AN APPROXIMATE PREDICTIVE TECHNIQUE FOR FREEZING OF THIN CONDENSATE FILMS ON VERTICAL PLATES

A Thesis

Presented to

The Faculty of the College of Engineering

The University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

---

by

Shiv Parkash Verma

December, 1972

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ABSTRACT

A technique suitable for engineering application has been developed for predicting the solid-phase history in freezing of a condensate film on a vertical plate. Utilizing the results of Beaubouef as well as Sparrow and Gregg a model was constructed approximating the flow features and freezing phenomena. The present model was also suitable for application to freezing, of noncondensing free-convective flow, for which good agreement was obtained with predictions of the "exact" analysis of Lapadula and Mueller.

In addition, the effect of turbulence in the liquid layer on the deposited solid was studied. It was found that the thickness of the deposited solid layer may decrease in the direction of the flow, which is just opposite to the situation when the liquid layer is laminar.

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NOMENCLATURE

c	Dimensional constant, defined in equation 16.
c_1	Dimensional constant, defined in equation 17.
C_p	Specific heat at constant pressure, $Btu/lb_m {}^\circ F$.
g	Acceleration due to gravity, ft/sec^2.
h_x	Local heat transfer coefficient, $Btu/hr\ ft^2\ {}^\circ F$.
h_{fg}	Latent heat of condensation, Btu/lb_m.
H	Dimensionless thickness of solid deposit, $(s/S)^2$.
L	Plate length, ft.
q_c	Convective heat flux, $(h_x)(\Delta T_L)$, $Btu/hr\ ft^2$.
$s(t)$	Local thickness of deposited solid phase, $S(H)^{\frac{1}{2}}$, ft.
S	Local steady-state thickness of deposited solid phase, $k_s \Delta T_s / q_c$, ft.
T	Temperature, ${}^\circ F$.
T_{sat}	Saturation temperature of vapor, ${}^\circ F$.
T_∞	Free stream temperature, ${}^\circ F$.
T_p	Temperature of cold surface, ${}^\circ F$.
T_f	Fusion temperature of solid phase, ${}^\circ F$.
ΔT	Temperature difference, ${}^\circ F$.
t	Time, hrs.
x	Dimensional coordinate parallel to cold surface.
y	Dimensional coordinate normal to the cold surface.
k	Thermal conductivity, $Btu/hr\ ft {}^\circ F$.

Nu_x	Nusselt number, $h_x \frac{x}{k_L}$.
Pr	Prandtl number, ν/α.
Gr_x	Grashof number, $g\beta(T_\infty - T_f)x^3/\nu_L$.

Greek Symbols

α	Thermal diffusivity, ft^2/hr.
β	Expansion coefficient, $1/{}^\circ F$.
γ	Dimensionless physical parameter, $\frac{2k_s \Delta T_s}{\rho_s \lambda \alpha_s}$.
λ	Latent heat of fusion, Btu/lb_m.
ν	Kinematic viscosity, ft^2/hr.
ρ	Density, lb_m/ft^3.
τ	Dimensionless time, $\alpha_s t/S^2$.

Subscripts

$(\ )_s$	Property value of/or across the solid layer.
$(\ )_L$	Property value of/or across the liquid layer.
$(\ )_v$	Property value of vapor.
$(\ )_x$	Local instantaneous value of ( ) based on x as the reference length in dimensional or dimensionless quantities.

CHAPTER I

INTRODUCTION

During the nineteenth century, Stefan [1]* conducted an analytical investigation of ice formation. Accordingly, the variety of related problems treated in recent literature have come to be recognized as "Stefan-like" problems. Such problems involve solution of unsteady heat conduction equation (the diffusion equation) in a region partly bounded by a moving liquid-solid interface having an unknown location to be determined as part of the solution. In general, the system of equations describing such phenomena is non-linear. In spite of their complicatedness, the practical importance of such problems has generated considerable interest in "Stefan-like" problems.

Recently Beaubouef [3] obtained a complete numerical solution for the exact "Stefan-like" problem in cartesian coordinates for those cases in which a solid deposited layer is formed in a steady plane flow over a cold surface, with the convective heat flux, q_c, known as a function of location on the cold surface, and independent of time.

Starting with the boundary layer equations, using mathematical techniques, Sparrow and Gregg [2] attacked the problem of laminar film condensation on a vertical plate. By means of a

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*Numbers in brackets designate identical numbers listed in the Bibliography.

similarity transformation, they reduced partial differential equations to ordinary differential equations. Solutions were presented for values of the parameter $C_p \cdot \Delta T_L / h_{fg}$ between 0 and 2 and for Prandtl numbers between 1 and 100, including the results of heat transfer as well as film layer thickness.

Lapadula and Mueller [4] obtained an analytical estimate for growth of deposited solid on a vertical isothermal plate based on a complete set of two-dimensional transient equations, with natural convection in the fluid phase. They showed that under certain conditions freezing or melting phenomena may be treated as involving a transient one-dimensional conduction process and a quasi-steady two-dimensional convection process coupled through the requirements of conservation of mass and energy at the moving phase interface.

The objective of the present study is to offer a technique for prediction of solid deposited layer thickness in freezing of a condensate film on a vertical surface. For this purpose one cannot use the work of Lapadula and Mueller, because their model does not incorporate the vapor-liquid conversion. Therefore, a model shall be constructed approximating the flow conditions and freezing phenomena using easily-applied predictive relations for convective heat transfer as well as those of Beaubouef for growth of the solid layer. Predictions of this model will be compared with the "exact" predictions of Lapadula and Mueller, for the case of free-convection on a vertical plate in a non-condensing liquid. In this connection, it should be noted that the model presented herein is flexible, in that solidification of free-convective flows may be considered whenever the convective heat flux, q_c, may be specified. Therefore, the same predictive technique offered here may be applied to laminar and turbulent liquid films, with and without the vapor-liquid conversion, to the direct solidification of vapors, etc.

CHAPTER II

FORMULATION OF ANALYTICAL MODEL

Consider a flat vertical plate of length "L" suspended in an infinite vapor at a uniform temperature, T_∞, less than or equal to saturation temperature, T_{sat}. For time greater than zero let the temperature of the plate have a uniform and constant value of T_p, less than the solidification temperature, T_f, of the surrounding fluid.

The following assumptions are being made: (a) the convective heat flux, q_c, transferred from the liquid to the solid phase is the same as predicted for steady film condensation without freezing, on an isothermal vertical plate; (b) the thickness of solid deposit is so small that heat conduction within the solid phase may be assumed as one dimensional in certesian coordinate system formed normal and parallel to the cold surface; (c) removal of liquid phase by freezing does not affect convection at the moving interface; (d) viscous dissipation has been neglected; (e) the solid adheres to the plate; (f) all physical properties of both fluid and solid phases (with the exception of fluid density) are uniform and constant; (g) there exists a definite interface between the fluid and solid phases; (h) thermodynamic phase equilibrium is maintained.

