A MODEL FOR ION HEATING IN MODULATED BEAM EXPERIMENTS

A Dissertation
Presented to
the Faculty of the Department of Physics
University of Houston

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

---

by
Theodore Varry Lautzenhiser

December 1970

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ABSTRACT

In long, narrow, electron beam formed plasmas energetic ions have been observed when the electron beam was modulated at frequencies ranging from one to ten times the ion cyclotron frequency.* Here a model has been developed which forms a basis for understanding the observed ion heating. The problem of finding the ion energy in a long, narrow, plasma column coaxially traversed by a strongly modulated electron beam is too difficult to handle theoretically at present. Consequently, the problem of understanding the observed ion heating is broken into two sections. Proton trajectories are calculated in ad-hoc fields - appropriate to a 60 ma 1 KeV electron beam 0.3 cm in radius, in a plasma dielectric rod - reveal a net energy transfer to the ions from the beam generated electric fields at frequencies ranging from one to ten times the cyclotron frequency. Typically, for a modulation frequency at an ion cyclotron harmonic and near the resonant frequency of the plasma dielectric rod, 6.2 MHz, ion energies increases of $\sim$100 eV per cyclotron period are calculated, and at a non cyclotron harmonic frequency $\sim$30 eV per cyclotron period.

To experimentally show the existence of radial electric fields similar to the ad-hoc electric fields used in the proton trajectory calculations, a new test ion beam diagnostic is developed and used. A heavy ion beam is passed transversly through a plasma column and the energy change of the ion beam is measured, determining the time

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* G.M. Haas and R.A. Dandl, Phys. Fluids 10, 678 (1967)

varying electric fields of the plasma column.

In the experiment to test for the electric fields, a 560 eV Argon beam is passed through a $\sim 5 \cdot 10^9/\text{cm}^3$ Helium plasma column of ~ 0.2 cm radius. Typically, a ~ 2 ma electron beam passing longitudinally through the plasma is modulated at ~ 7 MHz, corresponding roughly to the Helium plasma frequency, and energy shifts in the Argon test beam of 6 eV are observed, indicating plasma dielectric enhancement of the electron beam modulation fields by a factor of 7. Consequently, it is felt that the observed heating can be understood on the basis of the model.

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CHAPTER I

INTRODUCTION

Plasma, loosely a gaseous state with appreciable ionization, is the predominant state of matter in the universe. For instance, the interstellar plasma has "small" density perturbations, stars - which are also plasmas. Even on Earth, plasmas are important. For example, Earth plasmas include flames, the ionosphere, and gas discharges such as those used in fluorescent lamps.

Although there are many problems in plasma physics, the primary emphasis currently is on the problem of achieving a Controlled Thermonuclear Reaction (CTR). This problem has two formidable parts whose solution still eludes us; confining a hot dense plasma, and heating trapped plasma ions to "ignite" the CTR.

At present there appear to be only two practical ways to confine a CTR; gravitationally, such as in the sun, and magnetically. All laboratory schemes to confine a hot plasma have involved use of a magnetic "bottle", and consequently, heating plasmas confined in magnetic fields is of prime interest.

Because of the high mobility of electrons and ions, it is difficult to get an electric field into a plasma. The surface of the plasma carries the great majority of the current. Because of the mass ratios, electrons normally receive more energy from an externally impressed field than do the ions. Also, the collisional electron-ion thermalization time is much longer than the electron-electron, ion-ion, or ion-neutral thermalization times. This is readily observed in a fluorescent lamp where the electric fields result in electron temperatures of $10{,}000^{\circ}$ C. The ion thermalized with the neutrals are roughly at the same temperature as the lamp surface, which is cool enough so that it can be touched.

Nevertheless, many of the present efforts toward CTR involve massive ohmic heating of the electrons with the hope that the ions can be kept isolated from the walls long enough to be heated by electron-ion collisions. At this point the role of plasma diffusion to the walls of the machine enters in. If the diffusion rate of the plasma were that calculated for a stable plasma, ions indeed could be heated effectively. However, anomalous diffusion due to plasma instabilities seems to make the plasma lifetime, after the ohmic heating, too short for appreciable energy to be gained by the ions. It should be noted that the Tokomak machines developed by the Russians seem to have a diffusion rate which approaches the normal rate, when they operate at their present levels.[1]

It has been found that turbulence can increase the rate of the transfer of energy from the electrons to the ions after the electrons have been heated.[2] However, this same turbulence also increases the plasma diffusion rate to the walls, resulting in shorter plasma lifetimes than that of nonturbulent plasmas. Studies of stochastic heating, heating by random noise electric fields, show ion heating proportional to the Fourier component of the electric field at the ion cyclotron frequency, $\omega_{CI} = eB/m$, where m is the ion mass.[3,4,5] In turbulent heating, although there are Fourier components of the electric field over a large frequency range only a small portion of the spectrum is useful for heating, hence turbulent heating is inefficient.

In the Stellarator large external electric fields at the ion cyclotron frequency are used for ion heating with the hope that these fields can penetrate the plasma if they are sufficiently strong.[6] Significant ion heating has been achieved but only within a small volume.

Haas and Dandl[7], however, applied an impressed field from within a plasma, using the varying charge density of a modulated electron beam to impress time varying electric fields in the resulting beam formed plasma. They observed ion heating when the beam modulation frequency was in the vicinity of the ion cyclotron frequency. However, they observed much more ion heating when the modulation frequency was near the ion plasma frequency, $\omega_{PI} = (nq^2/m\epsilon_0)^{\frac{1}{2}}$, which was several times the ion cyclotron frequency for their experiment.

It is possible that this heating results from some special properties of beam plasma interactions in low density beam formed plasmas and as such would have little relevance to the CTR heating problem. On the other hand, if one can excite proper oscillation modes in dense highly ionized plasmas through the use of properly modulated beams, this arrangement could provide a vastly more efficient means of heating a plasma and could be of great importance for CTR. It is therefore considered to be of interest to understand the mechanism envolved in the Haas and Dandl experiment.

The problem of finding the ion temperature in a long, narrow, plasma coaxial with a strongly modulated electron beam in a self consistent manner is too difficult to handle theoretically at present, though foreseeably a new generation of computers might handle some aspects of the problem.

Consequently, the problem is broken into two sections. The problem in the first section is to find electric fields - not necesarily at the ion cyclotron frequency - capable of heating ions. Plausible electric field configurations (from a modulated electron beam in a plasma dielectric rod) at various frequencies are tested for their effects on ions by numerically calculating the appropriate ion trajectories. The electric fields associated with beam plasma interactions have been investigated[8,9,10] but in plasma, electron beam, and frequency regimes other than that where the ion heating was observed by Haas and Dandl. Consequently, the problem in the second section is to show experimentally the existence of electric fields similar to the ad-hoc fields used in the numerical calculations of ion heating when a modulated electron beam is passed through a non-beam formed plasma.

CHAPTER II

COMPUTER CALCULATIONS

Physics research is an effort to find a model similar enough to some aspect of the world so that calculations based on that model will sucessfully predict the course of the studied aspect of the world. Here we wish to construct a numerical model simulating the beam modulated heating experiments of Haas and Dandl[7] and Eisner and Haas.[11] The model is to contain the essential features of these experiments so that calculations based on the model will predict the results of the experiments, and that parameter studies of the model can be used to predict productive directions for future experiments.

The experiments which are to be simulated were both done on a long, narrow, electron beam formed plasma column confined in a longitudinal magnetic field. The purpose of the experiments was to heat plasma ions when the electron beam was modulated. In fact, energetic ions were observed when the electron beam modulation frequency was in a small range which depended on the plasma density. The experimental parameters were: plasma radius, $a = 0.3$ cm; plasma length, $L \sim 1$m; the electron beam, ~ 150 ma at 2 KV in the Haas and Dandl experiment and ~ 60 ma at 1 KV in the Eisner and Haas experiment; proton cyclotron frequency, $f_{CI} \sim 1$ MHz; proton plasma frequency, $f_{PI} \sim 7$ MHz, and the energetic ions were observed for a modulation frequency, f, on the order of 6 MHz.

The concept of the model is that the electric field of an energetic modulated electron beam are not much affected by ion heating.

The effect on hot ions can be obtained by single ion dynamics in the electric field of the beam and the cold background plasma. The impressed field is that calculated from the charge introduced by the beam, modified by the dielectric response of the plasma. This model is not self consistent, as any effect on the plasma dielectric function due to increased ion energy is ignored. Indeed, some skeptics have surmised the self consistent effect of the dielectric on the electron beam fields is to cancel these fields. Nevertheless the experimental observation of electric fields, similar to those postulated in the model, in a modulated beam experiment is our criterion for validating the model. Some simplification of the model results from geometric considerations of the experiment.

Since the plasma radius, a, is much smaller than the characteristic modulation wave length of the electron beam velocity divided by the modulation frequency, $\frac{a\,\omega}{v_e} \sim 10^{-3}$, the electron bursts are long and narrow. An ion near, or within the electron beam would then see the radial field of a long charged cylinder, except near the ends of the electron bursts. The longitudinal electric fields associated with the ends of the electron bursts would pass the ion at the electron beam velocity and would appear as a series of longitudinal impulses with Fourier frequency components several times that of the modulation frequency. The longitudinal impulse at the beginning of a burst would be opposite in direction to the impulse at the end of the burst and consequently, averaged over several bursts, the effects of these impulses on an ions motion would tend to cancel out, unless the ions longitudinal velocity was nearly that of the electron beam - a condiion requiring an ion energy of approximately an MeV. Since no MeV ions are presumed to exist in the experiments under consideration, the effect of the longitudinal electric fields associated with an electron burst can be neglected in the ion dynamics and the ion motion can be handled two dimensionally, in the transverse plane.

Assuming uniform charge density within both the electron beam and the plasma. and assuming only dielectric effects in the plasma response to the electron beam, the radial electric field can be written as

$$\begin{aligned} E\,(r,t) &= (\rho r/2\epsilon)\sin\omega t \qquad \text{for } r<a \\ &= (\rho a^2/2\epsilon_0 r)\sin\omega t \qquad \text{for } r>a \end{aligned} \tag{2-1}$$

where ρ is the electron beam charge density and, ϵ , the plasma dielectric response is[12]

$$\epsilon/\epsilon_0 = 1 - \sum_{j=I,e} \frac{\omega_{Pj}^2}{\omega^2 - \omega_{Cj}^2} \tag{2-2}$$

which for $\omega \ll \omega_{Pe} \ll \omega_{Ce}$ can be simplified to

$$\epsilon/\epsilon_0 = (\omega^2 - \omega_{CI}^2 - \omega_{PI}^2)/(\omega^2 - \omega_{CI}^2) \tag{2-3}$$

This dielectric response is calculated for an infinitesimal driving electric field, and near a resonance, $\overline{\epsilon \to 0}$, the experimental modulated beam current would drive the electric fields into saturation. Consequently, the results from the model, while qualitatively correct, are not quantitatively correct near the plasma resonance. This is also a result of the calculations not being self consistent.

Because the electric field is radial, angular momentum is conserved in a rotating coordinate system, and the transverse motion of an ion of charge q and mass m is given by the solution of

$$\ddot{r} = \frac{q}{m} E\ (r,t) + r\,\dot{\theta}\ (\dot{\theta} + \frac{q}{m} B)$$

$$\ddot{\theta} = - \dot{r}(2\dot{\theta} + \frac{q}{m} B)/r \qquad (2\text{-}4)$$

the Lorentz force components. For time varying electric fields, Eq. (2-4) can not be integrated analytically so the equation was integrated numerically on a computer in order to follow the motion of an ion in the assumed electric field, and to calculate the kinetic energy of such an ion, as a function of time, for a given set of initial conditions.

The initial conditions of the ions can be specified by their initial kinetic energy, their radial coordinate, and the value of corresponding to their angular momentum. The initial condition on the electric field is specified by giving its phase relative to an arbitrary reference at the start of the orbit. An ensemble consisting of 20 ions whose phasing with respect to the electric field at the start on the orbits was varied in 20 equal steps of 18° was used, and the average of the 20 energies so obtained was the value used for the ensemble kinetic energy gain for the given parameters. The motion was followed for a time of 5 ion cyclotron periods, a time comparable to the charge exchange lifetime of ions in the neutral background of the experiments.

Runge-Kutta[13] methods were used to allow variable time steps in the integration since singularities at r=0 and in the gradient of the electric field at r=a requires a time step size such that the run time would be enormous if the step size were not increased away from these regions. A step size of

$$\Delta t = A + B\, r\,|(r-a)/(r+a)| \tag{2-5}$$

was used, where A and B were adjusted so that over 5 cyclotron periods the error in the energy was less than 1% for test runs made with constant electric fields where the kinetic energy of the ions was analytically calculable.

