<u>The Effects of Interparticle Collisions on Magnetohydrodynamic Theory</u>

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INTRODUCTION

Magnetohydrodynamics is concerned with the motion of electrically conducting fluids or ionized gases under the influence of electromagnetic fields. The theory of magnetohydrodynamics (or hydromagnetics*) applies only when the ionized gas is considered to be a continuous, fluid-like medium. When the behavior or characteristics of the individual particles composing the gas cannot be ignored, the theory of ionized gases must be applied instead.

The purpose of this paper is to investigate the effects of collisional processes occuring within the gas. It will be shown that the applicability and validity of magnetohydrodynamic theory depends upon the frequencies and types of collisions of the gas particles. A brief introduction to ionized gas - and magnetohydrodynamic theory has been included in order to provide a general background.

* Elsasser has proposed that the term "hydromagnetics" be used instead of "magnetohydrodynamics" whenever the phenomena to be discussed can be described using purely classical electromagnetic and hydrodynamic theories. However, the distinction is not ordinarily made in most literature, and the two terms are used interchangeably in this paper.

See:
Walter M. Elsasser, "Some Dimensional Aspects of Hydromagnetic Phenomena", in <u>Magnetohydrodynamics</u>, a symposium edited by Rolf K. Landshoff, Stanford University Press, 1957. Hereafter all references to work contained in this book will be made by referring to the author, the title of his paper and the underlined letters <u>M.h.</u>

IONIZED GAS THEORY - BASIC ASSUMPTIONS

When a gas is ionized it becomes an electrical conductor due to the fact that there is a large number of free electrons present - electrons that belong to no particular atom. The gas may consist of electrons, negative ions or radicals, positive ions or groups of ions, atoms and molecules or any combination of these which allows the gas to be electrically neutral as a whole. Due to the fact that multiply-ionized particles are relatively more difficult to produce, only singly ionized particles will be treated in this paper. Furthermore, it will be assumed that:

(1) the gas is completely ionized (the number of neutral particles is negligible)

(2) the gas consists of only electrons and protons

(3) the gas is electrically neutral as a whole.

Such a gas is called a plasma.

The validity of the assumption that few neutral particles are present will be investigated in the latter part of this paper, for there certainly exists a possibility that if both positively and negatively charged particles are present in the plasma, neutral particles will occasionally be formed. We will also assume that if solid walls are used to contain the plasma, they will be far enough away that we may safely neglect the effects of collisions at the walls and consequent loss of energy of the plasma.

Very little generality is lost in making these assumptions. For instance, oscillations of the plasma do not depend on the presence of neutral particles or bound electrons, and magnetohydrodynamic theory is not affected since all that is assumed in this case is that the plasma is fluid-like and electrically conducting. In the case of the production of neutral particles, conditions will be established that require the probability of formation of neutral particles to be negligibly small.

EQUATIONS OF MOTION

Following Spitzer's[1] notation the simplified equations of motion for a plasma are*:

$$(1) \qquad \rho \frac{\partial \vec{v}}{\partial t} = \vec{j} \times \vec{B} - \vec{\nabla} p - \rho \vec{\nabla} \phi$$

$$(2) \qquad \frac{m_e c^2}{n_e e^2} \frac{\partial \vec{j}}{\partial t} = \vec{E} + \vec{v} \times \vec{B} + \frac{c}{e n_e} \vec{\nabla} p_e - \frac{c}{e n_e} \vec{j} \times \vec{B} - \eta \vec{j}$$

where

ρ = mass density	p = pressure
$\vec{v}$ = average velocity of gas	p_e = pressure due to electrons
$\vec{j}$ = current density	m_e = mass of electron
$\rho \vec{\nabla} \phi$ = gravitational force	n_e = number of electrons

1. Spitzer, Lyman, Physics of Fully Ionized Gases, Interscience Publishers, N. Y., 1956, P.21

* Refer to the list of symbols and physical constants at the end of this paper

and η is given by

$$(3) \qquad \eta = \frac{c}{en_e} \frac{P_{ei}}{j} .$$

where P_{ei} is the momentum transferred from electrons to ions by collisions.

