# Thin-Bed Interval Velocity Inversion

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A Thesis Presented to

the Faculty of the Department of Earth and Atmospheric Sciences

University of Houston

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science

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By

Brian Adrian Pack


May 2015

# Thin-Bed Interval Velocity Inversion

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## Abstract

For an isolated thin layer producing a composite seismic event exhibiting an apparent RMS velocity, the RMS velocities of the top and base reflectors, and thus the reflector time locations as a function of offset, can theoretically be determined if the zero-offset time thickness is known. For layers in the vicinity of tuning, the time thickness can be inferred using spectral decomposition. Thus, the interval velocity of the layer can be determined from its moveout curve, even when top and base reflectors cannot be resolved. This extends the range of layer thicknesses over which interval velocities can be measured using moveout from above to somewhat below tuning. It also provides a framework for simultaneous full-waveform inversion of thin layers for multiple offsets.

The interval velocity determination has been applied to single-layer synthetic seismic data derived from a range of different models. Results obtained under favorable conditions show a high degree of accuracy ranging from ninety-seven to ninety-nine percent in estimating the interval velocity of a thin layer, while those obtained in the presence of noise show a large degree of variability and accuracy ranges from forty-eight to eighty-five percent.

## Table of Contents

**Chapter 1**

**Introduction/ The Structure of This Thesis**

## 1.1 Introduction

Interval velocity estimation is often regarded as one of the most important problems in exploration geophysics. The implications of being able to extract an estimate of the subsurface velocity model directly correlate to the feasibility of performing accurate imaging, inversion and interpretation on seismic data. The current practice of extracting this velocity by inversion of the approximate RMS velocity derived from moveout analysis requires the geologic unit or bed over which an interval velocity is to be extracted to have separate moveout curve's corresponding to the top and base of that unit or bed. This, by definition, requires that the event be seismically resolvable. Many beds or geologic units of importance are however below seismic resolution. These beds or units, ubiquitously referred to as thin-beds are commonly the target of exploration interest and much work has been put forth to utilize their associated reflectivity characteristics in defining their properties. Despite this interest and subsequent body of work surrounding their reflectivity information one crucial parameter currently un-attainable by analyzing only the reflectivity spectrum or current velocity analysis method's is the interval velocity of these thin layers. A new method combining moveout velocity analysis and spectral-layer-thickness inversion capable of providing an estimate of a seismically thin layer's interval velocity will be put forward and its effectiveness, when implemented on a variety of synthetic seismic models, evaluated.

## 1.2 The Structure of This Thesis

- Chapter 2 develops the necessary background for this work including, (a) the connection between normal moveout, RMS and interval velocity for a seismic event (b) the time domain thin-bed response and (c) sub-resolution layer thickness estimation.

- Chapter 3 will present analytical equations for (a) the temporal zero-offset location of a thin layers upper and lower interface's, (b) the offset dependent temporal thickness of a thin layer and (c) the RMS values of a thin layers upper and lower interfaces. These equations establish the basis for this method.

- Chapter 4 presents the proposed method, tests it on synthetic examples including models representative of the Bakken Formation, and evaluates the methods performance in the presence of noise.

- Chapter 5 discusses the results, future work to be conducted in this research area and states conclusions.

- Appendix A presents the explicit equations for the RMS velocity's of a thin layers upper and lower interfaces.

- Appendix B discusses the undefined results obtained for model B4 utilizing single trace derived input parameters.

## Chapter 2

## Background

### 2.1 Normal Moveout RMS and Interval Velocity

Consider the simple case of one horizontal layer over a half space producing a single offset dependent reflection event in time (Fig. 2.1).

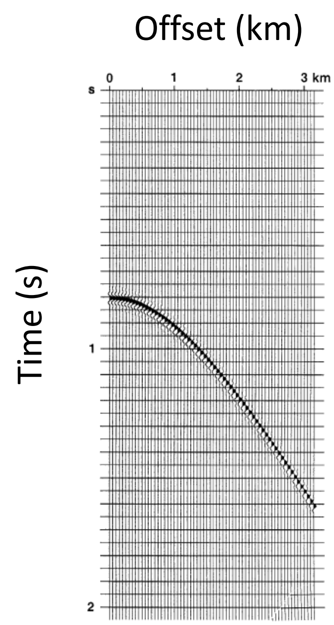



*Fig. 2.1 Synthetic CMP gather displaying the reflection from a single interface. From Yilmaz (1987).*

The reflection travel-time curve maps a hyperbola in the plane of two-way time. An equation describing this hyperbolic travel-time curve as a function of offset for the special case of a single layer can be derived from Pythagoras's theorem (Fig. 2.2, Eq. 2.1).

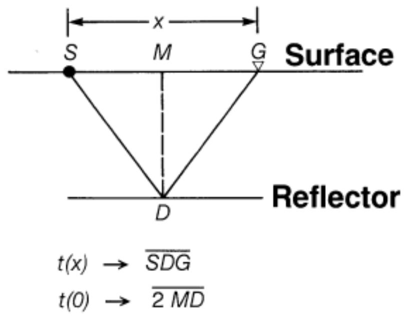



*Fig. 2.2 NMO geometry for a single horizontal reflector. From Yilmaz (1987).*

$$t^2(x) = t^2(0) + \frac{x^2}{v^2} \tag{2.1}$$

*Where $t(x)$ is the offset dependent travel-time, $t(0)$ is the zero offset or vertical trave-ltime, $x$ is the source receiver offset distance and $v$ is the layer velocity above the reflecting interface.*

For a stratified earth consisting of isovelocity layers the exact travel-time equation derived by Taner and Kohler (1969) becomes an expansion of the form.

$$t^2(x) = C_0 + C_1x^2 + C_2x^4 + C_3x^6 + \ldots\ldots \tag{2.2}$$

*Where $C_0 = t^2(0)$ and $C_1 = \frac{1}{V^2{}_{RMS}}$ the remaining terms are complex functions that depend on layer thickness and interval velocity.*

Substituting the first two coefficients into equation 2.2 and truncating thereafter yields equation 2.3.

$$t^2(x) = t^2(0) + \frac{x^2}{V^2{}_{RMS}} \quad \text{(2.3)}$$ **Hyperbolic Moveout Equation*

*Where $t(x)$ is the offset dependent travel-time, $t(0)$ is the zero offset or vertical travel-time, $x$ is the source receiver offset distance and $vRMS$ is the Root Mean Square velocity which describes the hyperbolic approximation to the true T-X curve.*

If offset is taken to be less than or equal to depth the travel-time curve can be considered approximately hyperbolic and the two term truncation of equation 2.2 given in equation 2.3 can be utilized to perform the zero-offset correction by providing an estimate the RMS velocity for a reflector in choosing the velocity which effectively flattens the reflection event (Fig. 2.3).

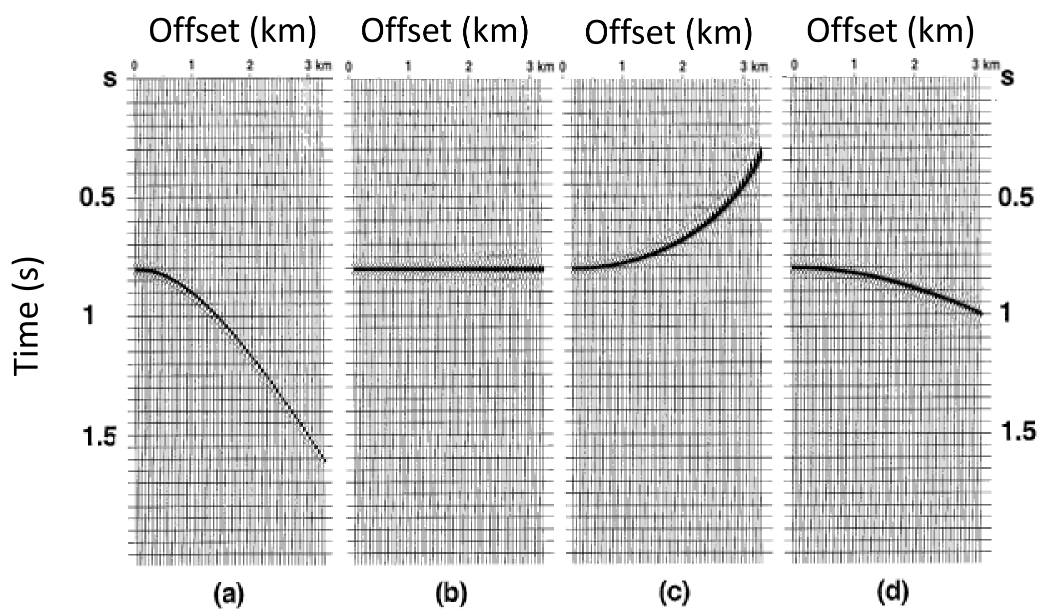



*Fig. 2.3 (a) CMP gather containing a single event with a moveout velocity of 2264 m/s; (b) NMO-corrected gather using the appropriate moveout velocity; (c) overcorrection because too low a velocity (2000 m/s) was used in equation (3.2); (d) undercorrection because too high a velocity (2500 m/s) was used in equation (3.2).From Yilmaz (1987).*

As offset becomes greater than depth the travel-time curve becomes increasingly non-hyperbolic. When the offset to depth ratios exceeds ~ 1.5 significant improvement of $vRMS$ estimation can be realized by using the three term expansion of equation 2.2 commonly referred to as the non-hyperbolic or fourth order normal moveout equation(s). In the form given by Taner et.al.

$$(2.4) \qquad t(x) = \sqrt{t^2(0) + \frac{x^2}{V^2RMS} + \frac{(1 + \frac{V_4^4}{V^4RMS})x^4}{4t(0)V^2RMS}}$$

$$Where \;\; V_4^4 = \frac{1}{t(0)} \sum V^4(i)\Delta t(i)$$

The RMS or root mean square velocity's physical significance is such that it is the velocity observed to a depth point D in a layered medium where i≥ 2 along a specific ray path. The mathematic expression being given by

$$(2.5) \qquad V^2rms = \frac{1}{\tau}\sum_i \; v^2i \; \Delta ti$$

*Where $\tau$ is the total travel-time to the ith layer, $v$ is the interval velocith of the ith layer and $\Delta t$ is the travel-time through the ith layer.*

In the context of normal moveout the RMS velocity estimated can be generalized as the velocity which properly corrects for an approximation to the travel-time curve resulting from the truncation of equation 2.2, the common forms of which are given by equation 2.3 and equation 2.4.

Examining equation 2.5 and noting its direct relationship to the interval velocity of the constitute layers that have been summed over we can extract the interval velocity of said layers by utilizing equation 2.5's analytic inverse. This is the Dix interval velocity equation Dix (1952) (Eq. 2.6).

$$V_n^2 = \frac{V^2RMS_n t(0)_n - V^2RMS_{n-1} t(0)_{n-1}}{t(0)_n - t(0)_{n-1}} \tag{2.6}$$

*Where* $t(0)_n$ *denotes vertical travel-time to the nth layer.*

Note the recursion form of Dix equation. This means we must know the value before the one we wish to find. Specifically for a layer of interest we must know t(0) and the RMS velocity of both the top and base interfaces.

## 2.2 Time Domain Thin-Bed Response

The seismic signature of a thin layer response is the result of interference from the top and base reflections of the layer. This interference imparts a 90 degree phase shift to the incident wavelet and an insensitivity of the composite waveform to changes in bed thickness. The amplitude response will continue to vary with changes in a thin layer's thickness however temporal separation of the top and base of the unit's interfaces is not quantifiable utilizing the separation between individual composite waveform peaks as they no longer coincide with the separate top and base interfaces of the unit. This, by definition, renders the layer seismically unresolvable Liner (2004). The lower limit of seismic resolution, or when a bed is said to be thin, is reached in practice when a layer's thickness falls below one quarter of the dominant seismic wavelength Widess (1973), (Eq. 2.7).

$$b = \frac{\lambda_b}{4} \tag{2.7}$$

*From Widess (1973). Practical limit of seismic resolution Where b=bed thickness and* $\lambda b = \tau V_b$ $\tau$ *is the predominant period of the wavelet and* $V_b$ *is the beds velocity.*

The quarter-wavelength limit also coincides with the maximum point of constructive interference for an opposite reflection coefficient pair known as the tuning thickness. The *theoretical* limit of resolution occurs when layer thickness falls below one eighth the dominant seismic wavelength. Below this threshold, the waveform approximates the derivative of the incident seismic wavelet and is completely unaffected by subsequent bed thickness changes (Eq. 2.8).

$$b = \frac{\lambda_b}{8} \qquad (2.8)$$

*From Widess (1973). Practical limit of seismic resolution Where b=bed thickness and* $\lambda b = \tau V_b$ $\tau$ *is the predominant period of the wavelet and* $V_b$ *is the beds velocity.*

Figure 2.4 illustrates these effects utilizing a wedge model with different bed thickness to incident wavelet wavelength ratios. Notice that at a bed thickness equal to one quarter the wavelength an increase in amplitude associated with tuning is observed and at bed thicknesses below one eighth the seismic wavelength the waveform remains unchanged.

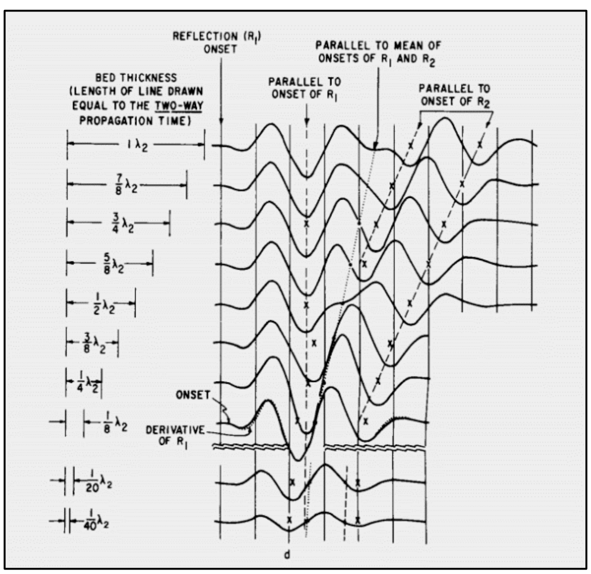



*Fig. 2.4 From Widess (1973).*

## 2.3 Sub-resolution layer thickness estimation

When utilized as a seismic analysis technique spectral decomposition is a process that transforms the composite time domain seismic signal into it's frequency domain representation. If the decomposition is constrained to a short time window the response approximates that of nonrandom geology and the decomposed spectra contains both the wavelet overprint and the spectral response of the layers contained within the window (Fig 2.5).

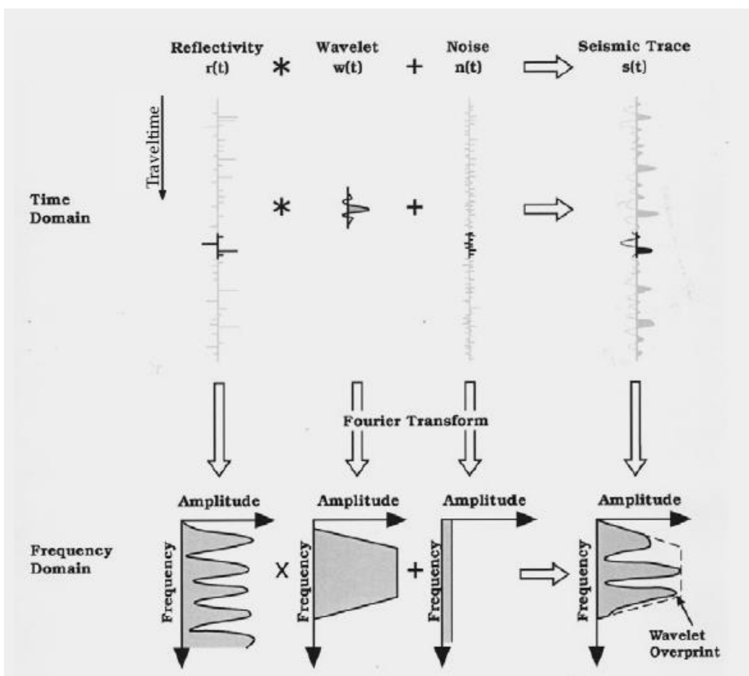



*Fig 2.5 Short-window spectral decomposition and its relationship to the convolutional model. A short temporal window samples ordered (nonrandom) geology that tunes the amplitude spectrum. From Partyka et al. (1999).*

Analysis of a geologic layers frequency spectrum allows for enhanced data interpretation of layer properties including inversion for the thickness of sub-resolution units. The basis for frequency domain layer-thickness inversion was introduced by Partyka et al. (1999) . They noted that the amplitude response from an isolated thin layer produces a predictable and periodic notch sequence in the layer's spectrum and that the frequency period of these notches is equal to the inverse of the layers temporal thickness (Fig. 2.6 and Fig. 2.7).

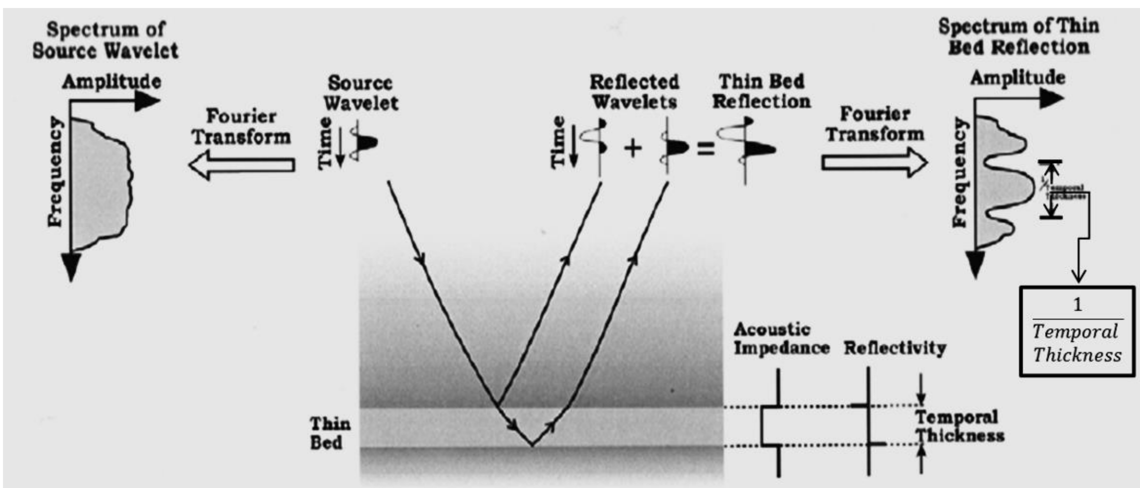



*Fig 2.6 Thin-bed spectral imaging. From Partyka et al. (1999).*

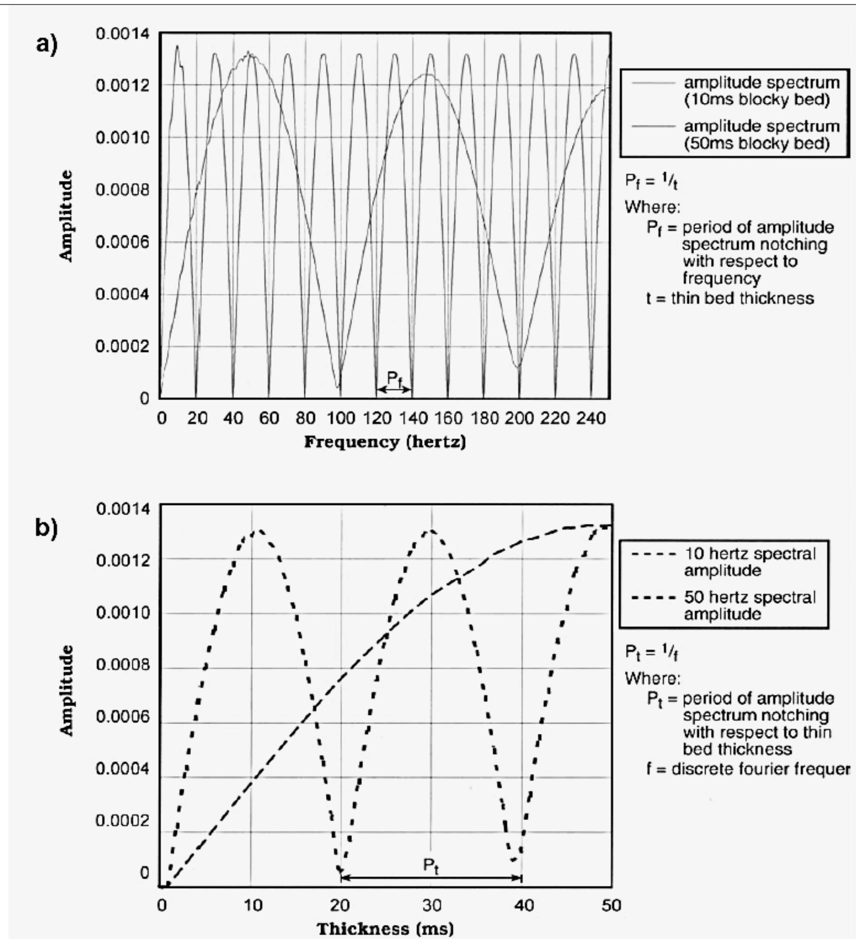



*Fig 2.7 Thin-bed tuning of amplitudes versus frequency (a) with respect to frequency and (b) with respect to bed thickness. From Partyka et al. (1999).*

This relationship between temporal thickness and spectral notch spacing allows the temporal thickness of a seismically thin layer to be inverted for.

## Chapter 3

## Theory

### 3.1 Thin-Bed Interval Velocity Inversion

When attempting to extract the interval velocity of a thin layer from seismic data utilizing the standard inversion method comprised of performing normal moveout on a layers top and base reflectors in order to attain an RMS velocity estimate for each, followed by substituting the RMS velocity's of both interfaces along with their respective zero offset times and the layers temporal thickness into Dix equation, it becomes clear that in the time domain no less than all the required parameters to derive an interval velocity estimate for a sub-resolution layer are un-attainable. This is due to the quantification of these parameters being directly or indirectly reliant upon temporal bed location and interface separation which by definition requires the layer be resolved. It can be shown that by utilizing the temporal midpoint and thickness of a thin layer and their respective change with offset we can solve for the interface locations, thickness and RMS velocity values that define an unresolved bed. Then, by substitution of this information into Dix equation the interval velocity can be inverted for.

We begin by locating the midpoint of a target thin layer at zero offset $t(0)_m$. Recall that the response from a layer below one eighth the seismic wavelength imparts a 90 degree phase shift to the incident wavelet. Therefore if the phase of our data is known we may locate the center (midpoint) of a thin-bed temporally by locating the time where the zero crossing of a 90 degree wavelet would occur. This can be conducted in a number of ways the most straight forward of which is to zero phase the data and locate the thin layers zero crossing (Fig 3.1).

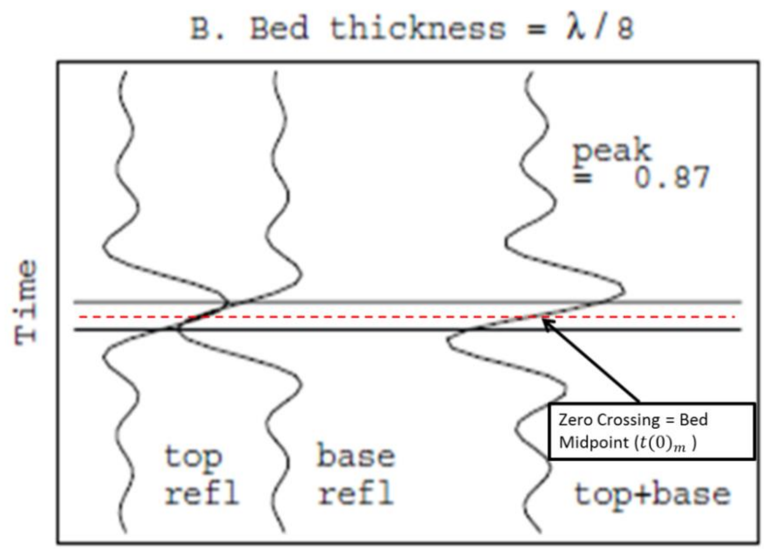



*Fig 3.1 Zero Phase Thin-Bed Response. Modified from Liner (2004).*

Once the midpoint is located, to define the zero offset top and base time of the layer we need to determine the temporal layer thickness at zero offset $\Delta t(0)$. The frequency spectrum of a thin-bed, which can be obtained by spectral decomposition of the layers local time domain response contains its discrete frequency amplitude information and associated notch spacing. By utilizing the relationship between layer thickness and notch period in the frequency spectrum defined by Partyka et al. (1999) an unresolved layers time thickness can be inverted for (Fig 3.2). If applied to the zero-offset trace the extracted thickness is the zero-offset time thickness of the layer $\Delta t(0)$ (Fig 3.3).

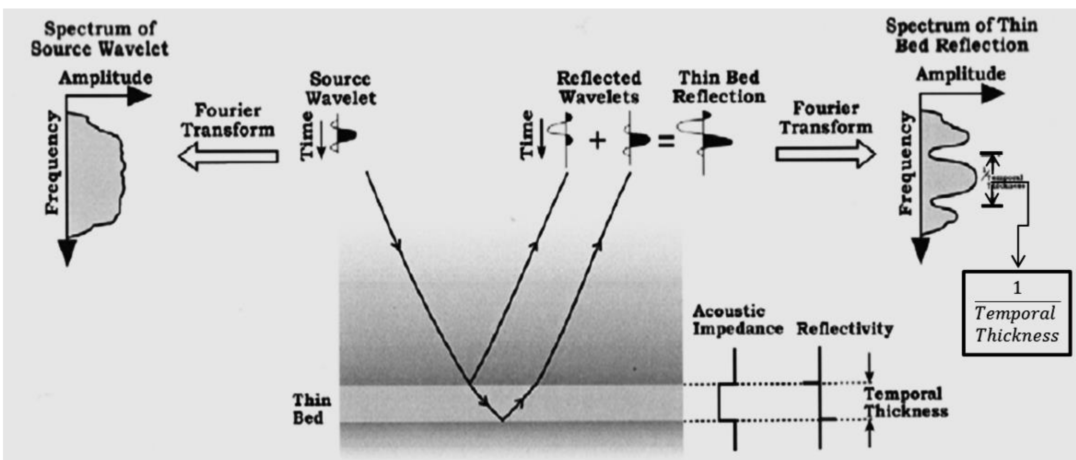
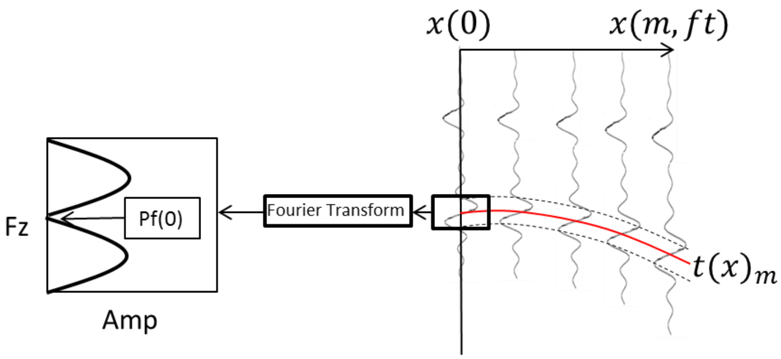


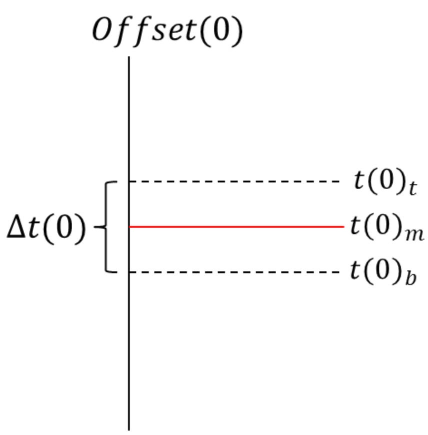

*Fig 3.2. From Partyka et al. (1999).*




$$\Delta t(0) = \frac{1}{Pf(0)}$$

*Fig 3.3 Sub-resolution temporal layer thickness determination utilizing inverse frequency notch spacing.*

After establishing a thin layers temporal zero-offset midpoint $t(0)_m$ and thickness $\Delta \mathrm{t}(0)$ the location of time zero for the upper surface $t(0)_t$ of the thin layer is taken to be half the total temporal thickness $\Delta \mathrm{t}(0)$ above the beds midpoint $t(0)_m$ and the base $t(0)_b$ one half below. In simple terms the bed is placed symmetrically about its midpoint. (Fig 3.4)




$$Eq. 3.1 \quad t(0)_t = t(0)_m - \frac{\Delta t}{2}$$

$$Eq. 3.2 \quad t(0)_b = t(0)_m + \frac{\Delta t}{2}$$

$$Eq.\ 3.3 \quad \Delta t\ (0) = \frac{1}{Pf_0} = t(0)_b - t(0)_t$$

*Fig 3.4 Solving for the top and base t(0) values of an unresolved layer utilizing its midpoint location and temporal thickness.*

Upon re-examination of Dix Equation (2.6) with substitution of the current coefficient nomenclature for clarity, we see the additional information missing for interval velocity inversion is that of the two RMS velocity's defining the top $t(0)_t$ and base $t(0)_b$ interfaces of the layer, denoted as $V^2RMS_t \ and \ V^2RMS_b$ respectively.

$$(3.4) \qquad V = \sqrt{\frac{V^2RMS_b(t(0)_b) - V^2RMS_{\ t}(t(0)_t)}{\Delta t(0)}}$$

*Where $V^2RMS_t$ and $t(0)_t$ are the layers upper interface RMS and zero offset time intercept values respectively and $V^2RMS_b$ and $t(0)_b$ are the equivalent values for the lower interface.*

Extraction of these velocity parameters will necessitate we obtain an estimate of the layers midpoint RMS velocity and define the relationship between an isovelocity layers midpoint RMS velocity and the RMS velocity of the top and base interfaces. The midpoint RMS velocity is defined by the layers midpoint travel-time curve as a function of offset. This curve can be observed as the offset dependent analog to the zero-offset midpoint and is similarly defined by the zero crossing equivalent of a 90 degree thin-bed response at offset (x) denoted as $t(x)_m$ (Fig 3.5). The midpoint RMS velocity for a thin layer then can be calculated by performing normal moveout on the midpoint travel-time curve via equation 2.3 or equation 2.4.

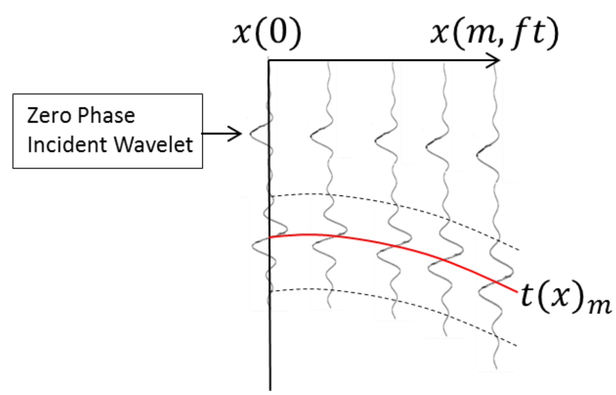
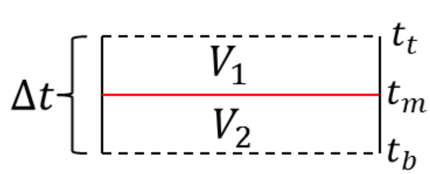



*Fig. 3.5 Midpoint travel-time curve for an isovelocity thin layer.*

The relationship between a layers midpoint RMS velocity and the RMS velocity's of the beds upper and lower interfaces can be derived mathematically by taking the layer to be isovelocity and setting the velocity from the midpoint to the top interface to be equal to the velocity from the midpoint to the bottom interface and likewise for the time separation (Fig 3.6)




$$V_1 = V_2$$

$$t_m - t_t = t_b - t_m$$

*Fig 3.6 Travel-time and velocity relationship between an isovelocity layers midpoint and bounding interfaces*

Setting Dix equation for each half of the layer equal we can write.

$$(3.5) \qquad \frac{V^2RMS_m * t(0)_m - V^2RMS_t * t(0)_t}{t(0)_m - t(0)_t} = \frac{V^2RMS_b * t(0)_b - V^2RMS_m * t(0)_m}{t(0)_b - t(0)_m}$$

*Where:* $V^2RMS_m$*=Midpoint RMS Velocity,* $V^2RMS_t$ *= Top RMS Velocity and* $V^2RMS_b$ *=Base RMS Velocity.*

Reducing equation 3.5

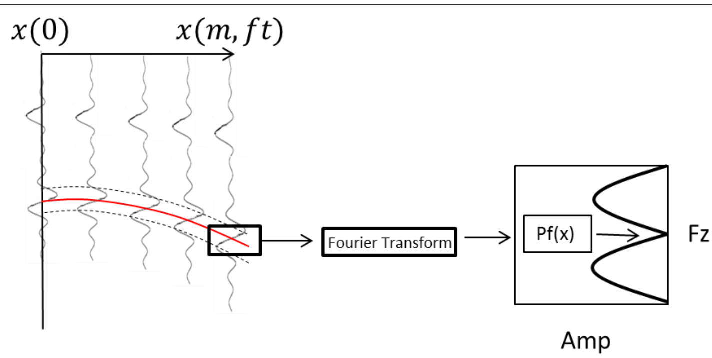
$$(3.6) \quad V^2RMS_m * t(0)_m - V^2RMS_t * t(0)_t = V^2RMS_b * t(0)_b - V^2RMS_m * t(0)_m$$

Rearranging equation 3.6 and collecting like terms.

$$(3.7)^* \quad 2V^2RMS_m * t(0)_m = V^2RMS_b * t(0)_b + V^2RMS_t * t(0)_t$$

Equation 3.7 shows by utilizing the Dix equation that for an isovelocity layer, the RMS velocities from the top and base multiplied by their respective zero-offset times combine are exactly equal to two times the midpoint RMS velocity multiplied by the zero-offset midpoint time value. This derivation establishes a direct connection between the observed midpoint RMS value and the two unknown top and base RMS curves for an unresolved layer. However the equation is non-unique due to the fact we have two unknowns ($RMS_t$, $RMS_b$) and with the above extractable parameters, only a solution for the midpoint contribution is attainable. Another equation is needed if we wish to extract the appropriate RMS values for each curve.

For the top and base reflections of a single isovelocity layer with a travel-time offset relationship defined by the moveout equation(s) for the top and base interfaces, the layers temporal thickness as a function of offset defines the separation between the top and base move out time correction for that offset. That is to say the relationship between layer thickness and offset establishes the time separation between the consecutive offset dependent travel-time values derived by the moveout equations for the top and base interfaces. We can invert for the temporal layer thickness of an unresolved isovelocity layer at any offset in precisely the same fashion as was utilized to determine the zero offset thickness by utilizing the spectral notch period of the layers frequency response at the desired offset (Fig 3.7, Eq 3.8).




$$Eq. 3.8 \quad \Delta t(x) = \frac{1}{Pf(x)}$$

*Fig 3.7 Offset dependent temporal thickness determination of an isovelocity sub-resolution layer utilizing inverse frequency notch spacing.*

The defined temporal separation between moveout corrections with offset in turn depends on the RMS velocity of each interface established by the offset dependent travel-time for that event. Therefore the time separation between the moveout corrections for the top and base of an event defines a relationship between RMS velocity, time zero and the offset dependent temporal thickness of a layer. We can write.

$$(3.9)^{*} \qquad \Delta t(x) = \sqrt{t(0)_b^{\,2} + \frac{x^2}{V^2RMS_b}} - \sqrt{t(0)_t^{\,2} + \frac{x^2}{V^2RMS_t}}$$

*And for the long offset, fourth order correction …*

$$(3.10)^{*} \quad \Delta t(x) = \sqrt{t(0)_b^{\,2} + \frac{x^2}{V^2RMS_b} + \frac{(1+\frac{V_4^4}{V^4RMS_b})x^4}{4t(0)_bV^2RMS_b}} - \sqrt{t(0)_t^{\,2} + \frac{x^2}{V^2RMS_t} + \frac{(1+\frac{V_4^4}{V^4RMS_t})x^4}{4t(0)_tV^2RMS_t}}$$

*Where* $V_4^4 = \frac{1}{t(0)} \sum V^4(i)\Delta t(i)$

By combining equation 3.7 with equation 3.9 or, as offset dictates equation 3.10 we now have two equations and two unknowns, $V^2RMS_t$ and $V^2RMS_b$ the layer top and base RMS values respectively. Therefore we can set these equations equal as follows.

$$(3.11)^{*} \qquad \left(\sqrt{t(0)_b{}^2 + \frac{x^2}{V^2RMS_b}} - \sqrt{t(0)_t{}^2 + \frac{x^2}{V^2RMS_t}}\right) - \Delta t(x) =$$

$$\sqrt{2V^2RMS_m * t(0)_m} - \left(\sqrt{V^2RMS_b * t(0)_b} + \sqrt{V^2RMS_t * t(0)_t}\right)$$

And incorporating the long offset fourth order correction…

$$(3.12)^{*} \qquad \left(\sqrt{t(0)_b{}^2 + \frac{x^2}{V^2RMS_b} + \frac{(1+\frac{V_4^4}{V^4RMS_b})x^4}{4t(0)_bV^2RMS_b}} - \sqrt{t(0)_t{}^2 + \frac{x^2}{V^2RMS_t} + \frac{(1+\frac{V_4^4}{V^4RMS_t})x^4}{4t(0)_tV^2RMS_t}}\right) -$$

$$\Delta t(x) = \sqrt{2V^2RMS_m * t(0)_m} - \left(\sqrt{V^2RMS_b * t(0)_b} + \sqrt{V^2RMS_t * t(0)_t}\right)$$

** Explicit solutions for $RMS_t$ and $RMS_b$ from equation 3.11 for models 1-7 and B1-B4 can be found in appendix A*

Substituting the known constants from equations 3.1, 3.2 and 3.8 combine with the midpoint location and RMS value extracted by performing normal moveout (on the midpoint curve), then solving for $V^2RMS_b$, $V^2RMS_t$, and finally taking the positive solution pair yields the top and base RMS values for a thin layer. Re-examination of equation 3.4 reveals we now have all the required input parameters to invert for the interval velocity of a (thin) layer.

$$(3.4) \qquad V = \sqrt{\frac{V^2RMS_b(t(0)_b) - V^2RMS_t(t(0)_t)}{\Delta t(0)}}$$

**Chapter 4**

**Method / Examples**

## 4.1 Method

A series of models representing a single target layer embedded in an isotropic background medium was created to test the theory (Table. 4.1). All examples were analyzed as uncorrected, pre-stack, offset-sorted common midpoint gather records. The full Zoeppritz equations were utilized to model the response corresponding to a 45 Hz zero phase Ricker source wavelet, only primary reflections were calculated. Conversion from the time to frequency domain was achieved via the Discrete Fourier Transform. Following the data's transformation into the frequency domain the wavelet overprint was removed by dividing the frequency spectrum of each trace by that of the source wavelet. The following sequence was utilized in gathering the necessary data parameters.

