CONSTRUCTION AND EVALUATION OF MEMBRANE ANALOGY APPARATUS FOR TORSION ANALYSIS

A Thesis

Presented to

the Faculty of the Department of Mechanical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

by

A. V. Hanumanth Rao

June, 1968

PREFACE

This research presents the design, construction, and evaluation of a membrane apparatus for the torsion analysis of prismatic bars. Torsional stresses and deflections in a non-circular sectional bar are difficult to determine analytically. The experimental results in this thesis were obtained from an experiment performed on the membrane analogy apparatus. Thin polyethlene sheet material and rubber were used instead of soap film in the experiment. The results were gratifying.

The researcher wishes to express his appreciation for the valuable advice and assistance given by members of the thesis committee, especially Prof. L. J. Castellanos of the Mechanical Engineering Department, who gave him all possible assistance and guidance in completing this research.

A. V. H. R.

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ABSTRACT

This research presents the design, construction, and evaluation of a membrane analogy apparatus for the torsion analysis of non-circular sections (prismatic bars). The purpose of this project was to construct and demonstrate the use of the apparatus. The degree of error obtained by the apparatus was determined by performing experiments with the following three sections:

1. Square (3" x 3")
2. Equilateral Triangle (4" side)
3. Rectangle (2" x 3")

The order of error was 1 to 5 percent.

Thin Plastic (Polyethylene) material (0.0015" thickness) and a rubber membrane (0.009" thickness) were used to carry out the membrane analogy experiment. Contours and graphs are presented for each section mentioned above.

The apparatus is easy to construct and is inexpensive. Experiments can be performed with the apparatus without much difficulty. The percentage error due to the apparatus can be reduced to a great extent by a few alterations in the design. A large amount of error in results is due to the membrane material which is not an ideal membrane.

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SYMBOLS USED IN THIS THESIS

E =Modulus of elasticity of material

G =Modulus of rigidity of material

θ =Twist of bar in radians per unit length

τ =Shear stress in lbs./sq. inch

T =Torque applied to bar

p =Pressure difference causing displacement (air pressure)

Z =Elevation of the membrane

S =Surface tension of soap film

A =Area of cross section of bar

P =Length of perimeter of cross section

β =Maximum angle at the edge of film covering the circular hole

R =Radius of the circle

Ψ =Angle at the edge of the film covering the opening representing the section

α =Maximum angle at the edge of film covering the test hole

$\tau_{c)max}$ =Maximum shearing stress in the circular hole

$\tau_{s)max}$ =Maximum shearing stress in any section under study

t =Thickness of the membrane

CHAPTER I

INTRODUCTION

The object of the project was to design, construct, and evaluate a membrane analogy apparatus for the purpose of demonstrating its use in experimental measurements. This apparatus is useful mainly for determining the stresses and torque in a twisted bar or shaft of any non-circular cross section, which are difficult to get analytically.

The Membrane Analogy was first proposed by Prandtl(1)* in 1903. It was not until 1917 that A. A. Griffith and G. I. Taylor (2) formulated a practical experimental approach with the use of a soap film for the elastic membrane as proposed by Prandtl. They used Autocollimator and Spherometer to obtain their experimental results. In the Autocollimator, a collimated bundle of light rays from a small source are directed on the surface of the film. The collimation axis is rotated about a horizontal axis until the reflected ray coincides with the incident ray. The angle between the collimation axis and the vertical measures the slope of the film. The

*No. refer to references in bibliography apparatus is carried on a pair of right-angle guides to locate the point at which the slope is measured. The chief error arrived from the fact that the film has a tendency to spread over the plate at the boundary. The error obtained is about 2 to 4 per cent. Spherometer consisted of a thin plate which contains a cut out of the desired cross section to be analyzed and a circular hole to be used as a standard. The plate is placed in a pressure applying fixture and soap film is stretched across the openings. Contours of the film surface are plotted by moving the micrometer on the glass plate, which is placed over the extended film. The volume of the film is computed with a contour map. Their error varied from 2 to 4 per cent, which was not quite as good as previous investigations, however, their experimental procedure had been greatly improved. Their method was lengthy and since soap film is subject to evaporation, time proved to be an important factor.

Other investigators (3) in the 1930's introduced their own techniques (Photogrammetric method, Quest Collimator, etc.) in determining the slopes on soap film, but their techniques were complicated and consequently less accurate. In 1934 E. Kopf and E. Weber (4) used a thin rubber membrane instead of the soap film. The rubber membrane was stretched over holes in an aluminum plate which was the top of a jar containing a mixture of paraffin. Their experimental procedure differed from previous investigators in that they used water to load the rubber membrane and made permanent wax impressions of the distended membrane. Measurements of slope and volume were obtained by slicing the wax impression and photographs were taken of the resulting section at every cut. Their techniques were time consuming with little or no improvement on accuracy.

The apparatus used for the present project was based on the Taylor-Griffith's Spherometer principle. In their apparatus the soap film was enclosed in a box and the micrometer was moved all around on the glass plate to obtain the contours of the extended soap film.

Rubber membrane and plastic material were used instead of the soap film to perform the experiment on the designed apparatus. The depth gauge was moved over the extended membrane and the contours were obtained. The membrane was not enclosed in any box because there was no evaporation problem like the soap film.

The designed apparatus was evaluated on the basis of the results obtained from it for simple non-circular cross sections by using rubber membrane and polyethylene plastic material. These results were compared with the known theoretical values of the sections. Non-circular cross sections like triangular, rectangular, and square with their respective standard circular sections were used for experimentation because their accurate torque ratio and shear stress ratio are known. The contours of the extended membrane over the non-circular section considered and the standard circular section are obtained on a drawing sheet with the help of the depth gauge. The volume under the extended membrane over the section and the circle is determined by graphical method. The ratio of the volumes give the torque ratio of the section considered and the circular section. This ratio is compared with the known theoretical ratio and the error in the apparatus in determining the torque ratio is obtained. The experimental error in results obtained by the designed apparatus is about 2 per cent. The maximum gradient of the extended membrane on the non-circular section and the circular section is obtained from the contour map by taking the ratio of the vertical spacing and the horizontal spacing of two contours at the edge. The maximum gradient ratio will give the maximum shearing stress ratio between the two sections. And this ratio is compared to the known theoretical ratio and the error of the apparatus in determining the maximum stress ratio is obtained. The experimental error in results obtained by the designed apparatus is about 4 per cent. Experiments were performed on the designed apparatus without difficulty. The apparatus is simple in construction and can be used for many different membrane materials. (except fluid membranes)

CHAPTER II

APPARATUS DESIGN CRITERIA AND DESCRIPTION

In the design and the construction of the apparatus the following requirements were taken into account:

1. Suitability for using non-fluid membranes.
2. Means for finding the contours of the extended membrane.

Because of this, the apparatus consisted of a depth gauge with its designed accessories and a recording board for contour measurements. The required results for torque ratio and shear stress ratio are obtained with the help of the contour mapping by graphical methods. The membrane used, being a non-fluid type, was not enclosed in any box like a soap film.

The photograph (Fig. 1) shows the designed apparatus. The layout assembly drawing of the designed apparatus with pertinent dimensions is shown in Fig. 2.

The apparatus consists of a square steel box, A, 12" x 12" x 2-7/8". The box is made out of 1/8" thick steel plate and is supported on four leveling screws. The circular test hole and the experimental hole are cut in two flat plates of any suitable metal (eg. steel) of thickness 1/8". The membrane material is placed between these two plates so that the membrane is held at the edges

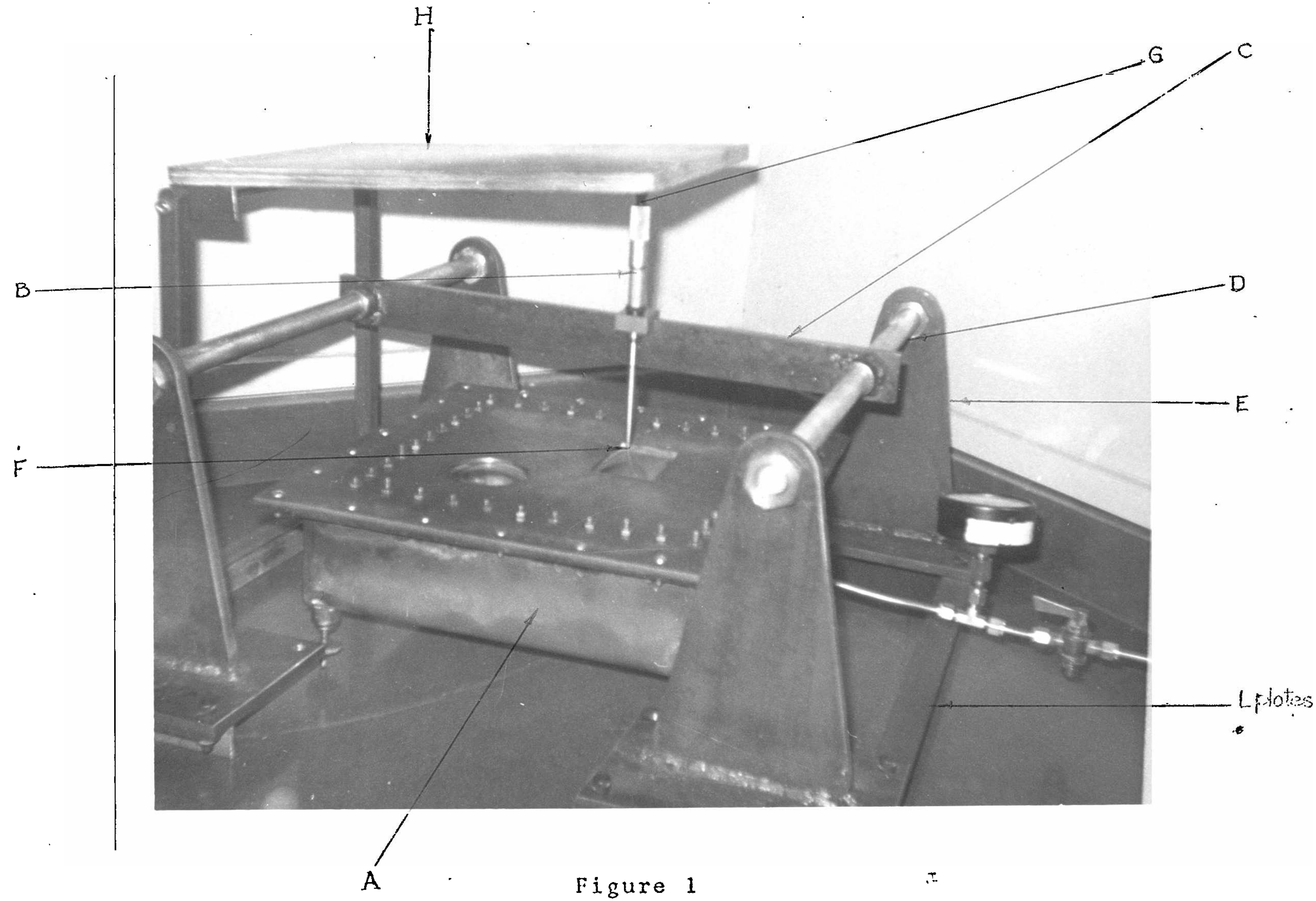


Figure 1

Membrane Analogy Apparatus

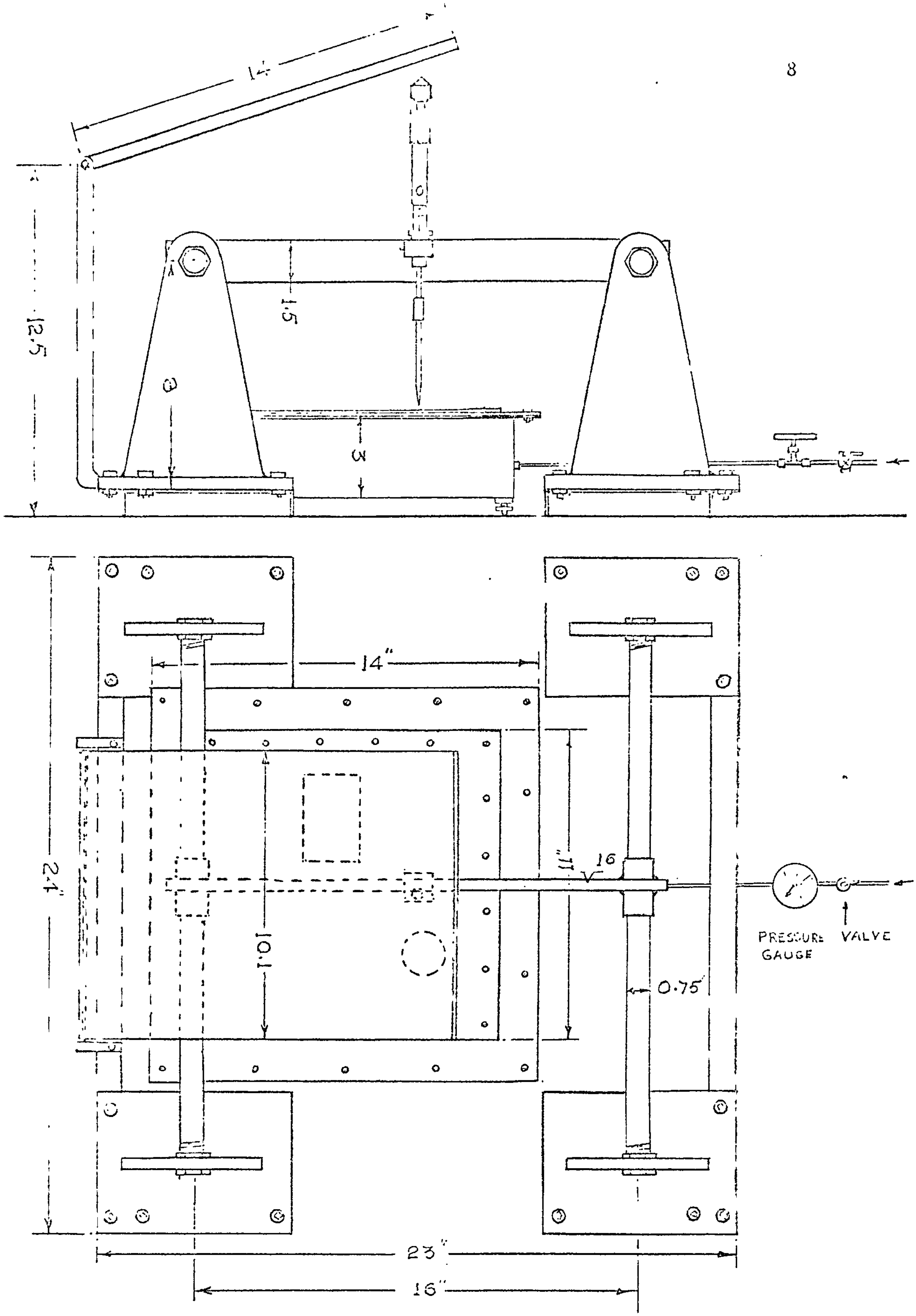


Figure 2
Apparatus with Pertinent Dimensions

of the holes. The plates are held in horizontal position during the experiment by leveling screws. These coupled plates are bolted to the frame of the square box. The joint is made air tight by means of a gasket. One side of the box has a 1/4" Copper tube connected to it with a pressure gauge (0-30 p. s. i.) and a stop valve in it.

The measuring device consists of a depth gauge ,B , (with the attachments), which slides freely on the upper face of the rectangular steel bar ,C. The rectangular steel bar is 18" x 1 1/2" x 3/8". The upper face of the bar (3/8" wide) has a 16 microinch finish for easy sliding of the depth gauge. The rectangular steel bar moves freely back and forth on two round steel bars ,D ,of 3/4" diameter, which are at either end of the rectangular bar. These bars, each 20" long, are 18" apart and are supported by means of four brackets ,E. The brackets are fixed to the L plates (Angle iron) for stability of the measuring device. The lower end of the depth gauge (an attachment) is a hard steel point ,F, tapering about 1 in 8 at the end. Fixed on top of the depth gauge and in its center line (an attachment) is the steel recording point ,G. The record of the point of contact of the depth gauge on the membrane is made on a sheet of graph paper fixed to the recording board,H,14" x 10 1/8" x 1/2", which can swing about a horizontal axis. The board is fixed to a brass rod and supported by vertical bars which are fixed to the angle irons. The vertical bars are 11 1/2" x 5/8" x 5/8". To mark any position of depth gauge point,F, it is merely necessary to prick a point on the paper by bringing the board down on the recording point,G. (The point of contact of the depth gauge point on the membrane is transferred to the graph paper). The depth gauge can be moved in x-y directions to touch the extended membrane at any desired point.

CHAPTER III
EXPERIMENTAL PROCEDURE

The object of the experiment on the apparatus is to determine the contour lines of the extended membrane. The slopes of the tangent line at the edge of the sections and the volume enclosed between the extended membrane and the plane of the plate are obtained by graphical method.

Method of Using Apparatus

The membrane is stretched by hand and placed between two clean steel plates so that it is held at the edges of the holes and has uniform tension in it. The two steel plates are bolted together. These coupled steel plates, in turn, are bolted to the frame of the test box. The joint between the coupled test plates and the box must be air tight. The membrane across the holes is now tested with the depth gauge; if it is not parallel to the plane of motion of the depth gauge, the test box is leveled by leveling screws. The extension of the membrane is done by allowing air into the test box at a very low pressure (about 1 p.s.i.g) through the copper tube. The pressure is kept with minimum variation by a stop valve. A graph sheet is fixed to the recording board, H, (Fig. 1). Measurements may now be made as desired.

