ADJUSTABLE MECHANISMS FOR REDUCING THE ERROR IN FUNCTION GENERATION AND PATH GENERATION

A Thesis

Presented to

the Faculty of the Graduate School

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

---

by

Ronnie Eugene Haws

August 1973

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This thesis is dedicated to my wife, Sue. She has performed tremendously in giving encouragement, understanding and financial support along with helping to prepare this thesis and typing it.

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ABSTRACT

An analytical procedure has been developed for the synthesis of cam link four bar mechanisms. This method has provided the capability to design function generating mechanisms and path generating mechanisms that are accurate over a limited range as compared to a few points by conventional linkage synthesis techniques. This method is used in conjunction with an optimization utilizing a grid search technique to develop a cam-link mechanism with minimum cam pressure angle over the range of operation.

The method has been applied for path generating linkages with the cam link as the input link. For the function generator the cam link has been positioned in all the links to demonstrate the solution technique.

The solutions have been compared with standard kinematic synthesis to demonstrate the accuracy and advantages of the cam link mechanism over the conventional four-bar linkage.

TABLE OF CONTENTS

LIST OF FIGURES

Figure Page

Figure | Page

Chapter 1

INTRODUCTION

The study of motions in machines may be considered from two different points of view identified as kinematic analysis and kinematic synthesis. Kinematic analysis is the determination of motion inherent in a given mechanism. In the past displacement analysis was of paramount importance, but with increases in rotational speed, velocity and acceleration analysis have become critical to the design of the machine. Kinematic synthesis is the reverse problem: it is the determination of mechanisms that are to fulfill certain motion specifications. Synthesis is the very foundation of design, for it permits the creation of new hardware to meet particular needs in motion as specified by displacements, velocities, or accelerations, one at a time or in combination. The two types of synthesis that are discussed in this paper are the major areas of interest today, function generation and path generation.

The four-bar linkage has been classified according to its design objective as either a function generator or a path generator. The function generator has been characterized as having a specified functional relationship between the input and output links. The ability to predict the location of the output link has enabled the design engineer to utilize the four-bar linkage as a computer mechanism, a positioning mechanism, and as a transport mechanism in many machines.

Classical kinematic synthesis methods have been developed to generate both types of devices. They generally are exact only at a limited number of precision points. Mathematical difficulties in obtaining a solution have generally limited these methods to a maximum of five points.

This thesis is concerned with the development of a cam-link mechanism that provides a design solution that gives accuracy over a range rather than at a limited number of accuracy points. The path generator developed in this study moves a coupler point along a prescribed path while also correlating an input shaft rotation to a particular position along the coupler curve. The function generator is controlled in a similar manner to produce the functional relationship between two shaft rotations.

The utilization of a cam-link mechanism complicates the simplicity that is inherent in the four-bar linkage, but has provided the capability of controlling the accuracy over a range rather than at specific points. For this case, the equations of constraint are more difficult than those of the four-bar linkage and require additional assumptions in order to provide an optimum solution.

The work done in this thesis develops a path generator and three types of function generators utilizing a cam-link to provide zero error.

Chapter 2

LITERATURE SURVEY

The subject of kinematics is not very old in science, although some of its phases are as old as recorded history. The story began with the random growth of machines and mechanisms under the pressure of necessity. This was the period of invention and establishment of basic forms, but there was neither unity nor plan. Hartenberg and Denavit, (1) in the first chapter of their book, cover the history of kinematics from the age of the Ancient Egyptians to the end of the nineteenth century.

With the progress in mathematics and the introduction of high speed computer machines, a considerable amount of work on analysis and synthesis of linkage mechanisms has been done, especially after the Second World War.

Four-bar linkages were the simplest mechanism to fulfill function generation, path generation, or coupler positioning. Hall (2) presented an analysis of this mechanism together with a brief introduction to some methods of synthesis. Freudenstein (3) developed several methods of varying degrees of accuracy and complexity for four-bar linkage synthesis. From these methods the designer could select the one best suited to his requirement. These methods were developed by using either several precision points (up to five points) or a single point with several precision derivatives. Freudenstein (4) completed tables of linkage types, functions, ranges, and accuracies possible using a conventional four-bar linkage. Freudenstein and Sandor (5) developed a general method for plane-linkage synthesis for path generation. This method applied complex numbers and matrix theory of linear systems on the four-link mechanism synthesis up to five precision points. They found that up to twelve possible solutions exist.

Several methods have been developed for minimizing the error between precision points. Freudenstein (6) developed methods for estimating and obtaining minimum error in the approximate synthesis of plane, function or path-generating mechanisms. He used successive respacing of precision points for minimization of the structural error. The method of successive improvements could be used with any optimization criterion. Lewis and Gyory (7) applied the extension of the method of damped-least-squares to provide a means for successive adjustment of parameters which define a four-bar linkage to result in a convergence toward an optimum approximation to the desired coupler curve. McLarnan (8) modified Freudenstein's method for respacing the precision points by reducing the number of points which had to be shifted to obtain minimum structural error. He was able to reduce the number of these points to half the total number of mechanism parameters used in the synthesis. Timko (9) established a computer method for synthesizing a four-bar linkage that approximated a desired position relationship between output and input cranks. The criterion he used is based upon a least-error-squared fit of the curves at a number of checking points, say, twelve along the desired curve.

Many mechanical design requirements involve inequality constraints rather than equations. The problem of synthesis of four-bar mechanisms subjected to constraints, such as, limiting the transmission angle or lengths of links, or restriction on the location of the pivot points, etc., was treated by Fox and Willmert (10). The solution was found by using an iterative technique with the aid of digital computer. Using the nonlinear programming approach Tomas (11) reduced the complexity of the mathematical expressions and reduced the computation time required to solve this system of equations.

When the required accuracy can not be attained by using a four-bar linkage, an obvious alternative is to use a cam-link mechanism. The cam-link mechanism was discussed by Nickson (12) (13) for uses requiring more accuracy than can be developed by a four-bar linkage.

Huey and Dixon (14) developed a cam-link function generator with the cam being between the input shaft and the coupler. The work presented herein shows that the method may be extended to other function generators of different configuration and extends the method to the design of a cam-link path generator.

Chapter 3

FORMULATION OF THE PROBLEM

This chapter includes the development of the design equations for the problems defined in the introduction. The complete formulation to be used in this work is subdivided into four distinct subproblems:

A. Cam-link mechamism for function generation with the cam located in the input link,

B. Cam-link function generator with the cam located in the output link,

C. Cam-link function generator with the cam located in the base link, and

D. Path generating cam-link mechanism with the cam located in the input link.

The equations derived in this chapter are based on the results of previous work (1) (5) that has made complex variables a standard kinematic analysis tool.

Huey and Dixon (14) developed a cam link function generator with the cam-link used as the coupler. In this chapter the equations are derived for a cam-link function generator and a cam-link path generator with the cam as part of the input link. Since the equations for the other function generators are similar, their results only will be included.

Cam-Link Function Generator
With the Cam in the Input Link

A schematic diagram of a cam-link function generator with the cam used on the input link is shown in Fig. 1. An enlargement of the pressure angle section from Fig. 1 is shown in Fig. 2. The design parameters available for optimization for this mechanism are r_1, r_3, r_4, θ_{2_o}, and θ_{4_o}, where θ_{2_o}, and θ_{4_o}, are the initial values of θ_2 and θ_4, respectively. The inputs required are the desired range of θ_2, the maximum pressure angle, α, the functional relationship $\theta_4 = f_1(\theta_2)$, and $\frac{d\theta_4}{d\theta_2} = f_2(\theta_2)$. In this problem r_1 is assigned the value unity and the solution is an optimization of $\frac{r_2}{r_1}$ and $\frac{r_3}{r_1}$.

The solution is begun by writing the closure equation

$$r_1 e^{i\theta_1} + r_4 e^{i\theta_4} - r_5 e^{i\theta_5} = 0, \tag{3.1.1}$$

where the real part is

$$r_5 \cos \theta_5 = r_1 + r_4 \cos \theta_4 \tag{3.1.2}$$

and the imaginary part is

$$r_5 \sin \theta_5 = r_4 \sin \theta_4 \quad . \tag{3.1.3}$$

Using (3.1.3)

$$r_5 = r_4 \frac{\sin \theta_4}{\sin \theta_5} \tag{3.1.4}$$

and using (3.1.2) and (3.1.3)

$$\theta_5 = \text{arc tan} \frac{r_4 \sin \theta_4}{r_1 + r_4 \cos \theta_4} \quad . \tag{3.1.5}$$

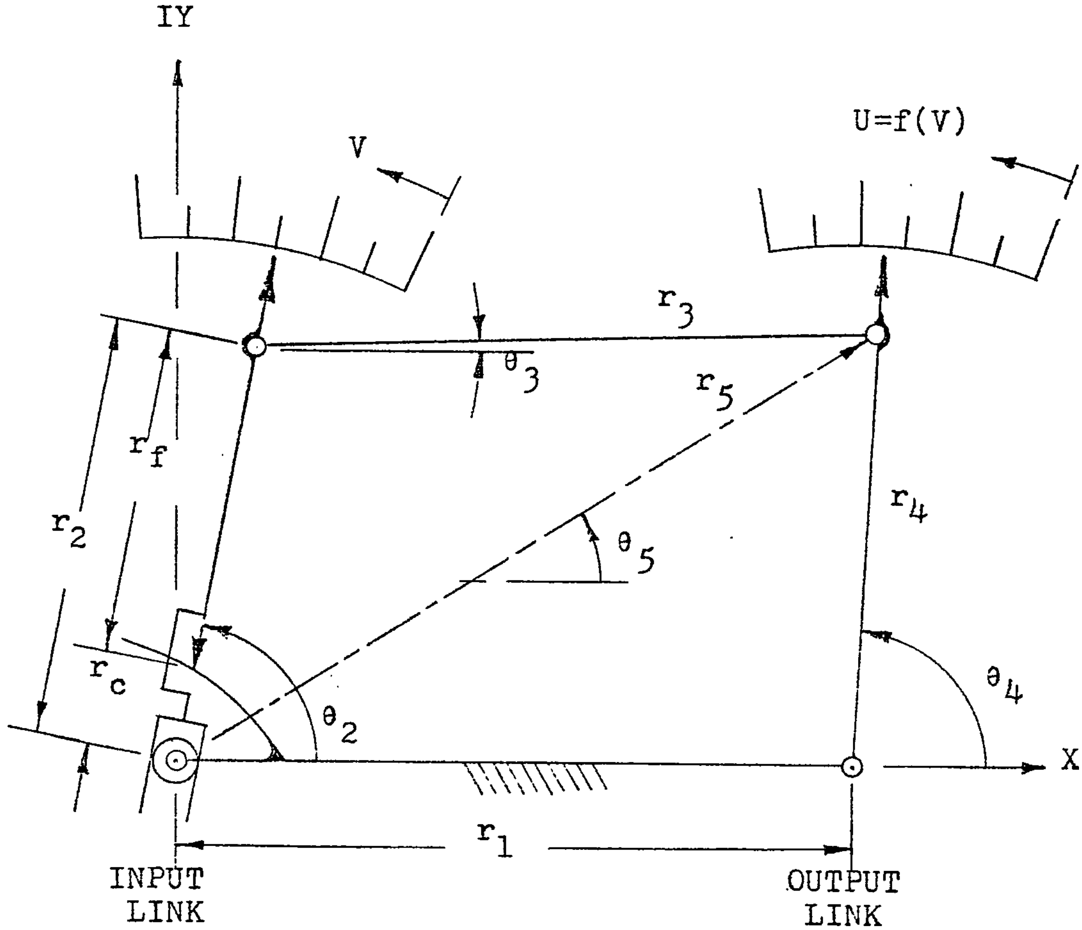

FIGURE 1

SCHEMATIC OF CAM-LINK MECHANISM
FOR FUNCTION GENERATION WITH CAM-LINK USED
IN INPUT LINK

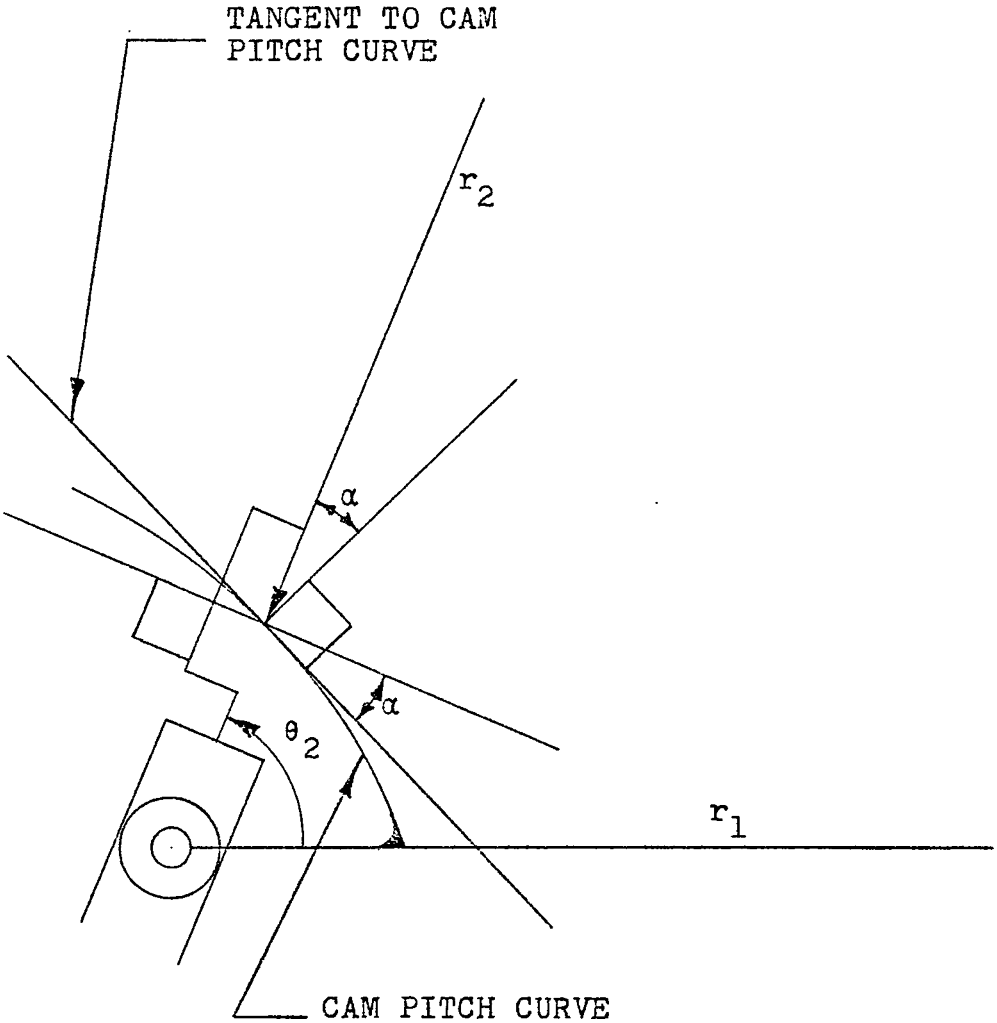

FIGURE 2

CAM PRESSURE ANGLE, α
SCHEMATIC SHOWN FOR FUNCTION GENERATOR
WITH CAM ON INPUT SHAFT,
OTHERS ARE SIMILAR

The angle, θ_5, must be placed in the proper quadrant.
Using the law of cosines

$$r_3^2 = r_5^2 - 2r_5 r_2 \cos(\theta_2 - \theta_5)$$

and changing the form to

$$r_2^2 + r_2[-2 r_5 \cos(\theta_2 - \theta_5)] + (r_5^2 - r_3^2) = 0,$$

the quadratic formula can be applied to obtain

$$r_2 = r_5 \cos(\theta_2 - \theta_5) \pm [r_5^2 \cos^2(\theta_2 - \theta_5) - r_5^2 + r_3^2]^{\frac{1}{2}} \qquad (3.1.6)$$

This gives two possible solutions, both will be solved and the solution with the lowest value for maximum cam slope will be selected.

Now to find the pressure angle, α, for a given set of design parameters.