The parameters used in the calculations were chosen to approximate the parameters of the Haas and Dandl[7] and the Eisner and Haas[11] experiments. Protons were assumed moving in a system with a= 0.3 cm, $f_{CI} = 1$ MHz, $f_{PI} = 6$ MHz, and $\rho = 1.1\cdot 10^{-10}$ coul/cm^3, appropriate to a 60 ma, 1 KV electron beam. The initial condition for the protons were $r_o = 0.1$ cm, $\dot{\theta} = 3.9 \cdot 10^7$/sec and with 8 eV of kinetic energy - appropriate to the Frank-Condon dissociation for the hydrogen molecule peaked at 8.6 eV.[9] The parameters are not crucial to the results, but are typical of those expected in the experiments.

To show the effect of the electric field enhancement on the ion energies due to a resonant dielectric, for the values of $\epsilon = 0.1\,\epsilon_o$

and $\epsilon = \epsilon_o$, the energy of the ion ensemble at t=5 cyclotron periods, $5t_{CI}$, was calculated for six values of modulation frequency between and including two ion cyclotron harmonic frequencies, and the results are shown in Figure 1. The maximum energy of the ions occurs when the modulation frequency is a cyclotron harmonic, while there are broad minimums in energy for modulation frequencies with values between the harmonics. It is the energy of the broad minima that reflects chiefly the difference in the value of the dielectric constant used in the calculation, with the energy of the ions in the resonant dielectric, $\epsilon_o/\epsilon = 10$, being significantly higher than the energy of the ions in the non-resonant dielectric, $\epsilon_o/\epsilon = 1$. These trends are also characteristic of the calculations at the other frequencies discussed.

The time evolution of the ion energy distribution for modulation at a ion cyclotron harmonic frequency and at a non-harmonic frequency are shown in Figure 2. The distribution at the cyclotron harmonic is not dispersed as the whole distribution increases in energy by approximately the same large step each period, ie. 100 eV per cyclotron period. Such an increase is appropriate for an ion which is in the correct phase of its orbit for each entry into the plasma such that it is both pulled into and pushed out of the plasma region. The ion motion is in phase with the electric field in the strong field region, ie. in the plasma, and though the phase lock might be lost after a sufficient time, the experimental ion lifetime is of the order of 5 cyclotron periods and this loss of heating phase is not seen and all the ions have large energy increases.

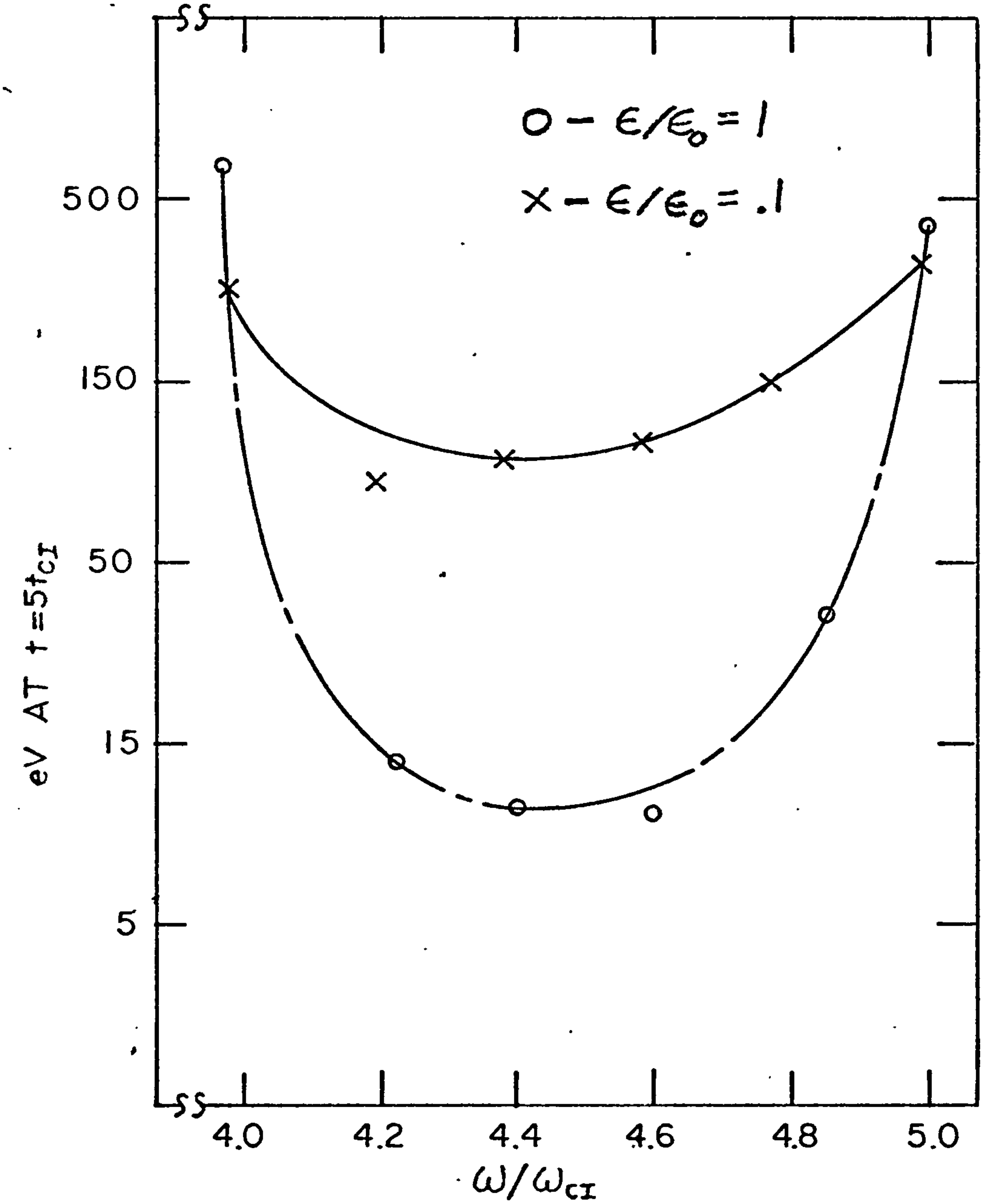


Figure 1. The ion ensemble energy at $t=5t_{CI}$ for a 60 ma, 0.3 cm radius electron beam for modulation frequencies from $4\omega_{CI}$ to $5\omega_{CI}$ with two dielectric values.

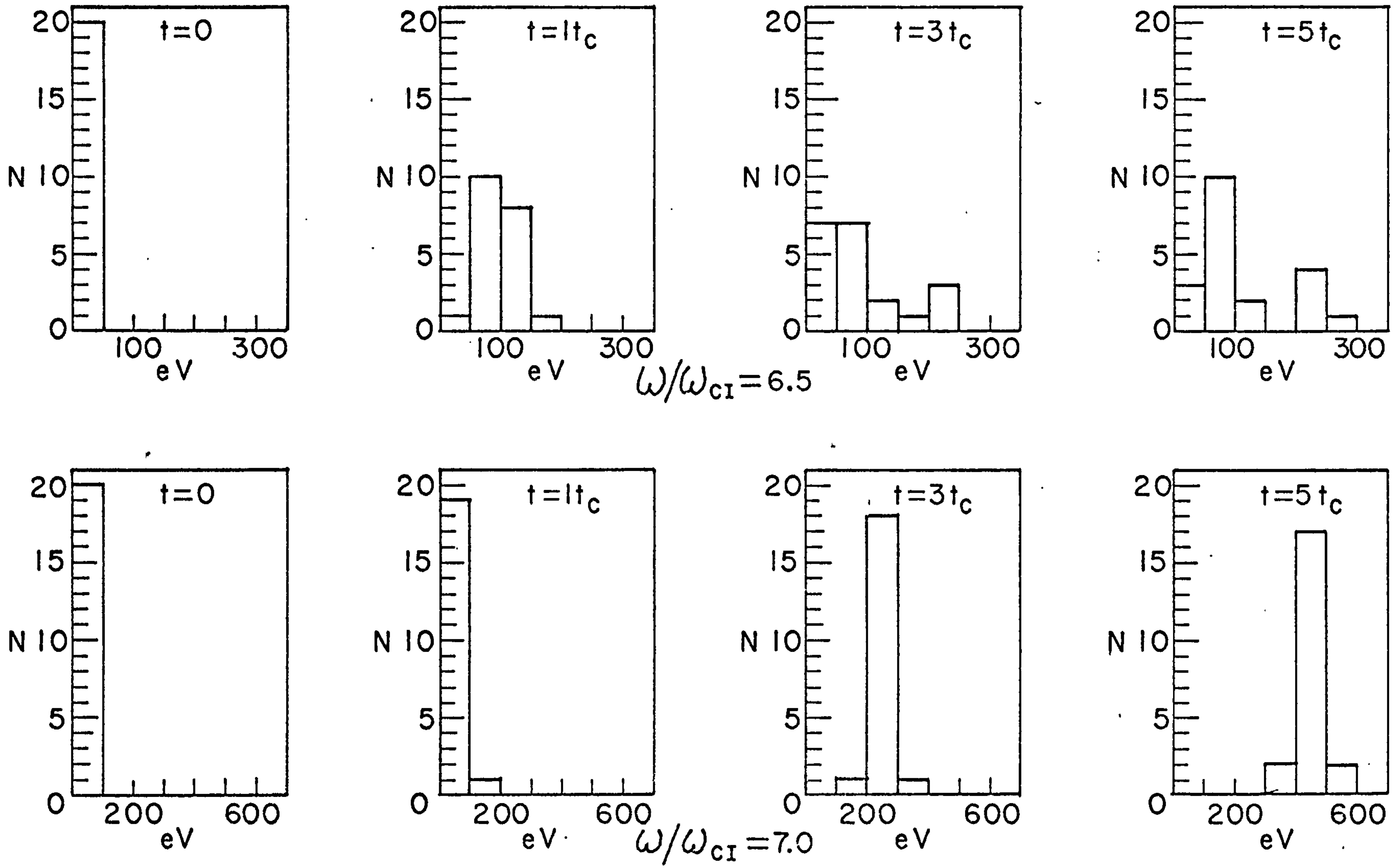


Figure 2. The time evolution of the ion energy distribution for ω/ω_{CI} = 6.5 with ϵ/ϵ_o =.14 and for ω/ω_{CI} = 7 with ϵ/ϵ_o = .25.

The distribution at the off harmonic frequency has an entirely different character. Here the ion can not keep the correct phase for a maximum increase in energy, and hence sometimes lose energy. However, there can be no negative kinetic energies, so as they gain or lose energy, on the average, they gain energy; akin to a one dimensional random walk problem with no negative values allowed, which results in a net change in the average displacement. The ion energy change is stochastic in nature which results in the ion distribution diffusing away from the forbidden area, KE < 0, but only a small portion of all the ions have large energy increases. The observed difference between the rate of energy gain for ions in modulation fields with frequencies at ion cyclotron harmonic to those at non-harmonic frequencies is then reasonable, according to this view.

In the experiments which we are trying to model, the magnetic field was that of a magnetic mirror, and was, then, not constant along the longitudinal axis. Since the ions would be expected to be moving longitudinally, it is expected that the ions would not remain in the magnetic field at which the modulation frequency was at a cyclotron harmonic frequency long enough to acquire the phase necessary to increase their energy at a rate predicted from the calculations for a cyclotron harmonic. The energies of the ions at the broad minima are then expected to correspond to the ion energies in the experiments utilizing a magnetic mirror configuration. Using the dielectric function of Eq. (2-2), and for comparison using $\underline{\epsilon = \epsilon_0}$, the energy of the ion ensemble was calculated over the frequency range 0.5 to 8.5 times the ion cyclotron frequencies and the results are shown in Figure

3, where also the dielectric function of Eq. (2-2) is plotted for the parameters used in the calculations. The large increase in the ion energies for modulation frequencies near the plasma resonance is clear, though the magnitude of the energy increase may not be valid because of the use of the cold dielectric function.

