A STUDY OF THE EFFECTS OF CYCLIC (ON-OFF) HEAT FLUXES ON CANNED FOOD HEATING TIMES DURING SKYLAB SPACE FLIGHTS

A Thesis

Presented to

the Faculty of the Cullen College of Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

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by

Michael Robb Mathias

August 1972

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ABSTRACT

A review of solutions for heat conduction with heat flux boundary conditions is presented for cylindrical geometries. A solution is selected that is sufficiently general to be applied to a food heating problem associated with the Skylab space flights which are to be conducted by the National Aeronautics and Space Administration.

The problem is to compute the total time required to heat food cans to a desirable temperature for consumption without having the temperature, at any point in the system, becoming higher than the boiling temperature of water for the given cabin environment.

This problem is solved, by using a piece-wise, analytical solution scheme programmed on the digital computer. The results of the analysis for a typical data case are presented graphically and in tabular form. In particular the effect of q_o, α, and R_o as parameters on a graph of $\overline{T}$ versus heating time, a typical family of dimensionless temperature profiles, and a table (Table 6.2) demonstrating the range of expected heating times predicted for Skylab are presented as primary analytical results.

The conclusions based on the results are stated and the possibility of system improvement is discussed.

NOMENCLATURE

English

A	Radial surface area of can contents, ft^2
C	Function name, Appendix D
Cp	Specific heat at constant pressure (BTU/lbm ^{o}R)
DR	Radial increment chosen to perform numerical integration in computer program, Appendix G
E	Function name, Appendix D
e	Exponential function
f	Functional representation
F_o	Fourier modulus, $\alpha t/R_o^2$, dimensionless
F_{om}	Modified fourier modulus, $\alpha t \rho_m^2/R_o^2$, dimensionless
I_m	Integral function, reference equation 2.7
J_o	Bessel Function of the first kind and of order zero
J_1	Bessel Function of the first kind and of order one
K	Thermal conductivity (BTU/ft hr ^{o}F)
M	Index
N	Index
Q	Flux conduction parameter, $\frac{q_o R_o}{K}$, dimensionless
q	General dimensionless heat flux
q_o	Applied radial heat flux (BTU/hr-ft^2)
r	Radius variable, ft
R_o	Outer radius of can, ft (unless otherwise stated)
R	Dimensionless radius variable, r/R_o

S	Partial sum of a series
s	Index
$T(r,t)$	Temperature at a particular radius and time, ^{o}F
$\overline{T}$	Mean temperature level of system, ^{o}F
T_{max}	Maximum allowable system temperature, ^{o}F
T_{min}	Minimum temperature at which food is consumed, ^{o}F
t	Time, hr
V	Volume of can contents, ft^3

Greek

α	Thermal diffusivity, ft^2/hr , $\frac{K}{\gamma Cp}$
γ	Material density, Lb_m/ft^3
ρ_m	Roots of Bessel Function J_1 multiplied by R
ρ_{om}	Roots of Bessel Function J_1
θ	Dimensionless temperature, $\frac{T(r,t) - \overline{T}}{T_{max} - \overline{T}}$

Miscellaneous

$(\ )_c$	Evaluated at the food centerline
$(\ )_I$	Initial
$(\ )_n$	nth derivative of a function with respect to time
$(\ )^n$	nth order partial sum of a series
$\|_{r = R_o}$	Evaluated at R_o
$(\ )_s$	Evaluated at the food surface
$\|_{wall}$	Evaluated at R_o

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CHAPTER I

INTRODUCTION

Historically food canning is a recent innovation, the practice was first started in 1809, by a Frenchman named Nicolas Appert.[1] Appert discovered that food could be preserved by heating it in heremetically sealed containers.[2] However, he did not understand why the food was preserved.

In the decades following Appert's discovery, much empirical data was produced which specified for how long a time and at what temperature a particular food container should be heated to preserve its contents without destroying their palatability and nutritional value.

Much of modern day food technology still depends on empirical data and on simple physical models. This lack of advancement in the state-of-the-art of canning is the result of no economic, social, or scientific demands for change. Since the standard methods of canning have presented no cost problems, no health problems, and have been adaptable to most new scientific requirements; the industrial procedures have changed very little.

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[1] G. Borgstrom, Principles of Food Science, Volume I, Food Technology (New York: The Macmillan Company, 1968).

[2] Appert's "cans" consisted of glass jars with cork end plugs sealed with wax.

However, several recent developments have upset the status quo of canning technology. Some of these developments are the increased resistance of virus and bacteria strains to the present thermal-death treatments, the increased variety of foods being canned, and the increased use of canned foods in environments other than the surface of the earth.

This last development has resulted in activity designed to produce the new thermal technology required to use canned products in high-altitude flight, in earth orbit, and, in the future, for interplanetary space flights.

Obviously, for space flight, new and improved thermal modeling is needed to replace the old, empirical methods and earth surface models that have sustained the canned food industry in the past. Some of the more important new environmental and design requirements encountered in space flight are the lack of a gravitational force, no free thermal convection, reduced atmospheric pressure, and the low-power consuming, light-weight heating systems consistent with spacecraft design criteria.

In conjunction with meeting the requirements stated above, new thermal models must have boundary conditions that correspond to given environments and that maintain the standard human comfort levels of crew members.

The first spacecraft environment where canned foods are to be heated and consumed by crewmen is that of the Skylab

Program flights produced by the National Aeronautics and Space Administration. The Skylab environmental conditions that are included in the thermal heating model of this thesis are zero gravity, maximum allowable system temperature of 150° F.,[3] and a storage temperature of 70° F.

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[3]This value is 10° F below the boiling point of water at a 5 psi cabin pressure.

CHAPTER II

THE SYSTEM ANALYSIS

THE PHYSICAL SYSTEM

The system to be modeled is that formed by a thin wall (1/64 inch thick) aluminum food can (see Figure 2.1)[1] containing various homogeneous and nonhomogeneous foods. An electrical resistance heater is wrapped around the can; resistance heating also occurs at the bottom of the container. The container's top is insulated. The can wall contains a thermocouple[2] placed such that the temperature at the radial boundary between the can and its contents is constantly measured. Because of the spacecraft environmental constraints mentioned earlier, the temperature, at any point in the system, cannot become greater than a given value, T_{max}. T_{max} is the upper limit used by the thermocouple to trigger the heating circuit. The heating procedure is as follows.

Initially the system is at a uniform constant temperature (can storage temperature). The system is activated and

[1]Figure 2.1 shows the Skylab Food Tray with several opened food cans in the various heating sockets and an orange juice container in the drink holder.

[2]The thermocouple actually measures the temperature at the exterior surface of the can. The small wall thickness (1/64 thk) and high termal conductivity of the aluminum can makes this a negligible thermal resistance.

FIGURE 2.1

THE PHYSICAL SYSTEM

a constant heat flux, q_o, is uniformly applied to the can's bottom and radial exterior. The heat flux (see graphical representation, Figure 2.2) is maintained until the thermocouple signals that the can/food interface is at T_{max}. At this time, the heater is shut off. Then, under the zero flux boundary condition, the can contents redistribute their energy by conduction (no free convection in the can contents in zero g). Heat is conducted inward from the regions of high temperature at the previously heated boundaries to the regions of lower temperature, the can top and center. The heater remains off until the temperature at one of the boundaries first reaches a value, T_{min}. The heater is turned on and a uniform heat flux q_o is again applied to the can bottom and circumference.

This sequence of events is repeated until the temperature of the can's contents at the geometrical centerline is at T_{min}.[3] Since T_{max} and T_{min} differ very little (15° R), the temperature profile in the food becomes approximately flat[4] as the heating cycles continue.

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[3]The thermal energy level at the can centerline is within some predetermined error of the value, T_{min}.

[4]The 15° R variation between T_{max} and T_{min} represents a 1½ percent variation in the value of T_{min} (595° R).

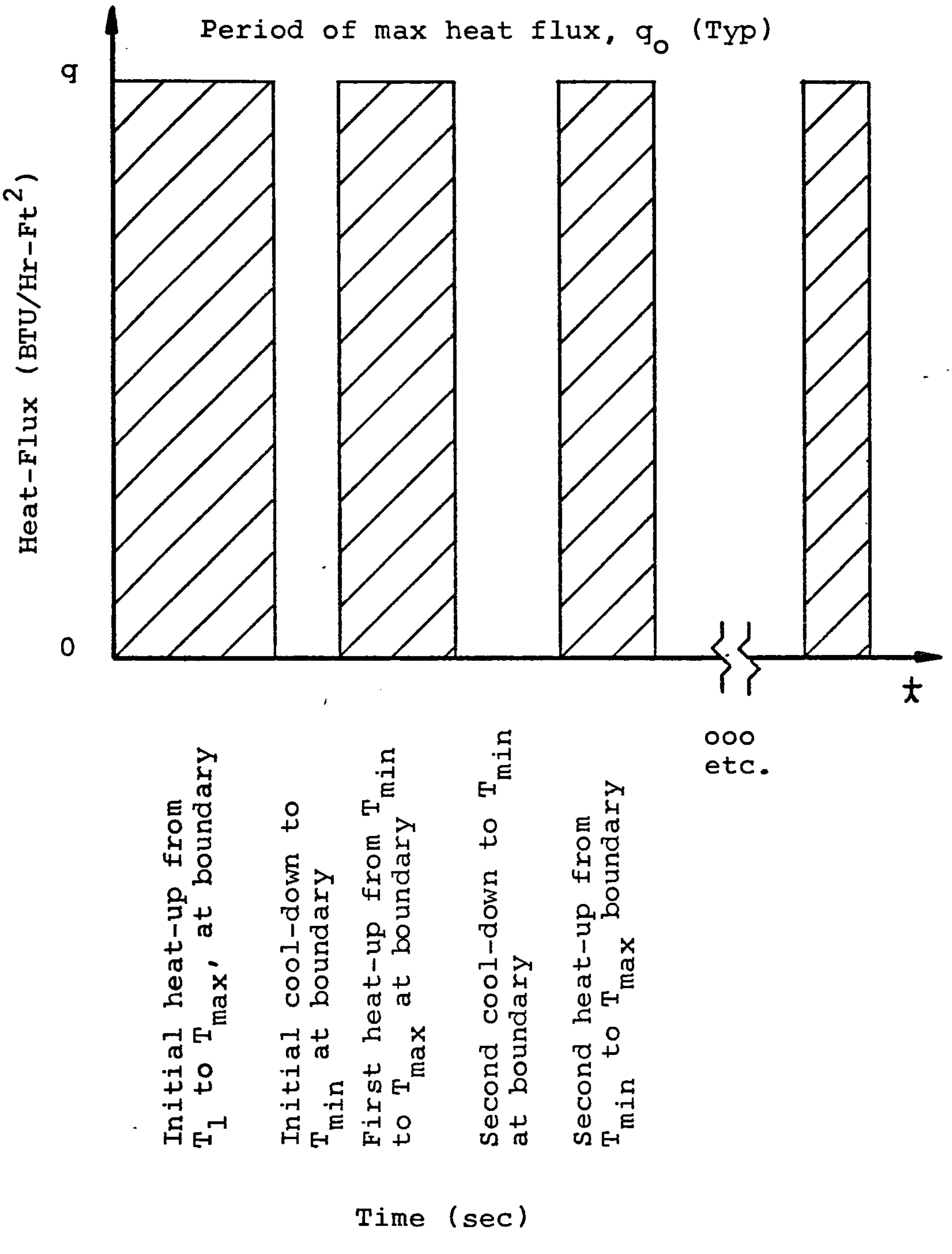

FIGURE 2.2

HEAT FLUX VS. TIME VARIATION

NOTE: (a) Decrease in heat-up times as the overall thermal energy level is increased.

(b) Increase in cool-down time as the overall thermal energy level is increased.

THE ANALYTICAL MODEL

The analytical model (see Figure 2.3) is the same as the physical model except for the following differences.

Both ends of the can are assumed to be insulated (infinite cylinder). The can contents are assumed uniform; that is no gas globules (voids) are mixed in with the food or separate the food from the can wall. The entire can is assumed to be perfectly insulated during the periods when the heat source is shut off.

The assumption of an infinite cylinder as compared to the actual cylinder is based on two considerations. First, the heating time computed using such an assumption is conservative. Secondly, the general relation[5] describing the actual physical system boundary conditions is so complex that the computer programming and estimated computer solution times are beyond the scope of this thesis.

The assumption of complete insulation during periods of no heat flux will reduce somewhat the conservatism of the infinite cylinder approximation.

The assumption of gas voids being absent during heating in zero gravity is justified by the earth gravity stowage of the cans prior to any space flight. It is felt that the

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[5]Nurettin Y. Olcer, "On the Theory of Conductive Heat Transfer in Finite Regions with Boundary Conditions of the Second Kind," International Journal of Heat and Mass Transfer, 8:529-56, 1965.

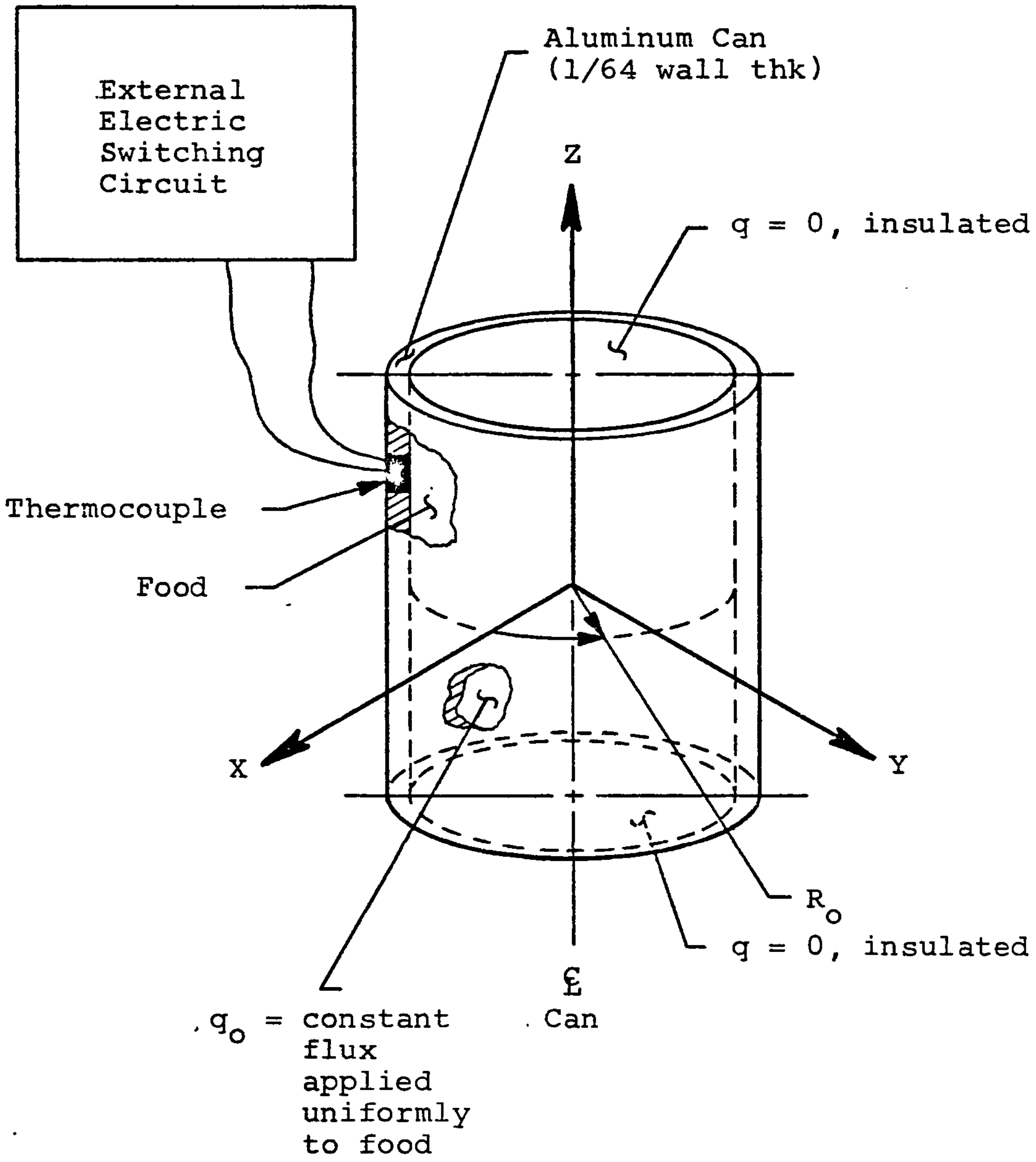

FIGURE 2.3

SYSTEM TO BE ANALYZED

gravity environment's stratification of the can contents and the interparticle bonds (e.g., coagulated grease) formed during stratification are strong enough to remain stable in the zero gravity environment. This, in the absence of any further disturbance, precludes any random arrangement of food and gas globules from occurring.

In the event an unexpected agitation induces a significant number of trapped gas pockets into the food, the conduction model breaks down. Gases have low conductivities and radiation will become an important heat transfer mode.

The differential equation and boundary conditions that describe the general analytical model are presented and discussed below.

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \tag{2.1}$$

The general boundary and initial conditions are:

$$T(r,t=0) = \text{nonzero, nonconstant function of radius} \tag{2.2}$$

$$\left[\left(\frac{dT}{dr}\right)\right]_{\text{wall}} = \frac{q_o}{kR_o} = \begin{cases} 0, & \text{heater off} \\ \text{constant}, & \text{heater on} \end{cases} \tag{2.3}$$

Equation (2.3) represents a constant heat flux at the boundary.

$$\lim_{r \to 0} \left[T(r,t)\right] = \text{finite value} \tag{2.4}$$

Thus, the solution needed is the one for an on-off, constant heat-flux (i.e., alternately on and then insulated $q_o = 0$) applied at the radial boundary of an infinite right circular cylinder with an arbitrary initial temperature distribution.

CHRONOLOGY OF RESEARCH PERFORMED IN THE LITERATURE

The lumped-heat-capacity method[6] (Newtonian heating or cooling) is a simple approximate method for solving unsteady state heat transfer problems for arbitrary geometries.

The approach's principle assumption is that the heated body possesses negligible internal resistance to conduction heat transfer (i.e., K is large). Thus, there are negligible temperature gradients within the body, and its temperature is essentially constant (i.e., one value of temperature characterizes the thermal state of the body).

The energy entering the body at its surface must all be stored within it, resulting in a temperature increase.

The equations associated with this method are:

$$Aq_o = C_p \gamma V \frac{d\overline{T}}{dt} = \text{constant} \tag{2.5}$$

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[6]J. P. Holman, Heat Transfer (New York: McGraw-Hill Book Company, Inc., 1963): see also F. Kreith, Principles of Heat Transfer (Scranton: International Textbook Company, April, 1963): and P. J. Schneider, Conduction Heat Transfer (Cambridge: Addison-Wesley Publishing Company, Inc., September, 1957).

Upon integrating (2.5), for a constant heat flux, the result is:

$$\overline{T}(t) - T_I = \frac{q_o A}{C_p V} (t - t_I) \tag{2.6}$$

The assumption of high thermal conductivity for all known canned foods is bad.[7]

The actual temperature distribution is in general not uniform, but it is nearly so at the end of the heating process. The assumption of a constant heat flux is not compatible with the on-off flux requirement of the physical system.

With all the necessary assumptions made to provide a solution, the results of the lumped-heat-capacity method can serve as the ultra-conservative solution to check, roughly, the more elaborate methods.

It is apparent that the total heating (on) time for the (on-off) cyclic heat-flux system would be greater than the time calculated from the lumped-heat-capacity model.

Another readily available method of approximating a solution is provided by the finite difference technique. This method is used largely for nonhomogeneous materials, for non-linear boundary conditions (radiation), for boundary conditions which are functions of space or time, and for unusual shapes. However, its use is not limited to those conditions.

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[7] Most canned foods have thermal conductivities ranging from (.37 to .18 BTU/hr-ft-^{o} R) compared to a range of (6 to 235 BTU/hr-ft-^{o} R) for metals.

For this problem, the method's utility is best judged by comparing desired solution accuracy to the time required to converge to a final answer. For a problem where several solutions of a set of finite difference equations are required to correspond to the various sets of boundary conditions, the amount of computation time[8] for an accurate answer becomes quite large.

Based upon the previous logic, it was decided to forego the finite difference technique and to seek an approximate or exact solution where accuracy and computation time are more easily controlled.

Another approximate solution method was considered; it is called the Schmidt Plot.[9] The method is used to solve unsteady state heat transfer problems of arbitrary geometry that are one-dimensional and have known initial temperature distributions (usually in the form of raw data). This method was dropped from consideration because it is graphical, it is primarily intended to be performed by hand, and it is not easily programmed.[10]

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[8]The time is large either for hand or computer computations using this method.

[9]Holman, loc. cit.; see also Schneider, loc. cit.

[10]Each computation starts over again with the same graphical techniques. The process is not set down in terms of general relations that can be iterated only by a change of input values.

The next type of solution considered was the analytical solution. Extensive collections of unsteady state conduction solutions can be found in the literature.[11]

The first special case solution was found in Carslaw and Jaeger's book[12] and is rewritten here in the terminology of this thesis (see Appendix A for change of variables).

The boundary conditions for which it is valid are a variable initial temperature distribution and a zero surface flux. These are the boundary conditions that exist when the heater is shut off and the can contents are cooling down to T_{min}.

$$\Theta = \frac{2}{T_{max} - T}\left[\frac{1}{R_o^2} \sum_{m=1}^{\infty} \frac{e^{-F_{om}} I_m \; J_o(\rho_m)}{J_o^2(\rho_{om})}\right] \qquad (2.7)$$

The auxiliary equations associated with (2.7) are:

$$F_{om} = \frac{\alpha t \rho_m^2}{R_o^2} \qquad (2.8)$$

$$I_m = \int_0^{R_o} T(r\ ,t=0)\ J_o\ (\rho_m)\ r\ dr \qquad (2.9)$$

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[11]H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids (2d ed.; Oxford: Clarendon Press, 1959); see also Schneider, loc. cit.

[12]Carslaw and Jaeger, loc. cit.

The physical significance of the auxiliary equation (2.8) is discussed later.

The second special case solution was also found in Carslaw and Jaeger's book and is written as follows (see Appendix B for change of variables).

The boundary conditions for which it is valid are a constant surface flux and a zero initial temperature distribution. These boundary conditions do not exist during the can's heating process.

$$\theta = \frac{2Q}{T_{max}} \left\{ F_o + \tfrac{1}{4} (R^2 - \tfrac{1}{2}) - R^2 \sum_{m=1}^{\infty} e^{-F_{om}} \frac{J_o (\sqrt{\rho_m})}{\sqrt{\rho_m}^2 J_o (\sqrt{\rho_{om}})} \right\} \tag{2.10}$$

The auxiliary equations associated with (2.10) are:

$$F_o = \frac{\alpha t}{R_o^2} \tag{2.11}$$

$$F_{om} = \frac{\alpha t \sqrt{\rho_m}^2}{R_o^2} \tag{2.12}$$

$$Q = q_o R_o / K \tag{2.13}$$

The physical significance of the auxiliary equations is discussed later.

The boundary conditions for which the third special case solution[13] is valid[14] are a constant surface flux and a constant, nonzero initial temperature profile. This is the set of boundary conditions that exists when the heater is first turned on. Equation (2.14) appears to be a valid solution from zero until the can wall first reaches T_{max}. It does not represent a total solution to the problem, only a solution for the original set of boundary conditions. It claims, as one of its special cases, equation (2.10).[15] The solution is written below (see Appendix C for change of variables).

$$\theta = \frac{2Q}{T_{max} - T_I} \left\{ F_o + \tfrac{1}{4} (R^2 - \tfrac{1}{2}) - R^2 \sum_{m=1}^{\infty} \left(e^{-F_{om}} \frac{J_o(\rho_m)}{\rho_m^{\,2} J_o(\rho_{om})} \right) \right\} \tag{2.14}$$

The auxiliary equations associated with (2.14) are equations (2.11), (2.12), and (2.13).

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[13] S. J. Lis and P. P. Nuccio, Method of Heating Food in Aerospace Flight, Techn. Doc. Report No. AMRL-TDR-63-135, Biomedical Laboratory, 6570th Aerospace Medical Research Laboratories, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, December, 1963.

[14] See discussion on page 21 before applying (2.14).

[15] See discussion on page 21 before applying equation (2.14).

Langford presents a different kind of solution to the unsteady heat conduction problem. The capability of the solution is best described by this abstract:

> New solutions of the heat equation are exhibited for the case in which both the temperature and heat flow rate are prescribed at a single fixed boundary. The prescribed temperature and heat flow rate may be any arbitrary infinitely differentiable functions of time. The new solutions are applicable for one-dimensional (radial) heat flow in spheres, cylinders, and slabs.
>
> Special solutions may be obtained by choosing special forms for the prescribed boundary temperature and boundary heat flow rate. These special solutions include the classical solutions of the heat equation, new sequences of polynomial and quasi-polynomial solutions of the heat equation, and new closed-form solutions to constant-velocity phase change problems with spherical and cylindrical symmetry.[16]

According to Langford, the temperature distribution in an infinite, right, circular cylinder as a function of time is:

$$\Theta(R,t) = \sum_{m=0}^{\infty} \left[\left(\Theta\ (R_o,t)\right)_m C^m\ (R^2/4) - \tfrac{1}{2}(q)_m(R_o,t)\ E^m\ (R^2/4) \right] \qquad (2.15)$$

The functions, E^m and C^m, are given for $m \geq 0$ by general relations (see Appendix D). E^m and C^m are functions only of the cylindrical geometry.

