

SUBCOOLED FORCED CONVECTION FILM BOILING
OVER
SPHERES AND CYLINDERS

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A Thesis
Presented to
the Faculty of the Department of Mechanical Engineering
University of Houston

.....

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

.....
.....

by
Satish Kumar Shoor
December 1970

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ABSTRACT

A theoretical treatment is presented for the heat transfer rates over spheres and cylinders in forced convection film boiling when the liquid bulk is subcooled to any degree. With the help of suitable assumptions, solutions for temperature profiles in liquid and vapour boundary layers are obtained which are then used to formulate a total energy balance on the sphere surface. The resulting equation is solved numerically to determine vapour film thickness at every point on the sphere body. Dimensionless Nusselt numbers are then computed for any given system parameters k_1 and k_2 (dimensionless), which can be looked upon as a general solution to the problem. It is confirmed that subcooling of the liquid bulk raises the heat transfer substantially.

A number of comparisons with experiment are made which in general show a reasonably good match between the theory and the experimental trend. Theoretical fluxes obtained are about one half the experimental values.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. LIQUID AND VAPOUR BOUNDARY LAYER SOLUTIONS OVER THE SPHERE.	5
III. ENERGY BALANCE AND THE NUMERICAL SOLUTION FOR NUSSELT NUMBERS.	18
IV. NUSSELT NUMBERS FOR CYLINDERS.	42
V. SOME COMPARISONS WITH EXPERIMENT AND DISCUS- SION OF RESULTS.	48

LIST OF FIGURES

FIGURE		PAGE
1	Model for Forced-Convection Film Boiling	
	Around a Sphere	6
2	Coordinate System for the Vapour Boundary Layer	13
3	Energy Balance on an Elemental Area of Sphere	
	Surface	19
4	Plot Between δ and ϕ	30
5	Heat Flux From a Differential Area on Sphere	
	Surface	32
6	Heat Transfer Results	33
7	Heat Transfer Results (cont.)	34
8	Heat Transfer Results (cont.)	35
9	The Model Used for Roughness Treatment.	37
10a	Potential Flow Around a Cylinder.	43
10b	Energy Balance Over an Elemental Area on the	
	Cylinder.	43
11	Comparison of Experimental and Theoretical Heat	
	Transfer Rates From Sphere to Liquid Sodium .	51
12	Comparison of Experimental and Theoretical Heat	
	Transfer Rates From Sphere to Liquid Sodium .	51
13	Comparison of Experimental and Theoretical Heat	
	Transfer Rates From Sphere to Liquid Sodium .	51
14	Graph Between Nusselt Number and k_2 at Constant	
	k_1	53

FIGURE	PAGE
15 Graph Between the Product $(Nu \times k_1)$ and k_1 . .	53
16 Average Heat Flux v/s Initial Temperature . . .	55

NOTATION

A	surface area
A_c	cross-sectional area of vapour film
C_p	specific heat capacity
D	diameter
h	heat transfer coefficient
H	heat transfer coefficient
k	thermal conductivity
k_1, k_2	dimensionless numbers
k'_1, k'_2	dimensionless numbers
M	$2R\alpha/3U_\infty$
Nu	Nusselt number
p	pressure
Pr	Prandtl number
q	heat flow rate
q''	heat flux
r	axisymmetric radial coordinate
r^*	radial coordinate
R	radius of sphere/cylinder
R'	radius of spherical vapor region
T	temperature
ΔT	temperature difference
u	velocity in x-direction
$\bar{u}$	average velocity in x-direction

U	velocity
v	velocity in y direction
w	mass flow rate
x	coordinate measured along surface
y	coordinate measured normal to surface
α	thermal diffusivity
δ	vapour film thickness
δ'	dimensionless film thickness
η	transformation variable
λ	latent heat of vaporization
λ'	effective latent heat of vaporization
μ	viscosity
ρ	density
τ	shear stress
ϕ	angle measured relative to the free stream velocity
ψ	transformation variable

Subscripts

B	bulk liquid
c	cross-section
exp	experimental
l	liquid
r^*	radial
sat	saturation
v	vapour

vap vaporization

w surface

ϕ angular

∞ free stream

CHAPTER I

INTRODUCTION

Film boiling is the type of boiling phenomenon where the heated surface is separated from the liquid by a continuous vapour blanket. In the classical boiling curve for liquids, the film boiling regime is marked by very high surface temperatures and relatively low heat fluxes. However this classical behavior is substantially changed if the liquid bulk is subcooled to some extent, i.e., if the liquid bulk temperature is kept below its saturation temperature at the given system pressure. Various experimental studies [1-5]* for subcooled film boiling have revealed that it is possible to obtain large heat transfer rates if the liquid bulk is below its saturation temperature; the heat flux will of course depend upon the degree to which the liquid is subcooled.

This circumstance may be used to advantage in several film boiling heat transfer situations; e.g., the operation of jets and rockets frequently involves the contact of a boiling liquid with hot surfaces and film boiling occurs. In thermal cracking operations, it might be desirable sometimes to use high temperatures and short contact times giving rise to the film boiling mode of heat transfer. Again in a boiling slurry reactor where the fuel particles generating heat are suspended

* Numbers in brackets indicate references.

in a liquid moderator and coolant, the heat transferred from particles to the liquid is restricted by the presence of a vapour blanket.

In all these cases the heat transfer characteristics can be considerably improved by suitably subcooling the liquid. It is thus important to develop a theory which would reasonably predict the heat transfer rates in subcooled film boiling.

The problem of film boiling of saturated liquids over a cylinder was analyzed by Bromley first for the free convection case [6] and then for forced convection [7]. The case of the sphere was then treated by Frederking [8], Kobayashi [9] and Witte [10]. These studies, however, do not include the effect of subcooling of the bulk liquid. Sparrow and Cess [11, 12] presented an exact boundary layer solution for subcooled forced convection film boiling over a flat plate.

In the present study it is attempted to explain theoretically some of the observed heat transfer rates for the subcooled film boiling from spherical and cylindrical geometries in forced convection. The basic assumption underlying the formulation of the problem is that there exists a continuous and smooth vapour film over the entire solid surface. Boundary layer equations are written for vapour

and liquid phases separately. The analysis is simplified by neglecting the inertia forces and energy convection terms thus linearizing the equations. These effects are accounted for reasonably well by replacing the latent heat of vaporization in the final result by an 'effective' latent heat [13].

For the flow of liquid over the sphere the velocity gradient in the liquid is taken to be negligibly small. This permits the use of potential flow theory for the calculation of velocity field in the liquid. This greatly simplifies the problem and a solution can be obtained which without this assumption might have been impossible.

The formulation of an energy balance at the interface provides a nonlinear differential relationship between the film thickness and the angular position on the sphere/cylinder, which is solved by a numerical technique. Dimensionless Nusselt numbers are then obtained for specific values of a set of dimensionless numbers, k_1 and k_2 , thus providing a general solution to the problem.

Unfortunately, only a very limited amount of data exists for this particular situation and that too is either not in a usable form or the experimental conditions are too uncertain. A comparison with Bromley's data [3] shows the theoretical values of flux about 2.5 times too low and that with Witte [2] about 2.5 times too high.

To take into account the effect of surface irregularities, a different treatment, the 'roughness' model was proposed. Though this model makes use of very simplified assumptions, it improves the results by about 20%.

CHAPTER II

LIQUID AND VAPOUR BOUNDARY LAYER SOLUTIONS OVER THE SPHERE

Physical Model and Governing Equations

Film boiling is normally characterized by the existence of a vapour film completely surrounding the heated surface. At some distance from the solid surface an interface exists between the vapour film and the surrounding liquid. Since the liquid is forced past the sphere, relative motion between the sphere, the vapour and the liquid will occur. The model used for this phenomenon is shown in Figure 1.

The problem may be thought of as one of two boundary layer systems occurring simultaneously. First, the vapour film moves past the solid sphere surface and is influenced in its motion by the presence of the sphere surface and also by the presence of the liquid at the liquid-vapour interface. Second, the liquid layer overriding the vapour film actually feels the influence of the vapour layer only. It cannot actually 'see' the sphere surface. Mass crosses the liquid-vapour interface from one boundary layer system to the other as the liquid flows past the sphere.

The problem is further complicated because of the fact that the liquid bulk is below its boiling temperature. This

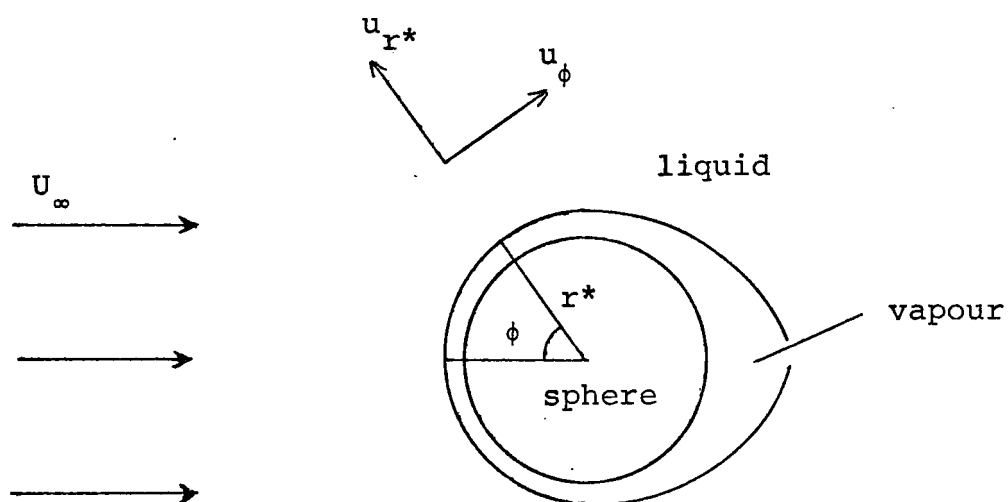


Fig. 1 Model for forced-convection film boiling around a sphere.

condition of the liquid allows heat to be transferred away from the liquid-vapour interface into the liquid bulk.

The problem has now been formulated and consists of two phases of a fluid flowing past a sphere with a simultaneous heat transfer occurring in both phase regions. The boundary layer treatment will be applied to both the vapour boundary layer and the liquid boundary layer to obtain the solution.

Liquid Boundary Layer

The liquid will be assumed to move around the sphere in potential flow. As stated previously it shall be assumed that the liquid feels only the influence of the vapour film. If it is assumed that the liquid-vapour interface is smooth and in dynamic equilibrium, then the shearing stress acting on the liquid-vapour interface in the liquid must be equal in magnitude to the shear stress acting upon the liquid-vapour interface in the vapour. The following relation can be written:

$$\tau_l = -\tau_v \quad (2.1)$$

where l and v refer to conditions in the liquid and vapour respectively, and τ stands for shear stress. Alternatively,

$$\mu_l \left(\frac{\partial u}{\partial y} \right)_l = -\mu_v \left(\frac{\partial u}{\partial y} \right)_v \quad (2.2)$$

This above equation relates the product of viscosity μ and velocity gradient $\frac{\partial u}{\partial y}$ in the liquid at the liquid-vapour interface to the corresponding product in the vapour. The velocity gradient in the liquid then may be written as,

$$\left(\frac{\partial u}{\partial y}\right)_l = - \frac{\mu_v}{\mu_l} \left(\frac{\partial u}{\partial y}\right)_v \quad (2.3)$$

For conditions sufficiently far from the critical point the ratio of viscosities $\frac{\mu_v}{\mu_l}$ is quite small. For water at 1 atm. and its boiling point, the ratio is .039. Further the velocity gradient $\left(\frac{\partial u}{\partial y}\right)_v$ will be assumed small so that $\left(\frac{\partial u}{\partial y}\right)_l$ remains small. Though this assumption gets progressively worse as the vapour film thickness goes down, it permits the use of potential flow theory for the calculation of the velocity field in the liquid bulk.

By using spherical coordinates, assuming axial symmetry and neglecting conduction in the ϕ direction as compared to convection in that direction, the differential equation describing the temperature distribution around the spherical vapour film becomes

$$u_{r^*} \frac{\partial T}{\partial r^*} + u_{\phi} \frac{1}{r^*} \frac{\partial T}{\partial \phi} = \alpha \frac{\partial^2 T}{\partial r^{*2}} + \frac{2}{r^*} \frac{\partial T}{\partial r^*} \quad (2.4)$$

where T is temperature and α is the thermal diffusivity of the liquid. The velocity components u_{r^*} and u_{ϕ} in accordance

with the assumption of frictionless flow can be written as

$$u_{r^*} = -3U_{\infty} \frac{r^*-R}{R} \cos\phi, \quad u_{\phi} = \frac{3}{2} U_{\infty} \sin\phi$$

These are obtained directly by differentiating the stream and potential functions for frictionless flow around a sphere.