If an experiment were to be made in a uniform magnetic field, a peak in ion energies when the modulation frequency was a cyclotron harmonic would be expected. Using the dielectric function of Eq. (2-2) shown in Figure 3, and for comparison using $\epsilon = \epsilon_0$, the energy of the ion ensemble was calculated over the frequency range 1 to 8 times the ion cyclotron frequency at the harmonic frequencies and the results are shown in Figure 4. As noted before, the order of magnitude increase in the ion energies when the modulation is at the ion cyclotron harmonic frequencies here, as opposed to the off harmonic frequencies shown in Figure 3, in obvious. There are also other differences. The calculations with $\epsilon = \epsilon(\omega)$ show a sharp rolloff in energy below the resonant frequency and a sharp increase in energy at the resonant frequency. The rolloff in energy below the plasma resonance is due to the opposite polarity of the field outside and inside the plasma, as signified by the negative sign of the dielectric function in this region. The effect of this field reversal is that the change in energy arising from the plasma electric field is offset by the external electric field, to some extent. If one field strength is much greater than the other, this effect is not important. For instance, at frequencies below the third ion cyclotron harmonic frequency, the plasma field is very small compared to the external field, and there is ion heating. However, in

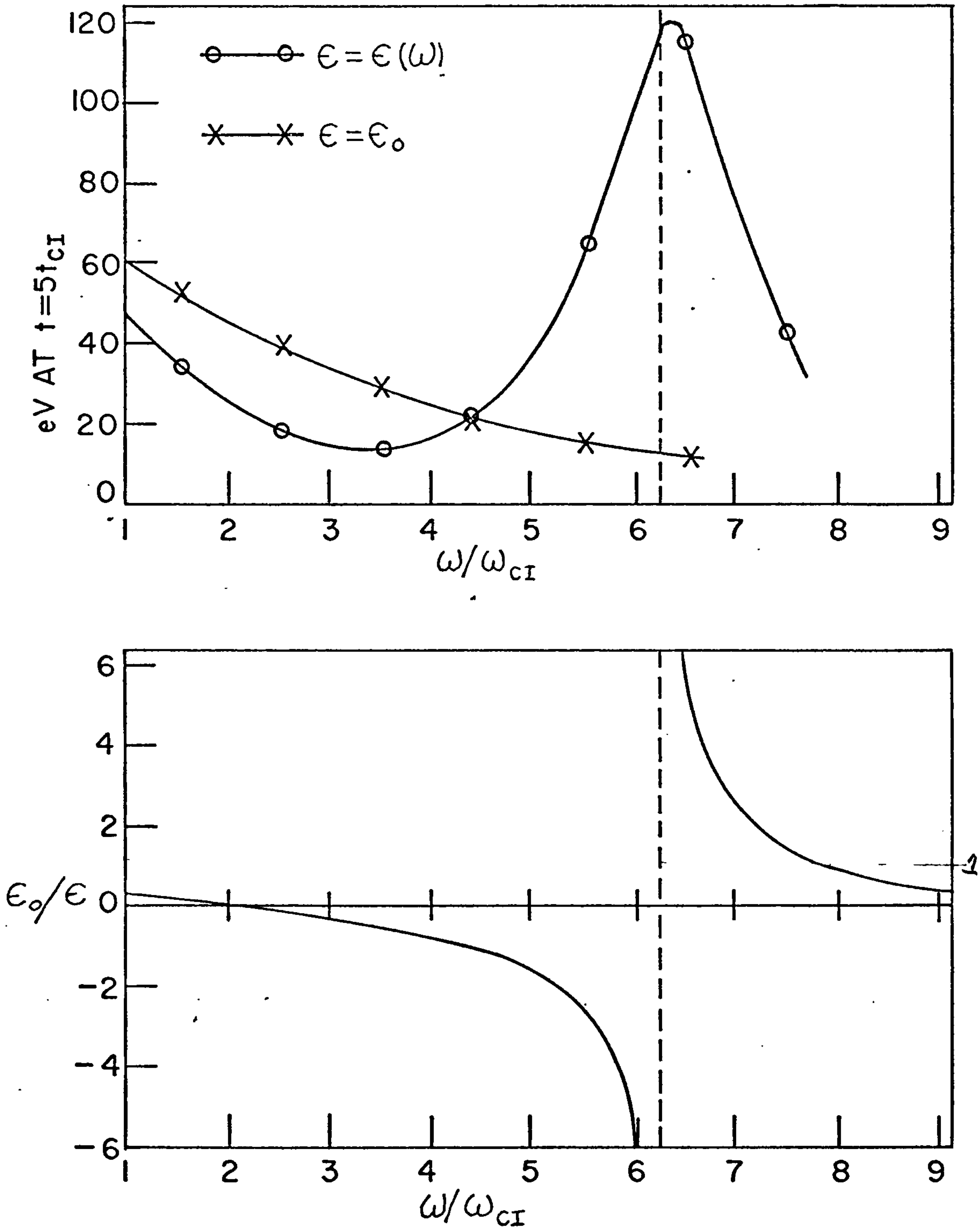


Figure 3. The ion ensemble energy at $t = 5\ t_{CI}$ at $\omega = (n+\frac{1}{2})\,\omega_{CI}$ with ϵ as plotted and with $\epsilon = \epsilon_o$.

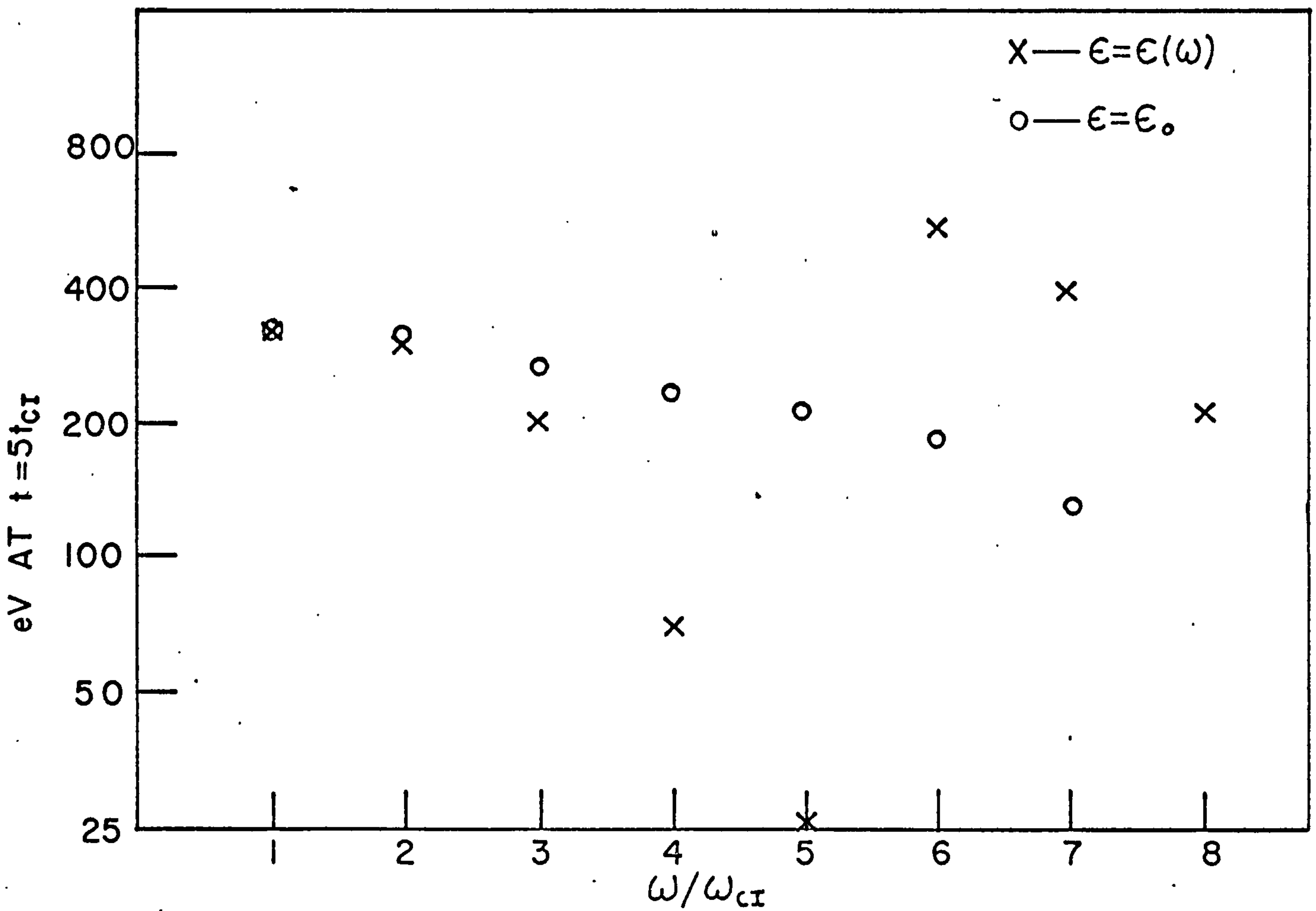


Figure 4. The ion ensemble energy at $t = 5\ t_{CI}$ at $\omega = n\,\omega_{CI}$ with ϵ as plotted in Figure 3 and with $\epsilon = \epsilon_o$.

the region where the fields are comparable, $\omega = 4\omega_{CI}$, there is reduced ion heating.

Contrary to what is found in most plasma heating methods, electrons are not readily heated by the electric fields heating the ions in this model. For the parameters used in the calculations, from Eq. (2-3), since the electrons have a radial acceleration due to the electric field of 1/60 the acceleration due to the magnetic field, the electron radial motion is barely perturbed by the electric field. Stated in another manner, the electric field strength is not sufficient to give an electron significant energy over a distance comparable to the electron Larmor diameter, and since the electron cyclotron frequency is ~ 2 GHz, compared to the modulation frequency of ~ 10 MHz, a guiding center approximation is valid for the motion of the electron. The electron guiding center velocity is E/B in the θ direction and for the experimental parameters the guiding center rotates back and forth about the axis approximately 1 revolution each half cycle.

In summary, the model developed to approximate the modulated beam heating experiments of Haas and Dandl and Eisner and Haas shows ion heating when the beam modulation frequency is near a plasma resonance where electric fields are strong for a given beam modulation. This calculated heating is at a frequency several times the ion cyclotron frequency. This corresponds to the experimental observations. The model is not self consistent. An ion is assumed to move in the field of a background plasma and modulated electron beam, but the field due to the ion is not fed back to the plasma. Therefore in the case of heating of a significant portion of the ions this model would require the use of a dielectric function which would apply to a warm plasma.

It remains to be shown experimentally that radial electric fields similar to those assumed in the model are in fact present in a plasma — the topic of the next chapter.

CHAPTER III

DIAGNOSTICS

The fact that the radial electric fields of Eq. (2-1) can accelerate ions has been demonstrated by numerical calculation. Some critics have expressed doubt; nay, some disbelief; that fields of this nature can be produced in a plasma column by modulating an electron beam. The model is not self consistent as the cold plasma dielectric is used in the calculation of the plasma electric fields when it is shown that the fields would heat the plasma ions. However, because of the difficulty of analyzing the problem of ion heating by electron beam modulation in a self consistent manner the ad-hoc electric field was used in the calculations of ion heating, and an experiment showing the existence of electric fields similar to those ad-hoc fields is then required. We here present the theory of the technique used in the experimental demonstration of these electric fields.

A technique using the energy change of a beam of test ions was used in the experiment rather than a probe technique because the plasma column would be expected to be seriously disturbed by the introduction of an electric field probe leaving the electric field probe measurements in doubt. Although Afrosimov and Gladkovskii[5] used a test ion beam to probe the external electric field of a plasma column in a study of the flute instability, it is believed that the technique has not been extended to probing electric fields inside the plasma.

The test particle method adopted consisted of shooting a test ion beam perpendicularly through a magnetically confined plasma column and measuring the change in the energy of the test particles induced by the local electric field. The test particle used is more massive than the plasma ions, and consequently, for equal speeds, has a greater Larmor radius. In the calculations, the energy of the test ion is chosen sufficiently large so that the departure of the test ion path from a straight line because of the magnetic field can be neglected.

The test ion might be affected by the electric field in two ways: the energy might be changed and the ion orbit might be changed. It will be shown that the deflection of the ion orbit due to the electric fields is too small to be easily measured so that the change in energy is the useful effect. The change in the energy of the test ion is

$$\Delta \mathcal{E} = \int_{l} q\overline{E} \cdot d\overline{l} \tag{3-1}$$

Because the velocity change of the test ion is negligible in determining the location along its trajectory, (the test ion initial energy is much greater than the change in energy due to the electric fields) and because the test ion Larmor radius is large compared to the plasma radius a, ie. the test ion energy and mass are much greater than those of the plasma ions, the radial position of the test ions with impact parameter b=0 can be written simply as $r=v/t/$, where v is the velocity of the test ion and t is measured from the time of arrival at the beam center. Using this to eliminate t in the expression for the electric fields of the model, Eq. (2-1), and substituting into Eq. (3-1), we have

$$\Delta \mathcal{E} = \left\{ \frac{a\mathcal{Z}_o}{\epsilon_o} \left[\int_{r_{wall}}^{a} \sin \frac{\omega r}{v} \frac{dr}{r} + \frac{\epsilon_o}{\epsilon} \int_{a}^{o} r \sin \frac{\omega r}{v} \, dr \right] \right\} \cos \phi = \Delta \mathcal{E}_{max} \cos \phi \tag{3-2}$$

where ϕ, the phase angle between the arrival of the test ion at the center of the beam and a zero of the electric field, is random. Although the calculations are made using electric fields of the functional form assumed in the model, the primary effect of using other functional forms, such as $J_0(Br)$, would be chiefly to make the calculations more difficult without changing the character of the result.

