EXECUTIVE FUNCTIONS AND SELF-REGULATED LEARNING AS PREDICTORS OF MATH ACHIEVEMENT: A PATH ANALYTIC FRAMEWORK

A Master's Thesis

Presented to

The Faculty of the Department

of Psychology

University of Houston

In Partial Fulfillment

Of the Requirements for the Degree of

Masters of Arts

By

Emily A. Huston-Warren

May, 2016

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ABSTRACT

Despite considerable evidence for the hierarchical nature of math learning and the influence of executive functions in early math development, few studies have investigated the role of executive functions and related skills (i.e. self-regulated learning skills) in later elementary years. The goal of the present study was to comprehensively evaluate the role of executive functions and self-regulated learning skills as predictors of mathematical outcomes, proposing ways in which these predictive relationships may differ across elementary grades 3 through 5. Directly examining the hierarchy of math learning, this study utilized a path analytic framework to assess the likely mediating role of early math skill mastery (e.g. fact fluency) and the hypothesized moderating effect of grade. Both direct and indirect effects were assessed in a large and diverse sample of students (*N* = 846) in third grade (*N* = 186), fourth grade (*N* = 484), and fifth grade (*N* = 176). While the moderating effect of grade was not significant, the final model showed good fit (χ^2 = 313.48, *df* = 256; CFI = 0.97; RMSEA = 0.028, 90% CI = 0.015 to 0.038; SRMR = 0.074) and demonstrated predictive power for several considered variables. Additionally, a strong mediating role of math fact fluency was observed. The results underscore the robust influence of executive functions and metacognition on math outcomes across grade level, thus supporting efforts to integrate findings across bodies of literature.

Keywords: math achievement, executive functions, self-regulated learning, path analysis

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Executive Functions and Self-Regulated Learning as Predictors of Math Achievement: A Path Analytic Framework

There is a wealth of insight regarding math skill development in the early elementary years, particularly the transition from informal math knowledge to formal, school-based education (Purpura, Baroody, & Lonigan, 2013; Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). Given the hierarchical nature of mathematics skill learning (Fuchs et al., 2005; National Mathematics Advisory Panel, 2008), it is understood that later math skills build on foundational skills, which can include both numerosity and earlier arithmetic skills. Outside of math-specific skills, executive functions such as working memory are among the most clear and strongest predictors of math performance (see Raghubar, Barnes, & Hecht, 2010; Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013 for reviews). However, less is known about other executive functions and related skills such as self-regulated learning, particularly in the later part of this hierarchy (e.g. later elementary grades). Therefore, it is not clear how these skills work together to predict math in the context of one another.

The goal of the present study is to comprehensively evaluate the role of executive functions (e.g. working memory, shifting, inhibition, planning) and self-regulated learning skills (e.g. effort/self-efficacy, strategy use, metacognition) as predictors of math outcomes (e.g. calculation skills), and how this relationship may differ across late elementary grades (e.g., 3 through 5). Directly examining the hierarchy of math learning, a path analytic framework is utilized to investigate the likely mediating effect of early math skill mastery (e.g. fact fluency). The above effects are examined in a large, diverse sample that controls for potential covariates. Understanding the role of specific executive functions and self-regulated learning skills, and identifying key predictors for math performance, may aid in determining potential targets of interventions to ameliorate math difficulties. A brief overview of current and relevant knowledge is provided below, focusing on theoretical and empirical work regarding the relationship of math with executive functions and with self-regulated learning. The specific study rationale and hypotheses follow.

Math: Background

A strong understanding of mathematical concepts is critical for college and career opportunities, with education through Algebra II correlated with graduation from college and an income in the top quartile (NMAP, 2008). Moreover, STEM (science, technology, engineering, and mathematics) career opportunities are growing and job growth in mathematics-intensive fields currently outpaces overall job growth 3:1 (NMAP, 2008). With increasing demands to comprehend quantitative concepts, a strong foundation in mathematics is critical for all individuals (NMAP, 2008). However, despite the clear importance of mathematics, a review of international and domestic studies revealed that American students demonstrate less than expected success in math (NMAP, 2008). On the National Assessment of Educational Progress, commonly known as the "Nations Report Card," only 27% of American students were found to be at or above the designated "proficient" level in Grade 8 (U.S. Department of Education, 2000). Even more alarmingly, this percentage falls to 17% in Grade 12, which translates to a rising demand for remedial mathematics education in college (U.S. Department of Education, 2000; NMAP, 2008). Overall, an estimated 5 to 8% of school-age children have some form of mathematical learning disability, with some studies estimating greater than 13% (Geary, 2004; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005).

To better understand math development and how math learning difficulties arise, it is informative to consider models of math learning. While some models emphasize the importance of domain-specific skills (e.g. numerosity) for math achievement (Butterworth, 2010; Fiegenson, Dehaene, & Spelke, 2004), much research has also focused on the influence of domain-general cognitive skills, the most prominent of which is executive functions (Passolunghi & Lanfranchi, 2012; Geary, 2004). For example, Geary (2004) proposed a theoretical framework in which a central executive, made up of attentional and inhibitory processes, supports the acquisition of procedural and conceptual mathematical competencies. In this framework, impairments in specific executive functions (i.e. working memory and inhibition) underlie the deficits observed in mathematical learning disability (Geary, 2004). As another example, von Aster & Shalev (2007) suggest that executive capacities such as working memory support the hierarchical progression from nonsymbolic to symbolic mathematical proficiency across development. That executive skills are emphasized in these models speaks to their conceptual importance for mathematics developmentally.

Building on such conceptual importance, the relation of executive skills to mathematical achievement has also been sufficiently demonstrated from an empirical standpoint. For example, studies investigating math learning difficulties have found that students who struggle with math display weaknesses in executive functions and self-regulated learning skills (Gathercole, Pickering, Knight, & Stegmann, 2004; Gathercole & Pickering, 2000; Swanson & Kim, 2007; Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; Cirino et al., 2013; Andersson & Lyxell, 2007; Passalunghi & Siegel, 2001; Schunk & Zimmerman, 1998; Fuchs et al., 2003; Blair et al., 2015). Similarly, these same skill domains have been related to math performance at a more continuous level using correlational designs (Bull, Espy, & Wiebe, 2008; Bull & Scerif, 2001; Fuchs et al., 2006; St Clair-Thompson & Gathercole, 2006; Blair & Razza, 2007; Clark, Pritchard, & Woodward, 2010).

Despite the above data, current knowledge regarding predictors of math performance, particularly the role of executive functions, remains incomplete. First, the constructs subsumed under the umbrella term "executive functions" are still quite varied in the literature (e.g. to include inhibition, shifting, planning, etc.). Additionally, those from the temperament perspective have focused less on executive functions, explicitly, and more on constructs such as "hot" and "cold" effortful control, which exhibit differential relations to academic performance at early ages (i.e. preschool; Kim, Nordling, Yoon, Boldt, & Kochanska, 2013). Others have emphasized the role of predictors such as behavioral regulation for growth in prekindergarten math skills (e.g. McClelland et al., 2007). Notably, these constructs are defined and discussed similarly in relation to academic performance (e.g. executive functions vs. "cold" effortful control). Much of this confusion stems from the overlap of executive functions (EF) with several separate but related constructs, such as self-regulated learning (SRL), though EF and SRL are rarely examined together (Liew, 2012). As one example, Best and colleagues (2011) refer to "complex EF", which includes strategy formulation and self-monitoring – topics that are prominent in the literature of SRL. While some have considered these constructs together as "executive self-regulatory functions" (e.g. Ylvisaker & Feeney, 2002), information about them remains largely divided across different bodies of literature and have not been examined together as predictors of math learning and development.

Furthermore, the hierarchical nature of math highlights the need to consider these relations in the context of already developed math skills. This is important because the relationship of domain-general skills to math may transition from direct to indirect as numerical competencies become mediators of math achievement (Passolunghi & Lanfranchi, 2012). Finally, our knowledge of predictors for math is underdeveloped at critical grades (e.g., later elementary school), where the emphasis shifts from foundational skills to more advanced computational and more applied math problem solving. Although some work has examined cognitive predictors of mathematics at this age range (e.g. Fuchs, Fuchs, Stuebing, Fletcher, Hamlett, & Lambert, 2008; Cirino et al., 2007; Vukovic et al., 2014), these studies vary in the predictors and outcomes utilized, and none focus specifically on executive functions and their relationship with self-regulated learning processes. Therefore, additional work is still needed, and the present study has the potential to expand our understanding in this area. An overview of both executive functions and self-regulated learning is presented below, with a focus on the potential mechanism of action and evidence for the factors considered in the current study as they relate to mathematical performance.

Executive Functions and Math

The term executive function (EF) is most closely associated with neuropsychology, though other literatures reflect on similar concepts. EFs can be defined as “those capacities that enable a person to engage in independent, purposive, self-directed, and self-serving behavior” (Lezak, Howieson, Bigler, & Tranel, 2012, p.37). Such capacities are considered core components of self-regulation ability (Mischel et al., 2011) and are thought to be necessary for formulating goals, identifying strategies, planning, and effectively carrying out cognitive tasks (Luria, 1973; Lezak, 1982). From a neuropsychological perspective, EFs were traditionally referred to as “frontal lobe skills”, given the observed loss of function following frontal lobe damage (e.g. Milner, 1963; Stuss & Alexander, 2000; de Oliveira-Souza, Moll, Moll, & de Oliveira, 2001; Koenigs & Tranel, 2007). With regard to math specifically, neuroimaging studies have provided evidence for the role of executive functions/frontal lobe tasks during math learning and problem solving (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Dehaene, Molko, Cohen, & Wilson, 2003; Rivera, Reiss, Eckert, & Menon, 2005; Ansari, Garcia, Lucas, Hamon, & Dhital, 2005). For example, children demonstrate greater activation of the prefrontal cortex during mental arithmetic relative to adults, which suggests a reliance on prefrontal skills (i.e. executive functions) as children learn math concepts and develop math proficiency (Rivera et al., 2005; for review, see Menon, Kadosh, & Dowker, 2014). Predictive studies support such neuroimaging findings, as executive functions have been identified as predictors of children's ability to successfully acquire novel math procedures, develop automatic arithmetic fact retrieval, and perform well on standardized math achievement tests (LeFevre et al., 2013; Neuenschwander, Röthlisberger, Cimeli, & Roebers, 2012).

One influential model of executive functions from this cognitive/neuropsychological perspective was proposed by Miyake and colleagues (2000). This model delineates three primary executive functions: mental set shifting ("shifting"), information updating and monitoring in working memory ("updating"), and inhibition of prepotent responses ("inhibition"). Examined among a sample of undergraduate students, this model demonstrated that these three executive functions represent clearly distinguishable constructs, although with a commonality likely related to the goal-related nature of executive function tasks (Miyake et al., 2000). More recent studies have supported the role of shifting, updating/working memory, and inhibition in math achievement (Bull & Scerif, 2001; Yeniad, Malda, Mesman, van Ijzendoorn, & Pieper, 2012; Lee, Ng, & Ng, 2009; Passolunghi & Lanfranchi, 2012; St. Clair-Thompson & Gathercole, 2006; D'Amico & Passolunghi, 2009). However, few studies have explicitly examined the influence of each construct on math development in later elementary grades (i.e. grades 3 through 5). Furthermore, as described by Miyake et al. (2000), the three constructs outlined in the model are considered relatively circumscribed and "lower level" functions. "Higher order" executive functions (e.g. planning) were considered, though as outcomes of the lower level skills. However, as students progress in their math development and learn to solve multistep calculations, it is theoretically plausible that such "higher order" skills support achievement across later elementary grades. In fact, research has shown that "complex EF" skills, such as planning, are correlated with academic performance in late childhood and adolescence, and such skills may in fact support the strategy formulation and implementation required for more advanced math problem solving (Best, Miller, & Naglieri, 2011). For this reason, this study includes both the set of executive functions highlighted by Miyake et al. (2000), but also planning, in the context of late elementary math skill development. These four executive functions are described below.

Working Memory. Traditionally conceptualized as a system that temporarily stores and processes information to aid cognitive operations and support problem solving (Baddeley, 1992), working memory has received substantial focus in the learning disabilities literature as a predictor of academic achievement, and specifically math competencies (Swanson, Jerman, & Zheng, 2008; Raghubar et al., 2010; de Smedt, Janssen, Bouwens, Verschaffel, Boets, & Ghesquiere, 2009; Passolunghi & Lanfranchi., 2012). Working memory appears important for mathematical development, as research suggests an increase in working memory capacity when children shift from learning number words to more advanced calculations and arithmetic thinking (von Aster & Shalev, 2007). Furthermore, growth in working memory up through grade three has been shown to predict math solution accuracy and problem solving, even in the context of other executive functions and skills in reading and calculations (Swanson et al., 2008). Specifically, in the study by Swanson et al. (2008), working memory accounted for 27% of the variance for problem solving accuracy. This makes sense in the context of earlier research, which proposed that poor working memory may underlie procedural deficits in those who struggle with math learning (e.g.

losing track while counting; Geary, 2004). This association between working memory and math performance is logical from a conceptual standpoint, as working memory should support a student's ability to maintain information (e.g. numbers, procedural rules, end goal) in mind without relying on concrete aids such as blocks, tokens, or fingers (Geary, 2004). However, more research is required to more fully comprehend the relationship of working memory in later elementary grades, particularly in the context of other cognitive skills and early math skill mastery.

Inhibition and Shifting. The two other "core" executive functions delineated by Miyake and colleagues (2000) are inhibition and shifting. In contrast to working memory, inhibition and shifting have received variable support with regard to their contribution to math skill development (Bull & Lee, 2014). Inhibition has been widely measured in the learning disabilities literature as a construct that allows individuals to selectively attend in a goal-driven manner to support learning, which itself is a goal-driven process (Diamond, 2013). Specifically, inhibitory processes support a math learner in suppressing unwanted responses and irrelevant information (e.g. distracting thoughts, fact retrieval errors) while engaged in a problem-solving task (Diamond, 2013; Geary 2004). In so doing, inhibitory processes may help minimize demands on working memory, thereby reducing errors (Diamond, 2013; Geary, 2400). There is empirical evidence supporting the role of inhibitory skills in math performance, with some estimates suggesting a moderate relationship (r = 0.36) between mathematics and inhibition (St. Clair-Thompson & Gathercole, 2006; Blair & Razza, 2007). Similar research has postulated a significant role of inhibition in the development of arithmetic skills (D'Amico & Passolunghi, 2009), and poor inhibition has been correlated with weak fact retrieval, increased errors, and impaired immediate recall of numerical information (Geary, 2004; Swanson & Jerman, 2006; Passolunghi & Siegel, 2001). However, others have not found a significant correlation between inhibition and mathematical achievement, particularly when considering different stages of math development (Van der Ven, Kroesbergen, Boom, & Lesemen, 2012).

Similar variability has been observed in studies of shifting, which has been regarded in the learning disabilities literature as a process necessary for switching between tasks, operations, or mental sets (Monsell, 2003). Some have found that shifting supports and predicts mathematical achievement, with moderate correlations of $r \sim 0.25$ (Bull & Scerif, 2001; Yeniad et al., 2013). These findings support the earlier findings of Rourke (1993), who found that children with arithmetic difficulties demonstrated difficulty shifting to new strategies or operations (e.g. addition to subtraction). From a theoretical standpoint, the role of shifting in math problem solving seems evident in the need for learners to alternate between strategies and procedures, particularly in multistep math problems. However, some have found no significant influence of shifting on mathematical performance (St. Claire-Thompson & Gathercole, 2006; Espy et al., 2004). It is important to note that much of this research has been limited by relatively small sample sizes and often the use of only one measure of a specified executive function (e.g. use of Trails B as only measure of shifting). Furthermore, some posit that the variability in findings may relate to changes in the relations between executive functions due to the effect of age- and ability-related differences on construct measurement (Willoughby, Wirth, & Blair, 2012; van der Ven et al., 2012; Bull & Lee, 2014). This supports further investigation concerning the relationship between math proficiency and the possible differential role of inhibition and shifting across math development, particularly in the context of other executive functions and multiple measures for each construct.

Planning. In the seminal article by Miyake and colleagues (2003), planning was excluded from the factor model because of the very nature of the construct. In contrast to working memory, inhibition, and shifting, planning is typically regarded as a relatively less circumscribed, perhaps "higher level" concept (Miyake et al., 2000). While difficulties regarding conceptualization and measurement may arise with less circumscribed constructs, planning has typically been conceptualized as the ability to identify and organize short- and long-term future behaviors, or steps, necessary to achieve a goal state (Heilman & Valenstein, 2012; Sternberg & Ben-Zeev, 2001). Planning has been further conceptualized from a neuropsychological perspective based on the tasks traditionally used to measure the construct, such as the Tower of London task (Shallice, 1982; Krikorian, Bartok, & Gay, 1994), and the brain regions typically activated during such tasks (e.g. prefrontal cortex, bilateral superior parietal regions; Newman, Carpenter, Varma, & Just, 2001). One study using the Tower of London task found that children with arithmetic difficulties exhibit significantly greater impairment on the task (Sikora, Haley, Edwards, & Butler, 2002). This makes sense theoretically, as successful math learners must demonstrate strategic planning abilities in order to effectively solve math problems, many of which require the integration of several steps, strategies, and operations. However, research investigating the role of planning in mathematics achievement remains sparse, and the study by Sikora et al. (2002) remains one of the few to explicitly examine this relationship. As such, much more knowledge is needed to address this gap.