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[16] David Langford, "New Analytic Solutions of the One-Dimensional Heat Equation for Temperature and Heat Flow Rate Both Prescribed at the Same Fixed Boundary (With Applications to the Phase Change Problem)," Quarterly of Applied Mathematics, 24:315-22, 1966.

In order to use equation (2.15) to compute temperature values at radial points for any given time, it is only required that complete time histories of both heat flux and temperature are known or can be accurately constructed at the surface, R_o (i.e., at the boundary). If these histories are available and analytic, then a complete time history of temperature at any point in the system can be generated using (2.15).

An attempt was made to construct the required time-history inputs for equation (2.15). The approximations constructed were read into a computer program. The results of the program indicated an increase in heating time of nearly 50 percent over the time determined from the lumped-heat-capacity method of solution.

It should be mentioned that the construction of the heat flux, surface temperature time histories for the computer program are somewhat arbitrary and of unknown accuracy. Hence the results obtained are, to say the least, questionable.

Olcer presents an analytical solution which is sufficiently general to be applied to the present problem.[17]

In this work general expressions are derived for unsteady temperature distributions in finite regions of arbitrary geometry, under conditions of prescribed heat flux on all

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[17] Olcer, loc. cit.

boundaries and with time-dependent heat sources and arbitrary initial conditions. The temperature fields are expressed in the form of uniformly convergent series solutions.

The general result of the paper specialized to the case of an infinite cylinder with a constant heat flux applied to its radial exterior and a nonzero, nonconstant initial temperature distribution is (see Appendix E):

$$\theta(F_o, F_{om}, R_o, R, I_m, Q, \rho_m, \rho_{om}) = \theta \tag{2.16}$$

$$= \frac{2}{T_{max} - \bar{T}} \left[Q\, F_o + \frac{Q}{4}\,(R^2 - \tfrac{1}{2}) - Q\, R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\rho_m)}{\rho_m^{\,2}\, J_o(\rho_{om})} \right) + 1/R_o^{\,2} \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}}\, I_m\, J_o(\rho_m)}{J_o^{\,2}(\rho_{om})} \right) \right]$$

The auxiliary equations associated with (2.16) are:

$$\bar{T} = (2/R_o^{\,2}) \int_0^{R_o} T(r, t=0)\, r\,dr \tag{2.17}$$

$$I_m = \int_0^{R_o} T(r, t=0)\, J_o(\rho_m)\, r\,dr \tag{2.18}$$

$$\rho_m = \text{roots of Bessel Function, } J_1 \text{ multiplied by } R \tag{2.19}$$

$$\rho_{om} = \text{roots of Bessel Function, } J_1 \tag{2.20}$$

$$F_o = \frac{\alpha t}{R_o^2}\text{, fourier modulus} \tag{2.21}$$

$$F_{om} = \frac{\alpha t \rho_m^2}{R_o^2}\text{, modified fourier modulus} \tag{2.22}$$

$$Q = \frac{q_o R_o}{K} \tag{2.23}$$

Some of the auxiliary equations have significant physical meanings.

$\overline{T}$ in equation (2.17) represents the initial, mean temperature level of the system.

F_o in equation (2.21) is the fourier modulus and F_{om} in equation (2.22) is a modified fourier modulus.

Q in equation (2.23) is the flux conduction parameter. It represents the ratio of heat per unit axial length crossing the system boundary to the ability of the system materials to conduct this heat into the interior of the cylinder. A large value of Q represents a conduction (K small) controlled process (i.e., changes in q_o do not affect the process to a large degree). A small value of Q indicates that the process is controlled by the imposed heat flux.

Several special cases of equation (2.16) and its auxiliary equations are needed to correspond to the different sets of boundary conditions present in this problem.

The first is for a constant boundary heat flux and a constant initial temperature distribution. These are the boundary conditions when the can is first heated up to T_{max}. The equation as specialized from the general equation (2.16) is:

$$\theta = \left\{ \frac{2Q}{T_{max} - \overline{T}} \left[F_o + \tfrac{1}{4}\left(R^2 - \tfrac{1}{2}\right) - R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\rho_m)}{\rho_m^{\,2} J_o(\rho_{om})} \right) \right] \right\} + \left\{ 1/R_o^{\,2} \sum_{m=1}^{\infty} \frac{e^{-F_{om}} I_m J_o(\rho_m)}{J_o^{\,2}(\rho_{om})} \right\} \qquad (2.24)$$

The auxiliary equations that change are:

$$\overline{T} = T_I = T(r,t=0) \qquad (2.25)$$

$$I_m = T_I \int_0^{R_o} J_o(\rho_m)\, r\,dr \qquad (2.26)$$

The remaining auxiliary equations are (2.19) through (2.23).

Since the same boundary and initial conditions were used in the derivation of equation (2.24) as were used for a previously discussed solution (equation (2.14)); equations (2.14) and (2.24) should be the same. They are not; they differ by the second bracketed term in equation (2.24). The term is rewritten.

$$1/R_o^{\,2} \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m \, J_o(\sqrt{\rho_m})}{J_o^{\,2}(\sqrt{\rho_{om}})} \right) \qquad (2.27)$$

Lis and Nuccio[18] lists Carslaw and Jaeger as their source for equation (2.14). The page referred to in Carslaw and Jaeger's book[19] contains only the solution for a zero initial temperature distribution (which is our equation (2.10) from Carslaw and Jaeger). Apparently the authors, Lis and Nuccio,[20] incorrectly made the assumption that the zero initial temperature condition of Carslaw and Jaeger's could be changed to a constant, nonzero initial temperature condition simply by adding in a constant initial temperature term. As it has just been shown, this is not true. The term indicated (equation 2.27)) must also be accounted for. Therefore, before any conclusions based on the results of Lis and Nuccio are made, the effects of (2.27) on the stated results should be considered. Also, it can be easily shown that by setting $T_I = 0$ in equation (2.24), $I_m = 0$. This causes the term (2.27) to be zero. Therefore, equation (2.10) is simply a special case of equation (2.24). Equation (2.10) is also a special case of the general

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[18] Lis and Nuccio, loc. cit.

[19] Carslaw and Jaeger, loc. cit.

[20] Lis and Nuccio, loc. cit.

theory, equation (2.16). This is as it should be.

In the physical model the heater is controlled by a thermocouple at the wall. Hence the wall temperature of the conduction model must be monitored. The wall temperature is determined from equation (2.24) with R = 1. Then, for a constant heat flux and for a constant, nonzero temperature distribution, the wall temperature is equation (2.28).

$$\theta\Big|_{r = R_o} = \theta\Big|\text{wall} = \left\{\frac{2Q}{T_{max} - T_I}\left[F_o + \frac{1}{8} - \sum_{m=1}^{\infty}\left(\frac{e^{-F_{om}}}{\rho_m^{\,2}}\right)\right]\right\} + \left\{\frac{1}{R_o}^{2} \sum_{m=1}^{\infty}\left(\frac{e^{-F_{om}}\, I_m}{J_o\,(\rho_{om})}\right)\right\} \qquad (2.28)$$

The auxiliary equations are (2.25), (2.26), and (2.19) through (2.23).

After the initial heat-up cycle, the wall temperature must again be monitored for the case of a nonuniform initial temperature distribution. The result from equation (2.16) with R = 1 is:

$$\theta\Big|_{r = R_o} = \theta\Big|\text{wall} = \left\{\frac{2Q}{T_{max} - T}\left[F_o + \frac{1}{8} - \sum_{m=1}^{\infty}\left(\frac{e^{-F_{om}}}{\rho_m^{\,2}}\right)\right]\right\} \qquad (2.29)$$

$$+ \left\{\frac{1}{R_o}^{2} \sum_{m=1}^{\infty}\left(\frac{e^{-F_{om}}\, I_m}{J_o\,(\rho_{om})}\right)\right\}$$

The form is the same as (2.28), but the auxiliary equations are different. The auxiliary equations are equations (2.17) through (2.23).

The next special case required of equation (2.16) is for the boundary condition of zero flux with an arbitrary initial temperature profile. This occurs physically during the cool-down periods of the food boundary from T_{max} to T_{min}. From equation (2.16) with Q = 0:

$$\theta = \frac{2}{T_{max} - \bar{T}} \left[\frac{1}{R_o}^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o(\lambda\rho_m)}{J_o^2(\lambda\rho_{om})} \right) \right] \quad (2.30)$$

The auxiliary equations are equations (2.17) through (2.22).

It is recalled that the boundary and initial conditions imposed in the derivation of equation (2.7) are the same as those used above; and, upon inspection, it is seen that the two equations are identical. Therefore, the general theory, equation (2.16), has as a special case equation (2.7). Also, it is shown (see Appendix D) that the superposition of equations (2.7) and (2.10) produces the general theory, equation (2.16).

The final special case of equation (2.16) is the one for which the boundary heat flux is zero, the radius is R_o, and the initial temperature profile is nonuniform. This gives the food boundary temperature, measured by the thermocouple, as a function of time during the cool-down periods of this analysis. Evaluation of equation (2.30) for R = 1 yields:

$$\theta = \frac{2}{T_{max} - T} \left[\frac{1}{R_o^{\,2}} \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om} I_m}}{J_o(\rho_{om})} \right) \right] \qquad (2.31)$$

The auxiliary equations are the same as those of equation (2.30).

It is noteworthy that many other solutions to the basic differential equation, equation (2.1), exist. However, these solutions are for different boundary and initial conditions than (2.2), (2.3), and (2.4). The primary sources of these other solutions are footnoted.[21]

Table 2.1 is presented as a summary of available solutions for constant heat flux boundary conditions in infinite

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[21]R. B. Bird, E. N. Lightfoot, and W. E. Stewart, Transport Phenomena (6th ed.; New York: John Wiley & Sons, Inc., 1965); see also Carslaw and Jaeger, loc. cit.; R. V. Churchill, Operational Mathematics (2d ed.; New York: McGraw-Hill Book Company, Inc., 1958); Holman, loc. cit.; W. M. Kays, Connective Heat and Mass Transfer (New York: McGraw-Hill Book Company, Inc., 1966); Kreith, loc. cit.; A. V. Luikov, "Heat and Mass Transfer Institute, Minsk, BSSR, USSR," Analytical Heat Diffusion Theory, James P. Hartnell, editor (New York: Academic Press, 1968); Schneider, loc. cit.; and C. R. Wylie, Jr., Advanced Engineering Mathematics (2d ed.; New York: McGraw-Hill Book Company, Inc., 1960).

TABLE 2.1

SUMMARY OF SOLUTIONS FOR HEAT FLUX BOUNDARY CONDITIONS ON THE SURFACE OF INFINITE RIGHT CIRCULAR CYLINDERS

Item	Problem Statement Boundary Condition(s) & Initial Condition(s)	Solutions Available	Literature[a] Source	Comments
1	(1) Constant initial & final temperature profiles. (2) Heat flux known as a time function.	Eq.'s (2.5) & (2.6) Lumped-Heat-Capacity Method	Bird, et al.; Holman; Kays; & Kreith	(1) Approximation best for high thermal conductivity materials.
2	(1) Arbitrary boundary and initial conditions.	Finite Difference Technique	Holman; and Schneider	(1) Best for unusual boundary conditions. (2) Unusual geometries.
3	(1) Arbitrary boundary and initial conditions.	Schmidt Plot No equation	Holman; and Schneider	(1) Graphical technique. (2) Useful for one dimension only. (3) Not easily programmed.
4	(1) Zero surface flux. (2) Arbitrary initial temperature distribution.	Eq. (2.7)	Carslaw and Jaeger	(1) Valid for cool-down sequence of thesis problem. (2) Special case of equation (2.16).
5	(1) Constant surface flux. (2) Zero initial temperature distribution	Eq. (2.10)	Carslaw and Jaeger	(1) Special case of equation (2.16).

TABLE 2.1 (continued)

Item	Problem Statement Boundary Condition(s) & Initial Condition(s)	Solutions Available	Literature Source[a]	Comments
6	(1) Arbitrary boundary conditions.	Eq. (2.15)	Langford	(1) Temperature and heat flux time histories must be known at radial surface. (2) Time histories must be analytic.
7	(1) Constant or zero flux boundary condition. (2) Arbitrary initial temperature distribution	Eq. (2.16)	Olcer	(1) Represents a general analytical solution to the problem.

[a]See Bibliography.

circular cylinders.[22] It also contains the respective boundary and initial conditions and selected comments.

[22] Some of the solutions presented in Table 2.1 are valid for semi-infinite and finite cylinders as well as other geometries.

CHAPTER III

DEVELOPMENT AND DISCUSSION OF A PROGRAMMING TECHNIQUE FOR APPLYING THE ANALYTICAL SOLUTION

Equation (2.16), its auxiliary equations, the special cases of (2.16), and their auxiliary equations are in effect a collection of analytic solutions, accounting for the changing boundary and initial conditions, of each segment of the problem. What is required is to put these equations together to form a piece-wise solution to the overall heat transfer problem.

The logic that underlies the assimilation of the piece-wise solution is based on two facts. The first fact is that five different sets of boundary conditions are applied to equation (2.16) generating five special cases. The second fact is that the criteria for choosing the appropriate set of boundary conditions (special case equation) for a given physical situation is whether the boundary temperature is T_I, T_{max}, or T_{min} and from what direction the boundary temperature is approaching T_I, T_{max}, or T_{min}. The five sets of boundary conditions are presented in Table 3.1.

The use of the given boundary conditions in conjunction with equations (2.16) through (2.23) to produce the required piece-wise solution is demonstrated in Table 3.2.

TABLE 3.1

BOUNDARY CONDITIONS

Boundary Condition	Description	Comments (See Figure 2.2)
A	Initial Temp. Dist.: Constant, nonzero Boundary Temp.: T_I Heat Flux: q_o	Initial system configuration.
B	Initial Temp. Dist.: Variable, nonzero Boundary Temp.: T_{max} Heat Flux: q_o	System configuration when boundary heats up to T_{max}.
C	Initial Temp. Dist.: Variable, nonzero Boundary Temp.: T_{max} Heat Flux: 0	System configuration when the boundary begins to cool down from T_{max}.
D	Initial Temp. Dist.: Variable, nonzero Boundary Temp.: T_{min} Heat Flux: 0	System configuration when the boundary cools down to T_{min}.
E	Initial Temp. Dist.: Variable, nonzero Boundary Temp.: T_{min} Heat Flux: q_o	System configuration when the boundary begins to heat up from T_{min}.

TABLE 3.2

PIECE-WISE SOLUTION SCHEME

Time Span	Initial & Final Boundary Conditions[a]	Equation used to iterate to req'd boundary condition & record system time	Equation used to generate temp. dist. in food at time req'd boundary condition is reached
1 (Initial heat-up)	Initial: A	N/A	N/A
	Final: B	Eq. (2.28)	Eq. (2.24)
2 (1st cool down)	Initial: C	N/A	N/A
	Final: D	Eq. (2.31)	Eq. (2.30)
3 (Heat-up)	Initial: E	N/A	N/A
	Final: B	Eq. (2.29)	Eq. (2.16)
4 (Cool down)	Initial: C	N/A	N/A
	Final: D	Eq. (2.31)	Eq. (2.30)
5 (Heat-up)	Initial: E	N/A	N/A
	Final: B	Eq. (2.29)	Eq. (2.16)

[a]See Table 3.1.

The piece-wise solution scheme in Table 3.2 has been programmed for the Univac 1108 computer and is executed by the computer in the sequence suggested by Table 3.2 Steps (4) and (5) of the sequence are repeated by the machine until, at either Step (4) or (5), the centerline temperature is greater than or equal to T_{min}.

The basic flow diagram used to execute the piece-wise solution technique is presented in Figure 3.1. A completely detailed flow chart, listing of the program, and an example of the program output are presented in Appendix G.

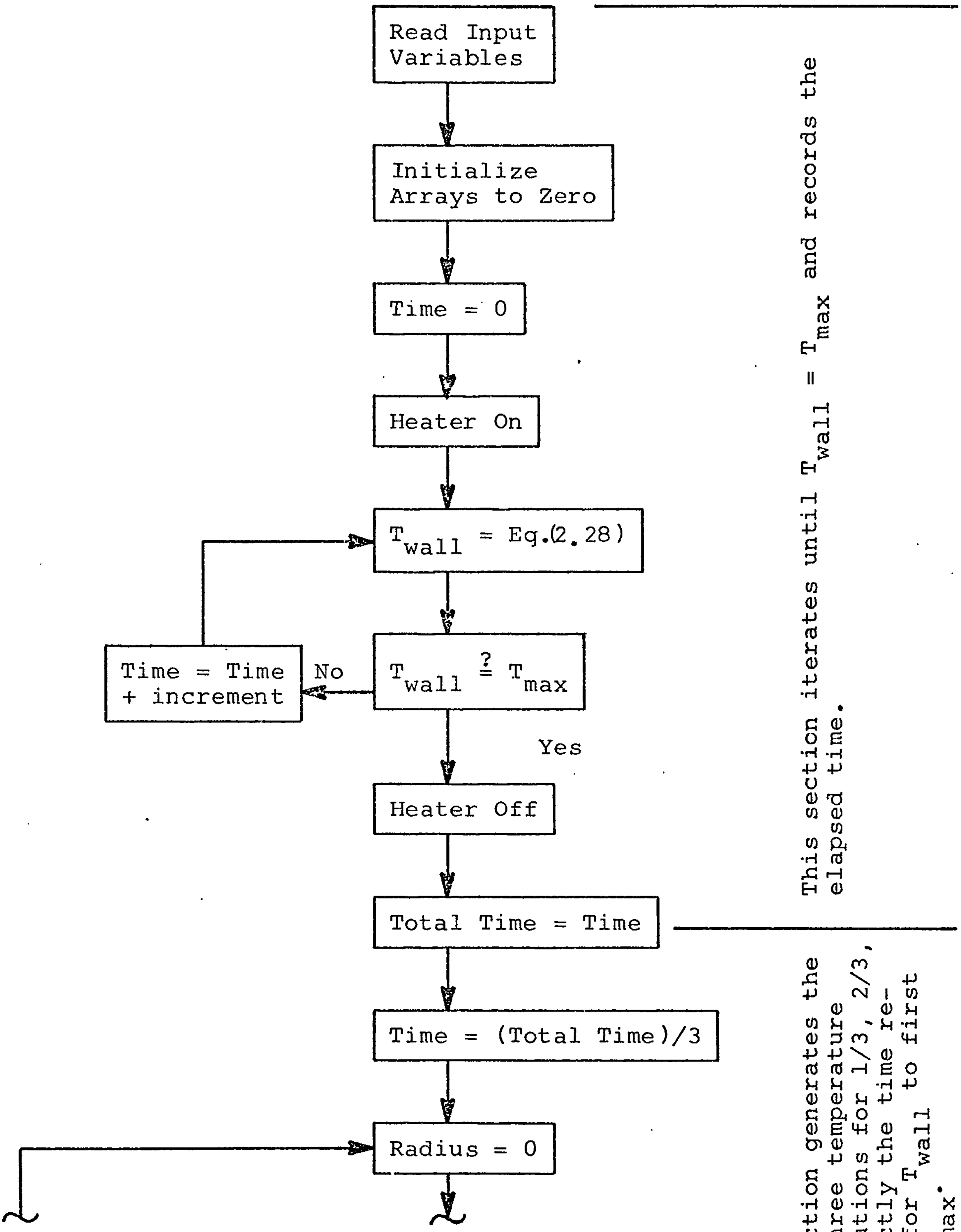
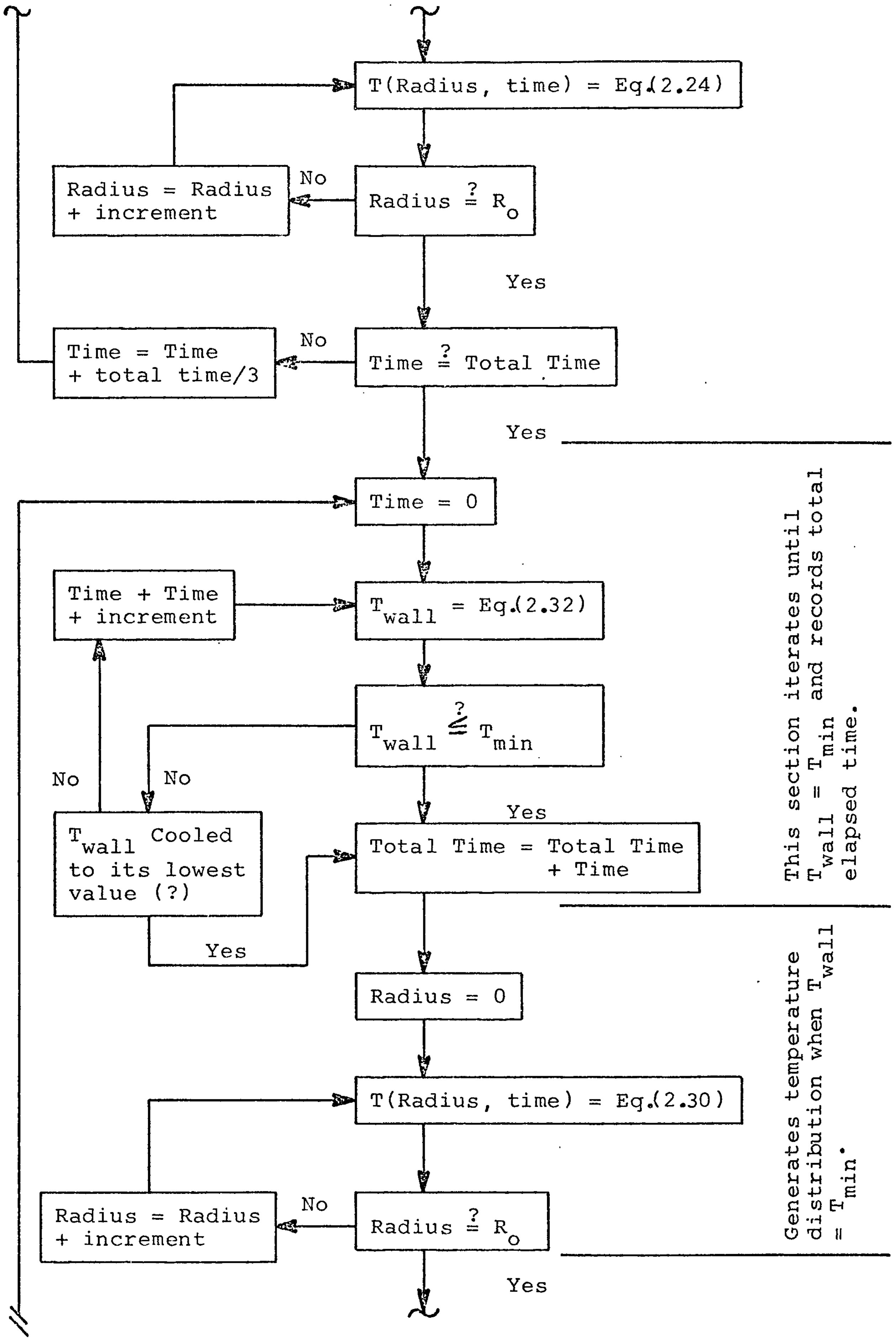
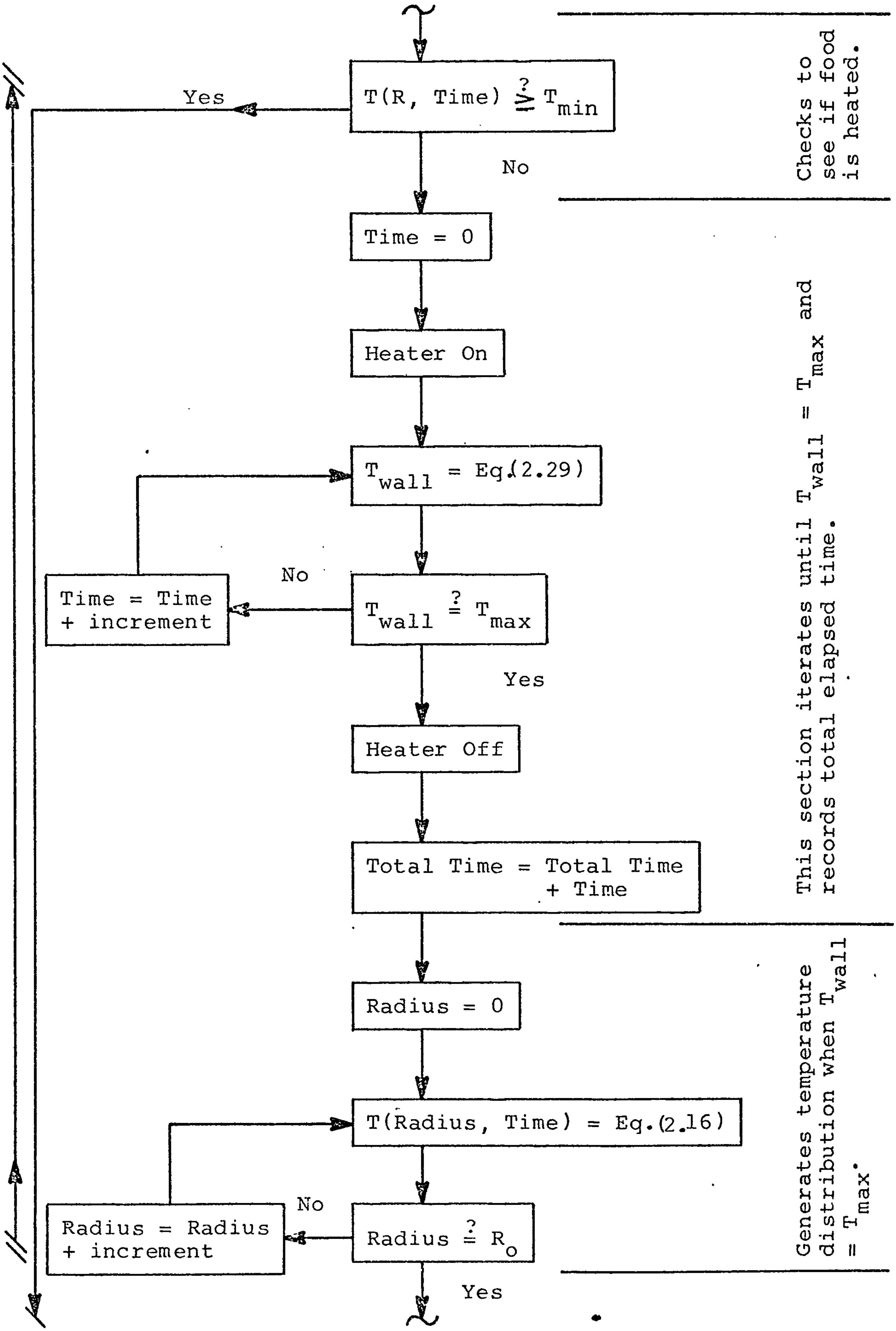


