THE EFFECT OF INDIVIDUALIZED VERBAL PROBLEM ASSIGNMENTS ON THE MATHEMATICAL ACHIEVEMENT OF FIFTH-GRADE STUDENTS

A Dissertation

Presented to

the Faculty of the College of Education

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Education

---

by

Cecil Thomas Nabors

June 1968

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C. T. N.

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PROBLEM

The purpose of the study was to determine the effect of individualized problem-solving assignments on the achievement of fifth-grade pupils in mathematical concepts and problem solving. Further information was sought concerning the effect of the experimental treatment on subgroups based on sex, reading ability, and intelligence.

PROCEDURES

The study involved 316 fifth-grade pupils in five elementary schools in Bryan, Texas. One-half of the members of each class were selected at random to serve as the experimental group. The other members of the twelve classes served as the control group. The arithmetic concepts and problem-solving sections of the Iowa Tests of Basic Skills were administered as a pretest to all members of the twelve mathematics classes.

Verbal problems selected from several series of mathematics textbooks were used in preparing assignments of verbal problems at varying levels of difficulty. Several sets of five problems were prepared for each of eleven levels of difficulty from second grade through seventh grade. The instructional readability level of each set matched the difficulty level assigned the set. Answer sheets and pupil personal record sheets were provided.

Three times a week for ten weeks, each member of the experimental group worked independently on verbal problem-solving assignments matched to measured problem-solving ability. The members of the control group used only assignments from the regular fifth-grade mathematics textbook.

After ten weeks all the pupils were given another form of the arithmetic section of the Iowa Tests of Basic Skills. The raw scores of the final test minus the raw scores of the pretest provided score gains for each pupil in the two areas of mathematics. For analysis the members of the experimental group and the control group were separated into subgroups based on sex, reading ability, and intelligence. The grade equivalent score in reading on the Iowa Tests of Basic Skills and the intelligence quotient determined by the Otis Mental Ability Test: Beta Test were used in the classification of pupils into three subgroups for reading and for intelligence.

With the score gains on the arithmetic concepts test as a dependent variable, one-way analysis of variance was used to test a series of null hypotheses that there was no difference in the score gains of the experimental group or subgroups and the score gains of the control group or corresponding subgroups. With problem-solving test score gains as a dependent variable, the procedure was repeated.

RESULTS

With the score gains of the arithmetic concepts test as the criterion measure, tests of significance failed to reject any of the null hypotheses of no difference between the experimental group or subgroups and the control group or subgroups.

With the score gains of the problem-solving test as the criterion measure, the null hypothesis of no difference in score gains of the pupils with average intelligence using individualized verbal problem assignments and score gains of pupils with average intelligence using regular fifth-grade materials was rejected with a five per cent level of confidence. Similarly, the null hypothesis involving the subgroups of boys with average intelligence was rejected with a five per cent level of confidence. Tests of significance failed to reject any other null hypothesis involving problem-solving score gains as a criterion.

CONCLUSIONS

Score gains in problem solving by pupils of average intelligence using individualized problem-solving assignments were significantly greater at the five per cent level of confidence than the score gains of pupils of average intelligence using regular fifth-grade mathematics textbook materials.

Score gains in problem solving by boys of average intelligence using individualized problem-solving assignments were significantly greater at the five per cent level of confidence than the score gains of boys of average intelligence using regular fifth-grade materials.

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CHAPTER I

INTRODUCTION

An important objective of the mathematics program of any school should be the development of a child's ability to deal with problem situations that arise in daily life. A program for developing the ability to solve verbal problems in a mathematics class might result in a generalized ability to solve many kinds of problems. Teachers of mathematics are usually concerned with what they can do to aid their pupils in developing problem-solving ability. In the many studies of problem solving, various phases of problem solving have been considered--(1) nature of problem solving, (2) verbal problem solving, (3) procedures associated with problem solving, (4) factors associated with problem-solving success, and (5) individual differences in problem-solving ability.

Nature of Problem Solving

A problem situation is a situation in which the individual has no readily appropriate behavioral response that will result in a solution to the problem. If the participant knows what to do immediately, no problem is involved. Marks, Purdy, and Kinney (34) considered a problem as a situation understood by the participant for which he must decide on an appropriate procedure to use for its solution.