The outline of the experimental hole and circular hole is marked on the graph paper by means of the recording point, G, of the depth gauge. The depth gauge is set and screwed down until the gauge point F touches the membrane and its height is noted. A magnifying glass is used to see whether the gauge point, F, touches the membrane or not and the recording board, H, is lowered to touch the recording point, G, to make a marking point. The depth gauge is moved to a neighboring point on the same contour and again a mark is made on the paper attached to the recording board. This is continued until the contour is completely mapped. Similarly, select another contour and note its height, and completely map it on the recording board. Select contours from very near the edge of the section to the top of the extended membrane with a contour interval from 0.010 to 0.075. The above procedure is done for the test section and for the standard circular hole. The mapped contours of the test section and circular section on the graph sheet are sufficient to proceed for further calculations (Fig. 3).

Results From Measurements

The volume is determined by graphical method after complete mapping of all the contours of the extended membrane of the test section and circular section. The area enclosed by each contour line is measured by a planimeter, and these areas are plotted against the corresponding elevation. The areas are plotted as ordinates and the heights as abscissa (Fig. 4). A smooth curve is drawn, enclosing an area that represents the volume under the extended membrane. This area is determined by a planimeter and this gives the volume. The volumes enclosed by the membranes over the test section and the standard circular section is determined by the above method. The ratio of the volumes of the test section and the circle gives the ratio of torque of the two above sections, under the condition that they have the same angle of twist per unit length and the same shearing modulus. This experimental ratio is compared with the existing theoretical ratio and the percentage error is determined.

The slopes of the membrane are calculated from the contour spacings (i.e., both the vertical and horizontal spacings). The vertical spacing is obtained from the depth gauge readings. The horizontal spacing of the contour is measured by a scale from the contour mapping on the graph sheet (which was attached to the recording board H). To determine the stress at any point (x,y) in the test section, the maximum slope at that point in the section is obtained and the maximum slope at the boundary in the standard circular section are taken into consideration. The ratio of these maximum slopes gives the shear stress ratio of the test section at the point (x,y) and the circular section at the boundary. Since the stress in the circle at the boundary is known, the stress in the section at the point (x,y) is determined. The maximum stress in a section is usually the only measurement of practical importance. Visualization of the shape of the extended membrane covering the section and circular hole shows that the maximum slope occurs at the edge of the sections. For estimating the slope of the extended membrane the horizontal spacings of the contours are plotted on x-axis and the vertical spacings of the contours are plotted on y-axis. The points are fitted with a smooth curve and the slope at any point is obtained by the graphical method. For maximum slope at the edge of the extended membrane, a tangent is drawn to the smooth curve through the origin (Fig. 5).

The maximum slope at the edge of the test section and the circular section is determined by drawing an accurate tangent to the curves through the origin. The ratio of these maximum slopes gives the maximum stress ratio between the two sections. This ratio is compared with the existing theoretical stress ratio and percentage error is determined.

Percentage error for the stress and torque ratios is calculated from the simple relation given below.

$$\left\{\frac{(\text{Experimental ratio}) - (\text{Theoretical ratio})}{\text{Theoretical ratio}}\right\} \times 100$$

WORK .. T

The procedure to calcul . the stress and torque ratios for a square (3" x 3', and a circle (3" diameter) is given below (refer to Fig. 3 and 4). The radius of the circular hole is made approximately equal to the value of 2 A/P where A is the area and P is the perimeter of the test hole, then Sin β=mean value of SinΨ (reference 2), where Ψ is edge angle at any point.

The area of each contour of the test section and circular section is given in Table I. Corresponding heights are also given in the Table I. The volumes of the test section and circular section are obtained from Fig. 4.

V_1 = Volume of the test section = 1.495 Cu. inches

V_2 = Volume of the circular section = 1.151 Cu. inches

$$\frac{V_1}{V_2} = \text{Volume ratio} = \frac{1.495}{1.151} = 1.299$$

Theoretical torque ratio of the two sections (Reference 5) = 1.432

$$\%\ \text{error } V_1/V_2 = \frac{1.299 - 1.432}{1.432} \times 100 = -9.2$$

The vertical spacings and horizontal spacings of the contours are plotted on Y-axis and X-axis (Fig. 5)

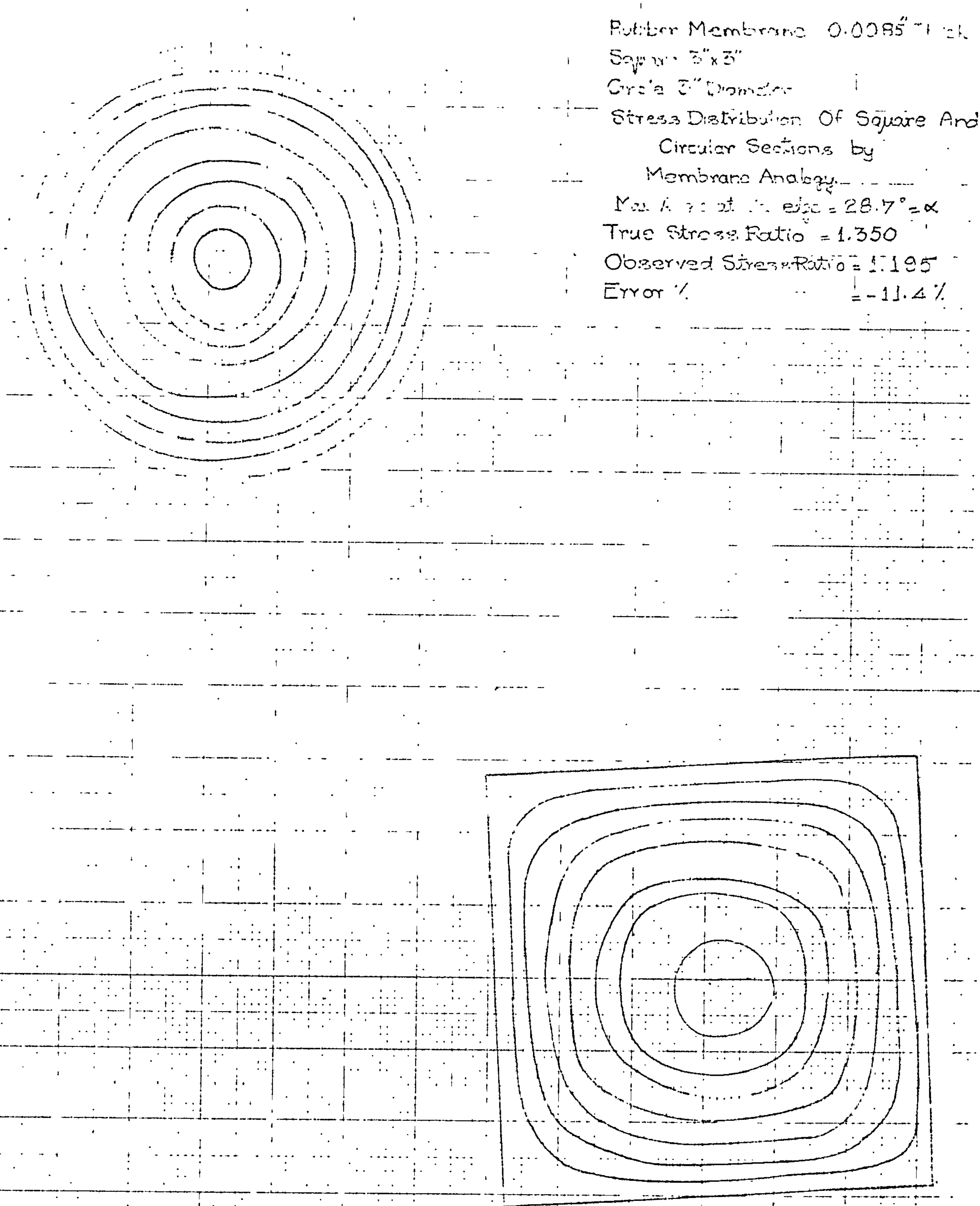


Figure 3
Membrane Contour Lines of the Square and Circular Sections Obtained from Rubber Membrane

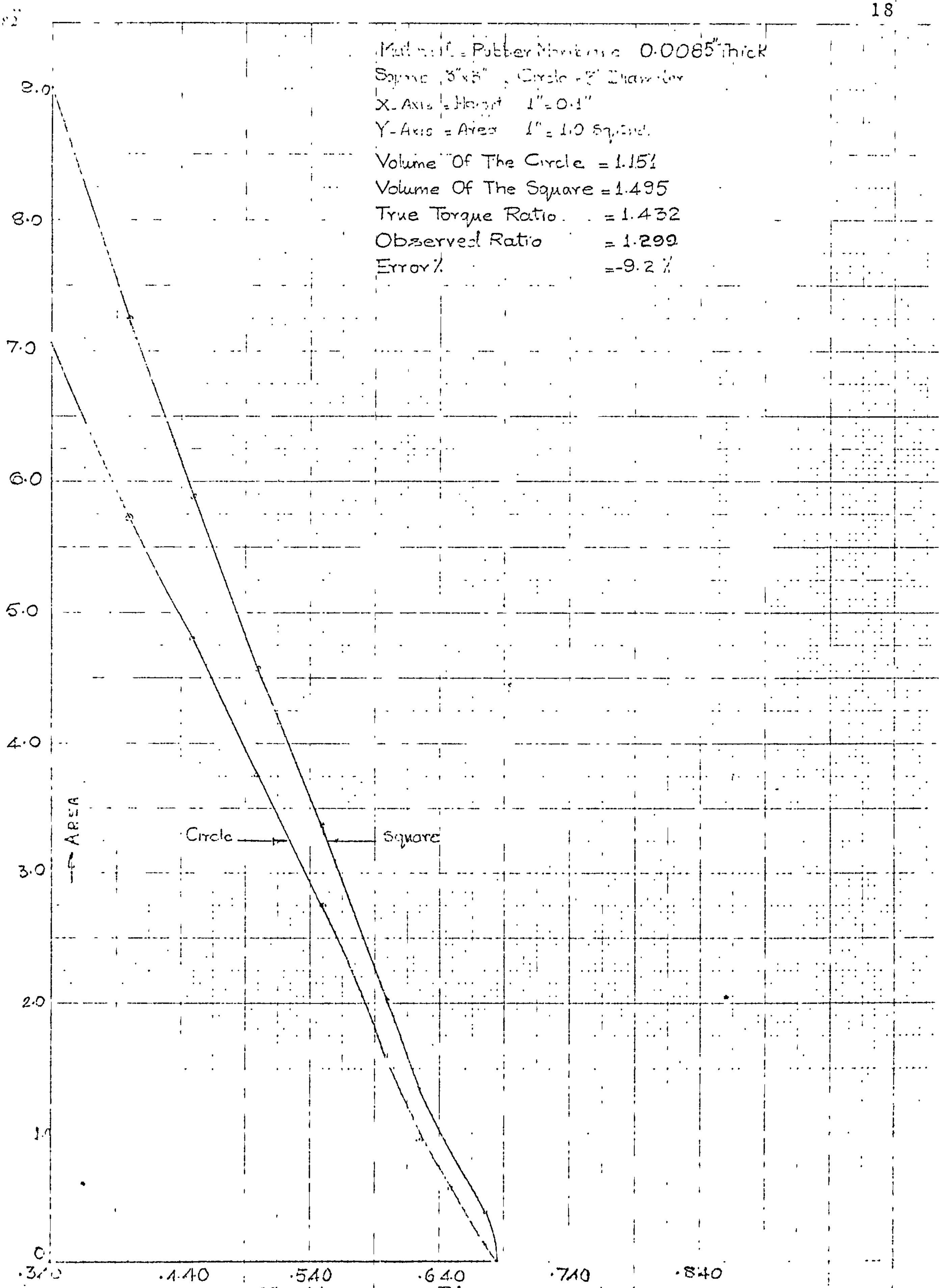


Figure 4
Volume Graphs for the Square and the Circle
(Based on data from Table I)

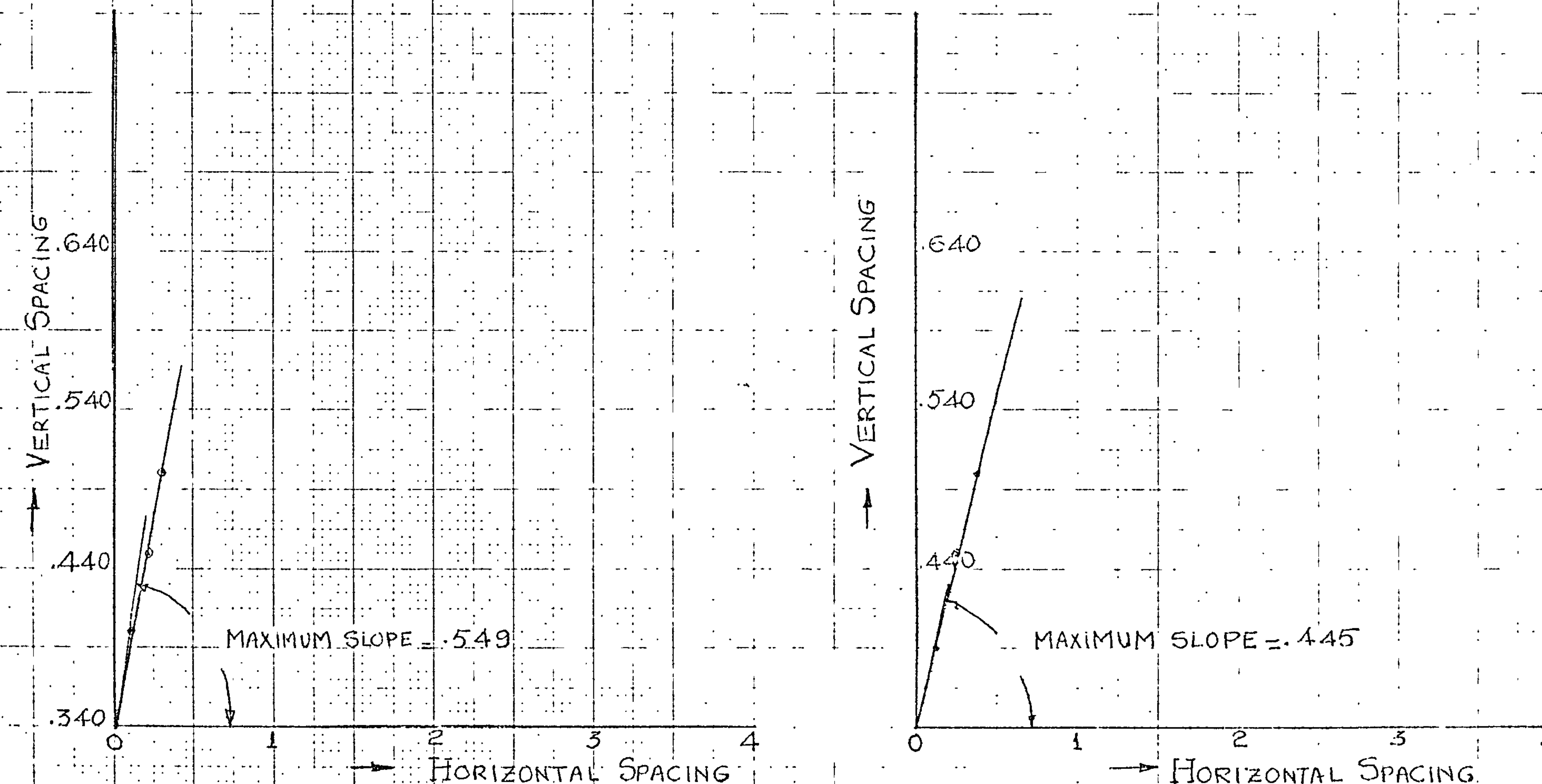
TABLE I

DATA TO DETERMINE THE VOLUMES OF SQUARE AND CIRCULAR SECTIONS

(RUBBER MEMBRANE)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
9	.340		7.07	.340				
7.25	.400		5.73	.400				
5.87	.450		4.80	.450				
4.56	.500		3.76	.500				
		1.495			1.151	1.300	1.432	- 9.2
3.37	.550		2.74	.550				
2.03	.600		1.58	.600				
1.35	.625		.95	.625				
.39	.675		.56	.650				
0	.685		.13	.675				
			0	.685				

Graph To Determine The Maximum Slope For The Two Sections



FOR SQUARE SECTION

FOR CIRCULAR SECTION

FIGURE 5

and the maximum slope of the . :es is determined by graphical method.

Maximum slope of test section $= \frac{.06}{0.1093} = 0.549$

Maximum slope of circular section $= \frac{0.06}{0.1347} = 0.4454$

Ratio of slopes $= \frac{0.549}{0.4454} = 1.232$

Theoretical Shear Stress ratio = 1.350*

% error in determining stress

ratio $= \frac{1.232 - 1.350}{1.350} \times 100 = -8.7$

Follow the procedure to determine the stress at any point in a section. Consider a point A in the rectangular section (refer to Fig. 6 and Table II).