Self-Regulated Learning and Math

The above section shows there to be ample evidence for the relationship between mathematics and EFs, particularly when assessed via cognitive tasks and when perceived from the cognitive and neuropsychological literatures. However, terms such as "EF" have not been prominently featured in the educational and developmental literature to date. Instead, there is a focus on skills subsumed under the umbrella terms self-regulation (SR) or self-regulated learning (SRL). Zimmerman (2000) defined self-regulation as a cyclical process by which self-generated thoughts, feelings, and actions are planned and adapted through three phases: forethought, performance/volitional control, and self-reflection. This cyclical process is inherent in SRL, which requires the purposive use of self-regulation processes to adjust performance and enhance learning (Zimmerman, 2000). Self-regulated learning emphasizes a metacognitive approach to learning, wherein students must establish realistic academic goals, utilize learning strategies, monitor their performance, and identify learning errors so they may adapt and correct their method of learning through feedback (Schunk, 1990; Pintrich, 2004). Research has provided evidence for the importance of SRL processes in mathematical development, with indications that self-regulated learning strategies are critical for productive math learning (de Corte, Mason, Depaepe, & Verschaffel, 2011). For example, high math achievers demonstrate higher and more specific proximal goal setting, frequent and more accurate monitoring of learning processes, and greater persistence (de Corte et al., 2000). Additionally, studies have posited a causal role of SRL processes in mathematics success among at-risk and minority college students (Bembenutty & Zimmerman, 2003; Fong, Zientek, Ozel, & Phelps, 2015; Zimmerman, Moylan, Hudesman, White, & Flugman, 2011). Specific components of self-regulated learning that appear particularly critical for math achievement include self-efficacy and effort (Peters, 2012, Carbonaro, 2005; Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014), strategy use (Murayama, Pekrun, Lichtenfeld, & von Hofe, 2013; Ramdass & Zimmerman, 2008), and metacognition (Desoete, Roeyers, & Buysse, 2001; Rosenzweig, Krawec, & Montague, 2011).

Self-Efficacy and Effort. One key aspect of self-regulated learning concerns perceived self-efficacy, which Bandura (1997) defined as referring to “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (p.3). As self-efficacy judgments involve a belief in one’s power to achieve, greater self-efficacy has been correlated with greater effort and perseverance (Bandura 1997; 1982). Due to this relationship, self-efficacy is often considered in conjunction with effort, which may be conceptualized as an individual’s “depth” or “thoughtfulness” when processing learning material (Salomon, 1983). The correlation between self-efficacy, effort, and academic achievement can be understood from a theoretical perspective, as students with stronger beliefs in their abilities are more likely to believe their efforts will lead to success and are more likely to persevere, thereby improving their understanding and reaching higher achievement levels (Wigfield, Klauda, Cambria, 2011; Pintrich & Zusho, 2002). There is also empirical evidence that both self-efficacy and effort promote mathematical achievement (Peters, 2013; Parker et al., 2014; Carbonaro, 2005). Self-efficacy beliefs have been positively correlated with math achievement, with significant correlations broadly ranging from 0.08 to 0.47 (Liew, McTigue, Barrois, & Hughes, 2008; Parker, 2014; Peters, 2013). Furthermore, math self-efficacy is a significant predictor of university entry (Parker et al., 2014), and math achievement gaps between ethnic minorities and Caucasian students appear to diminish with increased math self-efficacy (Kitsantas, Cheema, & Ware, 2011).

Research has also provided support for effort as an important predictor of achievement. Effort has been related to “deep learning,” which involves interest in acquiring new skills and the ability to relate knowledge back to prior knowledge (Phan, 2009). However, while some researchers have demonstrated a positive correlation between effort and mathematics (Carbonaro, 2005), most results center on the effect of effort on *general* academic achievement, leaving a gap in research regarding the relationship between effort and math achievement specifically. Furthermore, most research regarding self-efficacy and effort has focused on student achievement in high school and college, with little focus on the predictive value of these factors for late elementary achievement.

Strategy Use. Self-regulated learning also involves the use of effective learning strategies (Zimmerman & Martinez-Pons, 1988; Dignath, Buettner, & Langfeldt, 2008), which have been traditionally defined as behaviors that influence how a learner *processes* information (Mayer, 1988). However, learning strategies have been more recently conceptualized as behaviors that impact how learners *acquire new* information (Stroud, 2006). Learning strategies that have received support from the literature include setting goals, organizing, making associations when learning, identifying important information, summarizing, and referring to a variety of resources (e.g. books, Google) to strengthen understanding of a construct (Dunlosky, Rawson, Marsh, Mitchell, & Willingham, 2013). For example, Dunlosky et al. (2013) found that strategies promoting a student's ability to relate new information with prior knowledge had moderate utility, and summarization promoted learning by supporting student evaluation of what is and is not known. Other studies, although only a few, have specifically demonstrated the important role of learning strategies for math competencies (Murayama et al., 2013; Ramdass & Zimmerman, 2008; Fadlelmula, Cakiroglu, & Sungur, 2013; Fuchs et al., 2003). This relationship between math success and strategy use makes sense conceptually, as problem solving in mathematics often relies on the identification of a goal, organized and strategic implementation of procedures, and the generalization of concepts or algorithms across problems (Kilpatrick et al., 2001). For example, research has shown that deep learning strategies (i.e. elaboration of learned material) significantly predict math achievement in middle-school students (Murayama et al., 2013). However, such learning strategies were not significantly correlated with math achievement in grade 5, suggesting that strategy use may function differently across grade levels (Murayama et al., 2013). Additionally, recent research has demonstrated a relationship between improvements in math performance and interventions targeting math fluency and strategy use together (Carr, Taasoobshirazi, Stroud, & Royer, 2011). This indicates that the effect of strategy use on math achievement should be further examined in the context of math fluency skills. Overall, while the effects of SRL strategies have been demonstrated across a wide age range, including the preschool and elementary years, much of the research regarding cognitive strategies tends to focus on students in high school and college. Furthermore, the effects of SRL strategy use among younger students have been mixed, with variability associated with factors such as age and type of strategies taught (e.g. elaboration vs. organization; Fadlelmula et al., 2013; Dignath, Büttner, & Langfeldt, 2008). For example, a meta-analysis found the effect of student strategy use to be slightly higher in secondary school relative to primary school, although the effect on mathematics performance was higher in primary school samples than in secondary school samples (Dignath et al., 2008). These findings support the need for further research examining the efficacy of different SRL strategies, as well as the role of SRL strategies across different grade and skill levels.

Metacognition. Self-regulated learning also requires the learner to engage metacognitively, such that learners extend cognitive resources to monitor their problem-solving processes as well as their cognition itself (e.g. Winne, 1996; de Corte, Mason, Depaepe, & Verschaffel, 2011). Metacognitive processes, often measured by means of self-report and teacher/parent questionnaires (e.g. Metacognitive Index of the BRIEF; Gioia, Isquith, Guy, & Kenworthy, 2000), promote a learner's evaluation of strategy use and support the learner in adapting cognitive learning strategies to enhance learning (Wigfield et al., 2011). The importance of metacognition has been demonstrated in the literature, as research suggests that children with learning difficulties demonstrate less developed metacognition (Swanson, Christie, & Rubadeau, 1993; Desoete & Roeyers, 2002; Hessels & Schwab, 2015). Moreover, research has found that interventions that encourage metacognitive learning, often in conjunction with improved understanding of learning strategies, have the highest effect sizes (e.g. $d = .73$; Dignath, Buettner, & Langfeldt, 2008). Regarding mathematics performance in particular, metacognition allows students to monitor their progress as they solve math problems, identify "when and why" to apply a strategy/procedure, and identify what they don't know (Schoenfeld, 1987; Schneider & Artelt, 2010). In fact, more recently, metacognition has been shown to underlie the "expert" approach to mathematical problem solving in elementary school (Desoete et al., 2001). Another study found that children with math learning disability (MLD) were less accurate than children without MLD when evaluating their math performance and when predicting the problems they could solve correctly (Garrett, Mazzocco, & Baker, 2006). Importantly, research also suggests that metacognitive strategies are most important in helping learners apply cognitive strategies to novel and challenging math problems, and less important when students achieve automaticity (Rosenzweig, Krawec, & Montague, 2011). While this suggests a differential influence of metacognition on math performance across grade and ability level, few studies have investigated this relationship.

Overall, SRL techniques have been studied primarily in high school and college populations, thereby creating a gap in the literature regarding the use of SRL strategies in later elementary years. It is crucial to address this gap, as these later elementary years mark an important period during which students transition from basic fact mastery to more complex problem solving. While some studies have examined the influence of SRL interventions on academic achievement, some of this work has focused only on broad achievement (e.g. GPA) rather than improvement in specific skills (e.g. math computations). Furthermore, studies of SRL and math achievement have often focused narrowly on one or a few components of self-regulated learning, without analyzing the influence of SRL processes in the context of cognitive tests, such as measures of executive functions. As shown below, this is key given the similarity in the way these two types of skills are portrayed conceptually.

Overlap Among EF and SRL

The concepts of self-regulated learning/self-regulation and executive functions primarily exist in distinct bodies of literature, despite conceptual similarities and notable overlap among these concepts (Ilkowska & Engle, 2010; Ylvisaker & Feeney, 2002, 2008; Garner, 2009; for review, see Hunt, Turner, Polatajko, Bottari, & Dawson, 2013;). This overlap is apparent when one considers the important role of both EF and SRL in goal-directed behaviors, such as learning (Hunt et al., 2013; Ilkowska & Engle, 2010). In fact, Ilkowska and Engle (2010) described executive functions and self-regulation as non-distinct constructs with goal-directed information processing as a common denominator among the two. Also recognizing the substantial overlap between the constructs, Ylvisaker and Feeney (2002) stressed a comprehensive and holistic view of self-regulation and executive functions, and later presented a combined EF/SR construct (Ylvisaker & Feeney, 2008). With regard to academic outcomes, Ylvisaker and Feeney (2002) highlighted the importance of a comprehensive view of EF and SR to address goal achievement in academics and everyday meaningful tasks. Some researchers have contributed to this viewpoint with a call for greater communication between various disciplines of psychology, and have hypothesized that improvement of one construct (e.g. EF) could lead to improvements in the other (e.g SRL;

Hofmann, Schmeichel, & Baddeley, 2012). More knowledge regarding the overlap between EF and SRL may improve our understanding of mathematical learning, inform methods of identifying math difficulties, and/or strengthen interventions that address EF or SRL in the service of improving academic outcomes. The present study helps in this regard by evaluating both types of skills in relation to mathematics.

The Present Study

The present study extends the literature in two key ways. First, we comprehensively examined the role of cognitive predictors for math achievement (including both cognitive executive measures and behavioral self-regulatory measures), in a grade range that has been relatively neglected in the literature, and one that is later than many predictive studies. Second, we considered the moderating role that grade/academic experience has on the mediated relations of executive and self-regulatory skills on math learning via math fact fluency. Understanding the interplay of these skills and their relative impacts may eventually help guide later intervention efforts. We made several predictions:

1) Correlations. Given the evidence above, we hypothesized that both executive functions (working memory, inhibition, shifting, planning), and self-regulated learning factors (effort/self-efficacy, strategy use, metacognition), would have significant zero-order positive correlations with math fact fluency and math computation skills. Among executive function measures, the strongest correlations were expected for working memory. The empirical evidence presented above, while admittedly in older populations, also supported the prediction that effort and self-efficacy would have a modest predictive influence on math outcomes; similar relations were expected between math and metacognition.

2) Path Model. Given the hierarchical nature of math development and the differential contribution of EF and SRL predictors to math performance across developmental level, it

was predicted that these relations would be mediated by math fact fluency, and that this mediating effect would vary (i.e., be moderated by) grade (3 through 5). Specifically, we expected the *direct* influence of the EF and SRL predictors to increase with grade level such that indirect effects would predominate at grade 3, and direct effects would predominate by grade 5.

Methods

Participants

Participants included 842 students drawn from public school districts in two large southwestern metropolitan areas. Table 1 below (total and by grade) shows the sample was balanced with regard to sex and ethnically diverse, and economic disadvantage was relatively high across school locations. Fourth grade students made up a large portion of the sample. The majority of students were enrolled in mainstream classes and all students were instructed in English, with a substantial minority classified as limited English status. Overall, the demographic characteristics of the participants in this study were generally consistent with the individual schools and school districts. Several of these descriptive variables (e.g. reduced/free lunch status) were considered as covariates in the analyses (see below). All research procedures were approved by the institutional review board (IRB), informed consent was obtained from parents, and assent was also obtained from students.

Measures

Measures of Executive Function

Working Memory was assessed using the Listening Recall subtest of the Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001). The WMTB-C is an assessment of working memory based on the three component structure of the working memory model proposed by Baddeley and Hitch (Pickering & Gathercole, 2001). In the Listening Recall subtest, participants listen to a set of sentences and indicate whether each sentence is "true" or "false." When the final sentence in a set is read, the participant must then recall the last word of each sentence of the set in order. The number of sentences in each set increases as the subtest progresses, and discontinuation criteria are met when the participant responds incorrectly to three sets (Pickering & Gathercole, 2001). There are six

Table 1

Demographic Data for the Total Sample and by Grade

Variable	***Category/Scale***	***Total (n=842)***	***Grade 3 (n=184)***	***Grade 4 (n=483)***	***Grade 5 (n=175)***
Age	Years	9.93 (.90)	9.02 (.49)	9.80 (.52)	11.23 (.49)
Sex	Female	48.46%	54.30%	46.49%	47.73%
Free/Reduced Lunch	Yes	88.83%	84.71%	92.36%	83.02%
Special Education	Yes	19.91%	14.29%	18.53%	29.47%
Gifted/Talented	Yes	0.00%	0.00%	0.00%	0.01%
Limited English Proficiency	Yes	23.83%	0%	41.34%	0%
Race/Ethnicity	Hispanic	52.14%	32.80%	63.54%	41.48%
	African American	29.33%	39.25%	25.21%	30.11%
	Caucasian	16.63%	24.73%	10.21%	25.57%
	Asian	1.90%	3.23%	1.04%	2.84%

sets of sentences per span level, and the participant proceeds to the more difficult span after correctly completing at least four of the six sets in a given span. The dependent variable for this measure is the number of words correctly recalled. The test-retest reliability of the WMTB-C Listening Recall subtest ranges from 0.38 to 0.83 (Pickering & Gathercole, 2001).

The backward span of the computerized Corsi Block-Tapping Task was used as a second measure of working memory. Originally presented by Corsi (1972), this task is a widely used measure intended to assess the visuospatial working memory component of Baddeley's model of working memory (Kessels, van Zandvoort, Postma, Kappelle, & de Haan, 2000; Vandierendonck, Kemps, Fastame, & Szmalec, 2004). The current computerized version uses the Inquisit 3 software (Millisecond Software, 2012; www. millisecond.com). Participants view a computer screen with identical blue blocks on a black background and are instructed to watch as several blocks light up at a rate of one block per second. Following completion of the sequence, participants use the mouse to click on the blocks in the reverse order. The task begins with a sequence of only two blocks and becomes more challenging as the sequence increases in length (Berch, Krikorian, & Huha, 2008). The dependent variables for this task include span capacity and a total score. The span capacity is the highest level at which the participant correctly reproduces at least one sequence, and the total score reflects the sum of correctly recalled sequences across all levels (with a maximum total score of 54). The reliability for this task was assessed for a previous study, and reliability estimates yielded a Cronbach's alpha of 0.68.

The raw total scores of the Listening Recall and Corsi Backward subtests were used as indicators of a latent "working memory" variable, and it was this latent variable that was examined with the other predictors (and grade), and in terms of its relation to math fluency and computations.

Inhibition. The Stop-Signal Task (SST; Lappin & Eriksen, 1966) was used to assess inhibition. In this computerized task, participants must respond as quickly as possible when a specified stimulus (i.e. an arrow) appears on the computer screen. This is the "go task" (Verbruggen & Logan, 2008). On some trials, an auditory tone acts as a "stop-signal." After hearing the stop-signal, participants are instructed to inhibit their response to the specified stimulus. In this particular version, an empty circle appears on the computer screen, followed by an arrow pointing either left or right. The participant is instructed to press the "A" key when the arrow points left and the "L" key when the arrow points right. On the stop trials, the auditory stop-signal quickly follows the presentation of the arrow. Participants are expected to inhibit their prepotent response by refraining to press either the "L" or "A" key. In one study assessing the reliability of the stop-signal task in children with ADHD, the test-retest reliability was 0.72 for the stop-signal reaction time (SSRT), 0.62 for the "go" reaction time, and 0.74 for the "go" standard deviation (Soreni, Crosbie, Ickowicz, & Schachar, 2009). Overall, this task continues to receive support in the literature as a reliable measure of inhibitory control (Congdon et al., 2012).