$\Delta \mathcal{E}_{max}$ then consists of two integrals, the first being of the form sin y/y, giving the energy change due to the external electric field, and a second elementory integral (y sin y) giving the energy change due to the plasma electric field. The sin y/y integral is the well tabulated Sine integral denoted by Si(x),[16] which is the integral from 0 to x. Then integrating Eq. (3-2), we have

$$\Delta \mathcal{E}_{max} = \frac{a^2}{\epsilon_o}\left\{\left[\mathrm{Si}\left(x\,\frac{r_{wall}}{a}\right) - \mathrm{Si}(x)\right] + \frac{\epsilon_o}{\epsilon}\frac{1}{x^2}\left[\sin x - x\cos x\right]\right\} \tag{3-3}$$

where $x = \omega a/v$ which is π times the ion transit time of the plasma column divided by the modulation period. The two components of $\Delta\mathcal{E}_{max}$ are shown vs. x in Figure 5. For $x = .1$, or $v = 10\omega a$, the test ion transverses the plasma region in a small fraction of the modulation period and its energy is not sensitive to the internal plasma field, but is sensitive to the external electric field. For $x=2$, or $v=\omega a/2$ the test ion energy is sensitive to the internal plasma field, but is not sensitive to the external field. For $x>5$, or $v<\omega a/5$, the test ion energy is not sensitive to either electric field because the field reverses many times in transit. Therefore, the experiment to probe for the plasma electric field should have $v\simeq\omega a/2$, ie. the ions in the beam

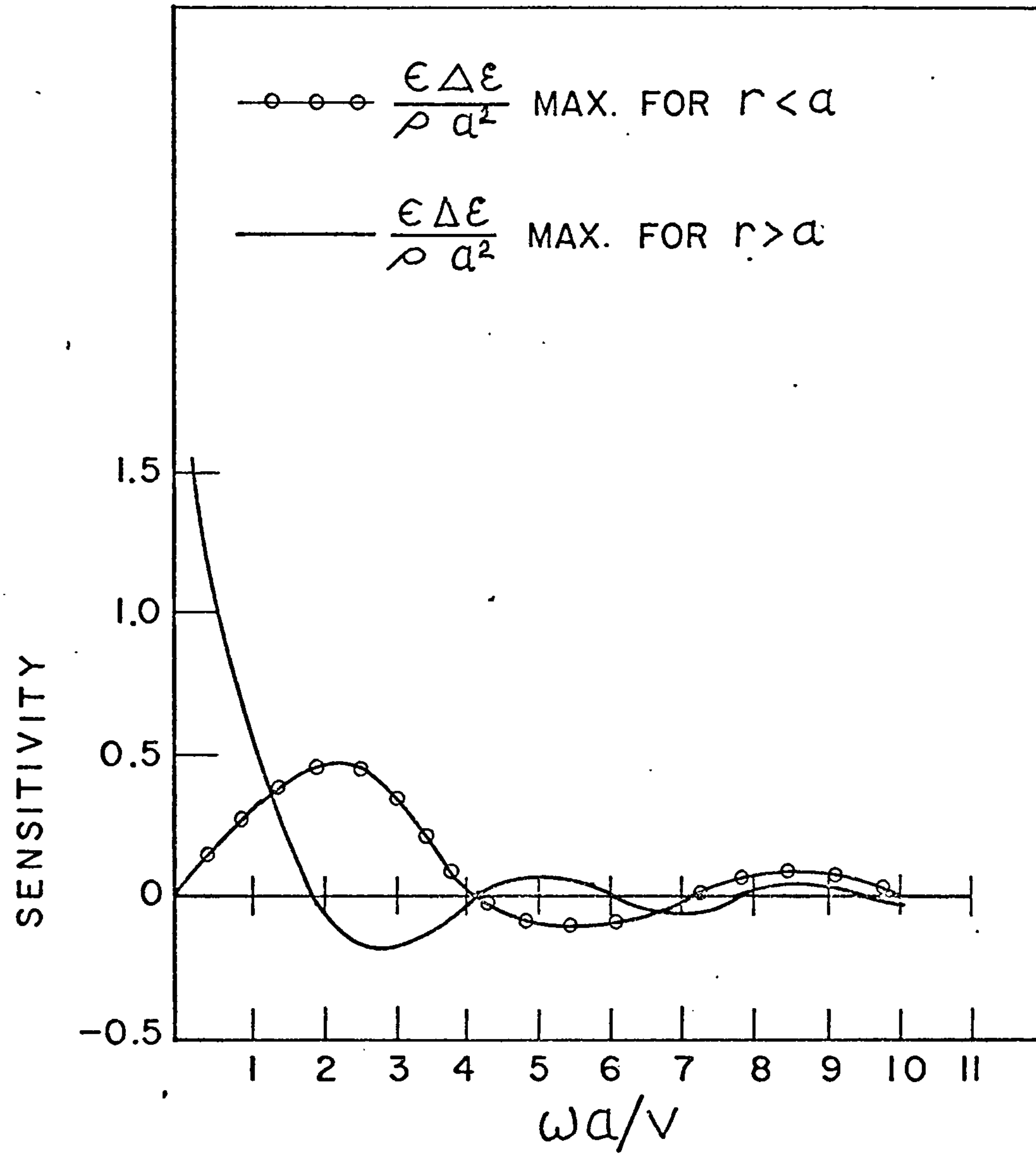


Figure 5. The sensitivity of the test ion energy to the electric fields in the regions $r > a$ and $r < a$ vs x.

should cross the plasma column in about 2/3 of a modulation cycle.

The other parameter determining $\Delta\mathcal{E}$ is ϕ. Since ϕ is random, the number of test ions of phase ϕ is

$$\delta N(\phi) = \text{const } \delta\phi \tag{3-4}$$

Now, from Eq. (3-2), we have

$$\phi = \cos^{-1} \Delta\mathcal{E}/\Delta\mathcal{E}_{max} \tag{3-5}$$

which when differentiated and combined with Eq. (3-4) yields

$$\delta N(\Delta\mathcal{E}) = \text{const } \delta(\Delta\mathcal{E})/ \left[1 - (\Delta\mathcal{E}/\Delta\mathcal{E}_{max})^2\right]^{\frac{1}{2}} \tag{3-6}$$

which diverges mildly at $\Delta\mathcal{E}_{max}$. This is the change in the energy distribution of the test ions that would occur for a given $\Delta\mathcal{E}_{max}$. This energy distribution change is folded into an initial 2 eV energy spread in an assumed test ion beam and the resulting energy distribution is shown in Figure 6. This is the expected ion energy distribution in an experiment.

Therefore, from analysis of the resultant test ion energy distribution, given the plasma radius, the electron beam density and modulation frequency, and for a appropriate test ion velocity, the radial electric field strength of a plasma column can be measured and hence, the transverse dielectric function of the plasma can be determined. In this analysis, it has been assumed that the test particle

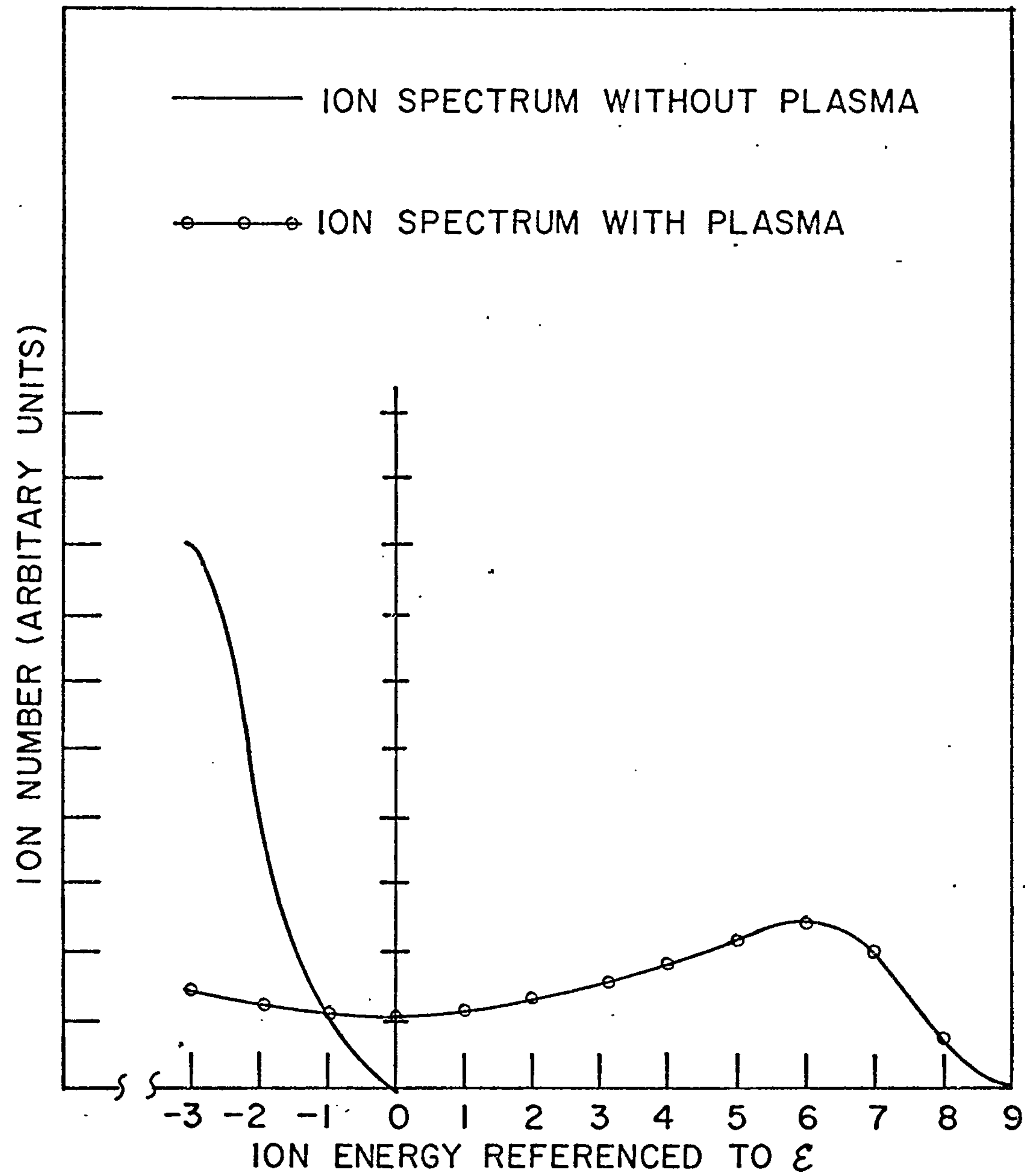


Figure 6. The test ion energy spectrum before and after passing through the plasma column.

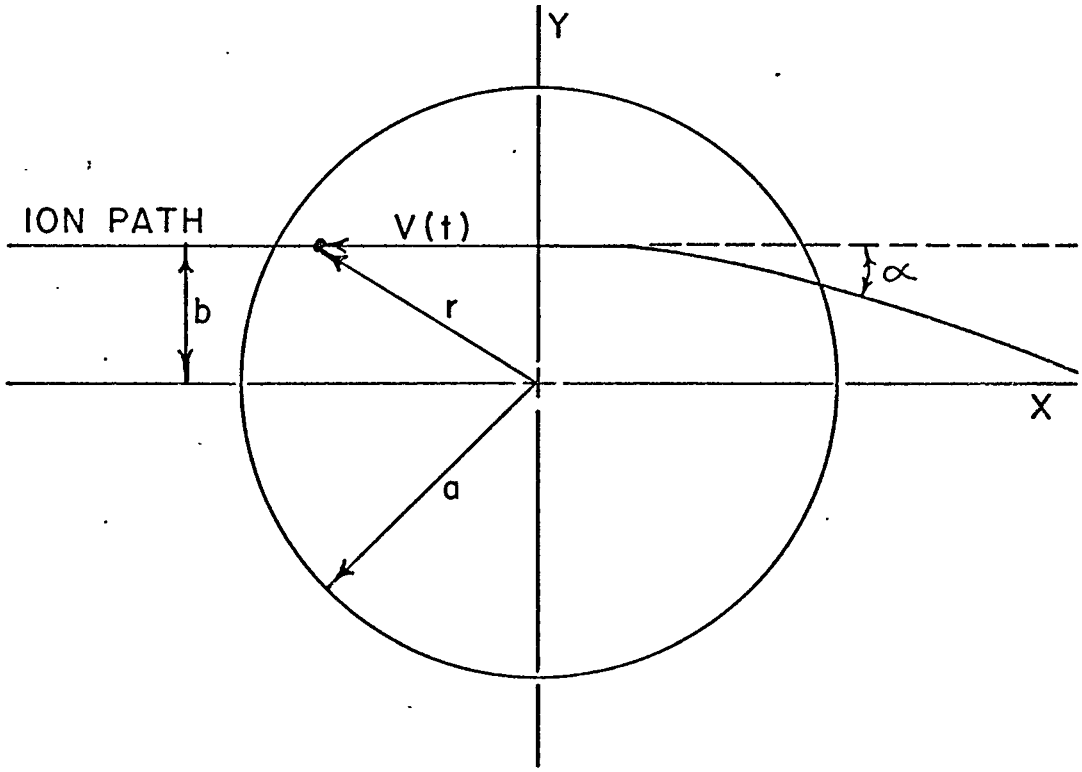


Figure 7. Coordinate frame used in calculation of ion deflection, α.

passed through the axis. If a test particle were to miss the beam axis there would be three possible effects. First, the radial electric field might deflect the test particle from its path. Second, the change in energy of the test particle might not accurately reflect the radial electric field strength. Third, a θ electric field might change the energy of the test particle. Because of the possibility that the test ion beam might not be focused on the plasma axis, these effects must be considered.