The derivative of θ_5 with respect to θ_2 gives

$$\frac{d\theta_5}{d\theta_2} = \left\{1 + \left(\frac{r_4 \sin\theta_4}{r_1 + r_4 \cos\theta_4}\right)^2\right\}^{-1} \Big[(r_1 + r_4 \cos\theta_4)(r_4 \cos\theta_4 \frac{d\theta_4}{d\theta_2}) + (r_4^2 \sin^2\theta_4 \frac{d\theta_4}{d\theta_2})\Big] (r_1 + r_4 \cos\theta_4)^{-2}. \qquad (3.1.7)$$

The derivative of r_5 with respect to θ_2 gives

$$\frac{dr_5}{d\theta_2} = r_4\left\{\frac{\sin\theta_5 \cos\theta_4 \frac{d\theta_4}{d\theta_2}}{\sin^2\theta_5} - \frac{\sin\theta_4 \cos\theta_5 \frac{d\theta_5}{d\theta_2}}{\sin^2\theta_5}\right\} . \qquad (3.1.8)$$

The derivative of r_2 with respect to θ_2 gives

$$\frac{dr_2}{d\theta_2} = \frac{dr_5}{d\theta_2} \cos(\theta_2 - \theta_5) - r_5 \sin(\theta_2 - \theta_5)(1 - \frac{d\theta_5}{d\theta_2}) \pm [r_5^2 \cos^2(\theta_2 - \theta_5) - r_5^2 + r_3^2]^{-\frac{1}{2}} \left\{ r_5 \frac{dr_5}{d\theta_2} \cos^2(\theta_2 - \theta_5) + r_5^2 \cos(\theta_2 - \theta_5)[-\sin(\theta_2 - \theta_5)(1 - \frac{d\theta_5}{d\theta_2})] - r_5 \frac{dr_5}{d\theta_2} \right\} . \quad (3.1.9)$$

Now the cam pressure angle, α, can be shown to be

$$\alpha = \text{arc tan}[(\frac{dr_2}{d\theta_2})(\frac{1}{r_c})] \quad (3.1.10)$$

The optimization of the cam slope $\frac{dr_2}{d\theta_2}$ proceeds until the mechanism with the lowest maximum value is found. The value for α is a specified design parameter. α is chosen as a specific value for the pressure angle and the radius of the cam is calculated to be a minimum for this value. The minimization of the cam is based on the cam slope at each point on the cam surface. Since the optimization is based on the cam slope the chosen value of α may be exceeded. If the value of α is so important that exceeding the chosen value slightly is intolerable, the problem can be recycled with a change of radius at this point until the necessary reduction is obtained.

Cam-Link Function Generator With the Cam in the Output Link

A schematic diagram of a cam-link function generator with the cam in the output link is shown in Fig. 3. Since this mechanism is very similar to the previous one, only the results will be given.

The length of the output link is given by

$$r_4 = r_5 \cos(\theta_5 - \theta_4) \pm [r_5^2 \cos^2(\theta_5 - \theta_4) - r_5^2 + r_3^2]^{\frac{1}{2}} \quad .$$

Proceeding towards the solution,

$$\frac{d\theta_5}{d\theta_4} = [1 + (\frac{r_2 \sin\theta_2}{r_2 \cos\theta_2 - r_1})^2]^{-1} (r_2 \cos\theta_2 - r_1) \frac{(r_2 \cos\theta_2 \, d\theta_2/d\theta_4) + (r_2 \sin\theta_2)^2 \, d\theta_2/d\theta_4}{(r_2 \cos\theta_2 - r_1)^2} \, ,$$

$$\frac{dr_5}{d\theta_4} = \frac{r_2 \cos\theta_2 \, d\theta_2/d\theta_4}{\sin\theta_5} \frac{-r_2 \sin\theta_2 \cos\theta_5 \, d\theta_5/d\theta_4}{\sin^2\theta_5} \, ,$$

$$\frac{dr_4}{d\theta_4} = -r_5 \sin(\theta_5 - \theta_4)(\frac{d\theta_5}{d\theta_4} - 1) + \frac{dr_5}{d\theta_4} \cos(\theta_5 - \theta_4) + [r_5^2 \cos^2(\theta_5 - \theta_4) + r_3^2 - r_5^2]^{\frac{1}{2}} [r_5 \frac{dr_5}{d\theta_4} \cos^2(\theta_5 - \theta_4)$$

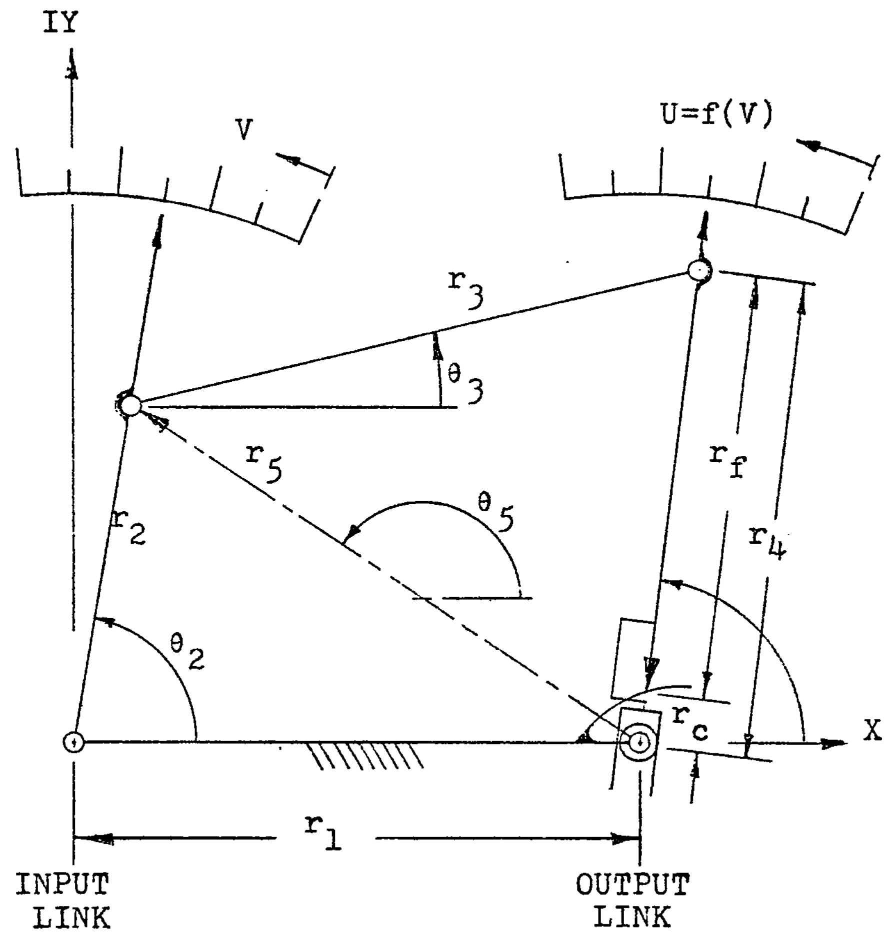

FIGURE 3

SCHEMATIC OF CAM-LINK MECHANISM FOR FUNCTION GENERATION WITH CAM-LINK USED IN OUTPUT LINK

$$-r_5^2 \cos(\theta_5 - \theta_4) \sin(\theta_5 - \theta_4) \left(\frac{d\theta_5}{d\theta_4} - 1\right) - r_5 \frac{dr_5}{d\theta_4} \Big] ,$$

and

$$\alpha = \arctan \left[\left(\frac{dr_4}{d\theta_4}\right) \left(\frac{1}{r_c}\right)\right] .$$

Cam-Link Function Generator
With the Cam in the Base Link

A schematic of a cam-link function generator with the cam used on the base link is shown in Fig. 4 . Since this mechamism is very similar to the previous two derivations only the results will be given.

The base length is given by

$$r_1 = r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 ,$$

where $\theta_3 = \text{arc} \sin(r_4 \sin \theta_4 - r_2 \sin \theta_2)/ r_3$.

Note that θ_3 has two possible values.

Following through,

$$\frac{d\theta_3}{d\theta_2} = \left\{ 1 - [(-r_2 \sin \theta_2 + r_4 \sin \theta_4)/r_3]^2 \right\}^{-\frac{1}{2}} [(-\frac{r_2}{r_3}) \cos \theta_2 + (\frac{r_4}{r_3}) \cos \theta_4 \frac{d\theta_4}{d\theta_2}] ,$$

$$\frac{dr_1}{d\theta_2} = -r_2 \sin \theta_2 - r_3 \sin \theta_3 \frac{d\theta_3}{d\theta_2} + r_4 \sin \theta_4 \frac{d\theta_4}{d\theta_2} ,$$

and

$$\alpha = \text{arc} \tan [(\frac{dr_1}{d\theta_2}) (\frac{1}{r_c})] .$$

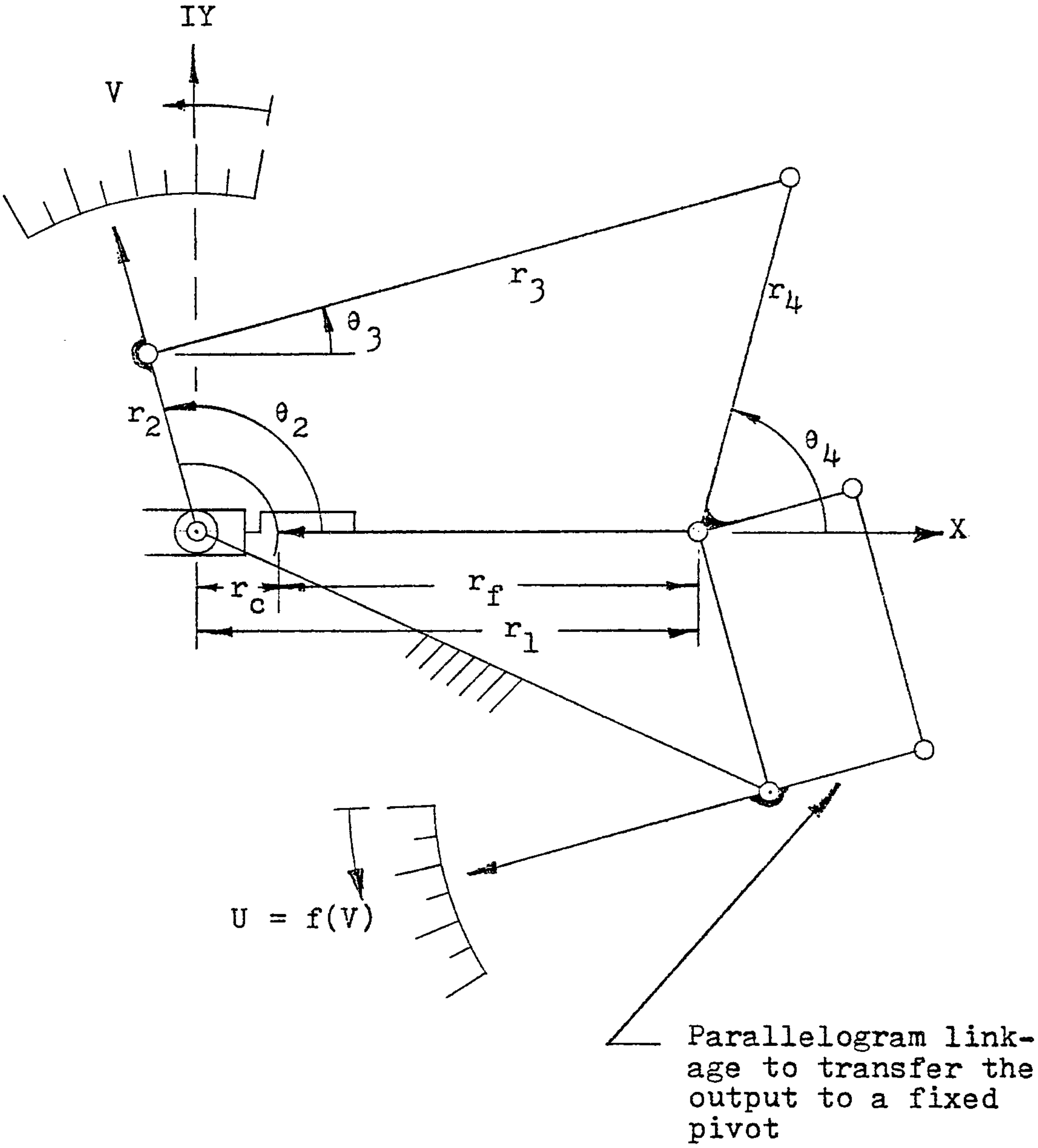

FIGURE 4

SCHEMATIC OF CAM-LINK MECHANISM
FOR FUNCTION GENERATION WITH CAM LINK USED
IN BASE LINK

Cam-Link Path Generator

A schematic diagram of a cam-link path generator with the cam in the input link is shown in Fig. 5 . The design parameters available for optimization for this mechanism have been defined as r_1, θ_1, r_3, r_4, r_5, r_6, ptl2x, and ptl2y. Where ptl2x is the x coordinate for the point 12 which is the point where link r_1 joins link r_2. The point, ptl2y, is similar and this usage with other points continues throughout this problem. The inputs required are the coupler path $p = f(\beta)$, $\frac{dp}{dx} = f_3(\beta)$, and the range of β. Where β is an input angle and if the path generator is optimum, the drift angle, DA, approaches zero.

The drift angle, DA, is defined as

$$DA = (\theta_2 - \theta_{2_o}) - \beta \tag{3.3.1}$$

where θ_{2_o} is the initial value of θ_2. In other words the change in β should in the optimum case equal the change in θ_2.

Although it is discussed later the merit function used by the optimizing scheme will be mentioned now. The merit function places a numerical value upon the quality of the linkage formed with a particular set of design parameters. It is important to note that not all sets of design parameter will work as a mechanism so they must be eliminated from the comparison early by a low merit value. The best mechanism

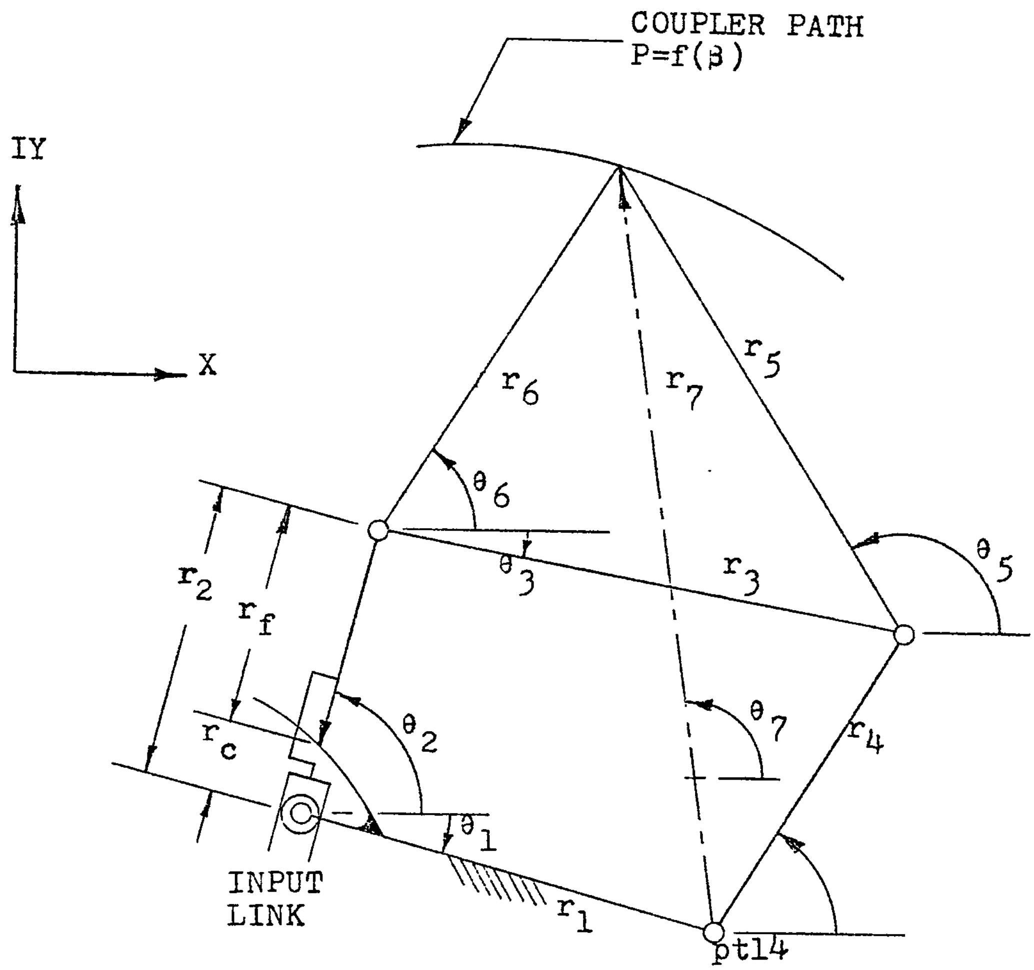

FIGURE 5

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION

will have the largest positive merit value. Once the mechanism is working it is optimized in this manner. The linkage is run through a complete cycle $\beta = 0$ to $\beta = \beta_{MAX}$. The maximum value of the cam pressure, α, is found and the largest value of the drift angle is found. The final merit value for the function is equal to the individual inverse of these two multiplied together. This way the lowest maximum cam angle and lowest maximum angle of drift may be found.

$$\text{Merit Value} = (\frac{1}{\alpha})\ (\frac{1}{DA}) \qquad (3.3.2)$$

A preliminary calculation is first run on the mechanism. As viewed in the schematic r_3, r_5, and r_6 form a triangle. The angle, θ_{56}, shown in Fig. 7 , will be needed later, so it is used to check if a triangle can exist.