Sideman [14] showed that if the assumption is made that the heat transfer takes place in a thin layer near the interface, the term $(\frac{2}{r^*} \frac{\partial T}{\partial r^*})$ may be neglected in comparison with the term $\frac{\partial^2 T}{\partial r^{*2}}$ in the energy equation. This further implies that the product of the Reynolds number and the Prandtl number called the Peclet number is much larger than unity. The Reynolds number and the Prandtl number are defined by the equations.

$$Re = \frac{\rho U D}{\mu} \quad \text{and} \quad Pr = \frac{\mu C_p}{k}$$

where ρ is density, U is velocity D is sphere diameter, μ is viscosity, C_p is specific heat capacity and k is the thermal conductivity of the liquid.

The energy equation is then reduced to

$$u_{r^*} \frac{\partial T}{\partial r^*} + u_{\phi} \frac{1}{r^*} \frac{\partial T}{\partial \phi} = \alpha \frac{\partial^2 T}{\partial r^{*2}}$$

The boundary conditions are

$$\begin{aligned}
T &= T_B & r^* &= \infty & \phi &\geq 0; \\
T &= T_{\text{sat}} & r^* &= R' & \phi &> 0; \\
T &= T_B & \infty &\geq r^* \geq R' & \phi &= 0
\end{aligned} \tag{2.5}$$

T_B represents the temperature in the bulk liquid flowing past the spherical region, T_{sat} is the saturation temperature of the liquid and R' is the radius of the spherical region. If the vapour film thickness is assumed thin, the radius of the solid sphere may be used for R' .

Defining y by $y = r^* - R$ one may write

$$u_{r^*} = -3U_{\infty} \frac{y}{R} \cos\phi, \quad u_{\phi} = \frac{3}{2} U_{\infty} \sin\phi$$

Transformation variables can be defined as

$$\begin{aligned}
\Delta T &= T - T_{\text{sat}} \\
\psi &= y \sin^2\phi
\end{aligned} \tag{2.6}$$

and

$$\eta = \int_0^{\phi} \sin^3\phi d\phi$$

The energy equation under these transformations becomes

$$\frac{\partial \Delta T}{\partial \eta} = M \frac{\partial^2 \Delta T}{\partial \psi^2} \tag{2.7}$$

where

$$M = \frac{2}{3} \frac{Ra}{U_{\infty}}$$

The boundary conditions become

$$\begin{aligned} \Delta T &= T_B - T_{\text{sat}}, & \psi &= \infty & \eta > 0 \\ \Delta T &= 0, & \psi &= 0 & \eta > 0 \\ \Delta T &= T_B - T_{\text{sat}} & \infty > \psi > 0 & & \eta = 0 \end{aligned} \quad (2.8)$$

This system of equations is similar to the heat conduction equations for which solutions are known. A solution is [15];

$$\frac{T - T_B}{T_{\text{sat}} - T_B} = \text{erfc} \left(\frac{\psi}{2\sqrt{M\eta}} \right) \quad (2.9)$$

Where the complimentary error function is defined by

$$\text{erfc}(a) = 1 - \text{erf}(a) = 1 - \frac{2}{\sqrt{\pi}} \int_0^a e^{-\lambda^2} d\lambda$$

The temperature gradient $\frac{\partial T}{\partial y}$ can be written as

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \psi} \frac{\partial \psi}{\partial y} + \frac{\partial T}{\partial \eta} \frac{\partial \eta}{\partial y}$$

However, $\frac{\partial \eta}{\partial y}$ is zero and the gradient becomes

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \psi} \frac{\partial \psi}{\partial y}$$

or

$$\frac{\partial T}{\partial y} = \frac{\partial \Delta T}{\partial y} = \frac{\partial \Delta T}{\partial \psi} \frac{\partial \psi}{\partial y}$$

the term $\frac{\partial \Delta T}{\partial \psi}$ is evaluated as

$$\frac{\partial \Delta T}{\partial \psi} = - \frac{\Delta T_B}{(\pi M \eta)^{1/2}} \quad \text{where } \Delta T_B = T_{\text{sat}} - T_B$$

and

$$\frac{\partial \psi}{\partial y} = \sin^2 \phi$$

The temperature gradient thus becomes

$$\frac{\partial T}{\partial y} = \frac{\Delta T_B \sin^2 \phi}{(\pi M \eta)^{1/2}} \quad (2.10)$$

we shall later use this gradient to make an energy balance at the interface.

Vapour Boundary Layer

The boundary layer equations for axisymmetric flow of a fluid past a sphere are as follows [16].

$$\frac{\partial (ur)}{\partial x} + \frac{\partial (vr)}{\partial y} = 0 \quad (2.11)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho_v} \frac{\partial P}{\partial x} + \frac{\mu_v}{\rho_v} \frac{\partial^2 u}{\partial y^2} \quad (2.12)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_v \frac{\partial^2 T}{\partial y^2} \quad (2.13)$$

These are the equations of conservation of mass, momentum and energy, respectively; r , x and y are measured as

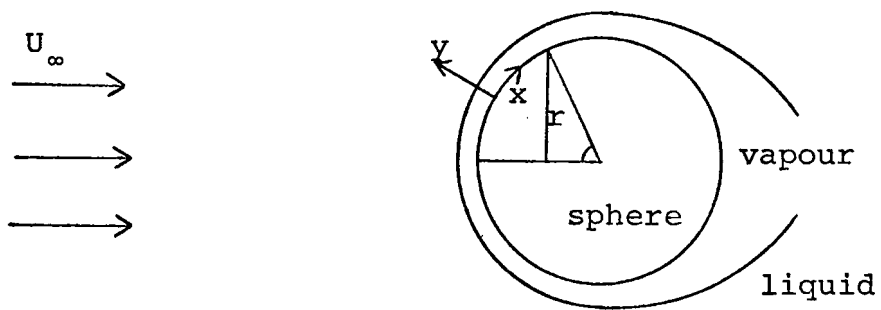


Fig. 2 Coordinate system for the vapour boundary layer.

shown in Fig. 2, u is the velocity in x -direction and v in y -direction. P is the pressure. These equations assume that vapour film thickness is much smaller than the sphere radius.

The following additional assumptions are made:

1. The vapour is incompressible.
2. The sphere surface is isothermal.
3. The average vapour temperature can be taken as the arithmetical mean of the T_{wall} and T_{sat} and the vapour properties can be evaluated at this temperature.
4. The liquid-vapour interface is continuous and smooth.
5. Inertia effects and energy convection effects in the vapour film can be neglected.
6. Heat is transferred across the vapour film only by conduction; i.e., radiative heat transfer is neglected.
7. The velocity of the liquid-vapour interface can be calculated from potential flow theory.
8. The liquid-vapour interface is at the saturation temperature.

The applicable boundary conditions are

$$\begin{aligned} y = 0 & \quad u = 0 & \quad T = T_w \\ y = \delta & \quad u = u(x) & \quad T = T_{sat} \end{aligned} \quad (2.14)$$

Where δ is the film thickness and $u(x) = 3/2 U_{\infty} \sin \phi$. The assumption that inertia effects and energy convection in the vapour boundary layer are negligible enables one to write the momentum and energy equations as

$$\frac{\partial P}{\partial x} = \mu_v \frac{\partial^2 u}{\partial y^2} \quad (2.15)$$

and

$$\frac{\partial^2 T}{\partial y^2} = 0 \quad (2.16)$$

The solution to the last equation is

$$T = C_1 y + C_2 \quad (2.17)$$

Evaluation of constants C_1 and C_2 from the boundary conditions, Equation (2.14), gives

$$T = T_w + (T_{sat} - T_w) y / \delta \quad (2.18)$$

Rearranging,

$$\frac{T - T_{sat}}{T_w - T_{sat}} = 1 - y / \delta \quad (2.19)$$

The solution to the momentum equation is

$$u = \frac{1}{2\mu_v} \frac{\partial P}{\partial x} y^2 + C_3 y + C_4 \quad (2.20)$$

C_4 is zero from the condition that $u = 0$ at $y = 0$.

Evaluation of C_3 gives

$$u = \frac{3}{2} U_\infty (\sin\phi) \frac{y}{\delta} + \frac{1}{2\mu_v} \frac{\partial P}{\partial x} (y^2 - y\delta) \quad (2.21)$$

If it is assumed that the pressure in the liquid layer is impressed upon the vapour layer, the pressure gradient term $\frac{\partial P}{\partial x}$ can be replaced by a velocity gradient term. Neglecting height changes, Bernoulli's equation for the vapour is

$$p_0 = p + \frac{1}{2} \rho_v u^2 = \text{const.}$$

differentiating with respect to x ,

$$\frac{\partial P}{\partial x} = - \rho_v u \frac{\partial u}{\partial x}$$

where u is the velocity at liquid-vapour interface given by

$$u = \frac{3}{2} U_\infty \sin\phi$$

Differentiating this with respect to ϕ and making use of $x = R\delta\phi$, one obtains for the pressure gradient

$$\frac{\partial P}{\partial x} = - \left(\frac{3}{2}\right)^2 \rho_v \frac{U_\infty^2}{R} \cos\phi \sin\phi \quad (2.22)$$

Substituting this into the expression for velocity, Equation (2.21) gives the velocity distribution in the vapour film as

$$u = \frac{3}{2} U_{\infty} \sin \phi \left[\frac{y}{\delta} + \frac{3}{4} \frac{\rho_v U_{\infty}}{\mu_v R} (\cos \phi) (y\delta - y^2) \right] \quad (2.23)$$

The average vapour velocity in the film is obtained from

$$\begin{aligned} \bar{u} &= \frac{1}{\delta} \int_0^{\delta} u \, dy \\ &= \frac{1}{\delta} \int_0^{\delta} \left[\frac{3}{2} U_{\infty} \sin \phi \left(\frac{y}{\delta} + \frac{3}{4} \frac{\rho_v U_{\infty}}{\mu_v R} (\cos \phi) (y\delta - y^2) \right) \right] dy \end{aligned}$$

Evaluation of this integral gives

$$\bar{u} = \frac{1}{\delta} \left[\frac{3}{2} U_{\infty} \sin \phi \left\{ \frac{3}{4} \frac{\rho_v U_{\infty}}{\mu_v R} \cos \phi \frac{\delta^3}{6} + \frac{\delta}{2} \right\} \right] \quad (2.24)$$

Thus we have solved for the temperature and velocity profiles.

CHAPTER III

ENERGY BALANCE AND NUMERICAL SOLUTION FOR NUSSELT NUMBERS

By writing an energy balance on a differential element of the vapour film shown in Figure 3, one obtains,

$$dq_c = dq_{vap} + dq_B \quad (3.1)$$

This equation states that the energy conducted through the vapour film arrives at the interface; is used partially to form vapour and is partially transferred to the bulk liquid. If the bulk liquid is at saturation temperature then all the energy arriving at the liquid-vapour interface goes into forming vapour and the last term dq_B is not necessary. The energy used to vaporize liquid at the liquid-vapour interface can be written as the product of the effective heat of vaporization and the increase in the mass flow rate in the differential film element.

$$dq_{vap} = \lambda' dw \quad (3.2)$$

The increase in the mass flow rate in the film element must come from the vapour which is formed at the liquid-vapour interface. Bromley [13] showed for laminar film condensation that the assumption of a linear temperature profile is valid

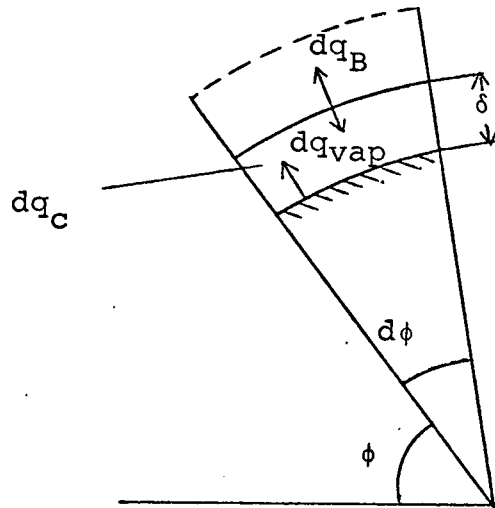


Fig. 3 Energy balance on an elemental area of sphere surface.

as long as an 'effective' latent heat of vaporization is used in an energy balance on an element of the film. He also used this concept in film boiling analyses with good success. The expression for the effective latent heat of vaporization λ' as derived by Bromley is

$$\lambda' = \lambda \left(1 + \frac{.4 C_p \Delta T}{\lambda} \right) \quad (3.3)$$

where $\Delta T = T_w - T_{sat}$. This expression accounts for the heat capacity of the superheated vapour.