Shifting. To assess shifting, the switch conditions from several measures of the Delis-Kaplan Executive Function System (D-KEFS; Delis, Kaplan, & Kramer, 2001a) were utilized. These measures included the (1) Switching condition of the Color-Word Interference Test, (2) the Number-Letter Switching condition of the Trails Task, (3) the Verbal Fluency Category Switching condition, and the (4) Design Fluency Switching condition. Published in 2001, the D-KEFS measures a wide range of executive functions in children and adults ages 8 to 89 years of age (Delis et al., 2001a). The D-KEFS was normed on a nationally representative sample stratified based on age, sex, race/ethnicity, years of education, and geographic region (Delis et al., 2001a). Reliability estimates are presented for each subtest below.

The Switching condition of the Color-Word Interference Test is an adapted version of the original Stroop task (Stroop, 1935), in which participants have to inhibit an overlearned, or prepotent, response. The Switching condition is the most difficult of the four conditions in the D-KEFS Color-Word Interference Test. In this condition, participants are provided with written color names printed in ink that differs from the color name. In this case, the verbal response (i.e. reading the word) acts as the prepotent response (Delis et al., 2001a). Participants are instructed to alternate between naming the dissonant ink color and reading the conflicting word (Delis et al., 2001a). Participants are provided with 180 seconds to complete this task. For the purpose of this study, the two dependent variables of interest are the accuracy score and the response latency. Raw scores are converted to scaled scores that are corrected based on sixteen age groups (Delis et al., 2001a). The test-retest reliability estimate for the Switching condition of the Color-Word Interference Test is 0.80 for the age range of 8 to 19 years, and the internal consistency coefficients for the subtest range from 0.62 to 0.86. (Delis, Kaplan, & Kramer, 2001b).

The Number-Letter Switching condition of the Trails Task measures cognitive flexibility through visual-motor sequencing (Delis et al., 2001a). Participants are instructed to switch between connecting numbers and letters on a page (Delis et al., 2001a). The participant's completion time is recorded in seconds, with 240 seconds representing the longest amount of time provided before discontinuation (Delis et al., 2001a). Immediately upon making an error, the participant is stopped, the examiner places an "X" over the incorrect letter-number connection, and the participant is prompted to continue (Delis et al., 2001a). Raw scores, in seconds, are converted to age-equivalent scaled scores using conversion tables provided in the D-KEFS Examiner's Manual (Delis et al., 2001a). Along with completion time, errors may also be recorded. The test-retest reliability of the Trails Test varies depending on the condition, with the test-retest reliability for the Number-Letter Sequencing condition estimated at 0.78 (Delis et al., 2001b).

The Category Switching condition of the Verbal Fluency Test measures a participant's ability to fluently generate words from two well-learned concepts (i.e. Fruits and Furniture). Participants are instructed to alternate between the two categories and are given sixty seconds to state as many words as possible that belong in the two semantic categories (Delis et al., 2001a). Administration requires the record form, stimulus book, and stopwatch. Prompts such as "Keep going" are permitted if a participant fails to provide another word after fifteen seconds. For those participants who provide three consecutive words that don't belong in the target categories, the examiner provides a reminder such as "The categories you are to switch between are fruits and furniture" (Delis et al., 2001a). Two scores are derived from the Category Switch condition. First, the Total Correct Response score represents the sum of the correct responses from each of the two categories (Delis et al., 2001a). Second, the Total Switching Accuracy score represents the correct across-category switches. For example, the participant would receive one correct score for accurately providing a *fruit* response followed by a *furniture* response (Delis et al., 2001a). The test-retest reliability coefficient of the Category Switch condition is 0.53 for school-aged children ages 8 to 19 years (Delis et al., 2001b).

Intended to measure both design fluency and cognitive flexibility, the Switching condition of the D-KEFS Design Fluency subtest requires participants to draw designs by connecting dots and alternating between filled and empty boxes (Delis et al., 2001a). Participants are provided only sixty seconds to draw as many designs as possible. Administration requires the record form, stimulus book, design fluency response booklet, a writing utensil, and a stopwatch. Examiners are permitted to provide prompts such as "That's ok. Try to get the next one right" following mistakes and "Keep going" following failure to respond for fifteen seconds (Delis et al., 2001a). To be scored as correct, designs must be constructed within the sixty-second time limit and must be constructed using only four lines, with each line connecting two dots and one dot at each endpoint. No repetitions within a condition are permitted (Delis et al., 2001a). The test-retest reliability estimates of the D-KEFS Design Fluency subtest range from 0.13 to 0.73 (Delis et al., 2001a).

As with working memory, a latent "shifting" variable was assessed in the path model together with the other predictors, the proposed moderator (grade), and the two math outcomes. The raw total scores of the four D-KEFS switching tasks were used as indicators of this latent variable.

Planning. The Tower of London Task, originally developed by Shallice (1982), is commonly used in both clinical and experimental contexts as a measure of planning (Krikorian, Bartok, &Gay, 1994). The version administered in this study was adjusted from a previous study to improve reliability and internal consistency. The current version includes the first three items from the Inquisit software (which may be downloaded at www.millisecond.com), as well as the last seven items from the Kaller, Rahm, Köstering, and Unterrainer (2011) manuscript. In this computerized task, participants are presented with three vertical pegs of different lengths and three colored balls. A "goal" configuration is located on the right side of the screen and participants are instructed to move the balls to match the target model in the number of moves indicated in the upper right corner of the screen. Participants must abide by the following rules: (1) move only one ball at a time; (2) do not move a ball anywhere other than on one of the three pegs; (3) only place three balls on the left peg, two on the middle peg, and one on the right peg; and (4) create the "goal" configuration in a designated number of moves. A total of ten test problems follow one practice problem, and participants are provided with two attempts to solve each trial in the number of moves allowed. The first problem permits two moves, the next two problems permit three moves, the next four problems permit four moves, and the final three test items permit five moves. As demonstrated by Kaller et al. (2012), the items increase in difficulty as the number of moves increases. The mean accuracy score represents the percentage of problems solved correctly in the specified number of moves. Movement execution time (i.e. the time from the movement of the first ball to trial completion), initial thinking time (i.e. the time from the presentation of the problem to the movement of the first ball), item difficulty, internal consistency, and split-half reliability values are also obtained. The reliability for this task was assessed within the sample for a previous study. This assessment yielded a reliability estimate of 0.69.

Measures of Self-Regulated Learning

Effort/Self-Efficacy and Strategy Use. The Contextual Learning Scale (CLS; Cirino, 2014) is a paper-and-pencil self-report questionnaire intended to evaluate student beliefs and behaviors consistent with self-regulated learning skills. The CLS was evaluated in a sample of students ($N = 896$) in Grade 3 and above. Additional information regarding the development of the CLS and its properties can be found at http://www.texasldcenter.org/projects/measures. The measure was read to all students, either in an individual or group setting, and the students responded to items on a four-point Likert scale ranging from "1= Not true about me/really disagree" to "4= Almost always true about me/really agree." After items were assessed for factor loadings, cross-loadings, and error covariances, the final version of the CLS consisted of 42 items. The CLS measured the following four latent variables: skill/preference (5 items), learning strategies (18 items), effort/efficacy (14 items), and reverse (5 items). In the present study, only the Learning Strategies and Effort/Efficacy subscales were considered applicable, as many items on the CLS are specific to reading and the 32 items on these specified subscales were more relevant to learning in general. The Effort/Efficacy subscale is a 14-item subscale that measures a student's likelihood to (1) persevere despite boredom or task difficulty, (2) go the extra mile to complete tasks, and (3) believe that efforts in learning will pay off. This subscale has a coefficient alpha of 0.84. The Learning Strategies subscale of the CLS is an 18-item subscale intended to assess student use of strategies (i.e. planning, performance, and reflection strategies). The coefficient alpha for the Learning Strategies subscale is 0.88.

Metacognition. The Metacognition Index of the Behavior Rating Inventory of Executive Function –Teacher Report (BRIEF-T; Gioia et al., 2000) is a behavioral rating scale that measures executive function behaviors in the school environment. Teachers complete 86 items, each rated on a 3-point Likert scale ranging from 1=never to 3=often (Gioia et al., 2000). Teacher ratings are intended to provide insight into a student's observable behaviors and therefore offer an estimate of the student's everyday functioning. The BRIEF-T includes both the Behavioral Regulation and Metacognition indices. For the purpose of this study, only the Metacognition Index is used. The Metacognition Index is intended to reflect a student's ability to self-monitor, plan, initiate, and organize (Gioia et al., 2000), and is thought to relate directly to a student's ability to problem-solve in a variety of settings. This index is composed of the following five scales: Initiate, Working Memory, Plan/Organize, Organization of Materials, and Monitor. The Internal consistency on the BRIEF ranges from 0.80 to 0.98 for the teacher and parent forms (Gioia et al., 2000). For the Metacognition Index, the retest correlation is 0.90 (Gioia et al., 2000).

The Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder Symptoms and Normal Behavior Scale (SWAN; Swanson et al., 2001) is an 18-item questionnaire for children and adolescents. In this study, the Inattention subscale of the SWAN will be used as an additional measure of metacognition, as the Inattention subscale correlates highly (about 0.80) with the Metacognition Index of the BRIEF (Cirino et al., 2016). The SWAN assesses symptoms of Attention-Deficit/Hyperactivity Disorder by asking parents or teachers to rate a child's behavior on a 7-point scale ranging from -3 = "Far Below" to 3 = "Far Above" relative to other children, with higher scores indicating greater Attention-Deficit/Hyperactivity Disorder symptomatology (Swanson et al., 2001). While information regarding the psychometric properties of the SWAN is limited, the test-retest reliability has been estimated to be 0.66 for the full scale and 0.61 for the Inattention subscale (Lakes, Swanson, & Riggs, 2012). Furthermore, the inattention items are consistent with each other in their measurement and response patterns (Young, Levy, Martin, & Hay, 2009), and one review suggests that the SWAN, which was developed as a modification to the SNAP-IV, better identifies children across the ADHD spectrum (Collett, Ohan, & Myers, 2003).

The raw total scores of the BRIEF Metacognition Index and the Inattention subscale of the SWAN were be used as indicators for a latent "metacognition" variable. This variable was then analyzed in the context of the predictors, grade, and the two math outcome variables.

Measures of Elementary Math Outcomes.

Two subtests from the Woodcock-Johnson III Tests of Achievement were used to assess mathematical outcomes. These subtests included the Math Fluency subtest and the Calculation subtest. The Woodcock-Johnson III Tests of Achievement was published in 2001 and was based on the Cattell-Horn Carroll theory (Woodcock, McGrew, & Mather, 2001; Horn, 1985; Carroll, 1993).

Math fluency. The Math Fluency subtest is a timed paper-and-pencil subtest in which participants are provided with only three minutes to complete two pages of addition, subtraction, multiplication, and division problems. This subtest is meant to assess a student's ability to quickly retrieve simple arithmetic facts (Woodcock, McGrew, & Mather, 2001a). The median reliability for the Math Fluency subtest is 0.85 for ages 5 to 19 (Woodcock, McGrew, & Mather, 2001). *W* scores were used as the dependent variable for this subtest. The *W* score is the unit on the *W* scale, which is an equal-interval scale based on the Rasch model of data analysis in which the person's ability level in a measured trait and the difficulty level of the items are represented on the same scale (Jaffe, 2009).

Math Calculation Skills. The Calculation subtest is an untimed measure of math achievement that measures a student's ability to access and apply his or her knowledge of calculation procedures (Woodcock, McGrew, & Mather, 2001). In this paper-and-pencil subtest, participants are provided with two pages of math problems that increase in difficulty from basic addition facts to more complex calculus problems. Participants must get six consecutive items correct to meet the basal criteria, and the subtest is discontinued following six consecutive incorrect responses. The median reliability statistic for the Calculation subtest is 0.85 (Woodcock, McGrew, & Mather, 2001). *W* scores were also used as the dependent variable for this subtest.

Procedures

The data from this study come from a larger study investigating the effects of an executive functions reading intervention on reading comprehension. Data collection took place in the schools during the Fall semester of 2012 at times that accommodated the schedules of both students and staff/personnel. Administration was conducted by experienced teachers and/or interventionists who received iterative training prior to data collection. Notably, many of these individuals also had substantial experience administering other academic and cognitive measures in school settings for this and other projects. Training included a full day session that incorporated didactic presentations and supervised administration practice. Examiners then practiced independently and with other examiners before completing a “check-out” procedure with the testing coordinator. Examiners also received on-site supervision. The order of administration was pseudo randomized. Cognitive EF data and SRL self-report data were collected over multiple assessment sessions. Executive function behavior ratings were collected from classroom teachers toward the end of the school year, after students were well known by their teachers. Due to testing time constraints in the larger project, students were randomized to six pre-determined patterns that determined the measures they would complete. While not all students received all measures, the students completed the measures in such a way as to provide estimates of relations among each pair of measures.

Analyses

Preliminary analyses were conducted in SAS (SAS, 2009), including evaluation of variable distributions and outliers. The covariates of sex, free/reduced lunch status, language status (i.e. limited English proficiency), and education status (i.e. receiving special education services or enrolled in the gifted/talented program) were considered. We did not expect interactive effects of the covariates with the key study variables, but considered their effects on the overall prediction and unique contribution of hypothesized variables. Age was examined separately, as grade was explicit to the model.

Primary analyses were conducted in a path analytic framework via Mplus (Muthen & Muthen, 2007). The path analytic approach permits the examination of causal relationships among a set of variables, with specified directionality of predicted causal effects (Klem, 1995; Kline, 2005). Direct and indirect effects may be analyzed simultaneously, with the sum of the direct and indirect effects representing the total effect, or the effect coefficient (Klem, 1995). Direct effects represent those involving only two variables, and are depicted with the use of straight arrows connecting one variable with the other. In contrast, indirect effects are those with three or more variables, and are depicted through chains of arrows connecting each variable of the path (Kline, 2005). Indirect effects are estimated statistically by multiplying the beta weights of the direct effects that comprise them (e.g. predictor → mediator, multiplied by mediator →outcome; Kline, 2005). Finally, total effects are calculated by summing the indirect effects and direct effects.

While similar to multiple regression, path analysis holds several advantages. First, path analysis allows for the consideration of more than one dependent variable in such a manner as “X causes Y and Y causes Z,” (Klem, 1995; Norman & Streiner, 2003). Furthermore, path analysis provides for instances in which a given variable may be an outcome for one variable and a predictor of another (Norman & Streiner, 2003). Overall, path analysis permits the simultaneous evaluation of all the relationships in a given model (Tabachnick & Fidell, 2007). In the present study, the intended path analysis will be a recursive model. This means that (1) the causal effects will be unidirectional and (2) the disturbances (representing omitted causes of an endogenous variable) are expected to be uncorrelated (Kline, 2005). Additionally, given the measures used to assess each variable, some variables (e.g. planning, math fluency, etc.) will be measured using one measure, while three variables (i.e. working memory, inhibition, shifting) will be measured using a composite score. Given the reality that all exogenous variables cannot be included in a given model, most path models run the risk of misspecification (Kline, 2005). In the present study, efforts were made to minimize potential bias through careful consideration of the included variables based on a thorough review of the relevant literature. The models were evaluated for goodness of fit, or the extent to which the models were consistent with the observed data

(Klem, 1995). Indices of both global (e.g. χ^2, CFI, TLI, RMSEA, SRMR) and local (variable parameter estimates) fit were assessed. As the X^2 statistic proves sensitive to relatively large numbers of parameters and large sample sizes (Little, 1997), the models were also assessed using practical fit indices (e.g. the comparative fit index/CFI and root mean square error or approximation/RMSEA). Nested models were compared using X^2 difference testing based on the loglikelihood values and scaling correction factors obtained with the MLR estimator in MPLUS. Procedures for X^2 difference testing with loglikelihood values can be found on the MPLUS website at https://www.statmodel.com/chidiff.shtml (Muthen & Muthen, 2016). Please see Figure 1 for a schematic version of the model.

As part of the preliminary analyses, confirmatory factor analysis was utilized to examine the measurement model (for the proposed latent variables), and evaluate its invariance across grade. Next, to test the mediating effect of math fact fluency across grade, the regression effects (of predictors to mediator, of mediator to outcome) were then constrained to be equivalent across grade level. This constrained model was compared to an alternative model in which all regression effects were free to vary across grade. To support our hypothesis, the unconstrained model was expected to demonstrate better fit relative to the constrained null model.