On the basis of the assumptions (b) and (c) the solid deposited layer can be treated as another isothermal vertical flat surface for the liquid layer.

A schematic presentation of the physical model and coordinate system is shown in Figure I.

Sparrow and Gregg [2] approximated their results of heat transfer across the liquid condensate layer with the following equation:

$$Nu_x \left[\frac{g C_p (\rho_L - \rho_v) x^3}{4 \nu_L k_L} \right]^{-\frac{1}{4}} = \left[0.68 + \frac{h_{fg}}{C_p \cdot \Delta T_L} \right]^{\frac{1}{4}} \tag{1}$$

The variation of h_x as $x^{-\frac{1}{4}}$ is a well-known result in condensation theory, which is also obtained in free-convection.

For heat transfer across the solid layer, using the heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} ,$$

the equation for energy balance at the free boundary

$$q_c + \rho_s \lambda \frac{ds}{dt} = k \frac{\partial T(s,t)}{\partial y} ,$$

and with the following boundary and initial conditions

$$T = T_p \quad @ \quad y = 0$$

$$T = T_f \quad @ \quad y = s(t)$$

$$S = 0 \quad @ \quad t = 0$$

Beaubouef [3] determined the local history of the solid layer deposited on an isothermal plate.

Treating the problem as involving a transient one-dimensional conduction process and a quasi-steady two-dimensional convection process, coupling them through the requirements of

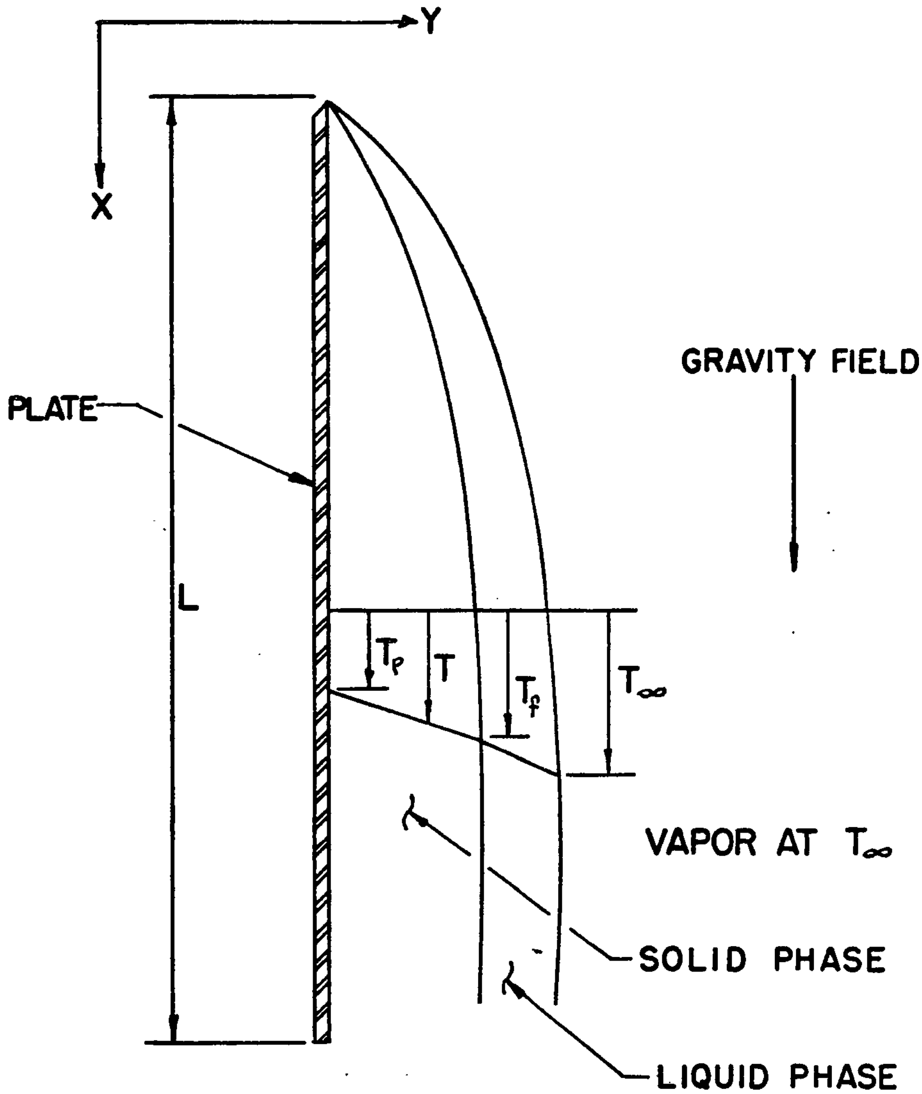

Y
X
PLATE
L
GRAVITY FIELD
T_p
T
T_f
T_∞
VAPOR AT T_∞
SOLID PHASE
LIQUID PHASE

FIGURE I

conservation of mass and energy at the phase interface, Lapadula and Mueller found out that

$$Nu_L = \frac{k_s(T_f - T_p) \cdot L}{S \cdot k_L(T_\infty - T_f)} \tag{2}$$

The results of this analysis, in terms of the local history of the solid layer, shall be determined using the results of Beaubouef and with q_c as determined by equation (1).

CHAPTER III

METHOD OF SOLUTION

I. Laminar Liquid Layer

The method of solution for the solid-phase history will be demonstrated using conditions employed by Lapadula and Mueller in their Numerical Example. This will be a common basis for comparison of the predictions of this analysis with those of Lapadula and Mueller.

It is first required to determine, for the purpose of this analysis, the functional description of convective heat flux, q_c, which would be obtained under the conditions set forth by Lapadula and Mueller. From Eckert and Drake [6], it is noted that the convective heat flux in free convection on a vertical surface varies as $(x)^{-\frac{1}{4}}$. Further, it is possible to determine the local value of q_c at a distance of 1.0 foot from the leading edge of the plate, using the results of Lapadula and Mueller; to wit,

$$Nu = \frac{h(1)}{k_L} = 97.3 \ , \tag{3}$$

$$\begin{aligned} q_c &= 97.3 \ k_L \ \ T_L \\ &= 931 \ \frac{\text{Btu}}{\text{hr-ft}^2} \ , \end{aligned} \tag{4}$$

with the value of k_L as taken from Eckert and Drake. Therefore, the function $q_x(x)$ to be used in this analysis is

$$q_c = 931(x)^{-\frac{1}{4}} \frac{\text{Btu}}{\text{hr-ft}^2} . \tag{5}$$

Proceeding according to Beaubouef's analysis while maintaining the same ΔT_s as that of Lapadula and Mueller

$$\Delta T_s = T_f - T_p = 32 - (-40) = 72^\circ F .$$

The steady-state thickness of the solid is found to be

$$S = \frac{k_s \Delta T_s}{q_c}$$
$$= \frac{1.235 \times 72 \ (x)^{\frac{1}{4}}}{931} \tag{6}$$

$$S = 0.0955(x)^{\frac{1}{4}} \tag{7}$$

At $x = 1.0'$,

$$S = 0.0955 \text{ ft.}$$
$$= 1.15 \text{ in.}$$

which is the same as that found by Lapadula and Mueller.

