FILTERING CHARACTERISTICS OF A LONG CYLINDRICAL STEEL BAR WITH DISCONTINUITIES IN CROSS SECTIONAL AREA

A Thesis

Presented to

the Faculty of the Department of Mechanical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

by

James Dennis Bruner

June, 1966

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ABSTRACT

When a continuous longitudinal wave is propagated along the length of a finite cylindrical steel bar the frequency of maximum response of the propagated wave depends on the length of the bar. This is due to the reflections of acoustic energy which occur at the ends. Any other abrupt change in cross sectional area is also a point of energy reflection and if there are several area discontinuities spaced along the bar length the possibility of acoustic resonances exists.

Typically, such a system is analyzed by means of mechanical-electrical analogies. A four-terminal electrical network is synthesized which has the characteristics of the mechanical system. The result is a Pi-section band pass filter composed of passive, non-dissipative components. These components are functions of the physical characteristics of the mechanical system and take into consideration such parameters as distance between discontinuities and area ratios at discontinuities.

A computer program is given by means of which quantitative information is obtained and discussed. The attenuation is presented as a function of frequency for a typical physical model. The pass and stop bands for this system are discussed.

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TABLES AND FIGURES

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SYMBOLS

A = Cross Sectional Area

c = Longitudinal Velocity of Sound

C = Capacitance

C_o = Compliance per Unit Length

C_{eq} = Equivalent Compliance for Longitudinal Vibrations

E = Young's Modulus

f = Frequency

f_n = Resonant Frequency

F = Force

I = Current

L = Inductance

ℓ = Length

M_o = Mass per Unit Length

M_{eq} = Equivalent Mass for Longitudinal Vibrations

n = Longitudinal Vibration Mode Number

N = Attenuation in Nepers

p = Pressure

Q = Quality Factor

t = Time

u = Longitudinal Displacement

V = Volume Velocity

x = Distance Along Longitudinal Axis

X_C = Capacitive, Compliance Reactance

X_L = Inductive Reactance

X_M = Mass Reactance

Z = Impedance

Z_A = Acoustical Impedance

Z_o = Characteristic Impedance

Z_{res} = Impedance at Resonance

α = Cross Sectional Area Ratio

λ = Wavelength

ρ = Mass Density

CHAPTER I

INTRODUCTION

The subject of this thesis is the attenuation suffered by a plane longitudinal acoustic wave propagated in a long steel bar with spaced discontinuities in cross sectional area. A sketch of the system being studied appears in Figure 1. The nominal dimensions for a typical system are; total length ℓ up to 10,000 feet; ℓ_b, 30 feet; ℓ_u, 2 feet; A_b, 5 square inches; A_u, 10 square inches. A sinusoidally varying excitation is

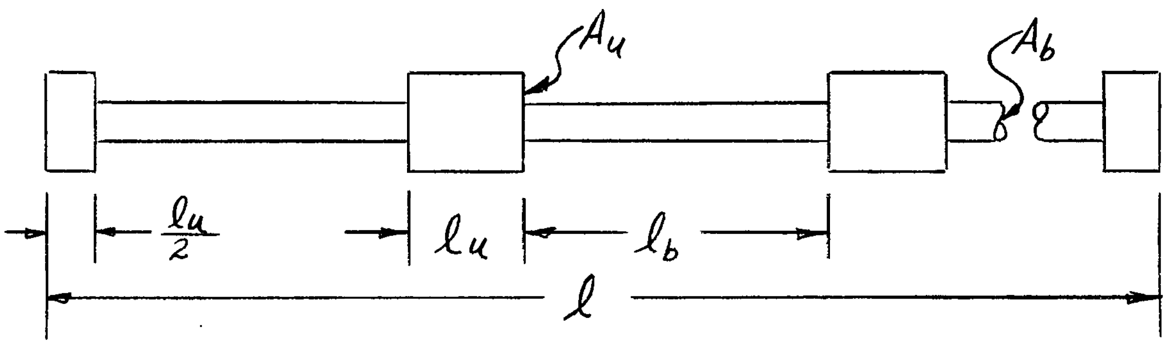

Figure 1

Steel Bar with Area Discontinuities

coupled to one end propagating a continuous wave in the bar. The attenuation then, is taken as a function of the ratio of the response at the far end to the excited end. The short sections of increased cross sectional area are called upsets and their length (ℓ_u) is constant along the total system length (ℓ). The length between upsets (ℓ_b)

is permitted to vary along the length ℓ. Usually the cross sectional areas, A_b and A_u, are constant in any specific study, but the possibility of using different values will be retained in the analysis. The excitation frequencies of interest are those for which the distance between upsets ℓ_b is approximately one wave length. The analysis is designed to study three effects on the passage of acoustic waves, namely, that of

- the presence of the upsets
- varying the bar length, that is, the distance between upsets (ℓ_b) and
- varying the cross sectional area ratio $\alpha = \frac{A_u}{A_b}$.

Several investigators [1, 2, 3, 4]* have studied similar problems, but none have studied the system described here. Some discussed single-area discontinuities or single restrictions (short sections of decreased cross sectional area) in infinite length bars [1, 2, 3]; others studied the effect of repeated sections at frequencies below those of interest here [4]. In the frequency range studied here the effect of repeated upsets may be significant.

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*Numbers in brackets refer to identically numbered items in the Bibliography.

This work is based on the work done by Mason, Miles and Karal [1, 2, 3], where use is made of electrical analogies. The values of the electrical components are functions of the mechanical and physical characteristics of the system. The analysis leads to a lumped-parameter electrical circuit which is analogous to a steel bar with repeated upsets.

The analysis is based on the following assumptions:

1) The bar material is assumed to be homogeneous, isotropic and perfectly elastic.

2) There is no external damping of the bar and the system is suspended horizontally in such a manner as to provide no restriction to axial motion. This restriction keeps the bar in a statically unstressed state.

3) In the actual physical system at points of energy reflection, there is a partial transformation of energy such that, say, longitudinal waves are coupled with, say, shear waves. However, for this analysis the assumption is made that no such coupling exists and that all energy remains in the plane, longitudinal wave mode.

4) The exciting function is coupled to the system in such a manner that only plane longitudinal waves are propagated. It is assumed further that the exciting function is applied at any one frequency a sufficient amount of time so as to allow the system to reach a steady-state condition.

5) The excitation-force impedance is equal to the characteristic impedance of the input to the system and the impedance at the free end of the system is equal to the characteristic impedance at the output. This assumption reduces the effect of the "end effects" mentioned by McNiven [5].

CHAPTER II

HISTORY OF PREVIOUS WORK

Plane longitudinal waves in a solid bar are described by the well known wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} , \tag{1}$$

where u is the displacement of an infinitesimal section of the bar, x is the distance along the longitudinal axis of the bar, t is time and c is the longitudinal velocity of sound propagation in the medium. For frequencies where one wavelength is much greater than the transverse dimensions of the bar, the velocity of propagation c is defined as

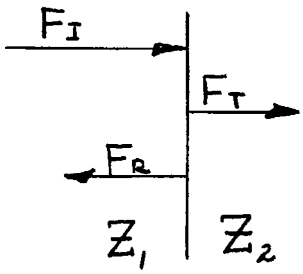
$$c = \sqrt{\frac{E}{\rho}} , \tag{2}$$

where E is Young's modulus and ρ is the mass density of the bar material. The solution to Equation (1) characterizes a longitudinal wave in an infinite length bar without dissipation.

If a wave traveling in an acoustic medium encounters an abrupt change in acoustical impedance, there is a reflection of energy. The acoustical impedance Z_A of a medium is defined as the ratio of the applied pressure, p, to the resulting volume velocity, V, that is,

$$Z_A = \frac{p}{V} \quad . \tag{3}$$

For a system such as the one studied here the acoustical impedance is a function of cross sectional area, acoustic velocity and density of the medium. At bar locations where the acoustical impedance is discontinuous the amounts of energy reflected and transmitted are functions of the impedances on either side of the discontinuity.



$$\frac{\text{Force Reflected } (F_R)}{\text{Force Incident } (F_I)} = \frac{Z_2 - Z_1}{Z_1 + Z_2} \tag{4}$$

$$\frac{\text{Force Transmitted } (F_T)}{\text{Force Incident } (F_I)} = \frac{2Z_2}{Z_1 + Z_2} \tag{5}$$

Figure 2

Wave Reactions at Impedance Discontinuity

For the case where the medium on either side of the discontinuity is the same, these ratios reduce to the following functions of area only,

$$\frac{F_R}{F_I} = \frac{A_2 - A_1}{A_1 + A_2} \quad , \qquad \frac{F_T}{F_I} = \frac{2A_2}{A_1 + A_2} \quad . \tag{6}$$

A resonance may occur if there are two or more acoustic discontinuities present. Resonance is possible when the distance between discontinuities is equal to an integral number of half-wavelengths. If the acoustical impedance relative to the bar is increased at adjoining discontinuities the pressure wave is reflected in phase with the incident wave. Each in-phase reflection results in increasing the magnitude of the pressure pulse. In the same manner the velocity wave is reflected 180° out-of-phase with the incident wave, resulting in a reduction of the velocity signal. Thus, at resonance the acoustical impedance of the bar between discontinuities becomes negligibly small.

This can be visualized as follows: If a driving pressure is applied to one end of the bar at a frequency equal to that of a resonance of the bar, all the pressure waves are reflected in phase. If these waves are permitted to reflect back and forth for a sufficiently long time, in theory the magnitude of the pressure wave within the bar will increase without limit. In the physical case, the ultimate pressure wave magnitude is limited only by the internal damping in the bar. Thus, a continuous pressure wave can be propagated in a finite bar with almost no reduction in magnitude. This is true only of the steady-state response of the system subjected to a single-frequency forcing function.

The studies of the effects of area discontinuities and restrictions have been limited to mechanical wave filters and delay lines, acoustic delay lines and strings of drill pipe. Mechanical wave filters and delay lines are manufactured commercially and have some characteristics which allow them to surpass their electrical counterparts in effectiveness. For example, the response of mechanical wave filters is steeper on either side of the pass band than comparable electronic devices. Acoustic delay lines, which take advantage of the relatively slow velocity of acoustic propagation (as compared to that of electrical signals) can be made small in size and are relatively simple in design. Systems of this kind are usually studied by means of mechanical-electrical analogies.