The increase in mass flow rate dw can be written as

$$dw = d(\rho_v A_c \bar{u}) \quad (3.4)$$

where A_c is the flow cross-section of the film. A_c can be written in terms of the film thickness and angular position as

$$A_c = 2\pi R \delta \sin \phi$$

The average velocity $\bar{u}$ as previously determined by Equation (2.24) is now used to write

$$dw = d[\rho_v 2\pi R (\sin \phi) \frac{3}{2} U_\infty \sin \phi \left\{ \frac{3}{4} \frac{\rho_v U_\infty}{\mu_v R} (\cos \phi) \frac{\delta^3}{6} + \frac{\delta}{2} \right\}] \quad (3.5)$$

The energy balance, Equation (3.1) becomes

$$\frac{k_v \Delta T}{\delta} dA = \lambda' 2\pi R \rho_v \frac{d}{d\phi} \left[\frac{3}{2} U_\infty \sin\phi \left\{ \frac{3}{4} \frac{\rho_v U_\infty}{\mu_v R} (\cos\phi) \frac{\delta^3}{6} + \frac{\delta}{2} \right\} \right] d\phi$$

$$+ \frac{k_\ell \sin^2 \phi}{\sqrt{\pi M \eta}} \Delta T_B dA$$

where dA refers to the area element on the sphere surface adjacent to the film element. dA can be written as

$$dA = 2\pi R^2 \sin\phi \, d\phi$$

Substituting this into the above equation and simplifying, one obtains,

$$\frac{k_v \Delta T}{\delta} = \frac{3}{2} \frac{\rho_v U_\infty \lambda'}{R \sin\phi} \frac{d}{d\phi} \left[\sin^2 \phi \left\{ \frac{3}{4} \frac{\rho_v U_\infty}{\mu_v R} (\cos\phi) \frac{\delta^3}{6} + \frac{\delta}{2} \right\} \right]$$

$$+ \frac{k_\ell \sin^2 \phi \, \Delta T_B}{\sqrt{\pi M \eta}} \quad (3.6)$$

This differential equation describes the variation of film thickness δ with angle ϕ . A solution of this equation for δ permits one to compute the heat transfer rate at any angle ϕ .

The equation is put in the derivative form as follows:

$$\frac{k_v \Delta T}{\delta} = \frac{3}{2} \frac{\rho_v U_\infty \lambda'}{R \sin\phi} \frac{d}{d\phi} \left[\sin^2 \phi \left(\frac{3}{4} \frac{\rho_v U_\infty}{\mu_v R} (\cos\phi) \frac{\delta^3}{6} + \frac{\delta}{2} \right) \right]$$

$$+ \frac{k_\ell \sin^2 \phi \, \Delta T_B}{\sqrt{\pi M \eta}}$$

Let $\frac{3\rho_V U_\infty \lambda'}{2R} = M_1$ and $\frac{1}{8} \frac{\rho_V U_\infty}{\mu_V R} = M_2$

$$\begin{aligned}
 \frac{k_V \Delta T}{\delta} &= \frac{M_1}{\sin \phi} \frac{d}{d\phi} [M_2 \delta^3 \sin^2 \phi \cos \phi + \frac{\delta}{2} \sin^2 \phi] \\
 &\quad + \frac{k_\ell \sin^2 \phi \Delta T_B}{\sqrt{\pi M \eta}} \\
 &= \frac{M_1}{\sin \phi} [M_2 \{ \sin^2 \phi \cos \phi 3\delta^2 \frac{d\delta}{d\phi} + \delta^3 \frac{d}{d\phi} (\sin^2 \phi \cos \phi) \} \\
 &\quad + \frac{1}{2} \{ 2\delta \sin \phi \cos \phi + \sin^2 \phi \frac{d\delta}{d\phi} \}] + \frac{k_\ell \sin^2 \phi \Delta T_B}{(\pi M \eta)^{1/2}} \\
 &= \frac{M_1}{\sin \phi} [M_2 \{ \sin^2 \phi \cos \phi 3\delta^2 \frac{d\delta}{d\phi} + \delta^3 (2\sin \phi \cos^2 \phi + \sin^2 \phi \\
 &\quad \cdot (-\sin \phi)) \} + \delta \sin \phi \cos \phi + \frac{\sin^2 \phi}{2} \frac{d\delta}{d\phi}] + \frac{k_\ell \sin^2 \phi \Delta T_B}{(\pi M \eta)^{1/2}}
 \end{aligned}$$

Solving for $\frac{d\delta}{d\phi}$ one obtains

$$\frac{d\delta}{d\phi} = \frac{\frac{k_V \Delta T}{\delta} + M_1 M_2 \delta^3 (1 - 3\cos^2 \phi) - M_1 \delta \cos \phi - k_\ell \frac{\sin^2 \phi \Delta T_B}{(\pi M \eta)^{1/2}}}{M_1 \sin \phi \{ 3M_2 \delta^2 \cos \phi + \frac{1}{2} \}}$$

$$\frac{d\delta}{d\phi} = \frac{\frac{k_V \Delta T}{M_1 \delta \sin \phi} + \frac{M_2 \delta^3 (1 - 3\cos^2 \phi)}{\sin \phi} - \frac{\delta \cos \phi}{\sin \phi} - \frac{k_\ell \sin \phi \Delta T_B}{M_1 \sqrt{\pi M \eta}}}{(3M_2 \delta^2 \cos \phi + \frac{1}{2})}$$

(3.7)

Let $C_1 = \frac{k_V \Delta T}{M_1}$, $C_2 = M_2$, $C_3 = \frac{k_\ell \Delta T_B}{M_2 \sqrt{\pi M}}$

then we have

$$\frac{d\delta}{d\phi} = \frac{\frac{C_1}{\delta \sin \phi} + \frac{C_2 \delta^3 (1 - 3 \cos^2 \phi)}{\sin \phi} - \frac{\delta \cos \phi}{\sin \phi} - \frac{C_3 \sin \phi}{\sqrt{n}}}{(3C_2 \delta^2 \cos \phi + \frac{1}{2})} \quad (3.8)$$

A solution to this equation in closed form was not found. However, the equation can be solved numerically for given values of constants, C_1 , C_2 and C_3 .

The fourth order system frequently called the 'Runge-Kutta-Simpson' [17] method was employed for the numerical solution of the problem. The method in brief is as follows:

Let $\frac{d\delta}{d\phi} = f(\delta, \phi)$ at any known point n , then find

$m_1 = h f(\delta_n, \phi_n)$ where h is an increment in ϕ .

Then find

$$m_2 = h f(\phi_n + \frac{h}{2}, \delta_n + \frac{m_1}{2}),$$

$$m_3 = h f(\phi_n + \frac{h}{2}, \delta_n + \frac{m_2}{2}),$$

$$m_4 = h f(\phi_n + h, \delta_n + m_3).$$

δ_{n+1} is then computed as,

$$\delta_{n+1} = \delta_n + \frac{1}{6} \{m_1 + 2m_2 + 2m_3 + m_4\}$$

The process is then carried on to δ_{n+2} .. and so on.

We can thus compute the vapour thickness δ for the entire periphery and then determine the heat transfer rates. However, since the differential Equation (3.8) contains three constants C_1 , C_2 and C_3 , the results cannot be put in a convenient form. This difficulty is overcome by making a further assumption that the velocity profile in the vapour film is linear. Because of the very small vapour thickness, this assumption is seen to introduce very little error in the final results.

The vapour velocity at any distance y is now given as

$$u = \frac{3}{2} U_{\infty} (\sin \phi) \frac{y}{\delta} \quad (3.9)$$

The average velocity becomes

$$\bar{u} = \frac{3}{4} U_{\infty} \sin \phi \quad (3.10)$$

The mass of vapour generated in the differential volume of vapour is

$$\begin{aligned} dw &= d(\rho_v \bar{u} A_c) \\ &= d\left(\rho_v \frac{3}{4} U_{\infty} \sin \phi \cdot 2\pi R \sin \phi \cdot \delta\right) \quad (3.11) \end{aligned}$$

Energy per unit area of sphere surface required to generate this much vapour is,

$$\frac{\lambda' U_{\infty}}{2\pi R^2 \sin\phi d\phi} d\left(\rho_V \frac{3}{4} \cdot 2\pi R \sin^2\phi \cdot \delta\right)$$

or

$$\frac{3}{4} \frac{\lambda' U_{\infty} \rho_V}{R \sin\phi} \frac{d}{d\phi} (\delta \sin^2\phi)$$

The energy balance now becomes

$$\frac{k_V \Delta T}{\delta} = \frac{3}{4} \frac{\lambda' U_{\infty} \rho_V}{R \sin\phi} \frac{d}{d\phi} (\delta \sin^2\phi) + \frac{k_L \sin^2\phi \Delta T_B}{\sqrt{\pi M \eta}}$$

$$\text{Let } \frac{3}{4} \frac{\lambda' U_{\infty} \rho_V}{R} = Z$$

$$\begin{aligned} \text{then } \frac{k_V \Delta T}{\delta} &= \frac{Z}{\sin\phi} \frac{d}{d\phi} (\delta \sin^2\phi) + \frac{k_L \sin^2\phi \Delta T_B}{\sqrt{\pi M \eta}} \\ &= \frac{Z}{\sin\phi} \left\{ \delta 2 \sin\phi \cos\phi + \sin^2\phi \frac{d\delta}{d\phi} \right\} + \frac{k_L \sin^2\phi \Delta T_B}{\sqrt{\pi M \eta}} \\ &= 2Z\delta \cos\phi + Z \sin\phi \frac{d\delta}{d\phi} + \frac{k_L \sin^2\phi \Delta T_B}{\sqrt{\pi M \eta}} \end{aligned}$$

$$\text{or } Z \sin\phi \frac{d\delta}{d\phi} = \frac{k_V \Delta T}{\delta} - 2Z\delta \cos\phi - \frac{k_L \sin^2\phi \Delta T_B}{\sqrt{\pi M \eta}}$$

$$\text{or } \frac{d\delta}{d\phi} = \frac{k_V \Delta T}{Z\delta \sin\phi} - \frac{2\delta \cos\phi}{\sin\phi} - \frac{k_L \sin\phi \Delta T_B}{Z\sqrt{\pi M \eta}}$$

By putting back the value of Z and for $M = \frac{2}{3} \frac{R\alpha}{U_{\infty}}$,

one obtains

$$\frac{d\delta}{d\phi} = \frac{4k_v R \Delta T}{3\delta \lambda' U_{\infty} \rho_v \sin\phi} - \frac{2\delta \cos\phi}{\sin\phi} - \left[\frac{8}{3} \frac{\rho_l C_{pl} R k_l}{\pi U_{\infty}} \right]^{1/2} \cdot \frac{\Delta T_B}{\lambda' \rho_v} \frac{\sin\phi}{\sqrt{\eta}} \quad (3.12)$$

Upon dividing by D to put the equation in dimensionless form, one gets

$$\frac{d(\delta/D)}{d\phi} = \frac{4k_v R \Delta T}{3 \frac{\delta}{D} \lambda' U_{\infty} \rho_v D^2 \sin\phi} - 2 \frac{\delta}{D} \frac{\cos\phi}{\sin\phi} - \left[\frac{8}{3} \frac{\rho_l C_{pl} D k_l}{3 \times 2 \pi U_{\infty} D^2} \right]^{1/2} \cdot \frac{\Delta T_B}{\lambda' \rho_v} \frac{\sin\phi}{\sqrt{\eta}}$$

Let $\delta/D = \delta'$

$$\frac{d\delta'}{d\phi} = \frac{2k_v \Delta T}{\delta' 3 \lambda' U_{\infty} \rho_v D \sin\phi} - \frac{2\delta' \cos\phi}{\sin\phi} - \left[\frac{4}{3} \frac{\rho_l C_{pl} k_l}{\pi U_{\infty} D} \right]^{1/2} \cdot \frac{\Delta T_B}{\lambda' \rho_v} \frac{\sin\phi}{\sqrt{\eta}} \quad (3.13)$$

$$\text{Let } k_1 = \frac{2}{3} \frac{k_v \Delta T}{\lambda' U_{\infty} \rho_v D} \quad (\text{Dimensionless}) \quad (3.14)$$

$$k_2 = \left[\frac{4}{3} \frac{\rho_l C_{pl} k_l}{\pi U_{\infty} D} \right]^{1/2} \frac{\Delta T_B}{\lambda' \rho_v} \quad (\text{Dimensionless}) \quad (3.15)$$

so that Equation (3.13) becomes

$$\frac{d\delta'}{d\phi} = \frac{k_1}{\delta' \sin \phi} - \frac{2\delta' \cos \phi}{\sin \phi} - \frac{k_2 \sin \phi}{\sqrt{\eta}} \quad (3.16)$$

$\sqrt{\eta}$ is evaluated as

$$\begin{aligned} \sqrt{\eta} &= \left[\int_0^{\phi} \sin^3 \phi d\phi \right]^{1/2} \\ &= \left[\left\{ -\frac{1}{3} \cos \phi + 2 \right\} \right]_0^{\phi}^{1/2} \\ &= \left(\frac{2}{3} - \cos \phi + \frac{\cos^3 \phi}{3} \right)^{1/2} \end{aligned}$$

We have finally,

$$\frac{d\delta'}{d\phi} = \frac{k_1}{\delta' \sin \phi} - \frac{2\delta' \cos \phi}{\sin \phi} - \frac{k_2 \sin \phi}{\left(\frac{2}{3} - \cos \phi + \frac{\cos^3 \phi}{3} \right)^{1/2}} \quad (3.17)$$

A solution to this equation by the previously described Runge-Kutta-Simpson method brings out an interesting property of the equation; that for a given k_1 and k_2 , it has a unique limit cycle as the solution. In other words the solution does not depend upon the initial value of δ' . Whatever value of δ' is chosen as an initial condition, the solution rapidly approaches the fixed limit cycle. Many nonlinear systems (such as the well-known Vander Pole Equation [18,19]) are known to exhibit such behaviour where the

solution is fixed due to the inherent properties of the system.