It has been assumed that

$$\frac{m_e}{m_i} \ll 1$$

It should be pointed out that neither of these equations are invalidated by the occurrence of collisions in the plasma. We have previously assumed that few neutral particles will be formed, and that the positive ions are exclusively protons, incapable of excitation or further ionization. If we now assume that radiative collisions do not occur, the only remaining effect of collisions is the transfer of momentum between particles. Equation (1) is unaffected since it applies to the whole gas, consisting of both types of particles, for which the net momentum transfer is zero. Equation (2) remains valid since the collisional effects are taken into account by the factor η.

It may readily be seen that a general solution for $\vec{j}$ or $\vec{v}$ would involve an enormous amount of labor and would result in a very cumbersome relation. For the sake of simplicity then, our investigations will be restricted to particular cases, and simplifying assumptions will be made as needed.

From equation (2) when $\vec{B}$, $\vec{\nabla} P_e$, and $\frac{\partial \vec{j}}{\partial t}$ all vanish, we see that

$$(4) \qquad \vec{E} = \eta \vec{j}$$

which is Ohm's law, the quantity η being the resistivity of the medium. For this reason equation (2) is referred to as the "generalized Ohm's law".

PLASMA OSCILLATIONS

Three fundamental types of wave motion may be simultaneously present in a plasma: electrostatic, electromagnetic and hydromagnetic. The following discussion will be restricted to cases where only one of the above types of oscillation exists at a time. Throughout this section it will be assumed that gravitational forces are negligible ($\vec{\nabla}\phi = 0$) and that collisional effects are not important ($\eta \approx 0$).

For the first case, that of electrostatic oscillation, let us consider a group displacement of electrons and the resulting oscillation. We will assume, in addition to the two assumptions above, that the frequency of oscillation is high enough that the positive ions may be considered as stationary since their mass is large compared to the mass of the electrons. Also, we assume that charge is conserved and that there is no magnetic field present.

We then obtain from equation (2) and from the relation[2]

(5) $$\vec{\nabla} P_e = - \frac{\gamma_e k T_e c}{e} \vec{\nabla} \sigma$$

the wave equation for electrostatic waves:

(6) $$\left(\nabla^2 - \frac{m_e}{3kT_e} \frac{\partial^2}{\partial t^2}\right)\sigma = \frac{\omega_p^2 m_e}{3kT_e} \sigma$$

where: k = Boltzman's constant

T_e = electron temperature

σ = charge density in e.m.u.

γ_e = ratio of specific heats = 3 (for this case)

ω_p ≡ "plasma frequency", $\left[\frac{4\pi n_e e^2}{m_e}\right]^{\frac{1}{2}}$

For the special case $\nabla^2 \sigma = 0$, the equation of simple harmonic motion results from equation (6).

The phase velocity of electrostatic waves is given by

(7) $$V = \left[\frac{\omega_p^2}{\mathcal{K}} + \frac{3kT_e}{m_e}\right]^{1/2}$$

where

$$\mathcal{K} = \frac{2\pi}{\lambda} : \text{ wave number}$$

Notice that at zero degrees Kelvin the plasma can have only one possible frequency of electrostatic oscillation, given by ω_p.

2. Ibid., P.59

Another interesting feature of electrostatic oscillations is that the waves are longitudinal, that is, they are propagated in the same direction as the electrostatic field which is the driving force. Very few longitudinal waves are known to exist in nature, sound waves being one of the few. Electromagnetic and hydromagnetic waves are of the more common transverse type, which behave like vibrations of a stretched string.

For electromagnetic waves, we assume that the gas is at steady state, that charge is conserved and that displacement currents may be ignored. By eliminating the magnetic field term from Maxwell's equations, we obtain

$$(8) \qquad \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_y = \frac{4\pi n_e e^2}{m_e c^2} E_y$$

for a wave travelling along the x-axis. Equation (8) may also be obtained from equation (2), but with somewhat greater difficulty. The wave described above is clearly transverse since at steady state the divergence of the electric field is zero.

