# COMPUPER EVALUATION <br> OF SELECTED POWER SYSTEM <br> STABIEIZER DESICNS 

A Thesis<br>Presented to the Faculty of The Department of Electrical Engineering University of Houston

In Partial Fulfillment of the Requirements for the Degree Master of Science in Electrical Engineering

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April 1977

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## ACKNOWLEDGEMENT

The author wishes to thank Professor Edgar $C$. Tacker of the Department of Electrical Engineering for his excellent guidance in planning and supervising this project. Without his help and encouragement this work could not have been completed.

Thanks are also due to Professors Charles W. Sanders and Thomas D. Linton for their valuable assistance. Finally, the author is thankful to his parents for their encouragement during the years of his graduate study.

## ABSTRACT

Dynamic stability problems of a generator connected to an infinite bus through a transmission line have been studied previously. Un this thesis an analysis of this stability problem is undertaken using modern control techniques. A nonlinear model of the generator together with its automatic voltage regulator is obtained, and is then linearized about the operating point. Four different types of power system stabilizers are designed on the basis of this linearized model. Dynamic stabilizer designs have been proposed by industry people, and these designs will serve as a standard of comparison. Modern control theory is used herein to obtain other stabilizers. The effectiveness of these stabilizers is tested by subjecting the system to a pulse disturbance. Time responses of the state variables are then compared to those resulting from the use of a dynamic stabilizer. It is found that there is a definite improvement in terminal voltage and internal frequency damping when stabilizers based on modern control are implemented. Cost functions are also compared in order to more completely specify the overall performance of the stabilizers.

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## CHAPTER 1

## INTRODUCTION

## I. Thesis Objectives

This thesis deals with the design of power system stabilizers (PSS). The development of PSS was prompted by a need to improve the dynamic stability of interconnected power systems.

Imbalance between the generated and demanded reactive power results in bus voltage aeviations. Recently, fast response autonatic voltage regulators (AVRs) have been used to return the terminal voltage to within the specified tolerance instantaneously following a disturbance. It has been indicated in [l]* that the damping of power system swings may be hampered rather than aided by very fast response excitation systems. It was also demonstrated [1,2] that an excitation systern could be employed to damp oscillations if the voltage regulator error signal is supplemented by an appropriate control signal.

In this thesis the feasibility of using modern control strategies to obtain the desired supplementary control signal has been explored. Basically two different types

[^0]of controllers are discussed. One is' a dynamic controller in the form of lead-lag circuits using one of the state variables as the input. The second is a controller with constant gains using either all the state variables or estimates of all the state variables as input. The output is the (constrained)optimal control signal applied to the input of the AVR.

## II. Background

The development of digital computers for transient stability studies of large power systems has led to a number of interesting developments. With modern computers it has been possible to include details of the exciters and voltage regulators. It was also possible to study system behavior for a longer time by using high speed computers. Some stability curves were found to diverge oniy after several oscillations, rather than on the first swing following a disturbance [2]. The voltage regulator was the chief contributor to this insufficient damping. Results of stability studies showed that a speed error signal applied to generators with static exciters would produce damping. Derivation of the equivalent rotor speed signal by measuring the frequency of the internal voltage (i.e., terminal voltage compensated for the quadrature reactance
drop) bypasses other measurement problems and yields a signal of sufficient high quality so that damping is effected.

The power system stabilizer designed in [1] uses this internal frequency as the only input. The first PSS developed therein is a combination of lead-lag circuits which can be thought of as a fixed structure controller with certain free parameters (gains and time constants). Frequency domain techniques are used in [1] to determine these parameters. This stabilizer has been redesigned in this thesis because Schleif's paper does not include sufficient details of the generator model and also does not use the IEEE standard exciter model [4]. The basic structure of their PSS has been retained but the optimum values of these parameters are obtained using a different approach [3]. This is done to be as fair as possible in comparing their results with PSS designed using the modern control approach. III. Foreword

Theoretical development of the model structure is discussed in Chapter 2. A system of equations to mathematically describe the model is then developed. Chapter 3 presents the different control algorithms to be used in the design of PSS. Simulation results illustrating the performance of each of the PSS designs are given in Chapter 4. In Chapter 5, performance of each PSS is discussed and interpretations
of the results are given. Finally, in Chapter 6 some conclusions are drawn and suggestions for further research are made.

## CHAPTER 2

MODEL STRUCTURE

## I. The Excitation System Model

The excitation system considered herein is of the high initial response category which is defined by the IEEE [4] as one capable of reaching ceiling voltage in less than 0.05 seconds. The system is capable of equal ceiling voltage in the boost and buck directions, yielding fast control to increase or decrease field current from its normal value. Field current in the negative direction is not possible, and no forseeable operating condition would necessitate this capability. Physically, the excitation system consists of only static components. Excitation power is obtained from the generator terminals through suitable transformers. The power is rectified by stationary thyristor modules which deliver current to the field winding through collector rings. A solid-state regulator receives voltage, current, and auxiliary information from the main generators, and controls the thyristor firing pulses. Auxiliary equipment allows for manual operation and startup capability.

Benefits to be derived from a high initial response excitation system include the high exciter ceiling voltages and short response times which can be utilized to force the main generator field current rapidly to a new level. During system faults the generator voltage is forced to maintain a high level to aid in system stability. Also, the terminal voltage can be maintained at normal levels during overspeed or load rejection.

## Pei Unit System

For the development of the excitation system, it has been useful to establish a per unit voltage base. For the model described herejn, one per unit generation is defined as the rated voltage. One per unit exciter output is that voltage required to produce rated generator voltage on the generator air gap line.

## Transfer Function Model

Figure 2.1 is the block diagram of the excitation system used in the computer simulation studies. In order to have a satisfactory representation all of the significant transfer functions are included. The transfer functions of Figure 2.1 will now be described in detail. The first summing point compares the regulator reference with the generator terminal voltage $e_{t}$ to determine the voltage error input to the regulator amplifier. The second summing point combines voltage error input with the


Fig.2.1 The excitation system model
excitation major damping loop signal. The regulator is characterized by a gain factor $K_{A}$ and a time constant $T_{A}$. Following this, the maximum and minimum limits of the regulator are imposed so that large input error signals cannot produce a regulator output which exceeds practical limits. The next input/output relation is that of the exciter, approximated by $1 /\left(\mathrm{K}_{\mathrm{E}}+\mathrm{sT}_{\mathrm{E}}\right)$. The saturation function of the exciter is neglected, as the operating point is such that the exciter does not saturate. The major damping loop signal is provided by the stabilizing transformer The transfer function $s K_{F} /\left(1+s^{\prime} T_{F}\right)$ is used to model the input/output relation of this device. Appendix A gives the values of the constants of Fig. 2.l actually used in the simulation.

## II. The Generator and Tieline Model.

A typical case of a synchronous generator connected to an infinite bus through an external reactance has been considered here. A nonlinear generator model is developed using direct and quadrature-axis representation with time constants given by Adkins [5], and simplifying assumptions made by Park [6]. Because only slow oscillations are studied, the transformer action between the direct and quadrature axes is assumed negligible. Armature resistance and saturation effects in both axes are neglected. No local loads are connected to the generator. Appendix $A$
presents the assumptions made and the pertinent equations used with the definitions of parameters included in the nonlinear generator model. Fig. 2.2 gives the complete exciter/nonlinear generator model connected to an infinite bus.

Following the analysis of deMello and Concordia [7],
a linearized small perturbation model of the generator and tieline can be developed. A rigorous treatment, to obtain a linearized model from the nonlinear model, is given in Appendix $B$. The constants $K_{1}$ through $K_{6}$ of Fig. 2.3 are defined as follows:

$$
\begin{aligned}
& \mathrm{K}_{1}=\frac{\Delta \mathrm{P}}{\mathrm{e}} \\
& \Delta \delta \mathrm{Eq}^{\prime} \quad
\end{aligned} \quad \begin{aligned}
& \text { change in electrical power for a } \\
& \\
& \\
& \\
& \\
& \text { flux linge in rotor angle with constant }
\end{aligned}
$$

$$
\begin{aligned}
K_{2}=\left.\frac{\Delta \mathrm{P}_{\mathrm{e}}}{\Delta \mathrm{Eq}}\right|_{\delta} \quad \begin{array}{l}
\text { change in electrical power for a } \\
\\
\\
\\
\\
\text { change in direct-axis flux }
\end{array} \\
\text { ling with constant rotor angle. }
\end{aligned}
$$

$$
K_{3}=\frac{x_{d}^{\prime}+x_{e}}{x_{d}+x_{e}}
$$

$$
\begin{aligned}
& \mathrm{K}_{4}=\frac{1}{\mathrm{~K}_{3}} \frac{\Delta \mathrm{Eq}}{\Delta \delta} \text { demagnetizing effect of a change in } \\
& \text { rotor angle. }
\end{aligned}
$$




Fig, 2,3 The linearized generator model

$$
\begin{aligned}
& K_{5}=\left.\frac{\Delta e_{t}}{\Delta \delta}\right|_{E q}, \quad \text { change in terminal voltage with } \\
& \text { change in rotor angle for } \\
& \text { constant Eq'. } \\
& K_{6}=\left.\frac{\Delta e_{t}}{\Delta E q^{\prime}}\right|_{\delta} \\
& \text { change in terminal voltage with } \\
& \text { change in Eq' for constant rotor } \\
& \text { angle. }
\end{aligned}
$$

Some facts about the linear model given in Fig. 2.3 will now be given. $\frac{\mathrm{K}_{3}}{\left(1+\mathrm{K}_{3} \mathrm{~T}_{\mathrm{do}}{ }^{\prime} \mathrm{s}\right)}$ is the transfer function of the generator field and is determined by $K_{3}$ and the open-circuit time constant $T_{\text {do }}{ }^{\prime}$. The feedback gain $D$ portrays the speed or frequency - dependent damping (such as load damping, friction, windage, etc.). The mechanical oscillations of the rotor of the synchronous machine are characterized by the fundamental oscillator formed by the synchronizing coefficient $K_{1}$, the inertia, and the tieline. The coefficient $K_{5}$, which may be positive or negative, is generally negative for machines prone to exhibit insufficient damping especially at operating points near full load.

The steady-state operating values of $\delta_{0}, \mathrm{Eq}_{0}, \mathrm{E}_{\mathrm{o}}$, $e_{\text {do }}$ and $e_{q o}$ are found by choosing real load current $I_{p o}$ $=1.0$ p.u., reactive load current $I_{q O}=0.0, e_{\text {to }}=1.0$ p.u., and utilizing the equations in Appendix B. Values
of the constants $K_{1}$ through $K_{6}$ are then evaluated using the relations for these constants given in Appendix B. Appendix A presents the numerical values of the constants $K_{1}$ through $K_{6}$.

## III. Formulation of the System Equations

The combined model of the exciter and linear generator shown in Fig. 2.4 can be modeled in the following state vector form:

$$
\dot{x}=A x+B u+D v . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad(2.1 a)
$$

where

$$
x=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
x_{3} \\
x_{4} \\
x_{5} \\
x_{6}
\end{array}\right]=\left[\begin{array}{c}
\Delta V_{R} \\
\Delta E_{f d} \\
\Delta E_{s t} \\
\Delta e_{t} \\
\Delta N \\
\Delta \delta
\end{array}\right]
$$

$u$ is an input vector and $v=P_{M}$
It is now required to put the state variables of the system in the above standard form. From the block diagram of Fig 2.4, we have by inspection

$$
x_{1}=\left[\frac{k_{A}}{1+s T_{A}}\right]\left(-x_{3}-x_{4}+u+\Delta V_{r e f}\right)
$$



Fig.2.4 Combined model of the exciter and linear generator connected to an infinite bus.

$$
\begin{aligned}
& x_{2}=\left[\frac{1}{\mathrm{~K}_{\mathrm{E}}+\mathrm{sT}} \mathrm{E}_{\mathrm{E}}\right] \mathrm{x}_{1} \\
& x_{3}=\left[\frac{s K_{F}}{1+\mathrm{ST}_{\mathrm{F}}}\right] \mathrm{x}_{2} \\
& \Delta E_{q}^{\prime}=\left[\frac{K_{3}}{1+s K_{3} T_{d o}^{\prime}}\right]\left(x_{2}-K_{4} x_{6}\right) \\
& x_{5}=\frac{1}{T_{M}^{s}}\left(\Delta P_{M}-D x_{5}-K_{1} X_{6}-K_{2} \Delta E_{q}^{\prime}\right) \\
& x_{6}=\frac{377}{s} \quad x_{5}
\end{aligned}
$$

By inverse Laplace transformation of the above equations, we obtain the following set of differential equations:

$$
\begin{align*}
& \dot{x}_{1}=-\frac{x_{1}}{T_{A}}-\frac{K_{A}}{T_{A}} x_{3}-\frac{K_{A}}{T_{A}} x_{4}+\frac{K_{A}}{T_{A}} u+\frac{K_{A}}{T_{A}} V_{r e f}  \tag{2.1}\\
& \dot{x}_{2}=\frac{1}{T_{E}} x_{1}-\frac{K_{E}}{T_{F}} x_{2}  \tag{2.2}\\
& \dot{x}_{3}=-\frac{1}{T_{F}} x_{3}-\frac{K_{F} K_{E}}{T_{F} T_{E}} x_{2}+\frac{K_{F}}{T_{F} T_{E}} x_{1}  \tag{2.3}\\
& \dot{x}_{4}=\frac{K_{6}}{T_{d o}} x_{2}-\frac{1}{K_{3} T_{d o}^{\prime}} x_{4}+377 K_{5} x_{5}-\left(\frac{K_{4} K_{6}}{T_{d o}}-\frac{K_{5}}{K_{3} T_{d o}^{\prime}}\right)^{2} x_{6} \\
& \dot{x}_{5}=-\frac{K_{2}}{T_{M} K_{6}} x_{4}-\frac{D}{T_{M}} x_{5}+\left(\frac{K_{2} K_{5}}{K_{6} T_{M}}-\frac{K_{1}}{T_{M}}\right) x_{6}+\frac{1}{T_{M}} \Delta P_{M} \cdot(2.5)  \tag{2.5}\\
& \dot{x}_{6}=377 x_{5} \tag{2.6}
\end{align*}
$$

In standard form, the equations are

wherein $A, B, a n d$ D are easily identified by comparison with(2.la).
IV. The System Eigenstructure

The eigenvalues of the system* are shown in Fig. 2.5 It is clear that the open loop system is unstable since two of its eigenvalues are in the right-half of the complex plane. This instability can be explained using the concept of damping torque given in [7]. For this purpose consider the contribution coming through branch $\mathrm{K}_{5}$ accounting for the effect of angle on terminal voltage. In Fig. $2.6 \frac{-K_{\epsilon}}{1+s T_{\epsilon}}$ represents the modified block diagram of the entire excitation system. The exact expression for $\Delta T_{D}$ due to a change in the angle and its effect on voltage is given by,

$$
\begin{aligned}
& \frac{\Delta T D_{D}}{\Delta \delta}=\frac{-K_{2} K_{5} K_{\epsilon}}{\left(1 / K_{3}+K_{\epsilon} K_{6}\right)+s\left(T_{\epsilon} / K_{3}+T_{d o}^{\prime}\right)+s^{2} T_{d o}^{\prime} T} \cdot \text { Thus }^{* *} \\
& \frac{\Delta^{T} D}{\Delta \delta}(j w)=\frac{-K_{2} K_{5} K}{\left(1 / K_{3}+K_{6} K_{6}\right)+j w\left(T_{6} / K_{3}+T_{d o}^{i}\right)-w^{2} T_{d o}^{\prime} T} \\
& \Delta T_{D}=\frac{K_{2} K_{5} K\left(T_{\epsilon} / K_{3}+T_{d o}^{\prime}\right) W}{\left(1 / K_{3}+K_{\epsilon} K_{6}-w^{2} T_{d o}^{\prime} T\right)^{2}+\left(T_{\epsilon} / K_{3}+T_{d o}^{\prime}\right)^{2}{ }^{2}} \Delta \delta
\end{aligned}
$$

This component gives positive damping whenever $K_{5}$ is positive, but for a large number of cases $\mathrm{K}_{5}$ is negative

[^1]


Fig. 2.6 An illustration of constant $K_{5}$ affecting the damping torque.
(especially for moderate-to-high system transfer impedance and heavy loading). In Section 2.II, $I_{p o}=1.0$ p.u. and $X_{E}=$ 1.0 p.u. were chosen corresponding to a heavily loaded generator and a very high system reactance. These two quantities make $K_{5}$ negative which in turn provides a negative damping torque. This negative damping torque plays a prominent role in making the open loop system unstable.

## CHAPTER 3

DESIGN OF POWER SYSTEM STABILIZERS

In this chapter four different types of power system stabilizers are discussed. These stabilizers will be identified by Type I, II, III, and IV throughout the thesis. A full description of each of them is given below.

Stabilizer Type I : Dynamic stabilizer having the structure given in [1] (DS)

Stabilizer Type II : Deterministic optimal constani state feedback controller (DOFC)

Stabilizer Type III: Deterministic Observer with optimal controller (DOOC)

Stabilizer Type IV : Stochastic optimal controller (SOC)

This chapter is divided into four sections. Each section describes the algorithm used to develop one stabilizer. I. Design of Dynamic Stabilizer (Lead-Lag Circuits) (DS)

This type of power system stabilizer has been designed in [1]. As mentioned in the introduction, [1] does not use the standard model of the exciter and the generator. In this section this type of stabilizer has
been redesigned using the standard excitation system model given in an IFEE Committee Report [4] and the generator model given by Concordia [7]. The basic structure of this PSS has been retained but optimum values of the parameters are obtained using a different approach. The structure used in [1] is given by the transfer function,

$$
G_{p s s}(s)=\frac{G_{p}\left(1+T_{1} s\right)\left(1+T_{2} s\right) T_{R} s}{\left(1+T_{2} s\right)\left(1+T_{4} s\right)\left(1+T_{R} s\right)}
$$

wherein the parameters $G_{p}, T_{1}, T_{2}, T_{3}, T_{4}$, and $T_{R}$ are to be selected according to some design criteria. Accoraing to Schlief, $G_{p s s}(s)$ is ideally designed with a leading phase characteristic which precisely cancels the phase lag of $G_{R}(s)$ in order that the product $G_{p s s}(j w) G_{R}(j w)$ be positive and real throughout the spectrum of interest. $G_{R}(s)$ is defined as the closed-loop excitation system transfer function, i.e., $G_{R}(s)=\frac{\Delta E_{q}^{\prime}}{\Delta V_{r e f}}$. In other words, the entire effort of the PSS is devoted to providing positive damping torques if, and only if, the phase of the product $G_{p s s}(j w) G_{R}(j w)$ is zero. The entire design of the PSS is based on frequency domain considerations, and local frequency deviations provide one of the inputs to the PSS. More details are given in Appendix C.

The optimal parameters for the above $G_{p s s}$ can also be found via time domain methods. The pattern search technique [3] is used here to determine the values of the: design para:meters. The total system including the original system and the stabilizer are of the form:

$$
\begin{equation*}
\dot{x}=A x+D v \tag{3.2}
\end{equation*}
$$

where state variables $x_{1}$ through $x_{6}$ belong to the original system and state variables $x_{7}$ through $x_{9}$ to the stabilizer. A is the system matrix and $D$ is the disturbance vector. The system equations are developed in Appendix $C$.

The cost functional to be used herein is given by,

$$
\begin{equation*}
\mathrm{J}=\int_{\mathrm{t}_{\mathrm{o}}}^{\mathrm{t}_{\mathrm{f}}}\left[\mathrm{x}_{1}{ }^{2}+{x_{2}}^{2}+\mathrm{x}_{3}{ }^{2}+\mathrm{x}_{4}{ }^{2}+\mathrm{x}_{5}{ }^{2}+\mathrm{x}_{6}{ }^{2}\right] \mathrm{dt} \tag{3.3}
\end{equation*}
$$

Given the cost functional (3.3), the pattern search technique will search for a optimal set of parameter values of $G_{p}, T_{1}, T_{2}, T_{3}, T_{4}$ and $T_{R}$ which minimizes (3.3). Appendix $C$ gives more details of the pattern search technique. Fig. 3.1 shows the PSS designed in this section. Numerical values of the parameters are given in Appendix A. Performance of the system was checked by simulating the system with dynamic stabilizer and the results of this simulation are shown in Chapter. 4.


