THE RELATIONSHIP BETWEEN PROPORTIONAL THINKING AND ACHIEVEMENT OF SELECTED SCIENCE AND MATHEMATICS CONCEPTS AT THE KNOWLEDGE, COMPREHENSION, AND APPLICATION LEVELS

A Dissertation

Presented to

The Faculty of the Graduate School

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Education

John Wynn McBride

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ABSTRACT

Introduction

Two important considerations for improving student achievement are the sequence of the curriculum and the developmental readiness of the learner. Gagne has developed an instructional strategy for improving the sequence of curriculum. Piaget has identified various stages of cognitive development and has described the logical thinking skills characteristic of each stage. The task analysis strategy of Gagne and the developmental theory of Piaget should be combined to enhance student learning. This can be accomplished if logical thinking skills necessary for learning specific concepts can be determined.

Purpose

The purpose of this study was to determine the relationship between proportional reasoning and students' ability to demonstrate knowledge, comprehension, and application of simple machines, structure of matter, and equivalent fractions. If a relationship does exist, then a student's achievement of knowledge, comprehension, and application of the selected concepts should be a function of his level of proportional reasoning.

Procedure

A sample of 136 subjects was randomly selected from a population of 444 white, middle class students who were taking ninth-grade physical

science as a required class. Equal numbers of boys and girls were selected. These students had studied the selected science concepts as part of their regular science curriculum.

Piagetian tasks were administered to assess students' proportional reasoning skills and to group them into four levels of proportional reasoning: high and low quantitative proportional reasoners, and high and low qualitative proportional reasoners. A paper/pencil test was used to assess students' achievement of knowledge and understanding (comprehension and application) of the selected concepts.

Discriminant function analyses were applied to the data to determine if a relationship existed between proportional reasoning and achievement of the selected concepts at the knowledge, comprehension, and application levels.

Findings

In general, a positive relationship was found to exist between proportional reasoning and achievement of the selected science and mathematics concepts. Quantitative proportional reasoners (formal operational) achieved significantly greater knowledge of the science concepts and significantly greater understanding of equivalent fractions concepts than qualitative proportional reasoners (concrete operational). Specifically, high quantitative proportional reasoners achieved significantly ($p \leq .01$) greater knowledge and application of simple machines, knowledge, and comprehension of structure of matter and application of equivalent fractions than high and low qualitative proportional reasoners. Low quantitative proportional reasoners achieved significantly ($p \leq .01$) greater knowledge of simple machines and structure of matter than low qualitative proportional reasoners. Low quantitative proportional reasoners achieved significantly less application ($p \leq .01$) of simple machines and comprehension ($p \leq .05$) of structure of matter than low qualitative proportional reasoners. And low quantitative proportional reasoners achieved significantly greater application of equivalent fractions than high qualitative proportional reasoners ($p \leq .05$) and low qualitative proportional reasoners ($p \leq .01$).

Conclusions

The findings of this study generally support the hypotheses investigated: achievement of selected science and mathematics concepts is related to proportional thinking, and quantitative proportional reasoners achieve significantly greater knowledge, comprehension, and application of the selected concepts than qualitative proportional reasoners.

Analysis of the means suggests a particular interaction pattern between proportional reasoning and level of achievement (Bloom's taxonomy). It suggests that when achievement is low, proportional reasoning interacts at the knowledge level. When achievement is high, proportional reasoning interacts at the application level or higher.

Recommendations

Since a general relationship was found between proportional reasoning and achievement, it is recommended that experimental studies be conducted to investigate a cause and effect relationship between proportional reasoning and achievement.

Implications

Because most high-school-age students would not be expected to possess quantitative proportional reasoning, teachers instructing students in these concepts would probably be unwise to teach for mastery at the application level. By employing a sound instructional program, they could probably teach for mastery at the knowledge and comprehension levels and minimal achievement at the application level.

The identified relationship between proportional reasoning and achievement of the selected concepts permits the task analysis strategies of Gagne and the developmental theory of Piaget to be combined and utilized to enhance student achievement of these concepts.

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CHAPTER I

INTRODUCTION AND BACKGROUND

Two areas of research related to student learning which are currently receiving considerable attention in the literature are the task analyses of Gagne and the developmental theory of Piaget. Gagne has sought to enhance student learning by improving the internal logic and organization of the curriculum. Piaget has attempted to identify stages of cognitive development, describing the logical thinking skills at the various levels. Both researchers have an important contribution to make to education; however, the realization of these contributions has not been achieved to a great extent in American education, even though considerable effort has been made to discuss their techniques and theories. If the logical thinking skills necessary for learning specific concepts can be identified along with the necessary prerequisite learning tasks for these concepts, then student learning can be greatly enhanced. The following is a discussion of the Gagnean and Piagetian positions with emphasis on their strengths and limitations for enhancing student learning.

Gagnean Task Analysis

According to Gagne (1970), "Learning is a change in human disposition or capability, which can be retained and which is not simply ascribable to the process of growth" (p. 3). It produces in the child a new capability. Gagne defines knowledge as "...that inferred capability which makes possible the successful performance of a class of tasks that could not be performed before the task was undertaken" (Gagne, 1962, p. 355). Thus, learning derives from the acquisition of knowledge.

Gagne (1970) views the learning situation as consisting of the learner, the stimulus situation, and the response or performance. The learner brings to the learning situation a set of internal capabilities obtained from previous learning and from heredity. The stimulus situation is the event or events external to the learner which cause the learning to occur. Learning is inferred from the response which ensues from the stimulus situation.

Gagne (1970) has identified eight types of learning, each deriving from a different set of conditions. Different types of learning are classified according to the conditions from which they derive. The eight types of learning Gagne describes are as follows: signal learning, stimulus-response learning, chaining, verbal association, discrimination learning, concept learning, rule learning, and problem solving.

Gagne (1970) recognizes that the term "learning," in its most comprehensive sense, includes the learning of motivations and attitudes as well as subject-matter content. However, he limits his instructional theory to the acquisition of subject-matter knowledge. His techniques pertain to methods of structuring the curriculum to produce conditions that will obtain desired types of learning. In other words, he concerns himself with "what to teach." He does not concern himself with arranging learning conditions which captivate a student's interest or which motivate a student to pursue a learning task. Thus, he is not concerned with the "management of learning" or with "modes of instruction."

For Gagne (1970), learning is cumulative and hierarchical. Learning to solve a particular problem (type 8, problem solving) requires as prerequisites knowledge of certain rules (type 7, rule learning). Rule learning requires knowledge of concepts (type 6, concept learning). Readiness to learn any task depends only upon the possession of all the prerequisite subordinate knowledge and skills. Although Gagne recognizes the neurological and physiological limitations placed upon learning by maturation, he takes the position that learning subsumes development. He states, "...differences in developmental readiness are primarily attributable to differences in the number and kind of previously learned intellectual skills" (p. 290). Because learning is viewed as cumulative, complex learning such as problem solving and rule learning is thought to be possible for even young learners. He states:

> Limitations of intellectual growth do not prevent a young learner from solving an abstract problem, or from learning new higher-order rules that are symbolically represented. Such learning may be readily accomplished if the learner has acquired, or will undertake to acquire, the intellectual skills that are prerequisite to the task. (p. 290)

Because the acquisition of a learning objective requires the learning of subordinate intellectual skills, it becomes necessary to identify these skills and to organize them hierarchically. This is accomplished by a technique called task analysis. The learning hierarchy which results from a task analysis delineates "...an entire set of capabilities having an ordered relation to each other, in the sense that in each case prerequisite capabilities are represented as subordinate in position, indicating that they need to be previously learned" (Gagne, 1970, p. 238). Gagne (1962) describes the process of task analysis as follows:

> Beginning with the final task, the question is asked, "What kind of capability would an individual have to possess if he were able to perform this task successfully, were we to give him only instructions?" The answer to this question...identifies a new class of tasks which appears to have several important characteristics. Although it is conceived as an internal "disposition," it is directly measurable as a performance. Yet it is not the <u>same</u> performance as the final task from which it was derived. It is in some sense <u>simpler</u>, and it is also <u>more general</u>. In other words, it appears that what we have defined by this procedure is an entity of "subordinate knowledge" which is essential to the performance of more specific final tasks. (p. 356)

The hierarchy obtained from a task analysis becomes the scope and sequence of the curriculum. It is a progression of separate learning events that lead the learner to the acquisition of the final task. Gagne states (1970) that if the hierarchy is valid, there will be positive transfer from the prerequisite capabilities to higher-order capabilities. Thus, Gagne's task analysis provides a technique for logically organizing and sequencing subject matter. It provides a method for determining exactly "what to teach" to bring the learner to mastery of particular learning objectives.

Gagne's instructional strategies have been widely received by educators as an effective technique for organizing curriculums. Case (1975) describes Gagne's instructional theory as one of the most powerful models of learning. Tennyson and Merrill (1971) state that Gagne's model "...enables translation of a given set of essential conditions into instructional strategy that can then be manipulated for maximally efficient and effective learning" (p. 27).

Gagne's task analysis model is based upon the hypothesis that hierarchically sequencing of subject matter can increase the effectiveness and efficiency of learning (Tennyson, 1972). Many researchers, however, have not been able to empirically verify this hypothesis. As obvious as this hypothesis seems to be, a review of literature reveals a surprisingly large number of studies that do not support the sequencing hypothesis. Roe, Case, and Roe (1962) investigated the performance differences between logically sequenced topics and randomly sequenced topics with college students studying elementary probability. Their results showed no significant difference between the two treatment groups. Garvin and Donahue (1961) conducted an experiment in which they presented a logically sequenced curriculum to one group and a randomly sequenced curriculum to another group. Students were required to work until they reached a given criterion. They found no significant difference in retention of the two groups when tested a month later. Levin and Baker (1963) investigated the effects of presenting logically and randomly sequenced mathematics curriculums to second-grade students and found no significant difference between the performances of the two groups. Hamilton (1964) investigated the effects of logically vs. randomly sequenced subject matter under two covert response conditions: specific-reading and thinking-up-responses to fill-in-blanks vs. non-specific reading only. They found the random non-specific version to be the most successful. Payne and Krathwohl (1967) investigated the effect of logically vs. randomly sequenced subject matter on achievement of university psychology students. Their results showed no significant difference in achievement among the three groups.

Several researchers have expressed views which might help to account for the lack of empirical support for the sequencing hypothesis. Niedermeyer, Brown and Sulzen (1969), after reviewing the sequencing studies in the literature, hypothesized that the sequence effects depend upon the subject matter being taught and the individual learner's ability and age. They concluded that the sequencing studies have failed to consider these factors. Case (1975) suggests that the effectiveness of Gagne's sequencing strategies is limited by the cognitive ability of the learner. He points out that Gagne's strategy does not acknowledge the developing cognitive capacities of children and suggests that it is possible for a learner advancing along a hierarchy to be advanced into subject matter beyond his present cognitive capacity to learn. Piaget (1964) has advanced the position that the cognitive development of children is ontigenetical, hierarchical, and evolves through stages. He has demonstrated that there are cognitive characteristics, representative of each stage, which place limitations on learning. Therefore, attempts to teach a child subject matter requiring cognitive structures which he does not yet possess will not succeed. The work of Neidermeyer, Brown and Sulzen, Case, and Piaget emphasizes the notion that improper consideration for cognitive development might be contributing to the lack of empirical support of the sequencing hypothesis.

Gagne's task analysis techniques for determining the scope and sequence of subject matter hold great potential for curriculum development. It is especially helpful to the classroom teacher in determining "what to teach" in order to bring students to mastery of specific learning objectives. Although this technique has been used extensively in curriculum development by textbook writers, classroom teachers, and programmed instruction developers, its effectiveness in promoting increased student achievement has been disappointing (Tennyson, 1972). Perhaps its effectiveness could be increased if the cognitive development of the learner were accounted for in developing the hierarchy.

Piagetian Developmental Theory

Piaget (1969) views intelligence as deriving from psychological adaptation to the environment. To Piaget, contact with the environment results in the formation of psychological "schemes" which are "...the structures or organization of actions as they are transformed or generalized by repetition in similar or analogous circumstances" (p. 4). For example, he speaks of the operational schemata of ordering, measuring, and counting. As a child interacts with his environment, his experiences are either "assimilated" into existing schemes or existing schemes are modified to "accommodate" the new experiences. Cognitive structures are formed and organized as a child adapts psychologically to his environment by assimilating and accommodating experiences. Piaget (1973) defines intelligence as constituting

> ...the state of equilibrium towards which tend all the successive adaptations of a sensori-motor and cognitive nature, as well as all assimilatory and accommodatory interactions between the organism and the environment. (p. 11)

According to Piaget, the development of intelligence in the child passes through distinct stages, which are invarient in sequence but which may vary as to duration and chronological age. Wadsworth (1971) describes this mode of cognitive development as being

> ...a coherent process of successive qualitative changes of cognitive structures (schemata), each structure and its concommitant changes deriving logically and inevitably from the preceding one. (p. 25)

The first stage is the sensori-motor period, which extends from birth to about 18 months and is the period of development when the child acquires sensori-motor control through trial and error. The next stage, preoperations, extends from about 18 months to about age 7 and is the period when language develops. Thought and language are egocentric, and the child solves problems perceptually rather than logically. The stage of concrete operations begins somewhere around age 7 and estends to about age 11 or 12 and is manifest by the child's ability to solve concrete problems using logical operations. The final stage, formal operations, begins somewhere around ages 11 or 12 and is characterized by the student's ability to solve problems through logical deduction of possibilities and consequences.

Piaget (1964) suggests that, if learning is to occur, the conceptual difficulty of the cirriculum must parallel the learner's level of cognitive development. He has stated,

> The child can receive valuable information via language or via education directed by an adult only if he is in a state where he can understand this information. That is, to receive the information, he must have a structure which enables him to assimilate this information. This is why you cannot teach higher mathematics to a five-year old. He does not yet have structures which enable him to understand. (p. 180)

Therefore, it is not only necessary to determine "what to teach" to bring the learner to mastery of particular learning objectives, but from the Piagetian point of view, it is necessary to determine "when to teach" a given concept or principle. Many researchers are emphasizing the importance of determining the stage of cognitive development of the learner and of providing subject matter having a level of conceptual difficulty matching the learner's logical thinking skills. Hartford and Good (1976) suggest that Piaget's model of cognition provides the best framework for assessing the cognitive demands of instructional materials so that subject matter may be presented at the proper time or stage in the student's cognitive development. Brady (1970) states, "An essential component of any successful teaching situation is an awareness by the teacher of a pupil's level of comprehension, so that teaching is meaningful" (p. 765). Rowell and Hoffmann (1975) comment, "Any attempt to adapt curricula to the (cognitive) developmental levels of the intended population must be applauded as an important step toward improving the quality of our educational system" (p. 157). Bass and Montague (1972) have urged educators to organize instruction to parallel the sequence of development of the child's ideas.

Although Piaget has described in detail the logical thinking skills characteristic of each stage of cognitive development, his work has been of little help to the classroom teacher in providing subject matter appropriate to the cognitive capacities of students. The problem stems from the fact that Piaget used very few concepts commonly taught in the public schools to illustrate the logical thinking skills characteristic of each stage of cognitive development. Consequently, the cognitive demands of most concepts taught in the public schools are conjecture. Even though Piaget has provided educators with techniques for determining the developmental stages of children, researchers have not yet developed techniques for determining the developmental stage in which specific concepts can be appropriately taught. Bridgham (1969), commenting on the need for research to determine the cognitive demands of specific concepts, states,

> It is only after particular effects of development have been demonstrated in the classroom learning of children that we will be able to use the extensive Piagetian work with confidence. (p. 119)

Synthesis of Gagnean Instructional Strategies and Piagetian Developmental Theory

Student learning could be significantly enhanced if an effective technique for matching subject matter with students' cognitive capacities could be developed and implemented. Gagne's task analysis provides techniques for determining the subject matter necessary to enable students to learn specific concepts of the curriculum, or, in other words, "what to teach." However, it does not make adequate provision for determining the logical thinking skills necessary for learning concepts, or "when to teach" prescribed concepts. Piaget's theory provides direction regarding "when to teach" various concepts; however, researchers have not yet successfully utilized this theory and identified the logical thinking skills required to learn specific concepts of the curriculum. Thus, his theory has not been adequately utilized to determine when students are cognitively ready to learn specific subject matter.

If an effective technique could be developed for determining the logical thinking skills necessary for learning specific concepts, then the instructional strategies of Gagne and the developmental theory of Piaget could be combined and utilized to enhance student learning. Until the present time, almost all studies investigating these relationships have attempted to relate general levels of cognitive development to concept attainment. These studies have been helpful because they have shown that a positive relationship does exist between cognitive development and concept attainment. However, they have been of little utility in promoting student learning of specific concepts.

The present study attempts to determine the relationships among a specific logical thinking skill, proportional reasoning, and understanding of specific science and mathematics concepts taught in the public schools. Several concepts which appear to require the same logical thinking skill for their understanding were selected for this study. They are: the science concepts associated with the particulate structure of matter, simple machines, and the mathematics concept of equivalent fractions. The logical thinking skill which appears to be necessary for understanding these concepts is what Piaget terms "quantitative proportionality." According to Piaget, this logical thinking skill does not develop until after age 11 or 12, when formal operations begin to develop. The concept of equivalent fractions was selected because its structure is almost identical to the structure of proportions. The structure of matter concepts were selected because their abstract nature requires the use of models to teach them. It will be shown later that the use of models requires an analogous-type thinking similar in structure to proportional thinking. These concepts were also selected because structure of matter is the most commonly taught concept in elementary and junior high school science books. The concepts associated with simple machines were selected because they require the use of mathematical proportions for their understanding (Inhelder & Piaget, 1958).

Before proceeding, it is necessary to define more precisely what is meant by "understanding" these concepts. For the purposes of this study, understanding is defined according to Bloom's Taxonomy of Educational Objectives (Bloom, 1956).

Bloom's Taxonomy

According to Bloom (1956), knowledge is the lowest level of educational objectives that a student can achieve, and it does not represent understanding. Comprehension, the next level of educational objectives, is defined as the first level at which understanding occurs. Comprehension is evidenced when the student "...knows what is being communicated without necessarily relating it to other material or seeing its fullest implications" (p. 204). The next level of educational objectives is application, which is evidenced when the student can "...make use of abstractions in particular and concrete situations. The abstractions may be in the form of general ideas, rules of procedure or generalized methods" (p. 205).