The velocity of propagation is given by

$$(9) \qquad V = \left[\frac{c^2}{1 - \frac{\omega_p^2}{\omega^2}}\right]^{1/2}$$

If $\omega_p > \omega$ then the phase velocity is imaginary and the wave is damped as it travels further into the plasma. This damping principle may be used to measure plasma densities.

Hydromagnetic waves are of an entirely different nature. They are produced by the motion of the heavy positive ions rather than the electrons and appear only in the presence of a magnetic field and only for low frequencies of oscillation compared to the cyclotron frequency, ω_c *. If we assume the ionized gas to be a perfect conductor at steady state, and that the magnetic field is parallel to the x-axis, then η, $\frac{\partial \vec{j}}{\partial t}$, and $\vec{j} \times \vec{B}$ all vanish, and from equations (1), (2) and (8) we obtain

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_z = \frac{4\pi\rho}{B_0^2}\frac{\partial^2 E_z}{\partial t^2} \tag{10}$$

Here B_0 is the mean magnetic field parallel to the x-axis. Equation (10) is the wave equation for hydromagnetic waves and may also be written

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{K}{c^2}\frac{\partial^2 E_z}{\partial t^2} \tag{11}$$

where

$$K = 1 + \frac{4\pi\rho c^2}{B_0^2} \tag{12}$$

is the dielectric constant of the gas.

* See the Glossary of Symbols and Physical Constants at the end of this paper

The similarity of equations (8) and (11) has prompted Spitzer[3] and Alfvén[4] to regard hydromagnetic waves as a special case of electromagnetic waves, modified by the high dielectric constant of the gas.

Alfvén[5] first predicted the existence of magneto-hydrodynamic waves and has shown[6] that they may be considered as vibrations of the magnetic lines of force in the gas. If the gas is a perfect conductor, any small motion of the lines of force relative to the medium produces an enormous restoring force. Hence the lines of force may be considered as "frozen" in the conducting fluid. The gas particles (both electrons and protons) are rigidly attached to the force-lines just as small masses are attached to a stretched string. After taking into account the interaction between lines of force, the hydromagnetic wave equation (equation (10)) is obtained.

3. Spitzer, op. cit., P.56

4. H. Alfvén, Cosmical Electrodynamics, Clarendon Press, Oxford, 1950, P.84.

5. Alfvén, On the Existence of Electromagnetic - hydrodynamic Waves, Ark. f. mat., astr. o. fysik, 29 B, No. 2; Nature, 150, 405

6. Loc. cit., Cosmical Electrodynamics, P.82

The phase velocity for hydromagnetic wave is given by

$$V = \left[\frac{c^2}{1 + \frac{4\pi\rho c^2}{B_0^2}} \right]^{1/2} \tag{13}$$

and we see that for very low values of B_0 the phase velocity is considerably less than the speed of light.

MAGNETOHYDRODYNAMIC THEORY - BASIC ASSUMPTIONS

Magnetohydrodynamics is currently receiving a great deal of attention. Previously it was a subject which interested primarily astronomers, astrophysicists and geophysicists. Today it is hoped that the solutions to some of the problems associated with the fusion reactor may be found in this area where electromagnetics, hydrodynamics and gas dynamics are combined in a unified theory. Much experimental work is being done both with pinched plasmas and with ionized gases in shock tubes. Alfvéns theoretical waves have been observed[7] using liquid sodium and mercury and new insights into cosmic phenomena have been obtained, chiefly in the production of cosmic rays and in the theory of sun spots.