Maximum Slope at A of test Section $= \frac{\delta z}{\delta x} = \frac{.065}{.1562}$

Maximum Slope at the boundary of the circular section

$= \frac{.05}{.109}$ Ratio of Slopes $= \frac{.109}{.05} \times \frac{.065}{.1562} = 0.91$

Theoretical Shear Stress ratio = 0.942 (Reference 5)

% Error in determining Stress ratio $= \frac{.91 - .942}{.942} \times 100$

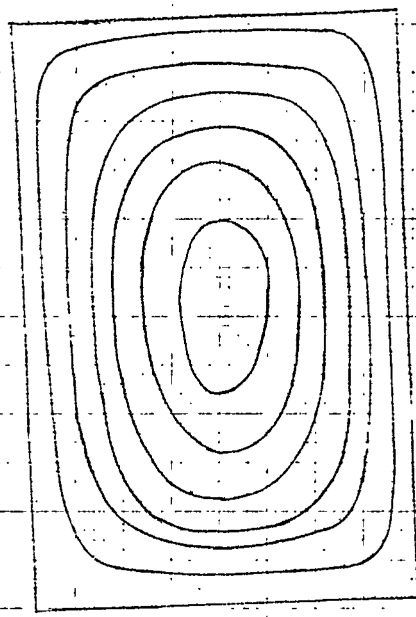
$= -3.4\%$

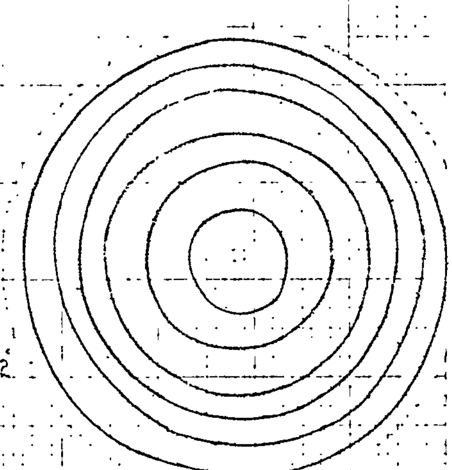
$$\frac{\tau_A}{\tau_{c)max}} = .91$$

$$\tau_A = .91\ \tau_{c)max}$$

where τ_A is the stress at A in the considered section

* See Appendix (page 71)





Stress Distribution Of The Two Sections

Rectangle 2" × 3"

Circle 2.40" Diameter

Material:

Polyethylene Sheet, Thickness = 0.0015"

Maximum Angle At The edge Of The Section = 32.62°

Maximum Angle At The edge Of The Circle = 24.6°

Observed Slope Ratio = 1.326

Stress Ratio = 1.414

Error = -6.2 %

Figure 6

Membrane Contour Lines of the Rectangle and Circular Sections Obtained from Polyethylene Material

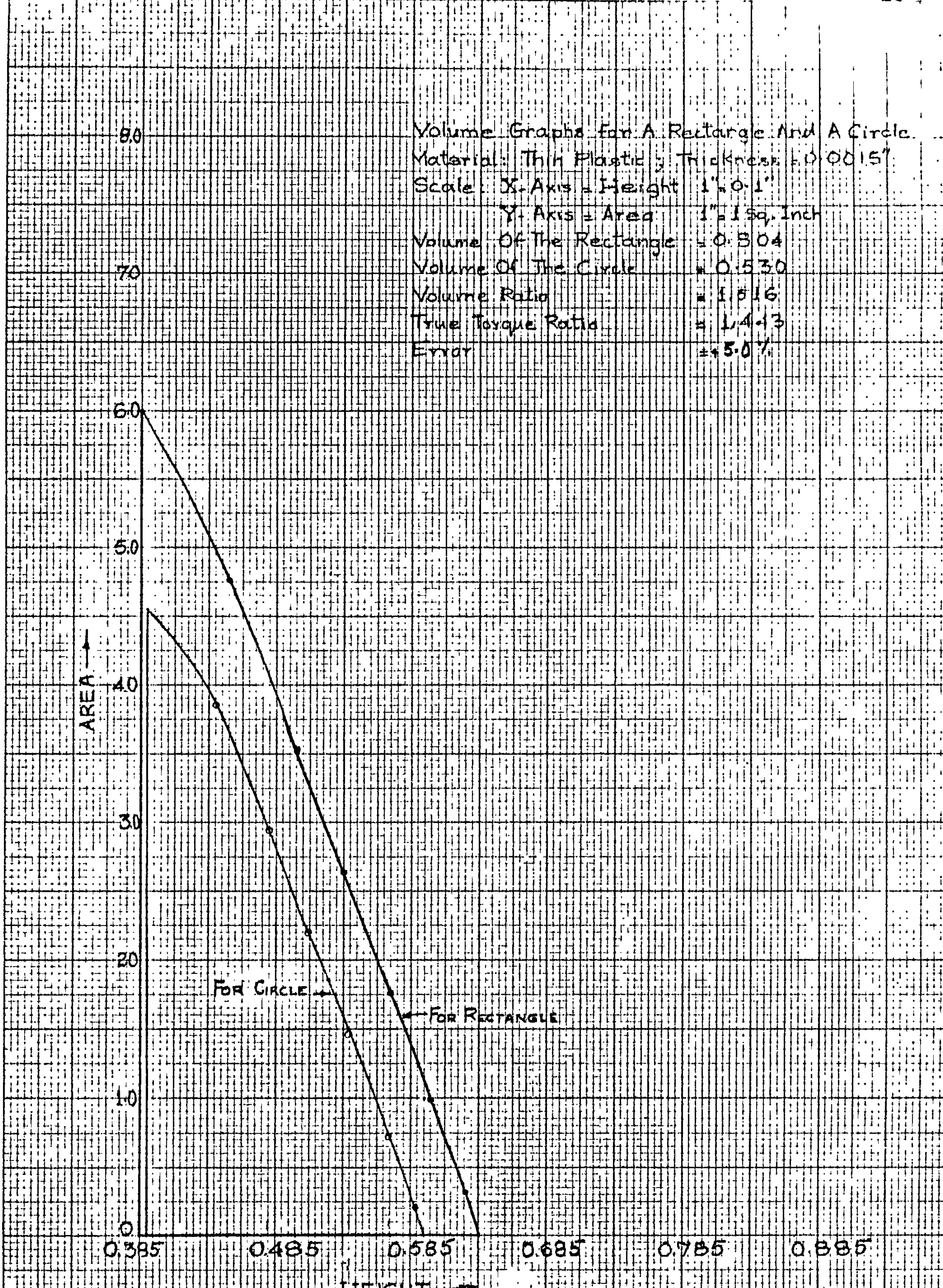


Figure 7
Volume Graphs for the Rectangle and the Circle
(Based on data from Table XIX)

TABLE II

DATA TO DETERMINE THE VOLUMES OF RECTANGLE AND CIRCULAR SECTIONS (POLYETHYLENE MATERIAL)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6	.385		4.52	.390				
4.76	.450		3.85	.440				
3.52	.500		2.94	.480				
		0.804			0.530	1.516	1.443	+ 5.0
2.63	.535		2.20	.510				
1.76	.570		1.46	.540				
0.98	.600		0.72	.570				
0.32	.625		0.20	.590				
0	.635		0	.595				

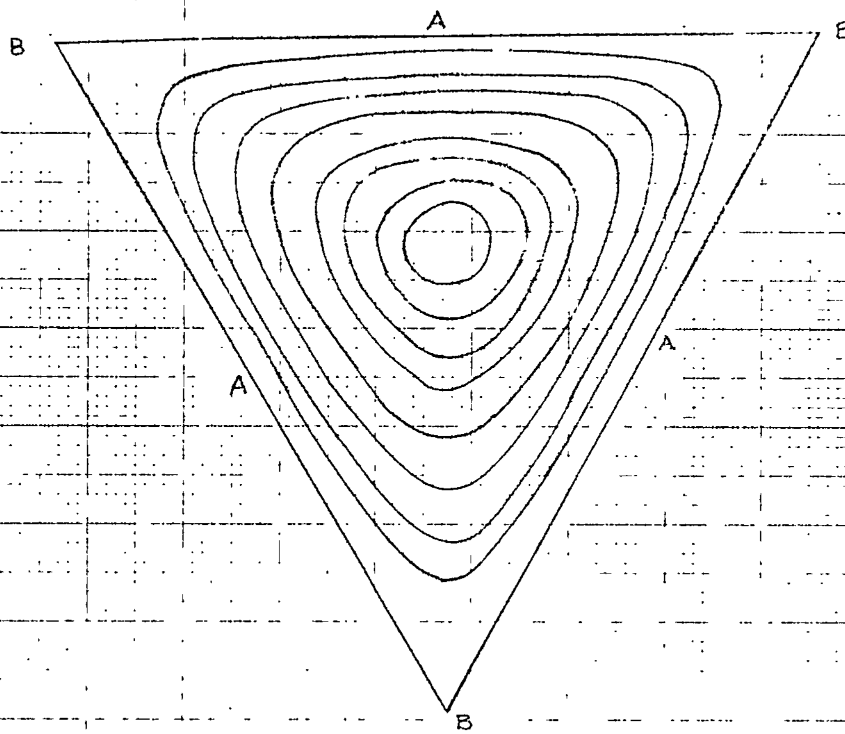
CHAPTER IV

EVALUATION, RESULTS AND DISCUSSION

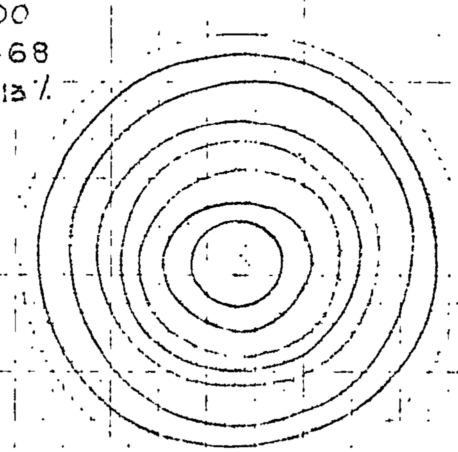
The results obtained from the experiments performed on the apparatus are given in Table I to Table XVII.

The values set down in Table III(page 28) indicate results of an equilateral triangular section (4" side). Fig. 8 is obtained directly from the experiment performed with rubber membrane on the apparatus. The contour lines of the section represent the lines of shearing stress. The spacing of contour lines tends to widen as they proceed towards the center of the section. Consequently, the slope at the middle point of the edge of the triangular section is maximum slope in the entire section. Since stress is proportional to slope, the maximum stress occurs at the middle point of the edge of the section. In order to eliminate the constant of proportionality a standard circular section of radius = 1.154 in. is taken into consideration. The ratio of maximum slopes obtained is 1.580. This gives the maximum stress ratio. The maximum stress for a circular section can be calculated so the maximum stress for the section is evaluated from the ratio of maximum stresses. The true maximum stress ratio is 1.500.* The maximum slope in the triangular section

*See Appendix (page 79)



Stress Distribution Of The Two Sections
Equilateral Triangle 4" Side
Circle 2.308" Diameter
Material:
Rubber Membrane; Thickness = 0.005"
Maximum Angle At The edge = 34.44°
Stress Ratio = 1.500
Observed Ratio = 1.468
Error = - 2.13 %



Note:
In the corner points marked B the stress is zero;
the points marked A are those of maximum stress

Figure 8
Membrane Contour Lines of the Equilateral Triangle and Circular Sections Obtained from Rubber Membrane

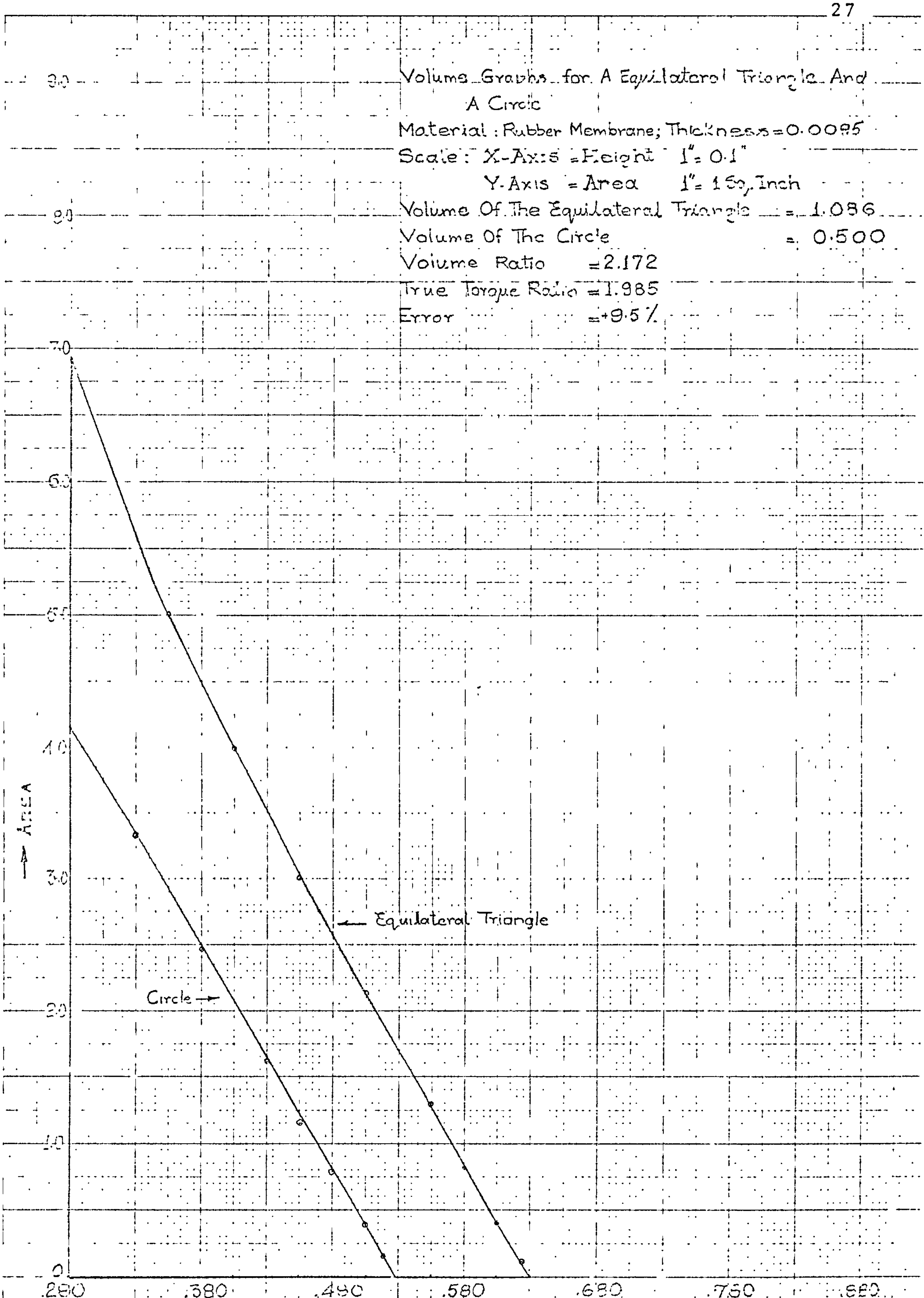


Figure 9

Volume Graphs for the Equilateral Triangle and the Circle

TABLE III

DATA TO DETERMINE THE VOLUMES OF EQUILATERAL TRIANGLE AND CIRCULAR SECTIONS (RUBBER MEMBRANE)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6.928	.280		4.17	.280				
5.01	.355		3.32	.330				
4.02	.405		2.48	.380				
3.02	.455		1.61	.430				
2.14	.505		1.14	.455				
		1.086			0.500	2.172	1.985	9.5
1.30	.555		0.78	.480				
0.82	.580		0.38	.505				
0.41	.605		0.16	.520				
0.13	.625		0	.530				
0	.630							

can be easily calculated from the contour mapping obtained from the apparatus. The experimental error in determining the maximum stress ratio by using a rubber membrane for a triangular and circular section is +5.3 per cent. The volume enclosed by the triangular and circular sections is obtained form Table III (page 28) by graphical construction (Fig. 9). The Volume ratio of the two above sections give the torque ratio of the said sections. The experimental volume ratio is 2.172. The true torque ratio is 1.985.* The experimental error in determining the volume ratio by using rubber membrane is + 9.5 per cent.

The error in the experimental results is due to the membrane material and a few drawbacks of the apparatus. Membrane material contributes a greater percentage of error than the apparatus drawbacks. The membrane used in the apparatus is not an ideal membrane. Rubber membrane and Polyethylene Material* used in the experiment have a certain amount of rigidity. Assuming the membrane to be a plate and finding the maximum deflection by plate theory equation for large deflection for a cir-

*See appendix (page 78)

cular plate with clamped edges an approximate amount of error involved in the experimental results due to shear is determined. Using the properties (E,P,μ) of the rubber membrane in the plate equation, the maximum deflection obtained is .306". Experimental value for the maximum deflection or elevation (Z_{max}) is .333" (Table X) and the maximum elevation by membrane theory (Spherical segment) with the same maximum angle at the edge as the experimental membrane is .3349". Similarly, using the properties of polyethlene material (.0015" thick) Z_{max} by plate theory equation is .3985" and Z_{max} by experimental analysis .405" (Table XI) and Z_{max} by membrane theory is .4614" (see page 42). The above numerical values for Z_{max} (plate theory) clearly show that about -13.6 per cent error and -8.6 per cent error is involved using the polyethylene sheet and rubber membrane as a plate and comparing with the membrane theory (Spherical Segment) results. And comparing with experimental values the per cent error was -1.62 and -8.1. The experimental error in the results by using rubber membrane and polyethylene sheet, is -.56 per cent and 12.2 per cent in determining Z_{max} (when compared with membrane theory results).

To determine the amount of error involved in slope calculation due to shear in the material, a deflection curve is drawn by using the plate theory equation. The configuration of the experimental elevated membrane and the spherical segment (ideal membrane) is drawn along with the above plate deflection curve (Refer to Fig. 26 and 27). Maximum slope at the same point (x,y) on all three curves is taken and compared. The results are given below:

	Polyethylene (.0015")	Rubber Mem.
From plate theory curve the maximum slope at (x,y)	.3185	.2362
From experimental curve the maximum slope at (x,y)	.3181	.260
From membrane theory curve the maximum slope at (x,y)	.3750	.2637

If rubber membrane behaved like a plate then the per cent of error involved in slope measurement is

$$\frac{.2362 - .2637}{.2637} \times 100 = -\ 10.4\%$$

$$\text{Experimental error} = \frac{.2600 - .2367}{.2367} \times 100 = -\ 1.4\%$$

The above numerical analysis clearly indicates that a small error in results is mainly due to membrane material.