This agreement indicates that one can achieve the results acceptable for most practical purposes, using the above simple approach.

In the same example, Lapadula and Mueller found out that after 1.12 hrs., midway of the plate, the local thickness of the solid deposited layer is 0.868 in.

To examine the validity of the above mentioned simple approach, further analysis was carried out.

Since the thickness of solid layer varies with time and position on the cold surface until it approaches it's steady-state value, Beaubouef non-dimensionalized the time as

$$\tau = \frac{\alpha_s t}{S^2} \tag{8}$$

and found the ratio of local thickness to the steady-state thickness as a function of τ as follows

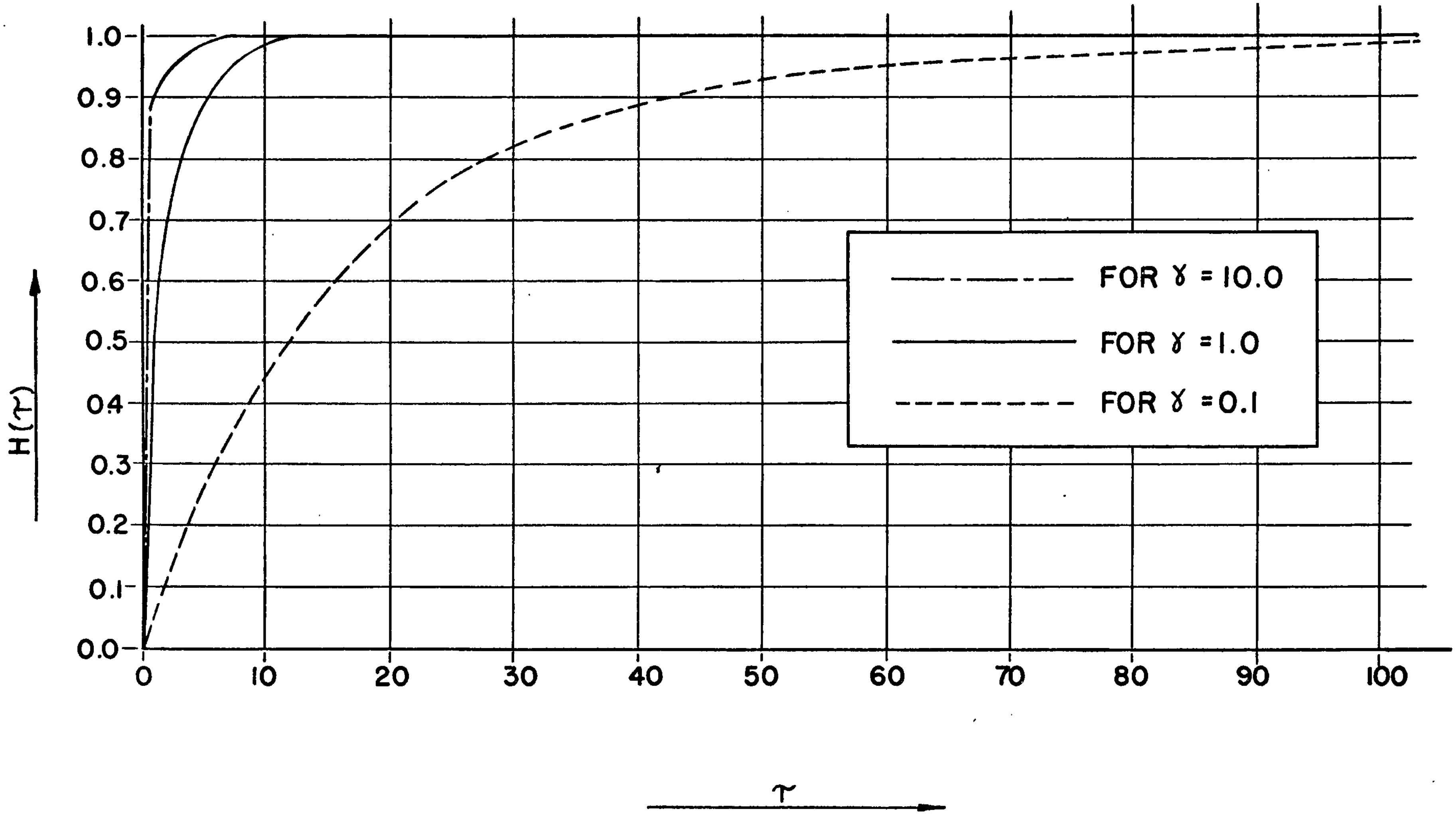
$$H(\tau) = \left(\frac{s}{S}\right)^2 . \tag{9}$$

Defining $$\gamma = \frac{2k_s \Delta T_s}{\rho_s \lambda \alpha_s} \tag{10}$$

which is nothing but the ratio of heat required to change the temperature of unit mass of the solid phase by an amount ΔT_s to the latent heat of solidification of unit mass, and using fixed values of γ = 0.1, 1.0 and 10 Beaubouef plotted three different graphs between τ and $H(\tau)$. These three graphs were combined in one as shown in Figure II.

A close look on these curves will reveal that for values of $1.0 \leq \gamma \leq 10.0$ it is safe to interpolate between the two curves, but for values of $0.1 \leq \gamma \leq 1.0$ the curves are so abrupt that it is not advisable to make any interpolation. For such conditions Beaubouef showed that the relationship between τ and $H(\tau)$ may be approximated from the following equation

$$\begin{aligned} \tau &= -\frac{\rho_s \lambda \alpha_s}{k_s \Delta T_s}\left[\ell n(1-(H)^{\frac{1}{2}}) + (H)^{\frac{1}{2}}\right] \\ &= -\frac{2}{\gamma}\left[\ell n(1-(H)^{\frac{1}{2}}) + (H)^{\frac{1}{2}}\right] . \end{aligned} \tag{11}$$



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
H(τ)
0
10
20
30
40
50
60
70
80
90
100
τ
FOR γ = 10.0
FOR γ = 1.0
FOR γ = 0.1

FIGURE II

Using the value $\Delta T_s = T_f - T_p = 72^\circ F$ in equation (10), there is obtained

$$\gamma = 0.468 .$$

Assuming different values of H, corresponding values of τ were calculated and plotted as shown in Figure III.