Bloom (1956) does not define knowledge as understanding. To him, it represents the "...recall of specific and isolated bits of information" (p. 201), which can be accomplished without an understanding of what is being recalled. For example, a student may be able to rotely compute the solution to the problem 2/3 = ?/12 by recalling and following a memorized procedure and yet have no comprehension of the equivalence relationship existing between 2/3 and 8/12. This is consistent with Piaget's description of true understanding of concepts. For example, Ginsburg (1969) states Piaget's position concerning the child's understanding of the concept of number as opposed to his ability to rotely compute as follows:

> The reason for Piaget's lack of interest in these matters (rote computation) is that simple addition and subtraction of whole numbers, as well as other manipulations of them, can be carried out entirely by rote and without understanding. The child can simply memorize the addition and subtraction tables and fail to comprehend the basic concepts underlying them. Piaget does not deny that it is necessary to memorize the facts of addition and subtraction; for purposes of computation, we all must do so. He asserts, however, that for the mature understanding of number, such rote memorization is not sufficient and must be accompanied by mastery of certain basic ideas. (p. 142)

For the purposes of this study, rote computation and recitation of fact were defined as being knowledge and not understanding, because they can be carried out rotely without understanding. Evaluation of student achievement of the science and mathematics concepts was made at the knowledge level and the understanding levels of comprehension and application. The logical thinking skill of quantitative proportionality was postulated as being necessary for understanding these concepts. This logical thinking skill is posited (by Piaget) as being necessary for understanding certain concepts associated with equivalent fractions, the particulate structure of matter, and simple machines, because of the similarity of structure between these concepts and the structure of proportions.

The following is a discussion of the structure of proportions and the logical thinking skills which underlie proportional thinking.

The structure of selected concepts associated with equivalent fractions, structure of matter, and simple machines will then be compared to the structure of proportions; and the logical skill thought to be related to the understanding of these two concepts will be discussed.

Proportions

To describe the structure of proportions, it is necessary to define ratio. A ratio is a comparison between two entities that expresses their quantitative relationship. For example, the quantitative relationship between six students and three pairs of scissors is expressed by the ratio 6/3. Two ratios are equivalent when they express the same quantitative relationship. For example, the quantitative relationship between two students and one pair of scissors, as expressed by the ratio 2/1, is the same relationship that exists between six students and three pairs of scissors or eight students and four pairs of scissors.

A proportion is defined by Inhelder and Piaget (1958) as being a statement of equality between the ratios. The structure of a proportion is given by: $X/Y = X'/Y'$. An example of a proportion would be the statement of equality between the two ratios describing the same relationship between six students and three pairs of scissors and eight students and four pairs of scissors: $6/3 = 8/4$. Both of these ratios express the quantitative relationship of 2/1.

Inhelder and Piaget (1958) describe the logical thinking skills necessary for understanding proportions as deriving from two schemata. The first is an anticipatory scheme for qualitative proportionality which can develop during the stage of concrete operations. This scheme is based upon logical multiplication, the idea that two factors acting together are equivalent to the action of two other factors acting together. For example, a student performing Piaget's balance task demonstrates that he has developed this scheme when he can visually balance two unequal weights without resorting to trial and error manipulations. Possession of this scheme enables the student to discover the inverse relationship between the size of a weight and the distance it needs to be from the fulcrum to balance a given weight placed on the opposite balance arm. The concrete operational child can develop this scheme when interacting with concrete material.

The second scheme is a numerical quantification deriving from the anticipatory scheme. This scheme enables the child to metrically quantify proportions. Once this scheme has developed, the child is able to metrically quantify the relationship between the two ratios comprising a proportion. According to Lunzer (1965), this requires the perception of three relationships: the relationship between the two terms of one ratio in the proportion, the relationship between the two terms of the other ratio in the proportion, and an "identity" relationship between the first two relationships. This identity relationship is a second-order operation characteristic only of formal operational thought. Concrete operational thinkers do not possess a scheme for quantitative proportionality. This scheme can develop during transition from concrete to formal operations and is fully developed in formal operational thinkers. A student performing the balance task demonstrates that he has developed this scheme when he can discover the quantitative relationship between weight and the distance it needs to be placed from the fulcrum to balance another weight placed in a particular position on the opposite end of the balance arm. This enables him to mathematically calculate size of weights and specific distances needed to achieve balance. The structure of equivalent fractions and the logical thinking skills necessary for understanding equivalent fractions will now be presented.

Equivalent Fractions

A fraction expresses a quantitative relationship between a part of a whole and the whole itself. For example, the quantitative relationship between a whole divided into two equal parts and one of its parts, as expressed by the fraction 1/2, is the same relationship that exists between a whole divided into six equal parts and three of its parts or a whole divided into eight equal parts and four of its parts.

Equivalent fractions are defined as fractions which name the same number (Keedy, 1970). The structure of two equivalent fractions is given by: $X/Y = X'/Y'$. For example, the fraction 1/2 is named by the equivalent fractions 3/6 and 4/8. These two equivalent fractions are expressed in the form $3/6 = 4/8$. Equivalent fractions name the same number because they express the same quantitative part/whole relationship.

The Relationship Between Proportional Thinking and Understanding Equivalent Fractions

A comparison between equivalent fractions and proportions reveals almost identical structures. If an understanding of proportions requires the concrete operational thinking skills of logical multiplication and the formal operational thinking skills of numerical quantification of qualitative proportions, then an understanding of equivalent fractions probably requires these same thinking skills. Since the minimum age posited by Piaget for the beginning of the development of formal operations is about 11 or 12, any instruction in equivalent fractions to students below this age will probably not succeed in producing understanding at the comprehension or application levels. Rote computation could be taught, and the students could become skillful in rotely computing answers to problems such as: 2/8 = ?/24; however, students would not be able to solve story problems or other tasks that required understanding at the comprehension or application levels.

The Particulate Structure of Matter

The structure of matter commonly taught in the school curriculum depicts matter as being composed of small particles called atoms and molecules. Because these particles are sub-microscopic and cannot be perceived directly by students, models are used to teach about them. These models are analogies which represent atoms and molecules (Pella and Ziegler, 1968). For a student to achieve understanding of the particulate structure of matter, he must possess the logical thinking skills necessary for understanding analogies.

Lunzer (1965) investigated the logical thinking skills necessary for students to understand verbal and numerical analogies. He constructed analogies of varying complexity and abstractness. Simple analogies such as "Leather is to shoe as wool is to ______" were assumed to require only the logical thinking skills of concrete operations. Lunzer (1965) reasoned that "...the child would simply 'read off' the relation between the first two terms and apply it to the third, so discovering (or selecting) the fourth" (p. 32). More complex and abstract analogies were thought to require the logical thinking skills of formal operations. After experimenting with students from age 9 to 17, he stated, "Analogies, whether verbal or numerical, demand a more complex process of reasoning than is available at the concrete level" (p. 39). He further stated:

> An analogy of the form, leather is to shoe as wool is to cardigan, necessarily involves three relations: a relationship between the first pair of terms, a relationship between the second pair of terms, and, finally, a third (of identity) between the first two relations. In point of fact, the logical structures of such a system exactly parallels that of a statement of proportionality. (pp. 40-41)

Pella and Ziegler (1968) investigated the effectiveness of two kinds of mechanical models in teaching elementary school children the particulate structure of matter. They described their models as analogies:

> Such models may be mechanistic, mathematical, or verbal, but they must function as analogies in either the hypothetical or the real world. (p. 138)

Students were organized into two treatment groups, modelers and nonmodelers. Modelers were students who spontaneously used "models" or analogies to explain natural phenomena observed in their day-to-day experiences. Both treatment groups viewed demonstrations illustrating the particulate structure of matter and explanations of the demonstrations using mechanical models. Students were post-tested at the recall, transfer, and invention levels of understanding by giving verbal explanations to a series of previously unobserved demonstrations. The recall, transfer, and invention levels of understanding as defined by the researchers are comparable to the knowledge, comprehension, and application levels of Bloom's taxonomy.

Pella and Ziegler's analysis of their data and conclusion that neither model was significantly superior to the other is of little significance to this paper; however, an analysis of their data concerning students' success in using models (i.e., analogies) to gain understanding of the particulate structure of matter is of considerable interest and is summarized as follows. The modelers explained 79% of all problems correctly. The nonmodelers explained 48% of all problems correctly. The percent of correct explanations by level of difficulty is shown below.

	Knowledge	Comprehension	Application
Modelers	96%	94%	63%
Nonmodelers	96%	56%	20%

It is important to note that both modelers and nonmodelers gained a high degree of knowledge of the structure of matter; however, modelers achieved significantly greater understanding of the concept at the comprehension and application levels than nonmodelers.

The Relationship Between Proportional Thinking and Understanding the Particulate Structure of Matter

From the research reviewed, it appears that the ability to understand analogies is necessary to understanding the particulate structure of matter. Since understanding analogies requires proportional thinking, it follows that understanding the particulate structure of matter should also require proportional thinking. If this is true, a review of the literature should reveal poor student understanding of the structure of matter in studies of students less than age 11 or 12, the minimum age postulated by Piaget for the beginning of proportional reasoning.

Simple Machines

Two kinds of simple machines commonly taught in physical science curriculums of junior high school textbooks are the lever and the inclined plane. One of the main concepts presented by most textbooks is that an object can be moved by a force less than its weight if a simple machine is used. The kind of problem students are commonly given to solve when studying simple machines is to calculate the force needed to move a given weight with a particular simple machine. The formulas for solving these kinds of problems are as follows.

For the lever:

$$\frac{L1}{L2} = \frac{W2}{W1}$$

L1	distance from force to the fulcrum
L2	distance from the fulcrum to weight
W2	weight associated with L2
W1	weight associated with L1

For the inclined plane:

$$\frac{D1}{D2} = \frac{W1}{W2}$$

D1	distance weight #1 moves up the inclined plane
D2	vertical distance moved by weight #1
W1	weight of object moved up the inclined plane
W2	weight needed to move weight #1 up the inclined plane

Since these formulas require proportions to set up and solve, proportional reasoning is probably required for a student to understand the concepts associated with levers and inclined planes.

Purpose and Significance of the Study

The purpose of this study was to allow the writer to investigate the relationship between proportional reasoning and students' ability to demonstrate knowledge, comprehension and application of simple machines, structure of matter, and equivalent fractions concepts.

The major research question to be answered was:

> Are the differences among students' knowledge, comprehension, and application test scores on simple machines, structure of matter, and equivalent fractions related to their level of proportional reasoning?

If the nature of the relationship between proportional reasoning and the achievement of knowledge, comprehension, and application of the concepts can be determined, it will lay the foundation for experimental research to determine the specific proportional reasoning skills necessary for achieving knowledge, comprehension, and application of the selected concepts. When this is done, it will then be possible to combine the task analysis strategies of Gagne to develop hierarchies of concepts to teach students to bring them to specific levels of achievement of knowledge and understanding and the developmental theory of Piaget to determine when the student possesses the logical thinking skills necessary to achieve knowledge and understanding of each concept of the hierarchy.

Definition of Terms

The following definitions, taken from Piagetian theory, will be used in this study.

Logical operations. An operation is a type of an action; it can be carried out either directly, in the manipulation of an object, or internally, when it is categories or (in the case of formal logic) propositions which are manipulated. Roughly, an operation is a means for mentally transforming data about the real world so that they can be organized and used selectively in the solution of problems. An operation differs from simple action or goal-directed behavior in that it is internalized and reversible (Inhelder & Piaget, 1958, p. xiii).

Concrete operations. These are operations characteristic of the first stage of operational intelligence and develop between 7 and 11 years of age. A concrete operation implies underlying general systems or "groupings" such as classification, seriation, and number. These mental operations are termed concrete because they operate on objects and not on verbally expressed hypotheses. They represent a means for structuring immediately present reality (Lawson, 1973, p. 3).

Formal operations. Formal operations typically manifest themselves in propositional thinking and a combinatory system that considers the real as one among other hypothetical possibilities. Formal operations are characteristic of the second and final stage of operational intelligence, which "reflects" on concrete operations through the elaboration of formal "group" structures (Furth, 1975, p. 157). This type of thinking begins to develop somewhere around 11 or 12 years of age.

Scheme. A scheme is the structure or organization of actions as they are transferred or generalized by repetition in similar or analogous circumstances (Piaget, 19). For example, Piaget speaks of a "sucking scheme" which is innate and a scheme for proportions which is constructed through experience and characteristic of formal operational thinking.

Stages. Stages are defined by Furth (1975) as being,

> ...successive developmental periods of intelligence, each one characterized by a relatively stable general structure that incorporates developmentally earlier structures in a higher synthesis. The regular sequence of stage-specific activities is decisive for intellectual development rather than chronological age. (p. 163)

Concept. Bruner (1956) defines a concept as:

> ...a continuum of inferences by which a set of observed characteristics of an object or event suggests a class identity, and then additional inference about other unobserved characteristics of the object or event. (p. 244)

According to Martorella (1972), concepts classify objects, events, and people according to common characteristics. They are categories of experiences having a rule which defines the relevant category and a set of positive instances or exemplars with attributes.

Quantitative proportionality. A logical thinking skill based upon the quantification of logical multiplication, quantitative proportionality enables comprehension of mathematical proportions.

Qualitative proportionality. This logical thinking skill is based upon logical multiplication, in which the idea that "two factors acting together are equivalent to the action of two other factors acting together" is comprehended (Inhelder and Piaget, 1958, p. 177).

CHAPTER II

REVIEW OF THE LITERATURE

This chapter presents a review of the literature pertinent to this study. The literature review is divided into three categories: (1) Piagetian logical thinking skill of quantitative proportionality, (2) mathematics concepts studies, and (3) science concept studies.

Piagetian Logical Thinking Skill of Quantitative Proportionality

The purpose of this section is to present an overview of Piaget's theoretical framework concerning this logical thinking skill. The findings of the research studies reviewed in the following sections will be compared to the framework to determine if this logical thinking skill might appear to underlie understanding of the selected science and mathematics concepts.

Describing the logical thinking skill of proportionality, Inhelder and Piaget (1958) state that proportionality does not develop until around age 11. Even though 8 - 11 year old children may be able to construct fractions and numerical ratios, they are not able to discover the equality between ratios and form proportions. They are also unable to quantify logical multiplications, two factors acting together which are equivalent to the action of two other factors acting together, which are characteristic of concrete operational thinking and form proportions. Thus, the quality of concrete operational thinking involved with logical multiplications is qualitatively different from the quality of formal operational thinking involved in proportional thinking.

Attempting to relate logical proportionality and numerical proportionality, Lunzer (1965) seriously questioned the Piagetian postulate that numerical quantification of logical multiplication produces proportional thinking at a level qualitatively different from logical multiplication of concrete thinking. He stated,

> The matter may be put even more sharply by asking whether Piaget is justified in postulating the existence of two successive levels in the development of logical reasoning or, whether, when the child has achieved the level of "concrete" reasoning and is, therefore, capable of argument in terms of fixed (because operationally definable) terms, his reasoning is *ipso facto* logical, insofar as the problem is *not unduly* complex and does not demand experience that he lacks. From this stage on, further progress would be a matter of quantitative gains rather than a difference in type of reasoning. (p. 31)

To test his hypothesis, Lunzer selected verbal and numerical analogies having structures identical to numerical proportions. He selected analogies ranging from very simple to very complex. The very simple analogies were thought to require only the concrete operational thinking skill of simple classification, while more complex analogies were thought to require formal operational thinking skills. If it could be demonstrated that simple analogies could be solved by concrete operational students, then it could be established that simple proportional thinking is characteristic of concrete operations and not qualitatively different from the more complex proportional thinking. The difference would therefore be quantitative.

Lunzer (1965) tested 153 students, ages 9 through 17. Summarizing his results, he stated,

> We are forced to conclude that the elementary analogies of group A represent something more than mere concrete reasoning. Indeed, the steep rise that occurs after the age of 10 strongly suggests that these problems involve a type of reasoning, that is, formal reasoning, that is not present at the earlier level and is only elaborated about the age of 11. (p. 38)

Lovell and Butterworth (1966) investigated the logical thinking skills underlying formal operational thinking of students ages 9 through 15. Many of their test items were verbal and numerical analogies similar to Lunzer's. Their results parallelled those of Lunzer's and they concluded that, "The scheme of proportionality depends on some central intellective ability which underpins performance on all tasks involving proportion" (p. 2).

Mathematics Concepts Studies

The purpose of this section is to review studies concerning students' understanding of equivalent fractions. If quantitative proportionality is necessary for understanding equivalent fractions, then few students before about 6th grade (11 - 12 year-olds) would be expected to demonstrate understanding.

Novillis (1976) sought to validate an hypothesized hierarchy of subconcepts leading to the understanding of the fraction concept.

Following Gagne's (1970) procedure, she developed a hierarchy of 16 subconcepts thought to be requisite to development of the fraction concept. Six of these subconcepts dealt with equivalent fractions. A test was developed for each subconcept of the hierarchy, and a criterion level of 75% was established for each subtest. The six equivalent fractions subconcepts were tested at the comprehension level of understanding. Two-hundred-and-seventy-nine fourth, fifth, and sixth-grade students were tested. Of the six equivalent fraction subconcepts tested, the largest percent of students to reach criterion was only 7%. Discussing these results, she states:

> Many students can associate the fraction 1/5 with a set of 5 objects, one which is shaded, but most cannot associate the fraction 1/5 with a set of ten objects, two of which are shaded, even when the objects are arranged in an array that clearly indicated that one out of every five is shaded. (p. 143)

Since the age level of most fourth, fifth, and sixth-graders is 9, 10, and 11 respectively, the majority of the students would probably be concrete operational, and would not possess the scheme of quantitative proportionality.

Steffe and Parr (1968) interpreted equivalent fractions as being proportions similar in structure to equal ratios. After reviewing Inhelder's and Piaget's (1958) and Lunzer's (1965) research on proportions, they questioned the ability of most fourth, fifth, and sixth-grade students to solve equivalent fractions and equal ratio problems. They selected 346 fourth, fifth, and sixth-grade students who had studied equivalent fractions and ratios as part of their regular mathematics curriculum and tested their ability to solve proportions composed of either equivalent fractions or ratios. Two kinds of tests were given, symbolic and pictorial. Symbolic test items were rote computation problems such as 2/3 = 4/?, which tested students at the knowledge level. Pictorial test items were problems which required the student to perceive and solve a proportional relationship between two arrays, and tested students at the application level.

They found that about 25% of the fourth-graders could correctly compute the computational test items, and about 71% of the fifth-graders, and about 83% of the sixth-graders could correctly compute these problems. Success in solving the pictorial test ranged from about 45% of the fourth-grade students to 55% of the fifth-grade students, and 65% for the sixth-grade students. They concluded by stating:

> These results seem to be very consistent with the facts that (1) Piaget sees formal thinking emerging at about 23 years of age---or at about the sixth-grade level, (2) Lunzer observes a steep rise in scores on verbal and numerical analogies after the age of 10, and (3) that Lunzer sees formal reasoning being elaborated only at about the age of 11. (p. 23)

Coburn (1974) compared the effectiveness of teaching equivalent fractions by using a ratio model which compared equivalent part/part relationships and a region model which compared equivalent part/whole relationships. Models were used to generate equivalent fractions describing particular regions or sets. This develops understanding at the comprehension level. The subjects consisted of 254 fourth-grade students (9 year-olds) from ten self-contained classrooms. Students were taught for four weeks and posttested. Coburn found no significant difference in effectiveness between either model and found that the level of achievement of both groups was only slightly greater than 50%. He stated that the students did not utilize the generalization used to generate equivalent fractions from models during the four weeks of instruction. Instead, they tended to rely on perceptual clues or counting. He concluded by questioning the appropriateness of teaching equivalent fractions in fourth grade. Since fourth-grade students are typically nine-year-olds, the majority of them were probably concrete operational and therefore would not possess the logical thinking skills of quantitative proportionality.