Using the law of cosines

$$\theta_{56} = \text{arc cos}\ (\frac{r_6^2 + r_5^2 - r_3^2}{2\ r_6\ r_5})$$

and

$$A_3 = \frac{r_6^2 + r_5^2 - r_3^2}{2\ r_6\ r_5} \qquad (3.3.3)$$

If the argument of arc cos, A_3, is not equal or not between +1 and -1, the triangle cannot exist and the problem aborts giving a merit value of $-1000\ A_3^2$.

The solution is started by locating points pt12 and pt14 from the design parameters. The coupler point pt56 is defined to be equal to pt56 = p = f(β). (3.3.4)

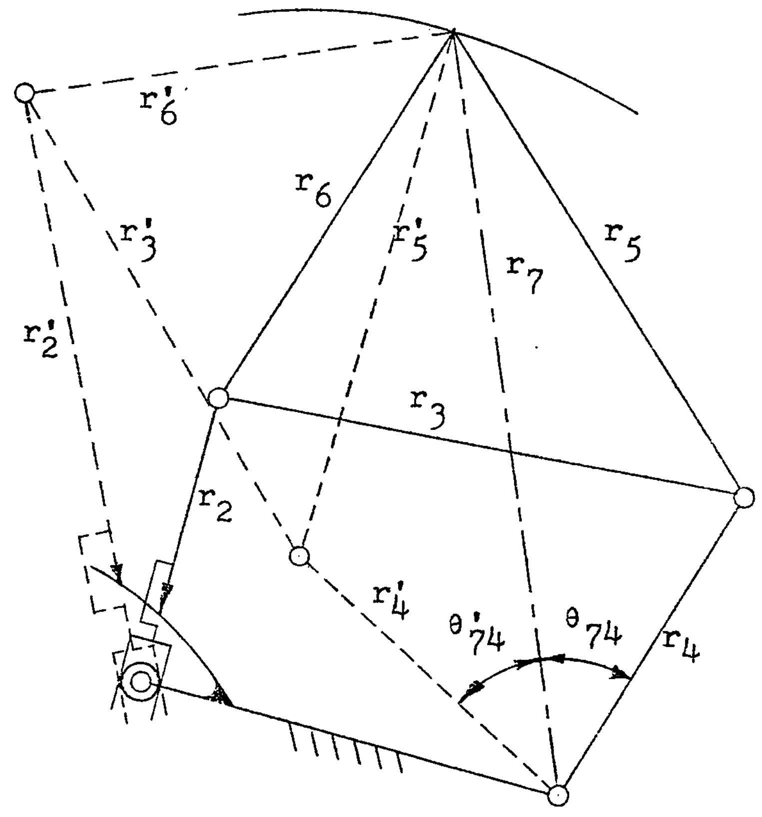

FIGURE 6

SCHEMATIC OF CAM-LINK MECHANISM FOR
PATH GENERATION
SHOWING TWO POSSIBLE SOLUTIONS
MADE AVAILABLE BY ROTATION ABOUT r_7

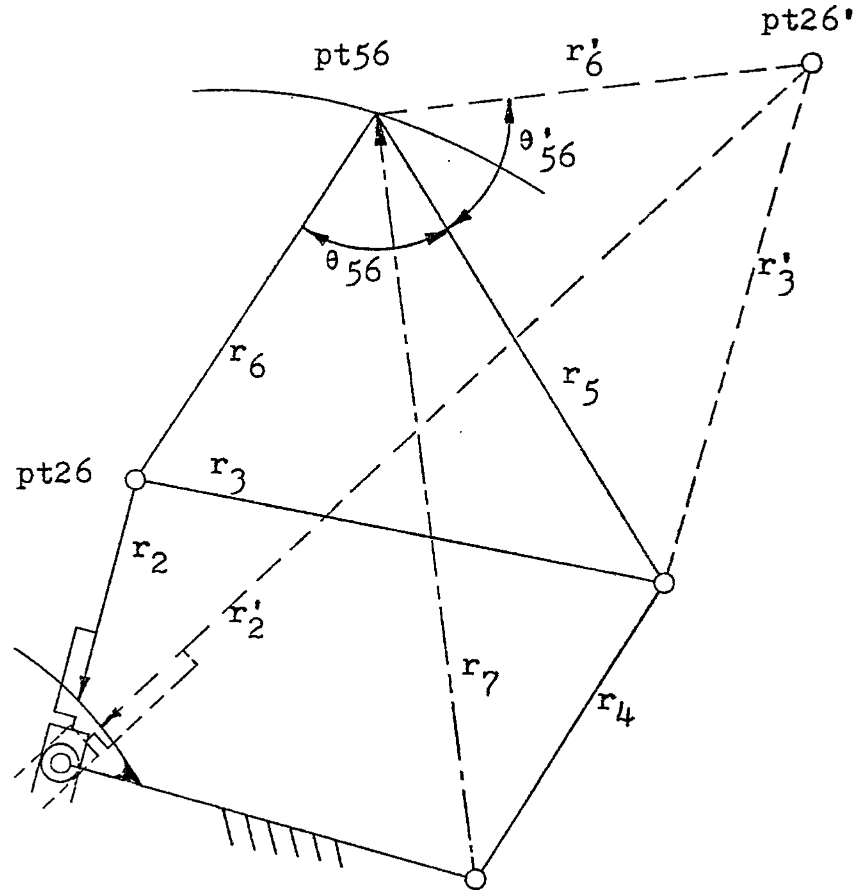

FIGURE 7

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION SHOWING TWO POSSIBLE SOLUTIONS MADE AVAILABLE BY ROTATION ABOUT r_5

The solution of r_7 is

$$r_7 = [(pt56x - pt14x)^2 + (pt56y - pt14y)^2]^{\frac{1}{2}} \quad (3.3.5)$$

$$\theta_7 = \arc\tan \frac{pt56y - pt14y}{pt56x - pt14x} \quad (3.3.6)$$

θ_7 must be placed in the proper quadrant.

Now to attach links r_4 and r_5. Using the law of cosines

$$\theta_{74} = \arc\cos\left(\frac{r_7^2 + r_4^2 - r_5^2}{2\, r_7\, r_4}\right)$$

where

$$A_7 = \frac{r_7^2 + r_4^2 - r_5^2}{2\, r_7\, r_4} \quad (3.3.7)$$

If A_7 is not between +1 and -1 or equal to them, the schematic triangle cannot exist. In this case the merit value is set to $-100\ A_7^2$ and the problem continues only calculating the value of A_7 at various locations to find the maximum value of A_7. This low merit value is then returned to the grid search program.

Once θ_{74} is found it can take either a positive or negative value as shown in Fig. 6. The angle θ_{75} is similar to θ_{74}. These two possibilities caused by the rotation about r_7 are not easily taken care of. The location of r_4 by θ_{74} can easily be started as either positive or negative by the choice of the initial conditions.

The problem arises when $\theta_4 = \theta_7 = \theta_5$ and two solutions degenerate into one solution. Later it breaks into two solutions again and there is the possibility that the solution desired could be either one. The proper solution is picked this way. The value of r_2 at the last cycle is incremented through the ideal increment of θ_2, and then the difference in distance between pt26 or pt36 and the tip of r_2 test is compared with the distance pt26' and the tip of r_2 test. The point, pt26 or pt26', whichever is closer to the tip of r_2 test, is chosen with the linkage containing it.

The angle θ_{56} has already been found. It may be either positive or negative as shown in Fig. . This creates no real problem though. The solution is tried both ways and the solution with the best merit value is chosen.

After the linkage configuration is decided upon by taking one of the four possibilities at a time, the merit value of the linkage is rather straight forward. It has been pointed out earlier how the merit values are arrived upon when the linkage is impossible to construct. The two items involved in the merit function of a good mechanism are the cam pressure angle, α, and the drift angle, DA. The maximum of each for the particular set of design parameters, and particular linkage configuration is found. Then each is inversed and the two are multiplied together to obtain the merit value. A final check is made to see that r_2 has never reversed its rotation. It would by a physical impossibility for the cam to have more than one radius at a particular angle. If the calculated linkage reverses itself, the linkage merit value is multiplied by a very small number 1×10^{-22}.

$$\text{Merit Value} = \left(\frac{1}{\alpha}\right)\left(\frac{1}{DA}\right)(AK) \tag{3.3.8}$$

where AK is a constant that is used as a flag. If the linkage input θ_2 is monotonic, increasing or decreasing, AK, is equal to one. If θ_2 reverses its rotation, AK is then set to 1×10^{-22} so that the merit value for this undesirable case will be very low.

The derivation of the cam pressure, α, is shown by the following procedure. The path to be generated $p = f(\beta)$ and the coupler path pt56 must be equal. So,

$$p = pt56 = f(\beta) \tag{3.3.4}$$

The function must be input in parametric form for this particular solution as

$$pt56x = f_1(\beta) \tag{3.3.9}$$

$$pt56y = f_2(\beta) \ . \tag{3.3.10}$$

Also required of the input function is

$$\frac{dpt56y}{dpt56x} = \frac{dy}{dx} = f_3(\beta) \ . \tag{3.3.11}$$

An auxilliary vector r_7 is used for this derivation and can be written as

$$r_7 = pt56 - pt14 \tag{3.3.12}$$

$$r_{7y} = pt56y - pt14y \tag{3.3.13}$$

$$r_{7x} = pt56x - pt14x \ . \tag{3.3.14}$$

Taking the derivative of the components

$$\frac{dr_{7x}}{dx} = \frac{dpt56x}{dx} - \frac{dpt14}{dx} \ .$$

Since pt14 is a stationary point, $\frac{dpt14x}{dx} = 0$,

and $\frac{dr_{7x}}{dx} = \frac{dpt56x}{dx} = \frac{dx}{dx} = 1$. (3.3.15)

Again

$$r_{7y} = pt56y - pt14y \qquad (3.3.13)$$

$$\frac{dr_{7y}}{dx} = \frac{dpt56}{dx} - \frac{dpt14y}{dx}$$

Since pt14 is a stationary point, $\frac{dpt14y}{dx} = 0$,

making

$$\frac{dr_{7y}}{dx} = \frac{dpt56y}{dx} = \frac{dy}{dx} = f_3(\beta) \ . \qquad (3.3.16)$$

Placing r_7 in easily obtainable form

$$r_7 = (r_{7x}^{\ 2} + r_{7y}^{\ 2})^{\frac{1}{2}} \qquad (3.3.17)$$

and taking its derivative

$$\frac{dr_7}{dx} = (r_{7x}^{\ 2} + r_{7y}^{\ 2})^{-\frac{1}{2}}(r_{7x} + r_{7y}\frac{dr_{7y}}{dx}) \ . \qquad (3.3.18)$$

The angle θ_7 is

$$\theta_7 = \text{arc tan}\ (\frac{r_{7y}}{r_{7x}}) \qquad (3.3.19)$$

and taking its derivative

$$\frac{d\theta_7}{dx} = [1 + (\frac{r_{7y}}{r_{7x}})^2]^{-1}[\frac{r_{7x}\ dr_{7y}/dx - r_{7y}\ dr_{7x}/dx}{r_{7x}^{\ 2}}] \ . \qquad (3.3.20)$$

Similar to θ_{74}, θ_{75} is found using the law of cosines as

$$\theta_{75} = \text{arc cos}\ \frac{r_7^{\ 2} + r_5^{\ 2} - r_4^{\ 2}}{2\ r_7\ r_5} \ . \qquad (3.3.21)$$

Taking its derivative

$$\frac{d\theta_{75}}{dx} = (-1)[1 - (\frac{r_7^{\ 2} + r_5^{\ 2} - r_4^{\ 2}}{2\ r_7\ r_5})^2]^{\frac{1}{2}} \quad [$$

$$\frac{4r_7^2\ r_5\ dr_7/dx - (r_7^2 + r_5^2 - r_4^2)(2r_5\ dr_7/dx)}{(2\ r_7\ r_5)^2}\ ]$$
(3.3.22)

Remembering from Fig. 5 and Fig. 6

$$\theta_5 \doteq \theta_7 \pm \theta_{75} \tag{3.3.23}$$

The plus or minus is chosen depending upon the final linkage desired. In practice both are tried and the one with the highest merit value is chosen.

$$\frac{d\theta_5}{dx} = \frac{d\theta_7}{dx} \pm \frac{d\theta_{75}}{dx} \quad . \tag{3.3.24}$$

With reference to Fig.

$$\theta_6 = \pm\ \theta_{56} + \theta_5 \tag{3.3.25}$$

$$\frac{d\theta_6}{dx} = \frac{d(\pm\ \theta_{56})}{dx} + \frac{d\theta_5}{dx}$$

and since θ_{56} is a constant,

$$\frac{d\theta_6}{dx} = \frac{d\theta_5}{dx} \tag{3.3.26}$$

r_6 is an input design parameter

$$r_{6x} = r_6 \cos \theta_6 \tag{3.3.27}$$

$$\frac{dr_{6x}}{dx} = -r_6 \sin \theta_6 \frac{d\theta_6}{dx} \tag{3.3.28}$$

From the geometry it can be seen that

$$pt26 = pt56 - r_6, \tag{3.3.29}$$

and for the x component

$$pt26x = pt56x - r_{6x} \tag{3.3.30}$$

Taking its derivative

$$\frac{dpt26x}{dx} = \frac{dpt56x}{dx} - \frac{dr_{6x}}{dx}$$

and remembering $\frac{dpt56x}{dx} = 1$ (3.3.15)

$$\frac{dp26x}{dx} = 1 - \frac{dr_{6x}}{dx} \qquad (3.3.31)$$

Continuing,

$$r_2 = pt26 = pt12 \qquad (3.3.32)$$

and its x components

$$r_{2x} = pt26x - pt12x \qquad (3.3.33)$$

Taking its derivative,

$$\frac{dr_{2x}}{dx} = \frac{dpt26x}{dx} - \frac{dpt12x}{dx}$$

And since pt12 is a stationary point,

$$\frac{dpt12x}{dx} = 0,$$

making

$$\frac{dr_{2x}}{dx} = \frac{dpt26x}{dx} \quad . \qquad (3.3.34)$$

Now for the y axis.

Again r_6 is an input parameter.

$$r_{6y} = r_6 \sin \theta_6 \qquad (3.3.35)$$

Taking its derivative,

$$\frac{dr_{6y}}{dx} = r_6 \cos \theta_6 \frac{d\theta_6}{dx} \qquad (3.3.36)$$

Taking the y components of pt26

$$pt26y = pt56y - r_{6y} \qquad (3.3.37)$$

Taking its derivative

$$\frac{dpt26y}{dx} = \frac{dpt56y}{dx} - \frac{dr_{6y}}{dx} \qquad (3.3.38)$$

and since

$$\frac{dpt56y}{dx} = \frac{dy}{dx} \qquad (3.3.11)$$

$$\frac{dpt26y}{dx} = \frac{dy}{dx} - \frac{dr_{6y}}{dx} \qquad (3.3.39)$$

Continuing through r_{2y}

$$r_{2y} = pt26y - pt12y \qquad (3.3.40)$$

$$\frac{dr_{2y}}{dx} = \frac{dpt26y}{dx} - \frac{dpt12y}{dx} \quad .$$

Since pt12y is a constant,

$$\frac{dpt12y}{dx} = 0.$$

And recalling

$$\frac{dpt26y}{dx}$$

making

$$\frac{dr_{2y}}{dx} = \frac{dy}{dx} - \frac{dr_{6y}}{dx} \quad . \qquad (3.3.41)$$

From the geometry

$$\theta_2 = \arc\tan\left(\frac{r_{2y}}{r_{2x}}\right) \quad , \qquad (3.3.42)$$

and taking its derivative

$$\frac{d\theta_2}{dx} = \left[1 + \left(\frac{r_{2y}}{r_{2x}}\right)^2\right]^{-1} \left(\frac{r_{2x}\ dr_{2y}/dx - r_{2y}\ dr_{2x}/dx}{r_{2x}^{\ 2}} \right) \quad . \qquad (3.3.43)$$

The link, r_2, may easily calculated as

$$r_2 = (r_{2x}^{\ 2} + r_{2y}^{\ 2})^{\frac{1}{2}} \qquad (3.3.44)$$

and taking its derivative

$$\frac{dr_2}{dx} = (r_{2x}^{\ 2} + r_{2y}^{\ 2})^{-\frac{1}{2}} \quad ($$

$$r_{2x} \frac{dr_{2x}}{dx} + r_{2y} \frac{dr_{2y}}{dx}) \quad . \qquad (3.3.45)$$

Finally

$$\frac{dr_2}{d\theta_2} = (\frac{dr_2}{dx}) (\frac{1}{d\theta_2/dx}) \quad . \qquad (3.3.46)$$

Now the cam pressure angle, α, can be shown as

$$\alpha = \text{arc tan} [(\frac{dr_2}{d\theta_2}) (\frac{1}{r_2 - r_F})] \quad . \qquad (3.3.47)$$

Where r_F is the length of the cam follower and $(r_2 - r_F)$ is the radius of the cam.

