects generated during the random sampling, then it is given a high ranking. Investment sets are formed from these heuristically ranked projects--set I contains the top "ranked" project only, set II, the top two "ranked" projects only, etc. Given these, the cash flows are again randomly simulated to get an expected value and variance for the heuristic project sets and an expected value versus variance curve is plotted similar to Markowitz's "Efficiency Frontier". Management can then select the heuristic project set which best suits their risk-return preferences.

Salazar and Sen's approach seems appealing except in the heuristic decision procedure for selecting the investment sets. These sets do not possess the optimal characteristics generated by LP runs and in many cases may even be infeasible for the original problem. Also, the Monte Carlo sampling techniques used repeatedly are a time-consuming affair in most instances, especially if "what if" analyses are desired by management (that is, multiple runs changing the basic data).

Hillier's book(69) presents a very thorough analysis of uncertainty in CB by adopting the expected utility maximization concept. His work deals with the derivation of the probability distribution of PV for different sets of investment alternatives and then transforms this to a utility function distribution in which expected utility for the investment set is to be maximized.

He develops theorems to improve the computational efficiency of search techniques by showing that feasible sets which are "dominated" by others may be discarded. Correlations between period-to-period project cash flows and project-to-project cash flows are developed with endogenous and exogenous variables of the system. Considerable attention is given to the probability distribution form of PV for the investment set, and thorough discussions on the Central Limit Theorem under strong and weak conditions are presented.

Hillier then develops several classes of prescriptive utility functions and examines their meaning in regards to when certain ones would be most appropriate. Finally, he integrates the first five chapters into a MP model in which exact and approximate solution techniques are proposed. The exact technique uses a modified (0-1) branch-and-bound algorithm Hillier developed(70) or he suggests Reiter's(71) method (also referenced in Weingartner's book) plus several others. His approximate technique involves linearizing the non-linear expected utility objective function and solving a series of LP problems; however, the 0-1 integer restrictions are dropped here to allow parametric studies, etc which can be used in LP codes. The companion volume in this series(72) shows some results of computational experiments on the exact and approximate techniques for various forms of the utility function.

The basis of Hillier's book was his earlier paper in 1964 (on the same subject) which was the TIMS-Office of Naval Research winner in "Capital Budgeting of Interrelated Projects", and in general his work has received very favorable response. One example of this is the book review by Wolf(73) who states "the book has stimulated a new interest in the area for himself".

The Appendix of Hillier's book contains several practical suggestions for gathering data on correlation patterns, cash flows, etc; and although the work is very thorough and mathematically elegant, he still must deal with the onerous task of defining a utility function. Also little attention is given to other forms of constraints besides those of complementary (contingency) or competitiveness (mutual exclusion).

Chance constrained programming (C^2P) is still another technique for including uncertainty into the CB selection process. Naslund(74) was one of the first to publish in this area relating to investments, although C^2P had been around for quite some time; Charnes and Cooper's work (75,76) along with Hillier's(77) are some examples of non-capital budgeting C^2P discussions.

Naslund took Weingartner's(52) basic horizon model and extended it to include probabilistic aspects on the liquidity constraints per period. The liquidity constraints are formed such that the net cash flow available from internal and external sources will be enough to match capital outlays and other cash commitments.

The cash flows for each project in each period are considered random variables of some known (normal) probability distribution, and the liquidity constraints are expressed as a sum of random variables involving the selection of specific projects. The resulting sum is itself a probability distribution and is changed to its deterministic form whereby the probability of not achieving the desired liquidity must not exceed some confidence level, β. Thus, these are the "chance constraints".

Naslund does not attempt to solve this model in this paper but uses the Kuhn-Tucker conditions for optimality to analyze the accept/reject criteria in view of the liquidity chance constraints. Points are made regarding how the uncertainty conditions lead to "certainty equivalent" adjustments of the cash flow streams, risk adjusted discount rates or other forms for a "risk premium". His work in this area is presented in much more detail (along with other related aspects) in his 1967 book(78).

Several other works along this line are also of mention. Those by Byrne, Charnes, Cooper and Kortanek(79,80) are basically extensions of some of their earlier work in C^2P and somewhat similar to Naslund's paper--with the exception of dealing more explicitly with solution techniques (non-linear, 0-1 integer programming) and other forms for the constraints (such as PP criteria constraints on projects).

Hillier(69) also introduced C^2P techniques into a two-stage "dynamic" decision model that he says was primarily motivated by Dantzig's(81) Linear Programming Under Uncertainty (LPUU) work.

The purpose of the dynamic formulation is to allow management to pick a set of strategies in project selection such that 1) if a specific value occurs after a period of time, the action(s) to be taken (corrective or complementary) would be predecided and prepared for, and 2) the projects chosen would have the best overall (timewise) economics. For instance, a project might be chosen such that its expected outcomes were high, but once the project is chosen and its behavior is observed to be less than expected, nothing can be done about it; this type of project might not be chosen in the portfolio under a "dynamic" formulation.

The decision making is thus made in two stages; one at the initial period and then others as the outcomes from the first stage are observed. Hillier proposes developing partitions for the various possible outcomes of the first stage investment decisions and assigning these probability values to include into the objective function, along with the second stage decision variables and their associated expected value PV "increments" to the first stage expected value PV.

The first and second stage decision variables are included into the overall liquidity and/or PP constraints which are sums of random variables resulting in "chance constraints".

There appears to be some practical problems, however, in defining meaningful disjoint partitioned sets for outcomes of the first period and especially in assigning <u>probability</u> values to these as proposed. Hillier does not elaborate on these aspects and the author of this thesis followed the dynamic approach as a first choice for a research topic; however, any meaningful partitioning appeared to make the problem very large and impractical for problems of any significant size. Hillier himself says the partitioning problem must be kept down to an appropriate size to stay "in bounds" computationally.

All of the C^2P works discussed above suffer from several impractical aspects; non-linear, strict 0-1 integer, or mixed 0-1 integer programming; probability distribution forms for the chance constraints are difficult to assess; estimation of variance and covariance terms period-to-period and project-to-project are difficult to obtain. Doubtlessly several other problems also exist. However, C^2P definitely has the distinct advantage of including, in a systematic MP framework, the uncertainties of management's assessments concerning future project outcomes; and conceptually they appear amenable both to well-developed theoretical works and management's perception of his "uncertain world".

Industry Response:

If the adoption of Weingartner-type deterministic MP models to CB is limited, then one could say the utilization of uncertainty in such models is virtually non-existent. None of the previously cited industry surveys or other publications show any applications of MP and uncertainty in CB portfolio selection.

In fact, the writer of this thesis knows of only one real-world industry application in this area. Robertson, et al(82,83,84,85) at Atlantic Richfield Corporation has implemented a C^2P/CB model within his company and published several papers on his work. These papers are based on his PhD dissertation(86) in which he did extensive research in developing C^2P models with debt-to-equity borrowing constraints, earnings per share constraints, dividend policy decisions, and income tax considerations. He states that his chance constrained model is being used on a regular basis by management to select optimal investment sets at budgeting time, and in his papers he presents tables showing the large magnitude of problems the model is capable of handling in an efficient manner.

Doubtlessly there are other large companies using some form of uncertainty analysis and MP to select investments, but they are probably still mostly in the "experimental stages"--hoping to evolve as management becomes more enlightened in this area. Nonetheless, just a single application using C^2P like Robertson's is very encouraging and lends some credence to the argument that perhaps C^2P is a likely area in which to concentrate research.

Some Conclusions from the Background Discussion:

Several things are evident so far from these CB surveys in regards to theoretical relevance, practice, and trends in acceptance of more advanced theoretical methods:

- In general the theorists are addressing "real problems; however, some of the problems such as utility theory and the discount rate controversy are, at present, "impractical".

- Capital budgeting procedures within industry are becoming increasingly more complex including DCF methods, uncertainty analysis, and in some cases MP.

- There can be a considerable time lag in implementation of new techniques, but when it begins to grow, it may rise very rapidly.

- Adoption of new techniques may be promoted by "bad industrial economic conditions".

- Management definitely needs to be convinced of the "benefit" of a new concept before it will be adopted.

- Keeping some of the older methods around enhances the acceptance of the newer ones--a sort of phasing-in/phasing-out or evolutionary concept.

In the earlier discussion of uncertainty and CB it was indicated that DCF methods were slow to gain industry acceptance because management felt the tools were too precise and time-consuming to fit the coarseness and uncertainty of the data. Maybe the problem with MP acceptance is in the same area. Just as risk adjusted discount rates, expected value, and even Monte Carlo probability distributions are being used to promote DCF methods for individual project evaluations, perhaps some "judiciously selected" form of uncertainty analysis might make MP more acceptable for optimal project portfolio selection.

Weingartner's earlier modeling work has been very good for the theorists, and his point that the deterministic case provides plenty of material for problems is probably valid--but the "material for problems" seems to be mostly picked up by those in academia and other theorists. Management may well visualize that, again, using such a fine-grained tool as MP for CB selection on projects that are not even well-defined (much less their individual cash flows) is inconsistent and not worthwhile.

It may be time for the deterministic models to be augmented and to concentrate on the "real world" uncertainties of management decision making--even if these uncertainties do introduce significant and challenging complexities in modeling. It may be that without such non-deterministic modeling industry will never adopt MP in CB practices, although conceptually MP is a very efficient and systematic project portfolio selection tool.

Of the uncertainty models promoted in the literature, C^2P, appears on the surface to offer some promise. Already there is a least one large industrial firm using such a technique. C^2P handles uncertainty in such a way that is fits in with previously developed probability theory and MP. Most of all it allows management to express their uncertainty in a way that seems easy to comprehend, i.e. "I want to have at least a 95% chance that my

EPS will not fall below $2.50 for the projects we select", etc. It allows inclusion of the uncertainties about known and unknown projects plus uncertainties about future profits of current on-going operations; this addresses, at least somewhat, the important problem of including ill-defined areas into a systematic structure for decision making, whereas normally they would be simply ignored or "subjectively" judged.

C^2P still suffers from all of the theoretical and practical complexities mentioned earlier. The models tend to be non-linear, strict 0-1 integer programming problems for which the state-of-the-art in computational techniques is rather limited, especially applied to large budget problems encountered in some major industries. Probability distributions must be defined in order to construct the chance constraints and this presents significant theoretical problems. The gathering of data on variances and correlation patterns necessary for the model may be difficult to obtain or estimate. And finally, but perhaps most important, is the communication and selling of the idea to management to convince them that "all the trouble is worthwhile"; and that the model can be incorporated into the existing procedures in some minimal impact fashion.

The following chapters in this thesis will attempt to explore some of these problem areas to see if they are insurmountable--or can "valid" approximations be made where necessary to overcome theoretical shortcomings; can models be constructed in this framework which are meaningful to management; is there a way to quickly and repeatedly solve such a model in the fast-paced, changeable environment that management faces; can the concepts be simplified enough to be explained and sold to management; where could such models fit into existing corporate CB procedures and their evolution; and finally, what human factor or "political" problems would be faced.

In short, is such an approach practical to be used in industry in the foreseeable future? If not, why?

CHAPTER II

AN OVERVIEW OF THE C^2P/CB APPROACH

General

The general MP/CB problem can be formulated as

$$\text{maximize:} \quad Z = f(e,x) \tag{2.1}$$

$$\text{subject to} \quad Cx \leqslant b$$
$$x = 0 \text{ or } 1 \tag{2.2}$$

where f(e,x) is some mathematical function which expresses the economic profitability of the capital budget items chosen. The e and x terms are vectors of size n; the e's are the economic worths of the individual items to be selected in the capital budget, and the x's are the decision variables specifically associated with the e's (i.e., an e for each x item). C is an mxn matrix of coefficients associated with the constraints to be met for the capital budget items (x's) selected; b is an m-vector defining what level the constraints must meet. The x's are normally restricted to be either 0 for rejection of the item or 1 for selection of the item; however, fractional values between 0 and 1 are sometimes permissable.

A very simple example might be

$$\text{maximize:} \quad Z = \$10MMx_1 + \$5MMx_2 - \$3MMx_3$$

$$\text{subject to} \quad \begin{aligned} \$2MMx_1 + \$3MMx_2 + \$2MMx_3 &\geq \$5MM \\ \$5MMx_1 + \$6MMx_2 - \$1MMx_3 &\leq \$10.1MM \\ \$5MMx_1 - \$1MMx_2 + \$2MMx_3 &\geq \$3.56MM \\ x_1, x_2, x_3 &= 0 \text{ or } 1 \end{aligned} \qquad (MM = \text{millions})$$

where the coefficients of Z are the e's,and the x's are items such as project or financing alternatives to be selected so as to maximize Z, the economic worth of the items. The constraints (whose coefficients form the C matrix) might be desired levels of cash available (cash balances) in a period, a limit on the debt-to-equity ratio (D/E), and a desired earnings per share (EPS) figure. The MP solution to this problem would find the combination of capital budget items which is best from an economic worth standpoint and also meets the requirements on cash balances, D/E, and EPS.

Conceptually, C^2P is very similar to the above formulations, except now some or all of the elements of C, b, and e in (2.1) and (2.2) are considered as random variables. This inclusion of uncertain terms into the problem is where management can express the very realistic aspect that the estimates for the capital budget items are just that--estimates; they are not known for certain and should not be treated as such.

The C^2P problem is expressed as

$$\text{maximize:} \quad Z = f(e,x)$$

$$\text{subject to} \quad \text{Prob}(Cx \leqslant b) \geqslant \beta \qquad (2.3)$$

$$x = 0 \text{ or } 1$$

where β is an m-vector designating a "confidence" factor for each constraint.

The "Prob" term associated with the constraint matrix (2.3) makes each individual constraint a probabilistic mathematical expression which includes uncertainty. That is, the probability of a constraint being met must be greater than or equal β (say .95); thus these are the "chance constraints". For example a cash balance constraint, with uncertain cash flow (c's) for the decision items to be chosen, might be expressed as

$$\text{Prob}(c_1x_1 + c_2x_2 - c_3x_3 \geqslant \$b\text{MM}) \geqslant .95 \qquad (2.4)$$

This type of decision making allowed by (2.3) will help management to avoid selecting a group of capital budget items whose "most likely" values would satisfy a very important cash balance constraint (say, money required to retire a debt issue), but that might also have a fairly large probability of <u>not</u> satisfying the cash requirement. If the "chance constraint" is satisfied, only capital budget items that have a 95% or greater chance of meeting the requirement would be selected (conversely, a 5% or less chance of <u>not</u> meeting the requirement).

Usually the objective function, Z, is simply a linear sum of the expected values of the random terms (e's). Given a 3-item problem for example

$$Z = E\left(\sum_{i=1}^{3} e_i x_i\right)$$

However, other non-linear probabilistic forms could be used; more will be said about this in the section of this chapter on "Examination of the Objective Function".

The chance constraints utilize the expected values plus the variances for the random terms (c's and b's) and are transformed into their deterministic equivalent much like that done for a single random variable. For example (2.4) can be expressed as

$$\sum_{i=1}^{3} E(c_i)x_i + K_{(1-.95)} \Big[\sum_{i=1}^{3} Var(c_i)x_i^2 + \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j\neq i}}^{3} Cov(c_i, c_j)x_i x_j \Big]^{\frac{1}{2}} \geqslant E(\$bMM) \qquad (2.5)$$

where $E(c_i)$ is the expected value for each random variable c_i

$Var(c_i)$ is the variance of each random variable c_i

$Cov(c_i, c_j)$ is the covariance between random variables c_i and c_j

$K_{(1-.95)}$ is the 5% fractile point of the general probability distribution (formed by the constraint) with mean zero and variance 1. The 5% fractile is a value such that 5% of the possible outcomes will be less than that value.

One of the basic assumptions behind most C^2P models is that neither the expected values or variances (as used in the objective function and constraints) of any of the terms associated with a capital budget item is affected by selection or rejection of other items. If this assumption is not made, the problem becomes highly non-linear and very difficult to solve. Ways of defining groups of interrelated items can be made to avoid this problem, if necessary, but it can expand the size of the constraint set significantly; this will be discussed briefly in the section of this chapter on "Non-Probabilistic Constraints".

Another problem area of the C^2P model is the form for the probability distribution for the constraint which in turn defines the β (or the $1 - \beta$) fractile values (i.e., K_{β_i}). For many chance constrained programming problems, the assumption is made that the random variables forming the constraint's probability distribution are identically distributed and independent. The constraint's probability distribution will thus approach that of a normal distribution due to the Central Limit Theorem (CLT); these conditions of independence and identical distribution are the weakest for the CLT to hold. For the problem where 1) the random variables are not all of the same distribution type and 2) may not be independent, the only recourse left is to again fall back on the Central Limit Theorem and assume that the sum of a large number of finite variance random variables asymptotically approaches a normal distribution.

For the case where the variables are independent but of different distributions, the Central Limit Theorem still holds for fairly weak conditions. The assumption of independence would remove most of the covariance terms such as in (2.5), and in some cases this may not be entirely unrealistic. There is usually dependence among some capital budget items, but there are cases where these dependencies are not very strong and/or there are not usually that many variables interrelated, compared to the entire decision variable set.

If the independence assumption is violated, the Central Limit Theorem holds only for fairly strong conditions or special conditions of "some" dependence with identical distributions, or restricted patterns of dependence and non-identical distributions.

The whole question of normality and the value for K_{β_i} involves the number of random variables involved in the constraint sum (which is not known until the solution is obtained), their dependence, and their individual distributions. As a last resort the one-sided Chebyshev inequality could be used to yield an upper-bound for K_{β_i} if the distribution is assumed entirely unknown. However, this bound is based on the worst possible distribution and will greatly overestimate K_{β_i}, giving a much tighter constraint than is necessary.

These two problems discussed above (the assumption of independence of expected values and variances with project selection and the normal distribution assumption) provide grounds for much of the theoretical criticism of C^2P. However, these assumptions would probably have less effect on the results than the data input approximations to the model (discussed in Chapter IV), and at least the C^2P formulation does include <u>some</u> form of uncertainty into the decision making process in an intuitively appealing way.

Earlier Works

As mentioned in the first chapter, one of the earlier works analyzing CB in a C^2P format was by Naslund(74) in 1966. His approach was to take Weingartner's(52) lending/constrained borrowing basic horizon model, which included constraints on cash balances, and incorporate uncertainty in the following way:

$$\text{maximize:} \quad Z = E\left(\sum_{j=1}^{n} \hat{a}_j x_j + v_T - w_T\right) \tag{2.6}$$

$$\text{subject to} \quad \text{Prob}\left(\sum_{j=1}^{n} a_{1j} x_i - v_1 + w_1 + D_1 \leqslant 0\right) \leqslant \beta_1 \tag{2.7}$$

$$\text{Prob}\left(\sum_{j=1}^{n} a_{ij} x_j + (1+r)v_{i-1} - (1+r)w_{i-1} - v_i + w_i + D_i \leqslant 0\right) \leqslant \beta_i \tag{2.8}$$

$$(i = 2, \ldots\ldots T)$$

$$x_j = 0 \text{ or } 1 \tag{2.9}$$

$$w_i,\ v_i \geqslant 0 \tag{2.10}$$

$$w_i \leqslant B_i \tag{2.11}$$

E is the expected value operator

$\hat{a}_j$ is the horizon(T) value of all cash flows <u>subsequent</u> to (i.e., past) the horizon associated with project j (discounted at rate r)

a_{ij} is the cash flow associated with project j in period i

x_j is the decision variable for project j

v_i is the cash lent in period i at interest rate r

w_i is the cash borrowed in period i at interest rate r

B_i is the limit on the cash borrowed in period i

D_i is the cash flow generated by activities other than the investment projects being considered

T is the horizon period

n is the number of projects

All the a_{ij} in (2.7) and (2.8) are assumed to be random variables, but D_i is deterministic.

The borrowing and lending are assumed to be one year (one period) constracts and are not intended to include the aspects of long-term financing. Also, the cash balance constraints are cumulative, which in effect, carries over unused funds (or deficits) from one period to the next.

The cumulative aspects are not readily apparent from the above formulation but are imbedded in the v_{i-1} and w_{i-1} terms of (2.8). That is, these terms are used almost in a sense of a slack or surplus variable; they will be chosen such as to make the inequality constraint an equality and thus are the surplus or slack amount for the period (as Weingartner points out, they cannot both be in solution at the same time). They are then carried over into the next period and combined with other variables to become a cumulative formulation. Naslund reformulates this cumulative aspect of (2.7) and (2.8) in a more straightforward fashion as

$$\text{Prob}\left(\sum_{j=1}^{n} a_{1j}x_j - v_1 + w_1 + D_1 \leqslant 0\right) \leqslant \beta_1 \qquad (2.12)$$

and

$$\text{Prob}\left(\sum_{j=1}^{n}\sum_{k=1}^{i} a_{kj}x_j + \sum_{k=1}^{i-1} v_k r - \sum_{k=1}^{i-1} w_k r - v_i + w_i + \sum_{k=1}^{i} D_k \leqslant 0\right) \leqslant \beta_i \quad (2.13)$$

$$(i = 2, \ldots\ldots T)$$

The compounding term $(1+r)v_{i-1}$, $(1+r)w_{i-1}$ used in (2.8) is still in the above equations. The interest terms are explicit, but the v_{i-1} and w_{i-1} values are also included <u>via</u> the double-summation a_{kj} term and the D_k term (since the v_{i-1} and w_{k-1} are slacks or surpluses of these items).

The w_i term would appear to be able to come into solution up to any value that is less than or equal to B_i (2.11) even if B_i is considerably past the amount required to satisfy the constraint. However, since v_T is in the objective function (2.6) to be maximized; and it has in it the cumulative effect of all periods cash balances, v_T implicitly will try to hold w_i down to only the amount required for a particular period's constraint (i.e., make it an equality). In other words, excessive borrowing past the amount required will tend to reduce v_T and thus the objective function.

Additional imperfections in the capital markets can be included here as in Weingartner's(52) treatment. He has the w_i's (2.11) divided into steps such that the borrowing rate is a function of the amount borrowed. The w_i's are each replaced by a sum of terms and their corresponding borrowing rates. For the cash balance constraints the borrowing term repayment in (2.13) is replaced by

$$\sum_{b=1}^{N_i} w_{bi}\, r_{bi} \qquad (2.14)$$

Limits on the borrowing are

$$\sum_{b=1}^{N_i} w_{bi} \leqslant B_i \qquad (2.15)$$

and

$$w_{bi} \leqslant M_{bi} \qquad b = 1 \text{ to } N_i \qquad (2.16)$$

where w_{bi} is the borrowed amount for the bth step in the ith period
M_{bi} is the limit on w_{bi}
r_{bi} is the borrowing rate associated with the bth step in the ith period
N_i is the number of borrowing steps in the ith period
B_i is the total borrowing limit for the ith period

In the cash balance constraints (2.13) borrowing will take place up to the limit of each step M_{bi} (2.16) until the constraint is satisfied or B_i is reached (2.15). A figure of this process helps in clarification.

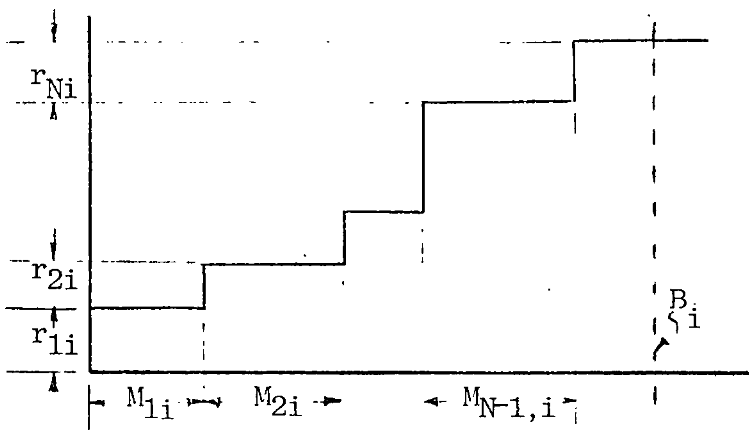

Figure 2.1

Borrowing Rates as a Function of Amount Borrowed

This is what Weingartner calls a sloping supply schedule of funds, and it definitely introduces more realistic aspects into the model. Theoretically one of these curves could be specified for each period i. The only problem with the concept is that, like Dean's(3) proposal (discussed in Chapter I) which used a similar approach, the determination of these curves may be difficult in practice.

The "Prob" nomenclature used in the liquidity formulations, as presented in (2.12) and (2.13), are the chance constraints; and the deterministic equivalent is formed in essentially the same fashion as illustrated in (2.5). For the multivariate chance constraints being considered, their deterministic form is (using Naslund's equal lending and borrowing rate model (2.12) and (2.13) for an example)

$$\sum_{j=1}^{n} \bar{a}_{1j}x_j - v_1 + w_1 + K\beta_1 \left[\sum_{j=1}^{n} \mathrm{Var}(a_{1j})x_j^2 \right]^{\frac{1}{2}} + D_1 \leqslant 0 \qquad (2.17)$$

and

$$\sum_{j=1}^{n} \sum_{k=1}^{i} \bar{a}_{kj}x_j + \sum_{k=1}^{i-1} v_k r - \sum_{k=1}^{i-1} w_k r - v_i + w_i$$

$$+ K\beta_i \left[\sum_{j=1}^{n} \sum_{k=1}^{i} \mathrm{Var}(a_{kj})x_j^2 \right]^{\frac{1}{2}} + \sum_{k=1}^{i} D_k \leqslant 0 (i = 2, \ldots .T) \qquad (2.18)$$

where $\bar{a}_{kj}$ and Var are the expected value and variance of the cash flow in period k for project j.

In this formulation the individual projects and cash flows per period for a specific project are assumed to be independent, and the D_k

are deterministic. It is clearly possible to extend (2.17) and (2.18) to include dependence between both project-project cash flows and period-period cash flows. Hillier(69) does this in his model, although he drops the short term borrowing and lending. His formulation includes co-variances between projects and period-to-period project cash flows, plus a probabilistic D_i. (The details of this covariance formulation are deferred until Chapter III.)

Given the value of $K\beta_i$ for the appropriate probability distribution, the deterministic equivalent formulation (2.17) and (2.18) becomes non-linear due to the x_j^2 terms (and cross terms if covariances are included). Also due to the nature of the project selection process the x_j's are usually strict 0-1 integers (2.9) while the v's and w's are not necessarily integers (2.10) but can be restricted as such without any loss of accuracy--since they will usually be large numbers and dropping the fractional portion will be insignificant. Thus, we are faced with a rather formidable task of solving an <u>integer</u> <u>non-linear</u> programming problem. The type of techniques used to solve these problems are usually "branch and bound", but they are frequently limited to problems of fairly small size (this will be discussed more in Chapter V).

Another way of approaching borrowing and lending is to have the v's and w's be expressed as continuous variables between 0 and 1. Then, the borrowing constraint would be included in (2.18); e.g., w_iB_i and v_iV_i would replace w_i and v_i where V_i and B_i would be some maximum limit on lendings and borrowings respectively and

$0 \leqslant w_i \leqslant 1$

$0 \leqslant v_i \leqslant 1$

However, in this formulation the problem now becomes a mixed 0-1 integer non-linear programming problem--an even more formidable computational task.

The main difficulty is in the non-linearities; if the problem were linear (as it would be without the probabilistic chance constraints) linear programming would probably be a suitable technique for solution, since Weingartner has shown(52) that the maximum number of non-integer x_i's must be less than or equal the number of functional constraints. If there are not too many of these constraints, manual rounding techniques might be possible. In Chapter IV a linearization of these chance constraints will be examined so more practical solution methods may be employed.

Although the C^2P formulation seems to provide some difficulty in computational aspects, conceptually the approach introduces facets of uncertainty analysis into the CB problem in a systematic, "practically appealing" framework. One of the approaches discussed earlier in Chapter I for including uncertainty and MP into CB was Hillier's(69). Here, he proposed a utility function for the objective which created a non-linear integer programming problem. His linear approximations to the utility formulation allowed practical solution techniques. However, he still is confronted with the onerous task of constructing and dealing with a utility function (these problems were partially discussed in Chapter I).

Hillier recognized this utility function problem and later presented a brief chapter on a chance constrained formulation. The chance constraints along with their β's are used to eliminate unacceptably risky decision variable sets rather than using the expected value of the non-linear utility function. In this sense, the "utility" of the decision maker is included in the chance constraints. Utility is placed on the pattern of the cash flows rather than the present value of the cash flows.

For instance, the possibility of a large negative present value might be more serious if the loss would occur in one year rather than being spread over a number of years, since this would provide less opportunity to recoup the losses elsewhere. Also a certain investment set might have an unacceptably large cash outflow (a cash balance problem) in a particular year which would cause rejection even though the present value was acceptable. Conceptually, C^2P can handle such problems.

Another favorable aspect of C^2P is that it allows the investor to specify the quantifiable aspects of changing this risk preferences. For example, suppose the decision maker initially has set his β in a particular cash balance constraint to 95% and obtains a solution. By examining another case in which he perhaps changes his β to 90%, the objective function might increase by $10,000. He can then assess the merit of changing his risk preferences in the chance constraints for the gain in the objective function.

Parametric programming or dual variable evaluations would be useful for this, but under the non-linear integer programming requirements they would be difficult to derive. A further exploration of this subject is made in Chapter V in which a heuristic integer linear programming algorithm may be used to quickly study at least "sensitivities" between the risk preferences in the constraints and the objective function.

Two other works by Byrnes, Charnes and Cooper _et al_(79,80), deserve special mention since they deal significantly with C^2P and CB; however, they also introduce Linear Programming Under Uncertainty (LPUU) to handle liquidity constraints. Constraints on payback period and "horizon posture" are considered (discussed in the section of this chapter on "Examination of the Constraint Set"); and in the later article(80) discrete probability distributions are introduced to better describe the form of the chance constraints. This results in a highly non-linear form which they attempt to transform to a 0-1 integer linear programming problem greatly expanded in size.

Much of their work seems very complex (such as introducing "signum" functions into the formulation) and beyond the scope of this thesis. However, it is obvious that a considerable amount of effort and thought has gone into their presentations, so doubtlessly this work contains meaningful contributions to the area of MP/CB. Perhaps their points might be an area for future research, but this thesis will choose a simpler, broader and (hopefully) more practical approach for the "near-term" analysis and evaluation.