Bohan (1971) investigated the effectiveness of three different instructional methods for teaching equivalent fractions to 171 fifth-grade students (ten-year-olds) during a six-week instruction period. One method utilized models and numberlines to generate equivalent fractions. A second method used a paper-folding technique, and the third method utilized multiplication of a fraction by different forms of "one" (e.g., 6/6) to generate equivalent fractions. He posttested students for understanding and administered a retention test three weeks later. He found that 72% achieved his criterion for understanding, and 53% reached criterion on the retention test. Since most fifth-graders are about ten years old, and would probably be concrete operational and would not possess the logical thinking skill of quantitative proportionality, his results are higher than would be expected.

Science Concepts Studies

The purpose of this section is to review studies illustrating the levels of children's understanding of the selected science concepts. If the logical thinking skill of quantitative proportionality is necessary for understanding of these concepts, then few students before sixth grade (ages 11 to 12) would be expected to demonstrate understanding.

Particulate Structure of Matter

Anderson (1965) investigated the extent to which children are able to form mental models to explain their observations of natural phenomena illustrating the particulate structure of matter. He showed 183 third through sixth-grade children five demonstrations illustrating natural phenomena such as the melting of ice and the decrease of total volume observed when alcohol is mixed with water. Each student was shown five demonstrations during a Piagetian-type interview, and his ability to formulate mental models to explain each phenomenon observed was assessed using problem-type questions. A mechanical model was then used to demonstrate to the student that marbles and BB's (steel air rifle shot) take up less total volume when mixed together than the sum of their individual volumes. Students were then re-shown the material used in the alcohol-water demonstration and asked, "Do the marbles and BB's give you any idea about what alcohol and water could be like so that alcohol and water take up less space when they are mixed together?" The models postulated by the students to explain the phenomena were classified as atomistic, non-atomistic, magical-animistic and no explanation.

Anderson (1965) found a significant difference between the models proposed for the alcohol and water demonstration when it was viewed initially and after observing the mechanical model. Only 13% of the students initially gave an atomistic model, whereas 30% gave atomistic models after viewing the model. From this, Anderson concluded that children can develop models to explain natural phenomena.

It is important to note that 70% of the students did not formulate an atomistic model even after having been individually taught by a teacher using a mechanical model. If the structure of matter concept does require the logical thinking skill of quantitative proportionality, as postulated, then Anderson's findings are fairly consistent with what would be expected. Since the students in his sample ranged from third-graders (8 year-olds) to sixth-graders (11 and 12 year-olds), most students would probably be concrete operational, and therefore lack the logical thinking skill of quantitative proportionality. This would account for the 70% who did not learn the particulate structure of matter concept. The 30% who were successful in acquiring the concept could probably be accounted for primarily by the 11 and 12 year-old sixth-graders, some of whom were in transition from concrete to formal operations and were developing the logical thinking skills of quantitative proportionality. Several other findings reported by Anderson are consistent with the above view point. Anderson (1965) reported that the ability of children involved in the study to formulate mental models increased with the age of the children, and the consistency of the children's explanations for the observed phenomena increased with age.

Harris (1964) investigated the ability of fourth, fifth, and sixth-grade children to understand concepts basic to the molecular or kinetic theory of heat. Seventy-four students were taught these concepts using audio-tutorial instruction. Pretests and posttests were administered individually by interviewing the student using a Piagetian-type clinical interview. Students' responses were analyzed in terms of content, accuracy of observation, understanding, and reasoning, derived in part from Bloom's and Broder's (1950) model for the analysis of oral responses to problem solving situations. The criterion for determining if a particular concept could be appropriately taught at a given grade level was established to be the 50% level of efficiency. That is, when at least 50% of the students at a given grade level correctly answered the posttest questions evaluating the concept, it was judged to be appropriate for the grade level. The average level of efficiency over all concepts was 62% for sixth-graders, 50% for fifth-graders, and 31% for fourth-graders.

Harris (1964) concluded that grade placement of most concepts of molecular-kinetic theory at the fourth-grade level was inappropriate for these students. If the logical thinking skill of quantitative proportionality is necessary for understanding the particulate structure of matter concept, then Harris' conclusion would be appropriate since fourth-grade students are typically nine and ten year-olds and few would be expected to have developed the logical thinking skill of quantitative proportionality. Because of the 62% and 50% levels of efficiency of the sixth-graders and fifth-graders respectively, Harris concluded that certain concepts of the molecular or kinetic theory of heat could appropriately be taught at these grade levels. Since fifth and sixth-graders range in age from about ten to twelve, it is conceivable that some of them might have developed the logical thinking skill of quantitative proportionality and partially account for the students who demonstrated understanding of these concepts. However, these percentages are still much higher than would have been expected.

Pella and Carey (1967) investigated levels of maturity and levels of understanding of concepts concerning the structure of matter. Their purpose was to determine the grade levels at which certain concepts could be learned at the knowledge, comprehension, and application levels as defined by Bloom's taxonomy. The criterion selected for determining grade placement of concepts was if the earned mean score (posttest) was significantly different from guessing and if more than 50% of the members earned a score of 65% or greater. The subjects selected for the study consisted of 82 second, third, fourth, and fifth-grade students randomly selected from an average ability group and a high ability group as defined by student I.Q. scores. Subjects were taught and tested over 16 concepts such as the particulate structure of matter, the structure of atoms and molecules, atomic weight, and atomic number.

Many concepts such as "molecules are composed of atoms" and "particles which make up matter are in motion" were found to be inappropriate for all grade levels when the comprehension and application levels of understanding were sought. Several concepts such as "particles which make up matter are in motion" were found to be appropriate at all grade levels if only the knowledge level of achievement was sought. These results would be expected since oldest students in the study were fifth-graders (10 and 11 year-olds) and few of them would be expected to possess quantitative proportionality. Some of the results are difficult to explain. For example, several concepts such as "A neutron does not have a charge" were found to be appropriate for high ability fifth-grade students at the comprehension and application levels of understanding. The concepts "Atoms may be composed of protons, neutrons and electrons" and "Atoms have the same number of protons and electrons" were appropriate at the comprehension level for third, fourth, and fifth grade. If quantitative proportionality is required for understanding these concepts, then success of these students, most of whom are less than age 11, is difficult to explain.

McNeil (1962) tested the ability of first-graders to form and use particular concepts related to molecular theory. In a prestudy, he analyzed first and third-graders' explanations of certain natural phenomena related to the particulate structure of matter, such as "Why do clothes dry?" He found that almost without exception, perceptual (direct sensory) approaches such as "Clothes dry because of the heat" rather than theoretical solutions predominated their explanations. In fact, he found essentially no explanations indicating an understanding of the particulate structure of matter. Few, if any, third-graders (ages 8 and 9) would be expected to possess the logical thinking skills of quantitative proportionality; therefore, McNeil's findings were exactly what would be expected.

Ward (1973) sought to determine if chronological age is a main factor in a child's ability to develop a particulate concept of matter. He used four physical demonstrations, such as gas diffusion and dissolving of substances, in a Piagetian-type clinical interview to determine the child's ability to understand the concept. He found that children under 10 years of age seemed to be unable to develop the abstract conceptualization. He found an interstage transitional "readiness" in some ten, eleven, and twelve year-old students, which he interpreted as indicating that the full particulate structure of matter concept requires the formal operational stage.

Simple Machines

A review of the literature revealed only a few articles pertaining to simple machines. Most of the articles described various techniques for teaching concepts about levers. None of these articles, however, were reports of research investigating children's learning of the concepts concerning simple machines.

Summary

The studies discussed in this review indicate that quantitative proportionality may be necessary for understanding the selected science and mathematics concepts. Students below the age of 11 or 12, the age postulated by Piaget as the beginning of quantitative proportional reasoning for most children, have in general shown poorer understanding of the selected science and mathematics concepts than the students above this age.

CHAPTER III

METHODS, PROCEDURES, AND HYPOTHESES

The purpose of this study was to allow the researcher to determine the relationship between proportional thinking and students' abilities to demonstrate knowledge, comprehension, and application of simple machines, structure of matter, and equivalent fractions. If a relationship does exist, then a student's achievement of knowledge, comprehension, and application of the selected concepts should be a function of his level of proportional reasoning. The nature of this relationship could be investigated by assessing the achievement of knowledge, comprehension, and application of the selected concepts by students differing in level of proportional reasoning who had studied these concepts.

Sample

To determine if a relationship does exist between proportional thinking and achievement of knowledge, comprehension, and application of the selected concepts, it was necessary to identify a population of students differing in proportional reasoning abilities, and who had studied the selected concepts. A population of ninth-grade physical science students was selected because students of this 14 and 15 year-old age group would be expected to range in level of proportional thinking from low concrete to high formal (Chiappetta, 1976). Ninth-grade was also selected because the concept of simple machines and structure of matter comprise a major part of the physical science curriculum in the high school selected for the study.

A sample of 136 students was randomly selected from a suburban, middle-class population of 444 ninth-grade physical science students who were taking physical science as a required course. The sample was composed of predominantly white students and a few Mexican-American students. The sample was randomly selected from 13 sections of physical science being taught by 7 different teachers. All students in each section used the same two physical science textbooks during the course. These textbooks were the Laidlaw Physical Science and the Ginn Modern Physical Science. All teachers followed the same sequence of chapters in the textbooks and generally followed the same time frame. Each teacher used a lecture/discussion approach in class and assigned homework problems from the textbook. Labs were interspersed with lecture sessions, and each teacher followed the same lab guide. Each teacher prepared and administered his/her own tests for grading purposes.

Instruments

The instruments used to assess students' knowledge, comprehension, and application levels of achievement of the selected science and mathematics concepts were developed by the researcher. Each instrument contained 12 test items which measured achievement at the knowledge level, 12 items which measured achievement at the comprehension level, and 12 items which measured achievement at the application level. The three instruments were composed primarily of four alternative multiple choice questions with some matching questions.

Validity of the instruments was provided for as follows: Content validity of the mathematics instrument was established by selecting test questions from elementary and junior high school mathematics testbooks and from the instrument developed by Steffe and Parr (1968). Content validity of the science instruments was established by examining the physical science textbooks used by the students in the sample and from this information, test questions were constructed.

A major task in constructing the instruments was to prepare items which assessed achievement at each of the three levels: knowledge, comprehension, and application. This was done by consulting Bloom's (1956) Taxonomy of Educational Objectives and constructing test items having the characteristics of each level as described in the taxonomy. The test items were then rated by a panel of judges consisting of public school teachers enrolled in graduate education programs at the University of Houston. Four science teachers rated the science questions, and four mathematics teachers rated the mathematics questions. Each judge was given a written description of the characteristics of knowledge, comprehension, and application-type questions and asked to rate the level of achievement assessed by each question. Items for the instruments were selected from questions which received the same rating by all four judges or by three out of four judges. Almost all items included in the instruments received unanimous rating by the judges.

The reliability of the instruments was estimated by administering them to a sample of 36 ninth-grade physical science students in a high school having a population of students very similar to the high school students investigated in the study. Kuder-Richardson formula 20 was used to obtain reliability estimates. A reliability estimate was obtained for each instrument and for each of the three sets of 12 items measuring knowledge, comprehension, and application. The results are given in Table 1. The reliability estimates for equivalent fractions and structure of matter were .91 and .90 respectively. The reliability estimate of simple machines was .79. The estimated reliabilities for all subtests were greater than .70 except for comprehension and application of simple machines, which had estimates of .40 and .43 respectively.

Results of the simple machines test were analyzed to determine why comprehension and application test reliabilities were low. It was found that most students scored very low on both subtests, and this probably accounted for the low reliability. It was hypothesized that most students in the sample were probably qualitative proportional reasoners and not capable of achieving a great deal of comprehension and application. Therefore, the only way to obtain an accurate reliability estimate would have been to use subtest scores of the quantitative proportional reasoners in the sample. This was not possible, since the students' proportional reasoning level had not been assessed. Therefore, the researcher decided to use test scores from the quantitative proportional reasoners in the main study to estimate the reliability of the two simple machines subtests.

Table 1

Reliability Estimates for Simple Machines,

Structure of Matter and Equivalent

Fractions Tests

Subtest	Equivalent Fractions	Simple Machines	Structure of Matter
Knowledge	.76	.80	.71
Comprehension	.87	.40	.80
Application	.74	.43	.71
Total Tests	.90	.80	.90

When the simple machines test scores in the main study were analyzed, it was found that only high quantitative proportional reasoners scored above the chance level. This made it necessary to use only the test scores of high quantitative proportional reasoners to estimate reliability. This was done, and the reliability estimates for comprehension and application were found to be .95 and .86 respectively.

Two Piagetian tasks, the Balance and the Quantification of Probabilities, were administered to students to assess their levels of proportional reasoning. Protocols for these tasks were developed by the researcher from the tasks described by Inhelder and Piaget (1958) in <u>The Growth of Logical Thinking</u> and Piaget and Inhelder (1975) in

The Origin of the Idea of Chance in Children. The tasks were administered to students by the investigator and a professor from the College of Education, University of Houston. The investigator administered about 82% of the tasks. Previous to this study, the two researchers trained together in administering these tasks and conducted a research study in which the same two tasks were administered to ninth-grade students. This background experience seemed to be adequate for insuring a high level of agreement on assessing students' levels of proportional reasoning. Other similar studies involving more than one investigator have reported high interrater agreement (Sayer and Ball, 1975).

Students were classified as having demonstrated either low qualitative, high qualitative, low quantitative or high quantitative proportionality and arranged into groups according to their classifications. Because all students did not demonstrate the same level of proportional reasoning on both tasks, four different procedures were used to group students. One procedure grouped students by their classification on the balance task, and another by their classification on the quantifications of probability task. A third procedure used their lowest classification on the two tasks, and a fourth procedure used their highest classification on the two tasks.

The reason for grouping students by their highest and lowest scores is as follows. If a student possesses cognitive structure enabling him to demonstrate proportional thinking on one of the two tasks, then perhaps that structure will be sufficient to enable him to achieve comprehension and application of the selected concepts. If a student does not possess enough cognitive structure to enable him to demonstrate proportional thinking on both tasks, then perhaps he will not yet possess enough proportionality to enable him to achieve comprehension and application of the selected concepts. When students were grouped according to these four procedures, the number of students classified as low qualitative proportional reasoners for the balance and the highest score grouping procedures were 5 and 3 respectively. Because of the small number of low qualitative proportional reasoners, their group was dropped from the discriminant function analyses for the balance and highest score grouping procedures.

Gathering the Data

The science and mathematics instruments were administered to students by a tester during their regular physical science period. All testing was given on the same day. The methods and procedures for test administration were standardized by the researcher and followed in each classroom tested. All ninth-grade physical science students in the high school received the science and mathematics instruments.

Sixty-eight boys and sixty-eight girls were randomly selected from each physical science classroom and were given the two Piagetian tasks. The tasks were administered to each student individually by a previously trained tester. Students were drawn from their regular physical science class and tested privately in a room near their class. All tasks were administered during a two-week period and begun the day following the administration of the science and mathematics instruments.

Statistical Analysis and Hypotheses

The relationship between proportional thinking and knowledge, comprehension, and application of the selected concepts was analyzed by applying discriminant function analyses. This statistical technique permits the researcher to classify individuals into groups on the basis of their scores on various tests (Kerlinger, 1973). With discriminant function analysis, the dependent or criterion variable is categorical and represents group membership. The independent variables are continuous and are called the predictor variables. Discriminant function analysis enables the researcher to analyze a student's scores on the predictor variables and determine which group of students his scores are the most like (Tiedeman, 1951).

With discriminant function analysis, the discriminating variables are linearly combined to form discriminant function equations which statistically force groups apart by maximizing differences between the groups relative to the differences within the groups (Kerlinger, 1973). Discriminant functions are regression equations having the general form of: $D=K_1X_1+K_2X_2...K_nX_n$, where D represents a discriminant function score, K_n represents weighted coefficients, and X_n represents the discriminating variables which significantly discriminated between the groups. Each discriminant function analysis equation derived in an analysis will contain at least one of the discriminating variables entered into the analysis. The maximum number of discriminant functions that can be derived in an analysis is either one less than the number of groups or equal to the number of discriminating variables, whichever is smaller (Nie, Hull *et al.*, 1975).

Discriminant functions can be used to categorize students and to determine group differences. When a student's scores on each of the discriminating variables are entered into the discriminant function equations, the resulting D scores can be used to categorize him into one of the criterion groups. When a group's means on each of the discriminating variables are entered into the equations, the D scores obtained collectively define the group's centroid in discriminant function space. A comparison of the D scores of all the groups for a particular function indicates the distance between group centroids along that dimension (Nie *et al.*, 1975). The efficacy of the set of discriminating variables, from which a particular function is derived, to discriminate between groups is indicated by the magnitude of differences between the D scores. The efficacy of each discriminating variable in a discriminant function equation is indicated by the magnitude of the coefficients of the discriminating variables.

In this study, knowledge, comprehension, and application test scores were used as discriminating variables and levels or proportional reasoning demonstrated by students on the Piagetian tasks were used as the criterion for forming groups. Discriminant function analysis was performed to investigate the degree to which knowledge, comprehension and application test scores discriminated among the four groups differing in proportional thinking. If a relationship exists between students' levels of proportional thinking and their achievements of knowledge, comprehension, and application for a given concept, then their test scores should be effective in discriminating among the groups.

If groups of students differing in proportional thinking do score differently on knowledge, comprehension, and application tests, then mean vectors[1] of the groups should be significantly different. These differences can be tested for significance by applying Wilks' lambda.[2] In this study, the primary research hypotheses stated the expected differences among the mean vectors of the groups and were tested by applying Wilks' lambda. Since quantitative proportionality was not thought to be necessary for achieving knowledge of the concepts, the expected differences among group means were anticipated to be attributable to comprehension and application score differences. The following primary hypotheses were tested:

H_1: There is a significant difference between the mean vectors of knowledge, comprehension, and application test scores of simple machines concepts for the four groups of students differing in proportional reasoning.

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[1]A group's mean vector consists of the group's means on the knowledge, comprehension, and application tests.

[2]Wilks' lambda is the portion of the variance in the criterion variable not accounted for by the regression of the criterion variable on the predictor variables.

H_2: There is a significant difference between the mean vectors of knowledge, comprehension and application test scores of structure of matter concepts for the four groups of students differing in proportional reasoning.

H_3: There is a significant difference between the mean vectors of knowledge, comprehension and application test scores of equivalent fractions concepts for the four groups of students differing in proportional reasoning.

Secondary hypotheses stated the expected differences among centroids of the four groups differing in proportional thinking. The centroids of the two qualitative proportionality groups were expected to differ from the centroids of the two quantitative proportion groups. These differences were investigated by application of discriminant function analysis in which discriminant functions were generated from knowledge, comprehension, and application test scores and were used to compute group centroids. Since knowledge of the concepts was thought to be attainable by both quantitative and qualitative proportionality students, knowledge test scores were not expected to discriminate between groups. Comprehension and application test scores, however, were expected to discriminate between qualitative and quantitative proportionality students and produce significantly different group centroids. Therefore, the following secondary hypotheses were devised.

H_4: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

H_5: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the low qualitative proportionality group.