Optimization Scheme

To insure that a good design results when rough data is used as input, the optimization technique must do its job properly. The optimization method used in this paper was developed by Mischke and is thoroughly discussed in reference (15). It is a grid type search within regional constraints in a hyperspace. The use of a merit value, or a numerical quality of goodness value informs the search routine of the quality of its choice for a particular set of design parameters and allows it to converge upon the optimum.

The merit value used for the function generators is maximum cam slope incountered during a cycle of the mechanism inversed.

$$\text{Merit Value}_1 = \frac{1}{(dr_2/d\theta_2)_{MAX}}$$

The two different merit values used with the path generators use drift angle and cam slope or cam pressure angle. The drift angle is a measure of the difference of where the input link r_2 is and where it is desired.

$$\text{Merit Value}_2 = \frac{1}{(dr_2/d\theta_2)_{MAX}} \left(\frac{1}{DA}\right)$$

$$\text{Merit Value}_3 = \left(\frac{1}{\alpha_{MAX}}\right) \left(\frac{1}{DA}\right)$$

These are the two merit functions that were used in different problems to show that different merit functions are available.

Using the above merit functions, the computer hunts for the maximum merit value and its design parameters.

This numerical technique is used to size the n variables design parameters in a n dimension grid search. The search technique employed finds the maximum point on the merit hypersurface and uses the values of the variable parameters corresponding to this maximum point in designing the mechanism. The ability of a grid search routine to negotiate surface peculiarities that tend to trap other search schemes made it a logical choice. A simple computer flow diagram is shown in Fig. 8.

While under the control of the grid search routine, the computer program developed a pattern of points at the corners and center of a hypercube in the hyperspace of normalized variable parameters. The merit function is evaluated at each point. The point corresponding to the maximum value of the merit function was used as the center point for a new hypercube of reduced size. This process continued until the hypercube was sufficiently small. In this manner, the program converged to the set of optimum variable parameters.

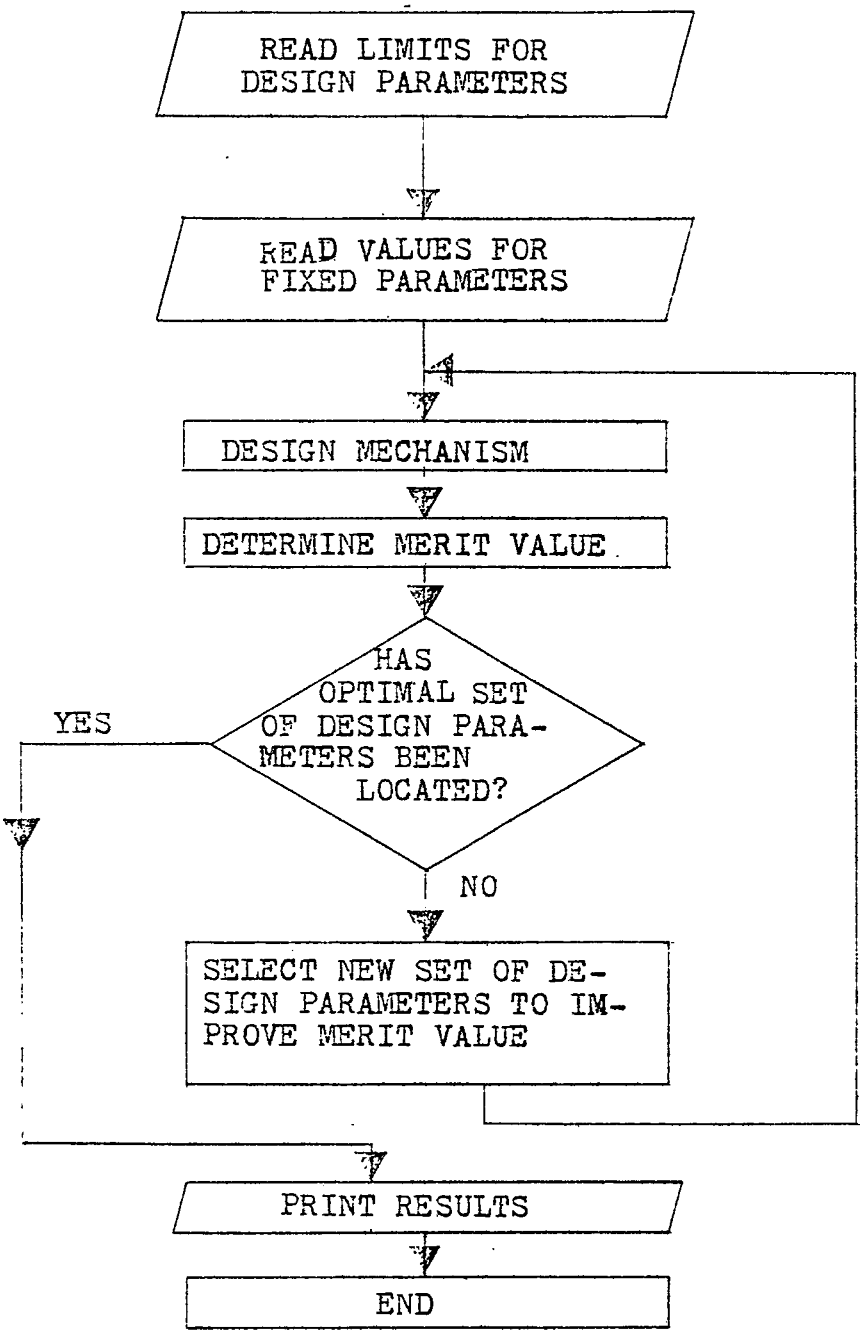

FIGURE 8

SIMPLIFIED COMPUTER FLOW DIAGRAM

Chapter 4

RESULTS

This section shows some of the possible types of problems that may be worked and the solutions that might be expected. Several problems were picked for demonstrating the function generators of different types.

The path generator is used to work two different coupler curves. The first problem is the upper right quarter of a square where its input angle rotated 90° as the coupler moved around the 90° of the coupler curve. This problem was worked several times to show what can be done to a particular problem. The second problem generates a cardioid showing that this method will generate a continuous and somewhat difficult form.

EXAMPLE PROBLEM NO. 1

This example illustrates the application of this synthesis technique to generate the function,

$$Y = X^2$$

Where X varies between -1 and +1

When this problem is worked using a standard four bar linkage, the accuracy point can be shifted to improve the mechanism's overall accuracy. The best fit with a standard non cam-link mechanism has an output error of 4.47% (1). These cam-link mechanisms developed here have no mathematical or theoretical error.

Part 1:

This mechanism uses the cam-link in its base and all dimensions are the same as the theoretical non cam-link four bar mechanism, that is except for the cam link. The mechanism is shown in Figure 9 and its computer output is shown in Figure 10. It may be noticed that the cam used on this mechanism is rather large to obtain the minimum maximum pressure angle of thirty degrees.

Part 2:

This mechanism is shown in Figure11 and its computer output is shown in Figure 12. It may be noticed that although this mechanism is fairly similar in appearance, the cam required is much smaller. On this mechanism the computer was given a broad range to optimize the link lengths and initial starting positions. The maximum radius of the cam was reduced from 1.165 to .290, a substantial reduction.

Part 3:

This mechanism is shown in Figure 13 and its computer output is shown in Figure 14. The mechanism uses the cam link in the input link. For a base length of one the maximum cam radius would be .284, which is the smallest cam yet, and this cam has a maximum pressure angle of only 15 degrees.

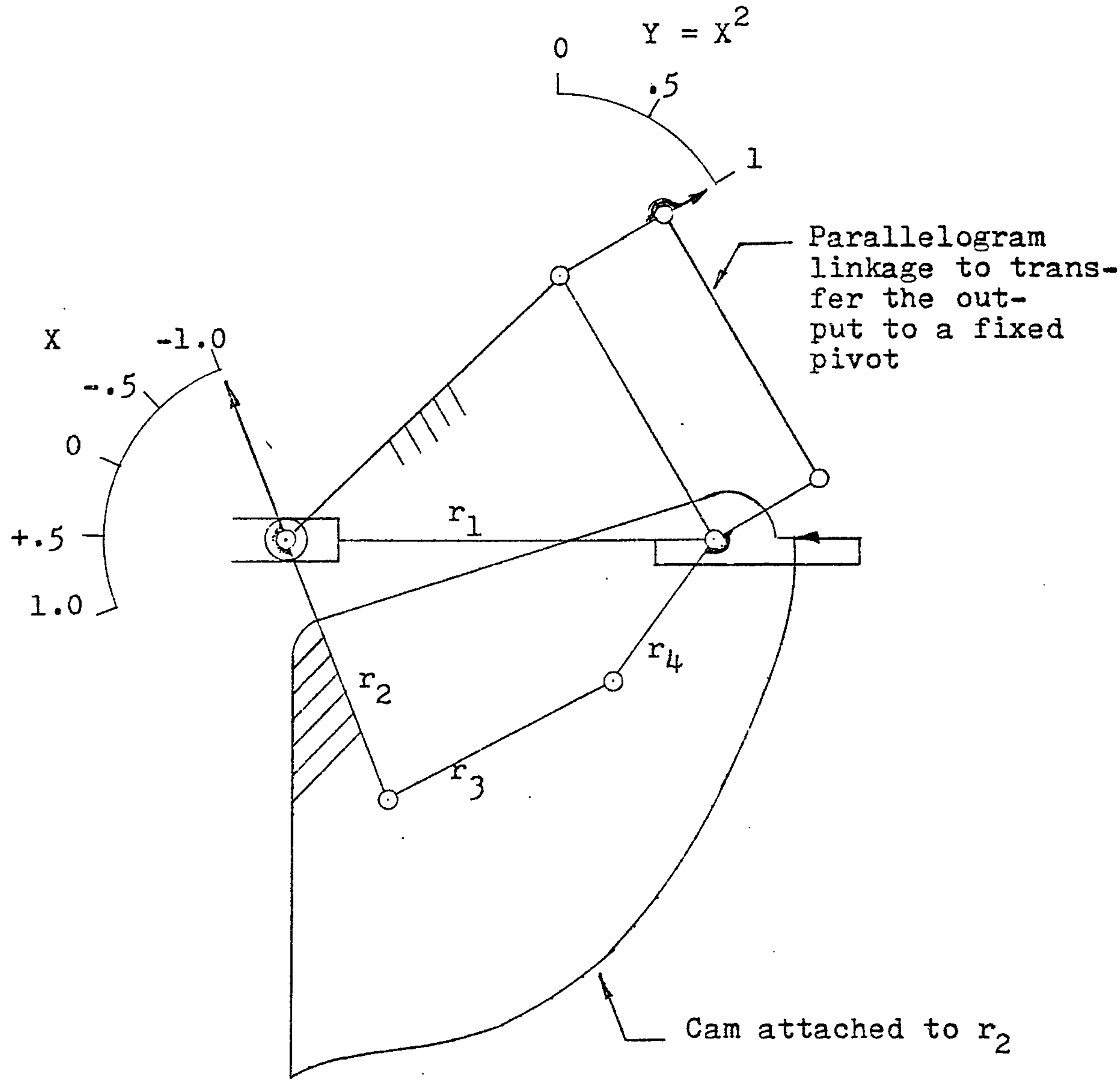

FIGURE 9

SCHEMATIC OF CAM-LINK MECHANISM TO GENERATE $Y = X^2$ WITH CAM USED IN BASE LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. 90.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
0.610	R2	0.610
0.566	R3	0.566
0.380	R4	0.380
-68.824	P20	-68.824
233.668	P40	233.668

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R2= 0.610 R3= 0.566
R4= 0.380 P20= -68.824
RF= -0.176 P40= 233.668

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR2/DP2	PRESSURE ANGLE ALPHA(DEG)
-68.824	233.668	1.123	0.182	9.216
-66.824	238.883	1.127	0.070	3.555
-64.823	243.861	1.128	-0.010	-0.501
-62.823	248.602	1.127	-0.064	-3.236
-60.823	253.106	1.124	-0.097	-4.911
-58.823	257.372	1.120	-0.113	-5.747
-56.823	261.402	1.116	-0.116	-5.924
-54.823	265.194	1.112	-0.109	-5.591
-52.823	268.750	1.108	-0.094	-4.870
-50.823	272.068	1.106	-0.075	-3.869
-48.823	275.150	1.103	-0.052	-2.678
-46.823	277.994	1.102	-0.026	-1.377
-44.823	280.602	1.101	-0.001	-0.035
-42.823	282.972	1.102	0.025	1.291
-40.823	285.105	1.103	0.049	2.552
-38.823	287.001	1.105	0.072	3.709
-36.823	288.661	1.108	0.092	4.733
-34.823	290.083	1.112	0.109	5.602
-32.823	291.268	1.116	0.123	6.302
-30.823	292.216	1.120	0.134	6.823
-28.823	292.927	1.125	0.141	7.162
-26.823	293.401	1.130	0.145	7.319
-24.823	293.638	1.135	0.145	7.298
-22.823	293.638	1.140	0.142	7.104
-20.823	293.401	1.145	0.135	6.744
-18.823	292.927	1.150	0.125	6.229
-16.823	292.216	1.154	0.112	5.568
-14.823	291.268	1.157	0.097	4.774
-12.823	290.083	1.160	0.078	3.860
-10.823	288.660	1.163	0.058	2.842
-8.823	287.001	1.164	0.035	1.738
-6.823	285.105	1.165	0.012	0.567
-4.823	282.971	1.165	-0.013	-0.645
-2.823	280.601	1.164	-0.038	-1.868
-0.823	277.994	1.163	-0.062	-3.060
1.177	275.149	1.160	-0.085	-4.168
3.177	272.068	1.157	-0.104	-5.116
5.177	268.749	1.153	-0.117	-5.797
7.177	265.194	1.149	-0.122	-6.054
9.177	261.401	1.145	-0.113	-5.648
11.177	257.371	1.141	-0.084	-4.227
13.177	253.105	1.139	-0.026	-1.284
15.177	248.601	1.140	0.075	3.785
17.177	243.860	1.145	0.230	11.351
19.177	238.882	1.156	0.439	20.766
21.177	233.668	1.176	0.679	30.000

FIGURE 10

COMPUTER OUTPUT FOR

$Y = X^2$

WITH CAM USED IN BASE LINK

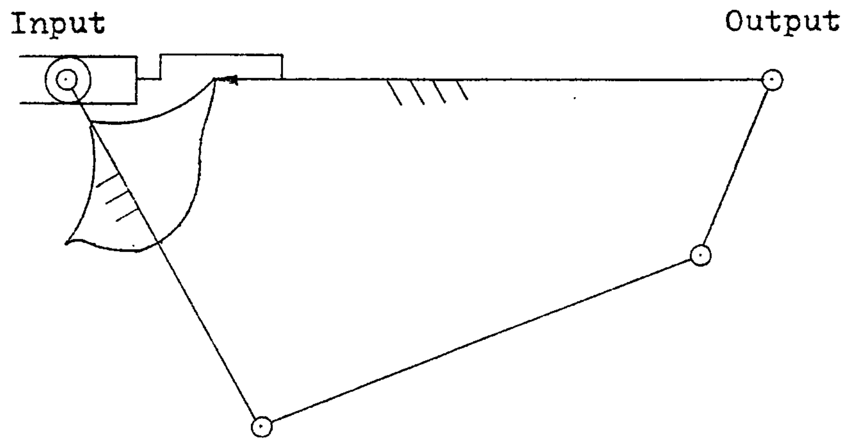

FIGURE 11

SCHEMATIC OF CAM-LINK MECHANISM
TO GENERATE $Y = X^2$ WITH CAM USED
IN BASE LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. 90.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
0.610	R2	0.610
0.300	R3	0.800
0.200	R4	0.700
-90.000	P20	0.0
200.000	P40	260.000

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R2=	0.610	R3=	0.736
R4=	0.292	P20=	-60.940
RF=	0.868	P40=	247.100