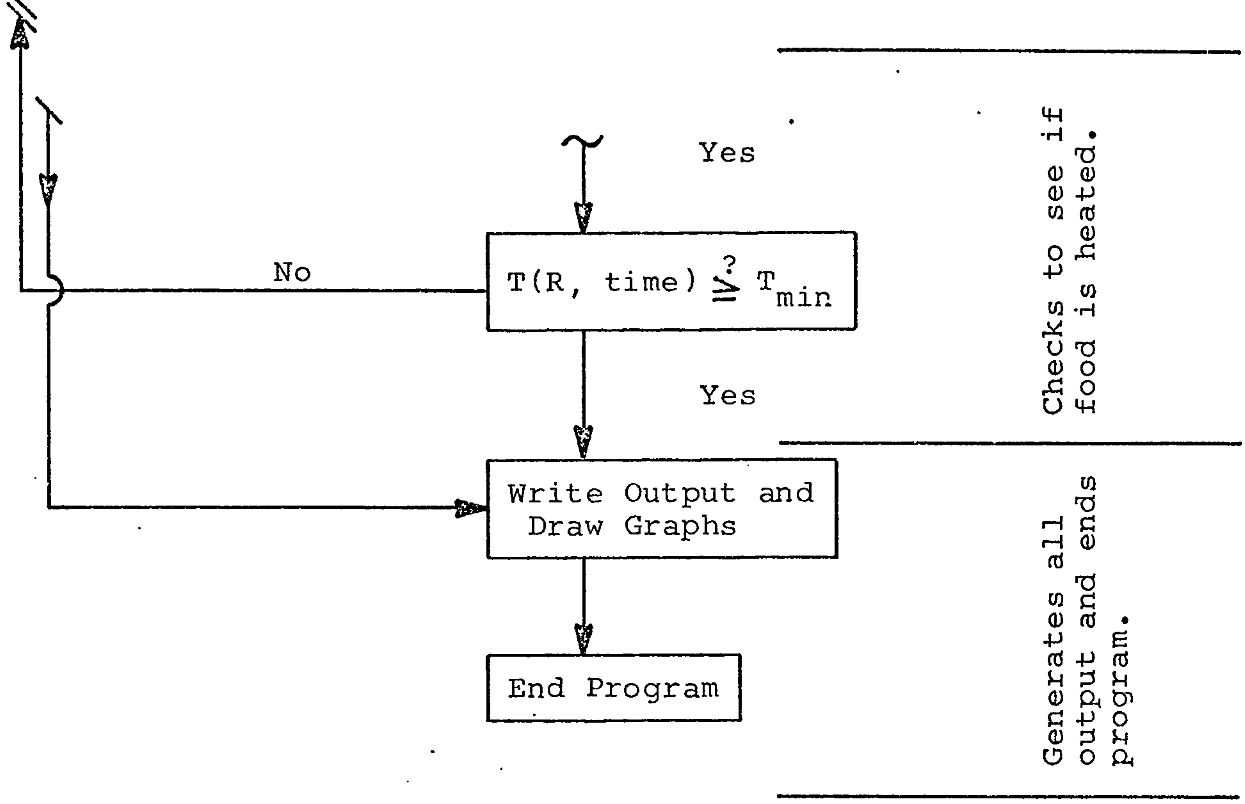
FIGURE 3.1

SIMPLIFIED FLOW CHART


T(Radius, time) = Eq.(2.24)
Radius = Radius + increment
No
Radius ?= R_o
Yes
Time = Time + total time/3
No
Time ?= Total Time
Yes
Time = 0
Time + Time + increment
T_wall = Eq.(2.32)
T_wall ?≦ T_min
No
No
T_wall Cooled to its lowest value (?)
Yes
Total Time = Total Time + Time
Yes
Radius = 0
T(Radius, time) = Eq.(2.30)
Radius = Radius + increment
No
Radius ?= R_o
Yes
This section iterates until T_wall = T_min and records total elapsed time.
Generates temperature distribution when T_wall = T_min.



T(R, Time) $\gtreqless^{?}$ T_{min}
Yes
No
Checks to see if food is heated.
Time = 0
Heater On
T_{wall} = Eq.(2.29)
Time = Time + increment
No
$T_{wall} \stackrel{?}{=} T_{max}$
Yes
Heater Off
Total Time = Total Time + Time
This section iterates until $T_{wall} = T_{max}$ and records total elapsed time.
Radius = 0
T(Radius, Time) = Eq.(2.16)
Radius = Radius + increment
No
Radius $\stackrel{?}{=} R_o$
Yes
Generates temperature distribution when $T_{wall} = T_{max}$.


Yes
No
T(R, time) ≥? Tmin
Yes
Write Output and Draw Graphs
End Program
Checks to see if food is heated.
Generates all output and ends program.

CHAPTER IV

DISCUSSION OF INPUT DATA TO THE COMPUTER PROGRAM

The determination of thermal properties of foods is not complete and is the subject of many studies today. However, several methods of estimating these properties exist and have been used for many years.[1] Most of the methods are based on the percentage of water a canned food contains and the fact that the dehydrated remainder of the can contents is a fibrous material similar to balsa wood or cork in its material properties. Using this logic a total homogeneous property is determined by multiplying the weight fraction of a particular component by its known property value and adding the results for all components together. The mathematical statement of the described operation is:

$$\text{Total Homogeneous Property} = \sum_{i=1}^{m} (\text{Weight Fraction})_i \times (\text{Property})_i \tag{4.1}$$

The requirement for the present study is to determine the range of material properties to be expected in canned foods. This was accomplished using equations similar to (4.1)

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[1] R. L. Earle, Unit Operations in Food Processing (1st ed.; Fairview Park, Elmsford, N.Y.: Pergamon Press, 1966).

(see Appendix H). The results of the work are presented in Table 4.1.

TABLE 4.1

MATERIAL PROPERTY RANGES TO BE EXPECTED IN CANNED FOODS

Property (Units)	Lower Limit	Intermediate Value[a]	Upper Limit
Thermal Conductivity (BTU/hr-ft-^{o}R)	.184	.235	.3081
Density (lbm/ft^3)	25.28	39.2	59.15
Heat Capacity at Constant Pressure (BTU/lbm-^{o}R)	.36	.60	.944
Thermal Diffusivity (Ft^2/hr)	.0055	.01	.02

[a]Representative of an average material property.

Along with the material property ranges exhibited in Table 4.1, it was decided that additional useful data would be produced by the material properties of pure water (H_2O). These properties are accurately known and readily available.[2]

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[2]Earle, loc. cit.; see also J. P. Holman, Heat Transfer (New York: McGraw-Hill Book Company, Inc., 1963).

The properties of water in conjunction with the values of Table 4.1 yield an expanded set of properties. The new property group is displayed by Table 4.2.

TABLE 4.2

MATERIAL PROPERTY RANGES CONSIDERED FOR ANALYSIS

Property (Units)	Lower Limit	Intermediate Values		Upper Limit
Thermal Conductivity (BTU/hr-ft-R)	.184	.235	.3081	.368[a]
Density (lbm/ft^3)	25.28	39.2	59.152	61.84[a]
Heat Capacity at Constant Pressure (BTU/lbm-R)	.36	.60	.944	.997[a]
Thermal Diffusivity (Ft^2/hr)	.0055	.0058[a]	.01	.02

[a]Values are for water at 110°F; see Holman, loc. cit.

The values in Table 4.2 are those that are entered in the digital computer program.

Another input variable to the program is the outer radius of the can. There are only two can sizes to be used in the Skylab Program. One size has an outer radius of approximately 1.34 inches and the other of 2.34 inches.

One more input variable that must be considered prior to executing the analysis is the applied heat flux, q_o.

A range of realistic boundary heat fluxes for this analysis was determined from Lis and Nuccio.[3] This reference suggested a nominal value of heat flux to be used. The nominal value's selection was based on the classic aircraft and spacecraft design considerations of availability of power and minimization of system weight. This nominal heat-flux value is 800 (BTU/hr-ft^2). The anticipated flux range has been arbitrarily built around the central flux value (see Table 4.3). The flux variation in Table 4.3 is considered to be representative of available spacecraft power levels.

TABLE 4.3

RANGE OF HEAT FLUXES CONSIDERED FOR ANALYSIS

Minimum Heating Rate	Nominal Heating Rate	Maximum Heating Rate
400 (BTU/hr-ft^2)	800 (BTU/hr-ft^2)	1200 (BTU/hr-ft^2)

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[3] S. J. Lis and P. P. Nuccio, Method of Heating Food in Aerospace Flight, Techn. Doc. Report No. AMRL-TDR-63-135, Biomedical Laboratory, 6570th Aerospace Medical Research Laboratories, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, December, 1963.

Three of the input variables–heat flux, can radius, and thermal conductivity–are combined to form a single input parameter (see page 20). This parameter is called the flux conduction parameter. It has been previously defined, but its definition is rewritten for clarity.

$$Q = \frac{q_o R_o}{K} \text{, flux conduction parameter} \tag{4.2}$$

Based on equation (4.2) Table 4.4 is generated which is analogous to Table 4.3 for heat flux.

TABLE 4.4

RANGE OF THE FLUX CONDUCTION PARAMETER

Can Radius R_o (In)	Minimum Flux Conduction Parameter (R)	Maximum Flux Conduction Parameter (R)
1.34	121.38	728.26
2.34	211.96	1271.74

From Table 4.4, it is seen that the maximum range of the flux conduction parameter is from 121.38 (R) to 1271.74 (R).

This important input variable is not read directly into the computer, but is calculated using (4.2) from other input data.

By introducing the parameter the number of unique input variables of θ is reduced to three. They are Q, α, and R_o.

After examination of the data presented in Tables 4.2, 4.3, and 4.4 and taking into account the knowledge that two sizes of cans are to be heated, it was decided that twenty-four data cases should be submitted to the computer.

Mathematically, with three values of q_o, four values of K and α, and two values of R_o; there are ninety-six possible combinations of the input variables. However, only twenty-four of these combinations have any physical meaning.

The twenty-four data cases submitted to the computer are summarized in Table 4.5 (they will generate adequate output data to draw the correct results and conclusions).

TABLE 4.5

SUMMARY OF DATA CASES FOR THE COMPUTER

Data Case	Can Radius (in)	Thermal Conductivity (BTU/HR-FT-R)	Thermal Diffusivity (FT^2/HR)	Flux Conduction Parameters (R)	Material Most Closely Represented
1	1.34	.184	.02	242.75	Bacon
2	1.34	.184	.02	485.51	Bacon
3	1.34	.184	.02	728.26	Bacon
4	1.34	.235	.01	190.07	Fat Beef
5	1.34	.235	.01	380.14	Fat Beef
6	1.34	.235	.01	570.21	Fat Beef
7	1.34	.368	.0058	121.34	Water
8	1.34	.368	.0058	242.75	Water
9	1.34	.368	.0058	364.13	Water
10	1.34	.3081	.0055	144.97	Asparagus
11	1.34	.3081	.0055	289.95	Asparagus
12	1.34	.3081	.0055	434.93	Asparagus
13	2.34	.184	.02	423.91	Bacon
14	2.34	.184	.02	847.83	Bacon
15	2.34	.184	.02	1271.74	Bacon
16	2.34	.235	.01	331.91	Fat Beef
17	2.34	.235	.01	663.83	Fat Beef
18	2.34	.235	.01	995.74	Fat Beef
19	2.34	.368	.0058	211.96	Water
20	2.34	.368	.0058	423.91	Water
21	2.34	.368	.0058	635.87	Water
22	2.34	.3081	.0055	253.16	Asparagus
23	2.34	.3081	.0055	506.33	Asparagus
24	2.34	.3081	.0055	759.51	Asparagus

CHAPTER V

DETERMINATION OF ERROR ENVELOPES ASSOCIATED WITH THE ANALYSES

The error is defined by equation (5.1); it is, in general, a function of the same arguments as the dimensionless temperature, θ (equation (2.16)). In addition the error is a function of M, the finite number of series terms taken.

$$ERR = f(F_o, F_{om}, R_o, R, I_m, Q, \rho_m, \rho_{om}, M) \tag{5.1}$$

For the material property ranges of canned foods and a given food can radius, the variation of only six of the nine arguments of (5.1) significantly effect the error. This six argument effect is demonstrated by (5.2).

$$ERR \simeq f(F_o, F_{om}, R, I_m, \rho_m, M) \tag{5.2}$$

The first two arguments in (5.2) are functions of t. The next three are functions of the size of the radial increment used in the digital computer program. The last argument is the number of series terms taken.

Based on the experience gained by use of the program to evaluate data range effects, equation (5.2) is replaced with equation (5.3).

$$ERR \simeq f(\alpha t, DR, M) \tag{5.3}$$

The most apparent and the hardest error type to control is the undamped error phenomena; it is associated with the first argument of (5.3). The error is produced in equation (2.16) and its special cases when the αt product in the exponential terms of these equations approaches zero. Equation (5.4) demonstrates the form of the terms under discussion for a given radius.

$$\text{General Form} = \sum_{m=1}^{M} \left(e^{-(\text{const})\alpha t}\right) \times \qquad (5.4)$$

(Bessel Functions = f(m))

The exponential term is the damping term, it damps out the error oscillations that occur when the series of Bessel Functions are evaluated for a finite number of terms. If the product αt approaches zero the exponential (damping) term approaches one. At this value, one, there is no damping; this causes the worst case error.

The second variable in (5.3) that effects the error is the radial increment size chosen for numerical integration performed by the computer. The form of the integration is demonstrated by equation (5.5).

$$\text{Integral} = \int_0^{R_o} J_o \; (\rho_m)\; r\,dr \qquad (5.5)$$

In equation (5.5) the radius, r, is an argument of the Bessel Function ($\varphi_m = f(r)$), appears by itself in the integrand, and is the variable of integration.

The smaller the size of the radial increments taken the more accurately the integral is evaluated and the smaller the error induced into equation (2.16) and its special cases.

The last argument of (5.3) to be discussed is the number of terms of the series used. The first few (one to seven) terms add great accuracy to the equation being evaluated, but the accuracy effect of adding terms dies out exponentially as M increases. Twenty terms of the series in equation (2.16) and its special cases were taken.

Proof of series convergency is furnished by two different methods. The first is the small (less than 1/2 a percent) accuracy improvement noted when forty terms of the series are compared to the twenty terms used. The second method[1] uses only five terms of the series and maintains the accuracy level of twenty terms. The method uses a simple algebraic closed form to converge a series by using three of its partial sums. A partial sum of the series is defined, generally, by equation (5.6).

---

[1]M. Abramowitz and A. Stegun, Handbook of Mathematical Functions (New York: Dover Publications, Inc., 1964).

$$S^n = \sum_{i=1}^{n} (\text{general series term})_i \qquad (5.6)$$

The convergency relation is:

$$\text{Series Sum} = \frac{S^n S^{n+2} - (S^{n+1})^2}{S^n + S^{n+2} - 2S^{n+1}} \qquad (5.7)$$

Both of the methods converge to the same temperature profiles.

An estimate of the total error involved in the analysis can be attained by comparing the theoretical requirement of zero heat flux during a cool-down cycle (heater shut off) to the actual numerical results. For the perfectly insulated condition the energy in the system remains constant as does $\overline{T}$, which is proportional to the system energy. Therefore, any variation in the value of $\overline{T}$, as numerically computed, at the beginning and end of a cool-down cycle is error.

$$\%\ \text{ERROR} = 100 \times \frac{\overline{T}\ \text{initial} - \overline{T}\ \text{final}}{\overline{T}\ \text{initial}} \qquad (5.8)$$

The maximum error encountered for any cool-down cycle and for any data case is less than one-half of a percent.

CHAPTER VI

RESULTS

The majority of the results of the computerized analyses are displayed graphically. Therefore, each figure (graph) is presented and discussed individually. For presentation purposes a nominal data case (Case 17, Table 4.5) has been chosen. The effect of each parameter is then demonstrated while all other parameters remain fixed at their nominal values.

The first typical result is demonstrated in Figure 6.1; it is a temperature profile for the nominal data case (Case 17, Table 4.5). Curves 1 and 2 are intermediate profiles for 1/3 and 2/3 of the time required for the can wall to first reach T_{max}; Curve 3 is the temperature when the can wall first does reach T_{max}. The remaining odd numbered curves correspond to times when the can wall heats to T_{max}, and the remaining even numbered curves correspond to the time when the can wall cools to T_{min}. Curve (16), the last profile, occurs when the can content's temperature at the centerline is greater than or equal to T_{min}) and the wall temperature is less than or equal to T_{max}. Notice the flatness of this final profile.

Figure 6.2 demonstrates that $\overline{T}$ remains constant (except for round off error) during a cool-down cycle (line

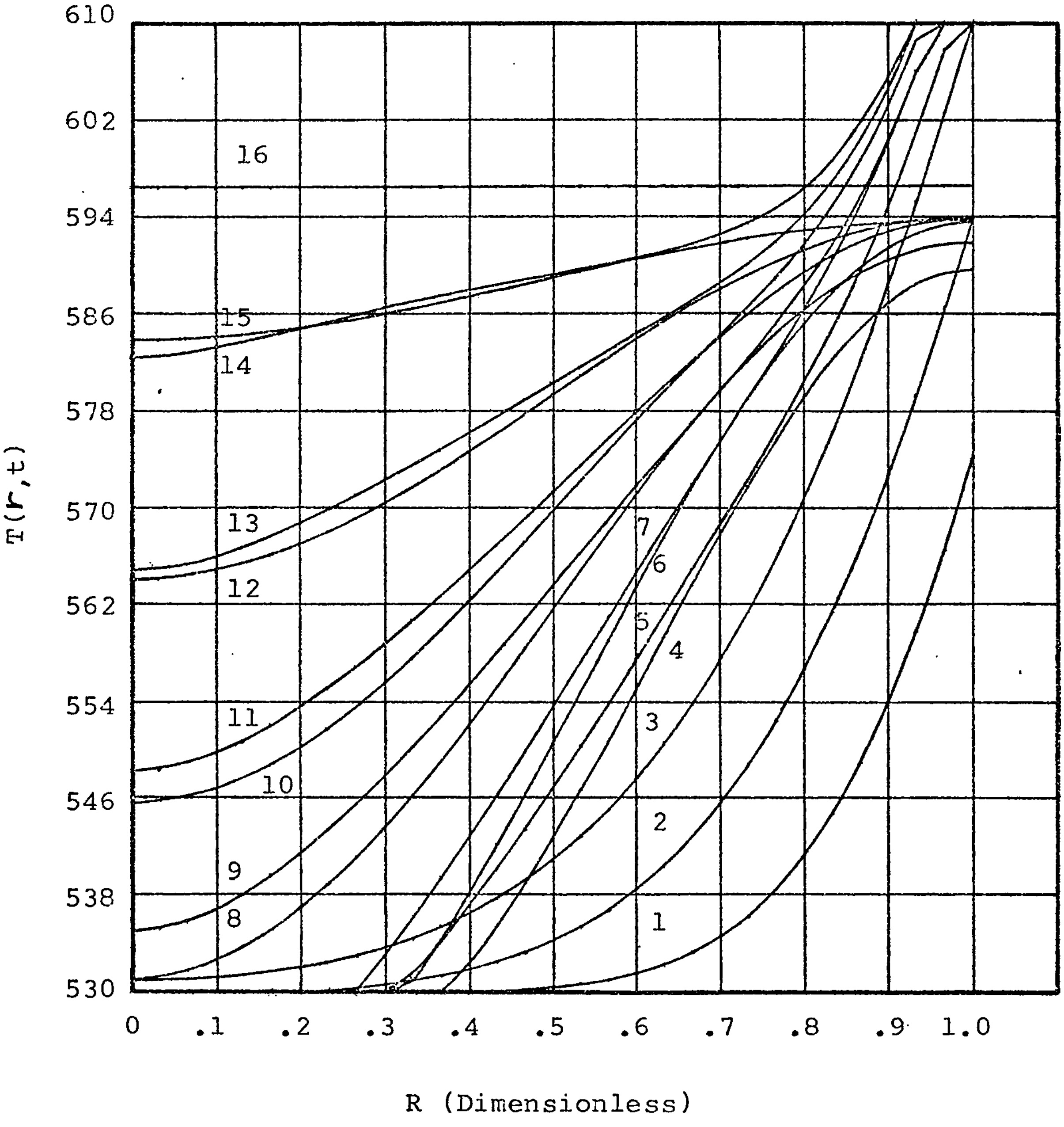
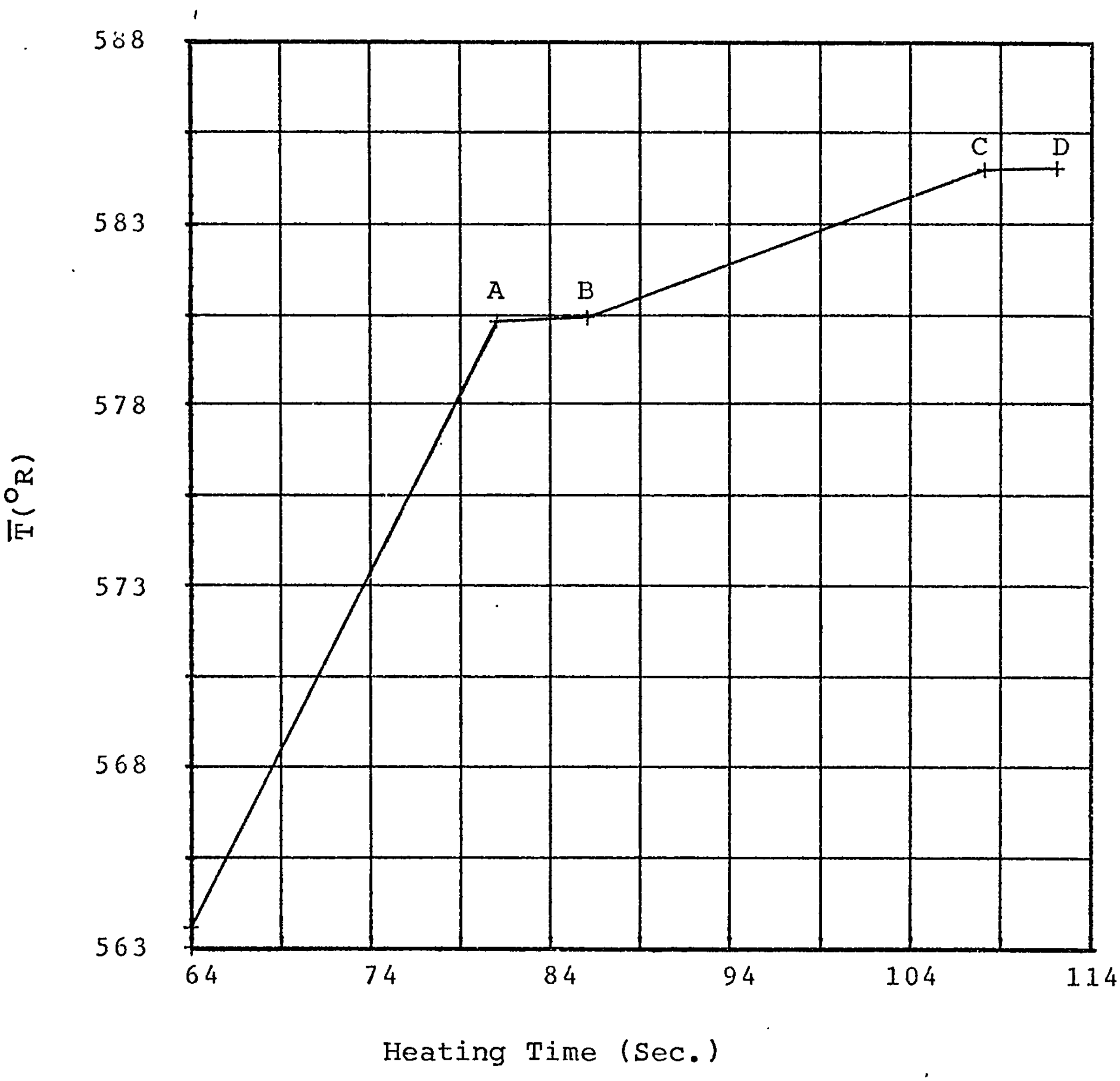