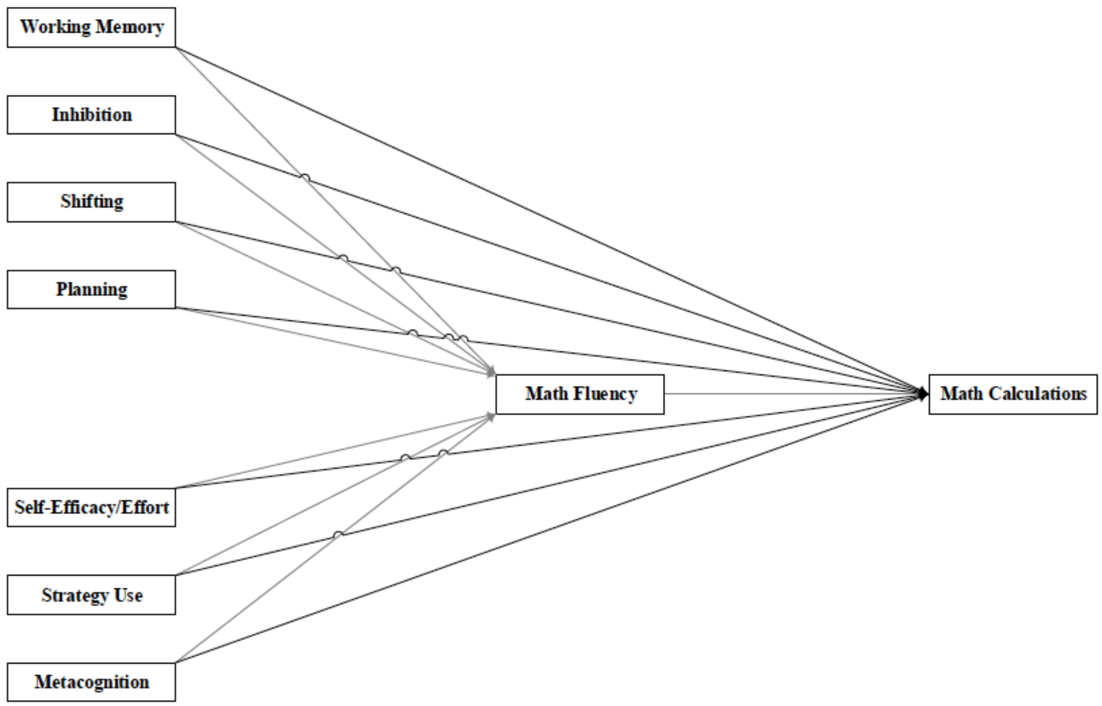

Figure 1. Schematic version of the proposed mediation model depicting both indirect and direct paths. To examine the moderating effect of grade, this proposed model was assessed for variance across grades 3 through 5.

Results

Preliminary Analyses.

Preliminary analyses first included an examination of variable distributions and a consideration of any significant threat to score reliability. As no significant threats were observed across measures, the final model included each of the specified measures of executive functioning and self-regulated learning. Descriptive statistics for each measure are provided in Table 2.

Potential covariates including sex, ethnicity, free/reduced lunch status, education status (i.e. gifted/talented or special education enrollment), and language status (i.e. English language proficiency) were examined. Sex was not significantly related to math fluency or calculations, so was no longer considered. Ethnicity was significantly related to each outcome, with the group mean for African-American students falling below that of the other ethnic/racial groups (i.e. White, Hispanic, and Other) for math fluency ($p = 0.003$) and for calculations ($p < 0.001$). Similarly, free/reduced lunch status was also related to math fluency ($p = 0.045$) and calculations ($p < 0.001$). Only one student was enrolled in the gifted/talented program, and special education as a designation was only available for approximately half the sample; therefore, these variables were no longer considered as covariates. Language status (i.e. English language proficiency) was not significantly related to calculations performance, the primary outcome, and was therefore no longer considered. Subsequently, only ethnicity and free/reduced lunch status were examined for their potential contribution in the constrained and unconstrained path models. As neither was a significant predictor of calculations performance and their inclusion did not change the overall pattern of results, they do not appear in the models below. Finally, age was considered separately for its influence beyond that of grade. While age was a significant predictor of calculations performance in the final constrained model, the overall pattern of effects for the predictors on the primary outcome (calculations) remained the same. Therefore, age was not included in the final model (in addition to grade).

Next, the three proposed latent variables (Working Memory, Shifting, and Metacognition) were assessed with confirmatory factor analyses (CFA). This model demonstrated rather poor model fit ($\chi^2 = 157.27$, $df = 34$; CFI = 0.87; RMSEA = 0.065). Furthermore, results revealed a very strong correlation between the Working Memory and Shifting latent variables ($r = 0.86$, $p < 0.001$), suggesting significant multicollinearity between the two; correlations of Working Memory and Shifting with the "Metacognition" latent variable were lower ($r = 0.42$ and $r = 0.34$, respectively). Based on these results, and on results from the larger parent project (Cirino et al., submitted), all cognitive performance measures, including those of planning and inhibition (which each had only a single indicator), were combined into an "Executive Function" latent variable. All behavioral/rating scales were combined into a separate latent variable. However, fit was poor ($\chi^2 = 525.67$, $df = 53$; CFI = 0.67; RMSEA = 0.103), and the factor loadings suggested that Effort/Self-efficacy and Strategy Use variables were separate from the "Metacognition" latent variable, also consistent with the parent project. As such, the final measurement model was assessed with the Metacognition and Executive Function latent variables, as well as separate Effort/Self-efficacy and Strategy Use "perfect" latent variables, with a single indicator for each. Fit statistics for all tested models appear in Table 3, including these preliminary models. In the constrained CFA (Model 1 of Table 3), all unstandardized loadings of indicator variables on latent variables were constrained to be equal across grade. This model was compared to an unconstrained CFA (Model 2 of Table 3) in which the unstandardized loadings of indicator variables were free to vary. As depicted in Table 3, both the constrained ($\chi^2 = 259.45$, $df = 190$; CFI = 0.95; RMSEA = 0.036) and the unconstrained ($\chi^2 = 225.58$, $df = 166$; CFI = 0.96; RMSEA = 0.036) CFA models fit the data well, and the unconstrained model did not significantly differ from the constrained measurement model ($\chi^2 = 33.79$, $df = 24$, $p > 0.05$). Consequently, final model comparisons were conducted using only the constrained measurement model.

Primary Analyses.

H1: Correlations. In addressing hypothesis one, zero-order correlations between the two math outcomes and indicators of predictor variables were examined. Correlations among the variables used in the final multigroup model are presented in Table 4. As predicted, working memory measures correlated moderately and significantly with calculations and math fluency performance, with correlations ranging from $r = .32$ to $r = .40$, all $p < 0.001$. These correlations were stronger than those observed for other measures of executive functions, with the exception of the D-KEFS Number-Letter Switching condition of the Trails Task, which correlated moderately and significantly with math fluency ($r = -.46$, $p < 0.001$) and calculations ($r = -.48$, $p < 0.001$). All other executive function variables correlated modestly and significantly with the two math outcomes, with correlations ranging from $r = .18$, $p < 0.001$ to $r = -.36$, $p < 0.001$. Regarding the self-regulated learning measures, correlations were strongest ($r = -0.35$ to $r = -.39$, all $p < 0.001$) between the metacognition measures and the math outcomes. Correlations between the CLS Self-Efficacy/Effort Subscale and the math outcomes were more modest (from $r = 0.14$ to $r = 0.18$, both $p < 0.001$), and the CLS Strategy Use Subscale did not correlate significantly with either math fluency ($r = -.06$, $p = 0.086$) or calculations ($r = -0.04$, $p = 0.208$). Notably, the two math outcome measures correlated strongly in all three grades, and their association appeared to increase across grade (G3 $r = 0.45$, $p < 0.001$; G4 $r = 0.57$, $p < 0.001$; G5 $r = 0.64$, $p < 0.001$).

H2: Multigroup Model. Primary analyses examined an unconstrained multigroup model where all regression parameters were estimated separately across grade. This was compared to a null model in which the following were constrained to be equivalent across grade: (1) all regression loadings of the predictor variables on the mediator [math fluency], and (2) the direct effect of the mediator on the outcome [calculations]. The hypothesized model (Model 4, Figures 3 to 5) with unconstrained paths was a good fit to the data ($\chi^2 = 288.43$, $df = 238$; CFI = 0.97; RMSEA = 0.027). However, as shown in Table 3, χ^2 difference testing indicated that this model did not significantly differ from the null constrained model

(Model 3; Figure 2: $\chi^2 = 24.99$, $df = 18$, $p > 0.05$). Consequently, the hypothesized unconstrained model failed to represent an *improvement* over the constrained model.

Parameter estimates for the primary constrained versus unconstrained models (3 vs. 4) can be found in Tables 5 and 6, respectively. The observed patterns of significance did not differ between standardized and unstandardized values, so unstandardized model parameter estimates are discussed to facilitate comparison between the constrained and unconstrained models. As depicted in Table 5, the unstandardized local fit parameters of the constrained Model 3 indicated that EF ($p < 0.001$), metacognition ($p = 0.003$), and math fluency ($p < 0.001$) were each uniquely predictive of math calculations. Self-efficacy ($p = 0.755$) and strategy use ($p = 0.346$) were not uniquely predictive of calculations. EF ($p < 0.001$) and metacognition ($p < 0.001$) also demonstrated significant indirect effects via math fluency, while this was not observed for self-efficacy or strategy use. Differing amounts of variance in the outcome variables of math fluency (G3 $R^2 = 0.27$; G4 $R^2 = 0.28$; G5 $R^2 = 0.31$; all $p < 0.001$) and math computations (G3 $R^2 = 0.39$; G4 $R^2 = 0.42$; G5 $R^2 = 0.49$; all $p < 0.001$) were observed across grade.

Table 6 presents the local fit parameters of the unconstrained Model 4, and Figures 3 to 5 depict the different parameter values at each grade level. When allowed to vary across grade, the local fit parameters indicated that EF ($p < 0.005$ all grades) and math fluency ($p < 0.002$ all grades) were uniquely predictive of calculations. While the constrained model indicated that metacognition was uniquely predictive of math calculations, this significant direct effect was not observed in grade 3 of the unconstrained model ($p = 0.724$). Just as executive functions had a significant indirect effect in the constrained model, executive functions acted significantly via math fluency in all three grades in the unconstrained model (G3 $p = 0.030$; G4 $p < 0.001$; G5 $p = 0.002$). This was also the case for metacognition (G3 p

= 0.038; G4 $p = 0.011$; G5 $p = 0.028$). When allowed to vary, self-efficacy and strategy use were still not uniquely predictive of calculations performance. Additionally, as depicted in Table 6, no indirect effects of self-efficacy or strategy use were observed.

In the unconstrained model, the predictors accounted for slightly differing amounts of variance in the outcomes of math fluency (G3 $R^2 = 0.22$; G4 $R^2 = 0.30$; G5 $R^2 = 0.37$; all $p < 0.001$) and calculations (G3 $R^2 = 0.37$; G4 $R^2 = 0.41$; G5 $R^2 = 0.57$, all $p < 0.001$). These results differed slightly from those observed in the constrained model, where variance in math fluency was better accounted for in grade 3 ($R^2 = 0.27$) and less well accounted for in grade 5 ($R^2 = 0.31$) relative to the unconstrained model. The constrained model accounted for slightly more variance in math calculations in grades 3 ($R^2 = 0.39$) and 4 ($R^2 = 0.42$) relative to grade 5 ($R^2 = 0.49$) when compared with the unconstrained model. Despite these differences, both the constrained and unconstrained models demonstrated a similar pattern such that variance in both math outcomes was better accounted for as grade level increased.

Discussion

The aim of the present study was to comprehensively evaluate the role of executive functions and self-regulated learning processes as predictors of math outcomes, with a particular focus on the moderating role of grade/academic experience on the mediational effect of math fact fluency. While the moderating role of grade was not significant, the final model showed predictive power for most considered variables, as well as a strong mediating role of math fact fluency.

H1: Correlations

Consistent with the first hypothesis, zero-order correlations between measures of executive and related functions and the two math outcomes were moderate and in the anticipated direction. Some of the strongest zero-order correlations were observed for measures of working memory (e.g. Listening Recall and Corsi-Blocks Backward), which is consistent both with expectation as well as with past findings of moderate to strong correlations between working memory capacity and math performance (Raghubar et al., 2010; Bull & Lee, 2014; Friso-van den Bos, Kroesbergen, & van Luit, 2014). Also in support of the first hypothesis and consistent with past findings (e.g. Dignath et al., 2008), the behavioral rating scale measures of metacognition correlated significantly and moderately with calculations and math fluency performance. These observed zero-order correlations were in a similar range as those of the executive function measures. This reflects prior research indicating that both performance-based executive function measures and rating scale measures significantly contribute to math achievement, and they appear to do so in a complementary rather than competing manner (Clark, Pritchard, & Woodward, 2010; Isquith, Roth, & Gioia, 2013; Gerst et al., 2016).

Regarding effects more specifically indicative of self-regulated learning, the finding that self-efficacy was related to math performance was consistent with expectation, although the size of the relationship was modest. Given that correlations between self-efficacy and math achievement range from small to large in the literature (Liew et al., 2008; Parker, 2014; Peters, 2013), this modest relationship was consistent with previous findings. To parse the findings out further, it was notable that the effects of self-efficacy were similar in magnitude for calculations and math fact fluency. It was expected that self-efficacy would relate more with calculations performance than with fluency, as complex tasks requiring confidence in one's ability to analyze, synthesize, and evaluate problems may rely more heavily on self-efficacy beliefs (Kitsantas et al., 2011). However, it is notable that most studies regarding the relationship between self-efficacy and math achievement have focused on general academic achievement or on older grades and later-learned skills (e.g. algebra; Parker et al., 2014;

Peters, 2013; Kitsantas et al., 2011). One of the few studies known to explicitly examine the effect of self-efficacy on math outcomes in elementary school combined math fluency and calculations into a broad math outcome measure, therefore neglecting to examine the relationship between self-efficacy and specific math skills (Liew et al., 2008). As such, the present study is one of the first known studies to specifically examine this relationship, and results suggest that self-efficacy supports performance in both skills to a similar, albeit modest, extent in this age range.

In contrast to self-efficacy, the hypothesis regarding strategy use was not supported – it did not significantly relate to either math outcome. Of the limited research regarding strategy use and mathematics achievement, the results have been modest and variable (Murayama et al., 2013; Fadlelmula et al., 2013; Dignath et al., 2008), and there has been some evidence for a significant relationship with math outcomes only when specific strategies (e.g. elaboration) are examined (Fadlelmula et al., 2013). In the present study, the *negative* rather than positive direction of the absolute relationship was somewhat surprising. It is possible that this finding reflects student awareness regarding strategy use in math problem solving, as students who struggle with math are more likely to require explicit instruction in math facts or in problem solving (Morgan, Farkas, & Maczuga, 2015). Therefore, it may be that students struggling with math were more cognizant of their strategy use and consequently reported greater use of strategies than their higher performing peers. Finally, it is also notable that the strategy use questions in the CLS were not specific to math, but instead pertained to general learning strategies (e.g. “I ask myself questions to make sure I know the material I have been studying”). It is possible that questions more specific to math (e.g. “I identify what kind of problem it is before solving”) would have better captured a relationship between strategy use and mathematics performance. Of course, such results would need replication given that the observed negative correlations were not significant.

H2: Invariance of Mediating Effect

The final model revealed a strong mediating effect for math facts on the relationship between EF and SRL on math calculation performance. While the predictors accounted for an increasing amount of variance in calculation performance across grade (from 39% to 49%), thus suggesting a developmental process, the mediational effect did not significantly vary by grade. As we expected the mediational effect to be larger at younger grades, our hypothesis regarding the moderating effect of grade was not supported. Three possibilities for this pattern of results are (1) the sample characteristics; (2) that the math skills examined were not differentially separable within this range; and (3) the mediational relationship between the predictors and math fluency does not differ over development.

First, it is important to consider characteristics of the present sample. A significant portion of the participants enrolled in this study were recruited based on identified reading difficulties. Children with reading problems often have weaknesses in phonological awareness (Melby-Lervåg, Lyster, & Hulme, 2012; Scarborough, 2009) and in rapid naming (e.g. Pauly et al., 2011; Donker, Kroesberger, Slot, van Viersen, & de Bree, 2015). Presumably due to weak phonological representations (Robinson, Menchetti, & Torgesen, 2002; Simmons & Singleton, 2008), weaknesses in these two areas are associated with poor math fact fluency (de Smedt & Boets, 2010; Koponen et al., 2016), as well as lower math performance more generally (Mazzocco & Grimm, 2013; Koponen, Salmi, Eklund, & Aro, 2013). Although the role of these phonological skills was beyond the scope of the present work, it is notable that the sample was weighted accordingly and students in this sample demonstrated average math achievement levels (see Table 2). Additionally, this study differs from previous studies that have compared typically achieving students with those demonstrating clear math difficulties. Such studies have found that poor fact mastery underlies significantly lower broad math achievement levels (Jordan, Hanich, & Kaplan, 2003), and deficits in basic fact retrieval abilities are associated with delayed growth in mathematics across elementary grades (Geary, Hoard, Nugent, & Bailey, 2012). Given these considerations, different mediational effects of math fact fluency might be evidenced in samples with other characteristics (e.g. different achievement levels), and in relation to additional math outcomes; future studies might elaborate the effect of the above conditions.