Fig. 3.1 Implementation of the dynamic stabilizer
II. Design of Optimal Constant Feedback Contoller (DOFC)

To design this type of controller all of the state variables should be available. Let the system, for which the optimal controller is to be designed be of the form

$$
\begin{align*}
& \dot{x}(t)=A x(t)+B u(t)  \tag{3.4}\\
& y(t)=H x(t) \cdot \tag{3.5}
\end{align*}
$$

where $x(t) \varepsilon E^{n}$ is the state of the system and $u(t) \varepsilon E^{m}$ and $y(t) \varepsilon E^{p}$ are the input and output of the system respectively. $A, B$ and $H$ are $n x n, n x m$ and $p x n$ matrices respectively, and are independent of time $t$. Matrices $A, B$ and $x(t)$ are defined in Section 2.III.

The cost functional which is to be minimized is given by*

$$
\begin{equation*}
J=\int_{0}^{\infty}\left[x^{T}(t) Q x(t)+u^{T}(t) R u(t)\right] d t \ldots . . \tag{3.6}
\end{equation*}
$$

where $Q$ is a positive semi-definite matrix of dimension (nxn) and $R$ is a positive definite matrix of dimension (nxm). In the design, $Q$ and $R$ are chosen to be

* T denotes 'transpose'

$$
\mathrm{Q}=\left[\begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right], \mathrm{R}=[1]
$$

## Problem Statement

Design a physically realizable control law $u=u(x(t))$ for the system (3.4) which minimizes the cost functional (3.6). The solution to this problem can be obtained [8,9] as follows:

$$
\begin{align*}
u(t) & =-K x(t)  \tag{3.7}\\
\text { where } K & =R^{-1} B_{B} T_{P} \tag{3.8}
\end{align*}
$$

and $P$ is the solution of the algebric matrix Riccati equation

$$
\begin{equation*}
P A+A^{T} P+Q-P B R^{-1} B^{T} P=0 \tag{3.9}
\end{equation*}
$$

The performance of the system was checked by simulating the system with this type of controller. It was then required to improve the damping of state variables of the system. This can be achieved by increasing the system stability margin. The basic idea used here is to make the real parts of the eigenvalues of the closed-loop system less than a constant $\alpha$, where $\alpha>0$.

## Linear Regulator Problem with a Prescribed Degree of Stability [9]

Let the system be given by:

$$
\begin{equation*}
\dot{x}(t)=A x(t)+B u(t) \tag{3.10}
\end{equation*}
$$

where matrices $A$ and $B$ are same as given in II.A. The associated cost functional is,

$$
\begin{equation*}
J=\int_{0}^{\infty} e^{2 \alpha t}\left[x^{T}(t) Q x(t)+u^{T}(t) R u(t)\right] d t . . . \tag{3.11}
\end{equation*}
$$

wherein $Q$ and $R$ are given in II.A and the non-negative constant $\alpha$, gives the desired minimum degree of stability of the closed-loop system. The minimization problem now becomes the task of finding the minimum value of the cost functional (3.11) and the associated optimal control.

Define

$$
\hat{x}(t)=e^{\alpha t} x(t)
$$

and

$$
\hat{u}(t)=e^{\alpha t} u(t) .
$$

Differentation yields

$$
\begin{align*}
\dot{\hat{x}}(t) & =\alpha e^{\alpha t} x(t)+e^{\alpha t} \dot{x}(t) \\
& =\alpha \hat{x}(t)+e^{\alpha t} A x(t)+e^{\alpha t^{B}} \mathbf{u}(t) \\
& =(A+\alpha I) \hat{x}(t)+B \hat{u}(t) \cdot \cdot \cdot \tag{3.12}
\end{align*}
$$

The initial state is defined by $\hat{x}\left(t_{0}\right)=e^{a t_{0}}$. The integrand (3.11) in teims of new variables $\hat{x}(t)$ and $\hat{u}(t)$ becomes,

$$
\begin{equation*}
J=\int_{0}^{\infty}\left[\hat{x^{T}}(\hat{t}) Q \hat{x}(t)+\hat{u}(t) R \hat{u}(t)\right] d t \cdot \cdot \cdot \cdot \cdot \tag{3.13}
\end{equation*}
$$

Hence the problem reduces to finding a control $\hat{u}($.$) for the$ system (3.12) that minimizes the cost functional (3.13). The solution to this problem can be obtained as

$$
\hat{u}(t)=-K \hat{x}(t)
$$

where

$$
K=R^{-I_{B}}{ }_{\bar{P}}
$$

and $\bar{P}$ is the solution of the equation

$$
\begin{equation*}
\bar{P}(A+\alpha I)+\left(A^{T}+\alpha I\right) \bar{P}-P B R^{-1} B^{T} \bar{P}+Q=0 \tag{3.14}
\end{equation*}
$$

For the system considered in this thesis, and choosing $\alpha=0.25$, the new matrix $(A+\alpha I)$ becomes,

$$
\left[\begin{array}{cccccc}
-49.75 & 0 & -20000 . & -20000 . & 0 & 0 \\
1.25 & -1.0 & 0 & 0 & 0 & 0 \\
0.0375 & -0.0375 & -0.75 & 0 & 0 & 0 \\
0 & 0.0638 & 0 & -0.053 & -125.91 & -0.1859 \\
0 & 0 & 0 & -0.255 & 0 & -0.1334 \\
0 & 0 & 0 & 0 & 377.0 & 0.25
\end{array}\right]
$$

It can be shown [9] that the eigenvalues $\lambda$ of the resulting closed-loop system. satisfy $\operatorname{Re}(\lambda)<-\alpha$.

The matrix Riccati equation can be solved using any of the standard algorithms.

The feedback gain matrix $K$ is determined to be

$$
K=\left[\begin{array}{llllll}
1.995 & 1.202 & -0.925 & 22.545 & -21.568 & 7.975
\end{array}\right]
$$

The system with optimal complete state feedback controller having gain matrix $K$ is then simulated to observe the behavior of the system state variables. The results of the simulation are presented in Chapter 4. Implementation of the feedback gains is given.in Fig. 3.2. In this case $\mathrm{K}_{\mathrm{v}}=1.995, \mathrm{~K}_{\mathrm{F}}=1.202, \mathrm{~K}_{\mathrm{S}}=-0.925, \mathrm{~K}_{\mathrm{t}}=22.545$, $\mathrm{K}_{\mathrm{N}}=-21.568$ and $\mathrm{K}_{\delta}=7.975$
III. Deterministic Observer with Optimal Controller (DOOC)

For the optimal complete-state-feedback controller described in II, it was assumed that all the state variables are directly measurable. It could be costly and often not possible to measure all the state variables, and for such cases a deterministic observer could be designed to obtain estimates of the unmeasurable state variables. These estimates are then used to implement the desired control. The overall problem can be separated into two subproblems, referred to as control and estimation [11].


Fig.3.2 Implementation of optimal constant state feedback controller

Design of the Controller:
In the control phase of the solution it is assumed that all the state variables can be measured exactly. Clearly this problem reduces exactly to the one described in section II., i.e., compute the optimal control law $u(t)=-K x(t)$ which would be applied if $x(t)$ were available and if (3.6) were the cost functional. The design of $K$ is exactly the same as in section II.

Design of Reduced Order Deterministic Observer:
According to Luenberger [ll] if $p$ noise-free measurements are available from an $n^{\text {th }}$-order system, then an observer of order ( $n-p$ ) can be formulated which, theoretically, can track the current state variables of the system as closely as desired. Hence an observer can be used to estimate the unmeasurable states. The observer design procedure described here is given in [12]. Consider a linear deterministic $n^{\text {th }}$ order system described by

$$
\begin{align*}
& \dot{x}(t)=A x(t)+B u(t) \cdot . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text { (3.15) }  \tag{3.15}\\
& y(t)=H x(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad(3.16)
\end{align*}
$$

It is assumed that $H$ is of full rank and hence we can write (by transformation or reordering of variables)

$$
y=\left[\begin{array}{ll}
I_{p} & 0
\end{array}\right] x(t) \quad \begin{aligned}
& \text { where } p \text { is the number of states } \\
& \text { measured. }
\end{aligned}
$$

Rearranging all the matrices such that the state variables conform to the above canonical structure of H, ie.,

$$
\begin{aligned}
& x=\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right] \begin{array}{l}
p \\
(n-p)
\end{array} \\
& B=\left[\begin{array}{ll}
\leftarrow p \rightarrow & \leftrightarrow-p \\
\mathrm{~B}_{1} \\
B_{2}
\end{array}\right] \begin{array}{l}
p \\
(n-p)
\end{array}
\end{aligned}
$$

$$
\begin{align*}
\text { yields } & \dot{x}_{1}=A_{11} x_{1}+A_{12} x_{2}+B_{1} u \cdot .  \tag{3.17}\\
\dot{x}_{2} & =A_{21} x_{1}+A_{22} x_{2} \quad B_{2} u \cdot .
\end{align*}
$$

According to Gopinath [12] if (H,A) is completely observable, then $\left[A_{12}, A_{22}\right]$ is completely observable, i.e.,
$\operatorname{Rank}\left(\left[A_{12}^{T}, A_{22}^{T} A_{12}^{T}, . . ., A_{22}^{T}{ }^{n-p-1} A_{12}^{T}\right]\right)=n-p$

Since $\left(A_{12}, A_{22}\right)$ is completely observable we can find an $L$ such that $\left(A_{22}-\mathrm{LA}_{12}\right)$ has arbitrary eigenvalues [12]. Hence if $\hat{\mathrm{x}}_{2}$ is defined by

$$
\begin{equation*}
\dot{\hat{x}}_{2}=A_{22} \hat{x}_{2}+L_{12}\left(x_{2}-\hat{x}_{2}\right)+B_{2} u+A_{21} x_{1} \tag{3.19}
\end{equation*}
$$

then $\quad \tilde{x}_{2}=x_{2}-\hat{x}_{2}$ implies that

$$
\dot{\tilde{x}}_{2}=\left(\dot{x}_{2}-\dot{\hat{x}}_{2}\right)
$$

However, from Eq. (3.18) and (3.19)

$$
\begin{align*}
& \dot{\tilde{x}}_{2}= A_{21} x_{1} \\
&+A_{22} x_{2}+B_{2} u-A_{22} \hat{x}_{2}-L A_{12} x_{2} \\
&+L A_{12} \hat{x}_{2}-B_{2} u-A_{21} x_{1} \\
&=\left(A_{22}-L A_{12}\right) x_{2}-\left(A_{22}-L A_{12}\right) \hat{x}_{2}  \tag{3.20}\\
& \therefore \dot{\tilde{x}}_{2}=\left(A_{22}-L A_{12}\right) \tilde{x}_{2} \cdot \cdots \cdot . . . \cdot . \cdot
\end{align*}
$$

Therefore, by choosing $L$ appropriately we can make $\tilde{x}_{2} \rightarrow 0$ as fast as desired. Using Eq. (3.17) to eliminate $\mathrm{x}_{2}$ from the Eq. (3.19) we obtain,

$$
\begin{array}{r}
\dot{\hat{x}}_{2}=A_{22} \hat{\mathrm{x}}_{2}+\mathrm{L}\left(\dot{\mathrm{x}}_{1}-\mathrm{A}_{11} \mathrm{x}_{1}-\mathrm{B}_{1} \mathrm{u}\right)-L A_{12} \hat{\mathrm{x}}_{2} \\
\\
+\mathrm{B}_{2} u+A_{21} \mathrm{x}_{1} \\
=\left(A_{22}-L A_{12}\right) \hat{x}_{2}-L A_{11} \mathrm{x}_{1}-L B_{1} u+B_{2} u  \tag{3.21}\\
\\
+A_{21} x_{1}+L \dot{x}_{1} \cdot . \cdot .
\end{array}
$$

Each of the terms on the right side of Eq. (3.21) can be observed except $L \dot{\bar{x}}_{1}$. Replacing $\dot{\mathrm{X}}_{1}$ by $\left(\mathrm{A}_{22}-\mathrm{LA}_{12}\right) \mathrm{L} \mathrm{x}_{1}$, ie.,

$$
\begin{array}{r}
\dot{\bar{x}}_{2}=\left(A_{22}-L A_{12}\right) \bar{x}_{2}-L A_{11} x_{1}-L B_{1} u+B_{2} u \\
A_{21} x_{1}+\left(A_{22}-L A_{12}\right) L x_{1} . . \tag{3.22}
\end{array}
$$

By integration by parts we can show that

$$
\begin{gathered}
\hat{\mathrm{x}}_{2}=\overline{\mathrm{x}}_{2}+\exp \left(\mathrm{A}_{22}-L A_{12}\right) \mathrm{t} \cdot\left[\hat{\mathrm{x}}_{2}(0)+L A_{11} \mathrm{x}_{1}(0)\right. \\
\left.+\mathrm{LB}_{1} \mathrm{u}(0)-\mathrm{B}_{2} \mathrm{u}(0)\right]+L \mathrm{x}_{1}(\mathrm{t})
\end{gathered}
$$

Hence by choosing initial conditions suitably for the system given by (3.22) we can make

$$
\hat{x}_{2}=\bar{x}_{2}+L x_{1}(t)
$$

or more compactly we can write the observer equations:

$$
\begin{align*}
\dot{\bar{x}}_{2}(t)= & \left(A_{22}-L A_{12}\right) \bar{x}_{2}(t)+\left(A_{22}-L A_{12}\right) L x_{1}(t) \\
& +\left(A_{21}-L A_{11}\right) x_{1}(t)+\left(B_{2}-L B_{1}\right) u(t) \cdot(3.23)  \tag{3.23}\\
\hat{x}_{2}(t)= & \bar{x}_{2}(t)+L x_{1}(t) \quad . \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot(3.24) \tag{3.24}
\end{align*}
$$

with $\bar{x}_{2}$ the solution to Eq. (3.22) (see Fig. 3.3).
DESIGN EQUATIONS:
In the observer design the main problem is to find an $L$ such that $\left(A_{22}-\mathrm{LA}_{12}\right)$ has the desired eigenvalues. From Thm. 1 , [12], if the given system is completely observable, then with $\gamma_{1}, \gamma_{2}$, . ., $\gamma_{n-p}$ denoting the coefficients of the polynomial of ( $\mathrm{A}_{22}-\mathrm{LA} \mathrm{A}_{12}$ ), i.e.,


Fig.3.3 Implementation of observer and controller

$$
x\left(A_{22}-L A_{12}\right)=s^{n-p}+\sum_{i=1}^{n-p} \gamma_{i} s^{n-p-i}
$$

there exists an $L$ of rank one satisfying

$$
\begin{aligned}
\gamma_{1} & =a_{1}+\operatorname{tr}\left(L A_{12}\right) \\
\gamma_{2} & =a_{2}+a_{1} \operatorname{tr}\left(L_{12}\right)+\operatorname{tr}\left(L A_{12} A_{22}\right) \\
\gamma_{n-p} & =a_{n-p}+a_{n-p-1} \operatorname{tr}\left(L A_{12}\right)+\ldots+\operatorname{tr}\left(L A_{12} A_{22}{ }^{n-p-1}\right)
\end{aligned}
$$

where the a's are coefficients of characteristic polynomial of $\mathrm{A}_{22}$, i.e.,

$$
x\left(A_{22}\right)=s^{n-p}+\sum_{i=1}^{n-p} a_{i} s^{n-p-i}
$$

Let

$$
\begin{aligned}
& \quad\left[\gamma_{1}, r_{2}, \ldots, \gamma_{n-p}\right] \triangleq \gamma^{T} \\
& \text { and } \quad\left[a_{1}, a_{2}, \ldots, a_{n-p}\right] \triangleq a^{T}
\end{aligned}
$$

Rewriting Eq. (3.25)

$$
\gamma=a+\left[\begin{array}{lllllll}
1 & 0 & \cdots & \cdots & \cdots & 0  \tag{3.25a}\\
a_{1} & 1 & \cdots & \cdots & . & 0 \\
a_{2} & a_{1} & 1 & \cdots & . & 0 \\
\vdots & & & & & \\
a_{n-p-1} & \cdots & \cdots & \cdots & . & 1
\end{array}\right]\left[\begin{array}{l}
\operatorname{tr}\left(\mathrm{LA}_{12}\right) \\
\operatorname{tr}\left(\mathrm{LA}_{12} A_{22}\right) \\
\vdots \\
\vdots \\
\operatorname{tr}\left(\mathrm{LA}_{12} A_{22}{ }^{n-p-1}\right)
\end{array}\right]
$$

To write Eq. (3.25a) in simplified form, let


0 bviously $\hat{A}^{-1}$ always exists and can be evaluated easily.

$$
\left[\begin{array}{l}
\operatorname{tr}\left(L A_{12}\right)  \tag{3.26}\\
\operatorname{tr}\left(L A_{12} A_{22}\right) \\
\vdots \\
\operatorname{tr}\left(L A_{12} A_{22} n-p-1\right)
\end{array}\right]=\hat{A}^{-1}(\gamma-a) \quad \cdot . .
$$

Now assuming $L=\alpha \beta^{T}(\alpha$ and $\beta$ are $((n-p) \times 1)$ and $(p \times 1)$ matrices respectively).

Then $\operatorname{tr}\left(\mathrm{LA}_{12} \mathrm{~A}_{22}{ }^{\mathrm{K}}\right)=\operatorname{tr}\left(\alpha \beta^{\mathrm{T}} \mathrm{A}_{12} \mathrm{~A}_{22}{ }^{\mathrm{K}}\right)$

$$
\begin{aligned}
& =\operatorname{tr}\left(A_{22}^{\mathrm{T}}{ }^{\mathrm{K}} \cdot \mathrm{~A}_{12}^{\mathrm{T}} \beta \alpha \alpha^{\mathrm{T}}\right) \\
& =\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22}^{\mathrm{K}} \alpha
\end{aligned}
$$

So Eq. (12) becomes

$$
\left[\begin{array}{l}
\beta^{\mathrm{T}} \mathrm{~A}_{12}{ }^{\alpha} \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22^{\alpha}} \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22}{ }^{\mathrm{n}-\mathrm{p}-1_{\alpha}}
\end{array}\right]=\hat{A}^{-1}(\gamma-\mathrm{a})
$$

or

$$
\left[\begin{array}{l}
\beta^{\mathrm{T}} \mathrm{~A}_{12} \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22} \\
\vdots \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22}{ }^{\mathrm{n}-\mathrm{p}-1}
\end{array}\right] \quad \alpha=\hat{A}^{-1}(\gamma-\mathrm{a})
$$

or

$$
w(\beta) \alpha=\hat{A}^{-1}(\gamma-a)
$$

where

$$
\begin{align*}
& \mathrm{w}(\beta) \triangleq\left[\begin{array}{l}
\beta^{\mathrm{T}} \mathrm{~A}_{12} \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22} \\
\vdots \\
\beta^{\mathrm{T}} \mathrm{~A}_{12} \mathrm{~A}_{22} \mathrm{n}-\mathrm{p}-1
\end{array}\right] \\
& \therefore \alpha=\mathrm{w}^{-1}(\beta) \hat{A}^{-1}(\gamma-\mathrm{a}) \tag{3.27}
\end{align*}
$$

According to Lemma 4 [12] rank $(w(\beta))=n-p$ for
almost all $\beta$, so a unique solution exists.
Thus the procedure to design the observer is as follows:
(1) Choose eigenvalues of $\left(\mathrm{A}_{22}-\mathrm{LA}_{12}\right)$ at any desired location in the left half of the complex plane.
(2) Compute $\gamma$, a and A matrices.
(3) Choose appropriate $\beta$.
(4) Compute w ( $\beta$ ).
(5) Compute $\alpha=w^{-1}(\beta) \hat{A}^{-1}(\gamma-a)$.
(6) Finally find $L=\alpha \beta^{T}$.