A small error in the results may also be due to the apparatus. The inaccuracy of the apparatus lies in the improper arrangement of the recording board. The recording board does not remain horizontal when the depth gauge readings at different heights are taken due to the fact that the pivot is fixed at one point permanently. The error in the linear measurement involved due to the recording board not being horizontal is about 1 per cent. This produces an errorneous effect in the results of slope.

For an ideal performance of the experiment there should be uniform tension in the membrane covering the test section and the circular section. The membrane was stretched by hand resulting in non-uniform tension in the membrane.

The values set down in Table IV and Table V indicate the degree of accuracy obtainable with the Membrane Analogy Apparatus in the determination of volume measurements in sections by using rubber membrane and polyethylene material. The minimum error, using a rubber membrane, was .62 percent for square section, + 2.98 percent for equilateral triangle section, and 12 percent for rectangle section. The minimum error, using a polyethylene material (.0015" thick) was .35 percent for square section, + 3.5 percent for equilateral triangle, and —.41 percent for rectangle section. The values set down in Table VI and Table VII indicate the degree of accuracy obtainable with the Membrane Analogy Apparatus in the determination of the maximum stress ratio in sections by using rubber membrane and Polyethylene material. The last column in Tables VI and VII show the error percent due to taking the ratio of inclinations as giving stress ratio. Column 8 in Table VI and Table VII show the error percent due to taking the ratio of maximum slopes as giving the maximum stress ratio. The minimum inclination ratio error was 1.1 percent for square section, -2.13 percent for equilateral triangle section, and -6.7 percent for rectangle section in the case of polyethylene material. Table VIII and Table IX show the best results obtained from the apparatus with the rubber membrane and Polyethylene material.

Table X to Table XVII are enclosed in the appendix, which are the experimental results obtained from the apparatus for square, rectangular, and triangular sections.

TABLE IV

EXPERIMENTAL ERROR IN DETERMINING TORQUE RATIOS BY MEANS OF

RUBBER MEMBRANE

1	2	3	4	5	6	7
SECTION	MAXIMUM INCLINATION α	VOLUME OF THE SECTION V_1(Cu.inch)	VOLUME OF THE CIRCLE V_2(Cu,inch)	EXPERIMENTAL VALUE V_1V_2	TRUE VALUE V_1/V_2	% ERROR
SQUARE 3" x 3"	33.6	1.689	1.172	1.441	1.432	+ 0.62
	28.7	1.495	1.151	1.300	1.432	- 9.2
EQUILATERAL TRIANGLE 4" side	43.80	1.486	0.727	2.044	1.985	+ 2.98
	34.44	1.086	0.500	2.172	1.985	+ 9.5
RECTANGLE 2" x 3"	72.30	2.766	1.704	1.622	1.443	+12
	36.20°	0.834	0.470	1.773	1.443	+22.8

TABLE V

EXPERIMENTAL ERROR IN DETERMINING TORQUE RATIO BY MEANS OF POLYETHYLENE MATERIAL

1 SECTION	2 MAXIMUM INCLINATION	3 VOLUME OF THE SECTION V_1 (Cu. inch)	4 VOLUME OF THE CIRCLE V_2 (Cu. inch)	5 EXPERIMENTAL VALUE $V_1 V_2$	6 TRUE VALUE V_1/V_2	7 % ERROR
SQUARE 3" x 3"	44.56	2.230	1.551	1.437	1.432	+ 0.35
	50.20	2.859	2.025	1.411	1.432	- 1.46
EQUILATERAL TRIANGLE 4" side	27.4	0.879	0.470	1.870	1.985	- 5.7
	40.3	1.155	0.562	2.055	1.985	+ 3.5
RECTANGLE 2" x 3"	33.78	0.844	0.587	1.437	1.443	- 0.41
	32.62	0.804	0.530	1.516	1.443	+ 5.0

TABLE VI

EXPERIMENTAL ERROR IN DETERMINING STRESS RATIO

BY MEANS OF RUBBER MEMBRANE

1 SECTION	2 RADIUS OF CIRCLE (INCH)	3 α	4 β	5 $\frac{\alpha}{\beta}$	6 SLOPE RATIO $\frac{\tan\alpha}{\tan\beta}$	7 TRUE VALUE $\frac{\tau_{smax}}{\tau_{cmax}}$	8 % ERROR IN $\frac{\tan\alpha}{\tan\beta}$	9 % ERROR IN $\frac{\alpha}{\beta}$
SQUARE 3" x 3"	1.5"	33.6	25.18	1.335	1.415	1.350	+4.6%	-1.1%
	1.5"	28.57	24.00	1.190	1.223	1.350	-8.7%	-11.9%
EQUILATERAL TRIANGLE 4" side	1.154	43.80	35.15	1.248	1.362	1.500	-9.0%	-16.8%
	1.154	34.44	23.46	1.468	1.580	1.500	+5.3%	-2.13%
RECTANGLE 2" x 3"	1.20	72.30	60	1.210	1.802	1.414	+27%	-14.5
	1.20	36.20	19.8	1.825	2.032	1.414	+43%	29%

TABLE VII

EXPERIMENTAL ERROR IN DETERMINING STRESS

BY MEANS OF POLYETHYLENE MATERIAL

1 SECTION	2 RADIUS OF CIRCLE (INCH)	3 α	4 β	5 $\frac{\alpha}{\beta}$	6 SLOPE RATIO $\frac{\tan\alpha}{\tan\beta}$	7 TRUE VALUE $\frac{\tau_{smax}}{\tau_{cmax}}$	8 % ERROR IN $\frac{\tan\alpha}{\tan\beta}$	9 % ERROR IN $\frac{\alpha}{\beta}$
SQUARE 3" x 3"	1.5"	44.56	34.22	1.302	1.447	1.350	+7.1%	- 3.5%
	1.5"	50.20	42.95	1.168	1.235	1.350	-8.5%	-13.4%
EQUILATERAL TRIANGLE 4" side	1.154	27.4	24.5	1.120	1.139	1.500	-24%	-25.3%
	1.154	40.3	27.4	1.472	1.636	1.500	+ 9%	- 1.86%
RECTANGLE 2" x 3"	1.20	33.78	25.61	1.319	1.396	1.414	-1.2%	+6.7%
	1.20	32.62	24.60	1.326	1.396	1.414	+1.2%	+6.2

TABLE VIII

A COMPARATIVE STUDY SHOWING EXPERIMENTAL ERROR IN DETERMINING STRESS RATIO BY MEANS OF RUBBER MEMBRANE AND POLYETHYLENE MATERIAL

1 MATERIAL	2 SECTION	3 RADIUS OF CIRCLE	4 $\frac{TAN\alpha}{TAN\beta}$	5 $\frac{\alpha}{\beta}$	6 TRUE VALUE $\frac{\tau_{smax}}{\tau_{cmax}}$	7 % ERROR IN $\frac{TAN\alpha}{TAN\beta}$	8 % ERROR IN $\frac{\alpha}{\beta}$
POLYETHYLENE MATERIAL	SQUARE 3" x 3"	1.50	1.447	1.302	1.350	+ 7.1	- 3.5
	EQUILATERAL TRIANGLE 4" side	1.15	1.636	1.472	1.500	+ 9.0	- 1.86
	RECTANGLE 2" x 3"	1.20	1.396	1.326	1.414	- 1.2	6.2
RUBBER MEMBRANE	SQUARE 3" x 3"	1.50	1.413	1.335	1.350	+ 4.6	- 1.1
	EQUILATERAL TRIANGLE 4" side	1.15	1.580	1.468	1.500	+ 5.3	- 2.13
	RECTANGLE 2" x 3"	1.20	1.802	1.210	1.414	+27.0	-14.5

TABLE IX

A COMPARATIVE STUDY SHOWING EXPERIMENTAL ERROR IN DETERMINING TORQUES BY MEANS OF RUBBER MEMBRANE AND POLYETHYLENE MATERIAL

1 MATERIAL	2 SECTION	3 MAXIMUM INCLINATION	4 OBSERVED VOLUME RATIO	5 CALCULATED TORQUE RATIO	6 % ERROR
	SQUARE 3" x 3"	44.56	1.437	1.432	+ 0.35
POLYETHYLENE MATERIAL	EQUILATERAL TRIANGLE 4" side	40.3	2.055	1.985	+ 3.5
	RECTANGLE 2" x 3"	33.78	1.437	1.443	-0.41
	SQUARE 3" x 3"	33.6	1.441	1.432	+ 0.62
RUBBER MEMBRANE	EQUILATERAL TRIANGLE 4" side	43.80	2.044	1.985	+ 2.98
	RECTANGLE 2" x 3"	72.30	1.622	1.443	+ 12

<u>Determination Of Error Due To Fact That Membrane Used Was Not A Theoretical Membrane (No Shear)</u>

Consider a circular section of 3" diameter. Assuming the maximum inclination at the boundary of the elevated ideal membrane be $\beta = 25.18°$ (same as in rubber membrane). The maximum elevation of the elevated membrane is given by $R \operatorname{Tan} \frac{\beta}{2}$

$$Z_{max} = R \operatorname{Tan} \frac{\beta}{2}$$

$$= 1.5 \times .2233 = .33495$$

Z_{max} obtained by experiment is $(.685 - .352) = .333$ Table X. The difference in the above two values of Z_{max} for the same $\beta = \frac{.333 - .33495}{.33495} \times 100 = -.56\%$

From the plate theory with large deflections Z_{max} is obtained from the equation (reference 6)

$$P = \frac{8}{3}\frac{E}{1-\mu} \cdot \frac{t}{R}\frac{(Z_{max})^3}{R^2} + \frac{64D}{R^3}\frac{(Z_{max})}{R}$$

where
$$D = \frac{E\,t^3}{12(1-\mu^2)}$$
$P = .85$ p.s.i.
$\mu = .5$
$E = 20{,}000$ lb./sq. inch
$R = 1.5"$
$t = .0085$ (Thickness of rubber membrane)
$Z_{max} = 0.167$

The value of Z_{max} obtained by plate theory equation is very low because t of the unstretched rubber membrane was used in the equation. However, in the experiment the rubber membrane was stretched. The thickness of the stretched membrane should be used in the plate equation. Assuming t of the stretched membrane = 1/2 t of the unstretched membrane. Modulus of elasticity in the rubber membrane is not constant so take average E = 6666.6 lbs./sq. in. in the Z_{max} calculations. Substituting the above values in plate equation, Z_{max} obtained is .3060.

Z_{max} = .33495 (From ideal membrane theory)
Z_{max} = .333 (Experimental Value)
Z_{max} = .3060 (From Plate Theory with large deflections)

Experimental error in Z_{max} (when compared to membrane theory results) $= \frac{.333 - .3349}{.3349} \times 100 = -0.56\%$

Using Polyethylene Material (.0015" Thick)

Consider a circular section of 3" diameter. The maximum elevation of an extended ideal membrane is given by R Tan $\frac{\beta}{2}$. Assuming (34.22°) the same for the ideal membrane and for the polyethylene material.

$$Z_{max} = R \operatorname{Tan} \frac{\beta}{2}$$
$$= 1.5 \times \operatorname{Tan} 17.11 = .4614$$

Z_{max} obtained from the experiment is (.810 - .405) = .405 (Table XI). The difference in the above two values of Z_{max} for the same $= \frac{.405 - .4614}{.4614} \times 100 = -12.1\%$

From the plate theory with large deflections Z_{max} is obtained from the equation (reference 6)

$$P = \frac{8}{3}\,\frac{E}{1-\mu}\cdot\frac{t}{R}\,\frac{(Z_{max})^3}{R^3} + \frac{64D}{R^3}\,\frac{(Z_{max})}{R}$$

where $D = \dfrac{Et^3}{12(1-\mu^2)}$

$P = 1.00$ p.s.i.

$t = \frac{3}{5} \times .0015'' = .0009$ (Thickness of the Stretched Polyethylene Sheet)

$R = 1.5$

$E = 20{,}000$ lbs./sq.in.

$\mu = .4$

$Z_{max} = .3985$

$Z_{max} = .4614$ (By ideal membrane theory)

$Z_{max} = .405$ (Experimental Value)

$Z_{max} = .3985$ (Plate theory for large deflections)

Experimental error in $Z_{max} = \dfrac{.405 - .4614}{.4614} \times 100 = -12.2\%$

In order to determine the error in slope measurement due to a slight shear in the membrane three deflection curves were drawn. The three curves are drawn from the data obtained from the experiment (Table X and Table XI), plate theory equation, and the ideal membrane theory. They give a relationship between deflection and horizontal axes. An arbitrary point A(x,y) was chosen on the three curves and maximum slope at that point was determined and the results were compared. The graph with results is given in Fig. 26 and Fig. 27.

CHAPTER V
CONCLUSION

The experimental apparatus is very simple and can be easily built. The results obtained from the apparatus have an error of about 4% average. And the ease of operation with it justifies its use in the membrane analogy experiments.

The experimental results of this thesis indicate that there was a considerable degree of error due to the membrane material. The error in the results was also added to by drawbacks in the apparatus. The apparatus can be made more accurate by following the recommendations which are based on practical difficulties encountered in performing the experiment and getting the results.

The recording board should be maintained horizontal whenever the readings are taken. A slot of considerable length (about 1 in.) should be made in the vertical bars at the pivot, so that the recording board can be moved up and down; and can be fixed by means of a nut. By this arrangement the board can be maintained horizontal whenever the readings are taken. The horizontal position of the board is obtained by taking the support of the depth gauge and with the help of the spirit level.

To develop an approximate magnitude of tension in the membrane see that a network of squares drawn on the unstressed rubber membrane should deform when stretched, into double the previous squares. Uniform tension may be produced to a small extent by placing the rubber membrane between two overlapping rings similar to wooden over rings employed for embroidery.

The purpose of this project was to construct and demonstrate the use of the apparatus. It was not until after the construction was finished that the short-comings mentioned above were realized. The two materials used for membranes were arbitrarily selected. It is recommended that as part of any further development of this apparatus that an investigation be made to optomize the selection of a more appropiate membrane material.

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BIBLIOGRAPHY

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APPENDIX

APPENDIX A

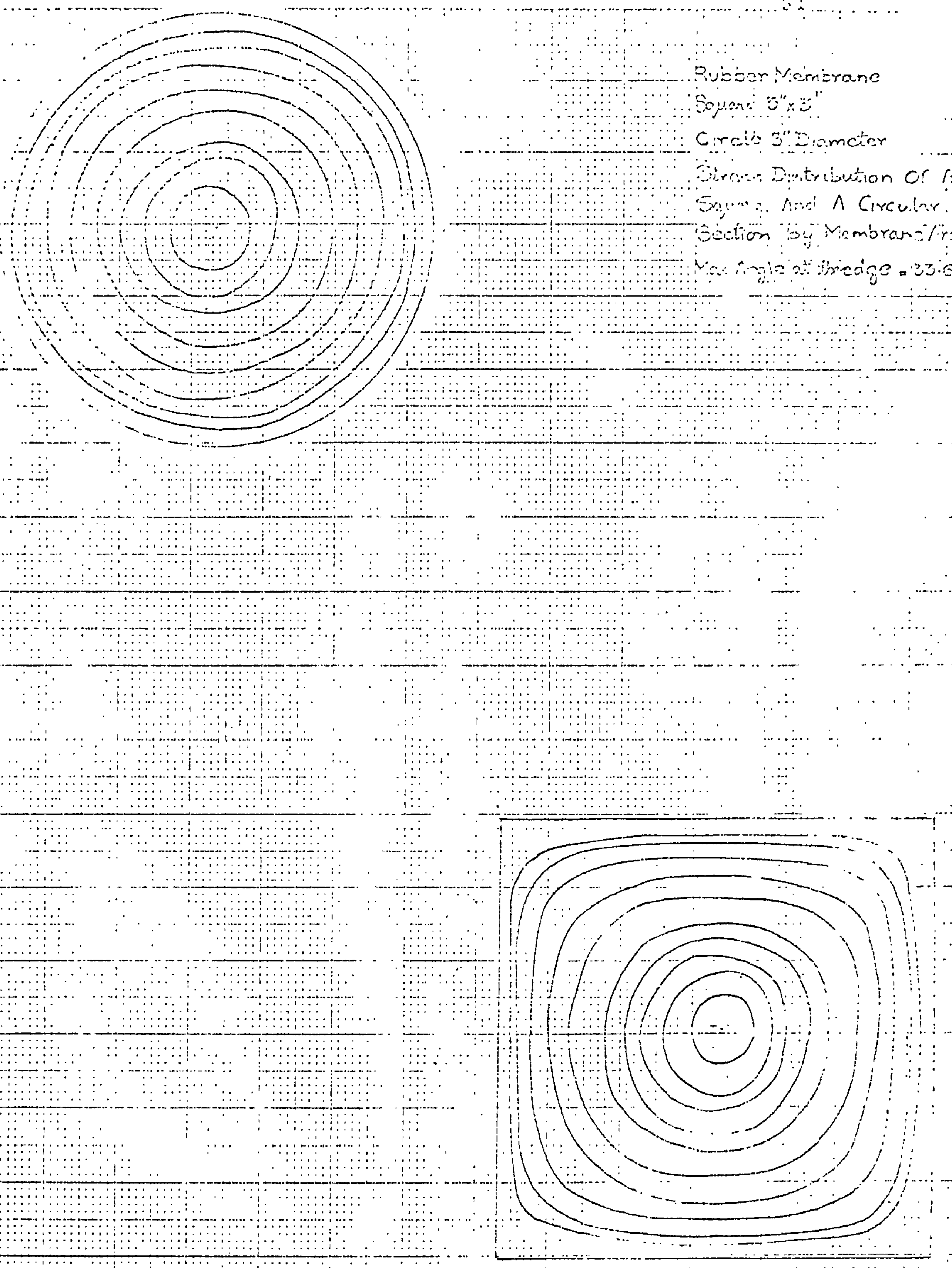


Figure 10

Membrane Contour Lines of the Square and Circular
Sections Obtained from Rubber Membrane