If the deflection, α , of the test ion from its initial path is small, several simplifying assumptions can be made. First, in the co-ordinate frame shown in Figure 7, $\dot{x}$ can be assumed constant and the y displacement can be neglected within the radius of the plasma column in calculating the field which will deflect the test ion. Because the field strength within the plasma can be much greater than the external field and because of the geometry, ie. the y component of the force $\frac{b}{r}$, the effect of the external field can be neglected. Measuring t from the arrival of the test particle at its closest approach to the axis, we have

$$r^2 = b^2 + v^2t^2 \qquad (3\text{-}7)$$

The y deflecting component of the acceleration is

$$\ddot{y} = \left(\frac{q}{m} \frac{\rho}{2\epsilon} \; r \cos \omega t \right) b/r \; , \qquad (3\text{-}8)$$

or letting $E(a) = \rho a/2\epsilon$,

$$\ddot{y} = \frac{q}{m} E(a) (\cos \omega t) b/a \tag{3-9}$$

and integrating with respect to time we have

$$\dot{y} = 2 \frac{q}{m} \frac{b}{a} \frac{E(a)}{\omega} \sin \frac{a^2-b^2}{v} \quad . \tag{3-10}$$

Then

$$\dot{y} < 2 \frac{q}{m} E(a) \frac{b \sqrt{a^2- b^2}}{av} \quad , \tag{3-11}$$

which has a maximum for $b^2 = \frac{1}{2} a^2$ and

$$\dot{y} < 2 \frac{q}{m} E(a) \frac{a}{v} \quad . \tag{3-12}$$

The maximum deflection is

$$\alpha = \frac{\dot{y}}{\dot{x}} < \frac{a E(a)}{\mathcal{E}} = \frac{\rho a^2}{2\epsilon} \frac{1}{\mathcal{E}} \tag{3-13}$$

where $\mathcal{E}$ is the energy of the test ion in volts. This deflection is negligible for the experiment, as will be discussed later.

The effect on $\Delta\mathcal{E}_{max}(b)$ as b is varied can be calculated in a manner similar to that used to calculate $\Delta\mathcal{E}_{max}(0)$. Using Eq. (2-1), (3-1), and (3-7), we have

$$\Delta \mathcal{E}_{max}(b) = \frac{\rho a^2}{\epsilon_o} \int_{r_{wall}}^{a} \sin \frac{r^2-b^2}{v} \frac{dr}{r} + \frac{\rho}{\epsilon} \int_{a}^{b} \sin \frac{\omega\sqrt{r^2-b^2}}{v} r\, dr \tag{3-14}$$

where the first integral is the energy change due to the external field and the second, the plasma field. The Sine integral is broken into three parts for evaluation.

$$\int_{r_{wall}}^{\sqrt{a^2-b^2}} \sin \frac{\omega u}{v} \frac{u du}{u^2+b^2} = \int_{\infty}^{0} - \int_{\infty}^{r_{wall}} - \int_{a^2-b^2}^{0} \tag{3-15}$$

where $a^2 = r^2-b^2$. The first integration from ∞ to 0 can be done by residues, the second integral is negligible provided that $r_{wall} >> a$, and the third integration $\sqrt{a^2-b^2}$ to 0 is done numerically using the value $\omega a/v = 2$, corresponding to the parameters desired in the experiment. The sensitivity, $\Delta \mathcal{E}_{max}(b) \left.\frac{\epsilon_o \rho}{a^2}\right|_{r<a}$ for the external field as a function of b is shown by the smooth curve in Figure 8. The sensitivity of the test ion energy to the external 1/r field is seen to be small and remain nearly constant for values of b/a <.5. For values of b/a > .5 the sensitivity rises uniformly achieving the value .21 at b/a = 1.0.

The integration of the second term of Eq. (3-14) yields

$$\Delta \mathcal{E}(b) \left.\frac{\epsilon_o \rho}{a^2}\right|_{\text{for } r<a} = \frac{1}{x^2} (\sin x' - x' \cos x') \tag{3-16}$$

Where $x' = \frac{\sqrt{a^2-b^2}}{a} x$ and as before $x = \omega a/v$. Taking $\omega a/v = 2$, the sensitivity, $\Delta \mathcal{E}(b) \epsilon \rho / a^2 \big|_{r<a}$, for the plasma field as a function

of b is shown by the dotted curve in Figure 8. Here the sensitivity of the test ion energy to the plasma field is seen to remain nearly constant having the value .44 for values of $b/a < 0.5$ above which the sensitivity falls uniformly, achieving the value 0 at $b/a = 1.0$. The effect on the test ion energy of the test ion missing the axis is then negligible for the accuracy of the system under consideration, but would need to be considered for larger values of the impact parameters.

From Eq. (3-1) we can see that a θ electric field would have no first order effect on the energy of a test particle that passed through the beam center, since to change the particle energy, the field would first have to change the particles velocity, so that it would have some θ component. This effect is negligible for a high energy test ion passing through fields whose strengths are comparable to those expected. However, if a test ion were to pass tangentially to the plasma column, a θ electric field could conceivably change its energy in a manner similar to that expected for a radial field. A simple test for this would be to probe for an enhanced electron current external to the plasma. The enhancement of the electron current would be due to $E_\theta \times B_z$ drift, since, the guiding center approximation for the electrons is valid. The effect of the θ field on the plasma ions is not as easily seen, but a probe biased at two different positive values with respect to the plasma floating potential would not collect the same ion current unless the ion energy were considerably above the biased levels. Assuming this not to be true, on observing decrease in

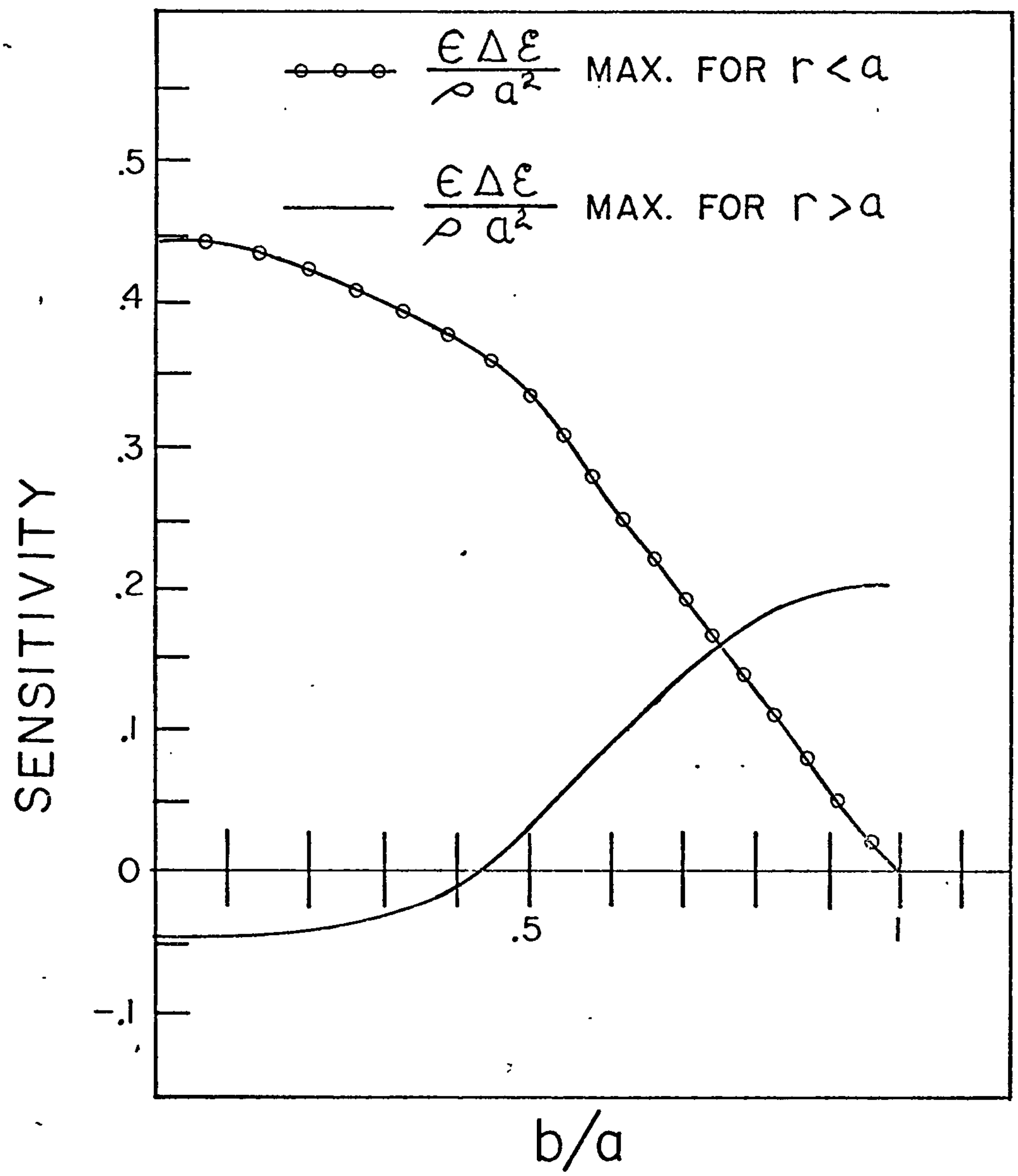


Figure 8. The sensitivity of the test ion energy to the electric fields in the regions $r<a$ and $r>a$ vs b, for $x = 2$.

electron current, for different positive probe bias levels, to the same level when the electron beam is modulated would effectively eliminate the possibility of θ electric fields.

In summary, the purpose of the experiment is to measure the radial electric field strength within a plasma column, confined by a longitudinal magnetic field, associated with the modulation of a coaxial electron beam. The method is that of injecting a test particle through the plasma column and analyzing the energy change of the test particle. Several criteria for the experiment have been established: the test ion motion within the plasma column must be negligibly affected by the magnetic field, the relation of ω, a, and v should be $\omega a/v \simeq 2$, for maximum sensitivity to fields in the plasma and discrimination against external fields. If the impact parameter is greater than half the plasma radius the effect on the test ion sensitivity to plasma fields needs to be considered, and the absense of θ electric fields must be shown. If these criteria are met, the energy change of the test particle determins the electric field within the plasma column and consequently the dielectric function of the plasma, assuming the functional form of the electric field is that of the model, ie. that described by Eq. (2-1).

CHAPTER IV

EXPERIMENT

Design and Parameters

The theory of the probe ion method for measuring plasma electric fields has been analyzed. An experiment using this technique to measure plasma electric fields is described. A long, narrow, magnetically confined plasma column is formed in a 6 inch diameter vacuum tank as shown in Figure 9.

The confining magnetic field is provided by a P.E.M. air core solenoid system having 8 coils with 10 inch ID. Six of the solenoid coils are used in a magnetic mirror configuration and the remaining two coils are used to collimate the electron beam. The coils are connected in series and a 50 amp current yields a maximum midplane in the magnetic mirror of $\sim$500 gauss.

