Reliability and Maintenance Modeling of Complex Systems Under Multiple Dependent Competing Failure Processes

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Presented to

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In Partial Fulfillment

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Doctor of Philosophy

in Industrial Engineering

By

Lei Jiang

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Reliability and Maintenance Modeling of Complex Systems Under Multiple Dependent Competing Failure Processes

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Abstract

To achieve commercialization and wide acceptance in industrial application, reliability analysis for complex evolving systems with multiple failure processes becomes increasingly important. The common assumption in analyzing reliability of such systems is that these multiple failure processes are independent, which may lead to the miscalculation of system reliability. To assist engineers with design, manufacturing and maintenance of complex systems, new reliability models that account for the dependence among multiple failure processes need to be developed to accurately predict the lifetime of these systems.

This research aims to develop probabilistic reliability models and analytical tools for systems with dependent competing failure processes, and explore cost-effective maintenance policies based on our reliability analysis. Different dependent patterns among competing failure processes are explored for single-component systems. When the arrival of external shocks diminishes the strength of material, we propose reliability and maintenance models for systems with a shifting, dependent hard failure threshold. When shocks impact the degradation process in different manners, we model zoned shock effects on stochastic degradation, and develop reliability functions for such dependent stochastic failure processes. Case studies of micro-electro-mechanical systems and stent devices are used to demonstrate our models, where Monte Carlo importance sampling is used to estimate system reliability.

We extend our models on single-component systems to a broader range of multi-component systems experiencing multiple failure processes, which presents more challenges on modeling the interaction and dependence among different components. A

new reliability model and a unique condition-based maintenance model are proposed for complex systems with dependent components subject to respective degradation processes, and the dependence among components is established through environmental factors. Another condition-based maintenance policy is developed for power transformers using Markov decision processes, where a power transformer with multiple components is modeled as a multi-state system.

The proposed reliability and maintenance models can be implemented to address the critical quality and reliability problems of evolving devices and many other systems with multiple dependent competing failure processes and multiple dependent components. The developed models and analytical tools can facilitate product design, manufacturing and maintenance, and enhance system reliability and availability.

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Chapter 1: Introduction

The rapid development of novel and evolving technologies brings a broad set of reliability challenges. Many aspects of product design and manufacturing are undergoing dramatic changes, which has a huge impact on product reliability. Even though the performance and complexity of products are increasing, customers have never lowered their expectation on product quality and reliability. The downscaling trend in the semiconductor industry greatly increases the complexity in design, manufacturing, and packaging. Reliability is always a critical factor that manufacturers cannot ignore, with an aim to increase their market share and make profit. As the market demand continues to push product performance to its technological limits, we need to find the tradeoff between product performance and lifetime according to the needs of different market segments.

To be most effective, reliability should be taken into account in the design stage of a product and consistently throughout the entire lifetime. Otherwise, companies or manufacturers will suffer a huge loss if any reliability problem happens to their products, both in profitability and reputation. Reliability has a great impact on the consumers' perception of a manufacturer. For example, consumers' experience with car recalls, repairs and warranties will affect the future sales of that manufacturer [1]. In February 2010, Toyota recalled 2.3 million vehicles, because they failed to test new cars and car parts under varying weather conditions, and the gas-pedal mechanism tended to stick as humidity increased. In October 2012, a new recall from Toyota covered more than 7.4 million cars and trucks worldwide, including 2.5 million in the U.S. The problem is a faulty power master switch that can become sticky. If the wrong kind of lubricant is used on the switch, it can be a fire risk. GM has recalled 1.3 million vehicles for the problem of the electric power-steering assist that can suddenly stop working, and 2.6 million small cars for an ignition switch defect since February 2014. These three cases demonstrate the importance of reliability analysis at the early stage of product development.

Among many quality characteristics, reliability is defined as “quality over time,” which is a critical dimension of quality [2]. Figure 1.1 shows the quality effort put in both reality and ideal cases during four stages of product life cycle: product development, design, manufacturing and assembly, and problem solving. Ideally, including quality at the early stage can tremendously reduce future effort on problem solving. However in reality, people usually ignore the importance of quality monitoring and only think to solve a problem when it occurs. In some cases, the problem can be relatively minor, such as a laptop battery causing fire and people suffering minor burns; while in other cases such as Toyota recall in 2010, the failure of gas pedal took life of several people tragically. Therefore, improving product quality and reliability should be a critical task to be implemented as early as possible during the product life cycle.

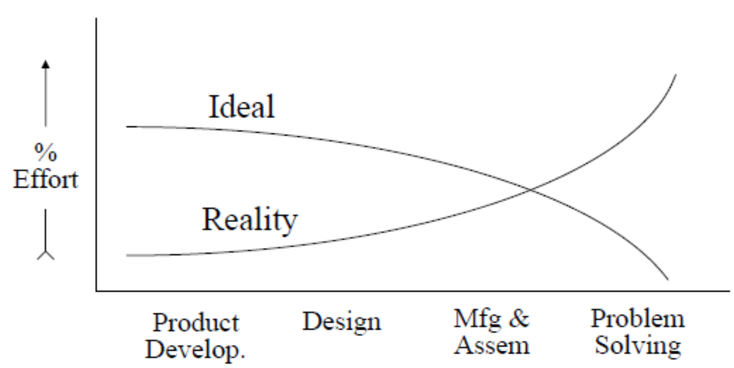

Figure 1.1: Quality effort in product life cycle

Reliability is one of the quality characteristics that consumers desire from the manufacturer of products. Nowadays global commercial competition forces manufacturers to produce products with high quality and reliability. However, reliability is not like other quality characteristics, such as geometric tolerances, weight, etc., which can be measured and controlled. It is a time-dependent quality characteristic and can only be predicted [3].

In traditional reliability study, failure-time-based reliability methods are implemented to predict the lifetime of devices, which is a straightforward way. In failure-time-based reliability approach, the distribution that fits the failure data needs to be found, which is then used to describe the lifetime of the device. To collect failure data, experiments are conducted to operate the devices till failure and record the failure time data. However, as devices become more reliable, it takes a long time to obtain failure data. In other cases, devices are very expensive, it is impossible or impractical to run failure test. Because of these challenging issues, degradation and shock based reliability analysis starts to attract a lot of attention. In this new approach, physics-of-failure mechanisms are investigated and combined into the reliability model. Degradation data is collected at different stages during the device operation and then analyzed to predict the overall degradation path and eventual failure time. It greatly reduces the time required to conduct the experiments, and no real failure is required to be recorded.

Based on these physics-of-failure mechanisms, our research mainly focuses on developing probabilistic models to predict the reliability performance for complex systems that may fail due to different types of failure processes, or multiple failure processes. Failure of devices may arise from forces generated internally in the devices, or from external sources. Common failure mechanisms and root causes are identified as wear degradation, stiction, shock loads, fatigue, etc., in mechanical and electrical systems. These multiple failure processes are competing with each other to fail the system, and may have some interactions with each other. For example, wear degradation may deteriorate the system resistance to future random shock loads. Complex systems usually have more than one component, and each of these components has their own failure processes. This creates challenging issues we need to address when predicting system reliability, such as the dependency among failure times of multiple components in a complex system. For example, as a component wears, its temperature increases, which causes the ambient temperature increases. In the meanwhile, the dependent components degrade over time and their degradation rates increase as the ambient temperature increases. Therefore, the degradation processes of the dependent components are statistically dependent on the degradation process of the dominant component via the environmental factors. It creates an interesting and challenging problem to analyze the reliability of the system, which is lacking in the literature. The dependence among multiple failure processes and the dependence among multiple components make the reliability prediction more complicated and difficult, which requires the assistance of new reliability models.

When a complex system is put into operation, it becomes necessary to conduct maintenance activities periodically to ensure continuous function, eliminate potential breakdown, and reduce production loss. For systems with multiple components, traditional time-based maintenance policies are not cost-effective when the multiple components are dependent. Maintenance actions come with costs that depend heavily on the operating condition of the system. Therefore, advanced condition-based maintenance policies designed for multi-component systems are greatly needed.

This chapter starts with the problem statement that describes the emerging reliability and maintenance issues and motivation of our research. Literature reviews are summarized on reliability modeling for systems experiencing degradation processes, random shock modeling, multiple failure processes, and condition-based maintenance modeling for systems with multiple failure processes. Finally, objectives and contributions of this research are discussed.

1.1 Problem Statement

Commercialization for many new and evolving technologies requires impressive advancements in materials science, electrical, and mechanical engineering. To achieve widespread usage in industrial applications, new devices must be highly reliable. For example, MEMS are a relatively new and fast-growing field in microelectronics. MEMS devices have diverse applications, ranging from heath care and military to consumer electronics and automotive. Each year, numerous MEMS designs and product concepts are proposed, among which only a small portion have actually succeeded in commercialization. The lack of proper understanding of MEMS reliability is one of the major challenges that prevent the commercialization of new product concepts. On the other hand, when complex systems are put into field operation, their health condition deteriorates over time, and appropriate maintenance strategies need to be designed to assist continuous functioning of complex systems. These new challenges need to be addressed to support further advancement and wider adoption, which requires industrial engineering skills and capabilities that associate with reliability, maintenance, statistics and operation research.

The goal of our research is to develop new tools and methodologies to understand the behavior of new and evolving devices for distinct applications, address critical reliability problems that have delayed the commercialization of certain new device types, and develop appropriate maintenance strategies to ensure smooth functioning of devices. We aim to develop general reliability models, maintenance policies, and analysis tools that can be customized and adapted for many existing and emerging design problems.

In reliability study, degradation is one of the common failure mechanisms that have been widely investigated [4-7]. Degradation is the reduction in performance, reliability and lifetime of systems. Examples of degradation include the wear on rubbing surfaces of a microengine, the decreasing light intensity of vacuum fluorescent displays (VFDs), and the growing crack size on microstructures [4]. Degradation-based reliability analysis is gaining extensive attention recently because degradation data can provide more information than failure time data, and can be used to predict the reliability performance even beyond the experiment time.

Another extensively explored area is the reliability of systems subject to random magnitude of shocks at random times. In the literature, four categories of shock models are typically proposed: extreme shock model [8-10], cumulative shock model [11, 12], run shock model [13, 14], and δ-shock model [15-17]. In general, a random shock process causes sudden impact to a system from the internal structure or external environment, such as the cyclic shocks due to mechanical operation, thermal shocks, random customer insurance claims in finance, etc.

With the advance in technology, products are highly integrated to be multifunctional, which has greatly benefited our daily life. However, it makes the reliability prediction of these products more difficult. Systems with complex structures have multiple failure processes and each of them competes with the others to fail the whole system over time. This phenomenon has attracted a lot of attention on reliability analysis of systems exposed to multiple failure processes. However, multiple failure processes that a system experiences are assumed to be independent, which is an appropriate assumption when no interaction exists among those multiple failure processes in a system. However, for complex systems where one failure process can affect others, the assumption of independence among multiple failure processes is not valid, and the reliability models may miscalculate the lifetime of the system. Therefore, new reliability models are needed to assess the lifetime of systems subject to multiple dependent failure processes, which is still lacking in the literature.

In this dissertation, we mainly focus on the reliability models for systems subject to multiple dependent failure processes. These multiple dependent competing failure processes include wear degradation processes that can cause soft failure of a system, and random shock processes that can cause catastrophic failure or hard failure of a system. The random shock process can affect the degradation process by causing abrupt damage on the degradation level, which speeds up system soft failure. Based on our reliability models, maintenance policies are proposed and compared to see which one is beneficial.

Furthermore, systems often operate under dynamic environment; therefore their degradation paths are stochastic rather than deterministic. Considering the impact of the operating environment into our reliability model makes it more flexible and applicable to cases where the devices are sensitive to their surrounding environment. Stochastic processes, such as Gamma process, Brownian motion process, can be used to model the stochastic behavior of degradation under dynamic environment. For the shock process, homogeneous Poisson process or nonhomogeneous Poisson process can be adopted to describe the randomness of the shock process.

In addition, the interactions between components in a complex multi-component system create many challenging research problems. In general, multiple components in a system operate under the same environment, which may cause interactions among these components. To discover the dependence among multiple components in a system, we need to understand the physics-of-failure mechanism of each component, and find the linkage of their failure processes. The dependency among multiple components may be due to various environmental factors, such as temperature, humidity, and vibration. For example, consider a system with multiple components where each component experiences its own degradation process. The degradation process of one dominant component increases the surrounding environmental temperature, while the elevated temperature speeds up the degradation processes of the remaining components.

Moreover, a complex system has multiple components as well as multiple failure processes, and these multiple failure processes damage the health condition of the system gradually over time. When the health condition of the system drops to a critical level, the system may not be able to perform smoothly and failure will happen sooner or later. System failure can cause severe consequences, such as downtime cost, production loss, and safety concerns. Therefore, it is essential to conduct maintenance activities on complex multi-component systems. However, traditional time-based maintenance policies, such as block replacement policy, do not apply to the complex multi-component system cases due to the complex dependency relationship among multiple components. Cost effective condition-based maintenance policies should be explored to assist continuous functioning of the system in the field.

Our research focuses on probabilistic reliability modeling for single-unit systems subject to different failure processes, and then extends to reliability and condition-based maintenance modeling for multi-component systems. Therefore, literature review on degradation modeling, random shock modeling, reliability modeling for systems experiencing multiple failure processes, and condition-based maintenance modeling is summarized as follows.

1.2 Literature Review

In this section, we review literatures on reliability modeling for systems experiencing degradation processes, random shock processes, and multiple failure processes. Literatures on condition-based maintenance policies for multi-component systems are also summarized.

1.2.1 Degradation Modeling

In the early stage of reliability prediction, reliability testing is conducted under normal operating conditions, and the time to failure data of units is collected and analyzed to estimate or predict reliability. Along with the development of high technologies and improvement of manufacturing processes, products become more reliable. We may not observe many failures of the units under normal testing conditions over a long period of time. Therefore, researchers have developed accelerated life testing based on the physics of failure of the units: to hasten their failure, units are tested under severe stresses until failure occurs or the test is terminated. This accelerated life testing approach works successfully for many cases. However for some cases, we may still not be able to collect enough failure data or it is very expensive to run the units to failure. Consequently, many researchers have been advocating degradation-based techniques as an alternative to the failure based approach. Degradation data often provide more information than failure time data for predicting the lifetime of the unit.

Lu *et al.* compared degradation analysis and traditional failure-time analysis in terms of asymptotic efficiency [18]. The comparisons considered a range of practical situations and provided an insight into the trade-offs between these two methods of estimating the quantiles of the time-to-failure distribution. In both cases, data can be used to assess the adequacy of the model within the data range. In terms of statistical efficiency, degradation has its advantages when estimating quantiles of failure probabilities beyond the range of the data. They suggested that whenever possible, physics of failure should be investigated in order to provide more confidence for the degradation models.

Elsayed classified degradation models as physics-based and statistics-based models [3]. The physics-based degradation models are those in which the degradation phenomenon is described by a physics-based relationship such as Arrhenius law, corrosion initiation equation, or experimentally-based results such as crack propagation or crack growth models. Statistics-based degradation models are those in which the degradation phenomenon is described by a statistical model such as regression. Their comparisons show that, except in extreme cases, degradation analysis provides more precision than traditional failure-time analysis does.

The limitations of physics-based degradation models are [3]:

(1) There is no universal physics-based or experimental-based relationship that describes the degradation phenomenon of all products, which makes it impossible to develop a general degradation model.

(2) It is time consuming to develop a physics-based (or experimental-based) relationship for new products.

(3) Physics-based (or experimental-based) degradation models may not be suitable for the development of closed form reliability functions. This is due to the fact that some of the parameters are random variables; the degree of difficulty in deriving a reliability function depends on the distribution of these random variables. Except for simple cases, the reliability function cannot be easily derived.

Statistics-based models are more general than physics-based models and have been investigated by many researchers in the literature. Lu and Meeker developed statistical methods for using degradation measures to estimate a time-to-failure distribution for a broad class of degradation models [19]. They used a nonlinear mixed-effects model and developed methods based on Monte Carlo simulation to obtain point estimates and confidence intervals for reliability assessment. Whitmore *et al.* presented a model based on a bivariate Wiener process, in which one component represents the marker process and the second component is a degradation process that is latent and determines the failure time [20]. Failure occurs when the latent component crosses a threshold level. Since the degradation process is latent or hidden, inference about the degradation process must be based on observation on the marker process. Lee and Whitmore used the threshold regression method to model the event times by a stochastic process reaching a boundary, which can be used in degradation progression [21].

Besides the above two types of degradation models, many researchers have investigated degradation models through other approaches. Advances in sensor technology have led to an increased interest in using degradation based sensory information to predict the remaining lives of partially degraded components and systems [22]. Gebraeel *et al.* have conducted experiments on bearing reliability and found out that the increase in the degradation signal level on bearing is associated with spall propagation along the surface of the raceway. They developed neural-network-based models from vibration-based degradation signals to predict bearing failure [23]. Next, they proposed two exponential degradation models, one with a random error term and the other with a Brownian motion error process [24]. Bayesian updating methods using degradation signals are presented to update the stochastic parameters of exponential degradation models. Later, they investigated two updating methodologies for computing and updating residual life distributions of partially degrading components with exponential degradation path [25]. The first method utilizes sensory signal values to update the distribution, while the second policy takes into account the entire history of sensory information. Elwany and Gebraeel introduced a stochastic degradation modeling framework for computing and continuously updating remaining life distributions using in situ degradation signals acquired from individual components during their operation [22].

Another research area is reliability modeling using both failure time and degradation data in accelerated life test. Padgett and Tomlinson put forward a general accelerated test model, in which failure times and degradation measures can be combined for inference about system lifetime, based on a continuous cumulative damage approach with a

Gaussian process describing degradation [26]. Park and Padgett presented accelerated degradation models for failure based on the geometric Brownian motion or gamma process [27]. Both failures and degradation measures were observed in their models and considered for parametric inference of system lifetime. Park and Padgett investigated new accelerated test models with several accelerating variables for inference based on both observed failure values and degradation measurements [28]. New accelerated test models were developed based on a generalized cumulative damage approach with a stochastic process characterizing a degradation phenomenon.

Many failure models assume that the operating environment is static, which is not applicable to the cases when environment can change and affect physics-of-failure of systems. Therefore the impact from dynamic environment has attracted a lot of attention in failure models. Singpurwalla gave an overview of failure models based on stochastic processes, which are suitable for describing the life length of items that operate in dynamic environments [29]. He summarized four strategies involved in failure modeling based on a stochastic process:

1. The item state (or equivalently, its wear) has been described by a diffusion process: typically a Wiener process, a gamma process or a deterministic diffusion.
2. The failure rate (also known as the hazard rate) of the item is described by a stochastic process: typically a gamma process, a shot-noise process, a function of a Wiener process, or in general, a Levy process.
3. The damage-causing environment is described by a stochastic process, typically a shock-inflicting Poisson process, and the resulting failure models are known as shock models.

4. A response variable that is strongly correlated with the life length, such as temperature, is described by a stochastic process, typically a stationary continuous-time Gaussian process.

Kharoufeh considered the reliability of a single-unit system whose cumulative damage over time is a continuous wear process that depends on an external environment process, and the external environment was modeled as a continuous time stochastic process [30]. Kharoufeh and Sipe used sensor data to estimate full and residual lifetime distributions for a single-unit system subject to a stochastically evolving environment [31]. The evolution of the random environment was characterized as stationary continuous-time Markov chain. Gebareel and Pan presented a degradation modeling for computing the condition-based residual life distribution of partially degraded systems functioning under time-varying environmental and/or operational conditions [32]. They modeled degradation-based signals from a population of components using stochastic models that combined three main sources of information: real-time degradation characteristics of component obtained by observing the component's in-situ degradation signal, the degradation characteristics of the component's population, and the real-time status of the environmental conditions under which the component is operating. Kharoufeh *et al.* extended their previous research on single-unit system reliability models under random environment to the case of semi-Markovian environments that place only mild restrictions on the dynamics of the evolving environment [33].

1.2.2 Random Shock Models

Random shock modeling has also been extensively studied for systems exposed to external shock environments. In the literature, four categories of random shock models are typically classified [34, 35]: (*i*) an extreme shock model, where failure occurs when the magnitude of any shock exceeds a specified threshold; (*ii*) a cumulative shock model, where failure occurs when the cumulative damage from shocks exceeds a critical value; (*iii*) a run shock model, where failure occurs when there is a run of k shocks exceeding a critical magnitude; and (*iv*) a δ-shock model, where failure occurs when the time lag between two successive shocks is shorter than a threshold δ. The shock arrival process is usually modeled into four types: 1) homogeneous Poisson process, that is, the times between two consecutive shocks are independent and identically distributed exponential random variables; 2) non-homogeneous Poisson process, that is, a counting process null at the origin with independent increments where the probability of a shock in $(t, t+\Delta t]$ is $\lambda(t)\ \Delta t+o(\Delta t)$, while the probability of more than one shock in $(t, t+\Delta t]$ is $o(\Delta t)$; 3) non-stationary pure birth process, that is, a Markov process where, given that k shocks have occurred in (0, t], the probability of a shock in $(t, t+\Delta t]$ is $\lambda_k\lambda(t)\Delta t + o(\Delta t)$, while the probability of more than one shock in $(t, t+\Delta t]$ is $o(\Delta t)$; and 4) renewal process, that is, the times between two consecutive shocks are independent and identically distributed random variables [36].

For the extreme shock model, Gut and Husler derived moment relations and asymptotic distribution of time to failure for the extreme shock model [8]. Cirillo and Husler introduced a new intuitive approach to generalized extreme shock models using urn processes, which can indirectly model the moving risky threshold of generalized extreme shock model [9]. Cirillo and Husler proposed an alternative model based on a special version of the reinforced urn process. This approach allows performing a

Bayesian nonparametric analysis of extreme shocks [10].

For the cumulative shock model, the life distribution is derived in [37] and shocks arrive according to a homogeneous Poisson process. Gut applied the stopped two dimensional random walk theory into the cumulative shock model and relaxed the assumption of non-negative damage due to the shock process [11].

For the run shock model, Mallor and Omey presented a model to study systems that fail when there are k consecutive shocks with critical magnitudes [13]. They obtained the limiting behavior when k tends to infinity or when the probability of entering a critical set tends to zero.

For the δ-shock model, Tang and Lam developed a δ-shock maintenance model for a deteriorating system, and assumed that the time lag threshold values of successive shocks were geometrically nondecreasing [14]. Bai *et al.* first stated a δ-shock model, and then introduced a generalized framework of shock models based on a cluster point process with cluster marks [15]. Li and Zhao proposed some reliability results for the δ-shock model of general complex systems with multiple components [17]. Tang and Lam proposed a δ-shock maintenance model for a deteriorating system and the shocks arrive according to a renewal process [14]. The interarrival time of shocks has a Weibull distribution or gamma distribution. Finkelstein discussed shock arrival processes following a renewal process and a nonhomogeneous process, respectively [38]. He came up with a unique approach to deal with δ-shock model and relaxed the assumption of the fixed critical threshold interarrival time between two successive shocks. Li and Zhao investigated the δ-shock model of complex systems consisting of n i.i.d. components, and considered coherent system structures including series, parallel and k-out-of-n [17].

Some mixtures of the classical four shock models are also investigated explicitly in the literature. Qian *et al.* proposed an extended cumulative damage model with two types of shocks from a nonhomogeneous Poisson process: one is the fatal shock at which a system fails, and the other is damage shock at which the system suffers only damage; basically, a combination of extreme shock model and cumulative shock model [39]. Mallor and Santos studied a general shock model that extends the extreme, cumulative and run shock models, allowing a correlation structure for the variables involved in the model definition [36]. The model was governed by a sequence of random vectors of correlated variables: the inter-shock time, the magnitude, and the damage caused by the shock. In their model, only critical shocks can cause damage, and system fails as soon as the additively accumulated damage exceeds a fixed threshold value.

Gut and Husler considered the generalized extreme shock model, generalized cumulative shock model, and generalized mixed shock model [40]. In the generalized extreme shock model, a system is harmed by some large but nonfatal shocks, which influence the maximum shock load the system can take; in the generalized cumulative shock model, only the sum of the most recent shocks implies a system failure; in the generalized mixed shock model, they combined both of the above two models with some link functions. Wang and Zhang considered a repairable system experiencing both extreme shock model and δ-shock model, and applied a replacement policy N based on the number of failures into the reliability model [41]. Cha *et al.* also extended the classical shock models to combined/mixed shock models of extreme shock model and cumulative shock model, where shocks arrive according to a nonhomogeneous Poisson process [42-44]. Finkelsteina and Zarudnij split the initial homogeneous Poisson shock process into three homogeneous Poisson processes: shocks with a small level of damage are harmless to a system; shocks with a large level of damage results in the system's failure; and shocks with an intermediate level of damage can result in the system's failure with some probability [45]. Other variations of shock models can be found in [12, 46].

1.2.3 Multiple Failure Processes

In the literature, single failure processes are often considered for complex systems, either degradation or random shock processes. The degradation damage may be caused by many factors: continuous wear, damage increments due to shocks, aging, etc. Klutke and Yang studied systems that deteriorate due to both shocks and graceful degradation, but they considered them in one failure process [47]. Shocks occurred according to Poisson process and between shocks, deterioration occurred at a constant rate. The reliability and availability measures for a single-unit system that suffers degradation due to its operating environment and the impact of shocks were also considered in [48]. The system experiences a soft failure when its cumulative level of degradation exceeds a fixed threshold value. In [49], transient and asymptotic reliability indices for a single-unit system were investigated, and the system degrades over time due to normal wear induced by its operating environment and randomly occurring shocks that cause additional damage to the system.

However, the single failure process model may not be applicable to systems that can fail due to multiple failure processes. Competing risk problems are becoming increasingly common and important in practice. Therefore, to make the reliability model more general and practical, researchers have started to consider multiple failure processes for a complex system. Li and Pham developed a generalized multi-state degraded system reliability model, in which the system was subject to multiple independent competing failure processes, including two degradation processes and random shocks [50]. Noortwijk *et al.* presented a unique method to combine two stochastic processes of deteriorating resistance and fluctuating load for computing the time-dependent reliability of a structural component [51]. The deterioration process was modeled as a gamma process, which is a stochastic process with independent non-negative increments having a gamma distribution with an identical scale parameter. The stochastic process of loads was generated by a Poisson process. The variability of random loads was modeled by a peaks-over-threshold distribution.

All the above studies share the same assumption that these multiple failure processes are independent, which limits their applications in dependent cases. For example, according to the reliability tests conducted by Sandia National Laboratories [52], a micro-actuator system may fail due to two competing yet dependent failure processes: catastrophic failures caused by extreme shocks from a random shock process, and a soft failures caused by wear degradation and cumulative wear damages from the same random shock process [53]. The dependency among the failure processes presents challenging issues in reliability modeling.

Peng *et al.* developed reliability models for systems subject to multiple dependent competing failure processes [54]. Specifically, two dependent/correlated failure processes were considered: soft failures caused jointly by continuous smooth degradation and additional abrupt degradation damage due to a shock process, and catastrophic failures caused by an abrupt and sudden stress from the same shock process. Ye *et al.* presented a convenient means of capturing both shock and degradation in a single model when the extent of degradation and the magnitude of shocks are not observable, but only the failure times and the corresponding failure modes are recorded [55]. Reliability models were developed considering three failure modes: catastrophic failure, degradation failure and failure due to shocks in [56]. The effects of shocks on performance were classified into two types: a sudden increase in the failure rate after a shock, and a direct random change in the degradation process at the arrival of a shock. They also considered two shock scenarios: shocks occur with a fixed time period; shocks occur with varying time periods. Further, they extended their work to a reliability model on competitive failure processes under fuzzy degradation data [57]. Keedy and Feng proposed a probabilistic reliability modeling framework for stent deployment and operation [58]. Two dominating failure processes of stents were evaluated: delayed failures or fatigue crack growth due to cyclic stresses, and instantaneous failures due to single-event overloads. Wang and Pham introduced a time-varying copulas technique into a dependent competing risk model for systems subject to multiple degradation processes and random shocks [59]. The proposed model allows for a more flexible dependence structure between risks in which (a) the dependent relationship between random shocks and degradation processes is modulated by a time-scaled covariate factor, and (b) the dependent relationship among various degradation processes is fitted using the copula method.

1.2.4 Condition-based Maintenance Policies

For deteriorating systems, reliability modeling helps to predict their health condition. The next step should be taking maintenance actions to repair or replace the system to prevent failure and reduce downtime costs. Maintenance activities come with costs, such as repair cost, replacement cost, inspection cost, downtime cost, etc. Therefore, optimal maintenance policies are needed to minimize the cost rate and/or maximize system availability. In this section, we present the literature review on condition-based maintenance policies for multi-component systems and multi-state systems, based on which we will develop our own condition-based maintenance strategies for different problems/applications in Chapter 4 and Chapter 5.

In the maintenance policies for multi-component systems, three types of dependence are typically considered: economic dependence, structural dependence, and failure dependence [60, 61]. Economic dependence in maintenance policies for multi-component systems considers that there are cost/time-savings to jointly perform maintenance on multiple components, instead of on individual components separately. Structural dependence implies that the components are structurally connected, and therefore, maintenance actions on a failed component require dismantling other components. Failure dependence in maintenance policies for multi-component systems refers to the dependence between the failure or state of one component and those of other components in the system. It also refers to situations when the components suffer from the common-cause failure from external sources.

After the first survey paper on maintenance policies for multi-component systems conducted by Thomas in 1986 [62], this topic has attracted increasing attention. Cho and Parlar [63] conducted a comprehensive survey on maintenance models for multi-component systems before 1991, including group, block, and opportunistic models. Another two survey papers on this topic are provided by Dekker *et al.* [64] with a focus on economic dependence, and Wang [61] with an emphasis on single-component systems. More recently, Tian and Liao [65] investigated condition-based maintenance of multi-component systems, where economic dependence exists among different components subject to condition monitoring. Castanier *et al.* [66] considered a condition-based maintenance policy for a system with two economically dependent components that deteriorate stochastically, independently and gradually. Laggounce *et al.* [67] proposed a preventive maintenance plan for a multi-component series system subject to random failures, where economic dependence is considered to reflect the influence of component operation/maintenance costs on the overall system costs.