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR2/DP2	PRESSURE ANGLE ALPHA(DEG)
-60.940	247.099	0.229	0.047	11.588
-58.940	252.314	0.230	0.004	1.104
-56.940	257.292	0.229	-0.022	-5.574
-54.940	262.033	0.228	-0.036	-9.051
-52.940	266.536	0.227	-0.040	-10.017
-50.940	270.803	0.225	-0.036	-9.019
-48.940	274.833	0.224	-0.026	-6.487
-46.940	278.625	0.224	-0.011	-2.804
-44.940	282.181	0.224	0.006	1.637
-42.940	285.499	0.224	0.025	6.434
-40.940	288.581	0.225	0.045	11.206
-38.940	291.425	0.227	0.064	15.640
-36.940	294.033	0.230	0.081	19.521
-34.940	296.403	0.233	0.098	22.737
-32.940	298.536	0.237	0.112	25.251
-30.940	300.433	0.241	0.123	27.076
-28.940	302.092	0.245	0.132	28.249
-26.940	303.514	0.250	0.137	28.816
-24.940	304.699	0.255	0.140	28.819
-22.940	305.647	0.260	0.140	28.296
-20.940	306.359	0.264	0.136	27.278
-18.940	306.833	0.269	0.130	25.787
-16.940	307.070	0.273	0.121	23.843
-14.940	307.070	0.277	0.109	21.461
-12.940	306.833	0.281	0.095	18.660
-10.940	306.359	0.284	0.079	15.464
-8.940	305.647	0.287	0.060	11.906
-6.940	304.699	0.288	0.041	8.032
-4.940	303.514	0.289	0.020	3.907
-2.940	302.092	0.290	-0.002	-0.388
-0.940	300.433	0.289	-0.024	-4.755
1.060	298.536	0.288	-0.046	-9.084
3.060	296.403	0.286	-0.067	-13.258
5.060	294.033	0.283	-0.087	-17.161
7.060	291.425	0.280	-0.106	-20.674
9.060	288.581	0.276	-0.121	-23.680
11.060	285.499	0.272	-0.133	-26.050
13.060	282.181	0.267	-0.140	-27.630
15.060	278.625	0.262	-0.140	-28.216
17.060	274.833	0.257	-0.134	-27.511
19.060	270.803	0.253	-0.118	-25.064
21.060	266.536	0.249	-0.092	-20.212
23.060	262.033	0.246	-0.053	-12.097
25.060	257.292	0.245	-0.000	-0.105
27.060	252.314	0.246	0.066	14.924
29.060	247.099	0.250	0.144	30.000

FIGURE 12

COMPUTER OUTPUT FOR

$Y = X^2$

WITH CAM USED IN BASE LINK

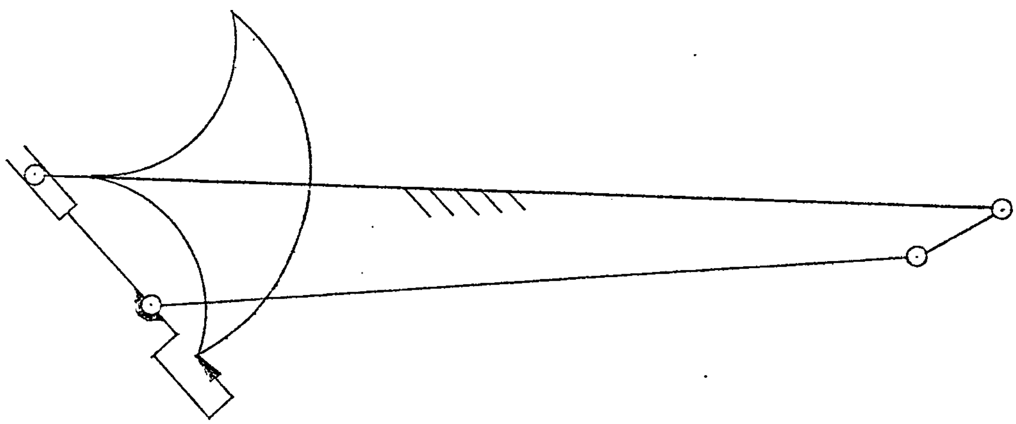

FIGURE 13

SCHEMATIC OF CAM-LINK MECHANISM
TO GENERATE $Y = X^2$ WITH CAM LINK USED
IN INPUT LINK

***** INPUT DATA *****

INPUT,P2,RANGE..................	90.000
PARAMETER RANGE REDUCTION.......	0.100
GRID SIZE REDUCTION INCREMENTS..	0.800
APPROXIMATE MAXIMUM	
CAM PRESSURE ANGLE,ALPHA........	15.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
10.000	R1	10.000
1.000	R3	10.000
1.000	R4	10.000
-100.000	P20	-40.000
200.000	P40	260.000

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	10.000	R3=	7.960
R4=	1.000	P20=	-46.722
RF=	-0.703	P40=	211.082

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR2/DP2	PRESSURE ANGLE ALPHA(DEG)
-46.722	211.082	2.486	-0.435	-9.925
-44.722	216.297	2.478	-0.023	-0.539
-42.722	221.274	2.483	0.261	5.996
-40.722	226.015	2.496	0.452	10.270
-38.722	230.519	2.514	0.575	12.894
-36.722	234.786	2.535	0.648	14.343
-34.722	238.815	2.559	0.684	14.963
-32.722	242.608	2.583	0.692	15.002
-30.722	246.163	2.607	0.681	14.638
-28.722	249.482	2.630	0.656	14.005
-26.722	252.563	2.652	0.622	13.200
-24.722	255.408	2.673	0.583	12.296
-22.722	258.015	2.693	0.540	11.348
-20.722	260.385	2.711	0.497	10.396
-18.722	262.519	2.728	0.455	9.468
-16.722	264.415	2.743	0.414	8.585
-14.722	266.074	2.757	0.375	7.757
-12.722	267.497	2.769	0.340	6.991
-10.722	268.682	2.780	0.306	6.289
-8.722	269.630	2.791	0.276	5.647
-6.722	270.341	2.800	0.248	5.061
-4.722	270.815	2.808	0.222	4.523
-2.722	271.052	2.815	0.198	4.021
-0.722	271.052	2.822	0.175	3.546
1.278	270.815	2.827	0.152	3.083
3.278	270.341	2.832	0.129	2.618
5.278	269.630	2.836	0.106	2.136
7.278	268.682	2.840	0.080	1.620
9.278	267.497	2.842	0.052	1.055
11.278	266.074	2.843	0.021	0.422
13.278	264.415	2.843	-0.015	-0.295
15.278	262.519	2.842	-0.055	-1.113
17.278	260.385	2.839	-0.102	-2.048
19.278	258.015	2.835	-0.154	-3.111
21.278	255.408	2.829	-0.213	-4.309
23.278	252.563	2.820	-0.279	-5.642
25.278	249.482	2.809	-0.350	-7.059
27.278	246.164	2.796	-0.425	-8.653
29.278	242.608	2.779	-0.503	-10.254
31.278	238.815	2.761	-0.578	-11.825
33.278	234.786	2.739	-0.645	-13.255
35.278	230.519	2.716	-0.696	-14.385
37.278	226.015	2.691	-0.721	-15.000
39.278	221.275	2.666	-0.705	-14.812
41.278	216.297	2.642	-0.631	-13.433
43.278	211.082	2.623	-0.478	-10.334

FIGURE 14

COMPUTER OUTPUT FOR

$Y = X^2$

WITH CAM USED IN INPUT LINK

EXAMPLE NO. 2

This example illustrates the application of this synthesis technique to generate the function,

$$Y = X^{2/3}$$

With X varying from zero to one

This mechanism in the optimum four bar linkage has an error of .162% (1). The cam link has reduced the error here to zero. This problem is worked twice using the cam in the output link. The first time the problem is worked using the dimension of the optimum four bar linkage. The mechanism formed is shown in Figure 15, and its computer output is shown in Figure 16. The cam has a maximum radius of .270. This same problem is tried again and the linkage is optimized to obtain the smallest maximum cam shape. The results are shown in Figure17, and its computer results are shown in Figure18. For the same maximum pressure angle on the cam the radius was reduced to .041.

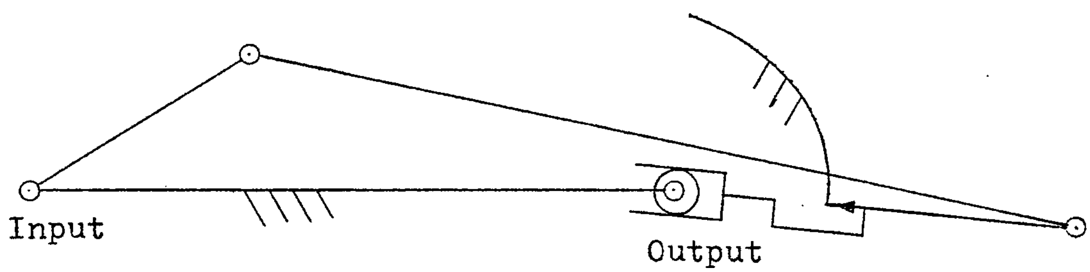

FIGURE 15

SCHEMATIC OF CAM-LINK MECHANISM
TO GENERATE $Y = X^{2/3}$ WITH CAM USED IN
OUTPUT LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................	90.000
PARAMETER RANGE REDUCTION.......	0.100
GRID SIZE REDUCTION INCREMENTS..	0.800
APPROXIMATE MAXIMUM CAM PRESSURE ANGLE,ALPHA........	30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
1.000	R1	1.000
0.401	R2	0.401
1.309	R3	1.309
31.689	P20	31.689
-5.171	P40	-5.170

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	1.000	R2=	0.401
R3=	1.309	P20=	31.689
RF=	0.390	P40=	-5.171

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR4/DP4	PRESSURE ANGLE ALPHA(DEG)
31.694	-5.028	0.234	0.056	13.336
33.694	1.956	0.237	0.003	0.744
35.694	6.132	0.237	0.001	0.152
37.694	9.636	0.237	0.002	0.428
39.694	12.763	0.237	0.004	0.947
41.694	15.638	0.237	0.006	1.524
43.694	18.326	0.238	0.009	2.084
45.694	20.868	0.238	0.011	2.598
47.694	23.291	0.239	0.013	3.050
49.694	25.615	0.239	0.014	3.435
51.694	27.855	0.240	0.016	3.752
53.694	30.020	0.240	0.017	4.004
55.694	32.122	0.241	0.018	4.192
57.694	34.165	0.242	0.018	4.323
59.694	36.157	0.242	0.019	4.399
61.694	38.102	0.243	0.019	4.427
63.694	40.004	0.243	0.019	4.411
65.694	41.867	0.244	0.019	4.358
67.694	43.694	0.245	0.018	4.272
69.694	45.487	0.245	0.018	4.161
71.694	47.249	0.246	0.017	4.028
73.694	48.982	0.246	0.017	3.883
75.694	50.687	0.247	0.016	3.732
77.694	52.367	0.247	0.015	3.582
79.694	54.023	0.248	0.015	3.443
81.694	55.655	0.248	0.014	3.322
83.694	57.267	0.248	0.014	3.229
85.694	58.857	0.249	0.014	3.175
87.694	60.429	0.249	0.014	3.175
89.694	61.981	0.250	0.014	3.237
91.694	63.516	0.250	0.015	3.381
93.694	65.034	0.250	0.016	3.624
95.694	66.535	0.251	0.017	3.982
97.694	68.022	0.251	0.020	4.479
99.694	69.493	0.252	0.023	5.132
101.694	70.949	0.252	0.026	5.977
103.694	72.392	0.253	0.031	7.030
105.694	73.822	0.254	0.037	8.330
107.694	75.239	0.255	0.045	9.905
109.694	76.643	0.256	0.053	11.775
111.694	78.035	0.258	0.064	13.975
113.694	79.417	0.259	0.077	16.523
115.694	80.787	0.261	0.092	19.422
117.694	82.146	0.264	0.110	22.658
119.694	83.494	0.267	0.131	26.204
121.694	84.832	0.270	0.156	30.000

FIGURE 16

COMPUTER OUTPUT FOR

$Y = X^{2/3}$

WITH CAM USED IN OUTPUT LINK

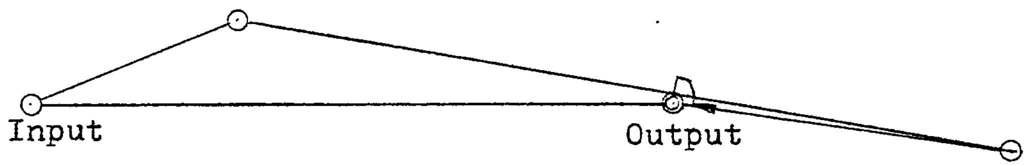

FIGURE 17

SCHEMATIC OF CAM-LINK
MECHANISM TO GENERATE
$Y = X^{2/3}$
WITH CAM USED IN OUTPUT LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. 90.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
1.000	R1	1.000
0.300	R2	0.500
1.100	R3	1.500
0.0	P20	60.000
-30.000	P40	30.000

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	1.000	R2=	0.345
R3=	1.220	P20=	21.995
RF=	0.493	P40=	-8.005

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR4/DP4	PRESSURE ANGLE ALPHA(DEG)
22.001	-7.862	0.034	0.004	5.843
24.001	-0.578	0.032	-0.019	-30.456
26.001	3.298	0.031	-0.014	-24.866
28.001	6.802	0.031	-0.009	-16.532
30.001	9.929	0.030	-0.004	-7.415
32.001	12.804	0.030	0.001	1.325
34.001	15.492	0.030	0.005	8.956
36.001	18.033	0.030	0.008	15.192
38.001	20.457	0.031	0.011	20.060
40.001	22.781	0.031	0.014	23.726
42.001	25.020	0.032	0.016	26.392
44.001	27.186	0.033	0.018	28.242
46.001	29.288	0.033	0.019	29.427
48.001	31.331	0.034	0.020	30.058
50.001	33.323	0.035	0.020	30.229
52.001	35.268	0.035	0.020	30.000
54.001	37.170	0.036	0.020	29.422
56.001	39.033	0.037	0.020	28.530
58.001	40.860	0.037	0.019	27.347
60.001	42.653	0.038	0.018	25.896
62.001	44.415	0.038	0.017	24.185
64.001	46.147	0.039	0.016	22.229
66.001	47.853	0.039	0.014	20.031
68.001	49.533	0.040	0.013	17.605
70.000	51.189	0.040	0.011	14.960
72.000	52.821	0.040	0.009	12.107
74.000	54.433	0.041	0.006	9.070
76.000	56.023	0.041	0.004	5.869
78.000	57.594	0.041	0.002	2.540
80.000	59.147	0.041	-0.001	-0.896
82.000	60.682	0.041	-0.003	-4.359
84.000	62.200	0.041	-0.006	-7.820
86.000	63.701	0.041	-0.008	-11.225
88.000	65.188	0.040	-0.010	-14.493
90.000	66.659	0.040	-0.013	-17.581
92.000	68.115	0.040	-0.015	-20.408
94.000	69.558	0.039	-0.017	-22.918
96.000	70.988	0.039	-0.018	-25.018
98.000	72.405	0.038	-0.019	-26.647
100.000	73.809	0.038	-0.020	-27.650
102.000	75.202	0.037	-0.020	-27.900
104.000	76.583	0.037	-0.019	-27.171
106.000	77.953	0.037	-0.017	-25.074
108.000	79.312	0.036	-0.014	-21.104
110.000	80.600	0.036	-0.009	-14.435
112.000	81.998	0.036	-0.003	-4.119

FIGURE 18

COMPUTER OUTPUT FOR

$Y = X^{2/3}$

WITH CAM USED IN OUTPUT LINK

EXAMPLE NO. 3

This example illustrates the application of this synthesis technique to generate the function,

$$Y = \sin X$$

Where the range of X is zero to ninety degrees

This example is worked using a cam link in the input link of the four bar mechanism. The optimum four bar for this function has an error of .21% (1).

The problem is worked the first time with the dimensions of the optimum four bar mechanism. The resultant mechanism is shown in Figure19 and its computer output is shown in Figure 20. The problem was then worked again allowing the computer to optimize the mechanism. The resulting mechanism is shown in Figure21 and its computer printout is shown in Figure 22. It can be noted that the maximum cam radius was reduced from .093 to .071.