FIGURE 6.1

TYPICAL FAMILY OF CURVES FOR DIMENSIONLESS TEMPERATURE VERSUS DIMENSIONLESS RADIUS



<u>Parameters</u>

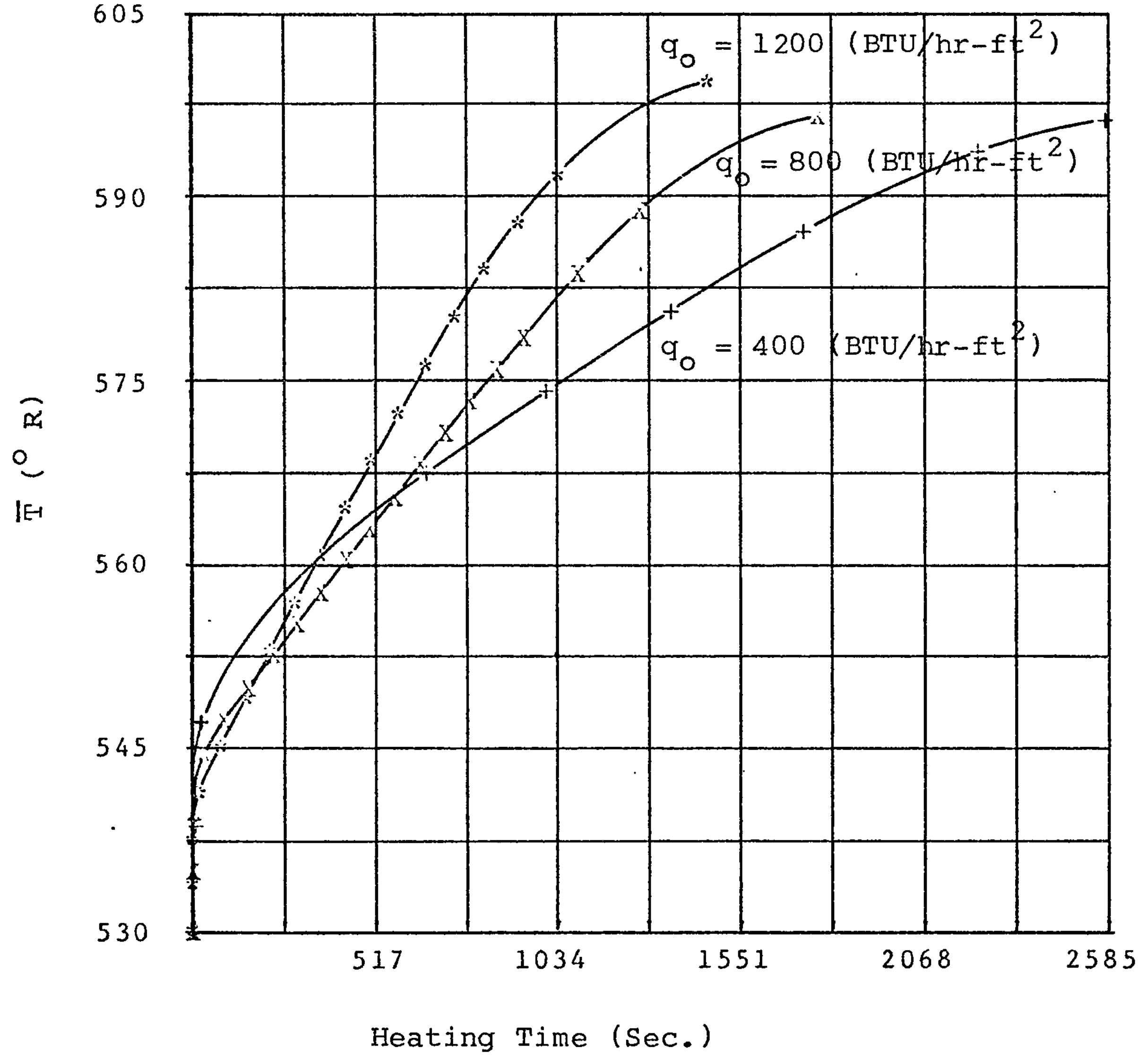
R_o = 1.34 (in.)
α_o = .0058 (ft^2/hr.)
q_o = 400 (BTU/hr-ft^2)

FIGURE 6.2

CONSTANT SYSTEM ENERGY DURING COOL-DOWN CYCLES segments AB, and CD in Figure 6.2) ($q_o = 0$). This is consistent with the criteria of zero heat flux (no energy leaving the system) that exists during cool-downs. Figure 6.2 (Case 7, Table 4.5) is plotted using a large scale to show more clearly the constant energy steps that occur during any data run for cool-down sequences.

Figure 6.3 represents the effects of heat flux as a parameter on a plot of $\overline{T}$ versus heating time for the nominal data case (Case 17, Table 4.5). The can size (R_o = 2.34 in.) and the thermal diffusivity (α = .01 Ft^2/hr) are held constant in Figure 6.3. As the heat flux applied at the radial surface increases, the total heating time decreases; and $\overline{T}$ increases at a faster rate. For $\overline{T} = T_{min}$ (595^o R), the heating time is approximately .40 hours (1420 seconds) with q_o = 1200 (BTU/hr-ft^2), and approximately .70 hours (2585 seconds) with q_o = 400 (BTU/hr-ft^2) a decrease of 80 percent in heating time for a corresponding increase of 200 percent in heat flux. The crossover of the curves of Figure 6.3 that occurs during low times is caused by the longer initial heating periods (initial time the heater is on and the initial time that is required for the wall to first reach T_{max}) required for the lower surface heat rates.

Figure 6.4 represents the effects of thermal diffusivity as a parameter on a plot of $\overline{T}$ versus heating time for the nominal data case (Case 17, Table 4.5). The can size (R_o =

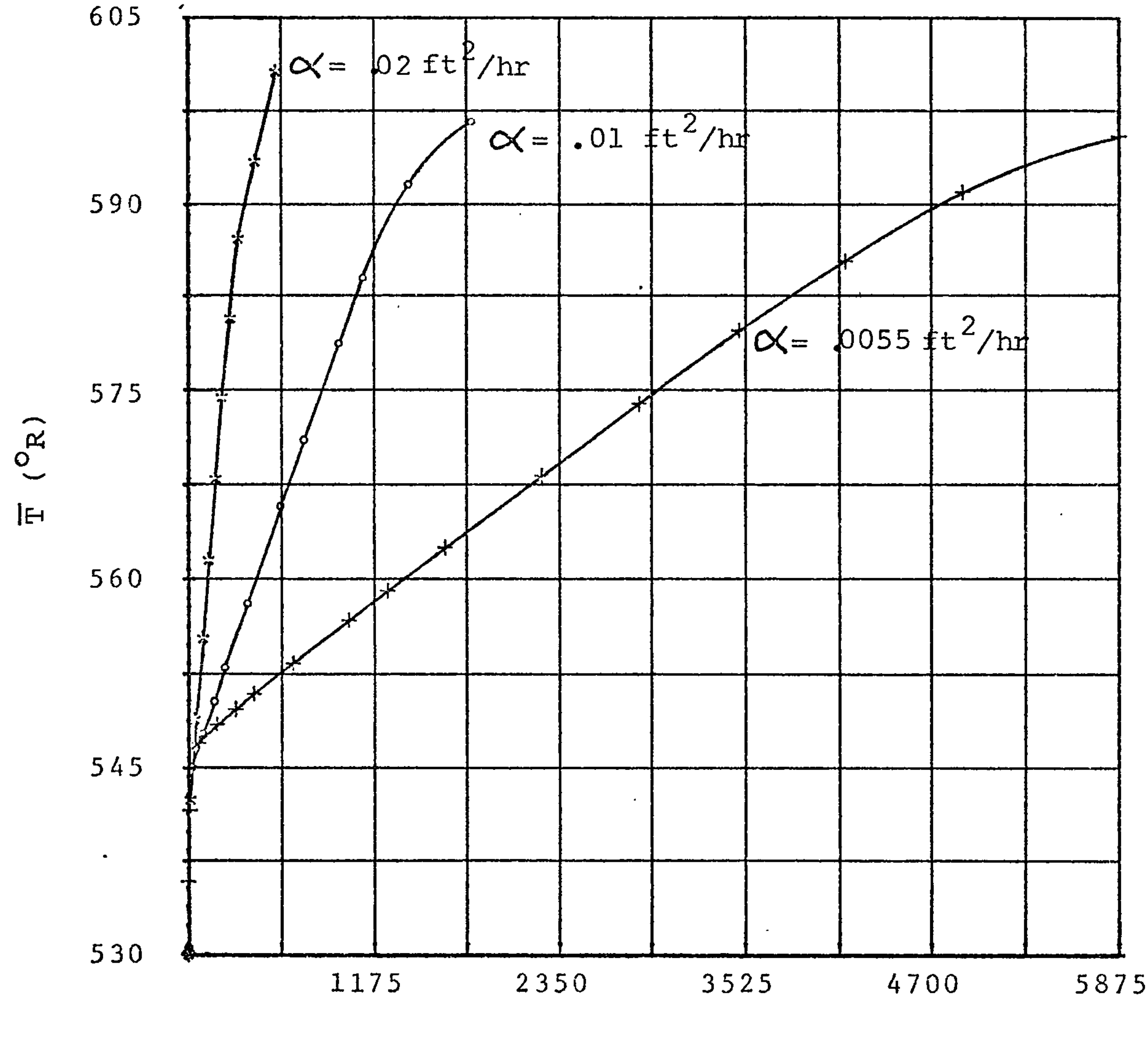


Parameters

R_o = 2.34 (in.)
α = .01 ft^2/hr.

FIGURE 6.3

MEAN SYSTEM TEMPERATURE VERSUS TIME WITH HEAT FLUX AS A PARAMETER FOR THE NOMINAL DATA CASE



Parameters

R_o = 2.34 (in.)
q_o^o = 800 (BTU/hr-ft^2)

FIGURE 6.4

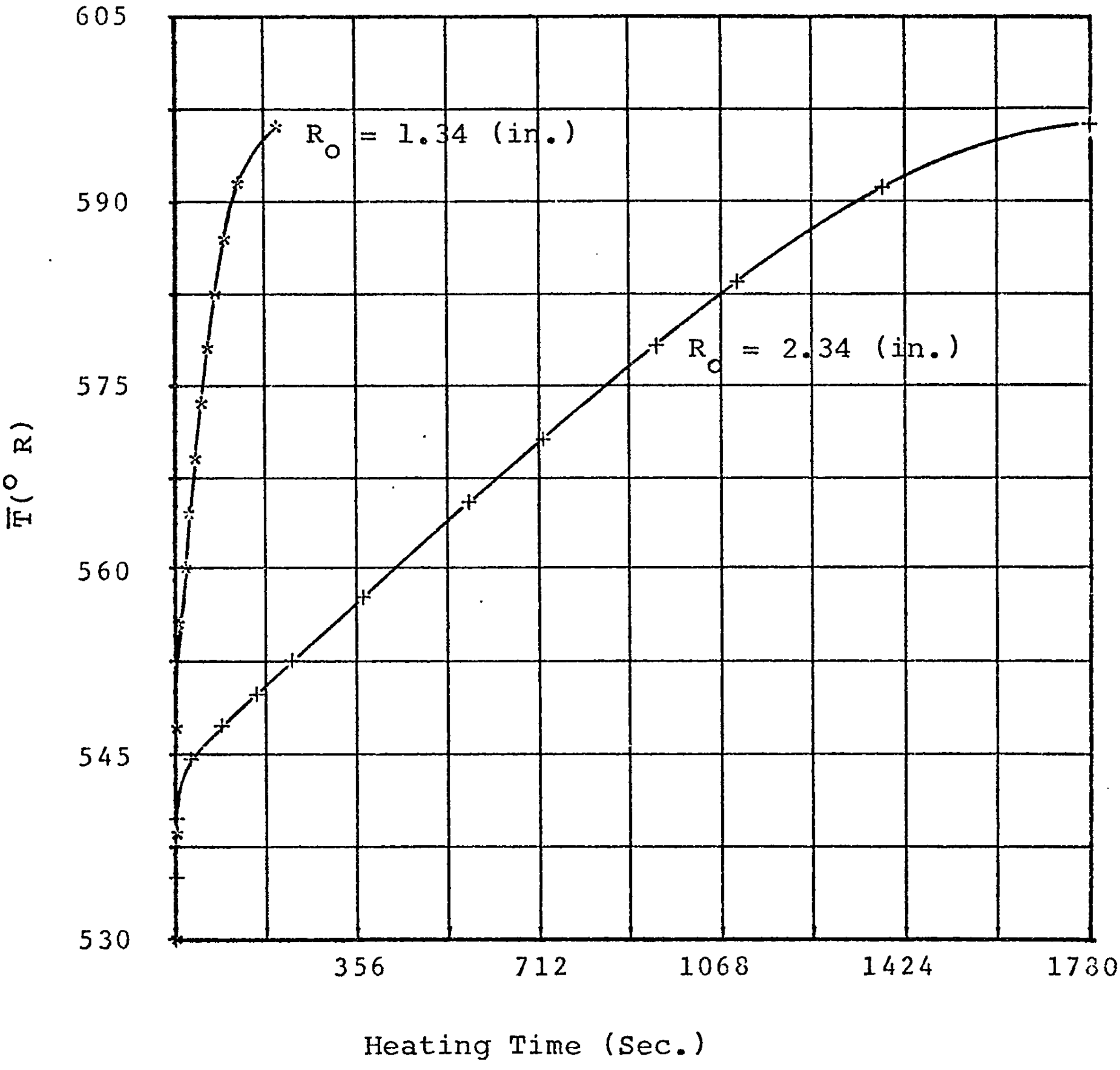
MEAN SYSTEM TEMPERATURE VERSUS TIME WITH THERMAL DIFFUSIVITY AS A PARAMETER FOR THE NOMINAL DATA CASE

2.34 in.), and the heat flux ($q_o^{\cdot}$ = 800 BTU/hr-ft^2) at the can's radial surface are held constant in this plot. As the thermal diffusivity increases, heating time decreases and $\overline{T}$ increases at a faster rate. For $\overline{T} = T_{min}$ (595^o R), the heating time is approximately .13 hours (470 seconds) for α= .02 ft^2/hr; and approximately 1.63 hours (5875 seconds) for α= .0055 ft^2/hr, an increase of 1150 percent in heating time for a corresponding decrease of 265 percent in thermal diffusivity.

Figure 6.5 represents the effects of can size (outer radius) as a parameter on a plot of $\overline{T}$ versus heating time with constant flux (800 (BTU/hr-ft^2-^{o} R)) and thermal diffusivity (.01 (ft^2/hr)). As the can radius increases, heating time increases, and $\overline{T}$ increases at a slower rate. For $\overline{T} = T_{min}$ (595^o R), it takes approximately .06 hours (225 seconds) to heat a small Skylab food can (R_o = 1.34 in.) and approximately .49 hours (1780 seconds) to heat up a large Skylab food can (R_o = 2.34 in.), an increase of 690 percent in heating time for a corresponding increase of 75 percent in can radius.

Figure 6.6 represents the variation of food centerline temperature with time with thermal diffusivity as a parameter for the nominal data case (Case 17, Table 4.5). As the thermal diffusivity increases heating time decreases and T_c increases at a faster rate.

An oscillation between consecutive data points is displayed in Figure 6.6; the oscillation generates two parallel

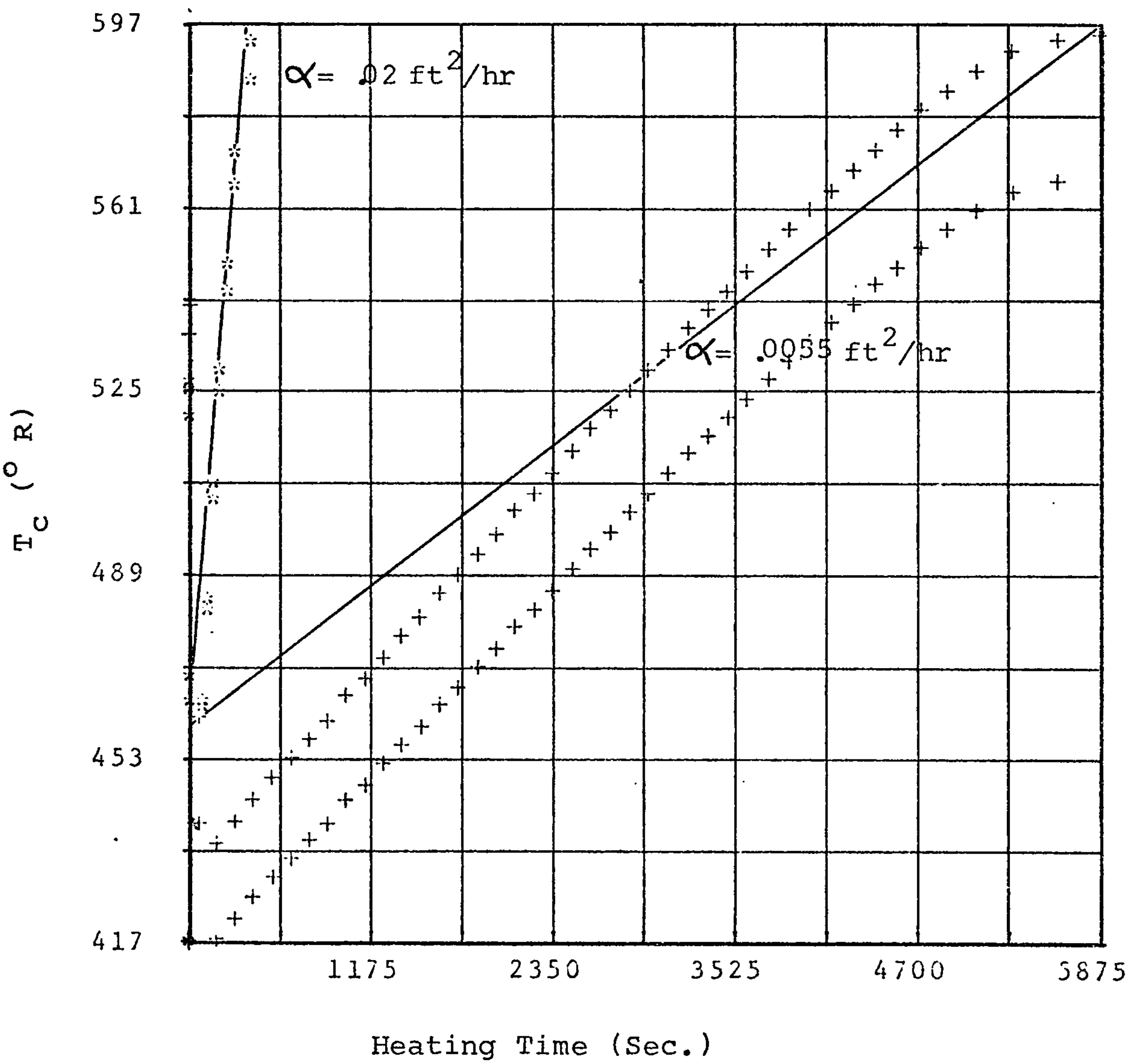


Parameters

q_o = 800 (BTU/hr-ft^2)
α = .01 ft^2/hr

FIGURE 6.5

MEAN SYSTEM TEMPERATURE VERSUS TIME WITH EXTERNAL RADIUS AS A PARAMETER FOR THE NOMINAL DATA CASE



<u>Parameters</u>

R_o = 2.34 (in)
q_o^o = 800 (BTU/hr-ft^2)

FIGURE 6.6

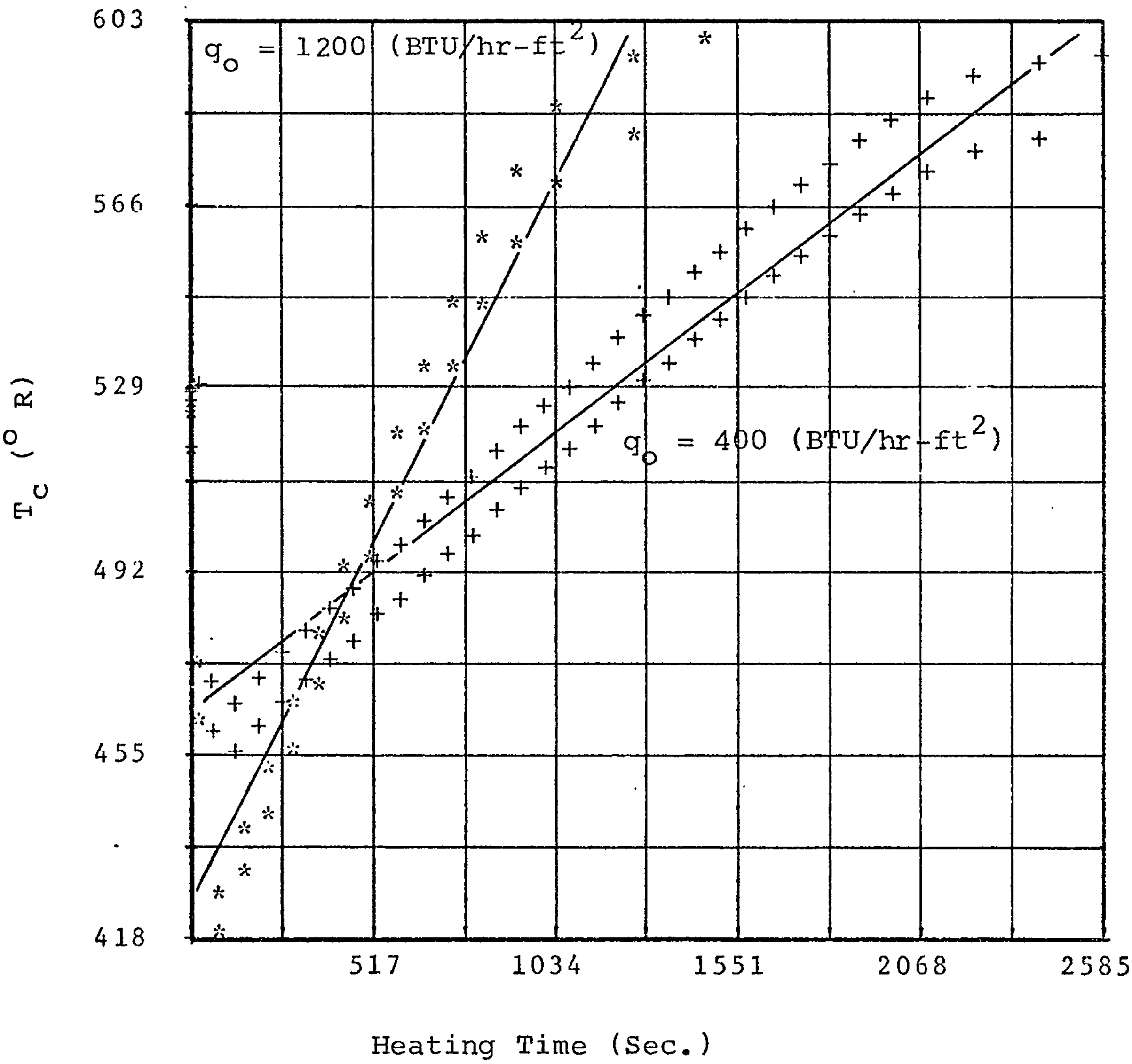
FOOD CENTERLINE TERMPERATURE VERSUS TIME WITH THERMAL DIFFUSIVITY AS A PARAMETER FOR THE NOMINAL DATA CASE curves. The data points in the higher valued curve correspond to heat-up solutions (equation (2.16)), and those in the lower valued curve correspond to cool-down solutions (equation 2.30)). Also, three additional data points for low times are plotted from equation (2.24) corresponding to the first three temperature distributions analogous to those of Figure 6.1. The random effect noticed for the individual data points at the centerline as contrasted to the smooth curves of the average property $\overline{T}$ versus time (e.g., Figure 6.3) is caused by the use of mean initial conditions rather than initial distributions when changing from a heat-up to a cool-down solution (or vice versa). The curve fitted to this data is linear. The differences between these three solutions (equations (2.16), (2.24), and (2.30)) are demonstrated by Table 6.1.

Figure 6.7 represents the variation of food centerline temperature with time with heat flux as a parameter for the nominal data case (Case 17, Table 4.5). As the heat flux increases, heating time decreases; and T_c increases at a faster rate. The comments, stated for Figure 6.6, about the oscillation of consecutive data points apply to Figure 6.7, as well.