Maintenance cost depends heavily on the operating condition of the equipment. To minimize the maintenance cost, it becomes necessary to have an equipment-state-dependent maintenance policy. As a matter of fact, maintenance optimization on multi-state systems has recently attracted a lot of attention in the literature. In Pandey *et al.* [68], a selective maintenance strategy was developed for a series-parallel multi-state system (MSS) that consists of multi-state components to maximize system reliability during the next mission. Three maintenance actions are considered for a component: do-nothing, imperfect maintenance, and replacement. This type of maintenance policy is called selective maintenance, because a subset of maintenance actions is performed on selected components such that the system is able to meet the next mission requirement. Pandey *et al.* [69] proposed a mathematical model to help in decision making for selective maintenance under imperfect repair, while the system and the components under study are in binary state, i.e., working or failed. Dao *et al.* [70] presented a study on selective maintenance for multi-state series-parallel systems with economically-dependent components. The optimization model takes into account the system reliability in the next operating mission, the available budget and the maintenance time for each component from its current state to a higher state.

Moghaddass *et al.* [71] studied a device with discrete multi-state degradation, which is monitored by a condition monitoring indicator through an observation process, and a nonhomogeneous continuous-time hidden semi-Markov process is employed to model the degradation and observation processes associated with this type of device. According to [71], in a multi-state degradation process, the degradation transition may depend on certain factors, such as the two states involved in the transition, the time that the device reaches the current state, the time already spent at the current state, the total age of the device, or any combination of these factors. A Markovian or a semi-Markovian structure can be used to model the degradation transition with the transition dependency on the mentioned factors between two states. Other relevant research on maintenance policy for multi-state systems can be found in [72-76].

1.3 Objectives and Contributions

In this study, we focus on developing new reliability models and cost-effective maintenance strategies for single-component systems with multiple failure processes and complex multi-component systems that can be applied to many current and evolving devices. Two types of dependency among multiple failure processes are considered: the arrival of each shock impacts on multiple failure processes, and the dependency between the random shock process and the threshold level of system resistance to shock loads. The dependency among multiple components within a complex multi-component system via environmental factors is studied, and we take temperature as an example application to demonstrate our model. We also study the cost effective maintenance policies and implement them based on our new reliability models. The detailed research objectives are listed as below:

1) ***Develop new reliability models for single-component systems with multiple failure processes.*** The failure processes can be wear degradation and fracture due to random shocks. In the new reliability model in Chapter 2, these two failure processes are correlated or dependent in two respects: (a) the arrival of each shock load affects both failure processes, and (b) the shock process impacts the hard failure threshold level. Three cases of dependency between the shock process and the hard failure threshold level are studied. In the future research, we are going to explore other potential dependence patterns among multiple failure processes and develop reliability models to assist the design, manufacturing and maintenance of systems with similar failure mechanisms.
2) ***Explore different dependent patterns among multiple failure processes.*** In Chapter 3, shocks are categorized into different shock zones which impact degradation differently. Three shock zones are considered: safety zone, where shocks with magnitude below W_0 are considered harmless; damage zone, where shocks with magnitude between W_0 and W_T can cause damage to the system; and fatal zone, where shocks with magnitude above W_T are considered fatal and fail the system immediately.
3) ***Investigate the reliability performance of systems with dependent components via environmental factors.*** In the literature, the dependence among components within a complex system via factors, such as the number of external shocks, shock arrival times, has been widely studied. However, little research has been done on the dependence among components via environmental factors. This becomes a crucial and challenging problem to system reliability performance. Chapter 4 presents our

work to address this problem, where we consider environmental temperature as the linkage among multiple dependent components in a complex system.

4) ***Explore and implement advanced maintenance policies based on our developed reliability models.*** Maintenance activities are essential to prevent potential system breakdown, reduce downtime cost and eliminate safety concerns. In Chapter 2, two classical maintenance strategies, block replacement policy and age replacement policy, are studied and compared with each other. Chapter 4 presents a unique condition-based maintenance policy designed for systems with multiple dependent components with the aim to reduce expected average cost rate. A condition-based maintenance model is proposed for the power transformer using Markov decision process in Chapter 5, where the deterioration process of the power transformer is modeled as multi-state.

Our proposed reliability and maintenance models can be implemented in applications of microelectronic devices and many other systems with similar failure mechanisms. They serve as tools in manufacturing industry to facilitate product design and to improve its reliability and availability.

1.4 Organization of the Dissertation

In Chapter 2, we present reliability and maintenance models for systems subject to multiple dependent competing failure processes with a changing, dependent failure threshold. In our model, two failure processes are considered: soft failure caused by continuous degradation together with additional abrupt degradation due to a shock process, and hard failure caused by the instantaneous stress from the same shock process. These two failure processes are correlated or dependent in two respects: 1) the arrival of each shock load affects both failure processes, and 2) the shock process impacts the hard failure threshold level. Three cases of dependency between the shock process and the hard failure threshold level are studied. The first case is that the hard failure threshold value changes to a lower level when the first shock is recorded above a critical value, or a generalized extreme shock model. The second case is that the hard failure threshold value decreases to a lower level when the time lag between two sequential shocks is less than a threshold δ, or a generalized δ-shock model. The third case is that the hard failure threshold value reduces to a lower level right after m shocks whose magnitudes are larger than a critical value, or a generalized m-shock model. Based on degradation and random shock modeling, reliability models are developed for these two dependent failure processes with a shifting failure threshold. Two preventive maintenance policies are also applied and compared to decide which one is more beneficial. Then a Micro-Electro-Mechanical System example is given to demonstrate the reliability models and maintenance polices.

In Chapter 3, we investigate reliability analysis of a system that experiences two dependent competing failure processes. In our new model, shocks are categorized into different shock zones which impact degradation differently. These two failure processes are a stochastic degradation process and a random shock process, and they are dependent because arriving shocks can impact the degradation process in the form of instantaneous damage. In our model, only shock loads that are larger than a certain level are considered to cause abrupt damages on the degradation process, which makes this new model realistic and challenging. Shocks are divided into three zones based on their magnitudes: *safety zone*, where shocks with magnitude below W_0 are considered harmless; *damage* *zone*, where shocks with magnitude between W_0 and W_T can cause damage to the degradation process; and *fatal zone*, where shocks with magnitude above W_T are considered fatal and the system fails immediately. We further model the abrupt damages using an explicit function of shock load exceedances (differences between load magnitudes and a given threshold). Due to the complexity in modeling these two dependent stochastic failure processes, no closed form of the reliability function can be derived. Monte Carlo importance sampling is used to estimate the system reliability. Finally, two application examples with sensitivity analyses are presented to demonstrate our models.

In Chapter 4, we introduce a new reliability model and a unique condition-based maintenance model for complex systems with dependent components subject to respective degradation processes, and the dependence among components is established through environmental factors. Common environmental factors, such as temperature, can create the dependence in failure times of different degrading components in a complex system. The system under study consists of one dominant component and n statistically dependent components that are all subject to degradation. We consider two aspects that link the degradation processes and environmental factors: the degradation of dominant component is not affected by the state of other components, but may influent environmental factors, such as temperature; and the n dependent components degrade over time and their degradation rates are impacted by the environmental factors. Based on the thermodynamic study of the relationship between degradation and environmental temperature, we develop a reliability model to mathematically account for the dependence in multiple components for such a system. Considering the unique dependent relationship among components, a novel condition-based maintenance model is developed to minimize the long run expected cost rate. A numerical example is studied to demonstrate our models, and sensitivity analysis is conducted to test the impact of parameters on the models.

In Chapter 5, we study the failure modes of power transformers and propose a condition-based maintenance policy for power transformers using Markov decision processes (MDP). Besides the weather-related random failure mode, we consider another three failure modes related to degradation processes: paper winding insulation, bushings, and tap-changer. The power transformer is modeled as a multi-state system, including four different operating states and four failure states (due to the four failure modes). Four maintenance actions are considered in this model: no action (NA), minimal maintenance (MM), preventive maintenance (PM), and corrective maintenance (CM). In the proposed maintenance strategy, periodic inspections are implemented, and the inspection interval is to be determined at each decision epoch. Therefore, the condition-based maintenance decision is a combination of two factors: maintenance action and the next inspection interval. A policy iteration algorithm is used to find the optimal policy that minimizes the average cost in a long run. A numerical example is given to demonstrate the proposed condition-based maintenance model.

Chapter 6 gives the summary and conclusions. Potential future directions are also included.

Chapter 2: Reliability and Maintenance Modeling for Dependent Competing Failure Processes with Shifting Failure Thresholds

We present reliability and maintenance models for systems subject to multiple dependent competing failure processes with a changing, dependent failure threshold. In our model, two failure processes are considered: soft failure caused by continuous degradation together with additional abrupt degradation due to a shock process, and hard failure caused by the instantaneous stress from the same shock process. These two failure processes are correlated or dependent in two respects: 1) the arrival of each shock load affects both failure processes, and 2) the shock process impacts the hard failure threshold level. In previous research, the failure thresholds are fixed constants, which is appropriate for most design and reliability problems. However, the nature of the failure threshold has become a critical issue for certain classes of complex devices. When withstanding shocks, the system is deteriorating, and its resistance to failure is weakening. In this case, it becomes more sensitive to hard failure.

In this chapter, three cases of dependency between the shock process and the hard failure threshold level are studied. The first case is that the hard failure threshold value changes to a lower level when the first shock is recorded above a critical value, or a generalized extreme shock model. The second case is that the hard failure threshold value decreases to a lower level when the time lag between two sequential shocks is less than a threshold δ, or a generalized δ-shock model. The third case is that the hard failure threshold value reduces to a lower level right after m shocks whose magnitudes are larger than a critical value, or a generalized m-shock model. Based on degradation and random shock modeling, reliability models are developed for these two dependent failure processes with a shifting failure threshold. Two preventive maintenance policies are also applied and compared to decide which one is more beneficial. Then a Micro-Electro-Mechanical System example is given to demonstrate the reliability models and maintenance polices [77].

ACRONYMS

MEMS	Micro-Electro-Mechanical Systems
MDCFP	Multiple dependent competing failure processes
ARP	Age replacement policy
BRP	Block replacement policy

NOTATION

D_1	Threshold level for hard failures
D_2	Lower threshold level for hard failures
D_0	Critical value in the m-shock model, i.e., when m shock loads are larger than D_0, the hard failure threshold level reduces to a lower level
D_k	Threshold level for hard failures D_k, k =1 or 2
B_l	l^{th} shock inter-arrival time
δ	Critical shock inter-arrival time
$N(t)$	Number of shock loads that have arrived by time t
λ	Arrival rate of random shocks
W_i	Size or magnitude of the i^{th} shock load
$F_W(w)$	Cumulative distribution function (cdf) of W_i
H	Threshold level for wear degradation failures
$X(t)$	Wear volume due to continuous degradation at t
$X_S(t)$	Total wear volume at t due to both continual wear and instantaneous damage
Y_i	Damage size on the wear degradation caused by the i^{th} shock load
$S(t)$	Cumulative shock damage size at t
$G(x, t)$	cdf of $X(t)$ at t

$F_X(x, t)$	cdf of $X_S(t)$ at t
$f_Y(y)$	Probability density function (pdf) of Y_i
$f_Y^{<k>}(y)$	pdf of the sum of k independent and identically distributed (i.i.d.) Y_i variables
$f_T(t)$	pdf of the failure time, T
ρ	Preventive replacement time interval in the block replacement policy
τ	Preventive replacement time interval (or age) in the age replacement policy
C_F	Corrective replacement cost
C_R	Preventive replacement cost
$\lfloor \cdot \rfloor$	Notation for floor function
$\lceil \cdot \rceil$	Notation for ceiling function

2.1 Introduction

In this chapter, a new model is presented to predict reliability and optimize maintenance when there are competing failure processes, and the failure thresholds can shift based on exposure to different patterns of shocks. This is an entirely new model that extends previous research in dependent competing failure processes, which offers some distinct advantages for design and reliability problems when the component resistance to failure reduces. Three different cases of shifting failure thresholds are presented together with two different maintenance policies. These developed reliability and maintenance models considering dependent competing failure processes can be applied to systems in which some components age with continuous degradation while other components are more vulnerable to discrete random shocks, as well as systems in which the same component experiences both continuous degradation and discrete shocks.

For complex systems, failure mechanisms and causes include wear-out, corrosion, fatigue, fracture, etc., which originate from either internal degradation or external sources.

For systems experiencing multiple failure mechanisms, failure processes are often dependent in various respects, which generate a set of interesting and challenging issues for analyzing system reliability. In this chapter, we study two dependent competing failure processes: soft failure caused by continuous degradation and additional abrupt degradation damage due to a shock process, and hard failure caused by instantaneous stress from the same shock process. These two failure processes are dependent in two ways: 1) the arrival of each shock affects both failure processes, and 2) the hard failure threshold can shift depending on the pattern of shocks.

In previous research on competing failure processes [34, 54, 78, 79], the failure thresholds are considered to be fixed constants, which is appropriate for most design and reliability problems. However, the nature of the failure threshold has become a critical issue for certain classes of complex devices, such as Micro-Electro-Mechanical Systems (MEMS). When withstanding shocks, the system is deteriorating, and its resistance to failure is weakening. For example, a component of a MEMS device becomes more sensitive to fracture or hard failure after exposure to a certain number of strong shocks [80].

In this chapter, we study three different cases of shock patterns that can shift the hard failure threshold: 1) a generalized extreme shock model, where the hard failure threshold changes to a lower level when the first shock is recorded above a critical value; 2) a generalized δ-shock model, where the hard failure threshold decreases when the time lag between two sequential shocks is less than δ; and 3) a generalized m-shock model, where the hard failure threshold reduces right after exposure to m shocks larger than a critical value. Based on degradation and random shock modeling, reliability models are developed for the two dependent failure processes when the hard failure threshold is dependent on the shock process.

2.1.1 Literature Review

The hard failure threshold essentially represents material strength in reality, and the relationship between stress and strength has been extensively studied by dynamic stress-strength models. Ellingwood and Mori [81] presented a probability-based methodology to evaluate the reliability of existing concrete structures in nuclear power plants, which includes models to predict structural resistance deterioration due to environmental stressors. Huang and Askin [82] proposed a generalized stress-strength interference reliability model considering both stochastic loading and strength aging degradation. Noortwijk *et al*. [51] developed a time-dependent reliability model combining two stochastic processes of deteriorating resistance and fluctuating load to compute the reliability of structural components. Huang and An [83] presented a discrete stress-strength interference model where the strength is dependent on the stress.

Many different maintenance strategies have been presented for deteriorating systems [61, 84], among which the block replacement policy (BRP) and age replacement policy (ARP) are widely studied for non-repairable systems. In the BRP, components are replaced at prearranged times preventively, and at failure correctively. As a result, very new components can sometimes be replaced if they had just been recently installed in response to a failure. To overcome this drawback, Alley and Lin [85] extended the basic BRP to a generalized BRP, where the time interval between successive planned preventive replacements is divided into two parts: items that fail in the first part are replaced with new ones, and items that fail in the second part are replaced with used ones.

Barlow and Hunter [86] first put forward the concept of ARP where the replacement age of components is a constant. The items are replaced at failure correctively, or at the constant age preventively. However, ARP is relatively difficult to implement because the time to perform the preventive replacement is not prescribed, and it becomes necessary to track the age of components. To address the disadvantages of both policies, Berg and Epstein [87] proposed a modified block replacement policy, where the components under a certain age are not replaced at prearranged block replacement points. Li [88] presented a more general age-dependent block replacement policy with an age limit for preventive replacements that allows imperfect repairs at failure. In this chapter, the basic BRP and ARP are implemented and applied to an application example to compare which one is more advantageous.

The remainder of this chapter is arranged as follows. Section 2.2 presents the reliability models for two dependent failure processes with three special cases of dependency between the shock process and the hard failure threshold. In Section 2.3, two maintenance strategies are derived based on the developed reliability models. Section 2.4 uses a numerical example to demonstrate the reliability models, and evaluate the maintenance policies. Section 2.5 summarizes the chapter.

2.2 Reliability Analysis

As shown in Figures. 2.1, 2.2, and 2.3, the failure of a unit depends on two competing dependent failure processes, soft failure and hard failure, whichever occurs first, results in failure. The dependence of these two failure processes are represented in two respects.

1) The two failure processes are dependent due to the same shock process. Soft failure

occurs when the total degradation exceeds H, accumulated by continuous degradation over time together with abrupt damages due to random shocks. The same random shock process can cause hard failure when one shock magnitude exceeds the material strength level [54].

2) The hard failure threshold is dependent on the shock process. When a unit sustains a series of shocks, it becomes more susceptible to hard failure. The hard failure threshold decreases according to a generalized extreme shock model, generalized δ-shock model, or m-shock model, shown in Figures. 2.1, 2.2, and 2.3, respectively. This aspect of the model represents a new research contribution that has never been adequately considered before.

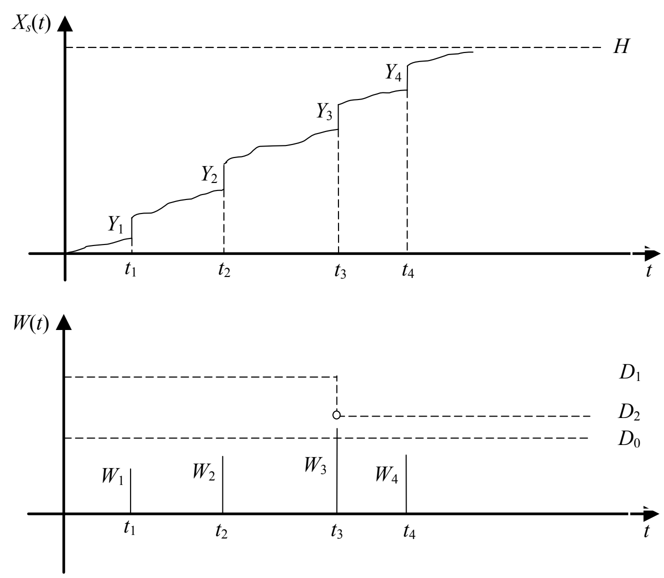

Figure 2.1: Case 1, generalized extreme shock model

Additional assumptions for reliability modeling are summarized as follows.

1) Hard failure occurs when the shock load exceeds the corresponding hard failure threshold (maximum strength of material). For the generalized extreme shock model

(Case 1), the hard failure threshold decreases from D_1 to D_2 when the first shock is recorded above a critical value D_0. For the generalized δ-shock model (Case 2), the hard failure threshold decreases from D_1 to D_2 when the time lag between two sequential shocks is less than δ. For the generalized m-shock model (Case 3), the hard failure threshold is reduced from D_1 to D_2 right after m shocks that are larger than a critical value D_0.

2) The arrival of random shocks follows a Poisson process. The mean damage size is proportional to the mean shock load, i.e., $\mu_Y = a\mu_W$, where a is a known constant.
3) In general, the wear degradation process can follow various paths. For a specific model, we use a linear path to model the wear degradation process.
4) For the specific model where W_i, Y_i, and β are normally distributed random variables, their standard deviations are assumed to be substantially smaller than the mean values, so that the probability that these random variables are negative values is negligible.

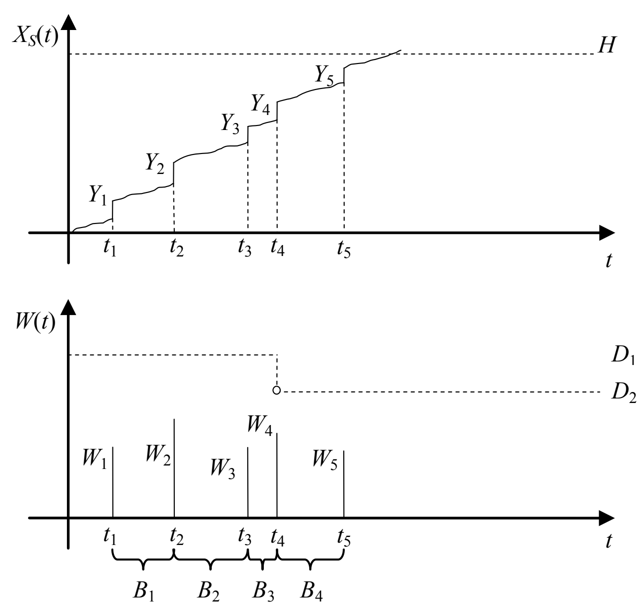

Figure 2.2: Case 2, generalized δ-shock model ($B_3 < \delta$)

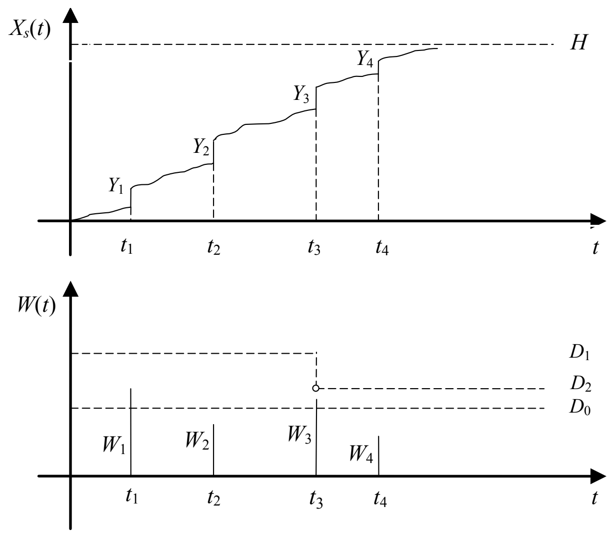

Figure 2.3: Case 3, generalized m-shock model (m = 2)

In this section, we first model the soft failure that is commonly used for all the three cases, followed by the detailed analysis for each of the three cases of hard failure, where the system reliability is also derived considering the dependent hard failure and soft failure processes.

2.2.1 Modeling for Soft Failure

Soft failure occurs when the overall degradation $X_s(t)$ exceeds a threshold level H. The graceful degradation due to wear, $X(t)$, may follow various degradation paths. In this chapter, a linear degradation path $X(t) = \varphi + \beta t$ is applied, where the initial value and degradation rate can be constants or random variables [54]. Random shocks arrive according to a Poisson process $\{N(t), t \geq 0\}$ with rate λ. Shock damage sizes are used to measure the instantaneous increase on degradation, denoted as Y_i for i = 1, 2, …, ∞, which are assumed to be i.i.d. random variables, independent of the Poisson process. The cumulative damage size due to random shocks by time t is given as

$$S(t)=\begin{cases}\sum_{i=1}^{N(t)} Y_i, & \text{if} \quad N(t)>0,\\ 0, & \text{if} \quad N(t)=0,\end{cases} \tag{2.1}$$

which implies a compound Poisson process [84]. The overall degradation of the system is $X_s(t)= X(t)+S(t)$. Then the cumulative distribution function (cdf) of $X_s(t)$ can be derived as

$$F_X(x,t)=P(X_s(t)<x)=\sum_{i=0}^{\infty} P\left(X(t)+S(t)<x \mid N(t)=i\right)P\left(N(t)=i\right). \tag{2.2}$$

Furthermore, if $G(x,t)$ denotes the cdf of $X(t)$ at t, $f_Y(y)$ denotes the pdf of Y_i, and $f_Y^{<k>}(y)$ denotes the pdf of the sum of k i.i.d. Y_i variables, then the cdf of $X_s(t)$ can be derived using a convolution integral:

$$F_X(x,t)=G(x,t)\exp(-\lambda t)+\sum_{i=1}^{\infty}\left(\int_0^x G(x-u,t)f_Y^{\langle i\rangle}(u)du\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!}. \tag{2.3}$$

If shock damage sizes are i.i.d. normal random variables, $Y_i \sim N(\mu_Y,\sigma_Y^2)$, the initial value φ is a constant, and the degradation rate β is normally distributed, $\beta\sim N(\mu_\beta,\sigma_\beta^2)$, then the cdf of $X_s(t)$ can be derived as

$$F_X(x,t)=\sum_{i=0}^{\infty}\Phi\left(\frac{x-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta^2 t^2+i\sigma_Y^2}}\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!}, \tag{2.4}$$

where $\mu_Y= a\mu_W$, and a is a known constant.

2.2.2 Case 1: Generalized Extreme Shock Model

Figure 2.1 shows a generalized extreme shock model; that is, the hard failure threshold value reduces from D_1 to D_2 when the first shock is greater than D_0 [89]. The system fails due to fracture when the shock load exceeds the corresponding hard failure threshold, D_k, k=1 or 2. The size of the i^{th} shock load is denoted as W_i for i= 1, 2, …,∞, which are i.i.d. random variables. The probability that a system survives the applied stress from the i^{th} shock is

$$P(W_i < D_k) = F_W(D_k), \quad \text{for } i = 1,2,\ldots,\infty,\ k = 1 \text{ or } 2, \tag{2.5}$$

where $F_W(w)$ denotes the cdf of W_i. If we assume the W_i follow a normal distribution, $W_i \sim N(\mu_W, \sigma_W^2)$, then the probability of survival in (2.5) is

$$F_W(D_k) = \Phi\left(\frac{D_k - \mu_W}{\sigma_W}\right), \quad \text{for } i = 1,2,\ldots,\infty,\ k = 1 \text{ or } 2, \tag{2.6}$$

where $\Phi(.)$ is the cdf of a standard normal random variable.

For systems that experience the two dependent competing failure processes, the reliability at time t is

$$R(t) = \sum_{i=0}^{\infty} R(t \mid N(t) = i) P(N(t) = i),$$

which is further derived considering the following situations.

1) When no shocks occur by time t, or $N(t) = 0$,

$$R(t \mid N(t) = 0) = P\left(X_S(t) < H \middle| N(t) = 0\right) = P\left(X(t) < H\right).$$

2) When there are shocks occurring by time t, or $N(t) > 0$, there are two mutually exclusive scenarios for successful operation without failure that must be considered, and their probabilities should be summed together for each possible value of $N(t)>0$.

   a) No shocks greater than D_0, or $|S_{D0}| = 0$ where S_{D0} is the set of all shocks greater than D_0 by time t,

$$R_{|S_{D_0}|=0}\left(t \mid N(t) = i > 0\right) = P\left(\bigcap_{j=1}^{N(t)} \{W_j < D_0\}, X_S(t) < H \middle| N(t) = i\right).$$

   b) There exists a $W_j > D_0$ for $j = 1, 2, \ldots, i$, or $|S_{D0}| > 0$, considering the case that the j^{th} shock is the first such shock greater than D_0,

$$R_{|S_{D_0}|>0}\left(t \mid N(t) = i > 0\right) = \sum_{j=1}^{i} P\left(\bigcap_{l=1}^{j-1} \{W_l < D_0\}, D_0 < W_j < D_1, \bigcap_{l=j+1}^{N(t)} \{W_l < D_2\}, X_S(t) < H \middle| N(t) = i\right).$$

Therefore, the reliability at time t is derived to be

$$
\begin{aligned}
R(t) &= P\big(X(t) < H\big)P\big(N(t)=0\big) \\
&\quad + \sum_{i=1}^{\infty}\left(P\left(\bigcap_{j=1}^{N(t)}\{W_j < D_0\}, X_S(t) < H \,\middle|\, N(t)=i\right)\right. \\
&\qquad \left. + \sum_{j=1}^{i} P\left(\bigcap_{l=1}^{j-1}\{W_l < D_0\}, D_0 < W_j < D_1, \bigcap_{l=j+1}^{N(t)}\{W_l < D_2\}, X_S(t) < H \,\middle|\, N(t)=i\right)\right) P(N(t)=i) \\
&= P\big(X(t) < H\big)P\big(N(t)=0\big) \\
&\quad + \sum_{i=1}^{\infty}\left(F_W(D_0)^i P\left(X(t) + \sum_{l=1}^{i} Y_i < H \right)\right. \\
&\qquad \left. + \sum_{j=1}^{i} F_W(D_0)^{j-1} P(D_0 < W_j < D_1) F_W(D_2)^{i-j} P\left(X(t) + \sum_{l=1}^{i} Y_i < H \right)\right) P\big(N(t)=i\big).
\end{aligned} \tag{2.7}
$$

By using Eqs. (2.3) and (2.7), the system reliability for the general case is

$$
\begin{aligned}
R(t) &= G(H,t)e^{-\lambda t} \\
&\quad + \sum_{i=1}^{\infty}\left[F_W(D_0)^i + \sum_{j=1}^{i} F_W(D_0)^{j-1}\big(F_W(D_1) - F_W(D_0)\big) F_W(D_2)^{i-j} \right] \\
&\quad \times \left(\int_0^H G(H-u,t) f_Y^{\langle i\rangle}(u)\,du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}.
\end{aligned} \tag{2.8}
$$

When W_i, Y_i, and β are normally distributed, the reliability function for the more specific case can be expressed as

$$
\begin{aligned}
R(t) &= \Phi\left(\frac{H - \mu_\beta t - \varphi}{\sigma_\beta t} \right) e^{-\lambda t}\big(-\lambda t\big) \\
&\quad + \sum_{i=1}^{\infty}\left[F_W(D_0)^i + \sum_{j=1}^{i} F_W(D_0)^{j-1}\big(F_W(D_1) - F_W(D_0)\big) F_W(D_2)^{i-j} \right] \\
&\quad \times \Phi\left(\frac{H - (\mu_\beta t + \varphi + i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2 + i\sigma_Y{}^2}} \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}.
\end{aligned} \tag{2.9}
$$

2.2.3 Case 2: Generalized δ-shock Model

Figure 2.2 shows a generalized δ-shock model; that is, the hard failure threshold is reduced from D_1 to D_2 when the time lag between two sequential shocks is less than δ. The system fails due to fracture when the shock load exceeds the corresponding hard failure threshold, D_k, k = 1 or 2. The probability that a system survives the applied stress from the i^{th} shock has the same formula as in Eqs. (2.5) or (2.6).