Second, it is notable that the zero-order correlations between math fluency and calculations were moderate to strong across grade. Although the zero-order correlations between the math outcomes appeared to increase from grade 3 to grade 5 (*r*'s = .45 to .67), even constraining versus unconstraining these 3 paths alone did not show differential model fit. While the predictive value of math facts for calculation performance is consistent with the hierarchical progression of math learning (i.e. early math skills support later math performance; Cirino et al., 2016; Fuchs et al., 2006), the relatively stable relationship across grade suggests that rather than calculations being more distinguishable from math fluency by grade five, the two skills are similarly related at both the beginning and the end of the grade range assessed. Therefore, it is possible that a greater difference across grade would have been observed if elementary math performance had been compared to later (e.g. middle school) math performance, when math fact fluency and calculation skills may be more distinguishable. This hypothesis is consistent with findings from the reading intervention literature, which suggest that components of reading are more sufficiently separable in middle school (Cirino et al., 2013) and can be targeted separately through intervention (Snowling & Hulme, 2012). Furthermore, it is possible that math fact fluency would have been more distinguishable from more specific assessments of complex math skills (e.g. fractions performance), rather than general computations. In one recent study, for example, researchers found that fact-based retrieval was directly predictive of procedural computations, but was more distinguishable from fraction skills and proportional reasoning abilities (Cirino et al., 2016).

Regarding the mediational relationship, a change might have occurred by either the indirect (mediational) effect being smaller with grade, or by the direct effect being larger with grade. Therefore, a potential explanation for the observed invariance is that executive functions and metacognition have a consistent, fundamental relationship to math fluency and calculations across developmental epochs (even as absolute math skill is higher in grade 5 relative to grades 3 and 4, see Table 2); these are examined below.

Executive Functions. In this study, executive functions were consistently related to math fluency and calculations across grade, rather than exhibiting a more pronounced direct effect in fifth grade, where math facts were expected to be better mastered. That executive functions strongly relate to math outcomes in elementary grades is not unique to the present study. For example, in a sample of younger elementary students, researchers found that their latent EF variable had a substantial *direct* effect on math achievement, as assessed via measures of quantity comparisons, mathematical sequence completion, and basic arithmetic (Roebers, Cimeli, Röthlisberger, & Neuenschwander, 2012). While the literature provides both empirical and theoretical support for the role of executive functions in supporting math calculation skills (Cragg & Gilmore, 2014; Berg, 2008; Chong & Siegel, 2008), there is also evidence for the role of executive functions for basic math fact fluency skills (Chong & Siegel, 2008; Andersson, 2008). In fact, in the elementary grade range, the associations of executive functions with math fluency and calculations appear to be similar in magnitude, with correlations ranging from $r = 0.23$ to $r = 0.58$, depending on the measure (Andersson, 2008). These findings imply a similar influence of executive functions for math fluency and calculation performance across elementary grades, which may contribute to the invariance observed in the present study. While this study confirms previous findings regarding the substantial role of executive functions for math outcomes across elementary grades, the present study expands on previous findings by (1) considering the role of executive functions across a relatively neglected grade range, and (2) examining the effect of executive functions on math fluency and calculations performance separately.

It is notable that multicollinearity among the executive functions resulted in a single cognitive/performance-based "Executive Functions" latent variable, rather than what was originally proposed (or as suggested by Miyake et al., 2000). More recent studies have also had difficulty clearly separating executive function variables. For example, Van der Ven and colleagues (2012) conducted a confirmatory factor analysis in which inhibition and shifting were indistinguishable in an early elementary school sample, and Bull and Lee (2012) found a three-factor solution when examining executive functions only in their oldest age cohort (14 years). Furthermore, in a more specific and comprehensive study of EF, Cirino and colleagues (submitted) found support for a bifactor model of executive functions that includes specific factors as well as one common EF factor. With regard to the invariance observed in the present study, it is possible that assessing the impact of EF on math outcomes with a common latent variable may make the observed effects more stable over grades.

Metacognition. Interestingly, a subtle qualitative difference in the effect of metacognition was observed across grade level (see Table 6). Metacognition acted only indirectly through math fluency at the third grade level, whilst exhibiting both indirect and direct effects in fourth and fifth grade. This suggests that metacognition, while influencing math outcomes in each grade, may do so differently in younger grades (i.e. 3rd grade) relative to later elementary grades in the context of numeric mediators (e.g. math fluency). As there are presently no other studies examining the relationship of metacognition with math outcomes in the context of a math mediator, it is unclear what drove this observed difference in the present study. However, this difference was not substantial enough to drive the mediational effects to significantly differ across grade level, and even constraining vs. unconstraining these paths alone did not result in differential model fit.

As described for executive functions, it is likely that the observed invariance in the mediational effect resulted from a fundamental and stable role of metacognition for math performance across developmental stages. This has been demonstrated in the literature, as metacognition appears to directly predict mathematics performance in primary school, even in the context of other internal (gender, math self-efficacy) and external contextual predictors (e.g. teacher beliefs, school location; Zhao et al., 2014). Evidence has also been gleaned from intervention research, which suggests that third and seventh grade students at risk for math difficulties demonstrate comparable improvement in math outcomes when provided with schema-based instruction utilizing metacognitive skills (Jitendra et al., 2013; Jitendra et al., 2016). Thus, while the extent to which metacognitive skills predict math achievement in the presence of math mediators remains understudied, there does appear to be growing support for the robust role of metacognition for primary math outcomes across development. As metacognition appears to have a significant influence on math outcomes even in early grades, this may explain the invariance in mediational effect across grade level.

Taken together, these findings communicate the importance of early math skill, executive functions, and metacognition for math learning across the late elementary school years. Self-efficacy and strategy use, however, evidenced no unique demonstrable influence on math outcomes. These results have some potential implications for math instruction. Strengthening even basic math skills (e.g. math fact fluency) even beyond the early elementary years is likely to be beneficial, at least until mastery is demonstrated rather than assumed. Given the influence of executive functions (e.g., working memory, shifting, planning, inhibition) on math performance, it is also likely that instruction that reduces demands in these areas is likely to have a positive impact. It may be more effective to incorporate such methods into math instruction directly rather than training underlying executive functions, given that transfer to academic achievement outcomes has been difficult (Redick, Shipstead, Wiemers, Melby- Lervåg, & Hulme, 2015; Melby-Lervåg & Hulme, 2013; Rapport, Orba, Kofler, & Friedman, 2013). Similarly, in light of the robust influence of metacognition on math outcomes, addressing teacher-reported metacognitive limitations will likely enhance math learning.

Limitations

Readers should note several limitations while considering the results of the present study. First, the cross-sectional design limits the extent to which conclusions can be drawn with respect to math development per se. While this design provides evidence for the importance of executive function and metacognition at different developmental epochs, an experimental and longitudinal design would facilitate more direct conclusions regarding causal changes in math learning in this understudied grade range.

Second, while multiple predictors were included and each was carefully selected based on support in the literature, it is recognized that not all potential variables were considered in the models. With respect to the outcome variables, it is notable that math computation skills still represent rather basic skills in the math hierarchy. Other skills often introduced in fifth grade (e.g. fractions), as well as skills introduced in later grades (e.g.

algebra), would have been interesting to consider and could have offered more separability from the basic math facts mediator.

Third, beyond the abovementioned concerns regarding general versus specific content in the SRL measures, additional measurement limitations may underlie the failure to observe expected associations between self-regulated learning factors and math outcomes. While SRL is primarily measured by self-report questionnaires, as it was here, it is important to consider issues inherent in the use of self-report measures. For example, Fulmer and Frijters (2009) delineated concerns such as item interpretation difficulties, inconsistent responding, and factor instability across age ranges. As such, some researchers have argued for "microprocess" approach to the student learning experience, with on-line monitoring tasks (Ainley & Patrick, 2006) and more comprehensive "in-vivo" assessments (i.e. think-aloud tasks) coupled with observation, structured interviews, and student descriptions (Winne and Perry, 2000; Fulmer & Frijters, 2009). As reviewed in the introduction, there is good reason to expect that strategy use and self-efficacy would be good determinants of academic performance, so the use of such alternative methodology (e.g., structured interviews, think-aloud tasks, physiological monitoring) might have been more productive in demonstrating that link empirically. However, due to time constraints in the schools and participant fatigue, such a comprehensive approach would have been infeasible in the present study.

Conclusions

This study brings together diverse bodies of literature by simultaneously examining both executive functions and self-regulated learning factors, and considering these in the context of the mathematical hierarchy. We expected that the mediational effect of math fluency would decrease for older students, but it instead remained constant. Even so, the collective predictive power of the factors considered here for math calculations did increase for older students. Moreover, even considering the strong mediating effect of math fluency, executive functions and (teacher-rated) metacognition were found to be robust predictors of math outcomes across elementary grades, thereby offering strong support for the influence of these skills in math learning. However, (student-rated) self-regulatory processes were unrelated or only minimally related to math outcomes, despite the zero-order relations not differing substantially from established literature. The results inform our understanding of the relationships among all these considered variables over this grade range, as well as provide avenues for future work and considerations regarding how these factors might influence math instruction and intervention.

References

Ainley, M., & Patrick, L. (2006). Measuring self-regulated learning processes through tracking patterns of student interaction with achievement activities. *Educational Psychology Review, 18*(3), 267-286. doi:10.1007/s10648-006-9018

Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: a general or specific deficit? *Journal of Experimental Child Psychology, 96*(3),197–228.

Andersson, U. (2008). Working memory as a predictor of written arithmetical skills in children: The importance of central executive functions. *British Journal of Educational Psychology, 78*(2), 181-203. doi:10.1348/000709907x209854

Ansari, D., Garcia, N., Lucas, E., Hamon, K., & Dhital, B. (2005). Neural correlates of symbolic number processing in children and adults. *NeuroReport, 16*(16), 1769-1773.

Aunola, K., Leskinen, E., Lerkkanen, M., & Nurmi, J. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*(4), 699-713.

Baddeley, A. (1992). Working memory. *Science, 255*(5044), 556-559.

Bandura, A. (1982). Self-efficacy mechanism in human agency. *American Psychologist, 37*(2), 122-147.

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Barbaresi, W., Katusic, S., Colligan, R., Weaver, A., & Jacobsen, S. (2005). Math learning disorder: Incidence in a population-based birth cohort, 1976–82, rochester, minn. *Ambulatory Pediatrics, 5*(5), 281-289.

Bembenutty, H., & Zimmerman, B. J. (2003, April). Relation of motivational beliefs and self-regulatory processes to homework completion and academic achievement. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Berch, D., Krikorian, R., & Huha, E. (1998). The corsi block-tapping task: Methodological and theoretical considerations. *Brain and Cognition, 38*(3), 317-338.

Berg, D. H. (2008). Working memory and arithmetic calculation in children: The contributory roles of processing speed, short-term memory, and reading. *Journal of Experimental Child Psychology, 99*(4), 288-308. doi:10.1016/j.jecp.2007.12.002

Best, J., Miller, P., & Naglieri, J. (2011). Relations between executive function and academic achievement from ages 5 to 17 in a large, representative national sample. *Learning and Individual Differences,* 327-336.

Blair, C., & Razza, R. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development, 78*(2), 647-663.

Blair, C., Ursache, A., Greenberg, M., & Vernon-Feagans, L. (2015). Multiple aspects of self-regulation uniquely predict mathematics but not letter–word knowledge in the early elementary grades. *Developmental Psychology,* 459-472.

Bull, R., Espy, K., & Wiebe, S. (2008). Short-Term Memory, Working Memory, and Executive Functioning in Preschoolers: Longitudinal Predictors of Mathematical Achievement at Age 7 Years. *Developmental Neuropsychology, 33*(3), 205-228.

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36-41.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19*(3), 273-293.

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences, 14*(12), 534-541.

Carbonaro, W. (2005). Tracking, students' effort, and academic achievement. *Sociology of Education, 78*, 27-49.

Carr, M., Taasoobshirazi, G., Stroud, R., & Royer, J. M. (2011). Combined fluency and cognitive strategies instruction improves mathematics achievement in early elementary school. *Contemporary Educational Psychology, 36*, 323–333.

Carroll, J.B. (1983). Studying individual differences in cognitive abilities: Through and beyond factor analysis. In R.F. Dillon (Ed.), *Individual differences in cognition* (Vol. 1, pp.1-33). New York: Academic Press.

Chong, S. L., & Siegel, L. S. (2008). Stability of computational deficits in math learning disability from second through fifth grades. *Developmental Neuropsychology, 33*(3), 300-317. doi:10.1080/87565640801982387

Cirino, P., Fletcher, J., Ewing-Cobbs, L., Barnes, M., & Fuchs, L. (2007). Cognitive arithmetic differences in learning difficulty groups and the role of behavioral inattention. *Learning Disabilities Research & Practice, 22*(1), 25-35.

Cirino, P. T., Fuchs, L. S., Elias, J. T., Powell, S. R., & Schumacher, R. F. (2013). Cognitive and Mathematical Profiles for Different Forms of Learning Difficulties. *Journal of learning disabilities*. doi:10.1177/0022219413494239

Cirino, P. T., Tolar, T. D., Fuchs, L. S., & Huston-Warren, E. (2016). Cognitive and numerosity predictors of mathematical skills in middle school. *Journal of Experimental Child Psychology, 145*, 95-119. doi:10.1016/j.jecp.2015.12.010

Cirino, P.T. (2014). Contextual learning scale. http://www.texasldcenter.org/projects/measures

Clark, C., Pritchard, V., & Woodward, L. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental Psychology, 46*(5), 1176-1191.

Collett, B., Ohan, J., & Myers, K. (2003). Ten-year review of rating scales. V: Scales assessing attention-deficit/hyperactivity disorder. *Journal of the American Academy of Child & Adolescent Psychiatry, 42*(9), 1015-1037.

Congdon, E., Mumford, J., Cohen, J., Galvan, A., Canli, T., & Poldrack, R. (2012). Measurement and reliability of response inhibition. *Frontiers in Psychology, 3*(37), 1-10. doi:10.3389/fpsyg.2012.00037

Corsi, P. M. (1972). Human memory and the medial temporal region of the brain. Dissertation Abstracts International, 34, 819B.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63-68. doi:10.1016/j.tine.2013.12.001

D'Amico, A., & Passolunghi, M. C. (2009). Naming speed and effortful and automatic inhibition in children with arithmetic learning disabilities. Learning and Individual Differences, 19, 170-180. doi:10.1016/j.lindif.2009.01.001

De Corte, L. Mason, F. Depaepe, and L. Verschaffel, "Self-regulation of mathematical knowledge and skills," in Handbook of Self-Regulation of Learning and Performance, B. J. Zimmerman and D. H. Schunk, Eds., pp. 155–172, Routledge, New York, NY, USA, 2011.

Dehaene, S., Molko, N., Cohen, L., & Wilson, A. (2004). Arithmetic and the brain. *Current Opinion in Neurobiology, 14*, 218-224. doi:10.1016/j.conb.2004.03.008

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284*(5416), 970-974.

Delis, D. C., Kaplan, E., & Kramer, J. H. (2001a). *Delis-Kaplan Executive Function System (D-KEFS) Examiner's Manual* (pp. 1-218). San Antonio, TX: The Psychological Corporation.

Delis, D. C., Kaplan, E., & Kramer, J. H. (2001b). *Delis-Kaplan Executive Function System (D-KEFS) Technical Manual* (pp. 1-132). San Antonio, TX: The Psychological Corporation.

de Oliveira-Souza, R., Moll, J., Moll, F. T., & de Oliveira, D. L. (2001). Executive amnesia in a patient with pre-frontal damage due to a gunshot wound. Neurocase, 7 (5), 383–389.

de Smedt, B. D., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: Evidence from developmental dyslexia. *Neuropsychologia, 48*(14), 3973-3981. doi:10.1016/j.neuropsychologia.2010.10.018

de Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of Experimental Child Psychology,* 186-201.

Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities, 34*(5), 435-447.

Diamond, A. (2013). Executive functions. *Annual Review of Psychology,* 135-168. doi:10.1146/annurev-psych-113011-143750

Dignath, C., Buettner, G., Langfeldt, H. (2008). How can primary school students learn self-regulated learning strategies most effectively? A meta-analysis on self-regulation training programmes. Educational Psychology Review, 3, 101 – 129.