- A. Extract $t(0)_m$ from time domain seismic (at zero crossing of 90 degree response)
- B. Estimate $RMS_m$ utilizing velocity analysis centered on midpoint
- Transfer data into the frequency domain via Discrete Fourier Transform
- C. Extract $Pf(0)$ from near offset trace, take inverse to estimate $\Delta t(0)$
- D. Extract $Pf(x)$ from far offset trace, take inverse to estimate $\Delta t(x)$

These parameters (Table. 4.2) were then combined with equations 3.1, 3.2, and 3.11 introduced in the previous section to derive the values necessary for interval velocity inversion (Table. 4.3). The values derived were then substituted into Dix equation (Eq. 3.4) to generate an interval velocity estimate. The results and comparison to the actual values are given for each model (Table 4.4). Illustrations of the four steps A-D for each model are shown with the annotation order first denoting model number and subsequently the letter corresponding to the step in the aforementioned sequence order.

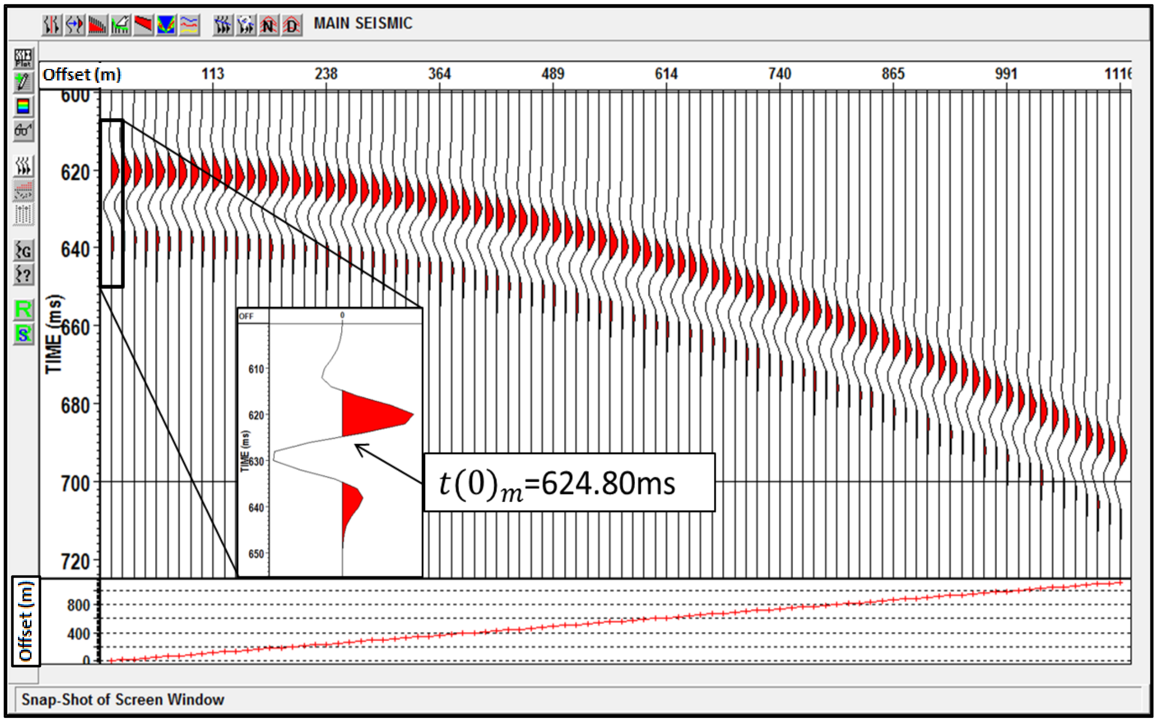
## 4.2 Single-Layer Model Synthetic Data

| Model Type/ Number | Vp m/s | Vs m/s | Density g/cc^3 | δ | ε | Layer Depth m | Layer Thickness m | Offset m | Offset Increment m |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| Isotropic: Model 1 | 5200 | 2888 | 2.63 | 0 | 0 | 1122 | 20 | 1116 | 12.5 |
| Anisotropic: Model 2 | 5200 | 2888 | 2.63 | -0.05 | -0.12 | 1122 | 20 | 1116 | 12.5 |
| Anisotropic: Model 3 | 5200 | 2888 | 2.63 | -0.10 | -0.12 | 1122 | 20 | 1116 | 12.5 |
| Anisotropic: Model 4 | 5200 | 2888 | 2.63 | 0.05 | 0.12 | 1122 | 20 | 1116 | 12.5 |
| Anisotropic: Model 5 | 5200 | 2888 | 2.63 | 0.10 | 0.12 | 1122 | 20 | 1116 | 12.5 |
| Low Velocity Layer: Model 6 | 3000 | 1667 | 2.63 | 0 | 0 | 1122 | 20 | 1116 | 12.5 |
| High Velocity Layer: Model 7 | 8000 | 4400 | 2.63 | 0 | 0 | 1724 | 20 | 1713 | 12.5 |
| | | | | | | | | | |
| Background Medium: Upper=Lower | 3615 | 2192 | 2.4 | 0 | 0 | N/A | N/A | N/A | N/A |

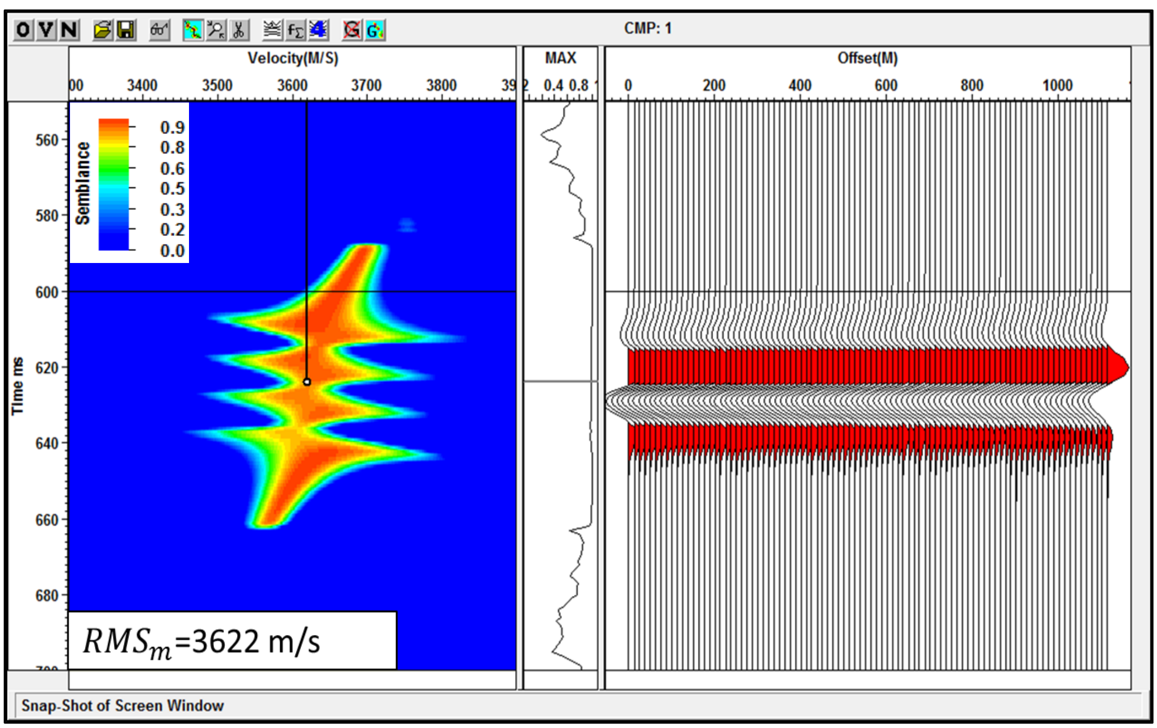
*Table 4.1*

**Model 1**

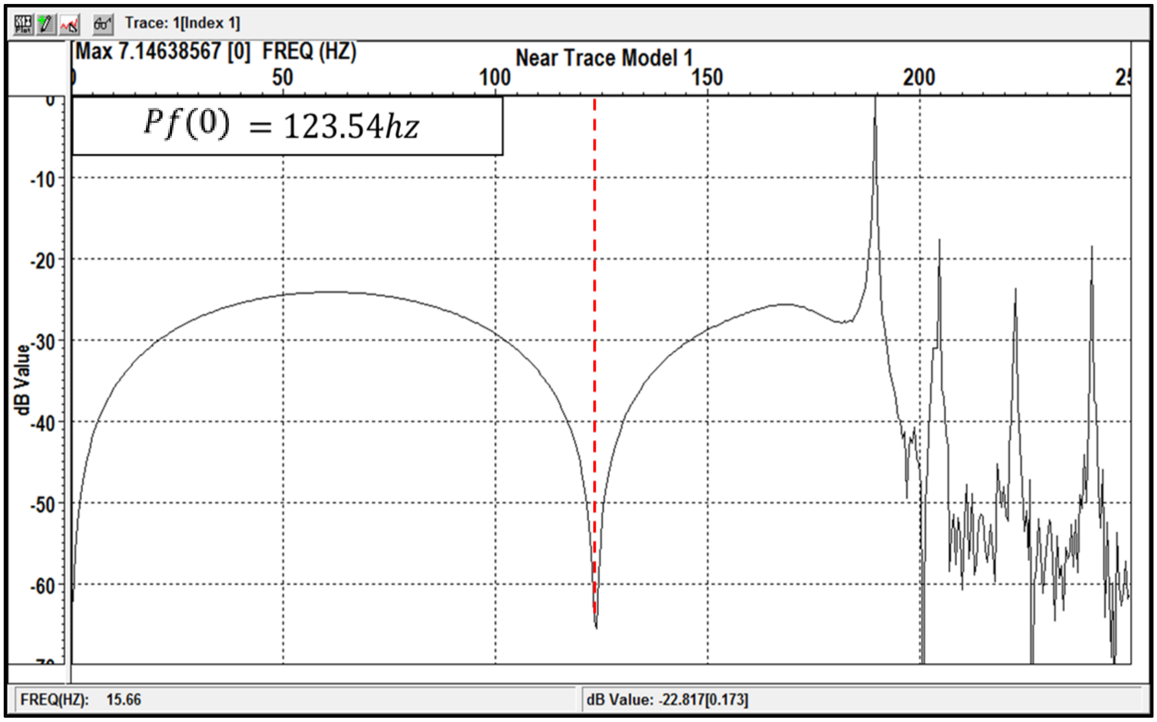
1: A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
TIME (ms)
600
620
640
660
680
700
720
$t(0)_m$=624.80ms
Offset (m)
800
400
0
Snap-Shot of Screen Window

1: B

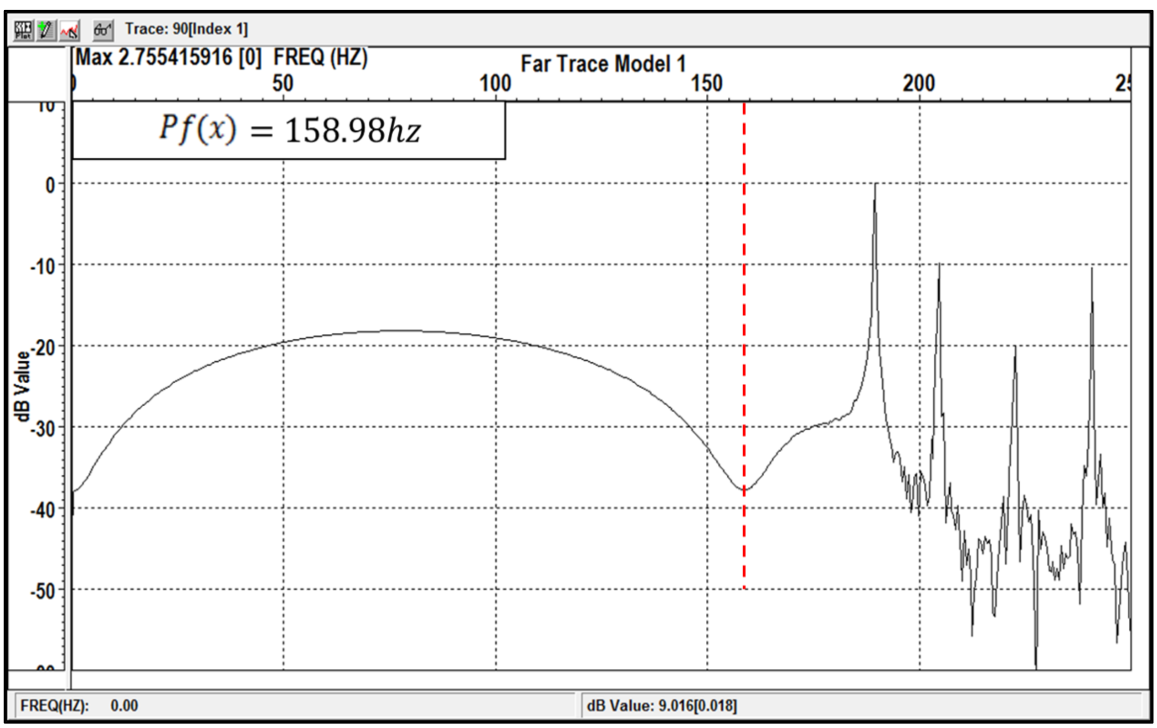

CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
0.4 0.8
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
Time ms
560
580
600
620
640
660
680
$RMS_m$=3622 m/s
Snap-Shot of Screen Window

1: C



Trace: 1[Index 1]
Max 7.14638567 [0] FREQ (HZ)
Near Trace Model 1
0
50
100
150
200
25
$Pf(0) = 123.54hz$
0
-10
-20
-30
-40
-50
-60
-70
dB Value
FREQ(HZ): 15.66
dB Value: -22.817[0.173]

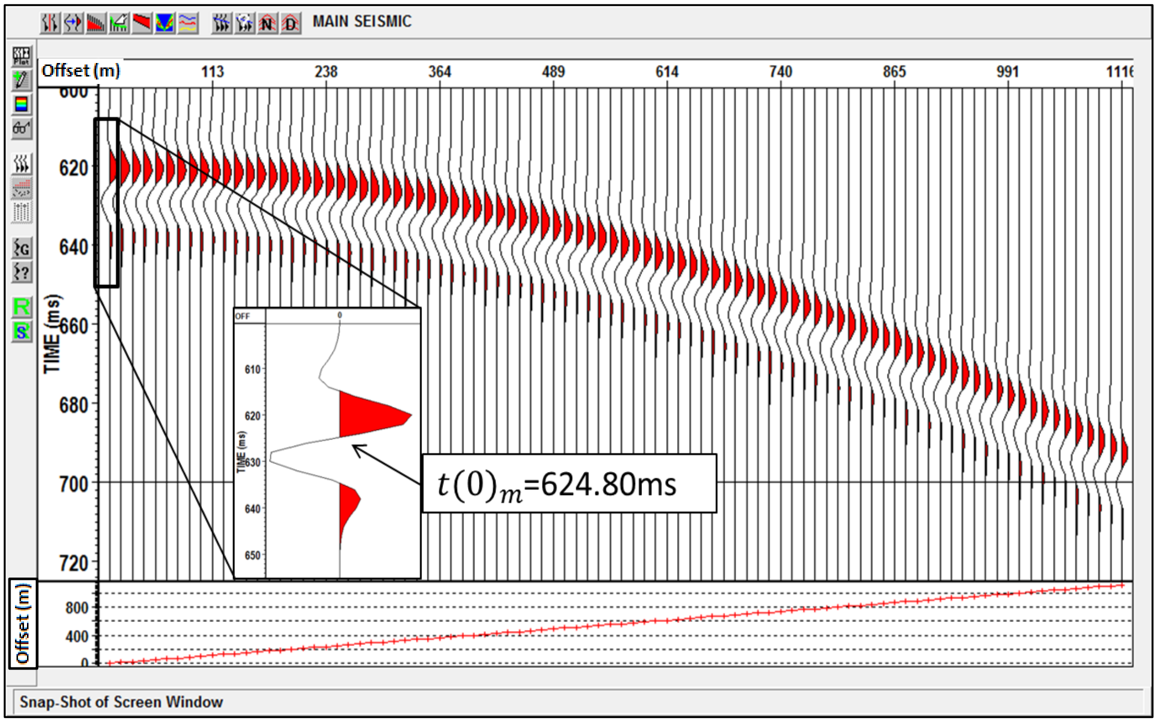

1: D



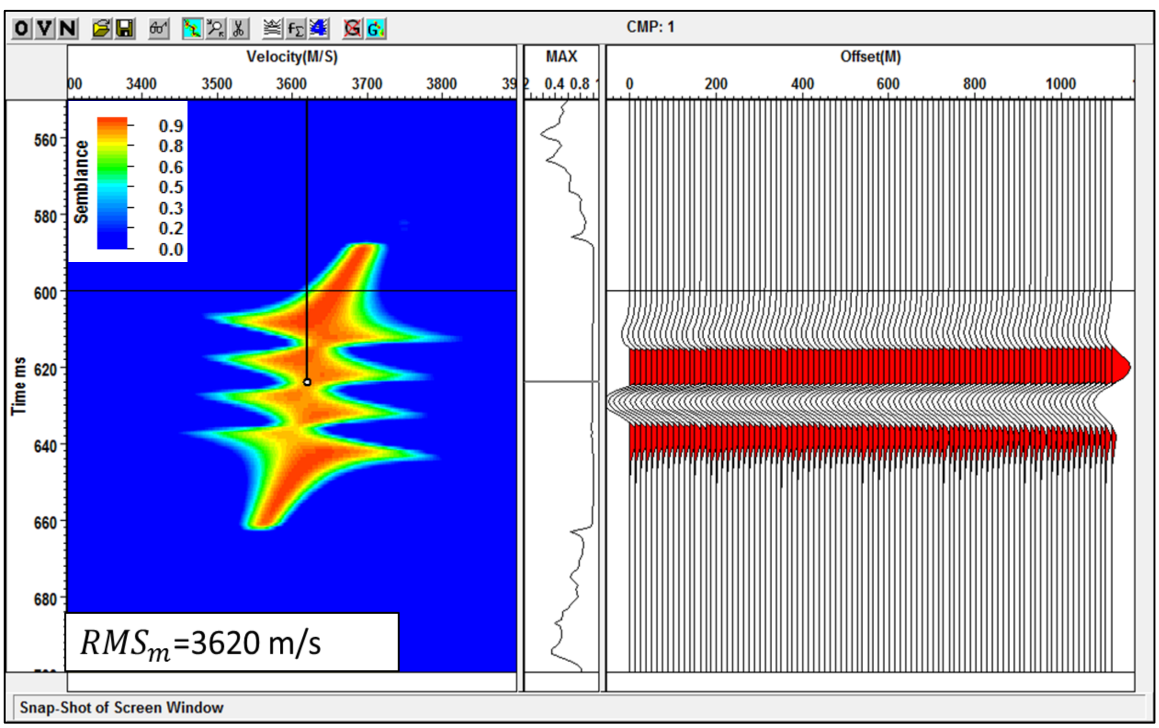
Trace: 90[Index 1]
Max 2.755415916 [0] FREQ (HZ)
Far Trace Model 1
0
50
100
150
200
25
$Pf(x) = 158.98hz$
10
0
-10
-20
-30
-40
-50
-60
dB Value
FREQ(HZ): 0.00
dB Value: 9.016[0.018]

**Model 2**

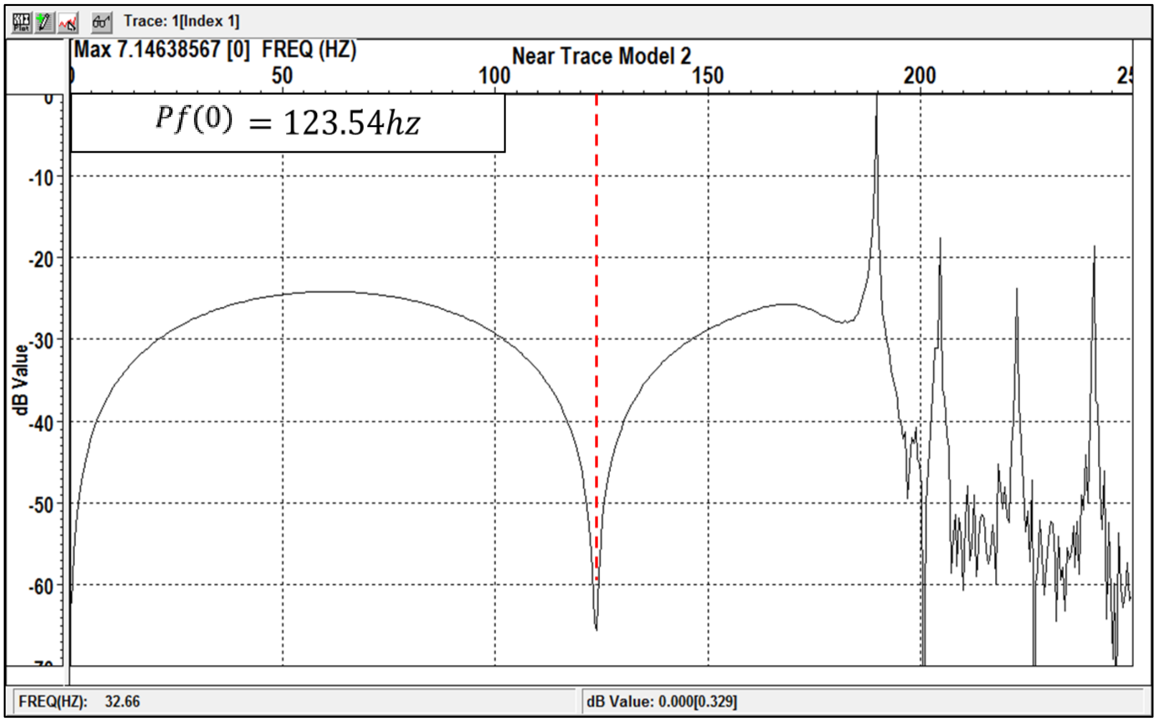
2:A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
TIME (ms)
620
640
660
680
700
720
Offset (m)
800
400
$t(0)_m$=624.80ms
Snap-Shot of Screen Window

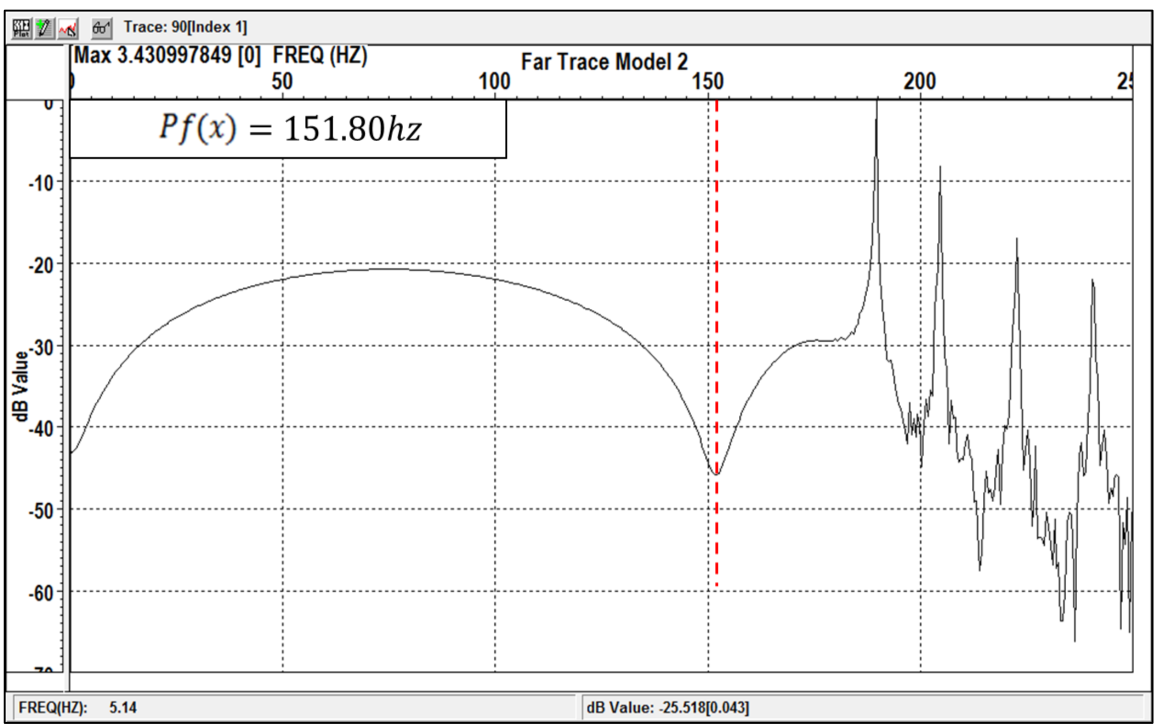
2: B


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
0.4 0.8
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
Time ms
560
580
600
620
640
660
680
$RMS_m$=3620 m/s
Snap-Shot of Screen Window

2: C

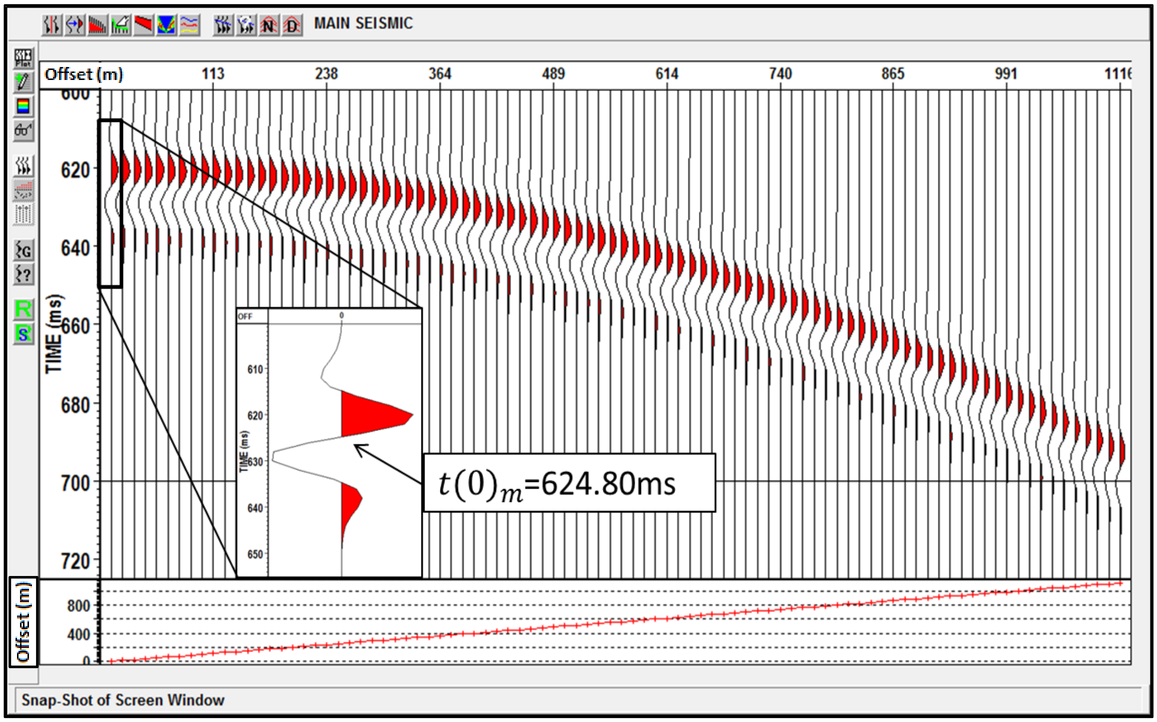

Trace: 1[Index 1]
Max 7.14638567 [0] FREQ (HZ)
Near Trace Model 2
$Pf(0) = 123.54hz$
dB Value
FREQ(HZ): 32.66
dB Value: 0.000[0.329]

2: D

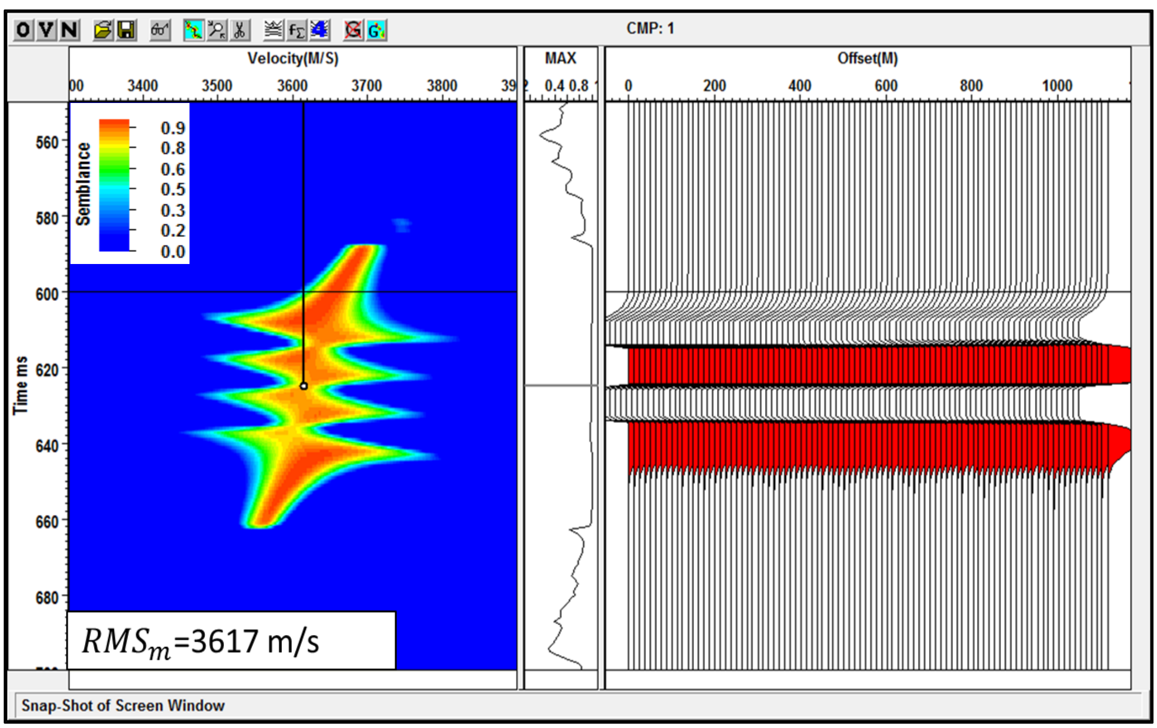

Trace: 90[Index 1]
Max 3.430997849 [0] FREQ (HZ)
Far Trace Model 2
$Pf(x) = 151.80hz$
dB Value
FREQ(HZ): 5.14
dB Value: -25.518[0.043]

## Model 3

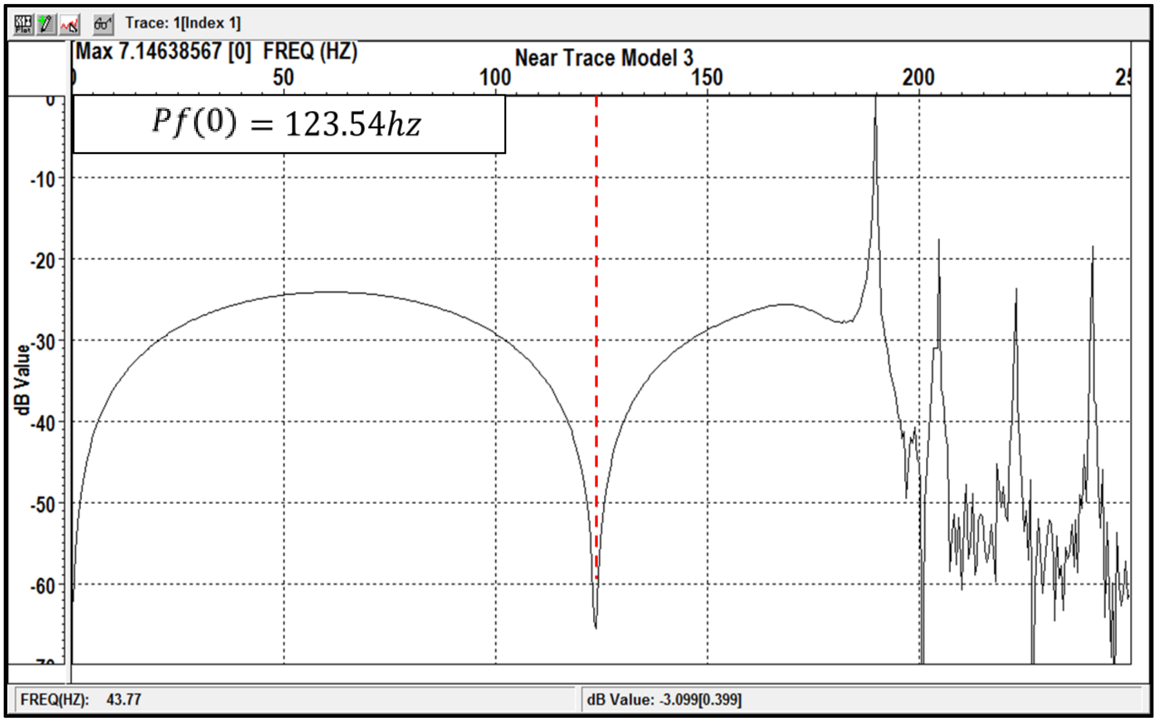
3: A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
TIME (ms)
620
640
660
680
700
720
$t(0)_m$=624.80ms
Offset (m)
800
400
Snap-Shot of Screen Window

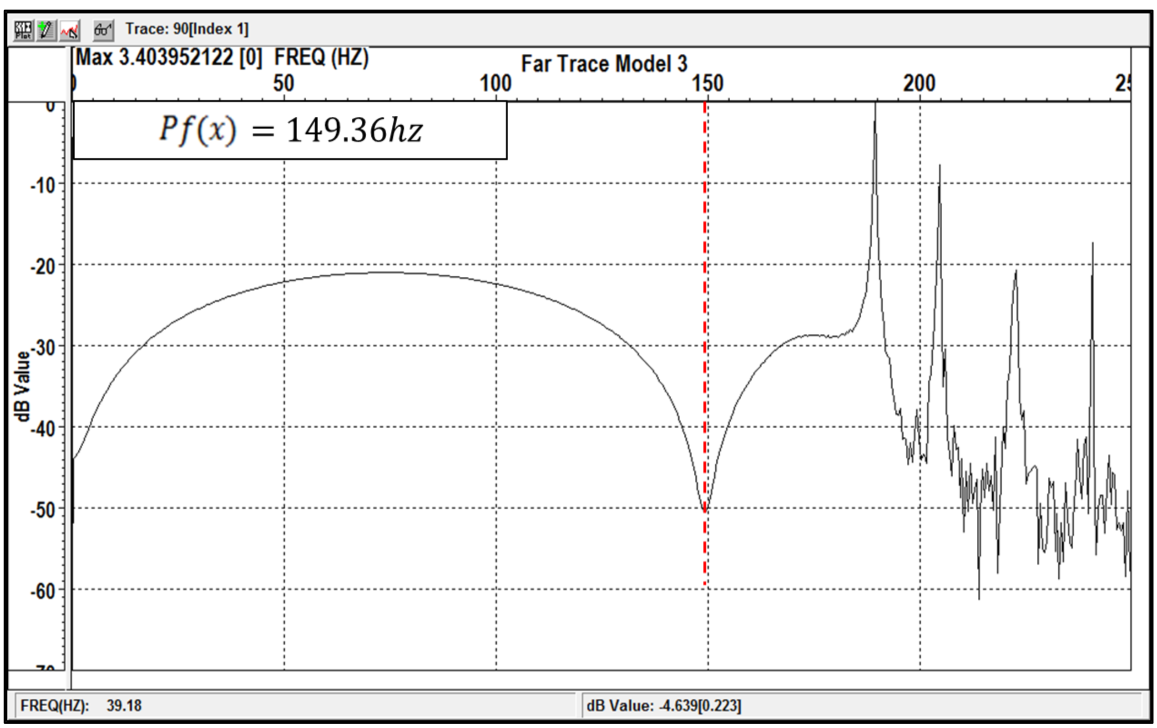
3: B


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
Time ms
560
580
600
620
640
660
680
$RMS_m$=3617 m/s
Snap-Shot of Screen Window