Another area of similar interest is concerned with the vibration of drill strings, such as those used in the oil industry [4, 6, 7]. In most cases these studies were undertaken to study the feasibility of using longitudinal waves to increase the efficiency of the drilling process and to study the stress levels which occur during drilling. Bradbury and Wilhoit [4] considered a problem similar to that studied here. However, they limited their interest to frequencies well below those corresponding to resonance, that is, to frequencies for which the wavelength was significantly greater than the distance between upsets. Their conclusion was that, at these low frequencies, the tool joints (upsets) had negligible effect on the passage of plane longitudinal waves.

The effect of acoustical impedance discontinuities in infinite length bars has been studied by several investigators [1, 2, 3, 8, 9]. Some have used electrical analogies [1, 2, 3] to study effects of single- and double-area discontinuities on the passage of plane longitudinal waves. Others [8, 9] have studied finite resonant bars in terms of electrical analogies. On the basis of these earlier studies the effect of repeated discontinuities on acoustical waves is studied here.

CHAPTER III

EFFECTS OF ARBITRARILY SPACED UPSETS ON PASSAGE OF PLANE LONGITUDINAL WAVES

It has long been known that mechanical and acoustical systems behave in a manner similar to electrical systems. The similarities in behavior stem from the close resemblance of the differential equations of motion for the systems studied. Two major analogies are used to describe mechanical and acoustical systems in terms of electrical components. These are the impedance and mobility analogies which are discussed in Appendix A.

Using the impedance analogy, Pollard [8] and Skudrzyk [9] have shown, that a finite resonant bar is analogous to a series-resonant circuit of an inductor and capacitor. The values of the electrical components are functions of the mechanical and physical characteristics of the system. Resonance in the bar is defined as a frequency at which there are an integral number of half-wavelengths present in the bar. A single series circuit model is valid for only one mode of longitudinal vibration. (The first mode is that for which the bar is one half-wavelength in length, the second for one wavelength, etc.). Thus, the response of a finite bar is analogous to that of the circuit shown in Figure 3 where, each resonant mode is represented by one series L-C branch.

The electrical analogy holds for an anti-resonance if the series circuit is replaced by a parallel-resonant circuit. Mechanical anti-resonance corresponds to the situation in which an odd multiple of quarter-wavelengths exist in the bar. An anti-resonant condition will always occur between any two peaks in response. At any anti-resonance the response is zero (or minimum) in the mobility analogy.

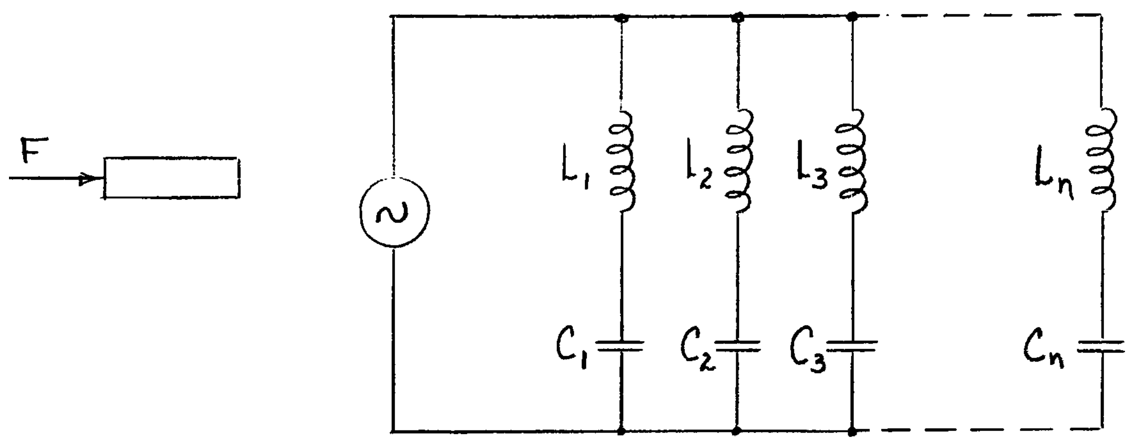


Figure 3

Finite Bar Analog

The effect of an abrupt change in cross sectional area in a long bar has been studied by Miles [2] and Karal [3] using the mobility and impedance analogies, respectively. Miles showed that the discontinuity could be represented by a shunt capacitance which is a function of the area ratio. Karal showed that the discontinuity may be represented by a series inductance which also is a function of the ratio of cross sectional areas. Note that the values of the analogous electrical components are not functions of frequency or mode but rather of the physical characteristics of the system (Figure 4).

Mason [1] and Miles [2] extended the results reported above and have shown that a double change in cross sectional area (upset or restriction) of a very long bar is analogous to a "T" or "Pi" network (Figures 5 and 18). Mason used the impedance analogy and Miles the mobility analogy. Again, the values of the inductors and capacitors are functions only of the physical characteristics of the upset.

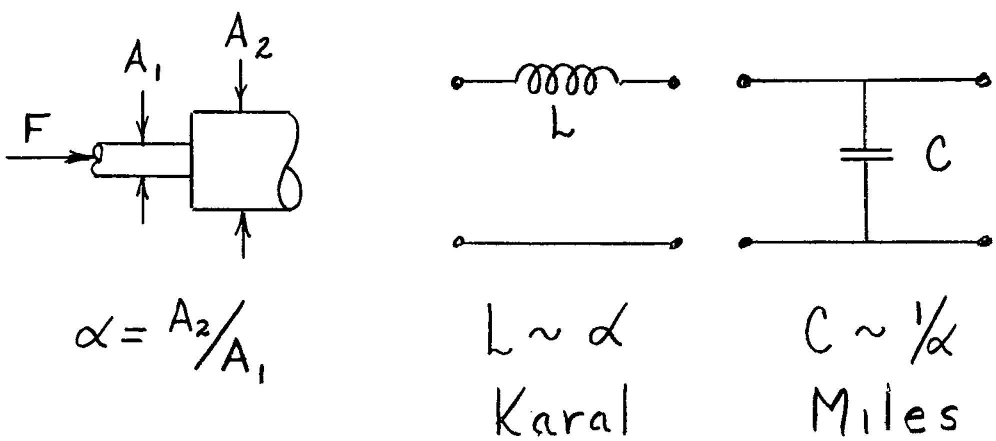


Figure 4

Single Discontinuity Analogies

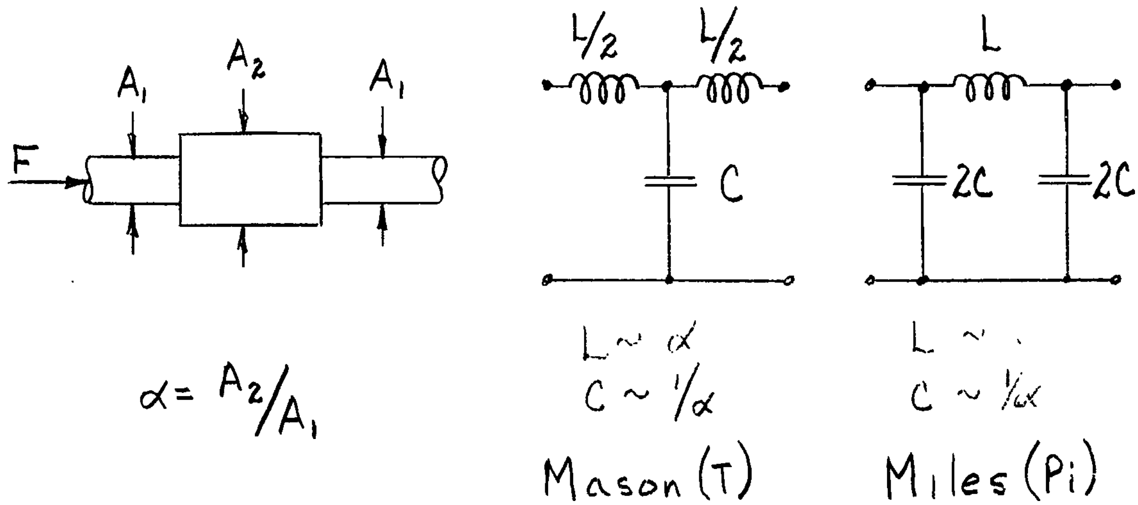


Figure 5

Upset Analogies

There are two important limitations placed on the above analyses, namely, that the frequency of the waves being propagated is sufficiently low that one wavelength is much greater than the transverse dimensions of the bar and that the length of the upsets is small compared to one wavelength. The first is required because the mode of acoustic propagation changes (from longitudinal to shear) when the ratio of the wavelength to the transverse dimensions of the bar approaches unity. Also, in the analysis of the single discontinuities, higher order modes are generated and are important to the pressure distribution at the discontinuity. It is important that the frequency of excitation is sufficiently below these higher order modes to ensure that the higher frequencies are rapidly attenuated.

As to the second limitation, Mason has stated that the T network representation of the upset is accurate to within 5 percent provided that the length of the upset (ℓ_u) is less than one-eighth wavelength. For the 2-foot upset shown in Figure 1, this implies that $f_{max} \approx 1000$ cps, that is, $f_{max} = C/\lambda_{min} = 16400/2 \cdot 8 = 1025$ cps.

If we wish to study the first four longitudinal modes between upsets, using a separation between upsets of 30 feet, it can be shown that this places f_{max} at approximately 1100 cps. Thus, we conclude that we can expect to use the proposed analysis in the frequency range from 100 to 1100 cps.

Within the above limitations the method of Mason is extended to determine the response of systems similar to that shown in Figure 1. If we consider the length of bar between any two upsets as a finite bar with increasing discontinuities in acoustical impedance at the ends, we are led to represent the system of Figure 1 as the circuit shown in Figure 6.

The elements of the circuit of Figure 6 may be rearranged as shown in Figure 7.

In turn, we note that the three sections in the center of the circuit in Figure 7 are repeated for each section of the system; thus, at resonance, the repeated sections reduce to Figure 8.