Magnetohydrodynamic theory is best described as a combination of electromagnetic and hydrodynamic theory, modified in order to account for the interaction of the magnetic field and the macroscopic motion of the medium, which is necessarily an electrical conductor. The theory assumes that the medium is continuous and that the individual particle characteristics need be considered only in so far as their electrical and thermal conductivities

7. S. Lundquist, Experimental Investigation of Magneto-Hydrodynamic Waves, Phys. Rev., 76, 1805 (1949)

are concerned. In addition, it is assumed that the medium is non-magnetic (relative permeability, $K_m=1$), that Maxwell's displacement currents are small enough to be neglected, and finally that the motions of the electrons do not contribute to the mass oscillation of the medium.

THE REYNOLDS NUMBER

As expected, numerous analogies may be drawn between hydromagnetics and hydrodynamics. Perhaps the best example is the existence of a magnetic Reynolds number, given by

$$R_m = \frac{4\pi l v}{\eta} = \frac{l v}{\eta_0} \tag{14}$$

where l = typical length

v = characteristic velocity of the medium

$\eta_0 = (4\pi)^{-1}$ times the resistivity, η .

For comparison, the viscous Reynolds number is

$$R = \frac{\rho l v}{\delta} = \frac{l v}{\nu} \tag{15}$$

where ρ = density

δ = viscosity

ν = combined density and viscosity, $\frac{\delta}{\rho}$

It should be pointed out that both (14) and (15) are greatly simplified. The microscopic expressions are

(16) $$R_m = \frac{2.38 \times 10^{-6} T^{3/2} v \ell}{\ln\left(\frac{3hkT}{e^2}\right)}$$

and

(17) $$R = \frac{v\ell}{\lambda}\left[\frac{\pi m_i}{2kT}\right]^{1/2}$$

where h = Debye shielding distance

m_i = ionic mass

λ = mean free path for collisions

The ratio of the Reynolds numbers gives the ratio of the viscosities

(18) $$\frac{R_m}{R} = \frac{\nu}{\nu_m} = \frac{\mu\nu}{\eta_0}$$

where μ is the permeability. Elsasser[8] has estimated this ratio to be $\frac{\nu}{\nu_m} = 2 \times 10^{-7} \frac{\alpha}{\rho}$ for ionized hydrogen, where α is the degree of ionization and ρ is the density in c.g.s. units. For a fully ionized gas of moderate density ($\alpha = 1$, $\rho \approx 10^{-3}$) we see that the energy dissipation due to

8. W. M. Elsasser, M.H., "Some Dimensional Aspects of Hydromagnetic Phenomena"

viscous effects is governed mostly by the magnetic viscosity ν_m, which for non-magnetic materials is directly proportional to η_o. As previously shown, η_o is a measure of the electron collisions in the gas and ν, the ordinary viscosity, is determined by the atomic or molecular collisions. Thus we may conclude that the majority of the energy dissipated in an ionized gas is due to collisions of the electrons with ions or with other electrons, rather than ion-ion collisions.

From hydrodynamic theory the viscous Reynolds number, which expresses the ratio of inertial and viscous forces, tells us whether to expect laminar flow or turbulence, depending on its magnitude. For water, if R is less than 2000, only laminar flow exists. Turbulence becomes evident only when R exceeds 2000.

The magnetic Reynolds number is more difficult to visualize. Landshoff[9] states that it is an indication of the extent to which the magnetic lines of force are frozen in the fluid, or that it is a measure of the influence of the moving, conducting fluid on the magnetic field lines in its path. If the magnetic Reynolds number is large, we expect the lines of force to be firmly frozen in the fluid, or that a strong interaction may be observed between

9. Landshoff, R.K.M., M.h., "Scaling Laws as an Aid to Experimental Studies", P.71

the motion of the fluid and the magnetic field in its path. This interaction may be expressed as[10]

$$\text{(19)} \qquad (B, v)\,\text{Interaction} \propto R_m \frac{E_M}{E_K}$$

where E_M = magnetic energy

E_K = kinetic energy

or $$I(B, v) \propto \frac{\ell v}{\eta_0} \cdot \frac{B^2/\mu}{\rho v^2} \propto \frac{B^2}{v}$$