H_6: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality group are significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the low qualitative proportionality group.

H_7: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality groups are significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

H_8: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high and low quantitative proportionality groups combined is significantly different from the group centroids of knowledge, comprehension and application test scores for the high and low qualitative proportionality groups combined.

H_{9-18}: Same for structure of matter and equivalent fractions.

To test the secondary hypotheses, Mahalonobis' test for pairwise differences between group centroids was applied. This test ascertained significant differences between all possible pairs of group centroids spread by either knowledge, comprehension, or application, or any combination of these test scores.

Directional differences between significantly different centroids were investigated by plotting the group centroids in reduced discriminant space and comparing their locations. Because quantitative proportionality students were expected to consistantly score higher on comprehension and application test items than qualitative proportionality students, the group centroids of quantitative proportionality students were expected to be located above and to the right of the group centroids of the qualitative proportionality students.

CHAPTER IV

RESULTS AND DISCUSSION

This study was concerned with the relationship between proportional thinking and students' abilities to demonstrate knowledge, comprehension, and application of simple machines, structure of matter, and equivalent fractions. If a relationship does exist, then a discriminant function analysis should spread the group centroids when level of proportional thinking is used as a four-group criterion variable and knowledge, comprehension, and application test scores are used as the discriminating variables. The nature of the relationship should be evident by determining which of the three variables, individually or collectively, spread the group centroids and by observing the direction in which the centroids are spread.

Two tasks, the Balance and Quantifications of Probabilities, were used to assess students' levels of proportional reasoning. Four different procedures were used to group students according to their assessed levels of proportional reasoning.

Four discriminant function analyses were performed on the data obtained on each of the three selected concepts. Although each analysis used a different procedure for grouping students, the same set of discriminating variable scores were used in each case. Sets of discriminating variable scores consisted of the knowledge, comprehension, and application test scores for a particular concept.

Findings Relative to Simple Machines

The primary hypothesis tested in this study is stated in full form as follows:

$H_0 1$: There is no significant difference between the mean vectors of knowledge, comprehension, and application test scores of simple machines concepts for the four groups of students differing in proportional reasoning.

The means of each proportionality group on the simple machines tests are summarized in Table 2 for each of the grouping procedures. Analysis of Table 2 reveals two expected patterns. First of all, group means are generally highest for knowledge and lowest for application, with comprehension means falling in between. A puzzling aspect of this pattern needs to be noted, however, Because comprehension and application were assumed to represent levels of understanding, it was anticipated that these means would be similar to each other and considerably lower in magnitude than the knowledge means. This was not the case. Comprehension means tended to be more similar to knowledge means than to application means.

A second expected pattern was that the magnitude of the group means would generally follow the level of proportional reasoning of the students: the higher the level of proportional reasoning, the larger the group means. This pattern was observed. For example, the low qualitative proportionality group (Group 1) tended to have the lowest group means for knowledge, comprehension, and application, while

Table 2

Means of the Four Groups Differing in Proportional Reasoning on Knowledge, Comprehension and Application of Simple Machines Concepts

Proportionality Group	Knowledge	Comprehension	Application
Quantifications of Probabilities Grouping Procedure			
1 (low qualitative)	6.8182	6.6212	3.7424
2 (high qualitative)	8.3571	7.3810	3.9762
3 (low quantitative)	9.5000	8.1875	3.0000
4 (high quantitative)	9.6364	9.2727	6.6364
Balance Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	7.2603	6.7808	3.8493
3 (low quantitative)	8.1304	7.5000	3.4783
4 (high quantitative)	9.8182	8.9091	6.5455
Lowest Score on Tasks Grouping Procedure			
1 (low qualitative)	6.9265	6.7353	3.7794
2 (high qualitative)	8.5102	7.3256	3.7755
3 (low quantitative)	9.2500	8.8750	3.1250
4 (high quantitative)	9.7000	9.2000	6.8000
Highest Score on Tasks Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	7.0303	6.7576	3.9848
3 (low quantitative)	8.3704	7.5000	3.3889
4 (high quantitative)	9.7500	9.0000	6.4167

*Group omitted from analysis because of insufficient number of subjects.

the high quantitative proportionality group (Group 4) tended to have the highest group means. The high qualitative proportionality group (Group 2) and the low quantitative proportionality group (Group 3) tended to have means which fell in order between the means of the two extreme groups. One exception to this pattern occurred with the means on the application test. Without exception, the means of the low quantitative proportionality group were the lowest of all the group means in each of the four analyses.

Hypothesis $H_0 1$ was tested for each of the four grouping procedures by the application of Wilks' Lambda. The results are summarized in Table 3.

Wilks' Lambda was found to be significant ($p \leq .01$) for each of the four grouping procedures. This indicated that there were significant differences between the mean vectors of the four groups differing in proportional reasoning. Therefore, proportional reasoning was found to be related to knowledge and understanding of the selected concepts. The researcher did not accept hypothesis $H_0 1$.

If understanding of the selected concepts is related to proportional reasoning ability, then the students' knowledge, comprehension, and application test scores should discriminate between groups of students differing in proportional reasoning when discriminant function analysis is applied. The group centroids generated from knowledge, comprehension, or application, or any combination of these test scores, should be significantly different. Therefore, the following secondary hypotheses stated in null form were developed to ascertain

Table 3

Wilks' Lambda Test of Primary Hypothesis

Grouping Procedure	Wilks' Lambda	Chi-Square	df	Significance
Balance task	.8288	23.740	6	.01
Quantification task	.7440	38.736	6	.01
Highest score on tasks	.7983	29.431	6	.01
Lowest score on tasks	.7581	36.136	6	.01

Table 4

Discriminant Function Analyses Results for

the Grouping Procedures

Analysis	Grouping Procedure	Discriminant Function	Eigenvalue	Relative Percentage	Canonical R
1	Balance task	1	.1704	84.67	.382
		2	.0309	15.33	.173
2	Quantification of probabilities task	1	.1974	61.50	.406
		2	.1225	38.80	.330
3	Highest score on tasks	1	.1620	66.35	.373
		2	.0821	33.65	.275
4	Lowest score on tasks	1	.1891	63.74	.399
		2	.0893	36.17	.286

the significance of differences in student achievement of knowledge, comprehension, and application of the concepts.

$H_0 4$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

$H_0 5$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension and application test scores of simple machines for the low qualitative proportionality group.

$H_0 6$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the low qualitative proportionality group.

$H_0 7$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

$H_0 8$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high and low quantitative proportionality groups combined is not significantly different from the group centroids of knowledge, comprehension, and application test scores for the high and low qualitative proportionality groups combined.

Discriminant function analysis was used to test these secondary hypotheses. Four analyses, each using a different grouping procedure, were performed. The step-wise method was used to enter the discriminating variables into the analysis. This method allows the computer to enter the variables into the analysis on the basis of their discriminating power. Variables that do not significantly discriminate between the criterion groups are not a part of the discriminant functions produced. Two discriminant functions were generated by each of the four analyses. These results are listed in Table 4.

The eigenvalues listed in Table 4 show the relative potency of each function in discriminating between groups. Therefore, the larger the eigenvalue, the greater the discrimination between groups (Nie, 1975). The relative percentage gives the percent of the total variance existing in the discriminating variables that are accounted for by the eigenvalue with which it is associated. With the exception of the first analysis, the first function in each analysis accounted for about twice as much variance in the discriminating variable than did the second function. In the first analysis, the first function accounts for about five times as much variance.

The canonical correlation indicates the correlation between the function and the criterion defining the groups. The canonical correlation squared is an estimate of the proportion of the variance in the discriminant function which is explained by the groups (Nie et al., 1975). With the exception of the two functions in analysis number 1, the canonical correlation of the function in each analysis is about one-third larger than the canonical correlation of the second function.

Functions 1 and 2 of all four analyses were derived from the knowledge and application variables. Comprehension did not discriminate between groups strongly enough to be included in the two functions. The relative contributions of knowledge and application to each function can be seen by examining the standardized discriminant function coefficients in Table 5. The absolute value of a coefficient represents its relative contribution to the function (Nie, 1975). In analyses numbers one and three, application test scores contributed about twice as much discriminating power to the first function as did knowledge test scores. The reverse pattern is seen in function number two. In analysis number four, knowledge and application test scores contributed about equally to the function.

Coefficients of the discriminating variables can be used to create a discriminant function prediction equation for each function. Table 6 summarizes these equations for each of the analyses.

By substituting the appropriate test scores of each student into the discriminant function prediction equations, group membership can be predicted for each student. The accuracy with which the equations correctly predict group membership reflects the utility of the discriminating variables. Predicted group memberships for each of the four grouping procedures are listed in Table 7. The actual number of students in each group, as determined by the Piagetian tasks,

Table 5

Standardized Discriminant Function Coefficients

Analysis	Grouping Procedure	Discriminating Variable	Function 1	Function 2
1	Balance task	Knowledge	.4740	.7819
		Application	.8038	-.5247
2	Quantifications of probabilities task	Knowledge	.7935	-.5325
		Application	.4877	.7531
3	Highest score on tasks	Knowledge	.4809	.7709
		Application	.7940	-.5325
4	Lowest score on tasks	Knowledge	.5317	-.4778
		Application	.5431	.6377

Table 6

Discriminant Function Prediction Equations

Analysis	Grouping Procedure	Function 1	Function 2
1	Balance task	D $.4740X_K$ $.8038X_A$	D $.7819X_K$ $-.5247X_A$
2	Quantifications of probabilities task	D $.7935X_K$ $.4877X_A$	D $.5323X_K$ $.7531X_A$
3	Highest score on tasks	D $.4809X_1$ $.7940X_2$	D $.7709X_K$ $.5325X_A$
4	Lowest score on tasks	D $.5317X_1$ $.5431X_2$	D $.4778X_K$ $.6377X_A$

K - Knowledge A - Application

is listed on the left. The number of predicted students in each group is listed in the remainder of the table. Percents are listed below the predicted numbers. For example, of the 66 students classified as the low qualitative proportionality group (Group 1) by the quantifications of probabilities task, the discriminant function analysis correctly classified 32 students (48.5%) into that group. Nine students (13.6%) were incorrectly classified into Group 2, and 16 students (24.2%) were incorrectly classified into Group 3. Nine students (13.6%) were incorrectly classified into Group 4.

As can be seen in Table 7, there is some overlap among the groups in level of achievement on the discriminating variables resulting in misclassification of students by the discriminant function analysis. Consequently, the percent of students correctly classified by the discriminant function analysis routines for the four grouping procedures ranged from 42.22% to 53.03%. Had little overlap occurred in level of achievement, the percent of correctly classified students would have approached 100%.

Analysis of Table 7 reveals that high quantitative proportionality students were classified more accurately in all four analyses than were the students in the other three groups. The number of high formal students correctly classified in the four analyses ranged from 63.6% to 70.0%. This means that there was less variability among the test scores of students in the high quantitative proportional group than among the test scores of students in each of the other three groups. The second-most accurately classified group of students were

Table 7

Predicted Group Memberships of 136 Students Differing in Proportional Reasoning for the Four Grouping Procedures

Actual Group	No. of Cases	Predicted Group Membership 1	2	3	4
1 (low qualitative)	66	32 48.5%	9 23.6%	16 24.2%	9 13.6%
2 (high qualitative)	43	9 21.4%	12 26.2%	13 31.0%	9 21.4%
3 (low quantitative)	16	4 25.0%	2 12.5%	8 50.0%	2 12.5%
4 (high quantitative)	11	1 9.1%	1 9.1%	2 18.2%	7 63.6%
% correctly classified:	42.96%				
Balance					
2 (high qualitative)	73		32 43.8%	28 38.4%	13 17.8%
3 (low quantitative)	46		10 21.7%	27 58.7%	9 19.6%
4 (high quantitative)	11		1 9.0%	3 27.3%	7 63.6%
% correctly classified:	50.77%				
Lowest Scores on Tasks					
1 (low qualitative)	68	31 45.6%	16 23.5%	13 19.1%	8 11.8%
2 (high qualitative)	50	11 22.0%	16 32.0%	14 28.0%	9 18.0%
3 (low quantitative)	8	1 12.5%	3 37.5%	4 50.0%	9 18.0%
4 (high quantitative)	10	1 10.0%	1 10.0%	1 10.0%	7 70.0%
% correctly classified:	42.27%				
Highest Scores on Tasks					
2 (high qualitative)	66		30 45.5%	24 36.4%	12 18.2%
3 (low quantitative)	55		11 20.0%	33 60.0%	11 20.0%
4 (high quantitative)	12		1 8.3%	3 25.0%	8 66.7%
% correctly classified:	53.03%				

the low quantitative proportional group. The number of these students correctly classified ranged from 50.0% to 60.0%.

The discriminating power of each derived function can be determined by computing group centroids. A centroid summarizes a group's location in the space defined by its discriminant functions (Nie, 1975). Group centroids were computed by substituting appropriate means from a group's mean vector into the two discriminant function prediction equations of each analysis and deriving a value on a standard scale. Table 8 lists the group centroids for analyses 1-4. Because functions number 1 and 2 were derived from knowledge and application scores, the differences between group centroids lies in the students' knowledge and application test scores. Mahalonobis' test for pairwise differences between group centroids was applied to determine which centroids were significantly different and to ascertain if the significant differences were caused by knowledge or application test scores or both. The F-test scores obtained and their levels of significance are listed in tables 9 and 10.

An examination of tables 9 and 10 reveals that the quantifications of prabab\ilities grouping procedure yielded the largest number of group centroids significantly spread apart, while the balance grouping procedure yielded the smallest number. The range was from 4 to 8. Because the quantifications of probability grouping procedure yielded the most potent discriminant function analysis, the researcher selected this analysis to test the secondary hypothesis.

Because quantitative proportionality was thought necessary for achieving understanding (comprehension and application), but not

Table 8

Group Centroids in Discriminant Space

Analysis	Grouping Procedure	Group	Function 1	Function 2
1	Balance task	2	- .1219	- .1248
		3	- .1015	.2010
		4	1.2331	- .0126
2	Quantifications of probabilities task	1	- .3364	.1244
		2	.1480	- .0932
		3	.2692	- .6231
		4	1.0620	.5157
3	Highest score on tasks	2	- .1287	- .2237
		3	- .1040	.2796
		4	1.0620	.5157
4	Lowest score on tasks	1	- .2674	.1343
		2	.0896	- .1679
		3	.2309	- .5518
		4	1.1968	.3511

Group 1 was eliminated in analyses one and three because of insufficient numbers.

Table 9

Pair-Wise F-Tests for Knowledge Differences

Between Group Centroids

Analysis	df	Grouping Procedure	Group	1	2	3
1	2,126	Balance task	3		1.9494	
			4		10.9973**	9.5463
2	1,130	Quantification of probabilities task	2	8,9375**		
			3	13,6023**	2.2224	
			4	10,9973**	2.0949	.0128
3	2,126	Highest score on tasks	3		5.2627**	
			4		10.1007**	9.7370
4	2,130	Lowest score on tasks	2	10.1438*		
			3	5.4878*	.5345	
			4	9.5235**	1.6696	.1278

* significant .05
** significant .01

Table 10

Pair-Wise F-Tests for Application

Differences Between Group Centroids

Analysis	df	Grouping Procedure	Group	1	2	3
1	2,126	Balance task	3		.7863	
			4		.0630**	16.9011**
2	1,130	Quantification of probabilities task	2	4.4386*		
			3	8.2043**	2.5608	
			4	11.8676**	6.7017**	8.7909**
3	2,136	Highest score on tasks	3		2.1406	
			4		12.2433**	18.3505**
4	2,120	Lowest score on tasks	2	5.1148*		
			3	3.3050	.6342	
			4	11.3307**	7.9187**	5.9987*

* significant .05
** significant .01

knowledge of simple machines concepts, it was anticipated that comprehension and application test scores would significantly discriminate between the four groups and produce the group centroids. This was not the case. The group centroids produced in the discriminant function analysis were derived from knowledge and application test scores. Tables 9 and 10 reveal five significant pair-wise differences between these centroids. These differences were between the centroids of groups: 1 and 3, 1 and 4, 2 and 4, 1 and 2, and 3 and 4. Three of these differences were predicted by research hypotheses H_4, H_5, and H_6.

Research hypothesis H_4, stated in null form, predicted no significant difference between centroids of groups 2 and 4. It was stated as follows.

$H_0 4$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 6.7017 for the application scores. In this instance, the significant differences between centroids of groups 2 (high qualitative proportional) and 4 (high quantitative proportional) resulted from differences in students' application test scores on the simple machines test. Because application test scores successfully discriminated between these students, it appeared that proportional reasoning ability was related to the achievement of application of simple machines concepts by high qualitative proportionality and high quantitative proportionality students. Therefore, $H_0 4$ was not accepted.

Research hypothesis H_5, stated in null form, predicted no significant differences between centroids of groups 1 and 4. It was stated as follows:

$H_0 5$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the low qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 10.9973 for knowledge score differences and a significant F ($p \leq .01$) of 11.8676 for application score differences. In this instance, the significant difference between centroids of groups 1 (low qualitative proportional) and 4 (high quantitative proportional) resulted from differences in both knowledge and application scores on the simple machines test. Because both knowledge and application test scores successfully discriminated between these students, it appeared that proportional reasoning ability was related to the achievement of application and knowledge of simple machines concepts by low qualitative proportionality and high quantitative proportionality students. Therefore, $H_0 5$ was not accepted.

Research hypothesis H_6, stated in null form, predicted no significant differences between centroids of groups 1 (low qualitative proportional) and 3 (low quantitative proportional). It was stated as follows:

$H_0 6$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the low qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 13.6023 for knowledge score differences and a significant F ($p \leq .01$) of 8.2042 for application score differences. Because knowledge and application test scores successfully discriminated between these students, it appeared that proportional reasoning ability was related to the achievement of knowledge and application of simple machines concepts by low qualitative proportionality and low quantitative proportionality students. Therefore, $H_0 6$ was not accepted.

Research hypothesis H_7, stated in null form, predicted no significant differences in centroids of groups 2 (high qualitative proportional) and 3 (low quantitative proportional). It was stated as follows:

$H_0 7$: The group centroids of knowledge, comprehension, and application test scores of simple machines for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of simple machines for the high qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a non-significant F score of 2.2224 and 2.5608 for knowledge and application scores, respectively. This means that Group 2 and Group 3 students did not score significantly different on the simple machines test for knowledge and application. If proportional reasoning ability is related to the achievement of application of these concepts, then Group 3 students (low quantitative proportional) would be expected to achieve significantly different from Group 2 students (high qualitative proportional) on the application test. This would result in significantly different group centroids. This was not the case; therefore, the researcher did not reject $H_0 7$.

In summary, three of the four secondary research hypotheses predicting a relationship between proportional reasoning ability and achievement of knowledge, comprehension, and application of simple machines concepts have been supported by these data. Specifically, the achievement of comprehension was found to be related to proportional reasoning ability in three of the four pair-wise comparisons: groups 1 and 3, groups 2 and 4, and groups 1 and 4. The achievement of knowledge was found to be related to proportional reasoning in two of the four pair-wise comparisons: groups 1 and 4, and groups 1 and 3. Achievement of comprehension proved to be insignificantly related to proportional reasoning ability.