Figure 6.8 represents the variation of food cylinder surface temperature (measured by the thermocouple) with time for the nominal data case (Case 17, Table 4.5) with thermal diffusivity as a parameter. The plot consists of data points

TABLE 6.1
COMPARISON OF PIECE-WISE SOLUTIONS USED IN THE ANALYSIS
TO THE GENERAL EQUATION BY TERM AND BY
AVERAGE PROPERTY INITIAL CONDITIONS

Primary Equation Number and Comments	Primary Equation	Auxillary Equations of Interest	Difference in Terms From the General Solution	Difference in Auxillary Equations From General Solution
(2.16) (Heat-up solution and general solution form)	$\Theta(F_o, F_{om}, R_o, R, I_m, Q, \varphi_m, \varphi_{om}) = \Theta$ $= \frac{2}{T_{max} - \bar{T}} \left[Q F_o + \frac{Q}{4}(R^2 - \frac{1}{2}) - Q R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\varphi_m)}{\varphi_m^2 J_o(\varphi_{om})} \right) + 1/R_o^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o(\varphi_m)}{J_o^2(\varphi_{om})} \right) \right]$	$\bar{T} = (2/R_o^2) \int_0^{R_o} T(r,t=0)\, r\,dr$ $I_m = \int_0^{R_o} T(r,t=0)\, J_o(\varphi_m)\, r\,dr$ $Q = \frac{q_o R_o}{K}$	None	None
(2.24) (Solution used to generate temperature profile when wall first heats to T_{max})	$\Theta = \left\{ \frac{2Q}{T_{max} - \bar{T}} \left[F_o + \frac{1}{4}(R^2 - \frac{1}{2}) - R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\varphi_m)}{\varphi_m^2 J_o(\varphi_{om})} \right) \right] \right\} + \left\{ 1/R_o^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o(\varphi_m)}{J_o^2(\varphi_{om})} \right) \right\}$	$\bar{T} = T_I = T(r, t=0)$ $I_m = T_I \int_0^{R_o} J_o(\varphi_m)\, r\,dr$ $Q = \frac{q_o R_o}{K}$	None	$\bar{T} = T_I = T(r,t=0)$ $I_m = T_I \int_0^{R_o} J_o(\varphi_m)\, r\,dr$
(2.30) (Cool-down solution)	$\Theta = \frac{2}{T_{max} - \bar{T}} \left[\frac{1}{R_o^2} \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o(\varphi_m)}{J_o^2(\varphi_{om})} \right) \right]$	$Q = 0$	$\Theta = \frac{2}{T_{max} - \bar{T}} \left[Q F_o + \frac{Q}{4}(R^2 - \frac{1}{2}) - Q R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\varphi_m)}{\varphi_m^2 J_o(\varphi_{om})} \right) \right]$	$Q = 0$

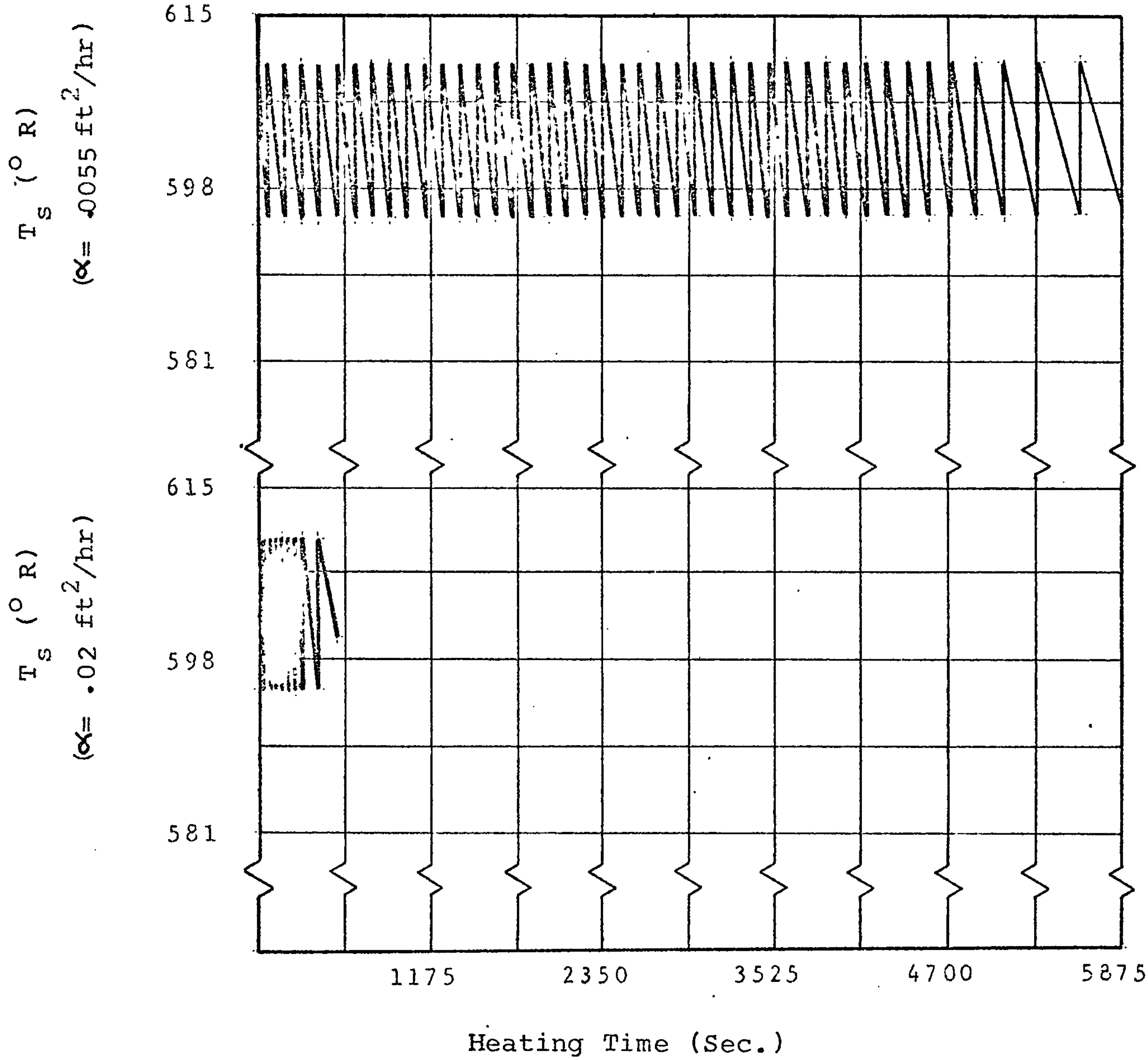


Parameters

R_o = 2.34 (in.)
α = .01 ft^2/hr

FIGURE 6.7

FOOD CENTERLINE TEMPERATURE VERSUS TIME WITH HEAT FLUX AS A PARAMETER FOR THE NOMINAL DATA CASE



<u>Parameters</u>

R_o = 2.34 (in.)
q_o = 800 (BTU/hr-ft^2)

FIGURE 6.8

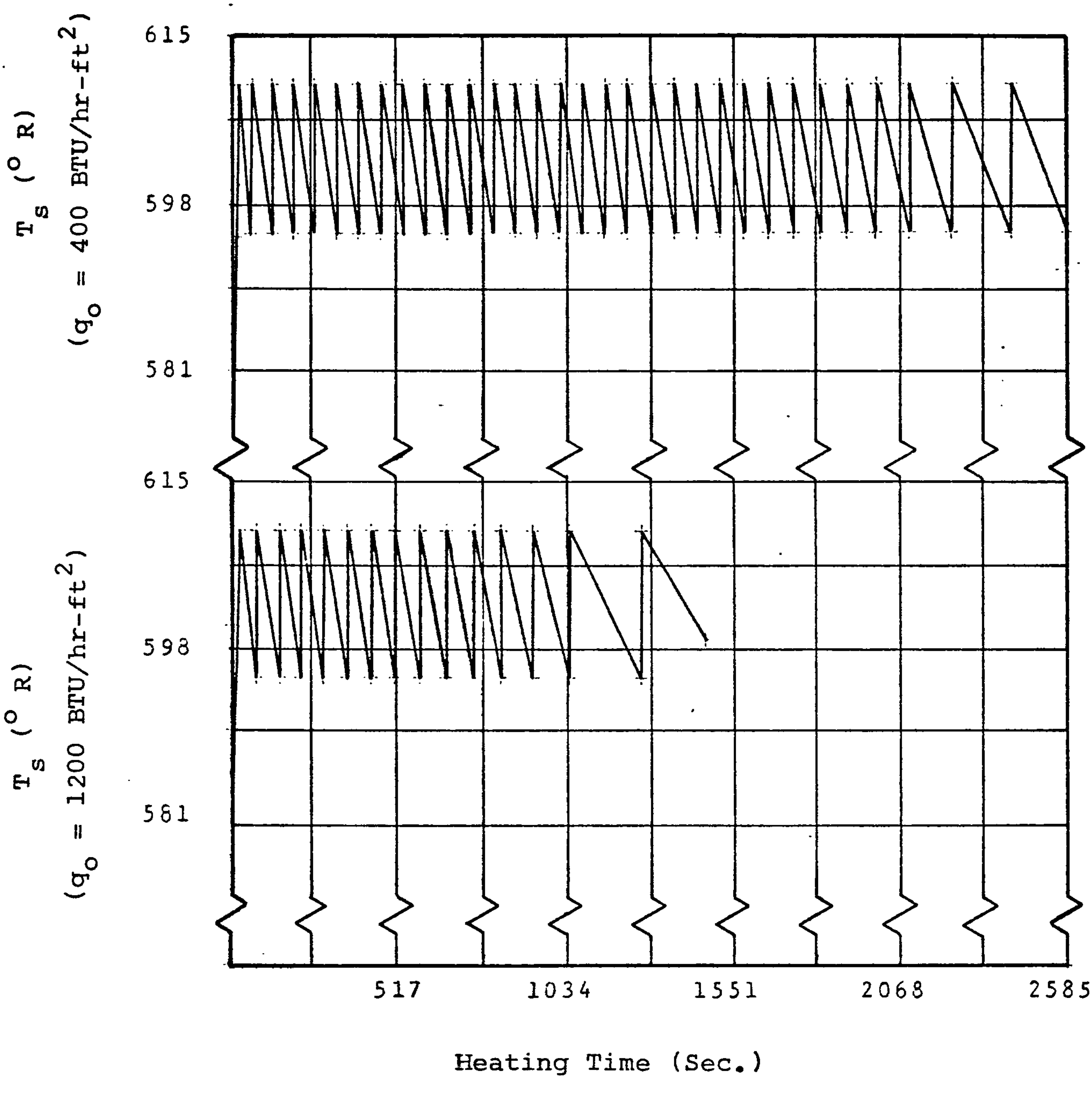
FOOD SKIN TEMPERATURE VERSUS TIME WITH THERMAL DIFFUSIVITY AS A PARAMETER FOR THE NOMINAL DATA CASE corresponding to the cyclic heat-up ($T|_{wall} = T_{max} = 610^{o}$ R) and cool down ($T|_{wall} = T_{min} = 595^{o}$ R) times of the can wall during the heating process. The general effects suggested by Figure 2.2 are present in this plot. They are the increase in time for each cool-down cycle as the number of cool-down cycles increases, and the decrease in the heat-up cycle time for each cycle as the number of cycles increases. The latter effect is diminished visually because of the overall scale in Figure 6.8; but, nevertheless, it is present in the graph.

The heat-up times are very short (less than 3 seconds) for the canned food data ranges because the low capability of foods to conduct heat allows the heat to remain at the can wall where it is first applied. This causes the wall temperature to climb rapidly to T_{max}, causes the heater to shut-off, and initiates a cool-down cycle.

Cool-down cycles are relatively long, again, because of the slow conduction of heat into the food's interior.

The relative size of cool-down and heat-up cycles is readily apparent from Figure 6.8. As α increases, heating times decrease in Figure 6.8.

Figure 6.9 represents the variation of food cylinder surface temperature (measured by the thermocouple) with time for the nominal data case (Case 17, Table 4.5) with applied heat flux as a parameter.



<u>Parameters</u>

R_o = 2.35 (in.)
α = .01 (ft^2/hr)

FIGURE 6.9

FOOD SKIN TEMPERATURE VERSUS TIME WITH HEAT FLUX AS A PARAMETER FOR THE NOMINAL DATA CASE

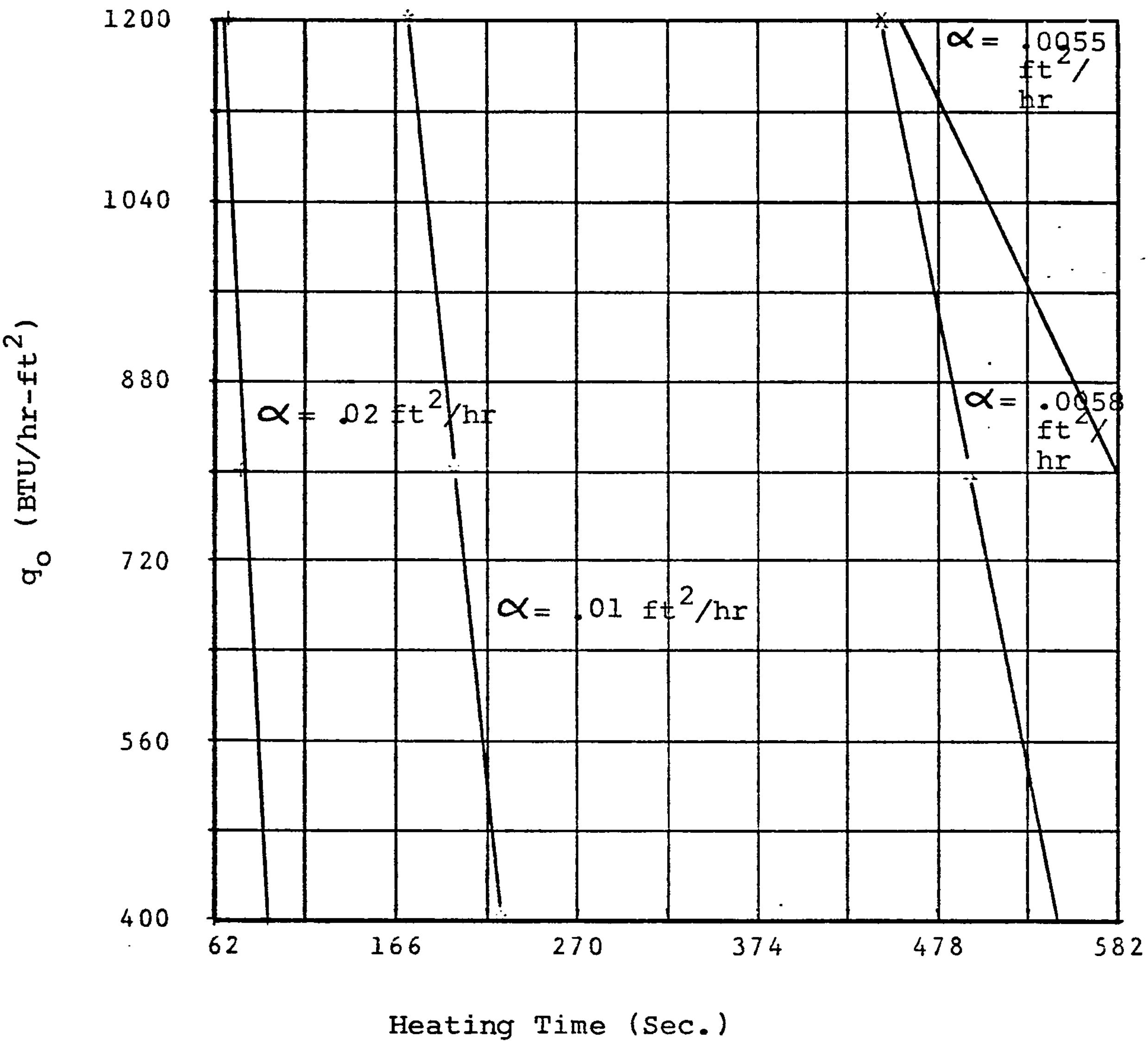
The same general effects noted for Figure 6.8 also apply to Figure 6.9.

Figure 6.10 represents the variation between heat flux and total heating time with thermal diffusivity as a parameter. The can size is held constant (R_o = 1.34 in., for the small Skylab can). The constant α curves are approximately linear in Figure 6.10. The general trend is that the larger values of q_o, for a given α, require a shorter heating time. Also, the larger constant α lines required shorter heating times for any given heat flux.

Figure 6.11 represents the variation between heat flux and total heating time with thermal diffusivity as a parameter for the large Skylab can size (R_o = 2.34 in.). The trends are the same for Figure 6.11 as they are for Figure 6.10.

From the twenty-four data cases submitted to the computer (Table 4.5) the range of heating times to be expected for the Skylab food system has been determined. The information is displayed in Table 6.2, page 65.

Finally, the impact of the infinite cylinder assumption on the results of this paper is examined. Since there are no available solutions to the finite cylinder with the constant flux boundary condition; the assumption is made that the heating time difference (error) between infinite and finite cylinders is approximately the same for constant

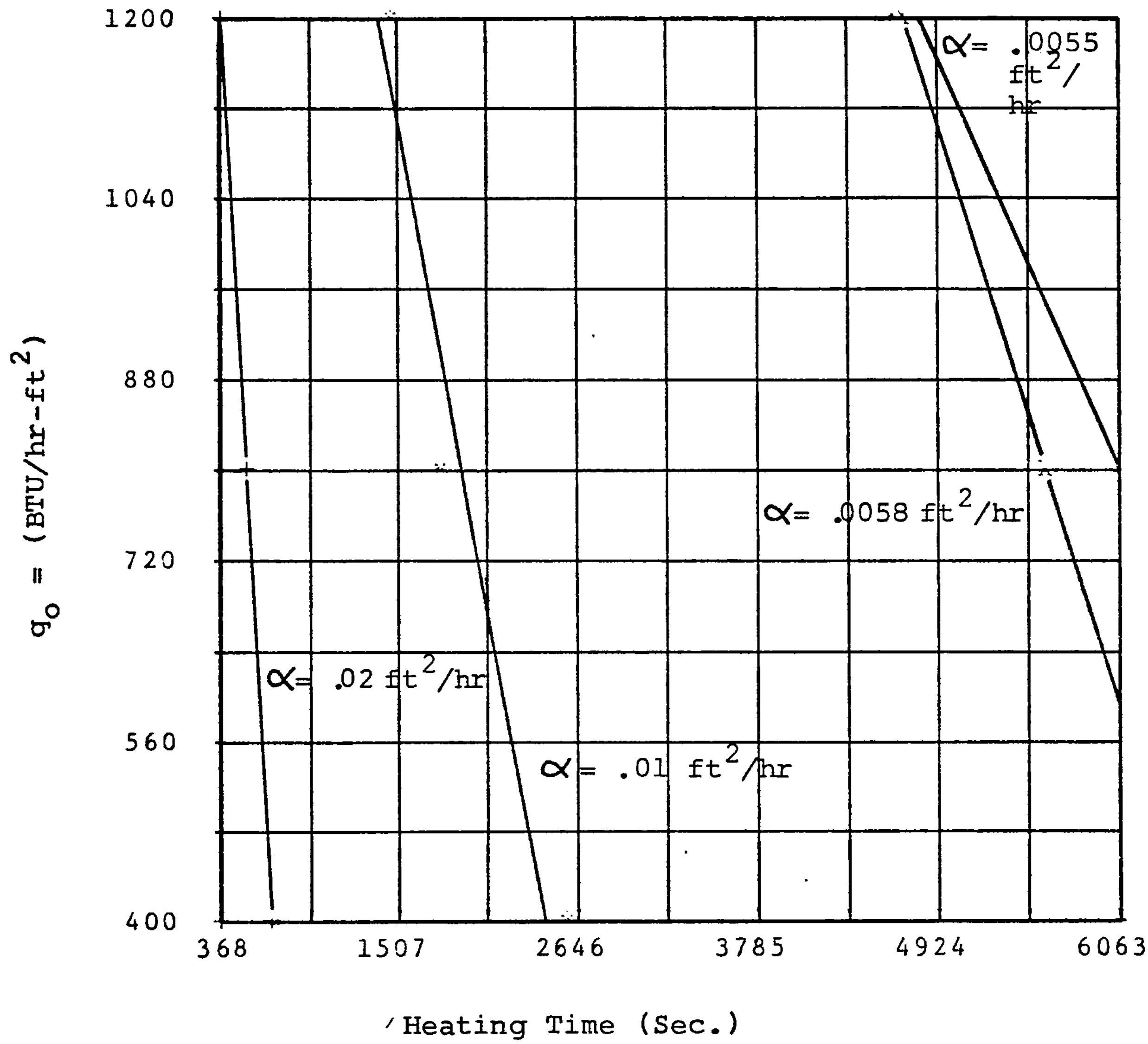


Parameter

R_o = 1.34 (in.)

FIGURE 6.10

HEAT FLUX VERSUS TOTAL HEATING TIME WITH THERMAL DIFFUSIVITY AND THE SMALL SKYLAB CAN AS PARAMETERS FOR THE NOMINAL DATA CASE



Parameter

R_o = 2.34 (in.)

FIGURE 6.11

HEAT FLUX VERSUS TOTAL HEATING TIME WITH THERMAL DIFFUSIVITY AND THE LARGE SKYLAB CAN AS PARAMETERS FOR THE NOMINAL DATA CASE

TABLE 6.2

RANGE OF HEATING TIMES TO BE EXPECTED FOR SKYLAB CANNED FOODS

Data Case[a]	Heating Time (Hours)	Comments
Case 3	.02 hours	Shortest Time
Case 17	.5 hours	Nominal Data Case
Case 22	2.1 hours	Longest Time

[a]Reference Table 4.5.

temperature boundary conditions (where the error is known[1]) as it is for the constant flux boundary condition (error unknown).

For the constant temperature boundary condition the error is derived from a plot of heat transfer percentage (finite flux/infinite flux) versus F_o with the cylinder slenderness ratio (height/radius) as a parameter.[2] The worst case error determined by employing this assumption is 20 percent.

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[1]F. Kreith, Principles of Heat Transfer (Scranton: International Textbook Company, 1963); also personal communication between this investigator and Charlton Chen, May 24, 1972.

[2]Chen, loc. cit.

CHAPTER VII

CONCLUSIONS

The range of heating times (Table 6.2) is acceptable for the detailed schedules presently incorporated into crew activity timelines. However, the requirement that a crewman, at some time in the future, might desire or require a quick, hot meal that is not on a schedule deserves consideration. Since the foods are extremely slow heat conductors and the cabin pressure limits the duration of the wall flux, other means need to be examined to optimize the system heating time or to create a more optimum heating system.

Reduction of the heating time range of the present system type would require changes in existing design parameters. Several paths of optimization could be considered. The first might be to optimize the area of the can (or container) exposed to heat flux (e.g., long thin cans, large flat rectangular containers, etc.). The second method might require a small pressure chamber where the environment pressure could be temporarily increased causing an increase in acceptable T_{max} levels and, consequently, causing the heater to not be shut off as frequently. This would shorten the heating time. Other methods should also be conceived and investigated that could improve the system performance.

The possibility of a different system opens a large spectrum of techniques some of which have been critiqued and discarded by the designers of this system and similar systems.[1] However, the possibility of radiation ovens (heaters), hot liquid (e.g., water) heating systems, and other devices should be investigated for future space flights.

Such a system, once developed, would allow much greater freedom in planning and scheduling prior to any lengthy mission, and would allow greater flexibility in spontaneous timeline changes during a space flight.

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[1] S. J. Lis and P. P. Nuccio, Method of Heating Food in Aerospace Flight, Techn. Doc. Report No. AMRL-TDR-63-135, Biomedical Laboratory, 6570th Aerospace Medical Research Laboratories, Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, December, 1963.

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APPENDICES

APPENDIX A

Change of Variables in Solution of Carslaw and Jaeger to the Variables of this Thesis

The equation (1) is from page 204 of Carslaw and Jaeger.

$$v = \frac{2}{a^2} \int_0^a r' \ f(r')dr' + \frac{2}{a^2} \sum_{n=1}^{\infty} e^{-K\alpha_n^2 t/a^2} \tag{1}$$

$$\frac{J_o(r \ {}_n/a)}{J_o^2 \ (\alpha_n)} \quad X \int_0^a r' f(r') J_o(r'\alpha_n/a)dr'$$

The boundary conditions associated with (1) are:

Initial temperature $f(r)$ (2)

Zero surface flux (3)

The variables in (1) are:

v = temperature at a particular radius and time (4)

a = outer radius of cylinder (5)

r = radius variable (6)

r' = dummy variable of integration (7)

t = time (8)

α_n = positive roots of J_1 (9)

The following substitutions for the variables are made in equation (1).

$$\theta = \frac{T(r,t) - \overline{T}}{T_{max} - \overline{T}} \tag{10}$$

$$\alpha = K \tag{11}$$

$$R_o = a \tag{12}$$

$$\sqrt{\rho_m} = \alpha_n \tag{13}$$

$$m = n \tag{14}$$

$$T(r,t=0) = f(r') \tag{15}$$

The following definitions are also substituted in equation (1):

$$\overline{T} = 2/R_o^{\,2} \int_0^{R_o} r \quad T(r,t = 0)\ dr \tag{16}$$

$$F_{om} = \frac{\alpha t \cdot \rho_m^{\,2}}{R_o^{\,2}} \text{, modified fourier modulus} \tag{17}$$

$$I_m = \int_0^{R_o} r \quad T(r,t=0) \quad J_o(\sqrt{\rho_m}) r dr \tag{18}$$

Then as a final result:

$$\theta = \frac{2}{T_{max} - \overline{T}} \left[\frac{1}{R_o^{\,2}} \sum_{m=1}^{\infty} \frac{e^{-F_{om}} I_m J_o(\sqrt{\rho_m})}{J_o^{\,2}(\sqrt{\rho_{om}})} \right] \tag{19}$$

Equation (19) is the same as equation (2.30) which is a special case of the general result equation (2.16).