Cowling[11] has shown that the ratio of magnetic and inertial forces is

$$\text{(20)} \qquad S = \frac{B^2}{4\pi\rho\mu v^2} \propto \frac{B^2}{v^2}$$

Also, S is the ratio of magnetic and kinetic energy densities. In the above reference, Cowling has also shown that S is in analogy with the Mach number occuring in aerodynamics,

$$\text{(21)} \qquad S = \frac{a^2}{V^2}$$

Here a is the velocity of a hydromagnetic wave and V

11. T. G. Cowling, Magnetohydrodynamics, Interscience Publisher, N. Y., 1957

is the material velocity. If S is large, the material velocity is small compared to the wave velocity, and small disturbances are dissipated as hydromagnetic waves, without causing any appreciable non-uniformities to develop in the field.

THE DEBYE SHIELDING DISTANCE

At the beginning of this section it was stated that magnetohydrodynamic theory assumes the existence of a continuous medium, where individual particle effects need not be considered. One justification for this assumption would be that the separation of the particles be small compared to lengths of interest. The Debye shielding distance (or Debye radius) is a measure of the charge separation allowable. It is defined as[12]

$$h = \left[\frac{kT}{4\pi n_e e^2}\right]^{1/2} = 6.9\left[\frac{T}{n_e}\right]^{1/2} \tag{22}$$

in c.g.s. units, where n_e is the number of electrons per cm.^{3} and the other symbols have their usual meaning. The Debye length may be obtained by solving Poisson's equation in a region free of positive charge and equating the mean kinetic energy per charged particle and the difference in

12. Spitzer, op. cit., P.17

electrical potential energies per particle across a region h centimeters wide. Debye has shown that the field of a point charge in an electrolyte is

$$E \propto r^{-1} e^{-r/h} \tag{23}$$

and that when $r > h$, the field is shielded by particles of opposite sign. According to Spitzer, the Debye length determines the extent to which the electron concentration $n_e e$ may differ from the positive charge concentrations $n_i Z$. Also, the thickness of the sheath formed when the plasma is in contact with a solid wall is given approximately by h.

COLLISIONS IN IONIZED GASES

Many different types of collision processes occur in ionized gases. There are elastic and inelastic collisions, "close" encounters, where momentum transfer is large, "distant" encounters, where scarcely any interaction is noticable, collisions between ions and electrons to form atoms which may then be excited or ionized, electron-electron collisions and so on. It would be an exceedingly difficult task to treat the most general case, where complex atoms or molecules are present. For our purposes we will assume that we have a "perfect" ionized gas, or a plasma, as described at the very beginning of this paper.

MEAN FREE PATH

The mean free path for collisions is obtained from the kinetic theory of gases and for neutral atoms or molecules of equal radii is

$$\lambda = \frac{1}{\sqrt{2}\,\pi \rho d^2} \qquad (24)$$

where $d = r_a + r_b = 2r_a$ is the particle radius. In general, the injected particle or particles have different radii and R.M.S. velocities than those of the gas and λ is given by[13]

13. J. H. Jeans, Dynamical Theory of Gases, Cambridge University Press, 1925, P.252

(25) $$\lambda_b = \frac{1}{\pi \rho_a d^2 \left(1 + \frac{v_a^2}{v_b^2}\right)^{1/2}}.$$

where the subscript b refers to the injected particles. Notice that the mean free path depends on the velocity as well as the size of the particles. It can be shown[14] that the mean free path for an electron is given approximately by

(26) $$\lambda_e = \frac{4}{\pi \rho_a r_a^2}$$

which is only $4\sqrt{2}$ times as great as the mean free path of an atom, while the radius of the electron is smaller than the atomic radius by a factor of 10^5.

In the actual case then, we are not justified in treating the particles as solid spheres according to gas kinetic theory. Not only are they predominately open space, but we must also consider the influence of gravitational and electrical fields which are ever present.

CROSS SECTION

In order to discuss collisions we will introduce the concept of total collision cross section and will define it as "an imaginary area normal to the trajectory of the incident particle which no other particle may penetrate without altering the paths or energies of either or both

14. F. L. Arnot, Collision Processes in Gases, Wiley, N. Y. 1950

particles". When a particle passes into or through the total cross section we will say that a collision has occured. The cross sections for different types of collisions are in general, of different sizes, which is to be expected since some types of collisions are more probable than others.