The nature of the relationship between proportional reasoning ability and achievement of knowledge and application of simple machines was determined by examining the directional differences between the pairs of significantly different centroids when plotted in reduced discriminant space. Group centroids are plotted by using D scores from the discriminant function analyses to define a point on a graph. These graphs are referred to as territorial maps. The D score derived from the first function of the analysis is plotted on the horizontal axis. A territorial map for the quantifications of probabilities analysis is given in Figure 1.

Examination of Figure 1 shows that the Group 4 centroid is above and to the right of the Group 1 and Group 2 centroids. This means that the Group 4 centroid has greater magnitude than the Group 1 and Group 2 centroids. To have a significantly greater centroid, Group 4 students had to score consistently higher on the simple machines test than the Group 1 and Group 2 students. Because the significant difference between the centroids of groups 1 and 4 was found to be attributable to both knowledge and application score differences, the Group 4 students had scores consistently higher on both knowledge and application test items than the Group 1 students. This indicates possession of high quantitative proportionality may facilitate achievement of both knowledge and application of simple machines. Because the significant difference between centroids of groups 4 and 2 were found to be attributable to application scores only, the Group 4 students had scores consistently higher on the application test items than the Group 1 students. This finding also indicates that high quantitative proportionality may facilitate achievement of application of simple machines concepts.

Figure 1

Territorial Map of the Quantifications

of Probabilities Task

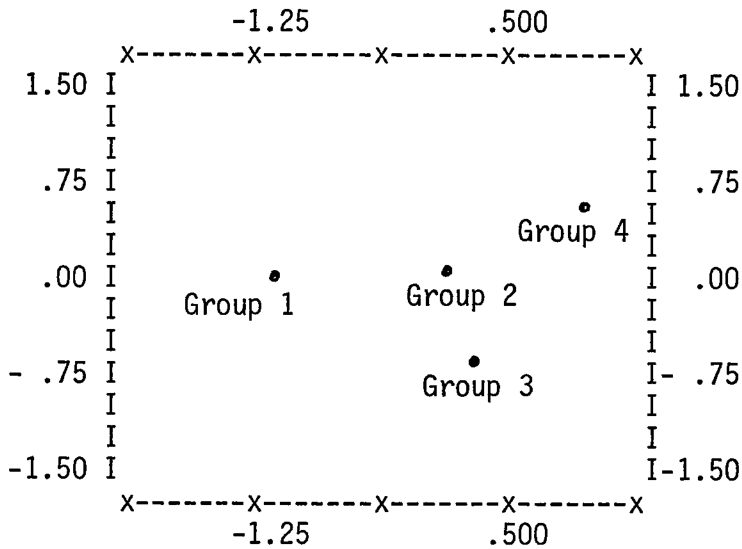


Figure 1 shows the Group 3 centroid to be below and to the right of the Group 1 centroid. Being to the right of Group 1 means that the Group 3 centroid was greater on the dimension defined by the first function than was the Group 1 centroid. Examination of the knowledge and application coefficients in the discriminant equation of the first function in Table 6 shows that the equation is primarily a knowledge dimension. Therefore, Group 3 students scored higher more consistently on the knowledge test questions than the Group 1 students. This finding indicates that low quantitative proportionality may also facilitate achievement of knowledge of simple machines concepts. Being below the Group 1 centroid means that the Group 3 centroid was not as great on the dimension defined by the second function. Table 6 shows that this equation is primarily an application dimension. Therefore, Group 3 students scored consistently lower than Group 1 students on the application test. This finding indicates that low quantitative proportionality may inhibit achievement of application of simple machines concepts. One possible explanation for this apparent inhibiting effect might be that low quantitative proportional students are in transition between equilibrium of qualitative proportionality and equilibrium of quantitative proportionality. When faced with tasks requiring proportional reasoning, they vacillate between the logic of qualitative proportional reasoning and their incomplete logic system of quantitative proportionality.

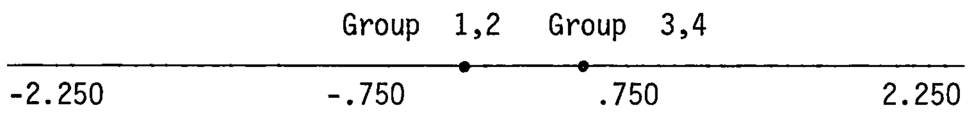
In summary, the significant difference between the centroids of Group 3 and Group 1 resulted from knowledge test scores of Group 3 students being consistently higher than the knowledge score of Group 1 students and the application test scores of Group 3 students being consistently lower than the application test scores of Group 1 students.

Research hypothesis H_8, stated in null form, predicted no significant difference in centroids between groups 3 and 4 combined and groups 1 and 2 combined. A discriminant function analysis produced one discriminant function using students' knowledge and comprehension scores. Application did not discriminate sufficiently to be included in the functions. The standardized discriminant function coefficients for knowledge and comprehension, .6837 and .4565, respectively, showed knowledge to be the more potent of the two discriminating variables. The group centroids in reduced discriminant space for groups 1 and 2 combined and for groups 3 and 4 combined were found to be -.1654 and .6612, respectively. Mahalonobis' test for pair-wise differences between group centroids produced a significant ($p \leq .01$) F score of 13.7920 with 1 and 133 degrees of freedom and a significant ($p \leq .01$) F of 8.3121 with 2 and 132 degrees of freedom for knowledge and comprehension, respectively.

These findings indicate that proportional reasoning may be related to achievement of both knowledge and comprehension of the simple machines concepts. Therefore, $H_0 8$ was not accepted.

Since only one discriminant function was produced, the resultant territorial map was unidimensional, as shown in Figure 2. The location of these centroids indicates that quantitative proportionality may facilitate achievement of knowledge and comprehension of simple machines

Figure 2



concepts. Because the significant difference between the centroids of Group 1,2 and Group 3,4 was found to be attributable to both knowledge and comprehension tests score differences, the Group 3,4 students must have scored higher more consistently on the knowledge and comprehension test questions than the Group 1,2 students. This finding indicates that quantitative proportionality may facilitate achievement of both knowledge and comprehension of simple machines concepts.

In summary, the location of the centroids of groups 1,2 and 4 plotted in reduced discriminant space indicated that quantitative proportionality may facilitate achievement of knowledge and application of simple machines concepts for qualitative proportional students and high quantitative proportional students. The location of the Group 3 centroid indicated that quantitative proportionality may facilitate achievement of knowledge, but inhibit achievement of application of simple machines concepts for low quantitative proportional students. The location of the centroids of groups 1,2 and 3,4 indicated that quantitative proportionality may facilitate achievement of knowledge and comprehension of simple machines concepts.

Findings Relative to Structure of Matter

The following primary hypothesis, stated in null form, was tested in this study:

$H_0 2$: There is no significant difference between the mean vectors of knowledge, comprehension, and application test scores of structure of matter concepts for the four groups of students differing in proportional reasoning. The means of each proportionality group on the structure of matter test are summarized in Table 11 for each of the grouping procedures.

Analysis of Table 11 reveals two expected patterns. First, group means are generally highest for knowledge and lowest for application, with comprehension means falling in between. A notable exception to this pattern, however, is the comprehension means of the low qualitative proportionality students (Group 2). Without exception, these means are slightly larger than the group's knowledge means. Because comprehension and application were assumed to represent levels of understanding, it was anticipated that these means would be similar to each other and considerably lower in magnitude than the knowledge means. This was not the case, as comprehension means tended to be more similar to knowledge means than to application means. The second expected pattern was that the magnitude of the group means would generally follow the level of proportional reasoning of the students: the higher the level of proportional reasoning, the larger the group means. This pattern is observed in Table 11.

Table 11

Means of the Four Groups Differing in Proportional Reasoning on Knowledge, Comprehension and Application of Structure of Matter Concepts

Proportionality Group	Knowledge	Comprehension	Application
Quantifications of Probabilities Grouping Procedure			
1 (low qualitative)	5.6970	6.3182	4.6212
2 (high qualitative)	7.1667	6.9048	4.5000
3 (low quantitative)	8.5000	7.1875	5.0625
4 (high quantitative)	9.9091	9.4545	6.1818
Balance Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	6.0548	6.1918	4.4795
3 (low quantitative)	7.5652	7.5000	4.8478
4 (high quantitative)	9.5455	9.1818	6.1818
Lowest Score on Tasks Grouping Procedure			
1 (low qualitative)	5.7206	6.2941	4.6029
2 (high qualitative)	7.3061	6.6980	4.6029
3 (low quantitative)	9.5000	8.2500	5.6250
4 (high quantitative)	9.9000	9.4000	6.4000
Highest Score on Tasks Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	5.8333	6.1212	4.4697
3 (low quantitative)	7.5556	7.2963	4.4963
4 (high quantitative)	9.5833	9.2500	6.0000

*Group omitted from analysis because of insufficient number of subjects.

Hypothesis $H_0 2$ was tested for each of the four grouping procedures by the application of Wilks' lambda. The results are summarized in Table 12.

Wilks' lambda was found to be significant ($p \leq .01$) for each of the four grouping procedures. This indicated that significant differences existed between the mean vectors in each of the four grouping procedures. Therefore, the researcher did not accept hypothesis $H_0 2$.

Secondary hypotheses, which predicted differences between the group centroids of groups possessing quantitative proportionality and groups possessing qualitative proportionality, were tested by application of discriminant function analysis. The following secondary hypotheses, stated in null form, were tested:

H_9: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the high qualitative proportionality group.

H_{10}: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the low qualitative proportionality group.

H_{11}: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the low quantitative proportionality group are not significantly different from the

Table 12

Wilks' Lambda Test of Primary Hypotheses

Grouping Procedure	Wilks' Lambda	Chi-Square	df	Significance
Balance task	.8548	19.843	4	.001
Quantification of probabilities task	.8068	28.123	6	.000
Highest score on tasks	.8364	22.954	4	.000
Lowest score on tasks	.7894	31.095	3	.000

Table 13

Discriminant Function Analysis Results

for Grouping Procedures

Analysis	Grouping Procedure	Discriminant Function	Eigenvalue	Relative Percentage	Canonical R
1	Balance task	1	.1698	100	.382
		2	.0001	0	.002
2	Quantification of probabilities task	1	.2080	88.97	.415
		2	.0258	11.03	.159
3	Highest score on tasks	1	.1929	98.81	.402
		2	.0023	1.19	.048
4	Lowest score on tasks	1	.2668	100	.459

group centroids of knowledge, comprehension, and application test scores of structure of matter for the low qualitative proportionality group.

H_{12}: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the low quantitative proportionality groups are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the high qualitative proportionality group.

H_{13}: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high and low quantitative proportionality groups combined are not significantly different from the group centroids of knowledge, comprehension, and application test scores for the high and low qualitative proportionality groups combined.

Two discriminant functions were generated for all analyses except number 4, in which only one function was produced. As evidenced by the eigenvalues, the relative potencies of the second functions are insignificant. Thus, the first function in each analysis accounts for almost 100% of the variance in the eigenvalues. Squaring the canonical correlations indicated that very little variance in the second discriminant functions is accounted for by the proportionality groups; however, the groups are accounting for about 16% of the variance in the first functions.

The two functions in all four analyses were derived from knowledge and comprehension scores. Application did not discriminate between groups strongly enough to be included in the two functions. The relative contribution of knowledge and comprehension to each function can be determined by examining the standardized discriminant function coefficients in Table 13.

Coefficients of the discriminating variables can be used to create a discriminant function prediction equation for each function. Table 14 summarizes these equations for each analysis. In analysis number 4, knowledge was the only variable that discriminated between groups strongly enough to be included in a function. Because the function was derived solely from one discriminant variable, only one standardized discriminant function coefficient was generated. This coefficient equals 1; therefore, the discriminant function prediction equation generated is equal to 1 times X_k , the knowledge scores of the students. In other words, a student's knowledge score predicts his group membership.

The accuracy with which these discriminant function prediction equations correctly predict group membership can be seen in Table 15. Because of overlap among groups in level of achievement on the discriminating variables, the percent of students correctly classified by the four discriminant function analyses ranged from 41.5% to 45.5%. Analysis of Table 16 reveals that high formal operational students were classified more accurately in all four analyses than were the students in the other three groups. The number of high quantitative proportional students correctly classified ranged from 70.0% to 81.8%. This means that there was less variability among the test scores of

Table 14

Standardized Discriminant Function Coefficients

Analysis	Grouping Procedure	Discriminating Variable	Function 1	Function 2
1	Balance task	Comprehension Knowledge	.5915 .5168	.5490 -.5881
2	Quantifications of probabilities task	Comprehension Knowledge	.1802 .7962	.6304 -.4577
3	Highest score on task	Comprehension Knowledge	.5162 .6042	.5690 -.5465
4	Lowest score on task	Knowledge	1.0000	.5690

Table 15

Discriminant Function Prediction Equations

Analysis	Grouping Procedure	Function 1	Function 2
1	Balance task	D $.5915X_C$ $.5168X_K$	D $.5490X_C$ $-.5881X_K$
2	Quantifications of probabilities task	D $.7962X_K$ $.1802X_C$	D $-.4577X_K$ $.6304X_C$
3	Highest score on task	D $.6042X_K$ $.5162X_C$	D $-.5465X_K$ $.5690X_C$
4	Lowest score on task	D $1.000X_K$	

K - Knowledge C - Comprehension

Table 16

Predicted Group Memberships of 136 Students Differing In Proportional Reasoning for the Four Grouping Procedures

Actual Group	No. of Cases	Predicted Group Membership 1	2	3	4
Quantification of Probabilities					
1 (low qualitative)	66	40 60.6%	4 6.1%	13 19.7%	9 13.6%
2 (high qualitative)	43	19 43.2%	5 11.9%	10 23.8%	9 21.4%
3 (low quantitative)	16	3 18.2%	1 6.3%	6 37.5%	6 37.5%
4 (high quantitative)	11	0 0%	1 9.1%	1 9.1%	9 81.8%
% correctly classified:	44.44%				
Balance					
2 (high qualitative)	73		39 53.4%	21 28.8%	13 17.8%
3 (low quantitative)	46		17 37.0%	9 19.6%	20 43.5%
4 (high quantitative)	11		1 9.1%	2 18.2%	8 72.7%
% correctly classified:	43.08%				
Lowest Scores on Tasks					
1 (low qualitative)	68	41 60.3%	11 16.2%	8 11.8%	8 11.8%
2 (high qualitative)	50	22 44.9%	6 12.2%	5 10.2%	16 32.7%
3 (low quantitative)	8	0 0%	1 12.5%	2 25.0%	5 62.7%
4 (high quantitative)	10	0 0%	2 20.0%	1 10.0%	7 70.0%
% correctly classified:					
Highest Scores on Tasks					
2 (high qualitative)	66		40 60.6%	18 27.3%	8 12.1%
3 (low quantitative)	55		23 40.8%	11 20.4%	21 38.9%
4 (high quantitative)	12		1 8.3%	2 16.7%	9 75.0%
% correctly classified:	45.45%				

students in the high quantitative proportional group than among the test scores of students in each of the other three groups. The second-most accurately classified group of students was the low qualitative proportional group. The number of these students correctly classified ranged from 60.3% to 60.6%.

The discriminating power of each derived function can be determined by computing group centroids. The group centroids, summarizing each group's location in reduced discriminant space, are summarized in Table 17.

Because functions number 1 and 2 were derived from knowledge and comprehension test scores, differences between centroids were caused by differences in these scores. F scores obtained from Mahalonobis' test for pair-wise differences between group centroids derived from knowledge and comprehension test scores are given in tables 18 and 19.

An examination of F values in tables 18 and 19 reveals that, of the four grouping procedures used to group students by their level of proportional reasoning, the quantifications of probabilities was the most effective grouping procedure for significantly spreading apart group centroids. Therefore, the quantifications of probabilities analysis was used to test the secondary hypotheses.

Because quantitative proportionality was thought to be necessary for achieving understanding (comprehension and application) but not knowledge of structure of matter concepts, it was anticipated that comprehension and application test scores would significantly

Table 17

Group Centroids in Discriminant Space

Analysis	Grouping Procedure	Group	Function 1	Function 2
1	Balance task	2	- .2915	- .0005
		3	.2402	.0015
		4	.9300	- .0027
2	Quantifications of probabilities task	1	- .3144	.0312
		2	.0857	- .0367
		3	.4318	- .1575
		4	.9624	.1823
3	Highest score on task	2	- .3432	.0125
		3	.2056	- .9264
		4	.9624	.0499
4	Lowest score on task	1	- .3415	
		2	.1467	
		3	.8223	
		4	.9455	

Table 18

Pair-Wise F-Tests for Knowledge Differences

Between Group Centroids

Analysis	df	Grouping Procedure	Group	1	2	3
1	2,126	Balance task	3		4.6466*	
			4		8.3065**	2.4599
2	1,131	Quantifications of probabilities task	2	6.1827**		
			3	11.2838**	2.2971	
			4	18.6551**	7.3112**	1.4434
3	1,129	Highest scores on tasks	3		9.5221**	
			4		15.4344**	4.3639*
4	1,131	Lowest scores on tasks	2	8.0367**		
			3	11.4772**	3.7157	
			4	17.0941**	6.2725	.0798

* significant .05

** significant .01

Table 19

Pair-Wise F-Tests for Comprehension Differences

Between Group Centroids

Analysis	df	Grouping Procedure	Group	1	2	3
1	1,127	Balance task	3		7.5492**	
			4		13.3597**	3.9250*
2	2,130	Quantifications of probabilities task	2	3.0992		
			3	5.8870	1.2919	
			4	10.6413	5.1525**	2.5670
3	2,128	Highest score on tasks	3		5.3008**	
			4		10.1174**	3.3875*

* significant .05
** significant .01

discriminate between the four groups and produce the group centroids. This was not the case. The group centroids produced in the discriminant function analysis were derived from knowledge and comprehension test scores. Tables 18 and 19 reveal four significant pair-wise differences between these centroids. These differences were between the centroids of groups 1 and 3, 1 and 4, 2 and 4, and 1 and 2. Three of these differences were predicted by research hypotheses H_9, H_{10}, and H_{11}.

Research hypothesis H_9, stated in null form, predicted no significant differences between centroids of groups 2 and 4. It was stated as follows:

$H_0 9$: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the high qualitative proportionality group. Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 7.3112 for knowledge score differences and a significant F ($p \leq .05$) of 5.1525 for the comprehension scores. In this instance, the significant difference between centroids of groups 2 (high qualitative proportional) and 4 (high quantitative proportional) resulted from differences in both knowledge and comprehension scores on the structure of matter test. Because both knowledge and comprehension test scores successfully discriminated between students demonstrating high qualitative proportionality and high quantitative proportionality, it appears that proportional reasoning ability is related to achievement of both knowledge and comprehension of structure of matter. Therefore, $H_0 9$ was not accepted.

