AN EQUILIBRIUM MODEL OF THE PRICING OF INTEREST RATE FUTURES CONTRACTS: THEORY AND EMPIRICAL TESTS

A Dissertation

Presented to

the Faculty of the College of Business Administration

University of Houston

In Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

by

Murad J. Antia

December, 1981

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ABSTRACT

This study develops and tests a model which determines the equilibrium price of interest rate futures contracts. The derivation of the model is based on the assumptions that all participants maximize expected utility of terminal wealth in a mean-variance framework and that expectations are homogeneous. A rationale is provided to divide the participants into two groups, hedgers and speculators. The function of hedgers is to reduce their exposure to interest rate risk and the function of speculators is to take advantage of transitory profit making opportunities that will arise if there is excessive hedging activity on either the long or the short side.

The demand for long contracts and the supply of short contracts for all the participants are determined. The total demand is equated to total supply to determine the equilibrium futures price. It is determined that the equilibrium futures price is a function of the expected futures price, the risk premium transfer from hedgers to speculators, and the costly guarantee which is the net cost of the margin maintenance requirements.

The equilibrium futures price will include a positive risk premium if there is excessive hedging activity on the long side (hedgers are net long), a negative risk premium if there is excessive hedging activity on the short-side (hedgers are net short), and no risk premium if there is equal hedging activity on both sides. The costly guarantee will have a positive effect on the equilibrium futrues price if hedgers are net long, a negative effect if hedgers are net short and no effect if there is equal hedging activity on both sides. This holds true only if the classification of participants as hedgers and speculators by this study is the same as the

classification defined by the CFTC. If the classifications differ, then the effect of the costly guarantee is independent of hedging activity.

The purpose of the empirical tests is to determine if (1) there is a significant positive or negative risk premium that tends to zero as the length to maturity decreases and (2) if the costly guarantee has a significant effect on the equilibrium futures price. The tests are performed with four different empirical specifications of the model. The four empirical specifications are due to two methods used to estimate the expected price - (1) the distributed lag approach and (2) the instrumental variable approach and two methods used to estimate the costly guarantee - (1) the absolute price differential approach and (2) the standard deviation approach. The results for the GNMA and T-bill futures contracts indicate that there is no significant risk premium included in the equilibrium futures price and that the costly guarantee has no significant effect. The results for the T-bond futures contracts are sensitive to the empirical specification of the model and therefore it is difficult to provide conclusive evidence.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

1.1 The purpose of futures markets and related theories.

The function of a futures market is to provide a mechanism to transfer the risk of price fluctuation. Producers and users of agricultural commodities, precious and semi-precious metals, and raw materials have been using commodity futures to reduce the uncertainty associated with their decisions and operations. In the recent past, the futures exchanges have extended their services to corporations, thrift institutions, commercial banks, mortgage banks, savings and loans associations, debt instruments dealers and institutional investors by offering interest rate futures contracts. These contracts are to be used to protect against the uncertainty of fluctuations in interest rates.

The participants in the futures market for whom the need is to reduce the uncertainty of future price fluctuations are known as hedgers. These hedgers supposedly transfer the risk of price fluctuations to other hedgers by taking opposite positions (opposite positions are the long and short positions). If hedgers are unable to find other hedgers to take opposite positions, it is hypothesized that they have to pay a premium to non-hedgers who assume the risk of hedgers for a price and therefore make a profit for services rendered. These participants are known as speculators.

Keynes [1930] espoused the theory of normal backwardation to explain the movement of commodity futures prices. He hypothesized that hedgers, on net, would take short positions in futures markets since they would at some future date, be selling the commodity in the market. The excess of short positions would create a situation in which the future price would be below the expected spot price. This would entice futures speculators to take long positions. If today's futures price falls below expected spot price, there exists then the opportunity for speculators to profit from the risk they are assuming by taking long positions. The risk premium paid to speculators would decline as the futures contract approaches maturity because hedgers would be willing to pay smaller premiums since the uncertainty of the events associated with an approaching future date would be declining. The gradual reduction in the risk premium would be a function of a gradual reduction in hedging activity associated with a futures contract as it approaches maturity. The implication of this theory is that the price of any futures contract can be expected to rise during its life.

The theory of normal contango is similar to the normal backwardation theory in concept but it takes an opposing view. The theory hypothesizes that hedgers, on net, are long. That is there exist more hedgers who wish to take long positions than hedgers who wish to take short positions. This excess of demand (long positions) over supply (short positions) would bid up the equilibrium futures price above the expected spot price that would then entice speculators to take on short positions. Again the incentive to take a short position would be the opportunity to make a profit in return for the risk that is being assumed. The futures price would decline over the life of the contract since hedgers would be paying a lower premium due to the reduction in uncertainty associated with the approaching future date.

The expectations theory hypothesizes that there is equal hedging activity on both the long side and the short side. In this case the equilibrium futures price will equal the expected futures price and there will be no incentive offered to speculators to enter the market since profit making opportunites do not exist. The entire demand for contracts by hedgers on the long side will be met by the supply of contracts by hedgers on the short side.

1.2 Purpose of this study.

The purpose of this study is to derive and empirically test a model which determines the equilibrium price of a futures contract. The study is not a theoretical effort in the sense that the model does not provide normative results. Rather, a set of hypotheses are developed within the framework of the model and the assumptions that are made to derive the model. The empirical tests that are performed on the model will determine the hypotheses that hold true. The derivations are based upon the assumption that all participants in the market maximize expected utility of terminal wealth in a mean-variance framework. A rationale is provided to divide the participants into two groups, hedgers and speculators. The demand for long contracts and the supply of short contracts for each set of participants is determined. Total demand is equated to total supply to determine the equilibrium futures price. It is determined that the equilibrium futures price is a function of the expected futures price, a risk premium transfer from hedgers to speculators, and the net cost of maintenance margin requirements which will be referred to as the costly guarantee.

The empirical tests to be performed on the model will determine if there exists a significant risk premium which tends towards zero as the futures contracts approaches maturity. The results will indicate if the normal backwardation hypothesis or the normal contango hypothesis or the expectations hypothesis holds true. To date, there have been no studies published that examine the existence of a risk premium associated with the equilibrium price of an interest rate futures contract. Dusak [1973] and Bodie and Rosansky [1979] have used the Sharpe [1964] - Lintner [1965] Capital Asset Pricing Model (CAPM) to determine the risk premium associated with the equilibrium price of commodity futures contracts. The obvious deficiency of the CAPM relevant to interest rate futures contracts is that the CAPM assumes that the risk free interest rate and therefore implicity, risky interest rates, remain constant over the holding period.

The empirical tests that are performed on the models will also determine if the costly guarantee has a significant impact on the equilibrium futures price. Kane [1980] introduced the concept of the costly performance guarantee, which as mentioned before is the cost of maintenance margin requirements. The methodology used to derive the model in this study requires the inclusion of the costly guarantee. Empirical estimates of the costly guarantee will be derived and will be included in the empirical specification of the model.

1.3 Outline

Chapter II will be devoted to defining a futures contract, describing the institutional aspects of the market and mentioning the types of participants in the market. Chapter III will briefly review the existing literature on interest rate futures contracts and other relevant literature. Chapter IV will begin with developing the framework that is necessary for the derivation of the models and presenting the assumptions that are necessary and will conclude with the derivations of the models Chapter V will provide the empirical methodology that will be used to test this model. The chapter will include justification for using the distributed-lag approach and an instrumental variable approach to estimate the expected futures price. The two empirical specifications of the costly guarantee, the absolute price differential approach and the standard deviation approach will also be presented. Chapter VI will be devoted to presenting the methodology used to create the data sets and to presenting the empirical results and their interpretation. Chapter VII will complete the study with a summary of the findings and concluding remarks.

Chapter II

AN OVERVIEW OF THE FINANCIAL FUTURES MARKET: INSTITUTIONAL FACTORS AND PARTICIPANTS

2.1 Introduction

This chapter will outline the essential differences between the futures and forward markets and, in brief, explain how transactions take place and how the market operates. Also presented will be the different types of participants, and reasons will be given for their participation. The last section of the chapter will present empirical observations on the participants in the market.

2.2 The Futures contract and the forward contract.

A forward contract is initiated when two parties agree to the purchase and sale of any commodity at some future date under conditions that are agreed upon by both the parties. The advantage of a forward contract is that it can be tailored to one's needs; one can pick the exact date and the precise commodity desired. The disadvantage of a forward contract is that it might be difficult to locate the other party with exactly the opposite needs. And because no guarantees exist other than what is included in the contract, both parties assume the risk of default by the other party.

A futures contract is defined as a standardized forward contract that is traded on one of many exchanges. The implication of standardization is that the type and grade of commodity or financial instrument and date of delivery are specified. Once an agreement is reached, the clearing corporation of the exchange intercedes and becomes the opposite party to the transaction. With the direct relationship between the two parties severed, each is subsequently free to buy and sell independently of the other. Therefore, it is the soundness of the clearing corporation rather than the credit-worthiness of the original opposite party to the transaction about which the participant is concerned.

Participants can take long and short positions. A long position is when the participant transacts to buy at the set future date and a short position is when the participant transacts to sell at the set future date.

2.3 How the markets operate

A participant places an order with a brokerage firm which is registered with the Commodity Futures Trading Commission (C.F.T.C.)and is permitted to take orders. The firm sends the order to the trading floor of the exchange where a member of the exchange, who represents the firm, enters the trading pit and announces his intention to fulfil the order. Another member of the exchange who had the exactly opposite order presents his offer and if the two can agree to a price, the trade is consummated. The offers are presented and agreed to by open outcry. After the trade is consummated the participant has a contract with, and is committed to, the clearing corporation of the exchange and not the opposing participant to the transaction.

To maintain the credit-worthiness and financial viability of the clearing corporation, the participants through the brokerage firm are required to put up margin deposits. The exchange specifies the minimum initial margin requirement, but the brokerage firm can increase this amount and require higher initial margins from its clients. The initial margin can be posted in the form of cash, selected securities or bank letters of credit.

For as long as the position is outstanding, the contract will be marked to market by the clearing corporation at the end of each business day. Clearing members with long positions have their margin accounts credited with a profit if prices rise or debited with a loss if prices decline, and members with short positions will have their accounts debited with a price increase and credited with a price decline. The prices used to determine the debits and credits are the final settlement prices which are determined by the exchange by examining the prices at which trades take place during the course of the day. The settlement price is basically an approximate average of the prices at which trades ocurred during the course of the day.

Profits in the margin account may be withdrawn immediately. If losses occur and reduce the margin in the account below a specified amount, then the brokerage firm must pay the difference, in cash, to the clearing corporation before trading opens the next business day. Relatively minor changes in the interest rates can result in rather substantial changes in the margin account due to the high leverage. For this reason, many regard uncovered investments in commodity futures to be speculative in nature.

The exchanges set limits for the maximum amount prices can change from one day to the next. When the price change within a day reaches the limit, trading is subsequently restricted. On the next business day trading is restricted to a price range that is within the daily price change limit from the settlement price of the previous day. If the daily limit restricts trading for a few days, then wider limits could be imposed on subsequent days. Margin requirements are temporarily increased during such periods.

When a customer wishes to close out his position he must take an offsetting position. To cancel the contract he bought he has to take a position exactly the opposite of his original position. For example, if the original position is a long position then the transaction required to close the contract would be a short position of the same amount as the long position. Once again the clearing corporation will interpose itself between the two parties, and this second transaction will be written off against the original transaction and the funds in the customer's margin account will be returned to him. The great majority of all contracts are terminated before maturity in this fashion. Only a very small percentage of contracts traded are delivered.

The traders are usually divided into two groups: (1) commission brokers who execute trades for customers, and (2) locals who basically trade for themselves. Commission brokers, also known as floor brokers, may operate on an independent basis making trades for firms that do not employ their own traders or assisting other firms during a heavy influx of orders. They could also be employees or officers of a corporation or government security dealer who holds a seat on the exchange.

The locals are composed of position traders, day traders, scalpers and spreaders. They perform the important function of providing liquidity to the market. Scalpers concentrate their attention on minimum fluctuations that occur as the price of the commodity changes during trading. They buy or sell continuously making small profits and losses on large amounts of trades. Day traders hold market positions only during the course of a day and rarely any positions overnight. In holding their positions longer than scalpers, they are concerned with same-day price changes hoping to profit from wider price swings. Position traders hold positions over periods of days, weeks or months. They tend to be concerned with extended price changes that occur as a result of shifts in demand and supply functions. Spreaders are concerned with the spread between prices between different delivery months of a single commodity or between prices of the same commodity on different markets or between prices of different commodities. They speculate that the spread will change in a manner that will enable them to earn profits.

The three interest rate futures contract that are to be examined in this study are

1. GNMA futures contracts, for which the financial instrument underlying the contract is the Government National Mortgage Association pass-through certificate. The market, called the CDR GNMA, uses mortgage backed certificates guaranteed by the GNMA as the unit for trading. These certificates have a principal amount of $100,000, a coupon rate of 8 percent and a maturity of 30 years with an assumed prepayment in the twelfth year. The price is stated as a percentage of par value with the fractional amount in multiples of 1/32 of a point. The minimum price fluctuation allowed is 1/32 of a point and the maximum daily fluctuation allowed is 24/32.

2. Treasury bond (T-bond) futures contracts for which the financial

instrument underlying the contract is a U.S. government T-bond with a par value of $100,000. The T-bond should mature at least 15 years from the delivery date if not callable; and if callable, is not so for at least 15 years from the delivery date. The price is stated as a percentage of par value with the fractional amount in multiples of 1/32 of a point. The minimum price fluctuation allowed is 1/32 of a point and the maximun daily fluctuation allowed is 32/32.

3. Treasury bill (T-bill) futures contracts, for which the financial instrument underlying the contract is a U.S. government T-bill with ninety days to maturity. The price quoted is in terms of an index. The relationship between the price and the index is provided in a later chapter. The minimun price fluctuation allowed is .01 of the index.

2.4 Participants in the market

The purpose of this section is to mention a few of the probable and potential participants in the interest rate futures market. This will provide the reader with an understanding of the participants in the industry, and this knowledge can be related to the model that will be derived in the next chapter. The participants are divided into two groups, hedgers and speculators. This grouping is the accepted norm in the industry and will also be used for this study. The mathematical proofs of why a hedger wishes to hedge and a speculator speculate will be provided in the next chapter.

As mentioned before, the purpose of the futures market is to satisfy the demand for hedging. For many institutions it could be the lowest cost mechanism, and in competitive markets the lowest transactions cost solution should dominate. For many types of businesses it could be the only legally allowed mechanism. For example, commercial banks and savings and loans institutions cannot reduce their risk by owning a well-diversified portfolio. Existing regulations prevent them from diversifying their portfolios. Presented below are examples of probable hedging activity, both long and short, for each of the three heavily traded financial futures contracts. Many of the cases provided below are examples of cross-hedging. If it is estimated that there is some degree of correlation with the spot market security and futures security, then the cross-hedge can reduce the uncertainty.

1. T-bill futures contracts.

(i) Short Positions

Commercial banks typically have more rate sensitive liabilities than rate sensitive assets. The liabilities have a shorter duration than the assets and therefore when interest rates vary, the cost of liabilities vary more than the rate of return earned on the assets.The largest asset account of a commercial bank is its loan portfolio, and the loan maturities are longer than the maturities of money market certificates and certificates of deposit. If interest rates rise, the cost of rolling over the rate sensitive liabilities will rise without a commensurate increase in rate of return on the asset side. By taking a short position in T-bill futures (and now probably certificates of deposit futures), banks can make their liabilities less sensitive to rate changes by virtually fixing the cost of re-issuing or rolling over the certificates by short-selling T-bill futures. If rates actually rise the costs of rolling over the certificates will increase, but this increase will be offset by a profit on the futures position. If rates actually fall the futures position will generate a loss but this will be offset by the reduced cost of their liabilities.

. Corporate treasurers planning to issue commercial paper in the future are concerned with the possibility that interest rates might rise, thereby increasing the cost of the issue. They could reduce this uncertainty by taking a short position that will mature at the time of the commercial paper issue.

(ii) Long Positions

. Money market fund managers may be concerned with rolling over their purchases of certificates of deposit, commercial paper money market certificates, etc., because they estimate that rates will fall; they can hedge against the possibility of falling rates by taking a long position in T-bill futures contracts whose maturity would coincide with the time of the roll-over.

. Insurance, trust and pension fund managers keep a portion of their assets in liquid assets. If they are concerned that rates of return will fall in the future, or if they simply wish to reduce uncertainty, they could attempt to "lock-in" the present rate by taking a long position in T-bill futures.

2. GNMA futures contracts

(i) Short positions

. Real estate developers who wish to "lock-in" the present rate to offer potential buyers of their homes attractive mortgage financing rates, could take a short position in GNMA futures. If rates do increase, the profits earned on the futures contract could be used to offer below-market mortgage rates.

. Savings and loans could hedge against rises in interest rates. Increases in the interest rates would reduce the value of their existing mortgages. Short positions in GNMA futures would offset the loss since a drop in interest rates would lead to a profit on the position when closed out.

(ii) Long positions

. Institutional investors who are committed to buying a pool of mortgages from mortgage banks could hedge against future uncertainty by taking a long position in GNMA futures. If rates decline, this opportunity loss could be offset by the gain on the futures side.

3. T-bond futures

(i) Short hedge

. Bond dealers who hold an inventory of bonds to be sold at a future date are concerned with the possibility of rising interest rates. They could hedge against this risk by taking short positions in T-bond futures. If rates do rise, they will incur a loss on the sale of the inventory but this loss will be offset by a profit on the futures contract.

. Corporations planning to issue bonds in the future, or their investment bankers underwriting the issue, can reduce uncertainty by taking a short position in T-bonds futures. If rates do rise, the increased cost will be offset by profits on the futures contracts.

(ii) Long hedge

. Institutional investors who are expecting to receive a sum of money in the future that is to be invested in T-bonds could hedge against future uncertainty by taking long positions in T-bond futures.[1]

Participants who are termed as speculators are those who are not hedging any cash market position and probably view their investment in futures independently of other considerations. The local category of traders defined in the previous section are classified by the industry as speculators since they are primarily concerned with very short term price movements. They are important because of the liquidity provided to the markets. In addition to the local category, individuals outside the system could be considered as speculators if they are not hedging any kind of cash market security. An example of speculative activity could be commodity pools that invest only in the futures market. Since these pools do not invest in cash market securities they could be considered as speculators. A significant portion of their investments would be used in various spreading strategies which cannot be considered as hedging since such strategies do not involve spot market securities.

2.5 Paticipants in the interest rate futures market

The importance of this section will be evident when the empirical results are analyzed.

Table 2.1 provides the results of surveys by the Commodity Futures Trading Commission. It lists the types of traders and the weight of their participation in the market. The survey indicates that the

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[1]The reader is directed to the various brochures and booklets published by the Chicago Board of Trade and the International Monetary Market. A list of these is presented after the bibliography.

majority of the participants tend to be non-commercial traders. It is safe to assume that only a minority of the non-commercial traders would be hedging. According to Arak and McCurdy [1979-80] the commercial traders are the only group that would use the futures market for hedging to any meaningful extent. The rationale provided is that the typical non-financial business will incur a higher cost of financing when interest rates rise. Since interest rates increase when inflation increases, the business will be compensated for the additional financing cost due to the increased price it will get on the product or service it sells. Thus, to some degree the firm is automatically hedged against inflation-induced changes in interest rates.

Also, some of the financial or commercial traders could be using the futures market for purposes other than hedging. Arak and McCurdy provide the example of security dealers who, besides using the futures market to manage their risk exposure, might try to arbitrage or speculate on interest rate changes. It is therefore concluded that the futures market is used for purposes other than hedging.

TABLE 2.1

FUTURES MARKETS PARTICIPANTS.
PERCENTAGE OF TOTAL PARTICIPATION

	GNMA		T-BOND		T-BILL	
	1977	1979	1977	1979	1977	1979
COMMERCIAL TRADERS						
Security dealers	17.1	7.2	50.9	18.2	18.3	12.5
Commercial banks	1.3	1.1	3.3	3.3	2.2	3.5
S & Ls	2.5	4.2	-	0.9	0.3	0.3
Mortgage bankers	6.1	2.5	5.1	0.7	0.3	2.2
Other	9.5	3.4	7.9	4.4	11.7	15.0
Total	36.5	18.3	67.2	27.4	32.8	33.6
NON COMMERCIAL TRADERS						
Futures industry	34.1	35.4	15.8	28.6	18.3	18.9
Commodity pools	14.4	18.6	8.4	21.0	10.1	12.6
Individual Traders	12.0	27.7	8.6	23.0	38.8	34.9
Total	63.5	81.7	32.8	72.6	67.2	66.4

CHAPTER III

REVIEW OF THE EMPIRICAL LITERATURE

3.1 Introduction

This chapter will review briefly the literature that is relevent to this study. The chapter will begin with a review of the literature that compares the futures rate to the forward rate that is implicit in the term structure of interest rate. A review of Kane's [1980] study will reveal the reasons for differences that exist between the forward rate and the futures rate. Subsequently the use of the CAPM in the futures literature will be examined together with Blacks [1976] rationale for the existence of a futures market. The last section of the chapter will provide a brief discussion of the hedging literature. This section will compare and contrast that literature with the methodology adopted by this study.

3.2 A Comparison between the Forward Interest Rate and the Futures Rate

We begin this section with a definition of the forward interest rate and its conceptual meaning according to different hypotheses. The forward rate is defined as a rate that is implicit in the term structure of interest rates. According to the unbiased expectations hypothesis,[2] at the margin, investors are indifferent between holding bonds of different maturities so long as the expected returns on all

[2]See Lutz [1940] for an explanation of the theory.

bonds are the same over any given holding period. This hypothesis is based on the premise that investors at the margin are not risk averse, and it leads to the conclusion that the long term interest rate is nothing more than the geometric mean of the current short term rate and the expected future short term interest rates. Therefore, the forward rate is the expected future short term rate according to the expectations hypothesis.