In 1937 Durkin (12) studied the nature of solutions to problems. She asked each subject to fit together card pieces to form a square. When the square was complete, the subject was asked to outline the steps taken in the solution of the problem. The experimenter decided whether the procedure was mainly trial and error or analysis. The subject was then given the pieces with which he had completed five squares and asked to form a maltese cross. If the subject realized that results of other problems could be used in this problem, then his solution was called sudden reorganization. Durkin considered these three forms of thought as points on a continuum rather than exclusive kinds of thinking.

Gagné suggested "that the underlying process of problem solving, whatever it turns out to be, must have a lot to do with transfer of training." (16:311) Problem-solving behavior is a form of learning which must be preceded by other forms of learning. Measuring problem-solving performance is not simple. Usually, the correct solution is accepted as a measure of success. Other things such as (1) time needed, (2) preliminary aids, and (3) kinds of errors involved in the solution may be considered in the evaluation of problem-solving performance.

In 1949 Johnson (25) attempted to measure the relationship of (1) number, (2) vocabulary, (3) space, (4) word fluency, (5) reasoning, and (6) memory to success in problem solving. He found a high correlation between both vocabulary and reasoning and problem-solving success. Johnson insisted that a good vocabulary was one of the main ingredients of problem-solving ability.

Piaget (37) investigated various stages of thinking by the young child with a series of problems. He used six distinct problems in his experiment. Two like containers contained the same amount of liquid. When one was poured into different shaped containers, the problem was to decide whether the volume changed. Problem II involved three different volumes of liquid in various shaped containers. Using other containers, the children were to discover some unit of measure for comparison purposes. The other problems involved variations of these. Performances on these problems were classified according to three stages of development in the child.

Getzels (17) pointed out that there were varied theories concerning the processes underlying the act of problem solving. According to Getzels, the famous five steps in problem solving of Dewey's were more of an outline of the scientific method than a way of thinking. Getzels reported that Guilford took his thinking factors and subdivided them into convergent-thinking and divergent-thinking processes. He also referred to Piaget's concept of developmental stages of thinking. The discussion by Getzels indicated that the nature of problem solving is not something that can be described in a few simple terms. Complete interpretation of the nature of problem solving was considered to be beyond the scope of this study.

Verbal Problem Solving in Mathematics

Problem solving in mathematics may be presented to pupils in the form of verbal problems. Verbal problems are written statements describing the conditions of the problem. A pupil's understanding of the problem is dependent on his ability to read and comprehend the statements concerning the situation. Since mathematics classes usually include pupils having different levels of reading ability, it seems logical that each pupil should have verbal problems assigned that match his reading level. It may be that individualized assignments of verbal problems will affect a pupil's achievement in mathematics. Shipp and Adams (49) noted that problems of various degrees of difficulty should be assigned to pupils so that all can have some degree of success. The textbooks reviewed for this study did not provide convenient materials for such assignments.

Verbal problems have been used in more than one way in many mathematics classes. Verbal problems have been used to introduce a process in mathematics. Banks stated, "Problem solving is learning," (2:419) and that verbal problems might point up the need for a new process in mathematics. Van Engen (47) noted that a new process in mathematics should be introduced with a verbal problem. An important use for the verbal problem is the illustration of the social use for a process in mathematics--a grocery bill involves addition. Spitzer (42) indicated that learning the application of a process in mathematics is the main use for verbal problems. According to Stokes (44), problem solving can be of value in developing one's thinking process. Pupils may need the experience in critical thinking necessary for the solution of some verbal problems. Practice in logical thinking was the phase of problem solving that had major consideration in this study. Verbal problems that stimulate a pupil to use his powers of reasoning will have a variety of processes involved in their solution and not just the recently learned process. Problems of this type require more careful reading and analysis.

When verbal problems are used to provide additional practice in a process, they usually involve only the one process. The pupil anticipates the use of the learned process in the solution of the verbal problem and no real problem-solving situation exists. Overman (36) indicated that too much time is spent in working problems of this type. Assignment of the same verbal problems to all members of a class may be suitable when the purpose is to illustrate the use of a particular process in mathematics.

Procedures Associated With Verbal Problem Solving

Methods of problem solving involve certain preliminary steps that a pupil takes in his approach to the solution of the problem. Dutton and Adams (13) listed the following as steps in the procedure of problem solving:

1. Read problem.
2. Decide what is given.
3. Decide what is to be found.
4. Decide on necessary operations.
5. Solve problem.
6. Check result. (13:178)

Thorpe mentioned the following steps in problem solving:

1. Reading carefully.
2. Analyzing to determine steps needed.
3. Applying known principles to determine relationships.
4. Formulating algorithm.
5. Performing algorithm.
6. Checking reasonableness of solution.
7. Deriving generalizations. (45:308)

A study in problem solving by Dwight led to the following steps in problem solving:

1. Careful reading.
2. Decide what is to be found.
3. Formulate a word sentence.
4. Writing mathematical sentence.
5. Finding solution set.
6. Check solution.
7. State word sentence using solution. (14:471)

New materials in elementary school mathematics emphasize the use of mathematical sentences as a preliminary procedure in the solution of verbal problems.