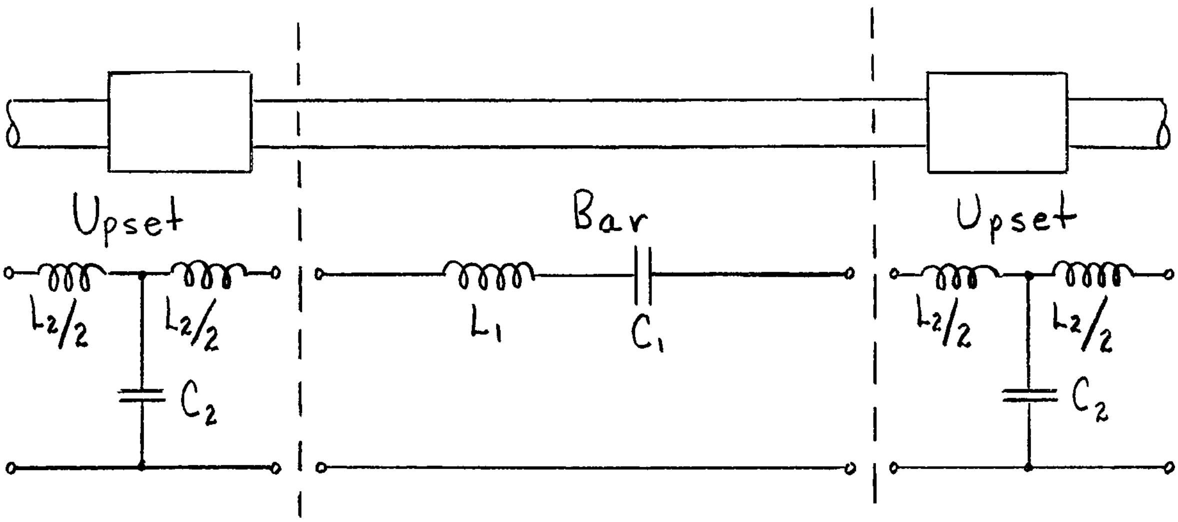


Figure 6

Electrical Analog of Figure 1

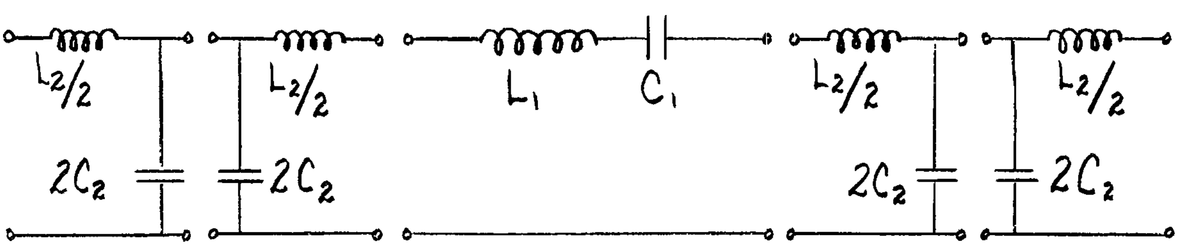


Figure 7

Rearranged Electrical Analog

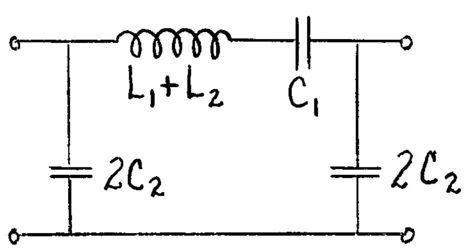


Figure 8

Electrical Analog of Repeated Section

At anti-resonance the circuit is the same as the above except that the series elements, L_1 and C_1, are replaced by a parallel arrangement of the same two elements.

As mentioned in Appendix B this circuit can be cascaded as many times as there are repeated sections in the physical system. The total system response will be governed by the response of the cascaded circuits. The only problem remaining is to determine the values of the components in the circuit.

Mason has shown that for longitudinal vibrations the value of the inductance L_1 is equal to the equivalent mass of the bar. Further, he showed that the equivalent mass is independent of the mode of vibration and has the value of

$$L_1 = M_{eq} = \frac{M_o \ell}{2} , \tag{7}$$

where M_o is the mass per unit length of a bar ℓ units long. The capacitance C_1 is (after Mason) equal to an equivalent compliance C_{eq} of the bar. This equivalent compliance is a function of mode number and is given by (8)

$$C_1 = C_{eq} = \frac{2 C_o \ell}{n^2 \pi^2} , \tag{8}$$

where C_o is the compliance per unit length, (1/AE), ℓ is the length of the bar and n is the mode number of longitudinal vibration. If the values of M_{eq} and C_{eq} are plotted as constant-mass and constant-compliance lines for each mode on impedance paper, as mentioned in Appendix A, the lines intersect at the resonant frequencies of the system and the corresponding values of impedance are called characteristic values.

The values of the inductance and capacitance of the T network representation of the upset may be calculated (after Mason) and are given by

$$L_2 = \frac{M_o' \ell'}{2} \quad , \quad C_2 = C_o' \ell' \tag{9}$$

respectively, where the primed notation refers to the upsets.

Thus far, we have neglected the effect of damping on the system. Since the system response at resonance is limited by the internal damping of the system, this quantity should be evaluated. In general, the internal dissipation of a bar vibrating longitudinally depends on the hysteresis cycle associated with the deformation of the material. This type of damping is independent of frequency and as a result can be represented by a series resistor in the circuit. Since the damping is independent of frequency, it can be described by the Q of the system (Appendix A). The Q of a system is a measure of the amount of damping present and is defined in terms of the ratio of the peak energy stored to the energy dissipated per cycle.

The Q of steel is a function of its microstructure; it is usually relatively large number, say 5000, signifying a small amount of internal damping. Typically the values of Q for the systems similar to that studied here are much less than this. For example, Hueter and Bolt [10] cite a value of Q = 200 for an unloaded drill string. Church [11] states that the Q of a similar structure is approximately 200-500. These values are associated with structural damping rather than any inherent energy dissipation mechanism in the material. The principal source of structural damping is the rubbing which occurs in the tool joints. It is similar to hysteresis damping in that it is independent of frequency.

In the circuit then, the major part of the resistance representing damping should be in that section which characterizes the upset. Since it appears as a series resistor, its precise placement within the section is unimportant. Thus, we conclude that the limiting effect of the structural damping in the drill string can be simulated by limiting the reactance value of the analogy at the resonant frequency.

The adequacy of an analogy can be checked by testing its response at some known limiting condition. In the system considered here this can be accomplished by observing the system response as the frequency approaches zero and as the area ratio at the upsets approaches unity. As the frequency approaches zero the shunt impedance of the T network approaches infinity. In turn, the impedance of the series inductor becomes negligibly small. The net result is that at low frequencies the upsets should have no measurable effect on the response of the system. This corresponds to the case considered by Bradbury and Wilhoit [4].

As the area ratio of the upsets approaches unity, the series inductance and shunt capacitance approach zero. Thus, again the series impedance becomes zero and the shunt impedance becomes infinite. The circuit at resonance then reduces to that in Figure 9.

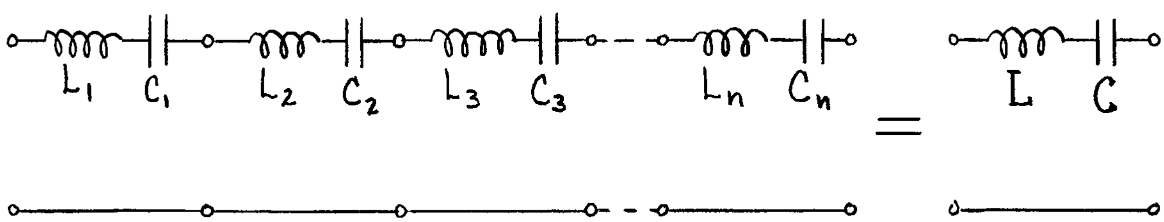


Figure 9

Unity Area Ratio Analog

Thus, the total system response reduces to that of a long, but finite, bar.

CHAPTER IV

RESULTS AND CONCLUSIONS

In the numerical analysis presented here the system parameters were taken to have the values or permitted to vary in the range stipulated below:

The cross sectional area of the bar A_b between upsets used was taken to be 5 square inches. The assumed cross sectional area ratios α between upsets and bar were 2:1, 3:1, 4:1, 5:1, and 10:1. A length between upsets of ℓ_b = 30 feet was used as a standard; however, this distance was permitted to vary as much as plus or minus one foot.

These physical factors were arranged so as to include several configurations, including the following:

1) All sections between upsets constant at either 360", 348" or 372".

2) One half of total number of sections each at 348" and 372".

3) One section of each length at increments of one inch from 348" to 372".

4) Ten times as many sections at 348" as 372" and conversely.

5) Reversing the order in which each of the above-mentioned section lengths occur in the system.

Details of the computation are given in Appendix C with typical numerical results. The machine output consisted of values for finite bar impedance between upsets and system attenuation versus frequency. Typical computer outputs are presented graphically in Figures 10 and 11. An examination of the detailed numerical results associated with the several system configurations cited earlier led us to conclude that

1) For systems in which all distances between upsets (ℓ_b) are constant, see Figure 1, there is no significant difference in the pass-and stop-band widths which can be attributed to varying ℓ_b from system to system. The only observed difference in the plots was a shift in the center frequency of the bands. As ℓ_b is increased the center frequencies of the stop bands are shifted downward.

2) The center frequency of t stop bands is related to the values of the anti-resonance frequencies of the finite bars between upsets.

3) For a constant length ℓ_b between upsets, an increase in pass-band width and a corresponding decrease in stop-band width was noted as the area ratio α is increased. This is shown graphically in Figures 12, 13 (For these plots all distances between upsets was

ℓ_b = 30 feet.)

The few points that do not fall precisely on the curves in the plots of stop- and pass-band widths can be explained in the following manner: The computer is programmed to select a frequency and then calculate an associated attenuation. A frequency increment of 2 cps was used in the program; this permitted deviations of 2 cps in the value of the band widths. The effects of the deviations are shown in the curves shown in Figure 14.

4) A decrease in attenuation at the low frequency end of the analysis (100 cps) was noted with increasing cross sectional area ratios at the upsets. This is plotted in Figure 14. A similar decrease in all attenuation peaks was found with increasing area ratios as shown in Figure 15.

In general, the points on which the attenuation curves are based follow a well-defined path. However, in a few cases some points do not fall precisely on the plotted curve. This is caused by the narrowness of band widths which span the anti-resonances. The computer program uses increments of frequency in steps of 2 cps. It is probable that in some cases the attenuation is affected significantly by the fact that large variations in response occur within the 2 cps frequency increments. Where this occurs, there is an observable deviation in the computed values and in the presumably smooth curve plotted in the figures.