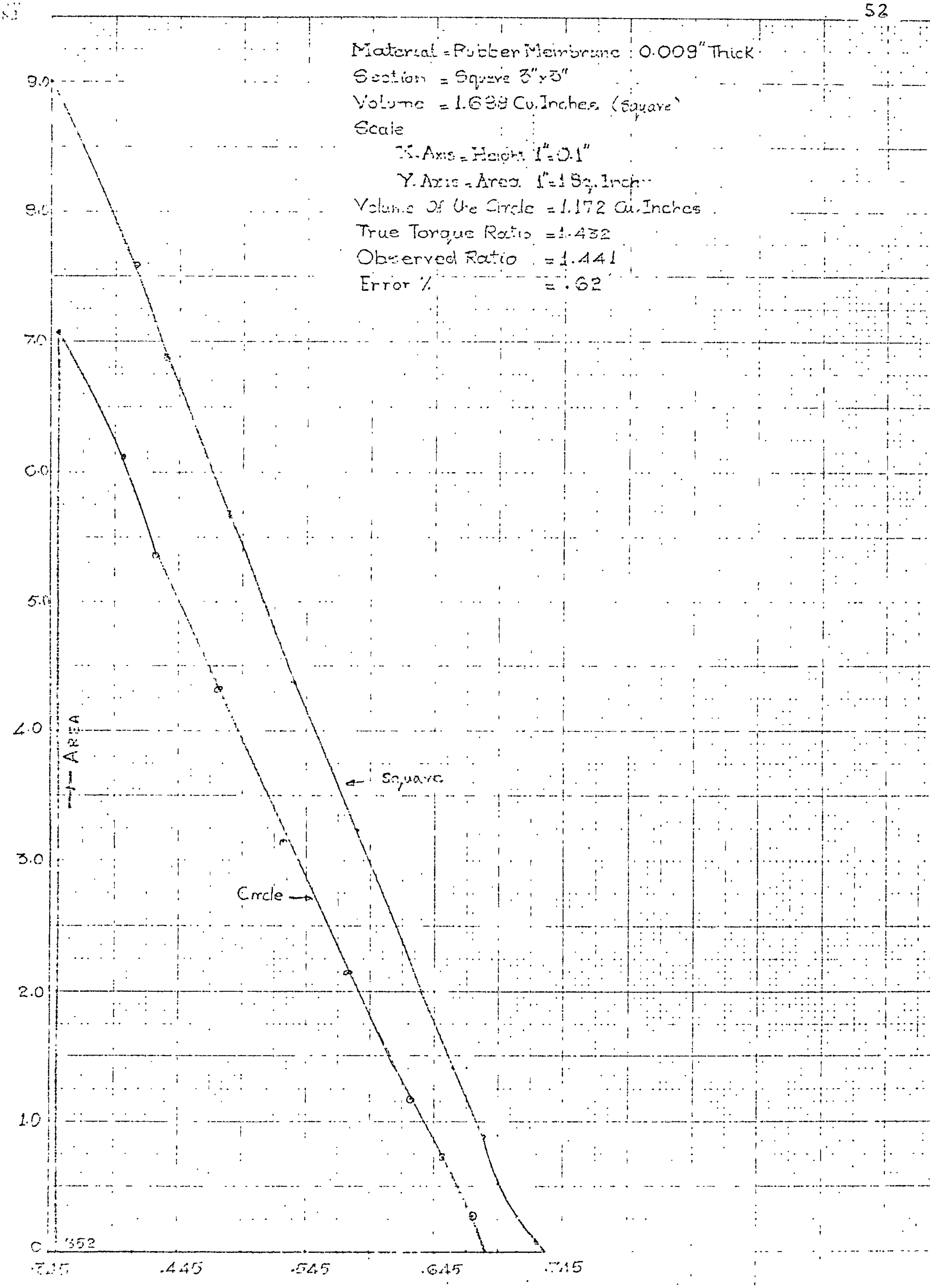


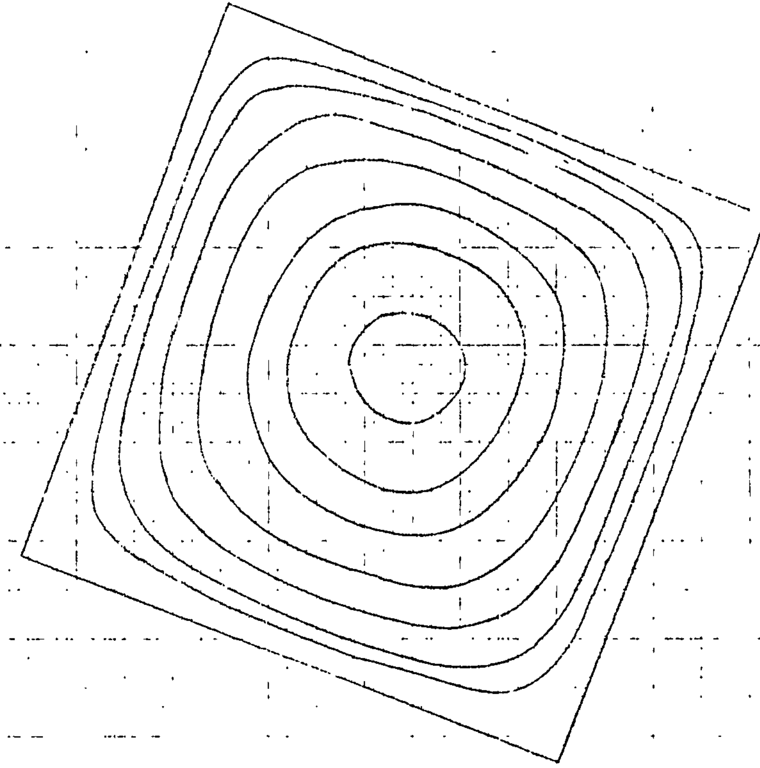
Figure 11
Volume Graph for the Square and the Circle

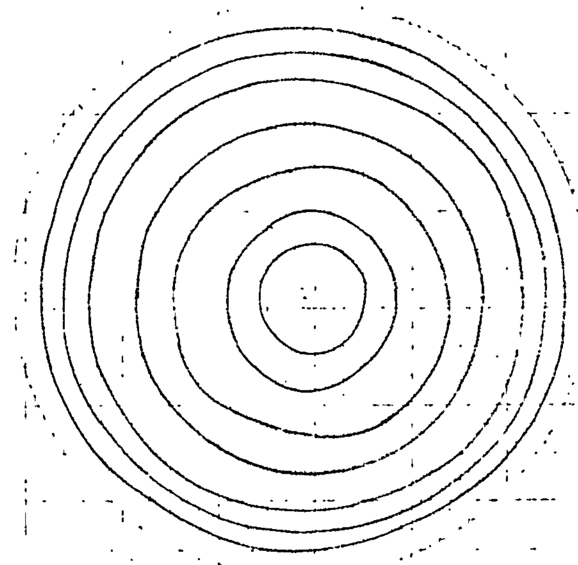
TABLE X

DATA TO DETERMINE THE VOLUMES OF SQUARE AND CIRCULAR SECTIONS

(RUBBER MEMBRANE)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
9.00	.345		7.07	.352				
7.59	.410		6.11	.402				
6.87	.435		5.36	.427				
5.68	.485		4.32	.477				
4.39	.535		3.14	.527				
3.24	.585		2.15	.577				
		1.689			1.172	1.441	1.432	+ 0.628
2.00	.635		1.17	.627				
1.40	.660		.73	.652				
.87	.685		.28	.675				
.53	.700		0	.685				
.17	.715							
0	.722							





Materal = Thin Plastic , Thickness = .0015"
Square 3" x 3"
Circle 3" Diameter
Stress Distribution Of The Two Sections
Maximum Angle at the Edge = 44.56° = α
Stress Ratio = 1.350
Observed Ratio = 1.302
Error % = -3.5 %

Figure 12
Membrane Contour Lines of the Square and Circular Sections
Obtained from Polyethylene Material

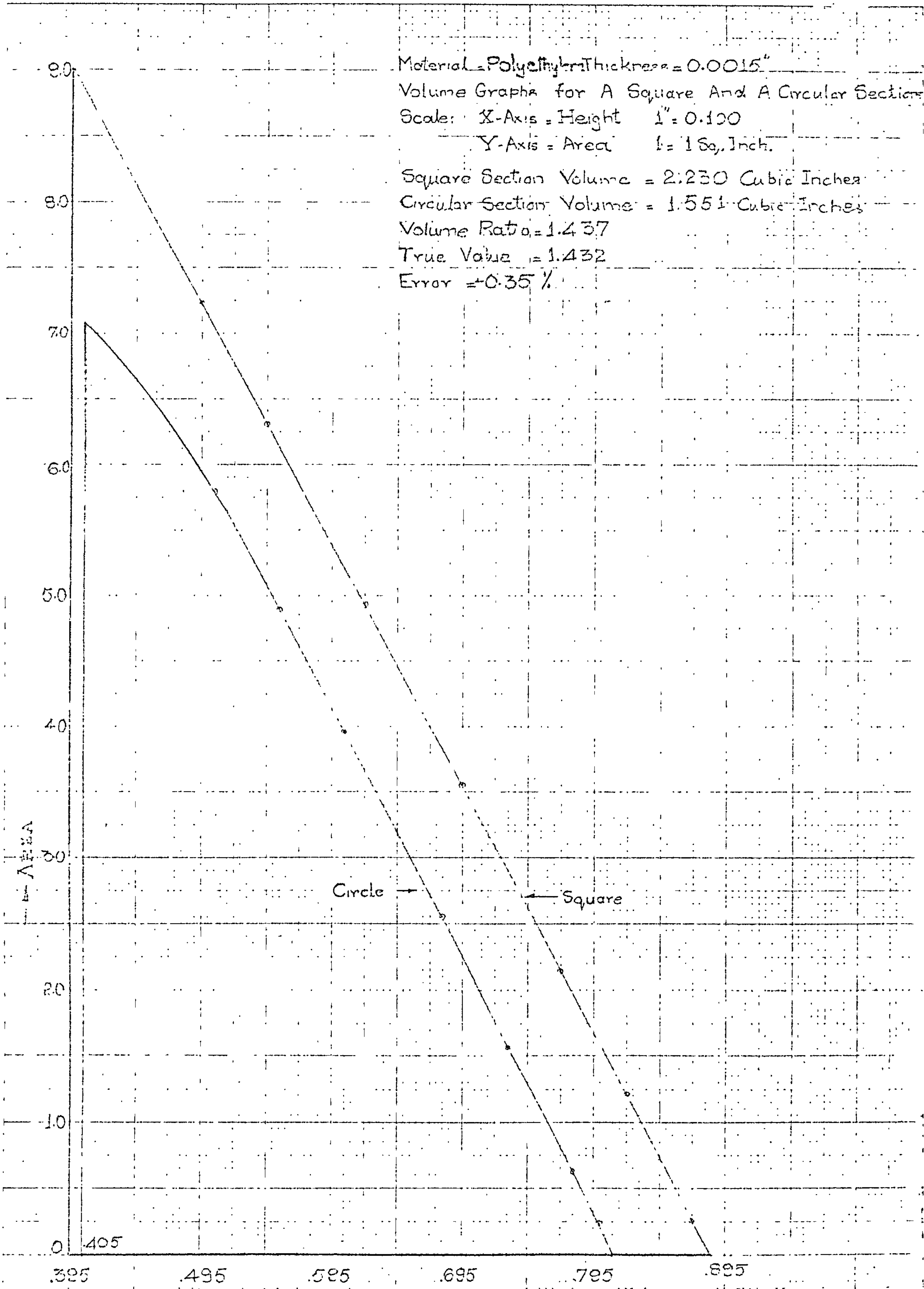


Figure 13
Volume Graphs for the Square and the Circle

TABLE XI

DATA TO DETERMINE THE VOLUMES OF SQUARE AND CIRCULAR SECTIONS (POLYETHYLENE MATERIAL)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
9	.395		7.07	.405				
7.26	.495		5.79	.505				
6.31	.540		4.89	.555				
4.93	.620		3.96	.605				
		2.230			1.551	1.437	1.432	+ .35
3.55	.695		2.55	.680				
2.14	.770		1.56	.730				
1.21	.820		.63	.780				
0.26	.870		.24	.800				
0	.880		0	.810				

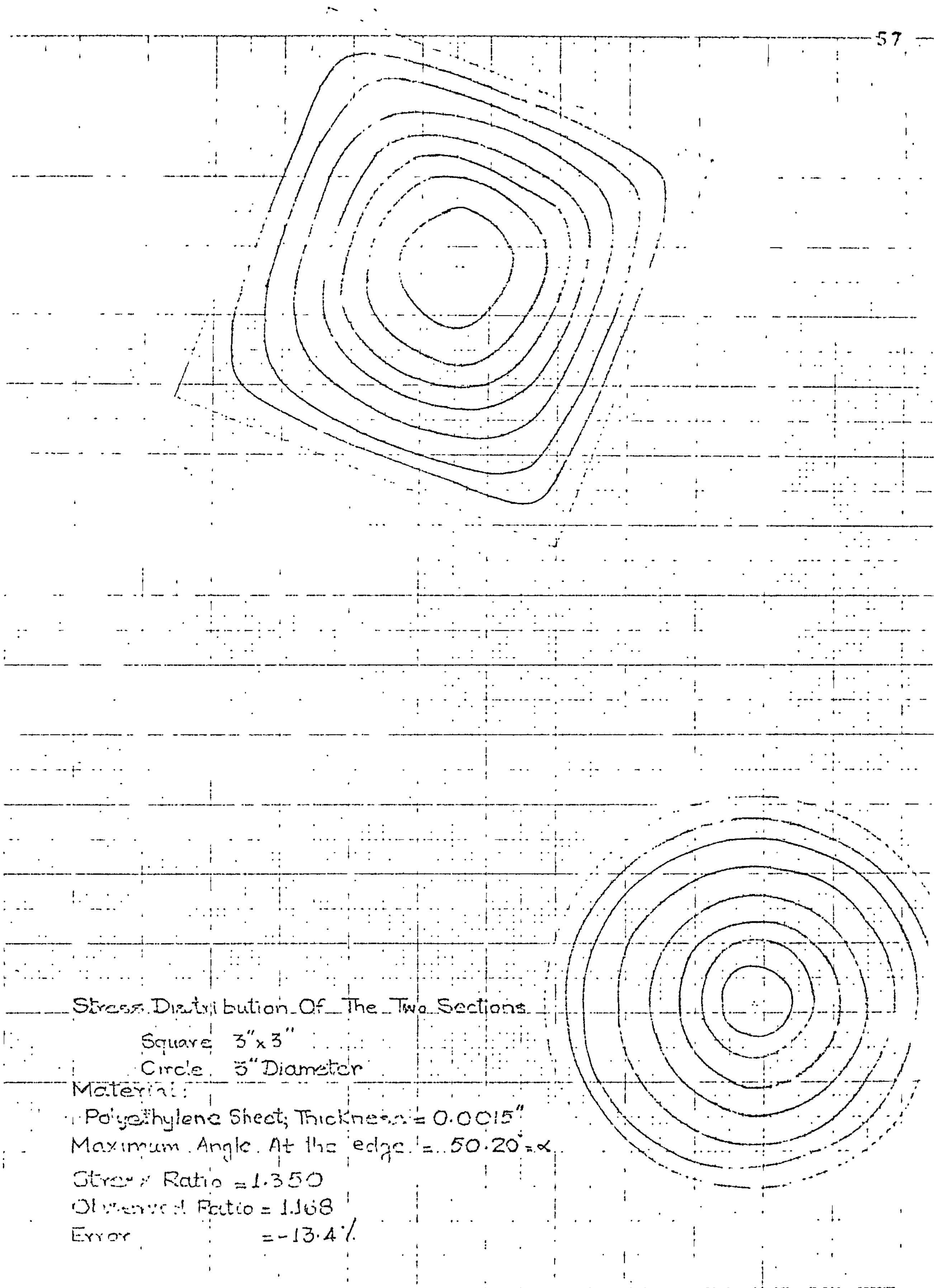


Figure 14
Membrane Contour Lines of the Square and Circular Sections
Obtained from Polyethylene Material