3: C

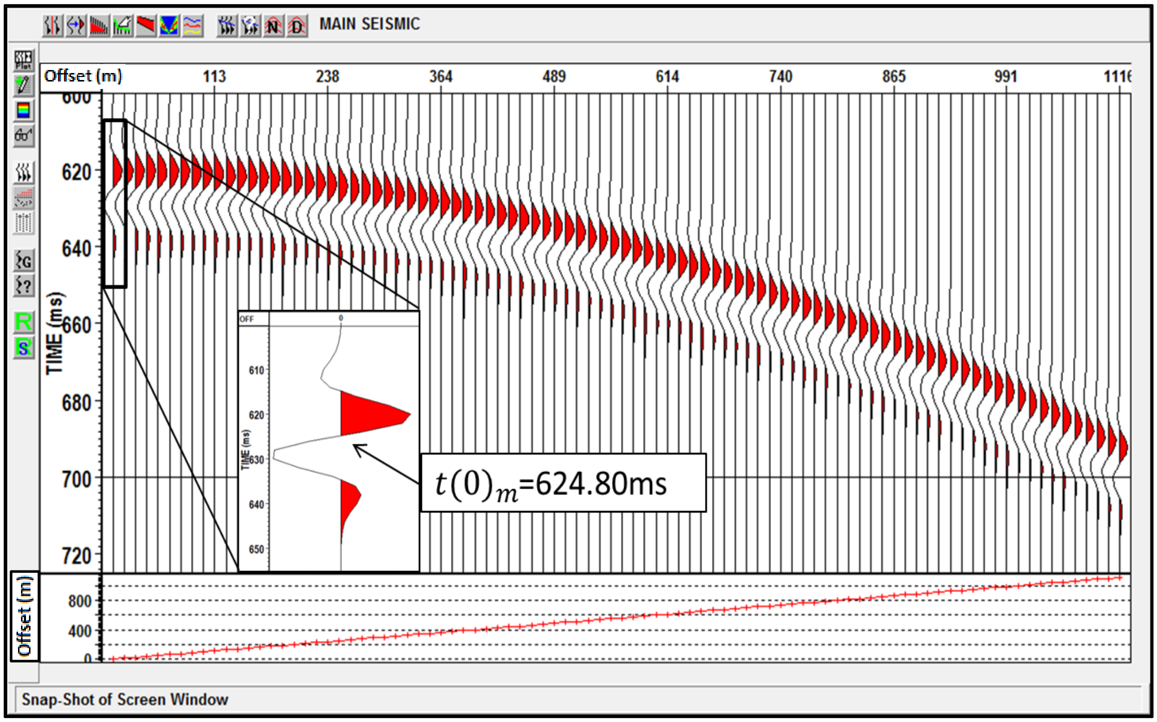

Trace: 1[Index 1]
Max 7.14638567 [0] FREQ (HZ)
Near Trace Model 3
50
100
150
200
0
-10
-20
-30
-40
-50
-60
dB Value
$Pf(0) = 123.54hz$
FREQ(HZ): 43.77
dB Value: -3.099[0.399]

3: D

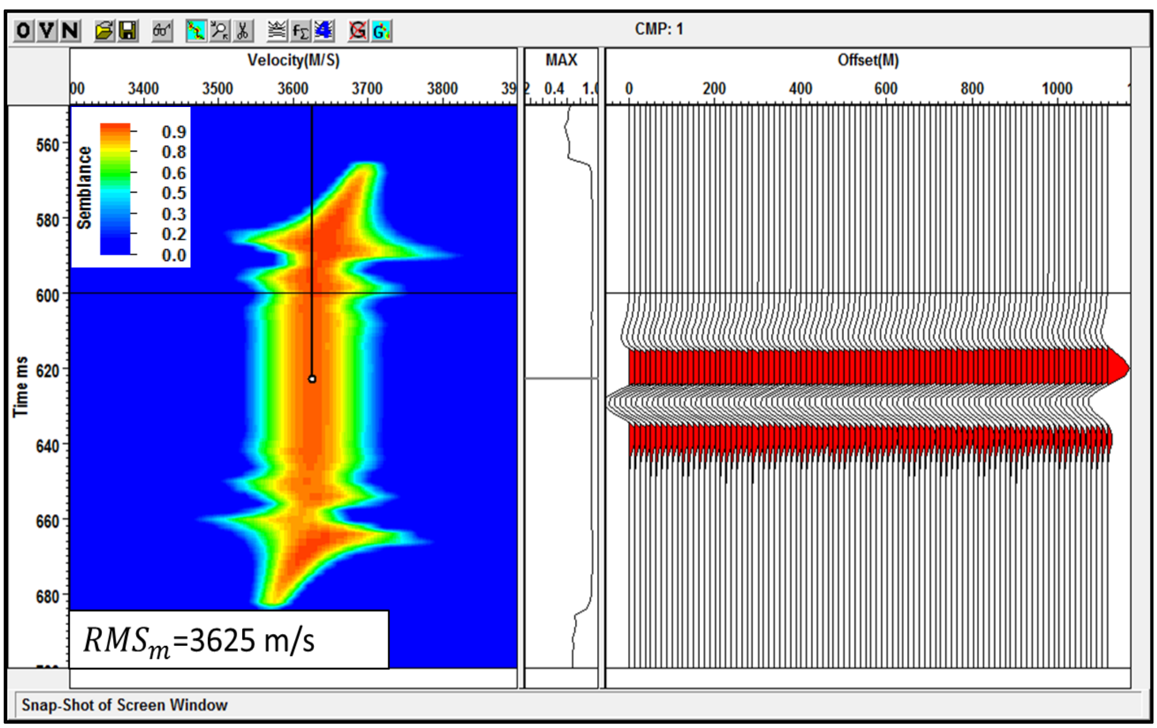

Trace: 90[Index 1]
Max 3.403952122 [0] FREQ (HZ)
Far Trace Model 3
50
100
150
200
0
-10
-20
-30
-40
-50
-60
dB Value
$Pf(x) = 149.36hz$
FREQ(HZ): 39.18
dB Value: -4.639[0.223]

**Model 4**

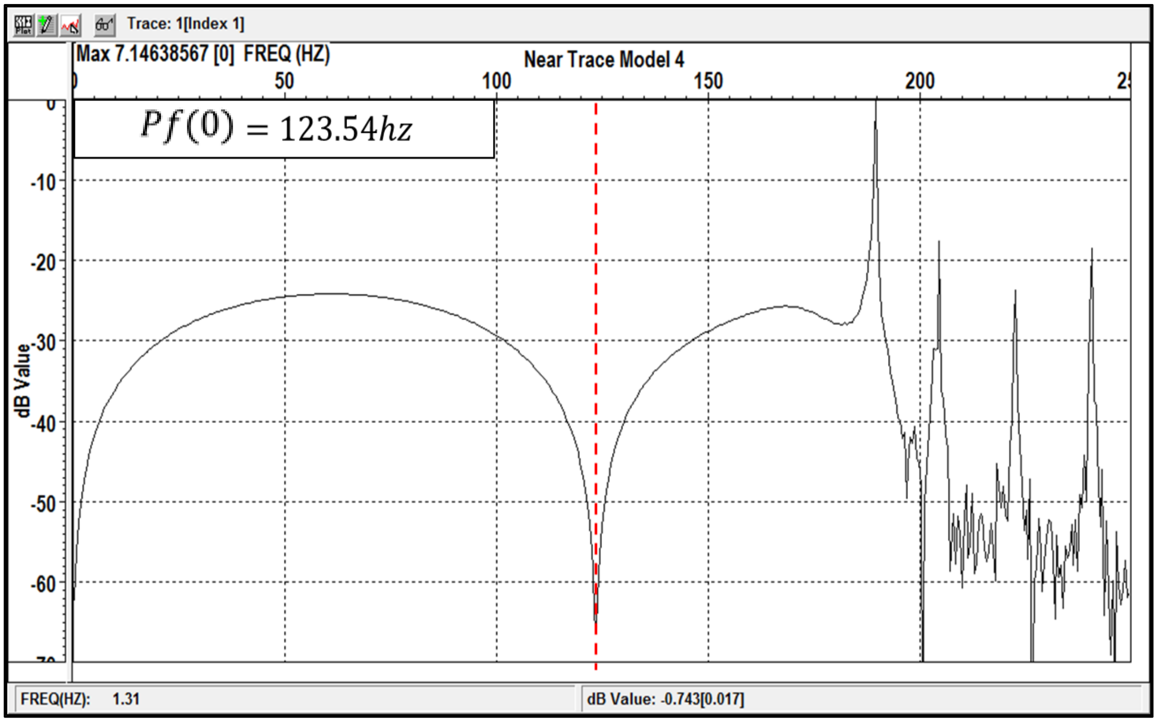
4: A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
620
640
660
680
700
720
TIME (ms)
$t(0)_m$=624.80ms
Offset (m)
800
400
Snap-Shot of Screen Window

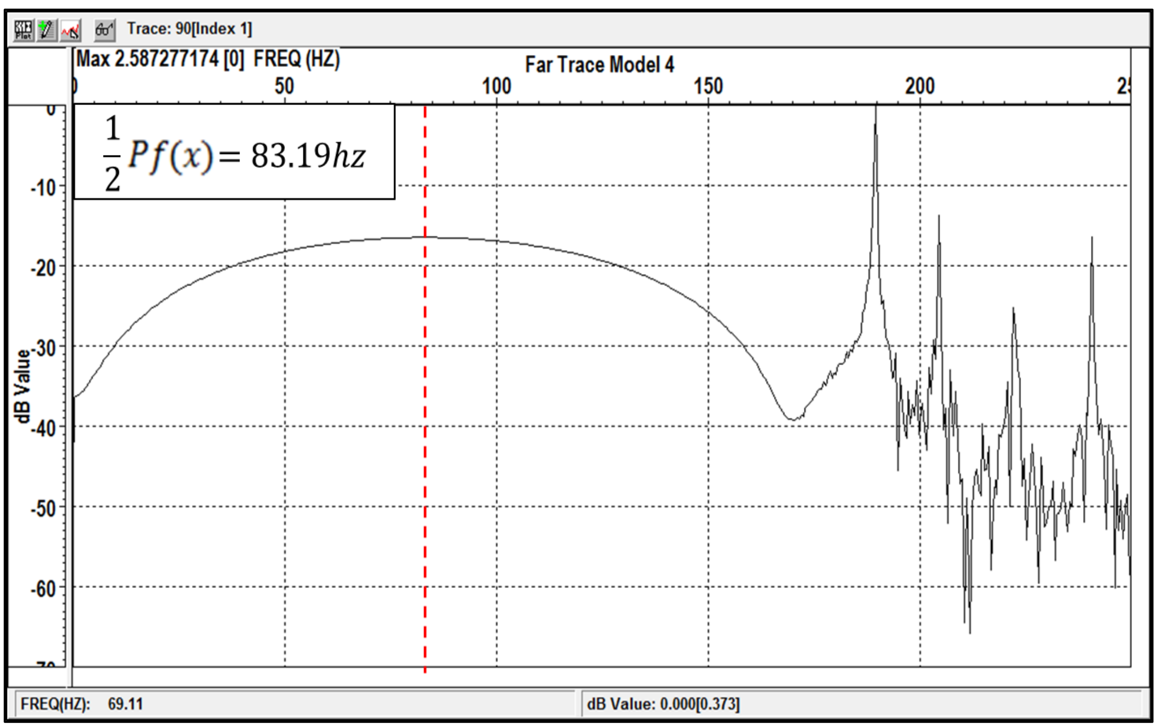
4: B


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
560
580
600
620
640
660
680
Time ms
$RMS_m$=3625 m/s
Snap-Shot of Screen Window

4: C

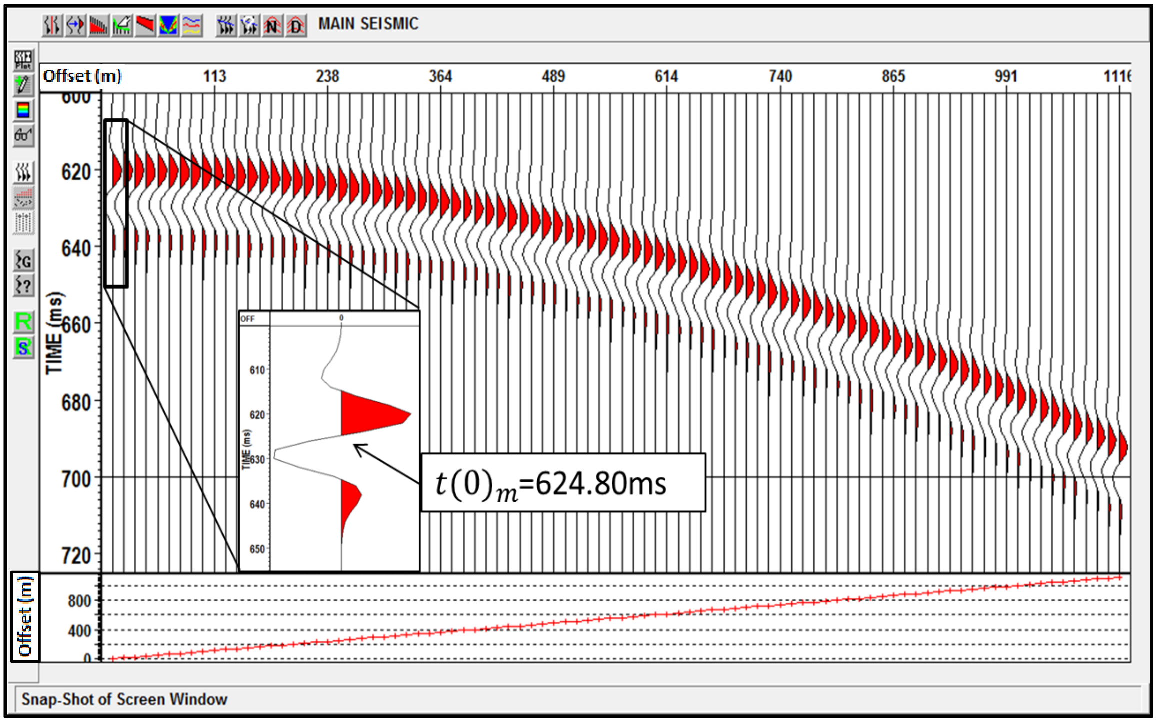

Trace: 1[Index 1]
Max 7.14638567 [0] FREQ (HZ)
Near Trace Model 4
$Pf(0) = 123.54hz$
dB Value
FREQ(HZ): 1.31
dB Value: -0.743[0.017]

4: D

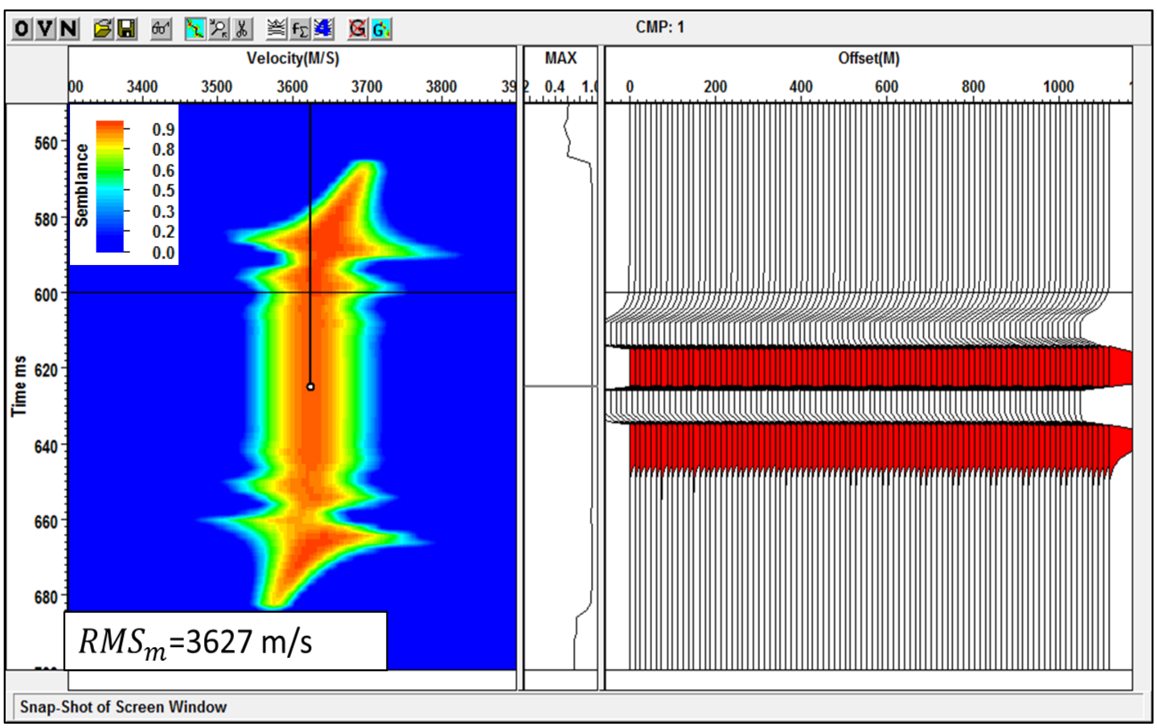

Trace: 90[Index 1]
Max 2.587277174 [0] FREQ (HZ)
Far Trace Model 4
$\frac{1}{2}Pf(x) = 83.19hz$
dB Value
FREQ(HZ): 69.11
dB Value: 0.000[0.373]

## Model 5

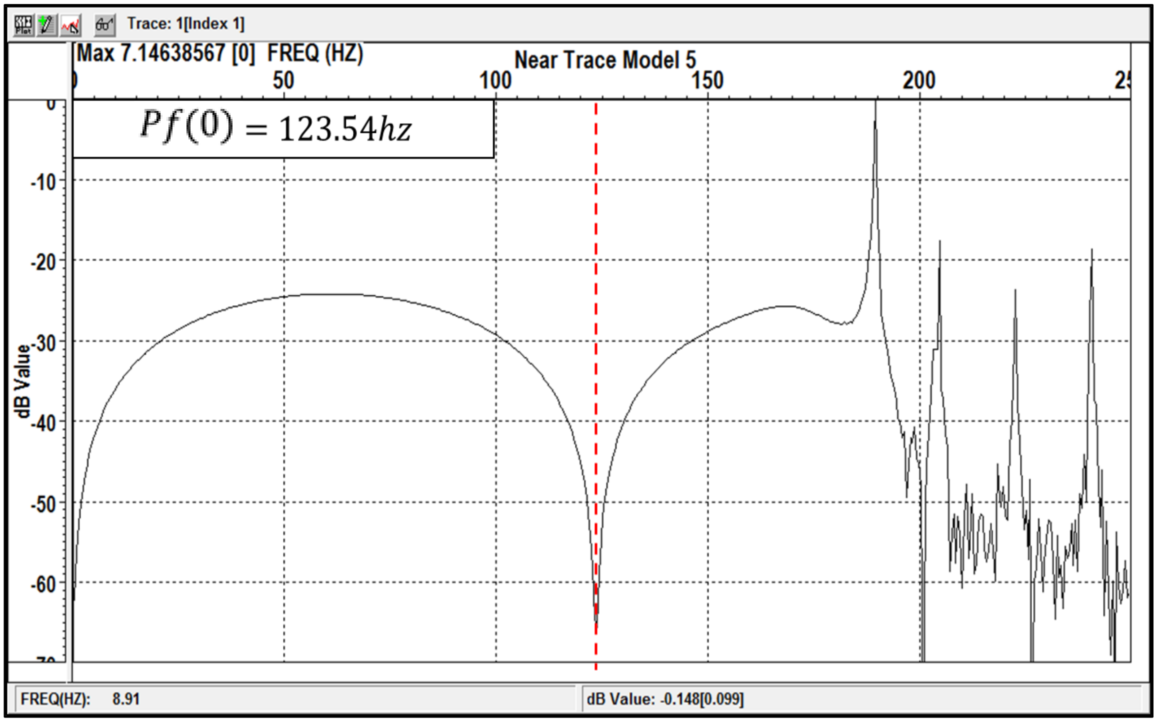
5: A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
600
620
640
660
680
700
720
TIME (ms)
$t(0)_m$=624.80ms
Offset (m)
800
400
0
Snap-Shot of Screen Window

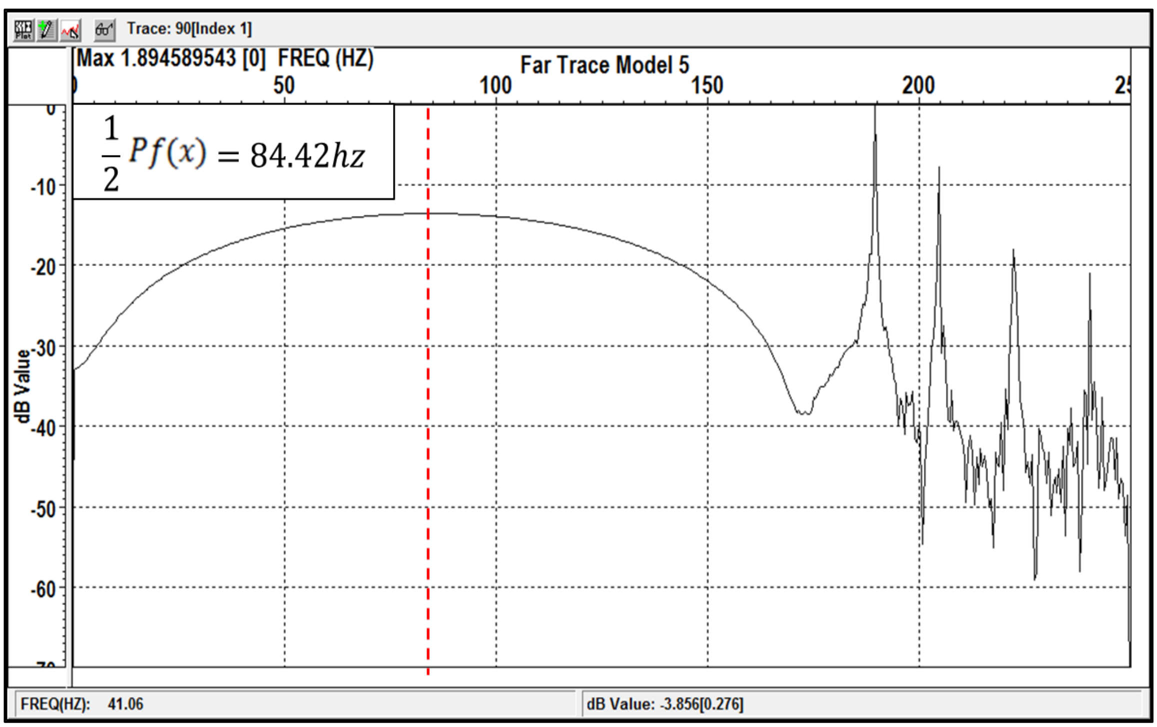
5: B


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
0.4
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
560
580
600
620
640
660
680
Time ms
$RMS_m$=3627 m/s
Snap-Shot of Screen Window

5: C

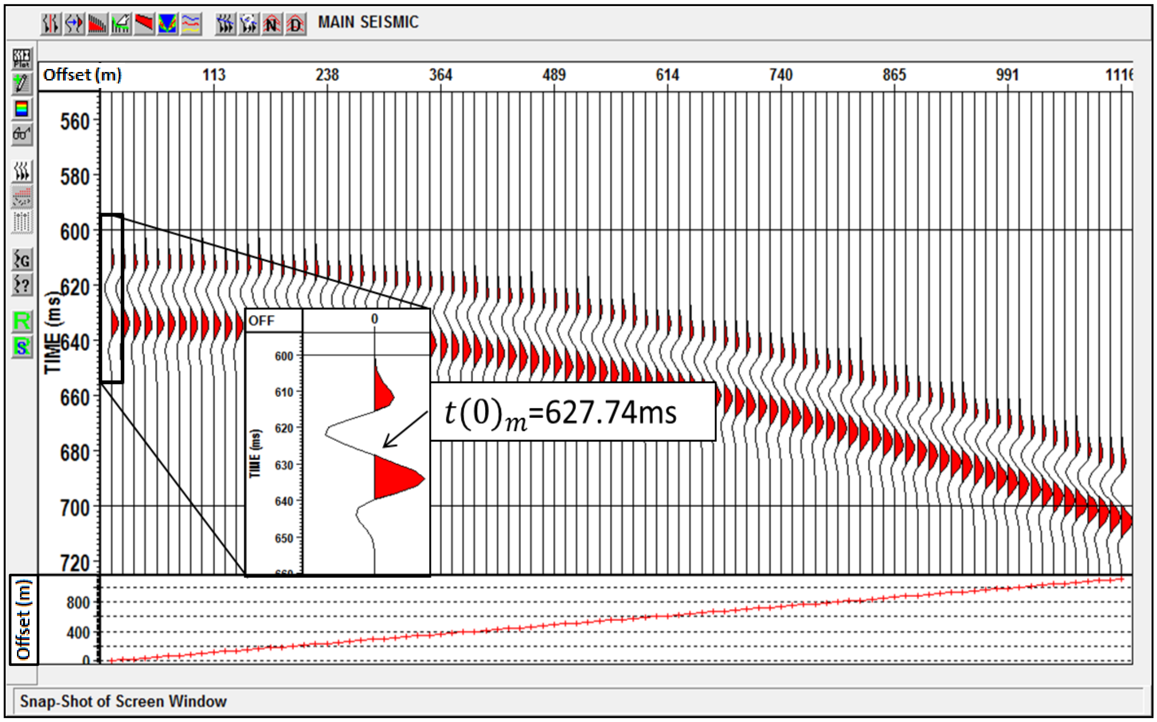

Trace: 1[Index 1]
Max 7.14638567 [0] FREQ (HZ)
Near Trace Model 5
$Pf(0) = 123.54hz$
dB Value
FREQ(HZ): 8.91
dB Value: -0.148[0.099]

5: D

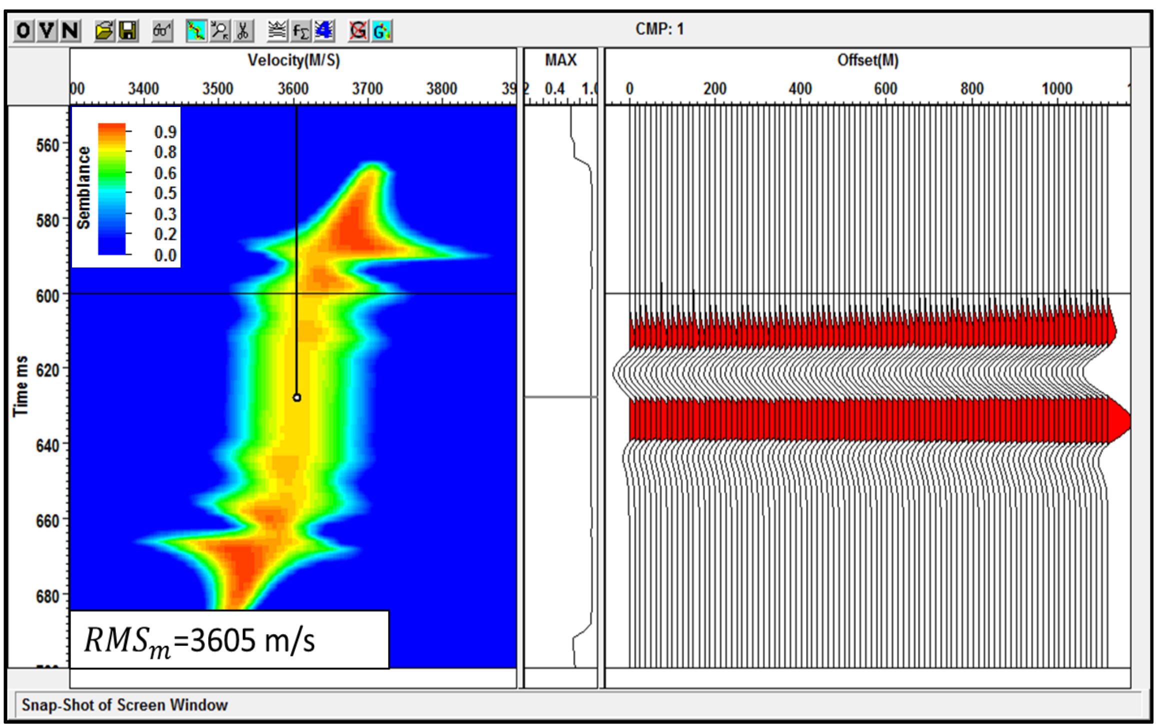

Trace: 90[Index 1]
Max 1.894589543 [0] FREQ (HZ)
Far Trace Model 5
$\frac{1}{2}Pf(x) = 84.42hz$
dB Value
FREQ(HZ): 41.06
dB Value: -3.856[0.276]

**Model 6**

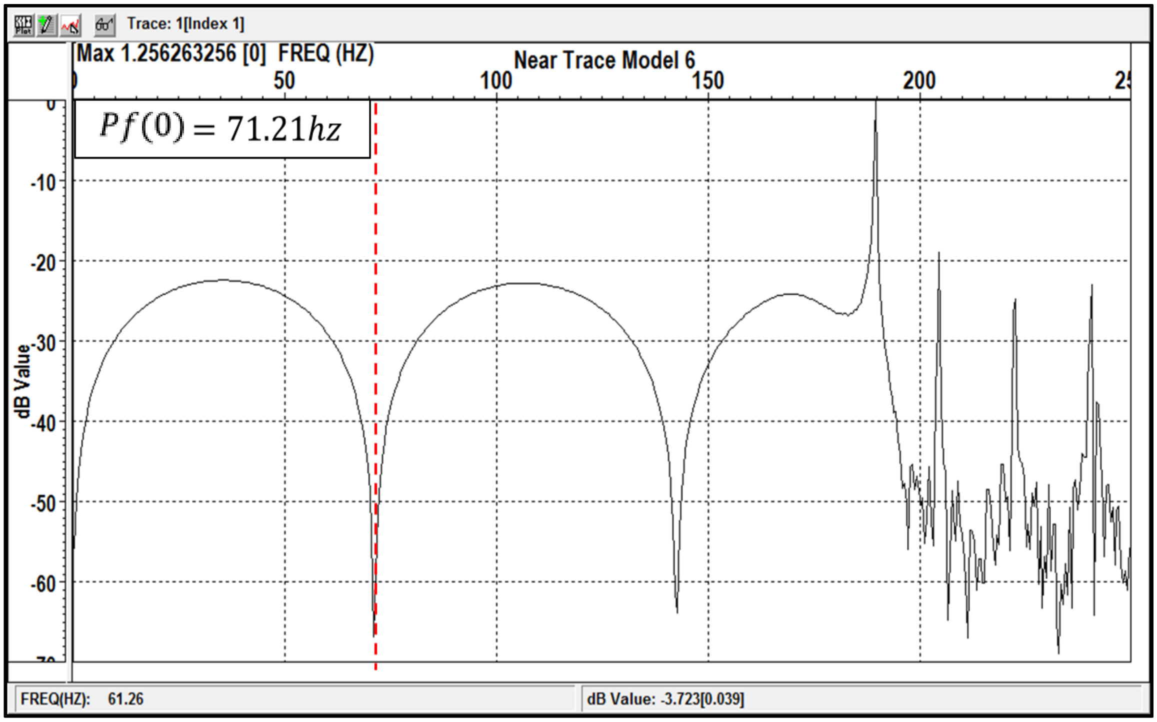
6: A


MAIN SEISMIC
Offset (m)
113
238
364
489
614
740
865
991
TIME (ms)
560
580
600
620
640
660
680
700
720
OFF
0
$t(0)_m$=627.74ms
Offset (m)
800
400
0
Snap-Shot of Screen Window

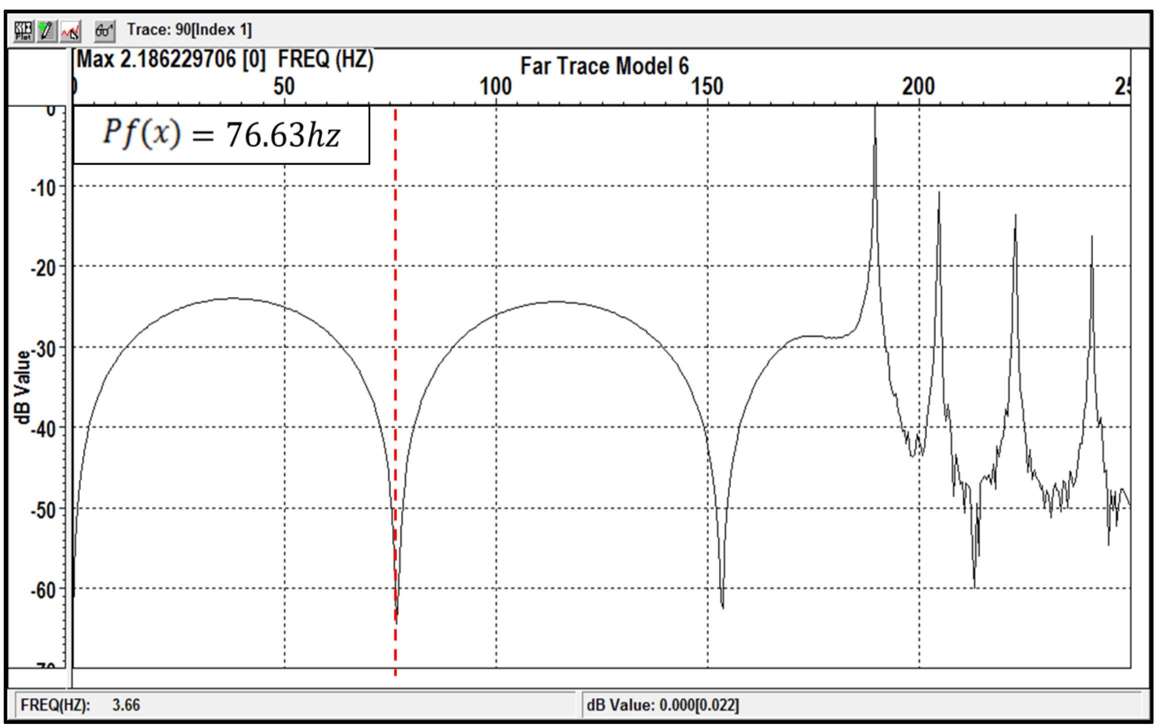
6: B


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
MAX
0.4
Offset(M)
0
200
400
600
800
1000
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
Time ms
560
580
600
620
640
660
680
$RMS_m$=3605 m/s
Snap-Shot of Screen Window

6: C

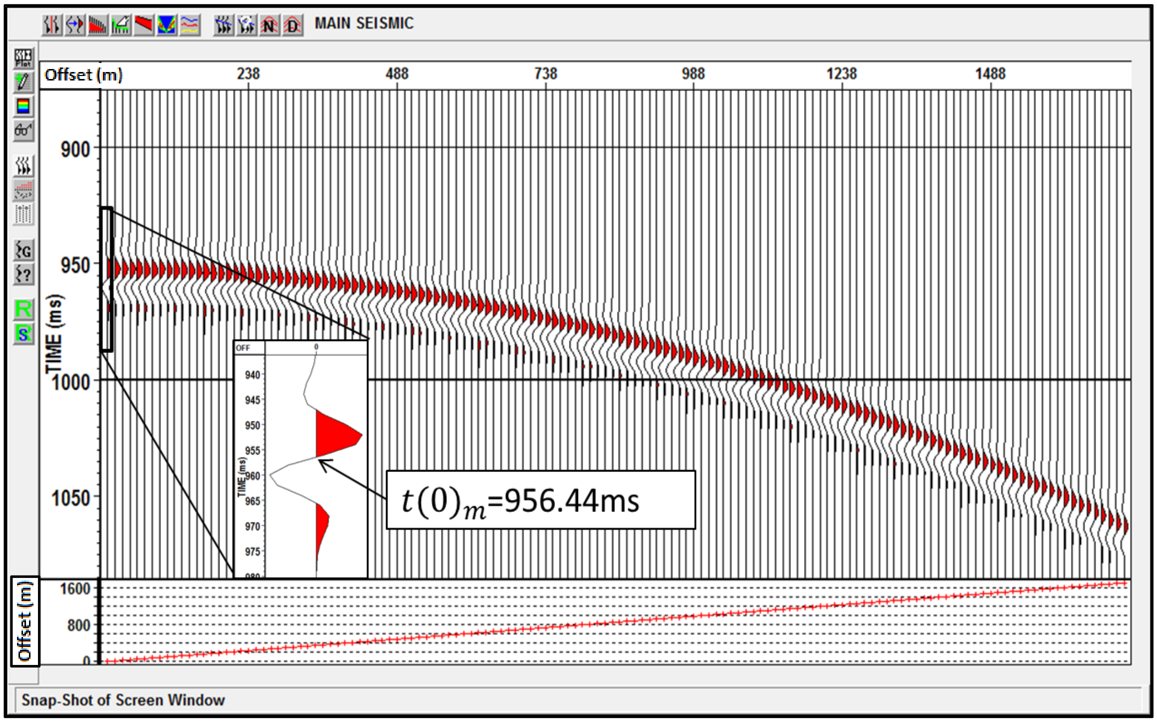

Trace: 1[Index 1]
Max 1.256263256 [0] FREQ (HZ)
Near Trace Model 6
$Pf(0) = 71.21hz$
dB Value
FREQ(HZ): 61.26
dB Value: -3.723[0.039]

6: D


Trace: 90[Index 1]
Max 2.186229706 [0] FREQ (HZ)
Far Trace Model 6
$Pf(x) = 76.63hz$
dB Value
FREQ(HZ): 3.66
dB Value: 0.000[0.022]