APPENDIX B

Change of Variables in Solution of Carslaw and Jaeger to Variables of this Thesis

The equation (1) below is equation (11), Chapter 13, page 329, of Carslaw and Jaeger. The equation is stated in terms of the variables of Carslaw and Jaeger.

$$v = \frac{-F_o a}{K} \left\{ \frac{2kt}{a^2} + \frac{r^2}{2a^2} - \frac{1}{4} - 2 \sum_{s=1}^{\infty} \exp(-k\alpha_s^2 t/a^2) \frac{J_o(r\alpha_s/a)}{\alpha_s^2 J_o(\alpha_s)} \right\} \qquad (1)$$

The boundary and initial conditions associated with (1) are:

a. Constant heat flux F_o

b. Zero initial temperature

The variables in equation (1) are:

$v = v(r,t) =$ temperature distribution (2)

$F_o =$ heat flux (3)

$k =$ thermal diffusivity (4)

$a =$ outside radius (5)

$r =$ radius (6)

$t =$ time (7)

$\alpha_s =$ positive roots of J_1 (8)

$K =$ thermal conductivity (9)

The substitutions required to convert equation (1) to the variables used in this paper are:

$$T = v \tag{10}$$

$$q_o = -F_o \tag{11}$$

$$\alpha = K \tag{12}$$

$$R_o = a \tag{13}$$

$$\sqrt{\rho}_{om} = \alpha_s \tag{14}$$

$$M = s \tag{15}$$

Applying equations (10) through (15) to equation (1) the following result is obtained:

$$T(r,t) = \frac{+q_o R_o}{K} \left\{ \frac{2\alpha t}{R_o^{\,2}} + \frac{1}{2} \left(\frac{r^2}{R_o^{\,2}} - \frac{1}{2} \right) - 2 \sum_{m=1}^{\infty} \exp\left(\frac{-\alpha \sqrt{\rho}_m^{\,2}\, t}{r^2} \right) \frac{J_o \ (\sqrt{\rho}_m)}{\sqrt{\rho}_{om}^{\,2}\ J_o(\sqrt{\rho}_{om})} \right\} \tag{16}$$

Applying the forthcoming definitions to equation (16) will complete the change of variables.

$$F_o = \frac{\alpha t}{R_o^{\,2}} \text{ , fourier modulus} \tag{17}$$

$$Q = \frac{q_o R_o}{K} \text{ , flux conduction parameter} \tag{18}$$

$$R = r/R_o \text{, dimensionless radius variable} \tag{19}$$

$$\overline{T} = 2/R_o^{\,2} \int_0^{R_o} T(r,t=0)rdr = 0 \tag{20}$$

$$\theta + \frac{T(r,t) - \overline{T}}{T_{max} - \overline{T}} \text{ , dimensionless temperature} \qquad (21)$$

$$F_{om} = \frac{\alpha t \rho_m^{\ 2}}{R_o^{\ 2}} \text{ , modified fourier modulus} \qquad (22)$$

Equations (17) through (22) applied to equation (16) give the result:

$$\theta = \frac{2Q}{T_{max}} \left\{ F_o + \frac{1}{4} (R^2 - \tfrac{1}{2}) - R^2 \sum_{m=1}^{\infty} \left(e^{-F_{om}} \frac{J_o(\rho_m)}{\rho_{om}^{\ 2} \ J_o(\rho_{om})} \right) \right\} \qquad (23)$$

This is equivalent to a special case of equation (2.16), of the thesis, for the same boundary conditions.

APPENDIX C

Change of Variables in Solution of Lis and Nuccio to the Variables of this Thesis[1]

The equation (1) is from page 14 of Lis and Nuccio.

$$\theta = \frac{2 F_o \alpha \tau}{k r_1} + \frac{F_o r_1}{k} \left\{ \frac{r^2}{2 r_1{}^2} - \frac{1}{4} - 2 \sum_{n=1}^{\infty} \frac{e^{(-\alpha \beta_n{}^2 \tau / r_1{}^2)} \; J_o \, (r \beta_n / r_1)}{n^2 \, J_o \, (\beta_n)} \right\} \qquad (1)$$

The boundary conditions associated with (1) are a constant heat flux and a constant, nonzero initial temperature distribution.

θ = temperature excess above initial temperature (2)

= $T(r,t) - T(r,t{=}0)$, for $T(r,t{=}0)$ = constant

F_o = heat flux (3)

α = thermal diffusivity (4)

τ = time (5)

k = thermal conductivity (6)

r = radius variable (7)

r_1 = outside radius (8)

β_n = positive roots of J_1 (9)

The following substitutions for the above variables are made.

---

[1]See discussion on page 21 before applying (1).

$$q_o = F_o \quad (10)$$

$$K = k \quad (11)$$

$$R_o = r_1 \quad (12)$$

$$\rho_m = \beta_n \quad (13)$$

$$M = N \quad (14)$$

The following definitions are used to complete the conversion of (1) to the variables of this thesis.

$$F_o = \frac{\alpha t}{R_o^2}, \text{ fourier modulus} \quad (15)$$

$$Q = \frac{q_o R_o}{K}, \text{ flux conduction parameter} \quad (16)$$

$$R = \frac{r}{R_o}, \text{ dimensionless radius variable} \quad (17)$$

$$\overline{T} = \frac{2}{R_o^2} \int_0^R T\ (r,t=0) r dr = T(r,t=0), \text{ for} \quad (18)$$

$$T(r,t=0) = \text{constant}$$

$$\theta = \frac{T(r,t) - \overline{T}}{T_{max} - \overline{T}}, \text{ dimensionless temperature} \quad (19)$$

$$F_{om} = \frac{\alpha t \rho_m^2}{R_o^2}, \text{ modified fourier modulus} \quad (20)$$

Equations (10) through (14) and then equations (15) through (20) applied to equation (1) give the result, (21).

$$\theta = \frac{2Q}{T_{max} - T(r,t=0)} \left\{ F_o + \frac{1}{4} (R^2 - \tfrac{1}{2}) \right. \tag{21}$$

$$\left. - R^2 \sum_{m=1}^{\infty} e^{-F_{om}} \frac{J_o(\gamma_m)}{\gamma_m^2 J_o(\gamma_{om})} \right\}$$

This is equivalent to a special case of equation (2.16) for the same boundary conditions.

APPENDIX D

Change of Variables in Solution of Langford to the Variables of this Thesis

The equations (1), (10), (11), (12), (14), (15), and (16) are written below and their variables defined as they appear in Langford. Equations (13) and (17) are written in a more programmable form than the form of Langford.

$$u(x,t) = \sum_{n=0}^{\infty} \left(f_n(t) \; C^n \; (x^2/4) - \tfrac{1}{2}\, q_n(t) \; E^n \; (x^2/4) \right) \tag{1}$$

The variables in (1) are:

$u(x,t)$ = dimensionless temperature at any point in the cylinder at any time (2)

$$x = \frac{r}{R_o} \tag{3}$$

$$f_n(t) = \frac{d^n}{dt^n} \left[f(t) \right] \tag{4}$$

$f(t)$ = dimensionless temperature distribution at the surface R_o for all time (5)

$$q_n(t) = \frac{d^n}{dt^n} \left[q(t) \right] \tag{6}$$

$q(t)$ = dimensionless temperature distribution at the surface R_o for all time (7)

$$Z = \frac{X^2}{r} \tag{8}$$

$$Z_o = \frac{1}{4} \tag{9}$$

$$E^o(Z) = \ln\ (Z/Z_o) \tag{10}$$

$$E^1(Z) = Z(\ln Z/Z_o - 2) + Z_o(\ln Z/Z_o + 2) \tag{11}$$

$$E^2(Z) = (\ln Z/Z_o - 3)\ \frac{Z^2}{4} + Z\ Z_o\ \ln \frac{Z}{Z_o}$$

$$+ (\ln Z/Z_o + 3)\ Z_o{}^2/4 \tag{12}$$

$$E^n(Z) = \frac{Z_o{}^n - Z^n}{(n!)^2} \sum_{i=1}^{n} \frac{2}{i} + \frac{Z^n}{(n!)^2} \ln Z/Z_o \tag{13}$$

$$+ \frac{Z_o^n\ e^o}{(n!)^2} + \sum_{j=1}^{n-1} \frac{Z_o{}^{n-j}\ e^j}{[(n-j)!]^2}$$

$$+ \sum_{j=1}^{n-1} \left[\frac{Z_o{}^{n-j}}{[(n-j)!]^2} \left(\sum_{i=1}^{n-j} 2/i \right) \left(c^j + \frac{Z_o\ e^{j-1}}{n-j+1} \right) \right]$$

for $n > 2$

$$c^0(Z) = 1.0 \tag{14}$$

$$c^1(Z) = (Z - Z_o) - Z_o \ln(Z/Z_o) \tag{15}$$

$$C^2(Z) = (Z^2 - Z_o{}^2)/4 + Z_o(Z - Z_o) \tag{16}$$

$$- Z_o(Z + Z_o/2)\ \ln(Z/Z_o)$$

$$C^n(Z) = -Z_o\ E^{n-1} + \frac{Z^n - Z_o{}^n}{(n!)^2} - \sum_{j=1}^{n-1} \frac{Z_o{}^j}{(j!)^2} \tag{17}$$

$$+ \left(C^{n-j} + \frac{Z_o\ E^{n-1-j}}{j+1}\right)$$

The following variable substitutions and definitions are made in equations (1) through (17).

$$R = X \tag{18}$$

$$R^2/4 = Z \tag{19}$$

$$1/4 = Z_o \tag{20}$$

$$\theta(R,t) = u(X,t) \tag{21}$$

$$\theta(R_o,t) = f(t) \tag{22}$$

$$\theta_n(R_o,t) = d^n/dt^n[f(t)] \tag{23}$$

$$q_n(R_o,t) = q_n(t) \tag{24}$$

$$\text{index } m = \text{index } n \tag{25}$$

Applying equations (18) through (25) to equation (1) and equations (10) through (17) to (1) gives the following results:

$$\theta(R,t) = \sum_{m=0}^{\infty} \Big[\theta_m(R_o,t)\ C^m(R^2/4) - \tfrac{1}{2}\ q_m(R_o,t)$$

$$E^m\ (R^2/4) \Big] \tag{26}$$

The auxiliary equations for E and C become:

$$E^0 = \ln R^2 \tag{27}$$

$$E^1 = R^2/4\ (\ln R^2 - 2) + \tfrac{1}{4}\ (\ln R^2 + 2) \tag{28}$$

$$E^2 = \frac{R^6}{256}\ (\ln R^2 - 3) + \frac{R^2}{16} \ln R^2 + 1/64$$

$$(\ln R^2 + 3) \tag{29}$$

$$E^m = \left(\frac{1 - R^{2m}}{4^m (m!)^2} \right) \sum_{i=1}^{m} \frac{2}{i} + \frac{R^{2m}\ \ln R^2}{4^m\ (m!)^2} \tag{30}$$

$$+ \frac{E^0}{4^m (m!)^2} + \sum_{j=1}^{m-1} \frac{E^j}{(4)^{m-j} \left[(m-j)! \right]^2}$$

$$+ \sum_{j=1}^{m-1} \left[\left(\frac{1}{(4)^{m-j} \left[(m-j)! \right]^2} \right) \left(\sum_{i=1}^{m-j} \frac{2}{i} \right) \right.$$

$$\left. \left(C^j + \frac{E^{j-1}}{4\ (m-j+1)} \right) \right] \quad ; \ m > 2$$

$$C^0 = 1.0 \tag{31}$$

$$C^1 = 1/4\ (R^2 - 1) - 1/4\ \ln R^2 \tag{32}$$

$$C^2 = \frac{R^4 - 1}{64} + 1/16\ (R^2 - 1) - 1/16\ (R^2 + \tfrac{1}{2}) \tag{33}$$

$$\ln R^2$$

$$C^m = -\frac{E^{m-1}}{4} + \frac{R^{2m} - 1}{4^m\ (m!)^2} \tag{34}$$

$$- \sum_{j=1}^{m-1} \frac{1}{4^j\ (j!)^2} \left[C^{m-j} + \frac{E^{m-1-j}}{4\ (j+1)} \right];$$

$m > 2$

APPENDIX E

Change of Variables in Solution of Olcer to the Variables of this Thesis

The general result from Olcer, specialized to the general boundary conditions of this thesis is:

$$T(r,t) = 2/a^2 \int_0^a F\,(r)\; r dr + \frac{2\,Qkt}{Ka} + \frac{aQ}{2K}\left(\frac{r^2}{a^2} - \frac{1}{2}\right) \tag{1}$$

$$- \frac{2Q}{Ka} \sum_{m=1}^{\infty} \left\{ \frac{\exp(-K\mu_m^{\;2}t) \quad J_o(\mu_m r)}{\mu_m^{\;2} J_o \quad (\mu_m a)} \right\}$$

$$+ \frac{2}{a^2} \sum_{m=1}^{\infty} \left\{ \frac{\exp(-K\mu_m^{\;2}t \quad J_o(\mu_m r)}{} \right.$$

$$\left. \frac{\left(\int_0^a F \quad (r)\; J_o \;(\mu_m r)\; rdr\right)}{J_o^{\;2}\;(\mu_m a)} \right\}$$

Where:

a = outer cylindrical radius (2)

F(r) = initial temperature distribution (3)

r = radius (4)

k = thermal diffusivity (5)

K = thermal conductivity (6)

$$\mu_m = (\text{positive roots of } J_1)/a \quad (7)$$

$$Q = \text{applied constant heat flux} \quad (8)$$

$$t = \text{time} \quad (9)$$

The following substitutions for the above variables will be made:

$$R_o = a \quad (10)$$

$$T(r,t=0) = F(r) \quad (11)$$

$$\alpha = K \quad (12)$$

$$\rho_{om} = R_o \mu_m \quad (13)$$

$$q_o = Q \quad (14)$$

Also the following variable groups are defined:

$$F_o = \frac{\alpha t}{R_o^2}, \text{ fourier modulus} \quad (15)$$

$$Q = \frac{q_o R_o}{K}, \text{ flux conduction parameter} \quad (16)$$

$$R = \frac{r}{R_o}, \text{ dimensionless radius variable} \quad (17)$$

$$\overline{T} = \frac{2}{R_o^2} \int_0^{R_o} T(r,t=0)\, r dr \quad (18)$$

$$\rho_m = r\mu_m \quad (19)$$

$$\theta = \frac{T(r,t) - \overline{T}}{T_{max} - \overline{T}}, \text{ dimensionless temperature} \quad (20)$$

$$I_m = \int_0^{R_o} T(r,t=0)\ J_o(\varphi_m) r dr \tag{21}$$

$$F_{om} = \frac{\alpha t \mu_m^2}{R_o^2}\ \text{, modified fourier modulus} \tag{22}$$

Then, upon substituting equations (2) through (22) into equation (1), we have:

$$\theta\ (F_o,\ F_{om},\ R_o,\ R,\ I_m,\ Q, \varphi_m, \varphi_{om}) \tag{23}$$

$$= \left[\frac{2Q}{T_{max} - T}\right] \left\{ F_o + \tfrac{1}{4}\ (R^2 - \tfrac{1}{2}) \right.$$

$$- R^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o\ (\varphi_m)}{\varphi_m^{\ 2}\ J_o(\varphi_{om})} \right)$$

$$\left. + \frac{1}{QR_o^{\ 2}} \sum_{m=0}^{\infty} \left(\frac{e^{-F_{om}} I_m\ J_o\ (\varphi_m)}{J_o^{\ 2}\ (\varphi_{om})} \right) \right\}$$

A more appropriate form of equation (23) for the special case of a zero heat flux boundary condition is:

$$\theta(F_o,\ F_{om},\ R_o,\ R,\ I_m,\ Q, \varphi_m, \varphi_{om}) \tag{24}$$

$$= \frac{2}{T_{max} - T} \left\{ QF_o + \frac{Q}{4}\ (R^2 - \tfrac{1}{2}) \right.$$

$$- QR^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o(\rho_m)}{\rho_m^2 J_o(\rho_{om})} \right)$$

$$+ \frac{1}{R_o^2} \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o(\rho_m)}{J_o^2(\rho_{om})} \right) \Bigg\}$$

The auxiliary equations associated with equation (24) are:

$$\overline{T} = \frac{2}{R_o^2} \int_0^{R_o} T(r,t=0) r dr \tag{25}$$

$$I_m = \int_0^{R_o} T(r,t=0)\, J_o(\rho_m r) r dr \tag{26}$$

The equation (24) and its auxiliary equations (25) and (26) have several special cases that are of interest. The simplifications are stated in the main body of the thesis.

APPENDIX F

Superposition of Solutions from Appendix A and Appendix C to Get the Constant Initial Temperature Result Equation (2.24)

From Appendix A, equation (19):

$$\theta = \frac{2}{T_{max} - \overline{T}} \quad \frac{1}{R_o^2} \left[\sum_{m=1}^{\infty} \frac{e^{-F_{om}} I_m \; J_o \; (\sqrt{\rho_m})}{J_o^2 \; (\sqrt{\rho_{om}})} \right]$$

$$\text{let } \overline{T} = T(r,t=0) \tag{1}$$

From Appendix C, equation (21):

$$\theta = \frac{2}{T_{max} - T(r,t=0)} \left\{ F_o + .\tfrac{1}{4} \; (R^2 - \tfrac{1}{2}) - R^2 \sum_{m=1}^{\infty} e^{-F_{om}t} \; \frac{J_o \; (\sqrt{\rho_m})}{\sqrt{\rho_m}^2 \; J_o(\sqrt{\rho_{om}})} \right\} \tag{2}$$

To get the total solution the following superposition is performed:

$$\theta_{total} = \theta_1 + \theta_2 = \theta = \text{Eq. (1)} + \text{Eq. (2)} \tag{3}$$

The result is:

$$\theta = \frac{2}{T_{max} - T(r,t=0)} \left\{ QF_o + \frac{Q}{4} (R^2 - ½) \right. \tag{4}$$

$$- QR^2 \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} J_o (\sqrt{\rho_m})}{\sqrt{\rho_m}^2 J_o (\sqrt{\rho_{om}})} \right) + \frac{1}{R_o^2}$$

$$\left. \sum_{m=1}^{\infty} \left(\frac{e^{-F_{om}} I_m J_o (\sqrt{\rho_m})}{J_o^2 (\sqrt{\rho_{om}})} \right) \right\}$$

This is equivalent to equation (2.24) which is the general result for a system with a constant, nonzero initial temperature distribution.