The liquidity premimum hypothesis[3] is based on the premise that investors are risk averse towards the longer term segment of the maturity spectrum. The long term rate incorporates both expectations about future short term rates and liquidity premiums. Therefore, the forward rate implied by the observed rate structure is the sum of the expected interest rate for that period and the liquidity or risk premium for that period.

According to the market segmentation hypothesis[4], the market for bonds is segmented due to the structural rigidities in the liabilities of bond investors. Therefore the structure of yields over the term structure is said to reflect primarily supply and demand at various maturities rather than expectations or liquidity premiums. Yields across the maturity spectrum are not constrained to have any systematic relationship. Short term rates would be determined in one submarket while intermediate and long term rates would be established in other submarkets.

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[3]See Roll [1970] for an explanation of the theory

[4]See Culbertson [1957] for an explanation of the theory

The tests of efficiency that have been performed on the futures market have assumed that there shoud be equality between the forward rate implicit in the term structure and the futures rate. If significant differences between the two existed, they would be arbitraged away if participants view a futures contract to be the same as the implicit contract underlying a forward position. This would imply that if the unbiased expectations theory holds true for the forward rate, the futures rate would also reflect the unbiased estimator of the future interest rate. Similarly, if the liquidity premium hypothesis holds true for the forward rate, the futures rate would include a risk premium equal to the liquidity premium. If the market segmentation hypothesis holds true for the forward rate, the equality implies that the segmentation effects have equal impacts on both rates.

According to Breeden [1979], two strategies are available for earning arbitrage profits with Treasury bills (T-bills):

1. Borrow the needed money and buy a T-bill maturing ninety days after the expiration of the futures contract and simultaneously take a short position in a T-bill futures contract. At the maturity of the futures contract, deliver the T-bill to honor the futures contract and use the proceeds to pay off the loan. An arbitrage profit is made if the futures rate was less than the corresponding forward rate when the contract was initiated. For example, suppose the forward rate, derived from using the rate on the loan as the spot rate, for the last 90 days for the T-bill purchased is 12 percent, which translates to a price of 97.09 on a

maturity value of \$100. And suppose that the return on the futures position is 9 percent, which translates to a price of 97.80. The difference between the two prices are the arbitrage profits. If the futures rate is greater than the foward rate then the second strategy is appropriate.

2. Take a long position in a futures contract and buy a T-bill maturing at the expiration of the futures contract. Borrow the money to buy the T-bill. When the futures contract matures, use the proceeds of the maturing T-bill to take delivery of the T-bill and when this T-bill matures use part of the proceeds to pay off the loan. The remainder of the proceeds are the arbitrage profits. For example, suppose the forward rate on the loan as estimated from the spot rate on the T-bill purchased presently is 9 percent and the rate on the futures position is 12 percent. Then the three percent difference that is earned are the arbitrage profits.

Several studies have been performed that use these or similar strategies to compare forward and futures rates.

Puglisi [1978] compared rates of return on two alternative strategies:

1. buy a short-term T-bill
2. Buy a long term T-bill and also take a short position on a T-bill futures contract. Make delivery on the futures contract when it matures with the longer term T-bill.

Futures market transactions costs were taken into consideration and T-bill purchases were assumed to have been made at ask prices. If the futures market is efficient, in the sense that arbitrage profits cannot be earned, the rates of return on the two strategies should not be significantly different. Using data from March 1976 to September 1977, Puglisi evaluated the differences between the returns on the two strategies using the t-test and the non-parametric sign test. He concluded that the futures market for T-bills is inefficient. For Puglisi's data, the major inefficiencies occurred early in the life of the T-bill futures market and lessened to some degree as indicated by tests on the data for the end of the test period.

Rendleman and Carabini [1979] determine the range within which the futures contract should trade if arbitrage opportunities are to be absent. They determine that the futures price should be between

$$(100\ P_n^A/P_m^B + .006) \text{ and } (100\ P_n^B/P_m^A - .006) \qquad (3.1)$$

where

m = the maturity date on the futures contract

n = the maturity date of the delivery vehicle for the futures contract.

P_m and P_n = are the spot prices per \$100 par value for T-bills maturing at time m and n respectively

A and B = asking prices for the bills and dealer bid prices respectively.

and .006 is the estimated transactions costs par \$100 per value.

The data included prices for transactions between January 1976 and March 1978. Only prices of futures in their last nine months prior to their maturity were used. The results indicated that deviations of futures prices from the range were not statistically significant. This led to the conclusion that pure arbitrage opportunities were not available and hence the futures market for T-bills was efficient. Rendleman and Carabini also tested whether quasi arbitrage opportunities were available. The difference between pure arbitrage and quasi arbitrage is that pure arbitrage requires that the investor has zero net investment. Quasi arbitrage can be defined as selling securities from an existing portfolio to fund an economically equivalent position at a lower price. The investor borrows to finance the arbitrage transactions. Rendleman and Carabini determined that a borrowing rate of ½% above the asked T-bill rate (which is a very conservative estimate) was sufficient to eliminate pure arbitrage opportunities. Quasi arbitrage does not require zero net investment on the investor's part. The results indicated that if the investor does not borrow, then the quasi arbitrage profits are statistically significant. These quasi arbitrage profits could be earned by using futures contracts to improve the return on an existing portfolio. However, it would not be worth exploiting given the indirect costs of educating traders and policy makers within financial institutions, the costs of monitoring futures markets and the inability in some cases to cover a futures obligation with the exact T-bill required.

Lang and Rasche [1978] calculated the forward rate for T-bills that is implicit in the term structure and compared it to the futures rate for T-bills to test for equality between the two yields. The comparison was performed by calculating the difference between the two and conducting t-tests to determine significant differences. Sample data from March 1976 to March 1978 was used. All futures contracts traded were examined with times to maturity up to 24 months. The differences were insignificant for contracts close to maturity and highly significant for contracts with distant maturities. For the distant maturity contracts, futures rates were significantly above implied forward rates by amounts that increased with the length to maturity of the futures contract. Quasi arbitrage opportunities were available for all contracts with considerably greater opportunities in the distant maturity contracts. Over time, distant contracts became more efficient but the near contracts became less efficient. This was evidence that the market had shown no general tendency to become more efficient over time.

Lang and Rasche explain the greater differential for the longer maturity contracts by pointing out that participating in the futures market entails a degree of default risk. This default risk is compensated for by a risk premium which is included in the futures rate. The default risk increases over time because the distant maturity futures contracts involve the delivery of T-bills that are not yet issued. Exchange guarantees are provided, but these guarantees are not risk free since the exchange is a private corporation and not part of the U.S. Government. Kane alludes to default risk as one of the reasons for the differences between the futures rate and the forward rate. Breeden finds this explanation rather unconvincing since both the individuals broker and the exchange guarantee the contract. In the event that T-bills are unavailable for delivery, a monetary settlement would seemingly cause very little inconvenience to the participants since the funds could be invested in other types of liquid investments which are close substitutes. This might not be true in the case of a wheat contract where the buyer truly is planning on receiving the wheat and can find no substitute for wheat in the event of a monetary settlement.

Poole [1978] performed tests similar to those of Lang and Rasche with T-bill futures data spanning the periods January 1976 to June 1977. He found a statistically significant tendency for futures rates to be less than the forward rate. Poole used only the data on the contracts nearest to maturity. He defined a critical range for profitable arbitrage for the futures rate given the spot yield, taking into account transactions costs. Futures rates that were within the range indicated that profitable opportunities did not exist. The differences, although statistically significant, were not large enough to make quasi-arbitrage profitable. He concluded that T-bill prices were efficient for his sample. Poole explained his results on the hypothesis that futures and forward rates should not be equal because of the effect of transactions costs. Transactions costs for futures contracts are negligible but significant for transactions in the spot market.

Branch [1978] compared yields on a composite security with equivalant cash market yields of the same maturity as the composite security. The composite security consisted of a security maturing on the delivery date of the futures contract and a long position in the futures contract itself. The differences between the two sets of yields were determined and the mean differences tested for significance. The study was conducted for the T-bill, GNMA and T-bond futures market for contracts with up to 32 months to maturity. The t-statistics for the differences in the means were significant for all maturities and all markets, and the mean differences increased with time to maturity. Transactions costs were not included in the analysis and Branch stated that if transactions costs were included a maximum 10 to 20 basis points differential advantage would have been available with the composite security. He concluded that the composite security, rather than the cash market security, should be used by portfolio managers to increase return without an increase in risk.

Cappoza and Cornell [1978] used weekly data for the first eighteen months of trading on T-bill futures contracts. They estimated the forward rate and compared it to the futures rate. The results indicated small differences for near delivery contracts and larger differences for the longer maturity contracts. For the longest contracts, with maturities of twenty-six to thirty-nine weeks, the differences were of the order of forty basis points. Since transactions costs would be not more than three to five basis points, they concluded that significant differences did exist. They provide a similar interpretation to Lang and Rasche that the differences could be accounted for by the added risk of the futures market since its guarantees are not equal to the U.S. government guarantees.

The literature reviewed above seems to indicate fairly consistent results. There are no significant differences between the forward and futures rate for contracts near maturity. For the distant maturity contracts the differences are significant to the extent that quasi arbitrage profits could be earned, although Rendleman and Carabini provided a qualitative argument that the costs of educating participants might make the profits insignificant. It seems they fail to understand that the costs of educating the participants is a one time fixed cost and that the arbitrage profits that accrue could be realized time after time in the future. The empirically observed differences were attributed to the default risk of delivery associated with futures contracts which bids up the futures rate above the forward rate. Kane subsequently provided a conceptual perspective to explain this difference.

Kane argued that the futures rate will equal the forward rate only when there is risk neutrality and identical expectations on the part of the investors, perfect divisibility, zero transactions costs, identical tax treatment of interest and capital gains income, and costly guarantees. The remainder of this section will provide the reasons for the differences.

1. The risk of default.

Even if the debt instruments underlying the futures contract are default free, that does not imply that futures commitments by participants to buy and sell these instruments are also default free. A participant has to recognize that participants on the other side of the contract act to maximize their own terminal wealth. In order to maximize they must be expected to default on their contracts whenever the benefits from reneging exceed the cost of the penalties of doing so. Similarly the participant, although concerned about default at the other end, also carries the option of defaulting. This risk of default does not exist in the bond market because the commitment is executed immediately which implies that the risk that besets a futures contract is non-existent in the implicit forward rate.

2. The differential tax treatment.

In 1977 the holding period necessary to qualify for long term capital gains increased from six months to nine months and in 1978 it increased from nine months to one year. This increase did not apply to futures contracts, which carry an anticipated capital gain because the tax law designates futures contracts as capital assets and participants are allowed to qualify for long term capital gains under certain conditions. The conditions are that the participant has to take a long position and that the participant is not a market-maker. A market-maker is one for whom these assets are deemed stock in trade. Profits earned on the short side are always regarded as short-term capital gain regardless of the holding period. Also, except for hedgers, net losses in futures transactions are tax-disadvantaged since there is a ceiling of $3000 on deductible losses. The tax differential will only have an impact if for the marginal participant the lure of expected capital gains outweighs the risk of unanticipated declines in the price. For example, some participants instead of investing in nine-month T-bills might invest in six-month T-bills and simultaneously buy a futures contract that matures after six months so as to realize the potential benefit of the tax gain on the futures position. The reduction in demand for the nine-month T-bills will decrease their price hence increasing the rate of return and the increase in demand for the futures position will increase their price which will reduce the rate of return. This will create a discrepency in the rates, with the forward rate being higher than the futures rate.

3. The costly guarantee.

The futures exchanges provide the participants in the markets with partial performance guarantees. The value of these guarantees are hidden in the escrowed deposits, net-worth screening, brokerage fees (which actually are insignificant compared to the size of the transaction) and the margin maintenance requirements. But there is also a cost that is to be borne by the participants who realize the value of these guarantees. The escrow deposits and initial margin requirements could mean an opportunity cost to the participant in the form of interest earned on other investments. The same argument applies for the margin maintenance requirements placed on participants since futures contracts are marked to market daily. Participants are allowed to put up interest bearing securities on which they keep the interest. This would impose only a minor cost to the participant. But maintenance margin requirements can only be met in the form of cash.[5]

With reference to the margin maintenance requirements it bears mention that Rendleman and Carabini also make a similar point:

> "Futures contracts differ from forward contracts however, in that day-to-day changes in the futures prices are either debited or credited to the customer's account with any deficits having to be debited as cash. Thus to be technically correct, any futures pricing model should take these day-to-day changes into account." (1979, p. 897)

3.3 The CAPM and Futures Markets.

To this day no published work has tested the interest rate futures market with the CAPM, perhaps because the CAPM is based on the assumption that the risk-free interest rate is constant and implicitly other interest rates are as well, over the time period. The CAPM has been used for empirical studies on commodity futures by Dusak, and Bodie and Rosansky. Black used the CAPM, but only theoretically.

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[5]A telephone conversation with the Chicago Board of Trade employee confirmed this point.

According to Black, the futures contract is a forward contract that is re-written each day. By forward contract he means a contractual agreement to deliver (and also receive) a commodity at some future date at the specified price. There will be no monetary settlement until that future delivery date. Since the futures contract is marked to market daily, the value of the contract is zero at the start of each day. According to the CAPM:

$$E[(P_1 - P_0)/P_0] - R_F = Cov[((P_1 - P_0)/P_0, R_m)/Var(R_m)] \times [E(R_m) - R_F] \qquad (3.2)$$

Multiplying through by P_0;

$$E(P_1 - P_0) - R_F P_0 = [Cov\ (P_1 - P_0,\ R_m)/Var(R_m)] \times [E(R_m) - R_F] \qquad (3.3)$$

Since the value of a futures contract is zero, Black sets P_0 equal to zero and arrives at the following result.

$$E(\Delta\ P) = [Cov(\Delta\ P,\ R_m)/Var(R_m)]\ [E(R_m) - R_F] \qquad (3.4)$$

Utilizing the CAPM framework Black indicated that hedgers have means other than the futures market to diversify away their risk. Furthermore stock holders can diversify away the risk of the hedging corporation by making adjustments to their well-diversified portfolios. But debt holders might require that the corporation hedge since the risk borne by the lenders is reduced if the corporation hedges.

It is his opinion that futures markets exist because in certain cases they might provide an inexpensive way to transfer risk and because many people like to gamble on commodity prices. He claims that neither of these is a major benefit to society[6]. The major benefit in his interpretation is that participants in the futures market can make production, storage and processing decisions by looking at the pattern of futures prices even if they do not take positions in the futures market.[7]

Dusak used the following CAPM relationship to derive the empirical relationship she used:

$$P_0 = \frac{E(P_1) - [E(R_m) - R_F]\ P_0 \cdot B}{1 + R_F}$$

where B is the Beta value.

Multiplying both sides by $(1 + R_F)$

$$P_0\ (1 + R_F) = E\ (P_1) = [E(R_m) - R_F]\ P_0 \cdot B \qquad (3.6)$$

$P_0\ (1 + R_F)$ is interpreted as the current futures price for delivery and payment of the spot commodity one period later. Buying a futures contract is like buying a capital asset on credit where the capital asset in this case happens to be the spot commodity. If P_0 is the current price for immediate payment, $P_0(1 + R_F)$ must be the price if the buyer buys on one period credit terms.

Setting $P_g = P_0\ (1 + R_F)$ the following is derived,

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[6]By being the lowest cost means of transferring risk it is of benefit to society if these lower costs lead to lower prices charged by the corporation.

[7]This reasoning cannot explain why interest rate futures markets exist since the expected futures rate could be derived.

$$\frac{E(P_1) - P_g}{P_0} = B[E(R_m) - R_F] \qquad (3.7)$$

From this equation it is inferred that there are two ways of measuring the risk premium. One way is the percentage change in the spot price over a given interval minus the riskless rate and the other is the percentage change in the futures price over the same interval. Dusak adopts the latter measure and regresses it against the S & P 500 minus the risk free rate. Futures data on three heavily traded agricultural commodities, wheat, corn, and soy beans spanning a fifteen year period from 1952 to 1967 were used. The results were quite interesting. The intercept terms were not significantly greater than zero, which was expected, but quite surprisingly the beta coefficients were not significantly different from zero. It was concluded that there was no systematic risk in holding futures contracts if they were part of a well-diversified portfolio. These results were interpreted to be a serious blow to the theory of normal backwardation.

Bodie and Rosansky determined the mean and variance of the rate of return earned from holding a portfolio of all twenty-three major commodities traded in the futures market. The rate of return on a commodity was based on a buy-and-hold strategy over a three-month holding period and was calculated according to the following measure:

$$R = \frac{P_{t+1} - P_t}{P_t} + \frac{R_{F,t}(P_t + P_{t+1})}{2P_t} \qquad (3.8)$$

where P_t = the future price at the start of the period,

P_{t+1} = end of period futures price and

$R_{F,t}$ = risk-free interest rate.

The first part of the right hand side of the equation is the return earned from posting 100% margin, and the latter part gives the risk-free interest earned on the daily adjusted margin requirement. Bodie and Rosansky compared the rate of return and variance estimates to the same for the stock market over the same time period and concluded that the means and variances were similar. Also, since the period-by-period rates of return between commodity futures and stocks were negatively correlated, one could earn the same rate of return with a significantly lower (one-third less) standard deviation by holding a portfolio of sixty percent stocks and forty percent commodity futures. They concluded from this that the market portfolio of common stocks as represented by the S & P 500 was not efficient in a mean-variance sense[8].

The study supported the normal backwardation hypothesis since the rate of return earned on the commodity futures portfolio was determined to be significantly greater than the risk-free rate. This conclusion contradicted Dusak's study of three commodity futures. To check the consistency of her data, Bodie and Rosansky restricted their sample to the same three commodities and the same time period as Dusak had used and indeed found that the average excess returns in this sub set equal to zero. They also checked their findings with the CAPM.

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[8]This conclusion is inconsistent with the theory of the CAPM since the market portfolio would include all long and short positions which would negate each other.

Since the mean excess return on the S & P 500 per year was approximatley equal to the mean excess return for the commodity futures portfolio, a regression of the excess rate of return on commodities, against the excess rate of return on the market should have given a beta equal to one. A regression between the excess return on the commodities and the excess rate of return on the S & P 500 gave a beta estimate of - 0.2614. Since this estimate was significantly different from one it was concluded that the empirical findings were inconsistent with the conventional form of the CAPM.

3.4 The Hedging Literature.

There exist three major theories of hedging:

1. The traditional theory
2. The theory of Holbrook Working
3. The portfolio theory.

The traditional theory[9] puts emphasis on risk avoidance. Hedgers take positions in futures equal in magnitude but of opposite sign to their position in the cash market. For example, an individual who has taken a spot position in T-bills would protect against increases in interest rates which would decrease the price of the T-bills by taking a short position in T-bills futures. If the spot T-bills are sold at a loss, the loss will be offset when the short position in the futures market is closed.

Working [1953], in a qualitative treatise, was of the opinion that hedgers are not minimizers of risk, rather they are maximizers of

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[9] See Ederington [1979] for an explanation of the theory.

expected profit. Hedgers are concerned with relative price changes and not absolute price changes. They would hedge only if there is a favorable change in the relationship between spot and futures prices.

The portfolio theory of hedging was first provided by Johnson [1960]. He derived the risk minimizing hedged position as follows:

If P_1 and P_2 are spot prices at times t_1 and t_2, respectively, then the gain or loss on an unhedged position U of X units is

$$X\,(P_2 - P_1) \qquad (3.9)$$

The gain or loss on a hedged position H is

$$X\,[(P_2 - P_1) - (F_2 - F_1)], \qquad (3.10)$$

where F denotes the futures price.

If spot and futures prices do move together, then

$$\mathrm{Var}(H) < \mathrm{Var}(U) \qquad (3.11)$$

This would depend upon the change in the "basis". The basis is defined as the difference between the futures and spot prices so that the change in the basis is

$$(F_2 - P_2) - (F_1 - P_1) \qquad (3.12)$$

The hedge is perfect if the change in the basis is zero.

The portfolio theory of hedging determines the proportion of the spot position which should be hedged so as to minimize variance.

$$b = -X_F/X_S \qquad (3.13)$$

where X_S and X_F denote spot and futures market holdings.

Let R represent the return on the hedged portfolio. Then

$$E(R) = X_s\,E(P_2 - P_1) + X_F\,E\,(F_2 - F_1) \qquad (3.14)$$

$$\mathrm{Var}\,(R) = X_s^2\,\mathrm{Var}\,(P_2) + X_F^2\,\mathrm{Var}\,(F_2) + 2\,X_s\,X_F\,\mathrm{Cov}\,(P_2, F_2) \qquad (3.15)$$

$$= X_s^2 \; [\text{Var } (P_2) + b^2 \text{ Var } (F_2) - 2b \text{ Cov } (P_2,F_2)] \qquad (3.16)$$

$$\frac{d \text{ Var } (R)}{db} = X_s^2 \; [\; 2 \text{ b Var } (F_2) - 2 \text{ Cov } \; (P_2, F_2)] \qquad (3.17)$$

where, d, is the partial derivative operator

The risk minimizing b, b* is

$$b^* = \frac{\text{Cov } (P_2,F_2)}{\text{Var}(F_2)} \qquad (3.18)$$

Ederington [1979] evaluated the performance of this theory. The measure of hedging effectiveness he used was the percent reduction in the variance

$$e = 1 - \frac{\text{Var}(R^*)}{\text{Var}(U)} \qquad (3.19)$$

where U is the return on the unhedged position and

$$\text{Var } (U) = X_s^2 \text{ Var } (P_2) \qquad (3.20)$$

Subsituting $b = X_F/X_s$

$$\text{Var } (R) = X_s^2 \; (\text{Var } (P_2) - \frac{\text{Cov}(P_2, F_2))}{\text{Var}(F_2)} \qquad (3.21)$$

Consequently

$$e = \frac{\text{Cov } (P_2, F_2)}{\text{Var}(F_2)} = R^2 \qquad (3.22)$$

where R^2 is the population coefficient of determination between the change in the cash price and the change in the futures price. The hedge used to estimate e is one in which futures positions were liquidated by offsetting trades prior to delivery. The spot market transactions were on the same instruments but perfect hedging was not possible since the price change risk could be eliminated entirely only if delivery were taken on futures bought and sold. The major problem with making or taking delivery is that there are only four delivery periods each year and they might not coincide with the timing of the spot market transaction.