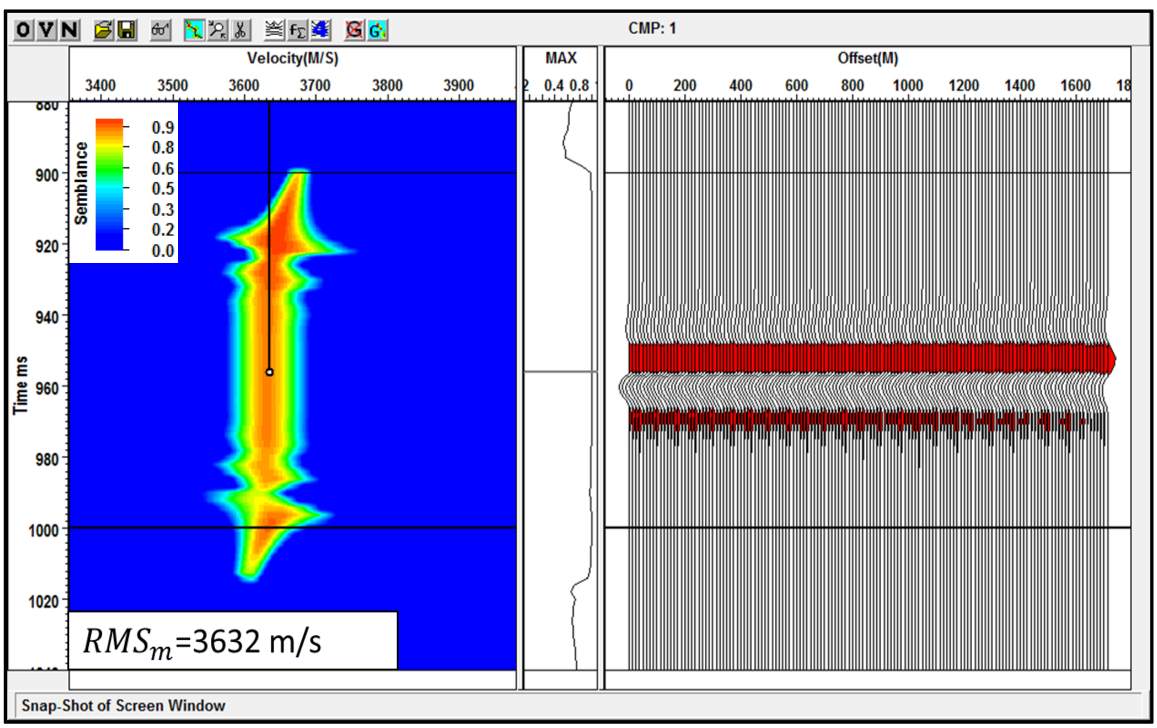
**Model 7:**

7: A



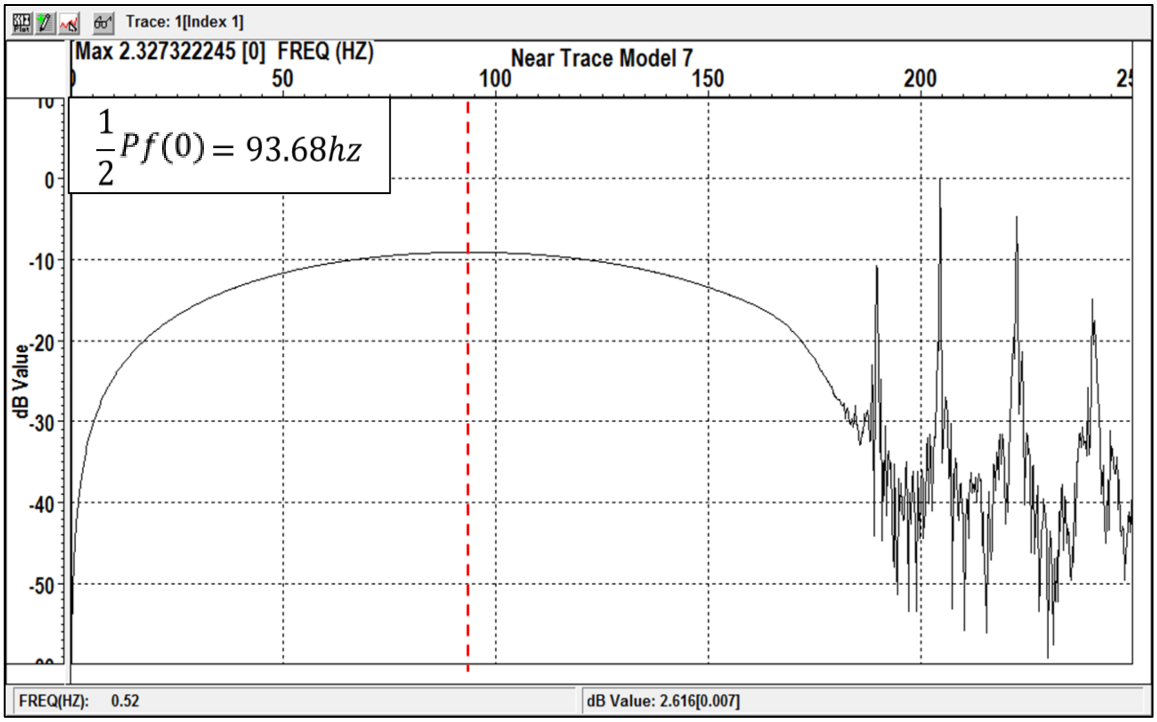
MAIN SEISMIC
Offset (m)
238
488
738
988
1238
1488
900
950
1000
1050
TIME (ms)
$t(0)_m$=956.44ms
1600
800
0
Offset (m)
Snap-Shot of Screen Window


7: B

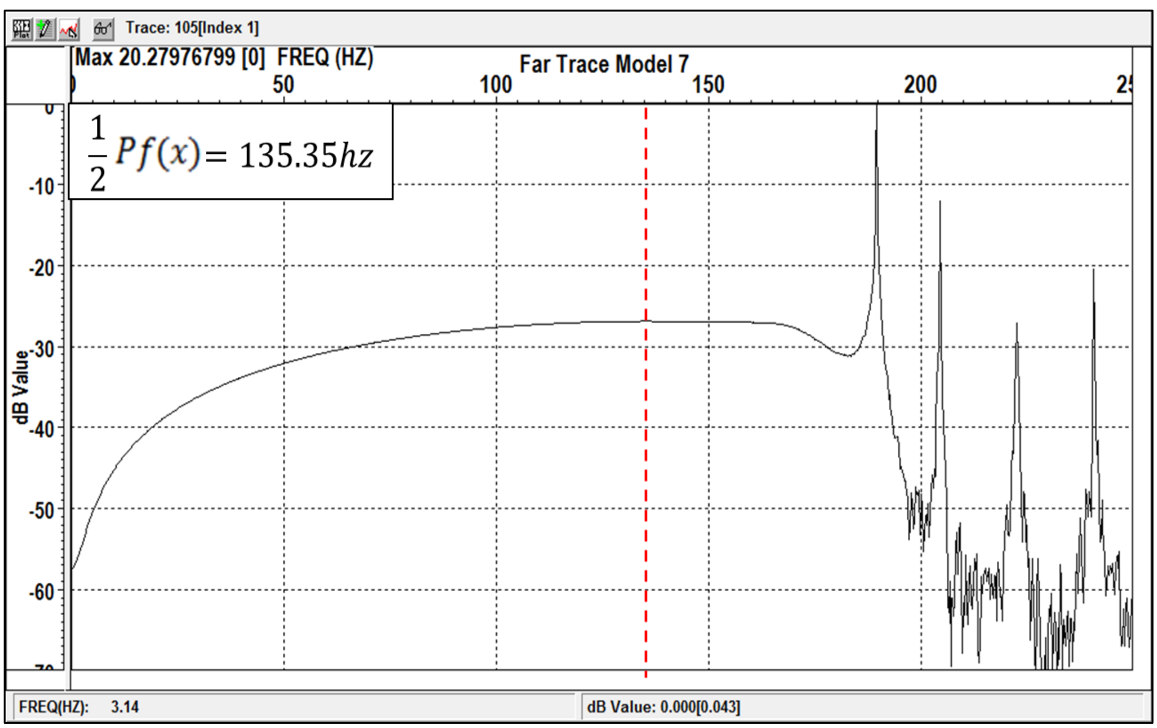


CMP: 1
Velocity(M/S)
3400
3500
3600
3700
3800
3900
MAX
0.4 0.8
Offset(M)
0
200
400
600
800
1000
1200
1400
1600
Semblance
0.9
0.8
0.6
0.5
0.3
0.2
0.0
Time ms
900
920
940
960
980
1000
1020
$RMS_m$=3632 m/s
Snap-Shot of Screen Window

7: C


Trace: 1[Index 1]
Max 2.327322245 [0] FREQ (HZ)
Near Trace Model 7
$\frac{1}{2}Pf(0) = 93.68hz$
dB Value
FREQ(HZ): 0.52
dB Value: 2.616[0.007]

7: D


Trace: 105[Index 1]
Max 20.27976799 [0] FREQ (HZ)
Far Trace Model 7
$\frac{1}{2}Pf(x) = 135.35hz$
dB Value
FREQ(HZ): 3.14
dB Value: 0.000[0.043]

### 4.3 Single-Layer Model Synthetic Data Results

Parameters extracted from data (Table. 4.2).

| Model | t(0)m (ms) | RMS m (m/s) | Pf(0) (hz) | Pf(x) (hz) | Delta t(0) (ms) | Delta t(x) (ms) |
|---|---|---|---|---|---|---|
| 1 | 624.8 | 3622 | 123.54 | 158.98 | 8.095 | 6.290 |
| 2 | 624.8 | 3620 | 123.54 | 151.80 | 8.095 | 6.588 |
| 3 | 624.8 | 3617 | 123.54 | 149.36 | 8.095 | 6.700 |
| 4 | 624.8 | 3625 | 123.54 | 166.38 | 8.095 | 6.010 |
| 5 | 624.8 | 3627 | 123.54 | 168.84 | 8.095 | 5.923 |
| 6 | 627.74 | 3605 | 71.21 | 76.63 | 14.043 | 13.050 |
| 7 | 956.44 | 3632 | 187.36 | 270.70 | 5.337 | 3.694 |

*Table 4.2*

Parameters derived utilizing equations 3.1, 3.2 and 3.11 (Table. 4.3).

| Model | t(0)top (ms) | t(0)base (ms) | RMS top (m/s) | RMS base (m/s) |
|---|---|---|---|---|
| 1 | 620.75 | 628.85 | 3608.94 | 3634.84 |
| 2 | 620.75 | 628.85 | 3610.96 | 3628.90 |
| 3 | 620.75 | 628.85 | 3609.50 | 3624.39 |
| 4 | 620.75 | 628.85 | 3608.13 | 3641.58 |
| 5 | 620.75 | 628.85 | 3608.92 | 3644.76 |
| 6 | 620.72 | 634.76 | 3611.09 | 3599.03 |
| 7 | 953.77 | 959.11 | 3612.98 | 3650.82 |

*Table 4.3*

Thin-bed interval velocity estimated vs. actual (Table 4.4)

| Model | Interval Velocity Estimated (m/s) | Actual Interval Velocity (m/s) | **% Error** |
|---|---|---|---|
| 1 | 5254 | 5200 | **1.03** |
| 2 | 4810 | 4940 | **2.63** |
| 3 | 4626 | 4680 | **1.15** |
| 4 | 5645 | 5460 | **3.27** |
| 5 | 5764 | 5720 | **0.76** |
| 6 | 2957 | 3000 | **1.43** |
| 7 | 7903 | 8000 | **1.21** |

*Table 4.4*

### 4.4 Bakken Formation Middle Member Synthetic Data

The middle member of the Bakken formation is a prolific hydrocarbon exploration target due in large part to its emplacement between the source rock quality upper and lower Bakken shale members and its reservoir class permeability encountered in many areas across the Williston Basin. While these factors combine have contributed significantly to the Bakkens status as a world class hydrocarbon play, like many other excellent petroleum system reservoirs the middle member unit at an approximate thickness of only about 50 feet and at a depth ranging from about 7000-10,000 ft is typically well below the limit of conventional seismic resolution. This leads to an inherent inability to extract a velocity estimate of the layer utilizing seismic data and normal interval velocity inversion practice. While knowledge of a reservoirs velocity is ubiquitously desired for use in accurate rock and fluid property characterization, in the Bakken like in many other cases permeability is heavily dependent on fracture presence and orientation. Subsequently azimuthal variation in the velocity field owing to the HTI anisotropy produced by open oriented vertical fractures is of great interest as well. To test the methods ability to extract, under ideal conditions an approximate velocity for the unresolved Bakken middle member the seismic response was modeled from a well log which perforates the formation (Fig 4.1). The Bakken Middle Member was isolated by utilizing the average density and velocity in the under and overburden to characterize two isotropic slabs encasing the unit (Fig 4.2). All modeling parameters used are identical to those employed in models 1-7 as are the analysis techniques A-D with the exception that random noise of varying percentage will be added to three models. For the cases containing noise Steps C and D will be repeated and modified to utilize an average-trace taken from adjacent traces in the near and far offset limit. The effects of using an average vs single trace derived parameters in the presence of noise will be illustrated in Tables 4.10 and 4.11.

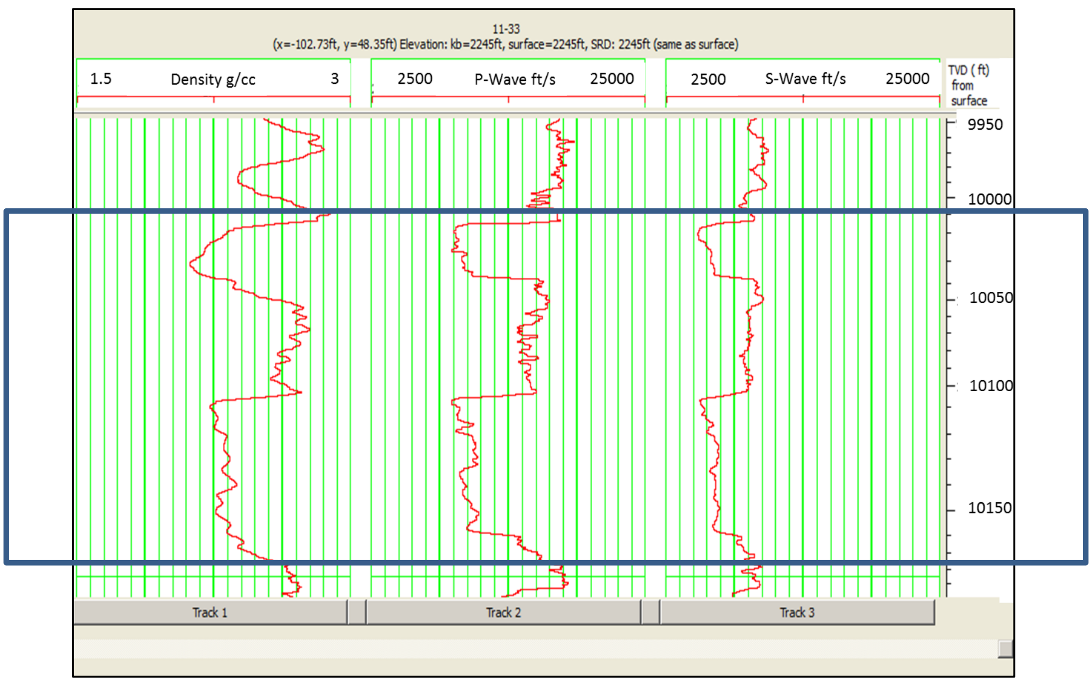



*Fig. 4.1 Well log response of Upper Middle and Lower Bakken Shale Formation. Well 11-33, log provided by Hess Oil Company.*

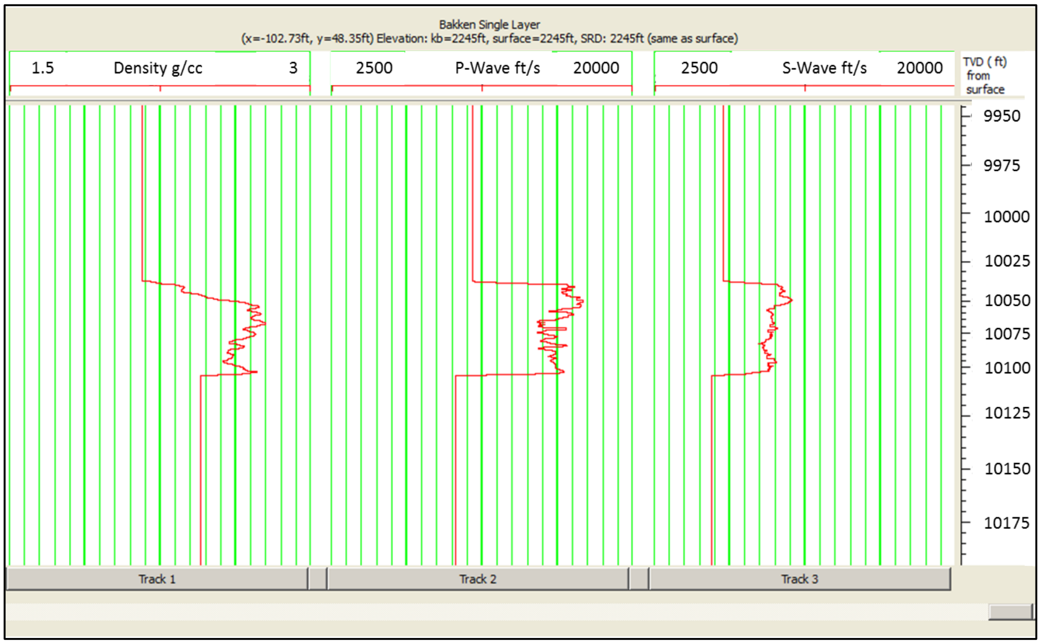


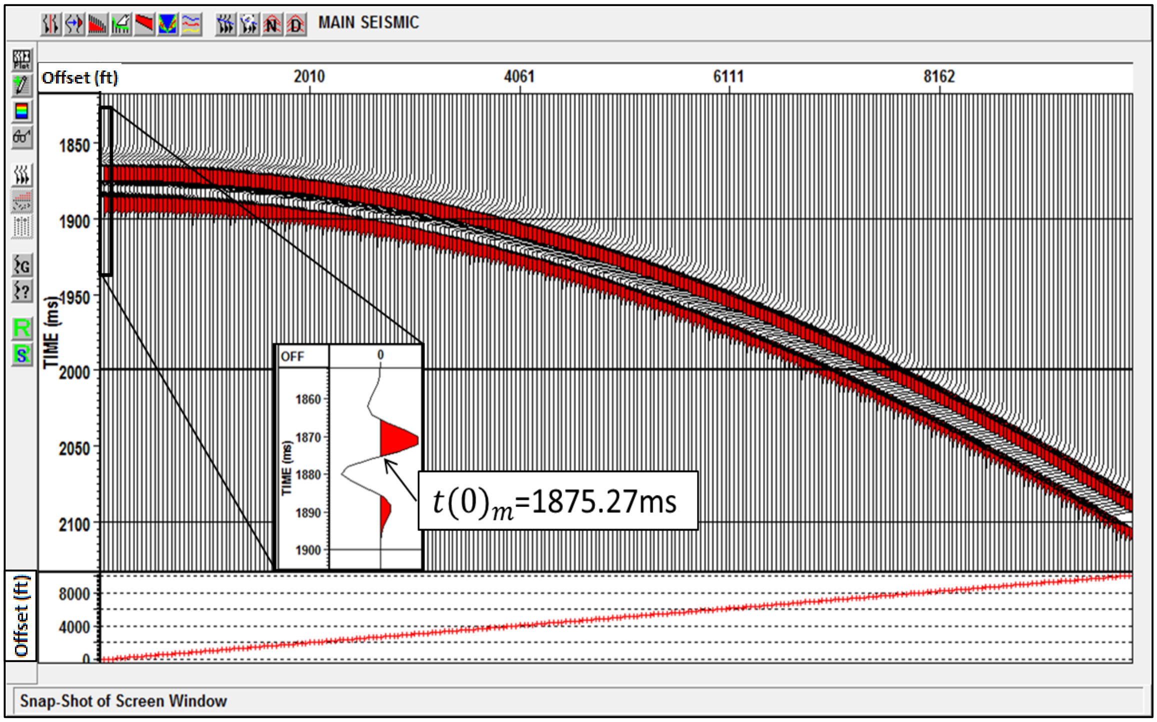

*Fig. 4.2 Isolated Bakken Middle Member Utilized to generate synthetic data.*

| Bakken Middle Member Model | S/N Ratio | Layer Depth (ft) | Layer Thickness (ft) | Offset (ft) | Offset Increment (ft) |
|---|---|---|---|---|---|
| B1 | Noise Free | 10036 | 70 | 10008 | 41.7 |
| B2 | 15/1 | 10036 | 70 | 10008 | 41.7 |
| B3 | 10/1 | 10036 | 70 | 10008 | 41.7 |
| B4 | 5/1 | 10036 | 70 | 10008 | 41.7 |

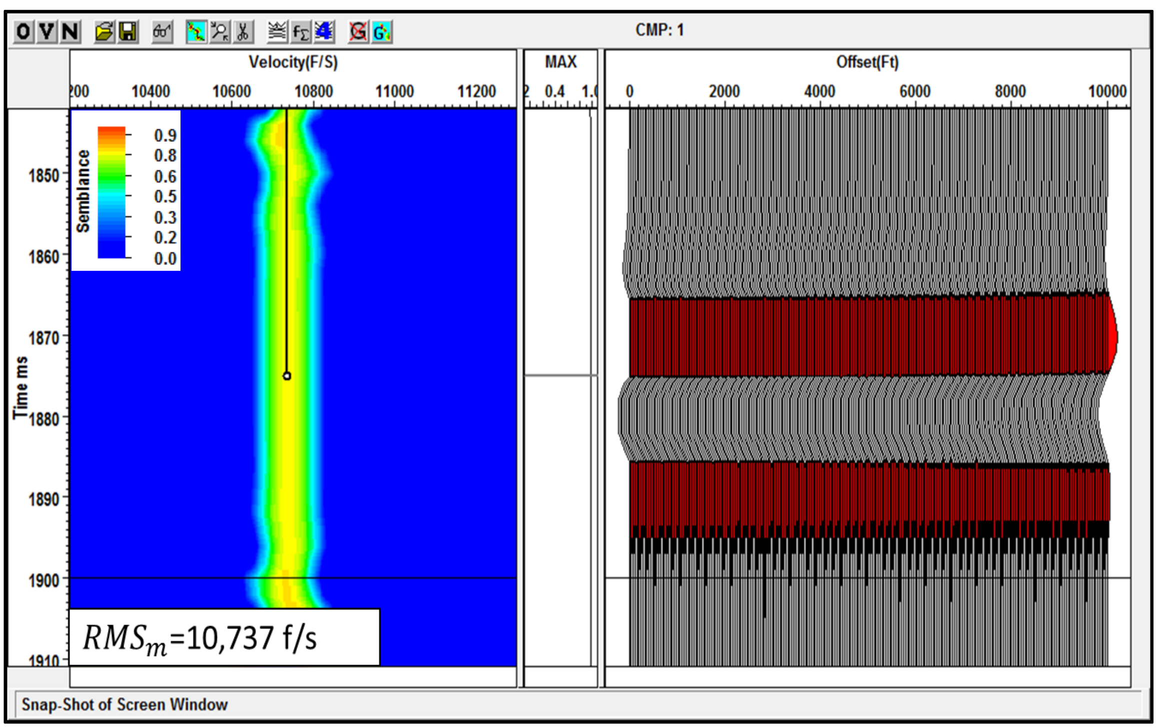
*Table 4.5*

**Model B1**

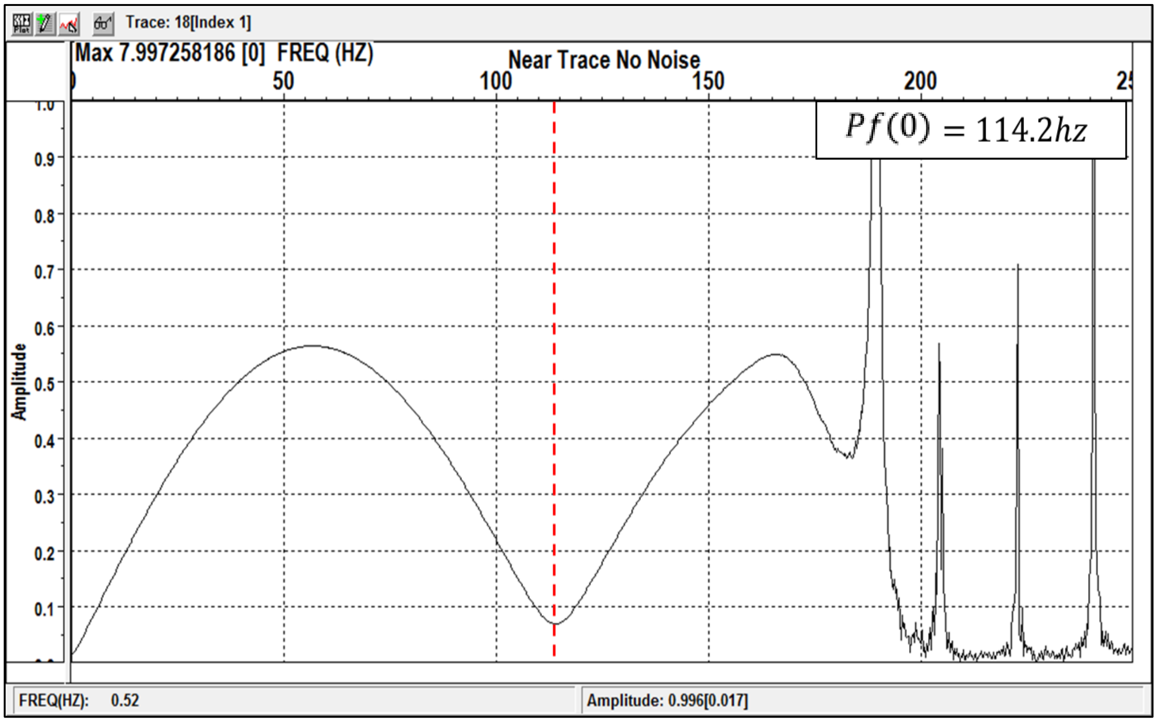
B1: A




B1: B

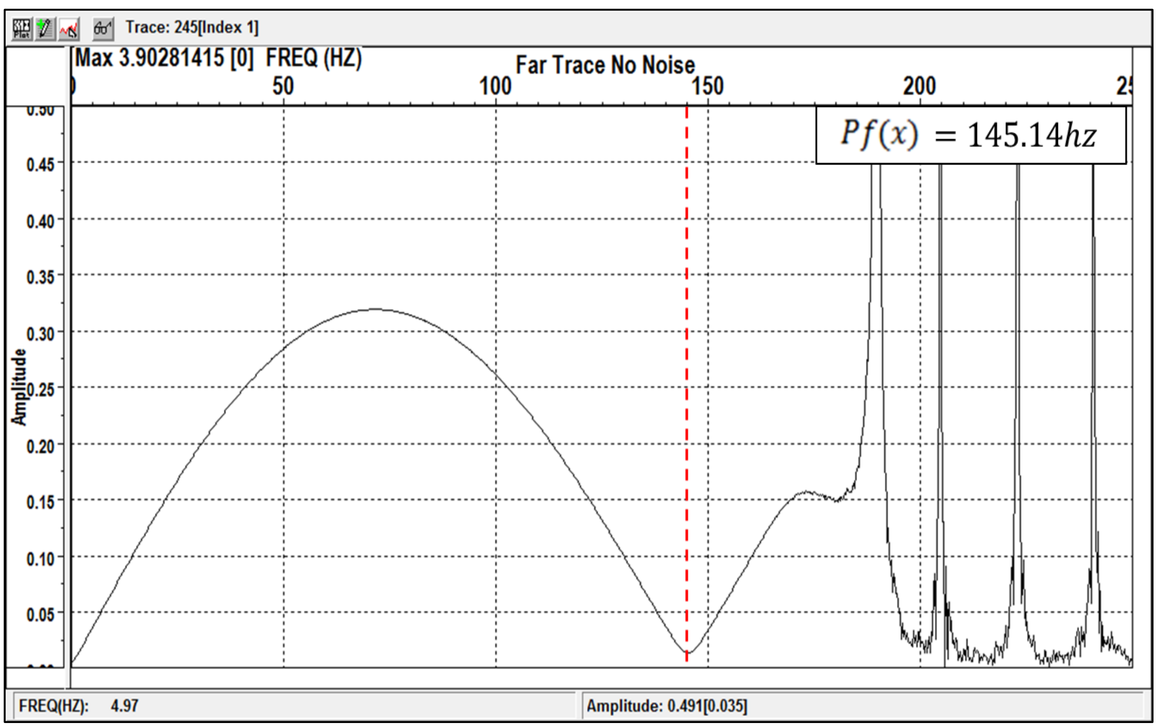


B1: C



Trace: 18[Index 1]
Max 7.997258186 [0] FREQ (HZ)
Near Trace No Noise
$Pf(0) = 114.2hz$
Amplitude
FREQ(HZ): 0.52
Amplitude: 0.996[0.017]

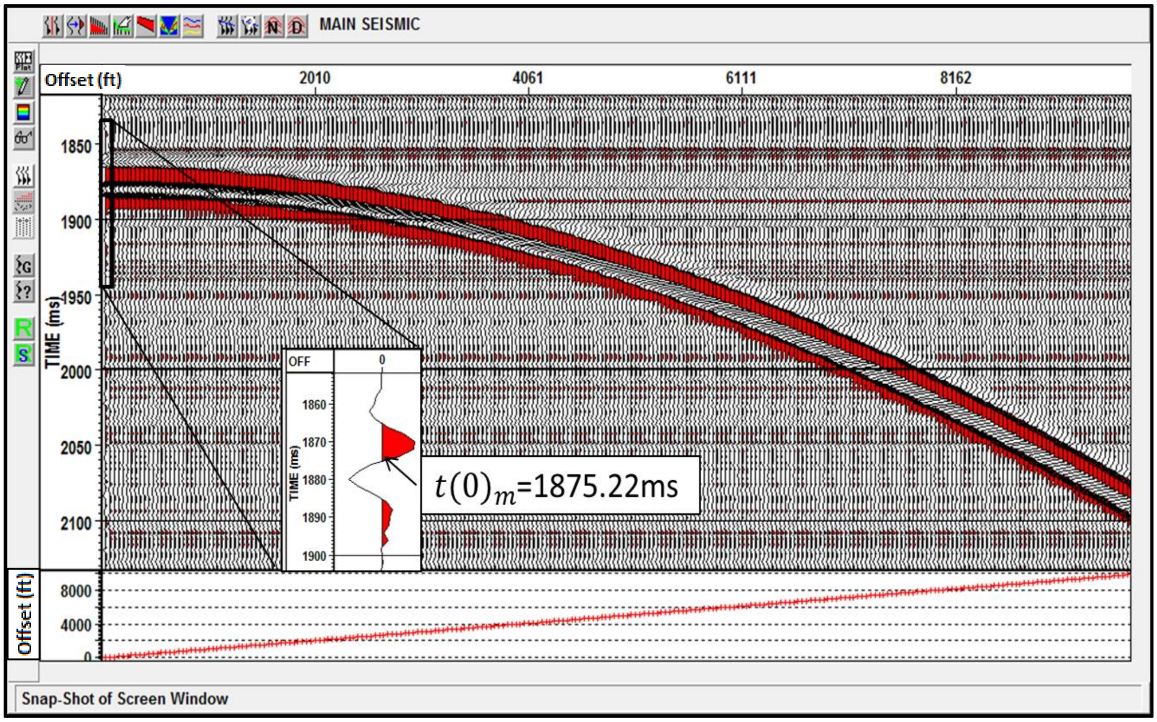

B1: D



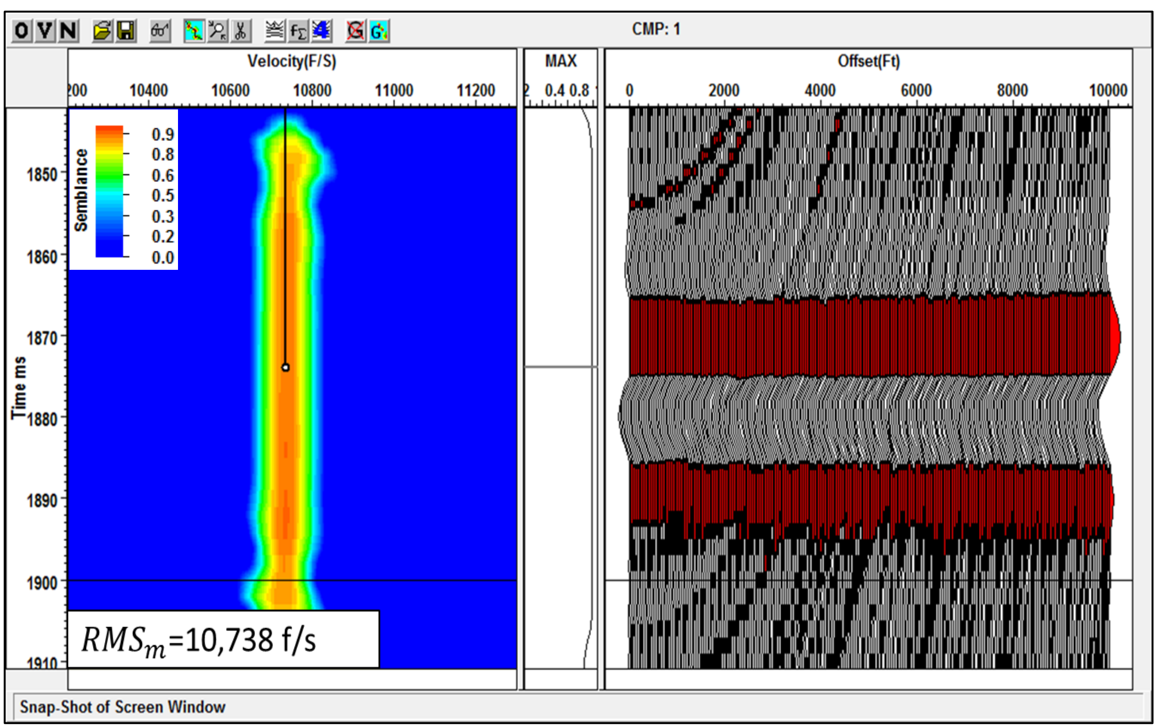
Trace: 245[Index 1]
Max 3.90281415 [0] FREQ (HZ)
Far Trace No Noise
$Pf(x) = 145.14hz$
Amplitude
FREQ(HZ): 4.97
Amplitude: 0.491[0.035]