Although many research efforts have been made to determine the best "method" to use in solving verbal problems, none have been very convincing in establishing the best "method." Requiring the pupil to always use a specific "method" in solving verbal problems is of doubtful value.

Banks (2) stated that pupils should not be restricted too much in procedure. He noted that they should be allowed to use any method they desire, most of the time. It may be, then, that the decision as to what procedure to use is an important phase of the problem-solving process. Marks, Purdy, and Kinney (34) indicated that a pupil should be encouraged to develop his own systematic approach. Riedesel (38) found that several systematic approaches to problem solving taught to certain sixth graders resulted in better achievement in problem solving by these pupils compared to those not having had such instruction in "methods." If a particular procedure or method were mastered by a pupil which enabled him to follow a pattern in solving a problem, the problem situation would not exist. It may be that the search for a better "method" of problem solving should be replaced by a search for better problems and better ways of providing the right problems for each individual. According to Overman (36), pupils should not work too many problems that are too easy for them.

Will certain experiences in solving problems result in better achievement in this area? A study by Maier (33) indicated that experience is not the "key" to success in problem solving.

Methods of Verbal Problem Solving

A study of problem-solving methods was made by Clark and Vincent (6) in 1933. Eighty pupils in the seventh and eighth grades were divided into two equivalent groups based on arithmetic reasoning and intelligence. Six sets of eight problems were used with each group. One group used the conventional-analysis method where they listed what was given and what was to be found. The other group used a procedure called graphical analysis which involved a picture or diagram of the facts in the problem. The conventional-analysis group did better at first. Later the graphical-analysis group was more successful. The investigators noticed that the brighter children wanted to discard any formal procedure as being necessary to the solution.

Keil (27) made a study in 1964 of the effect of the procedure of pupil writing and solving original verbal problems on their achievement in problem solving. Four sections of sixth graders were used as an experimental group and four sections were used as a control group. The experimental group received lessons in writing and solving their own verbal problems one day a week for sixteen weeks. The control group worked with regular materials supplied by the investigator. The time was held constant for both groups. The results for the two groups were compared. Further comparisons were made of three levels of intelligence, sex, and two socio-economic levels. The total experimental group scored higher at a significant level. Boys, girls, high intelligence, average intelligence, and low socio-economic groups having the experimental treatment scored higher.

Three methods of problem solving were involved in a study by Hanna (30) in 1930. Fourth and seventh graders used three methods of problem solving called (1) dependencies, (2) conventional analysis, (3) individual choice in the experiment. Twenty practice sheets were completed in six weeks. Each sheet had seven problems with the first used as an example of procedure to be used with others. Three ability levels in each grade were compared. In the fourth grade the average and below average did better with the dependencies method. In the seventh grade all levels did better with the dependencies and individual methods.

An experiment involving forty-one teachers in sixteen schools in Illinois in 1927 investigated the relative value of two methods of problem solving. Washburne (48) reported that each class of the second, fourth, and sixth grades were divided into two groups. One group was introduced to a new process in mathematics by the teacher using a verbal problem related to real experiences of the pupils. The other group was taught the process for four weeks and then worked related verbal problems for two weeks. Both groups used the same materials and devoted the same amount of time to the activity. A final test in problem solving was given all pupils at the end of six weeks. There was no significant difference in the performances of the two groups at any level.

A study of the structured-equation approach to problem solving was done by Lerch and Hamilton (29) in 1966 at Herrin, Illinois. An experimental section of twenty-eight fifth graders was compared with a control class of seventeen in the fifth grade. The experimental group was taught to use equations in analyzing problems from the regular textbook over a period of five months. The control class worked the same problems without using equations. A pretest and a final test were prepared by the experimenter. The pupils were graded on programming the problem as well as on the correct answer. The experimental group did better on the programming, but both groups had about the same gains on actual solutions. In this case the equation approach did not improve ability to get correct solutions.