Research hypothesis H_{10}, stated in null form, predicted no significant difference between centroids of groups 1 and 4. It was stated as follows:

$H_0 10$: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the low qualitative proportionality group. Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 18.6551 for knowledge score differences and a significant F ($p \leq .01$) of 10.6413 for comprehension scores. In this instance, the significant differences between centroids of groups 1 (low qualitative proportionality) and 4 (high quantitative proportionality) also resulted from differences in both knowledge and comprehension test scores. Because knowledge and comprehension test scores successfully discriminated between students demonstrating low qualitative and high quantitative proportionality, it appears that proportional reasoning ability is related to achievement of both knowledge and comprehension of structure of matter. Therefore, $H_0 10$ was not accepted.

Research hypothesis H_{11}, stated in null form, predicted no significant differences between centroids of groups 1 and 3.

It was stated as follows:

$H_0$11: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the low qualitative proportionality group. Mahalanobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 11.2838 for knowledge score differences and a significant F ($p \leq .05$) of 5.8870 for comprehension score differences. Here again, the significant difference between group centroids, resulting from knowledge and comprehension test scores, indicates that proportional reasoning ability is related to achievement of both knowledge and comprehension of structure of matter concepts. Because of these findings, $H_0$11 was not accepted.

Research hypothesis H_{12}, stated in null form, predicted no significant difference in centroids of groups 2 and 3. It was stated as follows:

$H_0$12: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of structure of matter for the high qualitative proportionality group. Mahalonobis' test for pair-wise differences between group centroids produced a non-significant F score of 2.2971 and

1.2919 for knowledge and comprehension scores, respectively. This means that Group 2 and Group 3 students did not score significantly different on the structure of matter test for knowledge and comprehension. If proportional reasoning ability is related to the achievement of comprehension of these concepts, then Group 3 students (low quantitative proportional) would be expected to achieve significantly different from Group 2 students (high qualitative proportional) on the comprehension test items. This would result in significantly different group centroids. This was not the case; therefore, the researcher did not reject $H_0 12$.

In summary, three of the four secondary hypotheses predicting a relationship between proportional reasoning ability and achievement of knowledge, comprehension, and application of structure of matter concepts have been supported by these data. Specifically, the achievement of comprehension was found to be related to proportional reasoning ability in three of the four pair-wise comparisons: groups 1 and 3, groups 2 and 4, and groups 1 and 4. The achievement of knowledge was found to be related to proportional reasoning in three of the four pair-wise comparisons: groups 1 and 4, groups 1 and 3, and groups 2 and 4. Achievement of application proved to be insignificantly related to proportional reasoning ability.

The nature of the relationship between proportional reasoning ability and achievement of knowledge and comprehension of structure of matter was determined by examining the directional differences between the pairs of significantly different centroids when plotted in reduced discriminant space. The resulting territorial map is given in Figure 3.

Examination of Figure 3 shows that the Group 4 centroid, being above and to the right of the other centroids, had the greatest magnitude. This means that Group 4 students scored consistently higher on the structure of matter than the students of groups 1, 2, and 3. Because the significant difference between the centroids of Group 4 and Group 1 and between the centroids of Group 4 and Group 2 was found to be attributable to both knowledge and comprehension score differences, the Group 4 students must have scored consistently higher on knowledge and comprehension test items than the Group 1 and Group 2 students. This indicates that possession of high quantitative proportionality may facilitate achievement of both knowledge and comprehension of structure of matter concepts. Although the centroid of Group 4 was larger than the centroid of Group 3, the difference was not statistically significant.

Figure 3 shows the Group 3 centroid to be below and to the right of the Group 1 and Group 2 centroids. Being to the right means that the Group 3 centroid was greater on the dimension defined by the first function than were the centroids of groups 1 and 2. Examination of the knowledge and comprehension coefficients in the discriminant equation of the first function in Table 15 shows that the equation is primarily a knowledge dimension. Because knowledge contributed to the significant difference between centroids of Group 3 and 1 and Group 3 and 2, Group 3 students must have scored

Figure 3

Territorial Map of the Quantifications

of Probabilities Task

```
        -1.00   -.500           .500    1.00
          X-------X-------X-------X-------X
   1.50 I                                 I 1.50
        I                                 I
        I                                 I
    .75 I                                 I  .75
        I                                 I
        I                       Group 4   I
        I    Group 1                      I
    .00 I                                 I  .00
        I          Group 2   Group 3      I
        I                                 I
  - .75 I                                 I- .75
        I                                 I
        I                                 I
  -1.50 I                                 I-1.50
          X-------X-------X-------X-------X
        -1.00   -.500           .500    1.00
```

higher more consistently on the knowledge test questions than Group 1 or Group 2 students. This finding indicates that low quantitative proportionality may facilitate achievement of knowledge of structure of matter concepts. Being below the Group 1 and Group 2 centroids means that the Group 3 centroid was not as great on the dimension defined by the second function. Table 15 shows that this equation is primarily a comprehension dimension. Because comprehension contributed to the significant difference between the centroids of groups 3 and 1 and groups 3 and 2, Group 3 students had to have scored lower more consistently on the comprehension test questions than Group 1 and Group 2 students. This finding indicates that low quantitative proportionality may inhibit achievement of comprehension of structure of matter concepts. This apparent inhibiting effect might be attributable to the fact that low quantitative proportional students are in transition between equilibrium of qualitative proportionality and equilibrium of quantitative proportionality. When attempting to solve problems requiring proportional reasoning, they vacillate between the logic of qualitative proportionality and their incomplete logic system of quantitative proportionality.

Research hypothesis H_{13}, stated in null form, predicted no significant difference in centroids between groups 1 and 2 combined (Group 1,2) and groups 3 and 4 combined (Group 3,4). It was stated as follows:

$H_0$13: The group centroids of knowledge, comprehension, and application test scores of structure of matter for the high and low quantitative proportionality groups combined is not significantly different from the group centroids of knowledge, comprehension, and application test scores for the high and low qualitative proportionality groups combined.

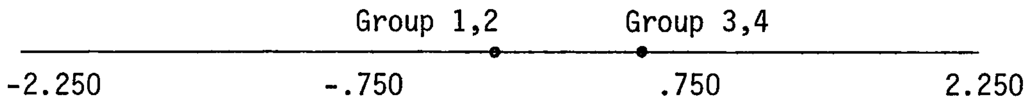
A discriminant function analysis produced only one discriminant function using the students' knowledge scores. Application and comprehension did not discriminate sufficiently to be included in the functions. The group centroids in reduced discriminant space for Group 1,2 and for Group 3,4 were found to be -.1728 and .6912, respectively. Mahalonobis' test for pair-wise differences between group centroids produced a significant ($p \leq .01$) F score of 18.1907 with 1 and 133 degrees of freedom. Because knowledge test scores successfully discriminated between qualitative proportional students (Group 1,2) and quantitative proportional students (Group 3,4), it appeared that proportional reasoning ability was related to the achievement of knowledge of structure of matter concepts. Therefore, $H_0 14$ was not accepted.

The nature of the relationship between proportional reasoning ability and achievement of knowledge of structure of matter was determined by examining directional differences between the centroids of groups 1,2 and 3,4 when plotted in reduced discriminant space. Because only one discriminant function was produced, the territorial map generated was unidimensional, as shown in Figure 4.

The location of these centroids indicates that quantitative proportionality may facilitate the acquisition of knowledge of structure

Figure 4

Territorial Map of Group Centroids



of matter concepts. Figure 4 shows that the Group 3,4 centroid, being to the right of the Group 1,2 centroid, has the greater magnitude. Therefore, Group 3,4 students must have scored consistently higher on the knowledge test than Group 1,2 students. This indicates that possession of quantitative proportionality may facilitate acquisition of knowledge of structure of matter concepts.

Findings Relative to Equivalent Fractions

Wilks' Lambda was applied to the following primary hypothesis, stated in null form:

$H_0 3$: There is no significant difference between the mean vectors of knowledge, comprehension, and application test scores of equivalent fractions concepts for the four groups of students differing in proportional reasoning.

The means of each proportionality group on the equivalent fractions are summarized in Table 20 for each of the grouping procedures.

Analysis of Table 20 reveals two expected patterns. First, group means are generally highest for knowledge and lowest for application, with comprehension means falling in between. Because comprehension and application were assumed to represent levels of understanding, it was anticipated that these means would be similar to each other and considerably lower in magnitude than the knowledge means. This was not the case. Comprehension means are virtually equal to the knowledge means, and these two sets of means are considerably lower than the application means.

A second expected pattern was that the magnitude of the group means would generally follow the level of the proportional reasoning of the students: the higher the level of proportional reasoning, the larger the group means. This pattern was observed. One significant aspect of this pattern needs to be noted. Although comprehension and application means of the two qualitative proportional groups are lower than for the quantitative groups, they are well above chance level. This is significant, because quantitative proportionality was thought to be necessary for achieving comprehension and application of these concepts.

Hypothesis $H_0 1$ was tested for each of the four grouping procedures by the application of Wilks' Lambda. The results are summarized in Table 21.

Wilks' Lambda was found to be significant for each of the four grouping procedures, indicating that significant differences exist between the mean vectors of the four groups. The researcher, therefore, did not accept Hypothesis $H_0 1$.

Secondary hypotheses which predicted differences between the group centroids of groups possessing quantitative proportionality and

Table 20

Means of the Four Groups Differing in Proportional Reasoning on Knowledge, Comprehension and Application of Equivalent Fractions

Proportionality Group	Knowledge	Comprehension	Application
Quantifications of Probabilities Grouping Procedure			
1 (low qualitative)	9.4091	9.2424	5.9697
2 (high qualitative)	9.5476	9.1905	6.4524
3 (low quantitative)	10.3125	11.3125	8.9375
4 (high quantitative)	11.7273	11.7273	10.2727
Balance Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	9.0685	9.1918	5.6712
3 (low quantitative)	10.3478	10.1087	7.7826
4 (high quantitative)	11.6364	11.5455	10.1818
Lowest Score on Tasks Grouping Procedure			
1 (low qualitative)	9.4412	9.2353	6.0000
2 (high qualitative)	9.6327	9.6735	6.8367
3 (low quantitative)	10.6250	10.8750	9.6250
4 (high quantitative)	11.7000	11.7000	10.1000
Highest Score on Tasks Grouping Procedure			
1 (low qualitative)*			
2 (high qualitative)	8.9545	8.8333	5.3030
3 (low quantitative)	10.2963	10.3519	7.8519
4 (high quantitative)	11.6667	11.5833	10.3333

*Group omitted from analysis because of insufficient number of subjects.

Table 21

Wilks' Lambda Test of Primary Hypotheses

Grouping Procedure	Wilks' Lambda	Chi-Square	df	Significance
Balance task	.7814	31.327	2	.000
Quantification of probabilities task	.7768	33.210	3	.000
Highest score on tasks	.7221	42.003	2	.000
Lowest score on tasks	.8101	27.200	3	.000

Table 22

Discriminant Function Analysis Results

for Grouping Procedures

Analysis	Grouping Procedure	Discriminant Function	Eigenvalue	Relative Percentage	Canonical R
1	Balance task	1	.2798	100	.467
2	Quantification of probabilities task	1	.2873	100	.472
3	Highest score on tasks	1	.3849	100	.527
4	Lowest score on tasks	1	.2344	100	.436

groups possessing qualitative proportionality were tested by application of discriminant function analysis. The following secondary hypotheses, stated in null form, were tested:

$H_0 14$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high qualitative proportionality group.

$H_0 15$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low qualitative group.

$H_0 16$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low qualitative proportionality group.

$H_0 17$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low quantitative proportionality groups are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high qualitative proportionality group.

$H_0 18$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high and low quantitative proportionality groups combined is not significantly different from the group centroids of knowledge, comprehension, and application test scores for the high and low qualitative proportionality groups combined.

Only one discriminant function was generated in each of the four analyses. The canonical correlations of all four analyses are quite similar and indicate that the groups are accounting for about 20% of the variance in the functions (Table 22).

The discriminant function generated in all four analyses was derived solely from the application variable. Knowledge and comprehension did not discriminate between groups strongly enough to be included in the function. Consequently, only one standardized discriminant function coefficient was generated in each analysis. This coefficient equals one in all analyses. Therefore, all the discriminant function prediction equations produced are equal to 1 times X_A , the students' application score (tables 23 and 24).

The accuracy with which the students' application scores correctly predicted their group membership can be seen in Table 25. Because of overlap among groups on the discriminating variable (application), the percent of students correctly classified by the four discriminant function analyses ranged from 39.3% to 53.8%. Analysis of Table 25 reveals that high formal operational students were classified more accurately in all four analyses than were students in

Table 23

Standardized Discriminant Function Coefficients

Analysis	Grouping Procedure	Discriminating Variables	Function 1
1	Balance task	Application	1.000
2	Quantification of probabilities task	Application	1.000
3	Highest score on tasks	Application	1.000
4	Lowest score on tasks	Application	1.000

Table 24

Discriminant Function Prediction Equations

Analysis	Grouping Procedure	Function
1	Balance task	D $1.000X_a$
2	Quantifications of probabilities task	D $1.000X_a$
3	Highest score on tasks	D $1.000X_a$
4	Lowest score on tasks	D $1.000X_a$

Table 25

Predicted Group Memberships of 136 Students Differing in Proportional Reasoning for the Four Grouping Procedures

Actual Group	No. of Cases	Predicted Group Membership 1	2	3	4
Quantification of Probabilities					
1 (low qualitative)	66	37 56.1%	9 13.6%	10 15.2%	10 15.2%
2 (high qualitative)	42	21 50.0%	4 9.5%	6 14.3%	11 26.2%
3 (low quantitative)	16	4 25.0%	0 0%	4 25.0%	8 50.0%
4 (high quantitative)	11	2 18.2%	0 0%	1 9.1%	8 72.7%
% correctly classified:					
Balance					
2 (high qualitative)	73		45 61.6%	11 15.1%	17 23.3%
3 (low quantitative)	46		15 32.6%	9 19.6%	22 47.8%
4 (high quantitative)	11		2 18.2%	1 9.1%	8 72.7%
% correctly classified:	47.69%				
Lowest Scores on Tasks					
1 (low qualitative)	68	38 55.9%	13 19.1%	7 10.3%	10 14.7%
2 (high qualitative)	49	22 44.9%	7 14.3%	5 10.2%	15 30.6%
3 (low quantitative)	8	2 25.0%	0 0%	1 12.5%	5 62.5%
4 (high quantitative)	10	2 20.0%	1 10.0%	0 0%	7 70.0%
% correctly classified:	39.26%				
Highest Scores on Tasks					
2 (high qualitative)	66		44 66.7%	14 21.2%	8 12.1%
3 (low quantitative)	54		17 31.5%	18 33.3%	19 35.2%
4 (high quantitative)	12		2 16.7%	1 8.3%	9 75.0%
% correctly classified:	53.79%				

the other three groups. The number of high quantitative proportional students correctly classified ranged from 70.0% to 75.0%. This means that there was less variability among the test scores of students in the high quantitative proportional group than among the test scores of students in each of the other three groups. The second-most accurately classified group of students were the low qualitative proportional group. The number of these students correctly classified ranged from 55.9% to 66.7%.

The discriminating power of each derived function can be determined by computing group centroids. The group centroids, summarizing each group's location in reduced discriminant space, are summarized in Table 26.

Because knowledge test scores were the only test scores that discriminated between groups, differences between group centroids were caused exclusively by differences in knowledge scores. F scores obtained from Mahalonobis' test for pair-wise differences between group centroids derived from knowledge test scores are given in Table 27.

Examination of Table 27 reveals that the quantifications of probabilities grouping procedure yielded the largest number of group centroids significantly spread apart. Therefore, the quantifications of probabilities analysis was used to test the secondary hypotheses.

Because quantitative proportionality was thought to be necessary for achieving understanding (comprehension and application), but

Table 26

Group Centroids in Discriminant Space

Analysis	Grouping Procedure	Group	Function 1
1	Balance task	2	-.3161
		3	.2752
		4	.9471
2	Quantifications of probabilities task	1	-.2401
		2	-.1042
		3	.5957
		4	.9717
3	Highest score on tasks	2	-.4223
		3	.2953
		4	.9939
4	Lowest score on tasks	1	-.2315
		2	.0041
		3	.7893
		4	.9320

Table 27

Pair-Wise F-Tests for Application Differences

Between Group Centroids

Analysis	df	Grouping Procedure	Group	1	2	3
1	1,127	Balance task	3			
			4			4.6972*
2	2,131	Quantifications of probabilities	2	.5445		
			3	10.3659**	6.5391*	
			4	15.9544**	11.6267**	1.0621
3	1,129	Highest score on tasks	3		19.2138**	
			4		25.5857**	6.0205*
4	1,131	Lowest score on tasks	2	1.7727**		
			3	8.3624**	4.7535*	
			4	13.0290	7.8628**	.0892

* significant .05
** significant .01

not knowledge of equivalent fraction concepts, it was anticipated that comprehension and application test scores would significantly discriminate between the four groups and predict the group centroids. This was not the case. The group centroids produced in the discriminant function analysis were derived from application scores only.

Table 27 reveals four significant pair-wise differences between the four groups in the quantifications of probabilities analysis. These differences were between the centroids of the following pairs of groups: 1 and 3, 1 and 4, 2 and 3, and 2 and 4. These four differences were predicted by hypotheses H_{14}, H_{15}, H_{16}, and H_{17}.

Research hypothesis H_{14}, stated in null form, predicted no significant differences between centroids of groups 2 and 4. It was stated as follows:

$H_0$14: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F of 11.6267 for the application score differences. Because application test scores successfully discriminated between Group 2 (high qualitative proportional) and Group 4 (high quantitative proportional) students, it appeared that proportional reasoning ability was related to the achievement of application of equivalent fractions concepts by high qualitative proportionality and high quantitative proportionality students. Therefore, $H_0 14$ was not accepted.

Research hypothesis H_{15}, stated in null form, predicted no significant differences between centroids of groups 1 and 4. It was stated as follows:

$H_0 15$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 15.9544 for the application score differences. This finding indicated that proportional reasoning ability was related to the achievement of application of equivalent fractions concepts by low qualitative proportionality and high quantitative proportionality students. Therefore, $H_0 15$ was not accepted.

Research hypothesis H_{16}, stated in null form, predicted no significant differences between centroids of groups 1 and 3. It was stated as follows:

$H_0 16$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F ($p \leq .01$) of 10.3659 for the application score differences. This finding indicated that proportional reasoning ability was related to the achievement of application of equivalent fractions concepts by low quantitative proportionality and low qualitative proportionality students. Therefore, $H_0 16$ was not accepted.

Research hypothesis H_{17}, stated in null form, predicted no significant difference in centroids of groups 2 and 3. It was stated as follows:

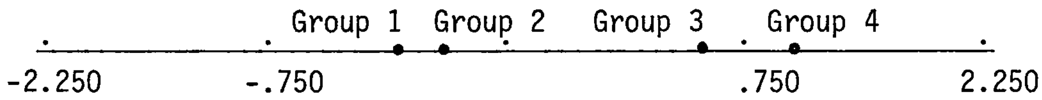
$H_0 17$: The group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the low quantitative proportionality group are not significantly different from the group centroids of knowledge, comprehension, and application test scores of equivalent fractions for the high qualitative proportionality group.