To compare the present analysis with that of Lapadula and Mueller at time t = 1.12 hrs., use equation (8) to determine

$$\tau = 5.89 ,$$

and find from Figure III the corresponding value of

$$H(\tau) = 0.81 .$$

By means of equation (7) and (9), one obtains the equation of the local solid layer thickness, as

$$S(x,t) = 0.0955(x)^{\frac{1}{4}}(H)^{\frac{1}{2}} .$$

At x = 0.5 ft., $S(t) = 0.0955 \times 0.841 \times 0.9$

$= 0.0723$ ft.

$= 0.868$ in.

This result is exactly the same as the one found by Lapadula and Mueller, using their more complex model.

To further analyze the growth of the solid deposited layer, the values of transient thickness of solid layer at distances x equal to 0.0′, 0.25′, 0.50′, 0.75′, and 1.00′ from the leading edge at time intervals of t = 0.25 hrs., 0.50 hrs., 0.80 hrs., 1.12 hrs., 1.50 hrs. and steady state were calculated and plotted as in Figure IV. It can be seen

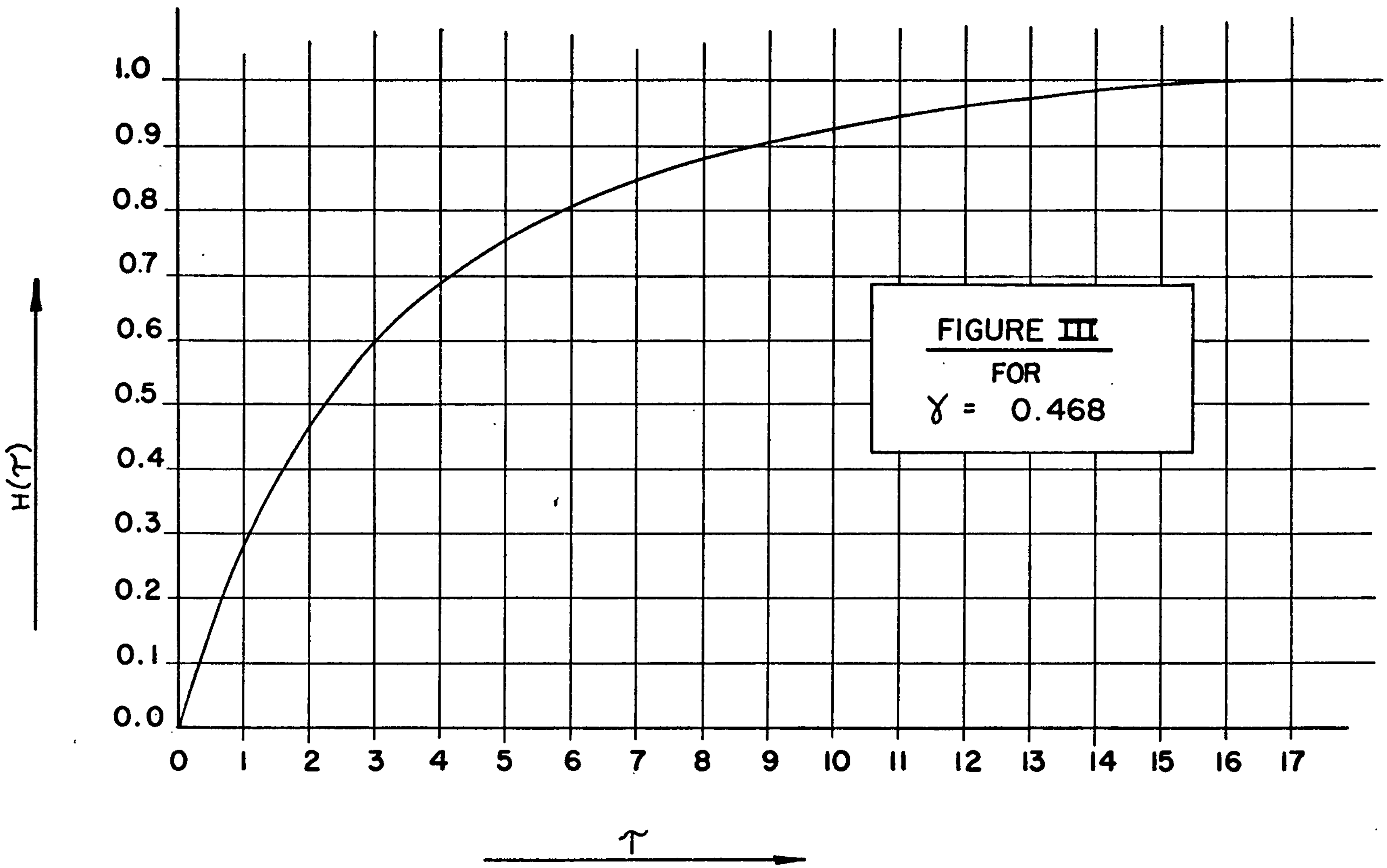
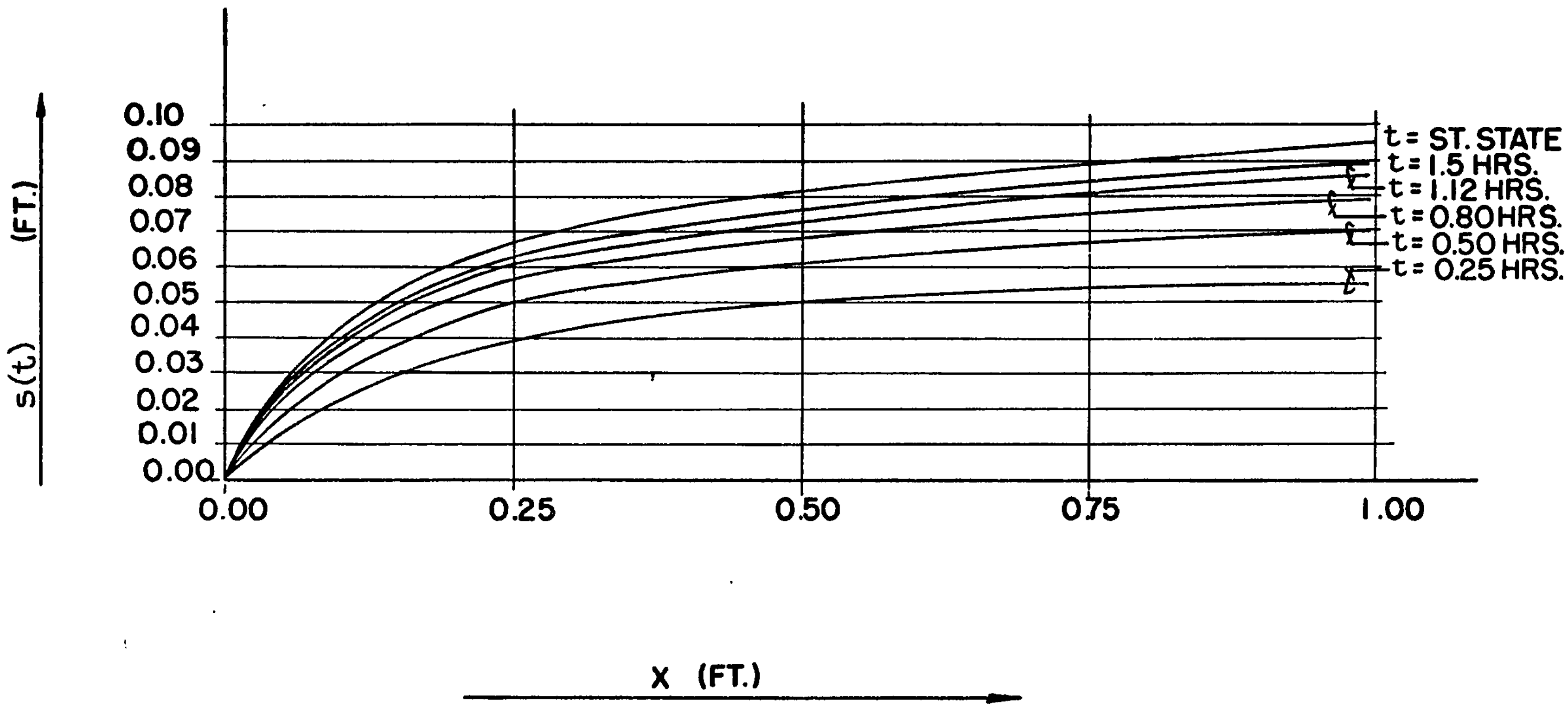