Design of an Observer for the PSS
Writing the system in the form described earlier
${ }^{A} 11$
${ }^{\mathrm{A}} 12$
$\left[\begin{array}{c}\dot{x}_{1} \\ \dot{x}_{2} \\ \hdashline \dot{x}_{3} \\ \dot{x}_{4} \\ \dot{x}_{5} \\ \dot{x}_{6}\end{array}\right]=\left[\begin{array}{cc:cccc}-50.0 & 0.0 & -20000.0 & -20000.0 & 0 \\ 1.25 & -1.25 & 0 & 0 & 0 & 0 \\ \hdashline-0375 & -.0375 & -1.0 & 0 & 0 & 0 \\ 0 & .0638 & 0 & -.303 & -125.91 & -.1859 \\ 0 & 0 & 0 & -.255 & -.25 & -.1334 \\ 0 & 0 & 0 & 0 & 377 . & 0\end{array}\right]$
$\mathrm{A}_{21}$
$\mathrm{A}_{22}$

$$
\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\cdots \\
x_{3} \\
x_{4} \\
x_{5} \\
x_{6}
\end{array}\right]+\left[\begin{array}{c}
20000.0 \\
0 \\
\cdots \cdots-\cdot \\
0 \\
0 \\
0 \\
0
\end{array}\right] u
$$

It is clear from the above arrangement of the system that states $x_{1}$ and $x_{2}$ are measured and the remaining four states are to be observed. Hence the observer is of fourth order.

Choosing eigenvalues of $\left(A_{22}-L_{12}\right)$ at $-5.0,-5.0$, $-5.0,-5.0$ and $\beta^{T}=[1.0 \quad 2.0]^{\mathrm{T}}$, steps $2,4,5$, and 6 were computed to find matrix $L$.

$$
L=\left[\begin{array}{ll}
5.998 \mathrm{E}-4 & 1.199 \mathrm{E}-3 \\
-1.52 \mathrm{E}-3 \\
3.528 \mathrm{E}-5 & -3.04 \mathrm{E}-3 \\
2.94 \mathrm{E}-3 & 5.87 \mathrm{E}-3
\end{array}\right]
$$

Finally, the system is simulated with the observer and controller using CSMP. Results are shown in Chapter 4. A scheme for implementing the observer along with the controller has been shown in Fig. 3.3. .
IV. Design of Stochastic Optimal Controller (SOC) If all the states are not available we can also design a Kalman filter [l0,13] with assumed statistics of system and measurement noises. Let the system be described by

$$
\begin{align*}
& \dot{x}(t)=A x(t)+B u(t)+w(t)  \tag{3.28}\\
& z(t)=H x(t)+v(t) \cdot \cdot \cdot \tag{3.29}
\end{align*}
$$

The matrices $A, B$ and vector $x$ are defined in 3.II.
The noise signals $w$ and $v$ have the following characteristics:
(1) Signals $w$ and $v$ are stationary gaussian process with zero mean;
(2) Signals $w$ and $v$ are uncorrelated, i.e., $E\left[w(t) v{ }^{T}(t)\right]=0$;
(3) Signals $w$ and $v$ are white noise and their correlation functions may be written as:

$$
\begin{aligned}
& E\left[w(t) w^{T}(T)\right]=\hat{Q} \delta(t-T) \\
& E\left[v(t) v^{T}(T)\right]=\hat{R} \delta(t-T)
\end{aligned}
$$

where $\delta(t-T)$ is the Dirac delta function. It is a common practice to make elements of $w$ and $v$ uncorrelated, so that $\hat{Q}$ and $\hat{R}$ become diagonal matrices. $\hat{Q}$ and $\hat{R}$ are chosen as:

$$
\hat{Q}=\left[\begin{array}{cccccc}
0.01 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.01 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.01 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.01 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.01 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.01
\end{array}\right], \hat{\mathrm{R}}=[0.001]
$$

The initial state $x\left(t_{o}\right)$ is taken as a zero mean gaussian random vector.

The problem now is to find a closed-loop controller for generating the optimal-control input $u(t)$ in terms
of the output $z(t)$. For this purpose, it is desired to determine $u(t)$ such that the cost functional

$$
\begin{equation*}
J=E\left\{\int_{t_{0}}^{\infty}\left[x^{T}(t) Q x(t)+u^{T}(t) R u(t)\right] d t\right\} \cdots \cdot \cdot \cdot \cdot \cdot \tag{3.30}
\end{equation*}
$$

is minimized, where expectation is over $x\left(t_{0}\right)$ and the stochastic processes $w$ and $v$ on the interval ( $\left.t_{0}, \infty\right)$. The matrices $Q$ and $R$ are defined in Section II.

Using the separation principle [13] this problem can separated into two subproblems the estimation problem and the control problem.

CONTROL PROBLEM:
To design the controller it is assumed that all the state variables are available, and hence that we can use the technique discussed in 3.II;i.e., compute the optimal control law $u(t)=-K x(t)$ which would be applied if perfect noise-free measurements of $x(t)$ were available and if(3.6) were the cost functional. The design of $F$ is exactly the same as in II.

## ESTIMATION PROBLEM:

In order to use the above controller it is necessary to reconstruct the state variables in some fashion from the noisy measurements which are the only actual outputs of the system. The device which accomplishes this task is the Kalman filter. Using the noisy measurements $z(t)$
as inputs, the Kalman filter generates estimates of all the state variables.

For this purpose Kalman [10] defined a linear dynamic system model very similar to the original system model. The input of the filter is $z(t)$ and the output is $\hat{x}(t)$. Specially, $\hat{x}(t)$ is the solution of

$$
\begin{equation*}
\dot{\hat{x}}(t)=A \hat{x}(t)+K_{e}[z(t)-H \hat{x}(t)]+B u(t) \tag{3.31}
\end{equation*}
$$

wherein $K_{e}$ is termed the Kalman gain matrix and is defined by

$$
\begin{equation*}
K_{e}=P H^{T} \hat{R}^{-1} \tag{3.32}
\end{equation*}
$$

where $P$ in (3.32) is obtained by solving the following algebric Riccati equation:

$$
\begin{equation*}
A P+P A^{T}+\hat{Q}-P H^{T} \hat{R}^{-1} H P=0 \tag{3.33}
\end{equation*}
$$

To compare the performance of the system given in Section 3.I the internal frequency deviation is used as the only measurement, i.e.,

$$
\mathrm{H}=\left[\begin{array}{llllll}
0 & 0 & 0 & 0 & 1.0 & 0
\end{array}\right]
$$

The matrix Riccati equation (3.33) was computed to determine the following Kalman gain matrix

$$
\mathrm{K}_{\mathrm{e}}=\left[\begin{array}{l}
1.847 \\
1.238 \\
-487.59 \\
-489.36 \\
351.58 \\
0.725
\end{array}\right]
$$

Finally, the system with Kalman filter and controller is simulated to see the behavior of the state variables of the system. The results are displayed in Chapter 4.

## CHAPTER 4

SIMULATION RESULTS

In this chapter the effectivenessミof the power system• stabilizers designed in Chapter 3 has been studied by subjecting the system to a pulse disturbance of -0.4 p.u. for a period of 0.025 sec . This is the same disturbance used by Schlief [1]. Digital simulation of these stabilizers is carried out using both the linear and the nonlinear models of the generator. This chapter has been divided into four sections. The first two sections present results using, respectively, the linear and nonlinear models when subjected to the pulse disturbance. The final two sections give results of the two models when system and measurement noises are taken into consideration.

## I. Results for the Linear Model

The time responses of the generator terminal voltage and the internal frequency are given in Figs. 4.1 to 4.4. First and second maximum deviations from operating values are presented in Table 4.1. The cost function data are compared in Table 4.2 .

## II. Results for the Nonlinear Model

Terminal voltage and internal frequency time res-
ponses for the nonlinear generator model when subjected to the pulse disturbance input for the dynamic stabilizer are given in Figs. 4.la and 4.2a. These responses for the other three stabilizers are depicted in Figs. 4.5 and 4.6.
III. Results of the Linear Model with Noise Considered

The system driving noise vector used in the simulation is a gaussian white noise with zero mean and covariance matrix $10^{-10}\left[I_{6}\right]$. The system measurement noise vector has the same properties but the dimensions of the noise vector and the covariance matrix vary according to the number of the state variables measured e.g., in the case of Stabilizer Type III(DOOC) only two state variables are measured. In that case, the dimension of the measurement noise vector will be ( $2 \times 1$ ) and that of the covariance matrix ( $2 \times 2$ ).

The time responses of the terminal voltage and internal frequency are given in Figs. 4.7 and 4.8. Table 4.3 gives the maximum daviations of these state variables from the nominal operating values. Cost function data are compared in Table 4.4.
IV. Results of the Nonlinear Model with Noise Considered The properties of the system driving noise and the measurement noise are the same as were given in the above section. Figs. 4.9 and 4.10 show the time responses of the terminal voltage and the internal frequency following the pulse disturbance.


Note:
a. Eb means $\mathrm{a} \times 10^{\mathrm{b}}$

Fig. 4.1 The terminal voltage response of the stabilizer Type I (DS) for the pulse disturbance input. (Linear model, noise-free case)








Fig.4.6 Comparison of internal frequency responses to the pulse disturbance input. (Nonlinear model, noisefree case)

Note:
a.Eb means $\mathrm{ax} 10^{\mathrm{b}}$


Fig.4.7 Comparison of the generator terminal voltage responses to the pulse disturbance input for the case of linear model with gaussian noise (Mean=0.0, S.D. $=0.00001$ )


Note:
a.Eb means $\mathrm{ax} 10^{\mathrm{b}}$

Fig.4.8 Comparison of internal frequency responses to the pulse disturbance input for the case of linear model with gaussian noise (Mean=0.0,S.D. $=0.00001$ )


Note:
a. Eb means ax10 ${ }^{\text {b }}$

Fig.4.9 Comparison of the generator terminal voltage responses to the pulse disturbance input for the case of nonlinear model with gaussian noise (Mean=0.0,S.D.=.00001)


Fig. 4.10 Comparison of internal frequency responses to the pulse disturbance input for the case of non-

Note:
a.Eb means $\mathrm{ax} 10^{\mathrm{b}}$
linear model with gaussian noise (Mean=0.0,S .D.=0.00001)

Table 4.1 Pulse disturbance responsesdeviations from steady-state values (Linear model, noise-free case)

| Stabilizer <br> Type | Terminal Voltage$100 \Delta \mathrm{e}_{\mathrm{t}} \text { (p.u.) }$ |  |  |  | Internal Frequency$10000 \Delta N \text { (p.u.) }$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Max. <br> First <br> Up <br> Swing | Max. <br> First <br> Down <br> Swing | Max. <br> Second <br> Up <br> Swing | Max. Seconr Down Swing | Max. <br> First <br> Down <br> Swing | Max. First Up Swing | $$ | Max. <br> Secon <br> Up <br> Swing |
| $\begin{gathered} \mathrm{I} \\ (\mathrm{DS}) \end{gathered}$ | 1.88 | -3.38 | 3.54 | -3.26 | $-11.3$ | 13.5 | -13.5 | 12.5 |
| $\begin{gathered} \text { II } \\ (\mathrm{DOFC}) \end{gathered}$ | 3.056 | -1.48 | 1.84 | -0.89 | -9.13 | 7.63 | -5.78 | 4.61 |
| $\begin{gathered} \text { III } \\ (\mathrm{DOOC}) \end{gathered}$ | 3.03 | -1.76 | 1.66 | - 1.23 | -9.12 | 7.87 | -6.01 | 4.81 |
| $\begin{gathered} \text { IV } \\ (\mathrm{SOC}) \end{gathered}$ | 3.22 | -0.89 | 2.08 | -0.63 | -9.14 | 7.04 | -5.33 | 4.42 |

Table 4.2 A comparison of cost functionals (Linear model, noise-free case)

| Stabilizer <br> Type | $\mathrm{J}_{\mathrm{x}}$ | $\mathrm{J}_{\mathrm{u}}$ | $J_{x}+J_{u}$ |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{I} \\ (\mathrm{DS}) \end{gathered}$ | 13.05 | $2.2 \mathrm{E}-3$ | 13.05 |
| $\begin{gathered} \text { II } \\ (\mathrm{DOFC}) \end{gathered}$ | 7.06E-3 | 6.81E-4 | $7.74 \mathrm{E}-3$ |
| $\begin{gathered} \text { III } \\ (\mathrm{DOOC}) \end{gathered}$ | 9.97E-3 | 6.73E-4 | 1.06E-2 |
| $\begin{gathered} \text { IV } \\ (\mathrm{SOC}) \end{gathered}$ | 3.64E-2 | $7.438 \mathrm{E}-4$ | 3.72E-2 |
| Mode1 <br> without stabilizer | 284.33 | - | 284.33 |

Table 4.3 Pulse disturbance responsesdeviations ${ }^{*}$ from steady-state values.
Linear model with gaussian noise (Mean=0.0,S.D $=0.00001$ )

| Stabilizer <br> Type | $\begin{gathered} \text { Terminal Voltage } \\ 100 \Delta \mathrm{e}_{\mathrm{t}} \text { (p.u.) } \end{gathered}$ |  |  |  | Internal Frequency$10000 \Delta N \text { (p.u.) }$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Max. <br> First Up Swing | Max. <br> First <br> Down <br> Swing | Max. Second Up Swing | Max. <br> Second <br> Down <br> Swing | Max. First Down Swing | Max. <br> First <br> Up <br> Swing | Max. Second Down Swing | Max. Second Up Swing |
| $\begin{gathered} \mathrm{I} \\ (\mathrm{DS}) \end{gathered}$ | 1.88 | -3.37 | 3.53 | -3.26 | $-11.3$ | 13.5 | -13.5 | 12.5 |
| $\begin{gathered} \text { II } \\ (\mathrm{DOFC}) \end{gathered}$ | 3.04 | -1.43 | 1.83 | -. 865 | -9.11 | 7.53 | -5.7 | 4.53 |
| $\begin{gathered} \text { III } \\ (\mathrm{DOOC}) \end{gathered}$ | 3.03 | $-1.76$ | 1.66 | -1.22 | -9.12 | 7.87 | -6.0 | 4.81 |
| $\begin{aligned} & \text { IV } \\ & (\text { SO C }) \end{aligned}$ | 3.21 | -0.87 | 2.08 | -0.62 | -9.12 | 7.01 | -5.32 | 4.41 |

[^2]Table 4.4 A comparison of cost functionals* for the case of. linear model with gaussian noise (Mean=0.0,S.D. $=0.00001$ )

| Stabilizer <br> Type | $\mathrm{J}_{\mathrm{x}}$ |  | $\mathrm{J}_{\mathrm{u}}$ |
| :---: | :---: | :---: | :---: |
| I <br> (DS) | 13.032 | $\mathrm{~J}_{\mathrm{x}}+\mathrm{J}_{\mathrm{u}}$ |  |
| II <br> (DO FC) | $6.76 \mathrm{E}-3$ | $6.21 \mathrm{E}-3$ | 13.035 |
| III <br> (DOOC) | $9.94 \mathrm{E}-3$ | $6.70 \mathrm{E}-4$ | $1.06 \mathrm{E}-2$ |
| IV <br> (SOC) | $3.63 \mathrm{E}-2$ | $7.42 \mathrm{E}-3$ |  |

* These cost functionals are time averages for particular sample paths of the noise.


## CHAPTER 5

INTERPRETATION OF RESULTS
The main objective of this thesis is to use modern control strategies to design different types of PSS. To show the effectiveness of these PSS it is necessary to compare them with the PSS designed earlier by using the structure suggested by Schleif [1]. The PSS designed earlier is designated as Type I, and its time responses of terminal voltage and internal frequency are shown in Figs. 4.1 and 4.2. Comparing these responses with those shown in Figs. 4.3 and 4.4 it can be clearly seen that the damping is much better in the case of stabilizer Type II, III and IV. By looking at Table 4.1 it is observed that cost functions of Type II,III, and IV are very much smaller than Type I. Hence the overall performance of PSS based on modern control would appear to be far better than that of PSS designed by classical approach used by Schleif [1]. The cost function obtained for the optimal controller with complete state feedback(Type II) is lower, as expected*, than that obtained via use of either the

[^3]Kalman filter or the observer. The performance of the observer plus optimal controller is comparable to the optimal controller with all the state variables available for measurement. Thus, it may be possible (in a very low-noise environment) to eliminate the necessity of measuring all the state variables of the system. Finally, the Kalman filter plus optimal controller behaves quite well and hence errors in measuring the state variables can be compensated for in the design of the controllers. Finally, it should be mentioned that the stabilizer Type I when applied to the nonlinear generator model makes the closed-loop system unstable. This instability is clearly seen by looking at Fig. 4.la. Also by looking at Fig. 4.5 it is observed that for the case of the stabilizer Type II(DOFC) the terminal voltage has a bias in it* i.e., it settles down at about 1.03 p.u. instead of the steady-state value of 1.0 p.u.. The mechanical power disturbance takes the terminal voltage from 1.0 p.u. to the new steady-state value of 1.03 p.u.. This means that once the disturbance is over the terminal voltage oscillates about the new steady-state value, and finally settles down at this value.

[^4]lizers, the dynamics of the real power-frequency controller were not included. Hence the mechanical power disturbance has no feedback to the governing system of the p-f loop. The rotor position will change, reducing the rotor angle, and thus, in the absence of such feedback, will create a steady-state offset in the terminal voltage. For the purpose of the research described herein, deletion of the p-f loop dynamics was deemed to be a cost-effective trade-off of (savings in computer time) versus (allowing the above mentioned predictable bias).

## CONCLUSIONS AND RECOMMENDATIONS

The research initiated in this thesis has provided the following specific results:
(I) From the transient stability point of view it has been found that PSS designed on the basis of modern control theory exhibit much better damping than the PSS designed earlier by Schlief et al [1].
(2) The need to measure all state variables can be eliminated by using either the observer or the Kalman filter to estimate these state variables. For low noise environments the former suffices, while in other cases the Kalman filter is the appropriate choice.

It is worthwhile here to compare the results obtained in this thesis with some previous work in this field. In [14] a comparison is made between voltage transients at a generator plant where the automatic voltage regulator is tuned for open-circuit and for on-line conditions. This technique is suitable only when the generator is delivering a particular load. Since the AVR is tuned at this load level, it will not damp voltage transients effectively when the generator is running at a different load level. On the contrary the PSS designed here will work equally well at various load levels.

In [15] a PSS of Type $I$ has been designed using
root-locus techniques. A comparison cannot be made since no transient responses were given in that paper. In [16] some output feedback controllers with dynamic and/or constant gains have been designed with the assumption that all the state variables are available for measurement. The results obtained therein are basically the same as given in Chapter 5. But this thesis goes a step further and explores the possibilities of estimating the state variables not available for measurement.

Some of the research topics not included in this thesis are given here as suggestions for further research:

1. Extend this research for a two-area or multi-area power system,
2. Extend the work done here for the case of multiple gen.erators connected to an infinite bus.
3. In the computer program for the pattern search, the integration to obtain state variables and the cost functional was carried out for a long period of time after the disturbance was over. A more efficient method would be to use a Lyapunov equation to evaluate the cost functional for times greater than $t_{d}$, where $t_{d}$ marks the time at which the disturbance vanishes.