Examination of the Objective Function

Previously the objective function has been discussed as the expected value of the terminal value of the projects to be selected (2.6). Some of the problems in using such an approach are discussed herein.

The expected value is a linear operator and since the objective function is simply a sum of terms, the objective is linear. This is a very desirable characteristic since non-linearities in the constraint set cause computational problems. Further non-linearities in the objective would only make matters worse.

However, as mentioned in the "General" section of this chapter, a key assumption in C^2P and expected value objectives like this is that the joint probability distribution for all projects is not effected by the selection or rejection of any other projects. In other words the expected value for the "yield" on project j is not affected by whether or not project j+1, etc is selected. This may be unrealistic for some types of projects where strong interrelationships exist such that acceptance or rejection of one project might shift the expected value of another project.

Hillier(69) includes this aspect in his utility model by assuming that the effect is "pairwise additive": that is, the only complementary or competitive effects are between pairs of investments or any joint effect involving more than two investments is merely the cumulation of the pairwise effects. This assumption is appealing but it introduces non-linearities in the form of cross terms between the "pairwise additive" projects.

The other problem in the previously presented objective function is the problem with terminal wealth itself. As pointed out in Chapter I, the earlier works in CB used discounted cash flow present value (PV) as the objective to be maximized. Even Weingartner's first approaches were formulated using this objective. However, the "Hershliefer Paradox"(12), of not knowing the discount rate before the optimal set is selected, seemingly motivated Weingartner's subsequent work to adopt the terminal value objective function.

In adopting this criterion there are still difficulties in 1) selecting the appropriate horizon and 2) determining the interest rate for discounting subsequent cash flows of the project to the horizon. Weingartner acknowledges these problems in his work(52) as does Bernhard(87).

The actual difficulty hinges on the discount rate for the post horizon cash flow, because the appropriate rate should be dependent on whether the firm is a borrower or lender during the period past the horizon (T). Given that it could be doing both, another question is raised "what discount rate should be used--the borrowing or the lending rate?" If the horizon is chosen large enough, then there are no post T cash flows and therefore no discount rate problem. However, for this case, the cash balance equations must extend all the way out to the horizon and the subsequent problem may be very large in size and difficult to solve.

As a solution to these problems, Weingartner attempts to include dividend policies into the objective function. Baumol and Quandt(53) (as discussed in Chapter I) proposed a similar approach in which they included a utility function for dividends as the objective. Weingartner(90) proposes to maximize the dividends at the horizon which is tantamount to maximizing the average growth rate of dividends. He includes individual period dividend payments in the cash balance constraints plus the terminal dividend in the objective function, although he acknowledges the effect of dividend policies with regards to investments is subject to considerable dispute and by no means settled.

He also points out that the average dividend growth rate criterion may undesirably produce a stream of dividends that are small over a large portion of the time horizon, but rise sharply at the end. Also, the dividend policy per time is presented as a single solution rather than a set of alternatives from which the optimal pattern of dividends within policy limits, can be chosen. His response to this is to specify in the constraint set a minimum growth rate per period for dividends and do parametric programming on this to define an "efficiency frontier" of average annual growth rate of dividends (d_A) versus minimum annual growth rate of dividends (d_m).

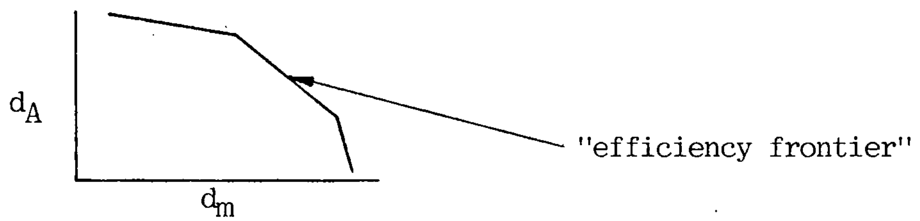

Figure 2.2: "Efficiency Frontier" for Average Dividends and Minimum Dividends

This efficiency frontier presents solutions for which no other alternatives have a higher average growth rate for a given minimum growth rate (or vice versa). Weingartner remarks that this approach has the obvious advantage over Baumol and Quandt's subjective utility approach in that the decision maker can examine his alternatives (efficient ones only) before expressing his preferences.

Even after all of this to avoid the discount rate problem, Weingartner still introduces a constraint which requires that at the horizon the value of remaining assets, physical and financial, be sufficiently large to maintain in the future the dividend rate attained in the horizon year. In formulating this constraint there is still difficulty in choosing the discount rate for calculating the residual values of the investment at the horizon. Weingartner says, "Fortunately, the investment decisions which the model is to determine are not likely to be sensitive to the actual rate utilized for discounting these residual values". This seems to be a rather peculiar statement since his dividend model apparently was formed to avoid using a discount rate which was considered a problem.

It appears that the terminal wealth "horizon model" still has its share of controversy; and although the approaches of distinguished authorities in the field, such as Weingartner, Bernhard, Baumol and Quandt and others most surely have valid contributions, the practicality of the proposals still seem to be in an embryonic state.

Although these theoretical aspects are of interest, industry continues to make investment decisions mostly based on DCF methods; and of these DCF techniques present value appears to have fewer theoretical problems that DCF-ROR. As shown in the Chapter I surveys of the industrial CB practices, DCF methods are gaining considerable support. Industry is just now beginning to "believe" in the more sophisticated DCF approach. To change the evaluation criterion to some dividend policy (or terminal wealth) and at the same time to try and sell the ideas of MP plus uncertainty in CB is to ask too much.

Therefore, the remainder of this thesis will concentrate on the expected value of present value (PV) for the CB portfolio as the objective function to maximize, while recognizing that it also has some unresolved problems associated with it.

Financing and the Objective Function:

As stated above the objective function to be considered in this thesis will be expected value of PV for the CB portfolio. However, a distinct advantage of the Weingartner horizon model objective function is that it includes borrowing in the constraint set and the objective function.

Chapter I discussed the capital rationing problem (budget restrictions) faced by industry and found that the budget was flexible enough to allow seeking of extra funds through borrowings if necessary. The previously presented horizon model did this and included the borrowings implicitly in the objective function by having the terminal wealth maximization minimize any borrowings (amount and interest rate) necessary to satisfy the constraints.

Borrowings should be included in any model, but since the terminal wealth objective function is being replaced by the PV of the CB portfolio, the borrowings do not enter into the objective either directly or indirectly. Hence, there is no mechanism to either select the most favorable (lowest cost) borrowings or to prevent the borrowings from going past what was actually needed to satisfy the cash balance constraints. That is, borrowing may enter into the solution, but it is only constrained, perhaps, by some type of upper bound limit. The cash balance constraints simply say the cash flows must be greater than or equal to zero; borrowings can satisfy this plus more--up to the borrowing limits.

To avoid this unecessary and/or inefficient borrowing, financing can be introduced as a decision variable in the objective function in such a way that least cost forms of financing will be chosen, only if needed, and only up to the point that it is needed. One approach to do this is to formulate the objective function as follows

$$\max: \quad Z = E\{ \sum_{j=1}^{n} [\sum_{i=1}^{m_j} \frac{c_{ij}}{(1+r)^i}] p_j + \sum_{j=1}^{n'} [\sum_{i=1}^{m'_j} \frac{q_{ij}}{(1+r)^i}] f_j \} \quad (2.19)$$

where E is the expected value operator
- c_{ij} is the net cash flow after taxes of the jth project in the ith period
- p_j is the jth project decision variable
- n is the number of projects under consideration
- m_j is the number of periods of cash flows for the jth project
- q_{ij} is the cash payback (including interest and after taxes) of the jth financing proposal in the ith period.
- f_j is the jth financing proposal decision variable
- n' is the number of financing proposals under consideration
- m'_j is the number of periods of cash payback for the jth financing proposals
- r is the discount rate assigned by management

In this form the financing proposals could be short or long-term arrangements, all having negative coefficients (due to the payback cash flows). Since maximization is being used, financing will be chosen only if required and only up to the point it is necessary.

From these various financing alternatives, the least cost ones will be chosen due to discounting the payback on the financing proposals at the discount rate, r, rather than discounting at the individual financing proposal borrowing rates. Thus, the financing with the best "leverage" will be chosen. The term leverage as used in this sense can probably be best addressed by an example.

A financing proposal, f_1, is to borrow \$1MM at 10% interest rate and the payback schedule is -\$263,800 for 5 years. When discounted at the discount rate of 12%(r) the payback amount is -\$950,940. Another proposal, f_2, for the same amount is at 8% interest with a payback schedule of -\$250,460 for 5 years. Its 12% discounted value is -\$902,852.

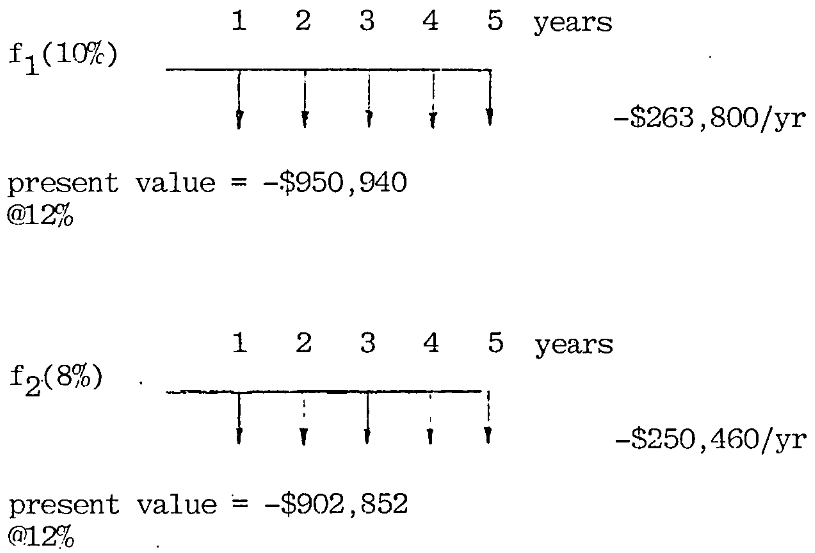

Figure 2.3: Illustration of "Leverage" Effect on Financings

Therefore in the objective function where the financing proposal payback schedules are discounted at the <u>firm's</u> discount rate, r, rather than the borrowing rate (8 and 10%, etc), the financings with the lowest interest rates (f_2 in the example) will be chosen since they have the highest present value (i.e., the least negative). A final correction to the objective must be made after the optimal program has been run since these financing proposals do not actually contribute profit.

As mentioned, the payback schedules for the financing proposals are general and could accommodate equity financing as well as other forms of long-term borrowings. However, the inclusion of equity issues delves into the very complicated and controversial area of what returns (dividends) are expected by the investment community for their contribution to the firm's equity capital.

The main questions involved are what should be the dividend payments (if any) and what magnitude of dilution of earnings (EPS--earnings per share ratio) is allowable? An equally difficult problem is what effect does additional equity financing have on the control of the firm by its owners? These types of problems have been somewhat addressed by Weingartner(52), Chambers(88), and especially Petersen(89). However, Bernhard's comment(87) that new financing, including stock issues "has not yet been effectively handled in a MP context" probably still stands.

For the chance constrained formulation, the c_{ij}'s are usually taken as random variables; the q_{ij}'s are assumed non-probabilistic, but with certain forms of financing such as bond or stock issues (if included) they could also be random. The decision variables for the project (p_j's) are usually considered strict 0-1 integers (0 for rejection, 1 for acceptance). However, the financing decision variables (f_j's) may be continuous between 0 and 1 since fractional borrowings would be allowed. This type of problem, as mentioned earlier, still leads into the difficulties of a mixed integer, 0-1, non-linear programming problem.

Probabilistic Objective Functions:

Although the approach discussed in this thesis will be to formulate the objective function as expected value and thus make it linear, other probabilistic (usually non-linear) objectives should at least be mentioned.

One of the earliest probabilistic forms for an objective function was proposed by Markowitz(65,66) in which the expected value and variance of the portfolio selected were included. His formulation was simply:

maximize Z = E(yield) - K x Var(yield)

where E is the expected value operator

yield is the measure of "return" for the portfolio selected (yield in our case is present value)

K is a coefficient of risk aversion

Var is the variance of the yield of the portfolio selected.

The variance term for the portfolio in general includes dependent projects which are handled by their covariances which in turn introduces non-linearities in the form of squared and cross proeuct terms. The requirements for estimates on covariances between projects has been one of the major criticisms of this approach and several authors have explored in more detail, simplifications to the formulation. This same type of problem appears in the chance constraints. Chapter IV will address the data gathering problem, which includes covariances.

Markowitz's objective function was briefly discussed in Chapter I and there the statement was made that his formulation included a utility function. This is "true" from the perspective that the decision maker is expressing a preference for portfolios of higher expected values and lower variances.

The unique factor of the Markowitz criterion is the inclusion in the objective of the variance term to complement the expected value. Some authors have proposed, that due to the variance term squares and cross products, the objective function represents a "quadratic utility function" in which the utility of the portfolios with higher variances fall off quickly (due to the negative coefficient associated with the variance term). This is true, but it has been shown by Borch(39) that using the Von Neumann/Morgenstern utility concept for a quadratic form, Markowitz's criterion can demonstrate irrational consumer behavior (that is, preferring less money to more). Thus, Markowitz's formulation is not truly a "utility function".

Hillier(69) introduced a more conceptually accurate utility function representation of the objective which includes variances and covariances. One of the main aspects of Hillier's work is his linear approximations to the resulting non-linear objective function, so linear and integer linear programming techniques can be used to solve the model. As mentioned in Chapter I, a detailed analysis of some computational experiments on his approach is presented in the first chapter of Byrne <u>et al</u>(72).

Additional approaches which include variance have been proposed, such as maximizing the expected value of the portfolio subject to a constraint on the variance. This was Cord's(67) approach in his dynamic programming formulation mentioned in Chapter I. Others have recognized that the variance term constraint or preference could be better represented by the "downside" portion of the variance--or "semi-variance". As pointed out in Chapter I, Mao's survey(43) and Petty's survey(22) both indicate an awareness by management that the downside risk is the most important.

Kataoka(91) presented an interesting formulation using an expected value, constrained semi-variance model. His approach was to

maximize: y (a dummy variable)

subject to $\text{Prob}(\text{yield} \leqslant y) = \beta$

where yield is the "return" of the investment set selected and is a probability distribution

This form is similar to a chance constraint on the projects portfolio yield but lets the limit on the probability distribution be a decision variable, y. The effect is to have the solution be selected such that the portfolio with the smallest (largest y value) "downside risk" is obtained. For example, if β is .05 then from the following figure, portfolio probability distribution A would be chosen over B.

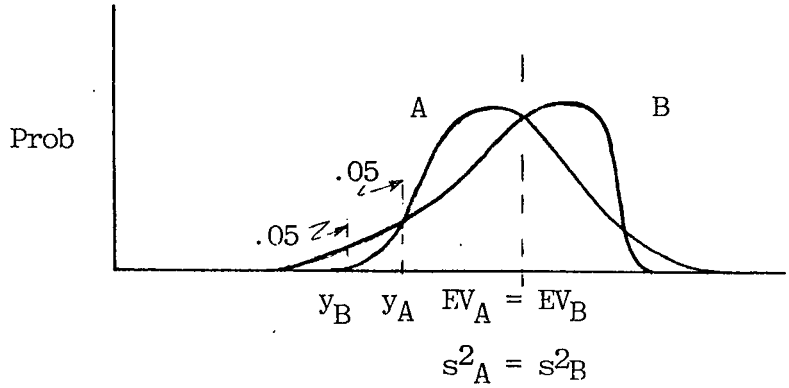

Figure 2.4: Example of Kataoka's "Downside Risk" Approach

The expected values(EV) and variances (s^2) for A and B are equivalent ($EV_A = EV_B$; $s^2_A = s^2_B$), but the 5% fractile for A(y_A) is larger than the 5% fractile for B(y_B); therefore, A will be chosen over B--it has the smallest downside risk.

The practicality of determining specific forms for the probability distributions for different portfolio yields restricts the full usefulness of this approach (since normally some assumption must be made about the form of the yield distribution and it should be the same for all portfolios). Still, the concept provides an approach for avioding portfolios with large deviations below the mean or most likely values.

Although this thesis will primarily consider expected value as an objective function, the merit of including some measure of dispersion into the criterion is fully recognized. The above discussions were intended to be a brief review. A more comprehensive treatment is afforded in Freeman and Gear's paper(92).

Examination of the Constraint Set

Probabilistic Constraints:

As discussed earlier, the chance constraint set of the capital budgeting formulation is where the uncertainty and "utility" of the model is imbedded. For the presentation of Naslund(74), discussed at the first of this chapter, the main consideration was on the cash balance and the pattern of the cash flows of the investments over time. There is no doubt that such constraints are of extreme importance to management who must guarantee that their firm is liquid enough to satisfy its creditors and other capital commitments. The inclusion of financing payback schedules (as discussed in the previous section) for long and short term borrowings into the cash flows of the objective function and constraints definitely enhance the realism of the cash balance requirements.

Just as these cash balance constraints act as sub-objectives or alternate criteria to the main objective function of present value, other constraints of differing forms can be imposed on the selected investment and financing proposals. The aspect of other criteria besides the primary economic indicator is a point that is frequently mentioned in the previously cited surveys of CB practices discussed in Chapter I.

Management is still struggling with what primary economic indicator should be adopted for project selection. As mentioned in Chapter I, DCF methods are gaining support, but other criteria such as payback and accounting rate of return (ROI) still remain in wide usage. Earnings per share (EPS) is another financial measure by which total project portfolios could be judged; Williams(19), Petty(22), and Bell(93) point out these as specific management considerations in the actual practice of CB. Also, the surveys by Christy(16), Kempster(17), Pflomm(18), and Davey(42) all mention some aspects of the multi-criteria problem associated with implementation of capital budgets.

Foremost among the items mentioned as being pertinent to management's decision making is the impact of selected portfolios on the firm's financial statement. EPS and ROI are examples, but others can be included. A company's debt-to-equity ratio (D/E) is an important consideration in undertaking new ventures and outside financing. Indeed, many creditors require that a firm maintain a certain posture with regards to D/E as covenants to loan agreements. Similar restrictions are placed on such things as the current ratio or quick ratio--the ratio of current assets to current liabilities of various forms. These measure how liquid the company is from a creditor's standpoint. Several others could be included by simply consulting a recent finance text such as Van Horne(94) to examine the long list of financial ratios, what they mean to a company's market stance, and how different projects and financing selections might affect these ratios.

This subject encompasses project/financing proposal constraints (or sub-objectives, multi-criteria, etc) on financial accounting or accounting reported results. Theorists have recognized this aspect for some time as evidenced by the works of Tilles(95), Chambers(96), Lerner and Rappaport(97), Otto(98) and Dyckman(99). Only recently has industry attention been devoted to handling this in a systematic framework such as MP. The work by Hamilton and Moses(62) demonstrate at least one industrial application covering most of these aspects, and Chapter III will explore in more detail the actual formulation of some of these constraints in a "fairly realistic" example problem.

Other forms of constraints or sub-objectives can be introduced such as payback period (PP) on the entire portfolio set selected or on specific projects. Byrne, et al(79) present the elements of this very clearly in their paper. It essentially involves a cash balance equation form such that the net cash balance must be greater than or equal to the net investment costs by period t, the designated payback period. A portfolio set payback constraint may be formulated as

$$\sum_{j=1}^{n} \sum_{k=1}^{t} p_{kj} x_j \geq \sum_{j=1}^{n} \sum_{k=1}^{t} I_{kj} x_j + \sum_{j=1}^{n'} \sum_{k=1}^{t} Q_{k_j} f_j$$

or

$$\sum_{j=1}^{n} \sum_{k=1}^{t} (p_{k_j} - I_{k_j}) x_j - \sum_{j=1}^{n'} \sum_{k=1}^{t} (Q_{k_j}) f_j \geq 0 \qquad (2.20)$$

where p_{k_j} is the income of the jth project in the kth period
I_{k_j} is the internally generated funds used for capital investment on the jth project in the kth period
n is the number of projects under consideration
Q_{k_j} is the net cash flow of the jth financing proposal in the kth period (includes interest).
n' is the number of financing proposals under consideration.

This constraint(2.20) says the net income from the project set selected must be greater than or equal to all investment costs for the set, by period t. The investment cost may include those financed externally in which interest charges are incurred.

A similar type of payback constraint can obviously be constructed for individual projects instead of the project set. Assigning financed capital to these individual projects may be a problem.

It is apparent that all of the above discussed constraints on accounting income or payback could be formulated in a fashion similar to the cash balance constraints discussed earlier in the Naslund model. That is, they involve "cash" flows for the projects and financings to be selected and due to their uncertain nature can be expressed as chance constraints.

For most of the above constraints a common problem is the inclusion of unknown projects and financing opportunities beyond the period for which the capital budget is being allocated. For example, if only a one-year budget is under consideration but the cash balance and accounting income constraints extend into the second or third year (or longer), how are these "yet-to-be-defined" investments and financings to be included? It is not unreasonable for management to ask what will be the effect on EPS for the next year if a certain budget is selected this year; or will there be enough cash to retire a long-term debt issue in the third year if the current year budget is allocated in a particular fashion?

Obviously, the answer to the questions is contingent upon what projects and financing proposals will be selected in the present year and future years. Weingartner(52), Hillier(69), and Lockett and Gear(98) all address this problem; and all come up with essentially the same conclusion--that future unknown investment/financing proposals must be included by examining historical activities and current trends and extrapolating these into the future periods.

Although forecasting of these projects and financings, like all projections, is subject to considerable uncertainty it is exactly this type of problem in which C^2P can be most useful. The projections are made from statistical historical data and adjusted by current trends; the mean projections are included into the constraints along with the uncertainties. The major effect on the chance constraints is that the dispersion is increased and the probability of achieving a specified level may be more difficult--especially if the future projections are very uncertain. At least this type of analysis will tell the manager what will be the <u>best</u> influence he can obtain in the <u>current</u> year's budget, if he predicts that future budgets will behave in a given fashion. He can also study the impact on current budgeting by changing the scenarios of future budget projections.

This aspect of including future investments and financing into the constraint set is one of the more formidable items in the approach of using MP and CB. More will be said about this in Chapter III.

Non-Probabilistic Constraints:

In addition to the probabilistic constraints discussed above, several other types of <u>non</u>-probabilistic constraints can be included into the model formulation. These constraints are usually referred to as mutually exclusive or contingent constraints and come mostly from Weingartner's(52) work.

If two projects, both of which cannot be accepted, are competing for limited funds and the aspect of multiple criteria (such as the objective function and the constraint set) are not resolved in a straightforward fashion favoring one or the other projects, then both projects may be put into the model formulation. By introducing a constraint of the form

$$x_1 + x_2 \leqslant 1$$

only one of the projects x_1 or x_2 will be chosen since they will, in most cases, be restricted to 0-1 integers. Obviously this type of formulation can be extended to include more than just two mutually exclusive projects.

Another probabilistic constraint can be formed when one project cannot be accepted before another project is accepted. This is a common occurrence in CB and can be represented by the constraint

$$x_1 \leqslant x_2$$

where x_1 is the contingent project, and x_1 and x_2 are strict 0-1 variables. That is, x_1 cannot be greater than x_2, so if x_2 is accepted then x_1 <u>may</u> be accepted (it also may not be accepted--it is not required, but only contingent upon x_2 being accepted). If x_2 is not accepted (i.e., equal to zero) then x_1 cannot be greater than 0 and therefore is not accepted. Again, this approach can be extended to more than one contingent project set.

The concepts of mutually exclusive project constraints can be used in several additional ways. One such use is suggested by Quirin(99) for including projects that my be "slipped" in time; that is, if a project may be postponed or accelerated, that alternative could be included into the model formulation. By defining the slipped project and different cash flows associated with it as a new project and making it mutually exclusive of the original project, then both projects may be included into the selection program.

Finally, the aspect of expected value and variance being dependent on the projects selected (p. 60) can be partially handled by mutually exclusive projects. If some projects strongly interact with one another as far as their expected values and variances go, then "new" projects composed of various combinations of the different projects can be defined with their associated combined expected values and variances. These new projects are then made to be mutually exclusive of one another. This approach might be tenable if not too many projects of the set are dependent such that the number of "new" project combinations could be held to manageable proportions.

CHAPTER III

SOME DETAILS OF A "FAIRLY REALISTIC" C^2P/CB FORMULATION

General Description

The above discussed (Chapter II) aspects of a C^2P/CB model can probably be clarified by going step-by-step through an example in order to formulate the objective function and constraint set; the following example problem provides such an explanatory tool. However, an additional and probably more important purpose of this example is to develop a "fairly realistic" model which can be used to evaluate the practical aspects of formulating, data gathering, and solving a problem using the proposed chance constrained techniques.

It is not intended for this example to include all of the aspects of a model which could be used in practice by a firm; it does not, in fact, include several important factors such as equity financing and imperfect capital market effects on borrowings. However, it does detail several areas with enough realism that a "feel" for the complexity of an actual problem can be gained.

First, the model is expressed in deterministic, expected value form (mentioning those variables which include uncertainty). Then the development of the probabilistic form is introduced for the chance constraints, forming a non-linear 0-1 integer programming problem. The data gathering and computational aspects of such a problem are discussed next in Chapter IV.

The example problem has a total of nine projects to be considered in the current CB period; they are listed descriptively in Table 3.1 and illustrate some typical projects for a petroleum company.

TABLE 3.1

Example Problem Project Alternative Descriptions

no.	decision variable	description
1	p_1	Refinery expansion
2	p_2	Petrochemical complex addition to refinery if p_1 accepted
3	p_3	Drilling company expansion--adding rigs, etc
4	p_4	Drilling supply company expansion
5	p_5	Major oil field workover
6	p_6	Major oil field secondary recovery
7	p_7	Service station and "quick-stop" food store construction
8	p_8	Truck company expansion (crude oil common carrier)
9	p_9	Coal company expansion

The CB period is limited to one year; that is, only decisions occurring in the current fiscal year are being considered. As the Chapter I discussion pointed out, longer period (2-10 years) capital budgets could be designated. However, longer periods are usually associated with strategic planning and more "macroscopic" than capital budgeting. The difficulty of a longer period consideration is in determining what projects will be available in those future years--frequently a difficult task. This topic was briefly mentioned in Chapter II, and more will be said about this in a later discussion in this chapter.

Theory suggests that during a CB process, <u>all</u> past, on-going projects as well as new projects should be considered as competing for funds; however, in practice this is seldom done. Usually firms consider mostly "new" projects with perhaps a few previous projects thrown-in which management suspects may not be "carrying their own weight". With this in mind, the majority of the nine investment projects of the example are new, with the exception of projects 5 and 6, which are "modifications" of an existing project to increase its productivity.

These nine projects are considered fixed in time;[6] that is, no project can be delayed, and also fractional projects are disallowed (a fairly realistic assumption as pointed out in Weingartner's book). This makes the decision variables strict 0-1 integers and thus we can use the integer linear programming solution technique to be presented in Chapter IV.

Also, from Table 3.1, there are several contingency and mutually exclusive constraints as discussed in Chapter II. Project 2 cannot be accomplished without first accepting project 1 (contingency):

$$p_2 \leqslant p_1$$

Also project 6 and project 5 cannot both be accepted (mutually exclusive):

$$p_5 + p_6 \leqslant 1$$

Other interactions among the projects in the C^2P model would be of a statistical nature such as correlations. All of the projects listed

---

[6]Various constraint forms can be set up to include non-fixed time items; see the section in Chapter II on 'Non-Probabilistic Constraints".

would probably be positively correlated with each other to some degree except project 9. Here a negative correlation pattern might emerge since coal and the other projects may be competing sources of energy.

As discussed in Chapter I, financing considerations are becoming recognized as highly interrelated with the capital budgeting project selection process. Therefore, there are 4 long-term financing alternatives considered in the example problem; they are described in Table 3.2. These proposals, like the investment projects, are also considered fixed in time; that is, the financing arrangements must be taken during the period offered, or not at all. This may be unrealistic since lending institutions and/or bond issues, etc are sometimes flexible as to when the lending (financing) is to be consummated. Provisions for this could be handled as discussed in Chapter II by defining a new financing proposal which is equivalent to an existing one except that its time-period cash flows are shifted.

Like Naslund's(74) model discussed in Chapter II, short-term borrowing could be included for each quarter to satisfy the constraints. However, for the computational procedures proposed later, this would introduce a fractional 0-1 variable (so that mixed integer programming is required) with a coefficient equal to the maximum short-term money amount thought to be available during the period. So, for computational reasons and the relative insignificance of such borrowings, short term financing has not been used in the example problem.

TABLE 3.2.

Example Problem Financing Alternative Descriptions

no.	decision variable	description
1	f_1	Term loan at 11.75 percent interest; balloon payments first 12 quarters; quarterly payments afterwards for 7 years.
2	f_2	Term loan at 12.0 percent interest; regular annual payments for 5 years.
3	f_3	Equipment loan at 10.75 percent interest; tied to project 1, the refinery expansion. Quarterly payments for 5 years.
4	f_4	Equipment loan at 10.5 percent interest; tied to project 7, the service station construction. Annual payments for 10 years.

For the same computational reason previously given for leaving out short-term financing, fractional long-term financing will not be allowed for the example problem; the financing decision variables will be strict 0-1 integers. Again, this may not be valid for all types of financing since a corporation would not always have to take the entire amount offered through lines of credit of a lending institution.

All of the financing alternatives considered are known value, long-term debt issues; however, stock and bond issues could have been included. Also, as discussed in Chapter II, stock and bond issues could be considered as random variables within the framework of the C^2P model, although this will not be done here.

There are several contingency constraints implicit in the financing proposals; alternative 3, an equipment loan is tied to the refinery expansion:

$$f_3 \leqslant p_1$$

Alternative 4 is contingent upon the acceptance of the service station construction:

$$f_4 \leqslant p_7$$

Note that these contingencies would hold even if the f's were fractional 0-1 variables.

The Objective Function

As discussed in the "Objective Function" section of Chapter II, DCF economic criteria currently appear to be the most meaningful to industry. Therefore, the coefficients associated with the project decision variables in the objective function are the yearly, expected value DCF net present value(PV) of after-tax cash flows for each project, discounted at some appropriate rate (r_d) chosen by management. Normally the cash flows of a project past 30 years are not included unless they are contributing significantly to the net present value.