5) When the system is made up of one-half short-length (29 feet) and one-half long-length (31 feet) bars, the response is similar to that of the case when ℓ_b is constant over ℓ. However, as α is increased a small pip is noted at the attenuation peaks, as shown in Figure 11. This can be attributed to beating between the anti-resonances of the different length bars.

6) For the case in which the system is composed of several different bar lengths, an attempt was made to determine the effect of the relative position of different sections on system response. It was determined that the frequencies at which the attenuation peaks occur are governed by the bar length at the output end of the system. This implies that, if the final bar length is different from those of the rest of the system, the factor which determines the center frequency of the stop and pass bands is the length of this final section. Apparently, this effect can be reduced slightly if many sections of different

lengths relative to the end section are present.

7) The attenuation peaks are greater in magnitude for systems in which all lengths between upsets are constant. As the number of different bar lengths in any given system is increased the attenuation peaks decrease slightly in amplitude.

An estimate of error in the analysis presented here is not possible due to the scantiness of experimental data available in commercial literature references; however, the results are quantitatively consistent and quantitatively reasonable.

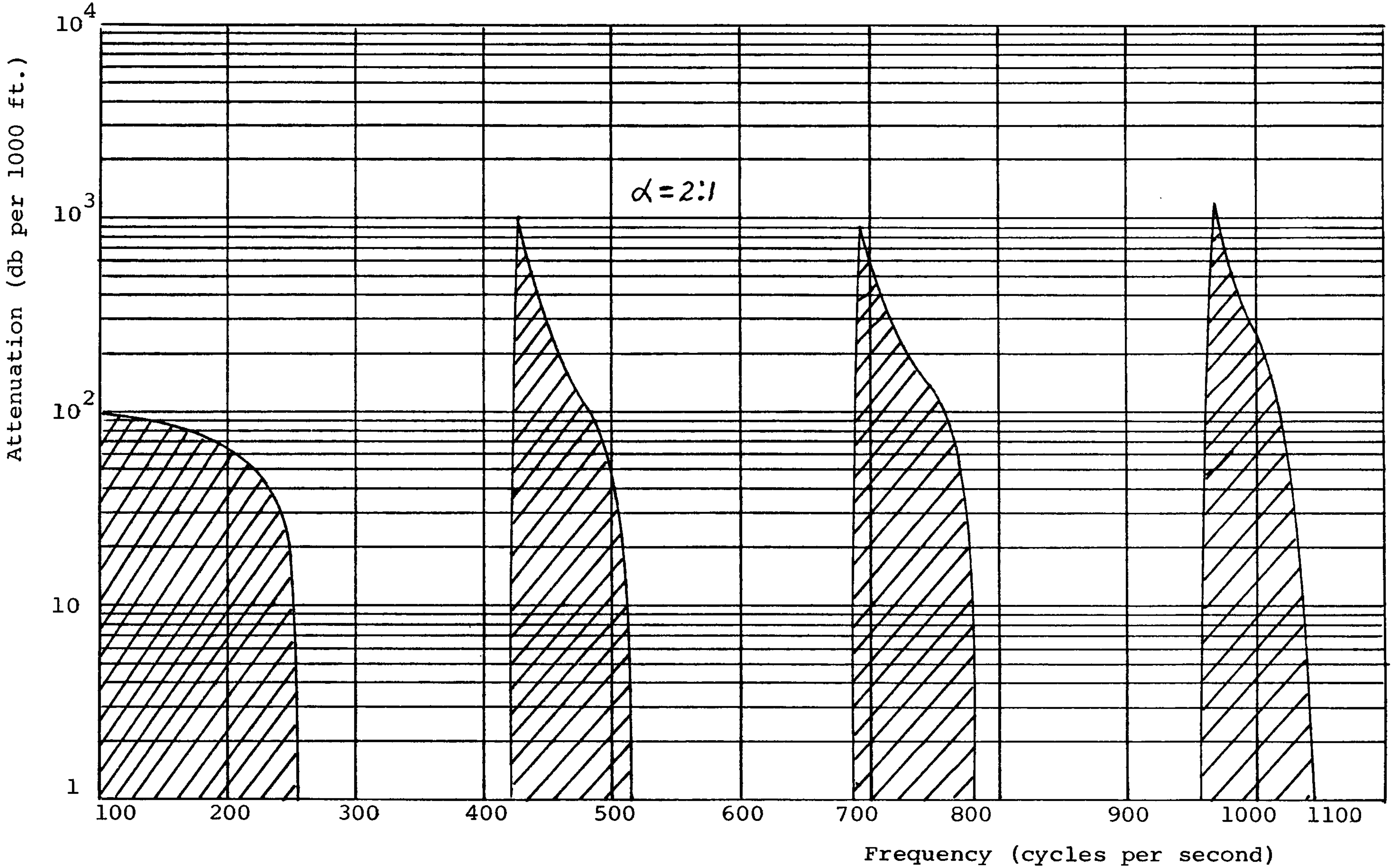


Figure 10

Attenuation versus Frequency, for ℓ_b = 360"

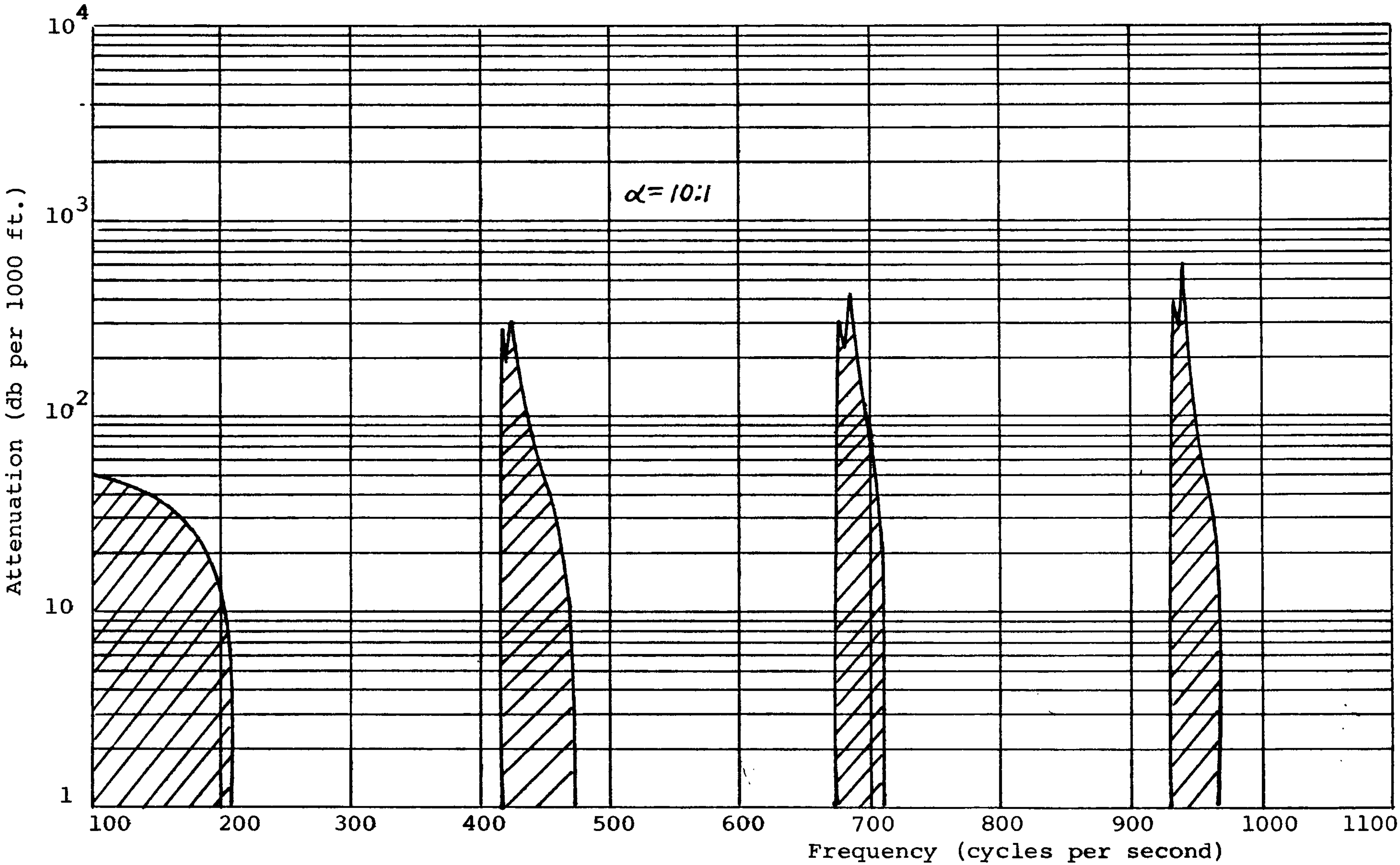


Figure 11

Attenuation versus Frequency, (1/2) ℓ_b = 348", (1/2) ℓ_b = 372"