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## APPENDIX A

Nomenclature

Table A. 1
CONSTANTS USED IN SIMULATION

| Symbol | Description | Value in per unit |
| :---: | :---: | :---: |
| Excitation | system |  |
| $\mathrm{K}_{\text {A }}$ | regulator gain | 400.0 |
| $\mathrm{T}_{\mathrm{A}}$ | regulator time constant | 0.02 |
| $K_{E}$ | exciter gain | 1.0 |
| $\mathrm{T}_{\mathrm{E}}$ | exciter time constant | 0.8 |
| $\mathrm{K}_{\mathrm{F}}$ | regulator stabilizer gain | 0.03 |
| T F | regulator stabilizer time constant | 1.0 |
| Generator | and tieline |  |
| T do | direct-axis transient open-cicuit time constant | 6.5 |
| $\chi_{\text {d }}$ | direct-axis synchronous reactance of generator | 1.6 |
| $X_{d}^{\prime}$ | direct-axis transient reactance of generator | 0.32 |
| $\chi_{q}$ | quadrature-axis synchronous reactance of generator | 1.55 |
| $\mathrm{X}_{\mathrm{e}}$ | equivalent system reactance | 1.0 |
| TM | generator mechanical time constant | 10.0 |
| D | damping factor | 2.5 |
| $\mathrm{K}_{1}$ |  | 0.483 |


| $\mathrm{K}_{2}$ | 1.0473 |
| :--- | :---: |
| $\mathrm{~K}_{3}$ | 0.5077 |
| $\mathrm{~K}_{4}$ | 1.34 |
| $\mathrm{~K}_{5}$ | -0.334. |
| $\mathrm{~K}_{6}$ | 0.4107 |
| $\mathrm{G}_{\mathrm{p}}$ | 7.532 |
| $\mathrm{~T}_{1}$ | 0.0425 |
| $\mathrm{~T}_{2}$ | 0.229 |
| $\mathrm{~T}_{3}$ | 4.145 |
| $\mathrm{~T}_{4}$ | 0.217 |
| $\mathrm{~T}_{\mathrm{R}}$ | 0.0836 |

Table A. 2
VARIABLES USED IN SIMULATION

Voltages

| $\mathrm{V}_{\mathrm{R}}$ | regulator output voltage |
| :---: | :---: |
| $\mathrm{V}_{\mathrm{R}} \mathrm{MAX}$ | maximum value of $\mathrm{V}_{\mathrm{R}}$ |
| $\mathrm{V}_{\mathrm{R}}$ MIN | minimum value of $\mathrm{V}_{\mathrm{R}}$ |
| $\mathrm{V}_{\text {ref }}$ | regulator reference voltage setting |
| $\mathrm{E}_{\mathrm{fd}}$ | exciter output voltage (applied to generator) |
| $\mathrm{E}_{\mathrm{q}}^{\prime}$ | voltage proportional to direct-axis flux |
|  | linkages |
| $e_{t}$ | generator terminal voltage |
| $e_{d}, e_{q}$ | armature voltage,direct and quadrature-axis components |
| E | infinite bus voltage |

Currents
$i_{d}, i_{q} \quad$ armature current, direct and quadrature-axis components
$I_{p}$
real load current
$I_{q} \quad$ reactive load current
Miscellaneous variables
Pe electrical power output of generator
Te electromechanical torque
N generator speed
$\mathrm{P}_{\mathrm{m}} \quad$ mechanical power or prime mover torque

## APPENDIX B

I. Derivation of Nonlinear Generator and Tieline Model

Assumptions:
(1) Zero-sequence currents and voltages are negligible.
(2) Armature resistance is negligible.
(3) Space wave harmonics are neglected.
(4.) Saturation in both axes is neglected.
(5) Induced voltages between the direct and quadrature axes are negligible.
(6) Changes in speed or frequency are assumed very small and do not affect voltages or impedances within the generator.
(7) Inertias from all rotating parts are treated as a single lumped constant.
(8) The infinite bus is a voltage reference and has a fixed frequency of 1 per unit or 60 HZ .
(9) Frequency, speed, and angle have meaning only when referenced to the infinite bus.
(10) No local loads are connected to the generator.

Development of Nonlinear Generator and Tieline Equations:

VOLTAGE EQUATIONS:
As derived in Adkins [5], the direct- and quadratureaxis voltages for a generator are

$$
e_{d}=s \psi_{d}-v \psi_{q}-r_{a} i_{d}
$$

and

$$
e_{q}=v \psi_{d}+s \psi_{q}-r_{a} i_{q}
$$

From the assumptions, $s \psi_{d}=0, s \psi_{q}=0, r_{a}=0$, and $v=1$. Therefore, above equations reduce to

$$
e_{d}=-\psi_{q}
$$

and

$$
e_{q}=\psi_{d}
$$

Adkins described the flux components as

$$
\psi_{d}=-\frac{x_{d}(s)}{w} i_{d}+\frac{G(s)}{w} E_{f d}
$$

and $\quad \psi_{q}=-\frac{x_{q}(s)}{w}$ iq
with the assumption that $w=1$. The frequency-dependent rectances and gains may be written as:

$$
\begin{aligned}
& x_{d}(s)=x_{d^{\prime}}^{\prime}+\frac{x_{d^{\prime}}-x_{d^{\prime}}^{\prime}}{1+s^{T} d^{\prime}} \\
& x_{q}(s)=x_{q} \\
& G(s)=\frac{1}{1+T_{d o}^{\prime} s}
\end{aligned}
$$

Combining equations we obtain,

$$
\begin{aligned}
e_{d} & =x_{q^{\prime}} i_{q} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
e_{q} & =-\left[x_{d}^{\prime}+\frac{x_{d}-x_{d}^{\prime}}{1+s T_{d o}^{\prime}}\right] i_{d}+\left[\frac{1}{1+s T}{ }_{d o}{ }^{\prime}\right] E_{f d} \\
& =\left[\frac{1}{1+s T_{d o}}\right] E_{f d}-\left[\frac{x_{d}-x_{d}^{\prime}}{1+s T_{d o}}\right] i_{d}-x_{d}^{\prime} i_{d}
\end{aligned}
$$

The voltage proportional to direct axis flux linkages Eq' is given by

$$
\begin{equation*}
\mathrm{Eq}^{\prime}=\left[\frac{1}{1+\mathrm{sT} \mathrm{do}^{\prime}}\right] \mathrm{E}_{\mathrm{fd}}-\left[\frac{\mathrm{x}_{\mathrm{d}^{-}} \mathrm{x}_{\mathrm{d}}}{1+\mathrm{ST}_{\mathrm{do}}}\right] i_{\mathrm{d}} \tag{B.2}
\end{equation*}
$$

Hence $\quad e_{q}=E q^{\prime}-x_{d}{ }^{\prime} i_{d}$

$$
e_{t}^{2}=e_{d}^{2}+e_{q}^{2}
$$

$$
\begin{equation*}
\mathrm{Eq}=\mathrm{Eq} \mathrm{q}^{\prime}+\left(\mathrm{x}_{\mathrm{q}}-\mathrm{x}_{\mathrm{d}}^{\prime}\right) i_{\mathrm{d}} \tag{BC}
\end{equation*}
$$

and

$$
E q=e_{q}+x_{q} i_{d}
$$

## POWER EQUATION

Electrical power is calculated from the equation

$$
\begin{equation*}
P_{e}=E_{q} i_{q} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \tag{B.6}
\end{equation*}
$$

CURRENT EQUATIONS:
Currents are calculated from the infinite bus voltage, $E_{q}$, and the torque angle.

$$
\begin{align*}
& i_{d}=\frac{E_{q}-E \cos \delta}{x_{e}+x_{q}} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot  \tag{B.7}\\
& i_{q}=\frac{E \sin \delta}{x_{e}+x_{q}} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \tag{B.8}
\end{align*}
$$

FREQUENCY AND ANGLE EQUATIONS:
Frequency is calculated as a deviation from synchronous frequency and represents in the simulation the shaft speed or "internal" frequency. Accelerating power is used to determine frequency by the relations:

$$
\begin{align*}
& P_{a}=P_{m}-P_{e}-D \cdot \Delta N  \tag{B.9}\\
& \Delta N=\frac{P_{a}}{T_{m} s} \cdot \cdot \cdot \cdot \tag{B.10}
\end{align*}
$$

The power angle between the infinite bus and the generator quadrature axis is calculated by the equation

$$
\begin{equation*}
\delta=\frac{377 \Delta N}{s} \tag{B.11}
\end{equation*}
$$

II. Linearization of the Nonlinear Model

In order to obtain a linear mathematical model for the nonlinear system, it is assumed that the state variables deviate only slightly from their operating condition.

Expressing Eqs. (B.1), (B.3), (B.4), (B.5), (B.7) and (B.8) in small oscillation form and giving subscript 'o' for their steady-state values we obtain,

$$
\begin{align*}
& \Delta e_{d}=x_{q} \Delta i_{q} \cdot \cdots \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot  \tag{B.12}\\
& \Delta e_{q}=\Delta E_{q}^{\prime}-x_{d}{ }^{\prime} \Delta i_{d}  \tag{B.13}\\
& 2 e_{t o} \Delta e_{t}=2 e_{d o} \Delta e_{d}+2 e_{q o} \Delta e_{q}  \tag{B.14}\\
& \Delta E_{q}=\Delta E_{q}^{\prime}+\left(x_{q}-x_{d}^{\prime}\right) \Delta i_{d}  \tag{B.15}\\
& \Delta i_{d}=\frac{\Delta E_{q}-E_{o} \sin \delta_{o} \Delta \delta}{x_{e}+x_{q}}  \tag{В.16}\\
& \Delta \dot{i}_{q}=\frac{E_{0} \cos \delta_{0} \Delta \delta}{x_{e}+x_{q}} . \tag{B.17}
\end{align*}
$$

First, establishing the relation between $\Delta e_{t}, \Delta \delta, \Delta E_{q}$, eliminate $\Delta \mathrm{E}_{\mathrm{q}}$ from Eqs. (B.15) and (B.16) and solve for $\Delta i_{d}$

$$
\begin{equation*}
\Delta i_{d}=\frac{\Delta E_{q}^{\prime}}{x_{e}+x_{d}{ }^{\prime}}+\frac{E_{0} \sin \delta_{0} \Delta \delta}{x_{e}+x_{d}} \tag{B.18}
\end{equation*}
$$

From (B.18) and (B.13) one obtains

$$
\begin{equation*}
e_{q}=\frac{x_{e}}{x_{e}+x_{d}} \Delta E_{q}^{\prime}-\frac{x_{d}^{\prime} E_{o} \sin \delta_{o}}{x_{e}+x_{d}^{\prime}} \Delta \delta \tag{B.19}
\end{equation*}
$$

Eliminating $\Delta i_{q}$ from (B.12) and (B.13)

$$
\begin{equation*}
\Delta e_{\mathrm{d}}=\frac{\mathrm{x}_{\mathrm{q}} \mathrm{E}_{\mathrm{o}} \cos \delta_{\mathrm{o}}}{\mathrm{x}_{\mathrm{e}}+\mathrm{x}_{\mathrm{q}}} \Delta \delta \tag{B.20}
\end{equation*}
$$

and substituting (B.19) and (B.20) into (B.14) and solving for $\Delta e_{t}$ yields

$$
\begin{aligned}
\Delta e_{t}= & {\left[\frac{x_{q}}{x_{e}+x_{q}} \frac{e_{d o}}{e_{\text {to }}} E_{o} \cos \delta_{o}-\frac{x_{d}^{\prime}}{x_{e}+x_{d}^{\prime}} \frac{e_{q o}}{e_{\text {to }}} E_{o} \sin \delta_{o}\right] \Delta \delta } \\
& +\frac{x_{e}}{x_{e}+x_{d}} \frac{e_{q o}}{e_{\text {to }}} \Delta E_{q}^{\prime}
\end{aligned}
$$

Letting

$$
\begin{align*}
& K_{5}=\frac{x_{q}}{x_{e}+x_{q}} \frac{e_{d o}}{e_{\text {to }}} E_{o} \cos \delta_{o}-\frac{x_{d}}{x_{e}+x_{d}^{\prime}} \frac{e_{q o}}{e_{\text {to }}} E_{o} \sin \delta_{o} \text { and } \\
& K_{6}=\frac{x_{e}}{x_{e}+x_{d}} \frac{e_{q o}}{e_{\text {to }}} \quad \text { one obtains } \\
& \Delta e_{t}=K_{5} \Delta \delta+K_{6} \Delta \mathrm{E}_{\mathrm{q}}{ }^{\prime} . \tag{B.21}
\end{align*}
$$

Next we need to obtain the relation between $\Delta E_{q}^{\prime}, \Delta E_{f d}$ and $\Delta \delta$

Rewriting Eq. (B. 2)

$$
\left(1+s T_{d_{o}}{ }^{\prime}\right) \Delta \mathrm{E}_{\mathrm{q}}^{\prime}=\Delta \mathrm{E}_{\mathrm{fd}}-\left(\mathrm{x}_{\mathrm{d}}-\mathrm{x}_{\mathrm{d}}^{\prime}\right) \Delta \mathrm{i}_{\mathrm{d}}
$$

and substituting the value of $\Delta i_{d}$ from.Eq. (B.18) gives

$$
\begin{equation*}
\left[\frac{\mathrm{x}_{\mathrm{e}}+\mathrm{x}_{\mathrm{d}}}{\mathrm{x}_{\mathrm{e}} \mathrm{tx}_{\mathrm{d}}{ }^{\prime}}+\mathrm{sT} \mathrm{do}^{\prime}\right] \Delta \mathrm{E}_{\mathrm{q}}^{\prime}=\Delta \mathrm{E}_{\mathrm{fd}}-\frac{\mathrm{x}_{\mathrm{d}}-\mathrm{x}_{\mathrm{d}}^{\prime}}{\mathrm{x}_{\mathrm{e}}+\mathrm{x}_{\mathrm{d}}{ }^{\prime}} \mathrm{E}_{\mathrm{o}} \sin \delta_{0} \Delta \delta . \tag{B.22}
\end{equation*}
$$

Now letting

$$
K_{3}=\frac{x_{e}+x_{d}^{\prime}}{x_{e}+x_{d}}
$$

and $\quad K_{4}=\frac{x_{d}-x_{d}{ }^{\prime}}{x_{e}{ }^{+x_{d}}{ }^{\prime} E_{o} \sin \delta_{o} .}$

Eq. (B.22) gives

$$
\begin{equation*}
\Delta \mathrm{E}_{\mathrm{q}}{ }^{\prime}=\frac{\mathrm{K}_{3} \Delta \mathrm{E}_{\mathrm{fd}}}{1+\mathrm{s}^{\mathrm{T}}{ }_{\mathrm{do}}{ }^{\prime} \mathrm{K}_{3}}-\frac{\mathrm{K}_{3} \mathrm{~K}_{4}}{1+\mathrm{sT}_{\mathrm{do}}{ }^{{ }^{\prime} \mathrm{K}_{3}} \Delta \delta} \tag{B.23}
\end{equation*}
$$

Finally to establish the relation between $\Delta \mathrm{P}_{\mathrm{e}}, \Delta \delta$ and $\Delta \mathrm{E}_{\mathrm{q}}{ }^{\prime}$, write Eq. (B.6) in small oscillation form

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{e}}=\Delta \mathrm{E}_{\mathrm{q}} \mathrm{i}_{\mathrm{qo}}+\mathrm{E}_{\mathrm{qo}} \Delta \mathrm{i}_{\mathrm{q}} \tag{B.24}
\end{equation*}
$$

and substitute the values of $\Delta \mathrm{E}_{\mathrm{q}}$ and $\Delta \dot{q}_{\mathrm{q}}$ from Eqs.(B.15) and (B.17) into Eq. (B.24) to yield

$$
\begin{aligned}
& \Delta P_{e}=\left[\frac{x_{q}-x_{d}{ }^{\prime}}{x_{e}+x_{d}{ }^{\prime}} i_{q o} E_{o} \sin \delta_{o}+\frac{E_{o} E_{q o} \cos \delta_{o}}{x_{e}+x_{q}}\right] \Delta \delta \\
& +\frac{E_{o} \sin \delta}{x_{e}+x_{d}} \Delta_{E^{\prime}}{ }^{\prime}
\end{aligned}
$$

Now letting $K_{1}=\frac{x_{q}-x_{d}^{\prime}}{x_{e}+x_{d}^{\prime}} i_{q o} E_{o} \sin \delta_{o}+\frac{E_{o} E_{q o} \cos \delta_{o}}{x_{e}+x_{q}}$

$$
\begin{align*}
& K_{2}=\frac{E_{0} \sin \delta_{o}}{x_{e}+x_{d}{ }^{\prime}} \text {, we have the desired result } \\
& \Delta \mathrm{P}_{\mathrm{e}}=\mathrm{K}_{1} \Delta \delta+\mathrm{K}_{2} \Delta \mathrm{E}_{\mathrm{q}}{ }^{\prime} \tag{B.25}
\end{align*}
$$

All the values of constants $K_{1}$ through $K_{6}$ are now determined, which give the approximation used to obtain the linear model from the nonlinear model. The linearized generator model connected to an infinite bus is given in Fig. 2.4.

TO OBTAIN OPERATING VALUES
The steady-state operating values of $\delta_{o}, E_{q o}, E_{o}, e_{\text {do }}$, $e_{q o}$ are derived from a standard machine vector diagram [1]. Expressed as a function of steady-state terminal voltage $e_{\text {to }}$ and steady-state real and reactive load currents $I_{\text {po }}$ and $I_{\text {go }}$, these values are:

$$
\begin{aligned}
E_{q o} & =\left[\left(e_{t o}+I_{q o} x_{q}\right)^{2}+\left(I_{p o} x_{q}\right)^{2}\right]^{1 / 2} \\
E_{o} & =\left[\left(e_{t o}-I_{q o} x_{e}\right)^{2}+\left(I_{p o} x_{e}\right)^{2}\right]^{1 / 2} \\
\operatorname{Sin} \delta_{o} & =\frac{e_{t_{o}} I_{p o}\left(x_{q}+x_{e}\right)}{E_{q o} E_{o}} \\
\operatorname{Cos} \delta_{o} & =\frac{e_{t o}\left[e_{t o}-I_{q o}\left(x_{q}-x_{e}\right)\right]}{E_{q o} E_{o}}-\frac{x_{e} x_{q}\left(I_{p o}{ }^{2}+I_{q o}\right)^{2}}{E_{q o} E_{o}}
\end{aligned}
$$

$$
\begin{aligned}
& i_{q o}=\frac{I_{p o}\left(e_{t o}+I_{q o} x_{q}\right)-I_{q o} I_{p o} x_{2}}{E_{q o}} \\
& i_{d o}=\frac{I_{p o}{ }^{2} x_{q}+I_{q o}\left(e_{t o}+I_{q o} x_{q}\right)}{E_{q o}} \\
& e_{q o}=\frac{\left(e_{\text {to }}+I_{q o} x_{q}\right) e_{t o}}{E_{q o}} \\
& e_{d o}=i_{q o} x_{q}
\end{aligned}
$$

## Frequency Response Technique Used in [1]

In [1] frequency response curves of the loaded generator and its excitation system are developed to provide basic data for parameter selection. The frequency at which the electro-mechanical resonance between the machine and the closely coupled infinite bus occurs is determined. The damping influence of the AVR was then obtained from the frequency response curves by the relation $\frac{M \sin \theta}{w_{n}}$ where $M$ is the magnitude ratio of the terminal voltage response to the regulator input driving signal, e is the phase-lag of that response, and $w_{n}$ is the natural frequency of oscillation in $\mathrm{rad} / \mathrm{sec}$. It was found that the damping influence is negative over all the frequency range of interest. Negative damping was primarily due to phase-lag characteristics of the voltage regulator and excitation system. The overall phase-lag of the excitation system and transducer(used to measure frequency deviation) is compensated by two lead-lag stages of the PSS.

Hence an overall lead-lag circuit is designed which serves as a PSS,with frequency deviation serving as input and, supplies as an output a control signal which supplements the voltage error signal.