The modulated electron beam source employs a standard commercial BTI three element electron gun mounted behind the modulator section. The gun current depends on the extracting potential, the magnetic field, and the filament current. Typically, at 1 KV and 500 gauss, more than 30 ma can be extracted. The beam is modulated by sweeping the beam back and forth across an output aperture by giving the beam an E x B velocity by means of an RF electric field between two parallel plates. In the absence of the RF field the electron beam passes directly through the output aperture, however when a voltage is applied to the plates, the beam is deflected either to one side or the other of the aperture, depending on the direction of the electric field. Thus,

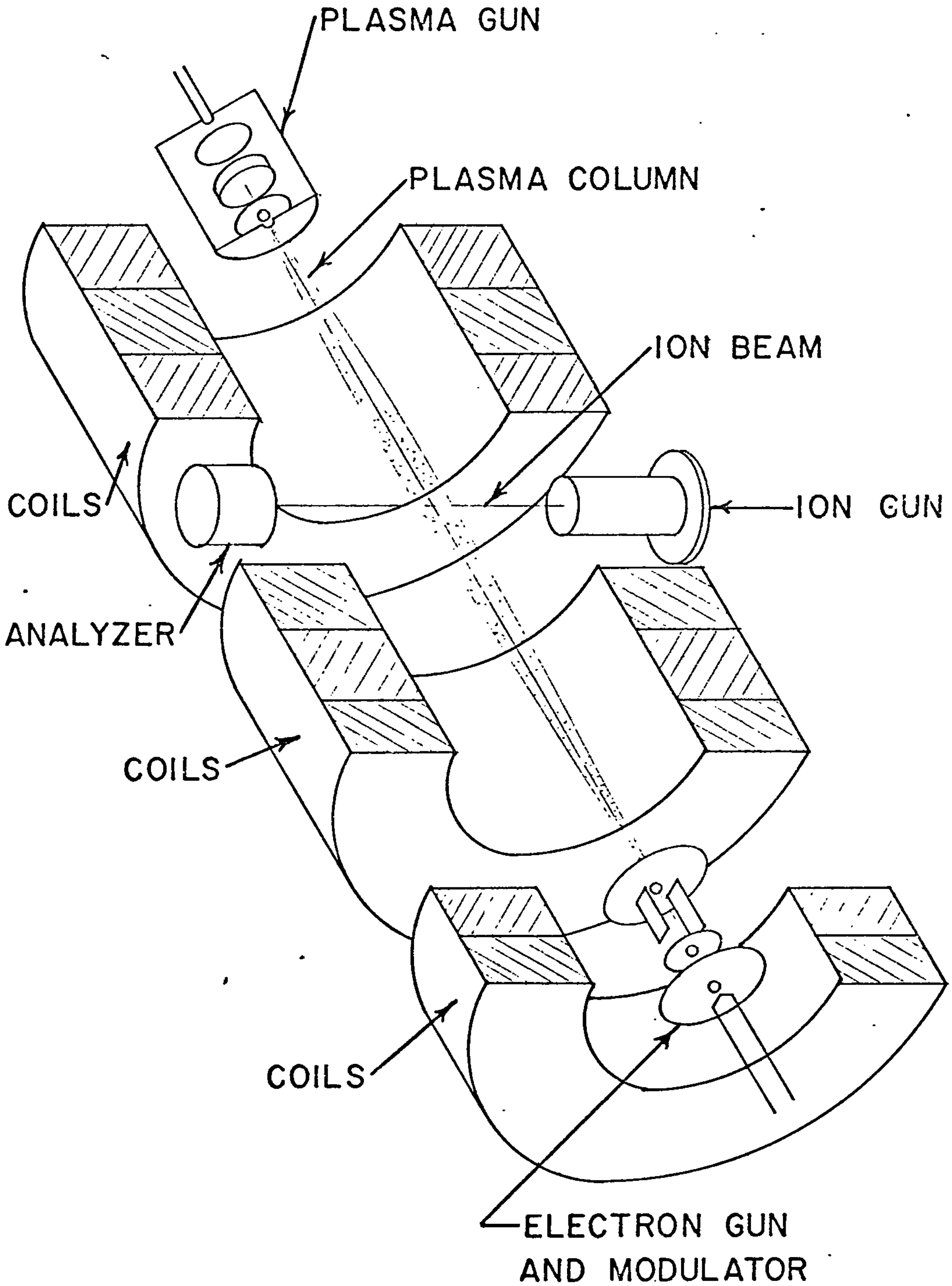


Figure 9. Experimental apparatus.

in one complete cycle of RF voltage, the beam is swept across the aperture twice, and is modulated at twice the frequency of the applied RF field. An alternate mode of operation is obtained by allowing the electron beam to hit one of the modulator plates, loading down the high impedence RF source so that the voltage on the plate is clamped for one half cycle. This mode produces beam modulation at the RF frequency. Except for some experimental checks, the frequency doubling mode of operation is used.

The plasms column is generated by a hot cathode reflex source,[17] located in the magnetic mirror opposite the electron gun, with the electron gun being used as the hot cathode source. The reflex oscillator section is enclosed and has an aperture aligned to receive the electron beam. The plasma is formed in Helium gas which is admitted into the reflex oscillator enclosure through a small leak. The plasma column formed is of the same diameter as the electron beam. The plasma density depends on the value of the positive DC voltage applied to the cylindrical anode of the reflex section and the maximum plasma density of the source when operated in a quiet mode is of the order of $\sim 5 \cdot 10^9/\mathrm{cm}^3$. At higher densities the source becomes unstable and ultimately it switches into an arc mode of operation, having densities greater than $\sim 10^{10}/\mathrm{cm}^3$. In the experiment, the plasma source is used in the low density mode. The addition of the external electron beam produces a stable and quieter plasma than can be obtained from the cold cathode reflex source.

The probe ion gun was designed to produce a beam with a narrow energy spread to facilitate the analysis of the energy shifts produced by plasma electric fields. In the test ion gun, patterned after that of Utterbach and Miller,[18] an electron beam in used to form a plasma in a plenum filled with Argon gas. Ions are extracted from this plasma through a ten thousandths inch hole and accelerated down a 5" long 1 ¼" ID tube at ground potential, emerging from this tube with an energy corresponding to the voltage applied to the plenum. An intermediate electrode normally used for focusing, is operated at ground potential resulting in a slightly defocused beam. The gun output current is a few nanoamperes having about a 3 eV energy spread. In the mode of operation used here it was found that about one half the ions are A^{+} and the other half A^{++}. In order to allow the probe ion beam to intersect the plasma column a sliding O ring seal is incorperated in the gun mounting flange.

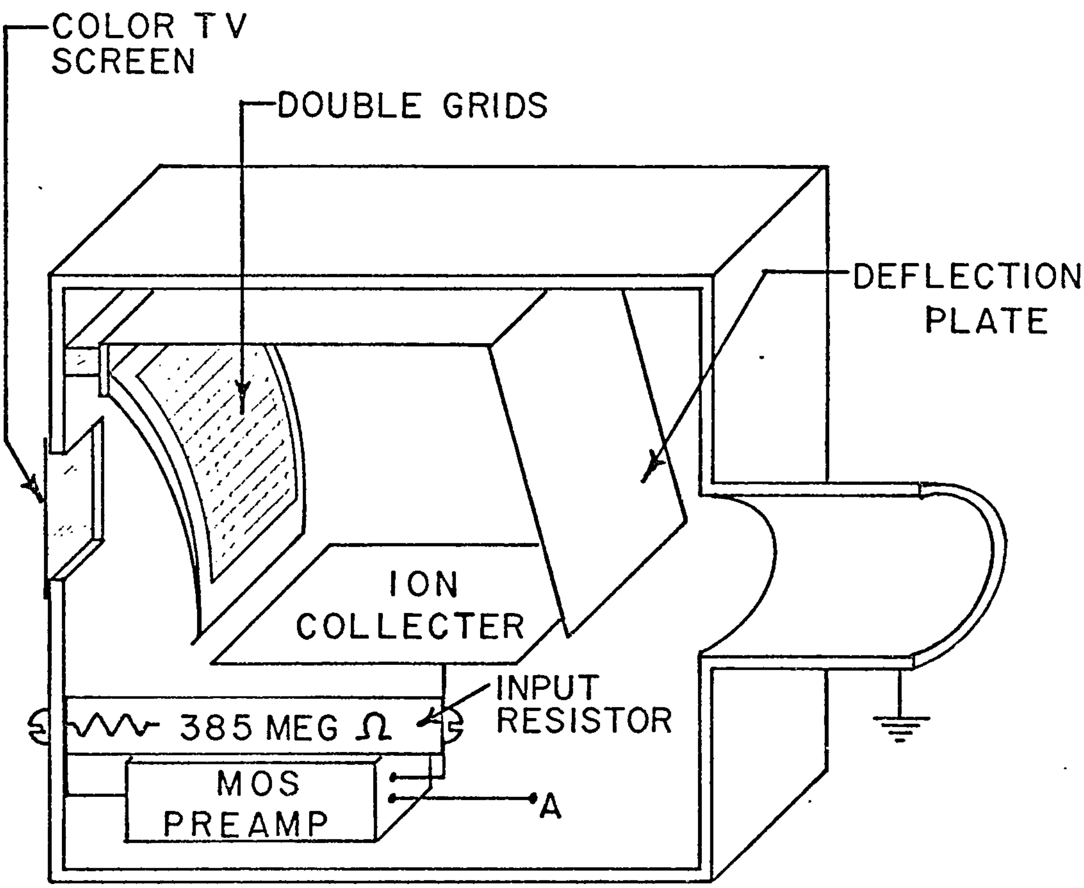
The most critical design considerations were required for the probe ion energy analyzer. The confining magnetic field was insufficient for this analysis and an electrostatic energy analyzer is used to analyze the Argon test ion beam. Several possible methods for test ion energy measurement were considered, among them were magnetic deflection and electrostatic deflection. Both of these were rejected because of the extremely small entrance aperture necessary to obtain the desired energy resolution of 1 eV.

Because the test ion beam as used produces a diverging ion beam, a biased Faraday cup is unsatisfactory for analysis of the beam because ions can be reflected from the cup even though they have sufficient energy to reach the biased cup surface.[19] This effect results in distortion of the ion energy spectra.

Although a biased grid can be shaped to reduce the effect of the beam divergence, the potential wells existing between the wires of the grid mesh, result in poor energy resolution of the transmitted ions. However, a field free region exists between the grids of a double grid structure, ie. two grids at the same potential separated by a few times the wire spacing of the mesh, and consequently good energy resolution can be obtained in this way. Biased double grid design was used to test the energy spread of the ion beam and, with little or no magnetic field, the energy spread of the gun and the analyzer was about 3 eV. However, in the 250 gauss field used in the experiment, the energy resolution was seriously reduced because the ions were approaching the double grid at an angle and some were reflected from the grids even though they had sufficient energy to be transmitted. Therefore, the final design utilizes double grids tilted at an angle so the 560 eV A^+ ions enter normally after passing through the magnetic field.

Also, in this design, shown in Figure 10, the transmitted ions are deflected into a collecting plate located out of the line of sight of the plasma, thereby decreasing the noise signal arising from secondary electrons produced by light and energetic neutrals. Ion species other than A^+ do not enter the retarding electric field of the grids normally and therefore they produce a low energy tail, since some of sufficient energy to be collected are reflected away. For the purpose of the experiment, the low energy tail due to the impurity ions can be tolerated, as will be seen below.

Because light, energetic neutrals from the ion gun, and stray plasma electrons can ionize the gas located in the volume



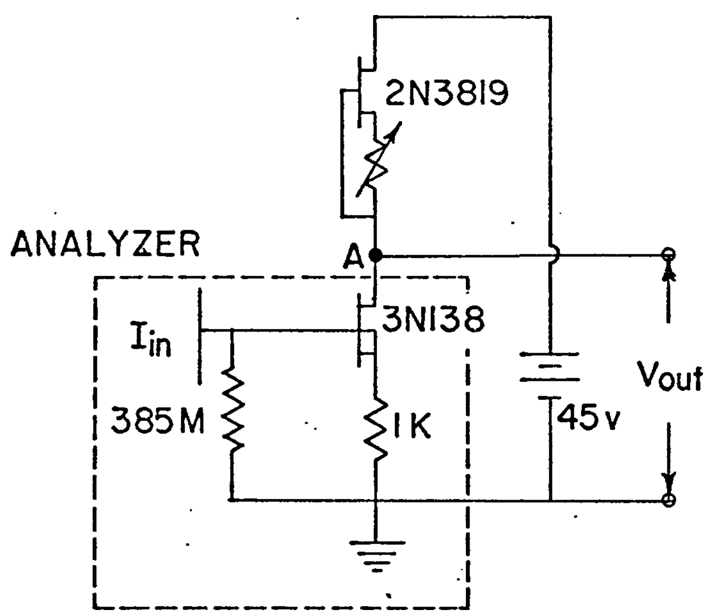


Figure 10. Ion energy analyzer and preamplifier.

behind the double grids there is a noise current to the MOS preamplifier. The use of the lock-in amplifier which is phase-locked to the signal, averages out the noise over a period of time. Besides reducing the noise, this technique simplifies the data presentation. In the experiment, an ion energy distribution is obtained by adding a sawtooth voltage ramp to the voltage applied to the plenum of the ion gun and monitoring the current collected by the plate in the electrostatic energy analyzer while holding the voltage on the double retarding grids constant. This would result in an integral energy spectrum. However, a PAR lock-in amplifier reference signal at 0.5 v at 800 Hz is added to the plenum voltage and a MOS preamplifier drives the input of the lock-in amplifier. The signal then is proportional to the change in the integral of the energy distribution. The data readout system is shown in Figure 11. The heart of the system is the PAR lock-in amplifier. This is used to modulate the ion gun bias voltage at 800 Hz and to detect the change in ion signal at that frequency from the MOS preamplifier. The MOS preamplifier signal is proportional to the integral of the ion energy spectrum, therefore the lock-in amplifier detects the change in the integral, or simply the ion energy spectrum. The time base sawtooth generator of a type 555 Tecktronix oscilloscope is used to sweep the ion voltage through 10 volts in 1.5 minutes and the vertical trace input is the 10 sec RC integrated output of the PAR lock-in amplifier. The oscilloscope trace obtained in this way represents the test ion energy distribution.