Mahalonobis' test for pair-wise differences between group centroids produced a significant F score of 6.5391 ($p \leq .05$) for the application score differences. Because application test scores successfully discriminated between Group 2 (high qualitative proportionality) and Group 3 (low quantitative proportionality) students, it appeared that proportional reasoning ability was related to the achievement of application of equivalent fractions concepts. Hypothesis $H_0$17 was not accepted.

The nature of the relationship between proportional reasoning ability and achievement of application of equivalent fractions was determined by examining the directional differences between the pairs of significantly different centroids when plotted in reduced discriminant space. The resulting territorial map is given in Figure 5. Because only one discriminant function was produced, the territorial map produced was unidimensional.

Figure 5

Territorial Map of Group Centroids



The location of the centroids in Figure 5 indicates that quantitative proportionality may facilitate achievement of application of equivalent fractions concepts. Figure 5 shows that the centroid of Group 4, being to the right of the other three centroids, had the greatest magnitude. Because the significant difference between the centroids of groups 4 and 1 and between the centroids of groups 4 and 2 was attributable solely to application score differences, Group 4 students must have scored consistently higher on the application test than students in groups 1 and 2. This indicates that possession of high quantitative proportionality may facilitate achievement of application of equivalent fractions concepts.

Figure 5 indicates that the centroid of Group 3 was of greater magnitude than the centroids of groups 1 and 2. Because the Group 3 centroid was found to be significantly different from the centroids of groups 1 and 2, Group 3 students must have scored consistently higher on the application test than the students in groups 1 and 2. This indicates that possession of low quantitative proportionality may facilitate achievement of equivalent fractions concepts.

Research hypothesis H_{18}, stated in null form, predicted no significant difference in centroids between groups 3 and 4 combined and groups 1 and 2 combined. It was stated as follows:

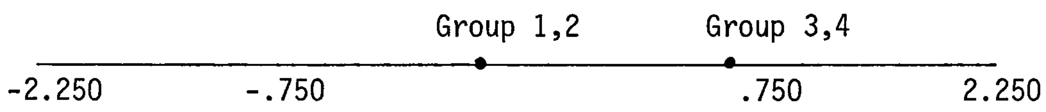
$H_0$18: The group centroids of knowledge, comprehension, and application test scores of simple machines for the high and low quantitative proportionality groups combined is not significantly different from the group centroids of knowledge, comprehension, and application test scores for the high and low qualitative proportionality groups combined. A discriminant function analysis produced one discriminant function using students' comprehension and application scores. Knowledge did not discriminate sufficiently to be included in the functions. The standardized discriminant function coefficients for comprehension and application, .3141 and .7400, respectively, showed application to be the more potent of the two discriminating variables. The group centroids in reduced discriminant space for groups 1 and 2 combined and groups 3 and 4 combined were found to be -.1872 and .7487, respectively. Mahalonobis' test for pair-wise differences between group centroids produced a significant ($p \leq .01$) F score of 21.8756 with 1 and 133 degrees of freedom for application score differences, and a significant ($p \leq .01$) F score of 11.5443 with 2 and 132 degrees of freedom for comprehension score differences.

The nature of the relationship between proportional reasoning ability and achievement of comprehension and application of equivalent fractions was determined by examining directional differences between the centroids of groups 1,2 and 3,4 when plotted in reduced discriminant space. Because only one discriminant function was produced, the territorial map generated was unidimensional, as shown in Figure 6.

The location of these centroids indicated that quantitative proportionality may facilitate the acquisition of comprehension and application of equivalent fractions concepts. Figure 6 shows that the Group 3,4 centroid, being to the right of Group 1,2 centroid, has the greater magnitude. Therefore, Group 3,4 students must have scored consistently higher on the application and comprehension test items than the Group 1,2 students. This indicates that possession of quantitative proportionality may facilitate acquisition of comprehension and application of equivalent fractions concepts.

Figure 6

Territorial Map of Group Centroids



Discussion of the Findings

The achievement of application by high quantitative proportional reasoners on simple machines, structure of matter, and equivalent fractions was found to be significantly ($p \leq .01$) greater than the achievement by high and low qualitative reasoners. This finding suggests that quantitative proportional reasoning may be related to and necessary for achievement of application of the selected concepts.

The expected relationship occurred between level of achievement and proportional reasoning for equivalent fractions. Quantitative proportionality was considered to be necessary for achieving application of the selected concepts; therefore, a significant difference in application level achievement was anticipated between qualitative and quantitative proportional reasoners. This was the case. Application was the primary discriminating variable between the groups for equivalent fractions.

An unexpected relationship occurred between level of achievement and proportional reasoning for the science concepts. Knowledge, rather than application or comprehension, proved to be the primary discriminating variable. Knowledge was not expected to discriminate between groups because quantitative proportionality was not considered essential for knowledge level achievement.

One possible explanation for why knowledge rather than application was the primary discriminating variable for the science concepts is as follows. The level of achievement was rather low on knowledge, comprehension, and application level test items. When this occurs proportional reasoning ability will probably interact with lower levels of achievement. On the other hand, achievement of equivalent fractions was generally at the mastery level for knowledge and comprehension, and generally below mastery for application. When this occurs, proportional reasoning will probably interact at the application level. This pattern of achievement is reflected in the group means for the science and mathematics concepts. These means are summarized in Table 20.

The means in Table 20 reveal that knowledge, comprehension, and application scores for science concepts are considerably lower than for equivalent fractions. Knowledge and comprehension means for science concepts were 5.70 to 9.91, respectively (with 12 points possible). Knowledge and comprehension means for equivalent fractions were 9.24 to 11.73, respectively. Application means for the science concepts were 3.00 (chance level) and 6.64 for comprehension, while application means for equivalent fractions were 5.97 and 10.27 for application means. The fact that application means for the science concepts were at the chance level and slightly above may account for why knowledge proved to be the primary discriminating variable for the science concepts. The fact that knowledge and comprehension scores of equivalent fractions were generally at the mastery level may account for why application was the primary discriminating variable for equivalent fractions.

Another important variable which may have contributed to the interaction between achievement and proportional reasoning with respect to mathematics and science concepts was the difference in the instructional programs in which the students participated. The science concepts were taught by the traditional group-paced lecture/discussion method. Because teachers were required to teach the entire content in their physical science textbook within two semesters, they presumably moved their classes very rapidly through the science concepts. On the other hand, students had definitely experience more classroom instruction in equivalent fractions.

Equivalent fractions are introduced in third-grade mathematics texts by virtually all textbook series and developed extensively in fourth, fifth, and sixth grades. These students undoubtedly had instruction in equivalent fractions over a several year period. In addition, they had certainly experienced numerous practical applications of equivalent fractions in their day-to-day experiences between third and ninth grades. The greater amount of time and experience with equivalent fractions may account for the higher achievement in equivalent fractions demonstrated by these students.

If this inference is correct, it has implications for classroom instruction. For instance, students may be capable of much greater achievement of knowledge and understanding of the science concepts if given more learning time and experience, regardless of their logical thinking abilities. Perhaps self-paced/modularized instructional programs would be an effective alternative to the group-paced lecture/discussion method for teaching the science concepts.

A study comparing the effectiveness of self-paced and group-paced instruction has recently been reported by Chiapetta et al. (1977). They investigated the difference between self-paced/modularized instruction and the traditional lecture/discussion method on achievement of the solubility concept by ninth-grade students differing in logical thinking skills. They found that concrete operational students in the self-paced/modularized program achieved significantly ($p \leq .05$) larger gain scores than formal operational students in the traditional lecture/discussion program. In fact, achievement of the concrete operational students in the self-paced/modularized program was at the level of the formal operational students in the traditional group-paced lecture/discussion program. These findings, in conjunction with the findings of this study, suggest that student achievement of the selected concepts might be improved if instruction provided students with self-paced learning rather than the group-paced learning given by the traditional lecture/discussion method.

Mastery learning may be another effective alternative to group-paced lecture/discussion methods. Bloom's (1976) "Mastery Learning Model" is based, in part, on Carroll's (1963) "School Learning Model." Carroll's model proposed that the degree to which a student could be expected to learn was a function of the ratio between the amount of time actually spent in learning and the amount of time needed. Bloom (1976) transformed Carroll's model into a three-part paradigm which states that student learning is a function of cognitive entry behaviors, affective entry characteristics, and quality of instruction. Quality of instruction increases when it is well designed and when feedback and corrective instruction are given to individual students. Considered in the light of mastery learning strategy, student achievement of the selected concepts might be significantly improved if mastery learning instruction were provided, regardless of their proportional reasoning ability.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Introduction

Two important considerations for improving student achievement are the sequence of the curriculum and the developmental readiness of the learner. Gagne has developed an instructional strategy for improving the sequence of curriculum. Piaget has identified various stages of cognitive development and has described the logical thinking skills characteristic of each stage. The task analysis strategy of Gagne and the developmental theory of Piaget should be combined to enhance student learning. This can be accomplished if logical thinking skills necessary for learning specific concepts can be determined.

This study had attempted to determine the relationship between a specific logical thinking skill and student achievement of selected science and mathematics concepts. The logical thinking skill of proportional reasoning was selected for investigation because it appears to be directly related to understanding of simple machines, structure of matter, and equivalent fractions concepts. An analysis of related literature suggests that quantitative proportionality may be necessary for achieving understanding of the selected concepts. Hypotheses were generated which predicted that quantitative proportional students would achieve greater understanding (comprehension and application) of the selected science and mathematics concepts than qualitative proportional students.

A population of students having a wide range in proportional thinking skills and who had studied the selected science and mathematics concepts in their ninth-grade physical science classes were selected for the study. Piagetian tasks were administered to assess students' proportional reasoning skills and to group them into four levels of proportional reasoning: high and low quantitative proportional reasoners and high and low qualitative proportional reasoners. A paper/pencil test was used to assess students' achievement of knowledge and understanding (comprehension and application) of the selected concepts.

Discriminant function analysis was applied to the data to determine if a relationship existed between proportional reasoning and achievement of the selected concepts. The effectiveness of knowledge, comprehension, and application test scores for discriminating between qualitative proportional reasoners and quantitative proportional reasoners indicated the nature of the expected relationship.

Summary of the Findings

The findings of this study are summarized as follows:

1. High quantitative proportional reasoners demonstrated significantly ($p \leq .01$) greater achievement on knowledge and application level test items for simple machines than high and low qualitative proportional reasoners.

2. High quantitative proportional reasoners demonstrated significantly ($p \leq .01$) greater achievement on knowledge and comprehension level test items for structure of matter than high and low qualitative proportional reasoners.

3. High quantitative proportional reasoners demonstrated significantly ($p \leq .01$) greater achievement on application level test items for equivalent fractions than high and low qualitative proportional reasoners.

4. Low quantitative proportional reasoners demonstrated significantly ($p \leq .01$) greater achievement of knowledge level test items for simple machines and structure of matter than low qualitative proportional reasoners.

5. Low quantitative proportional reasoners demonstrated significantly less ($p \leq .01$) achievement of application level test items for simple machines than low qualitative proportional reasoners.

6. Low quantitative proportional reasoners demonstrated significantly less ($p \leq .05$) achievement of comprehension level test items for structure of matter than low qualitative proportional reasoners.

7. Low quantitative proportional reasoners demonstrated significantly greater ($p \leq .05$) achievement of application level test items for equivalent fractions than high and low qualitative proportional reasoners.

8. In general, all groups achieved more knowledge of the concepts than comprehension and more comprehension than application.

9. Comprehension means of the four groups on all concepts tended to be more similar to knowledge means than application means.

10. Application means were in all cases much lower than knowledge and comprehension means.

11. Achievement of equivalent fractions was considerably higher at all levels than achievement of the science concepts.

Conclusions

The findings of this study are generalizable to the population sampled and, with reservation, to similar populations. Within these limitations, the findings of this study generally support the hypotheses investigated: achievement of selected science and mathematics concepts is related to proportional thinking, and quantitative proportional reasoners generally achieve significantly greater knowledge, comprehension, and application of the selected concepts than qualitative proportional reasoners.

Analysis of the means suggests a particular interaction pattern between proportional reasoning and level of achievement (Bloom's taxonomy). It suggests that as achievement increases, proportional reasoning may interact at the higher levels of achievement. Therefore, when achievement of knowledge, comprehension, and application is low, knowledge will yield the strongest interaction with proportional reasoning. When achievement of knowledge, comprehension, and application is high, application will yield the strongest interaction with proportional reasoning.

Recommendations

Since a general relationship was found between proportional reasoning and achievement, it is recommended that experimental studies be conducted to investigate a cause and effect relationship between proportional reasoning and achievement. The following studies are recommended:

1. A training study using a sound instruction program such as mastery learning or self-paced/modularized instruction in which ninth-grade students (14 or 15 year-olds) possessing qualitative proportionality and possessing little, if any, knowledge, comprehension, or application of the science and mathematics concepts are trained to determined to what extent they can achieve these concepts and to what extent qualitative proportionality limits their ability to achieve these concepts.

2. Similar studies investigating the relationship between proportional reasoning and other science concepts.

3. Similar studies investigating the relationship between other logical thinking skills and other concepts.

Implications

The positive relationship identified between proportional reasoning and achievement of the selected science and mathematics concepts has implications for classroom instruction. Because most high-school-age students would not be expected to possess quantitative proportional reasoning, teachers instructing students in these concepts would probably be unwise to teach for mastery at the application level. By employing a sound instructional program, they could probably teach for mastery at the knowledge and comprehension levels. Application could probably be developed, but only at a minimal level of achievement for most students.

The identified relationship between proportional reasoning and achievement of the selected concepts permits the task analysis strategies of Gagne and the developmental theory of Piaget to be combined and utilized to enhance student achievement of these concepts. Task analysis could be applied to develop hierarchical sequences of the selected concepts. Teachers applying sound instructional programs could teach the sequences of concepts at the knowledge and comprehension levels and could teach for mastery. Sequences of concepts could be followed until a concept requiring application level achievement is reached. If only a minimal level of application of the concept is required to continue the sequence, it could be taught and the sequence continued. If application at the mastery level is required, only students possessing high quantitative proportionality would be permitted to continue the sequence. Other students would be permitted to study the concept and expected to attain a minimal achievement of application level understanding. These students would then begin to study other concepts, rather than attempting to learn the given concepts at the application or higher levels.

By utilizing the findings of this study as just described, educators can employ the task analysis strategy of Gagne to determine "what to teach" and the developmental theory of Piaget to determine "when to teach" a given concept.

APPENDIX A

Quantification of Probabilities--Protocol

A. E presents S with

1. (2,2) (2,3); 2/4 and 2/5.

   E: Do you have a better chance of pulling out a red chip from here or there, or are the chances the same? (E circle S choice.)

   Why?

   What chance is there for getting a red chip out of each set?

   How do you know?

2. (1,2) (2,1); 1/3 and 2/3.

   E: ....Better chance?

   Why?

   What chance is there for getting a red chip out of each set?

   How do you know?

B. 3. (2,6) (1,3); 2/8 and 1/4.

   E: ....Better chance?

   Why?

   ....chance red chip from each set?

   How do you know?

****If S fails:

   E: Watch! (E separates (2,6) into two groups of (1,3).)

   What do you think now?

If S succeeds with the separated set, E mixes the set and asks,

Now? Why?

****If S succeeds in (B): E presents

4. (1,2) (2,4); 1/3 and 2/6.

E: ....Better chance?

Why?

(To S who say SAME): Wouldn't there be a better chance with this set (1,2) because there are only two blue chips?

S: Why?

****If S succeeds, E removes one blue chip from (2,4) and presents

C. 5. (1,2) (2,3); 1/3 and 2/5.

E: ....Better chance?

Why?

....chance red chip each set?

How do you know?

****If S succeeds, E presents

6. (4,5) (2,3); 4/9 and 2/5.

++N.B. (4,6) ≐ 4/10 = 2/5 ≐ (2,3)++

E: ....Better chance?

Why?

....chance red chip each set?

How do you know?

****If S says (4,5) has better chance, E says

Wouldn't they be the same? Here (4,5), there are 2 more red chips and 2 more blue chips than there (2,3).

S:

D. If S maintains that set (4,5) has a better chance than set (2,3), E presents

7. (2,3) (6,7); 2/5 and 6/13.

E: ....Better chance?

Why?

....chance red chip each set?

How do you know which one gives the better chance of getting a red chip?

8. (4,3) (5,4); 4/7 and 5/9.

E: ....Better chance?

Why?

How do you figure out which set gives you the better chance of getting a red chip?

****On close calls for levels IIIA or Level IIIB, E gives

9. (1,2) (2,1); 1/3 and 2/3.

E: How many blue chips must you add so that the chances are the same?

10. E: If you had to tell a friend how to figure out these kinds of problems, what would you say?

Why?

Balance Protocol

E: This is a balance. The arm goes up and down (demonstrate). And these are a bunch of weights we can hang on the balance like this (demonstrate).

***N.B. "Heavy" refers to the heavy weight, which is twice the weight of the light weight, "Light." Numbers without the apostrophe (') refer to left side of the fulcrum; numbers with the (') refer to page on the right side of the fulcrum.

1. E: Use some of these weights to make the arm level. Why does that balance?

2. (Heavy and Light weights) E: How can you make the arm level with these two weights? Why does that balance?

3. (Light at 10, Heavy at 5') E: If I move this (Light) weight to here (5), what will happen?

   How can you make it balance again?

   How else?

4. (Heavy at 5 and Light at 10') E: If I put another weight (Light) here (10'), what will happen?

   How can you make it balance again?

   How else?

5. E: If I put this one (Heavy) here (1) and give you this one (Light), how can you make it balance?

   Why does that work?

   E: I'll move mine (Heavy) out one peg. Where will you put yours (Light)? Why?

   ***E may repeat until Heavy is on 5 and S's Light is on 10'. E may use same sequence with Heavy and Light starting on 1, and S has Light starting on 3.

E: Can you figure out a rule for what we've been doing? How did you tell how many pegs to move yours? Why do you think a balance works like that?

6. (Light at 5 and Heavy at 5') E: How could you make the arm level?

   How else?

7. (Heavy at 3) E: Tell me without trying where you would put this weight (Heavy) to make the arm level.

   Why there?

8. (Light at 6') E: Tell me without trying where you would put this weight (Heavy) to make the arm balance.

   Why there?

9. E: If I put these two (Light) here (1), where would you put this one (Light) to make the arm level? Why?

   Repeat for three, four,..., nine, etc. Light weights. Why?

10. E: So, if I put this one (Heavy) on 3', where will you put this one (Light) to make the arm level?

    Why there?

    If you switched these weights around so that each one went to the opposite side, where would you put them to keep the arm level?

    Why?

    Is that the only way you can do it? Why? How do you know?

11. E: Now, suppose I put these four (Light) here (3). Tell me without trying where you will put these three (Light) to make it level. Why?

E: Repeat, giving S two (Light).

12. Suppose you want to tell a friend about all this. What would you say so that he would be able to get these questions right the first time he tried? What would be your general rule about how the balance works? Why?

APPENDIX B

Simple Machines

Date______________________ Name__________________________

Matching: Select the phrase on the right that matches the word on the left.

_____1. efficiency

_____2. lever

_____3. mechanical advantage

_____4. inclined plane

_____5. machine

(a) any device used to change the force a person is able to exert.

(b) basic kind of machine consisting of a rigid bar that pivots about a point.