Also, the objective function includes decision variables for selecting the most appropriate forms of financing to go with the project selections in satisfying cash balance constraints and other financial reporting requirements. Since financing *per se* lends no net economic gain to the firm, borrowings should occur only when needed to satisfy some constraint(s); and when needed, the least cost form of borrowing should be chosen--that is, the financing with the lowest interest and/or most time-wise favorable payback schedule. As presented in Chapter II, one approach to do this is to include, as coefficients to the financing decision variables, the expected value of the yearly cash flow payback schedule (negative numbers) discounted at the same rate (r_d) as the projects. Thus, since the objective is to be maximized and the financing decision variable coefficients are negative, they will be chosen only when required. Also, since the coefficients are discounted at r_d, the financing with the best "leverage" (smallest interest rate or best timewise payback schedule) will be chosen.

A practical, complicating factor overlooked in many theoretical treatments of MP/CB is that nearly all economic evaluations (such as PV) are based on after-tax cash flows, and most estimates of items contributing to net cash flows are made before taxes. Therefore, adjustments must be made to the cash flow estimates, and the complexity of formulation and data gathering can be increased significantly. The most straightforward manner is to adjust the before-tax cash flows by their tangible expenditures plus depreciation, depletion, and amortization (DD&A). The after-tax flows for a project can be shown to be

$$c_a = c - (c+t-d)TXR = c(1-TXR) - (t-d)TXR \qquad (3.1)$$

where c_a is the cash flow after tax of the project in a given year
c is the cash flow before tax of the project in a given year (which includes tangible, t, and intangible expenditures)
t is the tangible expenditure on the project (a positive number as used here, whereas all other expenditures are negative numbers)
d is the DD&A of the project (also a positive number; this could also be used to provide other adjustments to taxes such as investment tax credits)
TXR is the corporate tax rate

It is recognized that a project's tax accounting income which is used to actually calculate income taxes can differ from the before-tax cash flows and DD&A used here (3.1) to calculate income taxes. Tax accounting income for a project involves revenue, operating expenses, administrative expenses, and "tax accounting income" capitalized expenses (depreciation, depletion, and amortization, DD&A). The operating expense "booked" values can involve complicated inventory LIFO, FIFO expense charges. Also the revenues and operating/administrative expenses often include the accrual concept of accounting. That is, from an accrual accounting standpoint, revenue and operating/administrative expenses that are "booked" for tax accounting income (and thus taxes) may be significantly different in magnitude and timing than the actual cash flows of a project.

However, to calculate the tax liabilities and after-tax cash flow for the investment proposals in the example problem, it is assumed that the taxable income calculations involving their cash flows and DD&A as shown in (3.1) coincide reasonably well with their contributions to the periods tax accounting income. This presupposes that 1) the time delays between revenues and cash receipts as well as operating/administrative expenses and cash disbursements are relatively small and 2) there is no inventory "cost of goods sold" expense and 3) the DD&A of the projects is simply depreciation and depletion of tangible expenditures (i.e., no amortization of expenses). This DD&A is assumed to be the same for calculations in both the taxable income and financial income accounting.[7]

[7]The DD&A used for "tax accounting income" (i.e., calculating taxable income) may be different than the DD&A used for "financial accounting income" (i.e., financial reporting of before-tax income to the investment community). Frequently (but not always) both tax accounting and financial accounting income for a project use the same "operating accounting income" (operating profit) as a base (defined as revenues minus operating/administrative expenses); therefore, they usually differ only in their DD&A of tangible expenditures figure. This subject will come up again in this chapter in the discussion of the constraint set.

The above assumptions are mostly characteristic of the investment proposals chosen for the example problem. Such an assumption reduces the amount of input data required for the model. If other investment types were chosen, separate data would be required to calculate their tax accounting income and thus income taxes.

Investments such as acquisition of existing, "going-concern" companies (where for examples operating expenses involve deducing from LIFO or FIFO inventory pricings the cost of goods sold) might have significant variations between tax accounting income and taxable income calculated from actual cash flows (3.1). This additional (i.e., in addition to cash flows) accounting data required on each project would greatly expand the amount of data necessary for a C^2P/CB model analysis--and perhaps provide a significant problem in implementation. The section on "Data Gathering" of Chapter IV will address some aspects of this problem.

The same type of tax problem exists for the financing proposals' after-tax cash flows where a tax credit for interest is taken. For the financing alternative let q_a and q be the after-tax and before-tax cash flows, respectively. Then the after-tax cash flow is

$$q_a = q + \text{interest} \times \text{TXR} \qquad (3.2)$$

Certain accounting practices may not have the interest which was actually paid in the period charged against the period for booking, and additional accounting data would be required. However, this example will assume that the interest cash flow and tax accounting interest expense coincide.

Finally, (3.1) and (3.2) both implicitly assume that there will always be something positive, from previous on-going and current investments, to write-off any on-going operations and current year investment and financing proposal losses--otherwise the DD&A and interest tax credit would not be valid for all cases.

Considering the above discussed aspects, the objective function is similar to that presented in chapter II

$$\max: \quad Z = \sum_{j=1}^{9}\left[\sum_{i=1}^{m_j}\left[\frac{\hat{c}_{ij}(1-TXR) - (t_{ij} - d_{ij})TXR}{(1+r_c)^i}\right]\right]p_j$$

$$+ \sum_{j=1}^{4}\left[\sum_{i=1}^{m_j}\left[\frac{\hat{q}_{ij} + r_{ij}TXR}{(1+r_d)^i}\right]\right]f_j \qquad (3.3)$$

where the subscripts i and j refer to the ith period (year) of the jth project or finacing proposal and p_j and f_j are the decision variables for the project and financing proposals, respectively.

Also m_j - number of years for the project cash flows
$\hat{c}_{ij}$ - yearly before-tax expected value cash flow for the project
t_{ij} - expected value for the yearly tangible capital expenditure on the project
d_{ij} - expected value for the yearly DD&A for the project
m_j - number of years for the financing proposal cash payback flows
$\hat{q}_{ij}$ - yearly before-tax cash payback for the financing proposal
r_{ij} - yearly interest on the financing proposal
TXR - corporate tax rate
r_d - management-assigned discount rate

Cash Balance Constraints

The constraint set associated with the example problem includes both cash balances and financial reporting requirements. These types of "sub-objectives" were shown in Chapter I to be of rising importance in both the theoretical literature and industrial practice.

The cash balance requirements are referred to as cumulative, after-tax net cash flow(CNCF) constraints and are formulated for each quarter for two years so that cash surpluses or deficiencies are carried over (or accumulated) from quarter to quarter. These CNCF relations represent the "liquidity" of the firm in meeting debt payments or having ample cash for other requirements such as stock repurchases or acquisitions.

Quarterly periods can probably most realistically represent the actual "liquidity" requirements of the capital budgeting problem in that funds can, in most cases, be shifted within a quarter. That is, although the quarterly period cumulative constraints require cash flows to balance at the end of the period and not at specific points within the period, fund requirements usually are flexible enough to be shifted, at the most, three months.

For example say in some quarter, at the very beginning, retirement of a long-term debt issue is required; and although the quarterly cash flow requirement is met at the end of the period, not enough cash is available at the beginning to retire the debt. In this instance the creditors would most likely allow the debt retirement to be slipped, at the latest, to the end of the quarter where the cash flow constraint is assured to be met.

Yearly cash flow constraints would probably not be too realistic from this "slippage" standpoint. Monthly constraints would be even more accurate but significantly expand the size of the constraint set and pose more difficulty in cash flow parameter estimation—especially if more than two years were considered.

Several additional items must be included for the CNCF constraint formulations. They are defined as:

PIc - previous on-going investment cash flows; a <u>random</u> variable
M - miscellaneous cash requirements such as bond retirements, stock repurchase, acquisitions, tax carryforward credits, known future borrowings, etc; a <u>random</u> variable
PLp - previous debt repayment; deterministic
DIV - proposed dividend payments; deterministic
TX_o - tax payments based on the previous year's tax liability; deterministic

The first three quarterly CNCF constraints can be expressed as

$$\sum_{j=1}^{9} \sum_{i=1}^{n} (c_{ij})p_j + \sum_{j=1}^{4} \sum_{i=1}^{n} (q_{ij})f_j + \sum_{i=1}^{n} (PIc_i - M_i - PLp_i - DIV_i) - n(.25TX_o) \geq 0$$

for n = 1 to 3 (3.4)

where the c_{ij} and q_{ij} are quarterly cash flows for the projects and financings, respectively; with c_{ij} being a random variable and q_{ij} deterministic.

For the fourth quarter the CNCF constraint becomes somewhat more complicated due to the yearly tax payment which, for simplicity, is assumed payable in the last quarter of the year. This is unrealistic since taxes are usually paid in one or two installments following the end of the fiscal year. Also since the cash flows including the capital expenditures are random variables, the tangible expenditures per quarter will be random as will the DD&A for the year, as used in (3.1). One approximate way to consider this is to have the tangible expenditure and DD&A be expressed as a percentage of the random cash flows.[8] Introducing some more notation:

- $t\%_{ij}$ - is the tangible capital expenditure expressed as a percentage of the yearly total cash flow for the jth project in the ith year; deterministic
- $d\%_{ij}$ - is the DD&A expressed as a percentage of the yearly total cash flow for the jth project in the ith year; deterministic
- PIa_1 - the first year estimated operating accounting income (revenue-operating/administrative expenses)for tax purposes from previous on-going invesments; a random variable.
- $PItd_1$ - the first year tax accounting income DD&A, etc for all previous on-going investments; deterministic
- $PLIa_1$ - the first year tax accounting interest paid on all previous debt repayments; deterministic

The fourth quarter constraint can be manipulated and simplified to

$$\sum_{j=1}^{9}\left[\sum_{i=1}^{4} c_{ij}\left[1-(TXR)(1+t\%_{1j} - d\%_{1j})\right]\right]p_j + \sum_{j=1}^{4}\left[(\sum_{i=1}^{4} q_{ij}) + (r_{1j})TXR\right]f_j$$

[8]Again, as discussed earlier in this chapter, it is noted that the assumption is made here that the before-tax cash flows of the project can be used to calculate taxable income. Otherwise, an estimate of the random variable taxable income would be necessary; this would be a separate input from the before-tax cash flow although they would probably be highly correlated.

$$+\sum_{i=1}^{4}(PIc_i-M_i-PLp_i-DIV_i) - \underbrace{(PIa_1-PItd_1-PLIa_1)TXR}_{\text{taxes for "on-going" operations}} \geqslant 0 \qquad (3.5)$$

The cumulative nature of the constraint caused the previous quarterly tax pre-payments to drop out of the fourth quarter formulation.

Succeeding quarterly CNCF constraints (second year) include other items such as quarterly tax pre-payments from the preceding year; this was one reason for including all of the tax aspects calculations in the first year CNCF constraint. That is, taxes are included so explicit consideration of this can be made in the cash flows with respect to individual projects and financings in both the first and second year. Also, now in the second year, "unknown" investment and financing proposals are under consideration besides the current (first year) decision variables.

The inclusion of the aspects of new, yet-to-be-defined investments and financing is perhaps one of the most difficult and theoretically questionable facets of the MP/CB problem. In many theoretical works, constraints of various types are defined far into the distant future without explicit consideration of these unknown opportunities. It seems inadequate to select current year projects and financing and then to consider cash balance constraints in the following year without including at least some information that there will be additional cash requirements and generation from future investments and financing. A similar argument to include future activities would also apply to the financial accounting constraints such as EPS, ROI, and D/E ratios.

Management is concerned that the total operations of the firm (which includes all previous operations, current selections, and future opportunities) will meet the cash balance and financial accounting goals. Management cannot make the selection of yet-to-be-defined projects and financing, but it should at least consider them in some fashion while making the selection of current year opportunities.

One approach is to make these future investment and financing alternatives decision variables. That is, the mathematical program would determine how much would be required from new investments and how much from new financing to meet the constraints; upper bounds could be imposed on these decision variables based on past performance. One problem with this is that the constraint is almost guaranteed to be met regardless of the current year selection decisions; however, the magnitude of the investment required may give a very large, improbable "target" for management to reach in the next year.

An alternate approach is to use historical data to predict what the performance of future investments (projects) might be. This concept is well-suited to the C^2P formulation in which the new project activity is a random variable based on previous company investments and project trends. In this fashion, consideration would be given to the probabilities of various outcomes of the future investments. These would be combined with the probabilities of all on-going operations plus current year project selections to formulate the firm's total chance constraints in future years.

The future financing alternatives would be included in the constraints as a "management target" decision variable since the availability of borrowings could be upperbounded (by D/E,etc) such that the magnitude calculated would be a "probable" figure to obtain. That is, financing opportunities within the bounds specified can probably be secured with a reasonable amount of effort, if required; therefore, they may be included as decision variables.

Using this approach of including future project uncertainties, the β confidence limits as in (2.3) on the chance constraints of future years should probably be lowered from those of the current year. The variance of the future year constraint is definitely going to increase by including these additional uncertainties; and high confidence levels (such as 95%) might be impossible to obtain. Achieving a 95% confidence in the current year may be necessary since once the capital budgeting selection is made, only relatively minor alterations are usually available during the year to meet the constraints. However, for the next year, management has a significant "control variable"--a whole new budget to be selected. They may thus chose to select the current year capital budget so they get within some reasonable "confidence range" (say 75%) for the next year's constraints. Then they would rely on their initiative to get the new (next year) budget more precisely defined and of such a magnitude that higher confidence levels can be achieved for the coming year.

The extension of this concept could be made further into the future than the two years proposed in this example problem. However, anything past 3-5 years probably runs a significant chance in having technological breakthroughs or nation/world political events invalidate any future projections of investment performance.

Returning to the CNCF constraint for the second year and considering the above discussion, the following notation is introduced:

- N_i – an upper limit on the amount of net (inflow-payback) unknown lending available during quarter i of the second year; deterministic
- n_i – the decision variable for selecting the unknown financing in the ith quarter of the second year
- NIc_{2i} – the net cash flow predicted for the unknown (second year) investments in the ith quarter of the second year; random variable
- NIa_2 – the yearly operating accounting income for tax purposes predicted for the unknown (second year) investments; a random variable
- $NItd_2$ – the yearly tax accounting income DD&A predicted for the unknown (second year) investments; a random variable.

The notation for $t\%_{2j}$, $d\%_{2j}$, PIa_2, $PItd_2$, and $PLIa_2$ apply to current year capital budget items in the second year but are similar to the definitions for these variables for the first year. For simplicity the interest charges for the unknown lending in the second year have been ignored.

The decision variable for future financing is broken into three 0-1 integer segments ($n_i^{(k)}$; k = 1 to 3) in order to keep all decision variables consistent (0-1) so the computational procedure proposed in the next section can be used. These variables do not appear in the objective function, therefore they will not enter into solution unless required to satisfy a constraint.

As an example of one of the <u>simpler</u> of the second year CNCF constraints the 8th quarter CNCF can be rearranged and simplified to

$$\sum_{j=1}^{9}\left[\sum_{i=1}^{8} c_{ij}\left[1-(TXR)(1+t\%_{2j}+t\%_{1j}-d\%_{2j}-d\%_{1j})\right]\right]p_j$$

$$+\sum_{j=1}^{4}\left[\left(\sum_{i=1}^{8} q_{ij}\right)+(r_{2j}+r_{1j})TXR\right]f_j+\sum_{m=5}^{8}\left(.33N_m\sum_{k=1}^{3} n_m^{(k)}\right)$$

$$+\sum_{i=1}^{8}(PIc_i-M_i-PLp_i-DIV_i)$$

$$+\sum_{i=5}^{8}(NIc_i)-(PIa_2-PItd_2-PLIa_2+PIa_1-PItd_1-PLIa_1+NIa_2-NItd_2)TXR\geqslant 0 \qquad (3.6)$$

Financial Reporting Constraints

In addition to the quarterly cumulative net cash flow constraints, the example problem includes constraints on certain after-tax financial accounting income and other financial reporting criteria. As discussed in Chapter II, these constraints which become "sub-objectives" for the investments and financing proposals to meet, are becoming increasingly important to management in selecting the CB.

The accounting period over which these financial reporting requirements are measured is assumed to be one year, having the calendar year and fiscal year correspond (not a generally realistic assumption). Shorter periods, such as quarters, could be imposed on these constraints if management feels that interim financial reporting to stockholders and investors is of significant importance.

EPS:

As previously discussed in Chapter II, one of the indicators most commonly used by existing and potential investors in a company is the yearly earnings per share ratio (EPS). This is defined as the yearly after-tax financial accounting income (i.e., the after-tax income shown on the financial reports) divided by the number of shares of common stock outstanding.

For the example problem, the investment and financing proposals to be selected (plus other existing and "unknown" items) must yield an EPS greater than or equal to some value assigned by management. For this case the EPS at yearly financial reporting time for the first two years has been chosen to be related to the previous year's EPS plus some specified growth rate (g_e).

$$\text{After-Tax Financial Accounting Income/Number of Common Shares} \geq \text{EPS previous year } (1 + g_e) \tag{3.7}$$

The details of the EPS constraints can be defined from the items presented in the CNCF constraints, since the assumptions made in the section on the "Objective Function" of this chapter concerning projects and financings' cash flows and taxes will make the after-tax cash flows and after-tax financial accounting incomes be identical. Recall that it was assumed that for the projects under consideration, DD&A was the same for both taxable income and financial income accounting. However, the DD&A used in the CNCF constraint's taxable accounting income for previous "on-going" investments (PItd), will most likely be different from the DD&A necessary for calculating after-tax financial accounting income (see footnote 7, page 97).[9] To include this new DD&A, a new parameter (PIad) is defined for the constraints. The constraint contains this for the previous, on-going investments as

$$\underbrace{(PIa_1-PIad_1-PLIa_1)}_{\text{financial accounting income}}-\underbrace{(PIa_1-PItd_1-PLIa_1)}_{\text{tax accounting income}}TXR$$

simplifying

$$(PIa_1-PLIa_1)(1-TXR)-PIad_1+(PItd_1)TXR \tag{3.8}$$

Including this form (3.8) into the full constraint (3.7) for the first year gives

$$\sum_{j=1}^{9}\left[\sum_{i=1}^{4} c_{ij}\left[1-TXR(1+t\%_{1j} - d\%_{1j})\right]\right]p_j - (1-TXR)\left[\sum_{j=1}^{4} r_{1j}\right] f_j$$

$$+ (PIa_1-PLIa_1)(1-TXR)-PIad_1+(PItd_1)TXR - (NCS_o)(EPS_o)(1 + g_e) \geqslant 0 \tag{3.9}$$

---

[9]As pointed out in footnote 7, the financial accounting and tax accounting incomes usually have the same base for "operating accounting income". Although this is not always true, the example problem will assume this to be valid; and thus the operating accounting incomes for on-going and "unknown" items used in the CNCF constraints for tax accounting purposes can also be used here for financial accounting calculations.

where EPS_o is the EPS for year zero (beginning) and NCS_o is the number of common stock shares outstanding at year zero. For the example problem this is assumed to be a constant (in other words, common stock issue is not considered as a financing alternative nor or any convertible debt issues converted during the two-year horizon period).

The second year EPS constraint is similar to year one, except it also includes the second year "unknown" projects and financing proposals' operating accounting income considerations (NIa_2) along with its financial accounting ($NIad_2$) and tax accounting income DD&A figures ($NItd_2$). $NIad_2$ and $NItd_2$ are both random variables estimated from historical data.

$$\sum_{j=1}^{9}\left[\sum_{k=5}^{8} c_{ij}\left[1-TXR(1+t\%_{2j}-d\%_{2j})\right]\right]p_j - (1-TXR)\left[\sum_{j=1}^{4} r_{2j}\right] f_j$$

$$+(PIa_2-PLIa_2)(1-TXR) - PIad_2 + (PItd_2)TXR + NIa_2(1-TXR)$$

$$- NIad_2 + (NItd_2)TXR - (NCS_o)(EPS_o)(1 + g_e)^2 \geq 0 \qquad (3.10)$$

ROI:

Another important financial ratio used by the investment community is the return on investment (ROI) or return on equity. Although there are several slightly different definitions of ROI, the example problem uses

---

ROI = After-Tax Financial Accounting Income for the Year/(Average Owners' Equity + Average Long-Term Debt for the Year)

To have the ROI be greater than the year zero ROI (ROI_o) plus a specified growth rate (g_r) results in

After-Tax Financial Accounting Income for the Year/(Average Owners' Equity for the Year + Average Long-Term Debt for the Year) $\geqslant$ $ROI_o\ (1 + g_r)$ (3.11)

where ROI_o is the year zero ROI and g_r is the specified ROI growth rate.

This formulation (3.11) can be further simplified since the after-tax financial accounting income is constrained by the EPS constraints (3.9) and (3.10) to be greater than or equal to $(NCS_o)(EPS_o)(1 + g_e)$ for the first year and $(NCS_o)(EPS_o)(1 + g_e)^2$ for the second year. Therefore, to satisfy the ROI constraint the average owners' equity and long-term debt is the only item to be constrained.

Average Owners' Equity for the Year + Average Long Term Debt for the Year $\leqslant (NCS_o)(EPS_o)(1 + g_e)^{1 \text{ or } 2}/(ROI_o)(1 + g_r)^{1 \text{ or } 2} \leqslant$ a Constant for the Year (year one or year two) (3.12)

A further breakdown of these terms gives the average owners equity to be

RE_{t-1} + .5 (After-Tax Financial Accounting Income for the Year-Dividend for the Year) + CS_o (3.13) where RE_{t-1} is the retained earnings through the previous year and CS_o is the year zero dollar amount of the common stock value (assumed a constant).

The average long-term debt becomes

$$PL_{t-1} + .5\ (\text{New Long Term Liability} - \text{Payback of New Liability} - \text{Payback of Previous Liability}) \tag{3.14}$$

where PL_{t-1} is the previous year's long-term debt

Substituting these breakdowns (3.13) and (3.14) into the constraint form (3.12) and using the after-tax financial accounting income of the first year EPS constraint (3.9), the first year ROI relationship is

$$.5\sum_{j=1}^{9}\left[\sum_{i=1}^{4} c_{ij}\left[1-TXR(1+t\%_{1j}-d\%_{1j})\right]\right]p_j + .5\sum_{j=1}^{4}\left[\left(\sum_{i=1}^{4} q_{ij}\right) - r_{1j}(1-TXR)\right] f_j$$

$$+ .5\left[\sum_{i=1}^{4} (-DIV_i - PLp_i) + (PIa_1 - PLIa_1)(1-TXR) - PIad_1 + (PItd_1)TXR\right]$$

$$+ RE_o + PL_o + CS_o - \left[(NCS_o)(EPS_o)(1+g_e)/(ROI_o)(1+g_r)\right] \leqslant 0 \tag{3.15}$$

where CS_o, RE_o, and PL_o are the common stock, retained earnings and previous liability for year zero (i.e., the beginning).

For the second year, the "unknown" investment and financing considerations are included, along with the previous year's retained earnings to determine the ROI

$$\sum_{j=1}^{9}\left[.5\sum_{i=5}^{8}c_{ij}\left[1-TXR(1+t\%_{2j}-d\%_{2j})\right]+\sum_{i=1}^{4}c_{ij}\left[1-TXR(1+t\%_{1j}-d\%_{1j})\right]\right]p_j$$

$$+\sum_{j=1}^{4}\left[.5(\sum_{i=5}^{8}q_{ij})+(\sum_{i=1}^{4}q_{ij})-(.5r_{2j}+r_{1j})(1-TXR)\right]f_j$$

$$+.5\sum_{m=5}^{8}\left[(.33)(N_m)\sum_{k=1}^{3}n_m^{(k)}\right]+.5\left[\sum_{i=5}^{8}(-DIV_i-PLp_i)+(PIa_2-PLIa_2)\right.$$

$$\left.(1-TXR)-PIad_2+(PItd_2)(TXR)\right]+\sum_{i=1}^{4}(-DIV_i-PLp_i)+(PIa_1-PLIa_1)$$

$$(1-TXR)-PIad_1+(PItd_1)(TXR)+.5\left[NIa_2(1-TXR)-NIad_2+(NItd_2)TXR\right]$$

$$+RE_o+PL_o+CS_o-\left[(NCS_o)(EPS_o)(1-g_e)^2/(ROI_o)(1+g_r)^2\right]\leqslant 0 \quad (3.16)$$

D/E:

A third financial ratio of importance is the debt-to-equity (D/E) ratio. As discussed in Chapters I and II, this indirectly constrains the borrowing in a fashion most representative of an actual industry situation. D/E is defined here as

$$D/E = \text{Long Term Debt/Owners' Equity} \leqslant \text{Desired D/E} \quad (3.17)$$

The constraint thus becomes

$$\text{Long Term debt} - (\text{Owners' Equity})(\text{Desired D/E}) \leqslant 0 \quad (3.18)$$

The components of long-term debt and owners equity are again defined for the D/E constraint in a fashion similar to (3.12), (3.13) and (3.14)

D/E = PL_{t-1} + New Liability - Payback of New Liability - Payback of Previous Liability - $\left[RE_{t-1}\right.$ + After-Tax Financial Accounting Income for Year - Dividends for Year + $\left.CS_o\right]$ [Desired D/E Ratio] ≤ 0 (3.19)

The D/E constraint becomes, for year one, using the desired D/E ratio to be the year zero D/E (D/E_o) plus a growth rate (g_d)

$$-D/E_o(1+g_d)\sum_{j=1}^{9}\left[\sum_{i=1}^{4}c_{ij}\left[1-TXR(1+t\%_{1j}-d\%_{1j})\right]\right]p_j$$

$$+\sum_{j=1}^{4}\left[\left(\sum_{i=1}^{4}q_{ij}\right)+D/E_o(1+g_d)(r_{1j})(1-TXR)\right]f_j$$

$$-\sum_{i=1}^{4}PLp_i+D/E_o(1+g_d)\sum_{i=1}^{4}DIV_i - D/E_o(1+g_d)\left[(PIa_1-PLIa_1)\right.$$

$$\left.(1-TXR) - PIad_1 + (PItd_1)TXR\right] - D/E_o(1+g_d)(RE_o) +$$

$$PL_o - D/E_o(1+g_d)CS_o \leq 0 \qquad (3.20)$$

The D/E for year two is

$$-D/E_o(1+g_d)^2\sum_{j=1}^{9}\left[\left(\sum_{i=1}^{8}c_{ij}\left[1-TXR(1+t\%_{2j} + t\%_{1j} - d\%_{2j} - d\%_{1j})\right]\right]p_j$$

$$+\sum_{j=1}^{4}\left[\sum_{i=1}^{8}q_{ij}+D/E_o(1+g_d)^2(r_{2j}+r_{1j})(1-TXR)\right]f_j$$

$$+\sum_{m=5}^{8}(.33N_m\sum_{k=1}^{3}n_m^{(k)}) - \sum_{i=1}^{8}PLp_i+D/E_o(1+g_d)^2\sum_{i=1}^{8}DIV_i - D/E_o(1+g_d)^2$$

$$\left[(PIa_2+PIa_1-PLIa_2-PLIa_1)(1-TXR)-PIad_2-PIad_1+(PItd_2+PItd_1)TXR+NIa_2\right.$$

$$\left.(1-TXR)-NIad_2+(NItd_2)TXR\right] -D/E_o(1+g_d)^2RE_o+PL_o-D/E_0(1+g_d)^2CS_o \leqslant 0 \quad (3.21)$$

Several other types of financial reporting constraints could be introduced besides those presented above, and some are discussed in more detail along with some aspects of equity financing in Hamilton and Moses(62) and Peterson(89).

The main point to be made of all the above discussion is that these additional criteria (cash balances, financial reporting, etc) that management needs to evaluate in CB decisions can be incorporated into a MP framework for an efficient , systematic appraisal. However, the actual formulation of these "sub-goals" (3.4) through (3.21) are not as straightforward as might be indicated in some theoretical works--especially considering the complexities of financial and tax accounting.

Including Uncertainty into the Fomulation

As previously discussed in Chapter II, the constraint set is where the uncertainty of the model is included. Each of the constraints presented in this chapter can be expressed in a common probabilistic form

$$\text{Prob}\left[\sum_{j=1}^{n}(a_{ij})z_j - B_i \leqslant 0\right] \geqslant \beta_i \qquad i = 1 \text{ to } m \qquad (3.22)$$

In our case n = 25 (9 projects, 4 current year financings, 12 2nd year financings)

m = 14 (8 CNCF constraints; 2 EPS, 2 ROI, 2 D/E constraints)

The z_j's take the place of the p's, f's, and $n^{(k)}$'s in the previously formulated constraints, and B_i is actually composed of the PIc, M, NIc, PIa, NIa, NItd, NIad, random variable terms (for quarters and different years where applicable)plus other non-decision variable items.

This probabilistic expression (3.22) can be transformed into its deterministic form as done earlier in (2.5) forming a non-linear 0-1 integer programming problem.

$$\sum_{j=1}^{n}E(a_{ij})z_j + K_{\beta i}\left[\text{Var}\left(\sum_{j=1}^{m}a_{ij}z_j - B_i\right)\right]^{\frac{1}{2}} - E(B_i) \leqslant 0 \quad i = 1 \text{ to } m \quad (3.23)$$

Assuming the a_{ij}'s and B_i's are not independent the variance term can be expressed as

$$\sum_{j=1}^{n}\text{Var}(a_{ij})z_j^{\,2} + \text{Var}(B_i) + \sum_{k=1}^{n}\sum_{\substack{j=1\\ j\neq k}}^{n}\text{Cov}(a_{ij},a_{ik})z_jz_k - \sum_{j=1}^{n}\text{Cov}(a_{ij}, B_i)$$

(3.24)

For this form the variance of a_{ij} and B_i must be calculated; this calculation is not straightforward and is embodied in the specific constraint under consideration. For example, consider the 8th quarter CNCF constraint (3.6).

The variance of a_{81} in (3.24) is actually composed of a sum of random variables itself.

$$\mathrm{Var}(a_{81}) = A^2\left[\sum_{k=1}^{8}\mathrm{Var}(c_{k1}) + \sum_{k'=1}^{8}\sum_{\substack{k=1\\k\neq k'}}^{8}\mathrm{Cov}(c_{k1},c_{k'1})\right] \qquad (3.25)$$

where A is the coefficient of the project random variable terms in (3.6); $1\text{-}TXR(1+t\%_{2j}+t\%_{1j}-d\%_{2j}-d\%_{1j})$.