The study estimated, among other things, e and b* for two-week and four-week hedges for GNMAs and T-bills spot positions hedged with GNMA and T-bill futures contracts respectively. Four different delivery dates spanning a period of less than three months to one year were evaluated separately. For the two-week hedges, the T-bills performed poorly with e's of .272 and less, while the lowest e for the GNMAs was .661 and the highest was .677. For the four week hedges, the e's for T-bills ranged from .369 to .741 and for GNMAs .780 to .817. In most cases b* was significantly different from one and generally was less than one. These of course were ex-poste estimates of b*. The results lead to the conclusion that pure risk minimizers would wish to hedge only a portion of their portfolios and that the GNMA futures market appears to be a more effective instrument for risk avoidance than the T-bill futures market.

McEnally and Rice [1979] tested the efficacy of three cross-hedging strategies with GNMA futures and six different bond issues. Cross-hedging can be defined as the type of hedging performed when the spot market instrument to be hedged is different from the instrument underlying the futures contract. The three strategies were:

1. A naive hedge which as explained before, involves going

short a number of dollars equal to the dollar value of the bond offering.

2. A duration hedge which is computing the duration hedge ratio which will equate price volatilities.

$$(D_p{}^1\ YV_p) = HR\ (DF^1\ YV_F) \tag{3.23}$$

$$HR = \frac{D_p{}^1\ YV_B}{D_F{}^1\ Y.V_F} \tag{3.24}$$

where D^1 and D^1 are "adjusted" durations of the bond and futures instrument and HR is the hedge ratio.[10] The adjustment consists of dividing the conventional computed duration by its yield to maturity and is needed to maintain consistency with discrete compounding. $Y\ V_B$ and $Y\ V_F$ are the estimated yield volatility of securities with maturities equal to the bond and futures instruments respectively. The volatility values were estimated from fitting the equation

$$Y\ V_t = a + b_1\ (1/t) + b_2\ (1/t^2) \tag{3.25}$$

where t is the term to maturity.

A historically optimal hedge uses the hedge ratio that provides the minimum variance of the hedged outcome over a period immediately preceding the hedging decision. The length of the period used to calculate the historically optimal hedge ratio equals the time required to float the proposed issue. It is assumed that there is a time lapse of 12 weeks or more between the decision to issue and the date the securities came to market.

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[10]McEnally [1977] gives reasons for the adjustment.

The hedge ratio as explained before was:

$$HR = \frac{Cov\ (F,P)}{Var\ (F)} \tag{3.26}$$

The ex-poste optimal hedge is also estimated to determine the maximum reduction in variation possible. Commissions, margin requirements, and taxes were ignored. The six bond issues hedged were:

1. S & P AAA
2. S & P BBB
3. Salomon New Aaa
4. Salomon New Baa
5. Treasury Bond
6. S & P Municipals

The standard deviation of gain or loss in dollars indicated that hedging reduces the variation for all six bonds but the reduction was less for higher grade corporate bonds. Cross-hedging was more effective when the hedged issue was more like the hedging instrument, as would be expected. Prices of lower grade bonds were likely to be affected more by factors extraneous to the movements of basic interest rates such as changes in default premiums than are government and high grade corporate bonds prices. The historically optimal hedge and duration hedge strategies each did better than the "naive" strategy in only three out of six cases. This led to the conclusion that hedging did reduce the variance but no particular hedging strategy performed better than the others.

CHAPTER IV

DERIVATION OF THE MODEL

4.1 Introduction

This chapter will begin with the development of the framework that is necessary for the derivation of the model. The next section will be devoted to presenting the necessary assumptions and will include the utility function to be used in the derivation and a justification for the exclusion of the default risk. Subsequently, a section will be devoted to defining the rate of return measure to be used in this study and comparing it to rates of return measures adopted by other studies. The latter sections of the chapter will be devoted to providing a rationale for the existence of hedgers and speculators, providing the rationale for and the specification of the costly guarantee, and finally providing the derivation of the model. The section on the derivation of the model will also include derivations of the model with the demand and supply functions of hedgers as derived by the minimum variance hedge model.

4.2 The assumptions

All the participants in the market are assumed to be risk-averse. Their objective is to maximize the expected utility of their wealth at the end of the period. A period is the intended holding period of the contract by the participant and it spans a period from time 0 to time T. All decisions are made at the beginning of the period and are not changed until the end of the period. This assumption is made to maintain a one period scenario. A participant who takes a position is therefore assumed to maintain that position until the end of the period regardless of price changes that occur in the interim. This implicitly assumes that even though a wealth constraint exists partic ipants will be able to borrow money to meet margin calls. This issue will be addressed in detail in the section on the costly guarantee.

The utility function of all participants is assumed to be of the form

$$U(W) = -e^{-aW} \tag{4-1}$$

where W is terminal wealth and a is the investors index of absolute risk aversion as defined by Pratt (1964). As shown by Hakansson (1969), this exponential utility function, which exhibits constant absolute risk aversion and increasing relative risk aversion, is locally separable with respect to the investor's wealth. Thus, the investor can make decisions regarding the composition of his investment portfolio independent of his wealth by making decisions that maximize the expected value of

$$U(R) = -e^{-a(1 + R)} \tag{4-2}$$

where R is the rate of return on the portfolio.

Furthurmore, it has been shown by Tobin (1958), Lintner (1969) and other authors that the maximization of the mean variance function

$$U = a\ E(R) - b\ Var(R) \tag{4-3}$$

is equivalent to maximizing expected utility of the return on the investment, when investment return disbributions can be specified by two parameters, E(R) and Var(R). Thus for purposes of this study it will be assumed that returns are normally distributed and that investors make decisions so as to maximize expected utility over a mean-variance function. Specifically, if the utility function in equation (4-2) is used to maximize expected utility and assuming that returns are normally distributed, the maximization of expected utility is as follows

$$E[U(R)] = -e^{-a} \int_{-\infty}^{-\infty} \frac{-e^{-aR}}{\sqrt{2\pi Var(R)}} e^{\frac{-[R - E(R)]^2}{2Var(R)}} dR \qquad (4\text{-}4)$$

$$= -e^{-a} . e^{-a[E(R)-a \frac{Var(R)}{2}]} \qquad (4\text{-}5)$$

(See Hogg and Craig (1978) for the above derivation)

Therefore

$$\frac{d\, E[U(R)]}{d\, E(R)} = e^{-a} .a. \; e^{-a[E(R) - a.\frac{Var(R)}{2}]} \qquad (4\text{-}6)$$

which is greater than zero, and

$$\frac{d\, E(U(R)}{d\, Var\, (R)} = -e^{-a} . \frac{a^2}{2} \; e^{-a[E(R) - a.\, \frac{Var(R)}{2}]} \qquad (4\text{-}7)$$

which is less than zero.

Therefore, the risk aversion index specified by Bierwag and Grove [1967] which is measured as

$$- \frac{1}{2} \; \frac{d\, E(U(R)]/d\, E(R)}{d\, E[U(R)]/d\, Var(R)} = \frac{1}{a} \qquad (4\text{-}8)$$

is a constant. This study requires that the risk aversion index is a constant. This will be evident in Chapter V when the empirical form of the model is specified.

All participants are assumed to have the same estimates for the expected value and variance of the futures price at the end of the period. This assumption is probably quite unrealistic but is necessary for two reasons. The first is that with heterogeneous expectations it is impossible to quantify these different estimates for the empirical tests that are to be performed. Secondly, if heterogeneous expectations are assumed, a model cannot be derived that will test for the normal backwardation hypothesis, the normal contango hypothesis or the expectations hypothesis. With heterogeneous expectations, one cannot justify a market in which the majority of participants are hedgers and where the excess of hedging activity on one side, long or short, would be met by participants who are termed as speculators. With heterogeneous expectations, the market would be akin to a gambling casino in which the speculative activity would be a function of heterogeneous expectations and not a transfer of risk from hedgers to speculators for which the speculators are paid a premium. The issue of whether this assumption is realistic will be furthur discussed in the last chapter.

The default risk is assumed to be insignificant. It is assumed that there exist very few states of the world in which both the participant's broker and the exchange would lack the financial viability to meet the obligations of the participants. As mentioned before, Breeden is of the opinion that the default risk would not be significant since in the event of default by the opposing party the exchange and the broker would have to assume the liability of the defaulting party. Kane states that the brokerage firm is obligated to make good all defaults by its own customers and to bring suit in civil court against the defaulting party. The Chicago Mercantile Exchange operates an emergency fund which protects participants against individual broker bankruptcy, even to the extent of allowing the

Exchange to levy make-good payments on surviving members of the exchanges. Therefore, the protection the participant receives is not only from the participant's own brokerage firm and the Exchange but also from others brokerage firms that are members of the exchange.

It is assumed that price changes will occur at discrete time intervals. The changes are stochastically independent which implies that future price changes cannot be predicted from past price changes. These discrete intervals could be shorter than the holding period of the participant in which the participant could be subject to margin calls. No restrictions are placed on when a participant could close out a position. It could conceivably be at the time of, or before the maturity date of the contract. The end of the holding period for the participant does not necessarily coincide with the maturity data of the futures contract. Futures contracts are perfectly divisible and futures and spot market prices in the model are in terms of a $1 spot market position for hedgers and $1 of investable wealth for speculators.

4.3 The rate of return measure

The estimation of the rate of return for a futures contract has been subject to controversy. The estimate used in this study will be first provided and subsequently compared with Dusak's position on the issue.

In this study the rate of return will be measured on a pure cash flow basis. For example, the rate of return, on the long side, assuming no hedging is:

$$R = (F_T - F_0 - C)/K \qquad (4\text{-}9)$$

where R is the rate of return, F_0 and F_T are the futures prices at time 0 and time T respectively, C is the costly guarantee and K is a constant. The costly guarantee is the cost of margin maintenance requirements during the holding period and since price movements in the interim are uncertain the costly guarantee is an uncertain amount. The specification of the costly guarantee will be provided in a subsequent section. The numerator on the right hand side is the gain or loss that accrues to the participants and it is divided by a constant. This constant, K, includes the time zero cost of taking a position in the futures market. Even though participants are allowed to put up interest-earning assets as margin on which they keep the interest, there is some cost borne by the participant. Telser [1981] argued that the interest bearing assets that are pledged are highly liquid and can be used by the participants in the case of an emergency or to take advantage of profitable opportunities that may suddenly arise. They can be considered as part of the participant's precautionary balance. But once committed, they cannot be used elsewhere, and therefore are no longer a part of the precautionary balance. Hence the margin requirement does impose a cost on the participant even if interest-bearing assets are used to satisfy the margin requirements. Since the margin requirement is independent of the market price, the initial cost of taking a position is independent of the futures price and is taken to be a constant. For hedgers, the constant K includes the initial \$1 spot market position and for speculators it is \$1 which includes the cost of the initial margin requirement and the portion of each dollar of the speculators investable wealth that is not invested in futures contracts.

Dusak does not explicitly define a rate of return measure. She defines the current price of the futures contract, which can be considered as the initial investment, as F_0 $(1 + R_F)$. Since the transaction is initiated at time 0 and consummated at time T, the participant has to pay credit over the time period which amounts to $F_0 . R_F$ in addition to the current price F_0. Hence, F_0 $(1 + R_F)$ can be interpreted as the current futures price for delivery and payment of the spot commodity one period later.

This approach, although theoretically sound, seems to evade reality. For one, an investor evaluates an investment opportunity relative to the initial outlay committed by the investor. The initial outlay is not $F_0(1 + R_F)$. Furthermore, several of the participants in the futures market are not wealthy enough to consider an outlay as large as the price of a futures contract. Lastly, since the great majority of futures contracts are terminated before delivery, the estimation of the current futures price as F_0 $(1 + R_F)$ is unrealistic.

4.4 Hedging and Speculative Activity.

An issue that needs to be addressed is the rationale for hedging and speculative activity. Furthermore, there is a need to distinguish between the two kinds of activity. This section will critique the minimum variance hedge model which was presented in the review chapter and subsequently derive the demand and supply functions for futures contracts for the purposes of hedging. The latter part of the section will explain speculative actvity.

The portfolio theory of hedging put forth by Johnson [1960] will not lead to a global optimal solution. The minimum variance hedge model minimizes basis risk, or minimizes the variance of the spot market transaction to be hedged. The model does not take into consideration, expected prices. In a sense it maximizes utility in a variance world and not a mean-variance world.

In this study, expected utility will be maximized in a mean-variance world . The expected value and variance of the rate of return are determined and expected utility is maximized using the chain-rule of differentiation

$$\frac{dE(U)}{dE(R)} \cdot \frac{dE(R)}{db} + \frac{dE(U)}{d\,Var(R)} \cdot \frac{d\,Var(R)}{db} = 0 \qquad (4\text{-}10)$$

where E(U) is the expected utility of terminal wealth, R is the expected rate of return, Var(R) is the variance of the rate of return and, b is the decision variable which is the proportion of the spot market transaction to be hedged. The expected utility function can be denoted as U(E(R), Var(R)) where

$$\frac{d\,E(U)}{d\,E(R)} > 0 \text{ and } \frac{d\,E(U)}{d\,Var(R)} < 0.$$

Hedgers in the market own portfolios of securities. They find that there is a need to hedge a portion of that portfolio that is exposed to risk due to shifts in the interest rate. Accordingly, they will take positions in interest rate futures markets. One can rationalize a scenario in which the hedgers are in equilibrium, and then take a spot market position, or wish to take a position in the spot market at some future date which shifts them from their equilibrium position. They find that they are uncomfortable with this shift in their portfolio which has created a partially unhedged position. The implication of uncomfortable is that the risk is excessive and therefore the participant decides to hedge against this risk by taking a position in the futures market. Let Q be the dollar amount of the hedger's portfolio that is exposed to interest rate risk and therefore could be hedged by taking a position in the futures market.

Hedgers will take a short position in the futures to hedge if

1. They wish to issue debt in the future and find that this position is not hedged and wish to reduce the uncertainty associated with the issue. Taking a short position would reduce the uncertainty since interest rate increases which would increase the cost of the debt would be offset by the gain realized on the futures position.
2. They own a portfolio of securities part of which is unhedged, and the value of the portfolio would decline if interest rates would rise in the future. This unhedged portion of the portfolio would typically consist of fixed-income securities. They can hedge against this position by taking a short position. If interest rates rise, the value of the unhedged position would decline but this decline would be offset by the profits on the futures position.

The rate of return for hedgers going short is defined as:

$$R = [(P_T - P_0) + b(F_0 - F_T - C)]/K \qquad (4\text{-}11)$$

where P_T and P_0 is the price of the spot market security or portfolio of securities at time T and time 0, respectively, b is the proportion of the spot market transaction that is to be hedged, and the constant

K includes the initial spot market position and is considered as part of the constant since it is independently determined from F_0 or F_T. Define $K = M + P_0$ where M is the cost of the initial margin requirement. Recall that the prices are in terms of a \$1 spot market position and hence P_0 is equal to \$1. The other variables are as defined before. So

$$E(R) = [E(P_T) - P_0 + b(F_0 - E(F_T) - E(C))]/(M + P_0) \qquad (4\text{-}12)$$

and

$$\begin{aligned} Var(R) = \; & [Var(P_T)+b^2(Var(F_T)+Var(C)+2\,Cov(F_T,C))- \\ & 2b\,Cov(P_T,F_T+C)]/(M + P_0)^2 \end{aligned} \qquad (4\text{-}13)$$

The optimal proportion b to be hedged, according to the chain-rule of differentiation presented above is determined as follows.

$$\frac{d\,E(U)}{d\,E(R)} \quad \frac{[F_0 - E(F_T) - E(C)]}{(M + P_0)} \quad \frac{d\,E(U)}{d\,Var(R)} \frac{1}{(M+P_0)^2} [2b(Var(F_T)+Var(C)+2Cov(F_T,C))$$

$$-2(Cov(P_T, F_T + C))] \;=\; 0 \qquad (4\text{-}14)$$

Therefore

$$b = \frac{(M+P_0) \cdot \dfrac{-1}{2} \cdot \dfrac{d\,E(U)/d\,E(R)}{d\,E(U)/d\,Var(R)} [F_0 - E(F_T) - E(C)] + Cov(P_T,\, F_T + C)}{Var\,(F_T + C)} \qquad (4\text{-}15)$$

where

$$\frac{-1}{2} \cdot \frac{d\,E(U)/d\,E(R)}{d\,E(U)/d\,Var(R)}$$

is the hedgers risk aversion index and will henceforth be denoted as z. Since P_0 is equal to \$1 and it is added to M which is a constant, it follows then that the sum of the two, K, is also a constant and in all subsequent derivations K will be used.

Since b is the fraction of the spot market position to be hedged, the total supply of short contracts by hedgers is Q.b, where Q is the spot market position to be hedged. Therefore the dollar supply of futures contracts is

$$Q.b = \frac{Q[K.Z(F_0 - E(F_T) - E(C)) + Cov(P_T, F_T + C)]}{Var\ (F_T + C)} \qquad (4\text{-}16)$$

The supply function indicates that hedgers will supply more contracts

(i) The greater the difference between the time 0 futures price and the time T expected futures price

(ii) The smaller the expected costly guarantee

(iii) The greater the covariance between the time T spot price, and the time T futures price plus the costly guarantee. The covariance is a measure of the efficacy of hedging. Hedging is more effective if the basis risk is lower, and the greater the covariance, the smaller the basis risk. As mentioned before, the minimum variance hedge model minimizes basis risk and the position in futures contracts that minimizes basis risk is a function of the covariance between the time T futures price and spot price.

It is conceivable that for some participants, it might not be optimal to hedge in the futures market with short positions even though a portion of their portfolio is exposed to interest rate risk. For example, the covariance between the spot price, and the futures price plus the costly guarantee could be too low to warrant participation in the futures market. This covariance will differ among participants because they will be hedging different spot market securities. Or possibly, the positive magnitude of the covariance might be insufficient to negate K.Z.[$(F_0$ - $E(F_T)$ - E(C)], which could be a negative quantity. The latter could be a negative quantity if E(C) is too high or if $(F_0 - E(F_T))$ is negative because of excessive hedging activity on the short side which reduces F_0 to below $E(F_T)$. This could be the predicament for the marginal short side hedger when there is excessive hedging activity on the short side over the long side.

Hedgers will take a long position if:

1. They want to buy fixed-income securities in the future and wish to reduce the uncertainty of declining interest rates by taking a long position. If interest rates do decline then the opportunity loss associated with the purchase will be offset by the gain on the futures contract.
2. They borrow presently and wish to reduce the uncertainty of declining interest rates which will result in an opportunity loss. If interest rates do decline, the opportunity loss will be offset by the gain on the futures contract.

In the first case mentioned above, the problem can be interpreted as the hedgers attempting to minimize the outflow of funds or minimizing the price at which they would buy the security. In the second case the problem can be interpreted as the hedgers wanting to minimize the price at which they can buy back the securities they issued. The approach taken is to minimize the cost of the outflow in terms of the rate of return. The outflow is viewed as a negative quantity and so minimizing the cost is maximizing the negative quantity. Therefore the cost for hedgers going long is

$$R = -[(P_T - P_0) - b\,(F_T - F_0 - C)]/K \tag{4-17}$$

$$E(R) = [b(E(F_T) - F_0 - E(C)] + (P_0 - E(P_T))]/K \tag{4-18}$$

$$Var(R) = \frac{1}{K^2}[Var(P_T) + b^2\,(Var(F_T) + Var(C) - 2Cov\,(F_T,C)) - 2b\,Cov\,(P_T,F_T - C)] \tag{4-19}$$

Using the chain-rule of differentiation to maximize expected utility

$$\frac{d\,E(U)}{d\,E(R)}\quad [E(F_T) - F_0 - E(C)] + \frac{d\,E(U)}{d\,Var(R)}\quad [2b(Var(F_T - C) - 2\,Cov\,(P_T,F_T - C)] \tag{4-20}$$

Therefore

$$b = \frac{K.Z\,[E(F_T)\quad F_0 - E(C)] + \;Cov(P_T,F_T - C)}{Var(F_T - C)} \tag{4-21}$$

where z is the risk aversion index as defined previously. Since b is the fraction of the spot market position to be hedged, the total demand for long contracts is Q. b where Q is the spot market position to be hedged. Therefore the total demand for long positions is :

$$Q.b = \frac{Q.[K.Z.(E(F_T) - F_0 - E(C)) + Cov(P_T,F_T - C)]}{Var(F_T - C)} \tag{4-22}$$

The demand function indicates that hedgers will demand more contracts

(i) The greater the difference between the time T expected futures price and the time 0 futures price.

(ii) The smaller the expected costly guarantee

(iii) The greater the covariance between the time T spot price, and the time T futures price minus the costly guarantee. The covariance is a measure of the efficacy of hedging since increased covariance leads to lower basis risk.

It is conceivable that for some participants it might not be optimal to hedge in the futures market with long positions even though a portion of their portfolio is exposed to interest rate risk. The reasons are similar to the reasons provided for hedgers going short and will not be elaborated on.

Participation by speculators in the market is simply to satisfy the excess of hedging activity on one side over the other side. Since it is assumed that all participants have homogeneous expectations, the price of the futures contract presently will be the same as the expected price and there would be no reason for speculators to participate. They would not have the opportunity to earn a profit on the futures contract. But if there is greater hedging activity on one side over the other side, there is a momentary state of disequilibrium which induces speculators to enter the market. For example, if there is a greater demand for long contracts than supply of short contracts for hedging activity, the present futures price will be bid up because of the excess demand over supply. Since the price is greater than the expected price, speculators will enter the market because they can earn a profit by taking short positions. Speculators will continue to enter the market until the present futures price is equal to the expected futures price. The market then is in a state of equilibrium. Similarly, if there is a greater supply of short contracts than there is a demand for long contracts then there will exist an opportunity for speculators to earn profits by taking long positions.