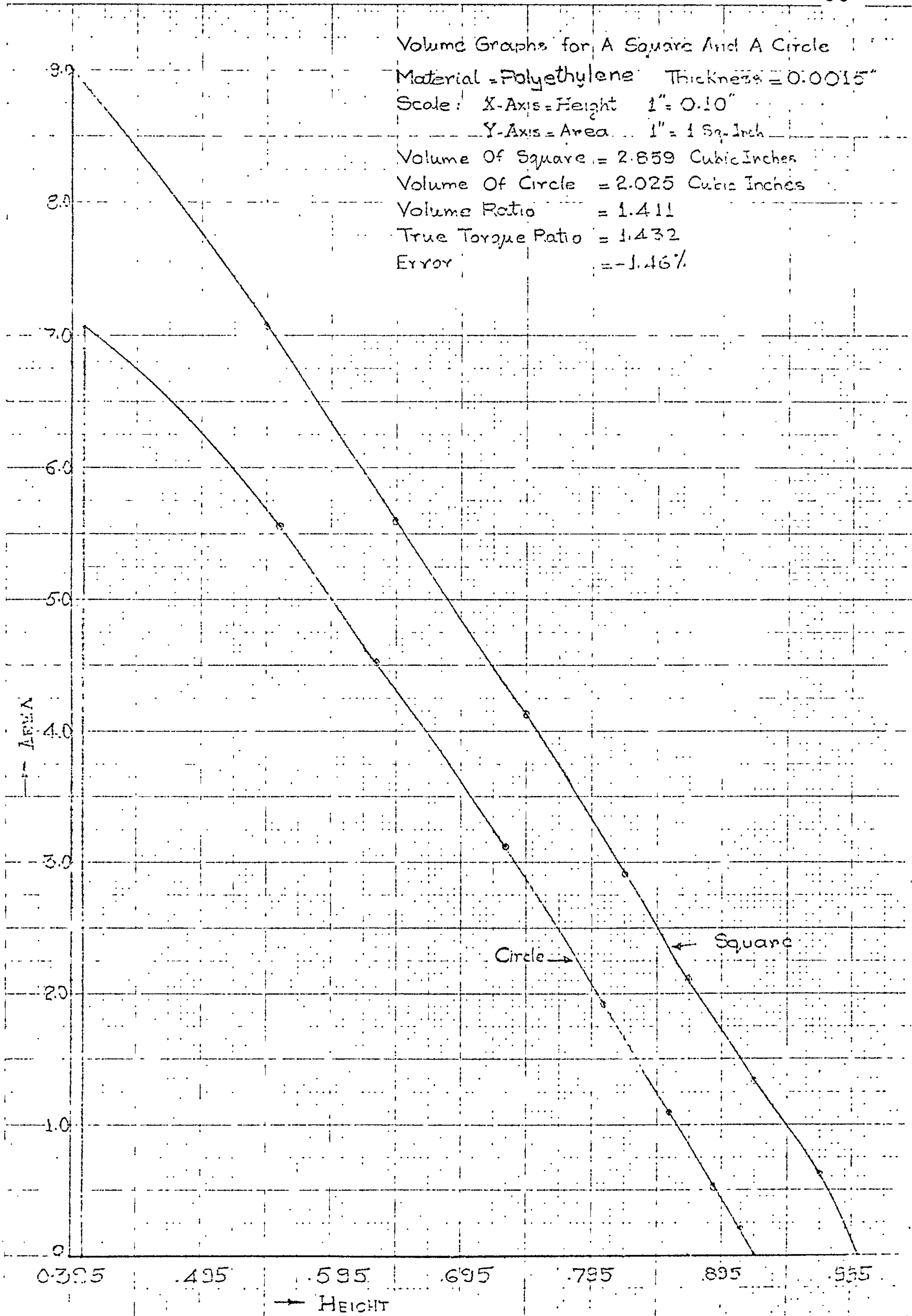
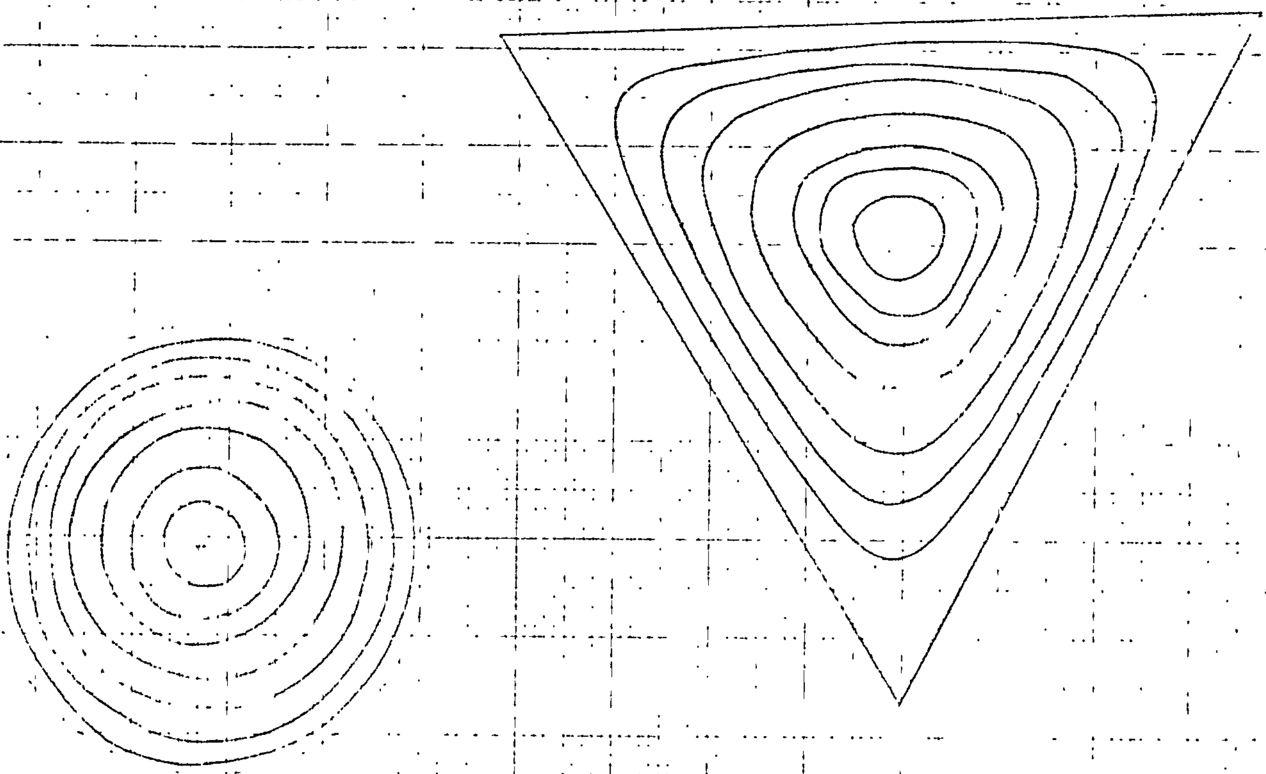


Figure 15
Volume Graphs for the Square and the Circle

TABLE XII

DATA TO DETERMINE THE VOLUMES OF SQUARE AND CIRCULAR SECTIONS
(POLYETHYLENE MATERIAL)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
9	0.395		7.07	0.405				
7.07	0.545		5.55	0.555				
5.59	0.645		4.52	0.630				
4.12	0.745		3.12	0.730				
2.92	0.820		1.92	0.805				
		2.859			2.025	1.411	1.432	1.46
2.10	0.870		1.09	0.855				
1.34	0.920		0.53	0.890				
0.63	0.970		0.20	0.910				
0	1.000		0	0.920				



Stress Distribution Of The Two Sections

Equilateral Triangle 4" Side

Circle 2.308" Diameter

Material:

Rubber Membrane; Thickness = 0.009

Maximum Angle At the edge = 43.80

Stress Ratio = 1.500

Observed Ratio = 1.248

Error = -16.8%

Figure 16

Membrane Contour Lines of the Equilateral Triangle and Circular Section Obtained from Rubber Membrane

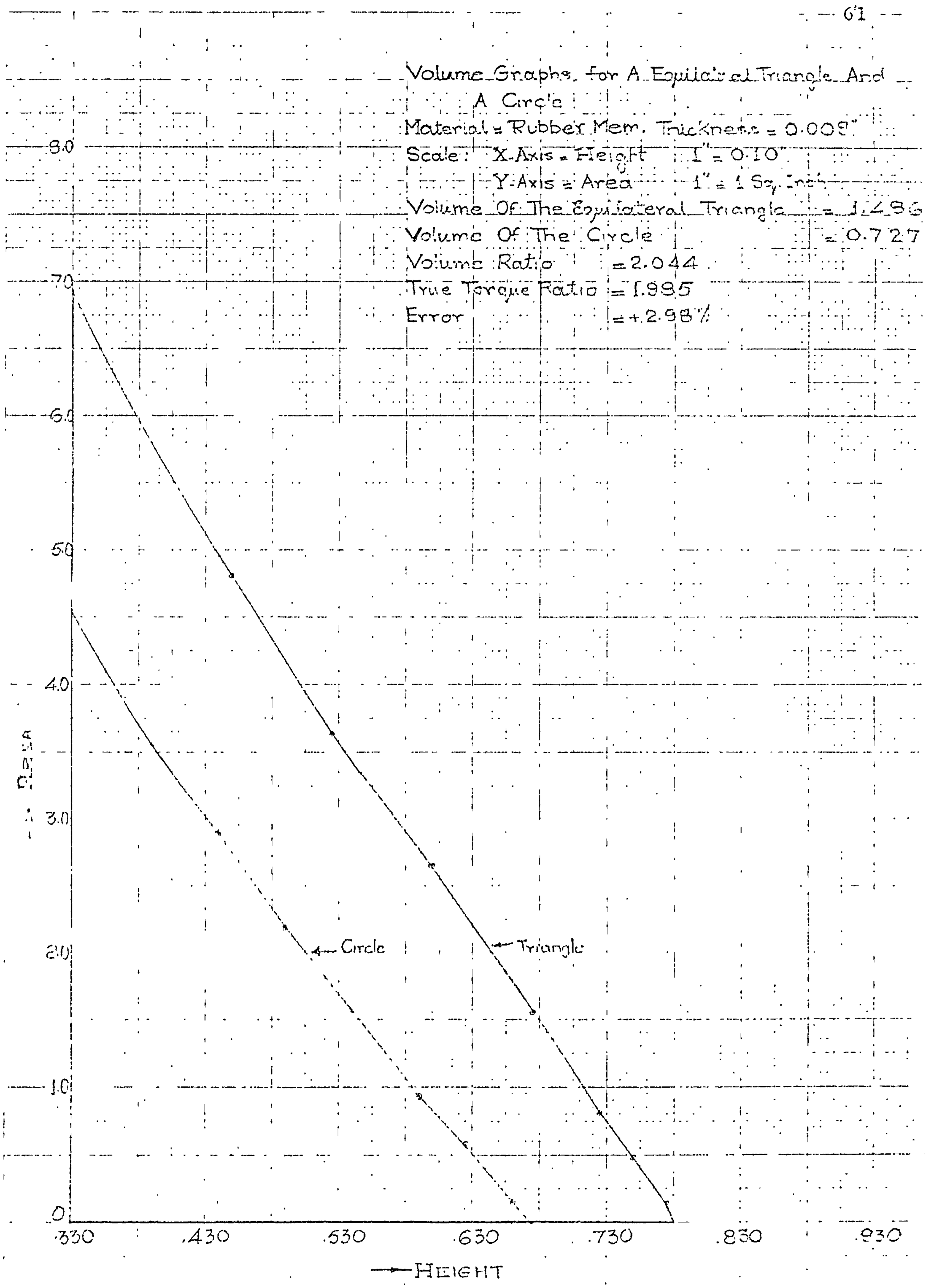


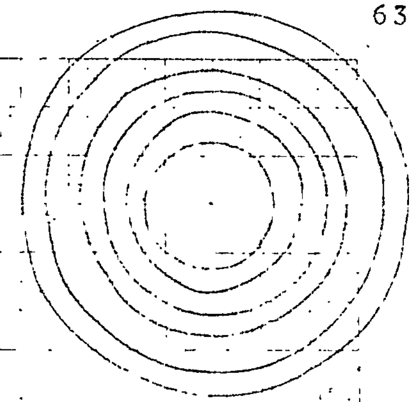
Figure 17
Volume Graphs for the Equilateral Triangle and the Circle

TABLE XIII

DATA TO DETERMINE THE VOLUMES OF EQUILATERAL TRIANGLE AND CIRCULAR SECTIONS

(RUBBER MEMBRANE)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6.928	0.330		4.17	.330				
4.80	0.450		3.55	.390				
3.64	0.525		2.89	.440				
2.65	0.600		2.18	.490				
1.55	0.675		1.56	.540				
		1.486			0.727	2.044	1.985	+ 2.98
0.81	0.727		0.93	.590				
0.48	0.750		0.57	.625				
0.15	0.775		0.15	.650				
0	0.780		0	.660				



Stress Distribution Of The Two Sections

Equilateral Triangle = 4" Side

Circle 2.308" Diameter

Material:

Polyethylene Mat., Thickness = 0.0015

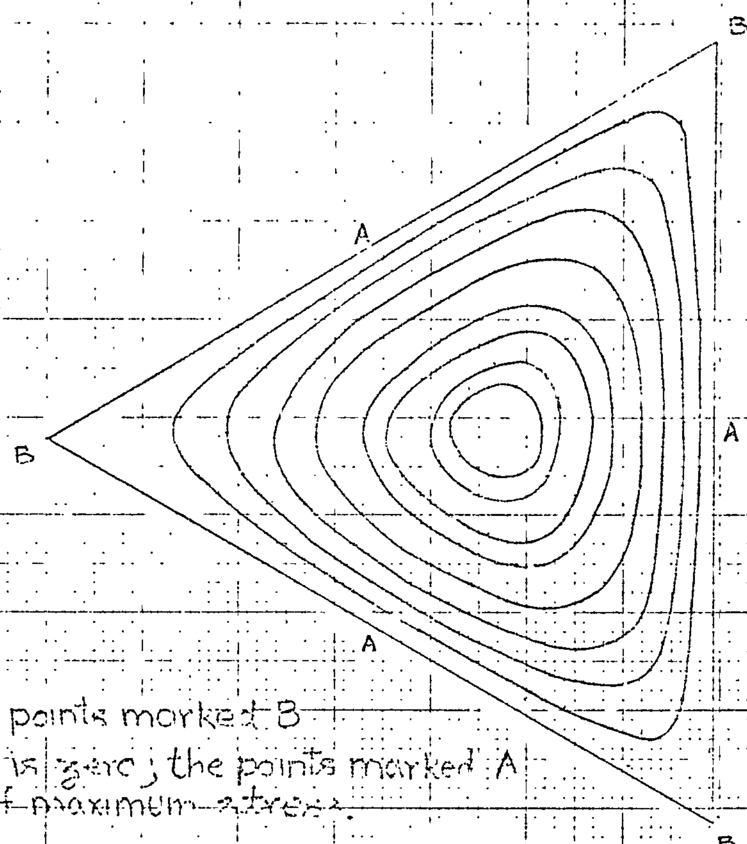
Maximum Angle At The Edge Of The Section = 40.3

Maximum Angle At The Edge Of The Circle = 27.4

[illegible] Ratio = 1.472

Stress Ratio = 1.500

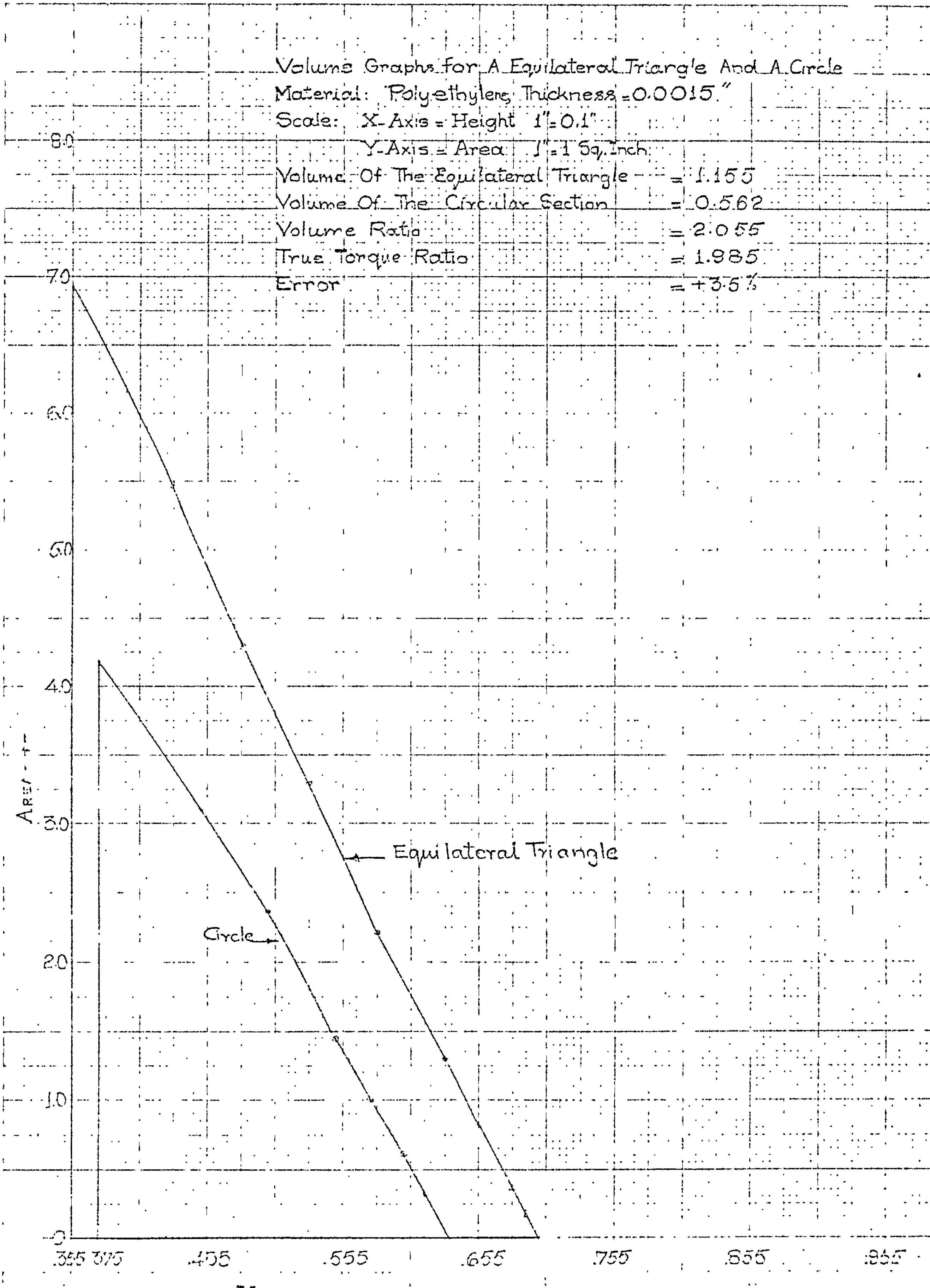
Error = –1.86%



Notes:

In the corner points marked B the stress is zero; the points marked A are those of maximum stress.