To illustrate this, solutions were obtained for a set of k_1 and k_2 values for initial values of $\delta = .00001$ and $\delta = .02$ (at $\phi = 1.5^\circ$). The results are in Table 1. In spite of the fact that the initial values are so widely different, both the solutions coincide just within $1/2^\circ$ increase in the angle, namely, at $\phi = 2.15^\circ$ both solutions exactly coincide and continue to be coincident.

A plot of δ v/s ϕ is shown on Figure 4 for a given system. As expected, δ uniformly increases with the angle ϕ .

Although the initial value of δ does not affect the results at all, this value is found by using the condition that $\frac{d\delta'}{d\phi} = 0$ at $\phi = 0$. To program this statement, any high value, say 1, is given to δ' at $\phi = 2.5^\circ$ and the slope $\frac{d\delta'}{d\phi}$ is computed. If this slope is negative then δ' is made $= .1$ and again the slope is found. If again it is negative it is multiplied by $.1$ and so on till the slope is positive. The value of δ' is then slowly incremented in the interval where it changes sign. We thus carry on the process say to 6 significant figures and then select the value of δ . This amounts to selecting a value of δ' such that $\frac{d\delta'}{d\phi} \approx 0$.

Having thus obtained the values of dimensionless

TABLE 1

ϕ (Degrees)	Initial Value of $\delta=1.239 \times 10^{-5}$ Ft. (at $\phi = 1.5^\circ$)	Initial Value of $\delta=.02$ Ft. (at $\phi = 1.5^\circ$)
	δ (Ft.)	δ (Ft.)
1.55	.00001062	.01735349
1.6	.00001088	.01498746
1.65	.00001110	.01286114
1.7	.00001129	.01094155
1.75	.00001145	.00920163
1.8	.00001159	.00761886
1.85	.00001170	.00617437
1.9	.00001181	.00485220
1.95	.00001189	.00363885
2.0	.00001197	.00252289
2.05	.00001204	.00149508
2.1	.00001210	.00055040
2.15	.00001215	.00001215
2.2	.00001220	.00001220
2.25	.00001224	.00001224
2.3	.00001228	.00001228
2.35	.00001231	.00001231

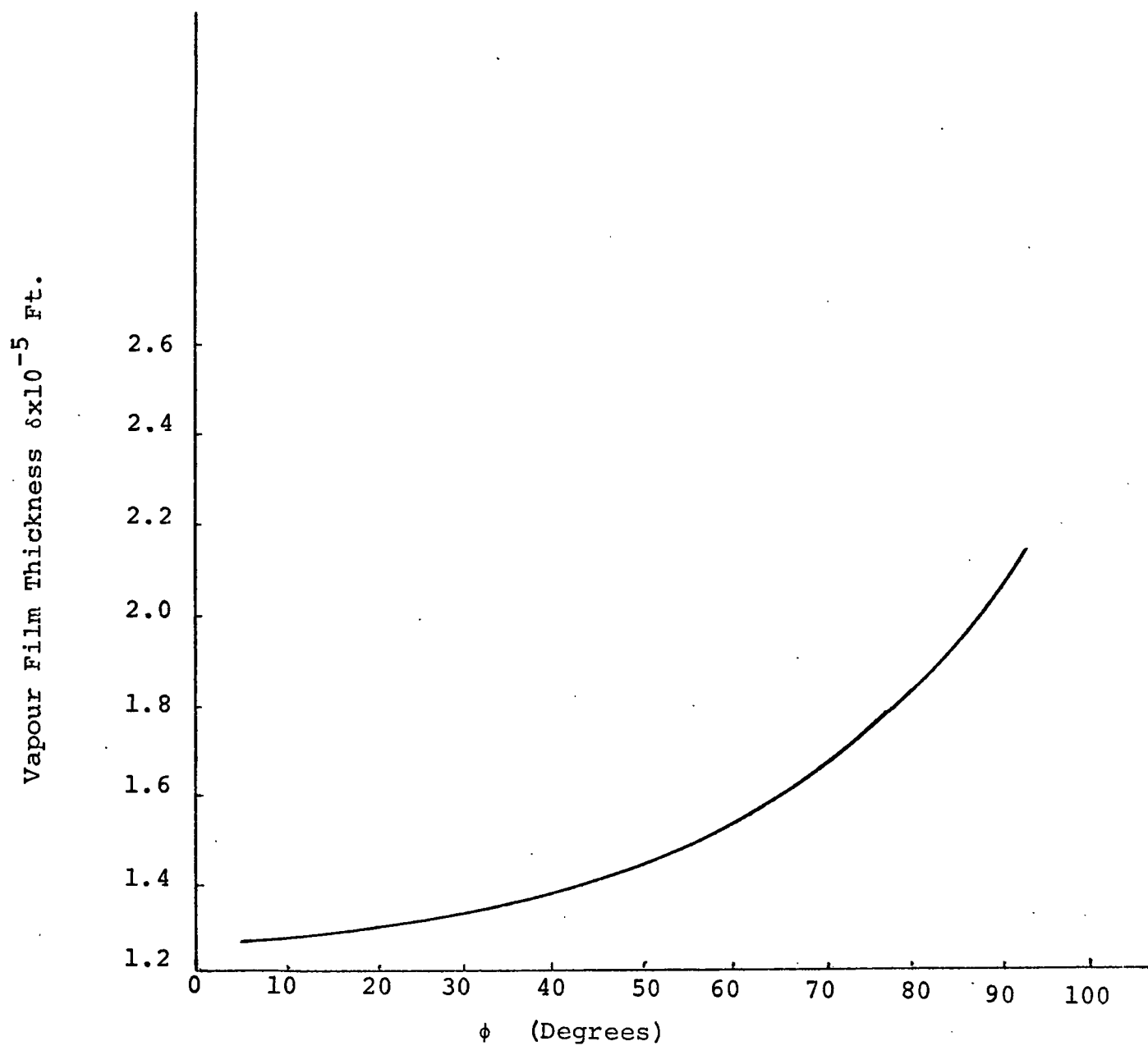


Fig. 4 Plot between δ and ϕ .

vapour film thickness δ/D for the whole sphere, we can proceed to compute the dimensionless Nusselt number as follows:

Heat transfer across the vapour film from elemental area dA subtended by angle $d\phi$ (Figure 5)

$$\begin{aligned} dq &= \frac{k_v \Delta T}{\delta} dA \\ &= \frac{k_v \Delta T}{\delta} 2\pi R^2 \sin\phi d\phi \end{aligned}$$

This quantity is calculated for a very small $d\phi$ say $.01^\circ$. The value of δ is assumed to be constant over this increment in angle. Further, we shall neglect the heat transfer beyond $\phi = 160^\circ$.

Total heat transfer is then written as

$$\begin{aligned} Q &= \sum_{\phi=0}^{\phi=160^\circ} \frac{k_v \Delta T}{\delta} 2\pi R^2 \sin\phi d\phi \\ &= k_v \Delta T 2\pi R^2 \sum_{\phi=0}^{\phi=160^\circ} \frac{1}{\delta} \sin\phi d\phi \end{aligned}$$

or

$$\frac{Q}{\Delta T (4\pi R^2)} = \frac{k_v}{2} \sum_{\phi=0}^{\phi=160} \frac{1}{\delta} \sin\phi d\phi$$

Noting that $\frac{Q}{\Delta T (4\pi R^2)}$ is the overall heat transfer coefficient, denoted by H , one obtains,

$$\frac{H}{k_v} = \frac{1}{2} \sum_{\phi=0}^{\phi=160} \frac{\sin\phi d\phi}{\delta}$$

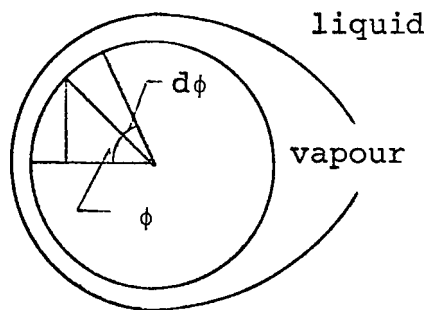


Fig. 5 Heat flux from a differential area on sphere surface.

$$k_1 = \frac{2}{3} \frac{k_v \Delta T}{\lambda' U_\infty \rho_v D}$$

$$k_2 = \left(\frac{4}{3} \frac{\rho_\ell C_{p\ell} k_\ell}{\pi U_\infty D} \right)^{1/2} \frac{\Delta T_B}{\lambda' \rho_v}$$

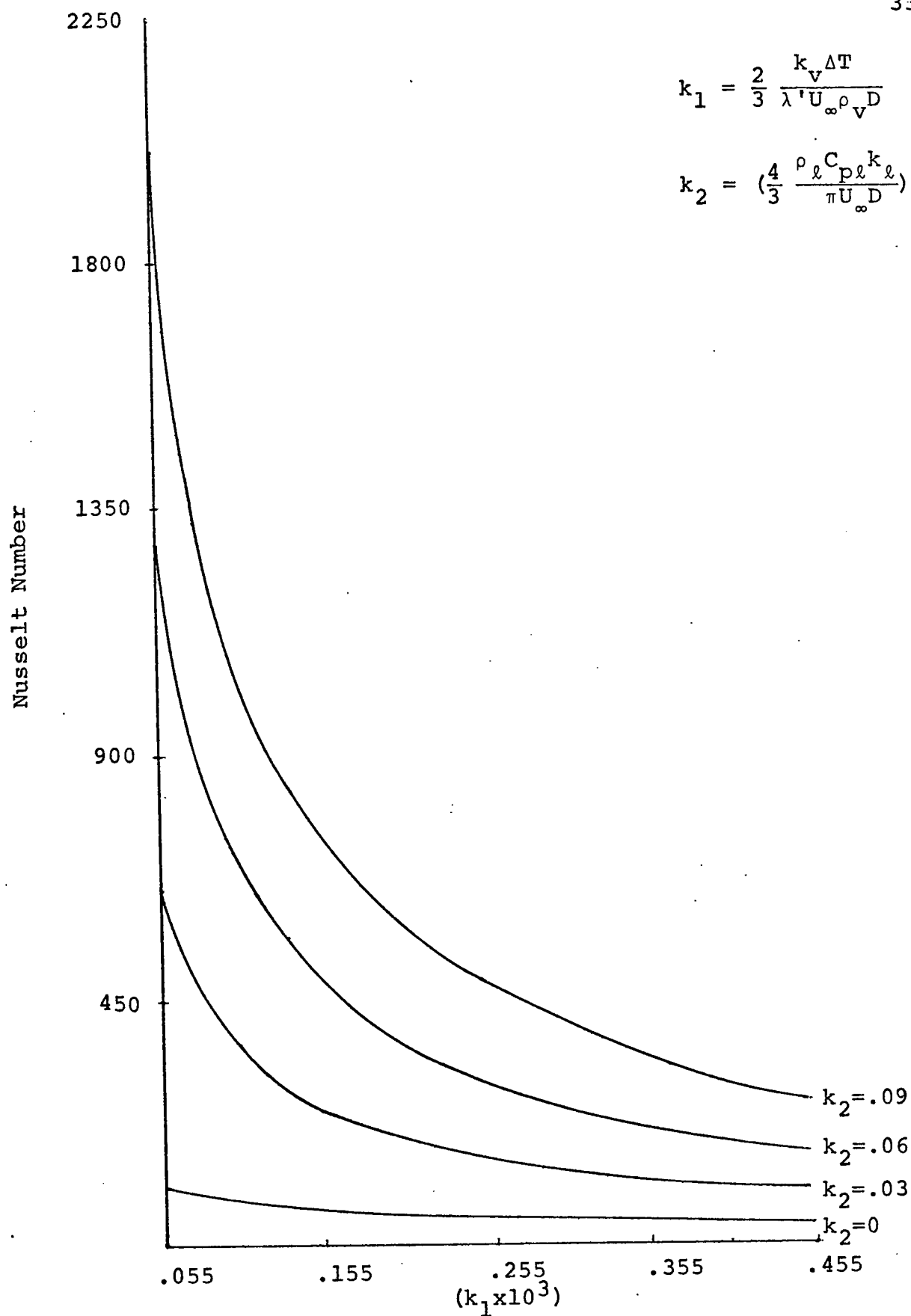


Fig. 6 Heat transfer results.