In this generalized δ-shock model, we denote the time lag between two sequential shocks (or inter-arrival time) as B_j, where $B_j = t_{j+1} - t_j$, and t_j is the arrival time of the j^{th} shock in the Poisson process for j= 1, 2, …,∞. Therefore, B_j, for j= 1, 2, …,∞, are i.i.d. exponential random variables with distribution parameter λ. For systems that experience the two dependent failure processes, the reliability at time t is

$$R(t) = \sum_{i=0}^{\infty} R(t \mid N(t) = i) P(N(t) = i),$$

which is further derived considering the following situations.

i) When no shocks happen by time t, or $N(t) = 0$,

$$R(t \mid N(t) = 0) = P\left(X_S(t) < H \middle| N(t) = 0\right) = P\left(X(t) < H\right).$$

ii) When only one shock happens by time t, or $N(t) = 1$,

$$R(t \mid N(t) = 1) = P\left(W_1 < D_1, X_S(t) < H \middle| N(t) = 1\right) = P\left(W_1 < D_1, X(t) + Y_1 < H\right).$$

iii) When there are more than one shock occurring by time t, or $N(t) > 1$, we have two scenarios to consider for successful operation without failure, and their probabilities can be summed together.

a) No $B_j < \delta$, for j = 1, 2, …, i-1, or $|S_\delta| = 0$, where S_δ denotes the set of all shock inter-arrival times less than δ,

$$R_{|S_\delta|=0}\left(t \mid N(t) = i, 2 \le i \le \lfloor t/\delta \rfloor + 1\right) = P\left(\bigcap_{l=1}^{N(t)} \{W_l < D_1\}, \bigcap_{l=1}^{N(t)-1} \{B_l \ge \delta\}, X_S(t) < H \middle| N(t) = i\right),$$

where $\lfloor t/\delta \rfloor$ takes the integer part of t/δ, and it is the maximum number of time lags by time t given that all shock inter-arrival times are larger than or equal to δ. Accordingly, the maximum number of shocks arrived by time t is $\lfloor t/\delta \rfloor + 1$.

b) There exists a $B_j < \delta$ for j = 1, 2, …, i-1, or $|S_\delta| > 0$, considering the case that the j^{th}

time lag is the first such time interval less than δ. Because it is not possible to have more than $\lfloor t/\delta \rfloor$ inter-arrival times before t with all $B_l \ge \delta$, $j - 1$ must be less than or equal to $\lfloor t/\delta \rfloor$, or $j \le \lceil t/\delta \rceil$ when t/δ is not an integer. When t/δ is an integer, j–1 should be less than or equal to $\lfloor t/\delta \rfloor - 1$, or $j \le \lfloor t/\delta \rfloor = \lceil t/\delta \rceil$. To consider this range of j in the reliability formulation, we need to split the range of i as

$$
\begin{aligned}
&R_{|S_\delta|>0}(t \mid N(t)=i, 2 \le i \le \lceil t/\delta \rceil + 1) \\
&= \sum_{j=1}^{i-1} P\left(\bigcap_{l=1}^{j+1} \{W_l < D_1\}, \bigcap_{l=j+2}^{N(t)} \{W_l < D_2\}, \bigcap_{l=1}^{j-1} \{B_l \ge \delta\}, B_j < \delta, X_S(t) < H \,\middle|\, N(t)=i \right)
\end{aligned}
$$

and

$$
\begin{aligned}
&R_{|S_\delta|>0}(t \mid N(t)=i, \lceil t/\delta \rceil + 2 \le i \le \infty) \\
&= \sum_{j=1}^{\lceil t/\delta \rceil} P\left(\bigcap_{l=1}^{j+1} \{W_l < D_1\}, \bigcap_{l=j+2}^{N(t)} \{W_l < D_2\}, \bigcap_{l=1}^{j-1} \{B_l \ge \delta\}, B_j < \delta, X_S(t) < H \,\middle|\, N(t)=i \right).
\end{aligned}
$$

By considering all situations from *i*) to *iii*), we have the reliability function at time t as

$$
\begin{aligned}
R(t) &= \sum_{i=0}^{\infty} R(t \mid N(t)=i) P\left(N(t)=i\right) \\
&= P\left(X(t) < H\right) P(N(t)=0) + P(W_1 < D_1, X_S(t) < H \mid N(t)=1) P\left(N(t)=1\right) \\
&\quad + \sum_{i=2}^{\lfloor t/\delta \rfloor + 1} P\left(\bigcap_{l=1}^{N(t)} \{W_l < D_1\}, \bigcap_{l=1}^{N(t)-1} \{B_l \ge \delta\}, X_S(t) < H \,\middle|\, N(t)=i \right) P\left(N(t)=i\right) \\
&\quad + \sum_{i=2}^{\lceil t/\delta \rceil + 1} \sum_{j=1}^{i-1} P\left(\bigcap_{l=1}^{j+1} \{W_l < D_1\}, \bigcap_{l=j+2}^{N(t)} \{W_l < D_2\}, \bigcap_{l=1}^{j-1} \{B_l \ge \delta\}, B_j < \delta, X_S(t) < H \,\middle|\, N(t)=i \right) P\left(N(t)=i\right) \\
&\quad + \sum_{i=\lceil t/\delta \rceil + 2}^{\infty} \sum_{j=1}^{\lceil t/\delta \rceil} P\left(\bigcap_{l=1}^{j+1} \{W_l < D_1\}, \bigcap_{l=j+2}^{N(t)} \{W_l < D_2\}, \bigcap_{l=1}^{j-1} \{B_l \ge \delta\}, B_j < \delta, X_S(t) < H \,\middle|\, N(t)=i \right) P\left(N(t)=i\right),
\end{aligned}
$$

or

$$
\begin{aligned}
R(t) &= P(X(t) < H)P(N(t)=0) + F_W(D_1)P(X(t)+Y_1 < H)P(N(t)=1) \\
&+ \sum_{i=2}^{\lfloor t/\delta \rfloor +1} F_W(D_1)^i P(B \ge \delta)^{i-1} P\left(X(t) + \sum_{l=1}^{i} Y_i < H \right) P\left(N(t)=i\right) \\
&+ \sum_{i=2}^{\lceil t/\delta \rceil +1} \sum_{j=1}^{i-1} F_W(D_1)^{j+1} F_W(D_2)^{i-j-1} P(B \ge \delta)^{j-1} P(B<\delta) P\left(X(t) + \sum_{l=1}^{i} Y_i < H \right) P\left(N(t)=i\right) \\
&+ \sum_{i=\lceil t/\delta \rceil +2}^{\infty} \sum_{j=1}^{\lceil t/\delta \rceil} F_W(D_1)^{j+1} F_W(D_2)^{i-j-1} P(B \ge \delta)^{j-1} P(B<\delta) P\left(X(t) + \sum_{l=1}^{i} Y_i < H \right) P\left(N(t)=i\right).
\end{aligned}
\tag{2.10}
$$

In this reliability formulation, certain combinations of B_l are not possible given the condition on $N(t) = i$. The reliability function in Eq. (2.10) actually provides a close approximation by constraining the limits on the sum to prevent combinations that cannot happen. By substituting Eq. (2.3) into Eq. (2.10), the system reliability for the two dependent failure processes considering the generalized δ-shock model is derived as

$$
\begin{aligned}
R(t) &= G(H,t)e^{-\lambda t} + F_W(D_1)\left(\int_0^H G(H-u,t) f_Y(u)du \right) \lambda t e^{-\lambda t} \\
&+ \sum_{i=2}^{\lfloor t/\delta \rfloor +1} F_W(D_1)^i e^{-(i-1)\lambda\delta} \left(\int_0^H G(H-u,t) f_Y^{\langle i \rangle}(u)du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!} \\
&+ \sum_{i=2}^{\lceil t/\delta \rceil +1} \sum_{j=1}^{i-1} F_W(D_1)^{j+1} F_W(D_2)^{i-j-1} e^{-(j-1)\lambda\delta} \left(1 - e^{-\lambda\delta}\right) \left(\int_0^H G(H-u,t) f_Y^{\langle i \rangle}(u)du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!} \\
&+ \sum_{i=\lceil t/\delta \rceil +2}^{\infty} \sum_{j=1}^{\lceil t/\delta \rceil} F_W(D_1)^{j+1} F_W(D_2)^{i-j-1} e^{-(j-1)\lambda\delta} \left(1 - e^{-\lambda\delta}\right) \left(\int_0^H G(H-u,t) f_Y^{\langle i \rangle}(u)du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}.
\end{aligned}
\tag{2.11}
$$

If we have normally distributed W_i, Y_i, and β, the system reliability in Eq. (2.11) is expressed as

$$
\begin{aligned}
R(t) &= \Phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta t}\right)e^{-\lambda t} + F_W(D_1)\Phi\left(\frac{H-(\mu_\beta t+\varphi+\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2+\sigma_Y{}^2}}\right)\lambda t e^{-\lambda t} \\
&+ \sum_{i=2}^{\lfloor t/\delta\rfloor+1} F_W(D_1)^i e^{-(i-1)\lambda\delta}\Phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2+i\sigma_Y{}^2}}\right)\frac{e^{-\lambda t}(\lambda t)^i}{i!} \\
&+ \sum_{i=2}^{\lceil t/\delta\rceil+1}\sum_{j=1}^{i-1} F_W(D_1)^{j+1}F_W(D_2)^{i-j-1}e^{-(j-1)\lambda\delta}\left(1-e^{-\lambda\delta}\right)\Phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2+i\sigma_Y{}^2}}\right)\frac{e^{-\lambda t}(\lambda t)^i}{i!} \\
&+ \sum_{i=\lceil t/\delta\rceil+2}^{\infty}\sum_{j=1}^{\lceil t/\delta\rceil} F_W(D_1)^{j+1}F_W(D_2)^{i-j-1}e^{-(j-1)\lambda\delta}\left(1-e^{-\lambda\delta}\right)\Phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2+i\sigma_Y{}^2}}\right)\frac{e^{-\lambda t}(\lambda t)^i}{i!}.
\end{aligned}
\tag{2.12}
$$

Based on the reliability function in Eq. (2.12), the pdf of the failure time, $f_T(t)$, can be derived and plotted.

2.2.4 Case 3: Generalized m-shock Model

Figure 2.3 shows a generalized *m*-shock model; that is, the hard failure threshold decreases from D_1 to D_2 right after m shocks larger than a threshold D_0. The system fails due to fracture when the shock load exceeds the corresponding maximal fracture strength, D_k, for $k = 1$ or 2. The probability that a unit survives the applied stress from the i^{th} shock has the same formula as in Eq. (2.5) or Eq. (2.6).

For systems that experience the two dependent failure processes, the system reliability at time t is expressed as

$$R(t) = \sum_{i=0}^{\infty} R(t \mid N(t)=i)P(N(t)=i),$$

which is further derived considering the following situations:

1) When no shocks occur by time t, or $N(t) = 0$,

$$R(t \mid N(t)=0) = P\left(X_S(t) < H \middle| N(t)=0\right) = P\left(X(t) < H\right).$$

2) When the number of shocks by time t is between 1 and m, or $N(t) = i$, $1 \le i \le m$,

$$R(t \mid N(t)=i, 1 \le i \le m) = P\left(\bigcap_{l=1}^{N(t)} \{W_l < D_1\}, X_S(t) < H \middle| N(t)=i \right).$$

3) When there are more than m shocks, or $N(t) = i > m$, the hard failure threshold value may or may not reduce from D_1 to D_2 based on the following two scenarios (see Appendix):

a) Less than m shocks greater than D_0, or $|S_{D_0}| < m$, where S_{D_0} is the set of all shocks greater than D_0,

$$R_{|S_{D_0}|<m}\left(t \mid N(t)=i>m\right) = \sum_{j=0}^{m-1} \binom{i}{j} P\left(\bigcap_{l=1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_0\}, X_S(t) < H \middle| N(t)=i \right).$$

b) More than or equal to m shocks greater than D_0, or $|S_{D_0}| \ge m$,

$$R_{|S_{D_0}|\ge m}\left(t \mid N(t)=i>m\right)$$
$$= \sum_{j=m}^{i} \binom{j-1}{m-1} P\left(\bigcap_{l=1}^{j-m} \{W_l < D_0\}, \bigcap_{l=j-m+1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_2\}, X_S(t) < H \middle| N(t)=i \right).$$

The reliability function by time t is therefore given as

$$\begin{aligned}
R(t) &= \sum_{i=0}^{\infty} R\left(t \mid N(t)=i\right) P(N(t)=i) \\
&= P\left(X(t) < H\right) P(N(t)=0) + \sum_{i=1}^{m} P\left(\bigcap_{l=1}^{N(t)} \{W_l < D_1\}, X_S(t) < H \middle| N(t)=i \right) P(N(t)=i) \\
&\quad + \sum_{i=m+1}^{\infty} \left(\sum_{j=0}^{m-1} \binom{i}{j} P\left(\bigcap_{l=1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_0\}, X_S(t) < H \middle| N(t)=i \right) \right. \\
&\quad \left. + \sum_{j=m}^{i} \binom{j-1}{m-1} P\left(\bigcap_{l=1}^{j-m} \{W_l < D_0\}, \bigcap_{l=j-m+1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_2\}, X_S(t) < H \middle| N(t)=i \right) \right) P(N(t)=i) \\
&= P\left(X(t) < H\right) P\left(N(t)=0\right) + \sum_{i=1}^{m} F_W(D_1)^i P\left(X(t) + \sum_{l=1}^{i} Y_l < H \right) P\left(N(t)=i\right) \\
&\quad + \sum_{i=m+1}^{\infty} \left(\sum_{j=0}^{m-1} \binom{i}{j} \left(F_W(D_1) - F_W(D_0)\right)^j F_W(D_0)^{i-j} + \sum_{j=m}^{i} \binom{j-1}{m-1} F_W(D_0)^{j-m} \left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_2)^{i-j} \right) \\
&\quad \times P\left(X(t) + \sum_{l=1}^{i} Y_l < H \right) P\left(N(t)=i\right).
\end{aligned} \tag{2.13}$$

By combining Eqs. (2.3) and (2.13), the system reliability for the two dependent failure processes considering the generalized m-shock model is derived as

$$R(t) = G(H,t)e^{-\lambda t} + \sum_{i=1}^{m} F_W(D_1)^i \left(\int_0^H G(H-u,t) f_Y^{\langle i \rangle}(u) du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}$$
$$+ \sum_{i=m+1}^{\infty} \left(\sum_{j=0}^{m-1} \binom{i}{j} \left(F_W(D_1) - F_W(D_0) \right)^j F_W(D_0)^{i-j} \right.$$
$$\left. + \left(F_W(D_1) - F_W(D_0) \right)^m \sum_{j=m}^{i} \binom{j-1}{m-1} F_W(D_0)^{j-m} F_W(D_2)^{i-j} \right) \tag{2.14}$$
$$\times \left(\int_0^H G(H-u,t) f_Y^{\langle i \rangle}(u) du \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}.$$

When W_i, Y_i, and β are normally distributed, the reliability function can be expressed as

$$R(t) = \Phi\left(\frac{H - \mu_\beta t - \varphi}{\sigma_\beta t} \right) e^{-\lambda t} + \sum_{i=1}^{m} F_W(D_1)^i \Phi\left(\frac{H - (\mu_\beta t + \varphi + i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2 + i\sigma_Y{}^2}} \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}$$
$$+ \sum_{i=m+1}^{\infty} \left(\sum_{j=0}^{m-1} \binom{i}{j} \left(F_W(D_1) - F_W(D_0) \right)^j F_W(D_0)^{i-j} \right.$$
$$\left. + \left(F_W(D_1) - F_W(D_0) \right)^m \sum_{j=m}^{i} \binom{j-1}{m-1} F_W(D_0)^{j-m} F_W(D_2)^{i-j} \right) \tag{2.15}$$
$$\times \Phi\left(\frac{H - (\mu_\beta t + \varphi + i\mu_Y)}{\sqrt{\sigma_\beta{}^2 t^2 + i\sigma_Y{}^2}} \right) \frac{e^{-\lambda t}(\lambda t)^i}{i!}.$$

Based on Eq. (2.15), the pdf of the failure time, $f_T(t)$, can be derived and plotted.

2.3 Maintenance Policies

In this section, we consider two preventive maintenance policies: the block replacement policy (BRP), and the age replacement policy (ARP). We compare them to determine which one is more beneficial for different reliability models. The basic assumptions for the two preventive replacement policies include the following.

1) The systems are packaged and sealed together, making it impossible or impractical to

repair, e.g., MEMS.

2) If the system fails, it can be detected immediately, and corrective replacement may take place.

3) Replacement is done instantaneously, and the replacement time is negligible.

An average long-run maintenance cost rate is implemented to evaluate the performance of the maintenance policies, where the maintenance cost considered includes the cost of unscheduled corrective replacement, and the cost of scheduled preventive replacement. Let $C(t)$ denote the cumulative maintenance cost until time t. From renewal theory, the average long-run maintenance cost per unit time, $\lim_{t\to\infty}(C(t)/t)$, can be evaluated by [54, 90]

$$\lim_{t\to\infty}\left(\frac{C(t)}{t}\right)=\frac{\text{Expected maintenance cost inccured in a cycle}}{\text{Expected length of a cycle}}, \tag{2.16}$$

where a cycle is defined as either a time interval between the installation of a system and the first replacement, or a time interval between two consecutive replacements. The successive cycles together with the costs incurred in each cycle constitute a renewal process.

2.3.1 Block Replacement Policy

Under the BRP, the system is preventively replaced with a new one at pre-scheduled times, $k\rho$ (k=1, 2…), independent of the failure history of the system. However, if failure occurs before a scheduled replacement time, corrective replacement is done. Based on Eq. (2.16), the average maintenance cost per unit time over an infinite time horizon for BRP, $B(\rho)$, is given by [91, 92],

$$B(\rho) = \lim_{t \to \infty} \left(\frac{C(t)}{t} \right) = \frac{C_F M(\rho) + C_R}{\rho}, \tag{2.17}$$

where $M(\rho)$ is the expected number of failures or unscheduled replacements within ρ, i.e., the renewal equation [91]:

$$M(\rho) = F(\rho) + \int_0^{\rho} M(\rho - u) f(u) du, \tag{2.18}$$

where $F(\rho)$ is the cdf of the failure time by ρ, and $f(\rho)$ is the pdf of the failure time at ρ.

A closed form solution for the renewal equation $M(\rho)$ is difficult to obtain for most failure time distributions. Various methods have been proposed to approximate the solution of the renewal equation [93, 94]. In this chapter, we apply the approximation method for the renewal equation in Bartholomew [93] as

$$M(\rho) = F(\rho) + \int_0^{\rho} F^2(t)dt \Big/ \int_0^{\rho} R(t)dt. \tag{2.19}$$

By minimizing $B(\rho)$ in Eq. (2.17), the optimal solution of the fixed time interval ρ can be obtained, i.e.,

$$\rho^* = \arg\min \{B(\rho)\}. \tag{2.20}$$

There are many optimization methods to solve this unconstrained nonlinear optimization problem, such as line search methods, gradient methods, Newton's method, and Quasi-Newton methods [95, 96]. BFGS Quasi-Newton method is one of the most popular methods for unconstrained nonlinear optimization problems, and has been used in our maintenance optimization.

2.3.2 Age Replacement Policy

Under the age replacement policy (ARP), the system is preventively replaced at its age τ, or correctively replaced at failure, whichever occurs first. Based on Eq. (2.16), the average maintenance cost per unit time over an infinite time horizon for ARP, $A(\tau)$, takes the form

$$A(\tau) = \frac{\text{Expected cost incurred in a cycle}}{\text{Expected renewal cycle}} = \frac{E[TC]}{E[U]}. \tag{2.21}$$

From [5, 91, 92], we have

$$E[TC] = C_F\left(1 - R(\tau)\right) + C_R R(\tau),$$

$$E[U] = \tau R(\tau) + \int_0^\tau tf(t) = \int_0^\tau R(t)dt .$$

Then the optimal solution at age τ can be found by minimizing $A(\tau)$:

$$\tau^* = \arg\min\left\{A(\tau) = E[TC]/E[U]\right\} . \tag{2.22}$$

We again use the BFGS Quasi-Newton method to solve this unconstrained nonlinear optimization problem.

2.4 Numerical Examples

A micro-engine developed at Sandia National Laboratories consists of orthogonal linear comb drive actuators mechanically connected to a rotating gear [80, 97]. The linear displacement of the comb drives is transformed into the rotation of the gear via a pin joint. The gear rotates about a hub, which is anchored to the substrate. Wear of rubbing surfaces is the dominant mode of failure for the micro-engine, especially the rubbing surface between the gear and the pin joint, which usually causes a broken pin. Micro-engine shock experiments show debris on the surface of the die when exposed to shocks, and the gear hub is broken when the magnitude of the shock is above a certain level [80]. Therefore, the micro-engine experiences two competing failure processes: soft failures due to the wear degradation and debris from external shocks, and hard failures due to hub fracture. Before the hub is broken, the shocks it sustains may initiate cracks in the hub, which reduces the material strength to survive further shocks, i.e., the dependency between the shock process and hard failure threshold. The values for the parameters in Eqs. (2.9), (2.12), and (2.15) for reliability analysis are given in Table 2.1.

Table 2.1: Parameter values for microengine reliability analysis

Parameters	Values	Sources
H	0.00125μm^3	Tanner and Dugger [97]
D_0	1.2Gpa	Assumption
D_2	1.4Gpa	Assumption
D_1	1.5Gpa (for polysilicon material used for the hub)	Tanner and Dugger [97]
φ	0	Tanner and Dugger [97]
β	$\sim N(\mu_\beta,\sigma_\beta^2)$ $\mu_\beta = 8.4823\times10^{-9}$μm^3 and $\sigma_\beta = 6.0016\times10^{-10}$μm^3	Tanner and Dugger [97] & Peng et al. [54]
λ	5×10^{-5}/ revolutions	Assumption
Y_i	$\sim N(\mu_Y,\sigma_Y^2)$ $\mu_Y = 1.2\times10^{-4}$μm^3 and $\sigma_Y = 2\times10^{-5}$μm^3	Assumption
W_i	$\sim N(\mu_W,\sigma_W^2)$ $\mu_W = 1.2Gpa$ and $\sigma_W=0.2Gpa$	Assumption
a	1.0×10^{-4}μm^3/Gpa	Assumption
m	2	Assumption
δ	0.2×10^4 revolutions	Assumption

2.4.1 Reliability Analysis for Case 1

For the generalized extreme shock model, the reliability function $R(t)$ in Eq. (2.9) and the pdf of failure time $f_T(t)$ are calculated and plotted in Figure 2.4. A sensitivity analysis was performed to evaluate the effects of the model parameters on $R(t)$ and $f_T(t)$. The model parameters evaluated include the ratios of D_0/D_1, and D_2/D_1. The results are shown in Figure 2.5, and Figure 2.6, respectively.

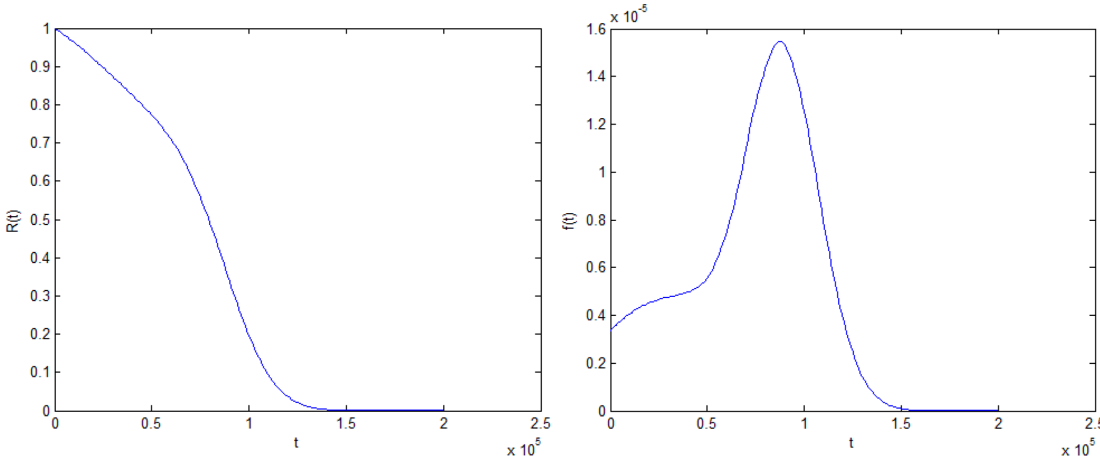

Figure 2.4: Plots of $R(t)$ and $f_T(t)$ for Case 1

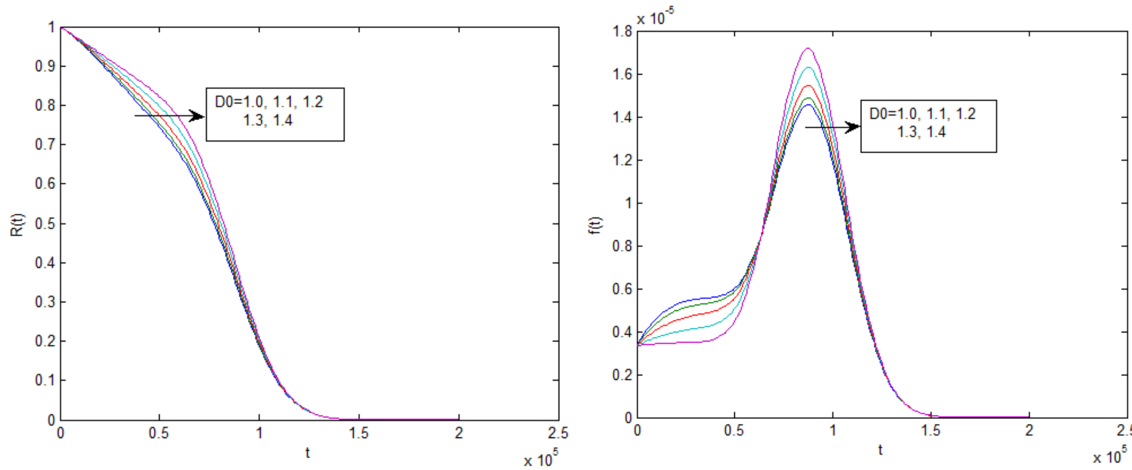

Figure 2.5: Sensitivity analysis of $R(t)$ and $f_T(t)$ on D_0/D_1 for Case 1 (D_1=1.5GPa)

In Figure 2.5, the ratio between the critical threshold and the hard failure threshold, D_0/D_1, has a significant impact on both $R(t)$ and $f_T(t)$. When the ratio D_0/D_1 increases (D_0 rises from 1.0Gpa to 1.4Gpa at a fixed D_1 of 1.5Gpa), $R(t)$ shifts to the right, and $f_T(t)$ is smaller first and then larger after a critical point. This result indicates that a larger ratio D_0/D_1 ensures a better reliability performance.

In Figure 2.6, we can observe that both $R(t)$ and $f_T(t)$ are susceptible to the ratio between the reduced hard failure threshold and the regular hard failure threshold, D_2/D_1. When the ratio D_2/D_1 increases (D_2 increases from 1.2Gpa to 1.5Gpa at a fixed D_1 of

1.5Gpa), $R(t)$ shifts to right, and $f_T(t)$ gets smaller first and then larger after a critical point. Therefore, units with a larger ratio D_2/D_1 can have a better reliability performance.

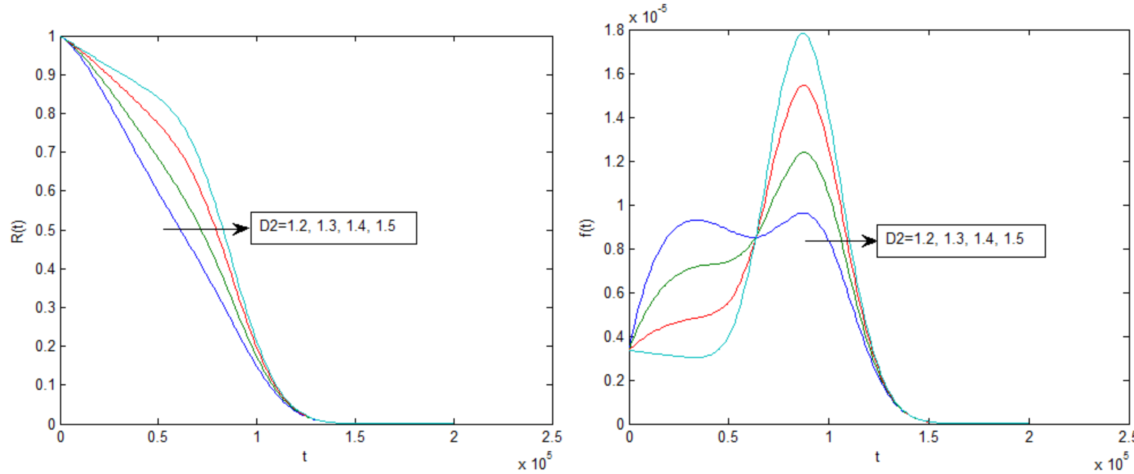

Figure 2.6: Sensitivity analysis of $R(t)$ and $f_T(t)$ on D_2/D_1 for Case 1 (D_1=1.5Gpa)

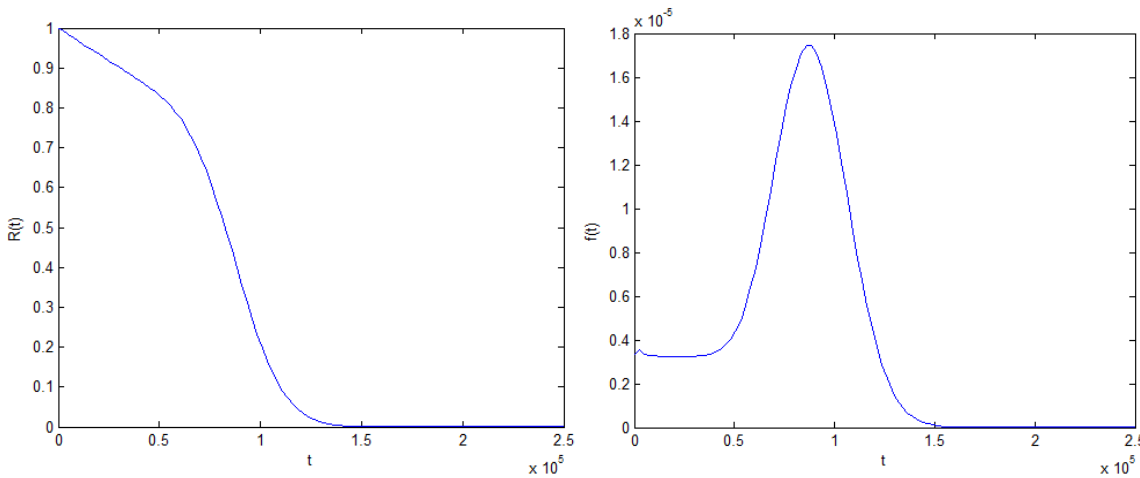

Figure 2.7: Plots of $R(t)$ and $f_T(t)$ for Case 2

2.4.2 Reliability Analysis for Case 2

For the generalized δ-shock model, the reliability function $R(t)$ in Eq. (2.12), and the pdf of failure time $f_T(t)$, are calculated and plotted in Figure 2.7. To assess the effects of the model parameters on $R(t)$ and $f_T(t)$, a sensitivity analysis was performed. The model parameters assessed include the critical time interval δ, and the ratio between the reduced hard failure threshold and the regular hard failure threshold D_2/D_1. The results are shown in Figure 2.8, and Figure 2.9, respectively.