Derivation of State Equations Associated with Power System

## Stabilizer

$$
\begin{aligned}
& x_{7}=\frac{\mathrm{sT}_{\mathrm{R}}}{1+\mathrm{sT}_{\mathrm{R}}} \mathrm{x}_{5} \\
& \mathrm{x}_{8}=\frac{1+\mathrm{sT}_{3}}{1+\mathrm{sT}_{4}} \mathrm{x}_{7} \\
& \mathrm{x}_{9}=\frac{\mathrm{G}_{\mathrm{p}}\left(1+\mathrm{sT}_{1}\right.}{\left(1+\mathrm{ST}_{2}\right.} \mathrm{x}_{8}
\end{aligned}
$$

By inverse Laplace transformation of the above equations, we obtain the following set of differential equations:

$$
\begin{align*}
& \dot{x}_{7}=-\frac{K_{2}}{T_{M} K_{6}}-\frac{\eta}{T_{M}} x_{5}+\left(\frac{K_{2} K_{5}}{K_{6} T_{M}}-\frac{K_{1}}{T_{M}}\right) x_{6}-\frac{x_{7}}{T_{R}}+\frac{p_{M}}{T_{M}}  \tag{C.1}\\
& \dot{x}_{8}=-\frac{T_{3} K_{2}}{T_{4} T_{M} K_{6}} x_{4}-\frac{T_{3} D}{T_{4} T_{M}} x_{5}+\frac{T_{3}}{T_{4}}\left(\frac{K_{2} K_{5}}{K_{6} T_{M}}-\frac{K_{1}}{T_{M}}\right) x_{6} \\
& +\left(\frac{1}{\mathrm{~T}_{4}}-\frac{\mathrm{T} 3}{\mathrm{~T}_{4} \mathrm{~T}_{\mathrm{R}}}\right) \mathrm{x}_{7}-\frac{1}{\mathrm{~T}_{4}} \mathrm{x}_{8}+\frac{\mathrm{T}}{\mathrm{~T}_{4}} \frac{\mathrm{~T}_{\mathrm{M}}}{} \mathrm{P}_{\mathrm{M}} \tag{C.2}
\end{align*}
$$

$$
\begin{aligned}
& +\frac{\mathrm{G}_{\mathrm{p}} \mathrm{~T}_{1}}{\mathrm{~T}_{2}}\left(\frac{1}{\mathrm{~T}_{4}}-\frac{\mathrm{T}_{3}}{\mathrm{~T}_{4} \mathrm{~T}_{\mathrm{R}}}\right) \mathrm{x}_{7}-\left(\frac{\mathrm{G}^{\mathrm{T}} 1}{\mathrm{~T}_{2} \mathrm{~T}_{4}}-\frac{\mathrm{G}}{\mathrm{~T}_{2}}\right) \mathrm{x}_{8} \\
& -\frac{1}{T_{2}} x_{9}+\frac{\mathrm{G}_{\mathrm{p}} \mathrm{~T}_{1} \mathrm{~T}_{3}}{\mathrm{~T}_{2} \mathrm{~T}_{4} \mathrm{~T}_{\mathrm{M}}} \mathrm{P}_{\mathrm{M}}
\end{aligned}
$$

pattern Search Technique
Hooke and Jeeves [3] have devised a logical method for staying on the crest of a sharp ridge while searching for an optimum. The pattern search technique is based
on the hopeful conjecture that any set of moves; that is, adjustments of the independent variables, which have been successful during early experiments will be worth trying again. This strategy is successful on straight ridges because the only way an early pattern of moves can succeed is if it lies along the crest. Hence further moves in the same direction will be worthwhile if the ridge is straight.

Although the method starts cautiously with short excursions from the starting point, the steps grow with repeated success. Subsequent failure indicates that shorter steps are in order, and if a change in direction is required the technique will start over again with a new pattern. In the vicinity of the peak the steps become very small to avoid overlooking any promising direction.

A description of a pattern search routine,
which has been applied, is given in the flow chart C.l. The sequence following the label (2) is the basic iterative loop consisting of a pattern move followed by a set of exploratory moves. The sequence following the label (1) is for an initial set of exploratory moves from a base point when a new pattern must be established. The sequence labeled (3) controls the reduction of step size and termination of search.

The remaining Charts C. 2, C. 3 give details of the procedure. Explicitly the procedure is carried out by sequentially transforming a set of variables. These variables and their value interpretations are given in Table C-I. Chart C. 2 has been drawn to parallel chart C.1. A detailed flow diagram in terms of the problem variables is exhibited. Notations are explained in Table C-I.


Chart C. 1 Descriptive flow diagram of pattern search


Chart C, 2 Detailed flow diagram for pattern search


Chart C. 3 Descriptive flow diagram for exploratory move (Program E)

## Table C-I

| $\theta$ | the previous base point |
| :--- | :--- |
| $\psi$ | the current base point |
| $\phi$ | the base point resulting from current move |
| $S(\psi)$ | the functional value at the base point |
| $S(\Phi)$ | the functional value for this move |
| $S$ | the functional value before this move |
| $\Delta$ | current step size |
| $\delta$ | minimum step size |
| $\rho$ | reduction factor for step size |

## APPENDIX I

The Computer Program to find Eigenvalues of the System and the Constants $\mathrm{K}_{1}$ through $\mathrm{K}_{6}$

```
    UlNELASION A( 6,6\()\)
    KEAL K1,K2,K3,K4,K5,Kb,
    EEAL IPO,IWU,ILOS,IDUS
    CuMi゙UW/II:JU/K, IN, KUUT
```



```
    CUMAUM/:MAINZ/UUK2(6, 6)
    \(K I N=5\)
    KOUT \(=6\)
    NOIH=t
    ETO \(=1.0\)
    RES = D.L
    \(x_{u}=1.2\)
    Xし1=し. 32
    \(x_{6}=1.55\)
    \(u=2.5\)
    TUOL=0.5
    \(r H=1 \cup \cdot U\)
    \(1 P O=1 \cdot 0\)
    \(1+0=6 \cdot 0\)
    XES \(=\mathrm{i} \cdot \mathrm{C}\)
    EGO=SORT( \(\left.\left.\mathrm{ETO}+\mathrm{IQ} Q * \mathrm{x}_{\mathrm{w}}\right) * * 2+(\mathrm{IPO} * \mathrm{XQ}) * * 2\right)\)
    EU=SG:T(1ETU-1UU*×ES)**2+(IFO*XES)**2)
    SINU=( (ETU*IFU)/(EGO*ED))*(XG+AES)
    \(C O S O=(E T O /(F+C * E O)) *(E T O-I Q U *(X Q-X E S))-(X E S * X Q) /(E Q U * E O))\)
```



```
        IUOS=(1*/EOU)*(IHO*IP!*XN+I世0*(ETO+IG6*X0))
        [G!
```



```
        \(E 0 r, 5=140 S * x_{4}\)
```



```
        \(K 2=(E O 4 S I\) INL; \(/(X E S+\times 01)\)
        K3 \(=(x \cup 1+X E S) /(x)+x \in S)\)
```




```
        1 *とU SINい
            Ko =(XES/.(X+S+XU1))*(EQOS/ETO)
            VRIIt(6,2ul) tQO,EO,SIND,COSD,IUOS,IDOS,EQOS,EDOS
```








```
    * \(\quad\) * \(6=\) *, Elら.ヶ1
        DU \(1 \quad 1=1.0\)
        \(101 \quad J=1,0\)
        A(1, J) \(=\) I.
    1 CU!! TIINUF
    \(A(1,1)=-5 \cdot(;\)
    A \((1,1)=-2\) Ju®u.
    \(A(1,4)=-2 \cup C\{U\).
    \(A(2,1)=1.25\)
    \(H(2,2)=-1.25\)
    \(A(3,1)=0.4375\)
```

$A(3,2)=-u \cdot 43 / b$
$A(3,3)=-1 . U$
$A(4,2)=K 6 / T D U 1$
A(4,4)=-1. $0 /(K 3 * T D O 1)$
$A(4, b)=377 \cdot u * K 5$
$A(4,6)=K 5 /(K 3 * T 0 O 1)-(K 6 * K 4) / T D O 1$
$A(5,4)=-K 2 /(T i \uparrow * K 6)$
$A(5, b)=-\cup / T M$
$A(5,6)=(K 2 * K 5) /(K b * T M)-K 1 / T M$
$A(6, b)=377 \cdot j$
wトITE(6,15)
I5 FORMAT(///, IX, 'MATKIXA•, /)
CALL EIGVAL( $6, A)$

EMD

## APPENDIX II

The Computer Program to Design Stabilizer Type I (DS)


```
C---- PKOGKAM PARSCH
C----- THIS PROGRAM USES THE PATTERN SEARCH TECHNIQUE TO DETERMINE --
C-..-.. IHE SIX PARANETERS OF THE PONER SYSTEM STABLIZER. --
    FEAL KA,KF,KF,K1,K2,K3,K4,K5,K6
    DIMENSION COMPAR(1U),NELTA(10),DIVI(10),X8(10)
    COMMON/SUUL/A(10,10),DIS(10,20GO)
    COMAON/SUE 2/W(1U,2GUU),NDIM,H,NOS
    COMMUN/SUB 3/KA,KE,KF,K1,K2,K3,K5,K6,TA,TDOI,TE,TF,TM,T,D,K4
    COM|uN/SUBS/0rM(2GOJ)
```



```
C-m- NOIM IS NO. OF POINTS
C---H IS STEP SIZE
C---- NOS IS THE NUMBER OF STATES
C---M IS THE NO. OF PARAMETERS TO RE SEARCHED
C-----------------
    H=U.U03
    NOS=y
C------------------------------------------------------------
C---SET PARAMETER VALUES UF THE SYSTEM -.--
C------------------------------------------------------------------
    D=\ddot{2.5}
    KA=4NC.L
    KI=L.4&27
    K2 = 1.0473
    k3 = ن.5477
    K4 = 1.34
    K5 = - [. .334
    K6 = 4.41:7
    KE=1.u
    KF=C:., 3
    TA=U.J2
    TOOL=6.5
    TE=0. % 
    TF=1.0
    TM=10.L
C
C---- SET UP SYSTEM MATRIX,DISTURBANCE AND INITIAL VALUES OF PARAMETERS
C---- TO EE SEARCHED
    M=6
    GP=12.766
    T1=U.丘36
    Tz=U.21
    T 3 = 3. 343
    T4=U.262
    TR=U.u624
    00 21 I=1,9
    21 OPN(1)=-0.4
        D0 22 I=1,NUS
        DO 22 J=1,NDIM
    22 UIS(I,J)=0.U
```

DO $23 \mathrm{~J}=1$,NDIM
DIS $(5, J)=0$ PH (J)/TM
DIS(7,J) $=$ UPM(J)/TM
DIS(à, J) = (T3*DPM(J))/(T4*TM)

```
23 LIS(G,J)=(GP*T1*T3*UP#(J))/1T2*T4*TM)
    DO 1 1=1,NOS
    DO 1 J=1,NOS
```

```
    A(I,J)= = 0
1 CONTINUE
    A(1,1)= -1.U/TA
    A(1,3)=-KA/TA
    A(1,4)=-KA/TA
    A(1,4)=KA/TA
    A(2,1) = 1.L/TE
    A(2,Z) = -KE/TE
    A(3,1)=KF/(TE*TF)
    A(3,2)=(-KE*KF)/(TE*TF)
    A(3,3)=-1.0/TF
    A(4,\angle)=K6/TDO1
    A(4,4)= -1.u/(K3*TUU1)
    A(4,5)=377.U*K5
    A(4,0)=K5/(K3*T0ن11)-(K6*K4)/TDG1
    A(5,4)=-K2/(TM*K6)
    A(5,b)=-0/TM
    A(5,6)=(K2*K5)/(N6*TM) - K1/TM
    A(6,5) = 377.U
    A(7,4)=-K2/(TM*K6)
    A(7,b)=-D/TM
    A(7,6)=-K1/TM+(K2*K5)/(K6*TM)
    A(7,7)=-1.U/TR
    A(8,4)= = (K2*T3)/(T**T4*K6)
    A(8,5)=-(0*T3)/(TM*T4)
    A(8,6)=-(K1*T3)/(T4*TM)+(T3*K2*K5)/(T4*K6*TM)
    A(8,7)=(1.U/T4)*(1.0-T3/TK)
    A(8,0)=-1.u/T4
        A(4,4)=-(GP*K2*T1*T3)/(T2*T4*TM*K6)
        A(9,5)=-(D*GP*T1*T3)/(T2*T4*TM)
        A(9,0)=((GP*TL*T3)/(T2*T4))*(((K2*K5)/(K6*TM))-K1/TM)
        A(4,7)=({GP*T1)/(T2*T4))*(1.0-T3/TK)
        A(4,०)=(GP/T2)*(1.U-T1/T4)
        A(4,9)=-1.0/T2
X0ं(1)=GP
XB(2)=T1
XE(3)=T2
xB(4)=T3
xts(5)=T4
XB(0) = TK
```

DU $5 \quad I=1, M$

DIVI(I) $=10.4$
5 continue
UO $6 \quad 1=1,14$
COMPAK(I) $=1 \cdot$ GEE- 6
6 DELTA(I) $=X$ B(I) 10.0
WRITE(6,11)
11 FURMATI//, INITIAL CHOICES OF PARAMETERS')
víite ( 6,101 ) (XB(1), I=1, M)
WRITE(6,12)
12 FGRMAT(//,' LELTA')
MKITE( 6,131 ) (DELTA(I), I=1,M)
AFITE(6,14)
14. FOKMAT(//, LUVI')

HRITE(6,1E1) (DIVI(1),I=1,M)
*RITE(6,13)
13 FOFMAT(//, COMPAF')
*RITE( $6,1 \cup 1)($ COMPAR(I), $I=1, M)$
1U1 FORMAT(6(5X,E15.8))
CALL PARSCH(H,XB,DELTA, UIVI, COMPAR)
WRITE(6,1U3)

1u3 FGRMATI//,' MatKIX A'I
GRITE( $6,1-2)((A(I, J), J=1, N O S), I=1$, NOS)

EHD

SUBROUTINE FIJNVAL(XB, N,RJ)
REAL KA,KE,KF,Kl,K2,K3,K4,K5,K6
DIfENSIUN Xa(1u), X(10)
COMmON/Ste:/A(1u,1u), DIS(1u, 2000)
COMMON/SUBZ/H(IU, ZUUO), NDIM, H, HOS
CUMMON/SUG3/KA,NE,KF,K1,K2,K3,K5,K6,TA,TDO1,TE,TF,TM,T,D,K4
COMMON/SUB5/DPM(2こUL)
CUMAON/SUGG/DJ(2JOE)
COMMON/SUET/INUEX,OVER
C--.- RESET Parametek Values, system matkix, and disturbance
$G P=X B(1)$
$T 1=x i s(2)$
$T 2=x \in(3)$
$T 3=X B(4)$
$T_{4}=X_{B}(5)$
$T_{K}=X B(6)$
$A(7,4)=-K</(T M * K 6)$
$A(7,5)=-0 / T M$
$A(7,6)=-K 1 / T M+(N \angle * K 5) /(K O * T M)$
$A(7,7)=-1 \cdot \overline{1} / \mathrm{TK}$
$A(0,4)=-(K 2 * T 3) /(T M * T 4 * K 6)$
$\mathrm{A}(\mathrm{*}, 5)=-(\mathrm{i} * \mathrm{~T} 3) /(\mathrm{TA*T4})$
$A(8,6)=-(K 1 * T 3) /(T 4 * T *)+(T 3 * K 2 * K 5) /(T 4 * K 6 * T M)$
$A(8,7)=(1 \cdot 0 / T 4) *(1 \cdot 0-T 3 / T$ K $)$

```
    A(3,8)=-100/T4
        A(9,4)=-(GP*K2*T1*T3)/(T2*T4*TM*K6)
        A(9,5)= -(L*GP*T1*T3)/(T2*T4*TM)
        A(9,6)=((GP*T1*T3)/(T2*T4))*(((K2*K5)/(K6*TM))-K1/TM)
        A(9,7)=((GP*T1)/(T2*T4))*(1.u-T3/TR)
        A(9,8)=(GP/T2)*(1•U-T1/T4)
        A(9,9) = -1.u/T2
    DO 23 J=1,ivNIM
    DIS (5,J)=URM(J)/TM
    DIS(7,J)= DPM(J)/TM
    DIS(&,J)=(T3*DPM(J))/(T4*TM)
<3
    DIS(4,J)=(GP*T1*T3*DPM(J))/(T2*T4*TM)
    DO 9 I=1,NOS
    9 X(1) = u.u
    DO 18 1=1,ivOS
18 w(1,1)=x(1)
    CALL KUNGE(x)
    IF(INDEX-EQE1) GO TO 14
DO \(7 \quad 11=1\), MOI IH
DJ(Ji) \(=\mathrm{N}(1,11) * a(1,11)+(2,11) * N(2,11)+i(3,11) *(13,11)+1(4,11) *\)
    1*(4,I|)+:(5,I1)*w(5,I|)+w(6,I|)*w(6,I|)
7 CUHTINUL
(ALL SIMPS(KJ)
6 CONTINUE
RETURN
14 INUEX = 1
RETUKN
ENG
SUBHUUTINE PAKSCH(N, XB, DELTA, DIVI, COMPAR)
REAL KA,KE,KF,K1,K2,K3,K4,K5,K6
OIMENSION XE(IU), OELTA(IU), OIVI(1U), COMPAR(10)
DIMENSION X1(10), XE(10),XP(10)
COMFUN/SU日I/A(1U,10), CIS(10,2000)
```



```
COMFON/SUB \(3 / K A, K E, K F, K 1, K 2, K 3, K 5, K 6, T A, T O D 1, T E, T F, T M, T, D, K 4\)
COMilloiv/SUB7/IHDEX, OVER
OVEK \(=1 \cdot U E+2 U\)
ITEK \(=0\)
1WDEX = U
CALL FUNVAL (XB, IV,YE)
1 INDEX \(=\) i
CALL EXPLO (XE, DELTA, N, XE,YE)
IF(INDEX.EQ.1) GO TO 3
\(I T E K=I T E R+1\)
シKITE (6,2u1)
```



```
HKITE(b, 1 U2) ITER, YE
```



``` जKITE \((6,1 \cup 3)\)
103 FORMAT (/,' VALUE OF \(X\) ')
```

C

WRITE（6，1U1）（XE（K），K＝1，N）
1U1 FORMAT（b（2X，E15．8））
WR1TE（6，2」2）
202 FURMAT（／，SYSTEM MATRIX＇）
NRITE（b，1U4）（（A（I，J），J＝1，NOS），I＝1，NOS）
1U4 FURMAT（ $9(2 x, E 10 \cdot 3))$
IF（YE－YB） $2,3,3$
$2 Y B=Y E$
DO $111=1, N$
$x \perp(1)=x_{13}(1)$
$X B(1)=x_{E}(I)$
11 CONIINUE
CALL FATTER $(X 1, X B, N: X P)$
CALL EXPLU（XP，DELTA，N，XE，YE）
IF（IINDEX．EQ．I）GO TO 3
IF（YB．GT．YE）GO TO 10
GOTUI
$101 T E R=1 T E R+1$
AHITE（6，2こ1）
WFITL（6，1こ2）ITER，YE
WKlTE（o，lu3）
NKITE $(6,1 \cup 1)(X E(K), K=1, N)$
wKITE（b，2U2）
WKITC（0，104）（（A（I，J），J＝1，NOS），I＝1，NOS）
GO TU 2
$300 b i=1, N$
IF（UELTAII）•GT．COMPAR（1））GO TO b
5 continue
GO TO 7
6 UO $8 \quad I=1$ ，N
DELTA（I）＝DELTA（I）／DIVI（I）
－CONTINUE
GO TU 1
7 bKITE（6，IC1）（XB（K），K＝1，N）
RETURIG
EinD
SURROUT INE PATTER（XI，XF，N，X）
DIMENSIOF XI（10），XF（1U），X（10）
DU $1 \quad 1=1, N$
$1 \times(I)=X F(I) * 2 \cdot 0-X I(1)$
RETUKiv
E．ND
SUBRUUTIIE EXPLU（X，DELTA，＇J，XI，Y）
DIMEWSIOH $X(1 u), X I(1 u)$ ，DELTA（1u）
REAL KA，KE，Nr，K1，K2，K3，K4，K5，K6
CUMNOi／SUd1／A（1U，10），UIS（10，2000）
COMROW／SUH2／a（10，2UUC），HOIM，H，NOS
CUMHOM／SUG3／KA，KE，KF，K1，K2，K3，K5，K6，TA，TUO1，TE，TF，TM，T，D，K4
COMFOU／SUG7／INOEX，OVER
CALL FUNVAL $(X, N, Y)$

```
    IF(INUEX.EQ.I) GO TO 6
    DO lu J=1,N
1GX1(J)= X(J)
    DO 1 1=1,iv
    XI(I)= XI(I) + DFLTA(I)
    CALL FUNVAL(X1,N,Y1)
    IF(INUFX.EQ.1) GO TO 6
    IF(YI-Y) 2,3,3
    2 Y = Y1
    Gu TU I
3 X1(I) = X1(I) -2.0*DLLTA(I)
    CALL FUNVAL(X1,N,Y1)
    IF(IINDEX.EQ. 1) GO TO O
    IF(Y1-Y) 4,5,b
    4 Y = Y1
    GO TU 1
5x1(I) = XI(I) + OELTA(I)
1.CONTINUE
6 CONTIINUE
    RETUKN
    EivD
    SURROUTINE XUUT(X,XDUO,L)
    OIMENSION X(1U),XUOO(10)
    REAL KA,KE,KF,K1,K2,K3,K4,Kb,KG
    COMMON/SUG1/A(10,1i),OIS(10,20UU)
    COMMOL,/SUE2/w11U,2UCIU),NDIM,H,NOS
    CUMMOi,/SUH3/KA,KE,KF,K1,K2,N3,K5,K6,TA,TDO1,TE,TF,TM,T,D,K4
    COMMUN/SUB7/INDEX,OVLF
    DO 2 1=1,NOS
    X000(1)=E.U
```