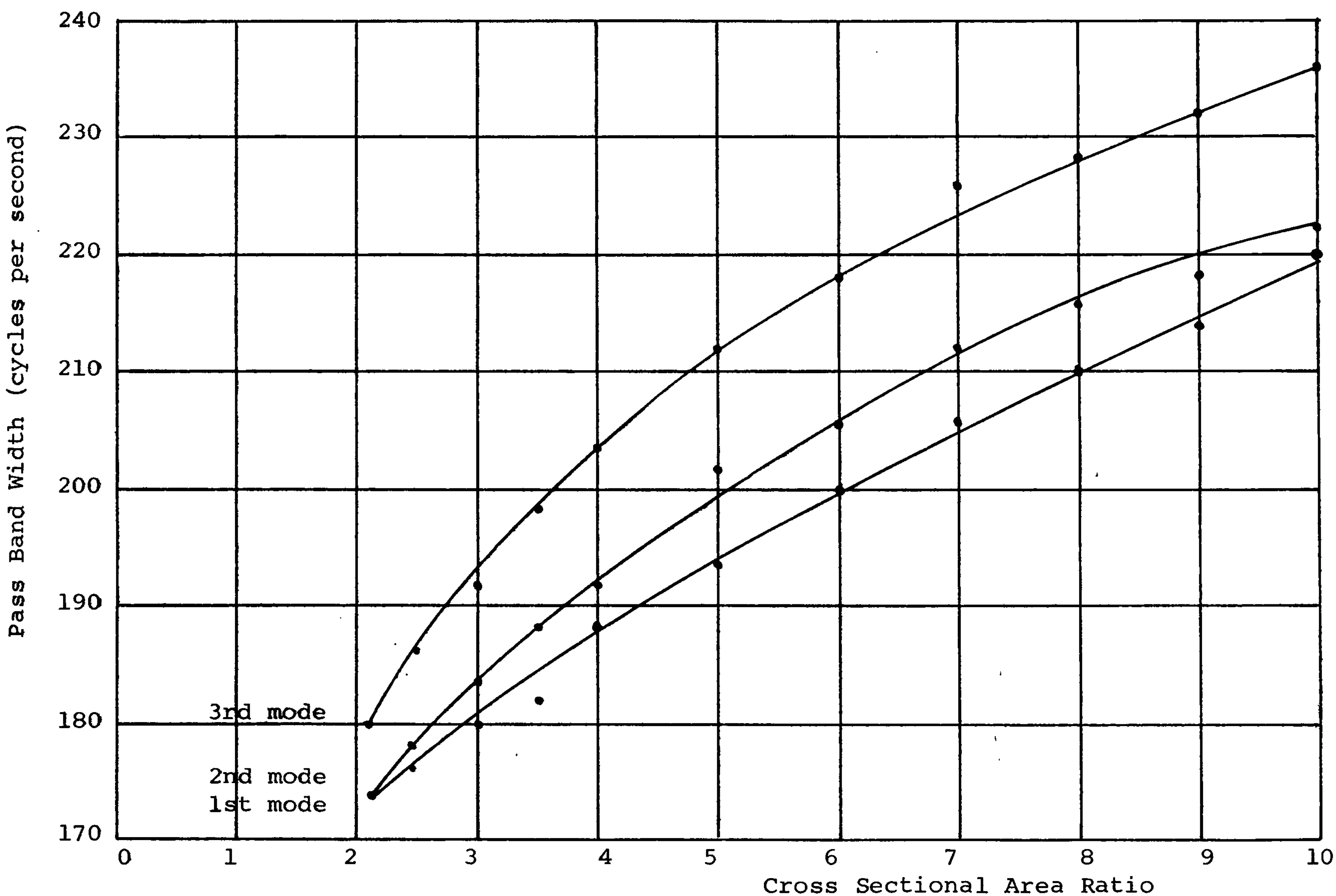


Figure 12

Pass Band Width versus Area Ratio, for ℓ_b = 360"

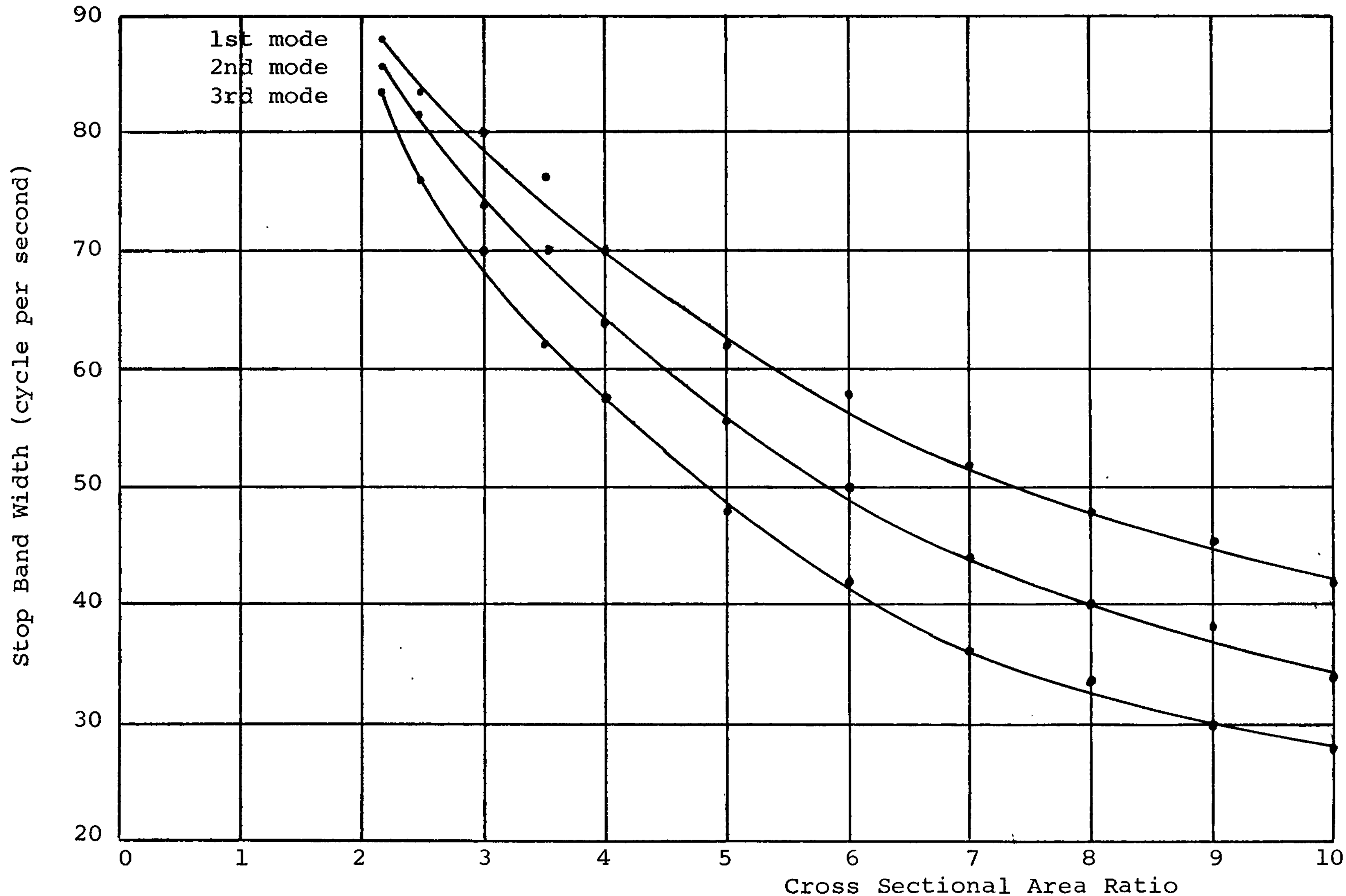


Figure 13

Stop Band Width versus Area Ratio, for ℓ_b = 360"

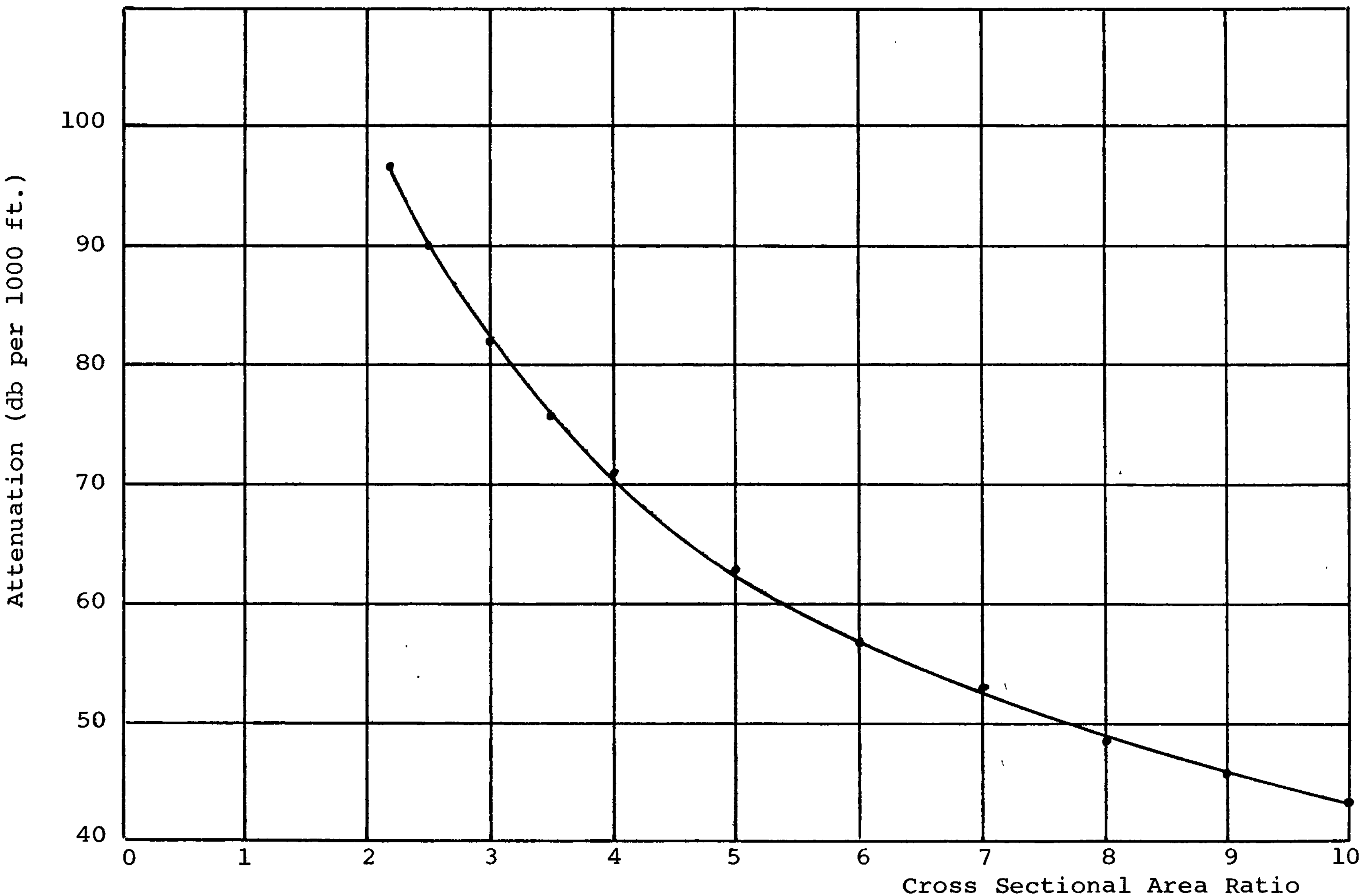


Figure 14

Attenuation at 100 cps versus Area Ratio, for ℓ_b = 360"

