Erratum and Comments on "Nonlinear oscillations of gas bubbles in liquids: An interpretation of some experimental results" [J. Acoust. Soc. Am. 73, 121–127 (1983)]

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Attention is called to an error in a previous paper. An explanation of the consequences of the error, as well as a reanalysis of the original data is given.

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In an earlier paper[1] we showed that the $n = 2$ harmonic resonance of an acoustically driven gas bubble could be studied in considerable detail by the technique of acoustic levitation. We reported further on an apparently successful attempt to explain the observed behavior in terms of the available theory. As we have extended our analysis to gas bubbles in other liquids and at temperatures sufficiently large that significant amounts of vapor are present in the bubble, we have encountered considerable difficulties in finding favorable agreement between theory and experiment. This difficulty has caused us to examine more closely our earlier results, which has led to the detection of an error in our previous analysis.

The earlier analysis is based on the use of Eq. (4) of Ref. 1 with the effective viscosity μ_e, which substitutes for the viscosity when thermal dissipation is included. An error, however, was made in multiplying μ_e by the factor $(\Omega/\Omega_0)^2$ where Ω is the acoustic frequency and Ω_0 is the linear resonance frequency for the gas bubble. Since in Ref. 1 this damping term is important only in the neighborhood of the $n = 2$ harmonic resonance where $\Omega/\Omega_0 = \frac{1}{2}$, this error amounts to approximately a fourfold reduction of the damping which one would expect to be appropriate on the basis of the linear theory.

Figure 1 shows the same experimental data for the levitation number as a function of normalized bubble radius as Fig. 2 in Ref. 1. The normalized pressure amplitude is $\eta = 0.155$ and the resonance radius has the value $R_{\text{res}} = 1.38 \times 10^{-2}$ cm. The dotted line is the incorrect theoretical prediction of Ref. 1, while the dashed line indicates the "corrected" theoretical result obtained from Eq. (4) in Ref. 1 with the effective viscosity defined as in Ref. 2. It is evident that the "correction" essentially destroys the characteristic structure (the rise and fall of the levitation number as a function of R) suggested by the data in the neighborhood of the $n = 2$ harmonic resonance, $R_0/R_{\text{res}} \cong \frac{1}{2}$. This fact indicates that the theoretical prescription used to include energy dissipation in the nonlinear oscillation of gas bubbles overestimates the damping by a factor of nearly 4 in this case. Similar behavior is found for other cases presented in Ref. 1.

This circumstance has prompted us to examine the problem in greater detail. The levitation number in the region $R_0/R_{\text{res}} \cong \frac{1}{2}$ is extremely sensitive to the harmonic component of R oscillating at the harmonic frequency 2Ω, and in particular to the damping affecting this component. This observation suggests that if the "effective" viscosity of linear theory is used, its value at the frequency 2Ω would be more appropriate than its value at the frequency Ω. This intuitive idea is indeed confirmed by the detailed analysis of Ref. 3, although other effects also come into play which cannot be accounted for in a simple way. The basic problem is that by the procedure indicated above, one is attempting to represent the very complex processes which determine P_i using only two parameters, the polytropic index and the "effective" viscosity. This difficulty is compounded by the fact that both these parameters are predicted by linear theory to depend on the driving frequency, so that it is far from clear which value of this quantity (Ω or 2Ω in the present case) is appropriate.