Figure 18
Membrane Contour Lines of the Equilateral Triangle and Circular Sections Obtained from Polyethylene Material



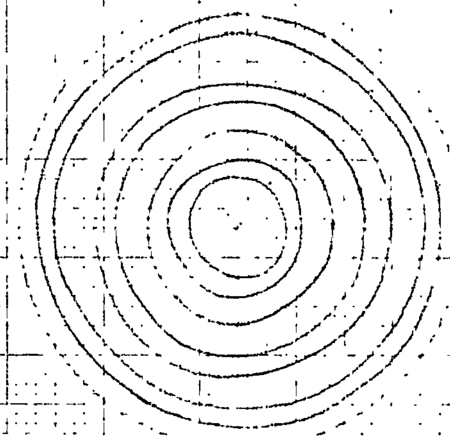
-Figure 19
Volume Graphs for the Equilateral Triangle and the Circle

TABLE XIV

DATA TO DETERMINE THE VOLUMES OF EQUILATERAL TRIANGLE AND CIRCULAR SECTIONS

(POLYETHYLENE MATERIAL)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6.928	.355		4.17	.375				
5.14	.420		3.44	.425				
4.22	.450		2.86	.450				
3.10	.490		1.87	.500				
2.22	.520		1.37	.525				
		.879			.470	1.870	1.985	- 5.7
1.50	.550		0.77	.550				
0.81	.575		0.39	.565				
0.40	.595		0.20	.575				
0.11	.605		0	.582				
0	.609							



Stress Distribution Of The Two Sections

Equilateral Triangle 4" Side

Circle 2.303 Diameter

Material:

Polyethylene Thickness = 0.0015

Maximum Angle At The Edge Of The Section = 27.4°

Maximum Angle At The Edge Of The Circle = 24.5°

Observed Ratio = 1.120

Stress Ratio = 1.500

Error = -25.3 %

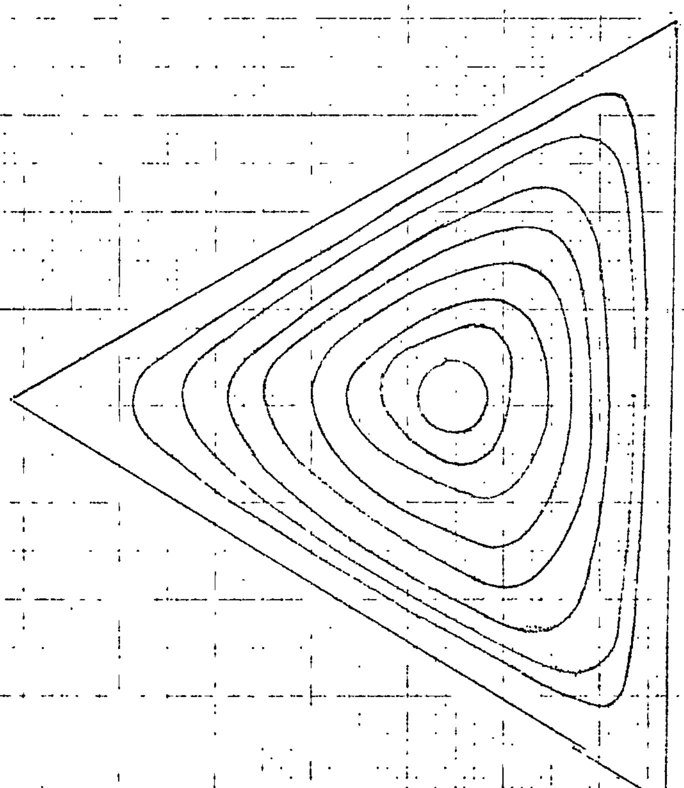


Figure 20
Membrane Contour Lines of the Equilateral Triangle and Circular Sections Obtained from Polyethylene Material

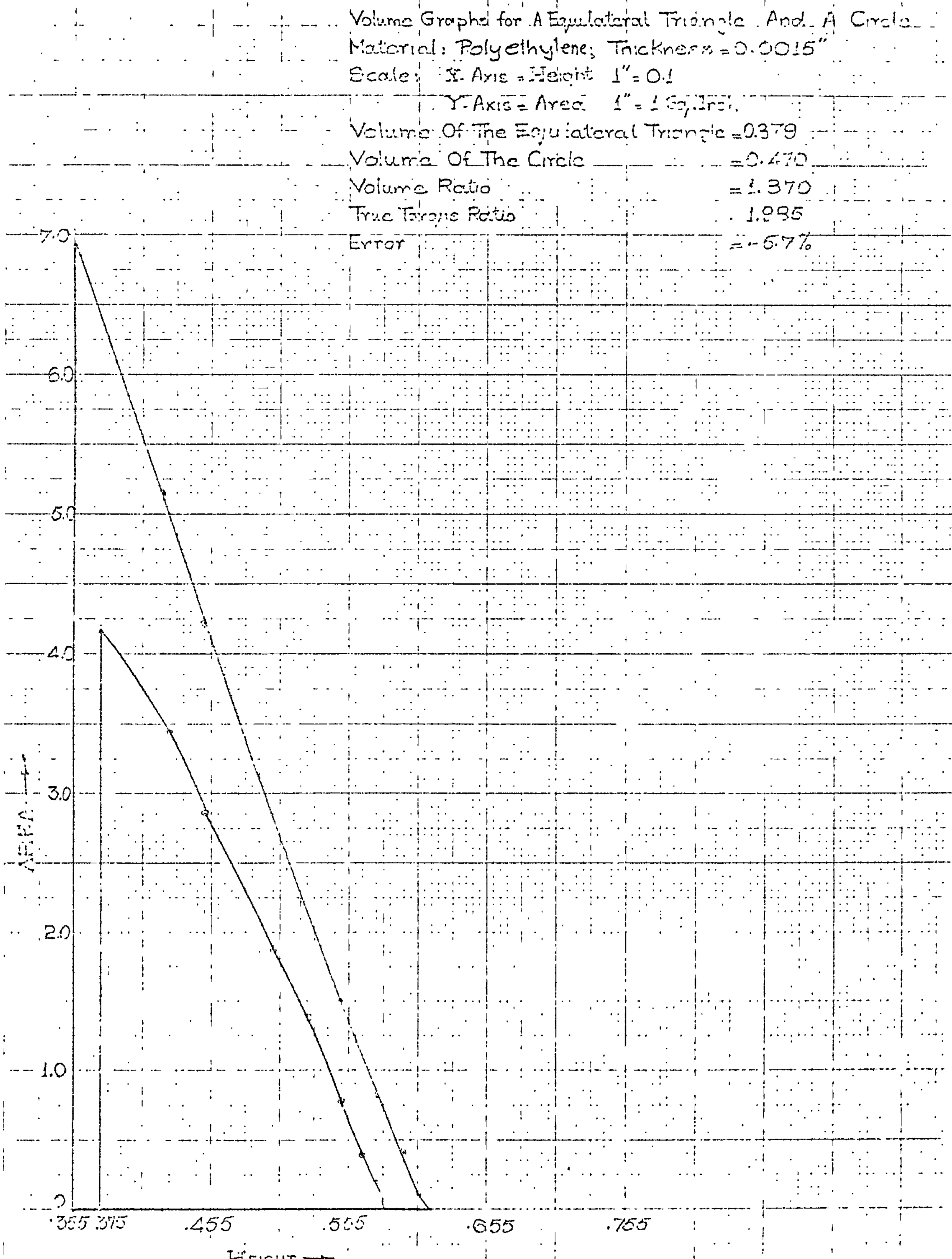


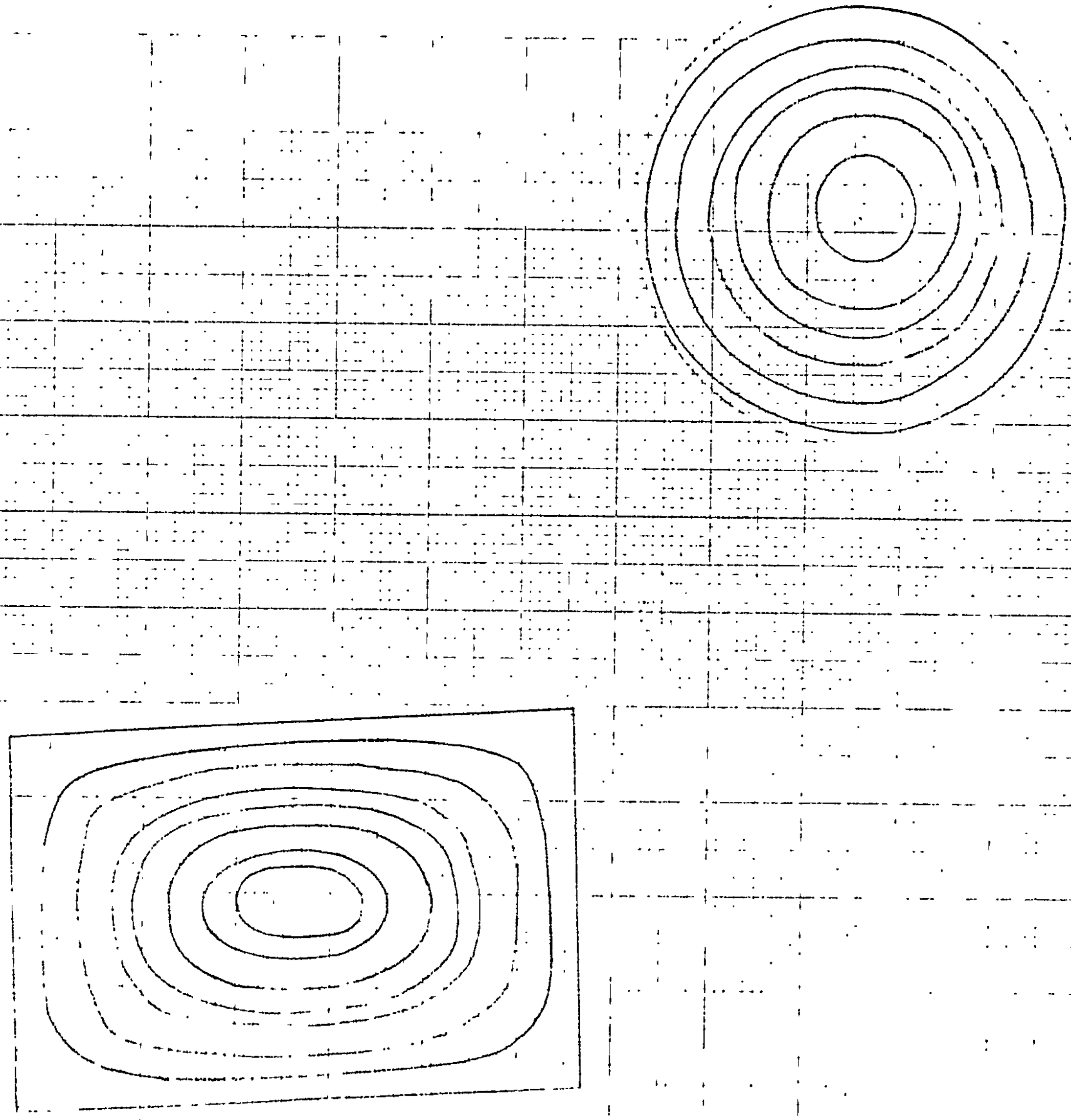
Figure 21
Volume Graphs for the Equilateral Triangle and the Circle

TABLE XV

DATA TO DETERMINE THE VOLUMES OF EQUILATERAL TRIANGLE AND CIRCULAR SECTIONS

(POLYETHYLENE MATERIAL)

1	2	3	4	5	6	7	8	9
AREA OF CONTOUR OF THE SECTION (Sq.inch)	HEIGHT (Inches)	VOLUME V_1 (Cu.inch)	AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	HEIGHT (Inches)	VOLUME V_2 (Cu.inch)	V_1/V_2	TRUE VALUE	% ERROR V_1/V_2
6.928	.355		4.17	.375				
5.46	.430		3.08	.450				
4.29	.480		2.36	.500				
3.27	.530		1.44	.550				
		1.155			0.562	2.055	1.985	+ 3.5
2.21	.580		1.00	.575				
1.29	.630		0.61	.600				
0.83	.655		0.32	.615				
0.35	.680		0	.633				
0.17	.690							
0	.700							



Stress Distribution Of The Two Sections
Rectangle 2"x 3"
Circle 2.40 Diameter
Material:
Rubber Membrane
Maximum Angle At the edge of the Section = 36.20
Observed Angle Ratio = 1.825
Stress Ratio = 1.414
Error = 29 %

Figure 22
Membrane Contour Lines of the Rectangle and Circular Sections
Obtained from Rubber Membrane

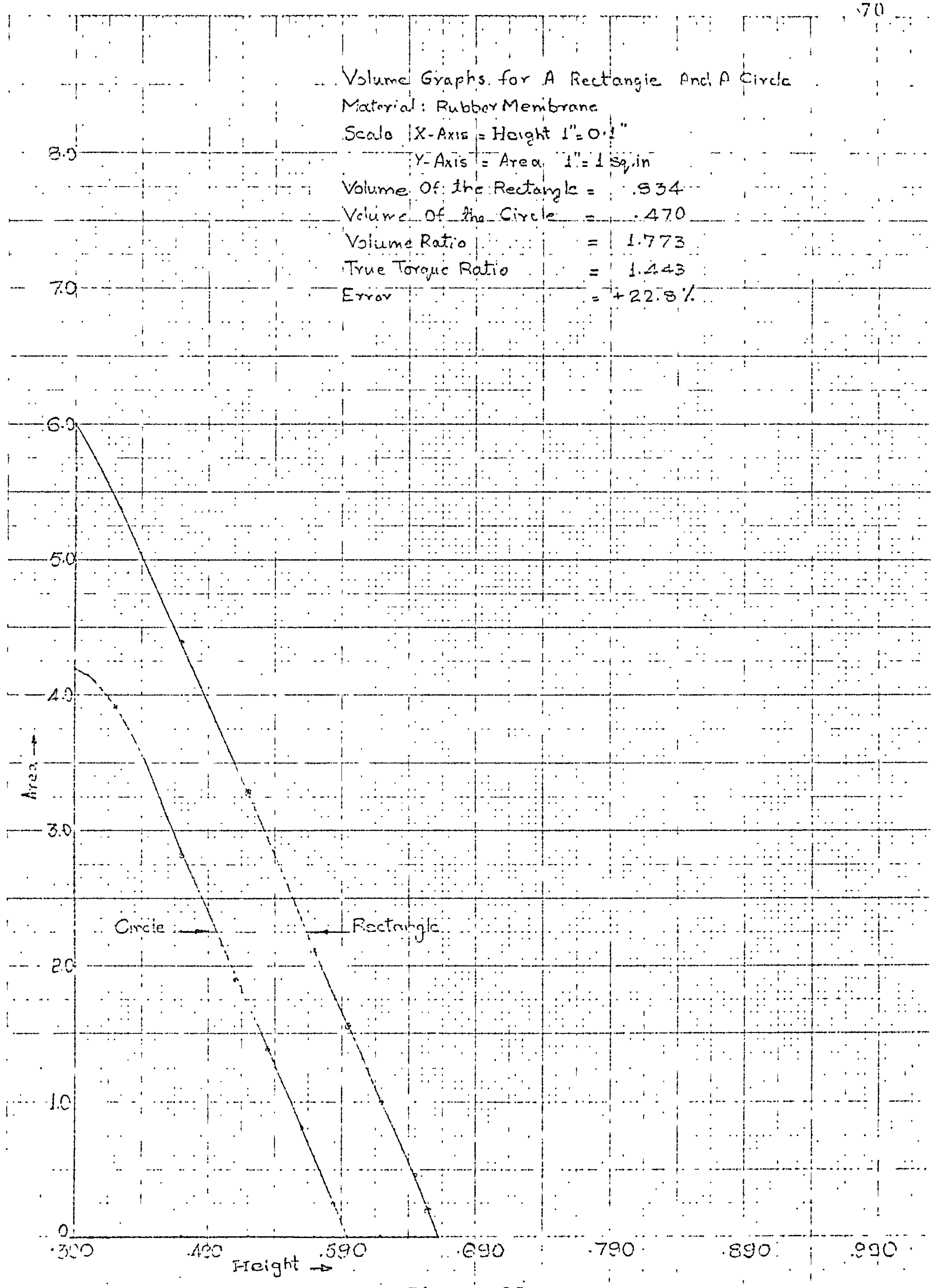


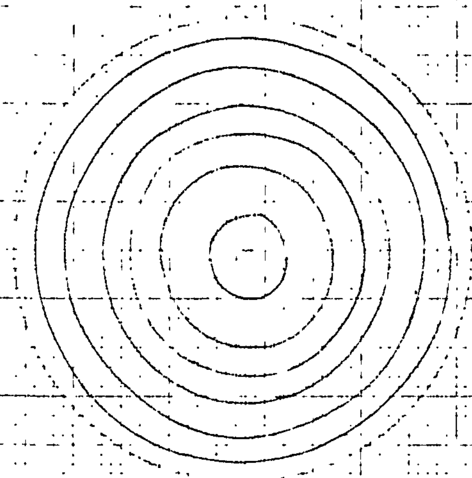
Figure 23
Volume Graphs for the Rectangle and the Circle