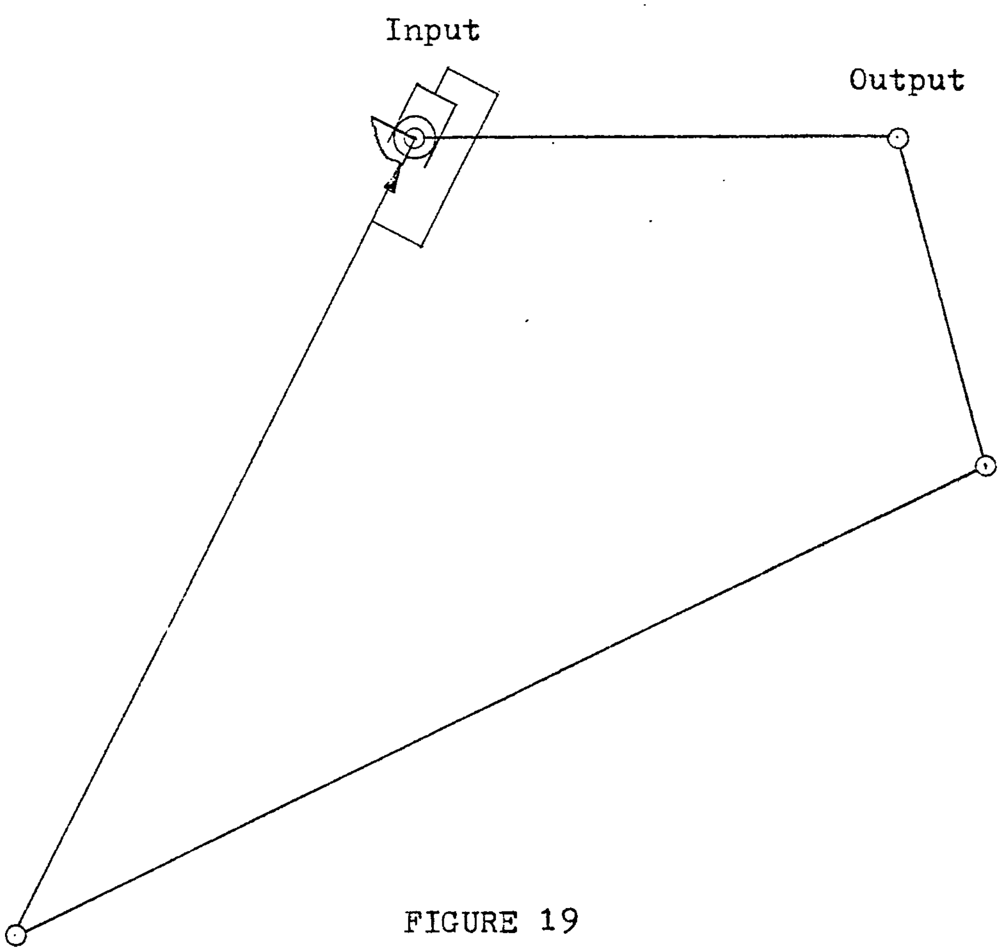

FIGURE 19

SCHEMATIC OF CAM-LINK
MECHANISM TO GENERATE
Y = SIN X
WITH CAM USED IN INPUT LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. -90.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
1.000	R1	1.000
2.239	R3	2.239
0.694	R4	0.694
242.263	P20	242.263
-75.607	P40	-75.606

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	1.000	R3=	2.239
R4=	0.694	P20=	242.263
RF=	1.775	P40=	-75.606

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR2/DP2	PRESSURE ANGLE ALPHA(DEG)
242.263	-75.606	0.059	0.045	37.421
240.263	-78.747	0.058	0.027	24.817
238.263	-81.884	0.057	0.011	11.116
236.263	-85.014	0.057	-0.002	-1.505
234.263	-88.132	0.057	-0.012	-11.504
232.263	-91.234	0.058	-0.020	-18.590
230.263	-94.318	0.059	-0.025	-23.184
228.263	-97.379	0.060	-0.029	-25.847
226.263	-100.414	0.061	-0.031	-27.041
224.263	-103.418	0.062	-0.032	-27.118
222.263	-106.388	0.063	-0.031	-26.330
220.263	-109.321	0.064	-0.030	-24.875
218.263	-112.213	0.065	-0.027	-22.914
216.263	-115.060	0.066	-0.025	-20.598
214.262	-117.859	0.067	-0.022	-18.062
212.262	-120.607	0.067	-0.019	-15.445
210.262	-123.299	0.068	-0.016	-12.880
208.262	-125.934	0.069	-0.013	-10.488
206.262	-128.507	0.069	-0.010	-8.381
204.262	-131.016	0.069	-0.008	-6.654
202.262	-133.458	0.070	-0.007	-5.380
200.262	-135.829	0.070	-0.006	-4.613
198.262	-138.126	0.070	-0.005	-4.392
196.262	-140.347	0.070	-0.006	-4.731
194.262	-142.490	0.070	-0.007	-5.625
192.262	-144.551	0.071	-0.009	-7.045
190.262	-146.528	0.071	-0.011	-8.938
188.262	-148.418	0.071	-0.014	-11.222
186.262	-150.220	0.072	-0.018	-13.791
184.262	-151.931	0.073	-0.022	-16.523
182.262	-153.549	0.073	-0.026	-19.283
180.262	-155.072	0.074	-0.030	-21.942
178.262	-156.498	0.076	-0.034	-24.381
176.262	-157.826	0.077	-0.038	-26.502
174.262	-159.053	0.078	-0.042	-28.225
172.262	-160.179	0.080	-0.045	-29.489
170.262	-161.202	0.081	-0.047	-30.241
168.261	-162.120	0.083	-0.049	-30.430
166.261	-162.933	0.085	-0.049	-30.000
164.261	-163.640	0.086	-0.048	-28.878
162.261	-164.239	0.088	-0.045	-26.969
160.261	-164.730	0.090	-0.040	-24.142
158.261	-165.113	0.091	-0.033	-20.228
156.261	-165.387	0.092	-0.025	-15.026
154.261	-165.551	0.093	-0.014	-8.335
152.261	-165.606	0.093	-0.000	-0.040

FIGURE 20

COMPUTER OUTPUT FOR
Y = SIN X
WITH CAM USED IN INPUT LINK

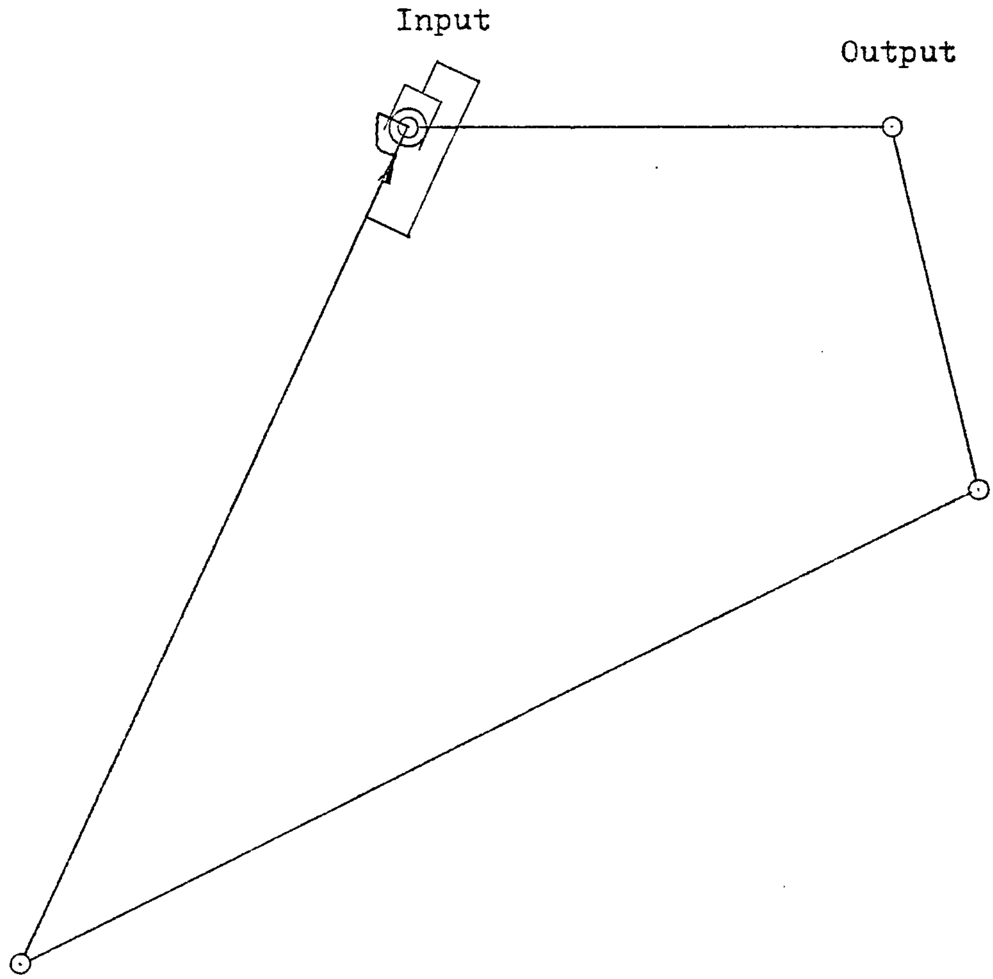

FIGURE 21

SCHEMATIC OF CAM-LINK
MECHANISM TO GENERATE
Y = SIN X
WITH CAM USED IN INPUT LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. -90.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
1.000	R1	1.000
2.000	R3	2.400
0.400	R4	0.900
200.000	P20	280.000
-100.000	P40	0.0

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	1.000	R3=	2.200
R4=	0.764	P20=	244.800
RF=	1.833	P40=	-77.334

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT, ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR2/DP2	PRESSURE ANGLE ALPHA(DEG)
244.800	-77.334	0.062	0.021	18.547
242.800	-80.475	0.062	0.005	4.480
240.800	-83.612	0.062	-0.008	-7.333
238.800	-86.741	0.062	-0.018	-15.901
236.800	-89.860	0.063	-0.025	-21.454
234.800	-92.962	0.064	-0.029	-24.608
232.799	-96.046	0.065	-0.032	-25.922
230.799	-99.107	0.066	-0.032	-25.804
228.799	-102.142	0.067	-0.031	-24.539
226.799	-105.146	0.068	-0.028	-22.329
224.799	-108.116	0.069	-0.024	-19.340
222.799	-111.049	0.070	-0.020	-15.719
220.799	-113.941	0.071	-0.015	-11.624
218.799	-116.788	0.071	-0.009	-7.225
216.799	-119.587	0.071	-0.003	-2.701
214.799	-122.335	0.071	0.002	1.765
212.799	-125.027	0.071	0.007	6.010
210.799	-127.662	0.071	0.012	9.896
208.799	-130.235	0.070	0.017	13.321
206.799	-132.744	0.070	0.020	16.212
204.799	-135.186	0.069	0.023	18.521
202.799	-137.557	0.068	0.025	20.221
200.799	-139.854	0.067	0.026	21.287
198.799	-142.075	0.066	0.026	21.702
196.799	-144.218	0.065	0.026	21.447
194.799	-146.279	0.065	0.024	20.502
192.799	-148.256	0.064	0.022	18.851
190.799	-150.146	0.063	0.019	16.492
188.799	-151.948	0.062	0.015	13.454
186.798	-153.659	0.062	0.011	9.806
184.798	-155.277	0.062	0.006	5.680
182.798	-156.800	0.062	0.001	1.277
180.798	-158.226	0.062	-0.003	-3.156
178.798	-159.554	0.062	-0.008	-7.346
176.798	-160.781	0.062	-0.012	-11.037
174.798	-161.907	0.063	-0.016	-14.017
172.798	-162.930	0.063	-0.018	-16.113
170.798	-163.848	0.064	-0.020	-17.191
168.798	-164.661	0.065	-0.020	-17.116
166.798	-165.368	0.065	-0.018	-15.734
164.798	-165.967	0.066	-0.015	-12.841
162.798	-166.458	0.066	-0.010	-8.181
160.798	-166.841	0.066	-0.002	-1.486
158.798	-167.115	0.066	0.009	7.380
156.798	-167.279	0.066	0.022	18.163
154.798	-167.334	0.065	0.037	30.000

FIGURE 22

COMPUTER OUTPUT FOR
Y = SIN X
WITH CAM USED IN INPUT LINK

EXAMPLE NO. 4

This example illustrates the application of this synthesis technique to generate the function,

$$Y = LOG\ X$$

Where X varies from 1 to 2

This example uses the dimension of the optimum four bar linkage without a cam. The mechanism is shown in Figure 23 and its computer output is shown in Figure 24.

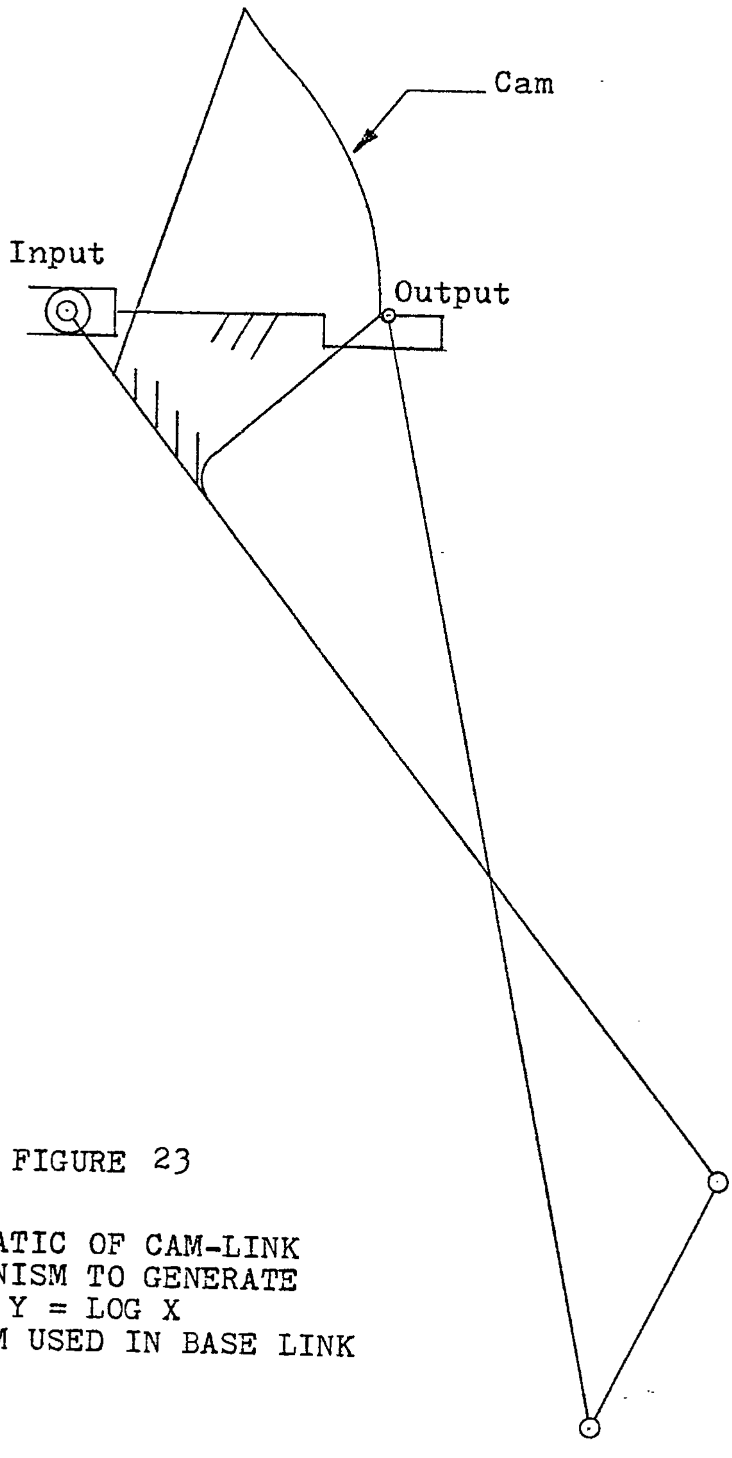

FIGURE 23

SCHEMATIC OF CAM-LINK
MECHANISM TO GENERATE
Y = LOG X
WITH CAM USED IN BASE LINK

***** INPUT DATA *****

INPUT,P2,RANGE.................. -60.000
PARAMETER RANGE REDUCTION....... 0.100
GRID SIZE REDUCTION INCREMENTS.. 0.800
APPROXIMATE MAXIMUM
CAM PRESSURE ANGLE,ALPHA........ 30.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETERS	UPPER LIMIT
3.352	R2	3.352
0.846	R3	0.846
3.486	R4	3.486
-52.628	P20	-52.628
-79.078	P40	-79.077

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R2= 3.352 R3= 0.846
R4= 3.486 P20= -52.628
RF= 0.026 P40= -79.077

** ONE CYCLE OF MECHANISM **

INPUT ANGLE P2(DEG)	OUTPUT ANGLE P4(DEG)	RADIUS OF CAM	CAM SLOPE DR1/DP2	PRESSURE ANGLE ALPHA(DEG)
-52.628	-79.077	0.974	-0.082	-4.810
-53.962	-80.980	0.976	-0.075	-4.404
-55.295	-82.841	0.977	-0.069	-4.020
-56.628	-84.664	0.979	-0.063	-3.675
-57.962	-86.449	0.980	-0.058	-3.368
-59.295	-88.197	0.981	-0.053	-3.102
-60.628	-89.911	0.983	-0.049	-2.874
-61.962	-91.592	0.984	-0.046	-2.682
-63.295	-93.241	0.985	-0.043	-2.522
-64.628	-94.859	0.986	-0.041	-2.393
-65.961	-96.447	0.987	-0.039	-2.292
-67.295	-98.007	0.988	-0.038	-2.216
-68.628	-99.539	0.989	-0.037	-2.163
-69.961	-101.045	0.989	-0.037	-2.132
-71.295	-102.524	0.990	-0.037	-2.120
-72.628	-103.979	0.991	-0.037	-2.128
-73.961	-105.410	0.992	-0.037	-2.153
-75.295	-106.817	0.993	-0.038	-2.194
-76.628	-108.203	0.994	-0.039	-2.251
-77.961	-109.566	0.995	-0.040	-2.325
-79.295	-110.908	0.996	-0.042	-2.413
-80.628	-112.229	0.997	-0.044	-2.517
-81.961	-113.531	0.998	-0.046	-2.636
-83.294	-114.813	0.999	-0.048	-2.771
-84.628	-116.077	1.000	-0.051	-2.924
-85.961	-117.323	1.001	-0.054	-3.094
-87.294	-118.550	1.002	-0.058	-3.284
-88.628	-119.761	1.004	-0.061	-3.496
-89.961	-120.955	1.005	-0.066	-3.731
-91.294	-122.133	1.007	-0.070	-3.995
-92.628	-123.295	1.009	-0.076	-4.289
-93.961	-124.441	1.010	-0.082	-4.621
-95.294	-125.573	1.012	-0.088	-4.996
-96.628	-126.690	1.015	-0.096	-5.423
-97.961	-127.792	1.017	-0.105	-5.913
-99.294	-128.881	1.019	-0.116	-6.481
-100.628	-129.957	1.022	-0.128	-7.146
-101.961	-131.019	1.025	-0.143	-7.934
-103.294	-132.068	1.029	-0.161	-8.881
-104.627	-133.104	1.033	-0.183	-10.038
-105.961	-134.129	1.038	-0.211	-11.476
-107.294	-135.141	1.043	-0.247	-13.304
-108.627	-136.142	1.049	-0.295	-15.690
-109.961	-137.131	1.057	-0.362	-18.902
-111.294	-138.109	1.066	-0.461	-23.400
-112.627	-139.077	1.079	-0.623	-30.000

FIGURE 24

COMPUTER OUTPUT FOR
Y = LOG X
WITH CAM USED IN BASE LINK

EXAMPLE NO. 5

Path generator to generate upper right quarter of a square

This mechanism is shown in Figure25 and its computer output is shown in Figure26. The mechanism was obtained early during the development of this study by optimizing a general set of data. It has a close resemblance to the schematic for the development of the equation and for this reason it makes a very good example problem. This example was optimized to obtain the minimum cam pressure angle and also to obtain the minimum drift angle.

