On passive backscatter cloaking in one dimensional oscillation phenomena

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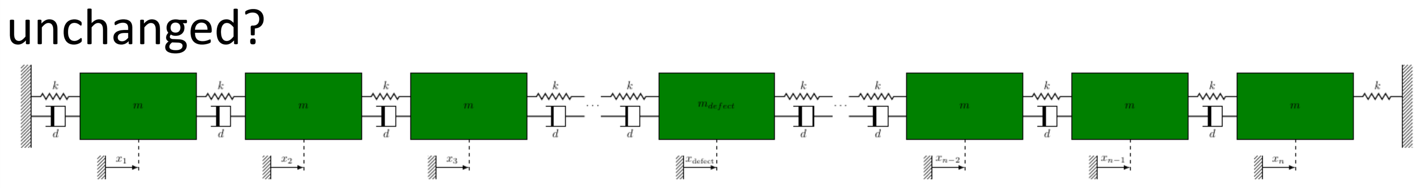
Introduction

Backscattered cloaking attempts to mask vibrations in the direction of measurement. This research only focuses on one dimensional backscattered cloaking. This problem can be described in terms of spring mass mechanics, circuits, acoustics, and atomic lattice vibrations.

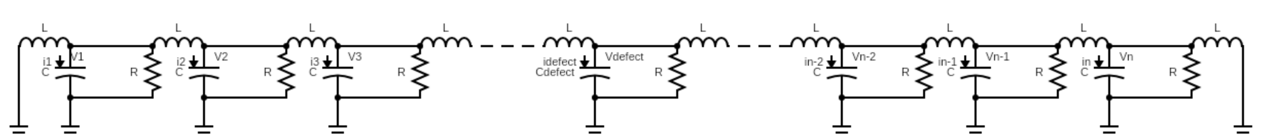
Mechanical version of the problem:

If we have a system of n-masses all with equal mass attached together via springs and dampers, is it possible to change one mass so that if we change the springs, dampers, and masses surrounding the defective mass, we can make it appear as if the motion of the first mass is unchanged?



Electric circuits version of the problem (using [1]):

If we have a system of n-capacitors all with equal capacitance attached together via inductors and resistors, is it possible to change one capacitor so that if we change the resistors, inductors, and capacitors surrounding the defective capacitor, we can make it appear as if the voltage of the first capacitor is unchanged?



Acoustics version of the problem (using [2]):

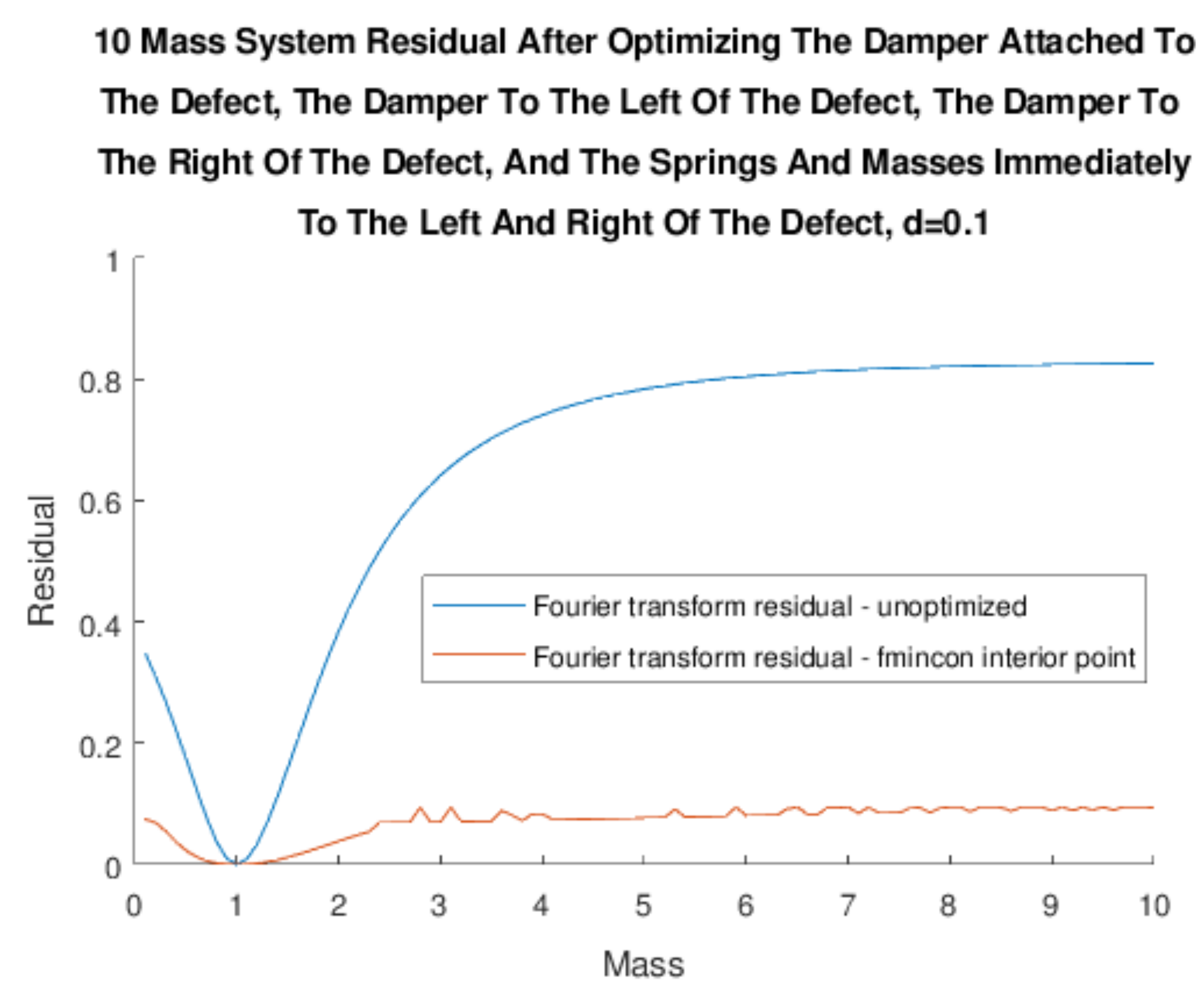
If we have a system of n-acoustic masses all with equal acoustic mass attached together via diaphragms and acoustic resistors, is it possible to change one acoustic mass so that if we change the diaphragms , acoustic resistors, and acoustic masses surrounding the defective acoustic mass, we can make it appear as if the pressure of the first acoustic mass is unchanged?