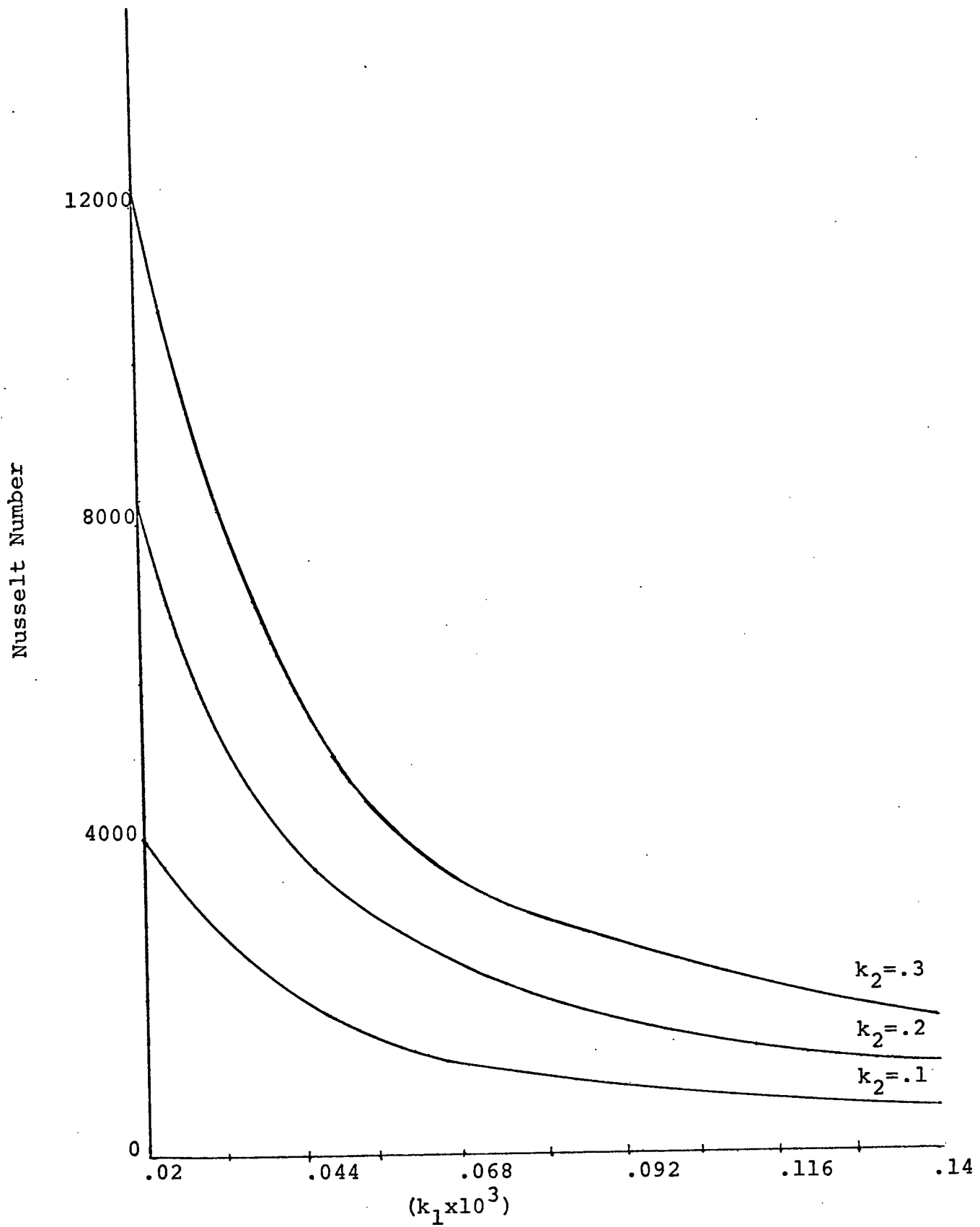


Fig. 7 Heat transfer results (cont.).

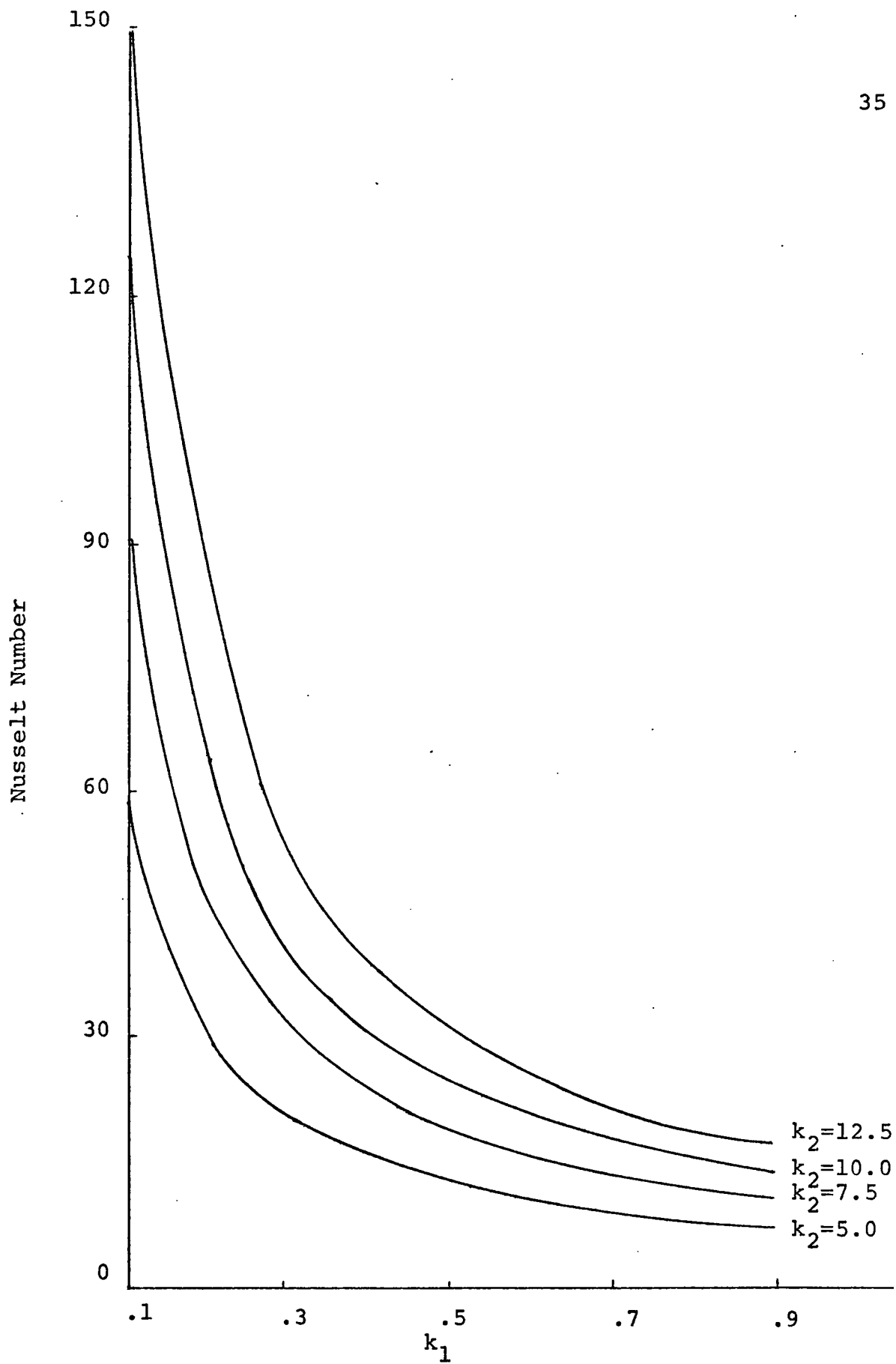


Fig. 8 Heat transfer results (cont.)

or

$$\frac{HD}{k_v} = \frac{1}{2} \sum_{\phi=0}^{\phi=160} \frac{\sin \phi d\phi}{\delta/D}$$

or

$$Nu = \frac{1}{2} \sum_{\phi=0}^{\phi=160} \frac{\sin \phi d\phi}{\delta'} \quad (3.18)$$

Thus we have been able to obtain the Nusselt numbers starting from dimensionless numbers k_1 and k_2 .

Computations were made for the Nusselt numbers for a limited range of numbers k_1 and k_2 and results plotted on Figures 6-8.

The Roughness Model

In our treatment so far it was assumed that the sphere surface was absolutely smooth i.e., it did not have any surface irregularities. But an ordinary machine finished sphere would definitely be expected to have some kind of surface roughness and if this roughness size is comparable to the vapour film thickness, it could interfere in the formation of a continuous and smooth vapour film. Such a size of roughness was indeed assumed, so that it would not allow the growth of a continuous vapour film over the sphere.

To deal with the problem quantitatively, it is necessary to assume some sort of roughness distribution. The roughness assumed was in the form of two dimensional discs

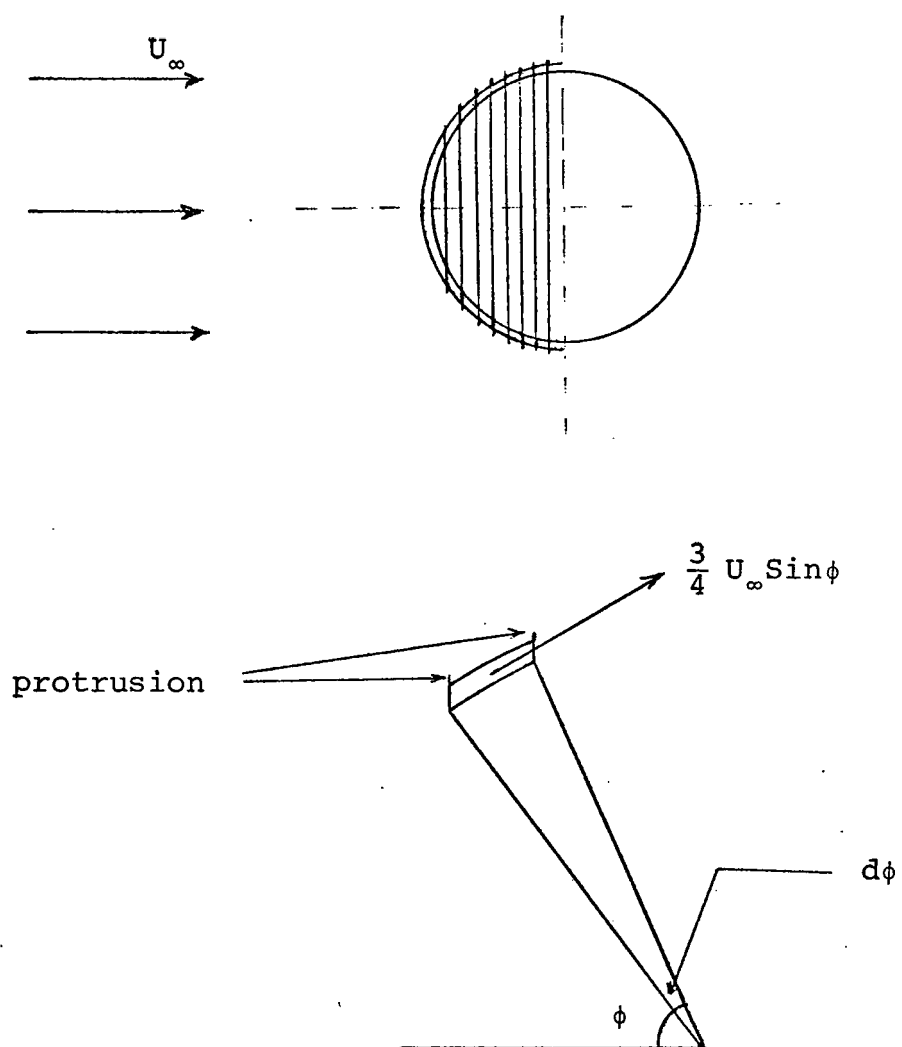


Fig. 9 The model used for roughness treatment.

having zero thickness placed on the sphere (Figure 9) in a direction perpendicular to the motion of the sphere and at a fixed distance apart. One can conceive the situation as that of film boiling of liquid contained in the individual 'troughs' formed by the consecutive protrusions. The problem then becomes one of making an energy balance over an individual 'trough' and computing the vapour film thickness in that region.