**Model B2**

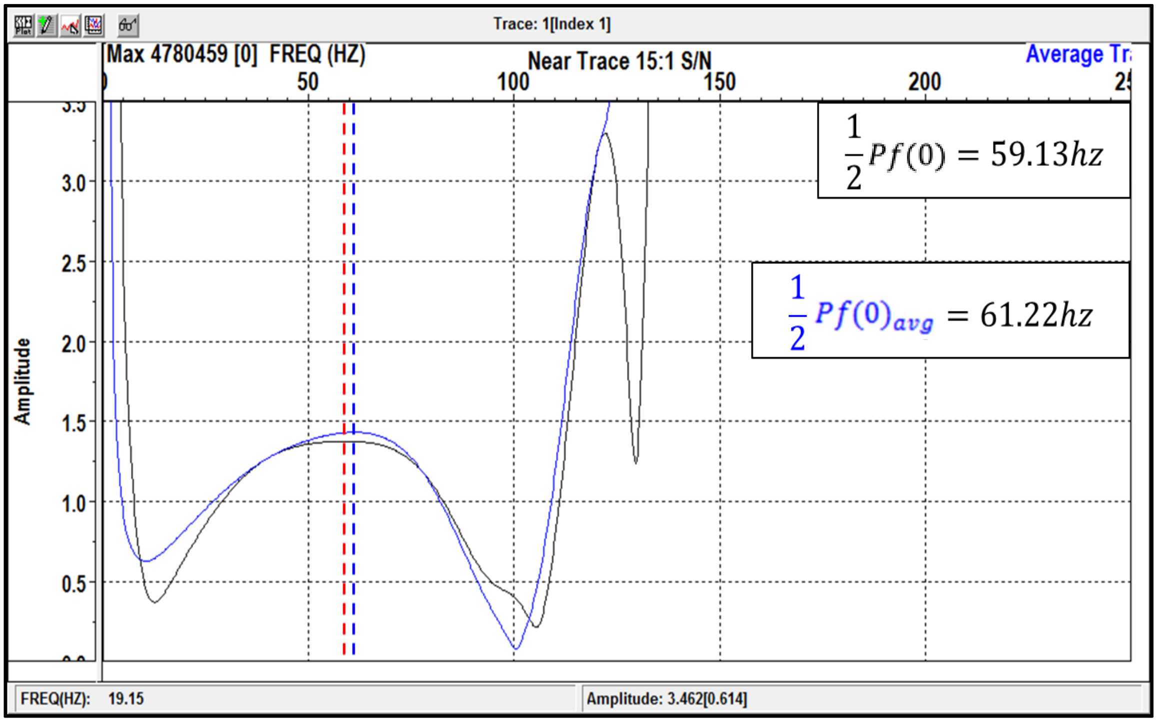
B2: A



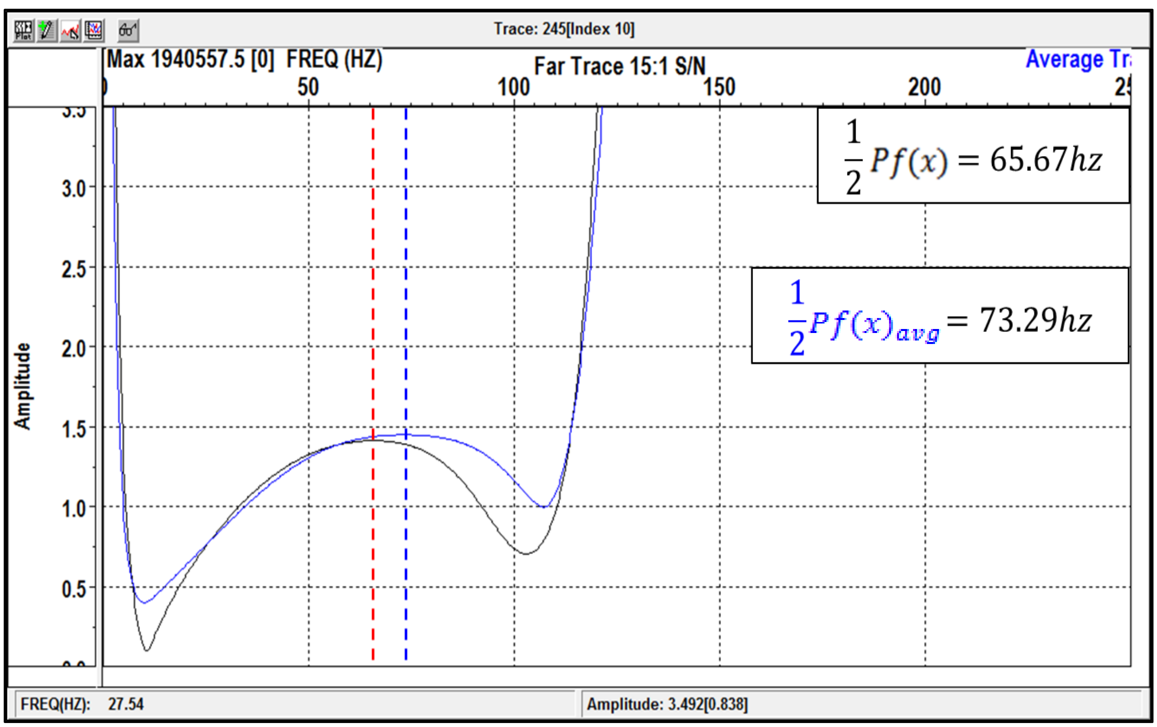

B2: B



B2: C

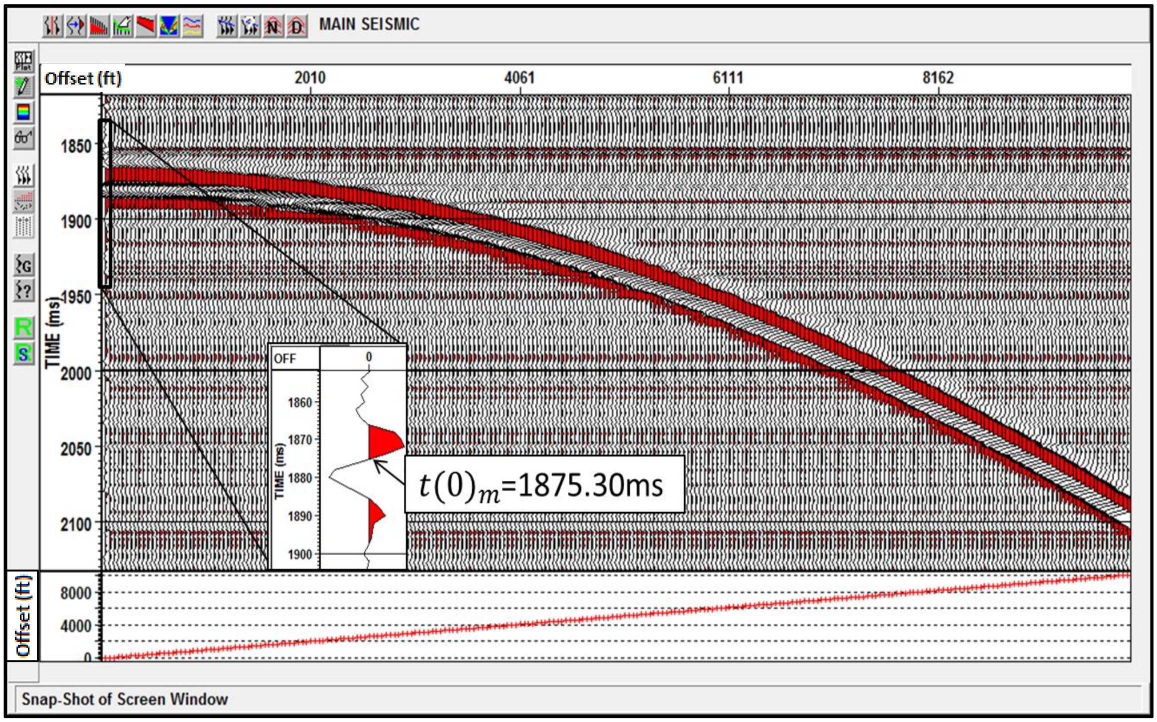

Trace: 1[Index 1]
Max 4780459 [0] FREQ (HZ)
Near Trace 15:1 S/N
Average Tr
50
100
150
200
3.0
2.5
2.0
1.5
1.0
0.5
Amplitude
$\frac{1}{2}Pf(0) = 59.13hz$
$\frac{1}{2}Pf(0)_{avg} = 61.22hz$
FREQ(HZ): 19.15
Amplitude: 3.462[0.614]

B2:D

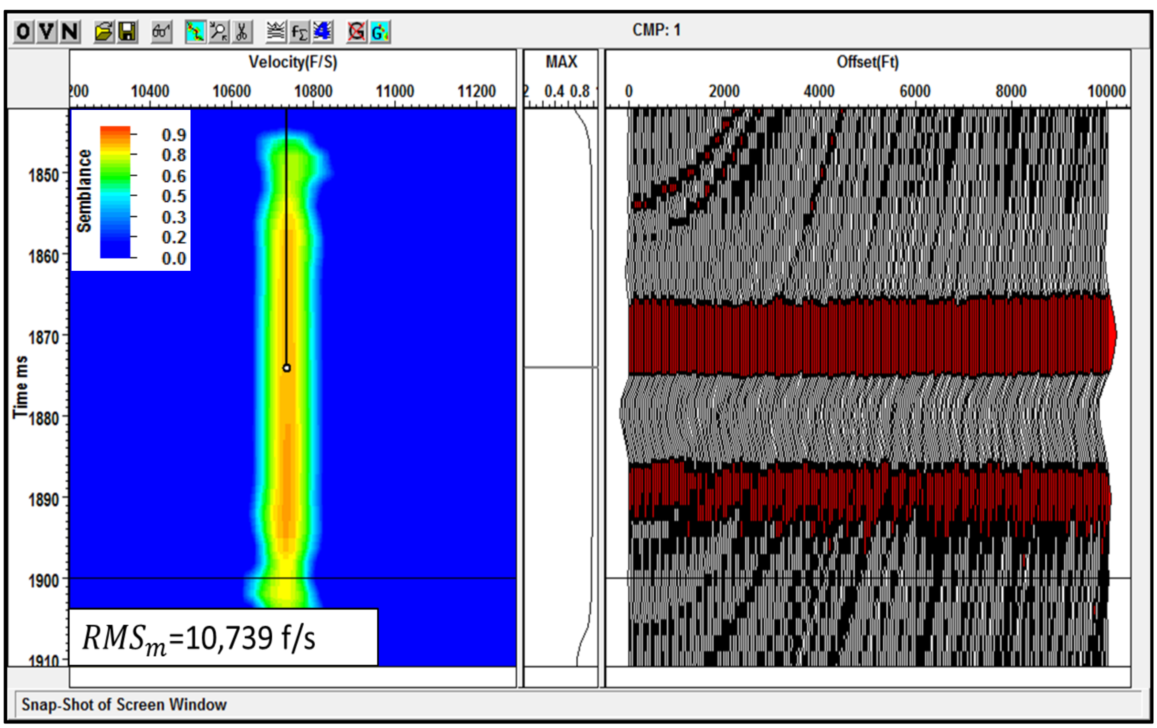

Trace: 245[Index 10]
Max 1940557.5 [0] FREQ (HZ)
Far Trace 15:1 S/N
Average Tr
50
100
150
200
3.0
2.5
2.0
1.5
1.0
0.5
Amplitude
$\frac{1}{2}Pf(x) = 65.67hz$
$\frac{1}{2}Pf(x)_{avg} = 73.29hz$
FREQ(HZ): 27.54
Amplitude: 3.492[0.838]

## Model B3

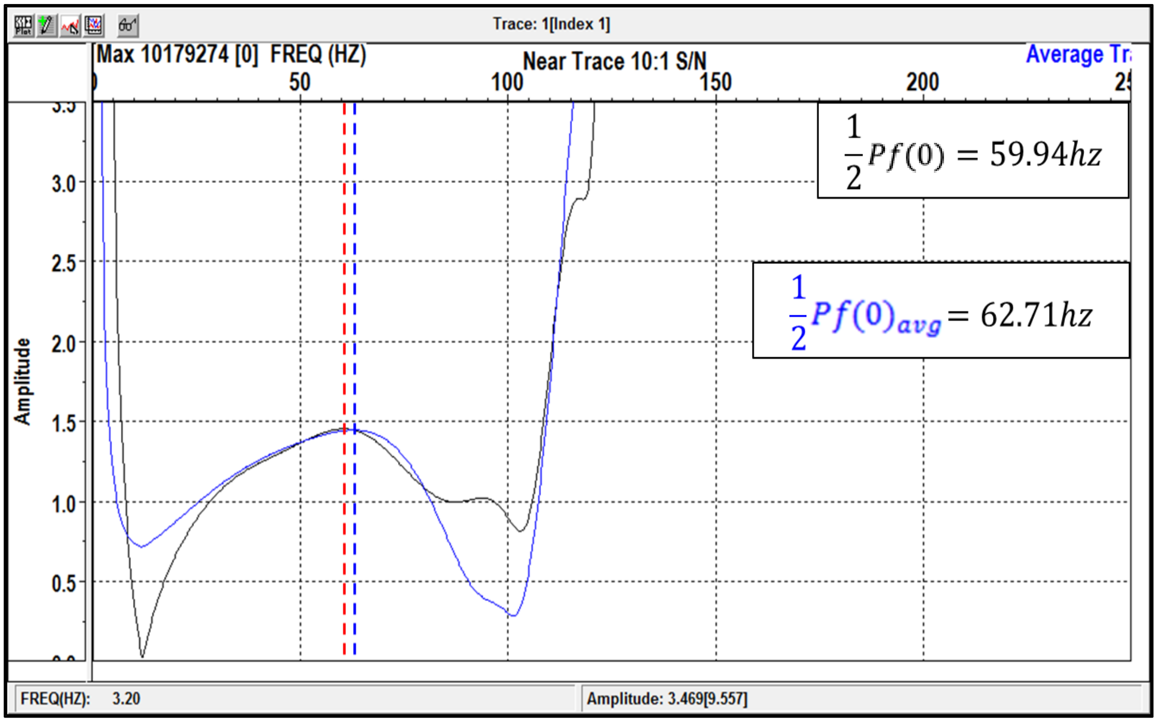
B3:A




B3: B

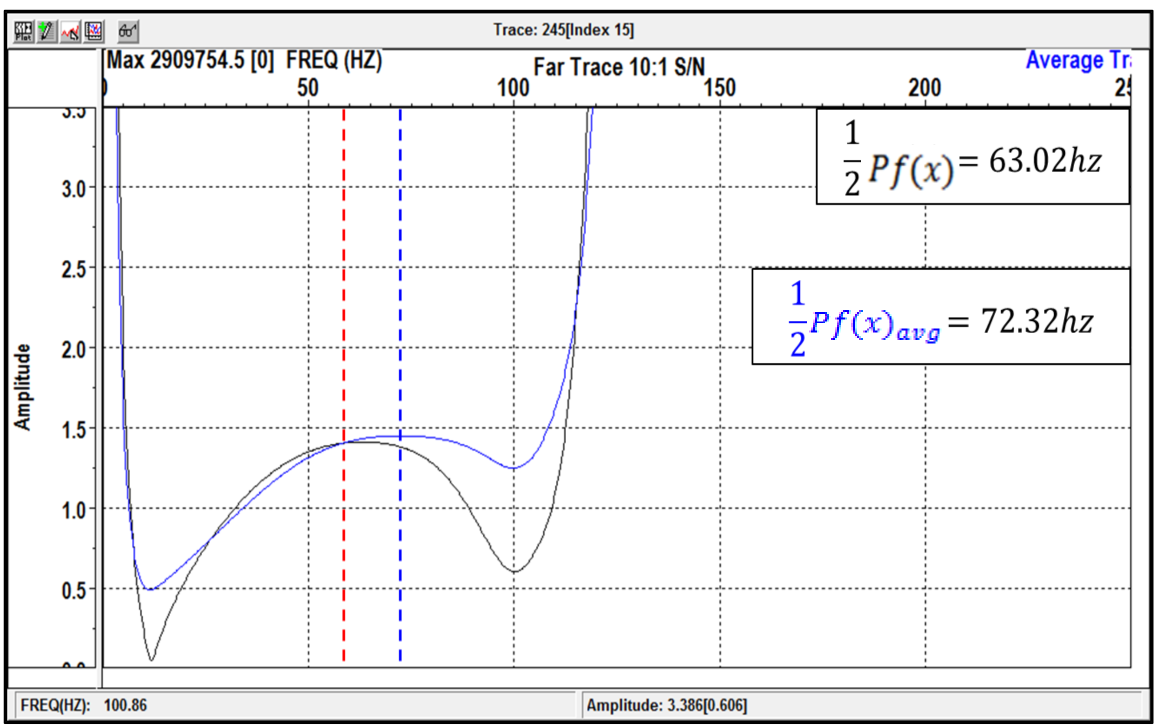


B3: C



Trace: 1[Index 1]
Max 10179274 [0] FREQ (HZ)
Near Trace 10:1 S/N
Average Tr
50
100
150
200
3.0
2.5
2.0
1.5
1.0
0.5
Amplitude
$\frac{1}{2}Pf(0) = 59.94hz$
$\frac{1}{2}Pf(0)_{avg} = 62.71hz$
FREQ(HZ): 3.20
Amplitude: 3.469[9.557]

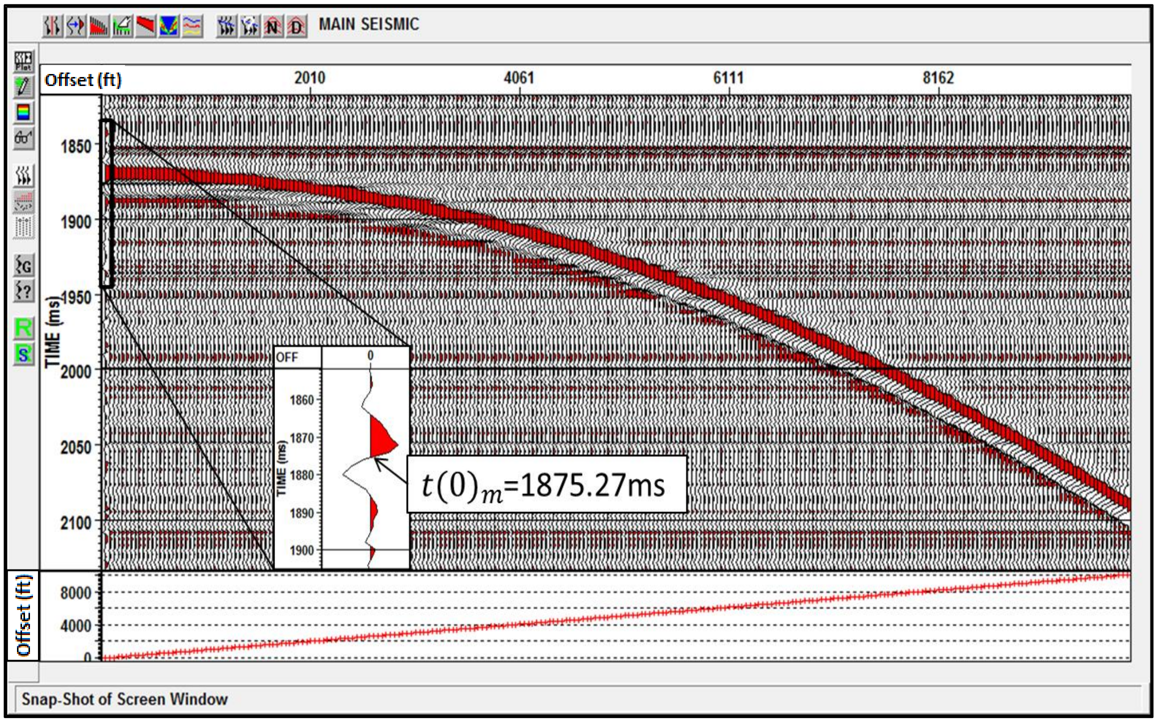

B3:D



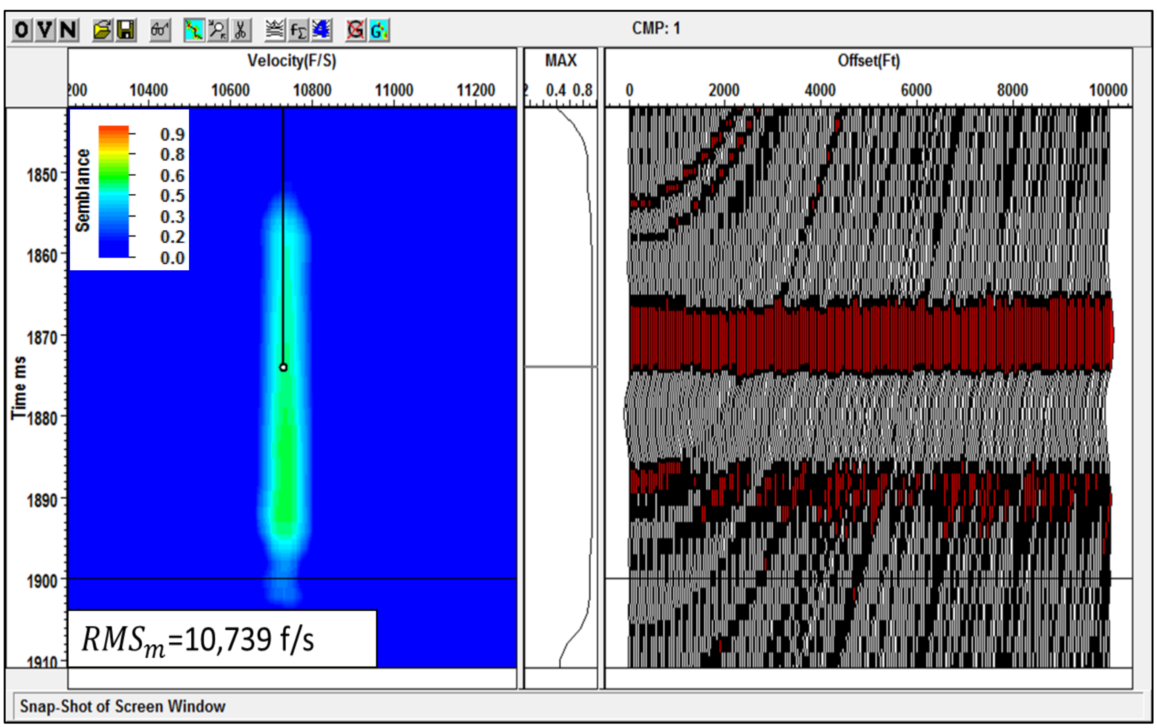
Trace: 245[Index 15]
Max 2909754.5 [0] FREQ (HZ)
Far Trace 10:1 S/N
Average Tr
50
100
150
200
3.0
2.5
2.0
1.5
1.0
0.5
Amplitude
$\frac{1}{2}Pf(x) = 63.02hz$
$\frac{1}{2}Pf(x)_{avg} = 72.32hz$
FREQ(HZ): 100.86
Amplitude: 3.386[0.606]

**Model B4**

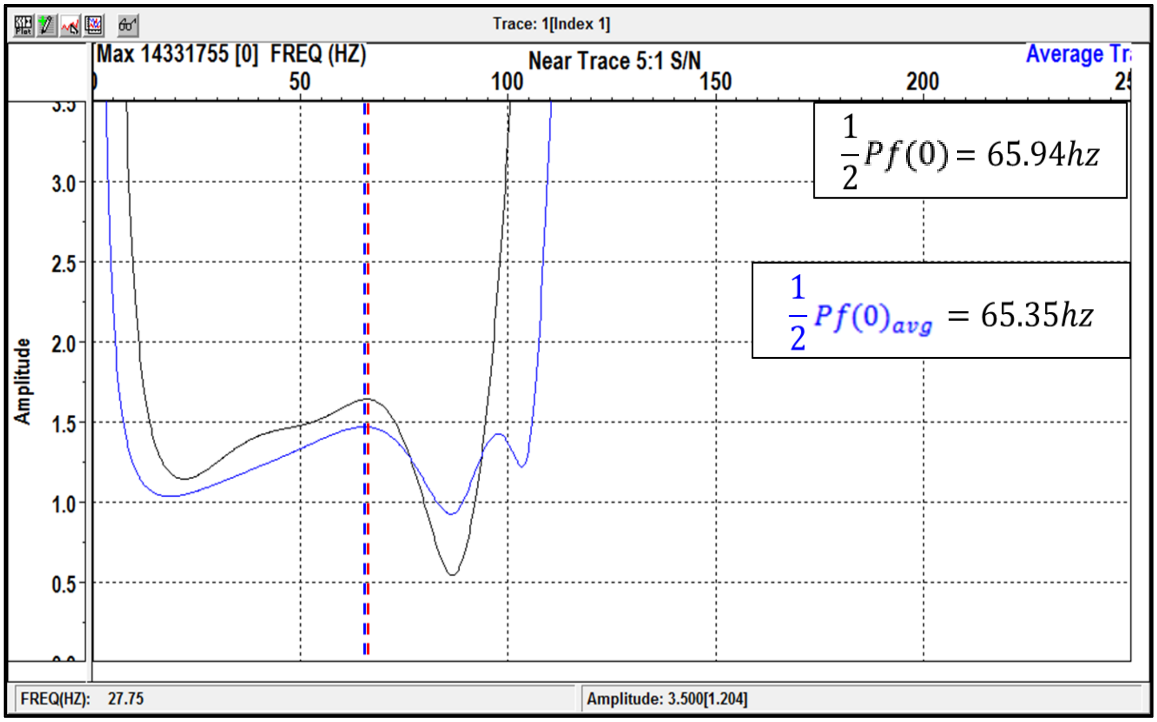
B4: A



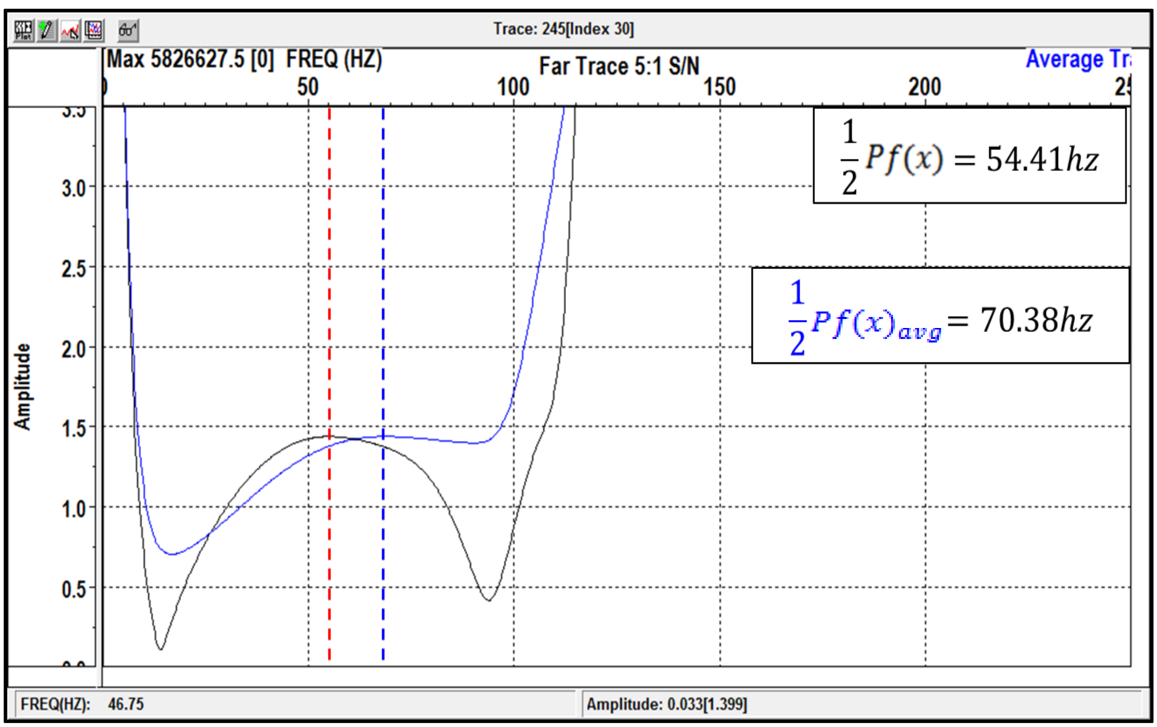

B4:B



B4: C


Trace: 1[Index 1]
Max 14331755 [0] FREQ (HZ)
Near Trace 5:1 S/N
Average Tr
50
100
150
200
Amplitude
3.0
2.5
2.0
1.5
1.0
0.5
$\frac{1}{2}Pf(0) = 65.94hz$
$\frac{1}{2}Pf(0)_{avg} = 65.35hz$
FREQ(HZ): 27.75
Amplitude: 3.500[1.204]

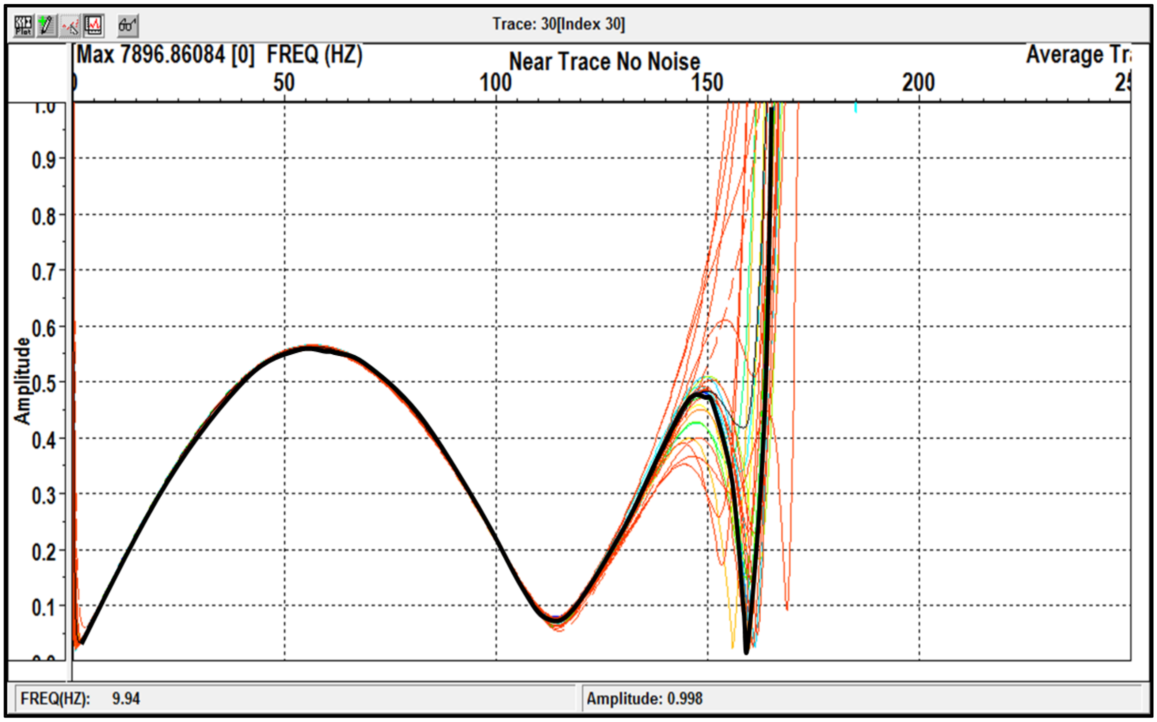
B4: D


Trace: 245[Index 30]
Max 5826627.5 [0] FREQ (HZ)
Far Trace 5:1 S/N
Average Tr
50
100
150
200
Amplitude
3.0
2.5
2.0
1.5
1.0
0.5
$\frac{1}{2}Pf(x) = 54.41hz$
$\frac{1}{2}Pf(x)_{avg} = 70.38hz$
FREQ(HZ): 46.75
Amplitude: 0.033[1.399]

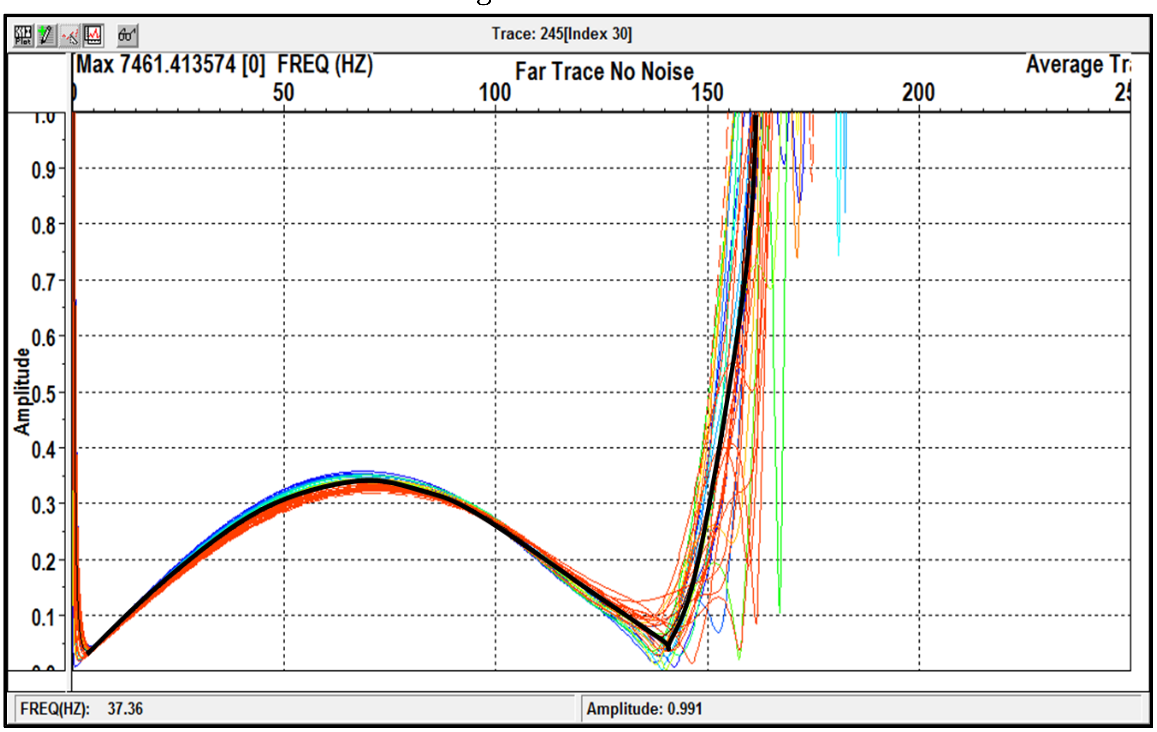
## 4.5 Average-Trace Extraction

The average-trace extracted for models B2, B3 and B4 was taken from the adjacent near and far offset traces. Trace quantities used in the average were 10, 15 and 30 respectively. The number of contributing traces was taken to be such that the peak frequency $\frac{1}{2}Pf$ was definable and an interpretable frequency spectrum was achieved. The quantity of traces required to achieve this goal increased with decreasing signal to noise as the amount of distortion in the signal increased with the addition of noise. The following plots show the average-trace, equal to the average-trace used in calculating $\frac{1}{2}Pf_{avg}$ in bold black with the traces contributing to the average plotted in thin multicolor lines. The increasing deviation about the mean with additional noise can be clearly observed. The noise free example B1 with a 30 trace average is also plotted to show the unaltered spectrum response for comparison. The illustrations are annotated with model number first followed by the offset location.