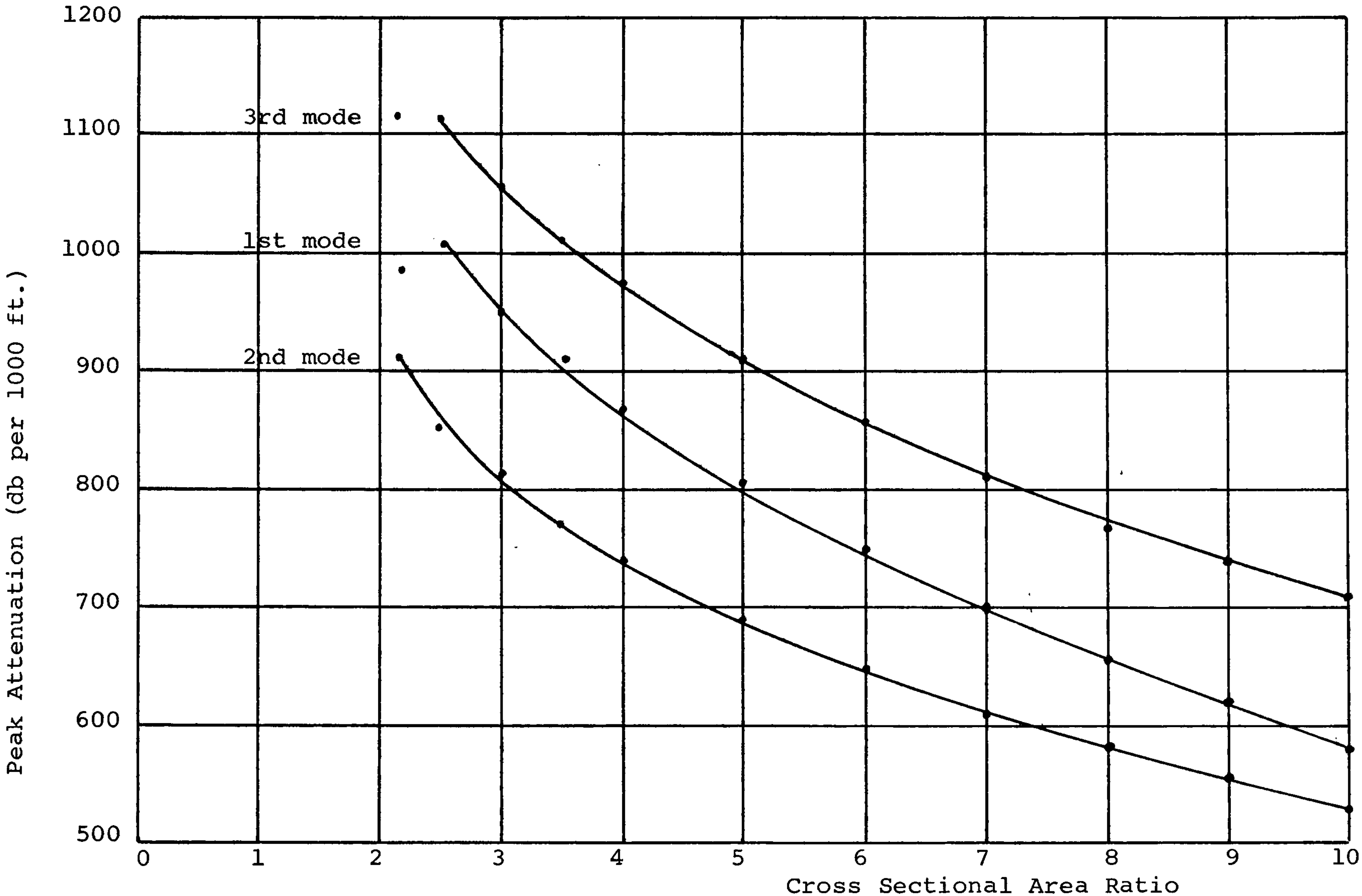


Figure 15

Peak Attenuation versus Area Ratio, for $\ell_b = 360"$

APPENDIX A

MOBILITY AND IMPEDANCE ANALOGIES

As is mentioned earlier, two separate analogies are normally used for vibration studies. They are both governed by the same set of differential equations and are thus called duals of each other [11]. The first, which is called the impedance (force-voltage) analogy, is based on Kirchoff's second law. This law states that in any network the algebraic sum of the voltage changes around any closed loop is zero. The second, which is based on Kirchoff's first (current) law, is called the mobility (force-current) analogy. This law states that the algebraic sum of all the currents flowing toward a point is zero. Circuit elements are classified as active (current or voltage generators) and passive (resistors, capacitors and inductors). A comparison of the corresponding elements in the two systems is given in Table I.

Each of the analogies may be classified further as based on the component-impedance or normal-mode impedance concept. In the component-impedance method the response of each component (spring, mass, or dashpot) is computed. These individual responses are then combined, according to the physical relationship of the components to determine the response at a particular point of interest in the system.

The normal-mode impedance method is, perhaps, more suited to the type of system studied here. The response at a point in the system is found in terms of the normal modes of vibration of the system. The net response at any frequency then, is calculated based on the responses for the individual normal modes. This concept makes use of such normal-mode parameters as the Q of the system, bandwidth, and resonance frequencies.

The impedance and mobility methods are developed for use with lumped-parameter multi-degree-of-freedom systems. The response of a one-degree-of-freedom system is plotted on an impedance grid in Figure 16. Such a grid includes lines of constant frequency, impedance, compliance (the inverse of stiffness) and mass, all plotted in logarithmic scales. The impedance used here, termed mechanical impedance, can be converted to acoustical impedance simply by dividing by the square of the cross sectional area over which the acoustic pressure acts. The response of a system with no external damping is shown in Figure 16 as a dashed line. At the resonant frequency, f_n, the impedance of the undamped system is zero. Note that at frequencies below the natural frequency the response is limited by the system compliance. Above this frequency the response is mass controlled. The constant-mass lines have positive coefficients and the constant-compliance lines have negative coefficients. Where the two lines of effective mass and effective compliance intersect the algebraic sum is zero and the impedance is zero. As the frequency is increased and approaches the resonant frequency the impedance is equal to the value of mass reactance multiplied by the factor [1 - $\left(f/f_n\right)^2$]. Similarly, as the frequency is increased to values greater than the natural frequency the impedance is equal to the compliance reactance multiplied by the factor [1 - $\left(f_n/f\right)^2$]. The mass and compliance reactance (X_m and X_c, respectively) are defined by

$$X_m = 2\pi f M_{eq} \quad , \quad X_c = -1/2\pi f C_{eq} \quad . \tag{10}$$

The response at resonance is controlled by the system damping, and characterized by the factor Q. For low damping, Q may be defined as

$$Q = \frac{Z_o}{Z_{res}} \tag{11}$$

where Z_o is the characteristic impedance and Z_{res} is the impedance at resonance. In terms of the natural frequency and bandwidth at the half-power points it can be shown that this is equivalent to the statement that

$$Q = \frac{f_n}{\Delta f} \; . \tag{12}$$

The response of a multi-degree-of-freedom system can be determined by extending the method already applied to the single-degree-of-freedom system. The response of each individual mode is plotted as though it existed as a one-degree-of-freedom system independent of all other modes. The total system response is then calculated based on the response of each mode.

Here we use the latter method to determine the impedance of the bar between upsets. We use the equivalent mass and compliances computed from Equations (7) and (8). There is a constant-compliance value C_{eq} associated with each normal mode. For longitudinal vibrations M_{eq} is the same for all modes. These lines intersect at the resonant frequencies of the bar between upsets. Any possible vibration of the system can be expressed by superposing the response in these normal modes [9].

The individual mode responses can be computed using the impedance method; as mentioned earlier, Q is taken to be 200. Impedances for a finite bar 30 feet long were computed and are shown in Figure 17 as a function of frequency.

Mechanical Component	Impedance Analogy (Force-Voltage)	Mobility Analogy (Force-Current)
Force (F) or Pressure (P)	Voltage (e)	Current (i)
Displacement (u)	Charge (q)	$\int e dt$
Velocity (v)	Current (i)	Voltage (e)
Mass (M)	Inductance (L)	Capacitance (C)
Resistance (damping)	Resistance (R)	Resistance
Compliance (C)	Capacitance (C)	Inductance (L)