Let $d\phi$ be the angle subtended by the two consecutive protrusions at the center of the sphere. We will consider the energy balance over the element $Rd\phi$ which makes an angle ϕ with the direction of sphere motion. If $d\phi$ is small we can assume that vapour film thickness δ is constant over it. From Equation (3.10) the average tangential velocity of vapour is $\frac{3}{4} U_{\infty} \sin\phi$. Now since the vapour motion in the tangent direction is restricted because of the presence of the protrusion, it would be reasonable to expect that the vapour flows along the direction of protrusion and escapes into the bulk liquid. It is premised here that the vapour flows along the direction of protrusion with a velocity which is the component of tangential velocity in that direction.

The amount of vapour generated over the elemental area $Rd\phi$ is

$$\begin{aligned} dw &= \rho_v V dA_c \\ &= \rho_v \cdot \frac{3}{4} U_{\infty} \sin\phi \cos\phi \cdot 2\pi R \sin\phi Rd\phi \end{aligned}$$

The energy required to generate this vapour is

$$dq_{\text{vap}} = \lambda' \cdot \rho_v \cdot \frac{3}{4} U_{\infty} \sin\phi \cos\phi \cdot 2\pi R \sin\phi R d\phi$$

Energy conducted through the vapour layer is $dq_c =$

$$\frac{k_v \Delta T}{\delta} \cdot 2\pi R \sin\phi R d\phi.$$

Energy conducted into the bulk liquid is $dq_b =$

$$k_l \left(\frac{\partial T}{\partial Y} \right)_l dA \text{ where from Equation (2.10)}$$

$$\left(\frac{\partial T}{\partial Y} \right)_l = \frac{\sin^2 \phi \Delta T_B}{(M\pi\eta)^{1/2}}$$

Writing an energy balance (Equation 3.1)

$$dq_c = dq_{\text{vap}} + dq_b$$

and substituting for the respective terms one obtains

$$\frac{k_v \Delta T}{\delta} dA = \lambda' \frac{3}{4} \rho_v U_{\infty} \sin\phi \cos\phi dA + \frac{k_l \sin^2 \phi \Delta T_B}{\sqrt{\pi M \eta}} dA$$

where $dA = 2\pi R \sin\phi \cdot R d\phi$

or

$$\frac{1}{\delta/D} = \frac{3}{8} \frac{\lambda' U_{\infty} D \rho_v \sin 2\phi}{k_v \Delta T} + \left[\frac{3 k_l U_{\infty} \rho_l C_{pl} D}{\pi} \right]^{1/2}$$

$$\cdot \frac{\Delta T_B}{\Delta T} \frac{\sin^2 \phi}{k_v (\eta)^{1/2}}$$

Let

$$\frac{3}{8} \frac{\lambda' U_{\infty} D \rho_v}{k_v \Delta T} = N_1 \quad (\text{Dimensionless})$$

$$\left[\frac{3k_{\ell} U_{\infty} \rho_{\ell} C_{p\ell} D}{\pi} \right]^{1/2} \frac{\Delta T_B}{\Delta T_{k_V}} = N_2 \quad (\text{Dimensionless})$$

and $\delta/D = \delta'$ (Dimensionless)

So that one has

$$\frac{1}{\delta'} = N_1 \sin 2\phi + \frac{N_2 \sin^2 \phi}{\left(\frac{2}{3} - \cos \phi + \frac{\cos^3 \phi}{3} \right)^{1/2}}$$

The dimensionless vapour film thickness δ' over every individual 'trough' is computed from the above equation and Nusselt number is found as in Equation (3.18), namely,

$$Nu = \frac{1}{2} \int_{\phi=0}^{\phi=90} \frac{\sin \phi \cdot d\phi}{\delta'}$$

Thus for every set of dimensionless numbers N_1 and N_2 a Nusselt number can be computed.

To determine how the results compare with the smooth sphere solution already obtained, a specific system was taken where

T_w	T_{sat}	Dia.	U_{∞}	λ	ρ_v	k_v	$C_{p\ell}$	ρ_{ℓ}
$^{\circ}\text{F}$	$^{\circ}\text{F}$	In.	$\frac{\text{Ft}}{\text{Sec}}$	$\frac{\text{BTU}}{\text{lbm}^{\circ}\text{F}}$	$\frac{\text{lbm}}{\text{Ft}^3}$	$\frac{\text{BTU}}{\text{Hr.Ft.}^{\circ}\text{F}}$	$\frac{\text{BTU}}{\text{lbm}^{\circ}\text{F}}$	$\frac{\text{lbm}}{\text{Ft}^3}$
1300	212	.5	.02	1178	.02	.032	1	62.4

so that $N_1 = .77$ and $N_2 = 22.2$. The Nusselt number obtained was 18.6 using the protrusion model, whereas for the same system for smooth sphere (with $k_1 = .322$ and $k_2 = 6.45$) the result was a Nusselt number of 15.6 (Figure 8), showing an increase of about 20%.

CHAPTER IV

NUSSELT NUMBERS FOR CYLINDERS

The Nusselt number for forced convection flow over a cylinder is given [20] as

$$\text{Nu}(x) = .57 \text{ Re}(x)^{1/2} \text{ Pr}^{.4} \quad (4.1)$$

To bring it in conformance with the expression used for a sphere, let us assume a Prandtl number dependence of .5th power (for Prandtl number range of water, this will introduce little error) then

$$\begin{aligned} \text{Nu}(x) &= .57 \text{ Re}(x)^{1/2} \text{ Pr}^{.5} \\ &= .57 \left(\frac{V x \rho_l}{\mu_l} \right)^{.5} \left(\frac{\mu_l C_{pl}}{k_l} \right)^{.5} \\ &= .57 \left(\frac{U_\infty x \rho_l C_{pl}}{k_l} \right)^{.5} \end{aligned}$$

where V is the free stream velocity and with the assumption of potential flow is given for a cylinder as (Figure 10a)

$$V = 2U_\infty \sin\phi$$

and also

$$x = R\phi$$

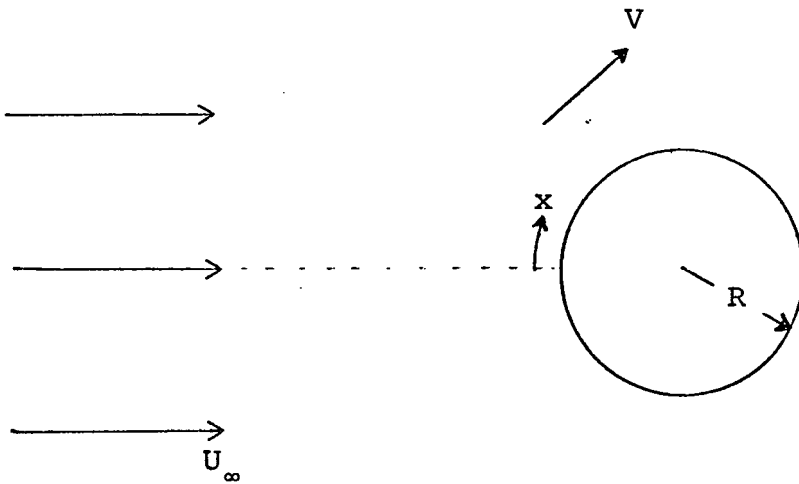


Fig. 10a Potential flow around a cylinder.

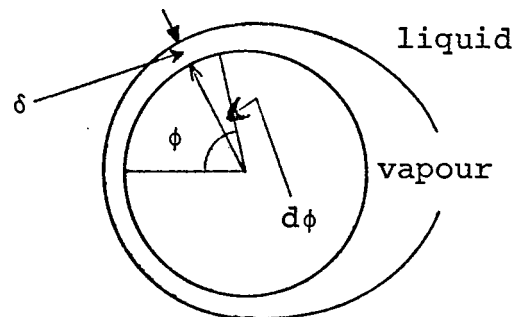


Fig. 10b Energy balance over an elemental area on the cylinder.

thus

$$\frac{hx}{k_\ell} = .57 \left(\frac{V_{\infty} \rho_\ell C_{p\ell} k_\ell}{k_\ell} \right)^{1/2}$$

$$h = .57 \left(\frac{V_{\infty} \rho_\ell C_{p\ell} k_\ell}{x} \right)^{1/2}$$

$$= .57 \left(\frac{2U_{\infty} \rho_\ell C_{p\ell} k_\ell \sin\phi}{R\phi} \right)^{1/2} \quad (4.2)$$

$$h_{(\phi)} = .57 \left(\frac{4U_{\infty} \rho_\ell C_{p\ell} k_\ell}{D} \right)^{1/2} \sqrt{\frac{\sin\phi}{\phi}}$$

$$= 1.14 \left(\frac{U_{\infty} \rho_\ell C_{p\ell} k_\ell}{D} \right)^{1/2} \sqrt{\frac{\sin\phi}{\phi}} \quad (4.3)$$

Energy Balance

We shall as before assume a linear velocity profile, the velocity varying from zero at the wall to $2U_{\infty} \sin\phi$ at the liquid vapour interface (Figure 10b). The average velocity is therefore

$$\bar{u} = \frac{0 + 2U_{\infty} \sin\phi}{2}$$

$$= U_{\infty} \sin\phi$$

Mass of vapour generated in an elemental volume of vapour is

$$dw = d(\rho \bar{u} A_c)$$

where A_c is the area of flow, so that

$$A_c = \delta \times 1 = \delta,$$

and,

$$dw = d(\rho \delta U_{\infty} \sin\phi)$$

The energy required to generate this vapour is $\lambda' dw$. Heat flux required for vaporization per unit area of cylinder is

$$dq_{\text{vap}} = \frac{\lambda' d(\rho U_{\infty} \delta \sin\phi)}{R d\phi} \quad (R d\phi \times 1 = \text{surface area})$$

$$dq_{\text{vap}} = \frac{\lambda' \rho U_{\infty}}{R} \frac{d}{d\phi} (\delta \sin\phi)$$

Upon writing an energy balance at the interface, one has, heat conducted across vapour film equalling heat required for vaporization plus heat going into the liquid bulk, or

$$\begin{aligned} \frac{k_v \Delta T}{\delta} &= \frac{\lambda' \rho_v U_{\infty}}{R} \frac{d}{d\phi} (\delta \sin\phi) + 1.14 \left[\frac{U_{\infty} \rho_l C_{pl} k_l}{D} \right]^{1/2} \frac{\sqrt{\sin\phi}}{\phi} \Delta T_B \\ &= \frac{\lambda' \rho_v U_{\infty}}{R} (\delta \cos\phi + \sin\phi \frac{d\delta}{d\phi}) + 1.14 \left[\frac{U_{\infty} \rho_l C_{pl} k_l}{D} \right]^{1/2} \frac{\sqrt{\sin\phi}}{\phi} \Delta T_B \end{aligned}$$

Letting $\frac{\lambda' \rho_v U_{\infty}}{R} = E$ one obtains

$$\frac{k_v \Delta T}{\delta} = E \delta \cos\phi + E \frac{d\delta}{d\phi} \sin\phi + 1.14 \left[\frac{U_{\infty} \rho_l C_{pl} k_l}{D} \right]^{1/2} \frac{\sqrt{\sin\phi}}{\phi} \Delta T_B$$

or

$$\frac{d\delta}{d\phi} = \frac{k_v \Delta T}{E \delta \sin\phi} - \frac{\delta \cos\phi}{\sin\phi} - \frac{1.14}{E} \left[\frac{U_{\infty} \rho_l C_{pl} k_l}{D} \right]^{1/2} \frac{\sqrt{1/\sin\phi}}{\phi} \Delta T_B$$

Dividing by D

$$\frac{d(\delta/D)}{d\phi} = \frac{k_v \Delta T}{E \delta_D^2 \sin\phi} - \frac{\delta \cos\phi}{D \sin\phi} - \frac{1.14}{E} \left[\frac{U_\infty \rho_\ell C_{p\ell} k_\ell}{D^3} \right]^{1/2} \sqrt{\frac{1}{\sin\phi \cdot \phi}} \Delta T_B$$

Let

$$\frac{k_v \Delta T}{E D^2} = \frac{k_v \Delta T D/2}{\lambda' \rho_v U_\infty D^2} = \frac{k_v \Delta T}{2 \lambda' \rho_v U_\infty D} = k_1' \text{ (Dimensionless)} \quad (4.5)$$

and

$$\begin{aligned} 1.14 \left[\frac{U_\infty \rho_\ell C_{p\ell} k_\ell}{D^3} \right]^{1/2} \frac{D}{2} \frac{\Delta T_B}{\lambda' \rho_v U_\infty} \\ = .57 \left[\frac{\rho_\ell C_{p\ell} k_\ell}{U_\infty D} \right]^{1/2} \frac{\Delta T_B}{\lambda' \rho_v} = k_2' \text{ (Dimensionless)} \end{aligned} \quad (4.6)$$

Hence

$$\frac{d\delta'}{d\phi} = \frac{k_1'}{\delta' \sin\phi} - \frac{\delta' \cos\phi}{\sin\phi} - \frac{k_2'}{\sqrt{\phi \cdot \sin\phi}} \quad (4.7)$$

We can now numerically solve this equation for δ' as a function of ϕ by using the same method as used for sphere.