Spencer and Brydegaard reported an experiment in the San Diego schools wherein certain procedures associated with problem solving were taught for four months. They reported unusual gains in problem-solving ability as a result. Some of the procedures taught were as follows:

1. Identifying "What is the question?"
2. Analysis of problem independent of the numbers.
3. Writing own problems.
4. Estimating answers.
5. Graphic structuring.
6. Labeling answer.
7. Identifying "On what does answer depend?"
8. Mental arithmetic. (41:353)

A study of three problem-solving programs by Wilson (53) was completed at Syracuse in 1964. Wilson's study involved eighty fourth graders who were divided into three levels according to intelligence. Pupils were selected at random from each of the three levels to form three groups for the experiment. Group one received three fifty-minute lessons a week for nine weeks in Action-Sequence structure. Group two received three fifty-minute lessons a week for nine weeks in Wanted-Given structure. Group three practiced verbal problems only for the same period. Tests showed that achievement of the Wanted-Given experimental group was better than the other two groups. Wilson (54) insisted that action meanings for operations should be avoided because they interfere with the pupil's problem solving.

A comparison of the exposition method and the discovery method in teaching problem solving was investigated by Scandura (39) in 1964. Two sixth-grade classes matched in ability were used in the experiment. The investigator taught each section the solution of problems using unusual cards for material. One group was taught by exposition; the other was taught by the discovery method. He taught both groups until they were equally successful in doing the problems. It took the "discovery" class 153 minutes and the "exposition" class 108 minutes to learn how to solve the problems. A test involving "routine" and "novel" problems was given to both groups. The discovery method was inefficient because it took longer to learn. However, the discovery method enabled the students to solve more "novel" problems.

Several writers discussed the results of experiments on problem solving in a general way. Hudgins (23) mentioned studies of group problem solving that did not improve individual ability in problem solving. He noted that some experiments showed that teachers should encourage a variety of approaches to problem solving. The value of the discovery method has not been established by research, according to Hudgins.

Riedesel (38) experimented with several procedures in problem solving. Eleven sections of sixth graders in Iowa were used as the experimental group. There were nine control sections used in this 1962 study. The experimenter prepared thirty lessons at two levels of difficulty. These lessons made use of several specific procedures--(1) writing number questions, (2) using drawings of diagrams, (3) pupil formulation of problems, (4) orally presented problems, and (5) problems without numbers. A tape-recorded problem-solving test was devised by the experimenter. A standardized achievement test was also used in measuring results of the thirty lessons given over a period of ten weeks. The gains made by the experimental group were significantly greater than the gains made by the group which had no special instruction in problem-solving procedures.

A review of the literature on research in problem solving by Johnson (24) included many studies with primary emphasis on methods of problem solving. Some of the studies did not show one method to be significantly better than another method. Johnson (24) considered many of the studies inconclusive because the experimental design was poor.

Problem solving was not an essential part of the mathematics program according to Wilson (52). He insisted that experiments in problem-solving procedures had been rather fruitless and that working verbal problems in mathematics wasted time.

Other writers were concerned with the possibility that problem-solving activity might become too mechanical. Overman (36) noted that too much emphasis could be placed on classifying problems according to type. Dutton and Adams (13) thought that classifying problems into types was helpful. According to Shipp and Adams (49), using given sets of steps in a mechanical way was not used very much anymore. Developing ability in problem solving involved a method of meanings provided by the teacher's procedure according to Stokes (44).

There was a wide variety of opinion as to what the teacher should do to develop greater problem-solving achievement by the pupil.

Factors Associated With Problem-Solving Success

A number of authors considered certain mental and personality factors as being important to success in problem solving. Some research has determined which factors showed a high correlation with success in problem solving.

Spitzer (42) indicated that to be a good problem solver one should be confident, persistent, and resourceful. He did not explain how a teacher might develop these traits in the pupils. Is it logical that certain personality traits are factors in success in the area of problem solving? To solve difficult problems must one have sufficient determination as well as skill?

In 1961 Klausmeier and Loughlin (28) studied certain behavioral traits of children as revealed in their solution of certain problems. A group of forty children were chosen at each of three levels of intelligence. Each group tried problems graded to fit the level of ability of the group. All problems required the selection of the right coins from a collection to form a given amount. Trained observers watched the behavior of each child and recorded the steps in his procedure. The behaviors were classed as (1) efficiency, (2) non-persistence, (3) selection of incorrect solution, (4) self-correction, (5) random approach, and (6) logical approach. There were no significant differences among groups or between groups in most cases except time and efficiency of method. The high intelligence group made a better showing in (1) correcting mistakes, (2) using logical approach, and (3) verifying solutions. The low intelligence group showed more (1) random approach, (2) non-persistence, and (3) offering wrong solution.