The variance of the c_{k1} terms are assumed as data inputs to the model; however, the covariance term is calculated from correlation coefficients (also assumed input) and the variances.

$$\mathrm{Cov}(c_{k1},c_{k'1}) = \sum_{k'=1}^{8}\sum_{\substack{k=1\\k\neq k'}}^{8} r_{kk'1}(s_{k1})(s_{k'1}) \qquad (3.26)$$

where $r_{kk'1}$ is the correlation coefficient between and c_{k1} and $c_{k'1}$ of project 1 and s_{k1} and $s_{k'1}$ are the standard deviations of c_{k1} and $c_{k'1}$ respectively.

The variance of B_8 is also complicated since it is composed of a sum of terms that are correlated with each other while also being sums themselves. The variances for each of the cash flow series terms for

PIc, M, NIc, and PIa in (3.6) would be handled as above using known correlation coefficients and variances (standard deviations). The random variable terms for NIa, NItd, and NIad would have their variances simply taken from their input data.

Using the variances for each term composing B_8 the covariances between these terms must be used to calculate the total variance of B_8. Again, for the covariances the correlation coefficients between the terms PIc, M, NIc, PIa, NIa, NItd, and NIad are assumed given and used with their "adjusted" standard deviations[10] derived from the form of the summation terms to determine the covariances.

The calculation remaining to be done for determining the variance of (3.23) using (3.24) is to compute the covariances between all the a_{8j} terms and the random terms of B_8. The same correlation coefficient and standard deviation approach discussed above is used, where the correlation coefficients between project-to-project cash flows are assumed known as well as the correlation coefficients between projects and individual random terms composing B_8. Again, the standard deviations of the summation terms of a_{8j} and B_8 must be "adjusted".

---

[10]The standard deviation of the summation terms to be used in the covariance calculation utilizing correlation coefficients is adjusted since it would be incorrect to use the true variances of the summation terms (previously calculated from individual items of the summation terms). It can be shown that the adjusted standard deviation for a summation term is simply the <u>sum</u> of the products of the standard deviation and its associated coefficient for all of the items composing the summation term.

The entire process discussed above for the CNCF of the 8th quarter is repeated for all of the other quarterly CNCF constraints and the financial reporting constraints of EPS, ROI, and D/E. Conceptually the approach is the same as for the above illustration; however, the forms of the equations will change--although in many cases there is some similarity in equation form.

From the preceding discussion it seems obvious that the inclusion of uncertainty aspects into the constraint formulations of a MP/CB model is not easy. The calculations are quite involved and the data input requirements on variances and correlation patterns are considerable. This problem of data gathering will be discussed in the next chapter, and then following this, attention will be directed to some computational aspects.

CHAPTER IV

DATA GATHERING AND COMPUTATIONAL CONSIDERATIONS

Data Gathering

The example problem discussed in Chapter III presented some aspects in the detailed formulation of a "fairly realistic" model. The items necessary to construct an objective function, cash balance constraints and financial reporting constraints were delineated; however, it remains to analyze how practical (or possible) it might be to collect these data items.

Project Data:

For the constraint set, eight quarterly, after-tax net cash flows are required. As shown in the CB practice surveys cited in Chapter I, the difficulty in getting any kind of cash flow estimates available for a project before capital budgeting time is one of the major obstacles to a MP approach. However, these same surveys [Kempster(17), Williams(19), and Davey(42)] show a definite trend towards gathering of this cash flow data but mostly on a yearly basis for project economic evaluations. If yearly data is available, gathering of quarterly data would not seem to be much additional trouble if it is only required for 2 or 3 years.

To use the deterministic form of the chance constraint equations (3.23), estimates on the expected values, variances and correlation coefficients of each quarterly cash flow is required. Several authors including Wagle(102) and Hillier(69) have addressed this issue, but most recently Bussey(56) has taken much of their work, summarized, simplified, and extended it to a more practical approach.

The expected value and variance are derived from a "pessimistic" (min) estimate, a "most likely" (ml) estimate, and an "optimistic" (max) estimate for each cash flow which is assumed to follow a Beta probability distribution (closely resembling a Normal distribution). For a Beta distribution the expected value and variance can be derived as

$$EV = 1/6\ (\min + 4ml + \max) \tag{4.1}$$

$$Var = 1/6\ (\max - \min)^2 \tag{4.2}$$

Bussey recognizes that the net cash flow can itself be composed of several random variable terms such as capital expenditures, sales revenues, operating costs, salvage, etc; and that this can complicate the "bottom line" estimation of the net cash flow expected value and variance. However, in most cases Bussey claims this problem must be "hurdled" to get a final net cash flow.

In practice, companies often require sensitivity or "range" analysis whereby all of the items composing a cash flow stream are set at their min, ml, and max values and economics are calculated for each case. This type of data, while being overly extended somewhat in min and max ranges would still suffice for the Beta distribution. Also, not all project evaluations are subject to sensitivity analysis--only those of a certain "risk class" and above a specified capital exposure. This same approach should be used in deriving the data for a C^2P model; not all projects need be analyzed for variance estimation.

The covariance in (3.26) using the correlation coefficients can be expressed in terms of auto-correlation coefficients. Bussey borrows some concepts from Box and Jenkins(103) to assume a first-order Markovian process in which the cash flow in a summation (such as those for a project in a CNCF constraint) is highly correlated with its immediately preceding period cash flow value, somewhat correlated with the next earlier cash flow, etc.

Using the min, ml, and max estimates given by the analyst, an approximate maximum likelihood estimator of the "one-period lag coefficient" (used in calculating the correlation coefficients) can be approximated as

$$\phi_1 = \frac{\frac{1}{n'-1} \sum_{t=1}^{n'} w_t w_{t-1}}{\frac{1}{n'-2} \sum_{t=1}^{n'-1} (w_t)^2} \qquad (4.3)$$

where w_t is the first difference in the estimated cash flow stream indexed at t (i.e., $max_t - max_{t-1}$; $ml_t - ml_{t-1}$; $min_t - min_{t-1}$)

w_{t-1} is the first difference (as above) indexed at t-1

n' is the number of total estimated cash flows when all three of the min, ml, and max estimates are used [the total should equal 3(n-1) where n is the number of periods in the cash flow = 8 in our example].

Once this "lag coefficient" (4.3) is estimated, the symmetrical matrix of correlation coefficients for the specific cash flow series under consideration can be estimated by assuming a first-order Markovian process whereby each correlation coefficient is calculated by

$$r_{kk'} = \emptyset_1^{|k'-k|} \tag{4.4}$$

where $r_{kk'}$ is the auto-correlation coefficients between period k and k' for a given cash flow series
$k,k' = 1,2,3,\ldots\ldots\ldots n$ (the number of periods for the cash flow series)

Bussey gives an esample of an entire 10 x 10 auto-correlation matrix; one column from this will help clarify the above discussion. The one-period lag coefficient is calculated as .40 and this holds for the auto-correlation coefficients between any period's cash flow and its <u>immediately</u> preceding period cash flow. For example, the cash flow in period 4 has the following correlation coefficients:

$$r_{34} = .4;\ r_{24} = (.4)^2 = .16;\ r_{14} = (.4)^3 = .064$$

Similar calculations would be performed for all of the cash flows in each period (of the series under consideration) to construct the matrix, and these would be used in an equation such as (3.26) to calculate the covariance of the summation of cash flows.

The yearly, expected value, net cash flow ($\hat{c}_{ij}$) used in the objective function (3.3) of the example problem can easily be determined for the first two years as the sum of the expected value quarterly data calculated from (4.1). A fairly large number of years (30 or so) is used in projecting cash flows for PV economics, but sensitivity study data (min, ml, max) may not be desired except in deriving the variances for the first two years' quarterly cash flows. However, past the second year, most likely estimates would probably suffice in lieu of expected value numbers calculated from min, ml, and max estimates.

The $t\%_{1j}$, $t\%_{2j}$, $d\%_{1j}$, $d\%_{2j}$ values used in the constraints for the first two years (3.5, etc) for calculating taxable income could be estimated from the tangible expenditure and DD&A values associated with the most likely net cash flow of the project for the given year. It would be assumed that these percentages would be constant even though the yearly net cash flow would vary randomly. This approach might be a decent approximation since random changes in capital expenditures during the year would probably have proportional changes in tangibles and DD&A. The changes in capital expenditures would cause proportional changes in any cash in-flow generation; therefore, the tangible and DD&A percentage of the entire sum of capital expenditures and cash in-flows for the year (i.e., net cash flow) might be fairly constant.

The yearly t_{ij}'s, d_{ij}'s used in the objective function (3.3) of the example would also be derived from the "most likely" cash flow estimates of the project for each year. For the first two years, these would be the same as the percentage numbers discussed above--except they would be expressed in (3.3) as absolute numbers, not percentages.

Financing Data:

The main problem with gathering financial data is similar to a major problem of the project data; that is, defining what are the financial proposals to be considered before CB time. Once this is done, the net cash received plus payback schedules ($\hat{q}_{ij}$'s and q_{ij}'s) and interest (r_{ij}'s) are known with a fair degree of certainty for the financial proposals in the example.

The interest rates for these borrowings are not considered to be influenced by the selection of the financing proposals. This may not be too bad of an assumption if the D/E or other financial "riskiness" measures of the company are not changed significantly by selection of these financings. Inclusion of a varying interest rate for an "imperfect capital market" requires more "formulation work" in order to be used in a MP model--especially one considering uncertainty (such as C^2P).

If bond or preferred stock issues were considered, the net cash received would be a random variable defined by an expected value and variance subjectively estimated by financial analysts again using a min, ml, and max Beta distribution assumption. The cash payback (and interest if applicable) would be assumed proportional to the random variable cash received with the timing of the payback schedule remaining constant.

Common stock financing could be handled similarly; however, the payback schedule would be defined only by the dividend payments declared by management and input to the MP formulation. Constraints on EPS, D/E, and perhaps some form of a price-to-earnings ratio (P/E) could be related to common stock issues to limit the amount of funds acquired in this manner. Again, as discussed in Chapter II, Weingartner(52), Chambers(88) and Peterson(89) are referenced in regards to common stock formulation. Hamilton and Moses(62) have also considered this issue, but more work needs to be done in this important area.

On-Going Operations:

Previous investments and financial commitments of the firm can usually be identified specifically by project and borrowing sources. "Profit plans" of a company include the estimation of all previous on-going operations frequently broken down by month or quarter for one or two years. The data for a project's tax and financial accounting DD&A ($PItd_1$, $PItd_2$, $PIad_2$) plus the data for a specific loan's payback and interest (PLp_i, $PLIa_1$, $PLIa_2$) should be obtainable from the profit plan.

Previous investment's cash flows and operating accounting income (PIc_i, PIa_1, PIa_2) would also be obtainable from the profit plan; however, since these are random variables, min, ml, max estimations would be required. Most profit plans are computerized, therefore the derivation of this data could be accomplished fairly efficiently by running three separate computer cases consisting of pessimistic, most likely, and optimistic estimates for all "uncertain" on-going projects which are "rolled-up" into a corporate summary. The expected values, variances, and auto-correlations for these items (especially the 8 quarters of data for PIc) can be calculated using Bussey's approach as discussed above in the section on "Project Data".

New "Unknown" Investments:

All of the items related to the second year unknown investments (NIc_i, NIa_2, $NItd_2$, and $NIad_2$) are random variables whose expected values and variances can be predicted from historical data of the firm (using regression analysis, averaging, exponential smoothing, etc). For the NIc_i quarterly cash flow series, covariances can be determined by standard statistical analyses of previous, historical data on cash flows.

The new financing for the second year (N_i) would be determined as a decision variable where N_i is determined for each quarter as the maximum amount of net financing cash flow "reasonably obtainable" in any one quarter. Finance departments usually have a good feel for numbers such as this.

Other Items:

The miscellaneous cash flows (M_i), which can include both inflows or outflows, are composed of items not covered by any of the above discussed categories Things such as stock repurchases, special tax credit rebates, liquidation of assets, known acquisitions or divestures, etc should be obtainable from corporate planning or some other centralized group. Subjective estimates on the min, ml and max ranges could be used in the approach suggested by Bussey to derive expected value, variance and auto-correlation coefficients.

Items of the model such as TX_O, NCS_O, EPS_O, D/E_O, ROI_O, and TXR are given from the previous year's ending conditions. All others such as DIV_i, r_d, g_e, g_r, g_d and the β's are parameters to be defined by management based on their subjective feelings for the problem. These items would be the most likely to be changed when "what if" cases are to be run.

Item-to-Item Correlation:

The remaining topic to be discussed in the data gathering section is that of the correlations between the various sets of random variables. The deterministic equivalent equation form (3.23) used these "cross-correlations" to determine the overall variance of the constraint. For example, the correlation between the various projects must be determined, plus correlations, between the projects and random variable terms such as $\Sigma_i PIc_i$, PIa_1, $\Sigma_i NIc_i$, etc are needed.

Again, Bussey(56) presents a synthesis of some earlier work by Hillier(69) in dealing with the cross-correlation problem. The approach is based on the linear regression between various pair-wise combinations of random variable items. The Pearsonian sample correlation coefficient

r_{jk} (which is assumed constant over time) can be found between the random variable terms j and k as

$$r_{jk} = \frac{\sum_{t=1}^{T}(Ytj - \bar{\bar{Y}})(Ytk - \bar{\bar{Y}})}{\left[\sum_{t=1}^{T} (Ytj - \bar{\bar{Y}})^2 \sum_{t=1}^{T} (Ytk - \bar{\bar{Y}})^2\right]^{\frac{1}{2}}} \tag{4.5}$$

where Y_{tj} and Y_{tk} are the expected values of random variable items j and k, respectively, in time period t, T is the horizon period, and

$$\bar{\bar{Y}} = \tfrac{1}{2}\left[\frac{\sum_{t=1}^{T} Ytj}{T} + \frac{\sum_{t=1}^{T} Ytk}{T}\right] = (\bar{Y}tj + \bar{Y}tk)/2 \tag{4.6}$$

and all summations are performed over all t periods for which the random variables are jointly defined.

These calculations would be performed for all the random variable items in the example model to complete the calculation of the covariance terms in (3.24).

Computational Considerations

With the introduction of the variance term into the chance constraints (3.23), the C^2P/CB formulation becomes non-linear due to $z_j^{\,2}$'s and z_{jk} cross terms (3.24). Thus, from a computational standpoint, the model now falls into the category of a 0-1 integer, non-linear programming problem.

Formulations of this type can be quite difficult to solve in the efficient manner necessary for quick response to management's "what if" questions. There are computational techniques available, involving "branch and bound" or "implicit enumeration" [see Mao and Wallingford (104) for example]; however, the state-of-the-art in these techniques in general leave much to be desired for practical CB applications. Robertson (85) briefly discusses what appears to be an integer, non-linear algorithm he has used on industry C^2P problems. But the details of his work have not appeared in the literature, and so for now, it may be considered as unavailable, company-proprietary material.

Hillier(77) has suggested an approach to linearly approximate the non-linear chance constraints so that at least the more "advanced" state-of-the-art integer, <u>linear</u> programming techniques can be used for solution. The deterministic form of the chance constraint (3.23) can be rearranged as

$$K \beta_i \left[\mathrm{Var}\left(\sum_{j=1}^{n} a_{ij} z_j - B_i \right) \right]^{\frac{1}{2}} \leqslant E(B_i) - \sum_{j=1}^{n} E(a_{ij}) z_j \tag{4.7}$$

and this is unaltered by squaring both sides, provided that

$$E(B_i) - \sum_{j=1}^{n} E(a_{ij})\, z_j \geqslant 0 \tag{4.8}$$

Thus the squared relationship is

$$K \beta_i^{\,2}\ \mathrm{Var}\left(\sum_{j=1}^{n} a_{ij} z_j - B_i \right) \leqslant E(B_i)^2 - 2E(B_i) \sum_{j=1}^{n} E(a_{ij}) z_j + \left[\sum_{j=1}^{n} E(a_{ij}) z_j \right]^2 \tag{4.9}$$

The expansion of the last term on the right-hand side will yield

$$\sum_{j=1}^{n} E(a_{ij})^2 z_j^2 + \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k \neq j}}^{n} E(a_{ij})E(a_{ij})z_j z_k \tag{4.10}$$

The variance term can be expanded as in (3.24) and combined with (4.10). Also, since for our problem the decision variables are strict (0-1) integers, the z_j^2's can be replaced by z_j in these squared relationships.[11]

Now defining the following terms (and ignoring for the moment that the a_{ij} and B_i are composed of sums of several random terms).

$$\begin{aligned} s^2_{ij} &= \mathrm{Var}(a_{ij}) \\ s_{jk} &= \mathrm{Cov}(a_{ij}, a_{ik}) \\ s_{jB} &= \mathrm{Cov}(a_{ij}, B_i) \\ s^2_{Bi} &= \mathrm{Var}(B_i) \end{aligned}$$

The chance constraint can be combined into terms involving single decision variables and those that do not involve any. Simplifying and defining some more new terms

---

[11]Hillier also discusses how piecewise, linear approximations can be made to replace a fractional 0-1 decision variable, so that the nonlinear z^2's are still removed.

$$\sum_{j=1}^{n}\Big[\overbrace{K^2\beta_i s_{ij}^2 - E(a_{ij})^2 + 2E(B_i)E(a_{ij}) - K^2\beta_i s_{jB}}^{C_{ij}}\Big]\, z_j$$

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n}\Big[\overbrace{K^2\beta_i s_{jk} - E(a_{ij})E(a_{ik})}^{D_{ijk}}\Big]\, z_j z_k \leqslant \overbrace{E(B_i)^2 - K^2\beta_i s^2_{Bi}}^{R_i} \quad (4.11)$$

Thus the deterministic equivalent of the chance constraint (3.23) can be replaced by two constraints, (4.11) and (4.8).

These forms of the chance constraint only contain non-linearities in the cross terms of (4.11). That is, if the

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{ijk} z_j z_k$$

terms for the ith constraint can be linearly approximated, the problem becomes entirely linear. Hillier's approach is to develop a uniformly tighter linear constraint to approximate the non-linear one. In this way any solutions found for the uniformly tighter linear constraint would also be feasible for the non-linear constraint.

Taking the non-linear term and noting that it can be rearranged as (dropping the i subscript):

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk} z_j z_k = \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk}\Big[1-(1-z_k) - (1-z_j)z_k\Big]$$

Expanding this

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk} - \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk}(1-z_k) - \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk}(z_k)(1-z_j) \qquad (412)$$

Now, since $D_{jk} = D_{kj}$, then

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk}z_k = \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk}z_j$$

A very simple illustration will clarify this:

Assume $n = 2$, then the above term becomes

$D_{12}z_2 + D_{21}z_1 = D_{12}z_1 + D_{21}z_2$; and since $D_{12} = D_{21} = D$

$Dz_2 + Dz_1 = Dz_2 + Dz_1$ Q.E.D.

Thus the second term in (4.12) can be changed to z_j instead of z_k and combining terms results in

$$\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} D_{jk} + \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n} (D_{jk} + D_{jk}z_k)(1-z_j)$$

Finally, this <u>non-linear</u> form can be shown to be less than or equal (i.e., uniformly tighter) to the form

$$\sum_{j=1}^{n} \sum_{\substack{k=1 \\ k\neq j}}^{n} D_{jk} - \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k\neq j}}^{n} (D_{jk} + \min[D_{jk}, 0])(1-z_j)$$

thus removing the z_k term. This results in a linear formulation for the ith constraint in the form of two inequalities

$$\sum_{j=1}^{n} C_{ij} z_j + \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k\neq j}}^{n} D_{ijk} - \sum_{j=1}^{n} \sum_{\substack{k=1 \\ k\neq j}}^{n} (D_{ijk} + \min[D_{ijk}, 0])(1-z_j) - R_i \leqslant 0 \qquad (4.13)$$

and

$$E(B_i) - \sum_{j=1}^{n} E(a_{ij}) z_j \geqslant 0 \qquad (4.14)$$

This linearization procedure is repeated on each of the i chance constraints thus forming a completely linear model.

The main point of the above linearization is in the $\min[D_{jk}, 0]$ term. Obviously, all other terms between the non-linear and linear form are equivalent except for $\min[D_{jk}, 0]$, which replaces $D_{jk} z_k$ in the non-linear form. The linearized constraint term is equivalent to the non-linear term whenever z_k is in solution (=1) provided the D_{jk} associated with it is negative or zero. The linear and non-linear terms are also equivalent whenever z_k is not in solution (=0) provided $D_{jk} \geqslant 0$. For the case where z_k is in solution but D_{jk} is >0, or $z_k=0$, but $D_{jk} \leqslant 0$, the non-linear term is always D_{jk} less than the linear expression. Therefore, a solution satisfying the linear approximation constraint (4.13) must always also satisfy the non-linear form (4.11).

In his paper, Hillier develops several more linear approximations to the non-linear constraints; however, he states that for interesting feasible solutions involving most of the decision variables associated with uncertain coefficients, the above-stated linear approximation tends to be a relatively tight upper-bound for the non-linear constraint. Thus, in a CB problem where hopefully a large percentage of the projects will be selected, this linear constraint may provide a fairly good approximation. If this assumption of having feasible solutions involving most of the decision variables is strongly violated, Hillier has developed another linear approximation that provides a tighter upper-bound when the number of decision variables selected is relatively small compared to the total number available.

In addition to these uniformly tighter linear approximations, Hillier also develops a uniformly looser linear constraint approximation. This can be used to partially evaluate the adequacy of the linear approximations by re-solving the integer linear programming problem to get "nearly feasible" solutions (uniformly looser linear constraints) and comparing these with the "definitely feasible" solution (uniformly tighter linear approximations). This approach will give some idea as to the sensitivity of the solution to the approxiate linear constraints.

Obviously, to calculate the C_{ij}, $D_{ijk'}$, and R_i terms of (4.13) in an efficient manner for quick response to "what if" questions by management on the basic input data, a "matrix generator" of some sort must be tied into the solution technique. Robertson(85) mentions such an approach in his work.

Integer Linear Programming:

If Hillier's linear approximations are used, then the C^2P/CB model becomes an integer linear programming (ILP) problem. As previously mentioned, the state-of-the-art for these computational procedures is more advanced than for non-linear problems, but still ILP behaves erratically at times in convergence to a solution. Weingartner(52) mentions this as a definite problem using the cutting plane constraint method for ILP.

A good discussion of ILP algorithms and their computational performance is presented by Geoffrion and Marsten(105). Their main conclusion seems to be that ILP methods using LP to supplement implicit enumeration or branch and bound techniques, show promise. However, some of their computational times quoted are in the 20 to 30 minute range for realistic CB sizes (30 to 200 integer variables). Hamilton and Moses(62) also support this data, although for doing sensitivity studies after the initial integer solution, they quote times around 1 minute.

Times such as 20 to 30 minutes may seem quite acceptable to an algorithm designer, but for a manager working in a real time "what if" environment (say from a timesharing terminal) a 20 to 30 minute solution may be unacceptable--he is looking for something in the range of 5 minutes or so.

The ILP computational effectiveness on realistic-size CB problems appears to still be the subject of some controversy; however, Hillier(106) and several others [see Kochenberger(107) for example] have done some interesting work in "heuristic" ILP, which obtains a feasible and oftentimes close-to-optimal solution in a very small amount of computational time. In many cases management would be willing to accept definitely feasible, "close-to-optimal" solutions in exchange for rapid computational response; this might be especially true if a large amount of "what if"-ing is done on the model's assumptions.

Hillier's heuristic approach is based on first obtaining a LP solution to the problem and then using some appropriate decision rules for integerizing the variables to maintain feasibility and stay as close to optimality as possible.

There are three phases in his technique with Phase 1 being a normal, non-integer solution of the LP problem. The next step in this phase is to select a nearby solution vector well within the interior of the binding constraints of the LP solution. This second feasible solution point (composed of the same variables as the first point, except with different values) should have the property that it can definitely be rounded to integer values and not violate any of the previously binding LP constraints.

Two optional methods are proposed in Phase 1 for finding this second point; both seem to perform well and both entail tightening the original constraints by a sufficient amount so that post-optimal LP procedures and rounding to the nearest integer will yield an all-integer solution still satisfying the original LP constraints.

Phase 2 has three optional methods (with the 2nd and 3rd showing best results) which search along the line segment between the first (optimal LP) and second (all-integer) solutions for "better" (i.e., closer to the first) all-integer solutions. The search must terminate at an all-integer feasible solution since it can end no worse than at the second solution.

Phase 3 attempts to further improve on the Phase 2 solution by using two optional methods (with the first being best) to adjust the integer solutions and examine the improvement in the objective function. Multiple "heuristic" solutions can be quickly generated for further examination to see if a better solution exists.

Hillier's heuristic procedure seems to show quite close-to-optimal results for his test problems which he claims were "designed to be difficult ones". And for a problem with 60 constraints and 300 variables the time required for integer solutions was only 43 seconds (in addition to the LP solution). The drawbacks to his approach are that it does not allow equality constraints, general integer variables (not just 0-1) are allowed, and an approximate feasible, integer solution is not <u>guaranteed</u> (although no failures were detected in the test problems).

Equality constraints present no real problem in capital budgeting, a modification to the algorithm to restrict it to 0-1 variables Hillier says would perhaps enhance the technique, and Hillier claims the extension to mixed integer programming (MIP) should be "relatively straightforward". The aspect of MIP capabilities holds very interesting possibilities for a CB problem (such as fractional acceptance of financings) and provides something that other "heuristic" methods do not seem to address.

In any case, whether some of the commercially available "branch and bound" MIP codes are used, or some form of a heuristic ILP technique, the computational aspects of the linearized C^2P/CB model do not seem to be of major concern. Computational results in the theoretical literature and practical applications indicate even relatively large problems can be solved in a "reasonable" amount of time. Whether "reasonable" is 20 to 30 minutes, or in the area of 5 minutes, must be decided as the requirements of the model are formulated.

CHAPTER V

IMPLEMENTING THE CONCEPTS IN AN INDUSTRIAL ENVIRONMENT

An Evolutionary Process

The preceding chapters have examined some of the details of formulating, data gathering, and solving a "fairly realistic" C^2P/CB problem. Some preliminary conclusions to be drawn from these discussions are 1) the basic concept is viable and that such a model may indeed introduce a much-needed aspect of uncertainty into project portfolio selection, 2) the actual acceptance by a significant portion of industry is still probably anywhere from 5 to 10 years away.

It seems that all new "practical" techniques must evolve into use over a period of time. Chapter I pointed out the general trend towards more sophistication in CB including the widespread adoption of DCF techniques and the growing utilization of uncertainty analysis in project evaluations. However, these concepts have taken 10 to 20 years (or more) to become widely accepted by industry.

The next major step in CB sophistication will most likely be the use of mathematical programming and then the use of uncertainty in this framework--such as C^2P. Some steps in this direction by industry have already been cited; however, it appears that these are just part of a general evolutionary process. In a manner similar to the acceptance of DCF and uncertainty techniques by industry, older accepted methods are kept active and used to gradually phase-in newer procedures.

Current Capital Budgeting Practice and Computerized Profit Plans:

One of these "older accepted methods" which is seen as a current phase of the evolution to mathematical programming use in capital budgeting is that of "computerized profit plans". This is basically a projection of the on-going operations for a firm, broken down into detailed levels of the company. Each manager submits forecasts for what he expects his portion of the business to do in the coming year or so based on already existing operations. These individual forecasts are input to a computer where they are combined or "rolled up" into higher levels such as divisions, and integrated with more forecasts for the division as a whole. The process continues all the way up to a corporate summary of projects' on-going financial operations. As mentioned in Chapter IV on "Data Gathering", it is from this source that much of the C^2P model's on-going data can be obtained.

The rest of the non-computerized portions of the capital budgeting process involve the financial officer of the company in determining what previous borrowings are due within the capital budgeting period and defining other external sources of funds available. Along with this, the various divisions, etc submit their requirements for capital projects (as well as perhaps new marketing, advertising, and research budgets). These projects may have been through several "screenings" at lower levels of the division in which various qualitative and quantitative economic criteria have been used to eliminate or pass the projects. The projects emerging as candidates are sometimes just in conceptual, qualitative form while others are in more detail, with accompanying economics.

The entire collection of profit plans, financial borrowings, and capital projects are eventually gathered together by a special committee composed of high ranking officers of the company. They then try to match available funds with capital requests in such a way so as to select the "best" set of projects and borrowings. Oftentimes, several iterations are required in re-defining the profit plans and capital budgets before a "feasible" project/financing set is selected. The use of a "computerized profit plan" facilitates these tedious iterations and is at least a first step towards computer-oriented capital budgeting methods (including the construction of computer "data bases" which are used in more advanced techniques).

Capital Budgets Tied to Long-Range Plans:

An additional advancement in CB procedure sophistication is where the project and financing proposals chosen are made compatible with some longer-range planning (LRP) objectives. Twenty of the twenty-eight firms studied in Kempster's survey(17) used this approach. LRP's may be extended 3 to 5 years into the future and strategic decisions are made, such as the type of businesses to be pursued and financial growth rates desired (which in turn spur sub-objectives on production volumes, marketing programs, research and development, acquisitions and divestures, etc)--all of these items being considered in the light of future economic climate predictions.

The capital budget is the current implementation of the LRP; at least it is the selection of the best available alternatives to meet, as close as possible (or surpass), the LRP objectives. These objectives are the most likely source of the financial accounting constraints such as ROI, EPS, D/E, etc used in a MP/CB problem as discussed in Chapters II and III.

The significant trend here is that the LRP's being developed by a large number of companies are computer-based deterministic simulation models used to perform analyses on income statements, balance sheets, and source and use of funds. These models are usually constructed from a "top-down" approach, or in other words, from an overall, macroscopic corporate standpoint. Frequently, they are referred to as "what if" models since management has access to them in a real-time environment (such as on time-sharing or other quick-response computer access), and items within the model such as projections on sales, prices, inventories, property plan and equipment, etc can be easily changed to see the effect on financial indicators of the firm--and thus help develop a strategic LRP.