A typical test ion distribution obtained without either plasma or electron beam is shown in Figure 12. The asymmetry which occurs

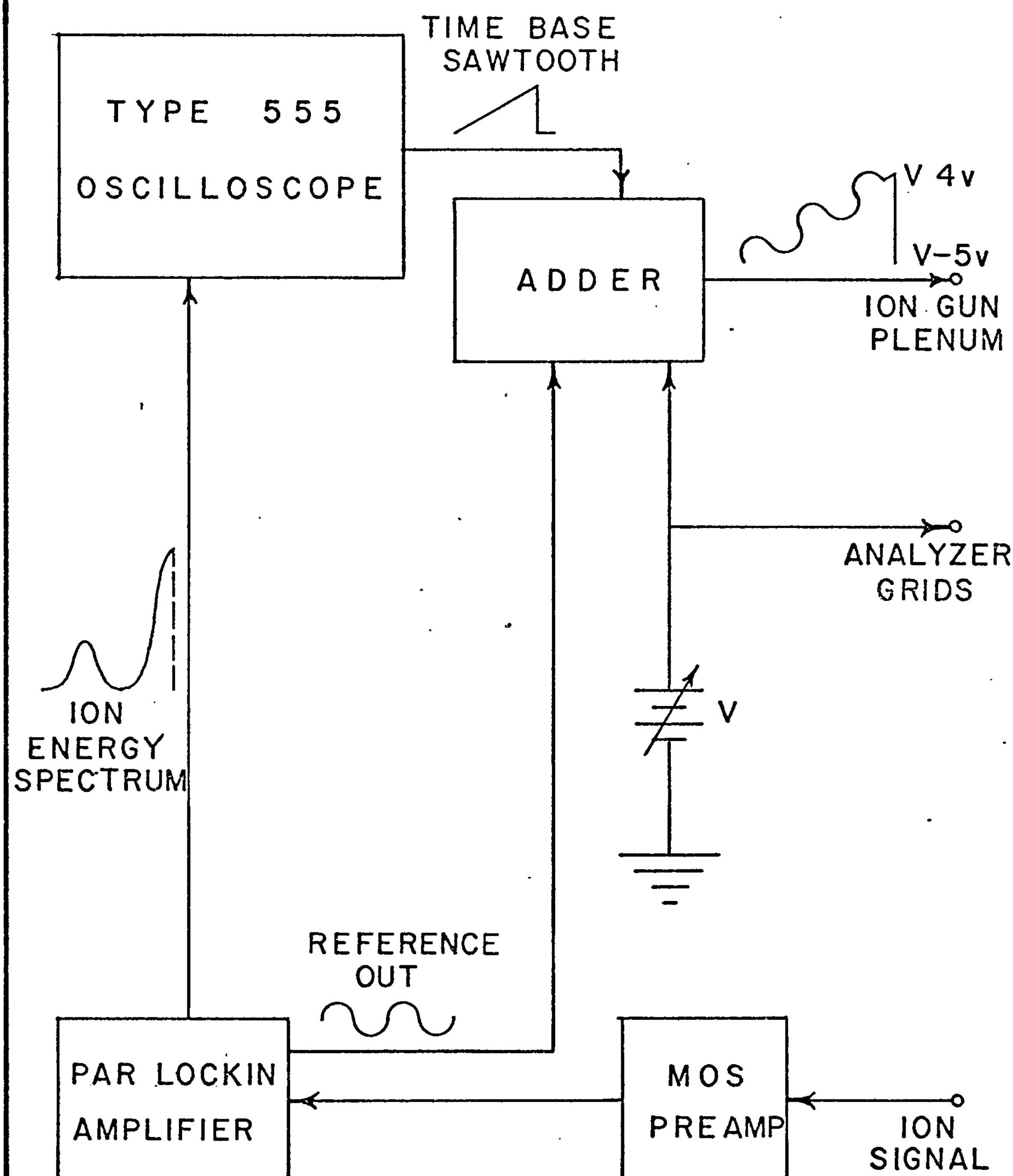


Figure 11. Data readout system.

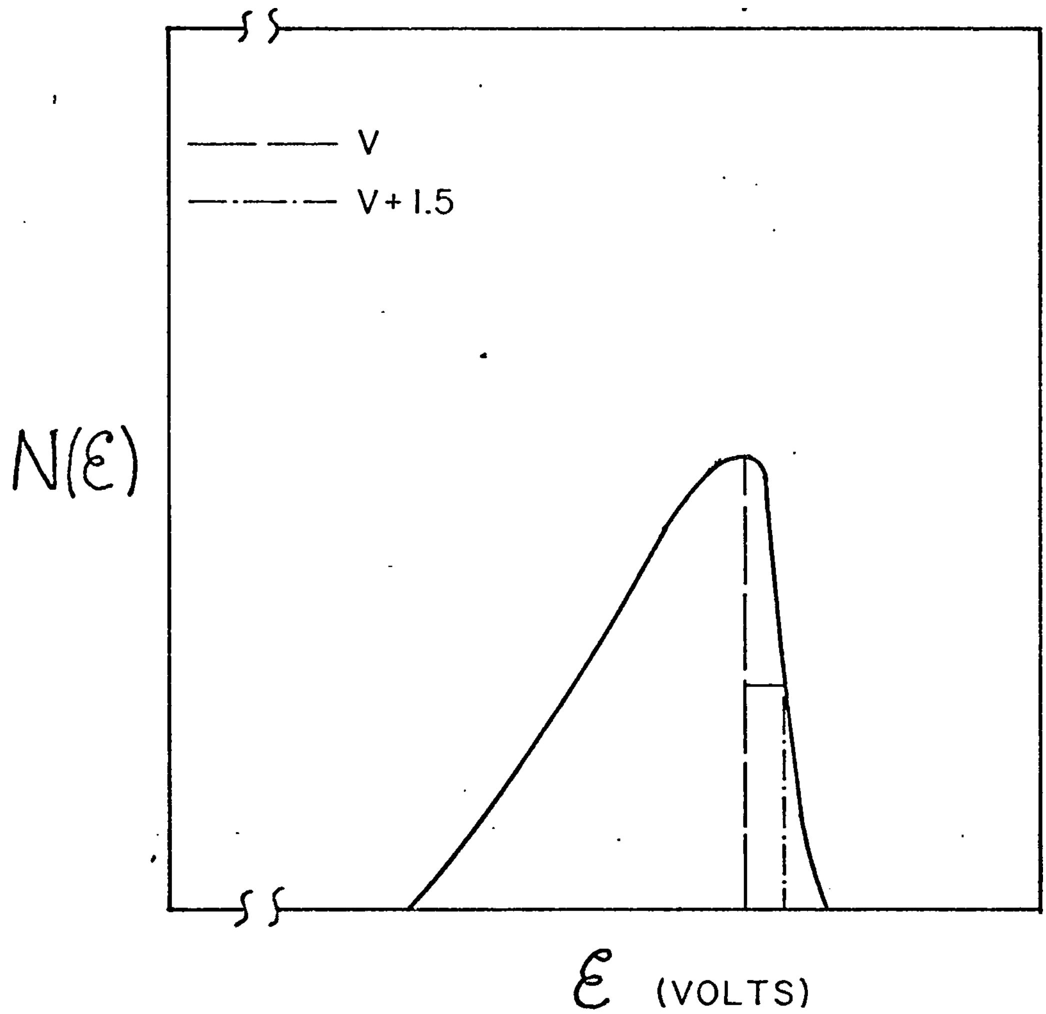


Figure 12. Typical test ion energy spectrum without plasma or electron beam.

arises from several sources. While there is no way for a test ion to have energy above that equivalent to the plenum voltage of the ion gun plus the ion temperature within the gun plasma, there are several ways an ion could have less energy. Secondary ions formed within the extracting region of the ion gun could have less energy than that of the primary ions. Primary ions which suffer small angle collisions with the background helium atoms have less energy than that of the primary ions. However, the principal reason for the change in the number of ions transmitted through the double grids of the electrostatic analyzer at the increased plenum voltage is that the impurity ions are not approaching the grid normally, ie. only A^+ makes a normal approach, and consequently as the excess energy of the impurity ions, above that necessary to pass through the double grids normally, is increased, fewer are reflected. The number of impurity ions is of the same order as the number of A^+ ions, so the low energy "tail" on the energy distribution is of the same order as the A^+, 3 eV energy spread peak. However, the impurity ions that are analyzed miss the plasma column because their different e/m leads to different orbits in the magnetic field, (see Figure 13), and consequently they suffer no energy change, so that the impurity ion distribution is unchanged by the plasma electric fields. Therefore, the ions whose energy has been increased by the presence of plasma electric fields must be A^+ ions. For these reasons, the increased ion energy portions of the ion spectra are analyzed to obtain the data on the plasma electric field in the experiment. The expected energy distribution of the collected test ions would contain a peak at the original test ion energy due to the A^{++} in addition to

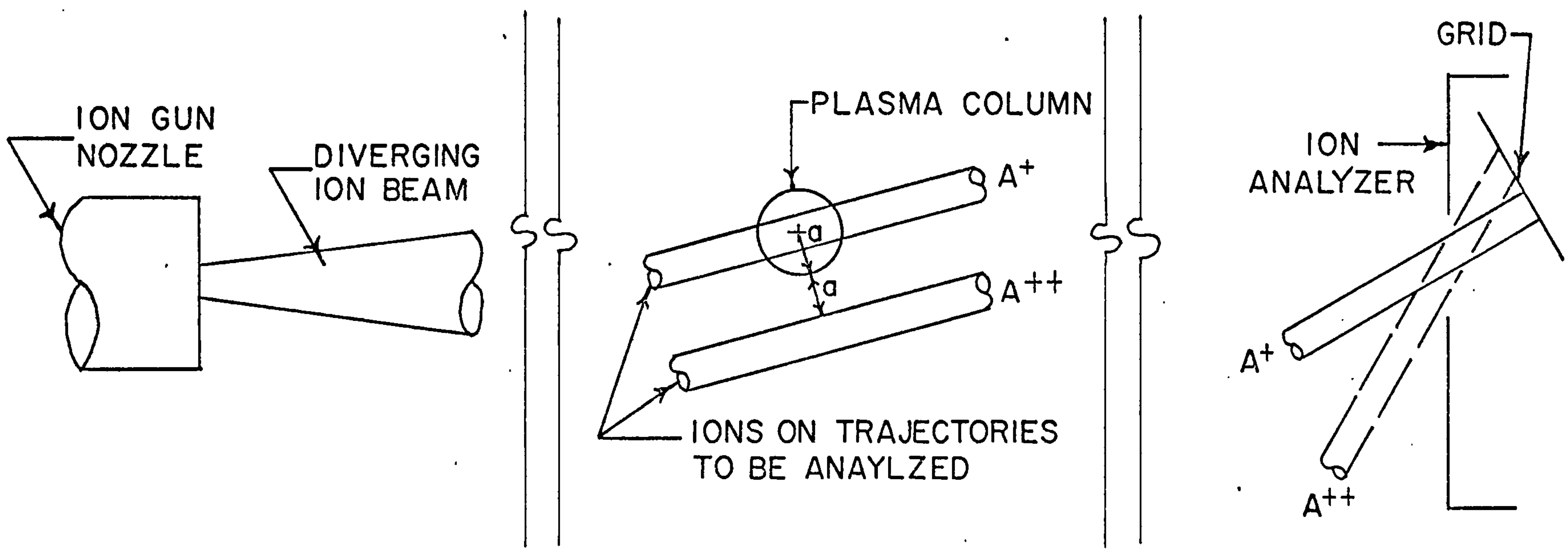


Figure 13. Geometry of ion gun, plasma column and ion analyzer.
The deflection angles have been exaggerated.

the peak at $\Delta\mathcal{E}_{max}$ as shown in Figure 6.

There are several parameters whose value can be adjusted in the experiment. Among these is the electron beam current, which could be as high as 60 ma, however, a current of only a few ma is used in this experiment. One reason for using this reduced electron beam current is so that the plasma is driven in a small signal mode, making the use of the plasma dielectric coefficient more justifiable than if a high electron beam current were used. For instance, in a $5\cdot10^9/cm^3$ density plasma, with $\epsilon_o/\epsilon=7$, so that the plasma charge modulation is 7 times the beam modulation , so that the propagation of a 100% modulated 60 ma beam would require the complete removal of either the plasma electrons or the plasma ions, depending on the phase of the oscillation. Clearly, this can not occur, and the dielectric effect of the plasma would display linear saturation effects. The result of this is that the electric fields in a plasma of constant density will tend to saturate for beam currents above a given level, particularly at a resonant frequency. This limiting beam current is of interest and should be studied further in other investigations.