The speculative function differs from the hedging function in the sense that speculators will participate in the market only for what can be called transitory profit making opportunities. They view their participation in the futures market independently of their portfolio of assets. If they are trying to diversify their portfolio by adding futures contracts then they will be considered as hedgers since they will be hedging some spot market position. The same individual or entity could perform both the hedging and speculative function. For example the individual could initially be a hedger by hedging the portion of the portfolio that is exposed to interest rate risk. At some later date the same individual discovers that the present futures price is different form the expected futures price, and even through he has a hedged position, he might take another position in the futures market to capitalize on profit making opportunities. He will invest part of his investable wealth in futures contracts. It is hypothesized that the remainder of the investable wealth is in the form of cash.

For speculators who go short the rate of return on their investable wealth is

$$R = b\,(F_0 - F_T - C)/K \tag{4-23}$$

where K is \$1 of the speculators investable wealth and b is the fraction of this dollar that is invested in the futures contracts.

$$E(R) = \frac{b}{K}\,[F_0 - E(F_T) - E(C)] \tag{4-24}$$

$$\mathrm{Var}(R) = \frac{b^2}{K^2}\,\mathrm{Var}(F_T + C) \tag{4-25}$$

According to the chain rule of differentiation

$$\frac{d\,E(U)}{d\,E(R)} \cdot \frac{1}{K}\,[F_0 - E(F_T) - E(C)] + \frac{d\,E(U)}{d\,\mathrm{Var}(R)}\,\frac{1}{K^2}\,2b\,\mathrm{Var}(F_T + C) = 0 \tag{4-26}$$

Therefore

$$b = \frac{K.Z.\ [F_0 - E(F_T) - E(C)]}{Var\ (F_T + C)} \qquad (4\text{-}27)$$

where Z is the speculator's risk-aversion index.

Since b is the fraction of investable wealth invested in futures contracts, the total supply of short contracts by speculators is Q.b where Q is defined as the amount of the speculator's investable wealth. Therefore, the dollar supply of futures contracts by speculators is

$$Q.b = \frac{Q.K.Z\ [F_0 - E(F_T) - E(C)]}{Var(F_T + C)} \qquad (4\text{-}28)$$

The supply function indicates that speculators will supply more futures contracts.

(i) The greater the difference between the time 0 futures price and the time T expected futures price.

(ii) The smaller the expected costly guarantee.

For speculators who go long, the rate of return is

$$R = \frac{b}{K}\ (F_T - F_0 - C) \qquad (4\text{-}29)$$

where all the variables are defined the same as in the previous case.

$$E(R) = \frac{b}{K}\ [E(F_T) - F_0 - E(C)] \qquad (4\text{-}30)$$

$$Var(R) = \frac{b^2}{K^2}\ Var(F_T - C) \qquad (4\text{-}31)$$

Again using the chain rule, expected utility is maximized when

$$\frac{d\,E(U)}{d\,E(R)}\,\frac{1}{K}\,[(E(F_T) - F_0 - E(C)] + \frac{d\,E(U)}{d\,Var(R)}\,\frac{1}{K^2}\,2b\,Var\,(F_T - C) \qquad (4\text{-}32)$$

Therefore

$$b = \frac{K.Z.[E(F_T) - F_0 - E(C)]}{Var(F_T - C)} \qquad (4\text{-}33)$$

where Z is the speculator's risk aversion index.

Since b is the fraction of the speculators investable wealth to be invested in long positions, the total demand for long contracts is Q.b. where Q is defined as the amount of investable wealth to be invested in long positions. Therefore the total dollar demand for futures contracts by speculators is

$$Q.b = \frac{Q.K.Z\,[E(F_T) - F_0 - E(C)]}{Var\,(F_T - C)} \qquad (4\text{-}34)$$

The demand function indicates that speculators will demand more futures contracts

(i) The greater the difference between the time T expected futures price and the time 0 futures price.

(ii) The smaller the expected costly guarantee.

4.5 The costly guarantee

Before the model is derived, it would be appropriate to provide the rationale for the inclusion of the costly guarantee and its specification. As mentioned before Kane introduced the concept of the costly guarantee as one of the reasons for the difference between the futures rate, and the forward rate implicit in the term structure.

The inclusion of the costly guarantee is required for this study, given the way the rate of return has been specified.

The rate of return specifies that the initial outlay or investment by the participant is a constant K which is the initial margin required plus transaction costs, and for hedgers it also includes their initial spot market positions. In the case where interest bearing securities are put up, it has been rationalized that there is some cost associated with pledging securities because such pledging (1) reduces the participants precautionary balances and (2) reduces the participant's liquidity since the securities are "tied-up" until the futures position is closed. It is important to remember that this cost is independent of the price of the futures contract. The initial margin requirement remains constant regardless of fluctuations in the price of the futures contract.

The participant has to estimate the gain to be derived from the futures position since the expected gain divided by the initial cost will determine the expected rate of return. Now, the expected gain to the participant is not simply the time T expected price minus the price at time 0. The participant between time 0 and time T is subject to margin calls if price movements are adverse to the position taken by the participant. A decline in the price is an adverse price movement for a long-side participant and an increase in the price is an adverse price movement for a short-side participant. If the price movement is adverse to the position taken, the participant is subject to a margin call, regardless of who the participant is, since the position is marked to market at the end of each day. If subject to a margin call, the participant within a specified time period has to put up the margin in the form of cash to maintain the position taken. Otherwise the broker will close out the position by taking an exactly opposite position. If the difference between the price at which the contract was initiated and the close out price is greater than the total margin put up by the participant then the participant is required to provide this deficit to the broker. If the participant does not provide the deficit, the broker is required to file suit against the participant in civil court. Conversely if price movements are advantageous to the position taken, the participant is allowed to withdraw the excess margin that has been built up in his account.

There are two options available to the participant for both adverse price movements and advantageous price movements. If price movements are adverse the participant either has to provide the additional margin or close out the position. In some cases the participant could be prevented form putting up additional margin due to a wealth constraint and the inability to borrow to make the payment. In this case the participant is forced to close out the position. Subsequent to the position being closed, price movements could be advantageous to the position taken and the time T price could be the price initially expected by the participant; however the participant does not earn the profits since the position was closed out. If price movements are advantageous to the position taken, the participant has the option of withdrawing the excess cash in the margin account or could use the cash to increase the size of the position taken. This is known as pyramiding.

Therefore, it seems logical that participants have to be mindful of price changes in the interim and so have to take these price changes into consideration to estimate the rate of return on the position taken. Suppose that participants will meet margin calls due to adverse price movements from their wealth by decreasing consumption and will use the excess margin built up in the account due to advantageous price movements. It can be argued that the decline in utility from the decreased consumption of a certain amount will be greater than the increase in utility from increased consumption of the same amount: since the participants are risk averse, utility increases at a decreasing rate. Suppose that participants will meet margin calls by borrowing money and will use excess margin to invest the money at the risk free rate. Since markets in reality are not perfect and participants can borrow at a rate higher than the risk-free rate the cost of borrowing for margin calls is greater than the rate of return earned on the excess margin. Suppose that participants do not have the ability to borrow and that their wealth is severely limited to prevent meeting margin requirements. In this case they would be forced to close out their positions. Again this imposes a cost on the participants since they will not be able to hold their position for the intended holding period. This prevents them for realizing the benefits that might accure due to advantageous price movements at some later time.

These examples indicate that the cost of margin calls exceed the benefits that can be gained from advantageous price movements. If participants perceive that there exist equal probabilities of advantageous price movements and adverse price movements of the same magnitude, that is, participants would assign equal probabilities to price increases and price decreases, then the costs of adverse price movements are greater than the benefits of advantageous price movements. Therefore there is a net cost to be borne by the participant due to the marked to market requirement. This cost is the costly guarantee. Speculators might assign higher probablities to advantageous price movements than to adverse price movements since they estimate the difference between the expected time T price and the present price to be favorable to the position taken. If this is the case then it is hypothesized that the expected cost of adverse price movements exceeds the benefit of advantageous price movements and hence there is a net cost. This might well be the case if the speculator has limited wealth and does not have the capability to meet margin requirements beyond a certain amount.

There is a need to specify the costly guarantee in a manner that is consistent with the methodology adopted to measure the rate of return, and which is empirically observable so as to permit empirical testing to determine if the guarantee has a significant impact on the equilibrium futures price. To remain consistent with a one-period world, there is a need to make some assumptions to be able to derive the specification.

In a one-period world, participants have no decision making capability in the interim. So participants, upon taking a position at time 0 will hold the position till time T regardless of price changes in the interim. Participants do not have the option to withdraw from the position in the interim. A world is visualized in which the broker will meet all the margin requirements for the participant and the participant will pay a fee to the broker for this service rendered. Likewise, if excess margin is build up in the account the broker keeps the excess margin and will pay a fee to the participant for the use of the money. The time period between time 0 and time T is divided into several sub-periods of equal length. For each sub-period that the broker has to meet margin requirements the broker charges a fee per dollar of the margin calls and for each sub-period there is excess margin in the account the broker pays a fee per dollar for the excess margin in the account during the time period. Payments are made at time T with the participant paying the broker a net amount or vice-versa. To simulate the effect of the cost of margin calls being greater than the benefits of excess margin to the participant, it is assumed that the fee paid to the broker for meeting margin calls is greater than the fee paid by the broker for excess margin in the account.

Let c_1 be the fee paid by the participant to the broker for each dollar of margin call for each sub-period, and let c_2 be the fee paid by the broker to the participant for each dollar of excess margin for each sub-period. The expected costly guarantee can be specified as

$$E(C) = C_1 \left| \sum_i (E(F_i) - F_0) \right| + C_2 \left| \sum_j (E(F_j) - F_0) \right| \qquad (4\text{-}35)$$

where the summation over i is the sum of the adverse price movements during the interim and the summation over j is the sum of the advantageous price movements. The absolute differences are measured and c_1 is positive since it is a cost and c_2 is negative since it is a benefit to the participant. This might seem to be contrary to one's intuition but it follows from the fact that the costly guarantee is subtracted from the gain.

The exchanges require that speculators have to maintain higher balances in their margin accounts than hedgers at all times.[11] This implies that speculators would be subject to higher margin calls than hedgers for adverse price movements but would have the same excess margin in the account for advantageous price movements. Therefore, the net cost to speculators will be greater than the net cost to hedgers. Within the framework of the above derivation, this is translated into a higher cost c_1 to be borne by speculators. This difference in costs will tend to have a significant impact on the equilibrium futures price as will be evident in the next section where the model is derived.

In summary, hedgers on both sides have the same estimates of the costly guarantee[12] and speculators have a higher estimate on the costly guarantee. The empirical specification of the costly guarantee is left to the chapter detailing the empirical methodology.

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[11]According to the CFTC, the hedgers are considered to be the commercial traders, who are the market makers and for whom these assets are deemed stock-in-trade. The speculators are considered to be non-commercial traders. According to the study, hedging and speculative activity are defined according to the function performed. Commercial traders could be performing the speculative function if they are not hedging interest rate risk, and probably non-commercial traders could be performing the hedging function if they are trying to reduce their exposure to interest rate risk. To facilitate the inclusion of the costly guarantee, it is assumed that the majority of hedgers according to the CFTC definition are performing the hedging function as defined by this study and the majority of speculators according to the CFTC definition are performing the speculative function as defined by this study.

[12]An issue that can be raised is that hedgers would not incur a costly guarantee because the adverse price movements that result in margin calls would be negated by an equal advantageous price movement on the spot market position. This is a fallacy because only with a perfect hedge can the loss on the futures position be offset by a gain on the spot position. Furthermore, depending upon the kind of hedging, the spot market position will be taken at the end of the

4.6 Derivation of the model

The derivation of the model requires equating total demand to total supply to determine the equilibrium futures price. The demand and supply functions which have been derived will be superscripted with L and S to denote long and short positions respectively. It bears mention again that speculators will take positions on only one side, and that side will depend upon whether there is excessive hedging activity on the long side or short side. The demand functions will include the demand function of speculators if there is excessive hedging activity on the short side, and the supply functions will include the supply functions of speculators if there is excessive hedging activity on the long side. Therefore, there is a need to derive two models, one with excess hedging activity on the long side and the other with excessive hedging activity on the short side. Since all participants have the same expected futures price $E(F_T)$, it will not be subscripted. Since all hedgers have the same estimate of the costly guarantee, it will be specified as $E^H(C)$ for hedgers, and for speculators it will be specified as $E^{SP}(C)$.

1. Excession hedging activity on the long side.

(contd.) holding period in which case no gain is realized. Finally, even if the spot market position is initiated at time 0, the gains that will occur on the spot market position will be realized at the end of the holding period while the margin calls due to adverse price movements on the futures contract have to be met during the holding period. The present value of the outflows due to the margin calls, since they occur in the interim, will be greater than the present value of the gain that is realized on the spot market position at the end of the period.

Assume that there are m hedgers on the long side, n hedgers on the short side and (n^1-n) speculators on the short-side. Setting total demand to total supply the equilibrium condition is

$$\sum_{i=1}^{m} Q_i^S \; b_i^S = \sum_{j=1}^{n} Q_j^S b_j^S + \sum_{j=n+1}^{n^1} Q_j^S b_j^S \tag{4-36}$$

The left hand side is the sum of the demand functions of all the hedgers and the right hand side is the sum of the supply function of hedgers and speculators. The variables Q and b are as they were defined when deriving the demand and supply function. They have been superscripted with L and S to denote long and short. Substituting the demand and supply functions that have been derived

$$\sum_{i=1}^{m} \frac{Q_i^L \; [K_i \; Z_i \; (E(F_T) - F_0 - E^H \; (C)) + Cov^L \; (P_T, F_T - C)]}{Var \; (F_T - C)} \tag{4-37}$$

$$= \sum_{j=1}^{n} \frac{Q_j^S \; K_j^S \; Z_j^S \; [(F_0 - E(F_T) - E^H \; (C)) + Cov_j^S \; (P_T, F_T + C)]}{Var \; (F_T + C)}$$

$$+ \sum_{j=n+1}^{n^1} \frac{Q_j^S \; K_j^S \; Z_j^S \; [F_0 - E(F_T) - E^{SP} \; (C)]}{Var(F_T + C)}$$

Therefore,

$$F_0 = \sum_{i=1}^{m} \frac{X^i}{K_i} Cov_i^L \; (P_T, F_T - C) - \sum_{j=1}^{n} \frac{Y_j}{K_j} Cov_j \; (P_T, F_T + C)$$

$$+ \sum_{j=1}^{n^1} Y_j \; E(F_T) + \left(\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i\right) E^H(C) + \sum_{j=n+1}^{n^1} Y_j \; E^{SP}(C) \tag{4-38}$$

where

$$X_i = \frac{1}{N} \frac{K_i Q_i^L Z_i^L}{Var(F_T - C)} \qquad Y_j = \frac{1}{N} \frac{K_j Q_j^S Z_j^S}{Var(F_T + C)}$$

and

$$N = \sum_{i=1}^{m} \frac{Q_i^L Z_i^L}{Var\ (F_T - C)} + \sum_{j=1}^{n} \frac{Q_j^S Z_j^S}{Var\ (F_T + C)}$$

Since $\sum_{i=1}^{m} X_i + \sum_{j=1}^{n} Y_j = 1$

the model simplifies to

$$F_0 = \sum_{i=1}^{n} \frac{X_i}{K_i} Cov_i^L (P_T, F_T - C) - \sum_{j=1}^{n} \frac{Y_j}{K_j} Cov_j^S (P_T, F_T + C)$$

$$+ E(F_T) + (\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i) E^H(C) + \sum_{j=n+1}^{n^1} Y_j E^{SP}(C) \qquad (4\text{-}39)$$

The model indicates that the equilibrium futures price is a function of

1. the expected futures price
2. The differences between the estimated covariance between spot positions to be hedged and futures prices on the long side and the short side. Since there is excess hedging activity on the long side over the short side this difference should be a positive quantity and it is the premium hedgers on the long side pay to speculators on the short side for assuming the risk.
3. The difference between the estimated costly guarantee for short side and long side hedgers, plus the estimated costly guarantee for short side speculators. The difference should be negative

since there is excessive hedging activity on the long side over the short side. But this difference should be less than the costly guarantee for speculators since they have a higher costly guarantee. Therefore the net effect of the costly guarantee on the equilibrium futures price should be positive.

2. Excessive hedging on the short side

Assume that there are m hedgers on the long side, n hedgers on the short side and (m^1-m) speculators on the short side. Setting total demand equal to total supply the equilibrium condition is

$$\sum_{i=1}^{m} Q_i^L b_i^L + \sum_{i=m+1}^{m^1} Q_i^L b_i^L = \sum_{j=1}^{n} Q_j^S b_j^S \qquad (4\text{-}40)$$

where the variables maintain the same definitions as before. Subsituting the demand and supply function in the above equations the model is derived to be

$$F_0 = \sum_{i=1}^{m} \frac{X_i}{K_i} \mathrm{Cov}_i^L(P_T, F_T - C) - \sum_{j=1}^{n} \frac{X_j}{K_j} \mathrm{Cov}_j^S(P_T, F_T + C)$$

$$+ E(F_T) + \left(\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i\right) E^H(C) - \sum_{i=m+1}^{m^1} X_i E^{SP}(C) \qquad (4\text{-}41)$$

The model indicates that the equilibrium futures price is a function of

1. The expected futures price
2. The differences between the estimated covariance between the spot positions to be hedged and the futures prices, on the long side and the short side. Since there is excess hedging activity on the short side over the long side, this difference should be a negative quantity and it is a premium hedgers on the short side pay to speculators on the long side for assuming the risk.

3. The difference between the estimated costly guarantee for short side and long side speculators, minus the estimated costly guarantee for speculators on the long side. The former difference should be a positive quantity since there is excessive hedging activity on the short side over the long side. But this difference should be less than the estimated costly guarantee for speculators on the long side and so when the latter is subtracted from the former, the net effect on the equilibrium futures price should be negative.

The model can also be derived on the assumption that the objective of hedgers is to minimize variance. If b is the proportion of the spot position to be hedged to minimize variance, the optimal b, is

$$b = \frac{\mathrm{Cov}\ (F_T,\ P_T)}{\mathrm{Var}(F_T)} \qquad (4\text{-}42)$$

Within the framework of the rate of return definition used for this study, the minimum variance proportion for hedgers going short is derived as follows:

$$\mathrm{Var}(R) = \frac{1}{K^2}\ (\mathrm{Var}(P_T) + b^2\ \mathrm{Var}(F_T + C) - 2\ b\ \mathrm{Cov}(P_T,\ F_T + C)] \qquad (4\text{-}43)$$

Setting

$$\frac{d\ \mathrm{Var}(R)}{db} = 0$$

$$2b\ \mathrm{Var}(F_T + C) - 2\ \mathrm{Cov}(P_T,\ F_T,\ + C) = 0 \qquad (4\text{-}44)$$

Therefore

$$b\ =\ \frac{\mathrm{Cov}^{\ S}(P_T,\ F_T + C)}{\mathrm{Var}\ (F_T + C)} \qquad (4\text{-}45)$$

and the dollar supply of futures contracts Q.b, where Q is the spot position to be hedged, is

$$Q.b = \frac{Q.\ Cov^{S}\ (P_T, F_T + C)}{Var\ (F_T + C)} \qquad (4\text{-}46)$$

Similarly, for hedgers going long Q.b is derived to be

$$Q.b = \frac{Q.\ Cov^{L}\ (P_T, F_T - C)}{Var\ (F_T - C)} \qquad (4\text{-}47)$$

If it is assumed that there is excessive hedging activity on the long side over the short side which means that speculators, will take short positions to satisfy the excess demand, the equilibrium futures price can be derived to be

$$F_0 = X \left[\sum_{i=1}^{m} \frac{Cov(P_T, F_T - C)}{Var(F_T - C)} - \sum_{j=1}^{n} \frac{Cov\ (P_T, F_T + C)}{Var(F_T + C)} \right] + E(F_T) + E^{SP}_{T}(C) \qquad (4\text{-}48)$$

where $\quad X = Var\ (F_T + C) / \sum_{i=n+1}^{n^1} Q_i\ K_i\ Z_i$

The model indicates that the equilibrium futures price is a function of.

1. The expected futures price
2. The difference between the estimated covariance between the spot positions to be hedged and futures prices on the long side and the short side. Since there is excessive hedging activity on the long side over the short side, this difference should be a positive quantity, and it is a premium hedgers on the long side pay to speculators on the short side for assuming the risk.
3. The expected costly guarantee of speculators on the short side,

which has a position impact on the equilibrium futures price.

If it is assumed that there is excessive hedging activity on the short side over the long side, which means that speculators will take long positions to satisfy the excess supply, the equilibrium futures price can be derived to be

$$F_0 = X \left[\sum_{i=1}^{m} \frac{\mathrm{Cov}\,(P_T, F_T - C)}{\mathrm{Var}\,(F_T - C)} - \sum_{j=n+1}^{n} \frac{\mathrm{Cov}\,(P_T, F_T + C)}{\mathrm{Var}\,(F_T + C)} \right] + E(F_T) - E^{SP}(C) \tag{4-49}$$

where

$$X = \mathrm{Var}(F_T - C) / \sum_{i=m+1}^{m^1} Q_i K_i Z_i$$

The model is similar to the one mentioned above expect that the difference between the covariance terms should be negative because of the excessive hedging activity on the short side over the long side. Also the expected costly guarantee of long side hedgers has a negative impact on the equilibrium futures price.

CHAPTER V

HYPOTHESES TO BE TESTED AND EMPIRICAL METHODOLOGY

5.1 Introduction

This chapter will begin with presenting the hypotheses that can be tested from the models that were derived in the previous chapter. Subsequently, the empirical specification of the variables in the model and the model in its empirical form will be presented. The last section of the chapter will outline the data sources and the methodology used to test the hypotheses.