    DO \(1 \quad J=1,1405\)
    XDOO(I) = XOOO(I)+A(1, J)*X(J)
    \(C 1=X 000(1)\).
    IF(CI •GT• OVER) GO TO 3
    1 CONTITUE
XUOO(I) = XOOU(1) + DIS(I,L)
$C 1=X D O O(I)$
IF(CI •GT• OVER) GO TO 3
2 CONTITUE
RETUKiv
3 IT.DEX $=1$
NFITE $(6,4)$ CI
4 FORMAT (//, GVERFLON PROCTATION AT XDOT•, E20.8)
RETUK゙N
END

```
SUBRUUTINE RUNGE(XI)
hEAL KA,KE,KF,K1,K2,K3,K4,K5,K6
CIMEMSION XI(10),FX(10),XA(1U)
DIMENSION XDI(1U),XD2(1U),XD3(10),XD4(10)
COMMUN/SUB1/A(10,10),DIS(10,200G)
COMMUN/SUB2/V.(IU, 2LUU),NDIM,H,NOS
CLMMUN/SUE3/KA,KE,KF,K1,K2,K3,K5,K6,TA,TDO1,TE,TF,TM,T,D,K4
COMMON/SUE7/INDEX,OVEK
DO lu J=1,NOS
10 W(J.1)= X1(J)
DO 5C I=2,NOIM
I11 = I
CALL XDOT(XI,FX,I11)
IF(INDEX.EG.I) GO TO 8
DO 1 J=1,NOS
x[1(J)=h*FX(J)
```

$1 \quad x A(J)=x I(J)+0.5 * x 01(J)$
CALL XDO1 (XA,FX,I11)
IF(IINEX.E日.1) GO TO b
OU $2 J=1$,NOS
XO2 (J) $=H^{*} F \times(J)$
$2 \times A(J)=x I(J)+0.5 * X D 2(J)$
CALL XDOT (XA,FX,111)
IF(JNUEX.EG.1) GO TO \&
OU $3 \mathrm{~J}=1$, HOS
$\times 03(J)=H * F \times(J)$
$3 X A(J)=X I(J)+X 03(J)$
CALL XDOT(XA,FX,111)
IF(INUEX.EQ. 1 ) GO TO \&
DU $5 \mathrm{~J}=1$, NOS
$5 \times \cup 4(J)=H * F \times(J)$
DU $4, J=1$, NOS
XI(J) $=\operatorname{XI}(J)+(1 \cdot L / 6 \cdot 0) *(\operatorname{XD1}(J)+2 \cdot 0 * \operatorname{XD} 2(J)+2 \cdot 0 * \operatorname{XD} 3(J)+X D 4(J))$
$4 \%(J, I)=x I(J)$
5C CUNTINUE
6 CONTIHNE
KETUKN
8 IMDEX = 1
PETURN
Ef 0
SURKOUTINE SIMPS(RJ)
COMHUH/SU:6/DJ(2000)
COMMUH/SUB2/w(1U, 2GOU),NDIM, H,NOS
COMMON/SUB7/INDEX, OVER

```
        NOIMI=NDIM-1
        NCIH2 = NUIM-2
        RJ=(UJ(1)+4.U*OJ(2)+4.O*DJ(NDIMI)+DJ(NDIM))*(H/3.O)
        DO 5u I=3,NDIM2
        RJ=RJ + 2.G*(H/3.E)*UJ(I)
        IF(RJ.GT.OVEK) GO TO 51
5G CONTINUE
    RETUKN
51 IHDEX = 1
    *RITE(6,52)
52 FORMATI//,' UVEFFLON PROCTATION AT SIMPS')
    RETURIV
    END
```


## APPENDIX III

The Computer Program to find Optimal Feedback Gains for Stabilizer Type II

I ：PLICIT REAL＊8（A－H，O－Z）
DIMENSION．A $(12,12), B(12,12), D(12,12), E(12,12)$
UIME IvSIOA L $1(12), \mathrm{M} 1(12), L 2(12), \mathrm{M} 2(12)$
UIMENSION E2（12，12），L3（6），M3（6）

DIMEMSION $\mathrm{E}(1,1)$ ，BT $(1,6), \mathrm{KBT}(1,6)$
$M=6$
$N=12$
$M K=1$
REAL（5，1）MAX，EFS
1 FORMAT（15，F15．7）
wRITE（0，1）MAX，EPS
KEAU（כ，1二）（ETA（1，J），J＝1，M）
10 FURMAT（6F8．4）
औKITE（6，búU）（ETA（1，J），J＝1，M）
5LUU FGYMAT（ET13．5，1／）
KEAD（5，11）（ETAT（I，1），I＝1，M）
11 F（RNAT（Fと．（i）
wKITE（6，5んUl）（ETAT（1，1），J＝1，M）
SUUl FURNAT（16F13．5，／），／／）
$K$ K．AU（5，12）（（K゙（I，J），J＝I，MR），I＝1，MR）
12 FORAhT（FE：3）

5uu2 FUPtim（F13．5，1／）
KEAD（S，27）（（uT（I，J），J＝1，M），I＝1，MR）
27 FuRPAAT（6F10．4）
nf： 1 TE（ $6,5 . J U 3$ ）（（RT $(1, J), J=1, M), I=1, M R)$

RFAC（כ， 27 ）（（A（I，J），J＝1，N），$I=1, N)$

$00<6 \leq 1 T E P=1,10$
ntelif（ 6,5 uL4）（（A（I，J），J＝1，N），$I=1, N)$

DO ZL $1=1$ ，N
いO 2u J＝1，N
くひ 日（1，J）＝A（1，J）
UU $<y \quad k=1$ ，$M A X$
$0032 \quad I=1, N$
DU $32 \mathrm{~J}=1$ ， 1
$32 E(1: \cup)=8(1, J)$
CAILL MIHV（E，N，Z1，L1，H1）
DO $3 \mathrm{I} \quad \mathrm{I}=1$ ，H
uU د心 $J=1,1$ ：
$3 u$ E．（1，J）$=$ U．5＊（E（I，J）＋E（I，J））
CALL GrAFKU（E，E，E2，N，H，H）
せK＝U．u
Bム＝u・ル
UU 3uU $I=1$ ，ik
$P A=B A+R(I, I)$

1）$E L=L \cdot 4 弓 冫(b B-12 \cdot 0)$
अ尺1TE（6，5ứ5）DEL，BA
SEL5 FURMAT（F13．5，／1
If（utL •LE．EPS）GO TO 31
29 CONTINUE

31 inflte( $6,6 \dot{( }) \quad((A(I, J), J=1, N), I=1, N)$
WRITE $(6,61)((B(1, J), J=1, N), I=1, N)$
WRITE (6,1) K, UEL
60 FOKMAT (1X,9HMATRIX A: , /, $12(6(3 X, E 13.6), 1,6(3 X, E 13.6), 1), 1 /)$

61 FURHAT(1ג, 4HAATRIX B: , /, $12(6(3 X, E 13.6), 1,6(3 X, E 13.6), /), 1 /)$
DO 7 U $I=1, N$
LU $1: J=1, N$
$70 \mathrm{k}(1, \mathrm{~J})=\mathrm{G} \cdot \mathrm{J}$
00 ou $1=1, M$
6G [(1,1)=-1.U
00 Y , $I=0, N$
90 E(1.1)=1.j
00 4i $I=1, N$
DO $4 \mathrm{~L} \quad J=1$, N
$4 U E(1, J)=E(1, J)+Q(I, J)$
ChLL HIMV(E,N,Z2,L2,!12)
DO bue $1=1$, H
[) 1 u $J=1$, i
1んU ט(I, J) $=2 \cdot$ U*E(I, J)

63 FORMMT(1X, 9HMATRIX $0:, 1,12(0(3 X, E 13.6), 1,6(3 X, E 13.6), 1), 1 /)$ DU Luj I $X=1, M$
Du 2ui $I Y=1, M$
$P(I X, I Y)=0(I X, I Y+6)$
$2 \mathbb{U} Q(1 \lambda, 1 Y)=-0(1 \lambda+0, I Y)$
CALL HIWV(Q,M, Z3, L3, M3)
veITE(b, /ul) ((P) I, J), J=1,M), I=I,M)
2U1FORMAT(1X,9HMATRIXF:,/,6(6(3x,E13.6), /))

2 U2 FORIAT (1x,9H(MATKIX $4:, 1,6(6(3 x, E 13.6), 1))$
CALL GMPRO(FIA,P,ETAP, 1, M, M)
(ALL GMPKU(ETAP,ETAT,E゙PE, $1, M, 1)$
CALL GMPKUIETA, G, ETAU, $1, M, M)$
CALL GMPKD(ETAQ,ETAJ,EQE, 1, M, 1)
ANIIL (b, 2u.3) EPE, EWE

DO 25i $I=1,6$
$25 \mathrm{~J} A(1,1)=A(1,1)+0 \cdot 05$
DU $200 I=1,6$
$J=I+o$
260 A(J,J)=A(J,J)-0.05
265 CUNTIINUE
STOP
END

SUBíuUTIHE GMPRD（A，F，R，N，M，L）
IMPLICIT KEAL＊ 8 （A－M，O－Z）
UIMELSION A（1），E（1），K（1）
c
$1 n=1$ ．
$1 k=-11$
OU 1 u $k=1$ ，L
$I k=I k+i$
DO 1 し $J=1, N$
$1 k=1 k+1$
$J I=J-11$
$1 b=1 k$
$K(I K)=0$
いし Lu $I=1$ ，M
$J 1=J I+N$
$1 B=1 t i+1$
$10 R(I K)=F(I K)+A(J I) * B(I B)$
RETUKiN
EIND

Suproutine minv（a，N，$\dot{\text { Sig }}$ L，M）
DIMENSION A（1），L（1），M（1）

If a double frecision version of this routine is desired，the C In coluan 1 shuUld be remuved from the dourle precision STATEMENT ．HICH fullows．

DCUNLE PRECISIOA A，O，BIGA，HOLD

$$
\begin{aligned}
& i=1 \cdot u \\
& N K=-i H
\end{aligned}
$$

    DO 80 K=1,N
    NK=NK+N
    L(K)=K
    N(K)=K
    Kに=ivk+k
    BI IGA=A(KK)
    DO 2u J=K,N
    IZ=N*(J-1)
    UO <u I=K,N
    I j=IL + I
    IC IF(DHOS(BIGA)-DABS(A(IJ))) 15,20,20
    15 EIGA=A(IJ)
        L(K)=I
        M(K)=J
    20 conTINUE
    C
c
C.
35 I=M(K)
IF(I-K) 45,45,38
3% JF=1,*(1-1)
Du Yu J=1,N
JK=ivK +J
JI=JP+J
HOLS=-4(JN)
A(Jn)=A(JI)
4J A(JI)=HOLO
C
C DIvIDE COLUMil by minus pivot ivalue of pivot element is
4h IF(oIGA) 48,46,48
46 0 = C.u
HETUNiN
48 00 55 1=1,N
IF(I-K) 5u,5s,50
50 IN=NK+I
A(IN)=A(IK)/(-BIGA)
5b CUNIINUE
c
c
NEDUCE MATKIX
00 65 1=1,N
IK=1JN + I

```
HULD＝A（IK）
\(1 J=1-i+\)
UU bら J＝1，N
\(I J=I J+N\)
IF（I－N）6u，05，60
06
IF（J－K）02，65，62
62
\(A(I J)=H O L D * A(K J)+A(I J)\)
O5 CUNTINUE
\(K J=K-i d\)
DO \(75 \mathrm{~J}=1\) ，is
KJ＝k．j＋n
1F（J－K）7Ú，75，70
70 A（KJ）\(=A(K J) / E I G A\)
75 Cuntivue
```


## PRODUCT OF PIVOTS

```
\(0=0 * E 1 G A\)
K゙LPLACE PIVOT BY KECIPROCAL
\(A(K K)=1 . \mathcal{L} / E I G A\)
hu contimue
FINAL RON AND COLUNN INTERCHANGE
．\(k=?\)
1 iu \(k=(k-1)\)
IF（n）15u，156，105
\(1051=L(k)\)
IF（1－K）120，120，158
```



```
\(J F=N *(I-1)\)
OU \(110 \quad J=1, N\)
\(J K=J G+J\)
HOLD＝A（JK）
\(J I=J K+J\)
\(A(J K)=-A(J I)\)
11E A（JI）＝HOLD
12L J＝M（K）
1F（J－K）LLG，1 0 0，125
\(125 \mathrm{KI}=K-N\)
bu \(130 \quad 1=1\) ，\(N\)
\(K I=K 1+r\) ！
HしLi \(=A(K 1)\)
\(J I=K I-K+J\)
\(A(K \mid)=-A(J)\)
13JA（J1）＝HOLD
GO Tu lú
150 K゙ヒTUFは
ER．D
```

APPENDIX IV
The Computer Program to Design Deterministic Observer (Stabilizer Type III)

REAL LT(4,4)
DIMENSION A $(4,4)$, GAMA $(4,4), A S(4,4)$, BETA $(4,4), F 12(4,4), F 11(4,4)$
UIMENSION F $21(4,4), F 22(4,41, W(4,4)$, WI $(4,4), A I(4,4), A L P H A(4,4)$
DIMENSION DUMFI $(4,4)$, DUMM $2(4,4)$, DUMM $3(4,4)$, DUMM $4(4,4)$, DUMMS $(4,4)$,

* DUMAB $(4,4), G A B A 1(4,4), L L(4), M M(4)$

DIMEASION C(IU)
COMMGN/INUU/KIN,KOUT
COMFON/NAINI/HDIN,DUMI 4,4 )
COMMGN/MAIN2/UUM2(4,4)
Cuminulv/MAIN3/OUM3(4,4)
HOIM=4
$K J N=5$
$K O U T=6$
$M=2$
$N=4$
CALL MATIU(1, M, BETA,4)
CALL MATIO(N,I,GAMA,4)
CALL MATIU(M, H,F11:4)
CALL HATIU(M,N,F12,4)
CALL MATIU(N,N,F21,4)
CALL MATIO(N, IV,F22,4)
CALL CHREGA(F<2,N,C)
$N 11=N+1$
$00831=1, N 11$
WFITE(b,31) C(I)
31 FGRMAT(1X,E2G.8)
83 CCNTINUE
CALL MMUL (BETA,F12,1,M,N,DUMM1)
CALL HMUL (DUNM1,F22,1,N, N, DUMM2)
CALL MMUL (UUMin $2, F 22,1, N, N, O U M M 3$ )
(ALL MMUL (OUMM3,F22,1,N,N,DUMM4)
いO $10 J=1$, N
$N(1, J)=$ OUMMI $11, J)$
w $(2, J)=$ UUMM2 $(1, J)$
$n(3, J)=$ DUMM $3(1, J)$
$\because(4, J)=$ UUMM4 $(1, J)$
10 CONTINUE
jO \& $J=1, N$
DO 8L $I=1, N$
$A(1, J)=i \cdot 0$
$A(1,1)=1 \cdot$ is
80 CONTIINE
$A(2,1)=C(4)$
$A(3,1)=C(3)$
$A(3,2)=C(4)$
$A(4,1)=C(2)$
$A(4,2)=C(3)$
$A(4,3)=C(4)$
ARITE (6, 81$)$
81 FORMAT(//, MATKIXA')
CALL MATIO(N,N,A,3)
AS(1) $=C(4)$
AS $(2)=C(3)$
$\operatorname{AS}(3)=C(2)$

AS(4) =C(1)
CALL GMINV(N,N,W,WI,MR,I)
CALL GMINVIN,N,A,AI,MR,1)
WKITE (6, 27)'
27 FURMAT(//,' MATRIX $\operatorname{HI}$ ')

```
            CALL HATIO(N,N,WI,3)
            WKITE(b,28)
    28 FGRMAT(//,' MATRIX AI')
            CALL MATIO(N,N,AI,3)
            DO o l=1,N
            6 GAMA 1(I,1)=GAMA(1,1)-AS(1,1)
            CALL MMUL(NI,AI,N,N,N,DUMN1)
            CALL MMUL(UUMM1,GANAI,N,N,1,ALPHA)
            CALL MMUL(ALFHA,BETA,M,l,M,L)
            #k1TE(6,1UU)
1UU FORMATI//,* MATRIX L'I
            CALL MATIO(N,H,L,3)
            CALL HMUL(L,FLI,IV,M,M,DUMM1)
            DU 2u I=1,N
            00 2u J=1,11
    2U DU|!&2(I,J)=F21(I,J)-OUMM1(I,J)
            WRITE(6,1U1)
lul FORMAT(//,' MATKIX F21-LF11')
            (ALL MATIO(N,H,DUHM2,3)
            CALL HMUL(L,F12,N,M,N,DUMM3)
            00 3L I=1,N
            DO 3u J=1,N
    30 UUMBi4(I,J)=F22(1,J)-DUMM3(I,J)
            \RITE(6,1u2)
1U2 FORMAT(//,' MATRIX F22-LFl2')
            CALL MATIO(N,N,DUMM4,3)
    55 CONTINUE
        EMO
            SURROUTINE CHKEQA(A,N,C)
            DIMENSION J(5),C(5),E(4,4),A(4,4),D(300)
            NN=N+1
            DO 20 I=1,NN
    20C(I)=\Sigma.0
            C(NN)=1.U
            OO 14 N=1,N
            K=0
            L=I
            J(1)=1
            GO TO 2
1 J LL)=J(L)+1
    2 IF (L-M) 3,5,5u
    3 HH= H-1
        DO 4 I=L,MM
        I I= I + l
```

```
    4 J(II)= J(1)+1
    5 DU lu I=1,N
        DO 1u KK=1,M
        NK=J(I)
        NC=J(KK)
    1U B(I,KK)=A(NR,INC)
        K=K+1
        D(K)=DET(E,M)
        OO O I=1,M
        L=M-I +I
        IF(J(L)-(N-M+L)) 1,6,50
    6 CONTIHNE
        MI=N-M+1
        0O 14 I=1,K
    14C(M1)=C(M1)+D(I)*(-1.0)**M
        KETUKN
    50 wR\TE(b,2uご)
2Gこう FURMAT(1Hj,5\lambda,'eRRUR IN CHREQA*)
        RETURN
        EHD
    FUNCTION DET(A:KCT
    DIMENSION A (4,4),R(4,4)
    IREV=U
    DO 1 I=1,KC
    DO 1 J=1,KC
    1B(I,J)=A(I,J)
    DO 2u I =1,KC
    K=I
    9 IF(b(K,1)) IG,11,10
11 K=K+1
    IF(K-KC)F,9,51
10 IF(I-K) 12,14,51
12 DO 13 M=1,KC
    TEMP=6(I,M)
    B(I,M)=R(K,M)
    13B(K,M)=TEMP
    IFEV=IRFV +I
    14 11=I +1
    IF(II.GTEKC) GO TO 2i)
    00 17 M=1I,KC
18 IF(H(H,1)) 14,17,19
19 TEMP=B(M,I)/FS(I,I)
    OO 16 N=1,KC
    16 B (M,N)=B(N,N)-B(I,N)*TEMP
    1 7 \text { CONTlWUE}
    20 CONTINUE
    UET=1.U
    OO < I=1,KC
    2 DET=0ET*E(I,I)
    DET=(-1•()**1KEV*OET
    RETUKN
    51 UET=U•G
    KETUKiv
    END
```