From Figure 2.8, we can see that the time interval threshold, δ, has a significant impact on both the reliability function and the failure time distribution. When δ increases from 0.1×10^4 to 2.0×10^4 revolutions, both $R(t)$ and $f_T(t)$ are sensitive to δ: $R(t)$ shifts to the left, and $f_T(t)$ is smaller first and then larger after a critical point. This result implies that the reliability performance is better when δ is smaller, because the hard failure threshold is less likely to reduce from D_1 to D_2.

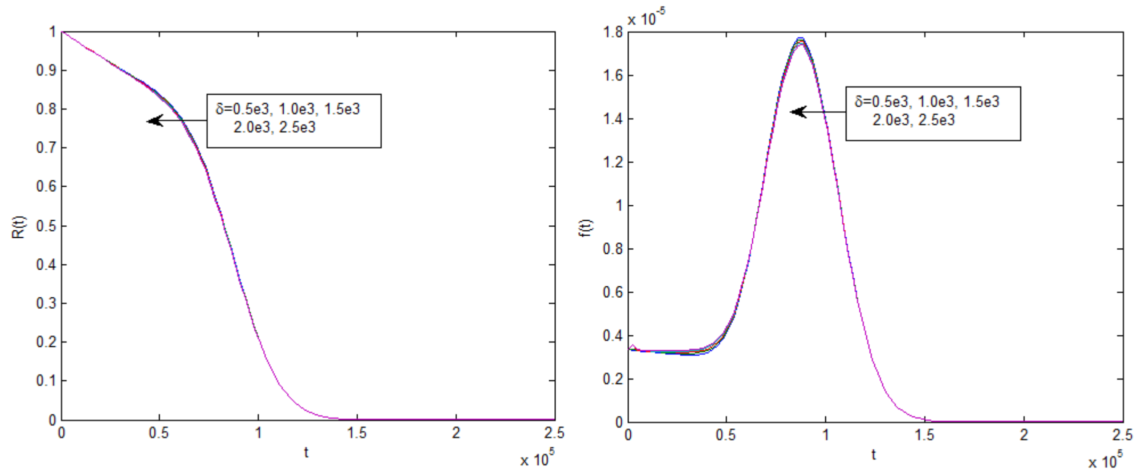

Figure 2.8: Sensitivity analysis of $R(t)$ and $f_T(t)$ on δ for Case 2

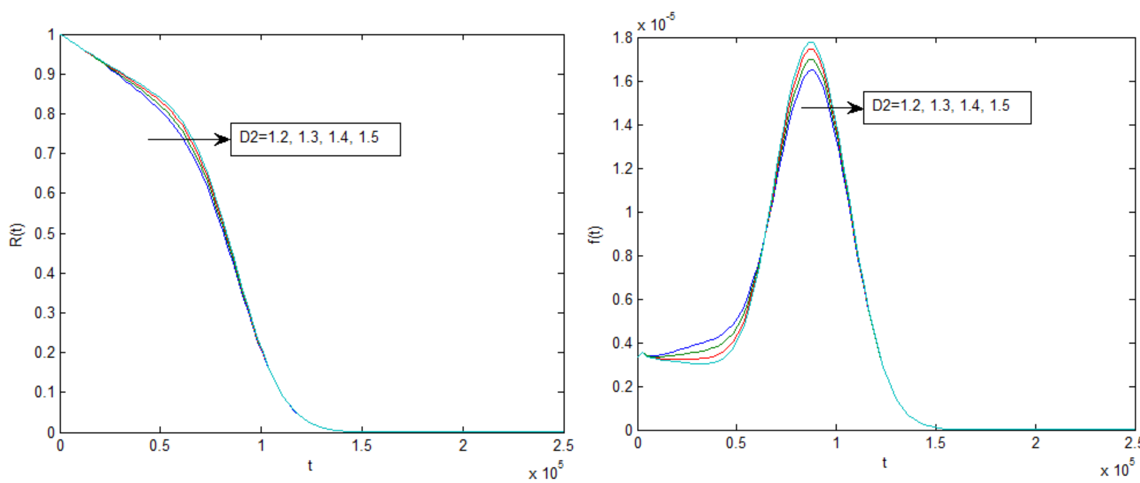

Figure 2.9: Sensitivity analysis of $R(t)$ and $f_T(t)$ on D_2/D_1 for Case 2 (D_1=1.5Gpa)

Figure 2.9 shows that both $R(t)$ and $f_T(t)$ are sensitive to D_2/D_1. When the ratio D_2/D_1

increases (D_2 raises from 1.2Gpa to 1.5Gpa at a fixed D_1 of 1.5Gpa), both $R(t)$ and $f_T(t)$ are sensitive to D_2: $R(t)$ shifts to right, and $f_T(t)$ is smaller first and then larger after a critical point, which means that the reliability performance is better when the ratio D_2/D_1 gets larger. Especially, when D_2 equals D_1 (1.5Gpa), plots of both $R(t)$ and $f_T(t)$ are the same as those in Peng *et al.* [54], in which the hard failure threshold value remains constant.

2.4.3 Reliability Analysis for Case 3

For the generalized *m*-shock model, the reliability function $R(t)$ in Eq. (2.15) and the pdf of failure time $f_T(t)$ are plotted in Figure 2.10. A sensitivity analysis was also conducted. The model parameters assessed include the number of shocks greater than a critical threshold m, and the ratios D_0/D_1 and D_2/D_1. The results are shown in Figures 2.11-2.13, respectively.

From Figure 2.11, we can see that the number of shocks greater than D_0, m, has a significant impact on both the reliability function and the failure time distribution. When m increases from 1 to 5, both $R(t)$ and $f_T(t)$ are sensitive to m: $R(t)$ shifts to right, and $f_T(t)$ is smaller first and then larger after a critical point. This result indicates that a larger m ensures better reliability performance. In addition, when m is greater than 3, both $R(t)$ and $f_T(t)$ tend to be almost the same shape, which is related to the average shock arrival rate of 5×10^{-5}/ revolutions.

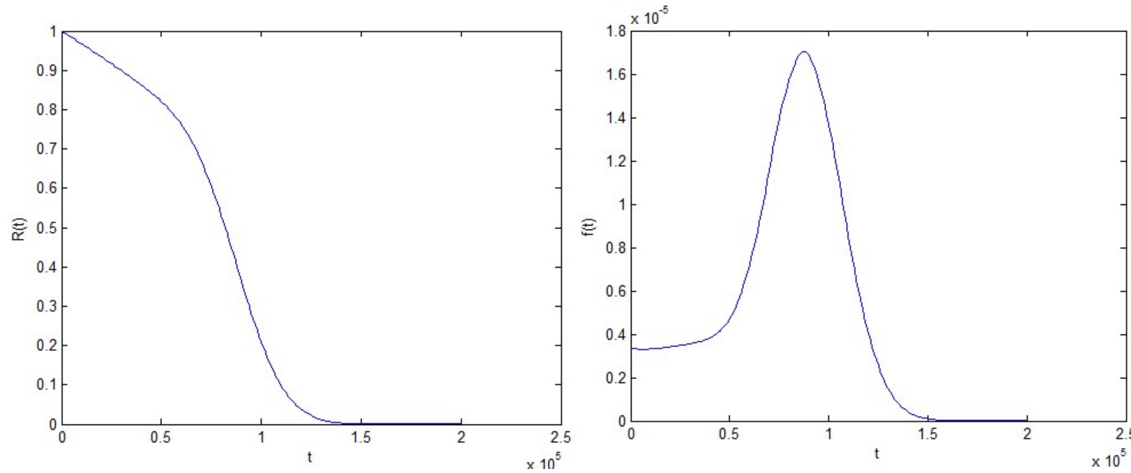

Figure 2.10: Plots of $R(t)$ and $f_T(t)$ for Case 3

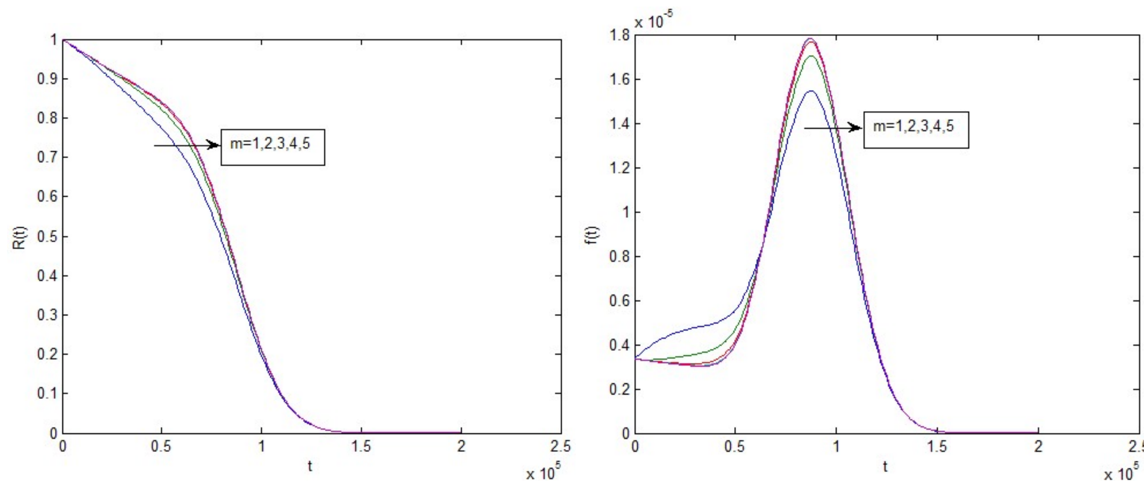

Figure 2.11: Sensitivity analysis of $R(t)$ and $f_T(t)$ on m for Case 3

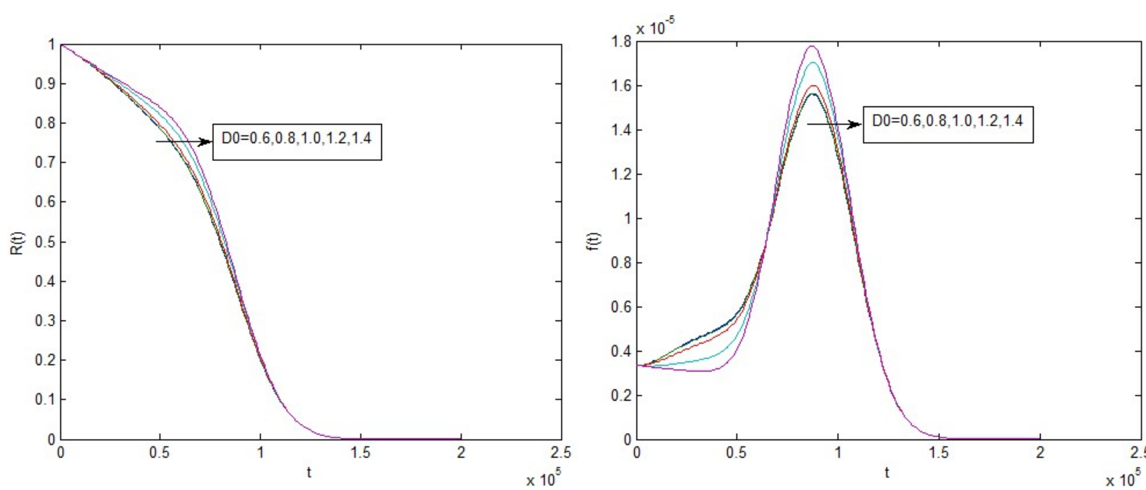

Figure 2.12: Sensitivity analysis of $R(t)$ and $f_T(t)$ on D_0/D_1 for Case 3 (D_1=1.5Gpa)

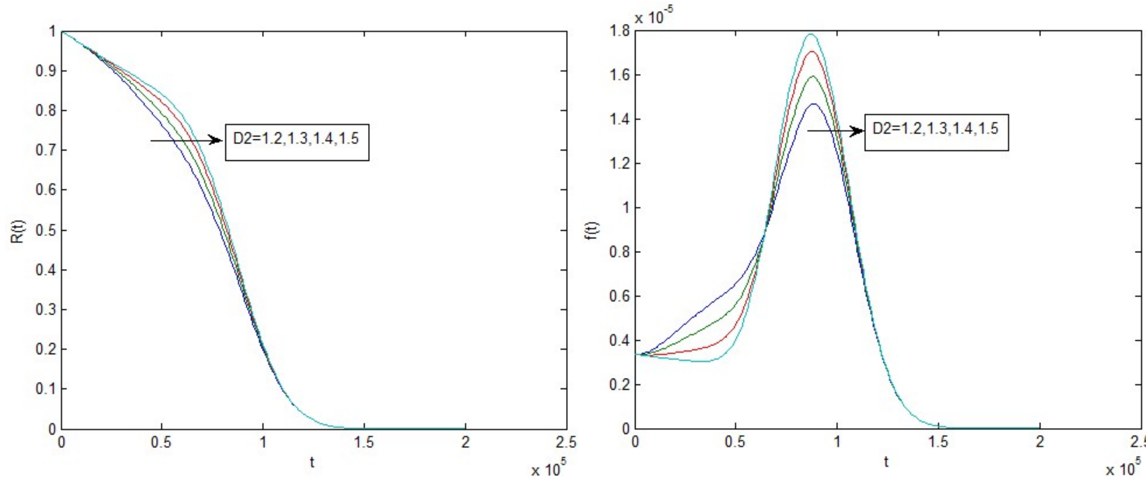

Figure 2.13: Sensitivity analysis of $R(t)$ and $f_T(t)$ on D_2/D_1 for Case 3 (D_1=1.5Gpa)

In Figure 2.12, the ratio D_0/D_1, has a great impact on both $R(t)$ and $f_T(t)$. When the ratio D_0/D_1 increases (D_0 rises from 0.6Gpa to 1.4Gpa at a fixed D_1 of 1.5Gpa), both $R(t)$ and $f_T(t)$ are sensitive to D_0: $R(t)$ shifts to right, and $f_T(t)$ is smaller first and then larger after a critical point. This result implies that, when the ratio D_0/D_1 is larger, the system reliability performance improves.

In Figure 2.13, we can observe that both $R(t)$ and $f_T(t)$ are susceptible to the ratio D_2/D_1: $R(t)$ shifts to right, and $f_T(t)$ is smaller first and then larger after a critical point. Therefore, systems with a larger D_2/D_1 can have a better reliability performance.

2.4.4 Optimal Maintenance Policies

Optimal maintenance decisions were determined for all cases under both maintenance policies. The results indicate that the new model can provide meaningful, useful reliability assessments for a new class of reliability problems with a behavior not adequately addressed previously.

For an example set of cost parameters, C_F=\$1, C_R=\$0.2, so C_R/C_F =0.2, we solve the unconstrained nonlinear problems for BRP in Eq. (2.17), and ARP in Eq. (2.21), using

BFGS Quasi-Newton method. The optimal solutions obtained for each case under each maintenance policy are shown in Table 2.2. For the three cases, age replacement policy provides a lower maintenance cost rate between two successive preventive replacements. Therefore, the age replacement policy outperforms the block replacement policy for the three cases.

Table 2.2: Optimal solutions for maintenance strategies (C_F=\$1, C_R=\$0.2)

	Generalized Extreme Shock Model (Case 1)	**Generalized δ-shock model (Case 2)**	**Generalized *m*-shock model (Case 3)**
BRP	ρ^*=57193.1128, $B(\rho^*)$= \$9.3643×10^{-6}	ρ^*= 58843.9306, $B(\rho^*)$= \$7.3129×10^{-6}	ρ^*= 58835.4787, $B(\rho^*)$= \$7.5807×10^{-6}
ARP	τ^*= 56937.9021, $A(\tau^*)$ =\$9.0026×10^{-6}	τ^*= 59387.3750, $A(\tau^*)$ =\$7.0233×10^{-6}	τ^*= 59301.6819, $A(\tau^*)$ = \$7.2728×10^{-6}

A sensitivity analysis for Case 1-3 was performed to analyze the effects of the cost parameters, C_R/C_F, on the optimal solutions of the maintenance models. Because the patterns in the sensitivity analysis are similar for the three cases, only the results for Case 2 are presented for BRP, and ARP in Figures 2.14, and 2.15, respectively.

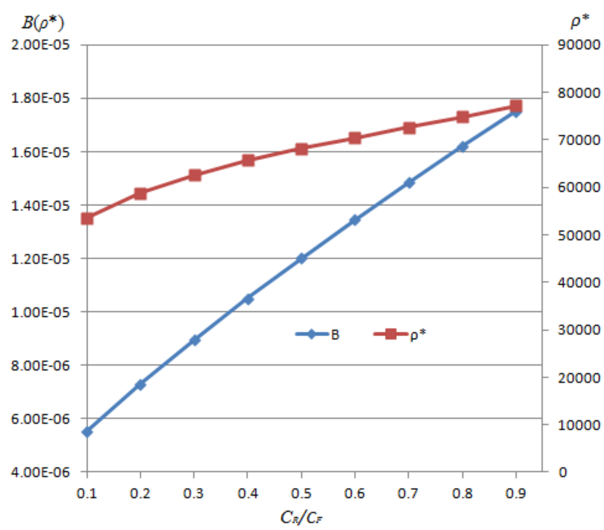

Figure 2.14: Sensitivity analysis of $B(\rho^*)$ and ρ^* on C_R/C_F

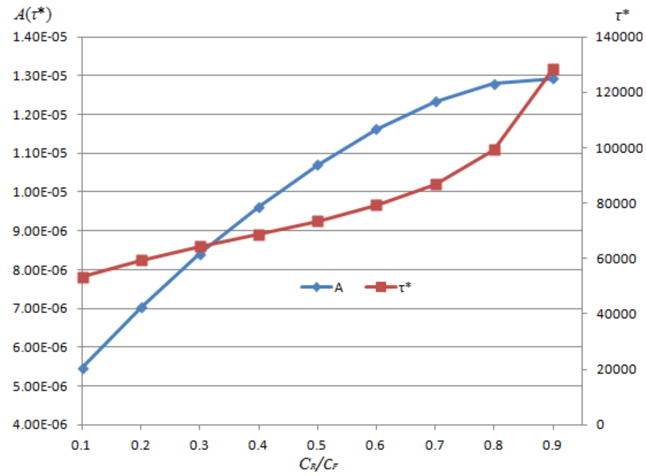

Figure 2.15: Sensitivity analysis of $A(\tau^*)$ and τ^* on C_R/C_F

When the ratio C_R/C_F changes from 0.1 to 0.9 (C_R increases from \$0.1 to \$0.9 while C_F=\$1), the minimum cost rate of the block replacement policy $B(\rho^*)$ increases from \$$5.54\times10^{-6}$ to \$$1.75\times10^{-5}$, and the optimal fixed replacement interval ρ^* increases from 53,589 to 77,239 revolutions. Similarly, for the age replacement policy, the minimum cost rate $A(\tau^*)$ increases from \$$5.46\times10^{-6}$ to \$$1.29\times10^{-5}$, and the optimal system age τ^* increases from 53,261 to 128,570 revolutions. This result implies that the unit should be replaced more frequently when the ratio C_R/C_F gets smaller. Besides, under the same failure replacement cost and preventive replacement cost, the optimal block replacement cost rate is always larger than the age replacement cost rate, and the optimal fixed replacement interval in BRP is always less than the optimal system replacement age in ARP.

2.5 Conclusions

In this chapter, we proposed new models to predict reliability and optimize maintenance for MDCFP when the failure thresholds can shift according to the patterns of shocks. This work is an entirely new research area that extends our previous work in dependent competing failure processes, which offers some distinct advantages for design and reliability problems when the component tolerance or resistance to failure reduces.

The two dependent failure processes considered in the MDCFP are soft failure caused by continuous degradation together with additional abrupt degradation due to a shock process, and hard failure caused by the instantaneous stress from the same shock process. These two failure processes are correlated or dependent in two ways: 1) the two failure processes are dependent due to the same shock process, and 2) the hard failure threshold level is dependent on the shock process. Most previous research considers constant failure thresholds, which may not be appropriate for some devices that are deteriorating due to shocks and degradation. In this chapter, three cases of dependency between the shock process and the hard failure threshold level are studied: 1) the hard failure threshold value changes to a lower level when the first shock is recorded above a critical value, or a generalized extreme shock model; 2) the hard failure threshold value decreases to a lower level when the time lag between two sequential shocks is less than a threshold, or a generalized δ-shock model; and 3) the hard failure threshold value reduces to a lower level right after m shocks larger than a threshold, or a generalized m-shock model. Reliability models are then developed for these three cases.

For maintenance models, we apply both block and age replacement policies to each of the three cases. Based on reliability analysis, the average long-run cost rate is evaluated and optimized for each policy. A numerical example is then used to demonstrate the reliability and maintenance models. From the optimization results, we can see that age replacement policy is better than the block replacement policy in all cases.

Chapter 3: Modeling Zoned Shock Effects on Stochastic Degradation in Dependent Failure Processes

In this chapter, we study a system that experiences two dependent competing failure processes, and in our new model, shocks are categorized into different shock zones which impact degradation differently. These two failure processes are a stochastic degradation process and a random shock process, and they are dependent because arriving shocks can impact the degradation process in the form of instantaneous damage. In existing studies, every shock causes an abrupt damage to the degradation process. However, this may not be the case when shock loads are small and within the tolerance of system resistance. In our model, only shock loads that are larger than a certain level are considered to cause abrupt damages on the degradation process, which makes this new model realistic and challenging. Shocks are divided into three zones based on their magnitudes: *safety zone*, where shocks with magnitude below W_0 are considered harmless; *damage zone*, where shocks with magnitude between W_0 and W_T can cause damage to the degradation process; and *fatal zone*, where shocks with magnitude above W_T are considered fatal and the system fails immediately. We further model the abrupt damages using an explicit function of shock load exceedances (differences between load magnitudes and a given threshold). Due to the complexity in modeling these two dependent stochastic failure processes, no closed form of the reliability function can be derived. Monte Carlo importance sampling is used to estimate the system reliability. Finally, two application examples with sensitivity analyses are presented to demonstrate our models.

NOTATION

$X_s(t)$	Overall degradation at time t
$X(t)$	Continuous degradation at time t

H	Soft failure threshold
W_i	Magnitude of the i^{th} shock load
W_0	Safety zone threshold
D_i	Load exceedance (difference between W_i and W_0)
Y_i	Damage on the wear degradation caused by the i^{th} shock load
W_T	Hard failure threshold
b	Coefficient in the linear relationship between D_i and Y_i
$f_{Wi}(w_i)$	Probability density function (pdf) of W_i
$F_{Wi}(w_i)$	Cumulative distribution function (cdf) of W_i
$g_{Wi}(w_i)$	pdf of W_i that is larger than W_0
$F_{Yi}(y_i)$	cdf of Y_i
λ	Arrival rate of random shocks
$N(t)$	Number of all shocks arrived by time t
$N_1(t)$	Number of shocks arrived in the damage zone by time t
$N_2(t)$	Number of shocks arrived in the fatal zone by time t
p_1	Probability of a shock arriving in the damage zone
p_2	Probability of a shock arriving in the fatal zone

3.1 Introduction

A complex device can fail due to multiple failure processes induced by internal and external sources, such as corrosion, fatigue, wear, and external shocks. These failure processes may practically interact in one way or another, and compete with each other to cause the system to fail. For instance, external shocks can impact a wear process by causing sudden increase in wear volume. This interaction between external shocks and graceful degradation has been studied as part of multiple dependent competing failure processes in the literature [54, 77, 98, 99]. However, not every shock has an impact on the degradation process, because devices in general have some resistance against small shock loads due to material strength and structures. Therefore, shock loads under a certain magnitude may not cause any additional degradation. For example, when microengines were tested at different levels of shocks at Sandia National Laboratories, they exhibited no damage and functioned smoothly under a low level of shock loads [80]. In this chapter, we conduct new reliability analyses of multiple dependent competing failure processes by considering that only shocks with magnitude larger than a certain threshold can impact the degradation process. This assumption is realistic for many applications with high reliability (e.g., microengines), and leads to new and useful reliability analysis models.

The dependency among failure processes presents challenging issues in reliability modeling. Extensive research has been performed concerning reliability analysis on dependent competing failure processes. Wang and Pham [100] studied an imperfect maintenance policy for systems subject to the dependent competing risks of degradation wear and random shocks. Two types of shocks are considered in their work: fatal shocks that can cause the immediate failure of the system, and nonfatal shocks that increase the system degradation level by the shock magnitude. Huynh *et al*. [101] developed a condition-based periodic inspection/replacement policy for single-unit systems that are subject to competing and dependent failures due to degradation and traumatic events. These two failure processes are dependent in a way that the shock arrival time is modeled by a nonhomogeneous Poisson process with a stochastically-increasing intensity that depends on the degradation level. Using the same dependency pattern between degradation and shock processes, Huynh *et al*. [102] studied two age-based maintenance policies with age-based minimal repairs and degradation-based minimal repairs for systems subject to competing and dependent failure processes.

There are two unresolved challenging issues in the exiting literature:

(1) Most previous research on dependent failure processes assumes that all shocks can affect the degradation process by causing an instantaneous increase to the degradation amount [54, 56, 77, 98, 99, 103]. However, shock loads with small magnitude may not impact the degradation process at all, because devices are typically designed to resist against small shock loads.

(2) Existing studies have not addressed the direct dependence between the abrupt damage and the shock magnitude in an explicit manner [54, 56, 77, 98, 99, 103]. In [54, 56, 77, 98, 99, 103], shocks can cause random damage on the degradation process, but no correlation is indicated between the damage size and shock load. The shock damage is mostly modeled by a normal distribution that is independent of the shock magnitude. Keedy and Feng [104] took a step further to model the relationship between the mean values of damage size and shock magnitude, by assuming that the mean damage size is proportional to the difference between the mean shock magnitude and a threshold W, only when the mean shock is greater than W. Although it is intuitive that the damage size caused by a shock is determined by the shock magnitude, no explicit model has been proposed to describe the mathematical relationship between these two random variables.

In this chapter, we address these two challenging issues by focusing on the influential large shocks, and explicitly modeling the dependence between the random damage size and shock magnitude. We study a system that fails when either of the two dependent failure processes reaches the respective failure threshold. The shock process impacts the degradation process, in a way that only shocks greater than a certain threshold can cause an instantaneous damage to the degradation process. The difference between the shock magnitude and the threshold is called the exceedance over the threshold [15]. By using the peaks-over-threshold (PoT) method [105] in extreme-value analysis, we explicitly model the abrupt damages as a linear function of shock load exceedances. Since a generalized Pareto distribution is typically used in the PoT method to model the exceedance over the threshold, the resulting shock damage follows a truncated generalized Pareto distribution, other than a normal distribution commonly used for shock damage in the literature [1-4]. If other distributions (e.g., exponential or gamma distribution) are selected to model the shock damage Y, the reliability function can be derived using convolution method. However, the truncated generalized Pareto distribution for the shock damage leads to a complicated reliability function that does not have a closed form. In addition, we use the gamma process to model the degradation in order to capture the temporal variability and the property of non-negative increments, which creates another dimension of complexity in modeling the reliability.

The remainder of this chapter is arranged as follows. Section 3.2 describes the system of interest in terms of the stochastic degradation process, shock damages and shock arrivals. Section 3.3 discusses the reliability modeling of systems subject to degradation and random shock processes, when only large shocks can impact on the degradation process. In Section 3.4, two application examples are presented to implement our models and sensitivity analysis is also discussed. Section 3.5 summarizes this chapter with concluding remarks.

3.2 System Description

As shown in Figure 3.1, we consider a system that experiences two dependent competing failure processes: soft failure due to degradation and hard failure due to random shocks. These two failure processes are dependent, because they are subject to the same random shock process. The system fails when the overall stochastic degradation exceeds the soft failure threshold level H, or when the magnitude of a shock is larger than the hard failure threshold level W_T.

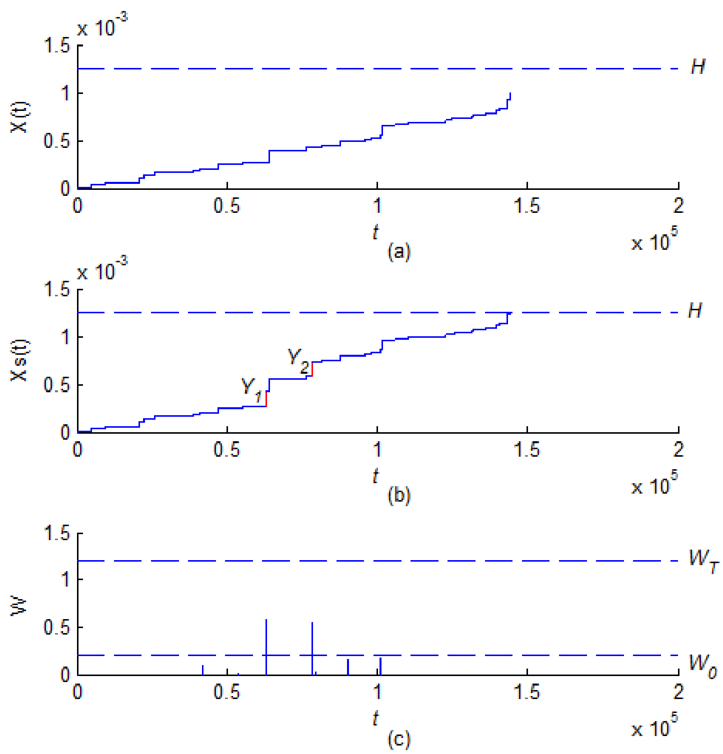

Figure 3.1: Relationship of two failure processes simulated for microengine example: (a) Continuous degradation, (b) Overall degradation process, (c) Random shock process

Figure 3.1(b) shows the overall degradation $X_s(t)$ that is composed of continuous degradation over time $X(t)$ (e.g., wear degradation, crack growth), and instantaneous increase of degradation (or shock damage) due to random shocks Y_i. The continuous degradation is simulated by a gamma process in Figure 3.1(a). The instantaneous damage only occurs when the magnitude of a shock is greater than a critical threshold W_0. The resulting damage size Y_i depends on the shock magnitude, W_i. Specifically, Y_i is proportional to the difference between W_i and W_0, or the shock load exceedance, D_i. Soft failure occurs when the overall degradation exceeds the failure threshold H.