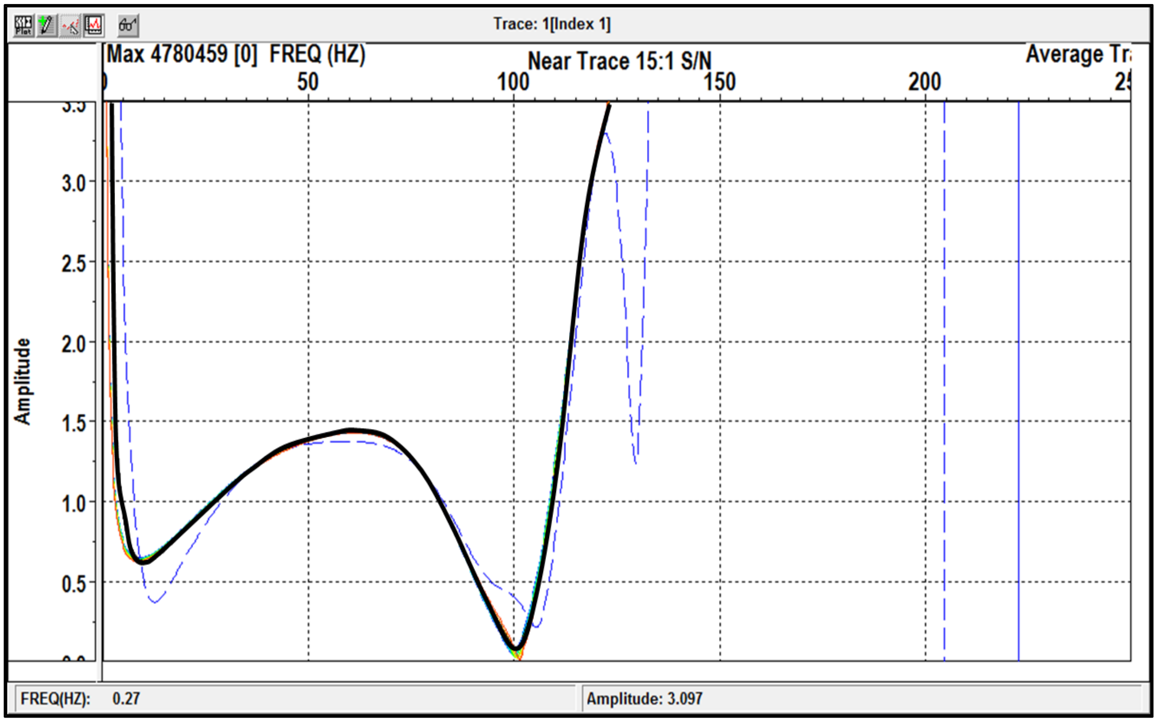
Model B1 Near Offset 30 Trace Average:



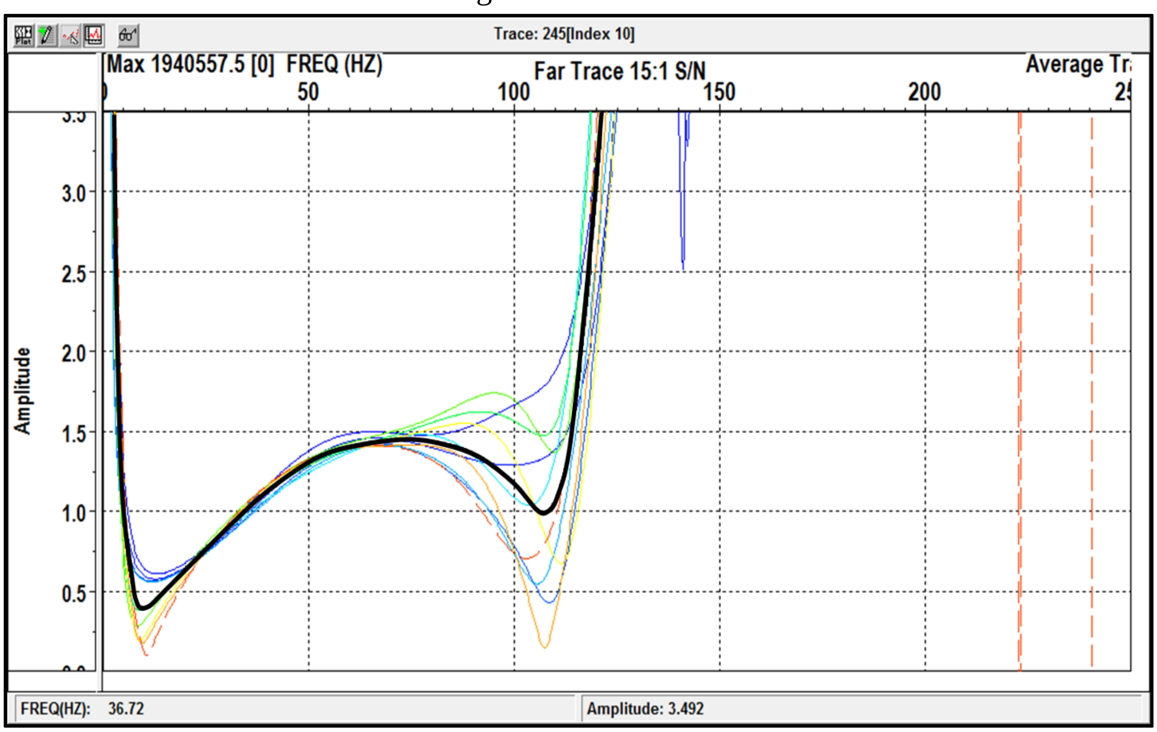

Model B1 Far Offset 30 Trace Average:



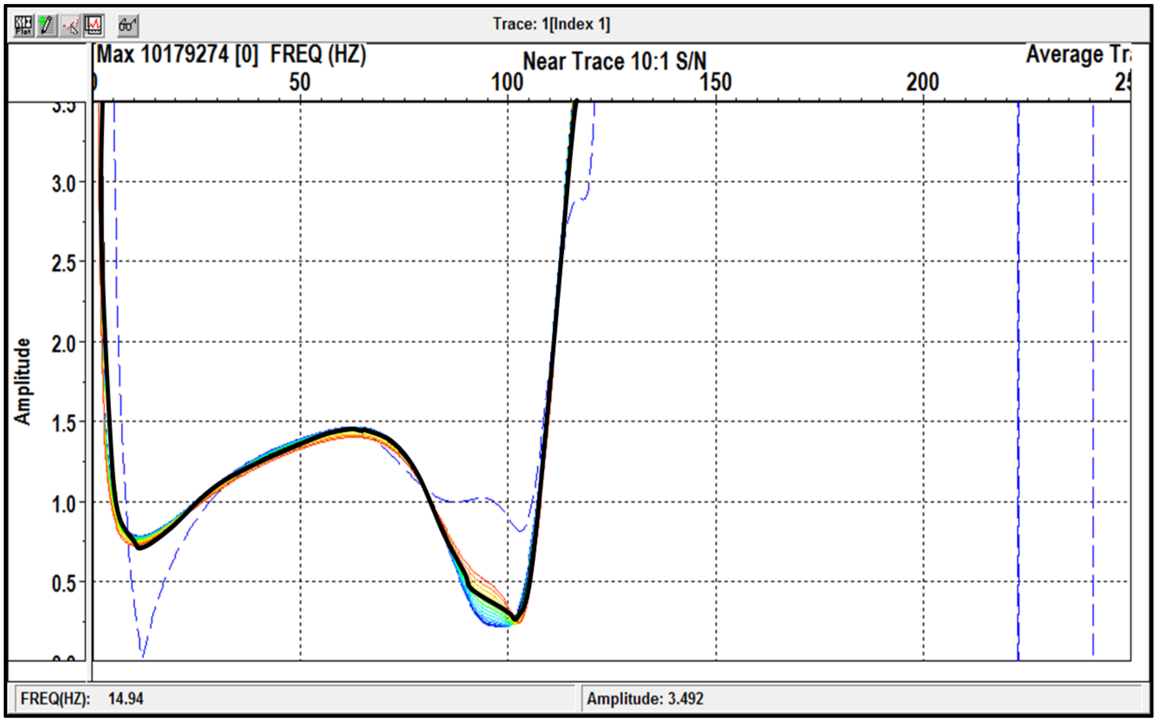
Model B2 Near Offset 10 Trace Average:



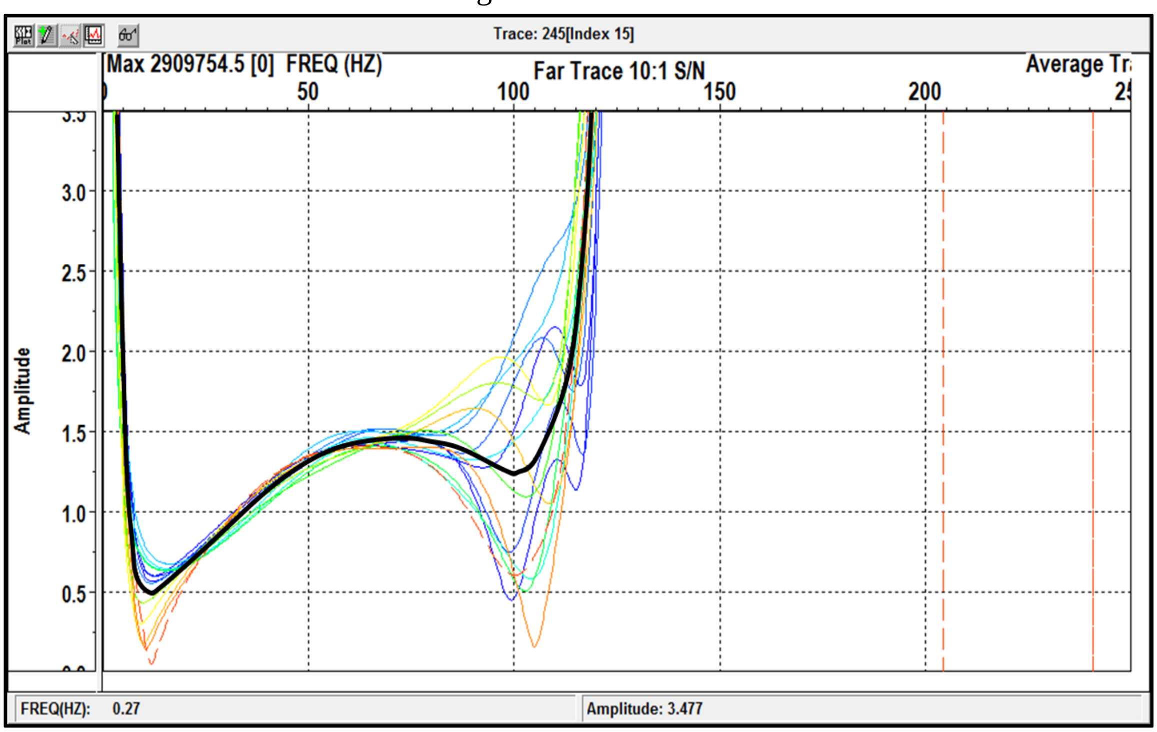

Model B2 Far Offset 10 Trace Average:



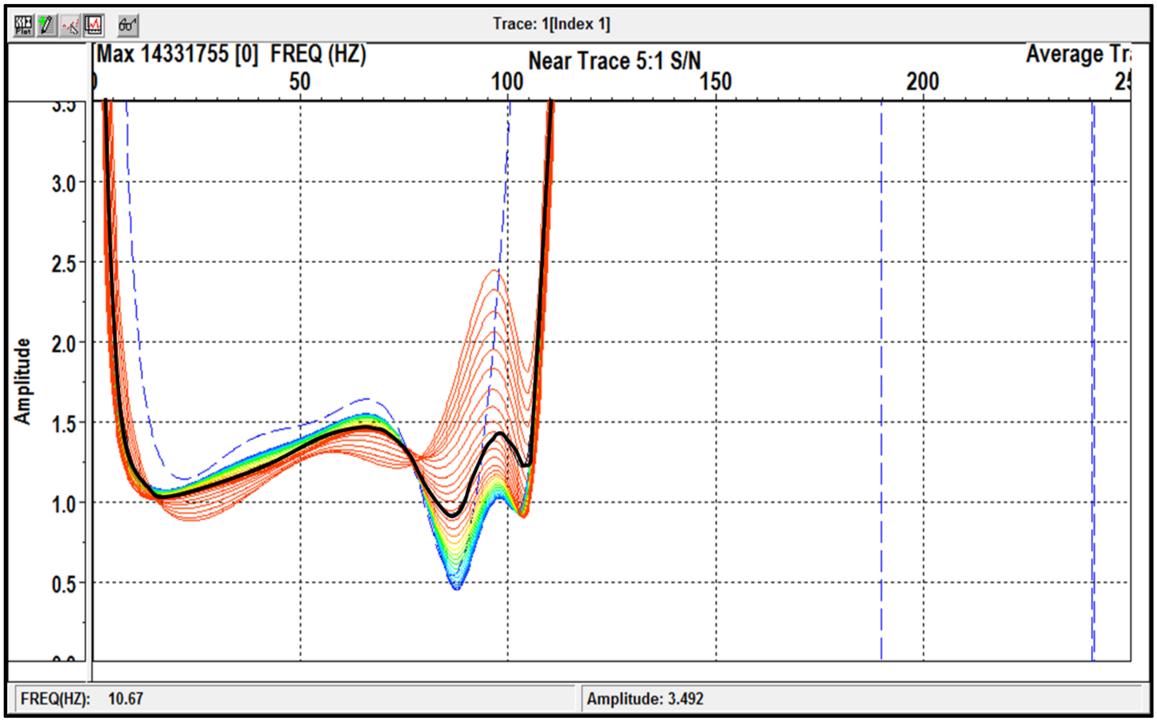
Model B3 Near Offset 15 Trace Average:



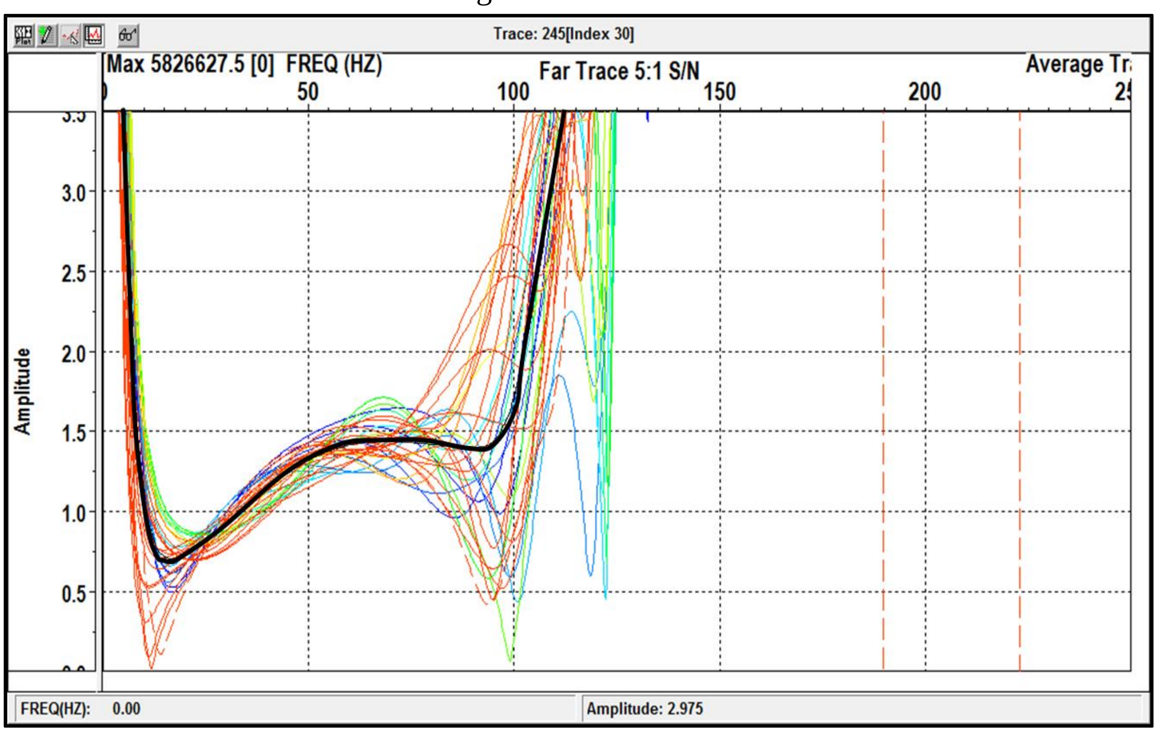

Model B3 Far Offset 15 Trace Average:



Model B4 Near Offset 30 Trace Average:




Model B4 Far Offset 30 Trace Average:



### 4.5 Bakken Formation Middle Member Synthetic Data Results

Single-trace parameters extracted from data (Table. 4.6).

| Model | S/N | t(0)m (ms) | RMSm (Ft/s) | Pf(0) (hz) | Pf(x) (hz) | Delta t(0) (ms) | Delta t(x) (ms) |
|---|---|---|---|---|---|---|---|
| B1 | Noise Free | 1875.27 | 10,737 | 114.20 | 145.14 | 8.757 | 6.890 |
| B2 | 15/1 | 1875.27 | 10,738 | 118.26 | 131.34 | 8.456 | 7.614 |
| B3 | 10/1 | 1875.30 | 10,739 | 119.88 | 126.04 | 8.342 | 7.934 |
| B4 | 5/1 | 1875.27 | 10,739 | 131.88 | 108.82 | 7.583 | 9.189 |

*Table 4.6*

Average-trace parameters extracted from data (Table. 4.7).

| Model | S/N | t(0)m (ms) | RMSm (Ft/s) | Pf(0)avg (hz) | Pf(x)avg (hz) | Delta t(0)avg (ms) | Delta t(x)avg (ms) |
|---|---|---|---|---|---|---|---|
| B1 | Noise Free | 1875.27 | 10,737 | N/A | N/A | N/A | N/A |
| B2 | 15/1 | 1875.27 | 10,738 | 122.44 | 146.58 | 8.167 | 6.822 |
| B3 | 10/1 | 1875.30 | 10,739 | 125.42 | 144.64 | 7.973 | 6.914 |
| B4 | 5/1 | 1875.27 | 10,739 | 130.70 | 140.76 | 7.651 | 7.104 |

*Table 4.7*

Single-trace parameters derived utilizing equations 3.1, 3.2, 3.11 (Table. 4.8).

| Model | S/N | t(0)top (ms) | t(0)base (ms) | RMS top (ft/s) | RMS base (ft/s) |
|---|---|---|---|---|---|
| B1 | Noise Free | 1870.89 | 1879.65 | 10724.6 | 10749.3 |
| B2 | 15/1 | 1871.04 | 1879.50 | 10738.5 | 10737.5 |
| B3 | 10/1 | 1871.13 | 1879.47 | 10745.0 | 10733.0 |
| B4 | 5/1 | 1871.49 | 1879.06 | *undefined* | *undefined* |

***For discussion of model B4's results see appendix B*

*Table 4.8*

Average-trace parameters derived utilizing equations 3.1, 3.2, 3.11 (Table. 4.9).

| Model | S/N | t(0)top (ms) | t(0)base (ms) | RMS top (ft/s) | RMS base (ft/s) |
|---|---|---|---|---|---|
| B1 | Noise Free | \ | \ | \ | \ |
| B2 | 15/1 | 1871.19 | 1879.35 | 10731.7 | 10744.3 |
| B3 | 10/1 | 1871.31 | 1879.29 | 10736.0 | 10742.0 |
| B4 | 5/1 | 1871.44 | 1879.10 | 10741.3 | 10737.0 |

*Table 4.9*

Bakken Middle Member single-trace interval velocity estimate vs. actual (Table. 4.10).

| Model | S/N | Interval Velocity Estimated (ft/s) | Actual Interval Velocity (ft/s) | % Error |
|---|---|---|---|---|
| B1 | Noise Free | 15129.6 | 15590 | **2.95** |
| B2 | 15/1 | 10516.5 | 15590 | **32.54** |
| B3 | 10/1 | 7573.6 | 15590 | **51.42** |
| B4 | / | / | / | **/** |

*Table 4.10*

Bakken Middle Member average-trace interval velocity estimate vs. actual (Table. 4.11).

| Model | S/N | Interval Velocity Estimated (ft/s) | Actual Interval Velocity (ft/s) | % Error |
|---|---|---|---|---|
| B1 | / | / | / | **/** |
| B2 | 15/1 | 13316.9 | 15590 | **14.58** |
| B3 | 10/1 | 12072.2 | 15590 | **22.56** |
| B4 | 5/1 | 9506.93 | 15590 | **39.01** |

*Table 4.11*

## Chapter 5

## Discussion/ Future Work/Conclusion

### 5.1 Discussion

The data presented support the theory that, for a seismically unresolved layer, an interval velocity estimate is attainable by utilizing the method presented here. The assumptions and limitations of both moveout velocity estimation and layer thickness determination by spectral inversion apply to this method and it is important to recognize that inversion for the interval velocity of a layer utilizing Dix equation becomes increasingly unstable as layer thickness is decreased. Therefore, the input parameters used must be sufficiently accurate if desirable results are to be realized. Results from the noise-contaminated models reinforce this idea as the distortion in the amplitude spectrum led to inaccuracies in layer thickness determination and subsequently incorrect interface RMS velocity's. The combination of these errors produced highly variable and inaccurate interval velocity estimates for the target layer. The effects of added noise were highly detrimental to the identifying of an accurate spectral notch with which to invert for layer thickness and suggest peak frequency to be far more resistant to distortion than the notch itself. This can be attributed to its higher amplitude and occurrence closer to the dominant frequency of the data leading to a relatively higher signal to noise ratio as mentioned by

Partyka et al. (1999). Occurrence of peak frequency at one half the frequency of the spectral notch also effectively doubles the bandlimited ability to invert for layer thickness. This ability was exploited in model 8 where the high velocity of the target layer led to very low temporal thickness and the spectral notch being outside the data's useable bandwidth. It should be noted however that the doubling of peak frequency value to estimate the spectral notch period assumes symmetry in the frequency spectrum which can be distorted by noise contamination and any inaccuracy in the peak frequency value will be doubled when used to estimate the notch frequency. In instances where noise resulted a single-trace peak frequency not being clearly interpretable and/or values varied greatly between offsets utilizing an average of adjacent traces added a degree of stability to the process. The inaccuracy of achieved results utilizing the average-trace value shows a correlation with decreasing signal to noise whereas when a single-trace value was utilized in the presence of noise results were less coherent due to the effects of additive noise varying greatly from trace to trace. The use of an average-trace also increased the accuracy of thickness inversion and subsequently results achieved by the method were improved, the error did however remain high. Moveout velocity analysis remained quite stable despite the addition of noise. This is to be expected as the semblance operation utilized operates specifically off the signal coherency of a moveout event over all pre-stack traces in a common midpoint gather. These results lead to the preliminary observation that the method will be very sensitive to noise contamination with the inversion for layer thickness suffering substantial inaccuracy's before the moveout velocity analysis is strongly affected. While very accurate interval velocity estimates are attainable data quality and expectations will need to be strongly considered before implementing this method.

## 5.2 Future Work

The case of a single isolated thin layer as studied in this work is in reality rarely observed in nature. In the presence of multiple thin layers which often exist within the portion of data from which the frequency spectrum is derived the spectral signatures of each layer combine, resulting in a complex interference pattern. The single-layer response for a target thin-bed must be separated from this interference pattern in order to extract an accurate thickness estimate for a horizon of interest. The spectral inversion method detailed by Puryear and Castagna (2008) put forth a novel means by which to separate this observed interference allowing for accurate layer isolation and analysis capable of utilizing only a limited portion of the layers spectral response. The basis for this method lies in the knowledge that the spectrum from a layer response can be separated into even and odd components (Fig 5.1)

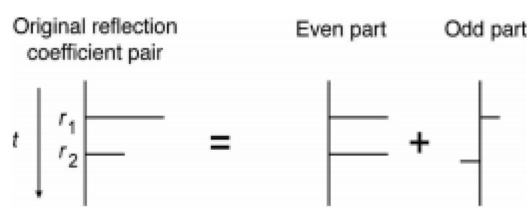



*Fig 5.1 Any arbitrary pair of reflection coefficients $r_1$ and $r_2$ can be represented as the sum of even and odd components. The even pair has the same magnitude and sign, and the odd pair has the same magnitude and opposite sign. (Puryear and Castagna 2008)*

The separated response allows for the reflections from the top and base of a layer to be properly characterized isolating individual layers from the observed interference pattern and subsequently allowing for accurate thickness determination. Another potentially useful consequence of separating the thin layer response into its even and odd components for the purposes of thin-bed interval velocity inversion is that a constant notch period in both the even and odd reflectivity spectra is observed when the analysis point is located symmetrically at the center of the layer, (Puryear and Castagna)(Fig 5.2). When the analysis point is shifted away from the layers midpoint a phase shift occurs between the two reflectivity spectrums.

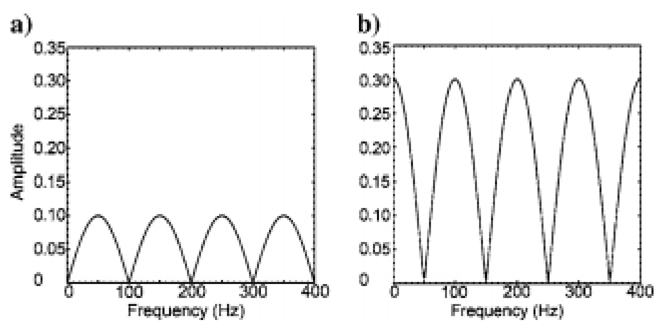



*Fig 5.2 Amplitude versus frequency plots for (a) even and (b) odd components of the reflection coefficient pair $r_1$=0.2 and $r_2$=0.1. In this example the even component is dominant.(Puryear and Castagna. 2008)*

This observation has the very attractive potential use as a phase independent means of locating the midpoint of a thin layer to be analyzed. Note that in the data presented in this work the phase was known and directly utilized in determining the target layers temporal midpoint location. In the case of multiple interfering layers in data where knowledge of the phase is limited or absent reflectivity inversion may hold the key to both accurate layer thickness determination and phase independent midpoint location for accurate midpoint RMS velocity calculation. Combine the information provided by thin-bed interval velocity and reflectivity inversion may form the basis for high resolution simultaneous inversion. This method also has the capability to provide a framework for full-waveform simultaneous inversion of thin layers, by linking the arrival times of their reflection events across offsets.

## 5.3 Conclusion

Thin-bed interval velocity inversion provides a novel means of extracting the velocity information of a seismically thin layer by combining information from both the time and frequency domains. Although the method has been shown to be highly sensitive to noise and prone to instability, under favorable conditions, good velocity estimates were achieved. While the degree of difficulty in extracting the necessary parameters under less than desirable conditions has proven to be challenging given the high level of accuracy required for the method to achieve desirable results, modern data analysis techniques may provide this necessary capability. As with most new methods, extensive testing under a variety of circumstances will be required to assess its feasibility of implementation given a wide range of conditions. Despite these challenges the benefits of achieving a seismically derived interval velocity estimate are vast and as such thin-bed interval velocity inversion provides for a promising area of future research and thin layer analysis.

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## Appendix A

### Equation 3.11 $Rms_t$ Explicit Solution

$$
\begin{aligned}
rmst = {} & \frac{1}{\sqrt{t1}} \Bigg( \sqrt{\Bigg(}\, 2\ rmsm^2\ tm - \frac{dt^2\ rmsm^2\ tm}{dt^2 - t1^2 + 2\ t1\ t2 - t2^2} + \\
& \frac{rmsm^2\ t1^2\ tm}{dt^2 - t1^2 + 2\ t1\ t2 - t2^2} - \frac{2\ rmsm^2\ t1\ t2\ tm}{dt^2 - t1^2 + 2\ t1\ t2 - t2^2} + \frac{rmsm^2\ t2^2\ tm}{dt^2 - t1^2 + 2\ t1\ t2 - t2^2} + \\
& \frac{t1\ x^2}{2\ (dt^2 - t1^2 + 2\ t1\ t2 - t2^2)} - \frac{t2\ x^2}{2\ (dt^2 - t1^2 + 2\ t1\ t2 - t2^2)} - \\
& \frac{1}{2}\ t2 \sqrt{\Bigg(}\Big( (2\ dt^2\ rmsm^2\ tm - 2\ rmsm^2\ t1^2\ tm + 4\ rmsm^2\ t1\ t2\ tm - 2\ rmsm^2 \\
& \quad t2^2\ tm - t1\ x^2 + t2\ x^2)^2 \Big/ \left(t2^2\ (dt^2 - t1^2 + 2\ t1\ t2 - t2^2)^2\right) - \\
& \quad (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2\ tm^2 - \\
& \quad\quad 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 - 4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + \\
& \quad\quad 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 + 8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - \\
& \quad\quad 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4) \,/ \\
& \quad (t2^2\ (dt^4 - 2\ dt^2\ t1^2 + t1^4 - 2\ dt^2\ t2^2 - 2\ t1^2\ t2^2 + t2^4)) + \\
& \quad (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2\ tm^2 - \\
& \quad\quad 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 - 4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + \\
& \quad\quad 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 + 8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - \\
& \quad\quad 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4) \,/ \\
& \quad (3\ (dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6)) + \\
& \quad \Big(2^{1/3}\ \Big(48\ rmsm^4\ (dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6) \\
& \quad\quad tm^2\ x^4 + (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 - \\
& \quad\quad\quad 8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 - \\
& \quad\quad\quad 4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 + \\
& \quad\quad\quad 8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 + \\
& \quad\quad\quad t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)^2 - 24\ (2\ dt^4\ rmsm^2\ t2\ tm - \\
& \quad\quad\quad 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm + 2\ rmsm^2\ t1^4\ t2\ tm - 4\ dt^2\ rmsm^2\ t2^3\ tm - \\
& \quad\quad\quad 4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm - dt^2\ t1\ t2\ x^2 + t1^3\ t2\ x^2 + dt^2\ t2^2 \\
& \quad\quad\quad x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)\ (2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2 \\
& \quad\quad\quad tm^2\ x^2 - 2\ rmsm^4\ t2^2\ tm^2\ x^2 + rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4)\Big)\Big) \Big/ \\
& \quad \Big(3\ t2^2\ (dt^4 - 2\ dt^2\ t1^2 + t1^4 - 2\ dt^2\ t2^2 - 2\ t1^2\ t2^2 + t2^4)\ \Big(432\ rmsm^4 \\
& \quad\quad tm^2\ x^4\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm + 2\ rmsm^2\ t1^4\ t2\ tm - \\
& \quad\quad\quad 4\ dt^2\ rmsm^2\ t2^3\ tm - 4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm - dt^2\ t1 \\
& \quad\quad\quad t2\ x^2 + t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)^2 - \\
& \quad\quad 288\ rmsm^4\ (dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - \\
& \quad\quad\quad 2\ t1^2\ t2^4 + t2^6)\ tm^2\ x^4\ (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + \\
& \quad\quad\quad 4\ rmsm^4\ t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 +
\end{aligned}
$$

$$4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 +$$
$$8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$
$$8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4) +$$
$$2\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$
$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$
$$8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^3 -$$
$$72\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm -$$
$$4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm - dt^2\,t1$$
$$t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)$$
$$(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$
$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 +$$
$$t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)\,(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2$$
$$tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4) +$$
$$432\,(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)$$
$$(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -$$
$$2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4)^2 +$$
$$\surd(-4\,(48\,rmsm^4\,(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2$$
$$t2^4 + t2^6)\,tm^2\,x^4 + (4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2$$
$$tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4$$
$$t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 +$$
$$4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2$$
$$t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 +$$
$$t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^2 - 24\,(2\,dt^4\,rmsm^2\,t2\,tm -$$
$$4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2$$
$$t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm - dt^2\,t1$$
$$t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 -$$
$$t2^4\,x^2)\,(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -$$
$$2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4))^3 +$$
$$(432\,rmsm^4\,tm^2\,x^4\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2$$
$$tm + 2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2$$
$$t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 +$$
$$dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)^2 -$$
$$288\,rmsm^4\,(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 -$$
$$2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)\,tm^2\,x^4\,(4\,dt^4\,rmsm^4\,tm^2 -$$

$$8\,dt^{2}\,rmsm^{4}\,t1^{2}\,tm^{2}+4\,rmsm^{4}\,t1^{4}\,tm^{2}-8\,dt^{2}\,rmsm^{4}\,t2^{2}$$
$$tm^{2}-8\,rmsm^{4}\,t1^{2}\,t2^{2}\,tm^{2}+4\,rmsm^{4}\,t2^{4}\,tm^{2}-4\,dt^{2}\,rmsm^{2}$$
$$t1\,tm\,x^{2}+4\,rmsm^{2}\,t1^{3}\,tm\,x^{2}+8\,dt^{2}\,rmsm^{2}\,t2\,tm\,x^{2}+$$
$$8\,rmsm^{2}\,t1^{2}\,t2\,tm\,x^{2}-4\,rmsm^{2}\,t1\,t2^{2}\,tm\,x^{2}-$$
$$8\,rmsm^{2}\,t2^{3}\,tm\,x^{2}+t1^{2}\,x^{4}+2\,t1\,t2\,x^{4}+t2^{2}\,x^{4})+$$
$$2\,(4\,dt^{4}\,rmsm^{4}\,tm^{2}-8\,dt^{2}\,rmsm^{4}\,t1^{2}\,tm^{2}+4\,rmsm^{4}\,t1^{4}\,tm^{2}-$$
$$8\,dt^{2}\,rmsm^{4}\,t2^{2}\,tm^{2}-8\,rmsm^{4}\,t1^{2}\,t2^{2}\,tm^{2}+4\,rmsm^{4}$$
$$t2^{4}\,tm^{2}-4\,dt^{2}\,rmsm^{2}\,t1\,tm\,x^{2}+4\,rmsm^{2}\,t1^{3}\,tm\,x^{2}+$$
$$8\,dt^{2}\,rmsm^{2}\,t2\,tm\,x^{2}+8\,rmsm^{2}\,t1^{2}\,t2\,tm\,x^{2}-4\,rmsm^{2}$$
$$t1\,t2^{2}\,tm\,x^{2}-8\,rmsm^{2}\,t2^{3}\,tm\,x^{2}+t1^{2}\,x^{4}+2\,t1\,t2\,x^{4}+$$
$$t2^{2}\,x^{4})^{3}-72\,(2\,dt^{4}\,rmsm^{2}\,t2\,tm-4\,dt^{2}\,rmsm^{2}$$
$$t1^{2}\,t2\,tm+2\,rmsm^{2}\,t1^{4}\,t2\,tm-4\,dt^{2}\,rmsm^{2}\,t2^{3}\,tm-$$
$$4\,rmsm^{2}\,t1^{2}\,t2^{3}\,tm+2\,rmsm^{2}\,t2^{5}\,tm-dt^{2}\,t1\,t2\,x^{2}+$$
$$t1^{3}\,t2\,x^{2}+dt^{2}\,t2^{2}\,x^{2}+t1^{2}\,t2^{2}\,x^{2}-t1\,t2^{3}\,x^{2}-t2^{4}\,x^{2})$$
$$(4\,dt^{4}\,rmsm^{4}\,tm^{2}-8\,dt^{2}\,rmsm^{4}\,t1^{2}\,tm^{2}+4\,rmsm^{4}\,t1^{4}\,tm^{2}-$$
$$8\,dt^{2}\,rmsm^{4}\,t2^{2}\,tm^{2}-8\,rmsm^{4}\,t1^{2}\,t2^{2}\,tm^{2}+4\,rmsm^{4}\,t2^{4}$$
$$tm^{2}-4\,dt^{2}\,rmsm^{2}\,t1\,tm\,x^{2}+4\,rmsm^{2}\,t1^{3}\,tm\,x^{2}+8\,dt^{2}$$
$$rmsm^{2}\,t2\,tm\,x^{2}+8\,rmsm^{2}\,t1^{2}\,t2\,tm\,x^{2}-4\,rmsm^{2}\,t1\,t2^{2}$$
$$tm\,x^{2}-8\,rmsm^{2}\,t2^{3}\,tm\,x^{2}+t1^{2}\,x^{4}+2\,t1\,t2\,x^{4}+t2^{2}\,x^{4})$$
$$(2\,dt^{2}\,rmsm^{4}\,tm^{2}\,x^{2}+2\,rmsm^{4}\,t1^{2}\,tm^{2}\,x^{2}-2\,rmsm^{4}\,t2^{2}$$
$$tm^{2}\,x^{2}+rmsm^{2}\,t1\,tm\,x^{4}+rmsm^{2}\,t2\,tm\,x^{4})+432\,(dt^{4}\,t2^{2}-$$
$$2\,dt^{2}\,t1^{2}\,t2^{2}+t1^{4}\,t2^{2}-2\,dt^{2}\,t2^{4}-2\,t1^{2}\,t2^{4}+t2^{6})$$
$$(2\,dt^{2}\,rmsm^{4}\,tm^{2}\,x^{2}+2\,rmsm^{4}\,t1^{2}\,tm^{2}\,x^{2}-2\,rmsm^{4}\,t2^{2}$$
$$tm^{2}\,x^{2}+rmsm^{2}\,t1\,tm\,x^{4}+rmsm^{2}\,t2\,tm\,x^{4})^{2})^{2}))^{1/3})+$$
$$(432\,rmsm^{4}\,tm^{2}\,x^{4}\,(2\,dt^{4}\,rmsm^{2}\,t2\,tm-4\,dt^{2}\,rmsm^{2}\,t1^{2}\,t2\,tm+$$
$$2\,rmsm^{2}\,t1^{4}\,t2\,tm-4\,dt^{2}\,rmsm^{2}\,t2^{3}\,tm-4\,rmsm^{2}\,t1^{2}\,t2^{3}\,tm+$$
$$2\,rmsm^{2}\,t2^{5}\,tm-dt^{2}\,t1\,t2\,x^{2}+t1^{3}\,t2\,x^{2}+dt^{2}\,t2^{2}\,x^{2}+$$
$$t1^{2}\,t2^{2}\,x^{2}-t1\,t2^{3}\,x^{2}-t2^{4}\,x^{2})^{2}-288\,rmsm^{4}$$
$$(dt^{4}\,t2^{2}-2\,dt^{2}\,t1^{2}\,t2^{2}+t1^{4}\,t2^{2}-2\,dt^{2}\,t2^{4}-2\,t1^{2}\,t2^{4}+t2^{6})$$
$$tm^{2}\,x^{4}\,(4\,dt^{4}\,rmsm^{4}\,tm^{2}-8\,dt^{2}\,rmsm^{4}\,t1^{2}\,tm^{2}+4\,rmsm^{4}\,t1^{4}\,tm^{2}-$$
$$8\,dt^{2}\,rmsm^{4}\,t2^{2}\,tm^{2}-8\,rmsm^{4}\,t1^{2}\,t2^{2}\,tm^{2}+4\,rmsm^{4}\,t2^{4}\,tm^{2}-$$
$$4\,dt^{2}\,rmsm^{2}\,t1\,tm\,x^{2}+4\,rmsm^{2}\,t1^{3}\,tm\,x^{2}+8\,dt^{2}\,rmsm^{2}\,t2\,tm\,x^{2}+$$
$$8\,rmsm^{2}\,t1^{2}\,t2\,tm\,x^{2}-4\,rmsm^{2}\,t1\,t2^{2}\,tm\,x^{2}-$$
$$8\,rmsm^{2}\,t2^{3}\,tm\,x^{2}+t1^{2}\,x^{4}+2\,t1\,t2\,x^{4}+t2^{2}\,x^{4})+2$$
$$(4\,dt^{4}\,rmsm^{4}\,tm^{2}-8\,dt^{2}\,rmsm^{4}\,t1^{2}\,tm^{2}+4\,rmsm^{4}\,t1^{4}\,tm^{2}-$$
$$8\,dt^{2}\,rmsm^{4}\,t2^{2}\,tm^{2}-8\,rmsm^{4}\,t1^{2}\,t2^{2}\,tm^{2}+4\,rmsm^{4}\,t2^{4}\,tm^{2}-$$
$$4\,dt^{2}\,rmsm^{2}\,t1\,tm\,x^{2}+4\,rmsm^{2}\,t1^{3}\,tm\,x^{2}+8\,dt^{2}\,rmsm^{2}\,t2\,tm\,x^{2}+$$
$$8\,rmsm^{2}\,t1^{2}\,t2\,tm\,x^{2}-4\,rmsm^{2}\,t1\,t2^{2}\,tm\,x^{2}-$$
$$8\,rmsm^{2}\,t2^{3}\,tm\,x^{2}+t1^{2}\,x^{4}+2\,t1\,t2\,x^{4}+t2^{2}\,x^{4})^{3}-72$$