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
H(τ)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
τ
FIGURE III
FOR
γ = 0.468

FIGURE III



0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
s(t) (FT.)
0.00
0.25
0.50
0.75
1.00
X (FT.)
t = ST. STATE
t = 1.5 HRS.
t = 1.12 HRS.
t = 0.80 HRS.
t = 0.50 HRS.
t = 0.25 HRS.

FIGURE IV

from this figure that the growth of deposited solid layer within the initial few minutes is comparatively very rapid, so that the solid achieves over one-half its steady-state thickness within the initial 15 minutes.

This behavior is of course explained by the fact that conductive resistance in the solid increases with time, and the rate of latent heat removal (i.e., solid deposition) is correspondingly diminished.

Transition thicknesses of solid deposited layer for a time interval of t = 0.25 hrs. were calculated by using this technique and that of Lapadula and Mueller which have been tabulated in Table I and plotted in Figure V.

II. Turbulent Liquid Layer

Expanding the analysis to find out what will happen to the growth of solid deposited layer as and when the liquid layer becomes turbulent, one can proceed in the following fashion.

Experiments have shown that condensate layers formed in free convection change from laminar state to turbulent state at about $Gr_x \cong 10^9$, as found in Chapman [5].

$$Gr_x = \left(\frac{x^3 g \beta \Delta T_L}{\nu_L^2}\right) \tag{12}$$

where x is the distance down the plate from the starting edge.

Equating $Gr_x = 10^9$ and taking the physical properties at T_∞, the value of x can be easily calculated.

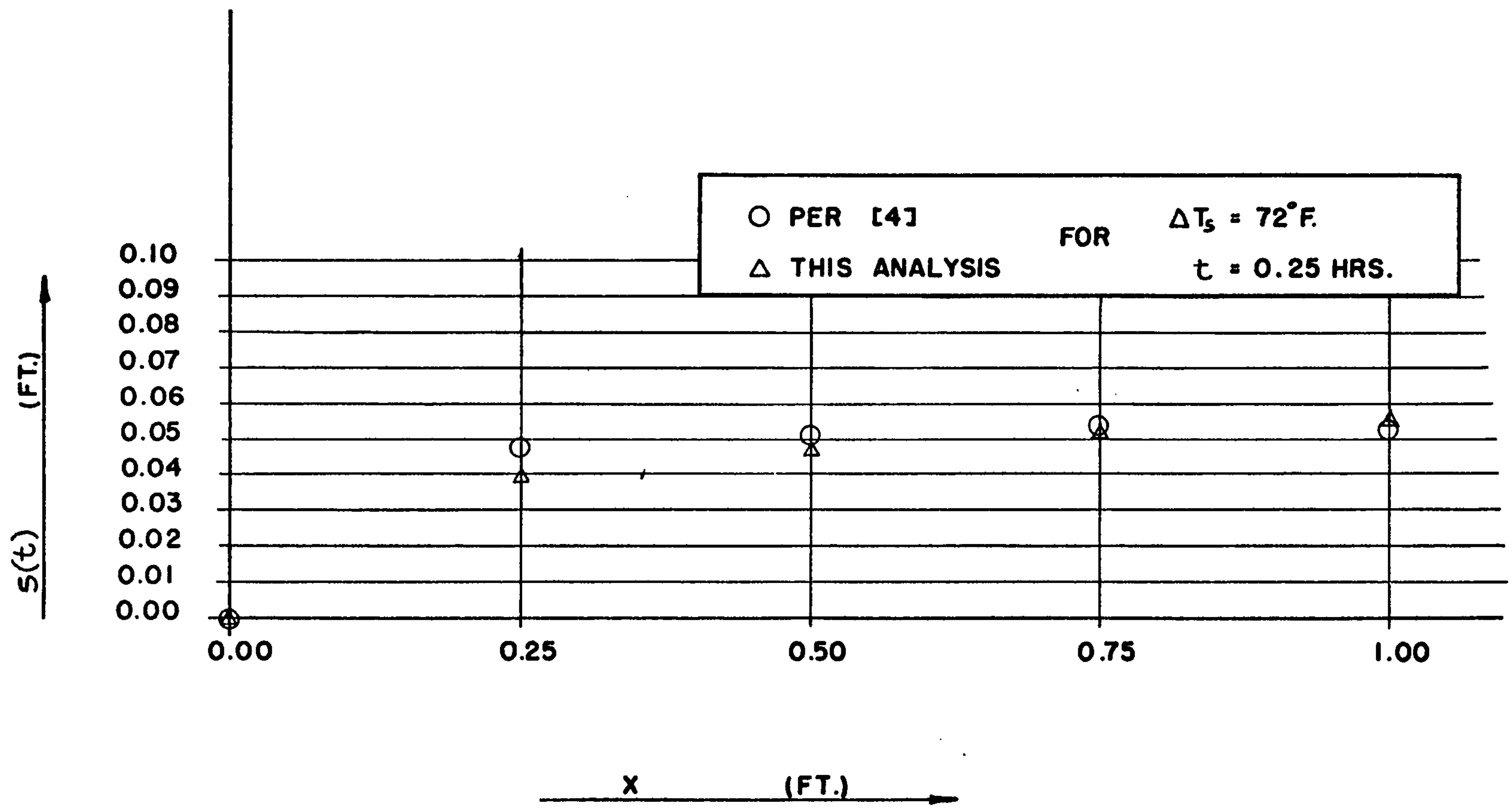
TABLE I

$t \quad = 0.25$ hrs.