As shown in Figure 3.1(c), hard failure occurs when a shock that is larger than the hard failure threshold W_T arrives. Random shocks arrive according to a homogeneous Poisson process (HPP) with a rate λ, and the shock size/magnitude, W_i, is an independent and identically distributed (*i.i.d.*) random variable. We classify shocks into three zones according to their magnitudes: *safety zone*, where shocks with magnitude below W_0 are considered harmless; *damage zone*, where shocks with magnitude between W_0 and W_T cause damage to the degradation process; and *fatal zone*, where shocks with magnitude above W_T are considered fatal and cause the system to fail immediately.

3.2.1 Stochastic Processes for Continuous Degradation

Under dynamic environmental conditions, a degradation process is stochastic and subject to influence from various factors, such as temperature, humidity, work load, etc. For systems operated under dynamic environments, a stochastic process is commonly used to describe the degradation process. Deterioration is usually regarded as a Markov process, where the future state of a process only depends on the current state, and is independent of the past states [84]. To model degradation processes, two general classes of Markov processes can be used: discrete-time Markov processes with a finite or countable state space (Markov Chains), and continuous-time Markov processes with independent increments such as Brownian motion with drift, the compound Poisson process, and the gamma process. Brownian motion with drift can have positive and negative increments alternately, which is inappropriate in modeling degradation that progresses in one direction, i.e., monotonic. The compound Poisson process can be used to model a stochastic process with *i.i.d.* jumps that occur according to a Poisson process. The gamma process is an effective and natural choice for describing degradation that progresses in one direction, because of its property of independent and non-negative increments [27, 84].

The gamma process has been widely used to model stochastic degradation processes in the literature. Tsai *et al.* [106] proposed a method to optimize the design of degradation tests based on a gamma degradation process with random effects. For a bivariate degradation involving two performance characteristics, Pan and Balakrishnan [107] applied a bivariate Birnbaum-Saunders distribution and its marginal distributions to approximate the reliability function, where the degradation is considered to be a gamma process. Tsai *et al.* [108] introduced a mixed gamma process to describe the degradation path of a product and presented a burn-in policy for highly reliable products. Park and Padgett [28] developed a new accelerated test model based on a generalized cumulative damage approach with a stochastic process characterizing a degradation phenomenon, where the degradation processes are described by Brownian motion, geometric Brownian motion, and gamma process for different conditions. van Noortwijk *et al.* [51] presented a method to predict reliability of systems with a gamma degradation process and a stochastic process of loads.

In our model, the overall degradation, $X_s(t)$, is a combination of continuous degradation, $X(t)$, and instantaneous shock damages from shocks in damage zone, Y_i:

$X_s(t) = X(t) + \sum_{i=1}^{N_1(t)} Y_i$, where $X(t)$ and Y_i are independent and $N_1(t)$ is the number of shocks that have arrived in the damage zone by time t. We use a gamma process to describe the continuous degradation at time t, $X(t)$, with the pdf:

$$f_{X(t)}(x\,;t) = \mathrm{Ga}(x\,;v(t),u) = \frac{u^{v(t)}}{\Gamma(v(t))} x^{v(t)-1} \exp(-ux), \quad x \geq 0, \quad t \geq 0, \tag{3.1}$$

where u >0 is the scale parameter, and $v(t)$ >0 is the shape parameter. The gamma process with positive scale and shape parameters is a continuous-time stochastic process with the following properties [51]:

1) $X(0)=0$ with probability of one,
2) $X(\tau) - X(t) \sim \mathrm{Ga}(v(\tau)-v(t), u)$ for all $\tau > t \geq 0$, and
3) $X(t)$ has independent increments.

The expectation and variance of the gamma process are:

$$E\left(X(t)\right) = \frac{v(t)}{u}, \quad \mathrm{Var}\left(X(t)\right) = \frac{v(t)}{u^2}. \tag{3.2}$$

The expected or mean degradation path is a key in modeling the trend of a degradation process that can follow various shapes, such as linear, exponential, or power law shape. For some degradation measures such as fatigue crack growth, the expected degradation at time t often follows a power law empirically [51]:

$$E\left(X(t)\right) = \frac{v(t)}{u} = \frac{\alpha t^{\theta}}{u}, \tag{3.3}$$

where α, θ, and u are positive constants. When $\theta = 1$, it reduces to a linear degradation path, e.g., wear amount on rubbing surfaces.

Let T_l denote the first time the continuous degradation level reaches the degradation level l. Then the distribution of a gamma degradation process is

$$P(T_l \le t) = P(X(t) \ge l) = \int_l^{\infty} f_{X(t)}(x)\mathrm{d}x = \frac{\Gamma(v(t), lu)}{\Gamma(v(t))}, \tag{3.4}$$

where $\Gamma(a,x) = \int_x^{\infty} t^{a-1}\mathrm{e}^{-t}\mathrm{d}t$ is the incomplete gamma function for $x \ge 0$ and $a > 0$. Gamma function $\Gamma(a)$ is obtained when we set x=0, namely $\Gamma(a) = \Gamma(a,0)$.

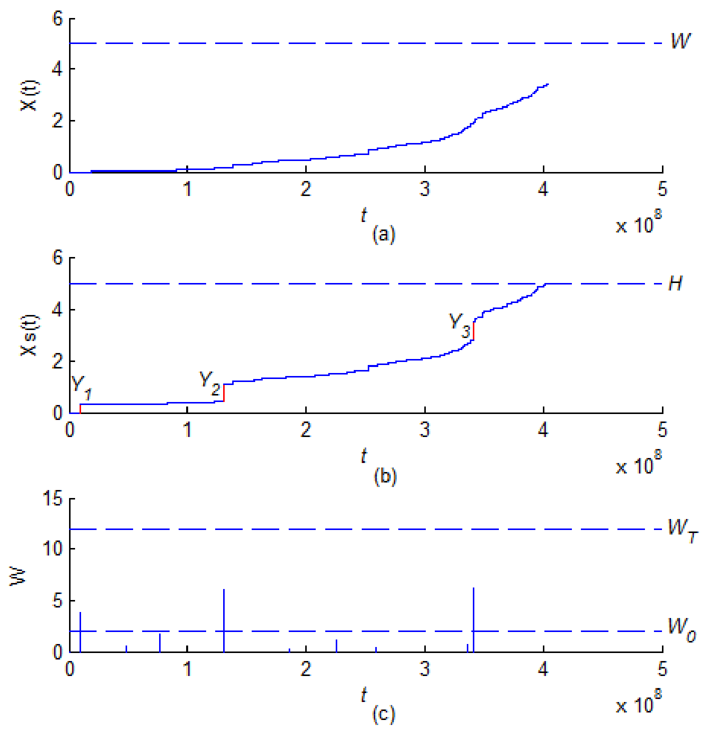

Figure 3.2: Relationship of two failure processes simulated for stent example: (a) Continuous degradation, (b) Overall degradation process, (c) Random shock process

For simulation of gamma processes, three methods are available: gamma-increment sampling, gamma-bridge sampling and approximating a gamma process as a limit of a compound Poisson process [84]. The last method is not efficient because there are infinitely many jumps in each finite time interval in a gamma process. Gamma-increment sampling and gamma-bridge sampling are better approaches to use, since they simulate independent increments with respect to very small units of time. Gamma-increment sampling is more straightforward, while gamma-bridge sampling allows the accuracy to be pre-defined and the old iteration results can still be used if we want to increase the accuracy [51]. We choose gamma-increment sampling to simulate two gamma processes in Figures 3.1(a) and 3.2(a) for the microengine and stent examples, respectively (see Section 4). In Figure 3.1(a), without the abrupt shock damages, the expected degradation is linear in time *t*, describing the increasing wear volume in microengines [97]. In Figure 3.2(a), without the abrupt shock damages, the expected degradation over time has a power law shape, which is appropriate for fatigue crack growth in stents [109, 110].

3.2.2 Truncated Generalized Pareto Distribution for Shock Damages

The continuous degradation over time experiences instantaneous damage from each shock load that is greater than W_0. The instantaneous damage size on degradation Y_i depends on the magnitude of the shock load W_i. We explicitly model Y_i as a linear function of the exceedances over the threshold, D_i:

$$Y_i = bD_i = b(W_i - W_0), \quad \text{for } W_i > W_0, \tag{3.5}$$

where $D_i = W_i - W_0$ is the exceedance over the threshold, which represents the shock loads with magnitude above the safety zone.

Extreme value analysis is commonly used to determine the probability distribution of the exceedance over the threshold, which generally has two approaches: block maxima (maxima in a given time period), and peaks over a given threshold [111]. The block maxima method is straightforward because it is applied to the variable of interest – block maxima. The observed maximal values are modeled by a generalized extreme value distribution. Due to the small number of observed block maxima, however, the obtained estimates may be sensitive to outliers [111]. In PoT method, the estimates of extreme values are based on all values that exceed a threshold, which takes advantage of all useful information leading to higher accuracy. When the PoT method is implemented, some parameters are required and often they are selected based on judgments that should consider the nature of the design and corresponding failures, e.g., the threshold W_0. There are often no general rules on how to choose them because the parameters are dependent on the characters of the studied system [111].

PoT method is widely used in extreme value analysis with applications to insurance, finance, hydrology and other fields [105]. van Noortwijk *et al*. [51] proposed a method to combine two stochastic processes of deteriorating resistance and fluctuating load for computing the time-dependent reliability of structural components, in which the magnitude of random loads is modeled by a PoT distribution. Katz *et al*. [112] summarized statistics of extremes used in hydrology, including a point process model that combines the block maxima and PoT techniques. Kysely *et al.* [113] presented a methodology for estimating high quantiles of distributions for daily temperature in a non-stationary context, based on the PoT analysis with a time-dependent threshold expressed in terms of regression quantiles.

A generalized Pareto distribution is typically used in the PoT method. We assume that shock loads W_i follow a generalized Pareto distribution with scale parameter σ ($\sigma > 0$) and shape parameter c. The pdf of W_i takes the form [51]:

$$
f_{W_i}(w_i) = \begin{cases} \dfrac{1}{\sigma}\left(1-\dfrac{cw_i}{\sigma}\right)^{\frac{1}{c}-1}, & c \neq 0, \\ \dfrac{1}{\sigma}\exp\left(-\dfrac{w_i}{\sigma}\right), & c = 0. \end{cases} \tag{3.6}
$$

The complementary cumulative distribution function (cdf) is:

$$
\overline{F}_{W_i}(w_i) = \Pr(W_i > w_i) = \begin{cases} \left(1-\dfrac{cw_i}{\sigma}\right)^{\frac{1}{c}}, & c \neq 0, \\ \exp\left(-\dfrac{w_i}{\sigma}\right), & c = 0, \end{cases} \tag{3.7}
$$

where $W_i > 0$ for $c \leq 0$, and $0 < W_i < \sigma/c$ for $c > 0$.

Therefore, the W_i that is larger than W_0 follows a truncated generalized Pareto distribution with a pdf

$$
g_{W_i}\left(w_i\right) = \frac{f_{W_i}\left(w_i\right)}{\overline{F}\left(W_0\right)}, \quad \text{for } W_i > W_0. \tag{3.8}
$$

Because of the linear relationship between W_i and Y_i for $W_i > W_0$ in Eq. (3.5), Y_i also follows a truncated generalized Pareto distribution, and its pdf is derived based on Eqs. (3.5)-(3.8):

$$
f_{Y_i}(y_i) = \frac{1}{b} g_{W_i}\left(\frac{y_i + bW_0}{b}\right) = \begin{cases} \dfrac{1}{\overline{F}\left(W_0\right)b\sigma}\left(1-\dfrac{c\left(y_i + bW_0\right)}{b\sigma}\right)^{\frac{1}{c}-1}, & c \neq 0, \\ \dfrac{1}{\overline{F}\left(W_0\right)b\sigma}\exp\left(-\dfrac{y_i + bW_0}{b\sigma}\right), & c = 0. \end{cases} \tag{3.9}
$$

The cdf of Y_i is derived to be:

$$F_{Y_i}(y_i) = \int_0^{y_i} f_{Y_i}(y)dy = \begin{cases} 1 - \dfrac{1}{\overline{F}(W_0)}\left(1 - \dfrac{c(y_i + bW_0)}{b\sigma}\right)^{\frac{1}{c}}, & c \neq 0, \\ 1 - \dfrac{1}{\overline{F}(W_0)}\exp\left(-\dfrac{y_i + bW_0}{b\sigma}\right), & c = 0, \end{cases} \tag{3.10}$$

where Y_i >0 for $c \leq 0$, and $0 < Y_i < b(\sigma/c - W_0)$ for c >0.

3.2.3 Decomposition of Homogeneous Poisson Process for Shock Arrivals

We consider random shocks that arrive according to a homogeneous Poisson process (HPP) with a rate λ. Let $N(t)$ denote the number of all random shocks arrived by time t. The probability of exactly n shocks occurring in the time interval [0, t] is

$$P(N(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0,1,..., \tag{3.11}$$

where we assume that no shock arrives at time zero.

We then use $N_1(t)$ and $N_2(t)$ to denote the numbers of shocks arrived in the damage zone and the fatal zone by time t, respectively. For an arriving shock, the probability it falls into the damage zone is $p_1 = P(W_0 < W_i < W_T) = F_{Wi}(W_T) - F_{Wi}(W_0)$, and the probability it is in the fatal zone is $p_2 = P(W_i > W_T) = 1 - F_{Wi}(W_T)$. Based on the decomposition of Poisson process [114], we have that the arrival of shocks in the *damage zone* follows a HPP with a rate λp_1, and the arrival of shocks in the *fatal zone* follows a HPP with a rate λp_2. Accordingly, the arrival of shocks in the *safety zone* follows a HPP with a rate $\lambda(1-p_1-p_2)$. We also prove that $N_1(t)$ and $N_2(t)$ are independent of each other.

3.3 Reliability Analysis of Dependent Failure Processes

For a system that experiences two dependent failure processes: soft failure due to degradation and hard failure due to random shocks, we develop and analyze the reliability function in this section.

3.3.1 Development of Reliability Function

In order to keep functioning, the system should experience no fatal shocks and the degradation level should be within its threshold. The reliability of the system by time t can be derived as

$$
\begin{aligned}
R(t) &= \Pr(\text{no failure happens by time } t) \\
&= \sum_{n=0}^{\infty} P\left(X(t) + \sum_{i=1}^{N_1(t)} Y_i < H, N_2(t) = 0 \middle| N_1(t) = n \right) P\left(N_1(t) = n\right) \\
&= \sum_{n=0}^{\infty} P\left(X(t) + \sum_{i=1}^{N_1(t)} Y_i < H \mid N_1(t) = n \right) P\left(N_1(t) = n\right) P\left(N_2(t) = 0\right) \\
&= \sum_{n=0}^{\infty} P\left(X(t) + \sum_{i=1}^{n} Y_i < H \right) P\left(N_1(t) = n\right) P\left(N_2(t) = 0\right).
\end{aligned}
\tag{3.12}
$$

The last step is valid because $N_2(t)$ is independent of $N_1(t)$. If we denote the conditional pdf of $X_s(t)$ given time t and $N_1(t)=n$ to be $f_{X_s}(x_s \mid t, n)$, we have

$$
\begin{aligned}
R(t) &= \sum_{n=0}^{\infty} \int_0^H f_{X_s}(x_s \mid t, n) dx_s \frac{e^{-\lambda p_1 t} (\lambda p_1 t)^n}{n!} \frac{e^{-\lambda p_2 t} (\lambda p_2 t)^0}{0!} \\
&= \sum_{n=0}^{\infty} \int_0^H f_{X_s}(x_s \mid t, n) dx_s \frac{e^{-\lambda (p_1 + p_2) t} (\lambda p_1 t)^n}{n!},
\end{aligned}
\tag{3.13}
$$

where $p_1 = P(W_0 < W_{\mathrm{i}} < W_T) = F_{Wi}(W_T) - F_{Wi}(W_0)$ and $p_2 = P(W_i > W_T) = 1 - F_{Wi}(W_T)$.

Next, we need to find $f_{X_s}(x_s \mid t, n)$ to solve this reliability function. The distributions of Y_i, i=1, … n, and $X(t)$ are assumed to be known, and we denote their summation $X(t) + \sum_{i=1}^{n} Y_i$ to be X_s. The distribution of X_s can be found by using the change-of-variable technique. By introducing a new set of variables Z_i, i=1, … n, we can get n+1 equations with n+1 unknown variables. Because the transformation is one-to-one, we can solve for $X(t)$ and Y_i, i=1, … n, in terms of X_s and Z_i, i=1, … n:

$$\begin{cases} X_s = X(t) + Y_1 + Y_2 + \ldots + Y_n \\ Z_1 = Y_1 \\ Z_2 = Y_2 \\ \quad \ldots \\ Z_n = Y_n \end{cases} \Rightarrow \begin{cases} X(t) = X_s - Z_1 - Z_2 - \ldots - Z_n \\ Y_1 = Z_1 \\ Y_2 = Z_2 \\ \quad \ldots \\ Y_n = Z_n \end{cases}.$$

Given the number of shocks in the damage zone, $N_1(t)$=n, the joint pdf of X_s and Z_i, i=1, … n, is

$$\begin{aligned} f_{X_s,Z_1,\ldots,Z_n}(x_s, z_1, \ldots, z_n) &= |J| f_{X(t),Y_1,\ldots,Y_n}(x_s - z_1 - z_2 - \ldots - z_n, z_1, z_2 \ldots, z_n) \\ &= |J| f_{X(t)}(x_s - z_1 - z_2 - \ldots - z_n) f_{Y_1}(z_1) f_{Y_2}(z_2) \cdots f_{Y_n}(z_n), \end{aligned} \tag{3.14}$$

where J is the Jacobian determinant,

$$J = \begin{vmatrix} \dfrac{\partial X(t)}{\partial X_s} & \dfrac{\partial X(t)}{\partial Z_1} & \ldots & \dfrac{\partial X(t)}{\partial Z_n} \\ \dfrac{\partial Y_1}{\partial X_s} & \dfrac{\partial Y_1}{\partial Z_1} & \ldots & \dfrac{\partial Y_1}{\partial Z_n} \\ \ldots & \ldots & \ldots & \ldots \\ \dfrac{\partial Y_n}{\partial X_s} & \dfrac{\partial Y_n}{\partial Z_1} & \ldots & \dfrac{\partial Y_n}{\partial Z_n} \end{vmatrix} = \begin{vmatrix} 1 & -1 & \ldots & -1 \\ & 1 & & \\ & & \ldots & \\ & & & 1 \end{vmatrix}_{(n+1)\times(n+1)} = 1. \tag{3.15}$$

Then considering the case that $c > 0$, we can find the pdf of $f_{X_s}(x_s|t,n)$ by

$$\begin{aligned} f_{X_s}(x_s|t,n) &= \underbrace{\int_0^{b(\frac{\sigma}{c}-W_0)} \int_0^{b(\frac{\sigma}{c}-W_0)} \cdots \int_0^{b(\frac{\sigma}{c}-W_0)}}_{n-fold} f_{X_s,Z_1,\ldots,Z_n}(x_s, z_1, \ldots, z_n) \mathrm{d}z_1 \mathrm{d}z_2 \ldots \mathrm{d}z_n \\ &= \underbrace{\int_0^{b(\frac{\sigma}{c}-W_0)} \int_0^{b(\frac{\sigma}{c}-W_0)} \cdots \int_0^{b(\frac{\sigma}{c}-W_0)}}_{n-fold} f_{Y_1}(z_1) f_{Y_2}(z_2) \cdots f_{Y_n}(z_n) f_{X(t)}(x_s - z_1 - z_2 - \ldots - z_n) \mathrm{d}z_1 \mathrm{d}z_2 \ldots \mathrm{d}z_n. \end{aligned} \tag{3.16}$$

There is no closed form of this reliability function in Eq. (3.13). We can use numerical analysis method to find the solution, and Monte Carlo simulation is one of the most effective and efficient numerical analysis methods to solve this problem.

3.3.2 Simulation of Reliability Function

Monte Carlo sampling is generally used to solve multiple integral problems [115]. Due to its important role in numerical analysis, extensive research has been dedicated to reducing Monte Carlo sampling errors and improving its efficiency [115-117]. We use one of the most commonly used Monte Carlo methods, variance reduction through importance sampling, to estimate the result. For an integral $A = \int_D f(x)dx$, the importance sampled Monte Carlo estimate can be written as follows [116]:

$$\tilde{A}_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{h(x_i)}, \quad X_i \sim h(x), \tag{3.17}$$

where $h(\cdot)$ is an importance function that mimics the behavior of $f(\cdot)$ over D, and it is either integrable analytically or can be easily integrated numerically. Also $h(\cdot)$ should be normalized to have $\int_D h(x)dx = 1$. The sampling procedure is then altered to generate points distributed according to $h(\cdot)$ instead of points that are uniformly distributed [116].

To solve our reliability function in Eq. (3.13), we first use Monte Carlo importance sampling to estimate $f_{X_s}(x_s \mid t, n)$ in Eq. (16), where Z_i, i=1, … n, follows the truncated generalized Pareto distribution on (0, $b(\sigma/c - W_0)$). By applying Eq. (3.17) to Eq. (3.16), the importance sampled Monte Carlo estimate of $f_{X_s}(x_s \mid t, n)$ is given as:

$$\begin{gathered} \hat{f}_{X_s}(x_s \mid t, n) = \frac{1}{N}\sum_{i=1}^{N} f_{X(t)}(x_s - z_{1i} - z_{2i} - \ldots - z_{ni}) \text{ and} \\ h(z_1, z_2, \ldots, z_n) = f_{Y_1}(z_1) f_{Y_2}(z_2) \cdots f_{Y_n}(z_n). \end{gathered} \tag{3.18}$$

Next, the integral of the estimated $f_{X_s}(x_s | t, n)$ can be further simplified by exchanging the order of summation and integral:

$$\begin{aligned}
\int_0^H \hat{f}_{X_s}(x_s | t, n)\mathrm{d}x_s &= \int_0^H \left(\frac{1}{N} \sum_{i=1}^{N} f_{X(t)}(x_s - z_{1i} - z_{2i} - \ldots - z_{ni}) \right) \mathrm{d}x_s \\
&= \frac{1}{N} \sum_{i=1}^{N} \int_0^H f_{X(t)}(x_s - z_{1i} - z_{2i} - \ldots - z_{ni}) \mathrm{d}x_s \\
&= \frac{1}{N} \sum_{i=1}^{N} \int_{-z_{1i} - z_{2i} - \ldots - z_{ni}}^{H - z_{1i} - z_{2i} - \ldots - z_{ni}} f_{X(t)}(x)\mathrm{d}x \\
&= \frac{1}{N} \sum_{i=1}^{N} \int_0^{H - z_{1i} - z_{2i} - \ldots - z_{ni}} f_{X(t)}(x)\mathrm{d}x \\
&= \frac{1}{N} \sum_{i=1}^{N} \left(1 - \frac{\Gamma(v(t), (H - z_{1i} - z_{2i} - \ldots - z_{ni})u)}{\Gamma(v(t))} \right).
\end{aligned} \tag{3.19}$$

The last step is based on Eq. (3.4). Thus, we can estimate the reliability function in Eq. (13) by substituting the integral of the estimated $f_{X_s}(x_s | t, n)$ in Eq. (3.19).

3.4 Case Study

To demonstrate our reliability model, two application examples are given in this section. The first example is on a microengine developed at Sandia National Laboratories, which consists of orthogonal linear comb drive actuators mechanically connected to a rotating gear [80, 97]. The second example is on stents implanted in human body, which are subject to failure due to a variety of overloads and cyclic stresses in manufacturing, deployment and operation phases [104]. Sensitivity analysis is also performed to analyze the effects of the model parameters on the system reliability in both examples.

Case 1: Micro-Electro-Mechanical System (MEMS) Devices

According to experimental studies conducted at Sandia National Laboratories, wear of rubbing surfaces between the gear and the pin joint is the dominant mode of failure for microengines, which usually causes a broken pin. In shock experiments on microengines, debris appears on the surface of the die when shocks larger than a certain threshold occur, and the gear hub is observed to be broken when the magnitude of the shock is above a certain level [80]. Therefore, the microengine experiences two competing failure processes: soft failures due to the wear degradation and debris from external shocks, and hard failures due to hub fracture.

In this example, the parameters in the model and their values are given in Table 3.1. The stochastic degradation process and random shock process are simulated in Matlab R2010a and shown in Figure 3.1, where the gamma process is simulated using gamma-increment sampling, and the shock magnitudes are simulated from a generalized Pareto distribution. The stochastic non-negative increments represent the graceful degradation, and the step increases are the shock damages from shocks larger than W_0. In this example presented in Figure 3.1, two out of six simulated shocks are greater than W_0, corresponding to two exceedances over threshold. Using Monte Carlo simulation, we obtain the plot of the importance sampled Monte Carlo estimate of $f_{X_s}(x_s \mid t, n)$ in Eq. (3.18). We also conduct the sensitivity analysis on parameters of interest: the safety zone threshold W_0, soft failure threshold H, and shock arrival rate λ.

Table 3.1: Parameter values in microengine example

Parameters	Values
H	0.00125 μm^3
λ	2.5×10^{-5}
u	1.2×10^{4}
θ	1.0
α	1.02×10^{-4}
c	0.055
b	0.0004
σ	0.33
W_0	0.2 *Gpa*
W_T	1.2 *Gpa*

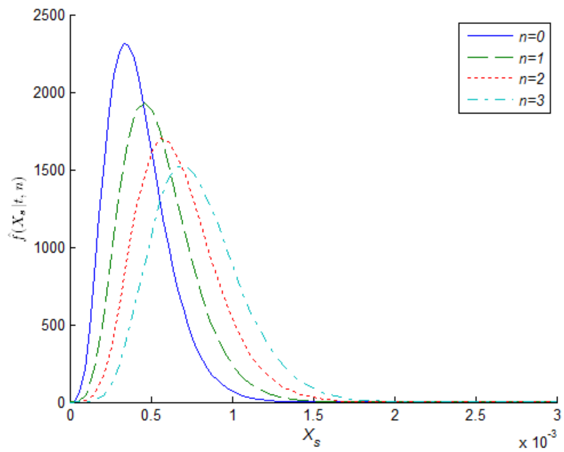

Figure 3.3: Probability plot of X_s at t=50,000.

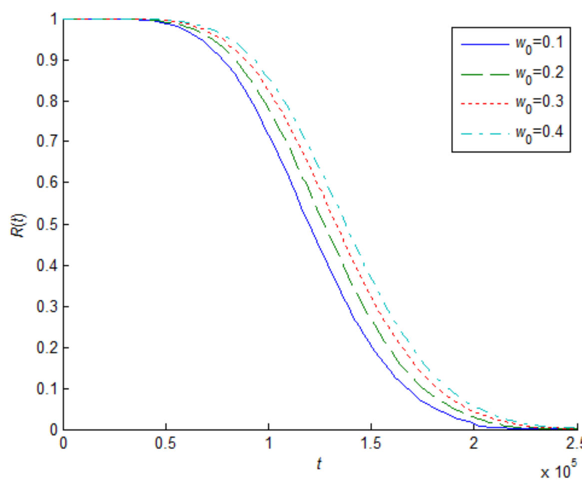

Figure 3.4: Sensitivity analysis of $R(t)$ on W_0.

The estimated $f_{X_s}(x_s|t,n)$ is plotted in Figure 3.3 at t=50,000 revolutions under different numbers of exceedance occurrences, n. As shown in the figure, the plot shifts to the right when n increases, implying that the system is more prone to soft failure when more shock load exceedances arrive.

In Figure 3.4, we can see that the safety zone threshold W_0 has a great impact on the system reliability. When W_0 increases from 0.1 to 0.4, the probability of shocks falling into the safety zone increases from 26.33% to 71.48%, indicating the improved reliability performance.

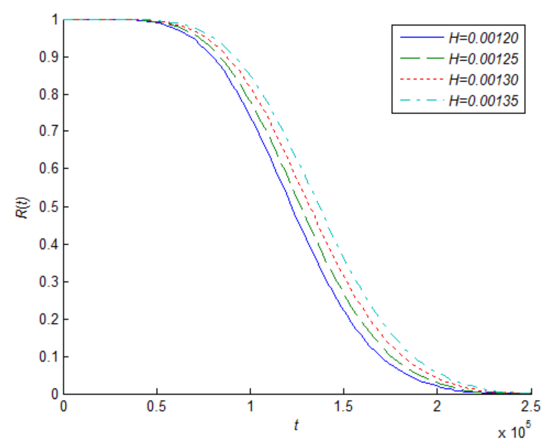

Figure 3.5: Sensitivity analysis of $R(t)$ on H.

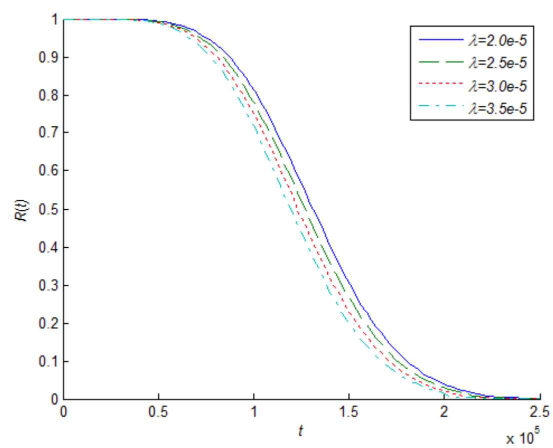

Figure 3.6: Sensitivity analysis of $R(t)$ on λ.

Figure 3.5 indicates that the soft failure threshold H has a significant effect on the reliability function. $R(t)$ shifts to right when H increases from $1.20\times10^{-3}\mu m^3$ to $1.35\times10^{-3}\mu m^3$, which implies that a larger value of H ensures better reliability performance. Figure 3.6 shows that the shock arrival rate, λ, has an inverse relationship with the system reliability. When λ increases from 2.0×10^{-5} to 3.5×10^{-5}, $R(t)$ shifts to the left, which indicates that a smaller value of λ provides better reliability performance.

Case 2: Stent Devices

A stent is a small wire mesh tube that acts as a scaffold to provide support inside arteries in treating coronary artery diseases. When implanted in human body, it experiences a variety of overloads and cyclic stresses. The overloads (e.g., external forces during stent deployment, patient excessive activities) can lead to immediate fracture of stents; and the cyclic stresses (e.g., contractions and dilations due to heartbeat) can lead to the generation of accumulated fatigue damage that may eventually result in the propagation of fatigue cracks [109, 110]. Experimental study of stent failure processes indicates that the crack growth of stents is not only the effect of fatigue stress, but also the result of single-event overloads in a form of sudden step increase of crack for certain patients who have excessive activities [104].