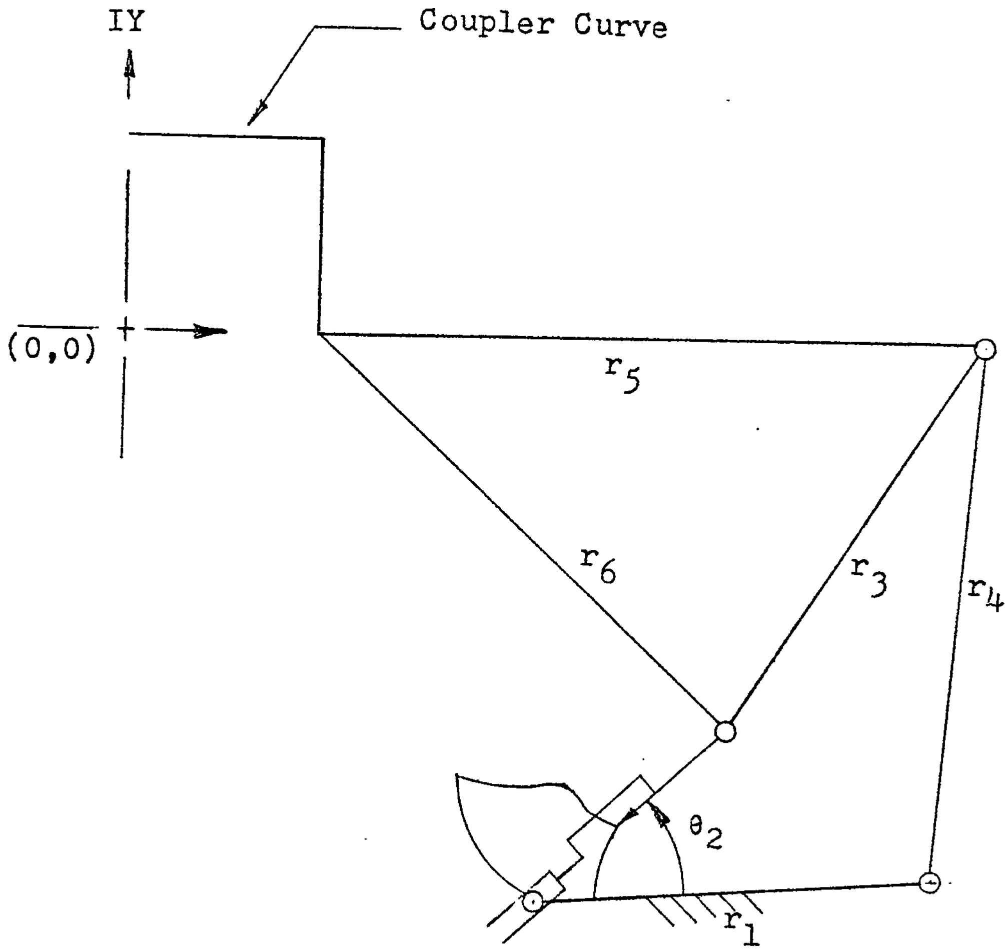

FIGURE 25

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION

***** PATH GENERATOR *****

***** INPUT DATA *****

INPUT,BETA RANGE..................	90.000
PARAMETER RANGE REDUCTION........	0.100
GRID SIZE REDUCTION INCREMENTS...	0.800
CAM FOLLOWER LENGTH,RF...........	0.750

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETER	UPPER LIMIT
2.059	R1	2.059
4.037	P1	4.037
2.381	R3	2.381
2.763	R4	2.763
3.446	R5	3.446
2.942	R6	2.942
2.169	PT12X	2.169
-2.885	PT12Y	-2.885

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	2.059	P1=	4.037
R3=	2.381	R4=	2.763
R5=	3.446	R6=	2.942
PT12X=	2.169	PT12Y=	-2.885

** ONE CYCLE OF MECHANISM **

INPUT BETA (DEG)	DRIFT ANGLE DA(DEG)	MECHANISM DRIVE P2(DEG)	RADIUS OF CAM RC	PRESSURE ANGLE ALPHA(DEG)	LOCATION OF COUPLER POINT X	Y
0.0	0.0	41.897	0.586	-32.087	1.000	0.0
8.182	-3.908	46.171	0.564	-23.136	1.000	0.144
16.364	-7.268	50.993	0.549	-11.487	1.000	0.294
24.545	-9.847	56.595	0.545	3.098	1.000	0.457
32.727	-11.334	63.290	0.558	19.430	1.000	0.643
40.909	-11.313	71.493	0.602	34.868	1.000	0.867
49.091	-9.250	81.738	0.618	-18.815	0.867	1.000
57.273	-7.565	91.605	0.599	-2.054	0.643	1.000
65.455	-6.983	100.369	0.608	13.750	0.457	1.000
73.636	-7.264	108.269	0.640	26.290	0.294	1.000
81.818	-8.179	115.536	0.691	35.531	0.144	1.000
90.000	-9.513	122.384	0.762	42.353	0.000	1.000

FIGURE 26

COMPUTER OUTPUT FOR
A CAM-LINK PATH GENERATOR
TO GENERATE UPPER RIGHT QUARTER OF A SQUARE
OPTIMIZING THE DRIFT ANGLE AND PRESSURE ANGLE

EXAMPLE NO. 6

Path generator to generate the right top quarter of a square

The optimization of this mechanism is shown in the computer output of Figure 29. The cam follower was short and the cam had very low pressure angles so the dimensions for the basic mechanism were put into the computer to generate a new mechanism with a longer cam follower. The mechanism is shown in Figure27 and its computer output is shown in Figure 28.

This mechanism was optimized for the cam pressure angle along with the drift angle. Notice that when optimized with a cam follower of length one, it is drawn with a cam follower of four, how the length of the input link grew to a tremendously long link. This will be discussed more later.

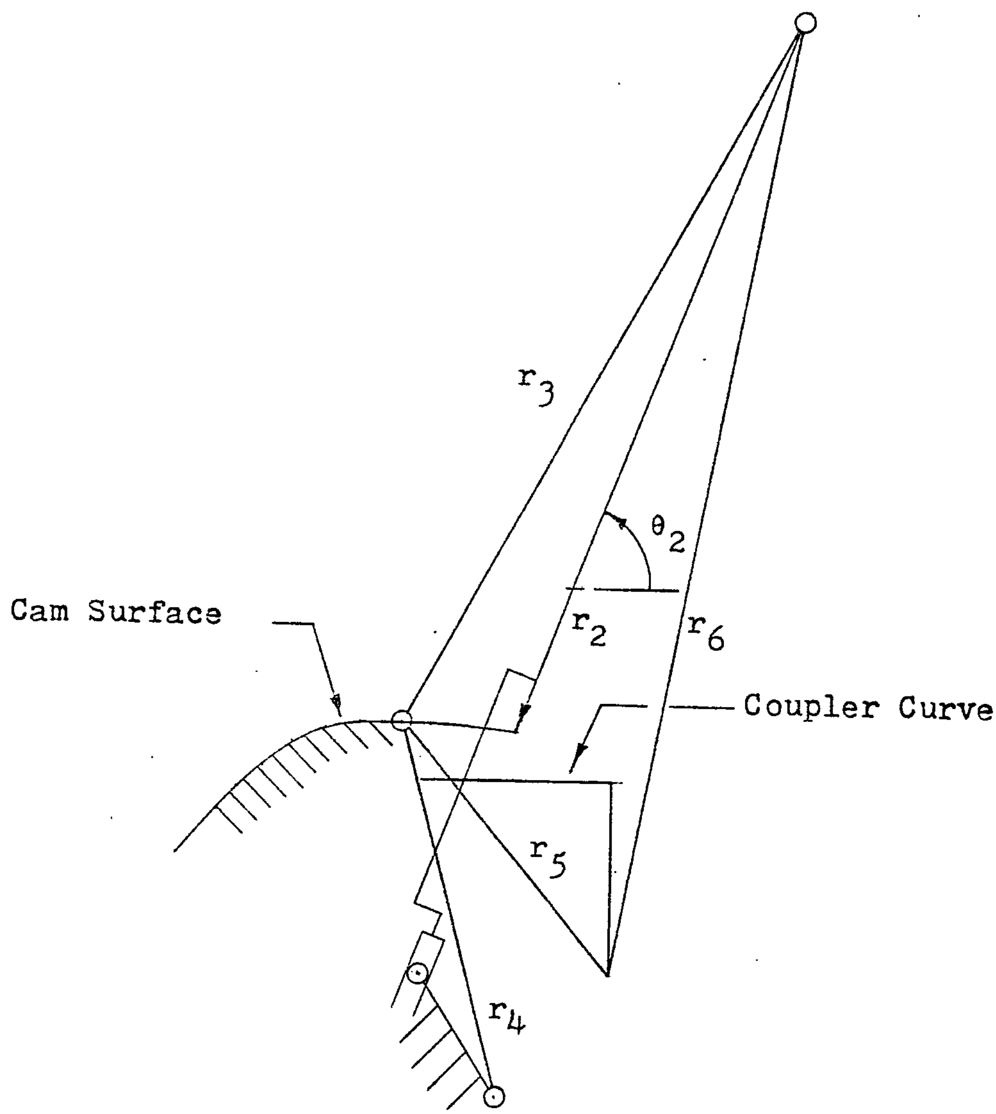

FIGURE 27

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION

***** PATH GENERATOR *****

***** INPUT DATA *****

INPUT,BETA RANGE..................	90.000
PARAMETER RANGE REDUCTION........	0.100
GRID SIZE REDUCTION INCREMENTS...	0.800
CAM FOLLOWER LENGTH,RF...........	4.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETER	UPPER LIMIT
0.746	R1	0.746
308.065	P1	308.065
4.167	R3	4.167
2.000	R4	2.000
1.697	R5	1.697
5.000	R6	5.000
0.0	PT12X	0.0
0.0	PT12Y	0.0

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	0.746	P1=	308.065
R3=	4.167	R4=	2.000
R5=	1.697	R6=	5.000
PT12X=	0.0	PT12Y=	0.0

** ONE CYCLE OF MECHANISM **

INPUT BETA (DEG)	DRIFT ANGLE DA(DEG)	MECHANISM DRIVE P2(DEG)	RADIUS OF CAM RC	PRESSURE ANGLE ALPHA(DEG)	LOCATION OF COUPLER POINT X	Y
0.0	0.0	64.948	1.341	-12.271	1.000	0.0
8.182	2.475	75.604	1.300	-6.176	1.000	0.144
16.364	3.432	84.744	1.289	0.297	1.000	0.294
24.545	3.408	92.902	1.300	6.309	1.000	0.457
32.727	2.831	100.507	1.327	11.031	1.000	0.643
40.909	2.089	107.946	1.366	13.302	1.000	0.867
49.091	2.127	116.166	1.365	-10.169	0.867	1.000
57.273	2.177	124.398	1.341	-3.843	0.643	1.000
65.455	1.445	131.848	1.338	1.885	0.457	1.000
73.636	0.078	138.662	1.350	6.804	0.294	1.000
81.818	-1.754	145.013	1.374	10.912	0.144	1.000
90.000	-3.880	151.068	1.407	14.278	0.000	1.000

FIGURE 28

COMPUTER OUTPUT FOR
A CAM-LINK PATH GENERATOR
GENERATES THE RIGHT TOP QUARTER OF A SQUARE.
THIS MECHANISM WAS PRODUCED BY TAKING THE DATA FROM
FIGURE29 AND INCREASING THE LINGTH OF THE CAM FOLLOWER RF.

***** PATH GENERATOR *****

***** INPUT DATA *****

INPUT,BETA RANGE................	90.000
PARAMETER RANGE REDUCTION........	0.100
GRID SIZE REDUCTION INCREMENTS...	0.900
CAM FOLLOWER LENGTH,RF...........	1.000

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETER	UPPER LIMIT
0.500	R1	5.000
0.0	P1	360.000
0.500	R3	5.000
0.500	R4	5.000
0.500	R5	5.000
0.500	R6	5.000
0.0	PT12X	0.0
0.0	PT12Y	0.0

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	0.746	P1=	308.066
R3=	4.167	R4=	2.000
R5=	1.697	R6=	5.000
PT12X=	0.0	PT12Y=	0.0

** ONE CYCLE OF MECHANISM **

INPUT BETA (DEG)	DRIFT ANGLE DA(DEG)	MECHANISM DRIVE P2(DEG)	RADIUS OF CAM RC	PRESSURE ANGLE ALPHA(DEG)	LOCATION OF COUPLER POINT X	Y
0.0	0.0	64.948	4.341	-3.844	1.000	0.0
8.182	2.475	75.604	4.300	-1.874	1.000	0.144
16.364	3.432	84.744	4.289	0.089	1.000	0.294
24.545	3.408	92.902	4.300	1.914	1.000	0.457
32.727	2.831	100.506	4.327	3.421	1.000	0.643
40.909	2.089	107.946	4.366	4.230	1.000	0.867
49.091	2.127	116.166	4.365	-3.210	0.867	1.000
57.273	2.177	124.398	4.341	-1.189	0.643	1.000
65.455	1.445	131.848	4.338	0.582	0.457	1.000
73.636	0.078	138.662	4.350	2.121	0.294	1.000
81.818	-1.753	145.013	4.374	3.465	0.144	1.000
90.000	-3.880	151.068	4.407	4.644	0.000	1.000

FIGURE 29

COMPUTER OUTPUT FOR
A CAM-LINK PATH GENERATOR.
GENERATES THE RIGHT TOP QUARTER OF A SQUARE.
THIS MECHANISM WAS PRODUCED BY THE OPTIMIZATION OF DRIFT ANGLE AND THE CAM PRESSURE ANGLE WHILE THE INPUT LINK WAS CONSTRAINED TO HAVE ITS BASE AT (0,0).

EXAMPLE NO. 7

Path generator to generate the upper right hand quarter of a square

This path generator is much the same as the last path generators with a minor change in optimization technique. This example does not optimize cam pressure angle, it optimizes the cam slope. If a mechanism is taken and only the cam and cam follower are changed, the pressure angle would change, but the cam slope would stay the same.

The path generator schematic is shown in Figure 30, and its computer output is shown in Figure 31.