$$
\begin{aligned}
&\left(2\,dt^4\,rmsm^2\,t2\,tm-4\,dt^2\,rmsm^2\,t1^2\,t2\,tm+2\,rmsm^2\,t1^4\,t2\,tm-\right.\\
&\quad 4\,dt^2\,rmsm^2\,t2^3\,tm-4\,rmsm^2\,t1^2\,t2^3\,tm+2\,rmsm^2\,t2^5\,tm-\\
&\quad \left.dt^2\,t1\,t2\,x^2+t1^3\,t2\,x^2+dt^2\,t2^2\,x^2+t1^2\,t2^2\,x^2-t1\,t2^3\,x^2-t2^4\,x^2\right)\\
&\left(4\,dt^4\,rmsm^4\,tm^2-8\,dt^2\,rmsm^4\,t1^2\,tm^2+4\,rmsm^4\,t1^4\,tm^2-\right.\\
&\quad 8\,dt^2\,rmsm^4\,t2^2\,tm^2-8\,rmsm^4\,t1^2\,t2^2\,tm^2+4\,rmsm^4\,t2^4\,tm^2-\\
&\quad 4\,dt^2\,rmsm^2\,t1\,tm\,x^2+4\,rmsm^2\,t1^3\,tm\,x^2+8\,dt^2\,rmsm^2\,t2\,tm\,x^2+\\
&\quad 8\,rmsm^2\,t1^2\,t2\,tm\,x^2-4\,rmsm^2\,t1\,t2^2\,tm\,x^2-\\
&\quad \left.8\,rmsm^2\,t2^3\,tm\,x^2+t1^2\,x^4+2\,t1\,t2\,x^4+t2^2\,x^4\right)\\
&\left(2\,dt^2\,rmsm^4\,tm^2\,x^2+2\,rmsm^4\,t1^2\,tm^2\,x^2-2\,rmsm^4\,t2^2\,tm^2\,x^2+\right.\\
&\quad \left.rmsm^2\,t1\,tm\,x^4+rmsm^2\,t2\,tm\,x^4\right)+432\\
&\left(dt^4\,t2^2-2\,dt^2\,t1^2\,t2^2+t1^4\,t2^2-2\,dt^2\,t2^4-2\,t1^2\,t2^4+t2^6\right)\\
&\left(2\,dt^2\,rmsm^4\,tm^2\,x^2+2\,rmsm^4\,t1^2\,tm^2\,x^2-\right.\\
&\qquad \left.2\,rmsm^4\,t2^2\,tm^2\,x^2+rmsm^2\,t1\,tm\,x^4+rmsm^2\,t2\,tm\,x^4\right)^2+\\
&\sqrt{}\Bigg(-4\,\Bigg(48\,rmsm^4\,\left(dt^4\,t2^2-2\,dt^2\,t1^2\,t2^2+t1^4\,t2^2-2\,dt^2\,t2^4-\right.\\
&\qquad\qquad \left.2\,t1^2\,t2^4+t2^6\right)\,tm^2\,x^4+\left(4\,dt^4\,rmsm^4\,tm^2-8\,dt^2\,rmsm^4\,t1^2\right.\\
&\qquad\qquad tm^2+4\,rmsm^4\,t1^4\,tm^2-8\,dt^2\,rmsm^4\,t2^2\,tm^2-8\,rmsm^4\\
&\qquad\qquad t1^2\,t2^2\,tm^2+4\,rmsm^4\,t2^4\,tm^2-4\,dt^2\,rmsm^2\,t1\,tm\,x^2+\\
&\qquad\qquad 4\,rmsm^2\,t1^3\,tm\,x^2+8\,dt^2\,rmsm^2\,t2\,tm\,x^2+8\,rmsm^2\,t1^2\,t2\\
&\qquad\qquad tm\,x^2-4\,rmsm^2\,t1\,t2^2\,tm\,x^2-8\,rmsm^2\,t2^3\,tm\,x^2+t1^2\,x^4+\\
&\qquad\qquad \left.2\,t1\,t2\,x^4+t2^2\,x^4\right)^2-24\,\left(2\,dt^4\,rmsm^2\,t2\,tm-4\,dt^2\right.\\
&\qquad\qquad rmsm^2\,t1^2\,t2\,tm+2\,rmsm^2\,t1^4\,t2\,tm-4\,dt^2\,rmsm^2\,t2^3\,tm-\\
&\qquad\qquad 4\,rmsm^2\,t1^2\,t2^3\,tm+2\,rmsm^2\,t2^5\,tm-dt^2\,t1\,t2\,x^2+\\
&\qquad\qquad \left.t1^3\,t2\,x^2+dt^2\,t2^2\,x^2+t1^2\,t2^2\,x^2-t1\,t2^3\,x^2-t2^4\,x^2\right)\\
&\qquad\quad \left(2\,dt^2\,rmsm^4\,tm^2\,x^2+2\,rmsm^4\,t1^2\,tm^2\,x^2-2\,rmsm^4\right.\\
&\qquad\qquad \left.t2^2\,tm^2\,x^2+rmsm^2\,t1\,tm\,x^4+rmsm^2\,t2\,tm\,x^4\right)\Bigg)^3+\\
&\quad\Bigg(432\,rmsm^4\,tm^2\,x^4\,\left(2\,dt^4\,rmsm^2\,t2\,tm-4\,dt^2\,rmsm^2\,t1^2\,t2\right.\\
&\qquad\qquad tm+2\,rmsm^2\,t1^4\,t2\,tm-4\,dt^2\,rmsm^2\,t2^3\,tm-4\,rmsm^2\\
&\qquad\qquad t1^2\,t2^3\,tm+2\,rmsm^2\,t2^5\,tm-dt^2\,t1\,t2\,x^2+t1^3\,t2\,x^2+\\
&\qquad\qquad \left.dt^2\,t2^2\,x^2+t1^2\,t2^2\,x^2-t1\,t2^3\,x^2-t2^4\,x^2\right)^2-288\,rmsm^4\\
&\qquad\quad \left(dt^4\,t2^2-2\,dt^2\,t1^2\,t2^2+t1^4\,t2^2-2\,dt^2\,t2^4-2\,t1^2\,t2^4+\right.\\
&\qquad\qquad \left.t2^6\right)\,tm^2\,x^4\,\left(4\,dt^4\,rmsm^4\,tm^2-8\,dt^2\,rmsm^4\,t1^2\,tm^2+\right.\\
&\qquad\qquad 4\,rmsm^4\,t1^4\,tm^2-8\,dt^2\,rmsm^4\,t2^2\,tm^2-8\,rmsm^4\,t1^2\,t2^2\,tm^2+\\
&\qquad\qquad 4\,rmsm^4\,t2^4\,tm^2-4\,dt^2\,rmsm^2\,t1\,tm\,x^2+4\,rmsm^2\,t1^3\,tm\,x^2+\\
&\qquad\qquad 8\,dt^2\,rmsm^2\,t2\,tm\,x^2+8\,rmsm^2\,t1^2\,t2\,tm\,x^2-4\,rmsm^2\,t1\,t2^2\\
&\qquad\qquad \left.tm\,x^2-8\,rmsm^2\,t2^3\,tm\,x^2+t1^2\,x^4+2\,t1\,t2\,x^4+t2^2\,x^4\right)+\\
&\qquad 2\,\left(4\,dt^4\,rmsm^4\,tm^2-8\,dt^2\,rmsm^4\,t1^2\,tm^2+4\,rmsm^4\,t1^4\,tm^2-\right.\\
&\qquad\qquad 8\,dt^2\,rmsm^4\,t2^2\,tm^2-8\,rmsm^4\,t1^2\,t2^2\,tm^2+4\,rmsm^4\,t2^4\\
&\qquad\qquad tm^2-4\,dt^2\,rmsm^2\,t1\,tm\,x^2+4\,rmsm^2\,t1^3\,tm\,x^2+8\,dt^2
\end{aligned}
$$

$$rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2$$
$$tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 -$$
$$72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm +$$
$$2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm +$$
$$2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 +$$
$$t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)\, (4\, dt^4\, rmsm^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 -$$
$$8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2$$
$$tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 +$$
$$2\, t1\, t2\, x^4 + t2^2\, x^4)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 -$$
$$2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) + 432$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 +$$
$$rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4)^2)^2))^{1/3} /$$
$$(3 \times 2^{1/3}\, t2^2\, (dt^4 - 2\, dt^2\, t1^2 + t1^4 - 2\, dt^2\, t2^2 - 2\, t1^2\, t2^2 + t2^4))) +$$
$$\frac{1}{2}\, t2\, \sqrt{}\, (((2\, (2\, dt^2\, rmsm^2\, tm - 2\, rmsm^2\, t1^2\, tm + 4\, rmsm^2\, t1\, t2\, tm -$$
$$2\, rmsm^2\, t2^2\, tm - t1\, x^2 + t2\, x^2)^2) /$$
$$(t2^2\, (dt^2 - t1^2 + 2\, t1\, t2 - t2^2)^2)) - (4\, dt^4\, rmsm^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 +$$
$$4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 +$$
$$8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 -$$
$$8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) /$$
$$(t2^2\, (dt^4 - 2\, dt^2\, t1^2 + t1^4 - 2\, dt^2\, t2^2 - 2\, t1^2\, t2^2 + t2^4)) -$$
$$(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 +$$
$$4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 +$$
$$8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 -$$
$$8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) /$$
$$(3\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)) -$$
$$(2^{1/3}\, (48\, rmsm^4\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)$$
$$tm^2\, x^4 + (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 -$$
$$4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 +$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 -$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)^2 - 24$$

$$(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} -$$

$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} -$$

$$\mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)$$

$$(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 - 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 +$$

$$\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)))\Big/$$

$$\Big(3\,\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)$$

$$\Big(432\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^4\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} +$$

$$2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} +$$

$$2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 +$$

$$\mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)^2 - 288\,\mathrm{rmsm}^4$$

$$(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)$$

$$\mathrm{tm}^2\,x^4\,(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 -$$

$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 -$$

$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 +$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 -$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) +$$

$$2\,(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 -$$

$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 -$$

$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 +$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 -$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)^3 -$$

$$72\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} -$$

$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}$$

$$\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)$$

$$(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 -$$

$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 -$$

$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 +$$

$$8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 +$$

$$\mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)\,(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2$$

$$\mathrm{tm}^2\,x^2 - 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4) +$$

$$432\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)$$

$$(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 -$$

$$2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)^2 +$$

$$\sqrt{}\Big(-4\,\Big(48\,\mathrm{rmsm}^4\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 -$$

$$2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\,\mathrm{tm}^2\,x^4 + (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4$$

$$t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2\ tm^2 -$$
$$8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 - 4\ dt^2\ rmsm^2$$
$$t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 -$$
$$8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)^2 -$$
$$24\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm +$$
$$2\ rmsm^2\ t1^4\ t2\ tm - 4\ dt^2\ rmsm^2\ t2^3\ tm -$$
$$4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm - dt^2\ t1\ t2\ x^2 +$$
$$t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)$$
$$(2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2\ tm^2\ x^2 -$$
$$2\ rmsm^4\ t2^2\ tm^2\ x^2 + rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4))^3 +$$
$$(432\ rmsm^4\ tm^2\ x^4\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2$$
$$tm + 2\ rmsm^2\ t1^4\ t2\ tm - 4\ dt^2\ rmsm^2\ t2^3\ tm - 4\ rmsm^2$$
$$t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm - dt^2\ t1\ t2\ x^2 + t1^3\ t2\ x^2 +$$
$$dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)^2 -$$
$$288\ rmsm^4\ (dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 -$$
$$2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6)\ tm^2\ x^4\ (4\ dt^4\ rmsm^4\ tm^2 -$$
$$8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2$$
$$tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 - 4\ dt^2\ rmsm^2$$
$$t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 -$$
$$8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4) +$$
$$2\ (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4$$
$$t1^4\ tm^2 - 8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 +$$
$$4\ rmsm^4\ t2^4\ tm^2 - 4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3$$
$$tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 + 8\ rmsm^2\ t1^2\ t2\ tm\ x^2 -$$
$$4\ rmsm^2\ t1\ t2^2\ tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 +$$
$$2\ t1\ t2\ x^4 + t2^2\ x^4)^3 - 72\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2$$
$$rmsm^2\ t1^2\ t2\ tm + 2\ rmsm^2\ t1^4\ t2\ tm - 4\ dt^2\ rmsm^2\ t2^3$$
$$tm - 4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm - dt^2\ t1\ t2\ x^2 +$$
$$t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)$$
$$(4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 -$$
$$8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4$$
$$tm^2 - 4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2$$
$$rmsm^2\ t2\ tm\ x^2 + 8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2$$
$$tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)$$
$$(2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2\ tm^2\ x^2 - 2\ rmsm^4\ t2^2$$
$$tm^2\ x^2 + rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4) + 432\ (dt^4\ t2^2 -$$
$$2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6)$$

$$\left(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\right.$$

$$\left.\left.\left.\left.\left.tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4\right)^2\right)^2\right)\right)^{1/3}\right) -$$

$$\left(432\,rmsm^4\,tm^2\,x^4\,\left(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm +\right.\right.$$

$$2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm +$$

$$2\,rmsm^2\,t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 +$$

$$\left.t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2\right)^2 - 288\,rmsm^4$$

$$\left(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6\right)$$

$$tm^2\,x^4\,\left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -\right.$$

$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$

$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$

$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$

$$\left.8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right) + 2$$

$$\left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -\right.$$

$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$

$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$

$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$

$$\left.8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right)^3 - 72$$

$$\left(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm -\right.$$

$$4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm -$$

$$\left.dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2\right)$$

$$\left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -\right.$$

$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$

$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$

$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$

$$\left.8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right)$$

$$\left(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 +\right.$$

$$\left.rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4\right) + 432$$

$$\left(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6\right)$$

$$\left(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -\right.$$

$$\left.2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4\right)^2 +$$

$$\sqrt{\left(-4\,\left(48\,rmsm^4\,\left(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 -\right.\right.\right.}$$

$$\left.2\,t1^2\,t2^4 + t2^6\right)\,tm^2\,x^4 + \left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\right.$$

$$tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4$$

$$t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 +$$

$$4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2$$

$$tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 +$$

$$\left.2\,t1\,t2\,x^4 + t2^2\,x^4\right)^2 - 24\,\left(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\right.$$

$$rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm -$$

$$4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 +$$
$$t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4$$
$$t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4))^3 +$$
$$(432\, rmsm^4\, tm^2\, x^4\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2$$
$$tm + 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2$$
$$t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 +$$
$$dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)^2 - 288\, rmsm^4$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 +$$
$$t2^6)\, tm^2\, x^4\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 +$$
$$4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 +$$
$$4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 +$$
$$8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2$$
$$tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) +$$
$$2\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4$$
$$tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2$$
$$rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2$$
$$tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 -$$
$$72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm +$$
$$2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm +$$
$$2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 +$$
$$t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)\, (4\, dt^4\, rmsm^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 -$$
$$8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2$$
$$tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 +$$
$$2\, t1\, t2\, x^4 + t2^2\, x^4)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 -$$
$$2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) + 432$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 +$$
$$rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4)^2)^2))^{1/3} \Big/$$
$$(3 \times 2^{1/3}\, t2^2\, (dt^4 - 2\, dt^2\, t1^2 + t1^4 - 2\, dt^2\, t2^2 - 2\, t1^2\, t2^2 + t2^4)) +$$
$$((8\, (2\, dt^2\, rmsm^2\, tm - 2\, rmsm^2\, t1^2\, tm + 4\, rmsm^2\, t1\, t2\, tm - 2\, rmsm^2\, t2^2\, tm -$$
$$t1\, x^2 + t2\, x^2)^3) \Big/ (t2^3\, (dt^2 - t1^2 + 2\, t1\, t2 - t2^2)^3) -$$
$$(8\, (2\, dt^2\, rmsm^2\, tm - 2\, rmsm^2\, t1^2\, tm + 4\, rmsm^2\, t1\, t2\, tm -$$
$$2\, rmsm^2\, t2^2\, tm - t1\, x^2 + t2\, x^2)$$
$$(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$

$$
\begin{aligned}
&8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + \\
&4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + \\
&8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - \\
&4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + \\
&2\,t1\,t2\,x^4 + t2^2\,x^4)) \Big/ \left(t2^3\,\left(dt^2 - t1^2 + 2\,t1\,t2 - t2^2\right)\right. \\
&\left.\left(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4\right)\right) + \\
&\left(32\,\left(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 + \right.\right. \\
&\left.\left.rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4\right)\right) \Big/ \\
&\left.\left(t2^2\,\left(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4\right)\right)\right) \Big/ \\
&\left(4\,\sqrt{\left(\left(2\,dt^2\,rmsm^2\,tm - 2\,rmsm^2\,t1^2\,tm + 4\,rmsm^2\,t1\,t2\,tm - 2\,rmsm^2\,t2^2\,tm - \right.\right.}\right. \\
&\left.t1\,x^2 + t2\,x^2\right)^2 \Big/ \left(t2^2\,\left(dt^2 - t1^2 + 2\,t1\,t2 - t2^2\right)^2\right) - \\
&\left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - \right. \\
&8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - \\
&4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + \\
&8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - \\
&\left.8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right) \Big/ \\
&\left(t2^2\,\left(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4\right)\right) + \\
&\left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - \right. \\
&8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - \\
&4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + \\
&8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - \\
&\left.8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right) \Big/ \\
&\left(3\,\left(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6\right)\right) + \\
&\left(2^{1/3}\,\left(48\,rmsm^4\,\left(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\right.\right.\right. \\
&\left.t2^4 + t2^6\right)\,tm^2\,x^4 + \left(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + \right. \\
&4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + \\
&4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + \\
&8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2 \\
&\left.tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4\right)^2 - \\
&24\,\left(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\right. \\
&t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2 \\
&t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - \\
&\left.t1\,t2^3\,x^2 - t2^4\,x^2\right)\,\left(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - \right. \\
&\left.\left.\left.2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4\right)\right)\right) \Big/ \\
&\left(3\,t2^2\,\left(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4\right)\right. \\
&\left(432\,rmsm^4\,tm^2\,x^4\,\left(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + \right.\right.
\end{aligned}
$$

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\begin{aligned}
&2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm}-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}-4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\\
&\mathrm{t2}^3\,\mathrm{tm}+2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm}-\mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2+\mathrm{t1}^3\,\mathrm{t2}\,x^2+\\
&\mathrm{dt}^2\,\mathrm{t2}^2\,x^2+\mathrm{t1}^2\,\mathrm{t2}^2\,x^2-\mathrm{t1}\,\mathrm{t2}^3\,x^2-\mathrm{t2}^4\,x^2)^2-288\,\mathrm{rmsm}^4\\
&(\mathrm{dt}^4\,\mathrm{t2}^2-2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2+\mathrm{t1}^4\,\mathrm{t2}^2-2\,\mathrm{dt}^2\,\mathrm{t2}^4-2\,\mathrm{t1}^2\,\mathrm{t2}^4+\\
&\mathrm{t2}^6)\,\mathrm{tm}^2\,x^4\,(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2+4\\
&\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2-8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2+\\
&4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2+4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2+\\
&8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2+8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2-4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\\
&\mathrm{tm}\,x^2-8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2+\mathrm{t1}^2\,x^4+2\,\mathrm{t1}\,\mathrm{t2}\,x^4+\mathrm{t2}^2\,x^4)+\\
&2\,(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2+4\,\mathrm{rmsm}^4\\
&\mathrm{t1}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2-8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2+\\
&4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2+4\,\mathrm{rmsm}^2\\
&\mathrm{t1}^3\,\mathrm{tm}\,x^2+8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2+8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2-\\
&4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2-8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2+\mathrm{t1}^2\,x^4+\\
&2\,\mathrm{t1}\,\mathrm{t2}\,x^4+\mathrm{t2}^2\,x^4)^3-72\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}-4\,\mathrm{dt}^2\\
&\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}+2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm}-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}-\\
&4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm}+2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm}-\mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2+\\
&\mathrm{t1}^3\,\mathrm{t2}\,x^2+\mathrm{dt}^2\,\mathrm{t2}^2\,x^2+\mathrm{t1}^2\,\mathrm{t2}^2\,x^2-\mathrm{t1}\,\mathrm{t2}^3\,x^2-\mathrm{t2}^4\,x^2)\\
&(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2+4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2-\\
&8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2-8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2+4\,\mathrm{rmsm}^4\\
&\mathrm{t2}^4\,\mathrm{tm}^2-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2+4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2+\\
&8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2+8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2-4\,\mathrm{rmsm}^2\,\mathrm{t1}\\
&\mathrm{t2}^2\,\mathrm{tm}\,x^2-8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2+\mathrm{t1}^2\,x^4+2\,\mathrm{t1}\,\mathrm{t2}\,x^4+\mathrm{t2}^2\,x^4)\\
&(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2+2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2-2\,\mathrm{rmsm}^4\\
&\mathrm{t2}^2\,\mathrm{tm}^2\,x^2+\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4+\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)+\\
&432\,(\mathrm{dt}^4\,\mathrm{t2}^2-2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2+\mathrm{t1}^4\,\mathrm{t2}^2-2\,\mathrm{dt}^2\,\mathrm{t2}^4-\\
&2\,\mathrm{t1}^2\,\mathrm{t2}^4+\mathrm{t2}^6)\,(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2+2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2-\\
&2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2+\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4+\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)^2+\\
&\sqrt{\,}(-4\,(48\,\mathrm{rmsm}^4\,(\mathrm{dt}^4\,\mathrm{t2}^2-2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2+\mathrm{t1}^4\,\mathrm{t2}^2-\\
&2\,\mathrm{dt}^2\,\mathrm{t2}^4-2\,\mathrm{t1}^2\,\mathrm{t2}^4+\mathrm{t2}^6)\,\mathrm{tm}^2\,x^4+\\
&(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2+4\,\mathrm{rmsm}^4\\
&\mathrm{t1}^4\,\mathrm{tm}^2-8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2-8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\\
&\mathrm{tm}^2+4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2+\\
&4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2+8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2+\\
&8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2-4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2-8\\
&\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2+\mathrm{t1}^2\,x^4+2\,\mathrm{t1}\,\mathrm{t2}\,x^4+\mathrm{t2}^2\,x^4)^2-\\
&24\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}+\\
&2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm}-4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}-4\,\mathrm{rmsm}^2\\
&\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm}+2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm}-\mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2+
\end{aligned}
$$

$$t1^{3}\ t2\ x^{2} + dt^{2}\ t2^{2}\ x^{2} + t1^{2}\ t2^{2}\ x^{2} - t1\ t2^{3}\ x^{2} -$$
$$t2^{4}\ x^{2})\ (2\ dt^{2}\ rmsm^{4}\ tm^{2}\ x^{2} + 2\ rmsm^{4}\ t1^{2}\ tm^{2}\ x^{2} -$$
$$2\ rmsm^{4}\ t2^{2}\ tm^{2}\ x^{2} + rmsm^{2}\ t1\ tm\ x^{4} + rmsm^{2}\ t2$$
$$tm\ x^{4}))^{3} + (432\ rmsm^{4}\ tm^{2}\ x^{4}\ (2\ dt^{4}\ rmsm^{2}\ t2$$
$$tm - 4\ dt^{2}\ rmsm^{2}\ t1^{2}\ t2\ tm + 2\ rmsm^{2}\ t1^{4}\ t2\ tm -$$
$$4\ dt^{2}\ rmsm^{2}\ t2^{3}\ tm - 4\ rmsm^{2}\ t1^{2}\ t2^{3}\ tm +$$
$$2\ rmsm^{2}\ t2^{5}\ tm - dt^{2}\ t1\ t2\ x^{2} + t1^{3}\ t2\ x^{2} +$$
$$dt^{2}\ t2^{2}\ x^{2} + t1^{2}\ t2^{2}\ x^{2} - t1\ t2^{3}\ x^{2} - t2^{4}\ x^{2})^{2} -$$
$$288\ rmsm^{4}\ (dt^{4}\ t2^{2} - 2\ dt^{2}\ t1^{2}\ t2^{2} + t1^{4}$$
$$t2^{2} - 2\ dt^{2}\ t2^{4} - 2\ t1^{2}\ t2^{4} + t2^{6})\ tm^{2}\ x^{4}$$
$$(4\ dt^{4}\ rmsm^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t1^{2}\ tm^{2} +$$
$$4\ rmsm^{4}\ t1^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t2^{2}\ tm^{2} - 8\ rmsm^{4}$$
$$t1^{2}\ t2^{2}\ tm^{2} + 4\ rmsm^{4}\ t2^{4}\ tm^{2} - 4\ dt^{2}\ rmsm^{2}\ t1$$
$$tm\ x^{2} + 4\ rmsm^{2}\ t1^{3}\ tm\ x^{2} + 8\ dt^{2}\ rmsm^{2}\ t2\ tm\ x^{2} +$$
$$8\ rmsm^{2}\ t1^{2}\ t2\ tm\ x^{2} - 4\ rmsm^{2}\ t1\ t2^{2}\ tm\ x^{2} -$$
$$8\ rmsm^{2}\ t2^{3}\ tm\ x^{2} + t1^{2}\ x^{4} + 2\ t1\ t2\ x^{4} + t2^{2}\ x^{4}) +$$
$$2\ (4\ dt^{4}\ rmsm^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t1^{2}\ tm^{2} + 4\ rmsm^{4}$$
$$t1^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t2^{2}\ tm^{2} - 8\ rmsm^{4}\ t1^{2}\ t2^{2}$$
$$tm^{2} + 4\ rmsm^{4}\ t2^{4}\ tm^{2} - 4\ dt^{2}\ rmsm^{2}\ t1\ tm\ x^{2} +$$
$$4\ rmsm^{2}\ t1^{3}\ tm\ x^{2} + 8\ dt^{2}\ rmsm^{2}\ t2\ tm\ x^{2} +$$
$$8\ rmsm^{2}\ t1^{2}\ t2\ tm\ x^{2} - 4\ rmsm^{2}\ t1\ t2^{2}\ tm\ x^{2} - 8$$
$$rmsm^{2}\ t2^{3}\ tm\ x^{2} + t1^{2}\ x^{4} + 2\ t1\ t2\ x^{4} + t2^{2}\ x^{4})^{3} -$$
$$72\ (2\ dt^{4}\ rmsm^{2}\ t2\ tm - 4\ dt^{2}\ rmsm^{2}\ t1^{2}\ t2$$
$$tm + 2\ rmsm^{2}\ t1^{4}\ t2\ tm - 4\ dt^{2}\ rmsm^{2}\ t2^{3}\ tm - 4$$
$$rmsm^{2}\ t1^{2}\ t2^{3}\ tm + 2\ rmsm^{2}\ t2^{5}\ tm - dt^{2}\ t1\ t2\ x^{2} +$$
$$t1^{3}\ t2\ x^{2} + dt^{2}\ t2^{2}\ x^{2} + t1^{2}\ t2^{2}\ x^{2} - t1\ t2^{3}\ x^{2} -$$
$$t2^{4}\ x^{2})\ (4\ dt^{4}\ rmsm^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t1^{2}\ tm^{2} +$$
$$4\ rmsm^{4}\ t1^{4}\ tm^{2} - 8\ dt^{2}\ rmsm^{4}\ t2^{2}\ tm^{2} - 8\ rmsm^{4}$$
$$t1^{2}\ t2^{2}\ tm^{2} + 4\ rmsm^{4}\ t2^{4}\ tm^{2} - 4\ dt^{2}\ rmsm^{2}\ t1$$
$$tm\ x^{2} + 4\ rmsm^{2}\ t1^{3}\ tm\ x^{2} + 8\ dt^{2}\ rmsm^{2}\ t2\ tm\ x^{2} +$$
$$8\ rmsm^{2}\ t1^{2}\ t2\ tm\ x^{2} - 4\ rmsm^{2}\ t1\ t2^{2}\ tm\ x^{2} -$$
$$8\ rmsm^{2}\ t2^{3}\ tm\ x^{2} + t1^{2}\ x^{4} + 2\ t1\ t2\ x^{4} + t2^{2}\ x^{4})$$
$$(2\ dt^{2}\ rmsm^{4}\ tm^{2}\ x^{2} + 2\ rmsm^{4}\ t1^{2}\ tm^{2}\ x^{2} - 2\ rmsm^{4}$$
$$t2^{2}\ tm^{2}\ x^{2} + rmsm^{2}\ t1\ tm\ x^{4} + rmsm^{2}\ t2\ tm\ x^{4}) +$$
$$432\ (dt^{4}\ t2^{2} - 2\ dt^{2}\ t1^{2}\ t2^{2} + t1^{4}\ t2^{2} -$$
$$2\ dt^{2}\ t2^{4} - 2\ t1^{2}\ t2^{4} + t2^{6})\ (2\ dt^{2}\ rmsm^{4}\ tm^{2}\ x^{2} +$$
$$2\ rmsm^{4}\ t1^{2}\ tm^{2}\ x^{2} - 2\ rmsm^{4}\ t2^{2}\ tm^{2}\ x^{2} +$$
$$rmsm^{2}\ t1\ tm\ x^{4} + rmsm^{2}\ t2\ tm\ x^{4})^{2})^{2}))^{1/3}) +$$

$$
\begin{aligned}
&(432\, rmsm^4\, tm^2\, x^4\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2 \\
&\quad tm + 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - \\
&\quad 4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + \\
&\quad t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)^2 - \\
&\quad 288\, rmsm^4\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - \\
&\quad 2\, t1^2\, t2^4 + t2^6)\, tm^2\, x^4\, (4\, dt^4\, rmsm^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - \\
&\quad 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + \\
&\quad 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&\quad 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) + \\
&\quad 2\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + \\
&\quad 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + \\
&\quad 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2 \\
&\quad tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 - \\
&\quad 72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + \\
&\quad 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - \\
&\quad 4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + \\
&\quad t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2) \\
&\quad (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&\quad 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2 \\
&\quad t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) \\
&\quad (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4 \\
&\quad t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) + \\
&\quad 432\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - \\
&\quad 2\, t1^2\, t2^4 + t2^6)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - \\
&\quad 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4)^2 + \\
&\quad \sqrt{\;}(-4\, (48\, rmsm^4\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - \\
&\quad 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)\, tm^2\, x^4 + \\
&\quad (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + \\
&\quad 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4 \\
&\quad t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1 \\
&\quad tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&\quad 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^2 - \\
&\quad 24\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm +
\end{aligned}
$$

$$2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2$$
$$t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2$$
$$x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4$$
$$t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4))^3 +$$
$$(432\, rmsm^4\, tm^2\, x^4\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2$$
$$t1^2\, t2\, tm + 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2$$
$$t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm -$$
$$dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 +$$
$$t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)^2 - 288\, rmsm^4$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 -$$
$$2\, t1^2\, t2^4 + t2^6)\, tm^2\, x^4\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2$$
$$rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2$$
$$tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2$$
$$rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2$$
$$tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 -$$
$$8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) +$$
$$2\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 +$$
$$4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4$$
$$t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1$$
$$tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 +$$
$$8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 -$$
$$8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 -$$
$$72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm +$$
$$2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2$$
$$t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 +$$
$$dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)$$
$$(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 +$$
$$4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2$$
$$t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 +$$
$$8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 -$$
$$8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4$$
$$t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) +$$
$$432\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 -$$
$$2\, t1^2\, t2^4 + t2^6)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2$$
$$tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 +$$
$$rmsm^2\, t2\, tm\, x^4)^2)^2))^{1/3} \Big/ (3 \times 2^{1/3}\, t2^2$$

$$\left(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4\right)\Big)\Big)\Big)\Big)\Big)\Big)$$

*Where* $t2 = t(0)_b$ *layer base time zero*, $t1 = t(0)_t$ *layer top time zero*, $tm = t(0)_m$ *layer midpoint time zero*, $dt = \Delta t(x)$, $x = offset$, $rmsm =$ *midpoint RMS velocity* , *rmst =layer top RMS velocity*

## Equation 3.11 $Rms_b$ Explicit Solution

$$
\begin{aligned}
\mathrm{rmsb} = \sqrt{} \Bigg( & \frac{2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}}{\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2} + \frac{\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{tm}}{\mathrm{t2}\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)} - \\
& \frac{\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{tm}}{\mathrm{t2}\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)} - \frac{\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}}{\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2} + \\
& \frac{x^2}{2\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)} - \frac{\mathrm{t1}\,x^2}{2\,\mathrm{t2}\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)} + \\
& \frac{1}{2} \sqrt{} \Big( \big(2\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{tm} + 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t2}^2\,\mathrm{tm} - \\
& \quad \mathrm{t1}\,x^2 + \mathrm{t2}\,x^2\big)^2 \big/ \left(\mathrm{t2}^2\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)^2\right) - \\
& \quad (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - \\
& \quad 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + \\
& \quad 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - \\
& \quad 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) \big/ \\
& \quad \left(\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)\right) + \\
& \quad (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - \\
& \quad 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + \\
& \quad 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - \\
& \quad 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) \big/ \\
& \quad \left(3\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\right) + \\
& \quad \Big(2^{1/3}\,\big(48\,\mathrm{rmsm}^4\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\,\mathrm{tm}^2 \\
& \qquad x^4 + (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4 \\
& \qquad \mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + \\
& \qquad 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - \\
& \qquad 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)^2 - \\
& \qquad 24\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - \\
& \qquad 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \\
& \qquad \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2) \\
& \qquad (2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 - 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \\
& \qquad \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)\big)\Big) \Big/ \\
& \quad \Big(3\,\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)\,\big(432\,\mathrm{rmsm}^4\,\mathrm{tm}^2 \\
& \qquad x^4\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2 \\
& \qquad \mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \\
& \qquad \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)^2 - 288\,\mathrm{rmsm}^4 \\
& \qquad (\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6) \\
& \qquad \mathrm{tm}^2\,x^4\,(4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - \\
& \qquad 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 -
\end{aligned}
$$

$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 +$$
$$t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4) + 2\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2$$
$$tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 +$$
$$4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 +$$
$$8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$
$$8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^3 - 72$$
$$(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm -$$
$$4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm -$$
$$dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)$$
$$(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$
$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 +$$
$$t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)\,(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -$$
$$2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4) + 432$$
$$(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)$$
$$(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 +$$
$$rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4)^2 + \sqrt{}\,(-4\,(48\,rmsm^4\,(dt^4\,t2^2 -$$
$$2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)\,tm^2\,x^4 +$$
$$(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4$$
$$tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2$$
$$rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm$$
$$x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^2 - 24$$
$$(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2$$
$$tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5$$
$$tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1$$
$$t2^3\,x^2 - t2^4\,x^2)\,(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -$$
$$2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4))^3 +$$
$$(432\,rmsm^4\,tm^2\,x^4\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2$$
$$tm + 2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2$$
$$t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 +$$
$$dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)^2 - 288\,rmsm^4$$
$$(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 +$$
$$t2^6)\,tm^2\,x^4\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 +$$
$$4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 +$$
$$4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 +$$
$$8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2$$

$$tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4) +$$
$$2\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4$$
$$tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2$$
$$rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2$$
$$tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^3 -$$
$$72\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm +$$
$$2\,rmsm^2\,t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm +$$
$$2\,rmsm^2\,t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 +$$
$$t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)\,(4\,dt^4\,rmsm^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - 8\,dt^2\,rmsm^4\,t2^2\,tm^2 -$$
$$8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 +$$
$$4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2$$
$$tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 +$$
$$2\,t1\,t2\,x^4 + t2^2\,x^4)\,(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 -$$
$$2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4) + 432$$
$$(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)$$
$$(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 +$$
$$rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4)^2)^2))^{1/3}) +$$
$$(432\,rmsm^4\,tm^2\,x^4\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2$$
$$t1^4\,t2\,tm - 4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm +$$
$$2\,rmsm^2\,t2^5\,tm - dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 +$$
$$t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)^2 - 288\,rmsm^4$$
$$(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)$$
$$tm^2\,x^4\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$
$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$
$$8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4) +$$
$$2\,(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$
$$4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + 4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 +$$
$$8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - 4\,rmsm^2\,t1\,t2^2\,tm\,x^2 -$$
$$8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4)^3 -$$
$$72\,(2\,dt^4\,rmsm^2\,t2\,tm - 4\,dt^2\,rmsm^2\,t1^2\,t2\,tm + 2\,rmsm^2\,t1^4\,t2\,tm -$$
$$4\,dt^2\,rmsm^2\,t2^3\,tm - 4\,rmsm^2\,t1^2\,t2^3\,tm + 2\,rmsm^2\,t2^5\,tm -$$
$$dt^2\,t1\,t2\,x^2 + t1^3\,t2\,x^2 + dt^2\,t2^2\,x^2 + t1^2\,t2^2\,x^2 - t1\,t2^3\,x^2 - t2^4\,x^2)$$
$$(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -$$
$$8\,dt^2\,rmsm^4\,t2^2\,tm^2 - 8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 -$$

$$4 dt^2 rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 tm x^2 + 8 dt^2 rmsm^2 t2 tm x^2 +$$
$$8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2 tm x^2 - 8 rmsm^2 t2^3 tm x^2 +$$
$$t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4) (2 dt^2 rmsm^4 tm^2 x^2 + 2 rmsm^4 t1^2 tm^2 x^2 -$$
$$2 rmsm^4 t2^2 tm^2 x^2 + rmsm^2 t1 tm x^4 + rmsm^2 t2 tm x^4) +$$
$$432 (dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + t1^4 t2^2 - 2 dt^2 t2^4 - 2 t1^2 t2^4 + t2^6)$$
$$(2 dt^2 rmsm^4 tm^2 x^2 + 2 rmsm^4 t1^2 tm^2 x^2 -$$
$$2 rmsm^4 t2^2 tm^2 x^2 + rmsm^2 t1 tm x^4 + rmsm^2 t2 tm x^4)^2 +$$
$$\surd(-4 (48 rmsm^4 (dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + t1^4 t2^2 - 2 dt^2 t2^4 - 2 t1^2$$
$$t2^4 + t2^6) tm^2 x^4 + (4 dt^4 rmsm^4 tm^2 - 8 dt^2 rmsm^4 t1^2 tm^2 +$$
$$4 rmsm^4 t1^4 tm^2 - 8 dt^2 rmsm^4 t2^2 tm^2 - 8 rmsm^4 t1^2 t2^2 tm^2 +$$
$$4 rmsm^4 t2^4 tm^2 - 4 dt^2 rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 tm x^2 +$$
$$8 dt^2 rmsm^2 t2 tm x^2 + 8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2$$
$$tm x^2 - 8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4)^2 -$$
$$24 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 t1^2 t2 tm + 2 rmsm^2$$
$$t1^4 t2 tm - 4 dt^2 rmsm^2 t2^3 tm - 4 rmsm^2 t1^2 t2^3 tm + 2 rmsm^2$$
$$t2^5 tm - dt^2 t1 t2 x^2 + t1^3 t2 x^2 + dt^2 t2^2 x^2 + t1^2 t2^2 x^2 -$$
$$t1 t2^3 x^2 - t2^4 x^2) (2 dt^2 rmsm^4 tm^2 x^2 + 2 rmsm^4 t1^2 tm^2 x^2 -$$
$$2 rmsm^4 t2^2 tm^2 x^2 + rmsm^2 t1 tm x^4 + rmsm^2 t2 tm x^4))^3 +$$
$$(432 rmsm^4 tm^2 x^4 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 t1^2 t2 tm +$$
$$2 rmsm^2 t1^4 t2 tm - 4 dt^2 rmsm^2 t2^3 tm -$$
$$4 rmsm^2 t1^2 t2^3 tm + 2 rmsm^2 t2^5 tm - dt^2 t1 t2 x^2 +$$
$$t1^3 t2 x^2 + dt^2 t2^2 x^2 + t1^2 t2^2 x^2 - t1 t2^3 x^2 - t2^4 x^2)^2 -$$
$$288 rmsm^4 (dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + t1^4 t2^2 - 2 dt^2 t2^4 -$$
$$2 t1^2 t2^4 + t2^6) tm^2 x^4 (4 dt^4 rmsm^4 tm^2 - 8 dt^2 rmsm^4 t1^2 tm^2 +$$
$$4 rmsm^4 t1^4 tm^2 - 8 dt^2 rmsm^4 t2^2 tm^2 - 8 rmsm^4 t1^2 t2^2 tm^2 +$$
$$4 rmsm^4 t2^4 tm^2 - 4 dt^2 rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 tm x^2 +$$
$$8 dt^2 rmsm^2 t2 tm x^2 + 8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2$$
$$tm x^2 - 8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4) +$$
$$2 (4 dt^4 rmsm^4 tm^2 - 8 dt^2 rmsm^4 t1^2 tm^2 + 4 rmsm^4 t1^4 tm^2 -$$
$$8 dt^2 rmsm^4 t2^2 tm^2 - 8 rmsm^4 t1^2 t2^2 tm^2 + 4 rmsm^4 t2^4 tm^2 -$$
$$4 dt^2 rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 tm x^2 + 8 dt^2 rmsm^2$$
$$t2 tm x^2 + 8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2 tm x^2 -$$
$$8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4)^3 -$$
$$72 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 t1^2 t2 tm + 2 rmsm^2$$
$$t1^4 t2 tm - 4 dt^2 rmsm^2 t2^3 tm - 4 rmsm^2 t1^2 t2^3 tm +$$
$$2 rmsm^2 t2^5 tm - dt^2 t1 t2 x^2 + t1^3 t2 x^2 + dt^2 t2^2 x^2 +$$
$$t1^2 t2^2 x^2 - t1 t2^3 x^2 - t2^4 x^2) (4 dt^4 rmsm^4 tm^2 -$$
$$8 dt^2 rmsm^4 t1^2 tm^2 + 4 rmsm^4 t1^4 tm^2 - 8 dt^2 rmsm^4 t2^2 tm^2 -$$