For stent crack propagation, the fatigue-crack growth can be well described in terms of a Paris power law formulation [109, 110]. The mean degradation path in Eq. (3.3) for the gamma process is conveniently used to describe the crack growth.

Table 3.2: Parameter values in stent example

Parameters	Values
H	5.0
λ	1.0×10^{-8}
u	20
θ	2.5
α	2.4×10^{-20}
c	0.3
b	0.15
σ	4
W_0	2
W_T	12

The parameters and their values for the stents example are listed in Table 3.2. The crack propagation process and random shock process are simulated in Matlab R2010a and shown in Figure 3.2. In this example, three shocks out of nine simulated shocks are larger than W_0, and cause step increases on the crack growth.

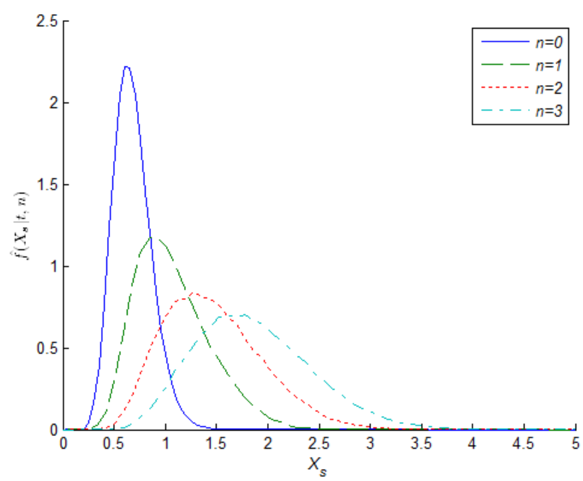

Figure 3.7: Probability plot of X_s at t=2×10^8.

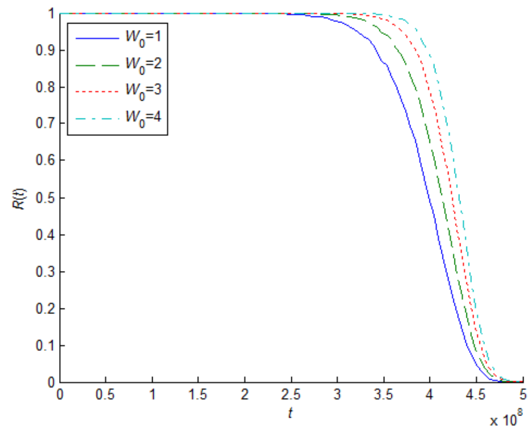

Figure 3.8: Sensitivity analysis of $R(t)$ on W_0.

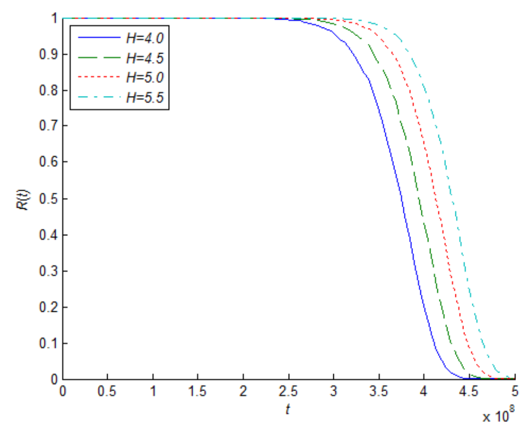

Figure 3.9: Sensitivity analysis of $R(t)$ on H.

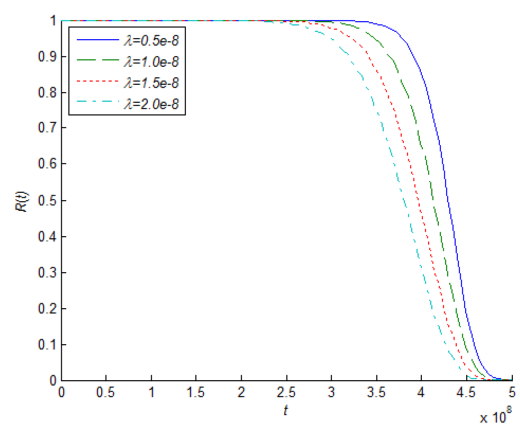

Figure 3.10: Sensitivity analysis of $R(t)$ on λ.

The estimated $f_{X_s}(x_s \mid t, n)$ is plotted in Figure 3.7 at t=2×10^8 under different numbers of exceedance occurrences, n. As shown in the figure, the plot shifts to the right when n increases, meaning that soft failure has a higher chance to occur when more shock load exceedances arrive, similar to Figure 3.3 in MEMS devices case. Sensitivity analysis are conducted on parameters of interest: the safety zone threshold W_0, soft failure threshold H, and shock arrival rate λ. In Figure 3.8, we can see that the safety zone threshold W_0 has a great impact on the system reliability, similar to the MEMS example. When W_0 increases from 1 to 4, the probability of shocks falling into the safety zone increases from 22.88% to 69.54%, which results in improved reliability performance. The results of sensitivity analysis on parameters H and λ (Figure 3.9 and 3.10) are similar to those in the MEMS example.

3.5 Discussion and Conclusions

In this chapter, we propose a reliability model for systems subject to degradation and random shock processes. These two failure processes are dependent because shock loads can cause instantaneous damage on the degradation process. To make the model more realistic, we consider that only shocks larger than a certain value can affect the degradation process, since systems usually have some resistance against small shock loads due to material strength and system structures.

Gamma process is used to model the stochastic degradation process, because it has non-negative increments properties. Peaks-over-threshold method is used to model random shock loads, where shock arrivals follow a homogeneous Poisson process. Because there is no closed form for the reliability function, we use Monte Carlo simulation method to estimate the results. Two application examples are given to illustrate our model, and sensitivity analysis is conducted to analyze the effects of parameters on system reliability performance.

The concept of three-zone shock effects resembles the three outcomes in the fault coverage model [118-120], although these two models are essentially not relevant from physics-of-failure and application point of view. In the imperfect fault coverage model, with the occurrence of a fault, transient restoration takes place when the fault is transient and can be handled without discarding any components; permanent coverage occurs when the fault is determined to be permanent and the offending component is isolated and discarded; single-point failure happens when the fault causes the system to fail. In our model, shocks in the safety zone are considered harmless and no actions need to be taken to recover the system, which is different from the transient restoration; shocks in the damage zone cause abrupt damage to the degradation process and no actions are taken here, which is also different from the permanent coverage; shocks in fatal zone cause system failure immediately, which is similar to the single-point of failure. Overall, the imperfect fault coverage model is used for systems with multiple components and each component can recover from a fault with certain probability. No recovery from faults is considered in our zoned shock model.

The shock damage effect on degradation considered in our model occurs instantaneously when the shock arrives in the damage zone. For the examples in this chapter, we do not consider the delayed damage that may potentially occur at a later time after the occurrence of damage shocks. The notion of delayed damage from shocks is an interesting and relevant idea that we will incorporate into extended versions of the model in future research. If the delay times are deterministic and equal, then this can be readily incorporated into an extended version of this chapter by a simple time transformation. If the delay times are changing and/or probabilistic, then this still could be incorporated into an extended version of the model using simulation or an additional level of conditioning and integration.

Chapter 4: Reliability Analysis and Condition-based Maintenance of Systems with Dependent Degrading Components based on Thermodynamic Physics-of-Failure

In this chapter, we present a new reliability model and a unique condition-based maintenance model for complex systems with dependent components subject to respective degradation processes, and the dependence among components is established through environmental factors. Common environmental factors, such as temperature, can create the dependence in failure times of different degrading components in a complex system. The system under study consists of one dominant component and n statistically dependent components that are all subject to degradation. We consider two aspects that link the degradation processes and environmental factors: the degradation of dominant component is not affected by the state of other components, but may influent environmental factors, such as temperature; and the n dependent components degrade over time and their degradation rates are impacted by the environmental factors. Based on the thermodynamic study of the relationship between degradation and environmental temperature, we develop a reliability model to mathematically account for the dependence in multiple components for such a system. Considering the unique dependent relationship among components, a novel condition-based maintenance model is developed to minimize the long run expected cost rate. A numerical example is studied to demonstrate our models, and sensitivity analysis is conducted to test the impact of parameters on the models.

NOTATION

H	Threshold level for wear degradation failures
$X(t)$	Wear volume due to continuous degradation at time t

β	Degradation rate
ΔT	Temperature rise at the interface during steady state operation
$r_i(t)$	Resistance of thin film resistor i at time t
ρ_i	The degradation rate of thin film resistor i
τ	Periodic inspection interval
D	Warning limit
N_{PM}	The inspection count at which a preventive maintenance/replacement is implemented
T_x	The time of the degradation path reaching a threshold x

4.1 Introduction

Complex systems, such as micro-electro-mechanical systems (MEMS), operate under dynamic environmental stresses that impair their functionality and life. Maintenance activities are essential to prevent unexpected sudden failures, and reduce downtime cost and production loss. For maintenance purposes, reliability analysis of such systems should incorporate an accurate description of the degradation evolution under these conditions [121], especially when the degradation processes of different components are not independent under common environmental conditions. In this chapter, we analyze the reliability of complex systems with failure-dependent components subject to respective degradation processes, where the dependence among components is established through environmental factors, such as temperature. Using the reliability analysis results, a unique condition-based maintenance scheme is developed for the complex system with an aim to minimize the expected total cost rate.

The degradation of many components can be affected by environmental factors, such as temperature and humidity, which either affect the degradation rate or change the relative frequency of different failure modes of sensitive components. On the other hand, environmental conditions are subject to change due to the degradation of certain components. For instance, the friction of two sliding surfaces in a component can cause wear degradation in the form of material loss in the wear tracks, and dissipation of frictional energy can result in the increase in local temperature [122]. Consequently, the elevated temperature can accelerate the degradation process of nearby temperature-sensitive components, such as resistors. This causal relationship generates a set of interesting and challenging research problems in reliability analysis and maintenance modeling of systems. In this chapter, based on the study of physics-of-failure mechanisms and the relationships between degradation and environmental factors, we analyze the reliability of complex systems with dependent components subject to respective degradation processes, and the dependency among components is established via environmental factors.

Reliability analysis for systems experiencing degradation has been extensively studied [5, 7, 19, 28]. For systems involving both degradation and shocks, Klutke and Yang [47] proposed a limiting average availability model for systems that deteriorate due to both shocks and graceful degradation. Wang *et al.* [78] derived a reliability model for systems involving dependent and competitive degradation and shocks. Peng *et al.* [54] proposed reliability models for systems subject to multiple dependent competing failure processes (MDCFP). Jiang *et al.* [77, 89] have done further research on reliability analysis for systems with dependent failure processes and dependent failure threshold. Rafiee *et al*. [99] proposed reliability models for devices subject to dependent competing failure processes of degradation and random shocks with a changing degradation rate according to four random shock patterns: generalized extreme shock model, generalized δ-shock model, generalized m-shock model, and generalized run shock model. Song *et al*. [98] extended the reliability study to multi-component systems with multiple dependent competing failure processes.

Extensive research has also been devoted to dependent components for systems. Schottl [123] developed a reliability model for systems with dependent components, where the dependence is caused by random environmental effects concerning all components, such as number of shocks, cracks or dust particles in the considered time interval. Coit and English [124] introduced a system reliability model based on proportional hazards models where components are dependent because of the shared environmental exposure within a system. Zhang and Horigome [125] presented reliability and availability analysis of systems that endure environmental shocks, which can result in the failure of one or more components due to a cumulative shock-damage process. Kotz *et al.* [126] investigated how the degree of correlation affects the increase in the mean lifetime for parallel redundant systems when the two components are positively quadrant dependent. Burkschat [127] proposed a model for describing the lifetimes of coherent systems, in which the failures of components may have an impact on the lifetimes of the remaining components.

Various types of dependence among components have been studied in the literature. However, very little research has been devoted to study the dependence among degrading components when the degradation processes, and therefore, the failure times of components are dependent due to environmental factors. In this chapter, we study a system with multiple components that include one dominant component and n statistically dependent components. The dominant component degrades over time, and its degradation rate or lifetime distribution is not affected by the state of other components. However, the degradation process of the dominant component may cause the change in environmental conditions, such as temperature. For example, as the component wears, its temperature increases, which causes the ambient temperature to increase. In the meanwhile, the dependent components degrade over time and their degradation rates increase as the ambient temperature increases. Therefore, the degradation processes of the dependent components are statistically dependent on the degradation process of the dominant component via the environmental factors. The dependence among different components creates an interesting and challenging problem to analyze the reliability of this type of system, which is lacking in the literature. In this chapter, we attempt to fill this void by investigating the inter-dependence between the degradation processes through the analysis of physics-of-failure mechanisms, especially thermodynamic analyses, and developing the reliability and maintenance models for such systems.

For systems with deteriorating components, maintenance activities are essential to prevent potential system breakdown, reduce downtime cost and eliminate safety concerns. Although the maintenance modeling of systems with single components has been extensively studied in the literature, the research on maintenance modeling of systems with multiple components is limited. The latter topic is more interesting and practical to industry applications, yet much more difficult due to the dependence among these components. As mentioned earlier in Section 1.2.4, there are three types of dependences considered in the maintenance policies for multi-component systems: economic dependence, structural dependence, and failure dependence [61, 62]. Most of the literature on maintenance policies for multi-component systems studies the economic dependence among the components. The failure dependence has rarely been considered in maintenance policies for multi-component systems. In this chapter, we develop a unique condition-based maintenance model for a complex system with multi-components that are failure dependent. Each of the components is subject to a respective degradation process and the dependence among the components is established through environmental factors.

The remaining sections are arranged as follows. Section 4.2 describes the thermodynamic study in analyzing the relationships between wear degradation and temperature. Section 4.3 presents the system reliability model. The proposed condition-based maintenance model for multi-component systems is introduced in Section 4.4. Section 4.5 gives a numerical example to demonstrate our models with the sensitivity analyses. Concluding remarks are summarized in Section 4.6.

4.2 Thermodynamic Study for Physics-of-Failure

For a system consisting of one dominant component and *n* statistically dependent components that are all subject to degradation, we consider two aspects that link the degradation processes and environmental factors [128]:

- The dominant component degrades over time, and its degradation rate or lifetime distribution is not affected by the state of other components. However, the degradation process of the dominant component may influent environmental factors, such as temperature. For example, the wear degradation of a microengine increases ambient temperature.

- The n dependent components degrade over time and their degradation rates are impacted by the environmental factors. For instance, the elevated temperature accelerates the degradation of resistors.

To demonstrate the thermodynamic analysis and reliability modeling, we use an example application. The dominant component in an example system is a microengine that experiences wear degradation over time, and the wear-out process increases the ambient temperature. In the system, there are n temperature-sensitive thin-film resistors whose resistances increase over time, and the degradation rates increase as the temperature elevates due to the wear-out process of the microengine. In order to analyze reliability performance of this system, we need to understand physics-of-failure mechanisms for these degradation processes, specifically through the study of thermodynamics.

The relationship between wear degradation and temperature has been of great interest to many researchers in thermodynamics. Bryant *et al.* [129] developed a thermodynamic characterization of degradation dynamics, which employs entropy, a measure of thermodynamic disorder, as the fundamental measure of degradation. Ramalho and Miranda [130] conducted experimental studies on the relationship between wear and dissipated energy in sliding systems using the energetic approach, and the results show that the dissipated energy is linearly related to wear volume. The experimental work on the relationship between wear and thermal response in sliding systems from Amiri *et al.* [131] shows that the temperature rise is linearly correlated with the material loss, and the slope of the linear relationship is a measure of the wear coefficient. On the other hand, the impact of elevated temperature on component degradation is usually modeled by the Arrhenius relationship in the literature. Tencer *et al.* [132] presented a method of assessing the effective temperature essential for predicting the temperature acceleration of the wear-out mechanism using the Arrhenius equation. Kuehl [133] developed a method for prediction of resistive value changes due to aging for any relevant condition in the temperature-time expanse, and the method is based on and derived from the Arrhenius equation.

4.2.1 Wear Degradation and Thermal Response

The degradation due to wear over time can follow various degradation path models, such as a linear degradation path with random coefficients or a randomized logistic degradation path [54]. For the dominant component (e.g., a microengine), we assume its wear degradation $X(t)$ follows a linear degradation path, $X(t) = \varphi+\beta t+\varepsilon_0$, where the initial value φ is a constant. The degradation rate β follows a normal distribution, $\beta \sim N(\mu_\beta,\sigma_\beta^2)$, characterizing the unit-to-unit variability; and ε_0 is the random error term following a normal distribution, $\varepsilon_0 \sim N(0,\sigma_{\varepsilon_0}^2)$, capturing the temporal variability. The microengine is considered to be failed when the wear degradation is greater than a failure threshold value H.

The degradation of the dominant component leads to the rise of ambient temperature. According to Amiri *et al.* [131], the temperature rise ΔT at the interface during steady state operation has a linear relationship with the wear degradation rate:

$$\Delta T = \frac{\Psi}{\xi}\beta, \tag{4.1}$$

where ξ is a constant, $\xi = \frac{K}{\eta\mu_{\text{ave}}h}$, K is the wear coefficient, η is the heat partitioning factor, μ_{ave} is the friction coefficient, h is the material hardness, and Ψ is a constant. Because β follows a normal distribution, $\beta \sim N(\mu_\beta,\sigma_\beta^2)$, the temperature rise ΔT at the steady state is also a normal random variable with mean of $\mu_\beta\Psi/\xi$ and variance of $\sigma_\beta^2\Psi^2/\xi^2$.

4.2.2 Arrhenius Relationship

Similar to the degradation process modeling of the dominant component, we want to incorporate both unit-to-unit variability and temporal variability in the degradation process modeling of the dependent components as well. For the n dependent components, such as thin film resistors, the resistance $r_i(t)$ increases linearly over time, $r_i(t)=r_{0i}+\rho_i t+\varepsilon_i$, where r_{0i} is the initial resistance of component i, ρ_i is the degradation rate of component i, ε_i is the random error with a normal distribution, $\varepsilon_i \sim N(0,\sigma_{\varepsilon i}^2)$ for component i, i=1, 2, ..., n. It is known that the degradation rate ρ_i is affected by the temperature via the Arrhenius relationship [132, 133]:

$$\rho_i = r_{0i} A \exp\left(-\frac{E_a}{kT}\right), \tag{4.2}$$

where E_a is the activation energy in eV, k is the Boltzmann constant, T is the temperature in Kelvin, and A is an experimental constant. Therefore, the resistance is expressed as

$$\begin{aligned} r_i(t) &= r_{0i} + \rho_i t + \varepsilon_i \\ &= r_{0i}\left(1 + At\exp\left(-\frac{E_a}{kT}\right)\right) + \varepsilon_i. \end{aligned} \tag{4.3}$$

A thin film resistor is considered to be failed when the resistance is beyond the failure threshold value, L_i, i=1, 2, ..., n. Figure 4.1 shows 30 pairs of simulated degradation processes of a dominant component and a dependent component. We can notice that the lifetime of the dependent component has a much larger variance than that of the dominant component, because the degradation rate of the dominant component significantly affects the degradation rate of the dependent component. For a series system with dependent components, we develop its reliability function and a unique condition-based maintenance policy in the following sections.

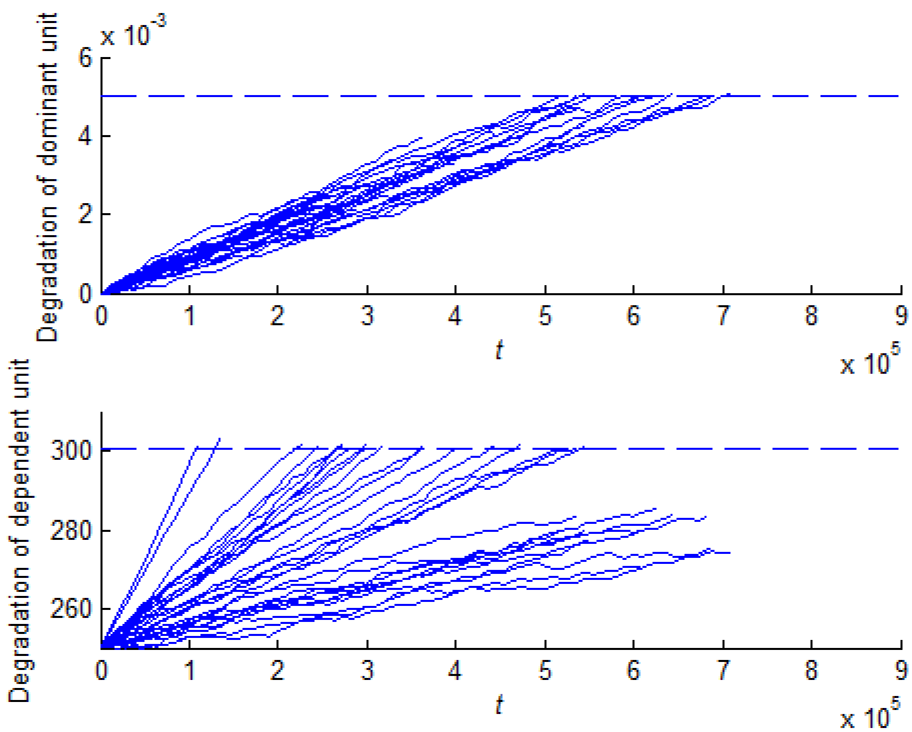

Figure 4.1: Simulation of the stochastic degradation processes for dominant and dependent components

4.3 System Reliability Modeling

Consider a series system with one dominant component and n dependent components, e.g., a microengine and n thin-film resistors connected in series. System reliability at time t is the probability that it survives by time t, that is, the degradation level of each component should be less than the corresponding failure threshold level [128]:

$$R(t) = P\left(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n\right). \tag{4.4}$$

Because the degradation processes of these components are dependent through temperature change, we need to compute it by finding the conditional probability given ΔT. Based on the law of total probability, we then integrate this conditional probability multiplied by the probability density function (pdf) of ΔT to derive the system reliability, as shown in Eq. (4.5):

$$\begin{aligned} R(t) &= P\left(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n\right) \\ &= \int_{-\infty}^{+\infty} P\left(X(t) < H, r_1(t) < L_1, \ldots, r_n(t) < L_n \mid \Delta T = s\right) f_{\Delta T}(s) ds \\ &= \int_{-\infty}^{+\infty} P\left(X(t) < H \mid \Delta T = s\right) \prod_{i=1}^{n} P\left(r_i(t) < L_i \mid \Delta T = s\right) f_{\Delta T}(s) ds, \end{aligned} \tag{4.5}$$

where the conditional probabilities of $X(t)$ and $r_i(t)$ given ΔT are derived, respectively:

$$\begin{aligned} P\left(X(t) < H \mid \Delta T = s\right) &= P\left(\varphi + ts\xi / \psi + \varepsilon_0 < H\right) \\ &= P\left(\varepsilon_0 < H - \varphi - \frac{\xi}{\psi} st\right) \\ &= \Phi\left(\frac{1}{\sigma_{\varepsilon_0}}\left(H - \varphi - \frac{\xi}{\psi} st\right)\right), \end{aligned} \tag{4.6}$$

$$
\begin{aligned}
P\left(r_i(t) < L_i \mid \Delta T = s\right) &= P\left(r_{0i} + r_{0i} A t \exp\left(-\frac{E_a}{k(T_0+s)} \right) + \varepsilon_i < L_i \right) \\
&= P\left(\varepsilon_i < L_i - r_{0i} - r_{0i} A t \exp\left(-\frac{E_a}{k(T_0+s)} \right) \right) \\
&= \Phi\left(\frac{1}{\sigma_{\varepsilon_i}} \left(L_i - r_{0i} - r_{0i} A t \exp\left(-\frac{E_a}{k(T_0+s)} \right) \right) \right).
\end{aligned}
\tag{4.7}
$$

The temperature rise ΔT is a normal random variable with mean of $\mu_\beta \Psi/\xi$ and variance of $\sigma_\beta^2 \Psi^2/\xi^2$, and its pdf can be expressed as

$$
f_{\Delta T}(s) = \frac{\xi}{\sigma_\beta \psi} \phi\left(\frac{s - \mu_\beta \psi / \xi}{\sigma_\beta \psi / \xi} \right).
\tag{4.8}
$$

Finally, the reliability function in (4.5) is expressed as

$$
\begin{aligned}
R(t) = \int_{-\infty}^{+\infty} & \Phi\left(\frac{1}{\sigma_{\varepsilon_0}} \left(H - \varphi - \frac{\xi}{\psi} s t \right) \right) \prod_{i=1}^{n} \Phi\left(\frac{1}{\sigma_{\varepsilon_i}} \left(L_i - r_{0i} - r_{0i} A t \exp\left(-\frac{E_a}{k(T_0+s)} \right) \right) \right) \\
& \times \frac{\xi}{\sigma_\beta \psi} \phi\left(\frac{s - \mu_\beta \psi / \xi}{\sigma_\beta \psi / \xi} \right) ds.
\end{aligned}
\tag{4.9}
$$

4.4 Condition-based Maintenance Modeling

Due to the unique relationship between the dominant and dependent components and their characteristics, we propose a new maintenance model for this type of system. Since the dominant component plays a key role in this system and it is typically expensive, we consider the case when the replacement cost of the dominant component is much higher than the replacement cost of all the dependent components combined (or the subsystem). We assume that the system is non-repairable or not worth repairing rather than replacing.

The replacement time for the whole system and the subsystem of all dependent components is negligible. With more attention on the expensive dominant component, the proposed maintenance strategy is designed as follows and illustrated in Figure 4.2:

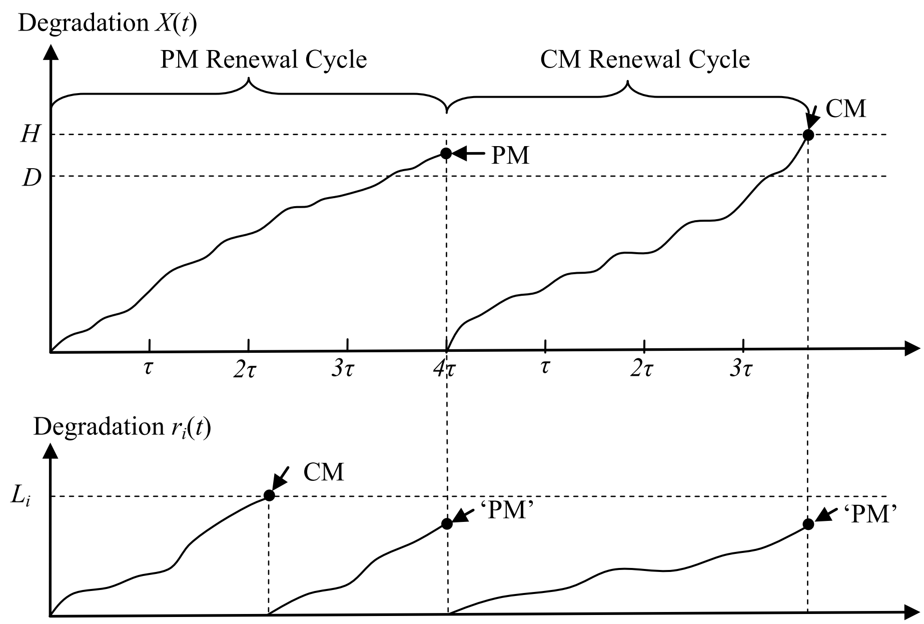

Figure 4.2: Proposed condition-based maintenance model

- Periodic inspection of length τ is carried out to observe or measure the degradation level $X(t)$ of the dominant component. If the degradation level is less than a warning limit, D, no action is taken; and if the degradation level is between the warning limit D and the failure threshold H, preventive replacement takes place.
- If the dominant component fails (the degradation level is beyond the failure threshold H) between two inspection actions, it is self-announcing and corrective replacement is implemented.
- Every time the dominant component is replaced preventively or correctively, the whole subsystem of dependent components is replaced preventively for the purpose

of saving time/labor, shown as 'PM' in Figure 4.2.

- The conditions of dependent components are not checked during the periodic inspection actions. However, the failure of any dependent component is self-announcing. If one of the dependent components in the subsystem fails, we replace the whole subsystem correctively for the purpose of saving time/labor.

To determine the inspection interval τ and the warning limit *D,* we need to derive and optimize the expected total maintenance cost per unit of time:

$$\text{Expected cost rate}=\frac{\text{Expected cost per cycle}}{\text{Expected cycle length}}=\frac{E(\text{Total Cost})}{E(\text{Cycle Length})}. \tag{4.10}$$

As illustrated in Figure 4.2, a renewal cycle of the dominant component can be terminated due to a preventive replacement (the degradation level is between *D* and *H*) or a corrective replacement (the degradation level is beyond *H*). To find the expected cost per renewal cycle and the expected renewal cycle length, we need to consider these two cases.

4.4.1 Renewal Cycle Terminated due to Preventive Replacement

We start with the case that a renewal cycle is terminated when the degradation level exceeds the warning limit, and therefore, preventive replacement is performed for the dominant component. Let N_{PM} denote the inspection count at which a preventive maintenance/replacement is implemented. The probability of performing preventive replacement is derived as follows.