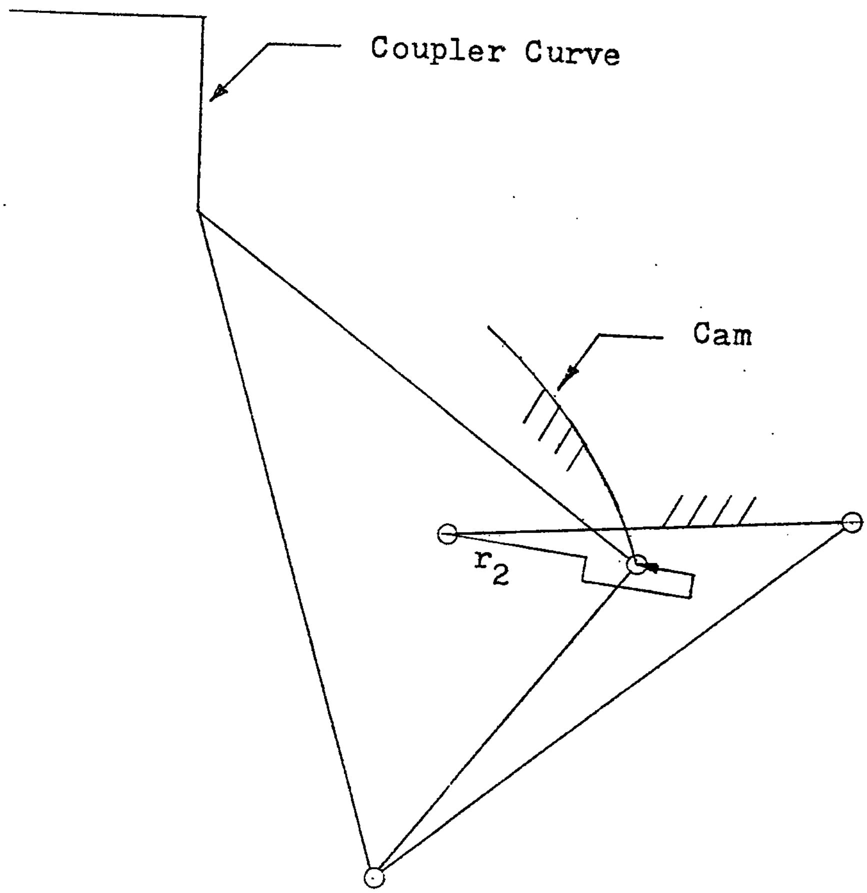

FIGURE 30

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION

***** PATH GENERATOR *****
***** INPUT DATA *****

INPUT.BETA RANGE.................	90.000
PARAMETER RANGE REDUCTION........	0.100
GRID SIZE REDUCTION INCREMENTS...	0.800
CAM FOLLOWER LENGTH,RF...........	0.0

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETER	UPPER LIMIT
0.001	R1	5.000
0.001	P1	5.000
0.001	R3	5.000
0.001	R4	5.000
0.001	R5	5.000
0.001	R6	5.000
-10.000	PT12X	10.000
-10.000	PT12Y	10.000

**** RESULTS ****
OPTIMUM DESIGN PARAMETERS

R1=	2.098	P1=	3.531
R3=	2.098	R4=	3.082
R5=	3.541	R6=	2.903
PT12X=	2.326	PT12Y=	-1.610

** ONE CYCLE OF MECHANISM **

INPUT BETA (DEG)	DRIFT ANGLE DA(DEG)	MECHANISM DRIVE P2(DEG)	RADIUS OF CAM RC	PRESSURE ANGLE ALPHA(DEG)	LOCATION OF COUPLER POINT X	Y
0.0	0.0	-8.326	0.997	-38.846	1.000	0.0
8.182	-3.549	-3.694	0.942	-31.638	1.000	0.144
16.364	-5.957	2.080	0.894	-23.221	1.000	0.294
24.545	-6.795	9.424	0.857	-13.067	1.000	0.457
32.727	-5.564	18.837	0.840	-0.548	1.000	0.643
40.909	-1.936	30.646	0.862	14.843	1.000	0.867
49.091	2.724	43.489	0.901	4.475	0.867	1.000
57.273	5.038	53.984	0.929	14.213	0.643	1.000
65.455	5.369	62.498	0.976	22.290	0.457	1.000
73.636	4.204	69.514	1.035	29.060	0.294	1.000
81.818	1.958	75.450	1.104	34.877	0.144	1.000
90.000	-1.050	80.623	1.183	40.034	0.000	1.000

FIGURE 31

COMPUTER OUTPUT FOR
A CAM-LINK PATH GENERATOR.
GENERATES THE RIGHT TOP QUARTER OF A SQUARE.
THIS MECHANISM WAS PRODUCED BY OPTIMIZATION OF THE
DRIFT ANGLE AND THE CAM SLOPE.

EXAMPLE NO. 8

Path generator to generate a cardioid

The path generator is shown in Figure 32, and its computer output is shown in Fig.33. The example shows that the cam link path generator has the ability to generate some curves that could not be approached using a standard four bar path generator.

Although this mechanism does show some rather high cam pressure angles these could be eliminated by making the cam larger and with more work the optimization could be improved considerably.

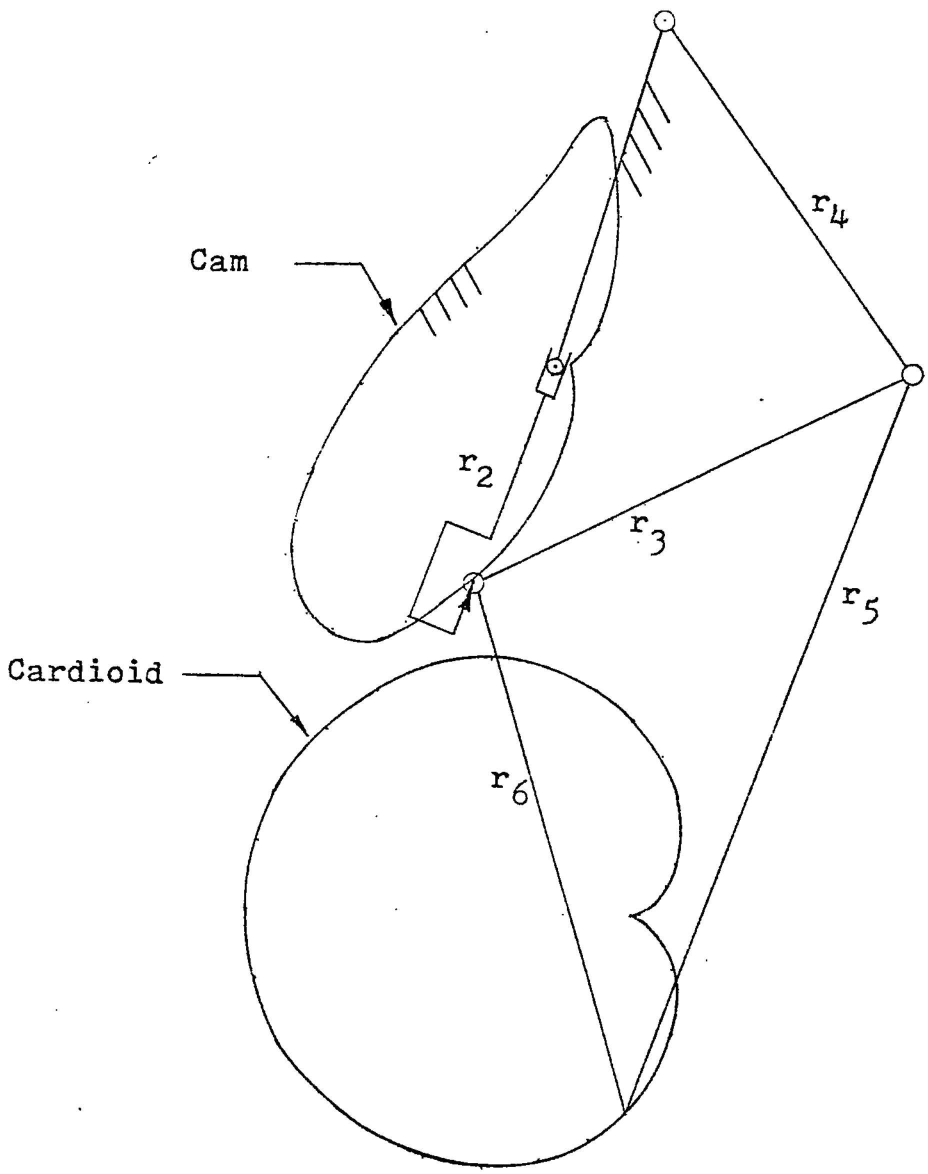

FIGURE 32

SCHEMATIC OF CAM-LINK MECHANISM FOR PATH GENERATION OF A CARDIOID

FIGURE 33

COMPUTER OUTPUT OF A CAM-LINK PATH GENERATOR TO GENERATE A CARDIOD. THIS MECHANISM WAS GENERATED BY OPTIMIZING THE DRIFT ANGLE AND THE CAM PRESSURE ANGLE. NOTE: A DRIFT ANGLE OF 361 DEGREES IS THE SAME AS 1 DEGREE.

***** PATH GENERATOR *****

***** INPUT DATA *****

INPUT,BETA RANGE.................	360.000
PARAMETER RANGE REDUCTION........	0.100
GRID SIZE REDUCTION INCREMENTS...	0.800
CAM FOLLOWER LENGTH,RF...........	0.0

DESIGN PARAMETER LIMITS

LOWER LIMIT	PARAMETER	UPPER LIMIT
0.500	R1	5.000
0.0	P1	360.000
0.500	R3	5.000
0.500	R4	5.000
0.500	R5	5.000
0.500	R6	5.000
-4.000	PT12X	4.000
-4.000	PT12Y	4.000

**** RESULTS ****

OPTIMUM DESIGN PARAMETERS

R1=	1.872	P1=	73.532
R3=	2.506	R4=	2.218
R5=	4.081	R6=	2.832
PT12X=	-0.433	PT12Y=	2.813

** ONE CYCLE OF MECHANISM **

INPUT BETA (DEG)	DRIFT ANGLE DA(DEG)	MECHANISM DRIVE P2(DEG)	RADIUS OF CAM RC	PRESSURE ANGLE ALPHA(DEG)	LOCATION OF COUPLER POINT X	Y
0.0	0.0	-109.586	1.172	-61.617	0.000	-1.000
8.000	-2.817	-104.403	0.978	-64.862	0.120	-0.852
16.000	-5.354	-98.941	0.789	-66.986	0.200	-0.696
24.000	-7.373	-92.959	0.613	-67.880	0.241	-0.542
32.000	-8.432	-86.019	0.456	-67.229	0.249	-0.399
40.000	-7.773	-77.359	0.325	-64.479	0.230	-0.274
48.000	-4.227	-65.813	0.223	-58.829	0.191	-0.172
56.000	3.335	-50.251	0.152	-49.803	0.142	-0.096
64.000	13.888	-31.698	0.110	-39.613	0.091	-0.044
72.000	22.321	-15.265	0.089	-35.593	0.047	-0.015
80.000	23.826	-5.761	0.078	-46.384	0.015	-0.003
88.000	18.462	-3.124	0.073	-73.966	0.001	-0.000
96.000	10.736	-2.850	0.075	76.182	0.005	0.001
104.000	6.460	0.874	0.086	59.315	0.029	0.007
112.000	7.606	10.020	0.110	55.169	0.068	0.027
120.000	11.633	22.047	0.150	57.656	0.116	0.067
128.000	15.297	33.711	0.212	61.629	0.167	0.131
136.000	17.177	43.591	0.299	64.775	0.212	0.220
144.000	17.259	51.673	0.409	66.682	0.242	0.333
152.000	15.922	58.336	0.540	67.591	0.249	0.468
160.000	13.497	63.910	0.685	67.839	0.225	0.618
168.000	10.182	68.596	0.836	67.695	0.165	0.775
176.000	6.074	72.488	0.986	67.263	0.065	0.928
184.000	1.210	75.624	1.120	66.194	-0.075	1.067
192.000	-4.356	78.058	1.226	62.370	-0.251	1.162
200.000	-10.462	79.952	1.289	42.949	-0.459	1.261
208.000	-16.725	81.689	1.295	-30.293	-0.690	1.297
216.000	-22.446	83.967	1.239	-54.098	-0.933	1.285
224.000	-26.551	87.862	1.128	-52.903	-1.177	1.219
232.000	-27.508	94.906	0.979	-44.626	-1.409	1.101
240.000	-23.259	107.154	0.829	-31.058	-1.616	0.933
248.000	-12.145	126.269	0.727	-11.683	-1.787	0.722
256.000	3.400	149.813	0.725	10.738	-1.912	0.477
264.000	16.453	170.866	0.831	29.262	-1.984	0.208
272.000	-336.246	-173.832	1.004	40.674	-1.998	-0.070
280.000	-333.484	-163.070	1.202	46.163	-1.955	-0.345
288.000	-333.496	-155.083	1.396	47.245	-1.856	-0.603
296.000	-335.127	-148.714	1.567	44.340	-1.707	-0.832
304.000	-337.699	-143.286	1.702	36.405	-1.516	-1.023
312.000	-340.792	-138.379	1.787	20.834	-1.295	-1.166
320.000	-344.118	-133.705	1.812	-2.866	-1.056	-1.258
328.000	-347.478	-129.064	1.773	-25.952	-0.811	-1.297
336.000	-350.763	-124.349	1.675	-41.363	-0.572	-1.285
344.000	-353.944	-119.531	1.533	-50.841	-0.352	-1.226
352.000	-357.026	-114.612	1.360	-57.126	-0.159	-1.128
360.000	-360.000	-109.586	1.172	-61.617	-0.000	-1.000

These examples clearly indicate that the cam-link mechanism can be successfully used to produce an error free function or path generator. Also shown was that constraints may easily be placed upon the mechanisms as might exist in actual practice and even with these constraints the results can be very good.

Chapter 5

CONCLUSION

This work is culminated with an analytical technique that provides error-free, optimized designs for path generation and function generation. The solution technique is briefly discussed for each solution in the following paragraphs.

Function Generator:

It has been shown with selected examples that a perfectly accurate function can be generated analytically utilizing a cam-link mechanism. But with this technique common sense must still be used because the computer did not check everything. Specifically in Figure 17 the transmission angle of the four bar cam link mechanism is too small to transmit any effective force.

The three different types of cam-link function generators developed in this thesis have the problem that the coupler link can never become perpendicular to the cam-link of the mechanism being used. When the cam-link is used for the input link and the input link is perpendicular to the coupler, it would take a large displacement in the length of the input cam-link to affect the angular position of the output. This requires a great change in the length of the cam-link in the input link with little rotation on the cam which requires a large pressure angle. A large pressure angle gives a low merit value and the computer optimization then selects another optimum.

Once this problem is realized, the proper type cam-link mechanism may be chosen. If the problem is of an unknown type, all the solutions may be tried and the best of the three selected.

Path Generator:

The path generator examples show that there are several acceptable solutions to a particular problem and that constraints may be easily placed upon the mechanism that would disable a normal precision point four bar generator. All of the examples here used the drift angle as a criteria of how good the linkage is along with another factor. The other factor is in most cases the cam pressure angle, but in one case it is the cam slope. This is to show that many parameters may be used to determine the merit value of a linkage and that once the factors determining merit value are determined, reasonable success may be expected by this method.

Interesting sidelights that should be mentioned when using this design scheme are that the design must be closely observed. It is essential that the computer solution be checked by some graphical visualization. Either the operator must use some plotting technique or output on a CRT so decisions on acceptable transmission angles and other important criterion can be judged. Also, it should be noted that even though analytically we have error-free function generation, this accuracy is now dependent on manufacturing accuracy.

SUGGESTIONS FOR FURTHER STUDY

The field of kinematics is broad and although this thesis study is narrow, it quickly opens up broad areas for further study. Some of the more pertinent areas are:

1. Inclusion of such items as limitation of transmission angle on the present study;
2. Determination of new and different types of merit function to see if better mechanisms may be produced;
3. A different optimization technique might be developed that would produce results much quicker;
4. Dynamic evaluation of the cam-link mechanism.

APPENDIX

Scaling of Function $Y = X^2$

For better fit the function will really be scaled as $-Y = + X^2$. Input Variable X, Range: -1 to + 1.

Represented by θ_2 whose range is 90 degrees

$$\text{Ratio} = \frac{\theta_{2\,final} - \theta_{2_o}}{X_{final} - X_{initial}} = \frac{90}{2} = 45 \text{ degrees}$$

$$45 = \frac{\theta_2 - \theta_{2_o}}{X - X_{initial}}$$

$$\theta_2 = 45\,(X - X_{initial}) + \theta_{2_o}$$

Now $X_{initial} = -1$

$$\theta_2 = 45(X \pm 1) + \theta_{2_o}$$

and

$$X = \frac{\theta_2 - \theta_{2_o}}{45} - 1$$

Y is output, represented by θ_4

Range of Y is 1 to 0 to 1 or 1

Range of θ_4 is 60 degrees.

$$\text{Ratio} = \frac{\theta_{4\,MAX} - \theta_{4\,MIN}}{Y_{MAX} - Y_{MIN}} = \frac{60}{1} = 60 \text{ degrees}$$

$$60 = \frac{\theta_4 - \theta_{4_o}}{Y - Y_o}$$

$$\theta_4 = 60(Y - Y_o) + \theta_{4_o}$$

Now $Y = -X^2$

$Y_o = +1$

$$\theta_4 = -60\left[\left(\frac{\theta_2 - \theta_{2_o}}{+45} - 1\right)^2 - 1\right] + \theta_{4_o}$$

Taking the derivative of θ_4

$$\frac{d\theta_4}{d\theta_2} = (24/405)\ (180/3.1415)\ (\theta_2 - \theta_{2_o}) + 8/3$$

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