There is considerable evidence that this type of model is becoming increasingly accepted by industry. Gershefski's(58) Sun Oil model was one of the first in this area, but since then many other companies have developed or are beginning to develop such models. Schrieber's book(57) and Naylor's forthcoming book(108) give several examples of industrial applications, while a recent survey by Naylor(109) and Traenkle of the Financial Executives Institute(110) present a detailed examination of the current trends in this area among businesses. An article on this subject in Business Week(111) shows the kind of widespread publicity these types of models are getting. Also, the number of computer software vendor/consultants [see(112) as one example], banks [see (113) for example], and schools [see (114) for example] active in this field give some indication it is catching on fast.

Companies are now taking the computerized LRP approach and adopting them to short range planning (SRP)--or CB--models. The same type of "what if" analysis is performed as in LRP models but on a more itemized detail and a shorter time frame, say one or two years, broken down into quarters or months. The typical approach is to "what if" various entries in a source and application of funds (or balance sheet) to simulate project/financing selection. The important financial indicators of the firm are then observed to obtain a "good" CB portfolio.

Again, this evolution from computerized profit plans of on-going operations to computerized, "real-time" models of a firm's entire financial operations, both new and old, indicates another important step towards increasing use of the computer in CB.

Simultaneous Equation Financial Models and Mathematical Programming:

The computerization and use of SRP models as discussed above is a significant step forward in efficiency and systemization of the CB process; however, another even more efficient approach is the use of simultaneous equations in the model.

In the process of using "what if" financial models, management often tires of the trial-and-error process of changing items to affect a desired result and wants to know "what should these items be in order to achieve my goals?". The process is then one of simultaneously altering such total corporate projections as assets, sales, inventories, financings, interest, etc to achieve some given objectives. Warren and Shelton (W-S) have proposed such a simultaneous equation approach to LRP modesl(115) and McDonald(116) of Memorex has applied the concept. The idea would be equally applicable to SRP or CB financial models.

The use of simultaneous equations in a financial model is but one step away from the use of MP (which is solved as a set of simultaneous equations) and is part of the continuing evolution. However, where the simultaneous equation approach is essentially solving for "feasibility" (i.e., achievement of some stated goals), MP not only does this but also "optimizes" some particularly important aspect of the problem.

In addition, the W-S approach does not solve for any specific project or financing opportunities (only the aggregates for each category are solved). Even if W-S did this, their method does not include items such as "real" cash balances; economic criteria (DCF present value, DCF rate of return, payback period); the proportion of projects chosen by company division (exploration, production, refining, etc); the proportion of projects chosen by asset type (land, building, machinery, etc) or investment type (replacements, improvements, expansions, new business, etc); manpower constraints associated with projects; and mutually exclusive or contingent projects and financing. All of these and more can be efficiently handled in a MP framework.

The tie-in between financial models, simultaneous equation financial models and MP seems apparent. The previously cited works of Hamilton and Moses(63), in which a MP "Optimizer" to select projects is used in conjunction with a financial simulation, indicates that this evolutionary process has already occurred in some cases.

The Extension to Uncertainty:

Once the MP/CB concept is accepted by industry, the concept of C^2P to introduce uncertainty into the decision making should not be of great difficulty. C^2P simply expands on the general MP framework and requires more data. By the time MP techniques have evolved through financial models, etc, sufficient data bases should be available to simplify the data gathering tasks.

Computationally, the inclusion of uncertainty of a linearized C^2P model would not be significantly different from that of a standard LP model. However, by using a C^2P model there is a definite gain in decision making from considering most of the very real uncertainties of current[12] and "unknown" (future year) decisions. The C^2P model becomes a "cross" between a SRP model and a LRP model (which includes relatively unknown future decisions) to facilitate better current year CB.

The C^2P model, with its inclusion of uncertainty, offers advantages in other day-to-day "follow-on" areas of the CB process--such as in authorization for expenditures (AFE's) submitted for capital appropriations, and changes (updates) on previous data used in the CB project set selection (for example changes in estimates of on-going operations or available financings).

AFE's sometimes have essentially the same data items as submitted at CB time; however, before spending the capital these items are supposed to be updated to reflect any changes which might have occurred since CB approval. Most of the time, there are not any significant changes. If there are changes, but they fall within the "uncertainty envelope" already established and <u>accepted</u> for the project, then the appropriation is still approved without a further re-evaluation. A deterministic model might require re-evaluation to see if the deviation (change) is still acceptable in the CB.

Changes outside the uncertainty envelope must be evaluated with regards to the magnitude of the change and what impact this particular

---

[12]For example, the evaluation of a petroleum exploration budget is very difficult because the amount of money to be spent or gained in a period is unknown. "Deals", or specific projects, may be generated at any time and are usually not too predictable. However, using historical and trend data, the expected values and variances of these cash flows can be forecasted and included into a C^2P/CB model; thus facilitating the evaluation of the exploration budget.

project's changes might have on the satisfying of the "chance constraints". The C^2P model can be quickly re-run to see if the updates on the project should cause it to be dropped and some other(s) added or dropped (the dropping of projects already "underway" could be forced to be considered only as a "last choice" by the model). A similar approach to this AFE evaluation could be taken for evaluation of changes to on-going operations and available financings.

New, unexpected projects which are proposed between formal CB cycles can be evaluated by simply placing their data estimates (including uncertainty) into a formulation of the chance constraints, along with all the other accepted projects. If the constraints are still not violated and the new project's economics are favorable, the project is accepted--otherwise it is not (or, if possible, the constraints might be "adjusted" so the project would be accepted).

Quarterly or mid-year CB reviews would have the latest data updates including those from previously submitted changes for AFE's, on-going operations, and financings. The model would be re-run for optimal project evaluation; but the data input would probably not be nearly as large as the yearly CB evaluation since only major changes would be included--the other data would remain as is.

Some "Human Factor" Considerations

The "Evolutionary Process" discussed in the preceding section is already underway as evidenced by the increasing acceptance of financial simulation models and in some cases MP models in CB. However, the process can be a very slow one, or more rapid, depending upon how well the management scientist (MS) and management communicate in this area.

The MS and management communication problem has become a quite popular subject in some of the MS literature. One of our most practically-oriented publications, Interfaces, has had a number of very interesting thoughts presented within its pages. Several other prominent journals such as The Harvard Business Review have also contributed significantly to addressing the problem. The survey of MS progress by Radnor(117) has several interesting points concerning communication/implementation aspects, and a soon-to-be-released survey by PoKempner(118) will probably provide some additional, more current information in this area. All of these referenced sources present items that are particularly applicable to selling the MP/CB concept--and especially the C^2P/CB approach--to management.

Education of Management:

A recurring theme in the "communication" literature is that management must become aware of the MS tools available to help them solve their problems. There is no doubt that this is a true statement, but the approach to its resolution can either make or break a MS effort.

The computerized profit plans and corporate financial models (SRP's and LRP's) are relatively easy to relate to management. These models mostly deal with computerization of calculations previously done manually on financial statements. The subjects of simulataneous equations, MP, and uncertainty definitely need more explaining and perhaps formal management training in linear programming plus probability and statistics.

The conducting of in-house MS seminars to "train" management has been widely proclaimed as a viable approach [see Braunstein(119) for example] . This may work in some cases, in some companies, but it appears that "general" MS education for management by an in-house MS group may be less than successful in most instances.

One possible way around this problem is to send some key management personnel to advanced management seminars conducted by various business schools and consulting firms, or to bring the outside firms into the company. Either way, there seems to be a more credible atmosphere created by the outside MS people in managment communications--particularly when MS efforts are being initiated in such a sensitive and important area as capital budgeting. Once initial communication has been favorably extablished, the in-house MS group may find themselves sharing part of the work with the consultants (which is probably a good idea anyway for first efforts), but MS/management communications will also have probably gained some internal credibility in the company.

Education of the Management Scientist:

It has also been stated in the literature that the lack of understanding by MS of the real-world problems faced by management may be of even more concern than the education of management to MS tools. Schyon(120) has pointed out that the most effective way of selling a MS concept to management is to concentrate on areas of "real need" in the company. Lewis Robertson(85) of Atlantic Richfield has supported this position in personal conversations with the author of this thesis in regards to how Robertson was able to sell his management on the C^2P concept in CB. The MS must find the areas of concern in the CB process; he must ferret-out and understand the details and intracacies of the current CB practices of his own company before he can even attempt to sell any MS improvements in this area.

One of the main problems is in the MS finding the real issues of the CB problem when the MS and manager operate in such a different environment and with different perspectives of a problem. Hammond(121) outlines many of the factors affecting MS implementation success; and one of his prevailing themes is the ill-defined nature of many of the manager's problems. The manager does not think in a structured form of his problems, and thus the MS has difficulty in relating to such an amorphous subject.

This aspect is paricularly true of the capital budgeting problems management faces. There are many factors of the decision making process which are very complex and interrelated; the manager handles these, but mostly "in his head". For the manager to communicate these "mental models" is difficult; for the MS to understand them is doubly difficult. Resorting to a detailed review of the surveys on capital budgeting practices which are published in the literature can definitely help the MS in obtaining some general perspectives on the manager's world--and thus facilitate communication.

There have been suggestions by some [see Grayson(122) for example] that MS get out into the management "firing line" to learn first-hand some of management's problems that are seldom communicated in textbooks. Woolsey(123), who is well-known for his candid and quite valid criticisms of the MS "realism" perspective, agrees in general with the "firing line" training. However, he points out that this can be an expensive proposition for a company if the MS falls on his face in performing the current duties of a manager's job that he is not suited for or trained to do--this can be a particular problem in dealing with very high-level decision making such as in CB.

If the MS were suited or trained for such a job he would no longer be a MS--he would be a manager, and a high-level one at that. A suitable compromise to this problem might be to let the MS be peripherally involved in the CB process and an observer of all aspects of the job, but not be directly on the "firing line".

The Bottom Line--Benefits:

Assume that somehow the MS and management have been able to communicate adequately with each other in regards to MS tools and management's "real" problems in CB. That is, the problem the MS has modeled is indeed a real one which management recognizes; management will not necessarily have to or want to understand all of the details of the MS tools behind the model, although they may have been made aware of some. The problem aspects would be emphasized to management, with the computational aspects de-emphasized. Assuming this then, the basic question to be answered is: Does the MS-proposed advanced CB technique deliver the benefits inherent in the recognized problem?

To try to convince management to use the more advanced CB concepts by presenting statistical studies on the benefits that other companies have derived from their utilization would be counter-productive. Indeed, studies by Christy(16), Fulmer(124), and Traenkle(110) show little correlation of improved performance of a company by adopting more sophisticated financial planning techniques.

The obvious difficulty in deriving statistical evidence of the "sophisticated financial planning benefits" without a significant number of samples over time in a controlled experiment (which has differences in management and varying economic conditions removed) preclude the conclusion of either positive, negative, or no performance benefits. Both Fulmer(124) and Istvan(15) allude to this. However, management generally knows most of their problems and seems to have a "gut feel" for whether or not a particular MS technique is beneficial to them on their problem. This appears to be the way most MS concepts are adopted--not by running controlled experiments and benchmark tests, but by management's gut feel.

Of course, this is not to say that some form of benefit is not necessary. In fact, without a benefit factor, it is extremely unlikely a more advanced CB concept will be adopted. However, there are some points to be made for benefits; the strongest cases in this area are the efficiency and manpower savings. Also, one can allude to and show by example the possible benefits of improvements in performance by being able to examine more complex alternatives that would be difficult if not impossible to do by current methods.

The Big Problem--Politics:

The MS may have overcome the MS and management education problems, built a model addressing a real management CB problem, and identified some significant benefits to adopting the model; however, implementation may never come about. Or the MS may be thwarted anywhere back along the lines in education or selling a benefit. Both of these "dead ends" can be caused by a very common phenomena in MS implementation called "politics"--or human self-interest in conflict with corporate objectives.

Naylor(125) says that politics is the most severe problem of all in the implementation of corporate financial models. Weingartner(126) also alludes to its importance in the use of MS in financial areas. The educational aspects of communicating MS ideas to managers can be completely voided if the manager has a "political perspective" on the MS effort. Similarly, the MS cannot very well obtain educational information for himself on management's real problems if management is involved in political games with the MS or other managers.

The basis for much of the politics problem is the manager's fear of MS. Baker(127) brings out several good points concerning this. Every time a new MS technique is proposed, the manager runs a risk that the very foundation on which he has built his career will be eroded. HIs ways are outdated and inadequate; knowledge unfamiliar to him now points to a better way. He may not feel capable in the area and think he is too old to learn new tricks, thus revealing a fatal weakness to the MS staff or his associates. He may lose control and credit for decision making procedures that have been in existence for many years and under his supervision.

The MS are guilty in their own way of perpetuating this by masking their own unsureness with technical jargon and implying to uncooperative managers that "disaster awaits if the manager does not keep up with the times and adopt more MS techniques". Some points in an article in Personal Report (128), a bi-weekly publication for general management, shows the defensive nature of management in dealing with a specialist who is trying to gain "one upmanship" on him through technical jargon.

It seems that the essence of this political problem is not completely resolvable; there will always be the "Old Guard" and the "Young Bucks" battering away at one another. However, the MS should recognize that in some cases he is battering away at a stone wall, and some compromises would be in order.

One "compromise" in this area is: Make it common knowledge and emphasize to all that will listen, that MS only deals with the quantifiable aspects of the decision making process. Management still must fit all the pieces together, both quantifiable and unquantifiable; and the unquantifiable aspects of a problem are often very important--frequently even more important than the quantifiable aspects.

Lewis Robertson(85) of Atlantic Richfield has made a good point in private conversations with the author on some of these problems. Robertson points out that a MS should never take the output of a model to management as "The Results--The Optimal". Rather he should present parametric studies and curves of results (perhaps of optimal solutions with different assumptions) so that management can still pick and choose alternatives; management still makes the final selections.

Another "compromise" is: Get rid of the jargon and learn to speak in management's terms. Both of these topics (management importance and technical jargon) are old themes but bear repeating because apparently it is still not sinking-in to all MS people.

Another aspect of the political problem, especially in corporate financial models (such as CB), is the conflict of interest among various managers of the company over who will and will not have control of the model. A corporate financial model can be a very big asset to a particular manager if it is under his control. For instance, if the Treasurer, Controller, Planning Officer and V.P. of Finance are all vying for control, each may sabotage the other (and thus the chances of a successful model) if control appears to be slipping from their grasp. Only the very highest management such as the President or Chief Executive Officer can resolve political problems of this nature; if they cannot be resolved, the effort should be postponed.

Organizing for Action

The type of organization adopted for an effort to implement a corporate financial model can alleviate some of the political problems discussed above plus enhance the possibilities for a success even in an "apolitical" environment. Naylor(125) and Hammond(121) have several interesting points on this subject. First of all, the roles of the various participants must be *precisely* defined. Managers make decisions, planners make recommendations, and MS make models. The entire effort should be an interactive, team approach with all parties equally sharing in the credit.

Naylor points out that a common political problem whereby a manager does not want to commit to a corporate financial model because he is afraid of being blamed for its failure, can be resolved. The manager is actually in a "fail safe" position: If the model succeeds, he shares in the credit; if it fails, the MS can be blamed for poor modeling. This is an inequitable arrangement for the MS but one he might be willing to accept in order to have management accept a new MS technique.

Also, the building of the model done by the MS should consider *all* of the objectives of the decision makers it is to serve. Oftentimes, there are many unstated "political" purposes behind the designs of a manager; it would behoove the MS to consider these in his formulation.

The MS should also make sure that the model's purposes are not overstated; it should not be represented to management as a panacea. In this same vein, all of the references in the area such as Hammond(121), Naylor(124), and Traenkle, *et al*(110) emphasize that one should start with a simple,relatively easy objective in modeling and build upon its success. As expressed by one manager familiar to the author of this thesis,the expression KISS (Keep It Simple, Stupid) would be appropriate, at least on first attempts.

These "simplistic" recommendations would seem to follow quite well the evolutionary process discussed in the preceding section in which simple financial/CB models are developed into more useful, sophisticated models as management is ready for them.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER RESEARCH

Summary and Conclusions

This thesis gives some indications that capital budgeting(CB) procedures within industry are becoming increasingly more complex and sophisticated. The utilization of discounted cash flow (DCF) techniques in project evaluation is becoming quite common, and its acceptance seems to have been aided by the selective inclusion of uncertainty into the analysis.

Firms are beginning to require more detailed project evaluation data (such as cash flows and DCF economics) before CB selection instead of waiting until afterwards at "authorization for expenditure" time. This "early evaluation" data has in turn aided in the limited but growing use within industry of mathematical programming (MP) to help select project portfolios with the "best profit"--and that also simultaneously satisfy many other constraints or "sub-goals" of management such as financial ratios, etc.

However, the growth rate in using the mathematical programming/ capital budgeting (MP/CB) approach has been rather slow since its introduction over 12 years ago. The hypothesis is proposed in this thesis that perhaps the inclusion of uncertainty data into the MP/CB problem would enhance its industry utilization by making the data better fit the sophistication of the MP tool. Since the introduction of uncertainty analysis appears to have aided the industry acceptance of DCF methods, perhaps the MP/CB technique might also benefit from this approach.

Chance constrained programming (C^2P), which is a special form of MP, appears to be a good candidate for introducing uncertainty into the problem since 1) it is similar in mathematical structure to earlier deterministic MP/CB models proposed in the literature, 2) the basic uncertainty theory for the model is well-developed, and 3) the form of the uncertainty being expressed in the constraints as desired "confidence levels" to be achieved[13] is amenable to an intuitive explanation to management without a lot of detailed mathematical background. There is even at least one industry application of C^2P in CB practice [see Robertson(85) of Atlantic Richfield Corp.].

However, there are a few relatively minor theoretical deficiencies in the C^2P approach. Mainly, it assumes that the expected value and variance (which enter into the chance constraints) of any given project are not altered by the selection or rejection of any other projects. Also, the probability function used in the mathematical description of the chance constraints is nearly always assumed to be a Normal distribution.

This thesis also explores some of the aspects of model formulation, data gathering, and solving a "fairly realistic" chance constrained programming/capital budgeting (C^2P/CB) model in order to get a feel

---

[13] Thus the term "chance constraints".

for its use within a practical industry environment. The formulation includes both selection of project and financing proposals, since management considers the interdependency of these two factors in determining a capital budget. The project data is considered uncertain and the financing data known.

Due to the pervasive use within industry of DCF techniques, DCF present value is considered as the most appropriate single criterion to optimize for the project set selected; thus, this is the "objective function" of the problem. Other "subgoals" or constraints on the projects and financings chosen are on after-tax cash balances per quarter and yearly, after-tax financial accounting ratios for two years (quarterly ratios could be considered if required). Both cash balances and financial ratios are items of definite concern to management in considering the capital budget selection.

The uncertainties of on-going operations and future, unknown CB selections are included in the model. However, the formulation does not include some realistic factors such as equity and bond financing (which could also be random variables), nor does it consider how financing interest rates would change as financing proposals are chosen. Also, uncertainty could be included in the objective function in a manner similar to the chance constraints. These areas need more work and then the formulation would probably be practical for industrial use. Several references are cited that address, at least in part, the resolution of some of these problems.

Even though "complete realism" is not covered in this model, the formulation illustrates how management may realistically incorporate a significant amount of uncertainty into the CB selection process—and thus improve the quality of their decision making. However, the data requirements of such a model are not small. Just "single-point" estimates of the data (i.e., no uncertainty) for on-going operations and current year selections--especially considering taxes--are formidable; but these should be reasonably available if a company has at least a "state-of-the-art- CB procedure.

The additional data requirements for uncertainty on these individual items add significantly to the load. Bussey(56) presents an appealing approach for gathering the uncertainty data just from pessimistic, most likely, and optimistic estimates on each uncertain item in the model. If, as expected, not <u>every</u> uncertain item is so uncertain that it requires these estimates, then the approach may indeed be tenable. That is, uncertain items are classified according to "risk category" and capital exposure; if this combination is above a management specified level, then uncertainty data is required--otherwise it is not.

The C^2P/CB model is generally a 0-1 integer, non-linear programming problem for which the state-of-the-art in computational techniques is somewhat lacking for problems of the size encountered within industry.[14]

---

[14] Robertson(85) reports some favorable results from what appears to be a 0-1, non-linear programming algorithm that he has adapted to use in his industry application of the C^2P/CB approach. However, the details of the algorithm have not been published in the literature and thus may be of a company-proprietary nature.

Hillier(77) has proposed a linearization of the model which gives uniformly tighter linear approximations to the non-linear "chance constraints". His linear constraints will always give a feasible solution (if one exists) to the original problem, and he proposes several types of linear approximations to be used under different situations to improve their accuracy.

Even with the problem now being an integer linear programming (ILP) problem, computational experience has shown that ILP algorithms do not behave predictably as far as convergence, optimal solution, etc is concerned. Again, Hillier(106) recommends a heuristic ILP technique which uses the speed and familiarity of linear programming to initially solve the model, and then uses a constrained rounding technique such that the variables will be integerized but not violate feasibility. His technique is one of several [see Kochenberger(107) for example] that show feasible, reasonably-close-to-optimal solutions with a considerable savings in computation time.

The linear approximations to the non-linear chance constraints and the heuristic ILP both seem to be acceptable compromises for adding the significant dimension of uncertainty to the CB decision making process. In fact, these compromises seem virtually required in order to present management a tool which includes uncertainty and can be used for the quick response needed in their real-time environment. Using an approach such as this, along with a computerized "matrix generator" that would allow rapid computations of the C^2P input data, management could quickly "what if" many aspects of the C^2P/CB model to examine trade-offs.

The general conclusion to be drawn from the above information is that the C^2P/CB approach is indeed feasible and probably within the practical grasp of a number of large industrial corporations. However, the majority of companies are still several years away from accepting this concept. C^2P and its inclusion of uncertainty may prompt some firms in adopting MP methods to CB, but in the short run, not very many. The main reasons for the current lack of use of the MP/CB approach seems mostly to stem from management and industry inertia--not a lack of appropriate management science techniques or data availability. Still, the utilization of C^2P/CB models, and in general MP/CB models, definitely appears to be in the future. As mentioned, the use of MP/CB models is rising somewhat and there is at least one industry user of a C^2P/CB model. As industry becomes more aware of the MP/CB approach and its general capabilities, then the inclusion of uncertainty *via* C^2P may indeed promote further usage.

The adoption by industry of these approaches is seen as some of the latter stages of an evolution in CB processes to more sophisticated techniques. The acceptance of DCF and uncertainty methods for project evaluation is one indicator of this trend. However, the computerization of corporate profit plans used in CB and growing acceptance of corporate simulation models as tools to guide budget decision making are even more indicative of this evolution. Simultaneous equation corporate financial models being promoted and used [see McDonald(~~73~~ 116) of Memorex Corp.] are but a step away from the simultaneous equation approach used in MP/CB models. The extension to uncertainty through C^2P has been shown to be a quite feasible process.

The latter stages of this evolution, such as the MP/CB and C^2P/CB approaches, appear to be anywhere from 5 to 10 years away—depending on many factors such as national and world economic climate. Poor economic conditions seem to promote the acceptance of the more sophisticated CB methods. However, another significant factor in the timing of this evolution is the education of management and management scientists to the ways of each other. Also, how the political factors involved are addressed—such as management and their fear that management science methods are taking over their jobs, or the power struggles between managers wanting to have control of the tools generated by management scientists—will have a great impact on the timing of the adoption by industry of the more sophisticated CB techniques.

All-in-all, the next 10 years should be of considerable interest and challenge to the management scientist involved in CB and corporate financial planning in general.

Recommendations for Further Research

The scope of this thesis has been rather broad in order to evaluate the practical aspects of most of the major areas involving C^2P/CB models. While this "broad approach" has provided a good over-view of the future potential for such models, there are several levels of detail which, by necessity, have been either ignored or just briefly mentioned.

It is felt that this thesis has determined that C^2P/CB models are now within the practical grasp of several large corporations and perhaps 5 to 10 years away from general industry acceptance. However, to realize this more general acceptance, theorists and researchers must begin to address some of the detailed items which have not been included in this work.

Foremost on this list of detailed items is the aspect of model formulation and the inclusion of equity financing issues plus effects of imperfect capital markets on borrowings. As mentioned in Chapters II, III, and IV, several authors [(52), (62), (88) and (89)] have been working in this area, but so far completely satisfactory approaches have not been achieved.

Most of the other detailed items to be investigaed concern computational aspects. The various forms of linearization of the non-linear constraints presented by Hillier(77) should be more closely examined for accuracy under different situations. Hillier discusses some of this in his work but more research could be done on the subject. In addition to Hillier's linear approximations, Whitmore and Darkazanli(129) plus Seppala(130) have also done work in linearization of non-linearities in MP models. Their approaches might be compared to Hillier's.

The heuristic, integer linear programming (ILP) algorithm proposed by Hillier(106) should be examined with regards to restricting it to 0-1 integers alone-and then possibly extending the procedure to include mixed 0-1 integers. Hillier claims both of these features appear to be attainable with a modest effort. There are several other fast, flexible heuristic ILP procedures by other authors [(107), (131), (132), and (133)] that could be addressed in this same manner and compared to Hillier's. The mixed 0-1 integer computational procedure would allow more realistic modeling by allowing short-term financing and fractional borrowings for longer term agreements. This is a very important feature for practical industry models.

Probabilistic objective functions like those of Markowitz(66), Kataoka(91), and Hillier(69) could be included along with the chance constraints. Linear approximations like those proposed by Hillier for non-linear utility functions might provide efficient solution procedures plus more meaningful information with regards to the risk nature of the primary economic indicator (PV, EPS, etc) for the portfolios being selected. Hillier's linear approximations in this area have been explored (72), but it might be interesting to see how the introduction of some form of uncertainty into the objective changes the optimal solution from that of considering only uncertainty in the constraints (the chance constraints).

Finally, some form of "goal programming" (134) formulation could be considered for the C^2P/CB model. Goal programming helps to avoid the prevalent problem of obtaining infeasibilities on constraints which are too tightly bound and can only be made feasible by some tedious trial and error processes. Using this approach, management can weight the relative importance of different objectives or goals (sometimes expressed as constraints) and obtain a solution which maximizes this weighted average criteria function while coming as close to "feasibility" as possible.

REFERENCES

1. Swalm, Ralph O., "Capital Expenditure Analysis--A Bibliography", Engineering Economist, Winter 1968, pp. 105-129.

2. Fisher, Irving, The Theory of Interest, 1930; reprinted 1965.

3. Dean, Joel, Capital Budgeting, 1951.

4. Lorie, James, H. and Savage, Leonard J., "Three Problems in Capital Rationing", Journal of Business, October 1955, pp. 13-37.

5. Solomon, Ezra (ed.), The Management of Corporate Capital, 1959.

6. Bierman, Harold J. and Smidt, Seymour, The Capital Budgeting Decision, 1960 (2nd edition, 1966, 3rd edition 1971).

7. Grant, Eugene L. and Ireson, William G., Principles of Engineering Economy, 1960.

8. Solomon, Ezra, "Measuring a Company's Cost of Capital", Journal of Business, October 1955, pp. 240-252.

9. Solomon, Ezra, "The Arithmetic of Capital Budgeting Decisions", Journal of Business, April 1956, pp. 124-129.

10. Bierman, Harold and Smidt, Seymour, "Capital Budgeting and the Problem of Reinvesting Cash Proceeds", Journal of Business, October 1957, pp. 276-279.

11. Modigliani, Franco and Miller, Merton H., "The Cost of Capital, Corporation Finance and the Theory of Investment", American Economic Review, June 1958, pp. 261-297.

12. Hirshleifer, Jack, "On the Theory of Optimal Investment Decisions", Journal of Political Economy, August 1958, pp. 329-352.

13. McLean, John G., "How to Evaluate New Capital Investments", Harvard Business Review, Nov-Dec 1958, pp. 59-69.

14. Baldwin, Robert H., "How to Assess Investment Proposals", Harvard Business Review, May-June 1959, p. 99.

15. Istvan, Donald F., Capital Expenditure Decisions: How They are Made in Large Corporations, Bureau of Business Research, Indiana University, 1961.

16. Christy, George A., Capital Budgeting Practices and Their Efficiency, Bureau of Business Research, University of Oregon, 1966.

17. Kempster, John H., Financial Analysis to Guide Capital Expenditure Decisions, National Association of Accountants, Research Report No. 43, July 1967.

18. Pflomm, Norman E., Managing Capital Expenditures, Business Policy Study No. 107, The Conference Board, 1963.

19. Williams, Ronald B., Jr., "Industry Practice in Allocating Capital Resources", Managerial Planning, May-June 1970, pp. 15-22.

20. Klammer, Thomas, "Utilization of Sophisticated Capital Budgeting Techniques in Industry", Journal of Business, July 1972.

21. Neuhauser, John J. and Viscione, Jerry A., "How Managers Feel About Advanced Capital Budgeting Methods", Management Review, November 1973, pp. 16-22.

22. Petty, William F., Scott, David F., Jr. and Bird, Monroe, "The Capital Expenditure Decision-Making Process of Large Corporations", The Engineering Economist, Spring 1975, pp. 159-172.

23. Fremgen, J.M., "Capital Budgeting Practices, A Survey", Management Accounting, May 1973, pp. 19-25.

24. Carter, Charles F., Uncertainty and Business Decisions, 1957.

25. Schlaifer, Robert, Probability and Statistics for Business Decisions, 1959.

26. Grayson, C. Jackson, Decisions Under Uncertainty, 1960.

27. Farrar, Donald E., The Investment Decision Under Uncertainty, 1962.

28. Hirshleifer, Jack, "Risk, the Discount Rate and Investment Decisions", American Economic Review, May 1961, pp. 112-120.

29. Lewellen, Wilbur C. and Michael S. Long, "Simulation Versus Single-Value Estimates In Capital Expenditure Analysis", Decision Sciences, October 1972, pp. 19-33.

30. Bower, Richard S. and Lessard, Donald R., "The Problem of the Right Rate: A Comment on Simulation versus Single-Value Estimates in Capital Expenditure Analysis", Decision Sciences, October 1973, pp. 569-571.

31. Gentry, James A., "Simulation Revisited", Decision Sciences, October 1973, pp. 572-574.

32. Lewellen, Wilbur G. and Long, Michael S., "Reply to Gentry and Bower", Decision Sciences, October 1973, pp. 575-576.

33. Hillier, F.S., "The Derivation of Probabilistic Information for the Evaluation of Risky Investments", Management Science, April 1963, pp. 443-457.