5.2 Hypotheses to be tested

In the last section of Chapter IV, four models were derived based on certain assumptions. The assumptions are (1) that there is either excessive hedging activity on the long side or on the short side and (2) that hedgers maximize expected utility in a mean-variance world or that they minimize basis risk using the minimum variance hedge model. For each of these assumptions, it was inferred that the net effect of the expected costly guarantee was either a positive or negative impact on the equilibrium futures price. The assumptions of either excessive hedging activity on the long side or on the short side and the net effect of the costly guarantee can be empirically tested. It is also conceivable that hedging activity on both sides would be equal, in which case there would be no risk premium payment to speculators. Also, there would be no net effect of the expected costly guarantee on the equilibrium futures price since the impact of the costly guarantee of hedgers on the long side would be the same as that of hedgers on the short side. The four sets of assumptions and the inferences made from the derived models are

1. Excessive hedging activity on the long side and hedgers maximizing utility in a mean-variance world.

Refer to equation (4-39) in Chapter IV and the subsequent inferences. It was inferred that because of excessive hedging activity on the long side, the difference between the covariance terms would be positive; this difference is the risk premium transfer from long side hedgers to speculators. The net effect of the costly guarantee would be positive since the positive impact of the expected costly guarantee of speculators on the short side, would be greater than the negative impact of the difference between the expected costly guarantee of hedgers on the short side and long side, respectively.

2. Excessive hedging activity on the long side and hedgers minimizing basis risk using the minimum variance hedge model.

Refer to equation (4-48) in Chapter IV. It was inferred that because of excessive hedging activity on the long side, the difference between the covariance terms would be positive; also, the effect of the costly guarantee would be positive since the expected costly guarantee of hedgers has no impact on the equilibrium futures price. Only the expected costly guarantee of speculators on the short side would have an impact on the equilibrium futures price, and this impact would be positive.

3. Excessive hedging activity on the short side and hedgers maximizing utility in a mean-variance world. Refer to equation (4-41)

in Chapter IV. It was inferred that because of excessive hedging activity on the short side, the difference between the covariance terms would be negative; this difference is the risk premium transfer from short side hedgers to speculators. The net effect of the costly guarantee would be negative since the negative impact of the expected costly guarantee on the long side would be greater than the positive impact of the difference between the expected costly guarantee of hedgers on the short side and long side, respectively.

4. Excessive hedging activity on the short side and hedgers minimizing basis risk using the minimum variance hedge model.

Refer to equation (4-49) in Chapter IV. It was inferred that because of excessive hedging activity on the short side, the difference between the covariance terms would be negative and the effect of the costly guarantee would be negative since the expected costly guarantee of hedgers has no impact on the equilibrium futures price. Only the expected costly guarantee of speculators on the long side would have an impact on the equilibrium futures price, and this impact would be negative.

In summary, it can be stated that if there is excessive hedging activity on the long side, both the risk premium and the costly guarantee will have a positive impact on the equilibrium futures price. If there is excessive hedging activity on the short side, both the risk premium and the costly guarantee will have a negative impact on the equilibrium futures price. The results of empirical testing should determine whether the risk premium and costly guarantee both have a positive or negative impact, thereby indicating excessive hedging activity on the long side and short side respectively. If both the risk premium and the costly guarantee would be insignificantly different from zero, equal hedging activity on both sides would be indicated.

There are two hypotheses that can be tested within the framework of the models derived. They are

1. (i) Normal backwardation holds true

   (ii) Normal contango holds true

   (iii) The expectations hypothesis holds true

2. The costly guarantee has a significant impact, if there is excessive hedging activity, in a manner consistent with the framework of the model derived. If there is no excessive hedging activity on either side, then the costly guarantee will have an insignificant impact on the equilibrium futures price.

Normal backwardation assumes that there is excessive hedging activity on the short side. Within the framework of the models derived, speculators will be induced to take short positions since the excessive hedging activity on the short side will lower the equilibrium futures price below the expected futures price. As the futures contract approaches maturity, it is assumed that there is less hedging activity on both sides and the excessive hedging activity on the short side is progressively reduced. Therefore, as the maturity date approaches the equilibrium futures price tends towards the expected price and the risk premium paid to speculators declines. Speculators are willing to accept the lower risk premium due to the reduced uncertainty associated with the approaching maturity date and also the progressively declining holding period. Within the framework of the models derived, the costly guarantee will have a negative impact on the equilibrium futures price. The impact of the costly guarantee should decline as the contract approaches maturity since the costly guarantee would decline as the holding period declines.

Normal contango assumes that there is excessive hedging activity on the long side. Within the framework of the model derived, speculators will be induced to take short positions since the execessive hedging activity on the short side will increase the equilibrium futures price above the expected futures price. As the futures contract approaches maturity it is assumed that there is less hedging activity on both sides and the excessive hedging activity on the long side is progressively reduced. Therefore, as the maturity date approaches, the equilibrium futures price tends toward the expected price and the risk premium paid to speculators declines. Speculators are willing to accept the lower risk premium due to the reduced uncertainty associated with the approaching maturity date and also the progressively declining holding period. Within the framework of the models derived, the costly guarantee will have a positive impact on the equilibrium futures price. The impact should decline as the contract approaches maturity since the costly guarantee would decline as the holding period declines.

The expectations hypothesis assumes that there is equal hedging activity on both sides. Therefore the equilibrium futures price is equal to the expected futures price, and there is no incentive for speculators to enter the market because there is no risk premium being paid by hedgers to induce their entry. Also, the costly guarantee will have an insignificant impact on the equilibrium futures price since the expected costly guarantees of hedgers will be equal and will negate each other. Therefore, with the expectations hypothesis, the equilibrium futures price is equal to the expected futures price.

5.3 Empirical specification of the variables

The model is to be tested with the use of multiple regression analysis. The first step is to provide empirical specifications for the variables in the model. The model will be tested in the following form:

$$F_T = B_0 + B_1\, E(F_T) + B_2\, E(C) + e \qquad (5\text{-}1)$$

The endogenous variable on the left-hand side, F_t, is the present price of the futures contract. The first term on the right-hand side, B_o, is the constant in the regression model: it measures the risk premium transfer from hedgers to speculators and it represents the difference between the covariance terms in the theoretical model. It is assumed that this risk premium is a constant, over time, for all futures contract with T periods to maturity. This risk premium will be zero if the expectations hypothesis holds true, negative if the normal backwardation hypothesis holds true, and positive if the normal contango theory holds ture. And for the latter two hypotheses, the risk premiums tend toward zero as the futures price approaches maturity.

B_1 is the coefficient of $E(F_T)$ and according to the theoretical model it should equal to 1. Two models are used to estimate $E(F_T)$ and empirical tests were performed with each of these. The two methods are

(1) Using an adaptive expectations model

(2) Using an instrumental variable approach.

The adaptive expectations model postulated by Nerlove [1958] assumes that expectations are related to past expectations and the realization of these expectations. The participants consistently modify their past expectations in light of their current experience. All future forecasts are adjusted in proportion to the forecast error just discovered. The following relationship is described by the model.

$$E_t\ (F_T) = (1 - d)\ [F_{t-1} - E_{t-1}\ (F_T)] + E_{t-1}\ (F_T) \qquad (5\text{-}2)$$

where $E_t(F_T)$ is the present time, t, expectation of the futures price at time T. The parameter (1-d) is the coefficient of expectations and it is interpreted as the permanant portion of the current error in prediction which is to be added to last period's expectations to develop this period's expectation. Accordingly, d measures the transitory portion of the difference. Solving the above difference equation recursively gives

$$E_t\ (F_T) = (1 - d) \sum_{i=0}^{\infty} d^i\ F_{t-1-i} \qquad (5\text{-}3)$$

which is a distributed lag model with the lag series being infinite. Since a futures contract has a finite life, the historical data avaliable on a futures contract is finite. A technique has been develope by Klein [1958] to estimate an infinite distributed lag with finite past data. Sargent [1968] and Maddala and Rao [1971] have discussed this technique. Equation (5-2) is reduced to the following form with this technique.

$$E_t\ (F_T) = (1-d)\ [\sum_{i=1}^{t-1} d^i\ F_{t-i}] + (1 - d) \sum_{i=0}^{\infty} (d^i\ F_{0-i})\ [d^t] \qquad (5\text{-}4)$$

The terms in parentheses are the parameters and the terms in brackets are the variables. Since equation (5-3) is non-linear, Klein developed a search technique which provides a maximum likelihood estimate of the coefficient the expected price in the regression model. The objective of the search procedure is to determine the optimal parameter d, which will minimize the sum of the squares of the residuals of the regression model represented in equation (5-1). It is assumed that the error term, e, are independent and identically distributed $N(0,\sigma^2)$. The procedure is to perform regression with values of d at intervals of .1 from .1 to .9. The value of d which produces the smallest sum of Squared residuals, referred to as d_{min}, is selected and then the region (d_{min} - .08) to (d_{min} + .08) will be searched at intervals of .02 to produce the optimal estimate of d.

The problems associated with this technique are that the standard errors of the coefficients of the variables cannot be estimated in the same way as in the case of a linear estimation. Sargent [1969] explains the procedure for the non-linear model and further adds that there is no guarantee that the variances estimated will be non-negative. Furthermore, the estimate of the standard errors are very unstable in the region of the optimal d. and are therefore of questionable value. Because of the shortcommings, Sargent used the standard errors of the coefficients of the variables for the regression which minimized the sum of squared residuals. For the same reason this study will also use the standard errors of the coefficients for the regression that minimizes the sum of squared residuals for determining the estimate of the t-statistic.

Because of the problems associated with non-linearity for the adaptive expectations model, it was decided to consider an instrumental variable approach as an alternative. The instrumental variable approach requires identification of a variable which is highly correlated with the independent variable, $E(F_T)$ and uncorrelated with the error term, e. It is important to remember that it is assumed that the instrument variable is highly correlated with the actual variable since it is impossible to measure that correlation. To some extent, the choice among different instrumental variables is arbitrary, and the choice will affect the results of the estimation process. The estimation technique guarantees consistent estimation[13] but it does not guarantee unbiased estimation. The approach adopted is a two step procedure similar to one discussed by Sargent [1968]. The first step is to perform the following regression:

$$F_t = a_0 + a_1F_{t-1} + a_2F_{t-2} + a_3\, F_{t-3} + A_4\, F_{t-4} + A_5\, F_{t-5} + e \qquad (5\text{-}5)$$

F_t is the equilibrium futures price and according to the instrumental variable approach adopted by this study, it is regressed on prices prevalent on five past dates. This procedure assumes that the expected price $E(F_T)$ is a linear extrapolation of past prices. The regression model provides the estimates, a_0, a_1, a_2, a_3, a_4, and a_5. The second step is to use the estimates a_1, a_2, a_3, a_4, and a_5 with the same data, the data that generated the regression equation, to come up with an estimate of $E(F_T)$. That is, the data that generated the model

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[13] If the probability that the estimator differs from the parameter by some small arbitrary amount tends to zero as the sample size tends to infinity, then the estimator satisfies the property of consistency. This implies that as the sample size increases, the estimators derived are more accurate.

is substituted into the derived model to provide the estimates. The estimates therefore are a linear extrapolation of past prices. The difference between the dependent variable F_t, and the estimated $E(F_T)$ is the residual, e.

Another method for estimating $E(F_T)$ is to use the ex-post realized price at time T. This approach is commonly followed in empirical studies. This assumes that markets are efficient and so expectations of participants on average, are realized over time. This technique was attempted but the results were inconsistent with the theory of the model. The literature does provide evidence for this inconsistency: Stein [1981] attempted to test the rational expectations hypothesis against what he calls the asymptotically rational expectations hypothesis. He used commodity futures data to test the hypotheses on a futures pricing model. Stein states the model to be the following

> "The market price of a futures contract is equal to the price that is anticipated to prevail at maturity less a positive or negative risk premium (depending on the balance of hedging pressure and risk aversion) plus a random term," (1981, p.140).

It should be noted that the model derived in this study is similar to Stein's except for the costly guarantee term which is part of the derivation of the model for this study. According to Stein, the rational expecations hypothesis states that a regression of the price of a maturing futures contract upon the futures price any number of months earlier should yield a slope which is not significantly different from unity and is significantly different from zero. The intercept term should be less significant since it reflects the average risk premium over the sample period. Tests were performed on wheat, corn and soy bean futures contracts and the results indicated that

(1) There were no significant differences among the means of prices on the maturing futures contract and the means of the same futures contracts at various distances from maturity. This result indicates that the mean futures at various distances from maturity were unbiased estimators of the subsequently maturing futures price.

(2) Regressions of the maturing futures on the futures prices successively further back in time before maturity yielded slopes (coefficient of the exogenous variable) that generally decreased monotonically as the distance to maturity increased. For contracts a month distant to maturity the slopes were often not significantly different from unity. On contracts longer than two months from maturity the slopes were either significantly less than unity or not significantly different from zero.

(3) The correlation between the maturing futures price and the futures price at successively furthur distances to maturity progressively declined.

(4) Correlations between the forecast error and the trend in the spot prices were positive.

The results led Stein to the conclusion that expectations were more in conformity with adaptive expectations rather than with rational expectations. The latter is synonymous with what he calls asymptotically rational expectations since the coefficient of the exogenous variable asymptotically tends to 1. According to Stein, the logic behind this hypothesis is similar to Holbrook Working's [1958] explanation of why a new piece of information generates a gradual price change rather than an instantaneous price change.

Refer to equation (5-1). The third variable on the right hand side is the costly guarantee, E(C). B_2 is the coefficient of the net effect of the costly guarantee. The costly guarantee, in the previous chapter, was specified as

$$E(C) = C_1 \left| \Sigma_i (E(F_i) - F_t) \right| + C_2 \left| \Sigma_j (E(F_j) - F_t) \right| \qquad (5\text{-}6)$$

where the summation over i is the sum of the adverse price movements and the summation over j is the sum of advantageous price movements. The advantageous price movements for long side participants will be the adverse price movements for short side participants and vice-versa. To make the specification empirically tractable, it is assumed that for all participants the expectations of adverse price movements are the same as the expectations of advantageous price movements over the holding period. Participants estimate the price differential from the time t price for each sub-period and would estimate one half of the differential to be advantageous price movements and the other half of the differential to be adverse price movements. The above specification of the costly guarantee is reduced to

$$E(C) = \frac{\left(\sum_{i=t+1}^{T-1} |E(F_i) - F_t|\right)}{2} . C_1 + \frac{\left(\sum_{i=t+1}^{T-1} |E(F_i) - F_t|\right)}{2} . C_2 \qquad (5\text{-}7)$$

$$= \frac{(C_1 + C_2)}{2} \left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right] \qquad (5\text{-}8)$$

where C_1 0, C_2 0 and (T - 1) is the number of sub-periods in the holding period. Withen the framework of this model, the cost of adverse price movements for speculators is interpreted to be greater than for hedgers. The cost for hedgers is specified as C_1^H and for speculators C_1^{SP}. Therefore the above specification for hedgers is

$$E^H(C) = (C_1^H + C_2) \left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right] \quad (5\text{-}9)$$

and for speculators it is

$$E^{SP}(C) = (C_1^{SP} + C_2) \left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right] \quad (5\text{-}10)$$

In the model (4-39) derived in Chapter 4, the terms including costly guarantee estimates are

$$\left(\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i \right) E^H(C) + \sum_{j=n+1}^{n^1} Y_j \, E^{SP}(C) \quad (5\text{-}11)$$

Substituting for $E^H(C)$ and $E^{SP}(C)$, the expression reduces to

$$\left(\left(\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i \right) (C_1^H + C_2) + \sum_{j=n+1}^{n^1} Y_j \, (C_1^{SP} + C_2) \right)$$

$$\times \left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right] \quad (5\text{-}12)$$

Similarly, for the model (4-48) derived in Chapter 4, the terms including the costly guarantee can be simplified to

$$\left(\left(\sum_{j=1}^{n} Y_j - \sum_{i=1}^{m} X_i \right) (C_1^H + C_2) + \sum_{i=m+1}^{m^1} X_i \, (C_1^{SP} + C_2) \right) \times$$

$$\left[\sum_{i=t+1}^{T-1} E(F_i) - F_t \right] \quad (5\text{-}13)$$

In the models (4-41) and (4-49) derived in Chapter 4 the costly guarantee is simply that of the speculators.

$$E^{SP}(C) = (C_1^{SP} + C_2) \left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right] \tag{5-14}$$

In each of the above expressions

$$\left[\sum_{i=t+1}^{T-1} |E(F_i) - F_t| \right]$$

is considered to be the variable and the remainder of the expressions are considered to be the parameters. These parameters therefore are assumed to be constants. Even though C_1 and C_2 could possibly be tied to prevailing interest rates, it can be assumed that for both hedgers and speculators, the sums of the two $(C_1^H + C_2)$ and $(C_1^{SP} + C_2)$, respectively can be assumed to be constant over time. The sums in each case could be interpreted as the difference between the borrowing and lending rates in imperfect markets which, over time, can be assumed to have remained constant.

The estimate of the variable $\left[\sum_{i=t+1}^{T-1} | E(F_i) - F_t | \right]$ will be performed using ex-post data for $E(F_i)$. Now there might seem to be an inconsistency if the estimate of $E(F_T)$ is performed using an adaptive expectations model which assumes that prices adjust gradually to new information rather than instantaneously. The use of ex-post data in this case can be justified since one is estimating differences between the ex-post price and the time 0 price. The differences with ex-post prices will be the same as with prices generated from an adaptive expectations model because the excessive estimated price movement in one direction for a particular sub-period will be negated by excessive price movements in the other direction during some other sub-period.

In addition to the estimate for the costly guarantee, the ex-post standard deviation will also be estimated and each of these will be used separately to test the model. The standard deviation (s.d.) is estimated as follows.

$$\text{s.d.} = \sqrt{\sum_{i=t+1}^{T-1} \frac{(F_i - \bar{F})^2}{T - 1}} \qquad (5\text{-}15)$$

where

$$\bar{F} = \sum_{i=t+1}^{T-1} \frac{F_i}{T-t-1}$$

The purpose of using the standard deviation also to test the model is to determine if the absolute differential estimate of the costly guarantee is a proxy for risk as estimated by the standard deviation.

5.4 Empirical tests of the hypotheses

In the previous section, two estimates of the expected price and two estimates of the costly guarantee were derived. This results in four different versions of the empirical model. They are as follows

(1) Using the adaptive expectations model for estimating the expected price and the absolute price differentials for estimating the costly guarantee.

$$F_t = B_0 + B_1 \left[\sum_{i=1}^{t-1} d^i F_{t-i}\right] + B_1' \left[d^t\right] + B_2 \left[\sum_{i=t+1}^{T-1} |F_i - F_0|\right] + e \qquad (5\text{-}16)$$

The search procedure will be used to determine the optimal level of the parameter d. The coefficient of the variable, B_1, should equal to (1-d).

(2) Using the adaptive expectations model for estimating the expected price and the standard deviation for estimating the costly guarantee

$$F_t = B_0 + B_1 \left[\sum_{i=1}^{t-1} d^i F_{t-i} \right] + B_1' \; [d^t] + B_2 \sqrt{\sum_{i=t+1}^{T-1} \frac{(F - F)^2}{T - t - 1}} + e \qquad (5\text{-}17)$$

The search procedure will be used to determine the optimal level of the parameter d. The coefficient of the variable, B_1, should equal to (1-d)

(3) Using the instrumental variable approach for estimating the expected price and the absolute price differential for estimating the costly guarantee

$$F_t = B_0 + B_1 \left[\sum_{i=1}^{5} a_i F_{t-i} \right] + B_2 \left[\sum_{i=t+1}^{T-1} |F_i - F_0| \right] + e \qquad (5\text{-}18)$$

where the parameters, a_i, are predetermined and the coefficient, B_1, should equal to 1.

(4) Using the instrumental variable approach for estimating the expected price and the standard deviation for estimating the costly guarantee.

$$F_t = B_0 + B_1 \left[\sum_{i=1}^{5} a_i F_{t-i} \right] + B_2 \sqrt{\sum_{i=t+1}^{T-1} \frac{(F_i - F)^2}{T - t - 1}} + e \qquad (5\text{-}19)$$

where the parameters, a_i, are predetermined and the coefficient, B_1, should equal to 1.

As mentioned before, there are two hypotheses to be tested. The first one tests for normal backwardation or normal contango or the expectations hypothesis. This hypothesis will be tested by observing the constant term and also using its t-statistic to test whether it is significantly different from zero. For normal backwardation to hold true, the constant should be significantly negative and should tend towards zero for regression runs that use data successively closer to maturity. The magnitude of the t-statistic should also decline and should be insignificant for regression runs close to maturity. For normal contango to hold true, the constant should be significantly positive and should tend towards zero for regression runs that use data successively closer to maturity. The t-statistic should also decline and should be insignificant for regression runs close to maturity. For the expectations hypothesis to hold true, the constant should be insignificantly different from zero for all regression runs regardless of the time to maturity.

The results of the tests to be performed on the costly guarantee should be consistent with the results of the tests on the constant. If normal backwardation holds true, then the coefficient of the costly guarantee, B_2, should also be significantly negative and should tend towards zero for regression runs that use data successively closer to maturity. Similarly the t-statistics of the coefficient should also decline in magnitude and should be insignificant for the runs close to maturity. For normal contango to hold true, the coefficient should be significantly positive and both the coefficient, and its t-statistic should tend towards zero for regression runs that use data successively closer to maturity. For the expectations hypothesis to hold true, the coefficient of the costly guarantee should be insignificantly different from zero for all regression runs regardless of the time to maturity.

Since lagged dependent variables serve as independent variables in this model, the Durbin-Watson statistic cannot be used to test for serial correlation in the residuals. Durbin [1970] has derived the statistic, h, that is to be used in such cases.

$$h = r\sqrt{\frac{n}{1-n\,\mathrm{Var}(B_1)}} \qquad \text{(5-20}$$

where

r = the serial correlation of the residuals

n = the sample size

$\mathrm{Var}(B_1)$ = the variance of the coefficient B_1. It is the coefficient of the variable that includes the lagged variables.

The statistic h is tested as a standard normal deviate. The hypothesis that there is no serial correlation would be rejected at the 10 percent level if h is greater than 1.645 in absolute value. If there is significant serial correlation then the Cochrane-Orcutt procedure is used to remove the serial correlation

5.5 Data sources and modification

The GNMA and the T-bond futures data were provided in machine readable form by the Chicago Board of Trade. The data for the GNMA futures contracts spans a period from October 1975, the inception date for GNMA futures, to December 1979. The data for the T-bond futures contracts spans a period from August 1977, the inception date for T-bond futures, to December 1979. The data did not provide the settlement prices, which reflect the average trading price over the course of the day determined by the clearing corporation to set the margin calls for the day. The closing price was used for performing the regressions.