Table 1

Mechanical-Electrical Analogies

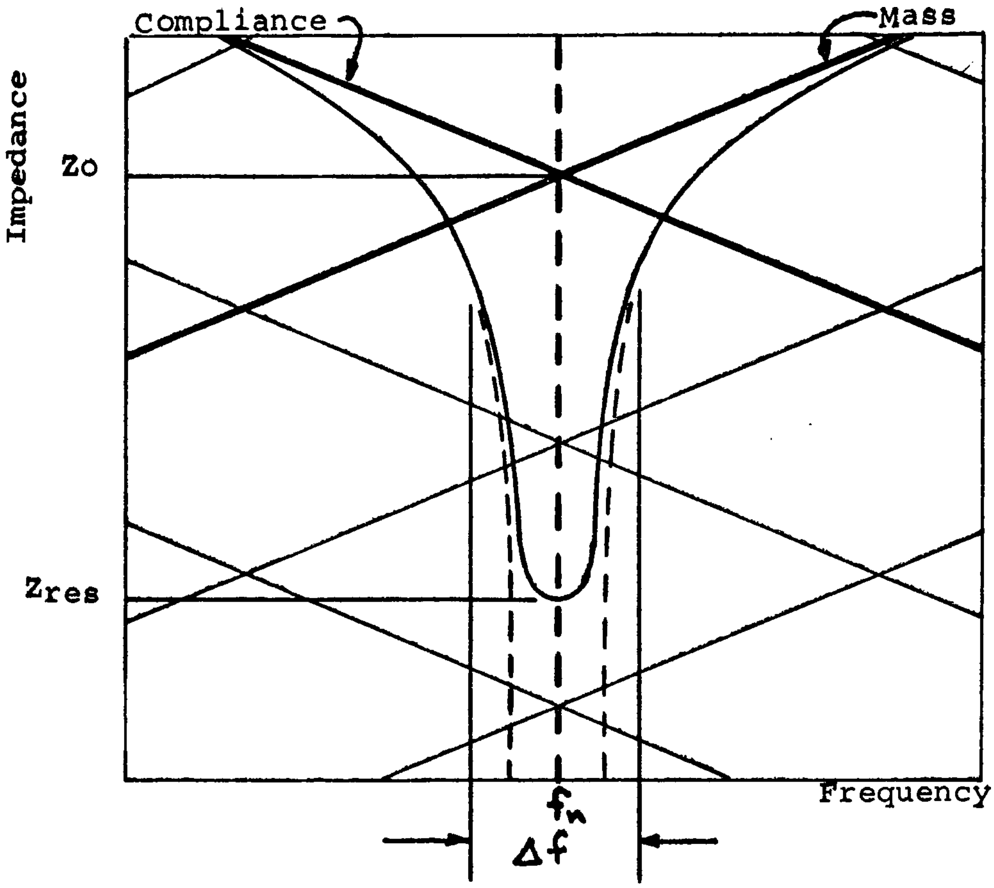


Figure 16

Impedance Plot, Single-degree-of-Freedom System

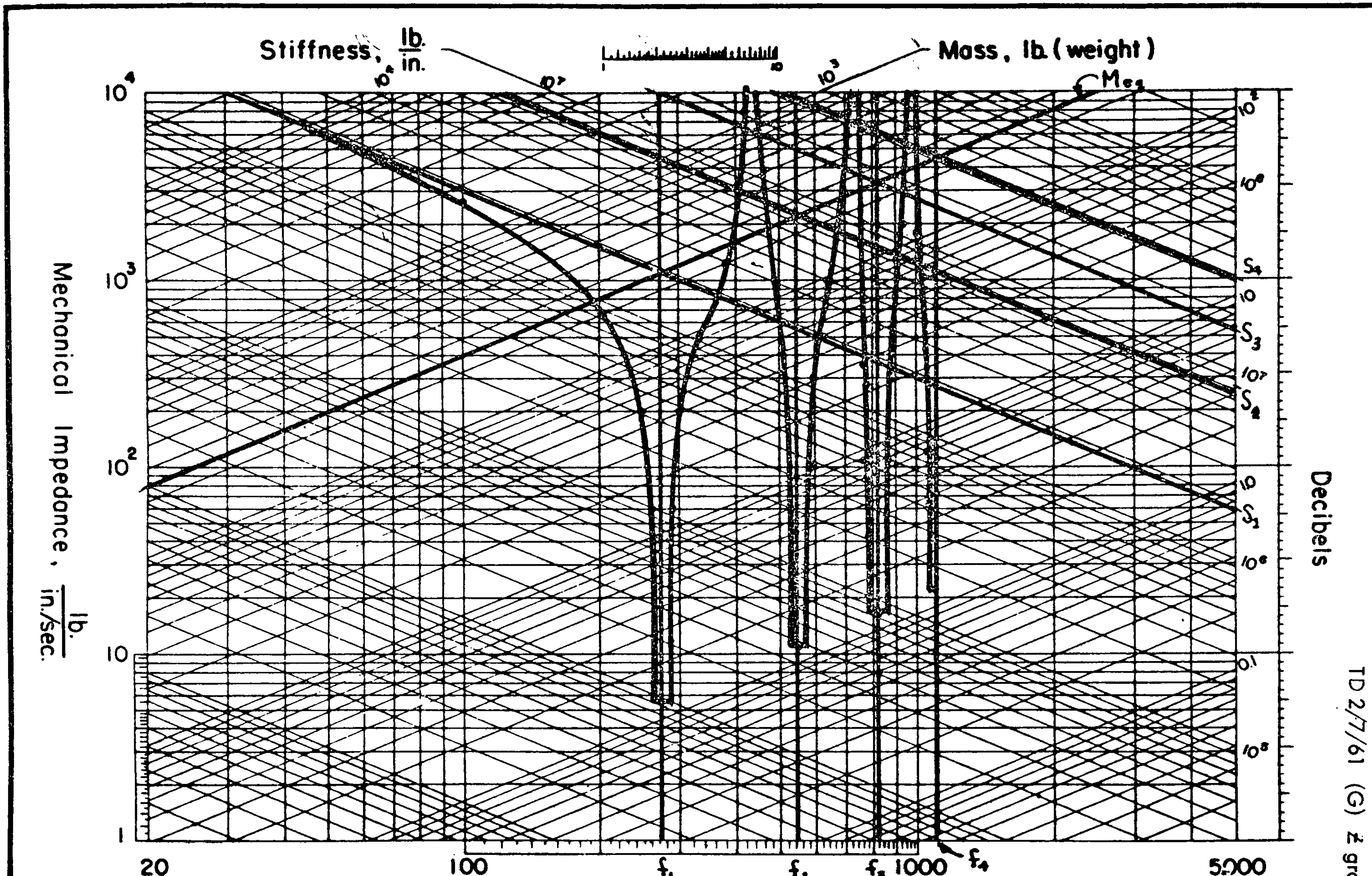


Figure 17

Finite Bar Impedance, ℓ = 30 ft.

APPENDIX B

ELECTRICAL NETWORKS (FILTER THEORY)

In order to make this report sensibly self-contained, certain elements of electrical filter theory will be cited.

In general, a passive filter is a network in which currents within certain continuous frequency ranges are transmitted freely while currents of other frequencies are suppressed. It can be defined as a four terminal network, composed of non-dissipative elements, which transmits continuous bands of frequencies and attentuates all others. Basic filters are usually thought of as one of three basic types, "L", "T" or "Pi". Here we will limit our discussion to Pi-type filters because their behavior is analogous to certain physical characteristics of our system.

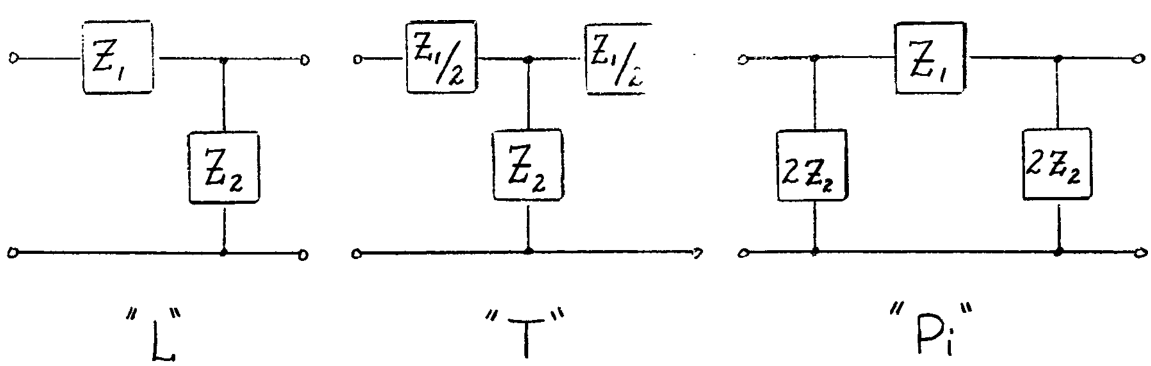


Figure 18

Basic Filter Types

The schematic diagram of a low pass filter is shown in Figure 19.

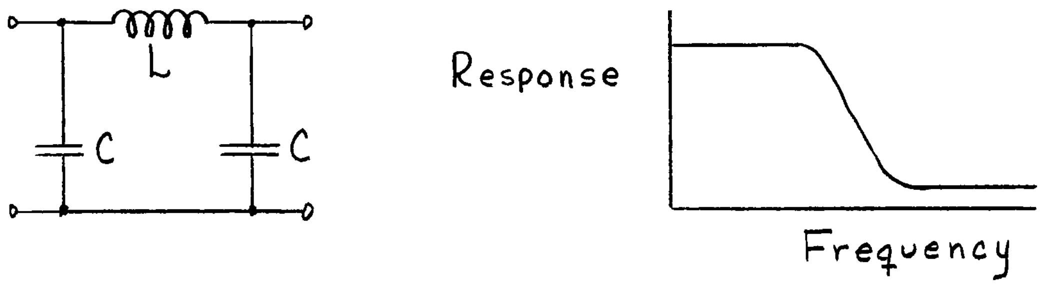


Figure 19

Low Pass Filter

At low frequencies the series-reactance is low and the shunt-reactance is high, giving the frequency response shown in the figure. This situation is reversed at high frequencies, and the signal is attenuated. Similarly, in a high pass filter, (Figure 20)

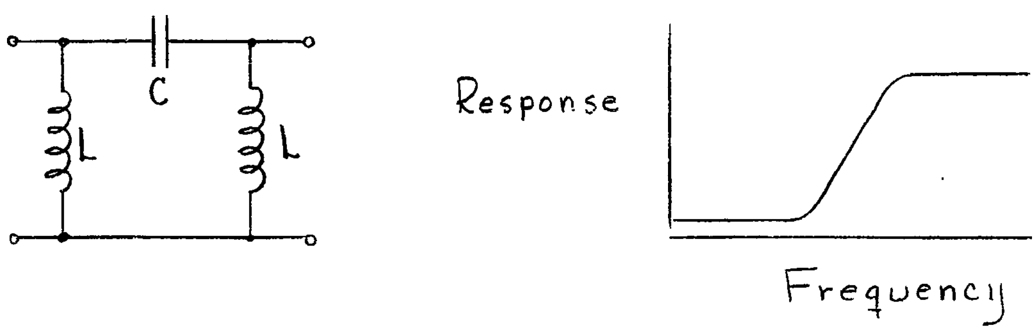


Figure 20

High Pass Filter

the shunt reactance at low frequencies is low, effectively shorting the input terminals. At higher frequencies the shunt reactance increases as the series reactance decreases.

A band pass filter combines the features of both of the above (Figure 21).