Having computed the dimensionless film thickness δ' for all ϕ , we can proceed to determine the Nusselt numbers as follows.

The energy conducted through the vapour from an elemental area subtended by an angle $d\phi$ is

$$dq = \frac{k_v \Delta T}{\delta} R d\phi \quad (4.8)$$

The total heat is then

$$Q = 2 k_v \Delta T R \int_{\phi=0}^{\phi=160} \frac{d\phi}{\delta} \quad (4.9)$$

if H is the overall coefficient of heat transfer then

$$H(2\pi R)\Delta T = 2k_v \Delta T R \sum_{\phi=0}^{\phi=160} \frac{d\phi}{\delta}$$

$$H = \frac{k_v}{\pi} \sum_{\phi=0}^{\phi=160} \frac{d\phi}{\delta}$$

or

$$\frac{HD}{k_v} = \frac{1}{\pi} \sum_{\phi=0}^{\phi=160} \frac{d\phi}{\overline{D}}$$

$$Nu = \frac{1}{\pi} \sum_{\phi=0}^{\phi=160} \frac{d\phi}{\delta} \quad (4.10)$$

This sum is easily computed numerically.

CHAPTER V

SOME COMPARISONS WITH EXPERIMENT AND CONCLUSION

As remarked earlier, very scant experimental data are available on this problem. Following is a brief discussion of comparisons which could be made.

First of all to check the validity of the numerical procedure, computations were made for the case when the dimensionless number k_2 is zero. Since k_2 represents the degree of subcooling, this case reduces to that of saturated film boiling, an analytic solution to which is available [10]. As shown below, the analytical solution matches very well with the numerical solution.

The properties of the system considered are given in Table 2 (No. 1)

TABLE 2

S.No	T_w (°F)	T_{sat} (°F)	U_∞ ($\frac{Ft}{Sec}$)	Dia (In)	k_v ($\frac{BTU}{Hr.Ft.^{\circ}F}$)	ρ_v ($\frac{BTU}{Ft^3}$)	λ ($\frac{BTU}{lbm.^{\circ}F}$)	C_{pl} ($\frac{BTU}{lbm.^{\circ}F}$)	ρ_l ($\frac{lbm}{Ft^3}$)
1	1000	212	20	.75	.027	.023	970	1	-
2	.762	174	8	.387	.012	.027	200	.762	46.2

The parameter k_1 from Equation (3.14) is

$$k_1 = \frac{2k_v \Delta T}{3\lambda' U_{\infty} \rho_v D} = .000121$$

Corresponding to $k_1 = .000121$ and $k_2 = 0$ one finds the Nusselt number from Figure 6 as 55. The heat transfer coefficient is then

$$h = \frac{\text{Nu } k_v}{D} = 23.4 \text{ BTU/Ft}^2\text{Hr. } ^\circ\text{F}$$

The total heat flux is found as

$$q'' = h\Delta T = 18400 \text{ BTU/Ft}^2\text{Hr.}$$

Now from film boiling theory for saturated case the heat flux is given by

$$q'' = .698 \left(\frac{\lambda' \rho_v k_v U_{\infty} \Delta T}{D} \right)^{1/2}$$

$$= 18500 \text{ BTU/Ft}^2\text{Hr.}$$

The two values thus very closely match.

A series of experiments were performed by Bromley [3] for the film boiling of various hydrocarbons over a cylinder in forced convection. The tube was electrically heated and the surface temperature was maintained constant. Different values of subcooling were taken.

We shall for our comparison take the tabulated data for ethyl alcohol for run No. 106. The system properties

are tabulated in Table 2 (No. 2)

The dimensionless numbers k_1 and k_2 are

$$k_1 = \frac{k_v \Delta T}{2 \lambda' \rho_v U_\infty D} = .000945$$

and

$$k_2 = .57 \left[\frac{\rho_\ell C_{p\ell} k_\ell \Delta T_B}{U_\infty D \lambda' \rho_v} \right]^{1/2} = .436$$

The Nusselt Number obtained using these k_1 and k_2 comes out to be 333.5

Whence

$$h = \frac{Nu k_v}{D} = 124.5$$

The experimental value of the heat transfer coefficient is

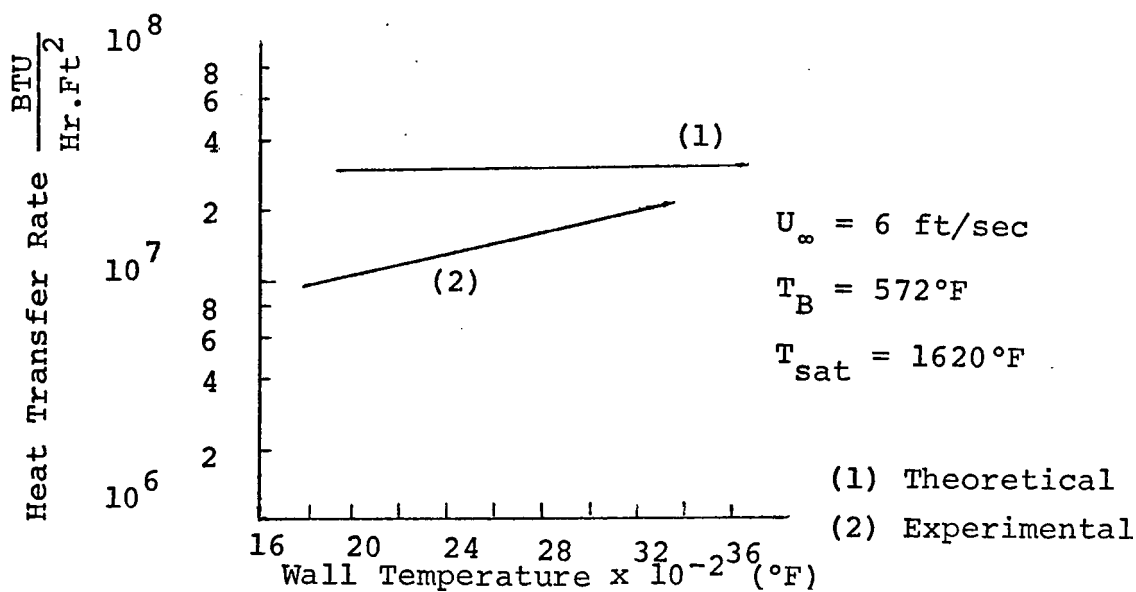
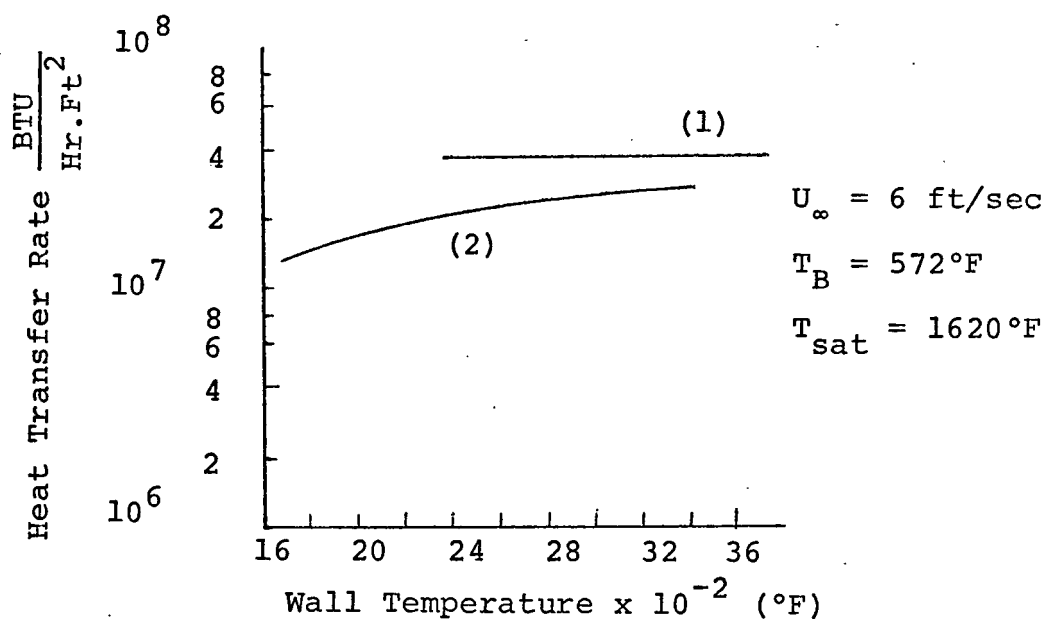
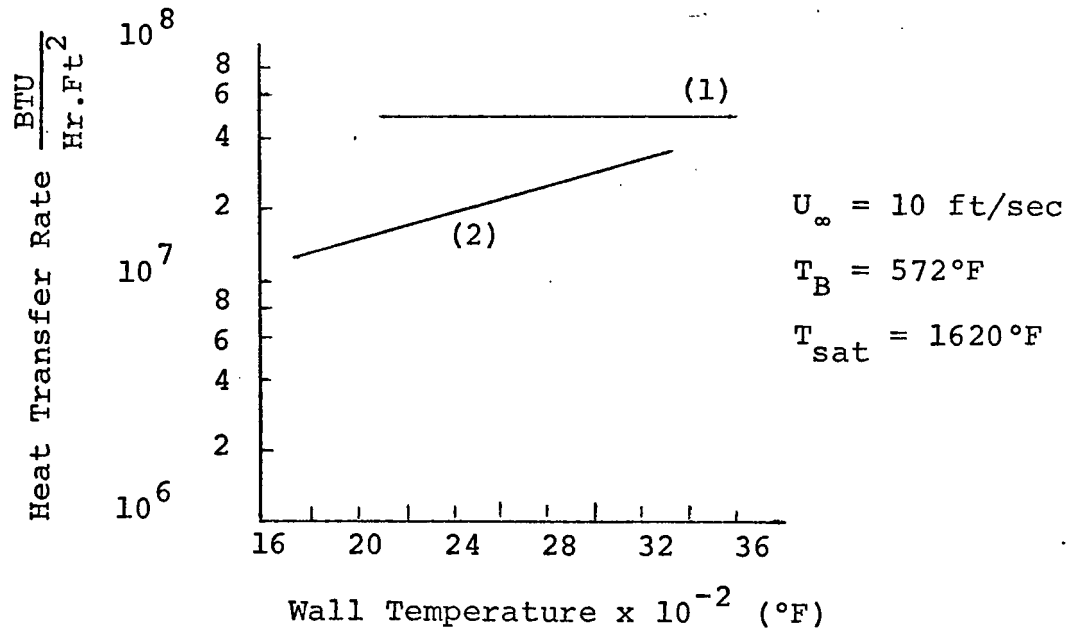
$$h_{\text{exp}} = 275$$

so that

$$\frac{h_{\text{exp}}}{h} = 2.2$$

Thus, the theoretical value is about half the experimental value.