Donker, M., Kroesbergen, E., Slot, E., Viersen, S. V., & Bree, E. D. (2016). Alphanumeric and non-alphanumeric rapid automatized naming in children with reading and/or spelling difficulties and mathematical difficulties. *Learning and Individual Differences, 47*, 80-87. doi:10.1016/j.lindif.2015.12.011

Dunlosky, J., Rawson, K., Marsh, E., Nathan, M., & Willingham, D. (2013). Improving students' learning with effective learning techniques: Promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest, 14*(1), 4-58. doi:10.1177/1529100612453266

Espy, K., McDiarmid, M., Cwik, M., Stalets, M., Hamby, A., & Senn, T. (2004). The contribution of executive functions to emergent mathematic skills in preschool children. *Developmental Neuropsychology, 26*(1), 465-486.

Fadlelmula, F., Cakiroglu, E., & Sungur, S. (2013). Developing a structural model on the relationship among motivational beliefs, self-regulated learning strategies, and achievement in mathematics. *International Journal of Science and Mathematics Education*.

Fasotti, L., Eling, P., & Bremer, J. (1992). The internal representation of arithmetical word problem sentences: Frontal and posterior-injured patients compared. *Brain and Cognition, 20*(2), 245-263.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314.

Fong, C., Zientek, L., Yetkiner Ozel, Z., & Phelps, J. (2015). Between and within ethnic differences in strategic learning: A study of developmental mathematics students. *Social Psychology of Education, 18*, 55-74. doi:10.1007/s11218-014-9275-5

Friso-van den Bos, I., van der Ven, S., Kroesbergen, E., & van Luit, J. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10*, 29-44.

Fuchs, L., Compton, D., Fuchs, D., Paulsen, K., Bryant, J., & Hamlett, C. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology, 97*(3), 493-513.

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., … Fletcher, J. M. (2006). The Cognitive Correlates of Third-Grade Skill in Arithmetic, Algorithmic Computation, and Arithmetic Word Problems. *Journal of Educational Psychology*, *98*(1), 29–43. doi:10.1037/0022-0663.98.1.29

Fuchs, L., Fuchs, D., Prentice, K., Burch, M., Hamlett, C., Owen, R., & Schroeter, K. (2003). Enhancing third-grade student' mathematical problem solving with self-regulated learning strategies. *Journal of Educational Psychology, 95*(2), 306-315.

Fuchs, L., Fuchs, D., Stuebing, K., Fletcher, J., Hamlett, C., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology, 100*(1), 30-47.

Fulmer, S. M., & Frijters, J. C. (2009). A review of self-report and alternative approaches in the measurement of student motivation. *Educational Psychology Review, 21*(3), 219-246. doi:10.1007/s10648-009-9107

Garner, J. (2009). Conceptualizing the relations between executive functions and self-regulated learning. *The Journal of Psychology, 143*(4), 405-426.

Garrett, A., Mazzocco, M., & Baker, L. (2006). Development of the metacognitive skills of prediction and evaluation in children with or without math disability. *Learning Disabilities Research and Practice 21*(2), 77-88.

Gathercole, S., & Pickering, S. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Educational Psychology, 70*(2), 177-194.

Gathercole, S., Pickering, S., Knight, C., & Stegmann, Z. (2004). Working memory skills and educational attainment: Evidence from national curriculum assessments at 7 and 14 years of age. *Applied Cognitive Psychology Appl. Cognit. Psychol., 18*(1), 1-16.

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology, 104*(1), 206-223. doi:10.1037/a0025398

Geary, D. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15.

Gerst, E. H., Cirino, P.T., Fletcher, J.F., & Yoshida, H. K. (2015). Cognitive and Behavioral Rating Measures of Executive Function as Predictors of Academic Outcomes in Children. *Child Neuropsychology,* 10.1080/09297049.2015.1120860

Gioia, G., Isquith, P., Guy, S., & Kenworthy, L. (2000). Test Review: Behavior Rating Inventory of Executive Function. *Child Neuropsychology, 6*(3), 235-238.

Heilman, K., & Valenstein, E. (1993). *Clinical Neuropsychology* (3rd ed.). New York: Oxford University Press.

Hessels, M., & Schwab, S. (2015). Mathematics performance and metacognitive behaviors during task resolution and the self-concept of students with and without learning disabilities in secondary education. *Insights into Learning Disabilities, 12*(1), 57-72.

Hofmann, W., Schmeichel, B., & Baddeley, A. (2012). Executive functions and self-regulation. *Trends in Cognitive Sciences, 16*(3), 174-180.

Horn, J.L. (1985). Remodeling old models of intelligence. In B.B. Wolman (Ed.), *Handbook of intelligence: Theories, measurements, and applications* (pp.267-300). New York: John Wiley.

Hunt, A., Turner, G., Polatajko, H., Bottari, C., & Dawson, D. (2013). Executive function, self-regulation and attribution in acquired brain injury: A scoping review. *Neuropsychological Rehabilitation, 23*(6), 914-932.

Ilkowska, & Engle. (2010). Working Memory Capacity and Self-Regulation. In *Handbook of personality and self-regulation*. Chichester, U.K.: Wiley-Blackwell.

Isquith, P. K., Roth, R. M., & Gioia, G. (2013). Contribution of rating scales to the assessment of executive functions. *Applied Neuropsychology: Child, 2*(2), 125-132. doi:10.1080/21622965.2013.748389

Jaffe, L.E. (2009). Development, interpretation, and application of the W score and the relative proficiency index (Woodcock-Johnson III Assessment Service Bulletin No.11). Rolling Meadows, IL: Riverside Publishing

Jitendra, A. K., Dupuis, D. N., Rodriguez, M. C., Zaslofsky, A. F., Slater, S., Cozine-Corroy, K., & Church, C. (2013). A randomized controlled trial of the impact of schema-based instruction on mathematical outcomes for third-grade students with mathematics difficulties. *The Elementary School Journal, 114*(2), 252-276. doi:10.1086/673199

Jitendra, A. K., Harwell, M. R., Karl, S. R., Dupuis, D. N., Simonson, G. R., Slater, S. C., & Lein, A. E. (2016). Schema-based instruction: Effects of experienced and novice teacher implementers on seventh grade students' proportional problem solving. *Learning and Instruction, 44*, 53-64. doi:10.1016/j.learninstruc.2016.03.001

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development, 74*(3), 834-850. doi:10.1111/1467-8624.00571

Kaller, C., Rahm, B., Köstering, L., & Unterrainer, J. (2011). Reviewing the impact of problem structure on planning: A software tool for analyzing tower tasks. *Behavioral Brain Research, 216*, 1-8.

Kessels, R., van Zandvoort, M., Postma, A., Kappelle, L., & Haan, E. (2000). The Corsi Block- Tapping Task: Standardization and Normative Data. *Applied Neuropsychology,* 252-258. doi:10.1207/S15324826AN0704_8

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Kim, S., Nordling, J., Yoon, J., Boldt, L., & Kochanska, G. (2013). Effortful control in “hot” and “cool” tasks differentially predicts children’s behavior problems and academic performance. Journal of Abnormal Child Psychology, 41(1), 43-56. doi:10.1007/s10802- 012-9661-4

Kitsantas, A., Cheema, J., Ware, H. (2011). The role of homework support resources, time spent on homework, and self-efficacy beliefs in mathematics achievement. Journal of Advanced Academics 22(2), 312-341.

Klem, L. (1995). Structural Equation Modeling. In L.G. Grimm and P.R. Yarnold (Ed.), *Reading and understanding more multivariate statistics*. Washington, DC: American Psychological Association.

Kline, R. (2005). *Principles and practice of structural equation modeling* (2nd ed.). New York: Guilford Press.

Koenigs, M., & Tranel, D. (2007). Irrational economic decision-making after ventromedial prefrontal damage: Evidence from the ultimatum game. *Journal of Neuroscience, 27*, 951-956.

Koponen, T., Salmi, P., Eklund, K., & Aro, T. (2013). Counting and RAN: Predictors of arithmetic calculation and reading fluency. *Journal of Educational Psychology, 105*(1), 162-175. doi:10.1037/a0029285

Koponen, T., Salmi, P., Torppa, M., Eklund, K., Aro, T., Aro, M., . . . Nurmi, J. (2016). Counting and rapid naming predict the fluency of arithmetic and reading skills. *Contemporary Educational Psychology, 44-45*, 83-94. doi:10.1016/j.cedpsych.2016.02.004

Krikorian, R., Bartok, J., & Gay, N. (1994). Tower of london procedure: A standard method and developmental data. *Journal of Clinical and Experimental Neuropsychology, 16*(6), 840-850.

Lakes, K., Swanson, J., & Riggs, M. (2011). The reliability and validity of the english and spanish strengths and weaknesses of ADHD and normal behavior rating scales in a preschool sample: Continuum measures of hyperactivity and inattention. *Journal of Attention Disorders, 16*(6), 510-516.

Lappin, J., & Eriksen, C. (1966). Use of a delayed signal to stop a visual reaction-time response. *Journal of Experimental Psychology, 72*(6), 805-811.

Lee, K., Ng, E., & Ng, S. (2009). The contributions of working memory and executive functioning to problem representation and solution generation in algebraic word problems. *Journal of Educational Psychology, 101*(2), 373-387.

Lefevre, J., Berrigan, L., Vendetti, C., Kamawar, D., Bisanz, J., Skwarchuk, S., & Smith-Chant, B. (2013). The role of executive attention in the acquisition of mathematical skills for children in Grades 2 through 4. *Journal of Experimental Child Psychology, 114*, 243-261.

Lezak, M., Howieson, D., Bigler, E., & Tranel, D. (2012). *Neuropsychological Assessment* (5th ed., p. 37). New York, New York: Oxford University Press.

Liew, J. (2012). Effortful control, executive functions, and education: Bringing self-regulatory and social-emotional competencies to the table. *Child Development Perspectives, 6*(2), 105-111.

Liew, J., McTigue, E., Barrois, L., & Hughes, J. (2008). Adaptive and effortful control and academic self-efficacy beliefs on achievement: A longitudinal study of 1st through 3rd graders. *Early Childhood Research Quarterly, 23*(4), 515-526. doi:10.1016/j.ecresq.2008.07.003

Luria, A. (1973). *The working brain: An introduction to neuropsychology*. New York: Basic Books.

Mazzocco, M. M., & Grimm, K. J. (2013). Growth in rapid automatized naming from grades K to 8 in children with math or reading disabilities. *Journal of Learning Disabilities, 46*(6), 517-533. doi:10.1177/0022219413477475

McClelland, M., Cameron, C., Connor, C., Farris, C., Jewkes, A., & Morrison, F. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and math skills. *Developmental Psychology, 43*(4), 947-959.

Melby-Lervåg, M., & Hulme, C. (2013). Is working memory training effective? A meta-analytic review. *Developmental Psychology, 49*(2), 270-291. doi:10.1037/a0028228

Melby-Lervåg, M., Lyster, S. H., & Hulme, C. (2012). Phonological skills and their role in learning to read: A meta-analytic review. *Psychological Bulletin, 138*(2), 322-352. doi:10.1037/a0026744

Menon, V., Kadosh, R., & Dowker, A. (2014). Arithmetic in the child and adult brain. *Oxford Handbooks Online*. doi:10.1093/oxfordhb/9780199642342.013.041

Milner, B. (1963). Effects of different brain lesions on card sorting: The role of the frontal lobes. *Archives of Neurology, 9*(1), 90-100.

Milner, B. (1971). Interhemispheric differences in the localization of psychological processes in man. *British Medical Bulletin*, 27, 272-277.

Millisecond Software. (2003). Tower of london task. (Version 3) [Inquisit]. Available from http:// www.millisecond.com/download/library/TowerOfLondon/

Mischel, W., Ayduk, O., Berman, M., Casey, B., Gotlib, I., Jonides, J., . . . Shoda, Y. (2011). 'Willpower' over the life span: Decomposing self-regulation. *Social Cognitive and Affective Neuroscience, 6*(2), 252-256.

Miyake, A., Friedman, N., Emerson, M., Witzki, A., Howerter, A., & Wager, T. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*, 49-100.

Monsell, S. (2003). Task switching. *Trends in Cognitive Sciences, 7*(3), 134-140.

Morgan, P. L., Farkas, G., & Maczuga, S. (2014). Which instructional practices most help first-grade students with and without mathematics difficulties? *Educational Evaluation and Policy Analysis, 37*(2), 184-205. doi:10.3102/0162373714536608

Murayama, K., Pekrun, R., Lichtenfeld, S., & Hofe, R. (2013). Predicting long-term growth in students' mathematics achievement: The unique contributions of motivation and cognitive strategies. *Child Development,* 1475-1490.

Muthen, L.K., & Muthen, B.O. (2007). *Mplus user's guide: Statistical analysis with latent variables*. Los Angeles, CA: Muthen & Muthen.

Muthen, L.K., & Muthen, B.O. (2016). *Chi-Square difference testing using the satorra-bentler scaled chi-square*. Retrieved from https://www.statmodel.com/chidiff.shtml

National Mathematics Advisory Panel. (2008). *Reports of the Task Groups and Subcommittees.* Washington, DC: Author.

Neuenschwander, R., Röthlisberger, M., Cimeli, P., & Roebers, C. (2012). How do different aspects of self-regulation predict successful adaptation to school? *Journal of Experimental Child Psychology, 113*, 353-371.

Norman, G., & Streiner, D. (2003). Path Analysis and Structural Equation Modeling. In *PDQ Statistics* (Vol. 1, pp. 156-176). Shelton, CT: People's Medical Publishing House.

Parker, P., Marsh, H., Ciarrochi, J., Marshall, S., & Abduljabbar, A. (2014). Juxtaposing math self-efficacy and self-concept as predictors of long-term achievement outcomes. *Educational Psychology, 34*(1), 29-48.

Passolunghi, M., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *British Journal of Educational Psychology, 82*, 42-63.

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of experimental child psychology*, *80*(1), 44–57. doi:10.1006/jecp.2000.2626

Pauly, H., Linkersdörfer, J., Lindberg, S., Woerner, W., Hasselhorn, M., & Lonnemann, J. (2011). Domain-specific rapid automatized naming deficits in children at risk for learning disabilities. *Journal of Neurolinguistics, 24*(5), 602-610. doi:10.1016/j.jneuroling.2011.02.002

Peters, M. L. (2012). Examining the relationships among classroom climate, self-efficacy, and achievement in undergraduate mathematics: A multi-level analysis. *International Journal of Science and Mathematics Education, 11*(2), 459-480. doi:10.1007/s10763-012-9347-y

Phan, H. (2009). Exploring students' reflective thinking practice, deep processing strategies, effort, and achievement goal orientations. *Educational Psychology, 29*(3), 297-313.

Pickering, S. J., & Gathercole, S. E. (2001). Working memory test battery for children. London: Psychological Corporation Europe.

Pintrich, P. (2004). A conceptual framework for assessing motivation and self-regulated learning in college students. *Educational Psychology Review, 16*(4), 385-407.

Pintrich, P., & Zusho, A. (2002). The development of academic self-regulation: The role of cognitive and motivational factors. *Development of Achievement Motivation,* 249-284.

Purpura, D., Baroody, A., & Lonigan, C. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology, 105*(2), 453-464. doi:10.1037/a0031753

Raghubar, K., Barnes, M., & Hecht, S. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences, 20*(2), 110-122.

Ramdass, D., & Zimmerman, B. (2008). Effects of self- correction strategy training on middle school students' self-efficacy, self- evaluation, and mathematics division learning. *Journal of Advanced Academics, 20*(1), 18-41.

Rapport, M. D., Orban, S. A., Kofler, M. J., & Friedman, L. M. (2013). Do programs designed to train working memory, other executive functions, and attention benefit children with ADHD? A meta-analytic review of cognitive, academic, and behavioral outcomes. *Clinical Psychology Review, 33*(8), 1237-1252. doi:10.1016/j.cpr.2013.08.005

Redick, T. S., Shipstead, Z., Wiemers, E. A., Melby-Lervåg, M., & Hulme, C. (2015). What's working in working memory training? An educational perspective. *Educational Psychology Review, 27*(4), 617-633.

Rivera, S.M., Reiss, A.L., Eckert, M.A., & Menon, V. (2005). Developmental changes in mental arithmetic: Evidence for increased functional specialization in the left inferior parietal cortex. *Cerebral Cortex, 15*(11), 1779-1790.

Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research and Practice, 17*(2), 81-89. doi:10.1111/1540-5826.00035

Roebers, C. M., Cimeli, P., Röthlisberger, M., & Neuenschwander, R. (2012). Executive functioning, metacognition, and self-perceived competence in elementary school children: An explorative study on their interrelations and their role for school achievement. *Metacognition and Learning Metacognition Learning, 7*(3), 151-173. doi:10.1007/s11409-012-9089-9

Rosenzweig, C., Krawec, J., & Montague, M. (2011). Metacognitive strategy use of eighth-grade students with and without learning disabilities during mathematical problem solving: A think-aloud analysis. *Journal of Learning Disabilities, 44*(6), 508-520.

SAS. (2009). SAS 9.4 Software [computer software]. Available from http://www.sas.com/en_us/software/sas9.html.