$$
\begin{gathered}
8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - 4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + \\
4\,rmsm^2\,t1^3\,tm\,x^2 + 8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + 8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - \\
4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + \\
2\,t1\,t2\,x^4 + t2^2\,x^4)\ (2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - \\
2\,rmsm^4\,t2^2\,tm^2\,x^2 + rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4) + 432 \\
(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6) \\
(2\,dt^2\,rmsm^4\,tm^2\,x^2 + 2\,rmsm^4\,t1^2\,tm^2\,x^2 - 2\,rmsm^4\,t2^2\,tm^2\,x^2 + \\
rmsm^2\,t1\,tm\,x^4 + rmsm^2\,t2\,tm\,x^4)^2)^2))^{1/3} \Big/ \\
(3 \times 2^{1/3}\,t2^2\,(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4))) - \\
\frac{1}{2}\sqrt{((2\,(2\,dt^2\,rmsm^2\,tm - 2\,rmsm^2\,t1^2\,tm + 4\,rmsm^2\,t1\,t2\,tm - 2} \\
rmsm^2\,t2^2\,tm - t1\,x^2 + t2\,x^2)^2)^2) \Big/ \\
(t2^2\,(dt^2 - t1^2 + 2\,t1\,t2 - t2^2)^2) - (4\,dt^4\,rmsm^4\,tm^2 - \\
8\,dt^2\,rmsm^4\,t1^2\,tm^2 + \\
4\,rmsm^4\,t1^4\,tm^2 - \\
8\,dt^2\,rmsm^4\,t2^2\,tm^2 - \\
8\,rmsm^4\,t1^2\,t2^2\,tm^2 + \\
4\,rmsm^4\,t2^4\,tm^2 - \\
4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + \\
4\,rmsm^2\,t1^3\,tm\,x^2 + \\
8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + \\
8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - \\
4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - \\
8\,rmsm^2\,t2^3\,tm\,x^2 + t1^2\,x^4 + \\
2\,t1\,t2\,x^4 + t2^2\,x^4) \Big/ \\
(t2^2\,(dt^4 - 2\,dt^2\,t1^2 + t1^4 - 2\,dt^2\,t2^2 - 2\,t1^2\,t2^2 + t2^4)) - \\
(4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 - \\
8\,dt^2\,rmsm^4\,t2^2\,tm^2 - \\
8\,rmsm^4\,t1^2\,t2^2\,tm^2 + 4\,rmsm^4\,t2^4\,tm^2 - \\
4\,dt^2\,rmsm^2\,t1\,tm\,x^2 + \\
4\,rmsm^2\,t1^3\,tm\,x^2 + \\
8\,dt^2\,rmsm^2\,t2\,tm\,x^2 + \\
8\,rmsm^2\,t1^2\,t2\,tm\,x^2 - \\
4\,rmsm^2\,t1\,t2^2\,tm\,x^2 - 8\,rmsm^2\,t2^3\,tm\,x^2 + \\
t1^2\,x^4 + 2\,t1\,t2\,x^4 + t2^2\,x^4) \Big/ \\
(3\,(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6)) - \\
(2^{1/3}\,(48\,rmsm^4\,(dt^4\,t2^2 - 2\,dt^2\,t1^2\,t2^2 + t1^4\,t2^2 - 2\,dt^2\,t2^4 - 2\,t1^2\,t2^4 + t2^6) \\
tm^2\,x^4 + (4\,dt^4\,rmsm^4\,tm^2 - 8\,dt^2\,rmsm^4\,t1^2\,tm^2 + 4\,rmsm^4\,t1^4\,tm^2 -
\end{gathered}
$$

$$8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 -$$
$$4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 -$$
$$8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)^2 -$$
$$24\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm + 2\ rmsm^2\ t1^4\ t2\ tm -$$
$$4\ dt^2\ rmsm^2\ t2^3\ tm - 4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm -$$
$$dt^2\ t1\ t2\ x^2 + t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)$$
$$(2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2\ tm^2\ x^2 - 2\ rmsm^4\ t2^2\ tm^2\ x^2 +$$
$$rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4)))\Big/$$
$$(3\ t2^2\ (dt^4 - 2\ dt^2\ t1^2 + t1^4 - 2\ dt^2\ t2^2 - 2\ t1^2\ t2^2 + t2^4)$$
$$(432\ rmsm^4\ tm^2\ x^4\ (2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm +$$
$$2\ rmsm^2\ t1^4\ t2\ tm - 4\ dt^2\ rmsm^2\ t2^3\ tm - 4\ rmsm^2\ t1^2\ t2^3\ tm +$$
$$2\ rmsm^2\ t2^5\ tm - dt^2\ t1\ t2\ x^2 + t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 +$$
$$t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)^2 - 288\ rmsm^4$$
$$(dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6)$$
$$tm^2\ x^4\ (4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 -$$
$$8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 -$$
$$4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 -$$
$$8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4) + 2$$
$$(4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 -$$
$$8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 -$$
$$4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 -$$
$$8\ rmsm^2\ t2^3\ tm\ x^2 + t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)^3 - 72$$
$$(2\ dt^4\ rmsm^2\ t2\ tm - 4\ dt^2\ rmsm^2\ t1^2\ t2\ tm + 2\ rmsm^2\ t1^4\ t2\ tm -$$
$$4\ dt^2\ rmsm^2\ t2^3\ tm - 4\ rmsm^2\ t1^2\ t2^3\ tm + 2\ rmsm^2\ t2^5\ tm -$$
$$dt^2\ t1\ t2\ x^2 + t1^3\ t2\ x^2 + dt^2\ t2^2\ x^2 + t1^2\ t2^2\ x^2 - t1\ t2^3\ x^2 - t2^4\ x^2)$$
$$(4\ dt^4\ rmsm^4\ tm^2 - 8\ dt^2\ rmsm^4\ t1^2\ tm^2 + 4\ rmsm^4\ t1^4\ tm^2 -$$
$$8\ dt^2\ rmsm^4\ t2^2\ tm^2 - 8\ rmsm^4\ t1^2\ t2^2\ tm^2 + 4\ rmsm^4\ t2^4\ tm^2 -$$
$$4\ dt^2\ rmsm^2\ t1\ tm\ x^2 + 4\ rmsm^2\ t1^3\ tm\ x^2 + 8\ dt^2\ rmsm^2\ t2\ tm\ x^2 +$$
$$8\ rmsm^2\ t1^2\ t2\ tm\ x^2 - 4\ rmsm^2\ t1\ t2^2\ tm\ x^2 - 8\ rmsm^2\ t2^3\ tm\ x^2 +$$
$$t1^2\ x^4 + 2\ t1\ t2\ x^4 + t2^2\ x^4)\ (2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2\ tm^2\ x^2 -$$
$$2\ rmsm^4\ t2^2\ tm^2\ x^2 + rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4) + 432$$
$$(dt^4\ t2^2 - 2\ dt^2\ t1^2\ t2^2 + t1^4\ t2^2 - 2\ dt^2\ t2^4 - 2\ t1^2\ t2^4 + t2^6)$$
$$(2\ dt^2\ rmsm^4\ tm^2\ x^2 + 2\ rmsm^4\ t1^2\ tm^2\ x^2 -$$
$$2\ rmsm^4\ t2^2\ tm^2\ x^2 + rmsm^2\ t1\ tm\ x^4 + rmsm^2\ t2\ tm\ x^4)^2 +$$

$$\surd\Big(-4\,\Big(48\, rmsm^4\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 -$$
$$2\, t1^2\, t2^4 + t2^6)\, tm^2\, x^4 + (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2$$
$$tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4$$
$$t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2$$
$$tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 +$$
$$2\, t1\, t2\, x^4 + t2^2\, x^4)^2 - 24\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2$$
$$rmsm^2\, t1^2\, t2\, tm + 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm -$$
$$4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 +$$
$$t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4$$
$$t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4)\Big)^3 +$$
$$\Big(432\, rmsm^4\, tm^2\, x^4\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2$$
$$tm + 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2$$
$$t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 +$$
$$dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)^2 - 288\, rmsm^4$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 +$$
$$t2^6)\, tm^2\, x^4\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 +$$
$$4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 +$$
$$4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 +$$
$$8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2$$
$$tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) +$$
$$2\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4$$
$$tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2$$
$$rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2$$
$$tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 -$$
$$72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm +$$
$$2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm +$$
$$2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 +$$
$$t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)\, (4\, dt^4\, rmsm^4\, tm^2 -$$
$$8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 -$$
$$8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 +$$
$$4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2$$
$$tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 +$$
$$2\, t1\, t2\, x^4 + t2^2\, x^4)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 -$$
$$2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) + 432$$
$$(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)$$
$$(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 +$$

$$\mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^4 + \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^4)^2)^2)))^{1/3}) -$$

$$(432\ \mathrm{rmsm}^4\ \mathrm{tm}^2\ x^4\ (2\ \mathrm{dt}^4\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm} - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm} + 2\ \mathrm{rmsm}^2$$

$$\mathrm{t1}^4\ \mathrm{t2}\ \mathrm{tm} - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm} - 4\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}^3\ \mathrm{tm} +$$

$$2\ \mathrm{rmsm}^2\ \mathrm{t2}^5\ \mathrm{tm} - \mathrm{dt}^2\ \mathrm{t1}\ \mathrm{t2}\ x^2 + \mathrm{t1}^3\ \mathrm{t2}\ x^2 + \mathrm{dt}^2\ \mathrm{t2}^2\ x^2 +$$

$$\mathrm{t1}^2\ \mathrm{t2}^2\ x^2 - \mathrm{t1}\ \mathrm{t2}^3\ x^2 - \mathrm{t2}^4\ x^2)^2 - 288\ \mathrm{rmsm}^4$$

$$(\mathrm{dt}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t1}^2\ \mathrm{t2}^2 + \mathrm{t1}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t2}^4 - 2\ \mathrm{t1}^2\ \mathrm{t2}^4 + \mathrm{t2}^6)$$

$$\mathrm{tm}^2\ x^4\ (4\ \mathrm{dt}^4\ \mathrm{rmsm}^4\ \mathrm{tm}^2 - 8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t1}^4\ \mathrm{tm}^2 -$$

$$8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2 - 8\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{t2}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t2}^4\ \mathrm{tm}^2 -$$

$$4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^2 + 4\ \mathrm{rmsm}^2\ \mathrm{t1}^3\ \mathrm{tm}\ x^2 + 8\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 +$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 - 4\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{t2}^2\ \mathrm{tm}\ x^2 -$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm}\ x^2 + \mathrm{t1}^2\ x^4 + 2\ \mathrm{t1}\ \mathrm{t2}\ x^4 + \mathrm{t2}^2\ x^4) +$$

$$2\ (4\ \mathrm{dt}^4\ \mathrm{rmsm}^4\ \mathrm{tm}^2 - 8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t1}^4\ \mathrm{tm}^2 -$$

$$8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2 - 8\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{t2}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t2}^4\ \mathrm{tm}^2 -$$

$$4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^2 + 4\ \mathrm{rmsm}^2\ \mathrm{t1}^3\ \mathrm{tm}\ x^2 + 8\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 +$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 - 4\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{t2}^2\ \mathrm{tm}\ x^2 -$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm}\ x^2 + \mathrm{t1}^2\ x^4 + 2\ \mathrm{t1}\ \mathrm{t2}\ x^4 + \mathrm{t2}^2\ x^4)^3 -$$

$$72\ (2\ \mathrm{dt}^4\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm} - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm} + 2\ \mathrm{rmsm}^2\ \mathrm{t1}^4\ \mathrm{t2}\ \mathrm{tm} -$$

$$4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm} - 4\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}^3\ \mathrm{tm} + 2\ \mathrm{rmsm}^2\ \mathrm{t2}^5\ \mathrm{tm} -$$

$$\mathrm{dt}^2\ \mathrm{t1}\ \mathrm{t2}\ x^2 + \mathrm{t1}^3\ \mathrm{t2}\ x^2 + \mathrm{dt}^2\ \mathrm{t2}^2\ x^2 + \mathrm{t1}^2\ \mathrm{t2}^2\ x^2 - \mathrm{t1}\ \mathrm{t2}^3\ x^2 - \mathrm{t2}^4\ x^2)$$

$$(4\ \mathrm{dt}^4\ \mathrm{rmsm}^4\ \mathrm{tm}^2 - 8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t1}^4\ \mathrm{tm}^2 -$$

$$8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2 - 8\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{t2}^2\ \mathrm{tm}^2 + 4\ \mathrm{rmsm}^4\ \mathrm{t2}^4\ \mathrm{tm}^2 -$$

$$4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^2 + 4\ \mathrm{rmsm}^2\ \mathrm{t1}^3\ \mathrm{tm}\ x^2 + 8\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 +$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 - 4\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{t2}^2\ \mathrm{tm}\ x^2 -$$

$$8\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm}\ x^2 + \mathrm{t1}^2\ x^4 + 2\ \mathrm{t1}\ \mathrm{t2}\ x^4 + \mathrm{t2}^2\ x^4)$$

$$(2\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{tm}^2\ x^2 + 2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2\ x^2 - 2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2\ x^2 +$$

$$\mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^4 + \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^4) +$$

$$432\ (\mathrm{dt}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t1}^2\ \mathrm{t2}^2 + \mathrm{t1}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t2}^4 - 2\ \mathrm{t1}^2\ \mathrm{t2}^4 + \mathrm{t2}^6)$$

$$(2\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{tm}^2\ x^2 + 2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2\ x^2 -$$

$$2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2\ x^2 + \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^4 + \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^4)^2 +$$

$$\sqrt{}\ (-4\ (48\ \mathrm{rmsm}^4\ (\mathrm{dt}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t1}^2\ \mathrm{t2}^2 + \mathrm{t1}^4\ \mathrm{t2}^2 - 2\ \mathrm{dt}^2\ \mathrm{t2}^4 - 2\ \mathrm{t1}^2$$

$$\mathrm{t2}^4 + \mathrm{t2}^6)\ \mathrm{tm}^2\ x^4 + (4\ \mathrm{dt}^4\ \mathrm{rmsm}^4\ \mathrm{tm}^2 - 8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{tm}^2 +$$

$$4\ \mathrm{rmsm}^4\ \mathrm{t1}^4\ \mathrm{tm}^2 - 8\ \mathrm{dt}^2\ \mathrm{rmsm}^4\ \mathrm{t2}^2\ \mathrm{tm}^2 - 8\ \mathrm{rmsm}^4\ \mathrm{t1}^2\ \mathrm{t2}^2\ \mathrm{tm}^2 +$$

$$4\ \mathrm{rmsm}^4\ \mathrm{t2}^4\ \mathrm{tm}^2 - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{tm}\ x^2 + 4\ \mathrm{rmsm}^2\ \mathrm{t1}^3\ \mathrm{tm}\ x^2 +$$

$$8\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 + 8\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm}\ x^2 - 4\ \mathrm{rmsm}^2\ \mathrm{t1}\ \mathrm{t2}^2$$

$$\mathrm{tm}\ x^2 - 8\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm}\ x^2 + \mathrm{t1}^2\ x^4 + 2\ \mathrm{t1}\ \mathrm{t2}\ x^4 + \mathrm{t2}^2\ x^4)^2 -$$

$$24\ (2\ \mathrm{dt}^4\ \mathrm{rmsm}^2\ \mathrm{t2}\ \mathrm{tm} - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}\ \mathrm{tm} + 2\ \mathrm{rmsm}^2$$

$$\mathrm{t1}^4\ \mathrm{t2}\ \mathrm{tm} - 4\ \mathrm{dt}^2\ \mathrm{rmsm}^2\ \mathrm{t2}^3\ \mathrm{tm} - 4\ \mathrm{rmsm}^2\ \mathrm{t1}^2\ \mathrm{t2}^3\ \mathrm{tm} + 2\ \mathrm{rmsm}^2$$

$$\mathrm{t2}^5\ \mathrm{tm} - \mathrm{dt}^2\ \mathrm{t1}\ \mathrm{t2}\ x^2 + \mathrm{t1}^3\ \mathrm{t2}\ x^2 + \mathrm{dt}^2\ \mathrm{t2}^2\ x^2 + \mathrm{t1}^2\ \mathrm{t2}^2\ x^2 -$$

$$\mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)\ (2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 -$$
$$2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4))^3 +$$
$$(432\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^4\ (2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} +$$
$$2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} -$$
$$4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 +$$
$$\mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)^2 -$$
$$288\,\mathrm{rmsm}^4\ (\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 -$$
$$2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\ \mathrm{tm}^2\,x^4\ (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 +$$
$$4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 +$$
$$4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 +$$
$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2$$
$$\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) +$$
$$2\ (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 -$$
$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 -$$
$$4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2$$
$$\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 -$$
$$8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)^3 -$$
$$72\ (2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2$$
$$\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} +$$
$$2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 +$$
$$\mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)\ (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 -$$
$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 -$$
$$8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 +$$
$$4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 -$$
$$4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 +$$
$$2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)\ (2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 -$$
$$2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4) + 432$$
$$(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)$$
$$(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 - 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 +$$
$$\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)^2)^2))^{1/3} \Big/$$
$$(3 \times 2^{1/3}\,\mathrm{t2}^2\ (\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)) +$$
$$((8\ (2\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{tm} + 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t2}^2\,\mathrm{tm} -$$
$$\mathrm{t1}\,x^2 + \mathrm{t2}\,x^2)^3) \Big/ (\mathrm{t2}^3\ (\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)^3) -$$
$$(8\ (2\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{tm} + 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t2}^2\,\mathrm{tm} -$$
$$\mathrm{t1}\,x^2 + \mathrm{t2}\,x^2)\ (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 +$$
$$4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 +$$
$$4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 +$$
$$8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 -$$

$$
\begin{aligned}
&8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)) \Big/ \\
&\big(\mathrm{t2}^3\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - \\
&\quad 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)\big) + \\
&\big(32\,(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 - 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \\
&\quad \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)\big) \Big/ \\
&\big(\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)\big)\Big)\Big) \Big/ \\
&\Bigg(4\sqrt{\Big(\big((2\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{tm} + 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}\,\mathrm{tm} - 2\,\mathrm{rmsm}^2\,\mathrm{t2}^2\,\mathrm{tm} -} \\
&\quad \mathrm{t1}\,x^2 + \mathrm{t2}\,x^2)^2 \Big/ \big(\mathrm{t2}^2\,(\mathrm{dt}^2 - \mathrm{t1}^2 + 2\,\mathrm{t1}\,\mathrm{t2} - \mathrm{t2}^2)^2\big) - \\
&\quad (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - \\
&\quad\quad 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - \\
&\quad\quad 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + \\
&\quad\quad 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - \\
&\quad\quad 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) \Big/ \\
&\quad\ \big(\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4)\big) + \\
&\quad (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - \\
&\quad\quad 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - \\
&\quad\quad 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + \\
&\quad\quad 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2\,\mathrm{tm}\,x^2 - \\
&\quad\quad 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4) \Big/ \\
&\quad\ \big(3\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\big) + \\
&\quad \Big(2^{1/3}\,\big(48\,\mathrm{rmsm}^4\,(\mathrm{dt}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t1}^4\,\mathrm{t2}^2 - 2\,\mathrm{dt}^2\,\mathrm{t2}^4 - \\
&\quad\quad\quad 2\,\mathrm{t1}^2\,\mathrm{t2}^4 + \mathrm{t2}^6)\,\mathrm{tm}^2\,x^4 + (4\,\mathrm{dt}^4\,\mathrm{rmsm}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2 + \\
&\quad\quad\quad 4\,\mathrm{rmsm}^4\,\mathrm{t1}^4\,\mathrm{tm}^2 - 8\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2 - 8\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{t2}^2\,\mathrm{tm}^2 + \\
&\quad\quad\quad 4\,\mathrm{rmsm}^4\,\mathrm{t2}^4\,\mathrm{tm}^2 - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^2 + 4\,\mathrm{rmsm}^2\,\mathrm{t1}^3\,\mathrm{tm}\,x^2 + \\
&\quad\quad\quad 8\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 + 8\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm}\,x^2 - 4\,\mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{t2}^2 \\
&\quad\quad\quad\ \mathrm{tm}\,x^2 - 8\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm}\,x^2 + \mathrm{t1}^2\,x^4 + 2\,\mathrm{t1}\,\mathrm{t2}\,x^4 + \mathrm{t2}^2\,x^4)^2 - \\
&\quad\quad 24\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + 2\,\mathrm{rmsm}^2 \\
&\quad\quad\quad\ \mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2 \\
&\quad\quad\quad\ \mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 + \mathrm{dt}^2\,\mathrm{t2}^2\,x^2 + \mathrm{t1}^2\,\mathrm{t2}^2\,x^2 - \\
&\quad\quad\quad \mathrm{t1}\,\mathrm{t2}^3\,x^2 - \mathrm{t2}^4\,x^2)\,(2\,\mathrm{dt}^2\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^2 + 2\,\mathrm{rmsm}^4\,\mathrm{t1}^2\,\mathrm{tm}^2\,x^2 - \\
&\quad\quad\quad 2\,\mathrm{rmsm}^4\,\mathrm{t2}^2\,\mathrm{tm}^2\,x^2 + \mathrm{rmsm}^2\,\mathrm{t1}\,\mathrm{tm}\,x^4 + \mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm}\,x^4)\big)\Big) \Big/ \\
&\quad \Big(3\,\mathrm{t2}^2\,(\mathrm{dt}^4 - 2\,\mathrm{dt}^2\,\mathrm{t1}^2 + \mathrm{t1}^4 - 2\,\mathrm{dt}^2\,\mathrm{t2}^2 - 2\,\mathrm{t1}^2\,\mathrm{t2}^2 + \mathrm{t2}^4) \\
&\quad\quad \Big(432\,\mathrm{rmsm}^4\,\mathrm{tm}^2\,x^4\,(2\,\mathrm{dt}^4\,\mathrm{rmsm}^2\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t1}^2\,\mathrm{t2}\,\mathrm{tm} + \\
&\quad\quad\quad\quad 2\,\mathrm{rmsm}^2\,\mathrm{t1}^4\,\mathrm{t2}\,\mathrm{tm} - 4\,\mathrm{dt}^2\,\mathrm{rmsm}^2\,\mathrm{t2}^3\,\mathrm{tm} - 4\,\mathrm{rmsm}^2\,\mathrm{t1}^2 \\
&\quad\quad\quad\quad\ \mathrm{t2}^3\,\mathrm{tm} + 2\,\mathrm{rmsm}^2\,\mathrm{t2}^5\,\mathrm{tm} - \mathrm{dt}^2\,\mathrm{t1}\,\mathrm{t2}\,x^2 + \mathrm{t1}^3\,\mathrm{t2}\,x^2 +
\end{aligned}
$$

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\begin{aligned}
&\quad dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2\big)^2 - 288\, rmsm^4 \\
&\big(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6\big) \\
&\; tm^2\, x^4\, \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&\quad 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2 \\
&\quad\; t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big) + \\
&2\, \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + \\
&\quad 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + \\
&\quad 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2 \\
&\quad\; tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big)^3 - \\
&72\, \big(2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + 2\, rmsm^2 \\
&\quad\; t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm + \\
&\quad 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + \\
&\quad t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2\big)\, \big(4\, dt^4\, rmsm^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - \\
&\quad 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + \\
&\quad 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2 \\
&\quad\; tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + \\
&\quad 2\, t1\, t2\, x^4 + t2^2\, x^4\big)\, \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - \\
&\quad 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big) + \\
&432\, \big(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - \\
&\quad 2\, t1^2\, t2^4 + t2^6\big)\, \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - \\
&\quad\; 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big)^2 + \\
&\sqrt{\Big(-4\, \Big(48\, rmsm^4\, \big(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - } \\
&\qquad\quad 2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6\big)\, tm^2\, x^4 + \\
&\qquad \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + \\
&\qquad\quad 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4 \\
&\qquad\quad\; t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1 \\
&\qquad\quad\; tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&\qquad\quad 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\qquad\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big)^2 - \\
&\qquad 24\, \big(2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + \\
&\qquad\quad 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2 \\
&\qquad\quad\; t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2 \\
&\qquad\quad\; x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2\big) \\
&\qquad \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4 \\
&\qquad\quad\; t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big)\Big)^3 +
\end{aligned}
$$

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\begin{gathered}
(432 rmsm^4 tm^2 x^4 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 \\
t1^2 t2 tm + 2 rmsm^2 t1^4 t2 tm - 4 dt^2 rmsm^2 \\
t2^3 tm - 4 rmsm^2 t1^2 t2^3 tm + 2 rmsm^2 t2^5 tm - \\
dt^2 t1 t2 x^2 + t1^3 t2 x^2 + dt^2 t2^2 x^2 + \\
t1^2 t2^2 x^2 - t1 t2^3 x^2 - t2^4 x^2)^2 - 288 rmsm^4 \\
(dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + t1^4 t2^2 - 2 dt^2 t2^4 - \\
2 t1^2 t2^4 + t2^6) tm^2 x^4 (4 dt^4 rmsm^4 tm^2 - 8 dt^2 \\
rmsm^4 t1^2 tm^2 + 4 rmsm^4 t1^4 tm^2 - 8 dt^2 rmsm^4 t2^2 \\
tm^2 - 8 rmsm^4 t1^2 t2^2 tm^2 + 4 rmsm^4 t2^4 tm^2 - 4 dt^2 \\
rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 tm x^2 + 8 dt^2 rmsm^2 t2 \\
tm x^2 + 8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2 tm x^2 - \\
8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4) + \\
2 (4 dt^4 rmsm^4 tm^2 - 8 dt^2 rmsm^4 t1^2 tm^2 + \\
4 rmsm^4 t1^4 tm^2 - 8 dt^2 rmsm^4 t2^2 tm^2 - 8 rmsm^4 \\
t1^2 t2^2 tm^2 + 4 rmsm^4 t2^4 tm^2 - 4 dt^2 rmsm^2 t1 \\
tm x^2 + 4 rmsm^2 t1^3 tm x^2 + 8 dt^2 rmsm^2 t2 tm x^2 + \\
8 rmsm^2 t1^2 t2 tm x^2 - 4 rmsm^2 t1 t2^2 tm x^2 - \\
8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + 2 t1 t2 x^4 + t2^2 x^4)^3 - \\
72 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 t1^2 t2 tm + \\
2 rmsm^2 t1^4 t2 tm - 4 dt^2 rmsm^2 t2^3 tm - 4 rmsm^2 t1^2 \\
t2^3 tm + 2 rmsm^2 t2^5 tm - dt^2 t1 t2 x^2 + t1^3 t2 x^2 + \\
dt^2 t2^2 x^2 + t1^2 t2^2 x^2 - t1 t2^3 x^2 - t2^4 x^2) \\
(4 dt^4 rmsm^4 tm^2 - 8 dt^2 rmsm^4 t1^2 tm^2 + 4 rmsm^4 t1^4 \\
tm^2 - 8 dt^2 rmsm^4 t2^2 tm^2 - 8 rmsm^4 t1^2 t2^2 tm^2 + 4 \\
rmsm^4 t2^4 tm^2 - 4 dt^2 rmsm^2 t1 tm x^2 + 4 rmsm^2 t1^3 \\
tm x^2 + 8 dt^2 rmsm^2 t2 tm x^2 + 8 rmsm^2 t1^2 t2 tm x^2 - \\
4 rmsm^2 t1 t2^2 tm x^2 - 8 rmsm^2 t2^3 tm x^2 + t1^2 x^4 + \\
2 t1 t2 x^4 + t2^2 x^4) (2 dt^2 rmsm^4 tm^2 x^2 + 2 rmsm^4 \\
t1^2 tm^2 x^2 - 2 rmsm^4 t2^2 tm^2 x^2 + rmsm^2 t1 tm x^4 + \\
rmsm^2 t2 tm x^4) + 432 (dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + \\
t1^4 t2^2 - 2 dt^2 t2^4 - 2 t1^2 t2^4 + t2^6) (2 dt^2 rmsm^4 \\
tm^2 x^2 + 2 rmsm^4 t1^2 tm^2 x^2 - 2 rmsm^4 t2^2 tm^2 x^2 + \\
rmsm^2 t1 tm x^4 + rmsm^2 t2 tm x^4)^2)^2))^{1/3}) + \\
(432 rmsm^4 tm^2 x^4 (2 dt^4 rmsm^2 t2 tm - 4 dt^2 rmsm^2 t1^2 t2 tm + \\
2 rmsm^2 t1^4 t2 tm - 4 dt^2 rmsm^2 t2^3 tm - 4 rmsm^2 t1^2 t2^3 tm + \\
2 rmsm^2 t2^5 tm - dt^2 t1 t2 x^2 + t1^3 t2 x^2 + dt^2 t2^2 x^2 + \\
t1^2 t2^2 x^2 - t1 t2^3 x^2 - t2^4 x^2)^2 - 288 rmsm^4 \\
(dt^4 t2^2 - 2 dt^2 t1^2 t2^2 + t1^4 t2^2 - 2 dt^2 t2^4 - 2 t1^2 t2^4 + t2^6)
\end{gathered}
$$

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\begin{aligned}
&tm^2\, x^4 \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&\quad 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&\quad 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big) + \\
&2 \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&\quad 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2 \\
&\quad t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\quad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big)^3 - \\
&72 \big(2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + 2\, rmsm^2\, t1^4\, t2\, tm - \\
&\quad 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1 \\
&\quad t2\, x^2 + t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2\big) \\
&\big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&\quad 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&\quad 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&\quad 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + \\
&\quad t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big) \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2 \\
&\quad tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big) + \\
&432 \big(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - \\
&\quad 2\, t1^2\, t2^4 + t2^6\big) \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - \\
&\quad 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big)^2 + \\
&\sqrt{\Big(-4 \Big(48\, rmsm^4 \big(dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - } \\
&\qquad 2\, t1^2\, t2^4 + t2^6\big)\, tm^2\, x^4 + \big(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2 \\
&\qquad rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2 \\
&\qquad tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2 \\
&\qquad rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2 \\
&\qquad tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&\qquad 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4\big)^2 - \\
&\qquad 24 \big(2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + \\
&\qquad 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - \\
&\qquad 4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + \\
&\qquad t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2\big) \\
&\qquad \big(2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - 2\, rmsm^4 \\
&\qquad t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4\big)\Big)^3 + \\
&\quad \Big(432\, rmsm^4\, tm^2\, x^4 \big(2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + \\
&\qquad 2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - 4\, rmsm^2\, t1^2 \\
&\qquad t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + t1^3\, t2\, x^2 +
\end{aligned}
$$

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\begin{aligned}
&dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2)^2 - \\
&288\, rmsm^4\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - \\
&2\, dt^2\, t2^4 - 2\, t1^2\, t2^4 + t2^6)\, tm^2\, x^4\, (4\, dt^4\, rmsm^4\, tm^2 - \\
&8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4 \\
&t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - \\
&4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2 \\
&t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4) + \\
&2\, (4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + \\
&4\, rmsm^4\, t1^4\, tm^2 - 8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2 \\
&t2^2\, tm^2 + 4\, rmsm^4\, t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + \\
&4\, rmsm^2\, t1^3\, tm\, x^2 + 8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + \\
&8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2\, t1\, t2^2\, tm\, x^2 - \\
&8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + t2^2\, x^4)^3 - \\
&72\, (2\, dt^4\, rmsm^2\, t2\, tm - 4\, dt^2\, rmsm^2\, t1^2\, t2\, tm + \\
&2\, rmsm^2\, t1^4\, t2\, tm - 4\, dt^2\, rmsm^2\, t2^3\, tm - \\
&4\, rmsm^2\, t1^2\, t2^3\, tm + 2\, rmsm^2\, t2^5\, tm - dt^2\, t1\, t2\, x^2 + \\
&t1^3\, t2\, x^2 + dt^2\, t2^2\, x^2 + t1^2\, t2^2\, x^2 - t1\, t2^3\, x^2 - t2^4\, x^2) \\
&(4\, dt^4\, rmsm^4\, tm^2 - 8\, dt^2\, rmsm^4\, t1^2\, tm^2 + 4\, rmsm^4\, t1^4\, tm^2 - \\
&8\, dt^2\, rmsm^4\, t2^2\, tm^2 - 8\, rmsm^4\, t1^2\, t2^2\, tm^2 + 4\, rmsm^4 \\
&t2^4\, tm^2 - 4\, dt^2\, rmsm^2\, t1\, tm\, x^2 + 4\, rmsm^2\, t1^3\, tm\, x^2 + \\
&8\, dt^2\, rmsm^2\, t2\, tm\, x^2 + 8\, rmsm^2\, t1^2\, t2\, tm\, x^2 - 4\, rmsm^2 \\
&t1\, t2^2\, tm\, x^2 - 8\, rmsm^2\, t2^3\, tm\, x^2 + t1^2\, x^4 + 2\, t1\, t2\, x^4 + \\
&t2^2\, x^4)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2\, tm^2\, x^2 - \\
&2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + rmsm^2\, t2\, tm\, x^4) + \\
&432\, (dt^4\, t2^2 - 2\, dt^2\, t1^2\, t2^2 + t1^4\, t2^2 - 2\, dt^2\, t2^4 - \\
&2\, t1^2\, t2^4 + t2^6)\, (2\, dt^2\, rmsm^4\, tm^2\, x^2 + 2\, rmsm^4\, t1^2 \\
&tm^2\, x^2 - 2\, rmsm^4\, t2^2\, tm^2\, x^2 + rmsm^2\, t1\, tm\, x^4 + \\
&rmsm^2\, t2\, tm\, x^4)^2)^2))^{1/3} \Big/ (3 \times 2^{1/3}\, t2^2 \\
&(dt^4 - 2\, dt^2\, t1^2 + t1^4 - 2\, dt^2\, t2^2 - 2\, t1^2\, t2^2 + t2^4)))))
\end{aligned}
$$

*Where* $t2 = t(0)_b$ *layer base time zero*, $t1 = t(0)_t$ *layer top time zero*, $tm = t(0)_m$ *layer midpoint time zero*, $dt = \Delta t(x)$, $x = offset$, $rmsm =$ *midpoint RMS velocity* , *rmsb =layer base RMS velocity*

## Appendix B

### Model B4 Undefined Result Discussion

Analysis of model B4 utilizing the parameters extracted from a single-trace returned an unrealistic RMS velocity pair, which in turn lead to a non-real interval velocity estimate. The origin of this error lies in the incorrect temporal layer-thickness derived from model B4's far offset trace, resulting from the effects of noise contamination in the spectral response leading to an inaccurate estimate of Pf(x). Examination of the input parameters detailed in table 4.6 reveals the far offset layer temporal thickness $\Delta t(x)$ (equation 3.8) is estimated to be greater than the layers near offset temporal thickness $\Delta t(0)$ (equation 3.3), this is unrealistic due to the flattening of moveout curves with depth which results in convergence of the top and base reflectors with offset and thus a reduction in temporal thickness with offset for an isovelocity layer. The effects of violating this condition cause the predicted interval velocity to fall below zero leading to an imaginary (non-real) result (Table A-B. 1).

| Model | RMS top | RMS base | Interval Velocity Estimated |
|---|---|---|---|
| B4 | 57551.8 | 2.03x10^13 | 8.21x10^5 **i** |

*Table A-B.1*

It is therefore recommended as an input parameter quality control measure that $\Delta t(0)$ is confirmed to be greater than $\Delta t(x)$ and the interval velocity estimate achieved corresponds to a real (non-imaginary) value. If either one of these conditions occur the accuracy of input parameters being utilized warrants investigation.