APPENDIX G

Computer Program, Program Flow Chart, and Sample Printout

The following pages consist of a combined program listing and flow chart (pp. 33-36) and a sample printout (pp. 93-119) for data case 3 (Table 4.5).

```
                         (ENTRANCE)
                              I
                              I
**************************************************************************
*C     AJO= BESSEL FUNC OF ZERO ORDER(JO) WITH ARGUMENT AMU*R            *
*C     AJ1= BESSEL FUNC OF ZERO ORDER(JO) WITH ARGUMENT AMU*RO           *
*C     AMU= ROOT OF BESSEL FUNC .JO, DIVIDED BY OUTSIDE RADIUS OF CAN(1/I*
*C     AX= ALPHANUMERIC ARRAY WHICH CONTAINS THE TITLE FOR THE ABSICCAS O*
*C         EACH GRAPH                                                    *
*C     AY= ALPHANUMERIC ARRAY WHICH CONTAINS THE TITLE FOR THE ORDINATES *
*C         OF EACH GRAPH                                                 *
*C     COUNT= FLOATING PT. VALUE OF INDEX, NRI , MINUS ONE               *
*C     DR= RADIAL COORDINATE INCREMENT [IN]                              *
*C     DT= TIME INCREMENT [SEC]                                          *
*C     ERR= MAX ALLOWABLE ERROR BETWEEN TEMP AT CAN/FOOD INTERFACE, AND T*
*C     ERR2= MAX ALLOWABLE ERROR BETWEEN TEMP AT FOOD CENTERLINE AND TMIN*
*C     ERR3= MAX ALLOWABLE ERROR BETWEEN SUCCESSIVE TERMS OF THE TEMPERAT*
*C     PROFILE                                                           *
*C     FO= FOURIER MODULUS(DLESS)                                        *
*C     N= NO. OF TERMS OF THE BESSEL SERIES REQUIRED                     *
*C     NRI= NO. OF RADIAL COORDINATE INCREMENTS, PLUS ONE                *
*C     Q= FLUX CONDUCTION PARAMETER (R)                                  *
*C     QUIKMV= UNIVAC 1108 SUBROUTINE THAT PRODUCES MICROFILM GRAPHS     *
*C     ROOT= ROOT OF THE BESSEL FUNC. J1(ROOT)=0                         *
*C     T= TIME[SEC]                                                      *
*C     TBAR= MEAN THERMAL LEVEL OF CAN CONTENTS, FOR A PARTICULAR INITIAL*
*C     TEMPERATURE DISTRIBUTION(R)                                       *
*C     TD= THERMAL DIFFUSIVITY OF CAN CONTENTS (IN2/SEC)                 *
*C     TEMP= TEMP. FOR A PARTICULAR RADIUS, TIME VALUE, SUMMED OVER ALL N*
*C     TEMPO= ARRAY OF TEMPERATURE ORDINATES TO BE PLOTTED ON MICROFILM F*
*C             THE FIRST(LOWEST) TIME VALUE (R)                          *
*C     TEMPW= TEMP AT CAN/FOOD INTERFACE FOR A PARTICULAR TIME(R)        *
*C     THETA= DLESS TEMPERATURE DISTRIBUTION                             *
*C     TIME= ARRAY OF TIME VALUES FOR WHICH WALL TEMP IS AT TMAX OR TMIN(*
*C     TIN= VALUE OF INTEGRAL FROM TRAP RULE APPROXIMATION (IN2-R)       *
*C     TK= THERMAL CONDUCTIVITY OF CAN CONTENTS (BTU/HR-FT-R)            *
*C     TMAX= MAX ALLOWABLE VALUE OF SYSTEM TEMP AT ANY POINT(R)          *
*C     TMIN= LOWER SWITCHING VALUE OF SYSTEM TEMP MEASURED AT CAN/FOOD   *
*C     INTERFACE(R)                                                      *
*C     TOI= VALUE OF INTEGRAL FROM TRAP RULE APPROXIMATION (IN2-R)       *
*C     X= RADIAL COORDINATE (DIMENSIONLESS), X=R/RO                      *
*C     XO= ARGUMENT OF BESSEL FUNC JO                                    *
*C     XI= ARGUMENT OF BESSEL FUNC JO                                    *
*C     Y= ARRAY OF TEMP ORDINATE VALUES TO BE PLOTTED ON MICROFILM, I.E. *
*C        FOR A PARTICULAR RADIUS AND TIME VALUE(R)                      *
*      DIMENSION       TEMP(31,399),AX(22),AY(22),AX1(22),AJ1(40),ROOT(40)*
*     1,AMU(40),X(31),Y(31),TEMPO(31),TO(31)   ,TOI(31)   ,TIN(40)        *
*     1,TIME(399),FO(399),THETA(31,399),TBAR(399),AJO(31,40),AY1(22)      *
*      READ(5,55) (AX(I), I=1,12)                                        *
*      READ(5,55) (AY(I), I=1,12)                                        *
*      READ(5,55) (AX1(I),I=1,12)                                        *
*      READ(5,55) (AY1(I),I=1,12)                                        *
*   55 FORMAT(12A6)                                                      *
**************************************************************************
                              I
```

```
                                       I
                                       I
                                       I
                                       I
                  ***************************************************************
                  *C-------------------------------------------------------------*
                  *C     READ THE ROOTS OF THE BESSEL FUNCTION, J1               *
                  *C-------------------------------------------------------------*
                  ***************************************************************
                                       I
                                       I
                                       I
                  ***************************************************************
A.................*     DO 130 I=1,36,5                                          *
A                 ***************************************************************
A                                      I
A                                      I
A                                      I
A                 ***************************************************************
A                 *     READ(5,230) ROOT(I),ROOT(I+1),ROOT(I+2),ROOT(I+3),ROOT(I+4) *
A                 ***************************************************************
A                                      I
A                                      I
A                                      I
A                 ***************************************************************
..................*  130 CONTINUE                                                *
                  *  230 FORMAT(5F10.5)                                          *
                  ***************************************************************
                                       I
                                       I
                                       I
                  ***************************************************************
                  *C-------------------------------------------------------------*
                  *C     READ IN SYSTEM PARAMETERS                               *
                  *C-------------------------------------------------------------*
                  ***************************************************************
                                       I
                                       V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
                                       I                                         D
                  ***************************************************************  D
                  *   31 READ(5,56) RO,TD,TMAX,TMIN,DR,DT,TI,NRI                 *  D
                  *   56 FORMAT(7F10.3,I10)                                      *  D
                  *   32 READ(5,57) N,TK,CO                                      *  D
                  *   57 FORMAT(I10,2F10.3)                                      *  D
                  ***************************************************************  D
                                       I                                         D
                                       I                                         D
                                       I                                         D
                  ***************************************************************  D
                  *C-------------------------------------------------------------*  D
                  *C     INITIALIZE ARRAYS TO ZERO                               *  D
                  *C-------------------------------------------------------------*  D
                  ***************************************************************  D
                                       I                                         D
```

```
                                        I                                      D
                                        I                                      D
                                        I                                      D
                                        I                                      D
                    ***************************************************************  D
A...................*     DO58 I=1,31                                         *  D
A                   ***************************************************************  D
A                                       I                                      D
A                                       I                                      D
A                                       I                                      D
A                   ***************************************************************  D
A B.................*     DO58 J=1,399                                        *  D
A B                 ***************************************************************  D
A B                                     I                                      D
A B                                     I                                      D
A B                                     I                                      D
A B                 ***************************************************************  D
A B C...............*     DO58 K=1,40                                         *  D
A B C               ***************************************************************  D
A B C                                   I                                      D
A B C                                   I                                      D
A B C                                   I                                      D
A B C               ***************************************************************  D
A B C               *     TIN(K)=0.                                           *  D
A B C               *     AJO(I,K)=0.                                         *  D
A B C               *     AJ1(K)=0.                                           *  D
A B C               *     AMU(K)=0.                                           *  D
A B C               *     X(I)=0.                                             *  D
A B C               *     Y(I)=0.                                             *  D
A B C               *     TEMPO(I)=0.                                         *  D
A B C               *     TO(I)=0.                                            *  D
A B C               *     TO1(I)=0.                                           *  D
A B C               *     TIME(J)=0.                                          *  D
A B C               *     TBAR(J)=0.                                          *  D
A B C               *     FO(J)=0.                                            *  D
A B C               *     THETA(I,J)=0.                                       *  D
A B C               *     TEMP(I,J)=0.                                        *  D
A B C               ***************************************************************  D
A B C                                   I                                      D
A B C                                   I                                      D
A B C                                   I                                      D
A B C               ***************************************************************  D
....................*  58 CONTINUE                                             *  D
                    ***************************************************************  D
                                        I                                      D
                                        I                                      D
                                        I                                      D
                    ***************************************************************  D
                    *C------------------------------------------------------------*  D
                    *C    WRITE HEADINGS AND PRINT OUT SYSTEM INFORMATION         *  D
                    *C------------------------------------------------------------*  D
                    *     WRITE(6,7777)                                       *  D
                    *     WRITE(6,7777)                                       *  D
                    *     WRITE(6,7778)                                       *  D
                    * 7778 FORMAT(54X,'NEW DATA CASE')                        *  D
                    *     WRITE(6,7777)                                       *  D
                    *     WRITE(6,7777)                                       *  D
                    * 7777 FORMAT(1X,'------------------------------------------*  D
                    *     1-------------------------------------------------------*  D
                    *     2-------')                                          *  D
                    *     WRITE(6,9811)                                       *  D
                    * 9811 FORMAT(55X,'ROOTS OF J1'//)                        *  D
                    *     WRITE(6,9812)                                       *  D
```

```
                     * 9812 FORMAT(44X,'K',30X,'ROOT(K)'/)                                   *    D
                     ***********************************************************************    D
                                                      I                                         D
                                                      I                                         D
                                                      I                                         D
                     ***********************************************************************    D
A....................*      DO 9813 J=1,40                                                   *    D
A                    ***********************************************************************    D
A                                                     I                                         D
A                                                     I                                         D
A                                                     I                                         D
A                    ***********************************************************************    D
A                    *      WRITE(6,9814) J,ROOT(J)                                          *    D
A                    * 9814 FORMAT(43X,I2,28X,F10.5)                                         *    D
A                    ***********************************************************************    D
A                                                     I                                         D
A                                                     I                                         D
A                                                     I                                         D
A                    ***********************************************************************    D
.....................* 9813 CONTINUE                                                         *    D
                     *      WRITE(6,7777)                                                    *    D
                     *      WRITE(6,1099)                                                    *    D
                     * 1099 FORMAT(61X,'SYSTEM PROPERTIES'//)                                *    D
                     *      WRITE(6,127)                                                     *    D
                     *  127 FORMAT(1X,'OUTER',5X,'MAX HEAT',4X,'INITIAL',6X,'MAXIMUM',5X,'MINI*    D
                     *     1MUM',7X,'THERMAL',6X,'THERMAL',5X,'RADIAL',7X,'TIME',7X,'NO. OF',6*    D
                     *     2X,'NO. OF')                                                      *    D
                     *      WRITE(6,128)                                                     *    D
                     *  128 FORMAT(1X,'RADIUS',6X,'FLUX',7X,'TEMP',7X,'ALLOWABLE',4X,'TEMP AT'*    D
                     *     1,8X,'COND.',8X,'DIFF',6X,'INCREMENT',3X,'INCREMENT',4X,'RADIAL',6X*    D
                     *     2,'TERMS')                                                        *    D
                     *      WRITE(6,129)                                                     *    D
                     *  129 FORMAT(2X,'(IN)',5X,'(BTU/HR-',6X,'(R)',7X,'TEMP OF',4X,'WHICH FOO*    D
                     *     1D',5X,'(BTU/HR-',6X,'(FT2/',7X,'(INCHES)',5X,'(SEC)',5X,'INCREMENT*    D
                     *     2S',2X,'OF BSSL')                                                 *    D
                     *      !!!!!ABOVE CARD NOT PROCESSED - PARENTHESES DON'T MATCH!!!!!     *    D
                     *      WRITE(6,2089)                                                    *    D
                     * 2089 FORMAT(14X,'FT2)',17X,'SYSTEM(R)',3X,'IS EATEN(R)',4X,'FT-R)',8X,'*    D
                     *     2HR )',30X,'(DMLESS)',6X,'SERIES'//)                              *    D
                     *      WRITE(6,7777)                                                    *    D
                     *      ED= TD*3600./144.                                                *    D
                     *      WRITE(6,158) RO,QO,TI,TMAX,TMIN,TK,ED,DR,DT,NRI,N                *    D
                     *  158 FORMAT(9(F8.3,4X), 2(I8,4X)//)                                   *    D
                     *      WRITE(6,7777)                                                    *    D
                     ***********************************************************************    D
                                                      I                                         D
```

```
                                        I                                      D
                                        I                                      D
                                        I                                      D
                                        I                                      D
          *****************************************************************    D
          *C---------------------------------------------------------------    D
          *C     SECTION OF PROG CORRESPONDING TO INITIAL TEMP DISTRIBUTION, AND TH*  D
          *C     TIME REQD FOR THE WALL TO FIRST HEAT UP TO TMAX.          *    D
          *C--------------------------------------------------------------*    D
          *      D= (DO+RO)/(TK*12.)                                       *    D
          *      T=DT                                                      *    D
          *****************************************************************    D
                                        I                                      D
                                        I                                      D
                                        I                                      D
          *****************************************************************    D
A.........*      DO3201 K=1,N                                              *    D
A         *****************************************************************    D
A                                       I                                      D
A                                       I                                      D
A                                       I                                      D
A         *****************************************************************    D
A         *      AMU(K)= ROOT(K)/RO                                        *    D
A         *      X1= RO*AMU(K)                                             *    D
A         *      AJ1(K)= BSSL(X1,1)                                        *    D
A         *****************************************************************    D
A                                       I                                      D
A                                       I                                      D
A                                       I                                      D
A         *****************************************************************    D
A B.......*      DO3301 J=1,NRI                                            *    D
A B       *****************************************************************    D
A B                                     I                                      D
A B                                     I                                      D
A B                                     I                                      D
A B       *****************************************************************    D
A B       *      COUNT= FLOAT(J)-1.                                        *    D
A B       *      R= COUNT*DR                                               *    D
A B       *      X0= R*ROOT(K)/RO                                          *    D
A B       *      AJ0(J,K)= BSSL(X0,1)                                      *    D
A B       *      TP(J)=AJ0(J,K) *R*DR                                      *    D
A B       *****************************************************************    D
A B                                     I                                      D
A B                                     I                                      D
A B                                     I                                      D
A B       *****************************************************************    D
A ........* 3301 CONTINUE                                                  *    D
A         *      SUM9=0.                                                   *    D
A         *      KK=NRI-1                                                  *    D
A         *****************************************************************    D
A                                       I                                      D
```

```
A                                         I                                            D
A                                         I                                            D
A                                         I                                            D
A                                         I                                            D
A                 **********************************************************************  D
A B...............*      DO 4301  J=2,KK                                              *  D
A B               **********************************************************************  D
A B                                       I                                            D
A B                                       I                                            D
A B                                       I                                            D
A B               **********************************************************************  D
A B               *      SUM9 = SUM9 + TO(J)                                          *  D
A B               **********************************************************************  D
A B                                       I                                            D
A B                                       I                                            D
A B                                       I                                            D
A B               **********************************************************************  D
A ................* 4301 CONTINUE                                                     *  D
A                 *      TIN(K)=  .5*( TO(1) + TO(NRI)) + SUM9                        *  D
A                 **********************************************************************  D
A                                         I                                            D
A                                         I                                            D
A                                         I                                            D
A                 **********************************************************************  D
..................* 3201 CONTINUE                                                     *  D
                  **********************************************************************  D
                                          I                                            D
                                          V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<<<<<
                                          I                                            D      L
                  **********************************************************************  D      L
                  *    3 SUM2=0.                                                      *  D      L
                  *      SUM3=0.                                                      *  D      L
                  **********************************************************************  D      L
                                          I                                            D      L
                                          I                                            D      L
                                          I                                            D      L
                  **********************************************************************  D      L
A.................*      DO 201 K=1,N                                                 *  D      L
A                 **********************************************************************  D      L
A                                         I                                            D      L
A                                         I                                            D      L
A                                         I                                            D      L
A                 **********************************************************************  D      L
A                 *      SUM2=  SUM2 +((DEXP(-1.*TD*AMU(K)*AMU(K)*T )*TIN(K)*T)/AJ1(K))  *  D      L
A                 *      SUM3=SUM3    + (DEXP(-1.*TD*AMU(K)*AMU(K)*T)) /(AMU(K)*AMU(K))  *  D      L
A                 **********************************************************************  D      L
A                                         I                                            D      L
A                                         I                                            D      L
A                                         I                                            D      L
```

```
  201 CONTINUE
      TEMPW= TI +  2./(RO**2) * SUM2 +(2.*QO*TD*T/(TK*RO*12.))+(RO*QO/(
     1 4.*TK*12.)) -  (2.*QO/(TK*RO*12.)  *  SUM3)
      ERR= 1. -    (TMAX- TEMPW)

      IF(ERR) 1,2,2

    1 T=T+DT

      GO TO 3

    2 SUM1= T

C------------------------------------------------------------------
C     SECT OF PROG USED TO GENERATE FIRST THREE TEMP DISTRIBUTIONS, THE
C     DISTRIBUTION OCCURS WHEN THE WALL FIRST REACHES TMAX.
C------------------------------------------------------------------
      TIME(3)= SUM1
      TIME(2)= (SUM1*2.)/3.
      TIME(1)= (SUM1/3.)

      DO 204 L=1,3
```

```
A                                    I                                      D
A                                    I                                      D
A                                    I                                      D
A                                    I                                      D
A            *****************************************************************  D
A            *     FO(L)= (TD*TIME(L))/(RO**2)                               *  D
A            *     T= TIME(L)                                                *  D
A            *****************************************************************  D
A                                    I                                      D
A                                    I                                      D
A                                    I                                      D
A            *****************************************************************  D
A B..........*     DO 4 I=1,NRI                                              *  D
A B          *****************************************************************  D
A B                                  I                                      D
A B                                  I                                      D
A B                                  I                                      D
A B          *****************************************************************  D
A B          *     COUNT = FLOAT(I)-1.                                       *  D
A B          *     R= COUNT*DR                                               *  D
A B          *     SUM2=0.                                                   *  D
A B          *     SUM3=0.                                                   *  D
A B          *****************************************************************  D
A B                                  I                                      D
A B                                  I                                      D
A B                                  I                                      D
A B          *****************************************************************  D
A B C........*     DO 1201 K=1,N                                             *  D
A B C        *****************************************************************  D
A B C                                I                                      D
A B C                                I                                      D
A B C                                I                                      D
A B C        *****************************************************************  D
A B C        *     SUM2= SUM2+  (DEXP(-1.*TD*AMU(K)*AMU(K)*T)*AJO(I,K)/(AMU(K)*AMU(K)*
A B C        *    1*AJ1(K)))                                                 *  D
A B C        *     SUM3= SUM3  +  (DEXP(-1.*TD*AMU(K)*AMU(K)*T)*AJO(I,K)*TIN(K)*TI/  *  D
A B C        *    1(AJ1(K)*AJ1(K)))                                          *  D
A B C        *****************************************************************  D
A B C                                I                                      D
A B C                                I                                      D
A B C                                I                                      D
A B C        *****************************************************************  D
A B .........* 1201 CONTINUE                                                 *  D
A B          *     TEMP(I,L)= TI +  (2./(RO*RO))*SUM3 - (2.*QO/(TK*RO*12.))*SUM2+(2.**
A B          *    1  QO*TD*T/(  TK*RO*12.)) +  (RO*QO/ (2.*TK*12.)) * ((R/RO)**2-.5) *
A B          *****************************************************************  D
A B                                  I                                      D
A B                                  I                                      D
A B                                  I                                      D
```

```
A B                                       I                                  D
A B                                       I                                  D
A B         *************************************************************    D
A B         *     IF(L.EQ.1) GO TO 450                                  *>>>>D>>>>>>>>V
A B         *************************************************************    D       L
A B                                       I                                  D       L
A B                                       I                                  D       L
A B                                       I                                  D       L
A B         *************************************************************    D       L
A B         *     TO(I)= TEMP(I,L-1)+R*DR                               *    D       L
A B         *************************************************************    D       L
A B                                       I                                  D       L
A B                                       V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<<<<<
A B                                       I                                  D
A B         *************************************************************    D
A B         * 450 CONTINUE                                              *    D
A B         *************************************************************    D
A B                                       I                                  D
A B                                       I                                  D
A B                                       I                                  D
A B         *************************************************************    D
A ..........*   4 CONTINUE                                              *    D
A           *************************************************************    D
A                                         I                                  D
A                                         I                                  D
A                                         I                                  D
A           *************************************************************    D
A           *     IF(L.EQ.1) GO TO 451                                  *>>>>D>>>>>>>>V
A           *************************************************************    D       L
A                                         I                                  D       L
A                                         I                                  D       L
A                                         I                                  D       L
A           *************************************************************    D       L
A           *     SUM8=0.                                               *    D       L
A           *     KK=NRI-1                                              *    D       L
A           *************************************************************    D       L
A                                         I                                  D       L
A                                         I                                  D       L
A                                         I                                  D       L
A           *************************************************************    D       L
A B.........*     DO 9204 I=2,KK                                        *    D       L
A B         *************************************************************    D       L
A B                                       I                                  D       L
A B                                       I                                  D       L
A B                                       I                                  D       L
A B         *************************************************************    D       L
A B         *     SUM8= SUM8+ TO(I)                                     *    D       L
A B         *************************************************************    D       L
A B                                       I                                  D       L
```

```
 B                                    I                                       D    L
A B                                   I                                       D    L
A B                                   I                                       D    L
A B                                   I                                       D    L
A B        ****************************************************************** D    L
A .........* 9204 CONTINUE                                                  * D    L
A          *       TBAR(L)=(.5*(TO(1)+ TO(NRI)) + SUMB) * (2./(RO**2))     * D    L
A          ****************************************************************** D    L
A                                     I                                       D    L
A                                     V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<<<<<
A                                     I                                       D
A          ****************************************************************** D
A          *  451 TBAR(1)= TI                                               * D
A          ****************************************************************** D
A                                     I                                       D
A                                     I                                       D
A                                     I                                       D
A          ****************************************************************** D
...........*  204 CONTINUE                                                  * D
           *       ERR2= TMIN- TEMP(1,3)                                    * D
           ****************************************************************** D
                                      I                                       D
                                      I                                       D
                                      I                                       D
           ****************************************************************** D
           *       IF(ERR2) 21,21,83                                        *>>>>D>>>>>>>>V
           ****************************************************************** D    L
                                      I                                       D    L
                                      I                                       D    L
                                      I                                       D    L
           ****************************************************************** D    L
           *   83 CONTINUE                                                  * D    L
           ****************************************************************** D    L
                                      I                                       D    L
                                      I                                       D    L
                                      I                                       D    L
           ****************************************************************** D    L
           *C---------------------------------------------------------------* D    L
           *C      SECTION OF PROGRAM GENERATING COOL-DOWN TIMES OF WALL TO TMIN. * D    L
           *C---------------------------------------------------------------* D    L
           *       J=4                                                      * D    L
           ****************************************************************** D    L
                                      I                                       D    L
                                      V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<<<<L<<<<<<<<
                                      I                                       D    L    T
           ****************************************************************** D    L    T
           *   22 T=DT                                                      * D    L    T
           ****************************************************************** D    L    T
                                      I                                       D    L    T
```

```
                                         I                          D     L     T
                                         I                          D     L     T
                                         I                          D     L     T
                                         I                          D     L     T
               ****************************************************  D     L     T
A..............*     DO2301 K=1,N                                 *  D     L     T
A              ****************************************************  D     L     T
A                                        I                          D     L     T
A                                        I                          D     L     T
A                                        I                          D     L     T
A              ****************************************************  D     L     T
A B............*     DO 504 I=1,NRI                               *  D     L     T
A B            ****************************************************  D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B            ****************************************************  D     L     T
A B            *     COUNT= FLOAT(I) -1.                          *  D     L     T
A B            *     R= COUNT*DR                                  *  D     L     T
A B            *     TO(I)  = TEMP(I,J-1)*R *DR                   *  D     L     T
A B            *     TOI(I)  = TEMP(I,J-1)*R*AJO(I,K)*DR          *  D     L     T
A B            ****************************************************  D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B            ****************************************************  D     L     T
A .............*  504 CONTINUE                                    *  D     L     T
A              *     SUM8=0.                                      *  D     L     T
A              *     SUM9=0.                                      *  D     L     T
A              *     KK=NRI-1                                     *  D     L     T
A              ****************************************************  D     L     T
A                                        I                          D     L     T
A                                        I                          D     L     T
A                                        I                          D     L     T
A              ****************************************************  D     L     T
A B............*     DO 9504 I=2,KK                               *  D     L     T
A B            ****************************************************  D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B            ****************************************************  D     L     T
A B            *     SUM8= SUM8 + TO(I)                           *  D     L     T
A B            *     SUM9= SUM9 + TOI(I)                          *  D     L     T
A B            ****************************************************  D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B                                      I                          D     L     T
A B            ****************************************************  D     L     T
A .............* 9504 CONTINUE                                    *  D     L     T
A              *     TOI= .5* (TO(1) + TO(NRI)) + SUM8            *  D     L     T
A              *     TBAR(J)=  TOI*(2. /(RO**2))                  *  D     L     T
A              *     TIN(K)= .5*(TOI(1) + TOI(NRI)) + SUM9        *  D     L     T
A              ****************************************************  D     L     T
A                                        I                          D     L     T
```

```
A                                        I                                            D    L    T
A                                        I                                            D    L    T
A                                        I                                            D    L    T
A                                        I                                            D    L    T
A                    ********************************************************         D    L    T
.....................* 2301 CONTINUE                                        *         D    L    T
                     *      TEMPW=0.                                        *         D    L    T
                     ********************************************************         D    L    T
                                         I                                            D    L    T
                                         V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<L<<<<T<<<<<
                                         I                                            D    L    T    2
                     ********************************************************         D    L    T    2
                     *    5 SUM2=0.                                         *         D    L    T    2
                     *      CHECK= TEMPW                                    *         D    L    T    2
                     ********************************************************         D    L    T    2
                                         I                                            D    L    T    2
                                         I                                            D    L    T    2
                                         I                                            D    L    T    2
                     ********************************************************         D    L    T    2
A....................*      DO 301 K=1,N                                    *         D    L    T    2
A                    ********************************************************         D    L    T    2
A                                        I                                            D    L    T    2
A                                        I                                            D    L    T    2
A                                        I                                            D    L    T    2
A                    ********************************************************         D    L    T    2
A                    *      SUM2=  SUM2  + ((DEXP( -1.*TD*AMU(K)*AMU(K)*T)*TIN(K))/AJ1(K))  *  D    L    T    2
A                    ********************************************************         D    L    T    2
A                                        I                                            D    L    T    2
A                                        I                                            D    L    T    2
A                                        I                                            D    L    T    2
A                    ********************************************************         D    L    T    2
.....................*  301 CONTINUE                                        *         D    L    T    2
                     *      TEMPW= (2.*SUM2/R0*R0) + TBAR(J)                *         D    L    T    2
                     *      ERR= 1. -    ( TMIN- TEMPW)                     *         D    L    T    2
                     ********************************************************         D    L    T    2
                                         I                                            D    L    T    2
                                         I                                            D    L    T    2
                                         I                                            D    L    T    2
                     ********************************************************         D    L    T    2
                     *      IF(ERR) 11,11,10                                *>>>>>D>>>V    L    T    2
                     ********************************************************         D  H L    T    2
                                         I                                            D  H L    T    2
                                         I                                            D  H L    T    2
                                         I                                            D  H L    T    2
                     ********************************************************         D  H L    T    2
                     *   10 ITEST= IFIX(T/DT)                               *         D  H L    T    2
                     ********************************************************         D  H L    T    2
                                         I                                            D  H L    T    2
                                         I                                            D  H L    T    2
```

```
                                    I
                                    I
                                    I
*****************************************************************
*     IF( ITEST.EQ.1) GO TO 89                                  *>>>>D>>>H>>>L>>>V
*****************************************************************
                                    I
                                    I
                                    I
*****************************************************************
*     ERR3=  .003 - ABS( TEMPW- CHECK)                          *
*****************************************************************
                                    I
                                    I
                                    I
*****************************************************************
*     IF(ERR3) 89,11,11                                         *>>>>D>>>V
*****************************************************************
                                    I
                                    V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<H<<<L<<<<
                                    I
*****************************************************************
*  89 T=T+DT                                                    *
*****************************************************************
                                    I
                                    I
                                    I
*****************************************************************
*     GO TO 5                                                   *>>>>D>>>H>>>L>>>>>>>T>>>>>>>A
*****************************************************************
                                    V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<
                                    I
*****************************************************************
*  11 SUMJ= SUMJ + T                                            *
*     TIME(J)= SUMJ                                             *
*     FO(J)= (TD*TIME(J))/ (RO**2)                              *
*****************************************************************
                                    I
                                    I
                                    I
*****************************************************************
*C--------------------------------------------------------------*
*C     GENERATION OF TEMP DISTRIBUTION AT TIME WHEN WALL IS AT THIN.  *
*C--------------------------------------------------------------*
*****************************************************************
                                    I
                                    I
                                    I
```

```
                                         I                                     D    L    T
                                         I                                     D    L    T
                     ***************************************************************  D    L    T
A....................*     DO 12 I=1,NRI                                        *  D    L    T
A                    ***************************************************************  D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                    ***************************************************************  D    L    T
A                    *     COUNT= FLOAT(I) -1.                                  *  D    L    T
A                    *     R= COUNT*DR                                          *  D    L    T
A                    *     SUM2=0.                                              *  D    L    T
A                    ***************************************************************  D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                    ***************************************************************  D    L    T
A B..................*     DO 2201 K=1,N                                        *  D    L    T
A B                  ***************************************************************  D    L    T
A B                                      I                                     D    L    T
A B                                      I                                     D    L    T
A B                                      I                                     D    L    T
A B                  ***************************************************************  D    L    T
A B                  *     SUM2= SUM2+  (DEXP( -1.*TD*AMU(K)*AMU(K)*T) *TIN(K)*AJO(I,K))/(AJ1*  D    L    T
A B                  *    1(K)**2)                                              *  D    L    T
A B                  ***************************************************************  D    L    T
A B                                      I                                     D    L    T
A B                                      I                                     D    L    T
A B                                      I                                     D    L    T
A B                  ***************************************************************  D    L    T
A ...................* 2201 CONTINUE                                            *  D    L    T
A                    *     TEMP(I,J)=  (2.*SUM2/RO*RO)  +  TBAR(J)              *  D    L    T
A                    ***************************************************************  D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                                        I                                     D    L    T
A                    ***************************************************************  D    L    T
.....................*   12 CONTINUE                                            *  D    L    T
                     *     ERR2= TMIN- TEMP(I,J)                                *  D    L    T
                     ***************************************************************  D    L    T
                                         I                                     D    L    T
                                         I                                     D    L    T
                                         I                                     D    L    T
                     ***************************************************************  D    L    T
                     *     IF(ERR2) 21,21,91                                    *>>>>D>>>>>>>V    T
                     ***************************************************************  D    L    T
                                         I                                     D    L    T
                                         I                                     D    L    T
                                         I                                     D    L    T
```

```
                                        I                                     D    L    T
                                        I                                     D    L    T
         ****************************************************************     D    L    T
         *  91 CONTINUE                                                 *     D    L    T
         ****************************************************************     D    L    T
                                        I                                     D    L    T
                                        I                                     D    L    T
                                        I                                     D    L    T
         ****************************************************************     D    L    T
         *C-------------------------------------------------------------*     D    L    T
         *C     SECTION OF PROGRAM THAT GENERATES HEAT-UP TIME OF WALL TO TMAX.  *     D    L    T
         *C-------------------------------------------------------------*     D    L    T
         *      J=J+1                                                   *     D    L    T
         *      T=DT                                                    *     D    L    T
         ****************************************************************     D    L    T
                                        I                                     D    L    T
                                        I                                     D    L    T
                                        I                                     D    L    T
         ****************************************************************     D    L    T
A........*      DO2501 K=1,N                                            *     D    L    T
A        ****************************************************************     D    L    T
A                                       I                                     D    L    T
A                                       I                                     D    L    T
A                                       I                                     D    L    T
A        ****************************************************************     D    L    T
A B......*      DO 1601 I=1,NRI                                         *     D    L    T
A B      ****************************************************************     D    L    T
A B                                     I                                     D    L    T
A B                                     I                                     D    L    T
A B                                     I                                     D    L    T
A B      ****************************************************************     D    L    T
A B      *      COUNT= FLOAT(I)-1.                                      *     D    L    T
A B      *      R= COUNT*DR                                             *     D    L    T
A B      *      TO(I) =  TEMP(I,J-1)+R *DR                              *     D    L    T
A B      *      TO1(I)  =  TEMP(I,J-1)+R*AJO(I,K)*DR                    *     D    L    T
A B      ****************************************************************     D    L    T
A B                                     I                                     D    L    T
A B                                     I                                     D    L    T
A B                                     I                                     D    L    T
A B      ****************************************************************     D    L    T
A .......* 1601 CONTINUE                                                *     D    L    T
A        *      SUM8=0.                                                 *     D    L    T
A        *      SUM9=0.                                                 *     D    L    T
A        *      KK=NRI-1                                                *     D    L    T
A        ****************************************************************     D    L    T
A                                       I                                     D    L    T
A                                       I                                     D    L    T
A                                       I                                     D    L    T
A        ****************************************************************     D    L    T
A B......*      DO 4601 I=2,KK                                          *     D    L    T
A B      ****************************************************************     D    L    T
A B                                     I                                     D    L    T
```

```
      SUM8= SUM8 + TO(I)
      SUM9= SUM9 + TO1(I)

 4601 CONTINUE
      TOI= .5*(TO(1) + TO(NRI)) + SUM8
      TBAR(J)=   TOI*(2. /(RO**2))
      TIN(K)= .5* (TO1(1) + TO1(NRI)) + SUM9

 2501 CONTINUE
      TEMPW=0.

   18 SUM2=0.
      SUM3=0.
      CHECK= TEMPW

      DO 501 K=1,N

      SUM2= SUM2 + DEXP(-1.*TD*AMU(K)*AMU(K)*T)/(AMU(K)**2)
      SUM3= SUM3+ (DEXP(-1.*TD*AMU(K)*AMU(K)*T)*TIN(K))/(AJ1(K))

  501 CONTINUE
      TEMPW=-1.*( 2.*DD/(TK*RO*12.))*SUM2+(2./(RO*RO))*(SUM3+TOI)+
     1(2.*DD*TD*T)/(TK*RO*12.)        + RO*DD/(4.*TK*12.)
      ERR= 1. -    (TMAX- TEMPW)
```

```
                                       I                                   D    L    T    2
                                       I                                   D    L    T    2
                                       I                                   D    L    T    2
                                       I                                   D    L    T    2
*************************************************************************  D    L    T    2
*     IF(ERR) 15,16,16                                                *>>>>D>>>V L    T    2
*************************************************************************  D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
*************************************************************************  D  H L    T    2
*  15 ITEST= IFIX(T/DT)                                                *   D  H L    T    2
*************************************************************************  D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
*************************************************************************  D  H L    T    2
*     IF(ITEST . EQ .1) GO TO 189                                     *>>>>D>>>H>>>L>>>V T 2
*************************************************************************  D  H L  P T    2
                                       I                                   D  H L  P T    2
                                       I                                   D  H L  P T    2
                                       I                                   D  H L  P T    2
*************************************************************************  D  H L  P T    2
*     ERR3= .003 - ABS(TEMPW- CHECK)                                   *   D  H L  P T    2
*************************************************************************  D  H L  P T    2
                                       I                                   D  H L  P T    2
                                       I                                   D  H L  P T    2
                                       I                                   D  H L  P T    2
*************************************************************************  D  H L  P T    2
*     IF(ERR3) 189,16,16                                              *>>>>D>>>V L  P T    2
*************************************************************************  D  H L  P T    2
                                       I                                   D  H L  P T    2
                                       V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<H<<<L<<<<  T    2
                                       I                                   D  H L    T    2
*************************************************************************  D  H L    T    2
*  189 T=T+DT                                                          *   D  H L    T    2
*************************************************************************  D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
                                       I                                   D  H L    T    2
*************************************************************************  D  H L    T    2
*     GO TO 18                                                        *>>>>D>>>H>>>L>>>>>>>T>>>>>>>Δ
*************************************************************************  D  H L    T
                                                                           D  H L    T
                                       V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<< L    T
                                       I                                   D    L    T
*************************************************************************  D    L    T
*  16 SUM1= SUM1 +T                                                    *   D    L    T
*     TIME(J)= SUM1                                                    *   D    L    T
*     FO(J)= (TD*TIME(J)) / (RO**2)                                    *   D    L    T
*************************************************************************  D    L    T
                                       I                                   D    L    T
```

```
                                        I                                        D    L    T
                                        I                                        D    L    T
                                        I                                        D    L    T
                                        I                                        D    L    T
                     **************************************************************   D    L    T
                     *C-----------------------------------------------------------*   D    L    T
                     *C     GENERATION OF TEMP DISTRIBUTION FOR TIMES WALL TEMP IS AT TMAX.   *   D    L    T
                     *C-----------------------------------------------------------*   D    L    T
                     **************************************************************   D    L    T
                                        I                                        D    L    T
                                        I                                        D    L    T
                                        I                                        D    L    T
                     **************************************************************   D    L    T
A....................*     DO 19 I=1,NRI                                          *   D    L    T
A                    **************************************************************   D    L    T
A                                       I                                        D    L    T
A                                       I                                        D    L    T
A                                       I                                        D    L    T
A                    **************************************************************   D    L    T
A                    *     COUNT= FLOAT(I) -1.                                    *   D    L    T
A                    *     R= COUNT*DR                                            *   D    L    T
A                    *     SUM2=0.                                                *   D    L    T
A                    *     SUM3=0.                                                *   D    L    T
A                    **************************************************************   D    L    T
A                                       I                                        D    L    T
A                                       I                                        D    L    T
A                                       I                                        D    L    T
A                    **************************************************************   D    L    T
A B..................*     DO 1501 K=1,N                                          *   D    L    T
A B                  **************************************************************   D    L    T
A B                                     I                                        D    L    T
A B                                     I                                        D    L    T
A B                                     I                                        D    L    T
A B                  **************************************************************   D    L    T
A B                  *     SUM2= SUM2 + (DEXP( -1.*TD*AMU(K)*AMU(K)*T)*AJ0(I,K))/(AMU(K)*AMU(*   D    L    T
A B                  *    1K)*AJ1(K))                                             *   D    L    T
A B                  *     SUM3= SUM3 +   (DEXP(-1.*TD*AMU(K)*AMU(K)*T)*AJ0(I,K)*TIN(K))/(AJ1*   D    L    T
A B                  *    1(K)*AJ1(K))                                            *   D    L    T
A B                  **************************************************************   D    L    T
A B                                     I                                        D    L    T
A B                                     I                                        D    L    T
A B                                     I                                        D    L    T
A B                  **************************************************************   D    L    T
A ...................* 1501 CONTINUE                                              *   D    L    T
A                    *     TEMP(I,J)=  (2.*Q0*TD*T/(TK*R0*12.))  +  (R0*Q0/(2.*TK*12.  ))*(( *   D    L    T
A                    *    1R/R0)**2 - .5) + (2./(R0*R0))* T0I - (2.*Q0/(TK*R0*12.)) * SUM2 + *   D    L    T
A                    *    2  (2./(R0*R0)) * SUM3                                  *   D    L    T
A                    **************************************************************   D    L    T
A                                       I                                        D    L    T
```

```
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                    *************************************************************  D      L      T
....................*  19 CONTINUE                                               *  D      L      T
                    *************************************************************  D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                    *************************************************************  D      L      T
                    *C----------------------------------------------------------*  D      L      T
                    *C     SECTION OF PROGRAM THAT CHECKS IF CALCULATED CENTERLINE TEMPERATUR*  D  L  T
                    *C     WITHIN PRESCRIBED ERROR OF DESIRED CENTERLINE TEMP( TMIN ).   *  D      L      T
                    *C----------------------------------------------------------*  D      L      T
                    *      ERR2= TMIN- TEMP(1,J)                                *  D      L      T
                    *************************************************************  D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                    *************************************************************  D      L      T
                    *      IF (ERR2) 21,21,20                                    *>>>>D>>>>>>>V      T
                    *************************************************************  D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                    *************************************************************  D      L      T
                    *  20 J=J+1                                                 *  D      L      T
                    *************************************************************  D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                                        I                                          D      L      T
                    ************************************************************T  D      L      T
                    *      GO TO 22                                              *>>>>D>>>>>>>L>>>>>>>Δ
                    *************************************************************  D      L
                                                                                   D      L
                                        V<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<D<<<<<<<<
                                        I                                          D
                    *************************************************************  D
                    *  21 WRITE(6,7777)                                          *  D
                    *************************************************************  D
                                        I                                          D
                                        I                                          D
                                        I                                          D
                    *************************************************************  D
                    *C----------------------------------------------------------*  D
                    *C     PRINT-OUT OF HEADINGS AND TEMP VS. RADIUS VALUES FOR SELECTED TIME*  D
                    *C----------------------------------------------------------*  D
                    *      WRITE(6,428) Q                                        *  D
                    *  428 FORMAT(42X,'FLUX CONDUCTION PARAMETER=',E10.5,'(R)'//)  *  D
                    *************************************************************  D
                                        I                                          D
```

```
                                              I                          D
                                              I                          D
                                              I                          D
                                              I                          D
                      ***********************************************    D
A.....................*      DO 25 K=1,J                             *   D
A                     ***********************************************    D
A                                             I                          D
A                                             I                          D
A                                             I                          D
A                     ***********************************************    D
A                     *      WRITE(6,9226) TIME(K),FO(K),TBAR(K),TEMP(1,K)        *   D
A                     * 9226 FORMAT(10X,'TIME=',E10.5,10X,'FOURIER MOD=',E10.5,10X,'TBAR=',E10.*   D
A                     *     15,10X,'TEMP AT CL=',E10.5//)            *   D
A                     *      WRITE(6,26)                             *   D
A                     *   26 FORMAT(22X,'(R/RO)(DLESS)',29X,'TEMPERATURE(DEG.R)',29X,'DLESS TEM*   D
A                     *     1PERATURE'/)                             *   D
A                     ***********************************************    D
A                                             I                          D
A                                             I                          D
A                                             I                          D
A                     ***********************************************    D
A B...................*      DO409 L=1,NRI                           *   D
A B                   ***********************************************    D
A B                                           I                          D
A B                                           I                          D
A B                                           I                          D
A B                   ***********************************************    D
A B                   *      COUNT= FLOAT(L) -1.                     *   D
A B                   *      X(L)= COUNT*DR/RO                       *   D
A B                   *      THETA(L,K)= (TEMP(L,K) - TI)  /  (TMAX-TI)   *   D
A B                   *      WRITE(6,427) X(L),TEMP(L,K),THETA(L,K)  *   D
A B                   *  427 FORMAT(29X,F10.3,35X,F10.3,35X,F10.3)   *   D
A B                   ***********************************************    D
A B                                           I                          D
A B                                           I                          D
A B                                           I                          D
A B                   ***********************************************    D
A ....................*  409 CONTINUE                                *   D
A                     *      WRITE(6,429)                            *   D
A                     *  429 FORMAT(1X,'    '////)                   *   D
A                     ***********************************************    D
A                                             I                          D
A                                             I                          D
A                                             I                          D
A                     ***********************************************    D
......................*   25 CONTINUE                                *   D
                      ***********************************************    D
                                              I                          D
```

```
                                        I                             D
                                        I                             D
                                        I                             D
                                        I                             D
                    ****************************************************   D
                    *C--------------------------------------------------*  D
                    *C    SECTION OF PROGRAM THAT GENERATES MICROFILM GRAPHS.  *   D
                    *C--------------------------------------------------*  D
                    *     TEMPO(1)= TI                                  *  D
                    ****************************************************   D
                                        I                             D
                                        I                             D
                                        I                             D
                    ****************************************************   D
A...................*     DO 2055 K=2,NRI                               *  D
A                   ****************************************************   D
A                                       I                             D
A                                       I                             D
A                                       I                             D
A                   ****************************************************   D
....................* 2055 TEMPO(K)=TMAX                                *  D
                    *     CALL QUIKMV(-1,1H*,AX,AY,31,X,TEMPO)          *  D
                    ****************************************************   D
                                        I                             D
                                        I                             D
                                        I                             D
                    ****************************************************   D
A...................*     DO 2066 K=1,J                                 *  D
A                   ****************************************************   D
A                                       I                             D
A                                       I                             D
A                                       I                             D
A                   ****************************************************   D
A B.................*     DO 2067 L=1,NRI                               *  D
A B                 ****************************************************   D
A B                                     I                             D
A B                                     I                             D
A B                                     I                             D
A B                 ****************************************************   D
A ..................* 2067 Y(L)= TEMP(L,K)                              *  D
A                   *     CALL QUIKMV(0,1H*,AX,AY,-31,X,Y)              *  D
A                   ****************************************************   D
A                                       I                             D
A                                       I                             D
A                                       I                             D
A                   ****************************************************   D
....................* 2066 CONTINUE                                     *  D
                    *     TEMPO(1)=0.                                   *  D
                    ****************************************************   D
                                        I                             D
```

```
      DO 4055 K=2,NRI
 4055 TEMPO(K)=1.
      CALL CUIKMV(-1,1H*,AX1,AY1,31,X,TEMPO)
      DO 4066  K=1,J
      DO 4067 L=1,NRI
 4067 Y(L)= THETA(L,K)
      CALL CUIKMV(0,1H*,AX1,AY1,-31,X,Y)
 4066 CONTINUE
      GO TO 31
      GO TO 31
```