34. Hillier, F.S., "Supplement to 'The Derivation of Probabilistic Information for the Evaluation of Risky Investments'", Management Science, January 1965, pp. 485-487.

35. Hertz, David B., "Risk Analysis in Capital Investment", Harvard Business Review, Jan-Feb 1964, pp. 95-106.

36. Bernoulli, Daniel, "Exposition of a New Theory on Risk", 1738 English translation by Louise Sommer, Econometrica, 1954.

37. Von Neumann, John and Morgenstern, Oscar, Theory of Games and Economic Behavior, 1947 (3rd edition, 1953).

38. Friedman, Milton and Savage, Lorie J., "The Utility Analysis of Choices Involving Risk", Journal of Political Economy, August 1948, pp. 279-304.

39. Borch, Karl, The Economics of Uncertainty, 1968.

40. Swalm, Ralph O., "Utility Theory--Insights into Risk Taking", Harvard Business Review, Nov-Dec 1966, pp. 123-136.

41. Smith, Marvin B., "Parametric Utility Functions for Decisions Under Uncertainty", paper presented to the Society of Petroleum Engineers, 1972.

42. Davey, Patrick J., Capital Investments: Appraisals and Limits, Conference Board Report No. 641, The Conference Board, 1974.

43. Mao, James C.T., "A Survey of Capital Budgeting: Theory and Practice", Journal of Finance, May 1970.

44. Proprietary document for major petroleum company "Procedure for Appraisal of New Capital Investment", 1966.

45. Proprietary document for major petroleum company "Project Evaluation Methods", 1963.

46. Proprietary document for major petroleum company "Measuring Project Profitability", 1970.

47. Proprietary document for major petroleum company, "Capital Budgeting Procedures", 1970.

48. Proprietary document for medium-sized petroleum company "Project Evaluation Procedures" , 1975.

49. Smith, Marvin B., "Probability Models for Petroleum Investment Decisions", Journal of Petroleum Technology, May 1970, p. 543.

50. Larson, Roy B., "Capital Budgeting/Program Selection Using Mathematical Programming--An Annotated Bibliography", Case, Western University Technical Memorandum #73, April 1971.

51. Charnes, A., Cooper, W.W. and Miller M.H., "Application of Linear Programming to Financial Budgeting and the Costing of Funds", Journal of Business, 1959.

52. Weingartner, H.M., Mathematical Programming and the Analysis of Capital Budgeting Problems, 1963.

53. Baumol, W.J. and Quandt R.E., "Mathematical Programming and the Discount Rate Under Capital Rationing", Economic Journal, June 1965, pp. 317-329.

54. Lusztig, Peter and Schwab, Bernhard, "A Note on the Application of Linear Programming to Capital Budgeting", Journal of Financial and Quantitative Analysis, December 1968, pp. 426-431.

55. Bernhard, Richard H., "Some Problems in the Use of a Discount Rate for Constrained Capital Budgeting", AIEE Transactions, September 1972, pp. 180-184.

56. Bussey, Lynn E. and Stevens, G.T., Jr., "Formulating Correlated Cash Flow Streams", The Engineering Economist, Fall 1972, pp. 1-30.

57. Schrieber, Albert N., Corporate Simulation Models, 1970.

58. Gershefski, George W., The Development and Application of a Corporate Financial Model, Planning Executives Institute Monograph, 1968.

59. Cohen, K.J. and Hammer, F.S., "Linear Programming and Optimal Bank Asset Management Decisions", The Journal of Finance, May 1967, pp. 147-165.

60. Woolsey, R.E.D., "A Candle to St. Jude, or Four Real-World Applications of Integer Programming", Interfaces. February 1972, pp. 20-27.

61. Woolsey, R.E.D., "A Novena to St. Jude, or Four Edifying Case Studies in Mathematical Programming", Interfaces, November 1973, pp. 32-39.

62. Hamilton, William F. and Moses, Michael A., "An Optimization Model for Corporate Financial Planning:, Operations Research, May-June 1973, pp. 677-692.

63. Hamilton, William F. and Moses, Michael A., "A Computer-Based Corporate Planning System", Management Science, October 1974, pp. 148-159.

64. Vandel, Robert F. and Stonich, Paul J., "Capital Budgeting: Theory or Results?", Financial Executive, August 1973, pp. 46-56.

65. Markowitz, Harry, "Portfolio Selection", Journal of Finance, March 1952, pp. 77-91.

66. Markowitz, Harry, Portfolio Selection: Efficient Diversification of Investments, 1959.

67. Cord, Joel, "A Method for Allocating Funds to Investment Projects When Returns are Subject to Uncertainty", Management Science, January 1964, pp. 335-341.

68. Salazar, Rudolfo C. and Sen, Subrata K., "A Simulation Model of Capital Budgeting Under Uncertainty", Management Science, December 1968, pp. 161-179.

69. Hillier F.S., Budgeting Interrelated Activities--Vol. 1: The Evaluation of Risky Interrelated Investments, 1969.

70. Hillier, F.S., "A Bound-and-Scan Algorithm for Pure Integer Linear Programming with General Variables", Operations Research, July-Aug 1969, pp. 638-679.

71. Reiter, Stanley, "Choosing an Investment Program Among Interdependent Projects", Review of Economic Studies, January 1963, pp. 32-36.

72. Byrne, R.F., Charnes, A., Cooper, W.W., Davis, O.A. and Gilford Dorothy (eds.), Budgeting Interrelated Activities Vol. 2: Studies in Budgeting, 1971.

73. Wolf, F.K., "Review of Hillier's 'The Evaluation of Risky Interrelated Investments'", Engineering Economist, Spring 1971, pp. 211-218.

74. Naslund, Bertil, "A Model of Capital Budgeting Under Risk", Journal of Business, 1966, pp. 257-271.

75. Charnes, A., Cooper, W.W., "Chance Constrained Programming", Management Science, October 1959, pp. 73-79.

76. Charnes, A. and Cooper, W.W., "Deterministic Equivalents for Optimizing and Satisficing Under Chance Constraints", Operations Research, Jan-Feb 1963, pp. 18-39.

77. Hillier, F.S., "Chance Constrained Programming with 0-1 or Bounded Continuous Decision Variables", Management Science, September 1967, pp. 34-57.

78. Naslund, Bertil, Decisions Under Risk; Economic Applications of Chance Constrained Programming, 1967.

79. Byrne, R., Charnes, A., Cooper, W.W. and Kortanek, K. O., "A Chance Constrained Programming Approach to Capital Budgeting with Portfolio Type Payback, Liquidity Constraints, and Horizon Posture Control", Journal of Financial and Quantitative Analysis, December 1967, pp. 339-364.

80. Byrne, R.F., Charnes, A., Cooper, W.W., Davis, O.A. and Gilford, Dorothy (eds.), "C^2 and LPU^2 Combinations for Treating Different Risks and Uncertainties in Capital Budgets", Budgeting Interrelated Activities Vol. 2: Studies in Budgeting, 1971.

81. Dantzig, George B., "Linear Programming Under Uncertainty", Management Science, April 1955, pp. 197-206.

82. Wood, Donald R., Blackburn, C. Ralph and Robertson, L., "Real-Time Capital Allocation", paper presented to the Institute of Management Science, October 1970.

83. Robertson,L., "An Application of Mathematical Techniques to Program Optimization", paper presented to Corporate Planning Council, September 1971, Dallas, Texas.

84. Robertson, L., Wood, D.R. and Blackburn, C.R., "Capital Budgeting and Sequencing of Projects with a Probabilistic Objective Function and Constraints on Risk", paper presented to Operations Research Society of America, October 1971.

85. Robertson, L., Wood, D.R. and Blackburn, C.R., "A Mathematical Programming Approach to Capital Allocation with Risk", National Petroleum Refiners Association Computer Conference, Houston, Texas, November 1971.

86. Robertson, L., Formulation and Analysis of Optimization Models for Planning Capital Budgets, PhD Dissertation, U. of Oklahoma, 1967.

87. Bernhard, Richard H., "Mathematical Programming Models for Capital Budgeting--A Survey, Generalization and Critique", Journal of Financial and Quantitative Analysis, June 1969, p. 111.

88. Chambers, David, "The Joint Problem of Investment and Financing", Operations Research Quarterly, 1971, pp. 267-295.

89. Peterson, D.E., A Quantitative Framework for Financial Management, 1969.

90. Weingartner, H.M., "Criteria for Programming Investment Project Selection", The Journal of Industrial Economics, November 1966.

91. Kataoka, Shinji, "A Stochastic Programming Model", Econometrica, 1963, pp. 181-196.

92. Freeman, P. and Gear, A.E., "A Probabilistic Objective Function for R&D Portfolio Selection", Operations Research Quarterly, 1962, pp. 253-265.

93. Bell, Kenneth S., "A Survey of Planning Practices in the Petroleum Industry", Managerial Planning, Sept-Oct 1974, p. 29.

94. Van Horne, Financial Management and Policy, 1971.

95. Tilles, Seymour, "Strategies for Allocating Funds", Harvard Business Review, Jan-Feb 1966, p. 72.

96. Chambers, David, "Programming the Allocation of Funds Subject to Restrictions on Reported Results", Operations Research Qtrly, 1967, p. 407.

97. Lerner, Eugene M. and Rappaport, Alfred, "Limit DCF in Capital Budgeting", Harvard Business Review, Sept-Oct 1968, pp. 133-139.

98. Otto, Gordon H., "The Use of Linear Programming in Corporate Budget Development", paper presented to the Southwestern Social Sciences Association, April 1969.

99. Dyckman, T.R. and Kinard, J.C., "The Discounted Cash Flow Criterion Investment Decision Model with Accounting Income Constraints", Decision Sciences, July 1973, pp. 301-313.

100. Lockett, Geoffrey and Gear, Anthony, "Programme Selection in Research and Development", Management Science, June 1972, pp. B575-B590.

101. Quirin, G. David, The Capital Expenditure Decision, 1967.

102. Wagle, B., "A Statistical Analysis of Risk in Capital Investment Projects", Operations Research Quarterly, March 1967, pp. 13-34.

103. Box, George E.P. and Jenkins, Gwilym M., Time Series Analysis, Forecasting and Control, 1970.

104. Mao, James C.T. and Wallingford, B.A., "An Extension of Lawler and Bell's Method of Discrete Optimization with Examples from Capital Budgeting", Management Science, October 1968, pp. 51-60.

105. Geoffrion, A.M. and Marsten, R.E., "Integer Programming: A Framework and State-of-the-Art Survey", Management Science, May 1972, pp. 465-491.

106. Hillier, F.S., "Efficient Heuristic Procedures for Integer Linear Programming with an Interior", Operations Research, July-Aug 1969, pp. 600-637.

107. Kochenberger, Gary A., McCarl, Bruce A. and Wyman, F. Paul, "A Heuristic for General Integer Programming", Decision Sciences, January 1974, p. 36.

108. Naylor, Thomas R., Corporate Simulation Models, forthcoming book.

109. Naylor, Thomas R., Corporate Planning Symposium, San Francisco, California, October 1974.

110. Traenkle, J.W., et al of Arthur D. Little, Inc., The Use of Computer Assisted Financial Planning Models in Industry, research report for Financial Executives Research Foundation, 1975.

111. "Corporate Planning: Piercing the Future Fog in the Executive Suite", Business Week, April 28, 1975.

112. Bonner and Moore Associates, Inc., PETROPLAN: Financial Planning for Petroleum Companies, March 1972.

113. First National City Bank of New York, MODFUN.

114. Carter, Eugene E., Finance Computer Programs, Harvard University, 1969.

115. Warren, James M. and Shelton, John P., "A Simultaneous Equation Approach to Financial Planning", Journal of Finance, December 1971, pp. 1123-1142.

116. McDonald, Joe, "Memorex Simultaneous Equation Financial Model", discussed at Social Systems, Inc. Corporate Planning Seminar, Houston, Texas, March 1974.

117. Radnor, Michael and Neal, Rodney N., "The Progress of Management Science Activities in Large U.S. Industrial Corporations", Operations Research, Mar-Apr 1973, p. 413.

118. PoKempner, Stanley, The Activities and Organization of Operations Research/Management Science Units in Business, 1976 (to be published by The Conference Board).

119. Braunstein, Daniel N., "Relating the Logic of Management Science to Executive Decision-Making", Interfaces, February 1975, pp. 44-46.

120. Schycon, Harvey N., "All Around the Model: Perspectives on MS Applications", Interfaces, August 1973, pp. 50-53.

121. Hammond, John S., "Management Scientist in Successful Implementation", Sloan Management Review, Winter 1974.

122. Grayson, C. Jackson, "Management Science and Business Practice", Harvard Business Review, July-Aug 1973, p. 41.

123. Woolsey, Gene, "O Tempora, O Mores, O C. Jackson Grayson, Jr.", Interfaces, May 1974, pp. 76-78.

124. Fulmer, Robert M. and Rue, Leslie W., The Practice and Profitability of Long-Range Planning, Research Report for the Planning Executives Institute, 1973.

125. Naylor, Thomas R., "The Politics of Corporate Modeling", preliminary chapter of Naylor's forthcoming book, Corporate Simulation Models, handout received at Corporate Planning Symposium, San Francisco, California, October 1974.

126. Weingartner, H. Martin, "What Lies Ahead in Management Science and Operations Research in Finance in the Seventies", Interfaces, August 1971, pp. 5-12.

127. Baker, John K. and Schaffer, Robert H., "Making Staff Consulting More Effective", Harvard Business Review, Jan-Feb 1969, pp. 62-71.

128. The Research Institute of America, New York, N.Y., "How to Handle Yourself with the Technical Specialist", Personal Report, July 1975.

129. Whitmore, G.A. and Darkazanli, S., "A Linear Risk Constraint in Capital Budgeting", Management Science, December 1971, p. B155.

130. Seppala, Yrjo, "Constructing Sets of Uniformly Tighter Linear Approximations for a Chance Constraint", Management Science, July 1971, p. 736.

131. Senju, S. and Toyoda, Y., "An Approach to Linear Programming with 0-1 Variables", Management Science, December 1968, p. B196.

132. Wyman, F.P., "Binary Programming: A Decision Rule for Selecting Optimal vs. Heuristic Techniques", The Computer Journal, 1973, pp. 135-140.

133. Toyoda, Y., "A Simplified Algorithm for Obtaining Approximate Solutions to Zero-One Programming Problems", Management Science, August 1975, pp. 1417-1427.

134. Three articles on goal programming and CB appear in Financial Management, Spring 1974.

APPENDIX

An Annotated Bibliography on Capital Budgeting

circa 1964 through 1972

This Appendix includes the results of a literature search performed prior to and during the completion of this thesis. The majority of the items listed contain a very brief abstract of the paper or book, and in most cases, this was taken from the International Abstracts in Operations Research (IAOR) Journal. In some cases, the IAOR Journals did not cover the referenced items; so the abstracts were either taken from the paper's summary or a brief survey of the paper was performed to determine its salient features. Also, some use was made of the Operations Research/Management Science Abstracts service provided by the Executive Sciences Institute, Inc.

The items in this annotated bibliography which do not contain abstracts were usually obtained as secondary or side references from papers personally reviewed by this author and abstracted. The title provides some clue, at least, to the general nature of the paper or book.

The primary journals reviewed through the IAOR Journal are listed below along with their abbreviation used in the bibliography.

Operations Research (OR)
Operations Research Quarterly (ORQ)
Management Science (MS)
AIIE Transactions (AIIET)

International Journal of Production Research (IJPR)
IEEE Transactions on Systems, Science and Cybernetics (IEET-SSC)
Journal of Industrial Engineering (IE)
Canadian Operations Research Journal (CORJ)

Other journals covered outside the IAOR abstracts are listed below along with their abbreviation and the time period covered.

The Engineering Economist (EE): Fall 1966 - 1972
The Journal of Financial and Quantitative Analysis (JFQA): March 1968 - 1972
The Journal of Finance (J. Fin): 1968 - 1972
The Harvard Business Review (HBR): 1968 - 1972
The Journal of Business (J. of Bus.): 1968 - 1972
IEEE Transactions on Engineering Management (IEET-EM): Feb 1970 - 1972
The Bell Journal of Economics and Management Science (BJEMS): Spring 1970 - 1972
Decision Sciences (DS): 1968 - 1972

Many outstanding journals dealing primarily with a more "economic theory" approach to capital budgeting (such as the Journal of Political Economy and the American Economic Review) were not reviewed. Also, journals related strongly to pure finance or accounting were not covered--such as the Accounting Review (AR). The motivating factor in not reviewing these publications was mostly time limitations; however, the desire to maintain a strong operations research/quantitative management science flavor in the literature search definitely influenced this choice.

It is felt that this bibliography along with references covered in the text of the thesis constitute an extensive (but certainly not exhaustive) survey of many of the important works in capital budgeting related to mathematical programming. Hopefully this information may be useful to future students of problems in this area.

1972

- Anderson, Dennis, "Models for Determining Least-Cost Investments in Electricity Supply", BJEMS, Spring 1972, p. 267.

  O.R. oriented with economic theory thrown in; formulates problem in L.P., D.P., and N.L.P. methods; many, many references relating to electricity economics, etc.

- Berger, Roger W., "Implementing Decision Analysis on Digital Computers", E.E., Summer 1972, p. 241-248.

  Monte Carlo techniques (packages currently available on computers; most consider financial/investment models; gives costs of programs).

- Bierman, Harold J., "Estimating the Cost of Capital, A Different Approach", D.S., Jan. - Apr. 1972, pp. 40-53.

  Estimates of cost of equity capital for a firm is analyzed by using information obtained directly from selected investors via questionaires. Four firms are studies as of May-June 1968.

- Bierman, Harold J., Jr. and Hausman, Warren H., "The Resolution of Investment Uncertainty Through Time", M.S., August 1972, pp. B654-B662.

  When investment opportunities have identical net present value and risk characteristics, the determination of which has the highest uncertainty resolution over time may be the decision variable. Definition of what this is, is presented; talks about Hillier's approach and Van Horne's approach; examples given.

- Bower, Joseph L., Managing the Resource Allocation Process: A Study of Corporate Planning and Investment, 1970, 363 pages, $8.00; Review in Vol. 17, no. 2, p. 146, E.E., Winter 1972.

- Bradley, Stephen P. and Crane, Dwight B., "A Dynamic Model for Bond Portfolio Management", M.S., Oct. 1972, B19-B31.

  Multistage decision model on buy, sell, and hold at discrete points in time. Normative models of this decision problem tend to become very large, particularly when its dynamic structure and the uncertainty of future interest rates and cash flows are incorporated. The paper here presents a new approach for solving reasonably large problems using the decomposition algorithm of mathematical programming.

- Bussey, Lynn E. and Stevens, G.T., Jr., "Formulating Correlated Cash Flow Streams", E.E., Fall 1972, pp. 1-30.

  Considers auto-correlation (same project) and cross-correlations (between projects) of cash flows for projects.

- Chambers, David, "The Joint Problem of Investment and Financing", O.R.Q., 1972, pp. 267-296.

  Programming model of project selection extended to include issue of new equity as one of activities; this allows marginal cost of capital, in each year up to a planning horizon, to be determined within the model. Interactions between project selection, financing policy, and dividend policy are illustrated and results contrasted with those derived from familiar decision rules.

- Chen, Andrew H.Y., Jen, Frank C. and Zionts, Stanley, "Portfolio Models with Stochastic Cash Demands", M.S., Nov. 1972, pp. 319-332.

  Four single period models are formulated using exogenous stochastic cash demands and 1) deterministic returns on earning assets and 2) stochastic returns on the earning assets. The resulting non-linear programming problems, two of which are computationally tractable, are discussed.

- Dudley, Carlton L., Jr., "A Note on Re-investment Assumptions in Choosing Between NPV and IROR", J. Fin., Sept. 1972, pp. 907-915.

- Fogler, H. Russell, "Investment Strategy for a Small Growth Company", D.S., Jan. 1972, pp. 31-46.

  Studies impact of simultaneous investments of a rather large nature compared to the size of the firm. Capital budgeting "rules of thumb" are clarified and the implicit assumptions for approximating a company's cost of capital are discussed. A relatively simple simulation model was constructed to study quantitative aspects of different capital budgeting strategies; and the combined strategy of rules of thumb and cost of capital criteria are related into what is believed to be a practical bridge between the "art" and science of finance.

- Hakansson, Nils H., "Mean-Variance Analysis in a Finite World", JFQA, Sept. 1972, p. 1873.

  Discussed single-period mean-variance approach and its relationship to empirical world. Theoretical and economic oriented.

- Haley, Charles W. and Schall, Lawrence D., "A Note on Investment Policy with ~~Imperfect Capital Markets", J. Fin., March 1972, pp. 93-96.~~

- Hogan, William N. and Warren, James M., "Computation of the Efficient Boundary in the E-S Portfolio Selection Model", JFQA, Sept. 1972, p. 1881.

Discussion of portfolio selection based on expected value and semi-variance. This tends to concentrate on reducing losses as opposed to variance which concentrates on extreme gains as well as extreme losses; in the presence of non-symmetrical probability distributions this equal weighting may not be valid. Has an example problem (fairly easy to read). Relates to math. programming--Wolfe's Algorithm to compute efficiency frontier.

- Huefner, Ronald J., "Sensitivity Analysis and Risk Evaluation", D.S., July 1972, pp. 128-135.

Addresses problem of what to do in evaluating effect of uncertain input variables on decision variables.

- Jensen, Michael C. and Long, John B., Jr., "Corporate Investment Under Uncertainty and Pareto Optimality in the Capital Markets", BJEMS, Spring 1972, p.151.

Investigates allocation of investment in new risky opportunities which results from the collective behavior of firms, each of which attempts to maximize the net increase in its market value. The allocation is compared to 1) maximizing nominal social wealth 2) maximizing social welfare.

- Kamien, Morton I. (Northwestern Univ.) and Schwartz, "A Direct Approach to Choice Under Uncertainty", M.S., April 1972, pp. B470-B477.

Related to portfolio selection and choice of action under uncertainty whereby one chooses action with most preferred distribution over payoffs via modification of an existing probability distribution at a direct monetary cost. Thus the family of distribution functions from which the decision maker can choose is implicitly defined by his initial wealth, the original distribution, and the modification cost function. Examples given.

- Kim, Sungwoo, "Investment Planning and Realization Functions with Capital Appropriations", E.E., Summer 1972, Vol. 17:4, p.777.

Macro-economic, econometric oriented.

- Keeley, Robert and Westerfield, Randolph, "A Problem in Probability Distribution Techniques for Capital Budgeting", J. Fin., June 1972, pp. 703-710.

Discusses risk adjusted discounting, certainty equivalence and relates final approach, single certainty equivalent, to Hillier's 1964 paper.

- Kenney, Ralph L. (MIT), 'Utility Functions for Multi-attributed Consequences", M.S., Jan. 1972, pp. 276-287.

Functional forms for multi-attributed utility functions which satisfy assumptions on decision maker's preferences. Procedure for verifying assumptions is included.

- Kryzanowski, Lawrence, Lustztig, Peter and Schwab, Bernard, "Monte Carlo Simulation and Capital Expenditure Decisions--A Case Study", E.E., Fall 1972, pp. 31-48.

- Lewellyn, Wilbur G. and Long, Michael S., "Simulation versus Single-Value Estimates in Capital Expenditure Analysis", D.S., Oct. 1972, pp. 19-33.

  Contention is challenged that simulation of capital investment provides more helpful measures of return and risk than single point discounted cash flow estimates.

- Lutzenberger, Robert J. and Rao, Cherukuri U., "Portfolio Theory and Industry Cost of Capital Estimates", Comment by James L. Bicksler, JFQA, Mar. 1972, p. 1463.

  Discusses cost of capital determination under risk and relates to capital budgeting (sort of Modigliani-Miller approach).

- Lockett, A. Geoffrey and Gear, Anthony E. (British), "Programme Selections in Research and Development", M.S., June 1972, pp. B575-B590.

  Math. programming difficulties encountered in practical application; case studies and numerical examples.

- Merton, Robert C., "An Analytic Derivation of the Efficient Portfolio Frontier", JFQA, Sept. 1972, p. 1851.

  Talks about how classic graphical technique for portfolio selection is incorrect under certain conditions.

- Meyers, Stewart C., "A Note on Linear Programming and Capital Budgeting", J. Fin., Mar 1972, pp. 89-92.

  Latest chapter in Baumol and Quandt's criticism of Weingartner's work (1963) on capital budgeting under capital rationing.

- Miller, Virgil V., Anderson, Leslie P. and Josephs, Spencer S., "A Probability Distribution of Discounted Payback for Evaluating Investment Decisions", JFQA, Mar. 1972, p. 1439.

  Relates payback period and its distribution to NPV and IROR using PERT-like assumptions and traditionally accepted academic techniques.

- Pellatt, Peter G., (University of Manitoba), "Real Estate Investments Under Uncertainty", J. Fin., Jan. 1972, pp. 459-471.

- Poliquen, Louis Y., Risk Analysis in Project Appraisal, review in E.E., Winter 1972, Vol. 17, no. 2, p. 141.

  Monte Carlo approach with case studies.

- Slovic, Paul, "Psychological Study of Human Judgments: Implications for Investment Decision Making", J. Fin., Sept. 1972, pp. 779-780.

- Souder, William E., "Comparative Analysis of R&D Investment Models", AIIET, March 1972, pp. 57-64.

  Compares various investment models (41 investment models chosen from literature); six generic types defined: linear, non-linear, zero-one, scoring, profitability index and utility models. Profitability index and scoring models were found to have easiest usability and lowest cost performance characteristics, while linear, non-linear, and 0/1 models had highest realism, flexibility and capability. Utility models generally were inferior.

- Tersine, Richard J. and Tudko, William, "A Bivariate Stochastic Approach to Capital Investment Decisions", E.E., Vol, 17,3,157, Spring 1972.

  Addresses why managers still cling to deterministic approaches. Develops model of future cash flows using Beta distribution such that managers can easily understand. Sort of analytical.

- Van Horne, James C., "Capital Budgeting under Conditions of Uncertainty as to Project Life", E.E., Spring 1972, pp. 189-199.

  Method proposed which allows the integration of uncertain project life into the information needed for overall assessment of the expected risk and return of the investment project.

1971

- Allen, D.H. and Johnson, T.F., "Realism in LP Modelling for Project Selection", R&D Management (U.K.), Vol. 1 and 2, 1971, pp. 95-100.

  Extent to which a L.P. Model can be made to realistically represent R&D situations. Uncertainty as to way project carried out can be incorporated as model is updated, but uncertainty in future potential benefits need to be incorporated. A method for doing this using Monte Carlo and linear programming is presented.

- Bernhard, Richard H., "A Comprehensive Comparison and Critique of Discounting Indices Proposed for Capital Investment Evaluation", E.E., Spring 1971, pp. 157-186.

  Article shows that for freedom to borrow and lend at one rate of interest and with certainty, the PV index and its equivalents are correct while other indices are inconsistent. However, with constraints on allowed borrowing or on available scarce material, the PV index is in general unsatisfactory and a complete mathematical programming solution is required.

• Bernhard, Richard, "Some Problems in the Use of a Discount Rate for Constrained Capital Budgeting", AIIET, Sept. 1971, pp. 180-184.

Paper refers to ideas of Baumol and Quandt that discount rate cannot be determined independent of the model itself. Recently ideas of Mao and Lusztig and Schwab contradict this thought but author supports Baumol and Quandt and by using dual model shows correct discounting factors not specifiable independent of external utilities.

• Buck, James R. and Hill, Thomas W., Jr., "Laplace Transforms for the Economic Analysis of Deterministic Problems in Engineering", E.E., Summer 1971, pp. 247-263.

Applied to capital budgeting such as PV and IROR.

• Bussey, Lynn E. and Stevens, B.T., Jr., "Net Present Value from Complex Cash Flow Streams by Simulation", AIIET, Mar. 1971, pp. 81-89.

References Hillier's earlier papers and Canada's paper.

• Byrne, R.F., Cooper, W.W., Charnes, A., Davis, O.A., Gilford, D.A. (eds.), "Budgeting Interrelated Activities", Studies In Budgeting, Vol. 2 of TIMS-ONR Monographs, 1971.

• Chambers, David, "Dividend Policy and Asset Allocation", London Business School Working Paper, 1971.

• Cochran, M.L., Pyle, E.B., III, Greene, L.C., Clymer, H.A. and Bender, A.D., "Investment Model for R&D Project Evaluation and Selection", IEEET-EM, Aug. 1971, pp. 89-99.

Math. Prog. model 0/1 ILP using discounted PV; implemented on timesharing systems and being used by industries (uses Lawler and Bell's method).

• Fleisher, Gerald A., Capital Allocation Theory, Review by Barnard E. Smith, E.E., Summer 1971, p. 280.

E.E. text not as illustrated as Grant and Ireson's. Brief and to the point (not too good of a review).

- Fleisher, Gerald and Cremer, Robert H., "On the Application of Cardinal Utility Theory to Engineering Economic Analysis", E.E., Winter 1971, pp. 117-130.

No abstract, but it looks like a good, easily-readable article on some interesting points about project selection.

- Gear, A.E., Lockett, A.G. and Pearson, A.W., "Analysis of Some Portfolio Selection Models for R&D", IEEET-EM, May 1971, pp. 66-76.

Looks at math. programming models related to R&D project selection: linear, integer, chance constrained, or dynamic. Representative examples for each class are presented and evaluated from standpoint of: Built-in assumption, ease of computation; usefulness of outputs; versatility of application.

Side refs: good list oriented toward R&D with MP flavor.

- Gear, A.E., "A Probabilistic Objective Function for R&D Portfolio Selection", O.R., Sept. 1971, pp. 153-165.

Various forms of objective functions are examined and parametric linear programming is used to yield a near-optimal allocation.

- Godfrey, James T. and Spivey, W. Allen, "Models for Cash Flow Estimation in Capital Budgeting", E.E., Spring 1971, pp. 187-210.

Develops models for cash flows to be used in models like Lorie-Savage and Weingartner; sort of forecasting oriented.

- Gray, Kenneth B. and Dewar, Robert B.K., "Axiomatic Characterization of the Time-Weighted Rate of Return", M.S., Oct. 1971, pp. B32-B35.