## APPENDIX V

The Computer Program to Design Kalman Gain Matrix $\mathrm{K}_{\mathrm{e}}$ (Stabilizer Type IV)

IMPLICIT KEAL*8 (A-H,O-Z)
DIMEINSION A $(12,12), 0(12,12), D(12,12), E(12,12)$
DIMENSION L1(12), M1 (12), L2(12), M2(12)
DIMEHSION B2(12,12),L3(6),M3(6)
DIMEIVSION $P(6,6),(10,6)$, ETA(1,6), ETAT(6, 1$)$, ETAP(1,6), ETAQ(1,6)
DIMEivSION R(1,1), BT $(1,6)$, RBT(1,b)
$M=6$
$N=12$
$M K=1$
READ(5,1) MAX,EPS
1 FURHAT (15,F15.7)
WHITE (b, 1 ) MAX,EPS
REAU(b, 1 U) (ETA(1,J),J=1,M)
10 FORMAT(6FO.4)
WKITE(b, SUUJ) (ETA(L,J),J=1,M)
5ULO FORMAT (6F13.5,//)
REA1) (5,11) (ETAT(I,1), $1=1, M)$
11 FURMAT(FB.0)
VKITE( 6, buG1) (ETAT(1,1),I=1,M)
SuUl FORNAT( (6F13.b,/),//)
REAU(b, 12) ( (K(I,J), J=1,MR), I=1,MR)
12 FURMAT(F8.3)
wRITE(6,buu2) ( (R(I,J),J=1,MR), $I=1, M R)$
GUU 2 FURMAT (F13.5,1/)
KEAU(b,2/) ((甘T(1,J),J=1,M), $1=1, M R)$
27 FURMAT (6F10.4)
WKITE(6, SJŪ3) ( (BTII,J),J=1,M),I=1,MR)
5iU3 FORMAT(b( $3 x, E 13 \cdot 6), 1 /)$
KEAD (5, 27) ( (A (I, J), J=1,N), $1=1, N)$
WRITE ( $6,5 \cup \hat{U} 4)((A(I, J), J=1, N), I=1, N)$
5uこ 4 FORMAT (12(6(3x,E13.6),/,6(3X,E13.6),/),//)
DO $2 \dot{\circ} \quad 1=1, N$
DO $2 \mathrm{E} \quad J=1, N$
$20 B(I, J)=A(I, J)$
DU $29 \mathrm{~K}=1$, MAX
DU $32 \quad 1=1, N$
UO $32 \quad J=1$, N
$32 E(I ; J)=B(I, J)$
CALL MINV(E,N,ZI,LI,M1)
UO 3 u $1=1$, $N$
00 3u $J=1, N$
30 b(I, J) $=0.5 *(B(I, J)+E(I, J))$
CALL GMPKD(B, $\triangle, B 2, N, N, N)$
$H H_{3}=U \cdot U$
$B \dot{A}=\dot{u} \cdot u$
DU 3ue $I=1$, v
$R A=B A+R(1,1)$
$3 C 0 \quad H B=5 \dot{B}+B 2(1,1)$
$D E L=0 A B S(B B-12.0)$
WRITE (6,5u05) DEL, BA
50L5 FGRMAT(F13.5,/)
IF (UEL •LE. EPS) GO TO 31
29 CONTINUE

```
31 WRITE(6,6:J) ((A(I,J),J=1,N),I=1,N)
    WRITE(6,61) ((B(I,J),J=1,N),I=1,N)
    WHITE (6,1) K,DEL
60 FORMAT(1X,9HMATRIX A:,/,12(6(3X,E13.6),/,6(3X,E13.6),/),//)
61 FORMMT(1X,9HMATRIX B:,/,12(6(3X,E13.6),/,6(3X,E13,6),/),//)
    DO 7u I=1,N
```

    DO 7ن \(J=1, N\)
    70 E(I,J)=ら・J
    DU \(8 \mathrm{G} \quad 1=1, \mathrm{M}\)
    8U E(1, I) \(=-1.0\)
    DO Y Y \(1=3, N\)
    $90 \mathrm{E}(\mathrm{I}, 1)=1 \cdot 0$
DO $4 \cup \quad 1=1, N$
DO $4, ~ J=1, N$
4u E (I, J) =E(I, J) +B(I, J)
CALL NINV(E,N,Z2,L2,M2)
DO luJ $1=1$, H
DO luu $J=1, N$
IU0 D (I, J) $=2 \cdot \omega * E(I, J)$
WRITE(b,63) ( (D(I,J), J=1,N), I=1,N)
63 FORMAT(JX, 9 HMATRIX D:,/, $12(6(3 X, E 13.6), /, 6(3 X, E 13.6), / 1, / /)$
OO 2UJ $1 X=1, M$
00 2id $I Y=1, M$
$P(I X, I Y)=0(I X, I Y+6)$
$2 C 0 \dot{U}(I \lambda, I Y)=-D(I X+6, I Y)$
CALL MINV(Q,N,Z3,L3,M3)
wR1TE( 6,2 U1) ( (P(1,J), J=1,M), $1=1, M)$
201 FURHAT(1X,9HMATRIX P:, /,6(6(3X,E13.6),11)

2 L 2 FOPRAT (IX,9HMATFIX $Q:, 1,6(6(3 X, E 13.0), 1)$
CALL GMPFU(ETA,P,ETAP,I,M,M)
CALL GMPKD (ETAP, ETAT,EPE, 1, M,1)
CALL GMPKU(FTA, U,ETAQ, $1, M, M)$
CALL GMPKD(ETAQ,ETAT,EQE, 1, M, I)
WKITE (6,2シ3) EPE,EQE
2 U3 FURMAT(1X, 'EPE = ', E15. B, 1 DX, 'EQE=', E15.8)
STOP
END

SUBROUTINE GMPRD(A,B,R,N,M,L)
IMPLICIT KEAL* 8 (A-H,O-Z)
DIMENSION A(1), H(1),K(1)
C
$1 k=0$
IK=-n
DO 1心 $K=1$, L
$I K=I K+N$
DO LG J=1,N
$1 k=1 k+1$
$J I=J-N$
$10=1 K$
R(IR) $=0$
DO 1 i $I=1, M$
$J I=J I+N$
It $=10+1$
$10 R(1 K)=R(1 F)+A(J 1) * R(1 B)$
RETUEN
EAD
SUBKO:TINE MIINV(A,N,D,L,M)
OIMENSION A(1),L(1),M(1)

IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIKED, THE C IN COLUMN I SHOULD BE KEMOVED FROM THE DOUBLE PRECISION STATEMENT WHICH FOLLONS.
dOUbLE PFECISION A, D, EIGA,HOLD
the c must also abe rehoved from double precision statements appearing in other routilves used in conjunction with this KOUTINE.

## SEARCH FOK lakgest element

```
D=1.0
NK=-N
DO 8u K=1,N
NG=NGK+N
L(K)=K
M(K)=K
```

$K K=N K+K$
BIGA=A(KK)
DO 2U J=K,N
$12=N \cdot(J-1)$
$002 L \quad 1=k, N$
$1 J=1 \angle+1$
10 IF (UAGS(BIGA)-DABS(A(IJ))) 15,20,20
15 bIGA=A(1J)
$L(K)=1$
$H(K)=J$
20 cuntinue
$351=M(K)$
IF (I-K) $45,45,38$
$38 \quad J P=r_{1} *(I-1)$
DU 4L $J=1, N$
$J K=N K+J$
$J I=J F+J$
HOLU=-A(JK)
$A(J K)=A(J I)$
$4 C A(J 1)=H O L D$
UIVIDE COLUMN GY MINUS PIVOT IVALUE OF PIVOT ELEMENT IS CONTAINED IN BIGA)

45 IF(BIGA) 48,40,48
$46 \quad 0=0 \cdot u$
ReTUKN
48 DU b勺 $I=1$, N
IF(I-k) 5u,55,50
Su $1 K=N K+I$
$A(I K)=A(I r) /(-B I G A)$
55 CUNTINJE
$0065 \quad 1=1, N$
$1 K=N K+1$
HULU=A(Ik)
I $J=I-i$
UO os J=1,N
$I J=1 J+N$

```
: IF(I-K) 6(, 65,60
    60 IF(J-K) 62,65,62
    62kJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
    6 CONTINUE
C
C DIVIDE RON BY PIVOT
    KJ=K-N
    DU 75 J=1,N
    KJ=KJ+N
        IF(J-K) 70,75,70
        7U A(KJ)=A(KJ)/BIGA
        75 CUNTINUE
            PKODUCT OF PIVOTS
        D=D*BIGA
    C
    C
    C
        A(KK)=1•U/EIGA
        BU CON1INUE
    C
    C
    C
            FINAL ROOW ANU COLUNIN INTERCHANGE
        K=N
        1wo k=(k-1)
        1F(K) 15u,15C,105
    1u5 I=L(K)
        IF(1-k) 120,120,108
    1U8 JG=N*(K-1)
        JF=iN*(1-1)
```



```
        JK=JG+J
        HOLD=A(JK)
        JI= JR+J
        A(JK)=-A(JI)
    110 A(JI)=HOLD
    120 J=M(k)
        &F(J-K) IUE,100,125
    125 KI=K-N
        DO 13U I=1,N
        Kl=Kl+N
        HOLD=A(KI)
        JI=KI-K+J
        A(KI)=-A(JI)
    130 A(JI)=HOLO
        GO TO IOC
    150 RETUKN
        EivD
```


## APPENDIX VI

Computer Programs to Simulate the System with Stabilizers Type I,II,III,IV respectively (Linear Model, Noisefree Case)

PARAMETER $G P=7,53, T 1=, 425, T 2=, 229, T 3=4,145, T 4=2175, T R=, ~ C 8366$ DYNAMIC

$\mathrm{X}=0 ., 25-\mathrm{TI}$



$\mathrm{D} \times 2=1.25 \div \times 1 \div 1.25 \times \times 2$
$\times 2=1 N T G R L(\therefore 0 \times 2)$
D $\times 3=3,375 \div 1-375 \times 2-\times 3$
$\times 3=1$ VTURL $(\because, 0 \times 3)$
$\mathrm{Q} \times 4=10-6316 \times 2-3-3 \div \times 4-125.51 * \times 5-1859 \div \times 5$
$X_{4}=1 N T U R L(, L X 4)$
$\mathrm{D} \times 5=-2=255 \times 4-25 \div \times 5-j .1334 \times 6 \times \mathrm{m}$ + $1 \div \mathrm{PM}$
$\times 5=I M T \hat{O} L(5,0 \times 5)$
OXG $=377$ * $\because=$
X6 = INTGRL ( $2,0 \times 6)$
$\mathrm{D} \times 7=-255 * \times 4=25 * \times 5-.133 * \times 6=12 * \times 7+0.1 * \mathrm{PN}$
$X 7=1 M T G R L(\therefore * U 7)$
$0 \times 8=-4,85 \pm 4 \times 4,77 \times \times 5-2,54 \div \times 5-225 . \div \times 7-4,6 \div \times 8+0,1 \div(\mathrm{T} 3 / \mathrm{T} 4) \div \mathrm{PN}$


$+\because=1 \times((C P: T 1-T: 1) /(T 2=T 4)) \times P N$
$U=\times 9$
$E_{1}=\times 1 \div \times 1+\times 2 \div \times 2+\times 24 \times 3+\times 4 \div \times 4+\times 5 \div \times 5+\times 5 \div \times 6$

$E 2=U 4$
JU = INTGRL ( $\therefore:, ~ E 2)$
$J=J X+J U$

PRINT JX, JU, J
PRTPLOT
PRTPLOT
PRTPLQT
PRTPLOT
$\times 2$

PRTPLOT
$\times 3$
$\times 4$
$\times 5$
PRTPLOT
$\times 6$

DYVAMI

$$
0 \times 1=-2 \times 1+20902 \times 10
$$

$$
C 1=I T B R L(,
$$

$$
\times 1=L 14107,7+2,1)
$$

$$
0 \times 2=1,26 \times 1-1 \cdot x=x
$$

$$
\begin{aligned}
& x 2=1414(1 \\
& 0 \times 3=1
\end{aligned}
$$

$$
0 \times 3=375 \times 1 \times 2=2
$$

$$
\mathrm{E}_{2}=1=\mathrm{U}
$$

$$
J U=I N T Q R(\therefore, E 2)
$$

$$
j=j x+j u
$$


PRINT JX:JU, J
PRTPLOT Cl
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT X

## EAD

STOP

$$
\begin{aligned}
& \text { TIM = } 1 \text { ITYRL(10, TIT) } \\
& x=\therefore .2 \text { に-TI } \\
& P M=-4 \% P \cup L S(\therefore 25, x)
\end{aligned}
$$

## DYNAMIC



```
    \(X_{P M}=\because 25-T I\)
```




```
    C1=INTGRL (, UX1)
    XI=LIMIT \((-7,3,7,3,21)\)
    \(0 \times 2=1,25 \div \times 1 \cdot 1,25 \div \times 2\)
    \(\times 2=\operatorname{INT} G L(\because, 0 \times 2)\)
    DX3 \(=375 \div \times 1-375 \div \times 2-\times 3\)
    \(\times 3=I N T ; R L(E \cdot E \times 3)\)
    D \(\times 4=3.6313 * \times 2=-3 \times 3 * x 4-125.51 * x 5-1.1859 * x_{6}\)
    \(\times 4=\) INTGRL \((, E \times 4)\)
    DX5=-2.255*x4-.25 \(=x 5-j .1334 * x 5+0.1 * P M\)
```



```
    DXE \(=377\). \(\because \times 5\)
    X6 \(=1 \mathrm{VTGRL}, 2,0 \times 51\)
```




```
    \(6 \mathrm{H}+3.442=\mathrm{U}\)
```



```
    \(-7.7 .50 \% 4\)
    \(026=139 \div \times 1 \pm, 735 \div \times 2+58.77 * \times 3 H+58.77 * \times 4 H+377 . * \times 511-58.67 * し\)
```



```
    \(\left.\begin{array}{l}24=1 N T G R L \\ 25=1010\end{array}\right)\)
    \(26=1 N T O R L(;, 026)\)
    \(\times 3 \mathrm{H}=23+.00 .6 \div \times 1+.0: 12 * \times 2\)
    \(\times 4 \mathrm{H}=74-, \cdots 152 \div \times 1-\quad 2 \quad 4=\times 2\)
    \(\times 5 \mathrm{H}=25+33.335 \times 1+, 0 \times 71 \times \times 2\)
    \(\mathrm{X} 6 \mathrm{H}=76+.0-294 \times x 1+. \therefore 589 \times x 2\)
    \(U=-1.995 * \times 1-1.2 .2 \div \times 2+3.925 * \times 3 H-22.545 * \times 4 H+21.568 * \times 5 H-7.975 * \times 6 \mathrm{H}\)
    \(E 1=X 1 \div \times 1+\times 2=\times 2+\times 3 \div \times 3+X 4 \div \times 4+X 5 * \times 5+\times 5 \div \times 6\)
    \(J X=I N T \cup R L(, ~ こ, ~ E 1)\)
    \(E 2=U \div(1)\)
    \(J U=I N T G R L(,, E 2)\)
    \(J=J X+J U\),
    TIMER PPDEL=`. 1, OUTDEL \(=\). 1, FINTIN \(=2.0\)
    PRINT JX,JU,J
    PRTPLOT \(\times 2\)
    PRTPLOT \(\times 2 \mathrm{H}\)
    PRTPLOT \(\times 3\)
    PRTPLOT X 3 ト
    PRTPLOT \(\times 4\)
    PRTPLOT X4H
    PRTPLOT \(\times 5\)
    PRTPLOT \(\times 5 \mathrm{H}\)
    PRTPLOT \(\times 6\)
    PRTPLDT X6H
```

    PRTPLOT
    PRTPLDT
    END
STCP

## DYNAMIC




C1 $=\operatorname{INTGRL}(, 2 \times 1)$
XI=LIMI I $-7,3,7,3,-11$
$0 \times 2=1.25 * \times 1.1 .25 * \times 2$
$\times 2=I N T G R L(, \quad, j x 2)$
$0 \times 3=375 \times \times 1-375 \div \times 2-\times 3$
$\times 3=I M T G R L(\therefore, C \times 3)$
$0 x_{4}=6318 \times 2-3: 3 \div \times 4-125.91 * \times 5-1859 * x 6$
$X_{4}=I N T G R L(\therefore 3,0 \times 4)$
$0 \times 5=-0.255 \% \times 4-25 \div \times 5-.1334 * \times 6+0.1 * P M$
$\times 5=1 N T G R L(2,0 \times 5)$
$\mathrm{DXS}=377 \mathrm{~F} L$
$\times 6=1 N T G(1,0 \times 6)$

$\times 1 \mathrm{H}=\mathrm{I} \vee \mathrm{GR} \mathrm{L}(, \quad \mathrm{DX1H})$
D $22 \mathrm{H}=1,25 \times \times 1 \mathrm{H}-1,25 \cdot \times 2 \mathrm{H}+1.238$ * (X5-XラH)
$\times 2 \mathrm{H}=I N T O R(L, 0 \times 24)$




DXSH=377. $\because \times 5+.725 \times(\times 5-X=H)$
X $6 H=I N T G R L(G \cdot U X S 1)$

$\mathrm{E}_{1}=\mathrm{X} 1 \times 1+\times 2 \div \times 2+\times 3=1 \times 3+\times 4+\times 4+\times 5 \div \times 5+\times 6 \div \times 6$
$j x=I N T S P L(J, j, 1)$
$E 2=U \div U$
$\mathrm{JU}=\mathrm{INTGRL}(\mathrm{J}, \mathrm{a}, \mathrm{c} 2)$
$J=7 x+j u$
TIMER PRDEL = 2. 1 , CUTDEL $=$ - 1 , FINTIN $=5.0$
PRINT JX, JU,J
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLO
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT
PRTPLOT 61
$\times 1$
$\times 1 H$
$\times 2$
$\times 2 H$
$\times 3$
$\times 311$
$\times 4$
$\times 4 H$
$\times 5$
$\times 5 H$
$\times 6$
$\times 6 H$ END STOP

## APPENDIX VII

Computer Programs to Simulate the System with Stabilizers Type I,II,III,IV respectively (Nonlinear Model, Noise-free case)




## DYNAMIC

```
            \(T \mathrm{IM}=1\)
            \(T=I N T G R L\)
\(X=O, ~ J, T I M)\)
            \(X=-25-T I\)
```

            \(P M=-64 * P U L S E(: 2 z, x)\)
    
$0 \times 2=1,25 * \times 1 \cdots 1025 \times 2$
$\times 2=1$
$\times 2=I N T G L($
$0 \times 3=-10 \times 2)$
$0 \times 3=5375 \cdots \times 1-1375 \div \times 2-\times 3$
$\times 3=1 M T=12 L(x, \dot{C})$

$E F D=2,436555+X 2$
$D Y I=(1,1 T D O 1) \div(E F D-Y 1)$
Y $1=I H T$ GRL (YI , $\mathrm{C} Y \mathrm{Y})$
$Y 2=I N T O R L(Y 2), G Y Z)$
$E Q 1=Y 1 \cdot Y 2$
$V D=E O 1-X D I: I 0$
$V D=X G * I G$
$I D=(E Q \cdot E+C S(X C)) /(X 5+X Q)$
$I Q=E=S I N(X 6) /\left(X E+X_{i j}\right)$
$P E=E Q_{1} \underset{C}{ }$
$X_{4}=S O R T\left(V U=V D+V S * V_{n i}\right)$
$P A=-P E \cdot 0 * \times 5+P M+5.5391$
$P A 1=P \Delta / T M$
$\times 5=I N T C R L(X 53, P A L)$
D X $6=377.6 \times 5$
X $6=I$ YGGRL (X6?, UXE)
$\times 1 H=\times 1-$
$\times 2 H=x 2-0$
$\times 3 H=x=-$
$\times 4 H=x 4-1$.
$\times 5 H=x-1$.
$\times 5 \mathrm{H}=\times 5-0 . j$
$\times 6 H=\times 6 \cdot 1.73323$
 PRTPLOT $\times 1$
PRTPLOT $\times \frac{1}{2}$
PRTPLOT $\times 3$
PRTPLOT $\times 4$
PRTPLOT $\times 5$
$\times$
PRTPLOT $x \in$