Classically the total collision cross section would include all space, since the gravitational and electrical fields of the particles extend to infinity, and a deflection could be measured at any separation provided the measuring device were sensitive enough. Quantum mechanical uncertainties, however, predict that the cross section may have a finite value if the attractive force between particles decreases faster than r^{-3} for large separations r. It has also been shown[15] that due to the Heisenberg uncertainty principle, classical theories are not applicable when the deviations of the particles are less than a few minutes of arc, depending on the particles in mind and the type of collision taking place.

CLASSIFICATION OF COLLISIONS AND COLLISIONAL ENERGY LOSSES

Collisional processes may be classified according to energy transfer and we may associate with each process an "effective cross section" for collision, which is the

15. Massey & Burhop, Electronic and Ionic Impact Phenomena, Clarendon Press, Oxford, 1952, P.3

total collision cross section multiplied by the probability that a specific type of collision will occur.

There are three basic types of collisions:

(1) Elastic - no net loss of kinetic energy of colliding particles

(2) Inelastic - some or all of the incident particles' energy is lost in exciting internal motion or causing emission of radiation

(3) Superelastic - incident particle gains energy upon collision with excited atom.

Superelastic collisions may be considered as a special type of inelastic collision and will not be discussed due to the assumed absence of neutral atoms.

There are three possibilities for elastic collisions:

(1) electron-electron

(2) electron-ion

(3) ion-ion.

Elastic collisions serve to distribute thermal and electrical energy and to impede the macroscopic motion of the particles. The energy loss due to electrical conductivity may be written

$$E_j = \eta j^2 \tag{27}$$

where j is the current density, and η is the resistivity, arising mainly from electron-ion collisions. Strictly speaking, the energy is not lost, but is converted to heat. The quantity E_j represents ohmic or Joule heating in the plasma. If the plasma is well isolated from the surroundings, as it would be if it were contained by a magnetic field, the heat loss is minimized and it is possible to produce plasmas of very high temperatures by the application of electric fields. Increasing the temperat ure of the plasma results in more complete ionization and a smaller cross section for collisions and is accomplished by applying to the plasma electric fields of the order of 10 volts/m.[16] The dominant collision cross section in a plasma is the coulomb cross section which is[17]

$$(28) \qquad Q_c \propto E_{rel}^{-2}$$

where E_{rel} is the relative energy of colliding particles. Hence raising the temperature lowers the losses due to collisions.

16. J. M. Berger, et. al., "On the Ionization and Ohmic Heating of a Helium Plasma", Second United Nations International Conference on the Peaceful Uses of Atomic Energy, A/CONF. 15/P/363, Geneva, 1958

17. R. F. Post, "Summary of UCRL Pyrotron (Mirror Machine) Program", Second United Nations International Conference on the Peaceful Uses of Atomic Energy, A/CONF. 15/P/377, Geneva, 1958

When a beam consisting of many particles is directed into a gaseous medium it is appropriate to refer to the resulting collisional processes as scattering. Inelastic scattering occurs as well as elastic scattering but to a much smaller degree. A beam of photons passing into a molecular gas is scattered through angles from 0 to 180° from the original direction and the intensity of scattered light is found to vary as ν^4 for frequencies in the visible range. Hence the high frequencies are scattered with greater intensity than the low frequencies. This effect is known as the Tyndall effect and accounts for the blue color of the sky.