$\tau \quad = 1.315$

From Figure III $\qquad H(\tau) = 0.34$

x (ft)	t (hrs)	τ	$H(\tau)$	s(t) (ft) This Analysis	s(t) (ft) Lapadula-Mueller
0.00	0.25	1.315	0.34	0.00	0.00
0.25	0.25	1.315	0.34	0.0394	0.0477
0.50	0.25	1.315	0.34	0.0468	0.0508
0.75	0.25	1.315	0.34	0.0518	0.0523
1.00	0.25	1.315	0.34	0.0557	0.0533



○ PER [4]
△ THIS ANALYSIS
FOR
ΔT_s = 72°F.
t = 0.25 HRS.
s(t) (FT.)
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.00
0.25
0.50
0.75
1.00
X (FT.)

FIGURE V

For turbulent region, which will start at or after the distance x, from the leading edge, calculated from equation (12), the heat flux can be calculated by using the following equation given in Eckert and Drake:

$$Nu_x = 0.0295(Gr_x)^{2/5} (Pr)^{7/15} \left[1 + 0.494(Pr)^{2/3}\right]^{-2/5} \qquad (13)$$

For a fixed value of ΔT_L

$$Nu_x = c(x)^{6/5} \qquad (14)$$

$$\text{or } q_c = c_1 (x)^{1/5} \qquad (15)$$

Following equations (6) and (8) through (11), and using q_c as given by equation (15), one can find the steady-state thickness of solid deposited layer at different locations on the plate.

CHAPTER IV

NUMERICAL EXAMPLES

Case I: Laminar Liquid Layer

Let us take an example of a cold plate at -50°F and surrounded by saturated steam at atmospheric pressure.

Proceeding as before and taking physical properties from tables in Chapman,

$$\Delta T_L = T_\infty - T_f = 212 - 32 = 180°F$$

$$\Delta T_s = T_f - T_p = 32 - (-50) = 82°F$$

$$Pr = 1.74 \ .$$

With all applicable values substituted into equation (1), the convective heat flux is found to be

$$q_c = 22{,}400\left(\frac{1}{x}\right)^{\frac{1}{4}} \tag{16}$$

= heat flux across liquid layer

= heat flux across solid layer, at steady state.

From equation (6), the steady-state thickness of the solid is

$$S = \frac{k_s \Delta T_s}{q_c}$$

$$= 0.00468(x)^{\frac{1}{4}} \text{ ft.}$$

At the location $x = 1.0$ ft.,

$$S = 0.00468 \text{ ft.}$$
$$= 0.0561 \text{ in.}$$

The value of γ is found from equation (10), as

$$\gamma = 0.532 \text{ .}$$

By using equation (11), a graph between τ and $H(\tau)$ can be plotted as shown in Figure VI.

From equation (8)

$$\tau = 2{,}190 \ t$$

and from equation (9)

$$s = S(H)^{\frac{1}{2}}$$
$$= 0.00468(x)^{\frac{1}{4}} \ (H)^{\frac{1}{2}} \text{ ft.}$$

Using different time intervals and at different locations on the plate from the leading edge, the transient and steady-state thickness of solid layer can be calculated, as per Table II and plotted in Figure VII.

Case II: Turbulent Liquid Layer

Let us take an example of a cold plate at -70°F and surrounded by saturated steam at atmospheric pressure.

Proceeding as before and taking physical properties from tables in Chapman,

$$\Delta T_L = T_{\infty}-T_f = 212 - 32 = 180^{\circ}F$$
$$\Delta T_s = T_f-T_p = 32 - (-70) = 102^{\circ}F$$
$$Pr = 1.74 \text{ .}$$