Another reason for using the low electron beam current is that it is not possible to get meaningful ion spectra at high beam currents, where enough plasma diffuses into the retarding field analyzer to make the energy analysis meaningless.

Because a and ω are chosen to approximate the values of the Haas and Dandl and Eisner and Haas experiments, the value of v for the test ion is here determined corresponding to a 560 eV $Argon^+$ ion. Because of a limited flexibility in the geometry of the experiment, if the test ions are to pass through the plasma column and then be energy analyzed, the magnetic field used must be limited to ~ 250 gauss.

Although the ion gun and the plasma source are extremely stable in operation, when they are turned off and then turned back on they do not always return to the same operating conditions and it is not always possible to come to the same operating point. For this reason, the data must be taken in a set order. First, the plasma, electron beam, and modulator are adjusted to the desired conditions. With the test ion gun off, a noise trace is taken. Then the test ion gun is turned on and the energy distribution of the test ions for the plasma, electron beam, and modulation conditions is taken. In this manner, neither the ion gun ncr the plasma source is operated with different conditions during a data run. The noise trace is taken first because, if there were noise observable on the PAR lock-in amplifier monitering meter, there would be no need to waste an extra 4 minutes (two runs at 1.5 minutes each plus recycling time).

To test the apparatus, several runs were made using the bare modulated electron beam with no plasma. The beam modulator was run in the clamped mode resulting in square wave modulation at the modulation frequency. Calculations similar to those make in Chapter 3 were made for the square wave field with only an attractive force, and showed that ion energy changes resulting for different values of ω a/v were similar to those shown in Figure 5 for the sine wave electric fields. The observed ion energy changes correspond to the calculated ion change of 7 eV for a 4 ma electron beam, for $\omega a/v = .25$, ie. f = 1 MHz. This is the energy change due to the 1/r external field. The energy change due to the internal electric field of the beam is calculated to be 1 eV, which is at the threshold of energy resolution for the present system, and in fact no energy change was observed. These test runs prove that the system is working properly and that the calculations are correct.

A probe measurement of the ion saturation current was used to measure the plasma density. Other techniques measure electron density which because of the multicomponent plasma would not accurately reflect the He plasma frequency unless the Argon and other impurity ion densities were known.[20] The ion saturation current can be written as

$$I_{ion\ sat} = ge\ n \sqrt{\frac{2e\ V_e}{m}}\ A \tag{4-1}$$

where V_e is the electron temperature in volts, n is the plasma density, A is the probe area, and $\frac{1}{4} < g < \frac{1}{2}$ depending upon the assumptions in the theory.[21,22,23] Because the helium plasma ions are less massive than the Argon and other impurity ions, the ion saturation current to a probe is less sensitive to the impurity ions than to the Helium ions.

Fortunately, the problem is to show the existence of the transverse plasma electric fields, and not the dependence of these fields on the ion density, so the purpose of the probe density measurement is to indicate the proper density regime of the plasma where enhanced plasma fields might be found. Taking g to be $\frac{1}{4}$ and V_e to be 1 volt, ion plasma density from the probe measurement is $\sim 5 \cdot 10^9/cm^3$ when the plasma source is operated in a quiet mode. The resonance calculated from Eq. (2-3) for this density for a helium plasma is at 7.5 MHz, and this frequency was the logical frequency to start searching for the test ion energy increase.

Results and Discussion

Using a 2.4 ma electron beam, increased test ion energy was found with $f = 7$ MHz, see Figure 14, typical of the increased ion energy traces. This has the expected shape, see Figure 6, where the peak at the original energy is due to the analyzed A^{++} missing the plasma column. The $\Delta\mathcal{E}_{max}$ of 6 eV corresponds to an ϵ_0/ϵ of 7. Increasing the density, but decreasing the electron beam current to 1.5 ma, increased test ion energy was found with $f = 8$ MHz. A $\Delta\mathcal{E}_{max}$ of 4 eV again corresponds to an ϵ_0/ϵ of 7. At an increased $\sim$+20% or at a decreased $\sim$-20% density, no test ion energy change was found.

Several points should be understood from this data. First, with increased density the resonant frequency increased, as expected from the dielectric function. Second, at a given frequency, departing from resonance by either increasing or decreasing the plasma density resulted in large decreases in plasma fields and no test ion energy change was observed under these conditions, again as expected from the dielectric function. Third, though not conclusive, the fact that the resonant amplification was 7 for two values of electron beam current indicates that the plasma is driven linearly at these drive levels of a few ma.

It is at this point that the inability to get precise values for the density of the plasma is the limiting factor. With precise density measurements, the dielectric response of the plasma could be tracked with changes in frequency, in density and in drive level. However,

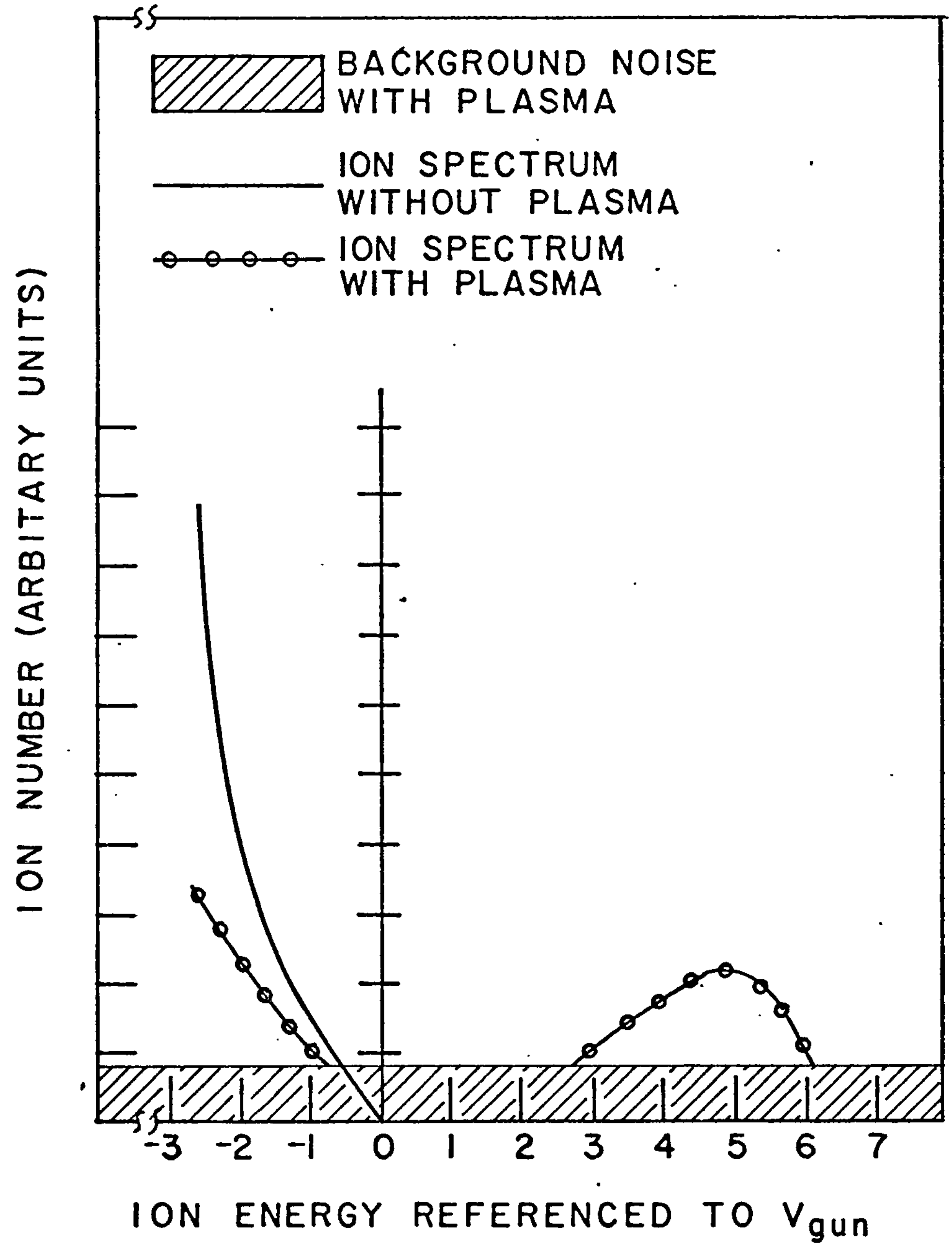


Figure 14. Test ion energy with 2.4 ma electron beam and f = 7 MHz.

the probe measurement, while certainly indicating the proper density range of the plasma, cannot be trusted to indicate the precise value. Of course, once the nature of the dielectric function was determined, the resonant frequency, as determined by the test particle method used here could be used to determine the density quite accurately.

Fortunately, precise density information is not crucial to this experiment since the principal aim was to show the existence of the resonance electric fields of the form used in the computer calculations. The test to determine the presence of θ electric fields as discussed in Chapter 3 showed no θ electric field for these experimental conditions. Consequently, we have shown that the modulated electron beam electric field does couple to the plasma producing amplified electric fields similar to those assumed in the computer calculations of Chapter 2, justifying the use of the ad-hoc electric fields in the calculations.

CHAPTER V

CONCLUSIONS

A model has been developed which forms the basis for understanding the ion heating observed in the experiments of Haas and Dandl. On the basis of this model one can conclude that the heating mechanism would be operable for a wide class of plasmas and might provide the basis for a technically feasible process for plasma heating in a CTR reactor.

Recapitulating, in long, narrow, electron beam formed plasmas energetic ions have been observed when the electron beam was modulated at frequencies ranging to ten times the ion cyclotron frequency. These modulation frequencies were several orders of magnitude lower than the electron plasma frequency. This was not understood for two reasons:

(1) Assuming there were electric fields within the plasma at the modulation frequency, homogeneous plasma theory and stochastic noise theory indicate ion heating proportional to the power in the electric field at the ion cyclotron frequency. The ion heating must then be related to the fact that plasma is inhomogeneous, a condition which makes the analysis of the plasma kinetic theory too difficult to handle analytically.

(2) Assuming that electric fields at frequencies other than the ion cyclotron frequency could heat ions because of a plasma inhomogeneity, the existence of these electric fields is questioned because electron mobility along the confining

magnetic field lines could cancel out the fields at frequencies much less that the electron plasma frequency if there is a sufficiently strong longitudinal electric field.

The approach taken was to handle each of these questions seperately, because the self consistent problem is too difficult to handle at present, though hopefully the new generation of computers should be able to handle the problem.

Computer calculations of single ion dynamics were made using ad-hoc fields of a dielectric column driven by the fields of a modulated beam. These indicated ion heating which did depend on the inhomogeneity of the plasma, more directly, on the large gradient in the electric fields at the plasma boundary. This calculated ion heating is a maximum for modulation frequencies near the ion plasma frequency as observed in the experiments.

A new test particle diagnostic was developed to test for the existence of electric fields similar to the ad-hoc fields used in the computer calculations. This new technique was necessary, because to study the electric fields in the presence of a high energy modulated electron beam without disturbing the plasma is impossible using other techniques. The test particle technique established the presence of electric fields similar to the ad-hoc fields used in the computer calculations.

There are several experiments yet to be done which would be of interest. The accurate determination of plasma density would make it possible to track the plasma dielectric function with respect to frequency, density, and driving field strength. A better test particle analyzer based on counting individual test particles rather than the gross current used in the experiment could have an input aperture small enough to reject impurity ions with their different orbits, and would have increased sensitivity. This would allow more accurate tracking of the plasma dielectric response. The primary consideration though was to improve the understanding of the heating experiments so that increased ion heating can be found through experimental parameter adjustment. The use of the computer calculation and the test particle diagnostic to achieve increased experimental ion heating is the most interesting future experiment.

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APPENDIX

The Runge-Kutta algorithms used in the numerical integration of the equations of motion are the forth order set of

$$\dot{r}_{n+1} = \dot{r}_n + \frac{\Delta t}{6}\left[\ddot{r}(t_n) + 4\ddot{r}(t_{n+\frac{1}{2}}) + \ddot{r}(t_{n+1})\right] \tag{A-1}$$

$$\dot{\theta}_{n+1} = \dot{\theta}_n + \frac{\Delta t}{6}\left[\ddot{\theta}(t_n) + 4\,\ddot{\theta}(t_{n+\frac{1}{2}}) + \ddot{\theta}(t_{n+1})\right] \tag{A-2}$$

and

$$r_{n+1} = r_n + \frac{\Delta t}{2}(r_n + r_{n+\frac{1}{2}}) \tag{A-3}$$

The discontinuity in electric field at r=a, see Eq. (2-1), and the singularity in $\check{E}$ at r=0, see Eq. (2-4), requires the use of the fourth order equation in Δt for $\dot{r}$ and $\dot{\theta}$. The same factors require the use of the incremental time step of Eq. (2-5)

$$t = A + B\left|r(r-a)/(r+a)\right| \tag{2-5}$$

The computer code is available upon request.