The T-bill data was provided by the Center for the Study of Futures Markets at Columbia University. This data spans a period from January 1976, the inception date for T-bill futures, to June 1980. The International Monetary Market, which is the exchange on which T-bills have been traded, does not quote the T-bill futures price but instead quotes an index which is the difference between 100 and the actual T-bill yield. The index data was initially converted into prices by using the formulation

$$\text{Price} = 10{,}000 - \quad \frac{90}{360} \times \text{T-bill yield} \times 10{,}000) \tag{5-21}$$

Settlement prices were used to perform the regressions.

CHAPTER VI

RESULTS AND INTERPRETATION OF EMPIRICAL TESTS

6.1 Introduction

The purpose of this chapter is to present the results of the regressions performed on each of the four empirical specifications of the models. The results are presented in the form of tables and are incorporated into the body of the chapter. The tables are presented at the end of the chapter. The latter part of the chapter is devoted to interpreting the empiricial results to determine if they are consistent with the theory of the model and to determine which of the hypotheses that are tested hold true. The implications of the results derived will be discussed in the concluding chapter. The chapter begins with the methodology used to set up the data for performing the regressions.

6.2 Data creation for regressions.

The data was read from the source computer tapes and appropriately transformed to perform the regression runs. Data sets were created for each of the regressions to be performed. For the T-bill and GNMA futures contracts the regressions spanned a period from 4 weeks to maturity to 76 weeks to maturity at four weeks intervals. For T-bond futures contracts the regressions spanned a period from 4 weeks to maturity to 52 weeks to maturity at four weeks intervals. Regressions beyond 52 weeks to maturity for T-bond futures were not performed due to the shorter time horizon over which data for T-bonds was available and the fact that when the contracts were first traded, only shorter maturity contracts were provided. These factors limited the sample size, and regressions with sample sizes of less than sixty observations proved too small to give meaningful results.

The number of weeks to maturity for the regression is interpreted as the holding period of participants. For example, a regression of four weeks to maturity would imply that a participant could concievably have purchased the futures contract fifty-two weeks from maturity with the intention of closing the position forty-eight weeks to maturity to give a holding period of four weeks. In effect, the maturity date for the participant is the intended closing date of the position. The reason for this interpretation is to provide large sample sizes for performing the regressions. Since the estimators are consistent they converge to the value of the parameter as the sample size increases. With this interpretation, for a regression run with, say, four weeks to maturity, all the past data can be used for a futures contract up to four weeks to maturity on the contract under the assumption that the participant intends to hold the contract for four weeks.

The data for the runs were set up as a pure time series since there are econometric problems associated with pooling time series and cross-sectional data[14]. The easiest way to explain this is with an example. Take the example of the regression run for GNMA futures data with four weeks to maturity. The GNMA futures data covers a time span from October 1975 to December 1979. The futures contracts mature at three-month intervals. The maturity dates are the third week of the

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[14]See Pindyk and Rubenfeld [1976] for an explanation of the problems.

months of March, June, September, and December. The regression will use data spanning the entire time period from 1975 to 1979. But for a particular time period in the interim it will use the prices for only one of the contracts and will not use the prices for the other contracts that are being traded during that time period.

The first set of data points for the four weeks to maturity regression will use prices from the inception date of the market, October 20, 1975 to up to four weeks to maturity for the December, 1975 contract. The second set of data points will use data for the contracts maturing March, 1976. The first data point from this set will use data from four weeks before the maturity of the December, 1975 contract up to four before the maturity of the March, 1976 contracts. Notice that data prior to four weeks before the maturity of the December, 1975 contract is not used for this data set because then it will include data that is concurrent with the first set of data points which used data for the December, 1975 contracts up to four weeks from maturity for the December, 1975 contract. The third set of data points will use data for the contract maturing June 1976. This set of data points will use data from four weeks before the maturity of the March, 1976 contract up to four weeks before the maturity of the June, 1976 contract. In this manner, data will be used for all the futures contracts ending with data for the contract maturing in December, 1979. This last data set will use data from four weeks before the maturity of the September, 1979 contract up to four weeks before the maturity of the December, 1979 contract.

Weekly data was used to generate the data for the regressions. That is, the original data provided was daily prices, and data points at intervals of five were selected. The expected price generated by the distributed lag method utilized all past data for the particular futures contract. Suppose the dependent variable is the equilibrium futures price for the March, 1976 contract on January 10, 1976. The expected price will be generated using prices from the beginning of trading for the March 1976 contract, at weekly intervals up to January 3, 1976. The expected price generated by the instrumental variable approach utilizes five data points prior to the data on which the expectation is generated. Since weekly data is used, these five points span a five week time period. The costly guarantee was generated by using ex-post data for both the absolute price differential method and the standard deviation method. That is, for the regression run with four weeks to maturity, four ex-post data points are used to estimate the costly guarantee.

The regressions were performed by using the Time Series Package[15] The versatility of this package facilitated the grid search procedure that was used with the distributed lag approach to minimize the sum of squared residuals. The package also includes the Cochrane-Orcutt procedure to correct for autocorrelation; this procedure was used if significant autocorrelation was present.

6.3 Presentation and interpretation of empirical results

The results of the regression provide the following.

(1) The estimated coefficients of the variables, B_0, B_1, B_1^1, and B_2.

(2) The t-statistics of the coefficients of the variables which are

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[15]Time Series Package is an econometric software package developed at Stanford University.

presented in parentheses below the estimated coefficients.

(3) The R-squared values of the regressions

(4) In the case of the distributed lag approach to estimate the expected price, (1-d), which should equal the estimated coefficient, B_1.

(5) The 95 percent confidence interval for the coefficient B_1 for both the distributed lag approach and the instrumental variable approach.

(6) The statistic, h, for testing for serially correlated errors.

The t-statistics for the constant and the coefficients that are less than -2. or greater than +2.0 are considered to be significantly different form zero at the 95 percent confidence level. If the statistic h is greater than 1.645 then it will be concluded that autocorrelation is present at the 95 percent confidence level and if it is greater than 2.33 then it will be concluded that auto correlation is present at the 99 percent confidence level. Since the theoretical value of the coefficient B_1 is known a test to evaluate whether the data fits the model is performed. The test is to determine if the 95 percent confidence interval of the estimated coefficient includes the theoretical value. In the case of the distributed lag approach, the theoretical value of B_1 is equal to (1-d); and in the case of the instrumental variable approach it is equal to one. The R-squared values of the regressions, although stated, really do not predict the "goodness of fit" since regressions performed on time series data in most cases have high values of R-square.

The results are presented separately for each of the three types of futures contract and for each of the four empirical specifications of the model.

(1) GNMA futures contracts

(i) The expected price is generated using the distributed lag approach, and the costly guarantee is generated using the absolute price differential approach. Table 6.1 presents the results . They indicate that:

(a) The constant term, B_0, is significant for the regression runs closer to maturity. For weeks 4, 8 and 12 to maturity, B_0 is significantly negative and declining in magnitude, and for the runs 68 weeks to maturity it is significantly positive. For all the other runs, the constant term is not insignificantly different from zero.

(b) The coefficient of the costly guarantee, B_2, is significantly greater than zero for the regressions, 32, 40, 56, 60 and 76 weeks to maturity and is not significantly different form zero for all of the other regressions. There is a lack of consistency between B_0 and B_2 in that the runs for which they are significant do not coincide[16].

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[16]When developing the hypotheses to be tested, it was stated that the costly guarantee would have a significant effect on the equilibrium price only if there is excess hedging activity on one side which leads to speculative activity on the other side. Since the estimated costly guarantee of speculators is greater than that of hedgers, the costly guarantee will have a significant impact on the equilibrium futures price. Furthermore, if there is excessive hedging activity on one side, ther will be a risk premium transfer which would be manifested by a significant B_0. Therefore, to maintain consistency with

(c) The coefficient of the expected price, B_1, is significantly greater than zero for all runs and its 95 percent confidence interval includes the theoretical value, (1-d), in 12 out of 19 cases and almost includes it for most of the remaining cases[17].

(d) For all the regressions, the statistic, h, indicate that there is no significant serial correlation at the 90 percent confidence level.

The results indicate that neither normal backwardation nor normal contango hold true. Since the constant, B_0, is not significantly different from zero in most cases, it can be said that the expectations theory holds true. Consistent with the expectations theory is the fact that the costly guarantee is insignificantly

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the model and the hypotheses developed, both the constant, B_0, and the coefficient of the costly guarantee, B_2, should either be insignificant or significant in a regression. It bears mention that theis point that the classification of participants as hedgers are speculators according to this study might be different from the classification defined by the CFTC. If this is the case, then the above arguement does not hold true. This issue will be discussed in detail in the last section of the chapter.

[17]The coefficient of the expected price is B_1 and in this formulation, B_1 must equal (1-d) [refer to equation (5-4) and (5-16]. However, the true value of $E(F_T)$ is not known, and an estimate [See equations (5-2) to (5-4)] is being used in its place. Thus the theoretical value of B_1 may differ from 1-d given that a proxy is being used for $E(F_T)$. Placing a confidence interval on B_1 to see if the interval includes the known value of (1-d) (recall that d is specified for each least squares estimation for the model) amounts to a test of the appropriateness of the model when a proxy is used for $E(F_T)$. Technically, the (1-d) term should have been included in the $E(F_T)$ term because a value is specified for d for each regression run; B_1 would then be an unknown parameter with a theoretical value of 1, and the interval estimate would be expected to include 1. The procedure adopted here produces identical results in terms of the interval including or not including the theoretical value, and it has the advantage of making the various components of the model easier to indentify.

different from zero in most cases.

(ii) The expected price is generated using the distributed lag approach and the costly guarantee is generated using the standard deviation approach. Table 6.2 presents the results. They indicate that:

(a) The constant term, B_0, is significant for the regressions closer to maturity. For weeks 4 and 12 to maturity, B_0 is significantly negative, and for the regression 76 weeks to maturity it is significantly greater than zero. For all the other runs, B_0 is not significantly different from zero.

(b) The coefficient of the costly guarantee, B_2, is significantly negative for the regression 16 weeks to maturity, significantly positive for the run 48 weeks to maturity and is not significantly different form zero for all the other runs. There is a lack of consistency between B_0 and B_2 in that the regressions for which the are significant do not coincide[18].

(c) The coefficient of the expected price, B_1, is significantly greater than zero for all the regressions and its 95 percent confidence interval includes the theoretical value in 10 of the 19 runs. For most of the remaining regressions (1-d) is quite close to the estimated confidence interval

(d) For all of the regressions, the statistic, h, indicate

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[18]See footnote 16 for an explanation.

that there is no significant serial correlation at the 90 percent confidence level.

The results indicate that neither normal backwardation nor normal contango hold true. Since the constant B_0 is not significantly different from zero in most cases, it can be said that the expectations theory holds true. Consistent with the theory is the observation that the costly guarantee is not significantly different from zero in most cases.

(iii) The expected price is generated using the instrumental variable approach, and the costly guarantee is generated using the absolute price differential approach. Table 6.3 presents the results. An examination of the tables indicates that

(a) The constant term, B_0, is not significantly different form zero for all regression runs.

(b) The coefficient of the costly guarantee is significantly negative for the 12 weeks to maturity regression and insignificantly different from zero for all the other regression.

(c) The coefficient of the expected price, B_1, is significantly greater than zero for all the regressions and its 95 percent confidence interval includes the theoretical value, 1, in 17 out of 19 cases.

(d) For all the regressions, the satistic, h, indicates that there is no significant serial correlation at the 90 percent confidence level for all of the regressions.

The results indicate that neither normal backwardation nor normal contango hold true. The weight of the evidence indicates that the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has no significant impact on the equilibrium futures price.

(iv) The expected price is generated using the instrumental variable approach and the costly guarantee is generated using the standard deviation approach. Table 6.4 presents the results, and an examination of the table reveals that:

(a) The constant term, B_0, is not significantly different from zero for all the regressions.

(b) The coefficient of the costly guarantee is significantly negative for the 16 and 76 weeks to maturity runs and is not significantly different from zero for all the other regressions.

(c) The coefficient of the expected price, B_1, is significant for all the regressions and its 95 percent confidence interval includes the theoretical value, 1, in 18 out of 19 cases.

(d) For all the regressions, the statistic, h, indicates that there is no significant serial correlation at the 90 percent confidence level.

The results indicate that neither normal backwardation nor normal contango holds true. The weight of the evidence indicates that the expectations hypothesis holds true, and consistent with this hypothesis is the observation that the costly guarantee has no significant impact on the equilibrium futures price.

(2) T-Bill futures contracts.

(i) The expected price is generated using the distributed lag approach and the costly guarantee is generated using the absolute price differential approach. Table 6.5 presents the results, and an examination of the table reveals that

(a) The constant term, B_0, is significantly positive for the 4 and 48 weeks to maturity runs, significantly negative for the 12, and 64 weeks to maturity runs, and is not significant for the remainder of the runs.

(b) The coefficient of the costly guarantee, B_2, is significantly positive for the regression runs, 16, 20, 24, 40, and 76 weeks to maturity, significantly negative for the regression with 32 weeks to maturity and is not significant for the remainder of the regressions. There is a lack of consistency between B_0 and B_2 in that the runs for which they are significant do not coincide[19].

(c) The coefficient of the expected price, B_1, is significantly greater than zero for all regressions and its 95 percent confidence interval includes the theoretical value (1-d) in 6 of the 19 cases. In some of the remaining regressions, (1-d) is quite close to the estimated interval.

(d) For 15 of the 19 regressions, the statistic, h, indicates that there is no significant serial correlation

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[19]See footnote 16 for an explanation.

at the 90 percent confidence level. For all the regressions, the statistic, h, indicates that there is no serial correlation at the 95 percent confidence level.

The results indicate that since the constant, B_0, is insignificant in most cases, the expectations hypothesis holds true, and consistent with this hypothesis is the fact that the costly guarantee has an insignificant impact on the equilibrium futures price in most cases.

(ii) The expected price is generated using the distributed lag approach and the costly guarantee is generated using the standard deviation approach. Table 6.6 presents the results. They indicate that

(a) The constant term, B_0, is significantly positive for the 8 and 48 weeks to maturity regressions significantly negative for the 64 weeks to maturity regressions, and is not significant for the remainder of the runs.

(b) The coefficient of the costly guarantee, B_2, is significantly negative for the 8, 12, 16, and 32 weeks to maturity regressions and is not significantly different from zero for the remainder. Ther is a lack of consistency between B_0 and B_2 in that the regressions for which they are significant coincide in only one case[20]. For the case in which they do coincide, 8 weeks to

[20] See footnote 16 for an explanation.

maturity they have oppostite impacts, positive and negative, respectively on the equilibrium futures price

(c) The coefficient of the expected price, B_1, is significant for all regressions, and its 95 percent confidence interval includes the theoretical value, (1-d) in 11 of the 19 cases. In some of the remaining regressions (1-d) is quite close to the estimated interval.

(d) For 14 of the 19 regressions, the statistic, h, indicates that there is no significant serial correlation at the 90 percent confidence level. For all the regressions, the statistic, h, indicates that there is no serial correlation at the 95 percent confidence level.

The results indicate that since the constant B_0, is insignificant in most cases, the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has an insginificant impact on the equilibrium futures price in most cases.

(iii) The expected price is generated using the instrumental variable approach and the costly guarantee is generated using the absolute price differential approach. Table 6.7 presents the results and they indicate that

(a) The constant term, B_0, is significantly positive for the 12 weeks to maturity regression and is not significant for the remainder of the regressions.

(b) The coefficient of the costly guarantee, B_2, is significantly negative for the 12, 16, 72, and 76 weeks to maturity regressions and is not significant for the remainder of the regressions. For the 12 weeks to maturity regressions both B_0 and B_2 are significant but they have positive and negative impacts respectively on the equilibrium futues price which is inconsistent with the theory of the model[21].

(c) The coefficient of the expected price, B_1, is significant for all the regressions and its 95 percent confidence interval includes the theoretical value, 1, in 16 out of 19 cases.

(d) For all the regressions, the statistic, h, indicates that there is no significant autocorrelation at the 90 percent confidence level.

The results lead to the conclusion that since the constant B_0 is insignificant in most cases the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has an insignificant impact upon the equilibrium futures price.

(iv) The expected price is generated using the instrumental variable approach and the costly guarantee is generated using the standard deviation approach. Table 6.8 presents the results and they indicate that

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[21]See footnote 16 for an explanation.

(a) The constant term, B_0 is significantly positive for the 12 weeks to maturity regression and is not significant for the remainder of the regressions.

(b) The coefficient of the costly guarantee, B_2, is significantly negative for the 12 and 16 weeks to maturity regressions and is not significant for the remainder of the regressions. For the 12 weeks to maturity run both B_0 and B_2 are significant but they have positive and negative impacts respectively on the equilibrium futures price which is inconsistent with the theory of the model.

(c) The coefficient of the expected price, B_1, is significant for all the regressions and its 95 percent confidence interval includes the theoretical value, 1, in 17 of the 19 regressions.

(d) For all the regressions, the statistic, h, indicates that there is no significant serial correlation at the 90 percent confidence level.

The results lead to the conclusion that since the constant, B_1, is insignificant in most cases the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has an insignificant impact on the equilibrium futures price.

(3) T-bond futures contracts.

(i) The expected price is generated using the distributed lag approach and the costly guarantee is generated using the absolute price differential approach. Table 6.9 presents the results and an examination of the table reveals that

(a) The constant term, B_0, is significantly negative for the 4, 36, and 44 weeks to maturity, significantly positive for the 52 weeks to maturity regression and is not significant for the remainder of the regressions.

(b) The coefficient of the costly guarantee, B_2, is significantly positive for all of the regressions. This is inconsistent with the observation that the constant B_0 is not significant in most cases, and when significant it takes a negative value in 3 of the 4 cases[22].

(c) The coefficient of the expected price, B_1, is significant in all cases and its 95 percent confidence interval includes the theoretical value, (1-d), in 10 of the 13 cases.

(d) For 7 of the 13 regressions, the statistic, h, indicates that there is no significant serial correlation at the 90 percent confidence level. For all the regressions, the statistic, h, indicates that there is no significant serial correlation at the 95 percent confidence level.

The results are inconsistent with the theory of the model because the costly guarantee has a significant effect in all the regressions while the constant B_0 which measures the risk premium transfer from hedgers to speculators, is insignificant in most cases. No meaningful conclusions can be derived.

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[22]See footnote 16 for an explanation.

(ii) The expected price is generated using the distributed lag approach and the costly guarantee is generated using the standard deviation approach. Table 6.10 present the results and they indicate that

(a) The constant B_0 is significantly positive for the 12, 16, 28, 32, 36, 40, 44, and 52 week regressions and is not significant for the remainder of the regressions.

(b) The coefficient of the costly guarantee, B_2, is not significant in all the regressions. This is inconsistent with the fact that the constant B_0 is significant in most cases for the close to maturity regressions[23].

(c) The coefficient of the expected price, B_1, is significant in all cases and its 90 percent confidence level includes the theoretical value, (1-d), in 8 of the 13 cases and in most of the remaining regressions the theoretical value is quite close to the estimated confidence interval.

(d) For all the regressions the statistic, h, indicates that there is no serial correlation at the 90 percent confidence level.

The results are inconsistent with the theory of the model since the constant B_0 although significant in most cases, does not show an increasing trend as the number of weeks to maturity increases and furthermore the costly guarantee has an significant impact on the equilibrium future price. No meaningful conclusions can be derived from these results.

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[23]See footnote 16 for an explanation.

(iii) The expected price is generated using the instrumental variable approach and the costly guarantee is generated using the absolute price differential approach. Table 6.1 presents the results and they indicate that

(a) The constant term, B_0, is not significant in all cases

(b) The coefficient of the costly guarantee, B_2, is significantly negative for the 4, 18, 12, and 48 weeks to maturity regressions and is not significant for the remainder of the regressions.

(c) The coefficient of the expected price, B_1, is significant in all cases and its 95 percent confidence interval includes the theoretical value, 1, in all cases

(d) For 12 of the 13 regressions, the statistic, h, indicates that there is no serial correlation at the 90 percent confidence level. For all the regressions, the statistic, h, indicates that there is no serial correlation at the 95 percent confidence level.

The results indicate that since the constant B_0 is insignificant in most cases the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has an insignificant impact on the equilibrium futures price.

(iv) The expected price is generated using the instrumental variable approach and the costly guarantee is generated using the standard deviation approach. Table 6.12 presents the results and they indicate that

(a) The constant B_0 is significantly positive for the 20 weeks to maturity regression and is not significant for the remainder of the regressions.

(b) The coefficient of the costly guarantee is significantly negative for the 20 weeks to maturity regression, significantly positive for the 24 weeks to maturity regression and is not significant for the remainder of the regressions.

(c) The coefficient of the expected price, B_1, is significant in all cases and its 95 percent confidence interval includes the theoretical value in 12 of the 13 cases.

(d) For all the regressions, the statistic h indicates that there is no serial correlation at the 90 percent confidence level.

The results lead to the conclusion that since the constant, B_0, is insignificant in most cases, the expectations hypothesis holds true and consistent with this hypothesis is the observation that the costly guarantee has an insignificant impact on the equilibrium futures price.

6.4 Summary

Table 6.13 presents a summary of the results. The table provides a tally of the number of cases in which the constant B_0 and the coefficient of the costly guarantee, B_2, are significant, and the number of cases in which the 95 percent confidence interval of the coefficient of the expected price includes the theoretical value. For all the empirical specifications of the model for GNMA and T-bill futures contracts, the tally indicates that the constant B_0 is not significant in most of the cases. This indicates that there is no evidence of a risk premium transfer from hedgers to speculators and therefore it is concluded that the expectations hypothesis holds true. Consistent with the expectations hypothesis is the fact that the costly guarantee also has an insignificant impact upon the equilibrium futures price. Therefore, the empirical results indicate that for GNMA and T-bill futures contracts, there seems to be equal hedging activity on the long side and short side, and the magnitude of speculative activity appears to be insignificant. Since there is equal hedging activity on both sides, the effect of the costly guarantee on both sides tends to cancel out thereby producing no significant effect on the equilibrium futures price.