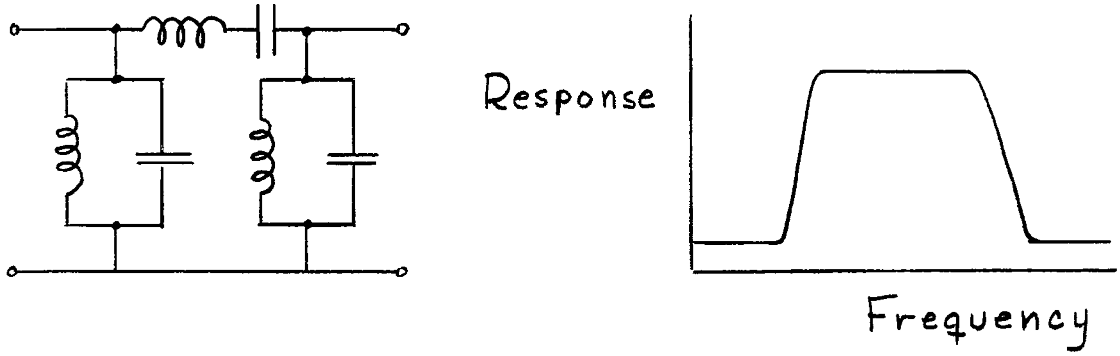


Figure 21

Band Pass Filter

If the filter is designed such that series and parallel resonances occur at the same frequency the shunt impedance is infinite and the series impedance zero, allowing the current to flow unimpeded. If the shunt inductor is omitted from the circuit in Figure 21, we have the model analogy we use here (Figure 8). Its response and impedance curves are shown in Figure 22.

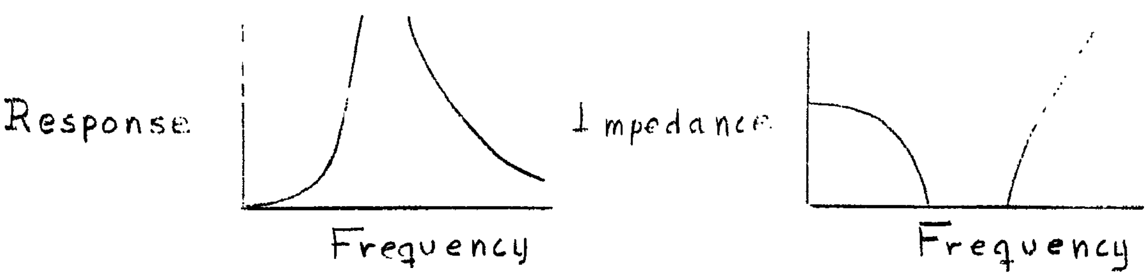


Figure 22

Impedance and Response Curves for Figure 8

In our system, since the series capacitor (compliance) is variable, we expect more than one band pass in the system response curve. Since these idealized filters are constructed from purely reactive components, no power is actually dissipated. In the pass band, energy is permitted to pass through to the load, outside the pass band, energy is not permitted to be transmitted by the filter. It is reflected to the source. If the circuit contains a resistor, its effect would be to dissipate energy at the band pass frequencies. Outside the pass band, the effect of the resistor would be negligible.

The attenuation characteristics of a Pi network filter are well known [12]. It can be shown [12] that the attenuation in either part of the stop band is given by

$$N = \operatorname{arccosh}\left|1 + \frac{Z_1}{2Z_2}\right| , \tag{13}$$

where N is the attenuation in nepers and Z_1 and Z_2 are the impedances of the Pi network. A neper is defined as the natural logarithm of the ratio of signal input to output. A study of Equation (13) discloses that

1) If $\left(1 + \frac{Z_1}{2Z_2}\right)$ lies between -1 and +1 there is phase shift but no attenuation.

2) If $\left(1 + \frac{Z_1}{2Z_2}\right)$ is greater than +1 there is attenuation but no phase shift.

3) If $\left(1+\frac{Z_1}{2Z_2}\right)$ is less than -1 there is attenuation and phase shift is 180°.

Most filter networks, including the Pi section, can be cascaded, or attached end to end, with no change in filtering characteristics if the characteristic or image impedances of adjacent filter sections are equal. If this condition is not satisfied an impedance discontinuity will be present and a reflection of energy will result.

In this study, since we have allowed the length of the bar between upsets to vary, this condition is not completely satisfied. However, the maximum variation is limited to three percent. In turn, this will cause a small, but not significant, variation in Z_1. Thus, we limit our study to the case where only small variations in distance between upsets is permitted, which is a realistic requirement. If the condition of equal characteristic impedances is met, the total system attenuation of any number of sections is merely the sum of the individual attenuations.

APPENDIX C

COMPUTER PROGRAM

The program used here consists of four major portions.

1) Compute total length of system studied.

2) Compute impedance of finite bar for each frequency.

3) Compute impedance of upsets for each frequency.

4) Using the above results, calculate the system attenuation at each frequency.

The program consists of the following MAD statements.

```
R  THESIS PROBLEM ATTENUATION VS. FREQUENCY
          INTEGER N, X, M
         DIMENSION L(25),NO(350),RF(5),CL(5),ATTEN(25),TOTATT(25),
        1 ZRF(5)
START     READ DATA
         PRINT RESULTS L(1)...L(X),NO(1)...NO(X)
         TOLEN = 0
         LEN = 0
         TONO = 0
         VEL = 1.968E5
         DENS = 3.57E-3
         C = 7.21E-9
         L2 = 9.38E-2
         C2 = 7.9E-8
         DEL1 = .68
         DEL2 = 1.37
         DEL3 = 2.06
         DEL4 = 2.78
         THROUGH LENTH, FOR N = 1, 1, N.G.X
         LEN = LEN + L(N)*NO(N)
LENTH    TONO = TONO + NO(N)
         TOLEN = (LEN + TONO*24)/12
         PRINT COMMENT CR THE TOTAL LENGTH IS CR
         PRINT RESULTS TOLEN
         THROUGH LOOP, FOR F=100,2,F.G.1100
         TOTAL = 0
         THROUGH RESULT, FOR N=1,1,N.G.X
         L1 = DENS*L(N)/2
         CE = C*L(N)
         WHENEVER  F.G.100
         TRANSFER TO GOON
         OTHERWISE
         END OF CONDITIONAL
         THROUGH RESO, FOR M = 1,1,M.G.5
         RF(M) = M*VEL/(2*L(N))
         CL(M) = 2*CE/(M*M*9.866)
RESO     ZRF(M) = 6.283*RF(M)*L1/200
GOON     WHENEVER F.L.RF(1)-DEL1
         B = 1-(F/RF(1)).P.2
         IMP = -B/(6.283*F*CL(1))
         OTHERWISE
         WHENEVER F.G.RF(1)+DEL1
         WHENEVER F.L.RF(2)-DEL2
         B = 1-(F/RF(2)).P.2
         R = 1-(RF(1)/F).P.2
         IND = R*6.283*F*L1
         CAP = -B/(6.283*F*CL(2))
         IMP = IND*CAP/(IND+CAP)
         OTHERWISE
```

```
WHENEVER F.G.RF(2)+DEL2
WHENEVER F.L.RF(3)-DEL3
B = 1-(F/RF(3)).P.2
R = 1-(RF(2)/F).P.2
IND = R*6.283*F*L1
CAP = -B/(6.283*F*CL(3))
IMP = IND*CAP/(IND+CAP)
OTHERWISE
WHENEVER F.G.RF(3)+DEL3
WHENEVER F.L.RF(4)-DEL4
B = 1-(F/RF(4)).P.2
R = 1-(RF(3)/F).P.2
IND = R*6.283*F*L1
CAP = -B/(6.283*F*CL(4))
IMP = IND*CAP/(IND+CAP)
OTHERWISE
WHENEVER F.G.RF(4)+DEL4
B = 1-(F/RF(5)).P.2
R = 1-(RF(4)/F).P.2
IND = R*6.283*F*L1
CAP = -B/(6.283*F*CL(5))
IMP = IND*CAP/(IND+CAP)
OTHERWISE
IMP = ZRF(4)
END OF CONDITIONAL
END OF CONDITIONAL
OTHERWISE
IMP = ZRF(3)
END OF CONDITIONAL
END OF CONDITIONAL
OTHERWISE
IMP = ZRF(2)
END OF CONDITIONAL
END OF CONDITIONAL
OTHERWISE
IMP = ZRF(1)
END OF CONDITIONAL
END OF CONDITIONAL
Z2 = -1/(6.283*F*C2)
Z1 = IMP + 6.283*F*L2
AL = .ABS.(1+Z1/(2*Z2))
WHENEVER AL.L.1
ATTEN(N) = 0
OTHERWISE
ATTEN(N) = ELOG.(AL+SQRT.(AL*AL-1))
END OF CONDITIONAL
TOTATI(N) = ATTEN(N)*NO(N)
```

```
RESULT      TOTAL = TOTAL + TOTATT(N)
            DBATT = 8.686*TOTAL*1000/TOLEN
            PRINT RESULTS F,DBATT
LOOP        CONTINUE
            TRANSFER TO START
            END OF PROGRAM
CR DATA
L(1)=372,348,372, NO(1)=1,1,1, X=3*
L(1)=348,372, NO(1)=1,2, X=2*
L(1)=372,348, NO(1)=2,1, X=2*
```

In this program the following symbols were used for system parameters.

F	= frequency
N	= section number
L (N)	= length of N^{th} section
NO (N)	= number of sections L (N) long
X	= number of different sections
M	= longitudinal mode number
L1	= equivalent mass between upsets
C	= compliance per unit length
CE	= equivalent compliance of L (N) section
RF (M)	= resonant frequency of M^{th} mode resonance
CL (M)	= equivalent compliance for M^{th} mode
ZRF (M)	= impedance at M^{th} mode resonance
IND	= mass reactance
CAP	= compliance reactance
IMP	= impedance of finite bar
R, B	= impedance factor near resonance
LEN	= length of NO (N) sections L (N) long
TOLEN	= total system length
DENS	= mass per unit length between upsets
L2	= equivalent mass at upsets
C2	= equivalent compliance of upsets

TONO = total number of sections used

VEL = velocity of sound

DEL 1,2,3,4 = one-half bandwidth of 1^{st} four modes

Z1, Z2 = impedance for "Pi" section

AL = factor for calculating attenuation

ATTEN (N) = attenuation of one section L (N) long

TOTATT (N) = attenuation of NO (N) sections L (N) long

TOTAL = total system attenuation in nepers

DBATT = total system attenuation in db per 1000 feet

The input data to the computer for this program consists of values for L (N), NO (N), and X. These correspond to the different lengths between upsets. To vary the cross sectional areas of the upsets, values for L2 and C2 are changed.

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