1) The preventive replacement is performed at the 1^{st} inspection, or $N_{PM} = 1$:

$$
\begin{aligned}
P\left(N_{PM}=1\right) &= P\left(D < X(\tau) < H\right) \\
&= P\left(D < \varphi + \beta\tau + \varepsilon_0 < H\right) \\
&= \Phi\left(\frac{H-\varphi-\mu_\beta\tau}{\sqrt{\sigma_\beta^2\tau^2+\sigma_{\varepsilon_0}^2}}\right) - \Phi\left(\frac{D-\varphi-\mu_\beta\tau}{\sqrt{\sigma_\beta^2\tau^2+\sigma_{\varepsilon_0}^2}}\right).
\end{aligned} \tag{4.11}
$$

2) The preventive replacement is performed at the i^{th} inspection, or $N_{PM} = i > 1$:

$$
\begin{aligned}
P\left(N_{PM}=i>1\right) &= P\left(D < X(i\tau) < H, X((i-1)\tau) < D\right) \\
&= \int_{-\infty}^{+\infty} P\left(D < X(i\tau) < H, X((i-1)\tau) < D \mid \beta = b\right) f_\beta(b)db \\
&= \int_{-\infty}^{+\infty} P\left(D < \varphi + bi\tau + \varepsilon_0 < H, \varphi + b(i-1)\tau + \varepsilon_0 < D\right) f_\beta(b)db \\
&= \int_{-\infty}^{+\infty} P\left(D-\varphi-bi\tau < \varepsilon_0 < \min\left(H-\varphi-bi\tau, D-\varphi-b(i-1)\tau\right)\right) f_\beta(b)db \\
&= \int_{-\infty}^{\frac{H-D}{\tau}} \left(\Phi\left(\frac{D-\varphi-b(i-1)\tau}{\sigma_{\varepsilon_0}}\right) - \Phi\left(\frac{D-\varphi-bi\tau}{\sigma_{\varepsilon_0}}\right)\right) f_\beta(b)db \\
&\quad + \int_{\frac{H-D}{\tau}}^{+\infty} \left(\Phi\left(\frac{H-\varphi-bi\tau}{\sigma_{\varepsilon_0}}\right) - \Phi\left(\frac{D-\varphi-bi\tau}{\sigma_{\varepsilon_0}}\right)\right) f_\beta(b)db.
\end{aligned} \tag{4.12}
$$

4.4.2 Renewal Cycle Terminated due to Corrective Replacement

When the degradation level of the dominant component exceeds the failure threshold H, a renewal cycle is terminated and corrective replacement is performed. To find the probability of performing corrective replacement upon failure, we need to derive the failure time distribution of the dominant component. The degradation process $X(t)$ follows a normal distribution with mean $\mu_{X(t)} = \varphi + \mu_\beta t$, and variance $\sigma_{X(t)}^2 = \sigma_\beta^2 t^2 + \sigma_{\varepsilon_0}^2$. If we denote T_x as the time of the degradation path reaching a threshold x, then the cumulative distribution function (cdf) of T_D is

$$P\left(T_D < t\right) = P\left(X(t) > D\right) = P\left(\varphi + \beta t + \varepsilon_0 > D\right)$$
$$= 1 - \Phi\left(\frac{D - \varphi - \mu_\beta t}{\sqrt{\sigma_\beta^2 t^2 + \sigma_{\varepsilon_0}^2}}\right). \tag{4.13}$$

Its pdf can be calculated by taking the first derivative of the cdf with respect to *t*, which is

$$f_{T_D}\left(t\right) = \frac{dP\left(T_D < t\right)}{dt} = \phi\left(\frac{D - \varphi - \mu_\beta t}{\sqrt{\sigma_\beta^2 t^2 + \sigma_{\varepsilon_0}^2}}\right)\frac{\left(D - \varphi\right)\sigma_\beta^2 t + \mu_\beta \sigma_{\varepsilon_0}^2}{\left(\sigma_\beta^2 t^2 + \sigma_{\varepsilon_0}^2\right)^{3/2}}. \tag{4.14}$$

In the case of a failure occurring between inspections, it indicates that at the previous inspection the degradation level of the dominant component does not reach the warning limit *D* yet. We need to include this condition in our derivation of the failure distribution for the dominant component. The cdf of the failure time T_H conditioning on T_D is

$$P\left(T_H < t \mid T_D = t_0\right) = P\left(X(t) > H \mid X(t_0) = D\right) = P\left(X(t) - X(t_0) > H - D\right)$$
$$= P\left(\beta(t - t_0) > H - D\right)$$
$$= 1 - \Phi\left(\frac{H - D - \mu_\beta (t - t_0)}{\sigma_\beta (t - t_0)}\right), \tag{4.15}$$

Similarly, the pdf of the failure time T_H conditioning on T_D can be derived as

$$f_{T_H|T_D}\left(t \mid t_0\right) = \frac{dP\left(T_H < t \mid T_D = t_0\right)}{dt}$$
$$= \phi\left(\frac{H - D - \mu_\beta (t - t_0)}{\sigma_\beta (t - t_0)}\right)\frac{H - D}{\sigma_\beta \left(t - t_0\right)^2}. \tag{4.16}$$

4.4.3 Optimization Model

The dominant component is either preventively replaced at inspection or correctively replaced upon failure between inspections. Based on Eqs. (4.11)-(4.16), the expected renewal cycle length can be found as:

$$E(\text{Cycle Length}) = \sum_{i=1}^{\infty} i\tau P(N_{PM} = i) + \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{t_0}^{i\tau} t \cdot f_{T_H|T_D}(t \mid t_0) dt \cdot f_{T_D}(t_0) dt_0 \ . \quad (4.17)$$

The overall maintenance cost includes preventive and corrective replacement costs for the dominant component, C_{PI} and C_{CI}, preventive and corrective replacement costs for the subsystem of all dependent components, C_{PD} and C_{CD}, and the inspection cost C_I. The system downtime cost is not considered, as we assume that the time for maintenance actions, such as inspection and replacement, is negligible.

When a renewal cycle is terminated at the i^{th} inspection due to preventive replacement, the incurred cost includes the preventive replacement cost of the dominant component, C_{PI}, the preventive replacement cost of the subsystem, C_{PD}, the cost for i inspection actions, and the cost to correctively replace the subsystem before $i\tau$. The subsystem can be correctively replaced multiple times whenever one of the dependent components fails before $i\tau$. The number of corrective replacements (or the number of failures) of the subsystem prior to $i\tau$ can be calculated by the renewal function, $M(t)$, which is derived in the next section. When a renewal cycle is terminated between $(i-1)\tau$ and $i\tau$ due to failure, the incurred cost includes the corrective replacement cost of the dominant component, C_{CI}, the preventive replacement cost of the subsystem, C_{PD}, the cost for $i-1$ inspection actions before failure, and the cost to correctively replace the subsystem before failure. Then the expected total maintenance cost is derived to be:

$$E(\text{Total Cost}) = \sum_{i=1}^{\infty} P(N_{PM}=i)\cdot\left(C_{PI}+C_{PD}+iC_I+M(i\tau)C_{CD}\right)$$
$$+\sum_{i=1}^{\infty}\int_{(i-1)\tau}^{i\tau}\int_{t_0}^{i\tau}\left(C_{CI}+C_{PD}+(i-1)C_I+M(t)C_{CD}\right)\cdot f_{T_H|T_D}(t\,|\,t_0)dt\cdot f_{T_D}(t_0)dt_0. \tag{4.18}$$

Based on Eqs. (4.17) and (4.18), we propose the following constrained non-linear optimization problem for the maintenance optimization:

$$\begin{aligned} &\text{Min} \quad c(\tau, D) = \frac{E(\text{Total Cost})}{E(\text{Cycle Length})} \\ &\text{Subject to:} \quad 0 < D < H \\ &\qquad\qquad\qquad 0 < \tau < t_{\max}, \end{aligned} \tag{4.19}$$

where t_{max} is the allowed upper bound of the inspection interval. The Sequential Quadratic Programming algorithm (Matlab optimization toolbox) is used to solve this constrained non-linear optimization problem.

4.4.4 Renewal Function of the Subsystem

To calculate the expected total cost per cycle, we need to have the number of corrective replacements (or the number of failures) of the subsystem in a renewal cycle, namely, the renewal function, which requires the reliability function of the subsystem of dependent components, $R_{Sub}(t)$:

$$\begin{aligned} R_{Sub}(t) &= P\left(r_1(t)<L_1,\ldots,r_n(t)<L_n\right) \\ &= \int_{-\infty}^{+\infty} P\left(r_1(t)<L_1,\ldots,r_n(t)<L_n \,|\, \Delta T = s\right) f_{\Delta T}(s)ds \\ &= \int_{-\infty}^{+\infty} \prod_{i=1}^{n} P\left(r_i(t)<L_i \,|\, \Delta T = s\right) f_{\Delta T}(s)ds \\ &= \int_{-\infty}^{+\infty} \prod_{i=1}^{n} \Phi\left(\frac{1}{\sigma_{\varepsilon_i}}\left(L_i - r_{0i} - r_{0i}At\exp\left(-\frac{E_a}{k(T_0+s)}\right)\right)\right)\cdot\frac{\xi}{\sigma_\beta\psi}\phi\left(\frac{s-\mu_\beta\psi/\xi}{\sigma_\beta\psi/\xi}\right)ds. \end{aligned} \tag{4.20}$$

The renewal function is calculated to be $M(t)=F_{Sub}(t)+\int_0^t M(t-u)f_{Sub}(u)du$, where $F_{Sub}(t)$ and $f_{Sub}(t)$ are the cdf and pdf of the subsystem, respectively. It is difficult to derive the closed form of the renewal function given the complicated subsystem reliability function. Estimation of the renewal function is typically applied [93]:

$$M(t)=F_{Sub}(t)+\int_0^t \frac{F_{Sub}^2(u)}{\int_0^u R_{Sub}(v)dv}du .$$

Even using the estimate of the renewal function, the complex non-linear optimization model is still difficult to solve mathematically. One approach demonstrated in the numerical example is to simplify the subsystem reliability function by fitting it to a simple regression model that could lead to a closed-form renewal function. For example, when the failure time follows an exponential distribution with arrival rate λ, its renewal function is simply $M(t)=\lambda t$.

4.5 Numerical Example

In this numerical example, a system consisting of one microengine (the dominant component) and two identical resistors (dependent components) is studied. The three components are dependent because the degradation of the microengine causes the temperature rise in the surrounding environment, while the temperature rise accelerates the degradation of both resistors. For this type of system, we are interested in determining reliability over time and the optimal maintenance strategies using the reliability and maintenance models that we developed. Figure 4.3 plots the reliability of the system over time. The parameters and their values used in our models are listed in Table 4.1.

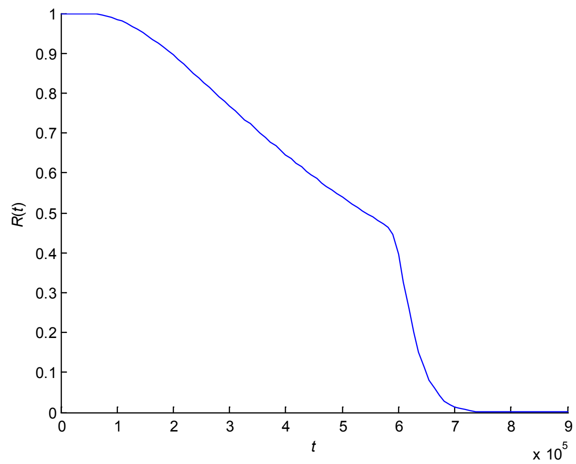

Figure 4.3: System reliability over time

Table 4.1: Parameters and values

Parameters	Values	Sources
k	8.6171×10^{-5} eV/K	
E_a	1.29 eV (for TaN)	[134]
A	1.162×10^{8} (for TaN)	[134]
T_0	293 K	
h	11.5 Gpa	[135]
K	1×10^{-4}	[135]
μ_{ave}	0.7	[135]
η	0.5	[131]
ξ	2.484×10^{-14}/pa	Calculation
Ψ	4.55×10^{14} K/W	Assumption
H	0.005 μm^3	Assumption
β	$\sim N(\mu_\beta,\sigma_\beta^2)$ $\mu_\beta = 8.4823\times10^{-9}\ \mu m^3$/time unit $\sigma_\beta = 6.0016\times10^{-10}\ \mu m^3$/time unit	[52, 54]
φ	0	Assumption
ε_0	$\sim N(0,\sigma_{\varepsilon0}^2)$ $\sigma_{\varepsilon0} = 5.0000\times10^{-5}\mu m^3$	Assumption
ε_1	$\sim N(0,\sigma_{\varepsilon1}^2)$ $\sigma_{\varepsilon1}= 5.0000\times10^{-1}\ \Omega$	Assumption
ε_2	$\sim N(0,\sigma_{\varepsilon2}^2)$ $\sigma_{\varepsilon2}= 5.0000\times10^{-1}\ \Omega$	Assumption
r_{01}, r_{02}	250.48 Ω	[134]
L_1, L_2	300.58 Ω	[134]

For the condition-based maintenance model, we assume that the preventive and corrective replacement costs for the subsystem of two dependent components are 40 and 50, respectively. Because the replacement cost of the dominant component is far more expensive than that of the dependent components, the preventive and corrective replacement costs for the dominant component are 400 and 500, respectively. The inspection cost is 10 per inspection for the dominant component.

It becomes difficult and inefficient to directly solve the optimization problem, because of the complex form of the subsystem reliability function and the resulting renewal function. An alternative way is to simplify the subsystem reliability function by using a regression model to approximate it. Using the set of parameter values provided in Table 4.1, we find that the exponential regression model fits the subsystem reliability well, as shown in Figure 4.4. The fitted exponential model is $\hat{R} = e^{-1.332\times10^{-6}t}$ and the corresponding R^2 value is 0.9869.

After fitting the subsystem reliability to an exponential regression model, the renewal function has a simple form, which is $M(t) = 1.332\times10^{-6}t$. Then we use the Sequential Quadratic Programming algorithm (in Matlab R2013a) to solve this constrained nonlinear optimization problem in (4.19) and obtain the minimum expected cost rate, 8.98×10^{-4}, when $\tau^* = 5.57\times10^{5}$ and $D^* = 0.0025$. The expected cost rates at different τ and D levels are plotted in Figure 4.5.

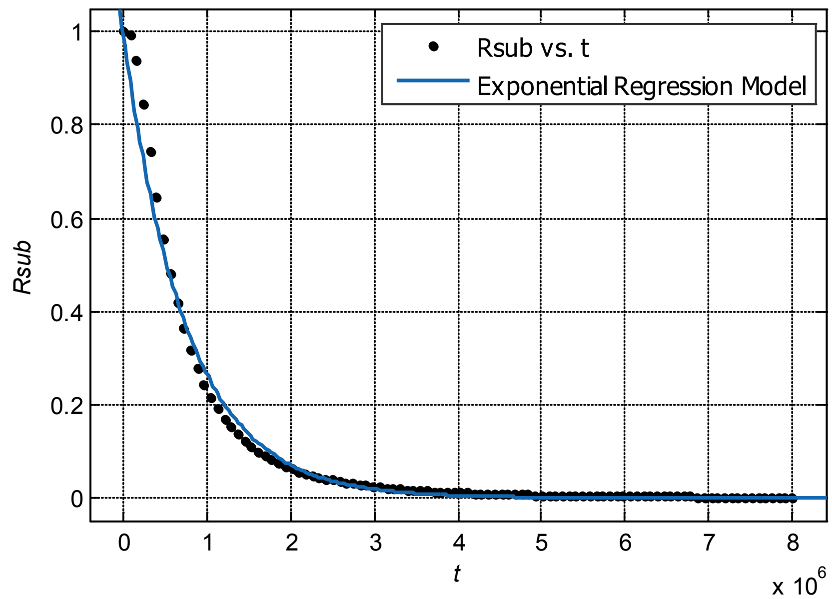

Figure 4.4: The subsystem reliability and exponential regression model

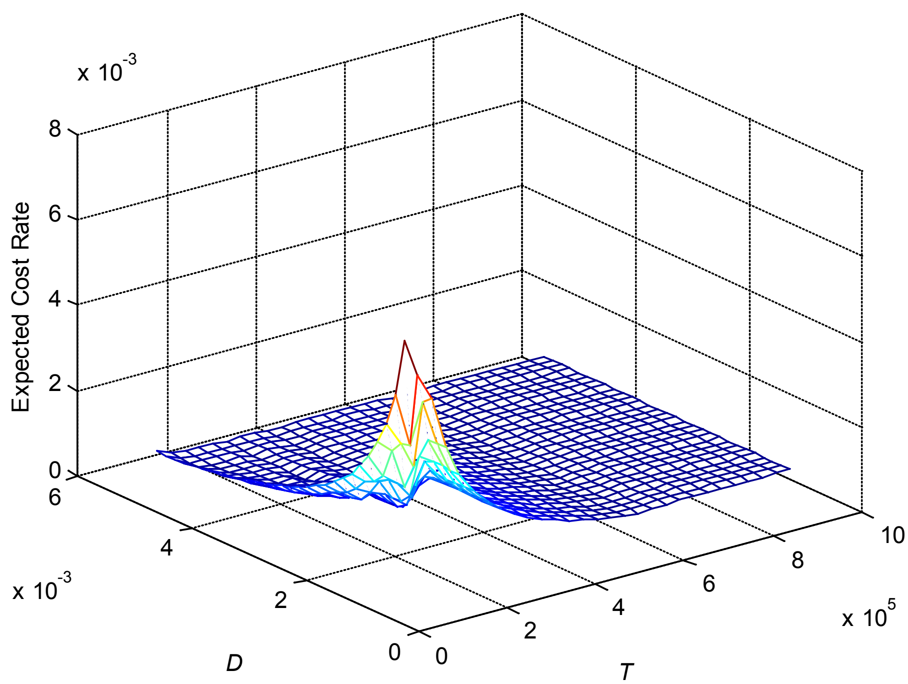

Figure 4.5: 3D plot of the expected cost rate vs τ and D

4.5.1 *Sensitivity Analysis*

Sensitivity analysis is conducted to see the sensitivity of the optimal results to the change of parameter values. The parameters of interest are the degradation failure threshold H, the ratio of resistor failure threshold to its initial value L/r_0, the ratio of preventive replacement cost for the dominant component to the preventive replacement cost for the subsystem of all dependent components C_{PI}/C_{PD}, the ratio of inspection cost to preventive replacement cost for the subsystem of all dependent components C_I/C_{PD}. The sensitivity analysis results are listed in Tables 4.2-4.5 and plotted in Figures 4.6-4.8.

Table 4.2: Sensitivity analysis result on H

$H(\mu m^3)$	τ^*	D^*	Min Expected Cost rate
0.0030	3.34E+05	1.51E-03	1.46E-03
0.0035	3.90E+05	1.76E-03	1.26E-03
0.0040	4.45E+05	2.01E-03	1.11E-03
0.0045	5.01E+05	2.25E-03	9.91E-04
0.0050	5.57E+05	2.50E-03	8.98E-04
0.0055	6.13E+05	2.75E-03	8.22E-04
0.0060	6.68E+05	3.00E-03	7.59E-04
0.0065	7.24E+05	3.25E-03	7.06E-04
0.0070	7.80E+05	3.50E-03	6.60E-04

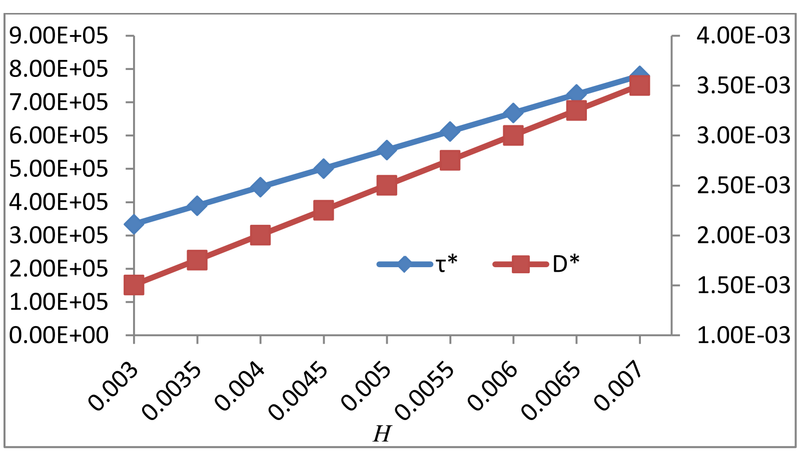

Figure 4.6: Sensitivity analysis of τ^* and D^* on H

Table 4.2 shows the values of the optimal inspection interval τ^* and warning limit D^*, the minimum expected cost rate at different H values from 0.003 to 0.007. When H increases from 0.003 to 0.007, τ^* and D^* linearly increases (also shown in Figure 4.6), while the minimum expected cost rate decreases. This is reasonable, since a higher failure threshold means the system can survive longer, requiring less frequent inspections and a higher warning limit, leading to a reduced cost.

Table 4.3: Sensitivity analysis result on L/r_0

L/r_0	L	λ	τ^*	D^*	Min Expected Cost rate
1.10	275.53	2.6810E-06	5.57E+05	2.50E-03	9.65E-04
1.15	288.05	1.7720E-06	5.57E+05	2.50E-03	9.20E-04
1.20	300.58	1.3320E-06	5.57E+05	2.50E-03	8.98E-04
1.25	313.10	1.0650E-06	5.57E+05	2.50E-03	8.85E-04
1.30	325.62	8.8640E-07	5.57E+05	2.50E-03	8.76E-04
1.35	338.15	7.5940E-07	5.57E+05	2.50E-03	8.69E-04
1.40	350.67	6.6440E-07	5.57E+05	2.50E-03	8.65E-04

In this numerical example, we consider two dependent resistors with identical failure threshold L and initial resistance r_0. The change of L/r_0 affects the subsystem reliability and further the fitted regression model parameter λ, shown in Table 4.3. From the sensitivity analysis result, we can see that the increase of the ratio of L to r_0 does not affect τ^* and D^*. This result implies that the L/r_0 of dependent components has no impact on determining τ^* and D^* on the dominant component.

In the sensitivity analysis on C_{PI}/C_{PD}, C_{PD} is fixed and the value of C_{PI} is changed, while maintaining the ratio of C_{PI} to C_{CI} at 4/5 (when C_{PI} increases, C_{CI} increases accordingly). In Table 4.4, when the ratio of C_{PI} to C_{PD} increases, τ^* decreases and the minimum expected cost rate increases (shown in Figure 4.7), while D^* stays at a constant value of 0.0025. With the increasing costs of preventive and corrective replacement, inspections should be performed more frequently to prevent failure and reduce cost. However, the increasing cost does not affect the optimal warning limit D^* notably.

Table 4.4: Sensitivity analysis result on C_{PI}/C_{PD}

C_{PI}/C_{PD}	C_{PI}	C_{CI}	τ^*	D^*	Min Expected Cost rate
4	160	200	5.65E+05	2.50E-03	4.51E-04
5	200	250	5.62E+05	2.50E-03	5.26E-04
6	240	300	5.60E+05	2.50E-03	6.00E-04
7	280	350	5.59E+05	2.50E-03	6.75E-04
8	320	400	5.58E+05	2.50E-03	7.49E-04
9	360	450	5.57E+05	2.50E-03	8.24E-04
10	400	500	5.57E+05	2.50E-03	8.98E-04
11	440	550	5.56E+05	2.50E-03	9.72E-04
12	480	600	5.56E+05	2.50E-03	1.05E-03
13	520	650	5.56E+05	2.50E-03	1.12E-03
14	560	700	5.56E+05	2.50E-03	1.20E-03
15	600	750	5.55E+05	2.50E-03	1.27E-03

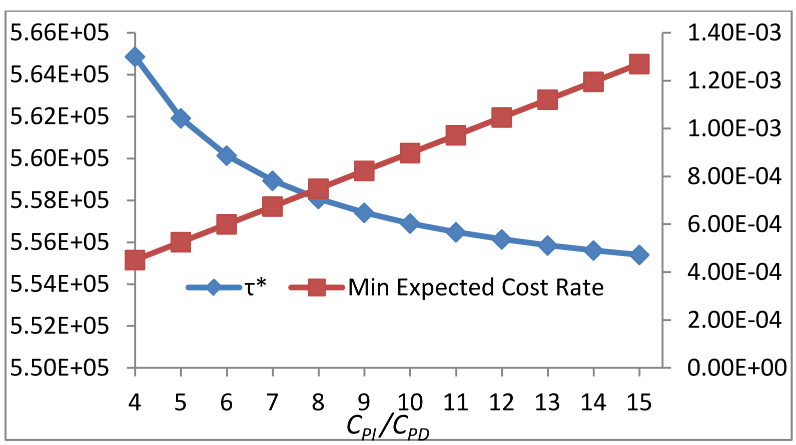

Figure 4.7: Sensitivity analysis of τ^* and D^* on C_{PI}/C_{PD}

Table 4.5: Sensitivity analysis result on C_I/C_{PD}

C_I/C_{PD}	C_I	τ*	D*	Min Expected Cost rate
0.1	4	5.55E+05	2.50E-03	8.88E-04
0.2	8	5.56E+05	2.50E-03	8.95E-04
0.3	12	5.57E+05	2.50E-03	9.01E-04
0.4	16	5.59E+05	2.50E-03	9.07E-04
0.5	20	5.60E+05	2.50E-03	9.13E-04
0.6	24	5.61E+05	2.50E-03	9.19E-04
0.7	28	5.63E+05	2.50E-03	9.25E-04
0.8	32	5.65E+05	2.50E-03	9.31E-04
0.9	36	5.67E+05	2.50E-03	9.37E-04
1.0	40	5.69E+05	2.50E-03	9.42E-04

The sensitivity analysis result in Table 4.5 shows that when the ratio of C_I to C_{PD} increases, τ* and the minimum expected cost rate increase (also shown in Figure 4.8), which indicates that the inspection cost has great impact on the optimal maintenance strategies. However, the inspection cost change has no impact on the optimal warning limit D*.

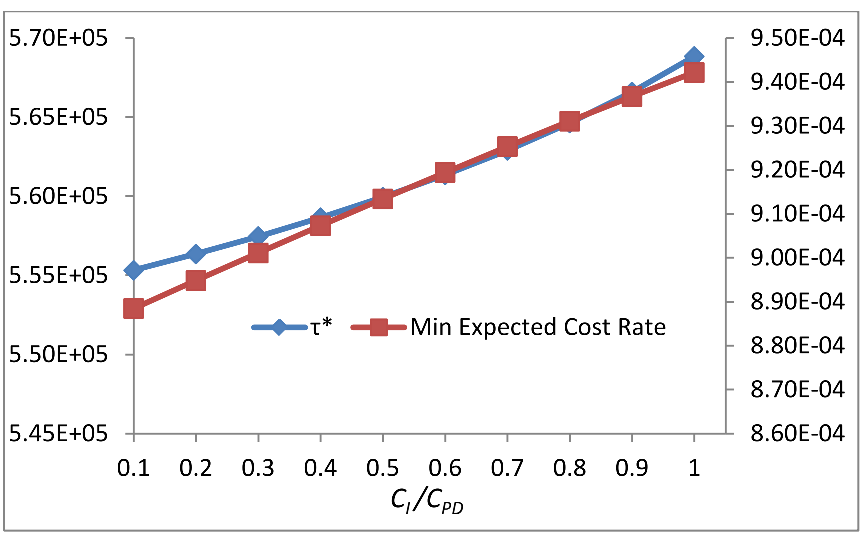

Figure 4.8: Sensitivity analysis of τ* and D* on C_I/C_{PD}

4.6 Conclusions

In this chapter, we study a complex system with dependent components subject to respective degradation processes, and the dependency among components is established via environmental factors. We develop a new reliability model for this type of system and use temperature as an example application to demonstrate our model. Relationships between degradation and environmental temperature are studied, and then, the reliability function is derived for such a system. Based on the unique dependent relationship among components within the system and the reliability analysis, a novel condition-based maintenance model is developed to assist system maintenance and minimize cost. To illustrate our reliability and maintenance models, a numerical example is used and sensitivity analysis is also conducted to test the model sensitivity to parameter changes.

Chapter 5: Condition-based Maintenance for Power Transformer using Markov Decision Process

In the previous chapters, we study reliability models and cost-effective maintenance policies for single-component systems and multi-component systems that have multiple failure processes. The degradation processes are usually modeled as linear paths that are the cases for many applications. However, when the degradation process has multiple states and the linear degradation path model is no longer applicable, new approaches should be studied to accurately predict system reliability and develop appropriate maintenance policies, such as Markov chain.

In this chapter, we study the failure modes of power transformers and propose a condition-based maintenance policy for power transformers using Markov decision processes (MDP). Power transformer is one of the most expensive assets in a power distribution system, and its health is very important to ensure continuous operation of the whole system. Failure mechanisms and reliability issues of power transformers have been studied in the literature, but research work on maintenance policies for power transformers is still lacking. Due to the increasing failure risk caused by their aging and deterioration global weather, there is an urgent need for maintenance actions to be taken on power transformers.

MDP method divides the system condition into different states, and dynamically determines the actions to take at the end of each decision epoch according to the current system state. This feature of MDP method is very useful, because an effective maintenance decision should be made based on the actual system condition revealed by periodic inspection. For systems with multiple failure processes/modes, we can use MDP method by adding additional failure states caused by these multiple failure processes/modes.

Power transformers can experience sudden failure due to weather-related random events. In addition, three major failure modes related to degradation processes have been identified: paper winding insulation, bushings, and tap-changer. The power transformer is modeled as a multi-state system, including four different operating states and four failure states (due to the four failure modes). Four maintenance actions are considered in this model: no action (NA), minimal maintenance (MM), preventive maintenance (PM), and corrective maintenance (CM). In the proposed maintenance strategy, periodic inspections are implemented, and the inspection interval is to be determined at each decision epoch. Therefore, the condition-based maintenance decision is a combination of two factors: maintenance action and the next inspection interval. A policy iteration algorithm is used to find the optimal policy that minimizes the average cost in a long run. A numerical example is given to demonstrate the proposed condition-based maintenance model.

NOTATION

S	Set of different states in the Markov Chain
S_f	Set of failure states where $S_f \subset S$
S_r	Set of failure states where the system condition is revealed without inspection, $S_r \subset S$
M_s	Set of different maintenance actions in the Markov decision process
$q_{ss'}(m)$	Probability that performing the maintenance action m changes the system from state s to state s'
$p_{ij}(t)$	Probability that the system deteriorates to state j by time t, when the initial state is i

$\tau_{ij}(t)$ Expected time that the system stays in state j, when the initial state is i

$P(s, s', a)$ Probability that performing action a brings the system from state s to state s'

$r(s, a)$ Expected cost of performing action a in state s

$y(s, a)$ Expected transition time of performing action a in state s

$C_M(s, a)$ Cost of performing a maintenance action $a \in A$ in state s

$C_F(s)$ One time cost of failure $S_f \subset S$

$C_I(s)$ Cost of inspection in state $s \in S$

$C_S(s)$ Cost of staying in state s per unit of time, $s \in S_f$, $s \notin S_r$

5.1 Introduction

For mechanical equipment based processes, when the output of the process depends on the proper functioning of the equipment, an effective maintenance policy is necessary to ensure the smooth operation of that equipment. Power transformer is one such piece of mechanical equipment where power distribution eventually depends on the proper functioning of the transformer, which in turn, depends on the maintenance actions implemented to ensure the optimal functioning. The power transformer is a crucial component of the power grid, and is often mentioned in relation to quality or capacity issues [136]. According to the failure statistics reported in [137], the main failure mechanisms of power transformers are related to: tap-changer (41%), windings (19%), leakage, bushings (13%), core (3%) and accessories (12%). Any failure to the transformer can lead to power outage, which causes financial losses and inconvenience to users. For the smooth operation of a transformer, it is imperative to have an effective maintenance policy to minimize unexpected failures and the losses due to these failures.