Previously, it was assumed that very few neutral atoms or molecules were formed as a result of collisions, or more precisely, that the rate of recombination was small. Recombination, which is always inelastic, may occur in several ways, the most probable being[18]

$$(1) \qquad A^+ + e \rightarrow A' + h\nu$$

$$(2) \qquad A^+ + e \rightarrow A'' \begin{cases} \nearrow A' + h\nu \\ \searrow A^+ + e \end{cases}$$

$$(3) \qquad A^+ + e + C \rightarrow A + C$$

For a "perfect" plasma there are equal numbers of ions and electrons and we may define the recombination rate for

18. Massey & Burhop, op. cit., P.631

low pressures as[19]

(29) $$\frac{dn}{dt} = a_0 n^2$$

where a_0 is the recombination coefficient. Von Engel has shown for a variety of elements that the total ion-electron recombination coefficients range from $10^{-6}\frac{cm^3}{sec}$ for H_2 to $10^{-10}\frac{cm^3}{sec}$ for Hg. According to von Engel[20], the probability of radiative recombination is of order 10^{-7} at ordinary temperatures, corresponding to a recombination coefficient of $10^{-13}cm^3/sec$. The three-body type of recombination has been shown to be the most probable under normal conditions[21] in which case the containing walls usually act as the third body in absorbing the extra energy.

In general, the recombination rate is low for low pressures. However, as the gas pressure is raised, the recombination rate increases and approaches a saturation value for pressures in the vicinity of 5×10^4 mm. Hg.

If the plasma is at equilibrium the rates of recombination and ionization are necessarily equal. However, since there is always an energy loss associated

19. A. von Engel, Ionized Gases, Clarendon Press, Oxford, 1955, P.141

20. von Engel, op. cit., P.141

21. F. Llewellyn - Jones, Ionization and Breakdown in Gases, Methuen and Co., N. Y., 1957

with recombination, the system will run down unless energy is constantly added to maintain equilibrium, or to enhance ionization.

The rate of ionization depends upon numerous quantities, some of which are:

(1) the mass and size of the atoms

(2) the electron's energy

(3) the pressure or density of atoms.

Massey and Burhop[22] have tabulated cross sectional values for the ionization of He, Ne, A, and Hg at different values of electron energy, and for low energy electrons a typical value would be $\sim 10^{-16}$ cm.^{2}

It should be noted that although the rates of ionization and recombination are equal there will remain, on the average, a certain number of neutral atoms. There are two courses of action for avoiding the difficulties encountered due to the presence of these particles:

(1) the temperature may be raised in order to increase the ionization rate or

(2) the plasma may be confined by a magnetic field to reduce recombination.

22. Massey and Burhop, op. cit., P.38

In a high temperature plasma of low density the principal energy loss is that of radiation of electromagnetic waves. Although this process is improbable compared to other types of inelastic collisions, it must be stressed that any energy which is converted to the form of radiation is generally irretrievable and over a period of time is accumulative. When an electron collides with an ion and is captured, it is compelled to emit a photon of energy $h\nu$, where h is Planck's constant and the frequency ν depends upon:

(1) the electron's energy

(2) the ion's energy and size

(3) the energy level into which the electron falls.

The probability that this photon will escape the plasma depends upon the sizes of the cross sections of the gas particles and the number of particles in the plasma between its point of origin and the containing walls. It is possible that the photon will be scattered by an electron or ion, but in order to impart any of its energy to the gas before it escapes, it must suffer a collision, either with an atom capable of ionization or with an electron. In the former event, since the photon has exactly the right energy for ionization, an electron is released from the atom and energy is conserved. Thus, for large volumes of plasma, where the probability of collision with a neutral atom is high, resonance occurs between radiative recombination and photo-ionization, and the energy losses are confined to the walls, where the emitted photon has less chance of ionizing an atom before it escapes.