(c) basic kind of machine consisting of a sloping, or slanting, surface.

(d) in a machine, the ratio of the useful work output to the work input.

(e) number of times a machine multiplies the force applied to the machine.

Matching:

_____6. fulcrum

_____7. effort arm

_____8. wheel and axle

_____9. resistance arm

(a) distance from the fulcrum to the applied force.

(b) pivot point of a lever.

(c) distance from the weight lifted to the fulcrum.

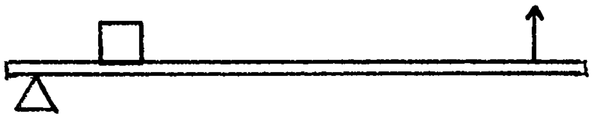
(d) a circular kind of 1st class lever.

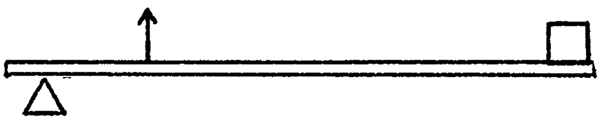
Matching:

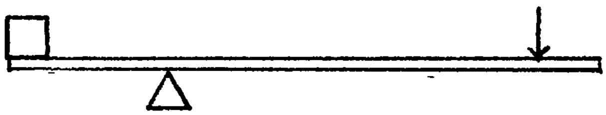
____10. 1st class lever

____11. 2nd class lever

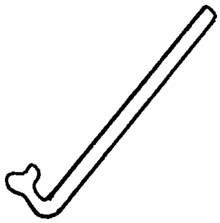
____12. 3rd class lever

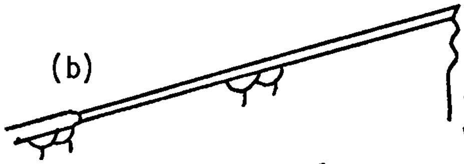
(a) 

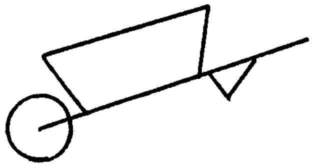
(b) 

(c) 

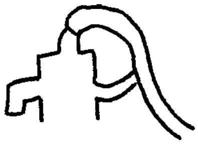
Matching:

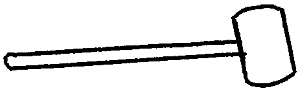
____13. 1st class lever (a)  prying with a crowbar

____14. 2nd class lever (b)  casting a fishing pole with two hands

____15. 3rd class lever (c)  moving a heavy load

Matching:

____16. 1st class lever (a)  pumping water

____17. 3rd class lever (b)  swinging a sledge hammer

Multiple choice:

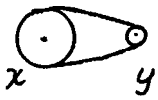
____18. Which of the following is not an example of an inclined plane? (a) wedge (b) wood screw (c) highway overpass (d) cylinder

____19. A 50 lb. rock could be lifted with the least effort using a lever with a mechanical advantage of (a) 5 (b) 3 (c) 10 (d) 8 .

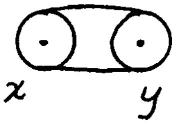
____20. A machine having which of the following inputs and outputs would be the most efficient? (a) input 3 lbs., output 6 lbs. (b) input 6 lbs., output 3 lbs. (c) input 3 lbs., output, 3 lbs. (d) input 6 lbs., output 6 lbs.

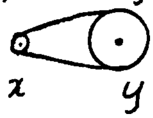
____21. A 25 lb. effort could lift the heaviest weight if which of the following is used: (a) first class lever (b) second class lever (c) third class lever (d) the lever with the largest mechanical advantage.

____22. The arrangement of pulleys that will give pulley Y the lowest speed of rotation is:

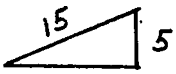
(a) 

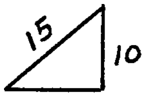
(c) 

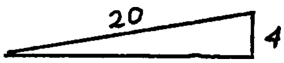
(b) 

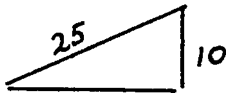
(d) 

____23. A box placed on rollers is pushed up each of the inclined planes below. Disregarding friction, for which inclined plane will the <u>least</u> force be required to move the box?

(a) 

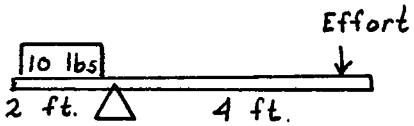
(c) 

(b) 

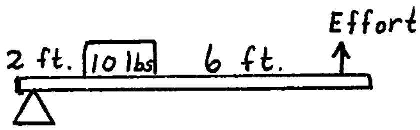
(d) 

____24. Which of the following is not an example of an inclined plane? (a) ramp (b) elevator (c) escalator (d) staircase

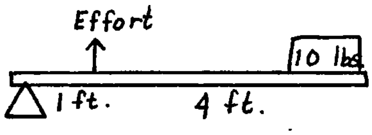
____25. When the effort moves 12 ft., how far does the resistance arm move? (a) 12 (b) 6 (c) 3 (d) 10



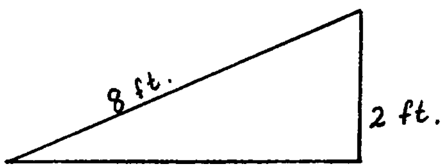
____26. What is the mechanical advantage? (a) 12 (b) 3 (c) 6 (d) 4



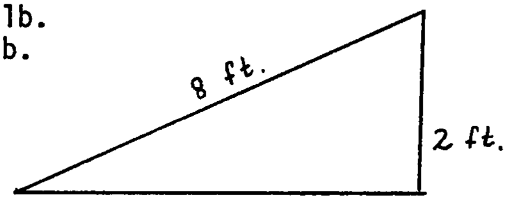
____27. What effort is required to lift the weight? (a) 40 (b) 2½ (c) 50 (d) 15



____28. What force is required to move a 100 lb. weight up the inclined plane? (a) 25 lb. (b) 50 lb. (c) 13 lb. (d) 30 lb.



____29. What is the heaviest weight that could be moved up the inclined plane by a 25 lb. force? (a) 20 lb. (b) 10 lb. (c) 100 lb. (d) 80 lb.



____30. What length would this inclined plane need to be for a 160 lb. weight to be moved up with a force of 80 lbs.? (a) 2 ft. (b) 4 ft. (c) 6 ft. (d) 8 ft.

____31. By using a lever, a 300 lb. rock is lifted 8 inches with an effort of 50 lb. How far does the effort move? (a) 8" (b) 100" (c) 48" (d) 96"

____32. What effort is needed to lift a 198 lb. box using a first class lever having an effort arm 6 feet long and a resistance arm 2 feet long? (a) 33 lb. (b) 26 lb. (c) 99 lb. (d) 66 lb.

____33. What is the maximum weight a 1st class lever can lift with an effort of 30 lbs. if it has a resistance arm of 2 and an effort arm of 10 ft.? (a) 150 (b) 300 (c) 60 (d) 600

____34. What is the maximum weight a 2nd class lever can lift with an effort of 10 lb. if it has a resistance arm of 3 and an effort arm of 9 ft.? (a) 30 lb. (b) 20 lb. (c) 35 (d) 45

____35. What effort is needed to lift a 300 lb. object using a 2nd class lever having an effort arm of 6 ft. and a resistance arm of 2 ft.? (a) 100 lb. (b) 300 lb. (c) 150 lb. (d) 200 lb.

____36. What force is required to move a 5 lb. object with a third class lever, having an effort arm of 1 ft. and a resistance arm of 4 ft.? (a) 10 lbs. (b) 15 lbs. (c) 20 lbs. (d) 25 lbs.

Structure of Matter

Use these relative atomic weights to answer questions 37 through 44.

Calcium 40	Oxygen 16	Potassium 19	Chlorine 35
Carbon 12	Hydrogen 1	Sulfur 32	Sodium 23

Find the atomic weight.

____37. $CaCl_2$ (a) 110 (b) 75 (c) 30 (d) 82

____38. H_2CO_3 (a) 71 (b) 58 (c) 29 (d) 62

____39. H_2SO_4 (a) 49 (b) 66 (c) 98 (d) 89

____40. NaOH (a) 40 (b) 49 (c) 24 (d) 17

Find the number of moles.

____41. 18 grams of KOH (a) 1½ (b) 2 (c) ½ (d) 1

____42. 68 grams of H_2S (a) 1 (b) 2 (c) 1½ (d) 1½

____43. 87 grams of NaCl (a) 2 (b) ½ (c) 3 (d)

____44. 132 grams of CO_2 (a) ½ (b) 3 (c) 2 (d) 1½

____45. If an atom has 17 protons, 17 neutrons, and 17 electrons, the arrangement of its electrons in the three energy levels would be (a) 2-8-7 (b) 8-2-7 (c) 8-7-2 (d) 7-2-8

____46. If an atom has five electrons, how many would be in the energy level farthest from the nucleus? (a) 1 (b) 2 (c) 3 (d) 5

____47. If carbon has an atomic number of 6 and oxygen has an atomic number of 8, then (a) carbon atoms have 2 more protons than oxygen atoms (b) oxygen has 2 more protons than carbon (c) carbon and oxygen atoms have the same number of atoms (d) oxygen has 1 more proton than carbon.

____48. If two atoms have the same atomic number but have different atomic weights, then they differ in (a) number of protons (b) number of electrons (c) size (d) number of neutrons.

____49. If two atoms of different elements have the same atomic weight, then they (a) differ in the number of protons (b) have the same number of electrons (c) are ions (d) do not have protons.

____50. An alpha particle would be repelled if it came near a nucleus because (a) the nucleus is very large (b) the alpha particle is very light (c) they both have positive charges (d) they both have negative charges.

____51. If an atom having an atomic number of 5 loses two electrons, its charge is (a) neutral (b) -2 (c) +2 (d) +3 .

____52. If two atoms have the same atomic number, they must be (a) the same element (b) different elements (c) have the same total number of electrons and protons (d) have the same total number of protons and neutrons.

Matching:

____53.	Molecule	(a) particles of matter that are too small to be seen and that combine to form molecules.
____54.	Nucleus	(b) tiny particle of matter having almost the same mass as a proton and having no electrical charge.
____55.	Electron	(c) smallest particle of a substance that retains all the properties of that substance.
____56.	Atom	(d) very small particle found within an atom and having a negative electrical charge.
____57.	Neutron	(e) central part of an atom.

Matching:

____58.	Atomic mass	(a) tiny particle of matter found within an atom and having the smallest possible positive electrical charge.
____59.	Isotopes	(b) average mass of the atoms in a natural sample of an element.
____60.	Protons	(c) forms of the same element that differ only in atomic mass.
____61.	Ion	(d) atom or atomic fragment that is electrically charged.

Use the valence chart to write the correct formula:

ammonium	NH_4	+1	carbonate	CO_3	-2
nitrate	NO_3	-1	sodium	Na	+1

____62. ammonium nitrate (a) NH_4NO_3 (b) $(NH_4)_2NO_3$ (c) $NH_4(NO_3)_2$ (d) $(NH_3)_2NO_3$

____63. sodium carbonate (a) $NaCO_3$ (b) $(Na)_2CO_3$ (c) $Na(CO_3)_2$ (d) $(Na)_3CO_3$

____64. sodium nitrate (a) $NaNO_3$ (b) Na_2NO_3 (c) $Na(NO_3)_2$ (d) $Na_2(NO_3)_2$

____65. ammonium carbonate (a) $(NH_4)_2CO_3$ (b) $NH_4(CO_3)_2$ (c) NH_4CO_3 (d) $(NH_4)_2(CO_3)_2$

____66. The molecular weight in grams of a compound is called (a) an ion (b) a mole (c) an isomer (d) a compound.

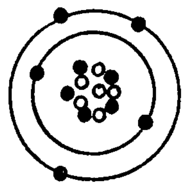
____67. Avagadro's number tells (a) the number of grams in a mole (b) the number of molecules in a mole (c) the number of molecules in a gram.

____68. Which of the following gives the three basic parts of the atom? (a) proton, neutron, electron (b) proton, mole, electron (c) neutron, amu, proton (d) ion, electron, proton.

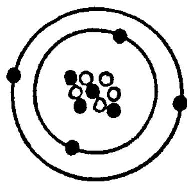
Matching:

	Element	Atomic Number	Atomic Weight
____69.	B	5	11
____70.	N	7	14
____71.	Be	4	9
____72.	Li	3	7

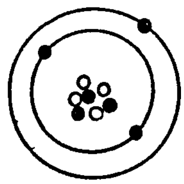
(a)



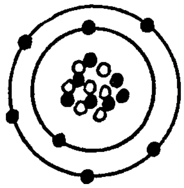
(b)



(c)



(d)



Equivalent fractions:

____73. $\frac{2}{3} = \frac{\quad}{6}$ (a) 4 (b) 2 (c) 3 (d) 5

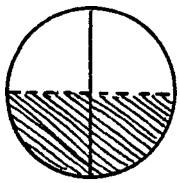
____74. $\frac{3}{15} = \frac{\quad}{30}$ (a) 9 (b) 18 (c) 14 (d) 6

____75. $\frac{5}{9} = \frac{\quad}{72}$ (a) 45 (b) 36 (c) 40 (d) 14

____76. $\frac{3}{4} = \frac{\quad}{16}$ (a) 12 (b) 4 (c) 18 (d) 9

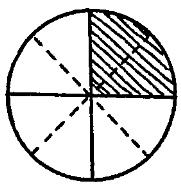
____77. $\frac{4}{7} = \frac{\quad}{56}$ (a) 30 (b) 32 (c) 14 (d) 24

Do the following example and then do the problems. Show your work.



Look at the circle. Notice the solid line and the dotted line. The solid line divides the circle into how many parts? ________. The solid line and the dotted line together divide the circle into how many parts? ________. Write the two fractions describing the shaded parts of the circle. ________ ________. (Answer: 1/2 and 2/4.) For each of the following circles, select the two fractions that describe the shaded part.

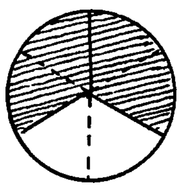
____78.



(a) 1/2, 1/4 (b) 1/4, 2/8

(c) 1/2, 2/3 (d) 2/4, 2/8

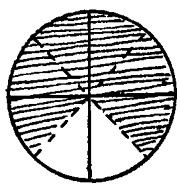
____79.



(a) 2/3, 2/6 (b) 4/6, 3/4

(c) 4/6, 2/3 (d) 4/6, 2/6

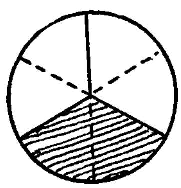
____80.



(a) 2/6, 4/10 (b) 6/8, 3/4

(c) 3/4, 2/6 (c) 6/10, 3/4

____81.



(a) 2/3, 2/6 (b) 2/6, 1/3

(c) 2/4, 2/6 (d) 1/3, 2/8

Matching:

____82. Fraction (a) top part of a fractional number

____83. Numerator (b) part of a whole

____84. Denominator (c) bottom part of a fractional number

____85. Equivalent (d) describes the same fractional part of a whole

____86. Equivalent fraction (e) same size

____87. The total number of parts into which a whole is divided is represented by the (a) numerator (b) denominator (c) the numerator and the denominator (d) neither the numerator nor the denominator.

____88. How many fourths of a pie are the same as half of a pie?

(a) 2 (b) 9 (c) 3 (d) 4

____89. George has two-thirds of a pie that is cut into ninths. How many pieces of the pie does he have?

(a) 4 (b) 9 (c) 3 (d) 6

____90. Julie cut a pie in eighths and gave her brother three-fourths of it. How many eighths of the pie did he get?

(a) 3 (b) 4 (c) 8 (d) 6

____91. How many fifths of a pie are the same as nine-fifteenths of a pie?

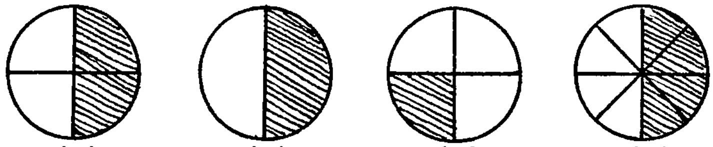
(a) 6 (b) 3 (c) 8 (d) 2

____92. Two sevenths of a pie are as much as how many twenty-firsts of a pie?

(a) 6 (b) 3 (c) 8 (d) 2

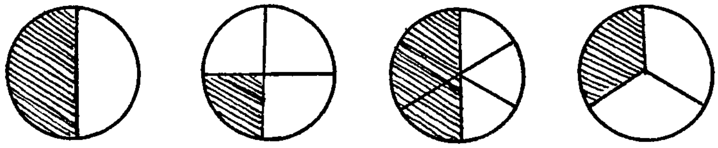
Look at the shaded part of each circle. For each of the following pairs of fractions, select the circle whose shaded part shows that the two fractions are equivalent.

_____93. 4/8 - 1/2



(a) (b) (c) (d)

_____94. 1/2 - 3/6



(a) (b) (c) (d)

_____95. Four-fifths (a) 5/4 (b) 4/5 (c) 4-5 (d) 45

_____96. ten-fifteenths (a) 1015 (b) 10/15 (d) 10-15 (d) 15/10

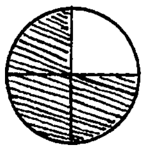
Matching:

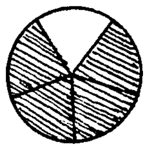
_____97. 1/2

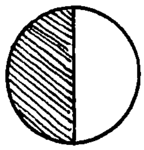
_____98. 3/4

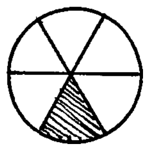
_____99. 4/5

____100. 1/6

(a) 

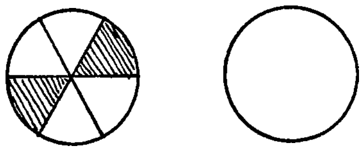
(b) 

(c) 

(d) 

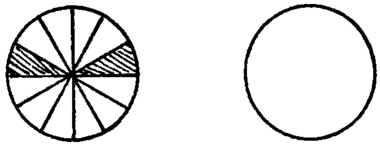
Look at each of the following pairs of circles. Into how many equal parts would you have to cut the blank circle so that if you shade one of its equal parts, the same amount will be shaded in both circles?

____101.



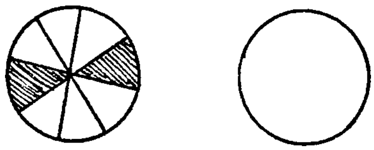
(a) 6 (b) 4 (c) 2 (d) 3

____102.



(a) 10 (b) 6 (c) 2 (d) 3

____103.



(a) 6 (b) 2 (c) 5 (d) 4

____104. In the fraction 2/5, the 2 is called the (a) denominator (b) numerator (c) equivalent (d) quotient.

____105. 9/12 = 12/?

(a) 10 (b) 16 (c) 8 (d) 12

____106. 16/28 = ?/35

(a) 20 (b) 18 (c) 35 (d) 28

____107. 10/15 = ?/21

(a) 16 (b) 20 (c) 14 (d) 30

____108. 12/16 = ?/24

(a) 18 (b) 16 (c) 24 (d) 20

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