Scarborough, H.S. (2009). Connecting early language and literacy to later reading (dis)abilities: Evidence, theory, and practice. In F. Fletcher-Campbell, J. Solar, & G.

Reid (Eds.), *Approaching difficulties in literacy development: Assessment, pedagogy and programmes* (pp. 23-38). London, England: Sage Publications.

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. ZDM, 42, 149–161.

Schoenfeld, A. (1987). *Cognitive science and mathematics education*. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Schraw, G., & Dennison, R.S. (1994). Assessing metacognitive awareness. *Contemporary Educational Psychology*, *19*, 460–475.

Schunk, D. H. (1990). Goal setting and self-efficacy during self-regulated learning. *Educational psychologist*, *25*(1), 71-86.

Shallice, T. (1982). Specific impairments of planning. *Philosophical Transactions of the Royal Society B: Biological Sciences, B298*, 199-209.

Sikora, D., Haley, P., Edwards, J., & Butler, R. (2002). Tower of london test performance in children with poor arithmetic skills. *Developmental Neuropsychology, 21*(3), 243-254.

Simmons, F. R., & Singleton, C. (2008). Do weak phonological representations impact on arithmetic development? A review of research into arithmetic and dyslexia. *Dyslexia, 14*(2), 77-94. doi:10.1002/dys.341

Snowling, M. J., & Hulme, C. (2011). Interventions for children's language and literacy difficulties. *International Journal of Language & Communication Disorders, 47*(1), 27-34. doi:10.1111/j.1460-6984.2011.00081.

Soreni, N., Crosbie, J., Ickowicz, A., & Schachar, R. (2009). Stop signal and conners' continuous performance tasks: Test-retest reliability of two inhibition measures in ADHD children. *Journal of Attention Disorders, 13*(2), 137-143.

St. Clair-Thompson, H., & Gathercole, S. (2006). Executive functions and achievements in school: Shifting, updating, inhibition, and working memory. *The Quarterly Journal of Experimental Psychology, 59*(4), 745-759.

Sternberg, R., & Ben-Zeev, T. (2001). *Complex cognition: The psychology of human thought*. New York: Oxford University Press.

Stroop, J. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology, 18*(6), 643-662.

Stroud, K. C. (2006). *Development of the school motivation and learning strategies inventory* (Doctoral dissertation, Texas A&M University).

Stuss, D., & Alexander, M. (2000). Executive functions and the frontal lobes: A conceptual view. *Psychological Research, 63*, 289-298.

Swanson, L., Christie, L., & Rubadeau, R. (1993). The relationship between metacognition and analogical reasoning in mentally retarded, learning disabled, average, and gifted children. *Learning Disabilities Research & Practice, 8*(2), 70-81.

Swanson, H. L. & Jerman, O. (2006). Math Disabilities: A selective meta-analysis of the literature. Review of Educational Research, 76, 249-274.

Swanson, H.L., Jerman, O., and Zheng, X. (2009). Math Disabilities and Reading Disabilities: Can They Be Separated?. *Journal Of Psychoeducational Assessment, 27* (3): 175–196.

Swanson, L., & Kim, K. (2007). Working memory, short-term memory, and naming speed as predictors of children's mathematical performance. *Intelligence, 35*, 151-168.

Tabachnick, B., & Fidell, L. (2007). *Using multivariate statistics* (5th ed.). Boston, MA: Pearson.

U.S. Department of Education. (2000). *The nation's report card: Mathematics 2000* (NCES Publication No. 2001-517). Washington, DC: Office of Educational Research and Improvement

Van der Ven, S., Kroesbergen, E., Boom, J., & Leseman, P. (2012). The development of executive functions and early mathematics: A dynamic relationship. *British Journal of Educational Psychology, 82*, 100-119.

Vandierendonck, A., Kemps, E., Fastame, M.C., & Szmalec, A. (2004). Working memory components of the corsi blocks task. *British Journal of Psychology*, *95*, 57-79.

Verbruggen, F., & Logan, G. (2008). Response inhibition in the stop-signal paradigm. *Trends in Cognitive Sciences, 12*(11), 418-424. doi:10.1016/j.tics.2008.07.005

von Aster, M., & Shalev, R. (2007). Number development and developmental dyscalculia. *Developmental Medicine & Child Neurology, 49*(11), 868-873.

Vukovic, R., Fuchs, L., Geary, D., Jordan, N., Gersten, R., & Siegler, R. (2014). Sources of individual differences in children's understanding of fractions. *Child Development,* 1461- 1476. doi:10.1111/cdev.12218

Wigfield, A., Klauda, S., & Cambria, J. (2011). Influences on the development of academic self-regulatory processes. In B.J. Zimmerman & D.H. Schunk (eds) *Handbook of Self-Regulation of Learning and Performance*. (p.33-48). New York: Routledge.

Willoughby, M., Wirth, R., & Blair, C. (2012). Executive function in early childhood: Longitudinal measurement invariance and developmental change. *Psychological Assessment, 24*(2), 418-431.

Winne, P. H., & Perry, N. E. (2000). Measuring self-regulated learning. *Handbook of Self-*

Regulation, 531-566. doi:10.1016/b978-012109890-2/50045-7

Woodcock, R., McGrew, K., & Mather, N. (2001). *Woodcock-Johnson III tests of achievement*. Itasca, IL: Riverside Publishing.

Yeniad, N., Malda, M., Mesman, J., IJzendoorn, M., & Pieper, S. (2013). Shifting ability predicts math and reading performance in children: A meta-analytical study. *Learning and Individual Differences, 23*, 1-9.

Ylvisaker, M., & Feeney, T. (2002). Executive functions, self-regulation, and learned optimism in paediatric rehabilitation: A review and implications for intervention. *Developmental Neurorehabilitation, 5*(2), 51-70.

Ylvisaker, M., & Feeney, T. J. (2008). Helping people without making them helpless: Facilitating the development of executive self-regulation for children with disabilities. In V. Anderson, R. Jacobs, & P. Anderson (Eds.), *Executive functions and the frontal lobes: A lifespan perspective*. London: Oxford University Press.

Young, D., Levy, F., Martin, N., & Hay, D. (2009). Attention deficit hyperactivity disorder: A rasch analysis of the SWAN rating scale. *Child Psychiatry and Human Development, 40*(4), 543-559.

Zimmerman, B.J. (2000). Attaining self-regulation: A social cognitive perspective. In M. Boekaerts, P.R. Pintrich, & M. Zeidner (Eds.) *Handbook of self-regulation* (13-39). New York, NY: Academic Press.

Zimmerman, B., & Martinez-Pons, M. (1988). Construct validation of a strategy model of student self-regulated learning. *Journal of Educational Psychology, 80*(3), 284-290.

Zimmerman, B., Moylan, A., Hudesman, J., White, N., & Flugman, B. (2011). Enhancing self-reflection and mathematics achievement of at-risk urban technical college students. *Psychological Test and Assessment Modeling, 53*(1), 108-127.

Zimmerman, B., & Schunk, D. (1989). Reflections on Theories of Self-Regulated Learning and Academic Achievement. In *Self-Regulated Learning and Academic Achievement Theory, Research, and Practice*. New York, NY: Springer New York.

Appendix A

Tables

Table 2

Descriptive Statistics for Each Measure (*n* = 408 - 832)

Domain	Measure	Scale Range	Skew	Kurtosis	*n*	Mean (SD) Total	Mean (SD) 3rd Grade	Mean (SD) 4th Grade	Mean (SD) 5th Grade
Math Outcomes	WJIII Calculations[1]	max = 539.6	-0.52	1.97	828	499.69 (13.88)	492.11 (12.76)	498.12 (11.12)	511.85 (14.09)
	WJIII Math Fluency[1]	max = 527.3	0.83	1.05	828	496.29 (7.35)	494.03 (6.47)	494.92 (5.92)	502.36 (8.46)
Executive Functions	WMTB-C Listening Recall[3]	0-22	-0.09	-0.02	434	11.37 (3.70)	11.33 (3.59)	10.63 (3.39)	13.35 (3.90)
	Corsi-Blocks Backward	0-47	0.10	-0.13	422	19.29 (9.45)	16.74 (8.88)	18.37 (8.74)	24.62 (9.99)
	Stop-Signal Task[2]	0-773	0.68	0.61	524	319.82 (109.24)	345.23 (117.39)	326.43 (110.91)	279.15 (83.94)
	D-KEFS Trail-Making Switch[2, 4]	0-240	0.05	-1.35	410	159.26 (58.96)	164.89 (52.47)	171.67 (56.89)	119.72 (53.66)
	D-KEFS Verbal Fluency Switch[4]	0-14	0.09	-0.43	419	5.02 (2.95)	5.44 (2.79)	4.24 (2.85)	6.72 (2.58)
	D-KEFS Design Fluency Switch[4]	0-26	0.52	-0.07	827	11.22 (4.17)	10.49 (4.06)	10.60 (3.70)	13.63 (4.55)
	D-KEFS Color-Word Interference[2,4]	0-177	1.31	1.99	408	91.55 (24.97)	99.38 (30.25)	92.15 (23.35)	81.78 (19.45)
	Tower of London	0-77	-1.17	1.61	832	51.04 (15.11)	49.96 (15.08)	49.61 (15.40)	55.97 (13.31)
Metacognition	SWAN Inattention Subscale[5]	-27-27	-0.21	-0.38	750	0.83 (12.70)	0.27 (12.40)	1.88 (12.75)	-1.58 (12.60)
	BRIEF-T Metacognition Index[6]	0-132	0.49	-0.81	533	75.47 (24.81)	74.95 (24.37)	78.27 (26.03)	71.97 (23.06)
Self-Efficacy/Effort	CLS Effort/Efficacy Subscale[7]	0-42	-0.99	0.86	826	30.86 (7.85)	30.80 (8.30)	30.58 (7.77)	31.66 (7.58)
Strategy Use	CLS Learning Strategy Subscale[7]	0-54	-0.06	-0.66	827	30.88 (11.30)	30.77 (12.14)	31.88 (11.20)	28.26 (10.25)

Note. [1]W scores used, [2]Reaction time (in milliseconds for the Stop-Signal Task, and in seconds for the D-KEFS Color-Word Interference Task and Trail-Making Switch Task), [3]WMTB-C = Working Memory Test Battery for Children, [4]DKEFS = Delis-Kaplan Executive Function System, [5]SWAN = Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder Symptoms and Normal Behavior, [6]BRIEF = Behavior Rating Inventory of Executive Function, [7]CLS = Contextual Learning Scale

Table 3

Model Fit Indices

Model	**AIC**	**BIC**	**ABIC**	χ^2	***df***	**RMSEA (CI)**	**CFI**	**TLI**	**SRMR**
1. CFA Constrained	55531.59	55910.84	55656.78	259.45	190	0.036 (0.024, 0.046)	0.95	0.95	0.09
2. CFA Unconstrained	55540.42	56033.44	55703.16	225.58	166	0.036 (0.023, 0.047)	0.96	0.95	0.08
3. Factors Constrained, Paths Constrained	23393.88	23872.67	23551.92	313.48	256	0.028 (0.015, 0.038)	0.97	0.97	0.07
4. Factors Constrained, Paths Unconstrained	23401.49	23965.61	23587.70	288.43	238	0.027 (0.013, 0.038)	0.97	0.97	0.07
Model Comparisons[a]	χ^2**diff**	***df***	***p***						
Model 1 vs. Model 2	33.79	24	> 0.05						
Model 3 vs. Model 4	24.99	18	> 0.05						

Note. [a] Model comparisons conducted using loglikelihood values and scaling correction factors obtained with the MLR estimator for maximum likelihood estimation

Table 4

Correlations of Final Measures Used in Path Models

Domain		Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Math Outcomes	1	WJIII Calculations	1.000													
	2	WJIII Math Fluency	0.649	1.000												
Executive Functions	3	WMTB-C Listening Recall	0.403	0.323	1.000											
	4	Corsi-Blocks Backward	0.376	0.320	0.413	1.000										
	5	Stop-Signal Task	-0.258	-0.312	-0.254	-0.206	1.000									
	6	D-KEFS Trail-Making Switch	-0.480	-0.461	-0.433	-0.424	0.286	1.000								
	7	D-KEFS Verbal Fluency Switch	0.295	0.258	0.364	0.055	-0.032	-0.413	1.000							
	8	D-KEFS Design Fluency Switch	0.312	0.278	0.296	0.291	-0.090	-0.377	0.247	1.000						
	9	D-KEFS Color-Word Interference	-0.306	-0.362	-0.157	-0.217	0.192	0.332	-0.110	-0.215	1.000					
	10	Tower of London	0.261	0.177	0.219	0.304	-0.064	-0.338	0.209	0.223	-0.084	1.000				
Metacognition	11	SWAN Inattention Subscale	-0.388	-0.389	-0.242	-0.238	0.195	0.268	-0.216	-0.126	0.230	-0.128	1.000			
	12	BRIEF-T Metacognition Index	-0.347	-0.377	-0.278	-0.294	0.227	0.276	-0.196	-0.127	0.175	-0.138	0.811	1.000		
Self-Efficacy/Effort	13	CLS Effort/Efficacy Subscale	0.178	0.141	0.095	-0.038	-0.068	-0.158	0.168	0.006	-0.071	0.038	-0.218	-0.285	1.000	
Strategy Use	14	CLS Learning Strategy Subscale	-0.044	-0.060	-0.154	-0.180	-0.021	0.038	-0.098	-0.179	0.027	-0.110	-0.043	-0.068	0.647	1.000

Note. Correlations with absolute values greater than 0.095 are typically $p < .05$; correlations > 0.110 are typically $p < .01$; correlations > 0.126 are typically $p < .001$.

Table 5

Constrained Unstandardized Indirect, Direct, and Total Effects Across Grade Level

Predictor		**Estimate**	***S.E.***	***p***	**95% Confidence Interval**
Executive Function	Indirect	0.125	0.024	0.000	0.078 to 0.171
	Direct	0.335	0.058	0.000	0.220 to 0.449
	Total	0.459	0.060	0.000	0.341 to 0.577
Metacognition	Indirect	0.081	0.022	0.000	0.039 to 0.123
	Direct	0.142	0.048	0.003	0.049 to 0.235
	Total	0.223	0.053	0.000	0.118 to 0.328
Self-Efficacy/Effort	Indirect	0.004	0.017	0.793	-0.028 to 0.037
	Direct	0.013	0.043	0.755	-0.070 to 0.097
	Total	0.018	0.046	0.702	-0.073 to 0.108
Strategy Use	Indirect	-0.004	0.018	0.802	-0.039 to 0.030
	Direct	0.045	0.048	0.346	-0.049 to 0.138
	Total	0.041	0.052	0.439	-0.062 to 0.143
Math Fluency	Direct	0.332	0.043	0.000	0.247 to 0.417

Table 6

Unconstrained Standardized Indirect, Direct, and Total Effects Across Grade Level

		Grade 3			
Predictor		**Estimate**	***S.E.***	***p***	**95% Confidence Interval**
Executive Functions	Indirect	0.058	0.027	0.030	0.006 to 0.111
	Direct	0.486	0.129	0.000	0.233 to 0.739
	Total	0.544	0.135	0.000	0.280 to 0.808
Metacognition	Indirect	0.074	0.035	0.038	0.004 to 0.143
	Direct	-0.044	0.124	0.724	-0.286 to 0.199
	Total	0.030	0.123	0.808	-0.211 to 0.271
Self-Efficacy/Effort	Indirect	0.016	0.028	0.571	-0.039 to 0.070
	Direct	0.014	0.111	0.896	-0.202 to 0.231
	Total	0.030	0.112	0.788	-0.190 to 0.250
Strategy Use	Indirect	0.016	0.030	0.591	-0.043 to 0.076
	Direct	0.041	0.123	0.742	-0.201 to 0.283
	Total	0.057	0.129	0.659	-0.196 to 0.311
Math Fluency	Direct	0.255	0.082	0.002	0.095 to 0.416
		Grade 4			
Predictor		**Estimate**	***S.E.***	***p***	**95% Confidence Interval**
Executive Functions	Indirect	0.164	0.044	0.000	0.078 to 0.249
	Direct	0.221	0.079	0.005	0.066 to 0.376
	Total	0.385	0.082	0.000	0.224 to 0.545
Metacognition	Indirect	0.076	0.030	0.011	0.018 to 0.135
	Direct	0.179	0.061	0.003	0.060 to 0.298
	Total	0.255	0.071	0.000	0.116 to 0.395
Self-Efficacy/Effort	Indirect	0.008	0.028	0.776	-0.048 to 0.064
	Direct	0.019	0.056	0.737	-0.091 to 0.128
	Total	0.027	0.063	0.672	-0.097 to 0.151
Strategy Use	Indirect	-0.035	0.031	0.251	-0.096 to 0.025
	Direct	0.039	0.069	0.566	-0.095 to 0.174
	Total	0.004	0.078	0.959	-0.149 to 0.157
Math Fluency	Direct	0.393	0.061	0.000	0.274 to 0.512
		Grade 5			
Predictor		**Estimate**	***S.E.***	***p***	**95% Confidence Interval**
Executive Functions	Indirect	0.100	0.032	0.002	0.037 to 0.163
	Direct	0.414	0.101	0.000	0.215 to 0.612
	Total	0.514	0.107	0.000	0.304 to 0.723
Metacognition	Indirect	0.118	0.054	0.028	0.013 to 0.224
	Direct	0.179	0.071	0.011	0.040 to 0.317
	Total	0.297	0.078	0.000	0.144 to 0.451
Self-Efficacy/Effort	Indirect	-0.008	0.025	0.754	-0.058 to 0.042
	Direct	0.042	0.078	0.596	-0.112 to 0.195
	Total	0.034	0.080	0.676	-0.124 to 0.191
Strategy Use	Indirect	0.028	0.026	0.290	-0.024 to 0.080
	Direct	0.038	0.071	0.597	-0.102 to 0.178
	Total	0.066	0.074	0.375	-0.079 to 0.211
Math Fluency	Direct	0.307	0.089	0.001	0.131 to 0.482