The results for the T-bond futures are inconsistent with the theory of the model when using the distributed lag approach and the absolute price differential approach to estimate the expected price and the costly guarantee respectively. The results are inconsistent because the constant term, which is the risk premium, has a significant effect on the equilibrium futures price in 4 of the 13 regressions while the costly guarantee has a significant positive impact in the 13 of the regressions. According to the theory of the model, both the risk premium and the costly guarantee should either have significant or insignificant impacts on the equilibrium futures price. The results are inconsistent with the theory of the model only if the classification of participants into the groups of hedgers and speculators by this study is the same as the classification of participants into these groups by the CFTC[24]. This was an assumption that was made when deriving the model. It might be that the classification defined by this study separates the participants into the two groups differently than the classification defined by the CFTC. In this particular instance, there might be equal hedging activity on both sides according to the hedging function defined by this study, but some of the participants on the short side, although hedgers by function, might be classified as speculators by the CFTC and therefore are subject to higher margin requirements. Thus the effect of the costly guarantee on the short side would be greater than the effect on the long side, and hence the costly guarantee has a significant positive impact on the equilibrium futures price even though all the participants on both sides are hedgers in the sense that their intention is to reduce their exposure to interest rate risk. Hence the results indicate no significant risk premium transfer and a significant costly guarantee.

The results for T-bond futures contracts are quite different when using the distributed lag approach and the standard deviation to measure the expected price and the costly guarantee respectively. In the majority of the regressions, the constant has a significant negative impact and the costly guarantee has no significant impact in all

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[24]Recall that the CFTC classifies the participants as hedgers if they are commercial traders and as speculators if they are non-commercial traders. This study classifies the participants as hedgers if their intention is to reduce their exposure to interest rate risk and as speculators if their intention is to take advantage of transitory profit making opportunities.

the regressions. The fact that there a significant negative risk premium together with an insignificant costly guarantee could be explained by the differences in the classifications of hedgers and speculators by this study and the CFTC. The negative risk premium does not increase as the number of weeks to maturity increase and therefore it cannot be concluded that normal backwardation holds ture.

The results for the T-bond futures contracts when using the instrumental variable approach to estimate the expected price indicate that the expectations hypothesis holds true because the constant term B_0 and the coefficient of the costly guarantee, B_2, have no significant impact in the majority of the regressions.

In summary, the results for the T-bond futures contracts are sensitive to the different empirical specification of the model The results are markedly different for the different empirical specification and hence it is difficult to derive meaningful conclusions from the results.

The R-square values for all the regressions are quite high, although as stated before, regressions utilizing time series data usually provide high R-square values and hence cannot be taken as evidence of "goodness of fit". The extent to which the data conforms to the model can be tested by observing whether the confidence interval of the expected price includes its theoretical value. In the majority of the cases, the confidence interval does include the theoretical value. It bears mention at this point that the results of the empirical tests and the conclusions derived are dependent upon the empirical specifications of the variable. The conclusions derived are correct, if the model is correctly specified and the estimated variables for the empirical study are accurate "estimators" of the true variables or that they are highly correlated with the true variables.

TABLE 6.1

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	-7400.79 (-2.6183)	.0360 (5.8790)	.0453 (6.0272)	.0453 (1.4708)	.9741	.02	.0240-.0480	.8053
8	-2844.74 (-2.7729)	.1442 (11.7772)	1.4163 (6.8467)	.0250 (1.6845)	.9731	.1	.1202-.1682	.7217
12	-2263.31 (2.3506)	.1077 (11.9889)	1.2731 (11.4308)	.0134 (1.4919)	.9719	.08	.0901-.1253	.1268
16	-671.81 (-0.6701)	.1190 (9.9628)	1.1603 (8.4482)	-.0033 (-1.9225)	.9777	.1	.0957-.1424	.6411
20	-358.997 (-0.7786)	.2922 (21.0661)	1.1674 (10.3035)	.0066 (1.3321)	.9677	.22	.2650-.3194	.4892
24	-752.83 (-0.7424)	.1197 (9.9093)	1.0904 (9.2178)	.0079 (1.8215)	.9687	.1	.0960-.1434	.1826

TABLE 6.1 (Contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
28	-173.516 (-0.3130)	.2870 (17.1587)	1.1570 (11.7615)	.0013 (.3163)	.9570	.22	.2542-.3198	.2395
32	-1366.03 (-0.9013)	.0726 (6.2149)	1.1415 (6.2139)	.0081 (2.4224)	.9632	.06	.0497-.0955	1.2172
36	56.83 (0.1784)	.9161 (29.2617)	1.8060 (27.8632)	.0017 (.6774)	.9477	.48	.8547-.9775	.2232
40	3593.19 (1.8691)	.0670 (2.9115)	.6738 (3.0808)	.0075 (2.7232)	.9502	.10	.0219-.112.	.8045
44	841.50 (.9328)	.2263 (9.3762)	1.0933 (9.5120)	.0035 (1.3133)	.9378	.20	.1790-.2736	.4632
48	17.11 (.2951)	.6104 (145.43)	1.7329 (5.2218)	.0022 (.9104)	.9970	.38	.6022-.6186	.2379

TABLE 6.1 (Contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
52	982.06	.3478	.8824	.0009	.9198	.28	.2783-.4173	.0216
	(1.1539)	(9.8115)	(2.9199)	(.4011)				
56	1365.52	.0172	.7977	.0064	.9339	.02	.0028-.0316	.3113
	(.4091)	(2.3379)	(2.3320)	(2.9254)				
60	125.52	.0199	.9236	.0062	.9969	.02	.0195-.0203	1.4204
	(1.2377)	(87.5753)	(73.7418)	(2.7847)				
64	78.5388	.0204	.9299	.0005	.9972	.02	.0199-.0209	.3473
	(.6994)	(87.2017)	(82.2423)	(.2320)				
68	2668.33	.2245	.8745	.0007	.9165	.24	.1401-.3089	.8978
	(2.1030)	(5.2124)	(5.2378)	(.3532)				
72	-488.44	.2308	1.1885	-.0001	.9186	.18	.1700-.2915	.1854
	(-0.3721)	(7.4370)	(7.3718)	(-0.0711)				

TABLE 6.1 (Contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	$B_1^{1'}$	B_2	R^2	(1-d)	C.I. for B_1	h
76	1209.59 (.3290)	.0735 (2.1389)	.8648 (2.1435)	.0074 (3.4448)	.9201	.08	.0061-.1409	.4085

TABLE 6.2

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	-7406.49 (-2.6225)	.0361 (5.8915)	1.7666 (6.0347)	-.0976 (-.5730)	.9739	.02	.0241 - .0481	.9138
8	-1100.32 (-1.8517)	.2799 (17.6957)	1.5558 (2.6247)	-.0588 (-1.8520)	.9737	.2	.2489 - .3109	.7870
12	-2104.02 (-2.1846)	.1064 (11.9130)	1.2562 (11.4578)	.0245 (.0879)	.9716	.08	.0889 - .1239	.0573
16	-268.65 (-0.5577)	.2259 (19.8809)	1.3218 (7.2146)	-.0911 (-4.1274)	.9792	.18	.2036 - .2482	.2581
20	-455.338 (-0.9754)	.3303 (21.3176)	1.2126 (10.1010)	.2161 (1.654)	.9677	.24	.2999 - .3607	.4289
24	-430.63 (-0.3748)	.1162 (8.6124)	1.0566 (7.9945)	-0.0138 (-0.0467)	.9681	.1	.0898 - .1426	.2924

TABLE 6.2 (contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_0	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
28	281.725 (.4402)	.2591 (14.5200)	1.0818 (10.4653)	-0.3672 (-1.5174)	.9576	.22	.2241 - .2941	.1549
32	-95.9843 (- 0.0634)	.0882 (6.3176)	1.0358 (6.2641)	-0.3592 (-1.0468)	.9622	.08	.0608 - .1156	1.1978
36	119.447 (.3463)	.9116 (27.3524)	1.7952 (25.9650)	-0.0317 (-0.2266)	.9476	.48	.8463 - .9769	.1618
40	2873.62 (1.5567)	.0767 (3.5185)	.7557 (3.6495)	-.1308 (-0.3209)	.9480	.10	.0340 - .1194	.8859
44	929.24 (1.1452)	.2851 (10.5995)	1.1446 (10.7766)	-0.1476 (-.5479)	.9374	.24	.2324 - .3378	.4701
48	-3.4097 (-.0603)	0.5327 (18.5370)	1.4071 (5.8666)	.4159 (2.7857)	.9971	.38	.4764 - .5890	.2024

TABLE 6.2 (contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
52	1027.86	0.3146	-0.2675	0.2675	0.9198	.26	.2468 - .3824	.1133
	(1.1259)	(9.0892)	(3.6202)	(-.4610)				
56	1503.21	0.0173	.7947	-0.2654	0.9301	.02	.0019 - .1347	.1358
	(.4156)	(2.2052)	(2.1644)	(-.5034)				
60	37.3081	0.1891	1.0912	.1489	.9963	.18	.1839 - .1943	1.6418
	(.4370)	(71.7706)	(59.7303)	(.4901)				
64	193.974	.0198	.9034	.7063	.9972	.02	.0187 - .0209	.7160
	(1.2574)	(36.0394	(38.2995)	(1.2757)				
68	1351.00	.2687	1.0461	.1342	.9164	.24	.1865 - .3509	.6133
	(1.0584)	(6.4038)	(6.3875)	(.3754)				
72	641.684	.2060	1.0560	-0.3575	.9190	.18	.1277 - .2843	.0742
	(.3655)	(5.1563)	(5.0352)	(-0.7962)				

TABLE 6.2 (contd.)

GNMA FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_0	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
76	4853.13 (1.9367)	0.1102 (1.9218)	.05575 (1.8378)	-0.9792 (-1.7707)	.9127	.18	-.0022 - .2226	.2170

TABLE 6.3

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
4	1.5715	.9999	.0148	.9700	.9600-1.0390	.1685
	(-.0083)	(50.1581)	(.4357)			
8	15.3719	.9982	.0102	.9748	.9548-1.0390	.0138
	(.0780)	(47.9724)	(.2962)			
12	214.7740	.9798	-0.0174	.9731	.9289-1.0307	.3303
	(.8758)	(37.7582)	(-2.4548)			
16	55.2722	.9940	-0.0005	.9778	.9665-1.0215	.1621
	(.4214)	(70.8883)	(-0.5896)			
20	38.9485	1.0338	-0.0017	.9695	1.0028-1.0648	.1836
	(.2724)	(65.2626)	(-1.8225)			
24	35.8323	.9971	-.0020	.9694	.9516-1.0426	.1627
	(.1641)	(42.9505)	(-0.5696)			
28	-522.25	1.0557	.0006	.9807	1.0128-1.0986	.6370
	(.0354)	(40.1733)	(-0.7783)			

TABLE 6.3 (contd.)

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
32	85.7780 (.2571)	.9945 (27.9236)	-0.0057 (-1.9261)	.9637	.9247-1.0643	.6244
36	40.5757 (.1502)	.9977 (34.6035)	-0.0027 (-1.0947)	.9526	.9412-1.0542	.5550
40	56.3050 (.2420)	.9968 (40.1915)	-.0031 (-1.5467)	.9486	.9482-1.0454	.3044
44	11.4977 (.0821)	1.0181 (66.9336)	-.0008 (-0.7696)	.9419	.9883-1.0479	.0161
48	-8.2144 (-0.0354)	1.0022 (40.1733)	-0.0012 (-0.7783)	.9213	.9533-1.0511	.0618
52	-3.1273 (.0.0134)	1.0007 (39.9167)	-.0004 (-0.3116)	.9225	.9516-1.0498	.1423

TABLE 6.3 (contd.)

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_2	R^2	C.I. for B_1	h.
56	76.3480 (.3224)	.9934 (39.0989)	-0.0012 (-1.1700)	.9276	.9436-1.0432	.2994
60	-2.7780 (-.0106)	1.0017 (35.3663)	-.0008 (-0.8145)	.9015	.9462-1.0572	.0931
64	-18.5442 (-0.0449)	1.0033 (22.4729)	-0.0007 (-0.5714)	.9198	.9158-1.0908	.1692
68	12.3234 (.0431)	.9997 (32.1402)	-0.0005 (-0.5517)	.9168	.9387-1.0607	.0735
72	-111.442 (-0.2880)	1.0144 (24.0649)	-.0012 (-1.0692)	.9350	.9318-1.0970	.0341
76	-342.063 (-0.6242)	1. 0416 (17.4882)	-.0021 (-1.8228)	.9199	.9248-1.1583	.4337

TABLE 6.4

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
4	2.7710 (.0147)	.9995 (49.9704)	.0778 (.3874)	.9770	.9603-1.0387	.1706
8	15.8535 (.0820)	.9982 (47.8058)	.0698 (.3476)	.9748	.9573-1.0391	.0152
12	101.66 (.4024)	.9904 (37.3689)	-0.1331 (-0.8640)	.9724	.9385-1.0423	.3928
16	191.499 (1.4627)	.9797 (69.9942)	-0.0218 (-2.1685)	.9782	.9523-1.0071	.1714
20	13.1539 (.0932)	1.0362 (66.0779)	-0.0118 (-1.8083)	.9695	1.0055-1.0669	.1936
24	-94.8946 (-0.4034)	1.0089 (41.0253)	0.1003 (0.8824)	.9694	.9607-1.0571	.0441

TABLE 6.4 (contd.)

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
28	-405.527 (-1.8301)	1.0451 (45.1396)	-0.1122 (-1.0561)	.9808	.9997-1.0905	.6370
32	231.058 (.6396)	.9791 (25.8856)	-0.2511 (-1.5531)	.9634	.9050-1.0532	.6844
36	84.2367 (.2899)	.9924 (32.5884)	-0.0897 (-0.6940)	.9523	.9327-1.0521	.3036
40	73.3741 (.2968)	.9937 (38.4443)	-0.0862 (-0.7233)	.9480	.9430-1.0444	.0047
44	-18.5376 (-.1247)	1.0201 (64.3995)	.0229 (0.3310)	.9417	.9891-1.0511	.0127
48	-1.5311 (-.0066)	.9989 (40.0640)	.0760 (.5077)	.9211	.9500-1.0478	.1125

TABLE 6.4 (contd.)

GNMA FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
52	-.6460 (-.0028)	1.0005 (39.8017)	-0.0323 (-0.2065)	.9225	.9512-1.0496	.3135
56	99.9443 (.4095)	.9903 (38.3918)	-.0584 (-0.6173)	.9271	.9397-1.0409	.0942
60	11.9114 (.0441)	.9992 (34.8903)	-.0206 (-0.1997)	.9010	.9431-1.0553	.2468
64	-108.893 (-0.2676)	1.0089 (23.4104)	.1356 (.9010)	.9201	.9244-1.0934	.0293
68	-88.0431 (-0.2998)	1.0069 (32.7579)	.1192 (1.0752)	.9174	.9467-1.0671	.0188
72	27.1982 (.0620)	.9971 (21.9553)	.0005 (.0028)	.9342	.9081-1.0861	.5065
76	205.73 (.3803)	.9824 (16.9975)	-0.1835 (-2.5742)	.9228	.8691-1.0957	.4642

TABLE 6.5

T-BILL FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	1339.68	.4856	1.3314	.0293	.9832	.36	.4158-.5537	.1680
	(2.1640)	(13.6799)	(13.5355)	(1.1982)				
8	164.19	.6025	1.5668	.0074	.9799	.38	.5492-.6546	.3724
	(.3774)	(22.1922)	(21.8779)	(.6642)				
12	-1065.07	.5218	1.6278	-.0022	.9843	.32	.4855-.5669	.4110
	(-2.7598)	(28.2132)	(25.5595)	(-.3574)				
16	-2385.60	.1694	1.4027	.0250	.9709	.12	.1352-.2028	.9446
	(-1.9088)	(9.7469)	(9.4558)	(3.9690)				
20	-872.474	.3069	1.3697	.0160	.9720	.22	.2725-.3405	.8936
	(-1.4297)	(17.5118)	(17.5519)	(3.4049)				
24	-1076.68	.1513	1.2346	.0101	.9669	.12	.1270-.1751	.1676
	(-1.2083)	(12.2202)	(12.2879)	(2.2833)				
28	-3067.82	.0547	1.3435	.0073	.9643	.04	.0404-.0687	.9853
	(-1.7905)	(7.5133)	(7.5254)	(1.7893)				

TABLE 6.5 (contd.)

T-BILL FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
32	278.746	.0846	1.0347	-.5783	.9613	.08	.0562-.1124	1.7926
	(.1705)	(5.8440)	(5.7998)	(-2.1584)				
36	174.345	.1339	1.0942	.0036	.9586	.12	.1123-.1579	2.1040
	(.1802)	(9.9640)	(9.9255)	(1.4122)				
40	38.5121	.2186	1.1899	.0048	.9544	.18	.1865-.2499	1.1662
	(.0528)	(13.3899)	(13.3416)	(2.3115)				
44	59.4986	.2803	1.2447	.0026	.9566	.22	.2438-.3160	1.8150
	(.0920)	(15.0923)	(15.0033)	(1.4203)				
48	4200.32	.0780	.6357	.0014	.9587	.12	.0300-.1249	1.1917
	(2.3927)	(3.1940)	(3.1826)	(.5679)				
52	2030.48	.1082	.8822	-.0001	.9539	.12	.0684-.1471	2.0479
	(1.3941)	(5.3389)	(5.3262)	(-.0909)				
56	-1468.45	.0733	1.2009	.0015	.9520	.06	.0534-.0928	1.4529
	(.9421)	(7.2319)	(7.2393)	(.6853)				

TABLE 6.5 (contd.)

T-BILL FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
60	-83.52	.3922	1.3731	.0008	.9333	.28	.3386-.4446	.9412
	(.1213)	(14.3795)	(14.3748)	(.3881)				
64	-2590.95	.1404	1.3789	.0033	.9554	.10	.1188-.1615	1.5662
	(-2.6645)	(12.7481)	(12.7605)	(1.5183)				
68	-1785.20	.1611	1.3189	.0036	.9286	.12	.1355-.1861	.8700
	(-1.9038)	(12.3602)	(12.3914)	(1.5794)				
72	-231.101	.3980	1.3968	-.0009	.9214	.28	.3479-.4470	1.3305
	(-.3591)	(15.6076)	(16.5459)	(-.5124)				
76	1865.93	.0699	.8575	.0159	.9738	.08	.0143-.1242	1.2976
	(.5863)	(2.4722)	(2.4716)	(8.7261)				

TABLE 6.6

T-BILL FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	1520.40 (0.3172)	.4753 (12.6329)	1.3030 (12.5140)	.1939 (.8997)	.9833	.36	.4014-.5268	.1358
8	1297.28 (2.2572)	.4086 (14.8481)	1.2624 (14.7305)	.6779 (-2.6743)	.9804	.32	.3545-.4614	.1045
12	-95.1111 (-.2883)	.5681 (30.1075)	1.6204 (13.3636)	-.5377 (-4.3624)	.9855	.38	.5210-.6043	.6289
16	-517.985 (.6414)	.2006 (12.8118)	1.2236 (13.0133)	-.3825 (-2.0370)	.9696	.16	.1698-.2307	.0630
20	299.365 (.4309)	.2735 (13.7197)	1.2243 (13.7802)	-.2179 (-1.2927)	.9707	.22	.2343-.3118	.8491
24	155.704 (.1855)	.1875 (11.5215)	1.1442 (11.6440)	-.1281 (-.7056)	.9662	.16	.1555-.2187	.3069

TABLE 6.6 (contd.)

T-BILL FUTURES
EXPECTED PRICE - DISTRIBUTED LAG
COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
28	-2001.42	.0501	1.2331	.0092	.9637	.04	.0332-.0666	1.1420
	(-.9849)	(5.8180)	(5.8211)	(.0323)				
32	-1749.18	.0492	1.2030	-.6611	.9614	.04	.0288-.0691	1.8816
	(-.7166)	(4.7483)	(4.7222)	(-2.1321)				
36	484.93	.1296	1.0591	.0925	.9582	.12	.0987-.1598	2.1039
	(.4279)	(8.2395)	(8.2061)	(.4516)				
40	951.55	.2258	1.1066	.0534	.9532	.20	.1829-.2677	.8298
	(1.1084)	(10.3351)	(10.2941)	(-.3375)				
44	759.410	.2603	1.1553	-.0346	.9563	.22	.2126-.3069	1.7513
	(.8980)	(10.7202)	(10.6483)	(-.2229)				
48	4899.49	.0683	.5566	-.1509	.9587	.12	.0172-.1182	1.2482
	(2.6175)	(2.6265)	(2.6159)	(-.4725)				
52	2022.18	.1083	.8831	-.0002	.9539	.12	.0685-.1472	2.0500
	(1.3878)	(5.3422)	(5.3295)	(-.0952)				

TABLE 6.6 (contd.)

T-BILL FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
56	1422.91	.0744	.9131	-.3479	.9522	.08	.0413-.1067	1.4435
	(.7485)	(4.4203)	(4.4195)	(-1.2364)				
60	825.802	.2894	1.1820	-.3046	.9332	.24	.2243-.3530	1.1082
	(.8013)	(8.7405)	(8.7210)	(-1.2396)				
64	-3773.12	.1203	1.4775	.1422	.9547	.08	.0914-.1485	1.7740
	(-2.2704)	(8.1819)	(8.1831)	(.6323)				
68	-639.60	.1734	1.2166	-.0421	.9273	.14	.1349-.2110	.6203
	(-.5410)	(8.8469)	(8.8595)	(-.2549)				
72	145.311	.3832	1.3445	-.1081	.9217	.28	.3178-.4471	1.3326
	(.1729)	(11.5102)	(11.5272)	(-.8521)				
76	11.2046	.2192	1.1986	-.0481	.9522	.18	.1738-.2635	1.1896
	(.1085)	(9.4935)	(9.5282)	(-.3470)				

TABLE 6.7

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_2	R^2	C.I. for B_1	h.
4	46.2314 (.0646)	.9746 (73.2318)	.0054 (.9214)	.9764	.9484-1.0008	.1161
8	-8.6359 (-.0686)	1.0010 (78.2445)	.0006 (.1035)	.9803	.9759-1.0261	.0729
12	308.569 (2.9771)	.9688 (91.9665)	-.0155 (-4.8048)	.9857	.9481- .9895	.1076
16	205.331 (1.5659)	.9793 (73.4235)	-.0076 (-3.3430)	.9720	.9531-1.0055	.2746
20	134.229 (.8518)	.9865 (61.5324)	-.0030 (-1.4896)	.9684	.9550-1.0180	.2568
24	34.5677 (.2493)	.9966 (70.6317)	-.0009 (-.6320)	.9678	.9689-1.0243	.2170
28	50.2681 (.3254)	.9949 (63.1119)	-.0009 (-0.7807)	.9623	.9639-1.0259	.0237

TABLE 6.7 (contd.)