Recently published banking industry study pointed out that internal rate of return was inappropriate for measuring performance of pension fund managers since it depends on fund contributions and withdrawals, and not, as it should, solely upon the way the manager of of the fund proportions it resources.

The referenced banking study proposed a new time-weighted rate of return which removes the dependency on contributions and withdrawals; however, this study was not complete in that it did not show that the well-behaved time-weighted rate of return is the only such type criteria available. The latter result is developed in this paper.

- Gustafson, David H., Pai, Gopinath K. and Kramer, Bary C., "A Weighted Aggregate Approach to R&D Project Selection", AIIET, Mar. 1971, pp. 22-31.

General model for setting priorities on R&D projects; reports on research that evaluates implementing the model. Specifically criteria weighting is addressed and methods for solving the problem of criteria independence and for estimating probability of project success are proposed, but not tested.

- Hakansson, Nils H., "Optimal Entrepreneurial Decision in a Completely Stochastic Environment", M.S., Mar. 1971, pp. 429–449.

Preferences: lifetime returns. Interest rate are all stochastic. Preferences sensitive to opportunities at each decision point plus other environmental factors. At each decision point must decide how to allocate resources between consumption, life insurance, investments, and lending/borrowing. Objective function is expected utility from consumption as long as he lives. Closed form solutions for class of utility functions.

- Hakansson, Nils H., "Participative Budgeting Under Uncertainty: A Decision Theoretic Approach", Consultant, The Rand Corp., Santa Monica, Cal., Rand Rpt. P-4496, Feb. 1971, 62 pgs.

Considers organizational budgeting problem when there are several decision makers. Whenever there is partial agreement on independence, statistical and value-wise, among the activities under consideration, a basis exists for a considerable reduction and division of labor in the budgeting effort using a decision theoretic approach; a scheme is proposed that offers the opportunity to utilize the organization's best talents in each area, without running much risk that cheating will appear worthwhile to any individual. Method facilitates determination of the size of the total budget.

- Hillier, F.S., "A Basic Model for Capital Budgeting of Risky Interrelated Projects', E.E., Fall 1971, p.1.

Discusses Hillier's monograph and extends somewhat for implementation.

- Hillier, F.S., The Evaluation of Risky Interrelated Investments, 1969, book review by F.K. Wolf, E.E., Spring 1971, pp. 211–218.

(Very favorable review.) Book is clear, well-referenced and contains related works. Keeps reader's attention focused on conceptual structure rather than becoming mired in methodological detail. A dynamic version of Hillier's model is presented as a chance constrained programming problem where the E(NPV) is maximized subject to the probability that a cash flow during period i is greater than some dollar value L. Such restraints can be made for the net cash flow for each period and for the cumulative cash flow through the periods. Reviewer states book has stimulated a new interest in this area for himself. (Linear programming, branch and bound, and chance constrained programming are all dealt with).

- Kennedy, Robert C., "An Annotated Bibliography of Estimation Procedures Useful in Engineering Economy", (Westinghouse Elec. Corp., Winston-Salem, N.C.), E.E., Spring 1971, pp. 211-218.

  Data and sources on estimated first cost, interest rate, project life, salvage value, net operating income (or cost).

- Klausner, Robert F., "Communicating Investment Proposals to Corporate Decision Makers", E.E., Fall 1971.

  Communication strategies suggested (for engineers).

- Laughhunn, D.J., "A Comment on 'Probility of Survival as an Investment Criterion' by Fred Hanssman", M.S., Aug. 1971, p. B772.

  Argues that although Hanssmann's previous article assumes normality for portfolio returns, it requires really a relatively weaker assumption; i.e. finite means and variances only.

- Lucas, Robert E., Jr., "Optimal Management of a R&D Project", M.S., July 1971, pp. 679-697.

  Considers fixed and random variable time for project completion; and fixed and variable costs/time with increasing expenditures resulting in decreased completion time. Each of four possible combinations treated in separate sections with last selection a summary and discussion of results.

- Mao, J.C.T., Quantitative Analysis of Financial Decisions, review by W. Beranek, E.E., Summer 1971, p. 179.

  Financial textbook directed at business school but with strong quantitative flavor and some knowledge of elementary calculus. Very good, readable textbook.

- Nagpaul, P.S. (Central Elec. Engineering, India), "R&D Project Selection", M.S., April 1971, pp. B553-B556.

  Ref's Moore and Baker's paper "Computational Analysis of Scoring Models for R&D Project Selection" (M.S., Vol. 16, No. 4, Dec. 1969) and claims it was significant contribution to literature on R&D

management. Says inclusion of factors like timing of streams of income payments and cost payments considerably improve the usefulness of the scoring model and bring it nearer to the "real world". Its consistency with economic models and constrained optimization models lends respectability to it and hopefully will stimulate further research in area. Partitioning of criterion measurement space for income stream timing appears artificial and presents practical difficulties; paper discusses this.

- Ochoa-Rosso, "A Branch and Double-Bound Algorithm for the Multi-Period Capital Budgeting Problem", Paper presented at 39th ORSA meeting in Dallas, March, 1971.

Paper considers independent projects requiring capital outlays in several time periods. Problem formulated as a maximal flow problem on a single-source, single-sink capacitated network, where flow on the arcs represents cash flow. Formulation facilitates branch and bound; experience is given.

- Peterson, D.E. and Laughhunn, D.J., "Capital Expenditure Programming and Some Alternative Approaches to Risk", <u>M.S.</u>, Jan. 1971, pp. 320-336.

Considers measures of risk besides variance such as Baumol's lower confidence limit and maximum probability of loss. Primary purpose of paper is to present a methodology which imposes certain "constraining relations" on acceptable investment programs rather than one which appeals to a specific utility function as the basis for ordering choices. Discussion of several utility functions along with usefulness when probability distrub. of NPV is known. Problems of constructing utility fcn is examined.

- Pyle, David H. and Turnovsky, Stephen J., "Risk Aversion in Chance Constrained Portfolio Selection", <u>M.S.</u>, Nov. 1971, pp. 218-225.

Discusses effects of changes in investable wealth on investment behavior when portfolio choices are subject to a chance constraint. Alternative specification of the chance constraint are shown to imply increasing, decreasing, or constant relative risk aversion with respect to changes in wealth.

- Sanathanan, Lalitha, "On An Allocation Problem with Multi-Stage Constraints", <u>O.R.</u>, Nov.-Dec. 1971, pp. 1647-1663.

Simple algorithm for finding optimal allocations subject to a hierarchy of limits when the loss function is separable strictly convex and the resources function is linear. Applications to capital budgeting and multi-stage sampling are pointed out. Previous work by Srikantan (<u>O.R.</u>, pp. 265-273, 1963) is special case of present article.

- Sharpe, W.F., Portfolio Theory and Capital Markets, 1970, review by Lawrence Fisher and James H. Lorie, E.E., Summer 1971, p. 277.

Good clearly written exposition of Sharpe's work on extending Markowitz's portfolio selection problem; almost all of material in book can be found in literature but book is of value because it contains in one package a systematic treatment (in uniform notation) of ideas that have appeared over the last 18 years. Treatment is thorough but concise; reader may want to refer to the literature for greater detail (bibliography is good but not all that helpful). Book extremely useful for textbook in modern courses in Finance.

- Smith, K.V., Portfolio Management: Theoretical and Empirical Studies of Portfolio Decision Making, 1971.

- Stapleton, R.C., "Portfolio Analysis, Stock Valuation, and Capital Budgeting Decision Rules for Risky Projects", J. Fin., Mar. 1971, pp. 95-118.

- Turban, Efram, "Utility Theory Applied to Multivariable System Effectiveness Evaluation", M.S., Aug. 1971, pp. B817-B828.

Complex system with several measures of performance. Various personnel asked to weigh measures and Churchman and Ackoff's method for checking the consistency of the method has been modified and applied; using this an overall utility index technique was developed to establish stopping rules for a simulation study.

- Weingartner, H.M., "What Lies Ahead in MS/OR in Finance in the 70's", M.S. Interfaces, Aug. 1971, pp. 5-12.

Discusses impact of developments in the external and internal environments on MS/OR in finance, the relationship of MS to management information systems and to the managerial process, and outlook for future development in M.S. relevant to finance.

- Weinwurn, E.H., "An Analysis of Applications of the Utility Concept", E.E., Winter 1971, pp. 131-140.

No abstract but looks like good exposition on corporate utility functions.

- Whitmore, G.A., "A Linear Risk Constraint in Capital Budgeting", M.S., Dec. 1971, pp. B155-B157.

Two methods for constructing linear restrictions to approximate risk constraints in capital budgeting are described. Practical applications discussed.

1970

- Bell, D.C. (The Gas Council, England) and Read, A.W., "The Application of a Research Project Selection Method", R&D Management (U.K.), Oct. 1970, pp. 35-42.

  Application of LP in two R&D labs, yielding information about outcomes of decisions under uncertainty. Simple examples illustrate elements of model and completed form. Benefit evaluation carried out using probabilistic networks and certainty equivalents. Practical applications discussed.

- Ben-Israel, A., Charne, A., Hurter, A.P. and Robers, P.C., "On the Explicit Solution of a Special Class of Linear Economic Models", O.R., May-June 1970, pp. 462-470.

  Explicit solutions for a class of linear (inequality) economic models heretofore treated by L.P. Applications shown in capital budgeting, production planning, and input-output analysis.

- Chidabaram, T.S. (India), "Optimal Funding Strategies in R&D Management", OPSearch, Mar. 1971.

  Problem of allocating budget of $C among n technical approaches, any of which if successful would realize the goal of the R&D effort. The cost of undertaking the ith project (which have a subjective probability P_i of succeeding) is a random variable C_i. Two approaches can be taken: a) Choose a large no. of projects and allocate a small amount of funds to each, or b) Choose a small no. of projects and allocate a large amount of funds to each. The mathematical analysis containted in the paper shows that under sufficient conditions b) is justified.

- Chidabaram, T.S., "Optimal Reallocation of R&D Money Under Budget Decrements", IEEET-EM, Nov. 1970.

  A simple algorithm is developed on the assumption that the current allocation is optimal at the origin. Using Kuhn-Tucker theory, it is shown that this algorithm reaches the new optimum in finite no. of iterations.

- Cootner, Paul H., "Rate of Return and Business Risk", BJEMS, Autumn 1970, pp. 211-226.

Study to measure relations between risk and rate of return; to determine empirical basis for implementing Supreme Court decisions that public utility is entitled to earnings sufficient to permit its rate of return to be similar to other businesses with "similar risk". Business risk functionally related to variability of earnings, a number of hypotheses concerning risk and rate of return are tested statistically. Reasonable and significant association is discovered between them for both a sample of industries and individual companies. Model only explains about a quarter of the variability of rate of return among industries and firms, suggesting that model has not fully captured some important determinants. Finally, results are related to recent developments in theory of financial risk.

- Elton, Edwin J., "Capital Rationing and External Discount Rates", J. Fin., June 1970, pp. 573-584.

- Fried, Joel, "Bank Portfolio Selection", JFQA, June 1970, pp. 203-228.

Considers variability of both income and gross asset levels as risks involved in banking; max exp. profit subject to risk constraints (chance constr. prog.) on wealth losses and the availability of liquid assets. Includes covariance matrices of rates of returns on bank assets and deposit changes. Basically an extension of Markowitz's work.

- Gepfert, Alan H., "Practical Financial Management Models", (McKinsey & Co., Inc., N.Y.), M.S., Apr. 1970, pp. B456-457.

Ogler's paper "An Unequal-Period Model for Cash Management Decisions" (M.S., Oct., 1969, P. B71-B92) provides thought-provoking and helpful framework for cash management models. It is also pertinent to commercial bank balance sheets models, especially as to handling the pre-maturity sale of assets and the "dimensionality problem", using time periods of various length. However, it omits some important details to make it "acceptable...to management", these problems are discussed.

- Gershefski, G.W. (Sun Oil Co.), "Corporate Models--The State of the Art", M.S., Feb. 1970, pp. B303-312.

By 1960's OR models being applied in inventory, scheduling and resource allocation. Past several years models have made such strides that it's become feasible to think of an entire corporate model. By early 1969, many companies developing corporate models or at least expressed intention to do so. Results of questionnaire sent out to companies on this subject is discussed.

- Greer, W.R., "Capital Budgeting Analysis with the Timing of Events Uncertain", A.R., Jan.1970, pp. 103-114.

- Hakansson, N.H., "Optimal Investment and Consumption Strategies Under Risk for a Class of Utility Functions", Econometrica, Sept. 1970.

  Sequential model of individual's economic decision problems under risk. Optimal consumption, investment, and borrowing-lending strategies are obtained in closed form for a class of utility functions. Necessary and sufficient conditions for long run capital growth are also given. Optimal investment strategies are independent of wealth, non-capital income, age, and impatience to consume.

- Hirshleifer, J., Investment, Interest, and Capital, 1970.

- Horowitz, I., Holt, Rinehart, Decision Making and the Theory of the Firm, Winston, N.Y., 1970.

- Hubert, J.M., "R&D and the Company's Requirements", R&D Management (U.K.), Oct. 1970, pp. 30-34.

  Current models select R&D portfolio based on returns from projects. In practice, financial data is so inaccurate as to make analysis a theoretical exercise. Selection of projects should be compromise between high investment return of corporation and maintenance of a scientifically balanced R&D involving some long term, unknown return investments. Method used at Unilever Research for doing this is described.

- Kabak, I.W. and Owen, Joel, "Random Variables, The Time Value of Money and Capital Expenditures", M.S., Nov. 1970, pp. 142-145.

  How much money should be invested at time t at interest rate i for a time T such that probability of the funds required D(T) exceeding those available x(T) equals at most p. I,T, D(T) are taken to be random variabels and theory to solve problem along with specific cases is given.

- Laughhun, D.J., "Quadratic Binary Programming with Application to Capital Budgeting Problems", O.R., May-June 1970, pp. 454-461.

  Algorithm useful in capital budgeting if returns are inter-correlated R.V.'s and if decision maker uses mean and variance returns; requires decision maker to have prior identification of mean/variance efficient set. Algorithm does this and solves problem based on implicit enumeration scheme of Egon Balas for solution of binary linear programming problem.

- Lesso, W.G., "An Extension of the Net Present Value Concept to Intertemporal Investments", E.E., Vol. 15, No. 1, 1970, pp. 1-8.

- Levy, H. and Sarnat,M., "The Portfolio Analysis of Multi-Period Capital Investment Under Conditions of Risk", E.E., Fall 1970, pp 1-20.

  Applies Markowitz-Tobin portfolio selection model to problems of multiperiod capital investments. Situations in which the use of the mean-variance criterion leads to the inclusion of projects with negative E(NPV) in the optimal decision are analyzed. It is shown that the firm may rationally accept a proposal with negative E(NPV) in the case of low risk projects or when that project has a sufficiently strong negative covariance with other projects to produce a favorable portfolio.

- Lippman, S.A., "Capital Accumulation in a Riskless Environment", J. of Economic Theory, Sept. 1970.

  Justification for using present value or internal rate of return in comparing riskless investments is inadequate. Model proposed to rectify this shows that if an investment A has the highest rate of return among all projects, then it is optimal to invest all available funds in A at time zero if we are to max. cash on hand at end of period N for all N sufficiently large. Withdrawals are not permitted in model.

- Litzenberger, R.H. and Budd, A.P., "Corporate Investment Criteria and the Valuation of Risk Assets", JFQA, Dec. 1970, pp. 395-420.

  Mostly on cost of capital and traces history of main elements of capital market theory as related to risk. Presents empirical data (regression studies) on this required risk rate of return in capital budgeting.

- Lockett, A.G.,(U. of England) and Freeman, P., "Probabilistic Networks and R&D Selection", ORQ, Sept. 1970, pp. 353-359.

  Mathematical programming methods suggested for R&D project selection but these are criticized due to ignoring stochastic effects. Paper presents a method for incorporating this by using probabilistic networks, simulation and mathematical programming. Case study presented and compared with expected value methods.

- "The Discount Rate Problem in Capital Rationing Situations: Comment by A. Geoffrey Lockett and Cyril Tomkins: Reply by Peter Lusztig and Bernhard Schwab", JFQA, June 1970, pp. 245-261.

  Comment: Refers to earlier paper by Lusztig and Schwab (L-S) about what to use for math. prog. problem (L.P.) discount rate because it depends on optimal solution (which is yet undetermined). L-S proposed a method and "Comment" checked it out with results that L-S is incomplete but with small modification it may be useful and also it may be a pointless exercise to even worry about the problem.
  Reply: Contested some of Lockett's interpretation of their earlier work.

- Mao, James C.T. and Brewster, "An E-S Model of Capital Budgeting", E.E., Winter 1970, p. 123.

  Develops new concept of semi-variance.

- Mao, James C.T., "Survey of Capital Budgeting: Theory and Practice", J.Fin, May 1970, pp. 349-360.

- Mao, James C.T., Quantitative Analysis of Financial Decisions, Review in M.S., May 1970, pp. 645-646.

- Mayer, R., "Capital Investment Analysis--Another Way", I.E., July 1970, p. 11.

  Discusses compromise between payback and theoretical approaches; tries to address which is best.

- McNichols, R.J. and Wortham, A.W., "Importance of Income Pattern In Economic Decision Making", IJPR, Vol. 8, 1970, pp. 85-91.

  Project appraisal assessment tachnique which is appealing to manager and easy to calcualte. Considers fact that income is not generated as an end of period, lump sum but rather is distributed over time in some manner. Utilizes concept of a time variable earning rate and continuous compounding. Examples given and comparisons made with conventional analyses.

- McNichols, R.J. and Wortham, A.W., "Other Income Patterns and Stochastic Considerations in Economic Decision Making", IJPR, Vol. 8, 1970, pp. 183-188.

  Describes method for evaluating projects which have an income pattern describable by a polynomial function of time, and which exhibit a yearly growth or decline in earnings while maintaining the ability to account for the time pattern of earnings. Also method

for assessing the possibility of attaining various levels of income for each period of interest. Methods utilized based on concept of dynamic return; model based on binomial distribution is developed for projects having two income levels. Extensions which permit any number of levels of profit for each year are included.

- Oakford, R.V., "The Prospective Growth Rate as a Measure of Acceptability of a Proposal", E.E., Fall 1970, pp. 207-216.

  Paper presents alternative for prospective rate of return as a measure of investment attractiveness. Shows formal relationship between present worth and the prospective growth rate criterion.

- Rondy, L.R., Birnberg, J.G. and Davis, C.L., "Effect of Three Voting Rules on Resource Allocation", M.S., Feb. 1970, pp. B356-B372.

  Different voting rules acopted by a capital budgeting committee have different characteristics as far as size of total budget allocation and interdivision variability. Various models attempt to explain laboratory experiment result.

- Sanathanan, L., "On an Allocation Problem with Multi-Stage Constraints", O.R., Nov. 1970, p. 1647.

- Schwab, B. and Lusztig, P., "A Note on Abandonment Value and Capital Budgeting", JFQA, Sept. 1970, pp. 372-379.

  Refers to earlier article about this by Van Horne and Robichek; proposes an additional abandonment criteria.

- Sharpe, W., Portfolio Theory and Capital Markets, 1970.

- Shubik, M., "Budgets in a Decentralized Organization with Incomplete Information", Rand Report P-4514, Dec. 1970.

  Problem of incentives in a decentralized organization where there is lack of information. It stresses the difficulty of designing incentive (or accounting) systems and formulates as an n-person game.

- Souder, W.E., "Budgeting for R&D", Business Horizons, June 1970, pp. 31-38.

  O.R. techniques for R&D project selection and control are described. Capital budgeting models, ranking techniques, cost prediction formulae, scoring models, and resource allocation methods are reviewed and assessed.

- Souder, W.E., "Suitability and Validity of Mathematical Models for Research Investment", Unpublished PhD Dissertation, St. Louis University, Aug. 1970.

- Unger, V.E., Jr., (Ga. Tech.), "Capital Budgeting and Mixed 0/1 Integer Programming", AIEET, Mar. 1970, pp. 28-26.

    Solution algorithms based on the work of Bender, Balas, and Geoffrion.

- Wortham and McNichols, "Comparison of Return Evaluation Techniques", IJPR, Vol. 8 1970, pp. 363-377.

    A new and alternate method of return assessment with computational, mathematical, and analytic advantage.

1969

- Adler, M., "The True Rate of Return and the Re-investment Rate", E.E., Spring 1969, pp. 185-188.

    Talks about Baldwin Rate of Return; good article.

- Agnew, N.H., Agnew, R.A., Rasmussen, J. and Smith K.R., "An Application of Chance Constrainted Programming to Portfolio Selection in a Casualty Insurance Firm", M.S., June 1969, pp. B512-B520.

    Single period model maximizing expected return subject to chance constraints on E.V. and liquidity. Related to Baumol's efficient portfolio. Example provided.

- Bell, J.A., France, H., Greer, J.C. and Watson, C., "Corporate Planning", ORQ, Apr. 1969, pp. 3-6.

    Summarizes four papers: "A Simulation Model for Business Planning", "Capital Project Planning and Evaluating Linear Programming", "Operational Research in I.C.L.'s Corporate Planning", "Planning for Profit: A Case Study of Capital Investment for a Multi-works Company".

- Bernhard, R.H., "Mathematical Programming Models for Capital Budgeting--A Survey, Generalization, and Critique", JFQA, June 1969, pp.111-158.

No abstract (use title). Author concludes chance constrained programming has handled several aspects of uncertainty but other possibilities such as multiple methods of new financing (stock and bond issues , of which uncertainty implies, are realistic complications not yet effectively handled. Also, uncertainty aggravates difficulty in choosing an appropriate objective function; finally, chance constrained programming has been substantially less satisfactory both from standpoint of meaningful model formulation and solving the model once it is formulated.

- Bickel, S.H. (Texas Instruments, Dallas, Texas), "Minimum Variance and Optimal Asymptotic Portfolios", M.S., Nov. 1969, pp. 221-226.

Bases optimal goal as long term maximization of wealth, whether by long or short term gains. Using this as goal, portfolio selection can be made by objective criteria rather than risk preference. Paper relates long run (or asymptotic portfolios to the set of efficient portfolios. Further insight into optimal policies can be developed from dynamic programming.

- Brennan, J.F.B., (Consulting Engr., San Francisco), "A Short Cut to Capital Budget Forecasting for Public Utilities", E.E., Spring 1969, pp. 151-158.

Method for forecasting capital budget on short or long term basis using LaPlace transforms. Input data required is present plant balance, average age, estimated average life, growth pattern in the recent past.

- Brockhoff, K. (Germany), "Some Problems and Solutions in the Selection of an R&D Portfolio", Proceeding of 5th Int'l Conference on O.R., June 1969, pp. 765-773.

Multi-period project selection model for mixed integer linear programming has certain drawbacks but has positive aspects as well. Advantages discussed and extension to chance constrained programming discussed—demonstrated for two-stage case.

- Byrne, R.F., Charnes, A. Cooper, W.W. and Kortanek, K.O., "A Discrete Probability Chance-Constrained Capital Budgeting Model--I", Opsearch (India), Sept. 1969, pp. 171-198.

Formulations developed for payback and liquidity constraints which provide for protection against risk of unplanned technological breakthroughs, or unplanned demands for cash. Model developed in non-linear programming model and an integer L.P. is also developed. Implications are compared to Weingartner's model. The imposition of risk constraints is included, and the unique set of dual variables give information on a project's "risk premium"; and the

direction of the dual variables can be associated with certain characteristics of the cash flow projections for individual projects. The existence of such portfolio effects, which may be characterized as risk subsidies and penalties, might be used for establishing preliminary screening rules for individual projects, or in delineating desirable project characteristics for use in the process of search for new investment opportunities.

"Part II" of above, Opsearch (India), Dec. 1969, pp. 225-261.

- Carleton, W.T., "Linear Programming and Capital Budgeting Models: A New Interpretation", J.Fin., Dec. 1969, pp. 825-833.

- Cernohorsky, J., "A Note to the Problem of Convergence of Periodic Reinvestments", M.S., Nov. 1969, pp. 187-194.

- Dearden, J., "The Case Against ROI Control", HBR, May-June 1969, pp. 124-135.

- Feldstein, M.S., "Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection", Review of Econ. Studies, Jan. 1969.

- Forsyth, J.D., "Utilization of Goal Programming and Capital Expenditure Planning", (French Summary), CORS Journal, July 1969, pp. 136-140.

  Single period math. prog. model linking prod. and cap. expenditure decisions. Management goal is to obtain at least a prespecified rate of return and treated as constraint (thus goal programming).

- Greenlaw, P.S., and Frey, M.W., FINASIM, review in M.S., Oct. 1969, p. B155.

- Hakansson, N.H., "Optimal Investment and Consumption Strategies Under Risk, An Uncertain Life-time, and Insurance", International Economic Review, Oct. 1969.

  Lifetime is R.V. with known probability distrib., utility function for individuals bequeath requests, possibility of buying life insurance. Model developed gives rise to an induced theory of the firm under risk. Also, when premium charged is "fair" individual can increase his expected utility by selling insurance to others.

- Hakansson, N.H., "Risk Disposition and the Separation Property in Portfolio Selection", JFQA, Dec. 1969.

Portfolio selection problem when investor is constrained to stay solvent. Presence of non-capital income is considered. Assumptions as to preferences, resources, and opportunities contained in Sections I and II. Portfolio problem stated in III and compared to other formulations. IV talks about utility functions and solutions to Cauchy equations. V shows Cauchy eqs. hold key to an enlarged class of utility functions with respect to which the so-called separation property holds; i.e., the optimal mix is independent of wealth. Also when solvency constraint is present, the separation property need not hold for all intervals of wealth, even if it holds for some.

- Hausman, W.H., "On the Correlation of Efficient Portfolios", M.S., Oct. 1969, pp. B15-B16.

  Further discussion of Hastie's article, "The Determination of Optimal Investment Policy", M.S., Aug. 1967.

- Jamieson, M., "Program Planning and Budgeting in the Federal Government", CORS Journal, July 1969, pp. 116-124.

  Progress report on new system being used and reviews past budget procedures; discussed problems encountered.

- Jensen, M., "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios", J. of Bus., Apr. 1969.

- Kendrick, D.A., Programming Investment in the Process Industries, 1967, 160 pgs and The Optimal Staging and Phasing of Multi-Product Capacity, H. Wein and V.P. Sreedharan, 1968, 131 pgs., Review by Donald Erlenkotter, E.E., Spring 1969, p. 187.

  Mathematical approaches to selection of investments from a standpoint of choosing production alternatives. L.P. models used in first book, D.P. in second book. Appendix F in first book gives excellent tabular presentation of model's structure.

- Klausner, R.F., (Esso Int'l), "The Evaluation of Risk in Marine Capital Investments", E.E., Summer 1969, pp. 183-214.

  Uses computer simulation and compares with traditional methods.

- Lesso, W.G., "An Extension of the NPV concept to Intertemporal Investments", E.E., Fall 1969, PP. 1-8.

- Lutzenberger, R.J. and Jones, C.P., "Adjusting for Risk in the Capital Budget of a Growth-Oriented Company: Comment", JFQA, Sept. 1969, pp. 301-304.

Refers to article by Vaughn and Bennet about risk adjusted capital budgeting framework which was laid by Gordon, Miller, Modigliani, Sharpe, Solomon and others. Comments says their approach is inconsistent with some of the basic tenents of business finance.

- Meyer, R.F., "On the Relationship Among the Utility of Assets, Consumption, and Investment Strategy in an Uncertain, but Time-Invariant World", Proceedings of the Fifth Int'l Conf. on O.R. (J. Lawrence, ed.), June 1969, pp. 627-648.

Decision theoretic framework for idealized investor. Utility functional for consumption streams derived for certain behavioral assumptions; results in set of differential equations for optimal consumption, investment, and utility for assets strategy. Equations solved for special case.

- Michelson, D.L., Commander, J.R. and Snead, J.R., "Risk Allowance in Original Capital Investments", E.E., Spring 1969, pp. 137-158.

Author suggests that PV of guaranteed depreciation cash flow be discounted at cost of capital and subtracted from total initial investment to give better measure of risk capital.

- Nemhauser, G.L. and Ullman, Z., "Discrete Dynamic Programming and Capital Allocation", M.S., May 1969, pp. 494-505.

Extension of Weingartner's work and Weingartner and Ness' work by including multi-level projects, re-investing returns, borrowing and lending, capital deferrals, and project interactions. Several state variables are handled because the optimal returns are monotone non-decreasing step functions. Computational experience is reported.

- Newnan, D.G., "Determining Rate of Return by Means of Payback Period and Useful Life", E.E., Fall 1969, pp. 29-40.

Relates easily-understood and widely used payback period to internal rate of return as an intermediate step. Conversion technique is simple in order to encourage the use of rate of return.

- Novick, C. (ed.), Program Budgeting, 382 pgs, 1969 (paperback), Review in M.S., Oct. 1969, p. B155.

- Seelenfreund, A., "Optimal Allocation for a Class of Finite Horizon Process", M.S., July 1969, pp. 728-738.

Derives fundamental frame of reference for capital budgeting problem of Dorfman, warehouse problem of Dreyful, price speculation model of Arrow and Karlin, and multi-stage linear programs by Dantzig. Results extended to stochastic problems (and infinite horizon models to be treated in a companion paper).

- Souder, W.E., "What Can C.R. Contribute to Market Planning and Budget Decisions?", Management Digest (U.K.), Spring 1969, pp. 16-19.

  Tutorial article on O.R. describes simulation, L.P., D.P., applications to marketing problems. Relevance shown to selecting distribution channels, allocating advertising budgets, and selecting total marketing mix. The interrelationships between O.R. and market research are discussed.

- Sutton, S.S., An Evaluation of Investment Criteria, Review by G.W. Smith E.E., Spring 1969, p. 174.

  PhD dissertation deals with multi-period capital budgeting criteria under varying cut-off rates of return, emphasis is placed on the capital-rationed situation; and cost of capital, under these circumstances, is shown to be an inappropriate budgeting criteria. Extends Fisher's theory to a complete L.P. version of the multi-period model and shows the equivalence of P.V. and IROR in perfect capital markets. Also, stated in terms of utility function. Interdisciplinary nature of book is good from engr. economists view.

- Thompson, R.G., "Optimal Production, Investment, Advertising, and Price Controls for the Dynamic Monopoly Firm", M.S. Nov. 1969.