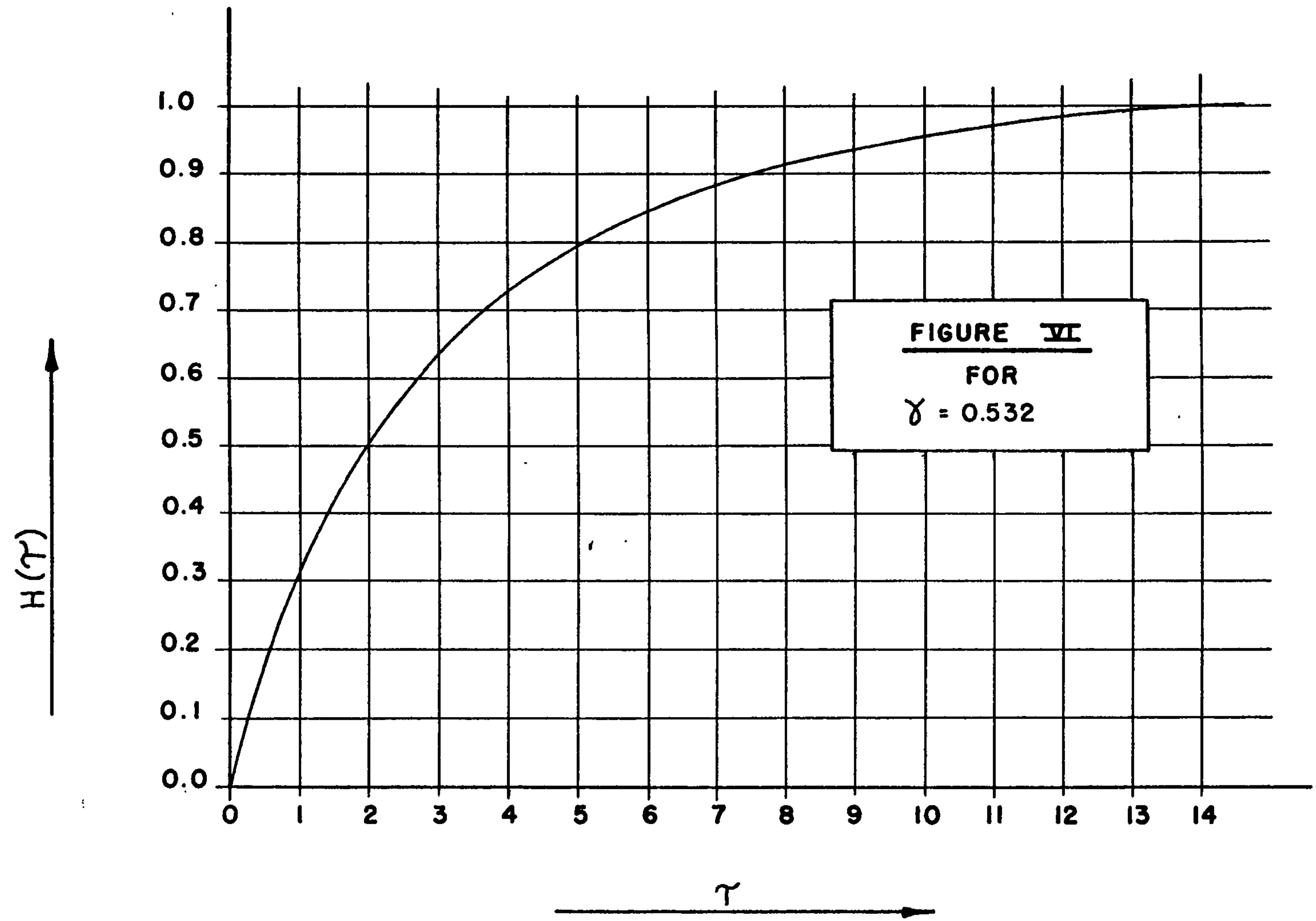

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
H(τ)
τ
FIGURE VI
FOR
γ = 0.532

FIGURE VI

TABLE II

x (ft)	t (hrs)	τ	H(τ)	s (ft)
0.00	0.001	2.19	0.535	0.00
0.25	0.001	2.19	0.535	0.00242
0.50	0.001	2.19	0.535	0.00287
0.75	0.001	2.19	0.535	0.00318
1.00	0.001	2.19	0.535	0.00342
0.00	0.0025	5.47	0.82	0.00
0.25	0.0025	5.47	0.82	0.003
0.50	0.0025	5.47	0.82	0.00356
0.75	0.0025	5.47	0.82	0.00394
1.00	0.0025	5.47	0.82	0.00424
1.00			1	0.00
0.25			1	0.00331
0.50	STEADY-STATE	STEADY-STATE	1	0.00394
0.75			1	0.00436
1.00			1	0.00468

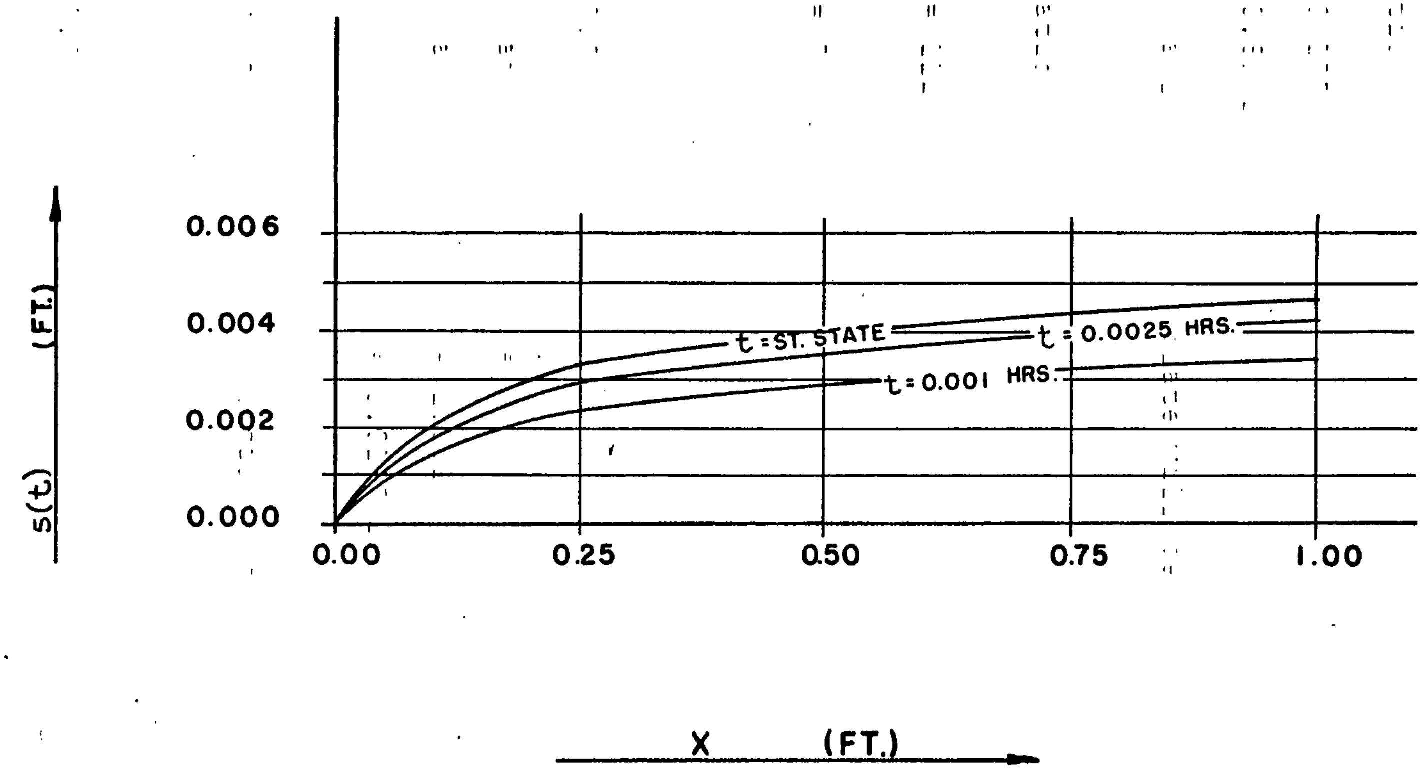

s(t) (FT.)
0.006
0.004
0.002
0.000
t = ST. STATE
t = 0.0025 HRS.
t = 0.001 HRS.
0.00
0.25
0.50
0.75
1.00
X (FT.)

FIGURE VII

Taking $Gr_x = 10^9$, let us find out the distance down the plate where the liquid layer will be in transition to turbulent state.

From equation (12)

$$x = 0.1575 \text{ ft.}$$
$$= 0.89 \text{ in.}$$

Therefore, a 1.0 ft. long plate will be more appropriate to use.

Using equation (13)

$$Nu_x = 1125.6(x)^{6/5}$$

$$\text{or } q_c = 79{,}625(x)^{1/5} \qquad (17)$$

From equation (6)

$$S = 0.00164(1/x)^{1/5} \text{ ft.}$$

$$\text{i.e., at } x = 0 \qquad S = \infty$$
$$\text{at } x = 1.0' \qquad S = 1.00164 \text{ ft.}$$
$$= 0.0197 \text{ in.}$$

Let us take the steady-state local thickness of the solid phase only,

$$S = 0.00164(1/x)^{1/5} \qquad (18)$$

If the liquid layer is laminar, from equation (16), we already know that

$$q_c = 22{,}400(1/x)^{\frac{1}{4}}$$

Therefore, $S = 0.00583(x)^{\frac{1}{4}}$ ft.

i.e., at $x = 0$ $\quad S = 0$

at $x = 1.0$ $\quad S = 0.00583$ ft.