Witte [2] performed a range of experiments on transient heat transfer from high temperature tantalum spheres moving in highly subcooled liquid sodium. Although presence of vapour blanket could not be visually observed due to the



Figs. 11-13 Comparison of experimental and theoretical heat transfer rates from sphere to liquid sodium.

opaqueness of liquid sodium, intuition would indicate that a vapour film would form around the sphere because of its high temperature. The transient data was suitably reduced to heat flux v/s instantaneous temperature variation.

A comparison with the above data is given in Figures 11-13 and shows the theoretical results to be 2 to 3 times the experimental.

If for a given T_{wall} , i.e. for a fixed k_1 , we plot Nu v/s k_2 (where k_2 is the subcooling parameter) we obtain a linear variation (Figure 14). Zuber and others [22] obtained a similar variation of critical heat flux with the degree of subcooling in nucleate boiling. Bromley [3] also gets almost a linear variation of flux with degree of subcooling in his experiments on subcooled film boiling of several hydrocarbons over a cylinder in forced convection.

Another interesting observation is that the product $(Nu \cdot k_1)$ for a given k_2 (subcooling parameter) is approximately constant as shown in Figure 15. The quantity $(Nu \cdot k_1)$ represents the total heat flux. That then means that the flux is quite independent of the sphere temperature in subcooled film boiling. Walford [4] also made this observation in his experiments.

The reason for this phenomenon is not difficult to

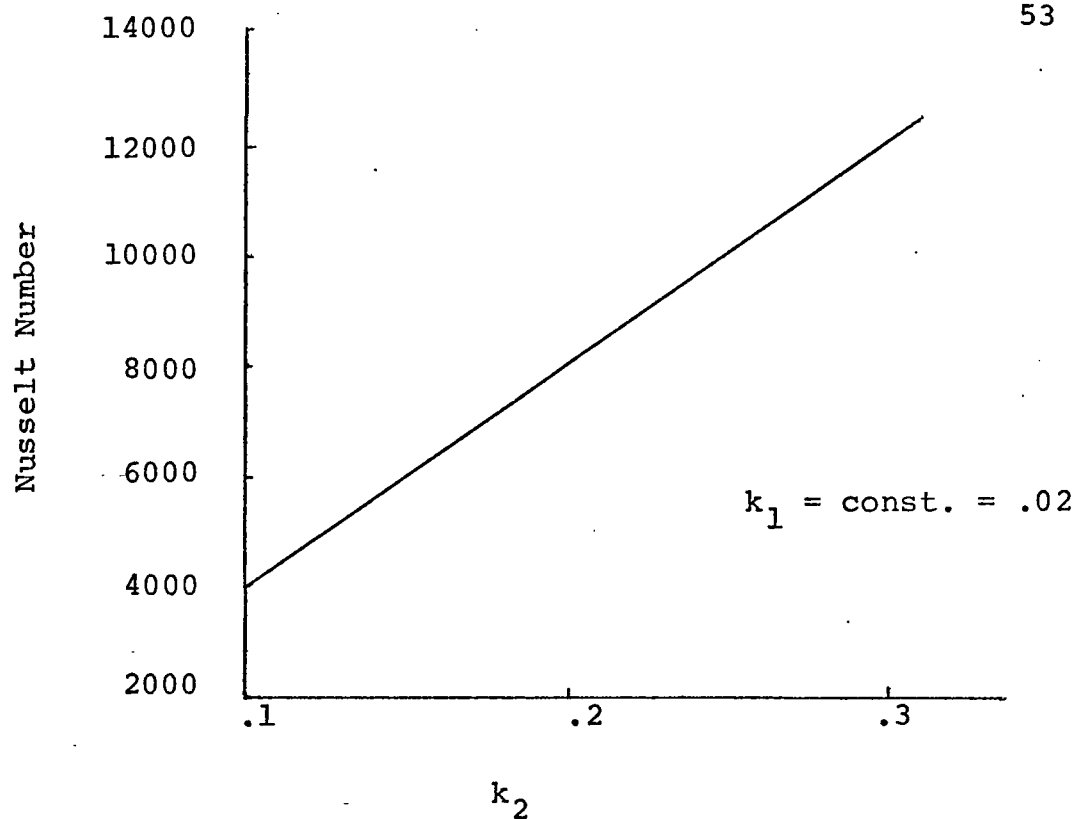


Fig. 14 Graph between Nusselt number and k_2 at constant k_1 .

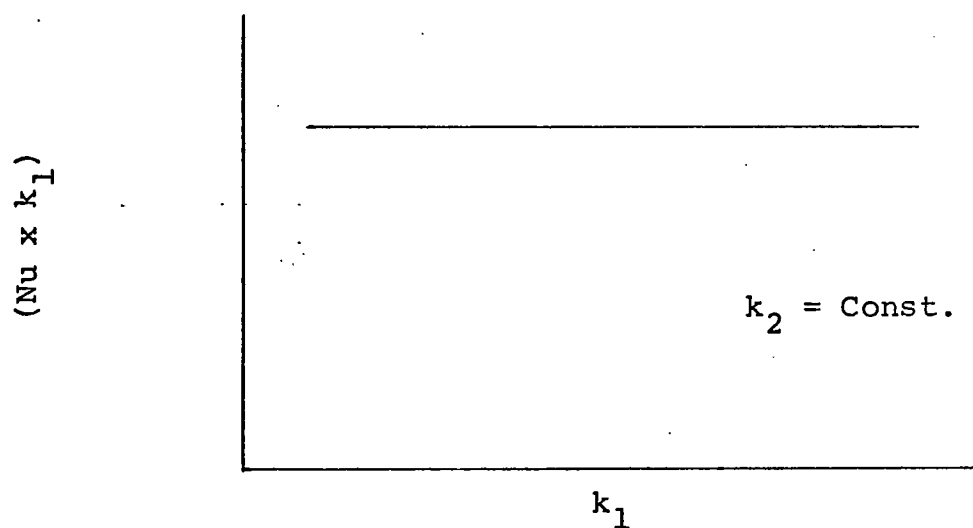


Fig. 15 Graph between the product $(Nu \times k_1)$ and k_1 .

see. If we look at the energy equation, the quantity of heat going into the bulk is independent of the sphere temperature, it depends upon only the temperature of the liquid bath. And since with substantial subcooling, Q_B is the major portion of the heat transfer, a change in sphere temperature does not affect the overall heat transfer.

Walford in his experiments with a zinc sphere also obtained a constant heat flux when plotted against wall temperature in the stable film boiling region. This plot is reproduced in Figure 16.

Jacobson and Shair [1] have also reported some experimental work regarding subcooled film boiling. However, the conditions under which the experiments were performed are not clear. Also their data show a steep rise in flux with the surface temperature of the sphere which is quite in contradiction with the results obtained by Walford who gets a constant flux with the said change in wall temperature. Moreover the degree of accuracy to which the temperature measurements were made by means of the optical pyrometer is not known.

However after giving suitable numbers to the system properties for the range of variables considered by them, the experimental value was found to be .8 to 7 times the theoretical value.

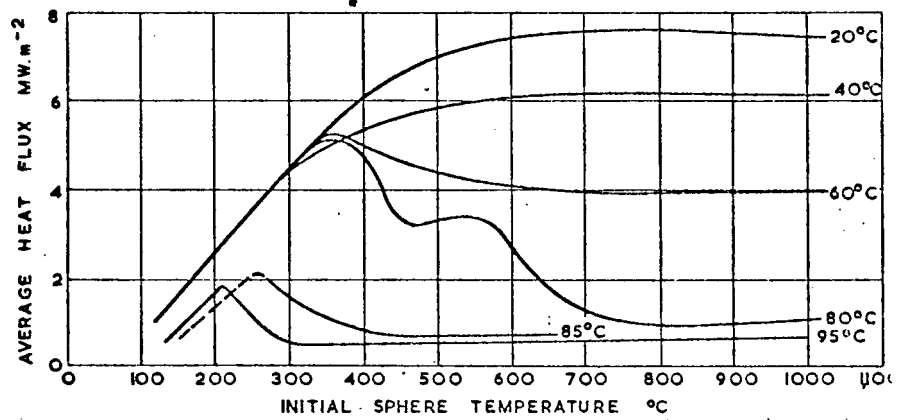


Fig. 16 Average heat flux v/s initial temperature.

Concluding Remarks

It is seen that the subcooling of the liquid bulk considerably reduces the vapour film thickness, thereby raising the heat transfer rates. Although higher transfer rates are obtained for the case of liquid sodium, it is premised that in general the theory underdepicts the heat fluxes; the higher values for liquid sodium being attributed to characteristics like surface wettability etc., peculiar of liquid metals. An attempt to explain the liquid sodium behaviour was not made and is left for future investigation.

BIBLIOGRAPHY

- [1] Jacobson, R. N., and Shair, F. H., 'Film Boiling From a Sphere During Forced Convection of Subcooled Water', I & EC Fundamentals, Vol. 9, No. 1, (1970), pp. 183-185.
- [2] Witte, L. C., Baker, L., and Haworth, D. R., 'Heat Transfer From Spheres Into Subcooled Liquid Sodium During Forced Convection', Trans. ASME Series C, November (1968), pp. 394-398.
- [3] Motte, E. I., and Bromley, L. A., 'Film Boiling of Flowing Subcooled Liquids', Ind. Eng. Chem. 49, No. 11 (1957), pp. 1921-1928.
- [4] Walford, F. J., 'Transient Heat Transfer From a Hot Nickle Sphere Moving Through Water', International Journal of Heat & Mass Transfer, Vol. 12 (1969), pp. 1621-1625.
- [5] Stevens, J. W., Bullock, R. L., Witte, L. C. and Cox, J. E., 'Transition Boiling From Spheres to Water', Tech. Rept. No. ORO-3936-3 (April 1970) AEC.
- [6] Bromley, L. A., 'Heat Transfer in Stable Film Boiling', Chem. Eng. Prog. 46, No. 5, (1950), pp. 221-227.
- [7] Bromley, L. A., et al., 'Heat Transfer in Stable Film Boiling', Ind. Eng. Chem. 45, No. 12 (1953), pp. 221-227.
- [8] Frederking, T. M. K., and Clark, J. A., 'Natural Convection Film Boiling on a Sphere', Advances in Cryogenic Eng., ed. K. D. Timmerhaus, Plenum Press, New York (1963), pp. 501-506.
- [9] Kobayashi, K., 'Film Boiling Heat Transfer Around a Sphere in Forced Convection', Jour. Nucl. Sc. Tech. 2, No. 2, (1965), pp. 62-67.
- [10] Witte, L. C., 'Film Boiling From a Sphere', I & EC Fundamentals, Vol. 7, No. 3, (1968), pp. 517-518.
- [11] Sparrow, E. M., and Cess, R. D., 'Film Boiling in a Forced Convection Boundary Layer Flow', Trans. ASME Series C 83, (1961), pp. 370-376.

- [12] Sparrow, E. M., and Cess, R. D., 'Subcooled Forced Convection Film Boiling on a Flat Plate', Trans. ASME Series C 83, (1961), pp. 377-379.
- [13] Bromely, L. A., 'Effect of Heat Capacity of Condensate', Ind. Eng. Chem. 44, No. 12, (1952), pp. 2966-2969.
- [14] Sideman, S., 'The Equivalence of the Penetration Theory and Potential Flow Theories', Ind. Eng. Chem. 58, No. 2, (1966), pp. 54-58.
- [15] Bird, R. B., Stewart, W. E., and Lightfoot, E. N., 'Transport Phenomenon' John Wiley & Sons, (1963), p. 539.
- [16] Schlichting, H., 'Boundary Layer Theory', 4th ed. McGraw-Hill Co., New York, N. Y. (1960), p. 464.
- [17] Lapidus, L., 'Digital Computation for Chemical Engineers', McGraw-Hill Co., New York, N. Y. (1962) p. 89.
- [18] Chen, C. F., and Haas, I. J., 'Elements of Control System Analysis', Prentice Hall, (1968), p. 378.
- [19] Gibson, J. E., 'Nonlinear Automatic Control', McGraw-Hill, (1963), p. 257.
- [20] Kays, W. M., 'Convective Heat and Mass Transfer', McGraw-Hill Co., New York, N. Y., p. 210.
- [21] 'International Critical Tables'.
- [22] Zuber, N., Tribus, M., and Westwater, J. W., 'The Hydrodynamic Crisis in Pool Boiling of Saturated and Subcooled Liquids', Int. Dev. in Heat Transfer, (1961), pp. 230-236.