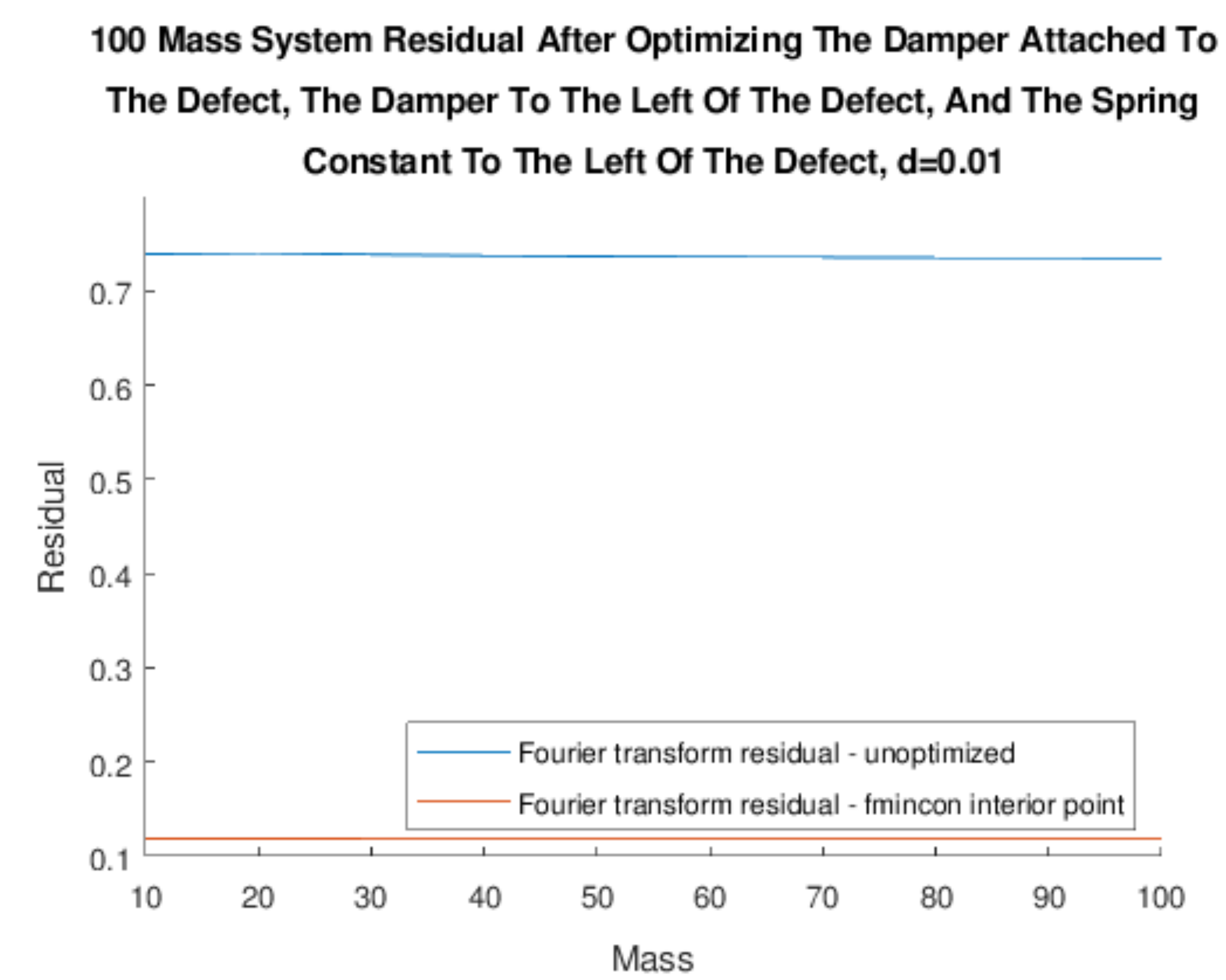
Atomic lattice version of the problem (using [3]):