NEW DATA CASE

ROOTS OF J1

K	ROOT(K)
1	3.83171
2	7.01559
3	10.17347
4	13.32369
5	16.47063
6	19.61586
7	22.76008
8	25.90367
9	29.04683
10	32.18968
11	35.33231
12	38.47477
13	41.61709
14	44.75932
15	47.90146
16	51.04354
17	54.18555
18	57.32753
19	60.46946
20	63.61136
21	65.19000
22	68.33150
23	71.47300
24	74.61450
25	77.75600
26	80.89760
27	84.03910
28	87.18060
29	90.32220
30	93.46370
31	96.60530
32	99.74680
33	102.88840
34	106.02990
35	109.17150
36	112.31310
37	115.45460
38	118.59620
39	121.73770
40	124.87930

SYSTEM PROPERTIES

OUTER RADIUS (IN)	MAX HEAT FLUX (BTU/HR-FT2)	INITIAL TEMP (R)	MAXIMUM ALLOWABLE TEMP OF SYSTEM(R)	MINIMUM TEMP AT WHICH FOOD IS EATEN(R)	THERMAL COND. (BTU/HR-FT-R)	THERMAL DIFF (FT2/HR)	RADIAL INCREMENT (INCHES)	TIME INCREMENT (SEC)	NO. OF RADIAL INCREMENTS (DMLESS)	NO. O TERMS OF BSS SERIE
1.340	1200.000	530.000	610.000	595.000	.184	.02	.134	1.000	11	20

FLUX CONDUCTION PARAMETER= .72826+03(R)

TIME= .33333+00 FOURIER MOD= .37128-02 TBAR= .53000+03 TEMP AT CL= .46564+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	465.638	-.80
.100	499.087	-.38
.200	526.456	-.04
.300	529.897	-.00
.400	529.998	-.00
.500	529.999	-.00
.600	529.999	-.00
.700	530.017	.00
.800	530.746	.00
.900	540.401	.13
1.000	586.015	.70

TIME= .66667+00 FOURIER MOD= .74256-02 TBAR= .53683+03 TEMP AT CL= .49917+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	499.165	-.38
.100	508.262	-.22
.200	522.372	-.09
.300	528.661	-.01
.400	529.881	-.00
.500	529.995	-.00
.600	530.045	.00
.700	530.640	.00
.800	535.120	.06
.900	554.657	.30
1.000	606.857	.96

TIME= .10000+01 FOURIER MOD= .11138-01 TBAR= .54221+03 TEMP AT CL= .50970+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	509.701	-.25
.100	513.870	-.20
.200	521.905	-.10
.300	527.432	-.03
.400	529.487	-.00
.500	529.977	-.00
.600	530.398	.00
.700	532.579	.03
.800	541.612	.14

.900	567.967	.47
1.000	623.684	1.17

TIME= .30000+01 FOURIER MOD= .33415-01 TBAR= .54763+03 TEMP AT CL= .48682+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	486.819	-.54
.100	489.614	-.50
.200	496.626	-.41
.300	505.087	-.31
.400	513.371	-.20
.500	522.196	-.09
.600	533.663	.04
.700	548.979	.23
.800	566.265	.45
.900	580.363	.63
1.000	585.755	.69

TIME= .40000+01 FOURIER MOD= .44553-01 TBAR= .54779+03 TEMP AT CL= .47767+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	477.674	-.65
.100	483.097	-.58
.200	494.920	-.43
.300	506.580	-.29
.400	516.721	-.16
.500	527.137	-.03
.600	539.362	.11
.700	554.598	.30
.800	576.063	.57
.900	611.457	1.01
1.000	670.444	1.75

TIME= .11000+02 FOURIER MOD= .12252+00 TBAR= .56555+03 TEMP AT CL= .50215+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	502.154	-.34
.100	504.309	-.32
.200	510.588	-.24
.300	520.446	-.11
.400	532.990	.03
.500	547.028	.21
.600	561.161	.39
.700	573.923	.54
.800	583.972	.67

.900	590.305	.75
1.000	592.425	.78

TIME= .12000+02 FOURIER MOD= .13366+00 TBAR= .56570+03 TEMP AT CL= .49199+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	491.995	-.47
.100	497.830	-.40
.200	510.924	-.23
.300	524.727	-.06
.400	537.505	.09
.500	549.999	.25
.600	562.431	.40
.700	575.396	.56
.800	592.922	.78
.900	624.668	1.18
1.000	682.205	1.90

TIME= .25000+02 FOURIER MOD= .27846+00 TBAR= .58345+03 TEMP AT CL= .55994+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	559.937	.37
.100	560.787	.38
.200	563.248	.41
.300	567.057	.46
.400	571.810	.52
.500	577.006	.58
.600	582.111	.65
.700	586.613	.70
.800	590.088	.75
.900	592.246	.77
1.000	592.963	.78

TIME= .26000+02 FOURIER MOD= .28960+00 TBAR= .58351+03 TEMP AT CL= .54203+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	542.027	.15
.100	547.156	.21
.200	557.738	.34
.300	566.816	.46
.400	573.027	.53
.500	577.959	.59
.600	582.726	.65
.700	588.763	.73
.800	600.830	.88

.900	629.136	1.23
1.000	685.522	1.94

TIME= .76000+02 FOURIER MOD= .84651+00 TBAR= .60118+03 TEMP AT CL= .60114+03

(R/RO)(DLESS)	TEMPERATURE(DEG.R)	DLESS TEMPERATURE
.000	601.145	.88
.100	601.146	.88
.200	601.150	.88
.300	601.156	.88
.400	601.164	.89
.500	601.173	.89
.600	601.181	.89
.700	601.188	.89
.800	601.194	.89
.900	601.197	.89
1.000	601.198	.89

APPENDIX H

Calculation of Material Property Ranges to be Used with the Computer Program

A. Discussion of Formulae

The following formulae were taken from page 334 of Earle.

The formula for specific heat at constant pressure is:

$$C_p \left(\frac{BTU}{LB_m - {}^oR}\right) = \frac{P}{100} + \frac{.2(100 - P)}{100} \qquad (1)$$

Where P is the percentage of water in the can contents.

The formula for thermal conductivity above freezing is:

$$K \left(\frac{BTU}{HR\text{-}FT\text{-}R}\right) = \frac{.32P}{100} + \frac{.15(100 - P)}{100} \qquad (2)$$

Where P is defined previously.

The density formula is based on the weight fraction method and is a standard computation procedure.

$$(\gamma)_{composite} = \sum_{i=1}^{m} (\gamma)_i \; (W)_i \qquad (3)$$

$(\gamma)_{composite}$ = average material density

$(\gamma)_i$ = food component density

$(W)_i$ = weight fraction of component present

M = number of components present

Usually canned foods are two component varieties (e.g., water and green beans). However some, such as beef stew, have many components.

The contents of a can, other than water, are typically a fibrous mass when they are dehydrated. It is well known that the material properties of the fibrous residues are very close to those of balsa wood or cork.

The material properties of such residues are taken to be those of cork in this analysis.

Thermal diffusivity is directly related to the properties just discussed. Therefore, the only formula required is:

$$\alpha = \frac{K}{\gamma C_p} \tag{4}$$

B. Computation of Material Property Ranges

1. Specific Heat:

   a. Highest Value (Asparagus (Lis and Nuccio)):

   From (1):

$$C_p = \frac{93}{100} + \frac{.2\ (100 - 93)}{100}$$

$$\underline{C_p = .944\ (BTU/LB_m - R)}$$

b. Intermediate Value (Fat Beef (Lis and Nuccio)):

From (1):

$$C_p = \frac{50}{100} + \frac{.2(100 - 50)}{100}$$

$$\underline{C_p = .6 \ (BTU/LB_m - {}^oR)}$$

c. Lowest Value (Bacon (Lis and Nuccio)):

From (1):

$$C_p = \frac{20}{100} + \frac{.2\ (100 - 20)}{100}$$

$$\underline{C_p = .36 \ (BTU/LB_m - {}^oR)}$$

2. Thermal Conductivity:

a. Highest value (Asparagus (Lis and Nuccio)):

From (2):

$$K = \frac{.32(93)}{100} + \frac{.15(100 - 93)}{100}$$

$$\underline{K = .3081 \ (BTU/HR\text{-}FT\text{-}{}^oR)}$$

b. Intermediate Value (Fat Beef (Lis and Nuccio)):

From (2):

$$K = \frac{.32(50)}{100} + \frac{.15(100 - 50)}{100}$$

$$\underline{K = .235 \ (BTU/HR\text{-}FT\text{-}{}^oR)}$$

c. Lowest Value (Bacon (Lis and Nuccio)):

From (2):

$$K = \frac{.32(20)}{100} + \frac{.15(100 - 20)}{100}$$

$$\underline{K = .184 \ (BTU/HR\text{-}FT\text{-}R)}$$

3. Highest Value (Asparagus (Lis and Nuccio)):

   a. Highest Value (Asparagus (Lis and Nuccio)):

Material	Density (LB_m/FT^3)
Water	62.4 (Holman)
Cork	16.0 (Weast)[1]

   From (3):

$$\gamma = .93(62.4) + .07(16)$$

$$\underline{\gamma = 59.152 \ (LB_m/FT^3)}$$

   b. Intermediate Value (Fat Beef (Lis and Nuccio)):

   From (3):

$$\gamma = .5(62.4) + .5(16)$$

$$\underline{\gamma = 39.2 \ (LB_m/FT^3)}$$

   c. Lowest Value (Bacon (Lis and Nuccio)):

   From (3):

$$\gamma = .2(62.4) + .8(16)$$

$$\underline{\gamma = 25.28 \ (LB_m/FT^3)}$$

---

[1]R. C. Weast (ed.), Handbook of Chemistry and Physics (45th ed.; Cleveland: The Chemical Rubber Co., 1964-1965).

4. Thermal Diffusivity:

a. Largest Value (Bacon (Lis and Nuccio)):

From (4):

$$\alpha = \frac{.184\ \text{BTU}}{\text{HR-FT-R}} \left| \frac{\text{LB}_m - \text{R}}{.36\ \text{BTU}} \right| \frac{\text{FT}^3}{25.28\ \text{LB}_m}$$

$$\underline{\alpha = .020\ (\text{FT}^2/\text{HR})}$$

b. Intermediate Value (Fat Beef (Lis and Nuccio)):

From (4):

$$\alpha = \frac{.235\ \text{BTU}}{\text{HR-FT-}^\circ\text{F}} \left| \frac{\text{FT}^3}{39.2\ \text{LB}_m} \right| \frac{\text{LB}_m - ^\circ\text{R}}{.6\ \text{BTU}}$$

$$\underline{\alpha = .01\ (\text{FT}^2/\text{HR})}$$

c. Smallest Value (Asparagus (Lis and Nuccio)):

From (4):

$$\alpha = \frac{.3801\ \text{BTU}}{\text{HR-FT-R}} \left| \frac{\text{LB}_m\text{-R}}{.944\ \text{BTU}} \right| \frac{\text{FT}^3}{59.152\ \text{LB}_m}$$

$$\underline{\alpha = .0055\ (\text{FT}^2/\text{HR})}$$

C. Summary of Material Properties

Along with the upper, intermediate, and lower values of the properties just defined, another data point of interest is useful. The most obvious set of properties to use are those of water. A tabulation has been constructed on page 39 of the main report summarizing all of these values.