- Van Horne, J.C., "The Analysis of Uncertainty Resolution in Capital Budgeting for New Products", M.S., Apr. 1969, pp. B376-B386.

  Investment in product lines of uncertain returns is approached using probability concepts. It is shown that the new products can be evaluated according to their marginal impact upon the resolution of the uncertainty pattern for a firm's total product mix.

- Verlag, Springer (Germany), Computing Methods in Optimization Problems, 1969, 191 pgs.

  Papers presented at Second Int'l Conf. on Computing Methods in Optimization Problems. (About 30 papers--one of which is "Optimization of a Quasi-stochastic Class of Multi-Period Investments", J.K. Skwirzynski).

- Wagle, B., "Models for Environmental Forecasting and Corporate Planning", ORQ, Sept. 1969, pp. 322-336.

  ~~Esso Petroleum Company tools in O.R. developed over years.~~ Describes three modesl: econometric short term environmental forecasting and economic analysis; a horizon year optimization model for long range corporate objectives; and a multi-time period simulation model for financial forecasting and planning both in short and long term. The structure, use and limitations of each of the models are discussed.

• Weingartner, H.M., "Some New Views on the Payback Period and Capital Budgeting", M.S., Aug. 1969, pp. B594-B607.

Business continues to utilize payback period even though it has been dismissed as misleading and worthless by most writers in capital budgeting. The reasons why management continues to hold on to this criterion is presented and from this knowledge possibly superior alternative criteria can be sold to management.

• Weinwurm, E.W., Financial Analysis Guide to Capital Expenditure Decisions, National Association of Accountants, Report #43, 1967, 194 pgs, Review by Weinwurm, E.E., Spring 1969, p. 171.

Accountant's point of view on capital budgeting in a field survey format. Interesting from point of view of 1) learning something about actual uses of the methods which are reported in academic writings, 2) opinions of financial executives as to their applicability. Study reports in considerable detail the no. of companies using particular methods and/or why executives feel that these methods, often hightly recommended in the professional literature, do not meet their practical needs.

• Williams, D.J., "A Study of Decision Model for R&D Project Selection", ORQ, Sept. 1969, pp. 361-374.

Report of study made in 1966 on Bristol Works of British Airport Corp., Guided Weapons Division into the evaluation and selection of company funded R&D projects. Analysis of objectives and selection criteria used by decision makers made to establish relative importance of and interactions between the various factors, with a view to deriving a model based on a project scoring system using a weighted sum of factor scores.

• Wilson, R., "Investment Analysis Under Uncertainty", M.S., Aug. 1969, pp. B650-B664.

Uncertain cash flows over time in an event tree. Seeks sufficient conditions for accepting and rejecting individual projects; formulated as mathematical programming problems which are amenable to routine application at subordinate levels of an organization.

• Wortham, A.W., McNichols, R.J., (I.E. Dept., Texas A&M), "Return Analysis on Equipment Payout", IJPR, Vol. 7, 1969, pp. 183-187.

Return ratio for use in investment analysis. Advantages of proposed method, especially in regards to competing investments of different time periods. Mathematical consistency of proposed method is shown.

- Ziemba, W.T., "A Myopic Capital Budgeting Model", JFQA, Sept. 1969, pp. 305-328.

Refers to Weingartner's model and talks about difficulty of determining the appropriate discount rate from the cost of capital and the L.P. formulation. Talks about Naslund's chance constrained model, etc. Paper presents a stochastic model in which investment returns and funds available from exogenous sources are random with essentially arbitrary distributions. Borrowing and lending are considered (Obj. Func. can be non-linear)--talks about some N.L.P. algorithms (SUMT). Objective function is expected discounted sum of dividends over T periods plus terminal wealth. Author is developing a non-myopic model.

1968

- Borch, K., "Economic Objectives and Decision Problems", IEEET-SSC, Sept. 1968, pp. 266-270.

Paper surveys some classical decision problems with and without uncertainty. From the survey, it is concluded that the natural generalization of these problems lead to the problem of describing preference orderings over sets of stochastic processes. It is shown that the decision maker will choose a decision which will minimize the probability of ruin. If this probability is equal to one, the natural objective is to maximize the expected time before ruin occurs.

- Brown, I. and Valentine, N., User's Manual for the Capital Risk Program, Control Data Corp., Data Center Division, Midwest District, review by J.M. White, E.E., Spring 1968, p. 195.

CDC's computer program for capital investments under risk and uncertainty. Reviewer thinks it's overly complex and that in view of the state-of-the-art of long range forecasting, a relatively simple model would be just as accurate and a company could develop it with its own resources.

- Byrne, R.F., Charnes, A., Cooper, W.W. and Fortaner, K., "Some New Approaches to Risk", Accounting Review, Jan. 1968.

Somewhat of a survey on more recent innovations in making investments under risky conditions. Covers decision trees, stochastic linear programming, chance constrained programming; has a good summary and conclusion section and a reference section of about 100 articles.

- Canada, J.R. and Wadsworth, H.M., "Methods for Quantifying Risk in Economic Analyses of Capital Projects", I.E., Jan. 1968, pp. 32-37.

  References Hillier's paper and his 1969 book (at that time a technical report at Stanford). States drawback to Hillier's approach is extreme difficulty of estimation and computation, especially when a large number of projects are under consideration. Paper proposes a simplified method for considering risk of proposed capital investment projects under a set of assumptions commonly realistic for such economic evaluation. The risk is measured by the probability distributions of present value measures of merit when several of the most important elements or quantities to be estimated are subject to variation. It includes a number of computational aids to facilitate the use of these quantitative techniques (method mostly deals with quantifying mean and variance of P.V. measure of merit for individual projects and also a procedure for comparing two or more mutually exclusive projects).

- Cohen, J.B. and Zinbarg, E.D., Investment Analysis and Portfolio Management, 1967, 792 pp., Review in M.S., Oct. 1968, p. B106.

- Fishburn, P.C., "Utility Theory", M.S., Jan.-Feb. 1968, pp. 355-378.

  Describes what is utility, why it is used in management and behavioral sciences. Also, summarizes a number of utility theories and gives a semi-technical survey of particular theories for readers interested in greater depth.

- Hanssmann,F., Operations Research Techniques for Capital Investment, 269 pp., 1968, Review in M.S., Oct. 1968, p. B105.

- Hanssmann,F., "Probability of Survival as an Investment Criterion", M.S., Sept. 1968, pp. 33-48.

  Investing firm primarily interested in achieving a specified minimum return critical to its economic survival. Appropriate to maximize the probability of exceeding the aspiration level. Application illustrated with several stochastic static investment models with budget constraints. Shows that as long as the aspiration level does not exceed the maximum expected return achievable with the given budget, the desired investment strategy must be sought among the efficient solutions in the Markowitz sense; for higher aspiration levels this is no longer true. For the special case of the Markowitz model we show that all investment projects with expected yield not exceeding the aspiration level of yield should be rejected.

- Henrici, S.B., "Eyeing the ROI", HBR, May-June 1968, pp. 88-97.

- Hertz, D.B., "Investment Policies That Pay Off", HBR, Jan.-Feb. 1968, pp. 96-108.

- Hester, D.D. and Tobin, J. (Cowles Monogram 19,20,21; Cowles Foundation, Yale Univ.), "Risk Aversion and Portfolio Choice, Studies of Portfolio Behavior, Financial Markets, and Economic Activity", 180 pp., 250 pp., 256 pp., Review in M.S., Oct. 1968, pp. B107-B108.

- Jeynes, P.H., Profitability and Economic Choice, Review by R.H. Sarikas, E.E., Fall 1968, p. 57.

- Kempster, J.H., Financial Analysis Guide to Capital Expenditures, National Association of Accountants, 1967, 193 pp., Review in M.S., Oct. 1968, p. B109.

- Kisler, Y. and Plessner, Y., "A Programming Model for Optimal Patterns of Investment, Production, and Consumption over Time", Israel Journal of Technology, Nov.-Dec. 1968, pp. 333-340.

  L.P. model includes production, investment, consumption, and credit. Alternative consumption functions are incorporated.

- Levy, H., "A Note on the Payback Method", JFQA, Dec. 1968, pp. 433-444.

  Compares discounted rate of return with payback method.

- Lusztig, P. and Schwab, B., "A Note on the Application of Linear Programming to Capital Budgeting", JFQA, Dec. 1968, pp. 422-432.

  Discusses mutual dependence between optimal solution and discount rate used. Says this is a severe limitation to initial solution of L.P. model--proposed sensitivity analysis.

- Mao, J.C.T. and Wallingford, B.A., "An Extension of Lawler and Bell's Method of Discrete Optimization with Examples from Capital Budgeting", M.S., Oct. 1968, pp. B51-B60.

  Extends Lawler and Bell's partial enumeration scheme to cover quadratic programming. Examples given on capital budgeting.

- Myers, S.C., "Procedures for Capital Budgeting Under Uncertainty", International Management Review, Mar. 1968.

- Oakford, R.V. and Theiesen, G.J., "The Maximum Prospective Value Criterion", E.E., Spring 1968, pp. 141-164.

  Analysis of capital budgeting decision in two different settings: 1) Complete information on current and future investment opportunities, 2) Complete information on current but only expectations about future investment opportunities. Analyzes logic underlying the selection of the capital growth (discount) rate that should be used in determining whether a marginal increment of investment should be taken. This leads to Maximum Prospective Value criterion (PV is shown to be a special case) and related to opportunity--cost and marginal--analysis concepts of classical economics (good).

- Pegels, C.C., "A Comparison of Decision Criteria for Capital Investment Decisions", E.E., Summer 1968, pp. 211-220.

  Computer simulation used and capital investment analyzed on seven different decision criteria rather than a single criterion.

- Porterfield, T.S., Investment Decisions and Capital Costs, Review by J. Morley English, E.E., Winter 1968, p. 133.

Paperback book; approached from an elementary point of view; very clear. Main point of book is that ultimate criterion is the way in which the stockholders'wealth can be increased; relates to dividend policy. Suggests an index of expected contributions to market value of stock over that of cash outlays. Last part of book has some very subtle and intricate points presented very simply and clearly. No references supplied.

- Quirin, G.D., The Capital Expenditure Decision, Review by J.E. Ullmann, E.E., Spring 1968, p. 189.

Quantitative by real-world oriented; assesses (somewhat) state of the art and how much still needs to be learned.(Good review)

- Robichek, A.A., Financial Research and Management Decisions, 1967, 232 pp., Review in M.S., Oct. 1968, p. B108.

- Salazar, R.C. and Sen, S.K., "A Simulation of Capital Budgeting Under Uncertainty", M.S., Dec. 1968, pp. B161-B179.

Two types of uncertainty which influence cash flows of the potential investment project. Techniques of simulation and stochastic linear programming are employed using Weingartner's Basic Horizon Model.

- Souder, W.E., "Experiences with an R&D Project Control Model", IEEET-EM, Mar. 1968, pp. 39-49.

Most R&D project cost control systems relate only expenditures with elapsed time rather than expenditures and achievement with time. Achievement reporting is generally considered separately from cost reporting. A theoretical control model relating cost and achievement with time for R&D projects is developed and described; provides early warning of impending project failures, a more conceptual pinpointing of the forces affecting these impending failures, and a detailed analysis of the achievement--per dollar spent. Application shown for Monsanto Co.

- Souder, W.E., "Management Science and Budgeting--Quo Vadis?", Budgeting, Mar.-Apr. 1968, pp. 1-8.

Discussion of practices of management science and budgeting; comparison of two areas reveals that all areas of budgeting such as PPBS and PERT are rudimentary forms of evolving systems in management science.

- Spetzler, C., "Establishing a Corporate Risk Policy", Proceedings of AIIE, 1968, p. 103.

Utility function for corporation.

- Swalm, R.O., "Capital Expenditure Analysis--A Bibliography", E.E., Winter 1968, pp. 105-129.

  Very good; goes back to about 1958 (or 1955); not too O.R.-oriented, but lots of good topics covered such as Industrial Practices, Utility Theory, Comparison of Various Approaches.

- Thorneycroft, W.T., Greener, J.W. and Patrick H.A., "Investment Decisions Under Uncertainty and Variability--Some Practical Experiences of Using Forecasts and Probabilities", O.R.Q., June 1968, pp. 143-160.

  Emphasizes interaction that must exist between management and O.R. analyst. Three examples on investment decisions is given.

- Tuttle, T.L. and Litzenberger, R.H., "Leverage, Diversification, and Capital Market Effects on a Risk-Adjusted Capital Budgeting Framework", J. Fin., June 1968, pp. 427-443.

- Van Horne, J.C., "The Analysis of Uncertainty Resolution in Capital Budgeting for New Products", M.S., Sept. 1968, B376-B386.

- Vaughn, D.E. and Bennet, H., "Adjusting for Risk in the Capital Budget of a Growth-Oriented Company", JFQA, Dec. 1968, pp. 445-462.

  Risk adjusted discount rate.

- Weingartner, H.M., Mathematical Programming and the Analysis of Capital Budgeting Problems, 1967, 265 pp., Review in M.S., Oct. 1968, P. B109.

1967

- Ben-Shahar, H. and Sarnat, Marshall, "Estimating the Cost of Capital Without the Social Expedient of a Security Market", International Management Review (W. Germany), Vol. 7, 1967 (#4-5), pp. 127-134.

  Cost of capital for firms whose securities are not traded in organized market.

- Bernhard, R.H., "Probability and Rates of Return: Some Critical Comments", M.S., Mar. 1967, pp. 598-600.

  Discusses conflict of re-investment rate of internal rate of return; especially in light of risky investments.

• Bernhard, R.H., "The Interdependence of Productive Investment and Financing Decisions", I.E., Oct. 1967, pp. 610-616.

Controversy over three-phase approach to analysis of capital investments on engr. economy is discussed. Bullinger's theory of the economy, financial, and intangible analyses being done separately and in sequence compared to Radnor's theory of analyzing jointly and concurrently. Normative models illustrating the optimality of separability and/or combination of analyses is considered.

• Brigham, E.F. and Smith K.V., "Cost of Capital to the Small Firm", E.E., Fall 1967, pp. 1-26.

• Byrne, R.F., Charnes, A., Cooper, W.W. and Kortanek, K.O., "A Chance Constrained Programming Approach to Capital Budgeting with Portfolio-Type Payback and Liquidity Constraints and Horizon Posture Control", JFQA, Dec. 1967, pp. 339-364.

• Canada, J.R., "The Consideration of Risk and Uncertainty in Capital Investment Analyses", International Management Review (W. Germany), Vol. 7, No. 6, 1967, pp. 47-55.

Survey of approaches to capital investment analysis. Included are risk discounting, sensitivity analysis, probability, decision tree analysis, utility, certainty equivalence, variable discounting, and statistical decision theory. Extensive bibliography.

• Chambers, D., "Programming the Allocation of Funds Subject to the Restricting of Reported Results", O.R.Q., Dec. 1967, pp. 407-432.

Allocation in a firm where 1) most investments are internally funded, 2) reasonable predictions can be made of investment opportunities rising several years ahead, 3) management wishes to take account of the way the allocation will affect other published financial results besides cash flow. Numerical example given on affects on current ratio, return on gross assets, and profit after tax in successive years. These measure related in L.P. formulation. Solution demonstrates how particular projects may attract funds by offering ways of altering published results. Criteria are also developed for whether and when to borrow. Related Solutions show how the existence and timing of future opportunities affect the optimal current allocation and value of the program.

• Cohen, K.J. and Elton, E.J., "Inter-temporal Portfolio Analysis Based on Simulation of Joint Returns", M.S., Sept. 1967, pp. 5-18.

Quadratic programming model for selecting portfolios of risky assets with detailed application to capital budgeting; also application in other areas. Paper develops a new, more efficient way of using simulation to calculate variance, co-variance elements required as input to Markowitz-type model.

- Crecine, J.R., "A Computer Simulation Model of Municipal Budgeting", M.S., July 1967, pp. 786-815.

  Paper presents a positive theory of municipal resource allocation, for large metropolitan communities, which have heretofore been neglected. Cleveland, Detroit and Pittsburgh used as data points.

- Champion, R.R. and Glaser, R.G., "Sugar Cane Irrigation: A Case Study in Capital Budgeting", M.S., Aug. 1967, p. 781.

- Demetriou, P.A., "The Present Value of Investments in Sinking Funds", M.S., Jan. 1967, pp. 336-343.

  Clarifies some basic concepts on P.V. analysis of sinking funds which have been incorrectly used. Also shows how methodology can be applied to investment policy whenever sinking funds serve as temporary storage for idle liquid resources.

- Grunewald, A.E., "Capital Budgeting Strategy", International Management Review (W. Germany), Vol. 7, No. 2-3,1967, pp. 109-116.

  An illustrative presentation of the decision--theoretic formulation of a capital budgeting problem.

- Hankansson, N.H., "Optimal Investment and Consumption Strategies Under Risk, and Uncertain Lifetime and Insurance", UCLA, Western Management Science Institute, Working paper 119, June 1967, (preliminary)(40 pp.).

  Individual investment and consumption strategies are derived for special class of utility functions.

- Hillier, F.S., "Chance Constrained Programming with 0-1 or Bounded Continuous Decision Variables", M.S., Sept. 1967, pp. 34-57.

  Computational methods both exact and approximate with application to work in capital budgeting.

- House, W.C., Jr., "Case Study No. 4: Sensitivity Analysis--A Case Study of the Pipeline Industry", E.E., Spring 1967, pp. 155-165.

  Capital investment sensitivity analysis to such things as cash flows, cost of capital, etc, case study on pipeline co.

- Kaplan, S. and Barish, N.N., "Decision Making Allowing for Uncertainty of Future Investments Alternatives", M.S., June 1967, p. 569.

- Lockett, A.G. and Freeman, P., "Probabilistic Networks and R&D Portfolio Selection", ORQ, Vol. 21, No. 3, 1967, pp. 353-359.

  Math. Prog. extended to use of probabilistic network simulation. Case study based on R&D lab is presented and compared with expected value method.

- Mao, J.C.T. and Know, D., "Analysis of Investment Returns by Computer", E.E., Summer 1967, pp. 229-239.

  Refers to article by Tiechroew, Robichek, and Montalbano in which they consider interdependence of investments. Mao presents a computer program for calculating IROR for pure investments and the functional relationship between IROR and cost of capital if investment is mixed.

- Newman, R.W., "Resource Allocation Under Uncertainty (Four Sets of Time Sharing Programs of the PARSIM Class)", (Report describes GE-265 timesharing, Monte Carlo Simulations, similar to PARSIM, Plant Appropriation Request Simulation, discussed in Decision Making Under Uncertainty, Information Systems Service, GE Co., NYC, April 1967, 31 pp.

- Peterson, C.C., "Integer L.P.", I.E., Aug. 1967, pp. 456-464.

  Overview of I.L.P. with some examples (specifically capital budgeting, etc).

- Seppalla, Y., "Choosing Among Investment Possibilities with Stochastic Pay-Off Minus Expenditure", O.R., Sept.-Oct. 1967, pp. 978-979.

  Multi-period investment problem with stochastic payoff minus expenditure is described. Problem can be solved by Lawler and Bell's discrete optimization method.

- Souder, W.E., "Selecting and Staffing R&D Projects Via Operations Research", Chemical Engr. Progress, Nov. 1967, pp. 27-37.

  Experiences with development, implementation and use of a resource allocation model at a large chemical company are presented. The algorithm used is a 2-0 (time and projects) dynamic programming scheme, with various budgetary constraints used as behavioral simulators. The model views the R&D managers problem as an interdependent triad of project selection, project funding, and resource allocation problems. A concept of R&D "Production" learning curves forms the basis of the model. The optimization algorithm is used parametrically and as a simulation tool, to help R&D managers to heuristically arrive at satisfactory organizational policies in reconciling what are often conflicting goals and objectives.

- Souder, W.E., "Solving Budget Problems with O.R.", Budgeting, July-Aug. 1967, pp. 11-19.

  Cases taken from two-year inter-company study whose objective was to test utility of O.R. in industrial budgeting problems. Author organized study and served as O.R. analyst and consultant to participating firms.

• Speedharan, V.P., (Math. Dept., Mich. State) and Wein, H.H., "A Stochastic, Multistage, Multiproduct Investment Model", SIAM J. Appl. Math, Mar. 1967, pp. 347-358.

N stage multi-product investment program. Given probabilistic estimate of future product(s) demand, seek minimum-minimorum of total expected costs. Solution by combination of dynamic prog. and num. method.

• Wagle, B., "A Statistical Analysis of Risk in Capital Investment Projects", O.R.Q., Mar. 1967, pp. 13-34.

Presents brief survey of various techniques used in measurement of risk in capital investment. Relates to approach presented in Hillier's paper and to his monograph (1964); claims these (Hillier's) approaches were theoretical and discusses approach and presents numerical examples on how this can be implemented in practice. Point of Hillier's analysis starts assuming means and variances of cash flows; these may not be known directly and technique for determining these is presented.

• Weingartner, H.M. and Ness, D.N., "Methods for the Solution of the Multi-dimensional 0/1 Knapsack Problem", O.R., Jan.-Feb. 1967, pp. 83-103. (Also, reprinted in Weingartner's 1967 edition of his book).

1966

• Cohen, K.J. and Fitch, B.P., "The Average Investment Performance Index", M.S., Feb. 1966, pp. B195-B215.

Paper is first step in producing an objective standard for use in measuring the investment performance of any portfolio of securities over some period of time. Compares with other methods for evaluating performance.

• Hankannson, N.H., "Optimal Investment and Consumption Strategies for a Class of Utility Functions", Working Paper, No. 101, Western Management Science Instritute, Doctoral Dissertation, UCLA, June 1966, 131 pp.

• Hillier, F.S. and Heebink, D.V., "Evaluating Risky Capital Investments", California Management Review, Winter 1965, pp. 71-80.

• Horowitz, I., "The Plant Investment Decision Revisitied", I.E., Aug. 1966, pp. 416-422.

P.V. and rate of return are criteria used for plant investment decision. Cash flow data are needed and it is important to assign the correct distribution (normal or non-normal) to each parameter.

- Mansfield, E. and Brandenburg, R., "The Allocation, Characteristics, and Outcome of the Firm's Research and Development Portfolio: A Case Study", J. of Bus., Vol. 39, No. 4, 1966, pp. 447-464.

- Mao, J.C.T. and Sarndal, E.C., "A Decision Theory Approach to Portfolio Selection", M.S., Apr. 1966.

  Brief summary of Markowitz's portfolio selection model and proceeds to reformulate it within the framework of modern statistical decision theory. Future returns from securities are veiwed as a function of the unknown state of nature of which there are certain *a priori* probabilities and following the Bayesian strategy the investor chooses that portfolio which maximizes the weighted average of payoffs; a computer program based on the critical line method is used to solve a simple illustrative problem.

- Mao, J.C.T., "An Analysis of Criteria for Investment and Financing Decisions Under Certainty: A Comment", M.S., Nov. 1966, pp. 289-291.

  Corrections to papers by Teichroew, Robichek, and Montalbano, M.S., Vol. 12, pp. 151-179 and 195-403, concerning simple and non-simple projects' rate of return.

- Paine, N.R., "A Case Study in Mathematical Programming of Portfolio Selections", Applied Statistics (U.K.), Vol. 15, No. 1, 1966, pp. 24-36.

  Analysis of 40 American Stocks for period 1946-54 to determine efficient portfolios and compared with hypothetical portfolio constructed by purchasing equal amounts of each stock in group of 40--Efficient group compared with hypothetical group for period 1955-1959.

- Peterson, C.C., "Computational Experience with Variants of the Balas Algorithm Applied to the Selection of R&D Projects", M.S., May 1966, pp. 736-750.

- Robichek, A. and Myers, S.C., "Conceptual Problems in the Use of Risk Adjusted Discount Rates", J. Fin., Dec. 1966.

- Rubenstein, A.H., "Economic Evaluation of R&D: A Brief Survey of Theory Practice", I.E., Nov. 1966, pp. 615-620.

  How it's done in practice as opposed to how it's done in theory; and how to bridge the gap.

- Schoomer, B.A., Jr., "Optimal Depreciation Strategy for Income Tax Purposes", M.S., Aug. 1966, pp. B552-B579.

  Use depreciation method which maximizes P.V. of cash flow due to depreciation; choice depends on no. of parameters and a flow chart for making decision is given.

- Souder, W.E., "Planning R&D Expenditures with the Aid of a Computer", Budgeting, Mar. 1966, pp. 25-32.

  Case studies at Monsanto using Dynamic Programming for R&D and budgeting.

- Swalm, R.O., "Utility Theory--Insights into Risk Taking", HBR. Nov.-Dec. 1966, pp. 123-126.

- Weingartner, H.M., "Capital Budgeting of Interrelated Projects: Survey and Synthesis", M.S., Mar. 1966, pp. 485-516.

  Survey of techniques available; lin. prog., dyn. prog., integer prog., Reiter's discrete optimization technique. A dyn. prog. code for the 0-1 knapsack (multi-dimensional) problem is presented. (Also reprinted in 1967 edition of his book).

- Weingartner, H.M., "Criteria for Programming Investment Project Selection", J. of Indus. Econ., Nov. 1966, pp. 65-76. (Also reprinted in 1967 edition of his book).

- Williams, A.C. and Nassar, J.I., "Financial Measurement of Capital Investments", M.S., July 1966, pp. 851-864.

  Set of axioms proposed for preference ordering over investments with different cash flows in each period, and where the level of these cash flows (not necessarily discounted) is the criterion for investment selection. The result is that the only "consistent" method for ranking is by P.V. with positive undetermined interest rates, possibly different for each time period. Finally, an axiom specifying "temporal consistency" leads to the use of a single interest rate. A consideration overlooked in the article is that utility of cash flow may not be the proper criterion for investment decisions of corporations.

1965 and earlier

- Baumol, W.J. and Quandt, R.E., "Investment and Discount Rates Under Capital Rationing-- A programming Approach", The Economic Journal, Vol. LXXV, 1965, pp. 317-329.

- Bernhard, R.H., "Discount Methods for Expenditure Evaluation, A Clarification of Their Assumptions", I.E., Jan.-Feb. 1962.

- Canada, J.R., "R of R: A Comparison Between the Discounted Cash Flow Model and a Model which Assumes an Explicit Re-investment Rate for the Uniform Income Flow Case, E.E., Spring 1964.

- Cord, J. (A.O. Smith Corp., Milwaukee, Wisconsin), "A Method for Allocating Funds to Investment Projects When Returns are Subject to Uncertainty", M.S., Jan. 1964, pp. 335-341.

  A method is developed for optimally selecting capital investments with uncertain returns, under conditions of limited funds and a constraint on the maximum average variance allowed in the final investment package. The concepts involved in the analysis are somewhat related to Markowitz's work on the portfolio problem. Dynamic programming is used along with a Lagrange multiplier for the variance constraint. Independence of cash flows and a single period budget outlay is assumed.

- Farrar, D.E., The Investment Decision Under Uncertainty, 1962.

- Hahn, F.H. and Brechling, F.P.R., editors, The Theory of Interest Rates, "The Theory of Portfolio Selection", 1965.

- Hertz, D.B., "Risk Analysis in Capital Investment", HBR, Jan.-Feb. 1964, pp. 95-106.

- Hespos, R.F. and Strassmann, P.A., "Stochastic Decision Trees for the Analysis of Investment Decisions", M.S., Aug. 1965, pp. B244-B259.

- Hirshleifer, J., "Efficient Allocation of Capital in an Uncertain World", Amer. Econ. Review, May 1964, p. 77.

  Most attention on portfolios, little or none to productive investments--good references and several comments by other authorities follow article.

- Litner, J., "The Evaluation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", Review of Economics and Statistics, Feb. 1965, pp. 13-37.

- Magee, J.F., "How to Use Decision Trees in Capital Investment", HBR, Sept.-Oct. 1964, pp. 79-96.

- Markowitz, Harry, Portfolio Selections: Efficient Diversification of Investments, 1959.

- Pollack, G.A., "The Capital Budgeting Controversy: Present Value vs Discounted Cash Flow Method", National Assoc. of Accountants Bulletin, Nov. 1961. (Also in Management of Capital Expenditure, R.G. Murdick and D.D. Deming, McGraw-Hill, 1968).

  Article points out that present value and discounted cash flow methods as criteria for equipment purchase decisions can give distinctly different results and illustrates with numerical examples. The argument that the present value method is a simpler and better method of evaluation is presented.

- Paine, N.R., "Uncertainty and Capital Budgeting", Accounting Review, Apr. 1964, p. 330.

- Reisman, A., "The Cost of Capital: A Reconciliation of Some Existing Theories Through Generalization", UCLA Working paper #57, Nov. 1964, 22 pp.

  Paper presents generalization of area and shows similarities and differences of existing theories.

- Sharpe, W.F., "A Simplified Model for Portfolio Analysis", M.S., Jan. 1963.

- Solomon, M.B., Jr., "Uncertainty and Its Effect on Capital Investment Analysis", M.S., Apr. 1966.

  Sensitivity analysis shows uncertainty really important. Discusses problem of uncertainty in costs, revenues, and project life and relates how classical measure of capital investment return are of limited value. It is shown by sensitivity analysis that relatively small overestimates and underestimates create relatively large errors in the discounted rate of return for different types of return schedules. Article concludes that businessmen are perhaps justified in seeking other methods of project evaluation.

- Souder, W.E. and Rosen, E.M., "A Method for Allocating R&D Expenditures", IEEET-EM, Sept. 1965, pp. 87-93.

  Paper shows how a slight modification of Hess' approach to project selection and an analogy to the theoretical economics of a multi-product factory have been used to aid management in projection selection.

- Teichroew, D., Robichek, A.A. and Montalbano, M., "An Analysis of Criteria for Investment and Financing Decisions Under Certainty", M.S., Nov. 1965, pp. 151-179.

  Paper investigates decision-making procedure for accepting or rejecting investment or financing alternatives available to the firm. The properties of the decision rules based on discounted present value and internal rate of return are studies for the class of projects described by a finite sequence of cash flows. The necessary and sufficient conditions under which the decision rules lead to unique solutions are derived. When unique solutions are not obtainable, two rates must be defined--the project investment rate and the project financing rate. The extension of the project analysis in terms of the two rates permits the derivation of unambiguous decision rules for all projects.