The solid line in Fig. 1 shows the theoretical results obtained when the damping is evaluated at the frequency 2Ω and the polytropic index at Ω. It is seen that the agreement between these analytical results and the data, although far from perfect, is much better than that of the "correct" theory portrayed by the dashed line. As a matter of fact, for $R_0(2\Omega/D_G)^{1/2} \gtrsim 20$, where D_G is the thermal diffusivity of

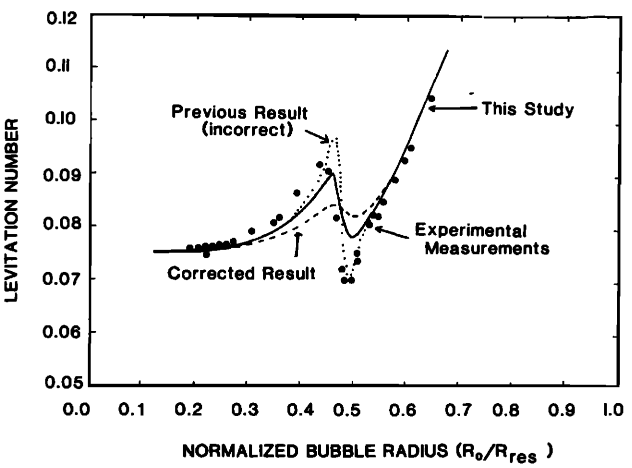

FIG. 1. Values of the levitation number as a function of the normalized radius for an air bubble in a glycerine–water mixture at 25 °C. The symbols are the experimental measurements; the dotted line is the incorrect result reported earlier (Ref. 1); the dashed line is the "corrected" result obtained with the damping constant and the polytropic index evaluated at Ω; the solid line is the result obtained when the damping is evaluated at 2Ω and the polytropic index at Ω. For this case, the acoustic pressure amplitude was 0.155 bar, the resonance radius 0.0138 cm, and the driving frequency 22.2 kHz.

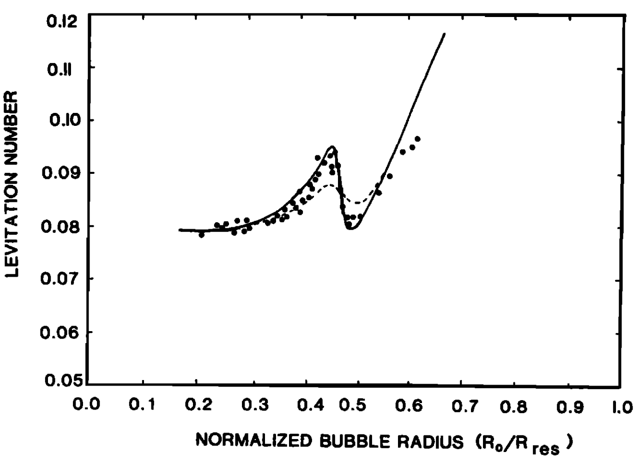

FIG. 2. Values of the levitation number as a function of the normalized radius for an air bubble in a glycerine–water mixture at 50 °C. The symbols are the experimental measurements; the dashed line is the result obtained with the damping constant and the polytropic index evaluated at Ω; the solid line is the result obtained when the damping is evaluated at 2Ω and the polytropic index at Ω. For this case, the acoustic pressure amplitude was 0.162 bar, the resonance radius 0.0138 cm, and the driving frequency 22.3 kHz.

the gas, the thermal damping constant β_{th} (related to the "thermal" viscosity μ_{th} by $\beta_{\text{th}} = 2\mu_{\text{th}}/\rho R_0^2$) can be approximated to within 20% by[4]

$$\beta_{\text{th}} \cong [9\gamma(\gamma - 1)/2\rho R_0^3](D_G/2\Omega^3)^{1/2}P_0,$$

where γ is the ratio of the specific heats, so that an increase in Ω by a factor of 2 reduces β_{th} by a factor of $(2)^{3/2}$ approximately. For the experimental conditions of Ref. 1, and $R_0/R_{\text{res}} \cong \frac{1}{2}$, the previous formula is not quite applicable, since $R_0(2\Omega/D_G)^{1/2} \cong 8$, and the decrease in damping is approximately 2. As stated above, the polytropic index used for these computations has been evaluated at Ω rather than 2Ω. This choice, which does not affect the computed results appreciably for several reasons, has been made because the frequency 2Ω would be inappropriate for use in the linear component which determines the major contribution to the levitation number on which the characteristic $n = 2$ harmonic structure is superimposed. In contrast, this component is very little affected by the value of damping away from linear resonance.