$= 0.06994$ in.

Local steady-state thickness of the solid phase is

$$S = 0.00583(x)^{\frac{1}{4}} \text{ ft.} \tag{19}$$

Using different values of x in equations (18) and (19), a graph was plotted as in Figure VIII.

CHAPTER V

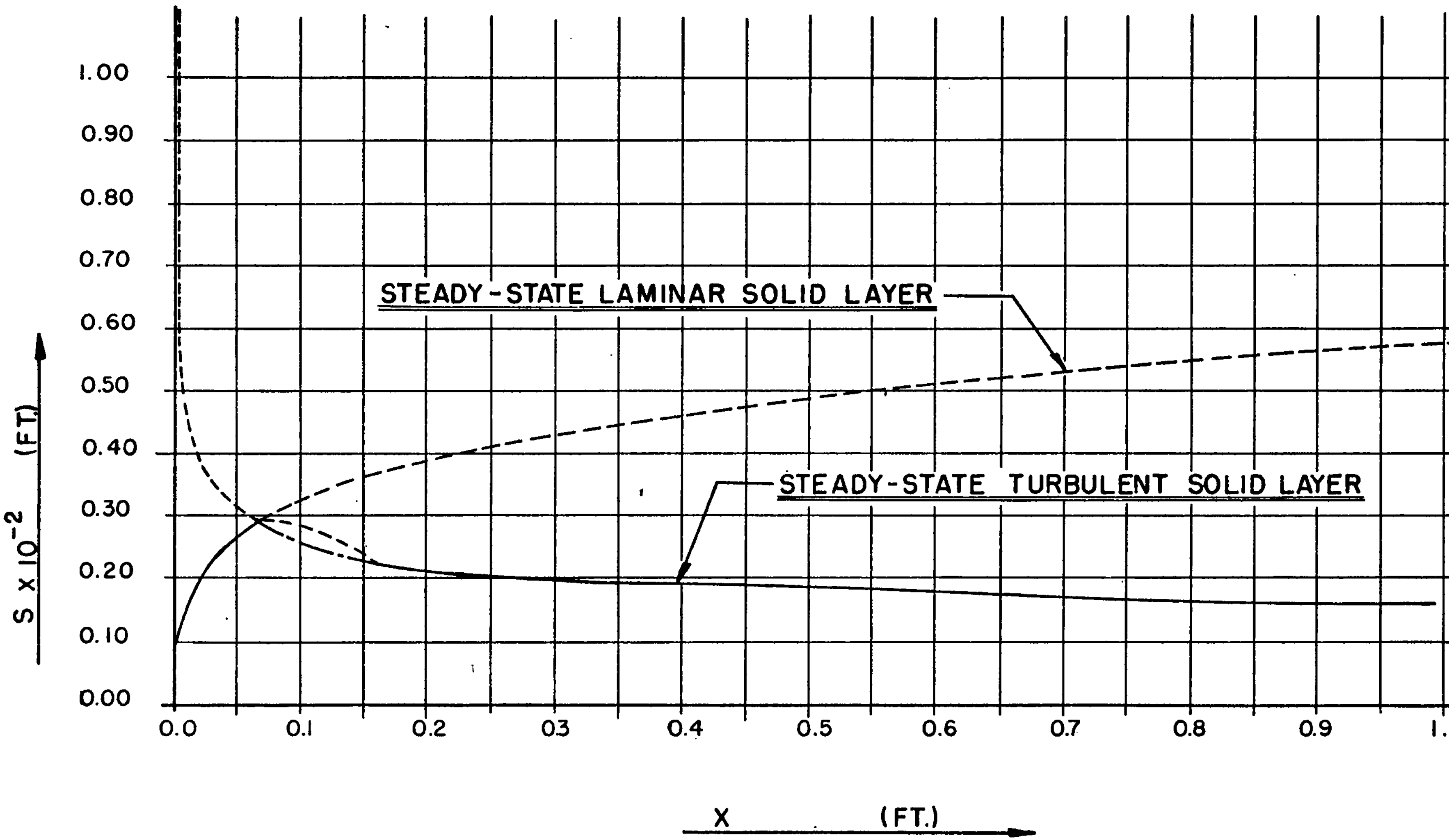
RESULTS AND CONCLUSIONS

As predicted earlier, it has been shown that with easy to use analyses of Sparrow and Gregg [2] and Beaubouef [3] for transient and steady-state thickness of solid deposited layer, one can achieve the same results as by the use of complicated "exact" predictions of Lapadula and Mueller [4].

The closeness of the results shown on Figure V clearly proves that the method developed for predicting the solid phase history in freezing of a condensate film on a vertical plate is most suitable for engineering applications.

This analysis can be applied to laminar as well as turbulent layers. Comparison between equations (16) and (17) indicates a higher convective heat flux in turbulent liquid layer for the same temperature differential across the liquid layer, and as evident from Figure VIII the turbulent liquid layer results in smaller steady-state solid layer as compared to the laminar steady-state solid layer.

It should be noted that the results of this analysis are not applicable to the situations where the liquid is at its fusion temperature as in the case of the experimental analysis carried out by Mullin and Renda [7]. During these experiments, although the liquid was flowing, it was not necessary to remove the sensible heat of the liquid, since the liquid was at its fusion temperature, with the result that there was no convective heat flux; ($q_c = 0$).



1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
S x 10^{-2} (FT.)
STEADY-STATE LAMINAR SOLID LAYER
STEADY-STATE TURBULENT SOLID LAYER
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
X (FT.)

FIGURE VIII

The effect upon convective heat flux, q_c, of liquid phase removal by solidification is measured by the parameter $c(T_\infty - T_f)/\lambda$, as shown by Lapadula and Mueller [4]. Fluids with relatively high values of λ will generally show negligible effect due to liquid removal.

With regard to the results shown in Figures VIII, it should be noted that the predicted solidification of the laminar condensate layer near to the leading edge of the plate is of doubtful validity. This situation arises because of leading-edge singularities in predictive relationships both for q_c and $H(\tau)$. It should not be expected that the configuration of the solid layer would show the "laminar hump" seen in Figure VIII. Similarly, the ambiguity concerning prediction of Nu_x for the turbulent film should be noted. Eckert and Drake [6] present both equation (13) and an empirical relationship $Nu_x = C(PrGr_x)^{1/3}$ which would predict uniform solid layer thickness, and claim an insufficient basis for choosing between them.

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