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_2	R^2	C.I. for B_1	h.
32	39.0427 (.2581)	.9961 (64.6845)	-.0007 (-.8061)	.9609	.9658-1.0264	.1038
36	-20.8425 (-.1267)	1.0021 (59.8069)	.0004 (.5348)	.9565	.9692-1.0350	.0958
40	-74.8449 (-.4348)	1.0741 (57.4702)	.0001 (.2178)	.9517	1.0374-1.1108	.0369
44	-46.1745 (-.2614)	1.0314 (54.9314)	.0006 (.2418)	.9614	.9945-1.0683	.1852
48	-34.7053 (-.1766)	1.0067 (50.1274)	.0008 (1.1098)	.9534	.9672-1.0462	.1214
52	-226.301 (-.9282)	1.0229 (41.2037)	.0017 (1.8387)	.9590	.9741-1.0717	.0698
56	176.788 (.5751)	.9821 (31.3909)	-.0012 (-.0008)	.9511	.9206-1.0436	.3832

TABLE 6.7 (contd.)

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_2	R^2	C.I. for B_1	h.
60	160.816 (.5922)	.9838 (35.6047)	-.0008 (-0.5545)	.9426	.9295-1.0381	.1416
68	380.934 (1.3036)	.9615 (32.3326)	-.0022 (-1.9344)	.9546	.9031-1.0199	.2522
72	450.778 (1.9365)	1.0546 (40.2606)	-.0028 (-3.0226)	.9639	1.0030-1.1061	.3782
76	264.868 (.9533)	.9748 (34.3652)	-.0024 (-2.5235)	.9525	.9191-1.0305	.0222

TABLE 6.8

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R_2	C.I. for B_1	h.
4	104.231 (.4324)	.9946 (78.2381)	-.0519 (-.1326)	.9866	.9696-1.0196	.1951
8	81.1442 (.5844)	.9919 (70.3179)	-.0679 (-.8002)	.9803	.9642-1.0196	.1630
12	298.19 (2.6085)	.9698 (83.4880)	-.2055 (-3.7891)	.9851	.9470- .9926	.0521
16	197.012 (1.3795)	.9801 (67.4929)	-.1247 (-2.4174)	.9713	.9516-1.0086	.2527
20	59.2650 (.3510)	.9940 (57.8872)	-.0198 (-.3601)	.9681	.9603-1.0277	.2711
24	15.7780 (.1028	.9984 (63.9329)	-.0084 (-.1881)	.9678	.9677-1.0291	.2021
28	49.6946 (.2813)	.9950 (55.3834)	-.0206 (-.4262)	.9623	.9597-1.0303	.0260

TABLE 6.8 (contd.)

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R_2	C.I. for B_1	h.
32	136.413 (.8162)	.9862 (58.0059)	-.0653 (-1.5662)	.9613	.9525-1.0196	.0908
36	-31.6654 (-.1747)	1.0032 (54.4079)	-.0161 (.3971)	.9564	.9670-1.0394	.0950
40	-55.3527 (-.2910)	1.0720 (51.9370)	-.0041 (-.1054)	.9517	1.0314-1.1126	.0716
44	-21.7615 (-.1718)	1.0648 (40.2320)	.0055 (.5854)	.9715	1.0193-1.1104	.0367
48	-42.1496 (-.1718)	1.0075 (40.2320)	.0291 (.5854)	.9532	.9583-1.0567	.1205
52	9318.879 (-1.0152)	1.0324 (32.3030)	.0819 (1.3222)	.9586	.9696-1.0952	.0599
56	549.835 (1.2620)	.9441 (21.3015)	-0.1275 (-1.4641)	.9515	.8570-1.0312	.4076

TABLE 6.8 (contd.)

T-BILL FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_0	B_1	B_2	R_2	C.I. for B_1	h.
60	160.816 (.5922)	.9838 (35.6047)	-.0008 (-0.5545)	.9426	.9295-1.0381	.1416
64	132.594 (.4650)	.9866 (33.9984)	-.0265 (-.4991)	.9675	.9296-1.0436	.0607
68	348.695 (.9331)	.9646 (25.3525)	-.0680 (-1.1104)	.9536	.8898-1.0394	.1028
72	398.991 (1.3214)	1.0601 (31.1996)	-.0644 (-1.3070)	.9599	.9933-1.1269	.0048
76	398.783 (1.1145)	.9609 (26.3223)	-.0808 (-1.4517)	.9499	.8892-1.0326	.0001

TABLE 6.9

T-BOND FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	-8855.77 (-2.1758)	.0382 (4.2583)	1.8760 (4.6114)	.0888 (3.0957)	.9779	.02	.0206-.0558	.9831
8	-3583.78 (-.8795)	.0264 (2.9570)	1.3683 (3.3421)	.0803 (5.1449)	.9746	.02	.0089-.0439	.4138
12	1211.84 (.6341)	.0533 (4.0974)	.9921 (4.5294)	.0418 (4.0735)	.9672	.06	.0277-.0789	2.1457
16	-3040.97 (-1.0552)	.0256 (4.0500)	1.2895 (4.4687)	.0514 (7.5518)	.9809	.02	.0132-.0380	1.6475
20	-3210.05 (-1.3467)	.0258 (4.9272)	1.3033 (5.4851)	.0518 (11.1050)	.9847	.02	.0155-.0361	.3368
24	-2052.85 (-1.2280)	.0231 (6.2779)	1.3079 (7.1796)	.0459 (12.6093)	.9850	.02	.0159-.0303	2.0352

TABLE 6.9 (contd.)

T-BOND FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
28	-2560.91	.0252	1.3572	.0416	.9884	.02	.0160-.0324	2.3341
	(-1.3379)	(5.7990)	(6.3952)	(14.0369)				
32	-879.225	.0203	1.0754	.0359	.9922	.02	.0143-.0263	.0821
	(-0.6328)	(6.6842)	(7.7190)	(16.3671)				
36	-4718.25	.0291	1.4266	.0333	.9948	.02	.0227-.0355	.6000
	(-3.1091)	(8.9843)	(9.1564)	(22.6895)				
40	-1529.93	.0218	1.1229	.0291	.9960	.02	.0170-.0266	2.0508
	(-1.3511)	(9.0080)	(9.6843)	(25.7896)				
44	-3065.76	.0251	1.2707	.0269	.9960	.02	.0194-.0308	1.2159
	(-2.2151)	(8.5793)	(8.8985)	(25.6497)				
48	-1828.42	.0226	1.1322	.0250	.9952	.02	.0149-.0303	2.4820
	(-.9844)	(5.7792)	(5.8835)	(25.3003)				
52	8130.08	.0180	1.1216	.0076	.9558	.14	.0160-.0254	.5082
	(31.5336)	(4.7684)	(3.5540)	(5.5786)				

TABLE 6.10

T-BOND FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
4	-8414.67	.0373	1.8306	.2757	.9763	.02	.0188-.0558	1.2104
	(-1.9620)	(3.9538)	(4.2528)	(1.0139)				
8	-2264.40	.0236	1.2557	.0427	.9684	.02	.0052-.0420	1.2012
	(-.5336)	(2.5255)	(2.9814)	(.1229)				
12	2545.01	.0619	.8634	-.0998	.9621	.08	.0419-.0819	1.5550
	(2.2943)	(6.0770)	(6.8360)	(-.3414)				
16	3612.03	.0518	.7710	.0694	.9713	.08	.0246-.0790	.7769
	(2.4150)	(3.7447)	(5.6915)	(.2344)				
20	2005.61	.0661	.9014	.5136	.9678	.08	.0448-.0874	.6895
	(1.6963)	(6.1013)	(8.0928)	(1.9555)				
24	2497.55	.0450	.8938	.3429	.9591	.06	.0198-.0702	.6775
	(1.3215)	(3.5052)	(4.6760)	(1.1121)				

TABLE 6.10 (contd.)

T-BOND FUTURES

EXPECTED PRICE - DISTRIBUTED LAG

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_0	B_1	B_1^1	B_2	R^2	(1-d)	C.I. for B_1	h
28	3936.70 (2.5043)	.0493 (3.4266)	.8017 (4.9410)	-.3706 (-1.1325)	.9643	.08	.0187-.0776	.2523
32	4715.45 (3.5992)	.0187 (3.2423)	.5969 (4.6508)	-.0972 (-.3963)	.9695	.04	.0074-.0300	1.2254
36	2832.04 (2.0778)	.0761 (4.8203)	.8580 (6.6850)	-.1156 (-.3417)	.9622	.10	.0451-.1071	.8793
40	3779.21 (2.3087)	.0365 (3.3286)	.7096 (4.3741)	-.1774 (-.4705)	.9610	.06	.0150-.0580	1.1582
44	3666.82 (2.4846)	.0670 (3.9308)	.7739 (5.2843)	-.4138 (-1.0104)	.9554	.10	.0335-.1005	.7669
48	1515.37 (.8599)	.0922 (4.5534)	.9400 (4.9212)	-.1831 (-.3327)	.9470	.10	.0524-.1320	1.2589
52	8617.18 (23.0616)	.0077 (1.8256)	.5345 (1.3476)	.0704 (1.7603)	.9327	.10	-.0006-.0160	.2104

TABLE 6.11

T-BOND FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
4	22.2370 (.0824)	1.0004 (34.7268)	-0.0651 (-2.2406)	.9737	.9439 -1.0569	.3081
8	220.761 (1.1283)	.9805 (47.3368)	-.0321 (-3.7813)	.9675	.9399 -1.1027	.2976
12	411.67 (.7264)	.9594 (15.7293)	-.0187 (-2.3900)	.9436	.8399 -1.0789	.5399
16	119.483 (.2290)	.9911 (17.6979)	-.0126 (-1.7322)	.9648	.8813 -1.1009	.4224
20	43.6900 (.4836)	.9927 (98.0057)	.0055 (.9188)	.9915	.9728 -1.0126	1.8762
24	-1.5301 (-0.6445)	1.0182 (38.6148)	-.0036 (-0.8647)	.9603	.9665 -1.0698	.3275

TABLE 6.11 (contd.)

T-BOND FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - ABSOLUTE PRICE DIFFERENTIAL

Weeks to Maturity	B_0	B_1	B_2	R^2	C.I. for B_1	h.
28	-20.4362 (-0.0910)	1.0012 (41.3666)	.0011 (.8388)	.9565	.9538-1.0486	.3490
32	-68.3397 (-.2852)	1.0082 (38.8525)	-.0008 (-0.6169)	.9551	.9573-1.0591	.3469
36	-246.231 (-1.1291)	1.0421 (23.7942)	-.0039 (-.7471)	.9437	.9560-1.1282	.3221
40	-380.581 (-0.9191)	1.0043 (22.4347)	-.0037 (-1.1702)	.9499	.9166-1.0920	.6088
44	-857.99 (-0.8683)	1.0995 (11.6846)	-.0082 (-1.9085	.9480	.9055-1.2705	.7011
48	-857.99 (-1.7702)	1.0995 (20.2033)	-.0059 (-2.2015)	.9446	.9928-1.2062	.9353
52	-704.00 (-.6488)	1.0852 (9.1701)	-.0063 (-1.8309)	.9452	.8527-1.3145	.9746

TABLE 6.12

T-BOND FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
4	-71.0035 (-0.2150)	1.0069 (28.7499)	.8396 (.2831)	.9727	1.0001-1.0755	.2199
8	59.6223 (.2678)	.9948 (42.6014)	-0.1001 (-0.6764)	.9633	.9490-1.9374	.3366
12	139.201 (.2420)	.9864 (16.0184)	-0.1178 (-0.7251)	.9403	.8657-1.1077	.5180
16	-38.9498 (.2347)	1.0030 (23.2349)	.0863 (.1016)	.9639	.9680 -1.0379	.3807
20	498.631 (5.3805)	.9477 (96.9747)	-0.0589 (-7.2884)	.9946	.9285- .9669	.1767
24	-122.16 (0.6070)	1.0093 (47.4570)	.2530 (2.4062)	.9624	.9676-1.0510	.8066

TABLE 6.12 (contd.)

T-BOND FUTURES

EXPECTED PRICE - INSTRUMENTAL VARIABLE

COSTLY GUARANTEE - STANDARD DEVIATION

Weeks to Maturity	B_o	B_1	B_2	R^2	C.I. for B_1	h.
28	-86.6156	1.0085	.0166	.9571	.9628-1.0542	.3851
	(-0.3952)	(43.2882)	(1.3958)			
32	-28.7558	1.0029	.0025	.9549	.9551-1.0507	.2299
	(-0.1249)	(41.0920)	(.2029)			
36	-35.1474	.9948	.0121	.9521	.8813 -1.1082	.2345
	(-1.042)	(17.2345)	(1.3432)			
40	-5.6173	.9975	.1694	.9486	.9443-1.0507	.0339
	(-.0219)	(36.7696)	(1.2156)			
44	-112.103	1.0067	.2632	.9476	.9128-1.0989	.2179
	(-.2489)	(21.4022)	(1.2287)			
48	3.9199	1.0016	-0.0950	.9407	.9399-1.0633	.5096
	(.0131)	(31.8028)	(-0.5778)			
52	64.4069	.9936	-.0240	.9432	.8592 -1.1280	.1215
	(.0980)	(14.5319)	(-.0840)			

TABLE 6.13

SUMMARY OF RESULTS

Type of contract and empirical procedure	Total number of runs	Number of runs in which H_o: $B_o=0$ is rejected at α =.05	Number of runs in which H_o: $B_2=0$ is rejected at α =.05	Number of runs in which 95 percent confidence interval of B_1 includes the theoretical value.
GNMA				
(i) $E(F_T)$=DL $E(C)$=APD	19	4	5	12
(ii) $E(F_T)$=DL $E(C)$=SD	19	3	2	10
(iii) $E(F_T)$=IV $E(C)$=APD	19	0	1	17
(iv) $E(F_T)$=IV $E(C)$=SD	19	0	2	18
T-BILL				
(i) $E(F_T)$=DL $E(C)$=APD	19	4	6	6
(ii) $E(F_T)$=DL $E(C)$=SD	19	3	4	11
(iii) $E(F_T)$=IV $E(C)$=APD	19	1	4	16
(iv) $E(F_T)$=IV $E(C)$=SD	19	1	2	17
T-BOND				
(i) $E(F_T)$=DL $E(C)$=APD	13	4	13	7
(ii) $E(F_T)$=DL $E(C)$=SD	13	7	0	8

TABLE 6.13

SUMMARY OF RESULTS

Type of contract and empirical procedure	Total number of runs	Number of runs in which H_0: $B_0=0$ is rejected at $\alpha=.05$	Number of runs in which H_0: $B_2=0$ is rejected at $\alpha=.05$	Number of runs in which 95 per-cent confidence interval of B_1 includes the theoretical value.
(iii) $E(F_T)$=IV $E(C)$ =APD	13	0	5	13
(iv) $E(F_T)$=IV $E(C)$ =SD	13	1	2	12

Legend

DL = Distributed Lag

IV = Instrumental Variable

APD = Absolute Price Differential

SD = Standard Deviation

CHAPTER VII

SUMMARY AND CONCLUDING REMARKS

The purpose of this study was to develop and test a model which determines the equilibrium price of interest rate futures contracts. The derivation of the model was based on a set of assumptions that are to be evaluated later. The derivations were based upon the assumption that all participants maximize expected utility of terminal wealth in a mean variance framework. A rationale was provided to divide the participants into two groups, hedgers and speculators. The demand for long contracts and the supply of short contracts for each set of participants were determined. Total demand was equated to total supply to determine the equilibrium futures price. It was determined that the equilibrium futures price is a function of the expected futures price, the risk premium transfer from hedgers to speculators and the estimated costly guarantee.

The effect of the risk premium on the equilibrium futures price would be positive if there were excessive hedging activity on the long side and it was shown that the costly guarantee would also have a positive effect on the equilibrium futures price. If there were excessive hedging activity on the short side, the effect of the risk premium on the equilibrium futures price would be negative and the effect of the costly guarantee on the equilibrium futures price would also be negative. If there were equal hedging activity on both sides, then there would be no risk premium transfer from hedgers to speculators; hedgers would be the only participants in the market and the costly guarantee would have an insignificant effect on the equilibrium futures price.

Empirical tests were performed on the model to test two hypotheses. The first was to determine if the risk premium has a positive, negative or insignificant impact upon the equilibrium futures price. A positive risk premium which tends towards zero as the futures contract tends toward maturity would indicate that normal contango holds true. A negative risk premium that tends towards zero as the futures contract approaches maturity would indicate that normal backwardation holds true. No risk premium would imply that the expectations hypothesis holds true. The second hypothesis concerned the effect of the costly guarantee on the equilibrium futures price. According to the theory of the model, the effect of the costly guarantee would be positive if there is a positive risk premium, negative if there is a negative risk premium, and insignificant if there is no risk premium paid by hedgers. The consistency between the impact of the costly guarantee and the risk premium on the equilibrium futures as indicated hold true only if the classification of participants into the groups, hedgers and speculators, by this study is the same as the classification defined by the CFTC. The lack of consistency that might arise from the empirical results would provide evidence that the participants are classified differently.

These hypotheses were tested on available data for GNMA, T-bond and T-bill futures contracts. Four empirical specifications of the model were tested. The four empirical specifications resulted from two specifications for the expected price and two specifications for the costly guarantee. The specifications for the expected price are the distributed lag approach and the instrumental variable approach, and the specifications for the costly guarantee are the absolute price differential approach and the standard deviation approach. The purpose of adopting the standard deviation approach is that the costly guarantee is a form of risk and the standard deviation is considered as a proxy for that risk.

The empirical results for the GNMA and T-bill futures provide evidence that the expectations hypotheses holds true. Consistent with this is the fact was that the costly guarantee has an insignificant impact on the equilibrium futures price. The results for T-bond futures contracts, when using the distributed lag approach and the absolute price differential approach to estimate the expected price and the costly guarantee respectively indicate no risk premium transfer and a significant costly guarantee. The results when using the distributed lag approach and the standard deviation approach to measure the expected price and costly guarantee indicate a significant risk premium and no significant costly guarantee. These results suggest that there may be a difference in classifications of participants into the groups of hedgers and speculators by this study and the CFTC. This would be due to the fact (See Table 2.1) that security dealers who constitute the largest segment of commerical traders and who are classified as hedgers by the CFTC might in reality be performing the speculative function. The results when using the instrumental variable approach to measure the expected price, indicate that the expectations theory holds true and that the costly guarantee has an insignificant impact upon the equilibrium futures price.

The results that have been derived are based upon two important sets of assumptions. The first is that the assumptions upon which the models derived are realistic and the second is that the empirical specification of the variables "accurately" estimate or proxy the true variable. Little can be said about the latter set of assumptions except that using the distributed lag approach to measure the expected price has a stronger theoretical foundation than the instrumental variable approach. The distributed lag approach to measure the expected price has been used quite frequently in the area of macroeconomics to estimate expectations. Stein provides evidence that for several commodity futures contracts, expectations tend to follow an adaptive expectations process rather than a rational expectations process. The adaptive expectations model postulates that the expectations of participants adjust due to errors in past expectations. The adaptive expectations model translates into a distributed lag model by differencing the former recursively. In summary, more weight should be given to the results derived from the distributed lag approach rather than the instrumental variable approach.

The derivation of the model is based upon the critical and controversial assumption that all participants have homogeneous expectations. The reason for making this assumption is twofold: (1) To define a scenario in which the majority of participants are hedgers and that the function of speculative activity is simply to meet excessive hedging on either the long side or short side. (2) To be able to empirically test the model. An empirical specification of a model based upon heterogeneous expectations would be intractable. An examination of Table 2.1 reveals that the majority of participants in the years 1977 and 1979 were non-commercial traders who are considered to be speculators by the CFTC. Only the commercial traders are considered as hedgers. If the non-commercial traders are in reality speculators, then one cannot justify the assumption of homogeneous expectations since the speculators in all probability would be taking both long and short positions, behavior which implies hetrogeneous expectations. If heterogeneous expectations are in fact reality, then the primary function of the market would be akin to a gambling casino in which the majority of participants are betting according to their estimates of future movements in the interest rate. It is quite possible that the majority of the non-commerical traders own diversified portfolios, and that their participation in the interest rate futures market is to hedge the portion of their portfolio that is exposed to interest rate risk. Such participation according to this study will satisfy the hedging rather than the speculative function. The speculative function requires that the participant's portfolio is not exposed to interest rate risk or that the participant does not own a diversified portfolio. If this really is the case, then the assumption of homogeneous expectations might well be correct.

A suggestion for future research is to test the model with the expected price generated from the forward interest rate that is implicit in the term structure of interest rates. There is substantial evidence in the literature that the forward rate includes a liquidity premium[25]. Therefore, the first step is to subtract the liquidity premium from the forward rate to determine the expected

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[25]See Tinic and West [1978] and Van Horne for a survey of the literature on the term structure of interest rates.

future interest rate[26]. This rate would then be converted into a price which would be the expected price of the futures contract at maturity. This procedure would probably provide the most accurate estimate of the expected price and the results might shed most light upon whether the equilibrium futures price includes a risk premium and or a costly guarantee.

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[26] Roll [1970] provides a procedure to determine the liquidity premium.

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