Appendix B

Figures

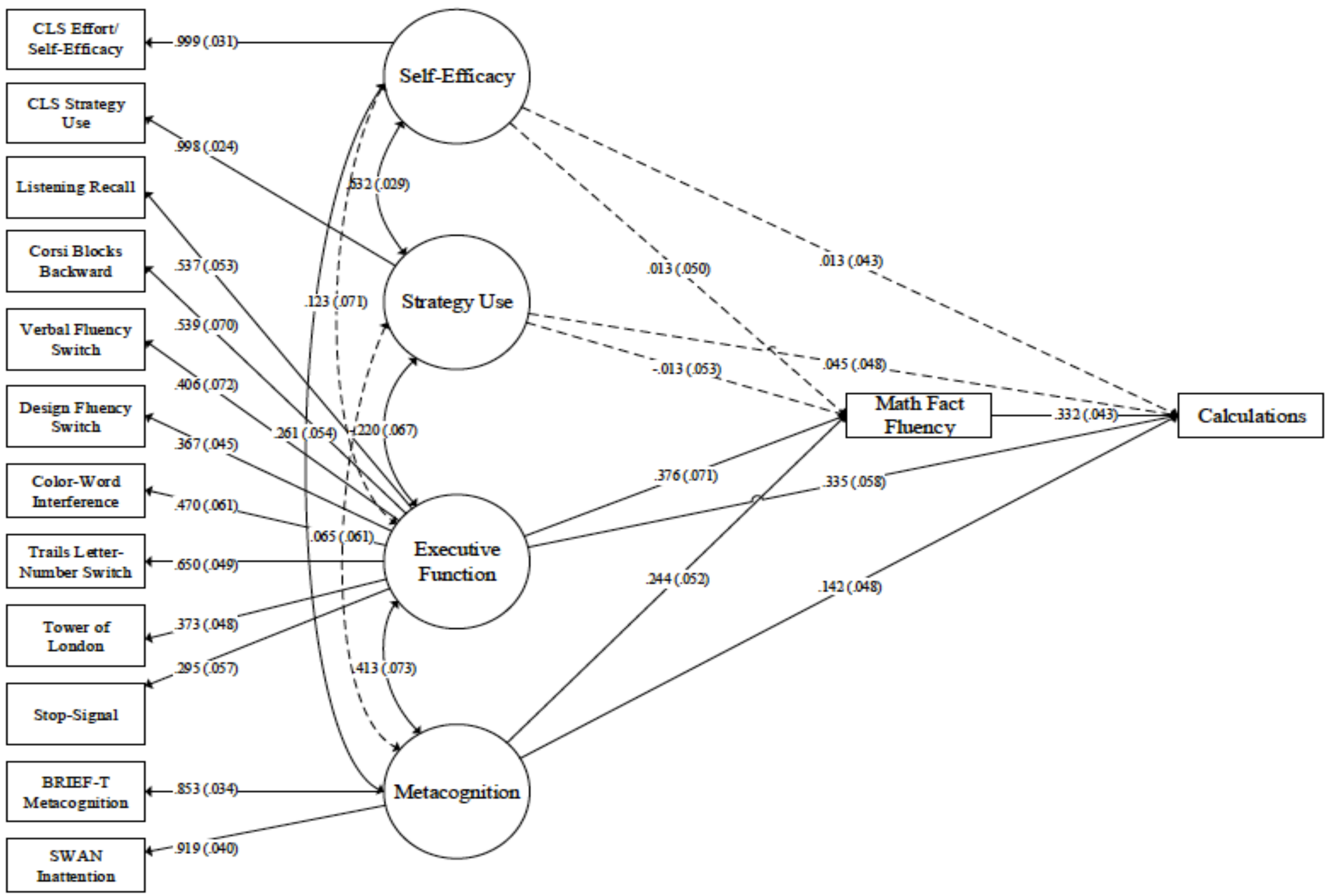

Figure 2. Constrained model with all unstandardized regression loadings and all loadings of indicator variables on latent variables constrained to be equal across grade. Correlations between latent variables are those for Grade 4, but these were similar for each grade.

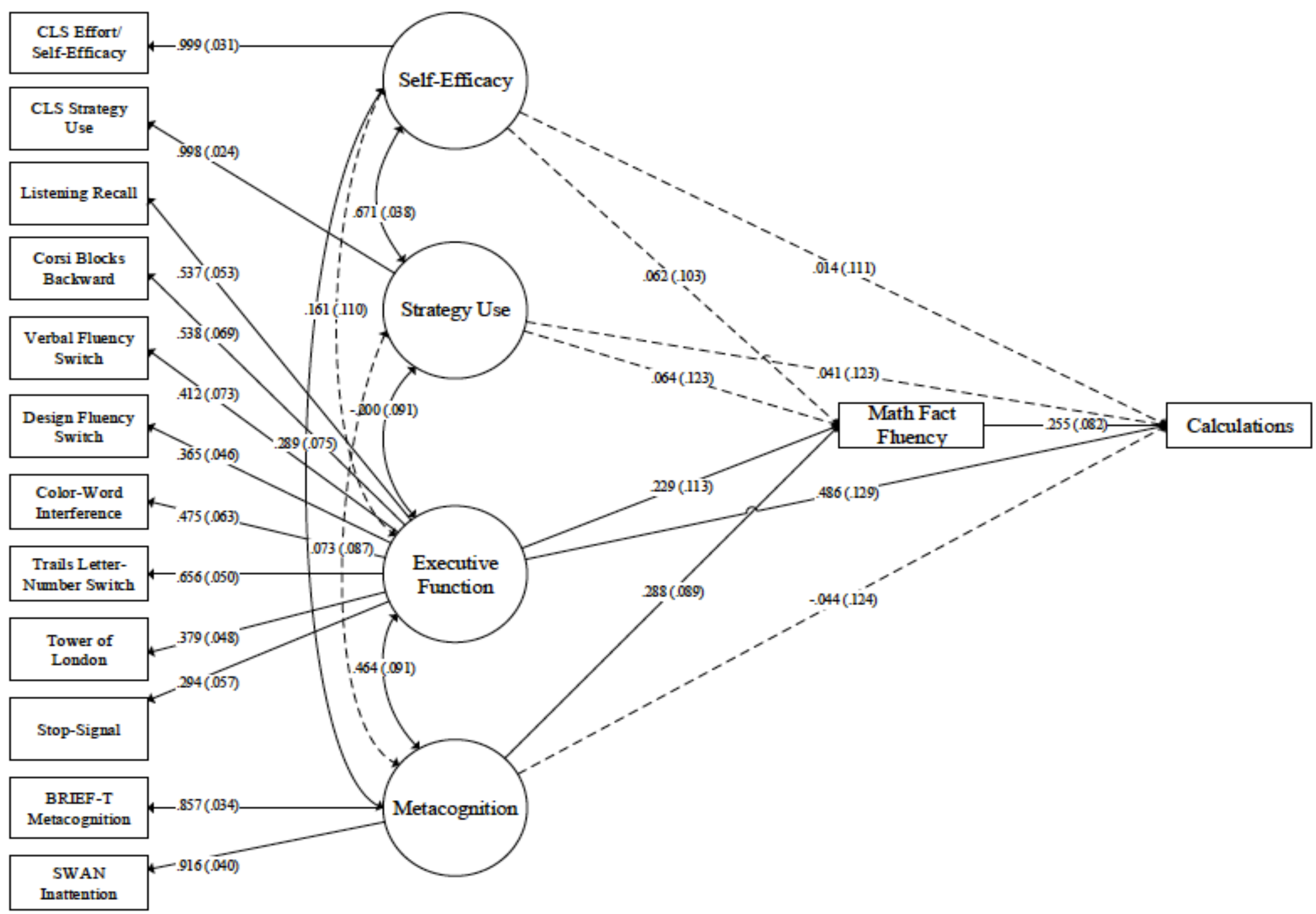

Figure 3. Unconstrained model for Grade 3. All unstandardized regression loadings were free to vary, while all loadings of indicator variables on latent variables were constrained to be equal across grade.

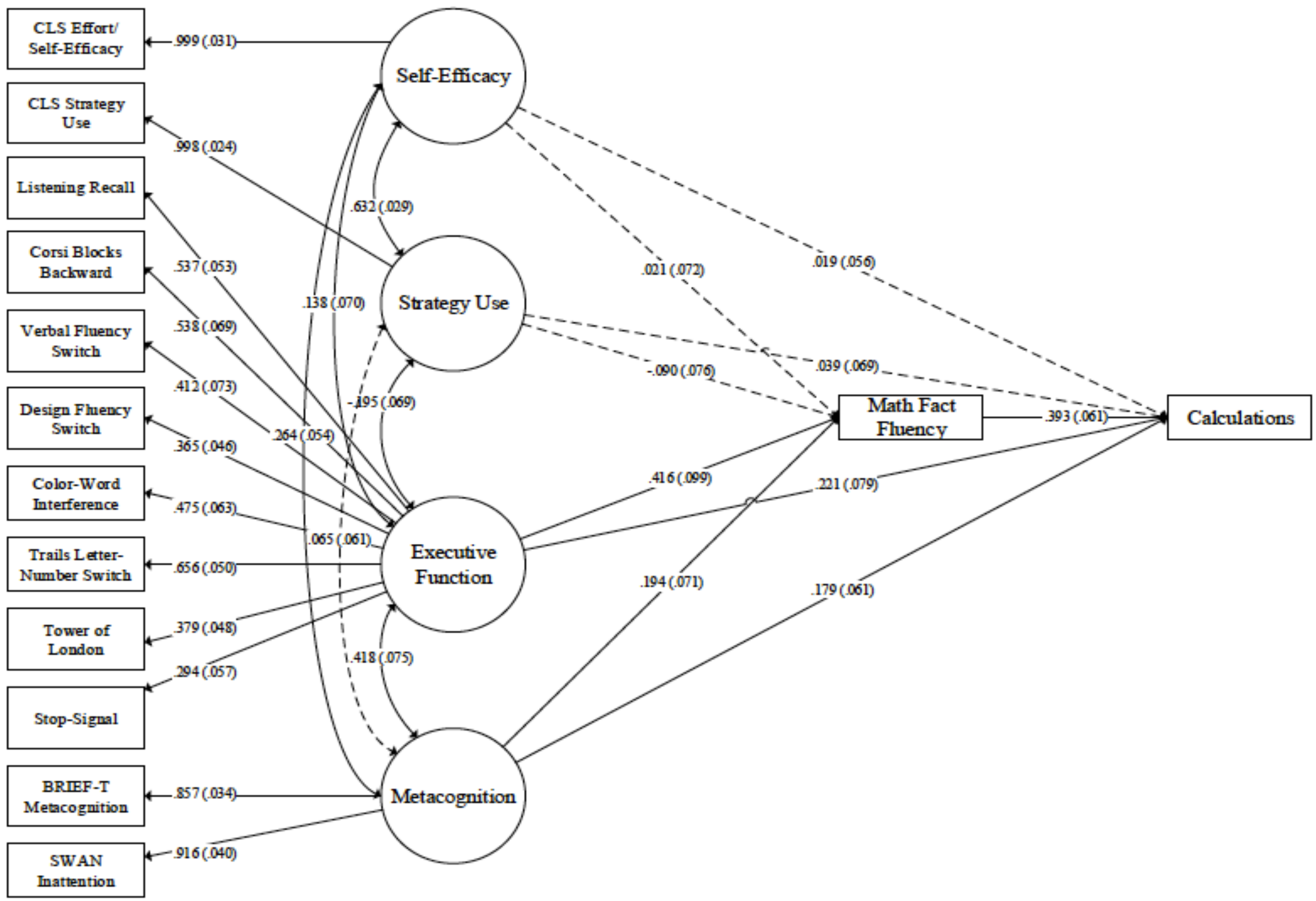

Figure 4. Unconstrained model for Grade 4. All unstandardized regression loadings were free to vary, while all loadings of indicator variables on latent variables were constrained to be equal across grade.

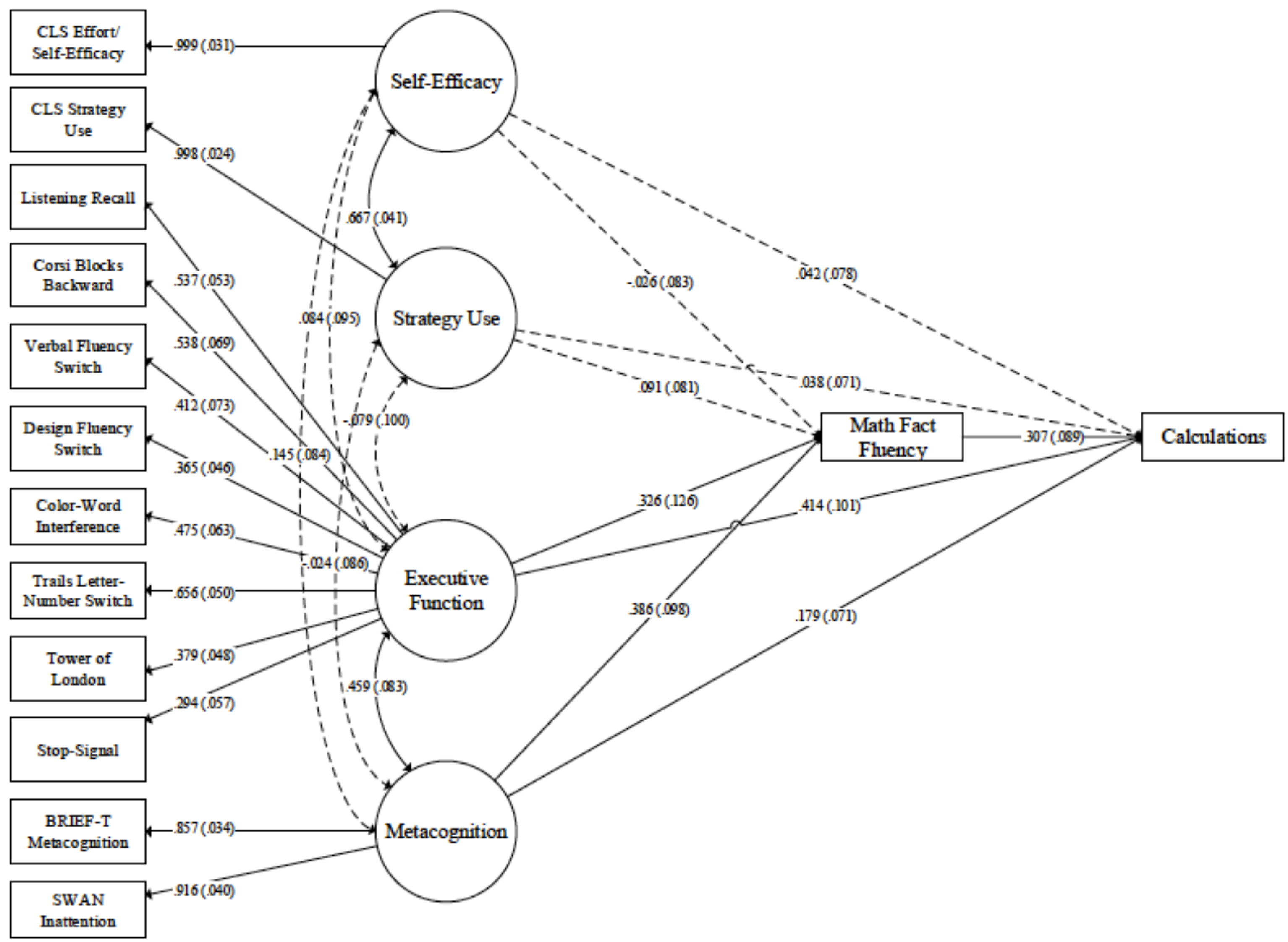

Figure 5. Unconstrained model for Grade 5. All unstandardized regression loadings were free to vary, while all loadings of indicator variables on latent variables were constrained to be equal across grade.