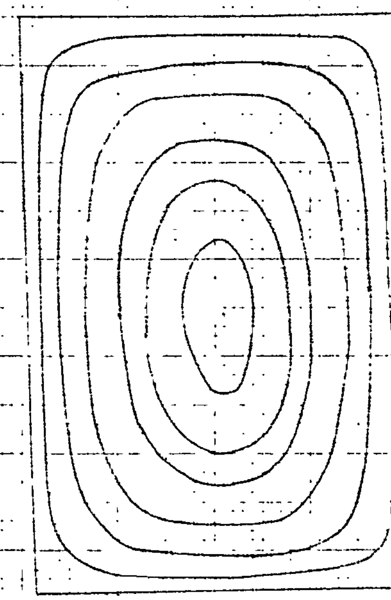
TABLE XVI

DATA TO DETERMINE THE VOLUMES OF RECTANGLE AND CIRCULAR SECTIONS

(RUBBER MEMBRANE)

1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR OF THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6.0	.390		4.52	.390				
4.38	.470		3.92	.420				
3.27	.520		2.82	.470				
2.11	.570		1.90	.510				
		0.834			0.470	1.773	1.443	+ 22.8
1.55	.595		1.37	.535				
1.00	.620		0.81	.560				
0.45	.645		0.24	.585				
0.20	.655		0	.595				
0	.662							





Stress Distribution Of The Two Sections

Rectangle 2"x3"

Circle 2.40 Diameter

Material:

Polyethylene Thickness = 0.0015"

Maximum Angle At The edge Of The Section = 33.78°

Maximum Angle At The edge Of The Circle = 25.61°

Observed Ratio = 1.319

Stress Ratio = 1.414

Error = -6.7%

Figure 24

Membrane Contour Lines of the Rectangle and Circular Sections Obtained from Polyethylene Material

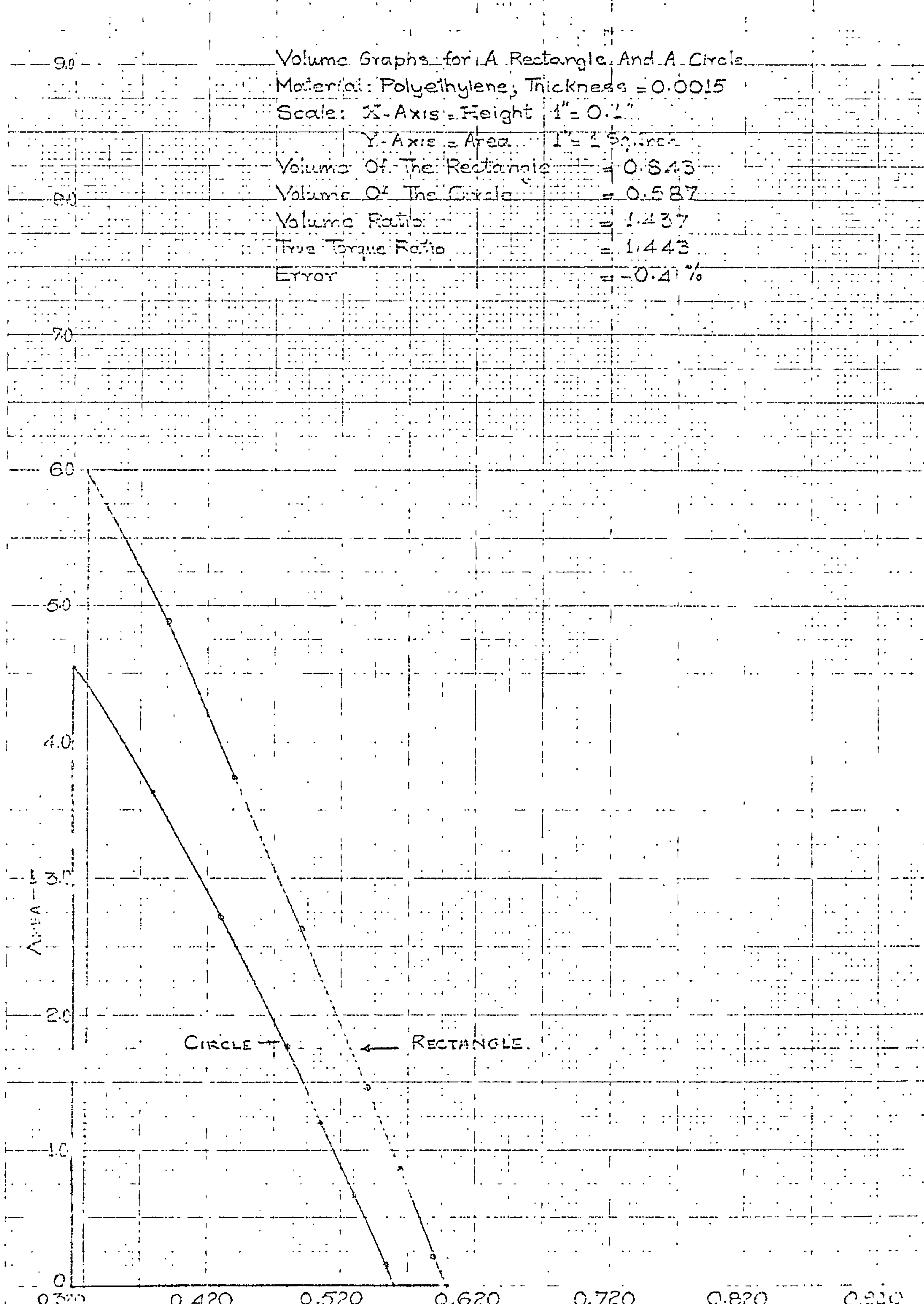


Figure 25

Volume Graphs for the Rectangle and the Circle

TABLE XVII

DATA TO DETERMINE THE VOLUMES OF RECTANGLE AND CIRCULAR SECTIONS (POLYETHYLENE MATERIAL)

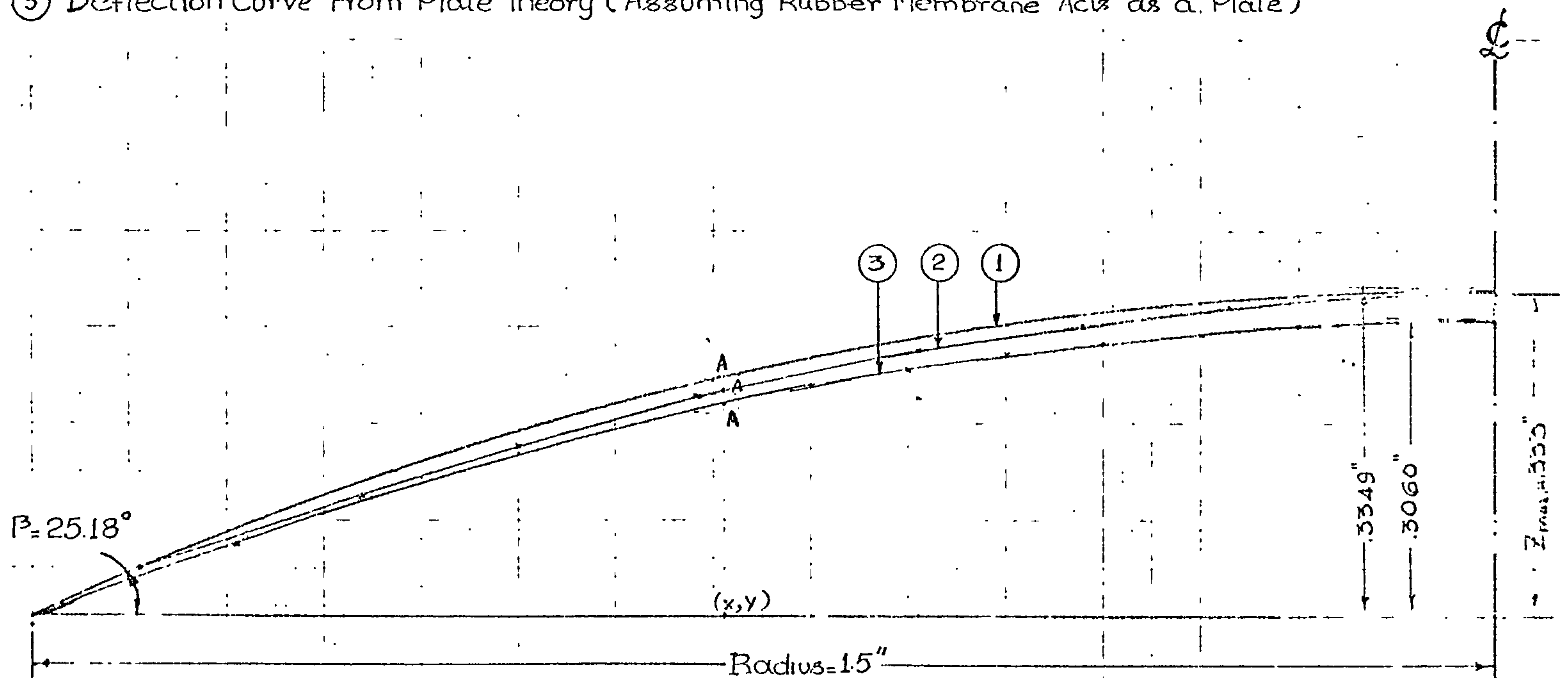
1 AREA OF CONTOUR OF THE SECTION (Sq.inch)	2 HEIGHT (Inches)	3 VOLUME V_1 (Cu.inch)	4 AREA OF CONTOUR THE CIRCLE (Sq.inch)	5 HEIGHT (Inches)	6 VOLUME V_2 (Cu.inch)	7 V_1/V_2	8 TRUE VALUE	9 % ERROR V_1/V_2
6	.330		4.52	.320				
4.88	.390		3.63	.380				
3.74	.440		2.72	.430				
2.62	.490		1.76	.480				
		0.844			0.587	1.4378	1.443	- .41
1.45	.540		1.19	.505				
0.85	.565		0.66	.530				
0.22	.590		0.15	.555				
0	.600		0	.560				

A Comparative Study of Deflection Curves (By using Rubber Membrane)

(1) Deflection Curve From Membrane Theory (Spherical Segment With a $Z_{max.} = R \tan \frac{\beta}{2}$)

(2) Deflection Curve From The Membrane Analogy Apparatus (Table X)

(3) Deflection Curve From Plate Theory (Assuming Rubber Membrane Acts as a Plate)



Maximum Slope at A on Curve (1) = 0.2637

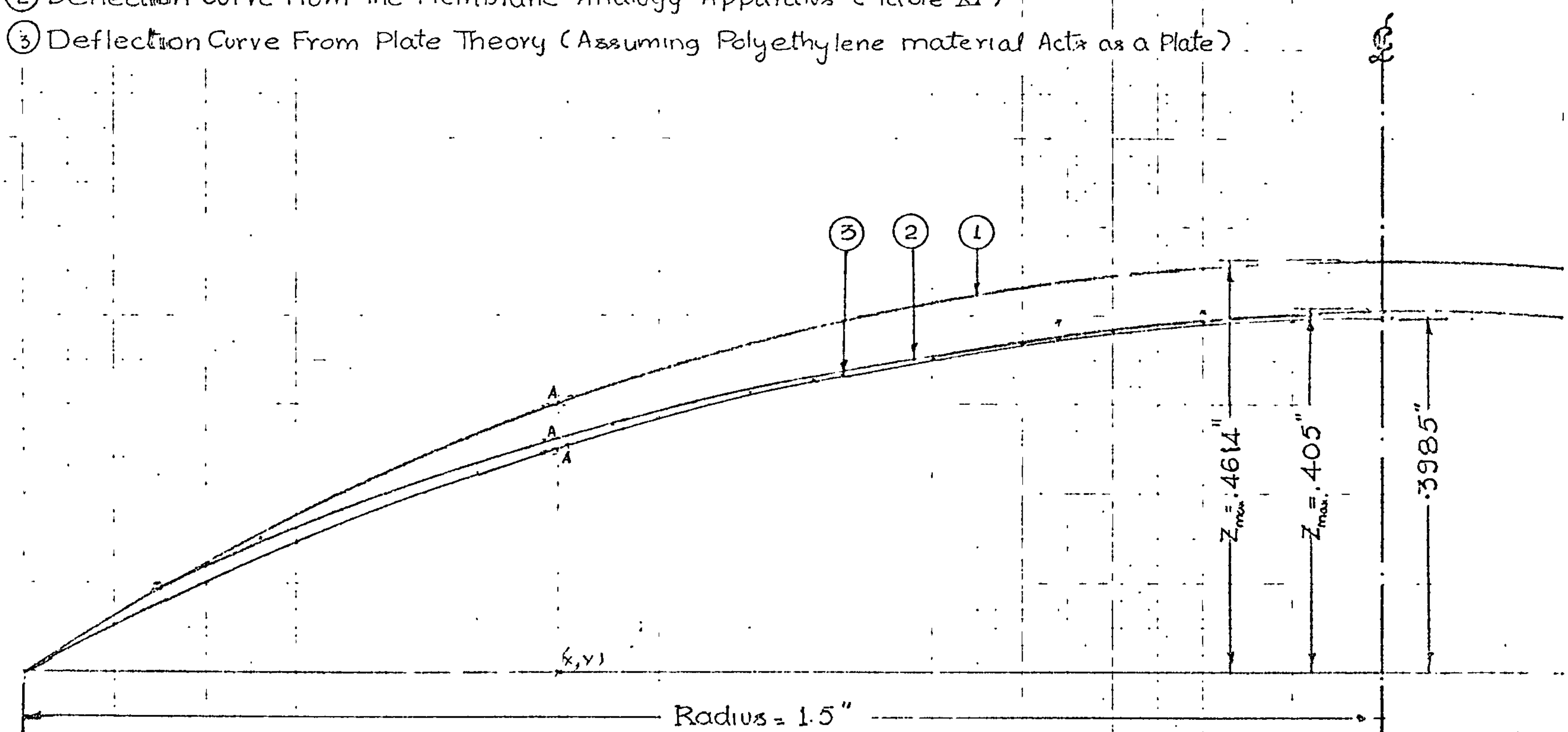
Maximum Slope at A on Curve (2) = 0.2600

Maximum Slope at A on Curve (3) = 0.2362

Figure 26

A Comparative Study of Deflection Curves (By using Polyethylene Material)

(1) Deflection Curve From Membrane Theory (Spherical Segment With a $Z_{max} = R \cdot \tan\frac{\beta}{2}$)
(2) Deflection Curve From The Membrane Analogy Apparatus (Table XI)
(3) Deflection Curve From Plate Theory (Assuming Polyethylene material Acts as a Plate)



Maximum Slope at A on Curve (1) = 0.3750
Maximum Slope at A on Curve (2) = 0.3181
Maximum Slope at A on Curve (3) = 0.3158

Figure 27

APPENDIX B

Torque Ratio And Stress Ratio Calculations

Analytical data for the stress and torque ratios of the three simple cross sections used in the experiment with their corresponding circles are obtained from the relations given in (5). The procedure to calculate analytically the stress and torque ratio for a triangular section (4" side) and a circle (2.308) is given below.

The maximum stress and torque in a circle is obtained from the relation

$$\frac{\tau_c}{R} = \frac{T_c}{J} = G\theta$$

$$\tau_c = \frac{T_c . R}{J}$$

$$T_c = G.\theta.J$$

where θ is the angle of twist per unit length.

The maximum torsional stress and torque in the triangular section is obtained from the empirical relation (5).

$$\tau_{s})_{max} = \frac{20\, T_s}{(Side)^3}$$

$$T_s = \frac{G.\theta.(Side)^4}{46.2}$$

Since the p/S value is the same for both sections because the same membrane is used, the corresponding $2G\theta$, is also same for the both sections.

Torque ratio is given by

$$\frac{T_s}{T_c} = \frac{G.\theta.(Side)^4}{46.2\ G.\theta.J} = \frac{(Side)^4}{46.2\ J} = \frac{4^4 \times 32}{46.2 \times \pi \times (2.308)^4} = 1.985$$

Maximum stress ratio is given by

$$\frac{\tau_s)_{max}}{\tau_c)_{max}} = \frac{20T}{(Side)^3.G.\theta.R} = \frac{20.(Side)^4.G.\theta}{46.2(Side)^3.G.\theta.R}$$

$$= \frac{20.\,Side}{46.2\,R} = \frac{20 \times 4}{46.2 \times 1.154}$$

$$= 1.500$$

Given the torque applied to a triangular section, the angle of twist per unit length, θ, can be determined from the relation given below.

$$\theta = \frac{1}{2G.A}\int \tau_{sz}\cdot dz = \frac{C}{2G.A}\int \frac{\delta z}{\delta n}.ds$$

(7) where $\frac{\delta z}{\delta n}$ is normal slope of the membrane around the boundary. The constant, C, is obtained from the relation given below.

$$\frac{\text{Volume under the extended membrane of the section}}{\text{Maximum slope at the boundary of the section}} = C \times K$$

$$\text{where } K = \frac{\text{Torque applied}}{\text{Maximum stress at the boundary of the section}}$$

The properties of the two materials are given below(8):

Polyethylene Material

Thickness = 0.0015" (Measured by micrometer)

Specific gravity = 0.910

Modulus of Elasticity in tension = 20,000 to 25,000 p.s.i.

Tensile stress = 1,400 to 2,000 p.s.i.

Elongation,% = 125-675

Shear stress, 1,400 to 1,700 p.s.i.

Poisson ratio = .4

Rubber Membrane

Thickness = .0085

Specific gravity = .93

Tensile stress = 2,500 - 3,500 p.s.i.

Modulus of Elasticity in tension = 20,000 p.s.i.

Elongation,% = 750 - 850

Poisson ratio = .5