In some critical applications, such as wind turbine, Markov decision process has been widely applied, because it takes into consideration various states of a system and different maintenance actions that can be taken for each state. Therefore, it is a useful approach to determine a cost effective maintenance policy that can be implemented to reduce the cost. Byon *et al.* [138] presented a condition-based maintenance policy for multi-state deteriorating wind turbines subject to several failure modes. Considering the dynamic weather conditions, the problem is formulated as a partially observed Markov decision process with heterogeneous parameters. Byon *et al.* [139] derived an optimal maintenance policy to minimize the expected average cost over an infinite horizon using a partially observed Markov decision process, and several critical factors unique to wind farm operations are considered, such as weather conditions, lengthy lead times, and production losses. Using Markov decision process, Amari *et al*. [140] provided an optimal cost-effective maintenance decision based on the condition revealed at the time of inspection.

The reliability issues of another important application in energy field, power grid, have attracted a lot of attention from researchers in electrical engineering due to the increasing failure risk caused by its aging and deterioration global weather. Schijndel *et al*. [136] developed an integral lifetime model for power transformer considering three failure modes that are related to the degradation of winding insulation, bushings, and tap-changers. Zhou *et al*. [141] presented two methods to model the failure rate of overhead distribution lines under weather impact: (1) a Poisson regression model to capture the counting nature of failure events of overhead distribution lines; and (2) a Bayesian network model uses conditional probabilities of failure given different weather states. Du *et al*. [142] took the weather impact into the reliability assessment of the power grid, and divided the weather state into normal weather, bad weather and catastrophic weather. Alvehag *et al*. [143] proposed a reliability model for power distribution systems, which counts in the stochastic nature of the severe weather intensity and duration to model variations in failure rate and restoration time. A non-homogeneous Poisson process (NHPP) is also applied to model the occurrence of severe weather. Although extensive research has been done on the topic of power grid reliability, none of them considers the maintenance actions for the power grid based on mathematical analysis. To fill the void, we introduce a semi-Markov decision process for the condition-based maintenance of power transformer by determining the optimal policy to reduce maintenance cost.

The remaining sections of this chapter are arranged as follows. The detailed study on the power transformer failure modes is given in Section 5.2. Section 5.3 describes the proposed condition-based maintenance for the power transformer application. Section 5.4 presents the Markov decision process methodology and the policy iteration algorithm to find the optimal policy. A numerical example is used to demonstrate our model in Section 5.5. Finally, Section 5.6 gives the conclusion.

5.2 Failure Modes of Power Transformer

Power grid is an electrical transmission and distribution network system that provides customers secure and reliable electricity. It consists of a variety of power components, including transformers, generators, and overhead lines. According to [144], power systems generally have two types of failure: (1) an insulation failure resulting in a short-circuit fault, which can occur as a result of insulation degradation over time or due to a sudden overvoltage condition; and (2) a failure that results in a cessation of current flow or an open-circuit fault. The majority of short-circuit faults are weather related followed by equipment failure. The weather factors that usually cause short-circuit faults are: lightning strikes, accumulation of snow or ice, heavy rain, strong winds or gales, salt pollution depositing on overhead lines and in substations, floods and fires adjacent to electrical equipment, etc. [144].

As one of the most expensive assets in a power distribution system, the health of power transformer is critical to ensure continuous functioning. Besides the weather related random failure mode, three degradation processes are considered for transformer failure: paper winding insulation, bushings, and tap-changer, identified in the study of [136]:

- Paper insulation – transformer paper provides electrical insulation between windings. The quality of the electrical insulation is mainly determined by its mechanical strength. The ageing of the paper is accelerated by high temperature, high water content, and acidity.
- Bushing – the high voltage conductor is insulated from its surroundings by a bushing. Within the bushing the electrical fields are capacitively controlled to prevent breakdown. Breakdown may occur due to insulation degradation and subsequent short circuits in between the capacitive layers. Another potential hazard is over-heating by increasing bushing losses.
- Tap-changer – with the tap-changer the output voltage of the transformer can be regulated. Its functioning may be endangered by unsynchronized switching of the tap-selector and power-switch due to a broken axis or malfunctioning engine.

To prevent the failure of power transformer and reduce the loss, we study a condition-based maintenance policy using a semi-Markov decision process with the aim to minimize the average cost in a long run in the following sections.

5.3 Condition-based Maintenance Model

We consider a system with *M* failure modes, and its deterioration level can be divided into a finite number (L) of conditions. Then the system health condition can be categorized into a series of states, 1, …, $M+L$ [138]. For the instance of a power transformer, it has 4 failure modes (paper winding insulation, bushings, tap-changer, and random failure). We assume that it has 4 deterioration levels. Then 8 states are used to describe the system health condition: State 1 stands for the new condition, while State 4 denotes the most deteriorated operating condition before the transformer fails; State 5, 6, 7, and 8 represent four failure states due to four different failure modes, i.e., paper insulation failure, bushing failure, tap-changer failure, and random failure, respectively.

Based on the physical relationships between different states, we propose the system state transition diagram illustrated in Figure 5.1. In this figure, the symbol on the arrow denotes the transition rate from the current state to the next state connected by an arrow. For example, λ_1 denotes the transition rate from State 1 to State 2. From State 1, 2, 3, or 4, the system can transit to State 8 with transition rate λ_0, indicating that the system experiences a sudden failure.

For this multi-state deterioration model, we introduce condition-based maintenance concepts by using periodic inspections and deterioration state-based maintenance actions including no action (NA), minimal maintenance (MM), preventive maintenance (PM), and corrective maintenance (CM). For the perfect state 1, the maintenance action is NA.

For failure state 5-8, CM is performed, and the corrective maintenance cost may be different depending on the failed part. While the random failure state 8 is self-announcing, the failure in State 5-7 is assumed to be non-self-announcing, and the downtime cost should be considered for State 5-7. The resulting state after a maintenance action does not need to be deterministic, and may depend on various factors that cannot be fully controlled. Therefore, there can be a set of possible system states that can be reached after a maintenance action [140]. Besides, the inspection interval is not fixed and needs to be determined at each decision epoch. Therefore, the condition-based maintenance decision is a combination of two factors: maintenance action and the next inspection interval.

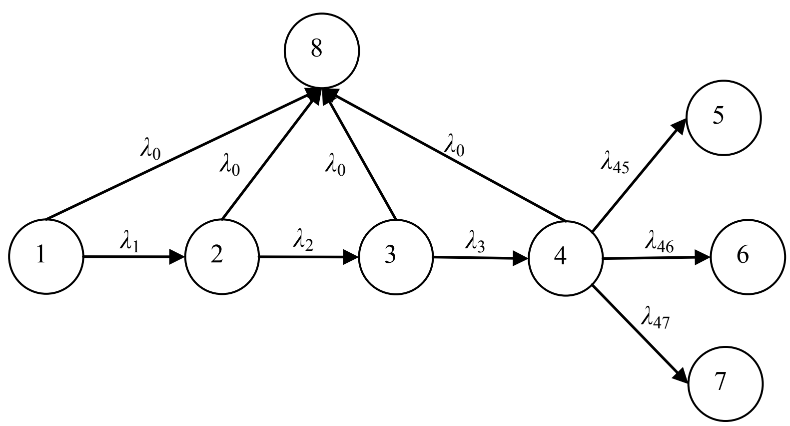

Figure 5.1: System state transition diagram

Different costs associated with different maintenance actions are considered in this model: inspection cost, minimal maintenance cost, preventive maintenance cost, corrective maintenance cost, failure cost, and downtime cost associate with State 5-7. The maintenance actions for State 1, 5-8 are predetermined; we need to determine the maintenance actions for State 2-4 and the inspection schedule for all the states.

5.4 Semi-Markov Decision Process

With continuous inspection intervals, standard Markov decision processes do not apply. We use a semi-Markov decision process (semi-MDP) for this type of CBM problem by applying the well-proven algorithms in [140]. In the semi-MDP, to find the optimal maintenance policy, we first need to compute the transition probability $P(s, s', a)$, expected transition time $y(s, a)$, and expected cost $r(s, a)$ for all $s, s' \in S$ and all a. Since the resulting state after a maintenance action is not deterministic, $P(s, s', a)$ is a product of maintenance action probability, $q_{sk}(m)$, and state transition probability, $p_{k\,s'}(t)$, for all $k \in S$. The detailed calculation steps are given in the following algorithm:

1. For each state s, select N values for the next inspection interval that are equally distributed over a possible range. Let T_s be the set of possible values in state s.
2. Solve the Markov Chain for $p_{ij}(t)$ and $\tau_{ij}(t)$ where

   $p_{ij}(t)$: P(state j at time t | initial state is i)

   $\tau_{ij}(t)$: E(time spent in state j in $(0, t)$ | initial state is i).
3. Each CBM action a in state s is a combination of maintenance action $m \in M_s$, and the next inspection time $t \in T_s$. Hence, $a = (m, t)$. Let A_s be the set of all possible CBM actions in state s: size(A_s)=size(M_s)·size(T_s).
4. For $s, s' \in S$, $a = (m, t) \in A_i$; compute $P(s, s', a)$: $P(s,s',a) = \sum_{k \in S} q_{sk}(m) \cdot p_{ks'}(t)$.
5. For $s \in S$, $a = (m, t) \in A_i$; compute $y(s, a)$: $y(s,a) = \sum_{s' \in S, s' \notin S_r} \sum_{k \in S} q_{sk}(m) \cdot \tau_{ks'}(t)$.
6. For $s \in S$, $a = (m, t) \in A_i$; compute $r(s, a)$:

$$r(s,a) = C_M(s,m) + \sum_{s' \in S_f} P(s,s',a) \cdot C_F(s')$$
$$+ \sum_{s' \in S, s' \notin S_r} q_{sk}(m) \cdot \tau_{ks'}(t) \cdot C_S(s') + \sum_{s' \in S, s' \notin S_r} P(s,s',a) C_I(s').$$

7. Using $P(s, s', a)$, $y(s, a)$, and $r(s, a)$, compute the optimal policy using policy iteration algorithm.

Standard Semi-MDP Solution

Let the policy π be a mapping from $s \in S$ to $a \in A_s$, $\pi(s) = a$. The objective is to find the optimal policy π^* that obtains the minimum average cost ρ in a long run [140]. $R_\pi(s)$ denotes the average cost associated with policy π in state s. $R_\pi(s)$ and ρ are different, as $R_\pi(s)$ is the average cost in state s, while ρ is the minimum average cost across all the states. Using the calculated $P(s, s', a)$, $y(s, a)$, and $r(s, a)$, a policy iteration algorithm is adopted to determine the optimal maintenance actions and the optimal inspection interval for each state.

Policy Iteration Algorithm [140]:

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1. Select an arbitrary policy: $\pi = \pi_0$.
2. Value determination: arbitrarily select a state $s_0 \in S$ and set $R_\pi(s_0) = 0$. Solve the following equations for unknowns: ρ_π and $R_\pi(s)$, $s \in S, s \neq s_0$.

$$R_\pi(s) = r(s,\pi(s)) - \rho_\pi \cdot y(s,\pi(s)) + \sum_{s' \in S} P(s,s',\pi(s)) \cdot R_\pi(s'), \quad s \in S.$$

3. Policy improvement: for each $s \in S$, determine the alternative action that yields

$$\arg\min_{a'} \left\{ r(s,a') - \rho_\pi \cdot y(s,a') + \sum_{s' \in S} P(s,s',a') \cdot R_\pi(s') \right\}, \quad s \in S.$$

---

4. The resulting optimum action for state $s \in S$ constitutes the new policy π'. If π and π' are identical, π is the optimum. Otherwise, set $\pi = \pi'$, and go to Step 2.

5.5 Numerical Example

For the studied power transformer, we consider three degradation processes (i.e., paper insulation, bushing and tap-changer) and a random failure process. As shown in Figure 5.1, there are 8 states to represent the health condition of the power transformer system. State 1 stands for the new condition, while State 4 denotes the most deteriorated operating condition before the transformer fails; State 5, 6, 7, and 8 represent four failure states due to four different failure modes, i.e., paper insulation failure, bushing failure, tap-changer failure, and random failure, respectively. We assume that the failure rates are: λ_0=0.001/yr, λ_1=λ_2=λ_3=0.1/yr, λ_{45}=0.33/yr, λ_{46}=0.22/yr, λ_{47}=0.44/yr. The maintenance action related costs are:

$$C_F = [0\ 0\ 0\ 0\ 500\ 500\ 500\ 500], \quad C_S = [0\ 0\ 0\ 0\ 20\ 20\ 20\ 0], \quad C_I = [10\ 10\ 10\ 10\ 10\ 10\ 10\ 0],$$

$$\mathrm{C}_M\left(s,\left(NA,t\right)\right) = 0,\ s \in S, \quad \mathrm{C}_M\left(s,\left(MM,t\right)\right) = 80,\ s \in S, \quad \mathrm{C}_M\left(s,\left(PM,t\right)\right) = 200,\ s \in S,$$

$$\mathrm{C}_M\left(5,\left(CM,t\right)\right) = 200, \quad \mathrm{C}_M\left(6,\left(CM,t\right)\right) = 250, \quad \mathrm{C}_M\left(7,\left(CM,t\right)\right) = 300, \quad \mathrm{C}_M\left(8,\left(CM,t\right)\right) = 350.$$

In this model, we consider four maintenance actions: NA, MM, PM, and CM. The maintenance actions in States 1, 5, 6, 7, and 8 are predetermined, where M_1 = {NA}, $M_5 = M_6 = M_7 = M_8$ = {CM}. The possible maintenance actions in States 2, 3, and 4 are: $M_2 = M_3 = M_4$ = {NA, MM, PM}. Regarding the resulting effect of different maintenance actions, NA brings no effect to the system state, and MM can improve the system condition by reducing one state, while PM and CM can bring the system to the perfect state. Namely,

$$q_{ss'}(NA) = \begin{cases} 1 & \text{for } s = s', \\ 0 & \text{for } s \neq s', \end{cases}$$

$$q_{ss'}(MM) = \begin{cases} 1 & \text{for } s = s'+1, \\ 0 & \text{for } s \neq s'+1, \end{cases}$$

$$q_{ss'}(PM) = q_{ss'}(CM) = \begin{cases} 1 & \text{for } s' = 1, \\ 0 & \text{for } s' \neq 1. \end{cases}$$

For the inspection interval, we consider 12 interval lengths, from 1 month up to 1 year, i.e. $t \in T_s = \{1, 2, \ldots, 12\}$. The initial policy is $\pi(s)$=(NA, 1) for s=1, 2, 3, 4, and $\pi(s)$ = (CM, 1) for s=5, 6, 7, 8. Using the given policy iteration algorithm, the solution is stabilized after four iterations. Table 5.1 shows the optimal policy. The minimum average cost ρ is 10.4838 per month.

Table 5.1: The optimal condition-based maintenance policy

State	Optimal Maintenance Action	Optimal Inspection Interval
1	NA	9 months
2	MM	9 months
3	MM	4 months
4	PM	9 months
5	CM	9 months
6	CM	9 months
7	CM	9 months
8	CM	9 months

The optimal policy indicates:

1) If the system is in State 1, NA is taken and the next inspection interval is 9 months.
2) If the system is in one of the failed states, 5-8, CM is taken and brings the system state to the perfect state 1. Therefore the next inspection interval is 9 months as well.
3) If the system is in State 2, MM is taken and brings the system state to the perfect state

1. Therefore the next inspection interval is 9 months again;

4) If the system is in State 3, MM is taken and brings the system state to a lower state 2. Then the next inspection interval is 4 months, shorter than that for the system starting from the perfect state 1;
5) If the system is in State 4, PM is taken and brings the system state to the perfect state 1. Therefore the next inspection interval is 9 months.

5.6 Conclusions

In this chapter, the failure modes of power transformer are investigated and a condition-based maintenance policy for power transformer using a semi-Markov decision process is proposed. Three degradation processes as potential causes of transformer failure are studied: paper winding insulation, bushings, and tap-changer. A weather-related random failure is also considered. The power transformer is modeled as a multi-state system, including four different operating states and four failure states due to the four failure modes. The proposed maintenance strategy has periodic inspection, and the inspection interval needs to be determined at each decision epoch. Therefore, the condition-based maintenance decision is a combination of two factors: maintenance action and the next inspection interval. A policy iteration algorithm is used to find the optimal policy that minimizes the average cost in a long run. A numerical example is given to demonstrate the proposed condition-based maintenance model.

This chapter is our attempt to fill the void of lack maintenance policies for power grid systems, where we implement Markov decision process in the condition-based maintenance modeling for such critical systems. To make the model more general and practical, there are many improvements that can be done for the future research. For example, some of the degradation processes, such as paper insulation, can be accelerated by high environmental temperature, and environmental temperature can be affected by seasonal weather. It is interesting and challenging to incorporate the impact of dynamic weather into the maintenance modeling for power transformers. For systems experiencing external sudden shocks, the shocks may speed up the degradation process in a way that the system state transfers to a more deteriorated state when a shock arrives, which is also very interesting to consider for power transformer maintenance modeling.

Chapter 6: Summary and Conclusions

In this dissertation, we focus on the reliability analysis and maintenance modeling for single component systems and multi-component systems. The proposed new models and analysis tools can be readily applied or customized to assist manufacturing and maintenance of evolving devices. The resulting models and optimization approaches represent fundamental research advancements that can be transformed or extended to other developing technologies, where traditional reliability methods do not apply.

6.1 Summary and Research Contributions

Chapter 2 to Chapter 5 discusses our work in detail. The main contributions of our work are summarized in the following.

In Chapter 2, new models are proposed to predict reliability and optimize maintenance for MDCFP when the failure thresholds can shift according to the patterns of shocks. In our model, two failure processes are considered: soft failure caused by continuous degradation together with additional abrupt degradation due to a shock process, and hard failure caused by the instantaneous stress from the same shock process. Three cases of dependency between the shock process and the hard failure threshold level are studied. This work is an entirely new research area that extends our previous work in dependent competing failure processes, which offers some distinct advantages for design and reliability problems when the component tolerance or resistance to failure reduces. For maintenance models, both block and age replacement policies are applied to each of the three cases. Based on reliability analysis, the average long-run cost rate is evaluated and optimized for each policy.

In Chapter 3, a reliability model is presented for systems subject to degradation and random shock processes. These two failure processes are dependent because shock loads can cause instantaneous damage on the degradation process. To make the model more realistic, we consider that only shocks larger than a certain value can affect the degradation process, since systems usually have some resistance against small shock loads due to material strength and system structures. Shocks are divided into three zones based on their magnitudes: safety zone, damage zone, and fatal zone. We further model the abrupt damages using an explicit function of shock load exceedances. Gamma process is used to model the stochastic degradation process, because it has non-negative increments properties. Peaks-over-threshold method is used to model random shock loads, where shock arrivals follow a homogeneous Poisson process. Due to the complexity in modeling these two dependent stochastic failure processes, no closed form of the reliability function can be derived. Monte Carlo importance sampling is used to estimate the system reliability.

In Chapter 4, a complex system with dependent components subject to respective degradation processes is studied, and the dependency among components is established via environmental factors. We develop a new reliability model for this type of system and use temperature as an example application to demonstrate our model. Based on the thermodynamic study of the relationship between degradation and environmental temperature, a reliability model is developed to mathematically account for the dependence in multiple components for such a system. A novel condition-based maintenance model is proposed based on the unique dependent relationship among components in the system.

In Chapter 5, the failure modes of power transformer are investigated and a condition-based maintenance policy for power transformer using a semi-Markov decision process is presented. Besides a weather-related random failure process, three degradation processes as potential causes of transformer failure are studied: paper winding insulation, bushings, and tap-changer. The power transformer is modeled as a multi-state system, including four different operating states and four failure states due to the four failure modes. In the proposed maintenance strategy, periodic inspections are implemented, and the inspection interval is to be determined at each decision epoch. Therefore, the condition-based maintenance decision is a combination of two factors: maintenance action and the next inspection interval. A policy iteration algorithm is used to find the optimal policy that minimizes the average cost in a long run.

6.2 Future Directions

Based on our research results, several research directions can be explored for the future work:

1. For multi-component devices that experience multiple failure processes, explore the reliability and maintenance problems for a broader range of complex and sophisticated devices, including multi-component systems with independent heterogeneous components, and systems with dependent homogeneous or heterogeneous components.
2. In the dynamic environment, the dependency among components in a system can be

attributed to different factors, such as temperature, humidity and random external shocks. In the future research work, it is necessary to investigate the physics of failure involving the impact from the dynamic environment on components' degradation behaviors and develop practical reliability models to facilitate manufacturing and maintenance in industry.

3. Incorporate the impact of dynamic weather into the maintenance modeling for power transformers and other critical devices/infrastructures.
4. Consider the effect of shocks on the degradation process in multi-state systems. For systems experiencing external sudden shocks, the shocks may speed up the degradation process in a way that the system state transfers to a more deteriorated state when a shock arrives. This is the case for power grid systems and other critical devices/infrastructures that operate under harsh environments.

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Appendices

Appendix I: Derivation of System Reliability Function for Case 3 in Chapter 2

To derive the system reliability function for the generalized m-shock model in Case 3, we consider the following three situations based on the number of shocks arrived at t, $N(t)$.

1) When $N(t)=0$, no shock happens, and

$$R(t \mid N(t)=i=0)=P\left(X_S(t)<H \middle| N(t)=0\right)=P\left(X(t)<H\right).$$

2) When $N(t)=i$, $1 \le i \le m$, and

$$R(t \mid N(t)=i, 1 \le i \le m)=P\left(\bigcap_{l=1}^{N(t)}\{W_l<D_1\}, X_S(t)<H \middle| N(t)=1\right).$$

3) When $N(t)>m$, the hard failure threshold value may reduce from D_1 to D_2 based on the following two scenarios.

   a) If there are no m shocks larger than D_0 ($N(t)=i>m$), then we design the following table to facilitate our derivation of the reliability function.

List of scenarios when no m shocks are larger than D_0

Scenarios		Number of combinations	For each combination, the probability that the system survives by the time t given this scenario
Number of shocks $>D_0$	Number of shocks $<D_0$		
m - 1	$N(t)$-m+1	$\binom{N(t)}{m-1}$	$\left(F_W(D_1)-F_W(D_0)\right)^{m-1}F_W(D_0)^{i-m-1}$ $\times P\left(X_S(t)<H\right)P\left(N(t)=i\right)$
…	…	…	…
1	$N(t)$ - 1	$\binom{N(t)}{1}$	$\left(F_W(D_1)-F_W(D_0)\right)^{1}F_W(D_0)^{i-1}$ $\times P\left(X_S(t)<H\right)P\left(N(t)=i\right)$
0	$N(t)$	$\binom{N(t)}{0}$	$\left(F_W(D_1)-F_W(D_0)\right)^{0}F_W(D_0)^{i}$ $\times P\left(X_S(t)<H\right)P\left(N(t)=i\right)$

For example, when only one shock is over D_0 and the remaining $N(t)$-1 shocks are less than D_0 (the second to the last row), the number of possible combinations is given in the third column. Assuming the first shock is greater than D_0 for i=m+1, m+2, …, ∞, we have

$$P\left(D_0 < W_1 < D_1, W_2 < D_0, \ldots, W_{N(t)} < D_0, X_S(t) < H, N(t) = i\right)$$
$$= \left(F_W(D_1) - F_W(D_0)\right)^1 F_W(D_0)^{i-1} P\left(X_S(t) < H\right) P\left(N(t) = i\right).$$

Because all the shock magnitudes are independent and identically distributed, the formula in the last step remains unchanged for each of the combinations.

Considering all the scenarios in this table, when there are no m shocks larger than D_0 for i=m+1, m+2, …, ∞, the reliability that the system survives by time t is

$$R(t \mid N(t) = i > m, \left|S_{D_0}\right| < m) = \sum_{j=0}^{m-1} \binom{i}{j} P\left(\bigcap_{l=1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_0\}, X_S(t) < H \middle| N(t) = i \right).$$

b) If there exist m shocks larger than D_0 ($N(t) = i > m$), we design the following table to facilitate our derivation of the reliability function.

List of scenarios when there are m shocks that are larger than D_0

Scenarios			Number of combinations	For each combination, the probability that a system survives by time t given the scenario that the j^{th} shock is the m^{th} shock larger than D_0
j^{th} shock (the m^{th} shock larger than D_0)	Number of shocks less than D_0 by the j^{th} shock	Number of shocks after the j^{th} shock (D_2 is the failure threshold)		
m	0	$N(t)$ - m	$\binom{m-1}{m-1}$	$\left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_0)^0 F_W(D_2)^{i-m} \times P\left(X_S(t) < H\right) P\left(N(t) = i\right)$
m+1	1	$N(t)$-m-1	$\binom{m}{m-1}$	$\left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_0)^1 F_W(D_2)^{i-m-1} \times P\left(X_S(t) < H\right) P\left(N(t) = i\right)$
…	…	…	…	…
$N(t)$ - 1	$N(t)$-m-1	1	$\binom{N(t)-2}{m-1}$	$\left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_0)^{i-m-1} F_W(D_2) \times P\left(X_S(t) < H\right) P\left(N(t) = i\right)$
$N(t)$	$N(t)$ - m	0	$\binom{N(t)-1}{m-1}$	$\left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_0)^{i-m} F_W(D_2)^0 \times P\left(X_S(t) < H\right) P\left(N(t) = i\right)$

In the above table, the j^{th} shock is defined as the m^{th} shock larger than D_0. For example, j = $N(t)$ - 1 implies that $N(t)$ - 1 is the m^{th} shock beyond the critical value D_0.

Therefore, m-1 shocks larger than D_0 occur by the $N(t)$ - 2 shock, and the number of possible combinations is given in the fourth column. Consequently, for the last shock, the failure threshold value decreases from D_1 to D_2. Without loss of generality, we assume that all the first m-1 shocks are greater than D_0, and then for i=m+1, m+2, …, ∞, we have

$$
\begin{aligned}
&P\begin{pmatrix} D_0 < W_1 < D_1, \dots, D_0 < W_{m-1} < D_1, W_m < D_0, \dots, W_{N(t)-2} < D_0, \\ D_0 < W_{N(t)-1} < D_1, W_{N(t)} < D_2, X_s(t) < H, N(t) = i \end{pmatrix} \\
&= \left(F_W(D_1) - F_W(D_0))\right)^m F_W(D_0)^{i-m-1} F_W(D_2) P\left(X_S(t) < H\right) P\left(N(t) = i\right),
\end{aligned}
$$

which is the formula given in the last column for j=$N(t)$ - 1.

The occurrence of m - 1 shocks larger than D_0 in the first $N(t)$ - 2 shocks does not impact the formula because the shock magnitudes are independent and identically distributed. Therefore, for the scenario of j=$N(t)$ - 1, we have

$$
\binom{i-2}{m-1}\left(F_W(D_1) - F_W(D_0)\right)^m F_W(D_0)^{i-m-1} F_W(D_2) P\left(X_S(t) < H\right) P\left(N(t) = i\right),
$$

for i= m+1, m+2,…, ∞.

Considering all the scenarios in this table, when there exist m shocks larger than D_0 for i=m+1, m+2, …, ∞, the probability that the system survives by time t is

$$
\begin{aligned}
&R(t \mid N(t) = i > m, \left|S_{D_0}\right| \geq m) \\
&= \sum_{j=m}^{i} \binom{j-1}{m-1} P\left(\bigcap_{l=1}^{j-m} \{W_l < D_0\}, \bigcap_{l=j-m+1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_2\}, X_S(t) < H \,\middle|\, N(t) = i \right).
\end{aligned}
$$

Finally, the reliability function of the system by time t is

$$
\begin{aligned}
R(t) &= \sum_{i=0}^{\infty} R(t \mid N(t) = i) P(N(t) = i) \\
&= P\left(X(t) < H\right) P(N(t) = 0) + \sum_{i=1}^{m} P\left(\bigcap_{l=1}^{N(t)} \{W_l < D_1\}, X_S(t) < H \,\middle|\, N(t) = i \right) P(N(t) = i) \\
&\quad + \sum_{i=m+1}^{\infty} \left(\sum_{j=0}^{m-1} \binom{i}{j} P\left(\bigcap_{l=1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_0\}, X_S(t) < H \,\middle|\, N(t) = i \right) \right. \\
&\quad \left. + \sum_{j=m}^{i} \binom{i-1}{j-1} P\left(\bigcap_{l=1}^{j-m} \{W_l < D_0\}, \bigcap_{l=j-m+1}^{j} \{D_0 < W_l < D_1\}, \bigcap_{l=j+1}^{N(t)} \{W_l < D_2\}, X_S(t) < H \,\middle|\, N(t) = i \right) \right) \\
&\quad \times P(N(t) = i).
\end{aligned}
$$

Appendix II: List of Publications

Journal Publications

L. Jiang, Q. Feng, and D. W. Coit, "Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds," *IEEE Transactions on Reliability,* vol. 61, pp. 932-948, Dec 2012.

L. Jiang, Q. Feng, and D. W. Coit, "Modeling zoned shock effects on stochastic degradation in dependent failure processes," Accepted by *IIE Transactions*, 2014.

L. Jiang, Q. Feng, and D. W. Coit, "Reliability analysis and condition-based maintenance of systems with dependent degrading components based on thermodynamic physics-of-failure," Submitted to *IEEE Transactions on Reliability*, 2014.

Conference Proceedings

L. Jiang, Q. Feng, and D. W. Coit, "Reliability analysis of systems with dependent degrading components via thermodynamic physics-of-failure analysis," *Proceedings of Industrial and Systems Engineering Research Conference*, Orlando FL, May 19-23, 2012.

L. Jiang, Q. Feng, and D. W. Coit, "Reliability analysis for dependent failure processes and dependent failure thresholds," *Proceedings of International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering*, Xi'an, China, June 17-19, 2011.

Conference Presentations

L. Jiang, Q. Feng, and D. W. Coit, "Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds," *INFOMRS*, Phoenix AZ, Oct 14-17, 2012

L. Jiang, Q. Feng, and D. W. Coit, "Reliability analysis of systems with dependent degrading components via thermodynamic physics-of-failure analysis," *ISERC*, Orlando FL, May 19-23, 2012.