If we have a system of n-atoms in a lattice all with equal mass attached together via interatomic forces approximated as springs and dampers, is it possible to change one atom's mass so that if we change the springs, dampers, and atoms' masses surrounding the defective atom, we can make it appear as if the motion of the first atom is unchanged?

Results

In this poster we demonstrate our best results for a 10 mass system and a 100 mass system with a mass changed at positions 5 and 50 respectively. We used the initial conditions $x_1(0) = 1$, $x_1'(0) = x_2(0) = x_2'(0) = x_3(0) = x_3'(0) = \dots = x_n(0) = x_n'(0) = 0$ for the simulation. We used some techniques from[6] to solve both problems. For the 10 mass system, we generated a polynomial of the Fourier transform of $x_{1nodefect}$ and x_{1cloak} ,while for the 100 mass system, we used Tikhonov regularization to approximate $x_{1nodefect}$ and x_{1cloak}. The 10 mass system was just a toy problem that we solved to use as a benchmark for later comparisons. We picked a small alpha value when using Tikhonov regularization vs. other methods (e.g. L curve[7]) to improve runtime efficiency. In both cases we used fmincon to minimize the residual but it is possible to create a mesh of optimization parameters and select the ones with the smallest residual to improve runtime efficiency (with a slight accuracy tradeoff).The code was also fully parallelized to improve speed.





We were able to cloak the defect quite well. It turns out that the damping constant in a mechanical system (and resistance in terms of an electrical system) contributes to cloaking the system the most. The leftmost spring and damping constants contribute to cloaking the system much more than the rightmost spring and damping constants.

As an explanation for why this is true, note that the dampers and resistors in the electrical and mechanical system dissipate energy. In the mechanical system, this slows motion, while in the electrical system, this reduces voltage oscillations. Since the masses are moving less (and the voltage is not changing as fast in the corresponding electrical system), the system is easier to cloak with larger dampers (or resistors).

Methods

To cloak the system, we want to make the cloaked system appear identical to the system with no defect. In the case of the mechanical version of the system, note the positions of the masses vary with time. Since we only care about the position of the first mass, we want to minimize the difference between the position of the first mass in the system with no defect and the cloaked system. This can be formulated mathematically as $\underset{\phi}{\operatorname{argmin}} \frac{\int_0^\infty (x_{1_{nodefect}}(t) - x_{1_{cloak}}(t,\phi))^2 dt}{\int_0^\infty (x_{1_{nodefect}}(t))^2 dt}$

Note the system is causal. By Plancherel's theorem[4], we can then optimize $\underset{\phi}{\operatorname{argmin}} \frac{\int_{-\infty}^\infty (\hat{x}_{1_{nodefect}}(\omega) - \hat{x}_{1_{cloak}}(\omega,\phi))^2 d\omega}{\int_{-\infty}^\infty (\hat{x}_{1_{nodefect}}(\omega))^2 d\omega}$

instead. Note that by using Tikhonov regularization[5], we can approximate $x_{1\ nodefect}$ and x_{1cloak} so that $\hat{x}_1 \approx (\hat{A}^T \hat{A} + \alpha I)^{-1} \hat{A}^T \hat{B}$. Note A is the coefficient matrix for the Fourier transformed system of ODEs that can be created from the spring mass system, and B is the constant matrix for that same system.

Conclusion

Overall our results demonstrated that it is possible to cloak a defect quite well in a large scale system. Tikhonov regularization is the best method overall for cloaking the system.

We could extend these results by attempting to fully cloak the system so that the defect is invisible from both ends of the system. We could also attempt to dynamically cloak the system, by allowing the spring constants, damping constants, and masses surrounding the defect to change over time.

Acknowledgements

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