PARAMETER TDOL=5.5,XE=1,O,E $=.99, T M=16, C=2.5, X C=1.6, X A=1.55, X D 1=C, 32$ DYiVAMIC

```
    \(T I^{M}=I N T G R L I, T I M^{M}\)
    \(X=3.25-T\)
```




```
    \(C 1=\operatorname{INTURL}(5,7,0 \times 1)\)
\(X 1=L I M I T(-7,3,7.3,1)\)
    \(\mathrm{D} \times 2=1,25 \times \times 1-1,25 \div x\)
    \(X 2=I A T G R L(j, J, U X 2)\)
    \(D \times 3=0,375 \div \times 1-6375 * \times 2-\times 3\)
```



```
    \(E Q=((X E+X G) /(X E+X C 1))\)
\(E F D=2.436565+X(1)\)
\(D Y 1=(1.1 T D O 1) *(E F D-Y 1)\)
    \(Y 1=I N T G R L(Y 1, L Y 1)\)
    \(\mathrm{OY} 2=(1 . / T D U 1) \div(((X)-X D 1) /(X E+X O)):(E Q-E \times \operatorname{COS}(X 6))-Y 2)\)
    \(Y 2=I N T O R L(Y 2 \because, 0 Y 2)\)
    \(E Q 1=Y .1-Y 2\)
    \(V Q=E Q 1-X O 1 \div I C\)
    \(V D=x O \operatorname{Li}\)
    \(10=(E Q-\Xi * \operatorname{COS}(X \dot{\theta})) /(X E+x Q)\)
    I \(Q=E=S I N(X S) /(A E+X G)\)
    \(\rho E=E O\) ※ 10
    \(X^{4}=\operatorname{SORT}\left(V O * V O+V V^{2} * V ;\right)\)
    \(P A=-P E-E x \times 5+P D+\because, 95391\)
    PAI \(=\mathrm{PA} / \mathrm{TM}\)
    X5 = INTSRL (X50,0A1)
```



```
    \(X 6=I N T G R L(\times 6, O X G)\)
    DZ \(3=066 \times 1-066 \times 2+10.996=\times 3 H+11.7967 \times 4 H-11.976 \neq 11\)
```



```
    \(6 \mathrm{H}+3-42\) 新
```



```
    \(-0,7356=\mathrm{U}\)
```



```
    \(Z 3=I N T G R L(\),
\(Z 4=I N T G R L\)
2,
    \(Z 5=I N T \subseteq 2 L(\because, \because, 025)\)
    \(26=I N T G R L(\cdots, \angle Z S)\)
    \(\times 3 H=23+0.66 \times 1+0.12 \pi \times 2\)
    \(\times 4 \mathrm{H}=24 \cdots 252 \times 10 ; 304 \times 2\)
```



```
    \(\times 6 \mathrm{H}=26+, 274 \times 1+, 2583+\times 2\)
    \(U=-1.995 \times 1-1,202 * 2+j .923 * \times 3 H-22.545 * \times 4 H+21.568 * \times 5 H-7.775 * \times 6 H\)
TIMER PMDEL \(=3.1\), GUTOEL \(=: 1\),FINTI \(4=5 \ldots\)
PRTPLOT CLI
\(\begin{array}{ll}\text { PRTPLOT } & \times 1 \\ \text { PRIPLOT } & \times 2\end{array}\)
PRTPLOT X 3 H
PRTPLOT \(X+H\)
PRTPLOT XVH
PRTPLOT X6
```

PRTPLOT
PRTPLOT
PRTPLDT
END
STUP
 DYNAMIC

$$
\begin{aligned}
& T I M=I N T R L(J,: T I N) \\
& X I=0.25-T I U S E 1
\end{aligned}
$$


$C 1=I T T I T$
$\times 1=1,3,7,2,-1)$
$0 \times 2=1 \cdot 25 \div \times 1-1.25 \times 2$
$\times 2=I N T G R L(0 \times 2)$
0×3=475*x1- $0375 * \times 2-\times 3$
$\times 3=\operatorname{INTGL}(, 0,03)$
$E Q=((X E+X G))(X=+X D 1)) * E 21-((X Q-X D 1) /(X E+X D 1)) \neq E \pi \operatorname{COS}(X 6)$

DY2=(1./TU: 1$) *(((X U-X 01) /(X E+X Q)) *(E Q-E * \operatorname{COS}(X S))-Y 2$.
$Y 2=$ IMTGRL (Y2, DYZ $)$
$E Q 1=Y 1-Y 2$
VO=EOL-XDI*ID
$V D=\bar{x} Q * 10$
$10=(E Q-E \times \operatorname{COS}(X E)) /(X E+X Q)$
$10=E=S I N(X B) /(K E+X G)$
$\times 4=$ SORT (VUSVD $+V Q=V 0)$
$P_{A}=-P E \cdots C \div X 5+P M+.9=391$
PAI=PA/TM
$\times 5=I A T G R L(\times 5$ 2, PA1)
$0 \times 6=377 \times$
$\times 6=1 N T G R L(\times 50,0 \times 6)$

$\times 1 H=I N T G L(\ldots, D \times 1 H)$
D $\times 2 \mathrm{H}=1,25 \times \times 1 \mathrm{H}-1,25 \times 2 \mathrm{H}+1.238 \div(\times 5-\times 5 \mathrm{H})$

$\times 3 \mathrm{H}=\mathrm{I} N \mathrm{GRL}(2, \cdots, \mathrm{C} \times 3 \mathrm{H})$
D $\times 4 \mathrm{H}=0.06318 \div 2 \mathrm{H}-3,303 \div \times 4 \mathrm{H}-125.91 \times \times 5 \mathrm{H}-0.1859 \div \times 6 \mathrm{H} \ldots$
-489.3 $3=(x 5-x=4)$


DX6F $=377 \quad=\times 5 \mathrm{H}+725 \times(\times 5-\times 5 \mathrm{H})$

TIMER PROEL=0.1, OUTUEL=. 1 , FINTIN=5.

|  | C |
| :---: | :---: |
| prtplot |  |
| PRTPLO |  |
| PRTPLOT |  |
| PRTPLOT | $\times 2$ |
| PRTPLOT |  |
| PRTPLOT |  |
| PRTPLOT |  |
|  |  |
|  |  |
|  |  |

## PRTPLOT $\times 6$ <br> PRTPLOT $X \in H$

END
STOP

# APPFNDIX VIIII <br> Compuetr Programs to Simulate the System with Type II,III, and IV Stabilizers Respectively. (Linear Model with Noise) 

```
    PARAMETER P1= . \because, 1%,P2=0, \because\because: 1;
    DYNAMIC
        TIM= IOOGRL(C.N,TIM)
    PM = -GU4%PULSE(O, -25,X)
    NSI=GAUSS(1, 盾, ,P1)
    Cl=INTGRL(J. ©,0X1)
```



```
    DX2 =1,25~X1-1,25:X2 +NS2
    X2=IMTSRL(%,0\times2)
    NS = = AUSSS(5,: ,P1)
    X3=INTSRL(%,ON3)
    NS4=GAUSS(7,O
```



```
    X5=INTGRL(% ', [X5)
    NSG=GAUSS(11, S,U,P1)
```



```
    NM1=CAUSS(1,:,P2)
```



```
    NM4=GAUSS(7,O,\ldots,P2)
    NMS=GAUSS(7,\therefore,P2)
    S1=X1+NN1
    S2=x2+NM2
    S3=x3+VM3
    S5 =X4+NN4
    S5=X5+NM5
    U=-1.975*S1-1.2, 2*S2+3. S25*S3-22.545*S4+21.568:55-7.975*S6
```



```
    JX=INTGRL(E.G,E1)
    E2=U*U
    JU=INTGRL(: , E2)
    J=JX+JU
    TIMER PRDEL=:.1,OUTUEL=0.1,FINTIN=5.3
    PRINT JX,JU,J
    PRTPLOT
        < +1
    PRTPLOT
    PRTPLOT
    PRTPLOT X5
    PRTPLOT X6
ENC
STCP
```

PAPAMETER D $1=0.0 C 0010, \mu 2=0.000010$
[jY JiMIC
TIM $=\therefore 0$
$T I=I T Y$ SRL $0.0, T I M)$
$X=0 . C 25-\mathrm{TI}$
$P M=-r .4 * P$ ULLS $S(n, r 25, x)$
$\because S i=i A \cdot 1 S S(1, r \cdot(1, P 1)$
$0 x:=-50.0 * x:-20000.0 * x 3-50000.0 * x 4+20000.0 \div U+v S 1$
CI=INTOXL ( 0 . $0,1 \times 1$ ).
X $1=L 1,1$ IT(-7. $3,7 \cdot 3,21)$
NS $2=51!5 S(3,0 \cdot \because, \mu 1)$
$0 \times 2=1.45 \times x_{1}-2 \times 2+152$

$\cdot \mathrm{dS} ;=115 \mathrm{~S}(2,0 . j, \vec{p} 1)$
$0 \times 3=0.1275 \times 1-0.0375+x 2-x 3+\sqrt{2} 3$
$\times 3=14 T=1(7, n,(\times 3)$




$0 \times 3=-0.255 * \times 4-3 \cdot 25 * x 5-0.13 .4 * x 6+0.1 * P M+N S 5$
$x=1$ NTUL (0.2, $4 \times 5$ )
$15=\operatorname{AnSS}(1,0.0, p)$
$0 \times \cdot=77.0 * x 5+105$


$51=x=+161$



が +20.4 +2

-0.7050*1

$L^{2}=1 \because T 0 L(0 . n, \therefore z 3)$
$24=1 \cdot 1 \mathrm{O} \mathrm{L}(0 . \mathrm{C}, \mathrm{C} 24)$



$$
x_{4}=24-10122,930=52
$$





$E 2=U * 1 \mathrm{~J}, 2 \mathrm{~L}(0,0, \div 2)$
$\mathrm{JU}=\mathrm{I} \because T$
$J=J x+J, J L(0,0, \div 2)$
T1:
MEI:JT Jx, J!!, J
PRTPLOT X
PRTPLUT $x 3$
PRTPLOT $\times 314$
PRTPLOT $\times 4$
PRTPLGT $\times 4 \mathrm{H}$

PRTPLOT $\times 5 \mathrm{H}$
PRTPLOT $\times 6$
PRTPLOT XSH

## Pُ.



```
            TI
                \(=\)
                        \(=\begin{array}{ll}\text { I } \\ \text { INR }\end{array}\)
    \(T I=I M T G R L(C \cdot(1, T I F)\)
    \(X=C . C 25-T 1\)
```



```
    \(V S:=01 L S S(1,0, n, p i)\)
```



```
    \(X 1=L I N T(-7,2,7.2,(1)\)
    \(\because \leq=3115(2,0,(p i)\)
```



```
    \(\because S==115=(=0,0,01)\)
```




```
    \(\therefore S t=\therefore+U S S(7, C, C, S 1)\)
```



```
    MS, =
```




```
    \(x 5=1 \cdot 1 T 2(10 \cdot 0, x \leq)\)
```






```
    \(\times 2 H=1 \times i=1(0.0,0 \times 2,1)\)
    ) \(2 H=C, ~\)
\(\times 3: 1\)
```






```
    UXit=3?7-
```






```
    \(32=10 \mathrm{Cl}\)
\(\mathrm{ju}=1-1\)
```




$\begin{aligned} & \text { PRTPLDT X, } \\ & \because \because G \\ & \text { 二TEP }\end{aligned}$

## APPENDIX IX

Computer Programs to Simulate the System with Stabilizers Type II,III,IV respectively ( Nonlinear Model with Noise)
i.

INCON $\times 5=, 3, \times 6=1,78223, Y 1=2.436565, Y 2:=1.323 .01$
 DYNAMIC
$T I M=1$.
$T I=I N T G R L(\therefore . O, T I N)$
$X=: .22-T I$
PM $M=-4 \neq P U L S E(, 2 \bar{*}, X)$
$N S I=G A U S(1)$

C1=INTGRL (, $\because, 10 \times 1)$
XI =LINIT $(-7,3,7,3,: 1)$
NS $2=G A U S S(3,0, ? 1)$
$D X 2=1+25 \times X 1-1=25=2+N S 2$


$\times 3=1 N T C R L(, \dot{S})$

$\mathrm{EFO}=2.4365 \dot{5} 5+\times 2$
$\mathrm{OY} 1=(\mathrm{I}, / \mathrm{T} \mathrm{CO} 1) \div(E \mathrm{FO}-Y 1)$
$Y 1=I N G Q L(Y 1, i \bar{Y} 1)$
$D Y 2=(1: / T U 1)=((1 X)-X D 1) /(X E+X Q)) *(E Q-E * C C S(X 6)) \rightarrow Y 2)$
$Y 2=1 \wedge T$ SRL (YZ:, UYZ)
$E Q 1=Y 1-Y 2$
$V Q=E Q 1-X D 1 \div I D$
$V D=X \cap * I$
$I D=(F Q-E \operatorname{COS}(X E)) /(X E+X Q)$
$I 0=E \therefore S I N(X O) /(X E+X \approx)$
$P C=E O * I$ i
$N S=G A U S S 17, \quad P 1$
X4 = SGQT(VOrVE+VGUV) +NS4
$P A=-P E-1) \times 5+P M+\therefore 7591$
NS $5=G A U S S(9, \%, P 1)$
PA1 = PA/TM
X5 = 1 NTURL (X5: PA1)
NS $6=G 4 U S(11,: P 1)$
$D \times 6=377,4 \times 5+56$
X6 = INTURL (X0 $\because$, DXE)
NM1 = GAUSS (1,
$N M 2=G A U S S(5$,
$N M 3=G \cap U S$
$N M$

$N M O=G A U S S$
$S 1=X 1+N M 1$
$S 1$
$S 2$
$S 3$
$S 4$
$S 5$
$S 6$
$\times 1 H$
$\times 2 H$
$\times 3 H$
$\times 4$
$\times 51$
$\times 6$
$\mathrm{U}=-1.99 \underset{5}{2} \times 1 \mathrm{H}-1,2: 2: \times 2 \mathrm{H}+.925 \pm \times 3 \mathrm{H}-22.545 \times \times 4 \mathrm{H}+21.568 * \times 5 \mathrm{H}-7.9754 \times 6 \mathrm{H}$

PRTPLOT X1
PRTPLOT $\times 2$
PRTPLOT
PRTPLOT
PRTPLOT $x$
PRTPLOT $\times 6$
END
STOP




```
EY:VA+11G
            TI F=1.0
    TI=INTS'RL(N,n,TIM)
    P:N=-0.4*TLLE=(n.025,x)
    iNS:=RAUSS(i,O.0,P1)
```



```
    Cl= I!TưL(O.C,rx1)
    XI=LI:IT(-7.3,7.3,C1)
    VS={3115S(2,0.0,01)
    DxE=1.5}3*x+1.こ5*42+v:
    x2=I vTG2L(7. त, 叹2)
    NS3=54115S(5,0.1), D1)
    Ox.3=0.0375*x1-3.0375*x2-x3 +y53
    X3=1NTGSL(0.0,0\times3)
```



```
    EFO=2.430555+X2
```



```
    Y1=INTUQL(Y1O,OY1)
```



```
    Y2=1'!TBRL(Y20, 二Y2)
    EO:=Y1-Y2
    VO=EO1-XO1*ID
    VO= 积:IO
```



```
    IO=E*SI j(xS)/(x三+人い)
    PE= 二6:%IG
    \becauseS4=自AUSS(7,O,O,P1)
    X4=S!2T(vu*VL+V!%v!:) +VS4
```



```
    VS5=t,NuSS(9,O.?, +1)
    PAI=PA/TN+NS;
    X5=INT心<L(X5O,FAl)
    MSj=r,14S5(17 ,0.7,2!)
    UX:=377,0*x5 +NS6
    ;NMi=;AUSS(3,0.0,p1)
    S1=X1+%N1
    NME=GAUSS(5,?.n,P1)
    S2=x2+14+2
```




```
    6H+30.442*U
```



```
    -0.7056%J
```



```
    Z3=I!TG`L(0,0,i)23)
    Z4=1NTGRL(O.O.VZL)
    Z5=INTSPL(?.0,1.2F)
```



```
    x 3u=73+.0ソC5*S-+.CN12*S?
    x414=24-.00152%51-.0n30%%52
    x5H=25+.000035*51+.000071%52
    x与H=25+.00204*51+.0058**S2,
    U=-1.735*Si-1.202*5? +0.725*x3H-2?.545*人411+21.5う.*入511-7.?7F*x5H
```

TIACR PFDEL＝0．1，OUTOEL＝0．1，FIJTIN＝5．0
PKI能 $\times 1$
PRTPLDT $\times 2$
PRTPLOT $\times$
PRIPLOT $\times 4$
PRTPLGT X4H
PRTHLOT $\times 5$
PETPLOT $\times$ SH
PRTPLOT $\times 6$
PRTPLOT $\times 64$


```
    PAOMETOP P1=0,OCON1C,P2=0. COOCiO
```



```
EY`ANIC
    TI隹=1.0
    Tl=IT!TGRL(0.0, (I'A)
    PN=-\hat{L}<~PULSE(r.02末,x)
    1151=GAL.SS(I,O.O,P1)
    OX:=-50.0*x1-20000.0*x3-20070.0*x4+20000.0*U + \51+20000.0
    X1=1)
    XI=LIM1T(-7, 3,7,2,61)
    NS?=(;1!SS (2,0.E,P1)
    D X2=1. E5* * 1-1. =5* \2 +NS2
    X2=1,NT心2L (0.0,: < ( ) 
    NSS=r, 11:S5(3,0.0,P1)
```





```
    CY1=(1, (TO_1)*(EFO-Y1)
    Y1=INTGKL(YlO,LYYI)
    CYZ=(1./TDO1)*(((XE-XDZ)/(X\overline{C}+X0))*(ER-E*COS(XS))-Yこ)
    Y2=INT'URL(YこO,[Y2)
    O1=Y1-Y2
```



```
    VD=X0:IS
    ID=(こと-5*COS (Xf) )/(XE+XO)
```



```
    PE=Eח*IG
    NS4=-10SS(7,0.0,P1)
```




```
    VSE=S1UGS(O,C, C,P1)
    PA I=OA/T:4 + \becauseS5
    XJ=IMT,arl(x50,iNAl)
    VS:=G4USS(11,0,C,P1)
    DYt=377.C*x5 +iS6
    XS=INT,RL(XNO, r.x会)
    #N",GAB,SS(9,O,n,P?)
    S5=X = + H15
```







```
    <3H=1 JTGRL(O.n,DX?+1)
```



```
    X+F=INTGDL(O.C,[X&H)
    DX5F=-0.255* X4,-0.25*X5H-0.1334*X6H+351.58*(Sラ-XEH)
    X5H=INTSQL(0.0, DX5H)
    Ox5F=377.0%x5F+0.7こう%(5%-XडH)
```




```
TINFR PRE=
PRTPLUT XlH
```

| PRTPLOT | $\times 2 \mathrm{H}$ |
| :---: | :---: |
| PRTPLOT | $\times$ $\times 3$ $\times 3$ |
| PRTPLCT | $\times 4$ |
| Pritplot | $\times 4 \mathrm{H}$ |
| PRIPLUT | $\times 5$ |
| PRTPLDT | $\times 5 \mathrm{H}$ |
| PRTPLOI |  |
| PRTPLOT | $\times 6 \mathrm{H}$ |

STOP


[^0]:    *Square brackets are used to denote reference numbers.

[^1]:    * Using the parameter values given in Appendix A. ** Throughout this thesis w will be used to denote the imaginary part of the complex variable s ; i.e., $s=\sigma+j w$

[^2]:    * These deviations are time averages for particular sample paths of the noise.

[^3]:    * The availability of noise-free measurements of all state. variables is a convenient fiction for benchmark purposes.

[^4]:    * In the simulations of the system with the different stabi-