As a final comment, we observe that the theory on which the computed results in Fig. 1 are based allows sufficient freedom to use different values of κ and μ_e for different components of R. However, this freedom is not available to someone using the Rayleigh–Plesset equation directly, as is commonly done. One of the aims of the approach followed in Ref. 2 was to present a way to evaluate κ and μ_e which in turn could be used to obtain less inaccurate values of the bubble radius than could be obtained from direct use of the Rayleigh–Plesset equation.

As a matter of interest and to dispel doubts that the multiplication of the effective viscosity by the factor of $(\Omega/\Omega_0)^2$, which seemed to work so well in Ref. 1, could in some way give an acceptable approximation to the damping affecting nonlinear oscillations, we include Fig. 2 which refers to the case where the vapor of the host liquid has a non-negligible effect. Specifically, Fig. 2 refers to the same glycerine–water mixture as in Fig. 1; however, in this particular case the temperature of the host fluid was raised to 50 °C. As in the previous case the dashed line refers to the "corrected" result, the solid line to the result obtained when the damping is evaluated at 2Ω rather than Ω. One sees that a reasonably good fit occurs in the latter instance. This good agreement is fortuitous, however, because we have discovered that higher temperatures apparently reduce the amplitude of the harmonic structure. It is seen in Fig. 1 that the predicted amplitude of this structure is significantly less than the measured amplitude for a temperature of 25 °C. We have discovered, and Fig. 2 demonstrates the effect, that as the temperature of the host fluid is raised and significant amounts of vapor are present in the bubble's interior, the measured amplitude of the harmonic structure is reduced until, at approximately 50 °C, it matches the predicted value. Of course, we anticipate a further reduction as the temperature is raised even further, but those data are not accessible with our current apparatus. We have attempted an explanation for this temperature dependence of the levitation number along the lines of Refs. 5 and 6, i.e., an additional damping term due to evaporation–condensation and other effects, but have not achieved much success.

When we began this investigation we had the impression that an appropriate application of the available theory to our data would have resulted in a satisfactory agreement. We have instead discovered that this theory is inadequate when nonlinear effects become significant. In view of their increasing importance, it is now time to develop a new theory adequate to interpret these effects. This new theory must be capable of describing in detail the physical processes which determine the internal pressure P_i. In the first place, the pressure in the bubble can be taken to be uniform if the radius is much smaller than the wavelength of sound in the gas. This approximation avoids the necessity of considering the momentum equation in the bubble's interior, leaving only the continuity and energy equations. Second, in view of the large heat capacity and conductivity of the liquid with respect to the gas, the temperature of the bubble interface can be taken to remain undisturbed, thus rendering the energy equation in the liquid superfluous. The Rayleigh–Plesset equation is already the solution of the combined liquid continuity and momentum equations.

Some preliminary analytical results using the previous formulation are to be reported in Ref. 3. We hope to be able to report more complete numerical results in the near future.

ACKNOWLEDGMENTS

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Erratum: "Normal mode identification for impedance boundary conditions" [J. Acoust. Soc. Am. 73, 1567–1570 (1983)]

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The end of the last sentence in the paragraph containing Eq. (14) should read "equals zero or unity."

The arc tangent on the left-hand side of Eq. (15) should begin with a lower case "*t*" to correspond with the arc tangent on the right-hand side of Eq. (14) and to distinguish these functions from the principal value of the arc tangent which appears on the right-hand side of Eq. (15).

The second and third sentences, which begin after Eq. (15), should read: "This occurs when the phase ϵ of the reflection coefficient transits even multiples $2n_s$ of π. The value of n_s is zero for $\epsilon = 0$ or $\epsilon = \pi$, increases to 1 for $\epsilon = 2\pi$, etc."

In the last sentence of Sec. IV "from π" should be inserted after "increments."