In the latter event, when the photon collides with an electron, it may lose all of its energy only if there is a third body present to absorb some of the momentum. The reverse process may also occur; a free electron may decelerate in a coulomb field and emit a photon, but only if momentum is transferred to the particle providing the coulomb field. If the electron remains free, this process is called bremsstrahlung, or braking radiation, and according to Heitler[23], at very high temperatures or energies the energy losses are almost entirely due to this process. The rate of energy loss due to bremsstrahlung is given by Spitzer[24] as

$$(30) \qquad \epsilon = 1.42\times10^{-27} Z^3 n_i T^{1/2} \ \text{ergs/cm}^3 \cdot \text{sec.}$$

and for the typical values $Z=4$, $n_i = 10^{13}$ (10^{-3} mm Hg), $T = 10^4$,

$$(31) \qquad \epsilon = 10^3 \ \text{ergs/cm}^3 \cdot \text{sec.} = 10^{-4} \ \text{watts/cm}^3.$$

23. W. Heitler, The Quantum Theory of Radiation, Clarendon Press, Oxford, 1953, P.375

24. Ibid. P.90

At first sight this seems like a small value. However the thermal energy density at 10^{-3} mm. Hg and 10^4 °C. is only

$$D = (10^{13} \text{ ions/cm.}^3)(\tfrac{3}{2} kT \text{ ergs/ion}) = 20 \text{ ergs/cm.}^3 \tag{32}$$

Hence unless energy is added to the plasma in some fashion, the system will rapidly run down.

A photon may collide with an electron and lose only part of its energy. In this case the photon is deflected through an angle θ and its wavelength decreases by an amount

$$\Delta\lambda = \lambda_o(1 - \cos\theta) \tag{33}$$

where λ_o is the Compton wavelength given by

$$\lambda_o = \frac{h}{m_e c} = 2.43 \times 10^{-2} \text{ angstroms.} \tag{34}$$

In this type of collision both momentum and energy are conserved without the aid of a third particle and the plasma regains part of the photon's energy.

CONCLUSION

We have attempted to show that considerable caution must be used in applying magnetohydrodynamic theory to ionized gases in order that the individual particle effects produce no barrier to our interpretation of experimental results, or lead us astray. The justification of the application of a continuous - fluid theory is determined by at least the following factors:

(1) All observations must be made over a region of radius $r \gg h$, the Debye length, and away from the walls by approximately the same distance.

(2) Inelastic collisional losses must be negligible. In general, this is true if: (a) the plasma is of low density and is at a high temperature (b) the volume of plasma is large (c) the atomic number of the element is small, and (d) there are no impurities of high Z, since the bremsstrahlung losses are proportional to Z^3.

GLOSSARY OF SYMBOLS AND PHYSICAL CONSTANTS

a = hydromagnetic wave velocity

a_o = recombination rate coefficient

A, A^+ = colliding particles

$B, \vec{B}$ = magnetic field

c = speed of light = 2.9979×10^{10} cm./sec.

C = colliding particle

d = solid sphere collision radius

D = thermal energy density

e = base of natural logarithms = 2.7182···, electronic charge = 1.602×10^{-19} coul., or as a subscript refers to electrons

$\vec{E}, \mathcal{E}$ = electric field

E = energy

h = Planck's constant = 6.625×10^{-27} erg sec., Debye radius

i = refers to ions or protons

$j, \vec{j}$ = current density

k = Boltzman's constant = 1.380×10^{-16} ergs/deg. C.

K = dielectric constant

K_m = relative permeability

$\mathcal{K}$ = wave number

ℓ = length

m = mass, or refers to magnetic quantity

M = refers to magnetic quantity

n = number, density, or refers to neutral particles

P = pressure

Q = cross section

r = length or radius

R = Reynolds number

S = ratio of magnetic and kinetic energy densities

t = time

T = temperature

v = velocity

V = velocity of propagation

x, y, z = coordinates

Z = atomic number

α = degree of ionization

γ = ratio of specific heats

δ = viscosity

ϵ = energy dissipation

η = resistivity

θ = angle of deflection

λ = mean free path or wavelength

λ_0 = Compton wavelength = 2.43×10^{-2} angstroms

μ = permeability

ν = frequency or combined density and viscosity

π = 3.14159...

ρ = density

σ = charge density

ω = frequency

ω_c = cyclotron frequency = $\frac{ZeB}{mc}$

ω_p = plasma frequency = $\left[\frac{4\pi n_e e^2}{m_e}\right]^{1/2}$

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* see footnote in Introduction

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