Lindgren and others (30) investigated the relation of pupils' attitudes toward problem solving and the pupils' success in problem solving. The team of investigators used as a sample 108 fourth-year pupils in five schools in Porto Alegre, Brazil. Pupils' attitudes toward problem solving were measured by a test devised by Carey. Other tests were devised by the research team. The results showed only slight correlation between attitudes and problem-solving success.

Other factors associated with successful achievement in problem solving were studied by Hansen (21) in 1944. Various tests were administered to 680 sixth-grade pupils in ten different communities. The highest achievers (twenty-seven per cent) were compared to the lowest achievers (twenty-seven per cent) in certain mathematical factors, mental factors, and reading factors. The two groups were balanced statistically so differences were not due to age or mental capacity. It was found that reading factors were not consistent with the problem-solving achievement. General mathematical factors such as skill in fundamental operations, estimating answers, and problem analysis were consistent with achievement in problem solving. General mental factors such as reasoning, non-language factors, and memory were consistent with problem-solving achievement.

In 1932 Englehardt (15) conducted a study with 568 fifth graders in Decatur, Illinois, concerning factors in the individual differences in problem-solving ability. He used as independent variables (1) intelligence scores, (2) computation scores, and (3) reading scores. The per cent of variance of problem-solving scores due to each of the independent variables was determined. Englehardt's figures showed that 25.69 per cent of the variance was due to variance in intelligence; 42.05 per cent of the variance in problem-solving scores was due to variance in computation scores; a negative 1.33 per cent of the variance was due to reading ability variance.

The relationship of reading skills to ability to solve arithmetic problems was the topic of a study by Treacy (46) in 1944. Treacy tested 244 seventh graders in Milwaukee using the following tests:

1. Analytical Scales of Attainment: Problems.
2. Public Schools Achievement Tests: Arithmetic Reasoning.
3. Otis Quick Scoring Mental Test: Beta, Form A.
4. Gates Silent Reading Test.
5. Diagnostic Examination of Silent Reading Abilities.

Treacy found that good achievers in problem solving also were high in intelligence and high in all reading skills considered in the study.

Johnson (25) in 1949 reported a study of the degree of correlation between certain mental factors and problem-solving success. Six schools in Chicago were involved in this study. Tests showed a high correlation between pupils' problem-solving success and vocabulary as well as reasoning. Johnson emphasized that vocabulary was one of the main ingredients in problem-solving ability.

A study conducted by Chase (4) in 1960 compared the level of various skills with achievement in problem solving. Skills such as (1) verbal, (2) reasoning, (3) spatial, (4) perceptual speed, (5) number, (6) computation, (7) arithmetic vocabulary, (8) problem analysis, (9) reading, and others were used as independent variables. The criterion measure was the problem-solving section of the Iowa Every Pupil Test of Arithmetic. Variables having greatest relationship to success in problem solving included computation, reading to note details, and fundamental knowledge of arithmetic.

In 1958 Corle (9) studied the importance of certain thought processes as factors in the successful solution of problems. He used the interview technique with seventy-four pupils in the sixth grade. The pupils read problems orally, re-read, and worked the problems on cards. Corle found a high correlation between understanding and accuracy in problem solving. Also, he found a high correlation between success and confidence in one's own accuracy. Mistakes in computation accounted for only twelve per cent of the errors made.

Alexander (1) made a study of the relationship of certain factors to ability to solve problems in arithmetic. The 1959 study involved 623 seventh graders in California. Factors found to have a close relationship to ability in arithmetic reasoning were (1) understanding (verbal), (2) mental age, (3) reading, (4) arithmetic concepts and computation, (5) ability to interpret data, and (6) recognition of limitations of data. Sex differences were not significant. A conclusion of Alexander's study included a suggestion for a differentiated program of instruction in problem solving.

In 1930 Maier (33) conducted an experiment with five groups of students at the University of Berlin and three groups at the University of Michigan. The problem was to devise a way to suspend two pendulums from the ceiling with limited pieces of equipment. Each student received suggestions after futile attempts. It was discovered that the previous experiences that were necessary did not insure success. Another factor called "direction" was necessary to the success in solving the problem. The experiences of the students aided in the solution only when they conformed to the "direction" taken in attempting a solution.

The thought processes of college students while attempting the solution of a problem were studied by Bloom and Broder (3) in 1950. Students at the University of Chicago were asked to think aloud as they solved the problems. The observer took notes on steps used by students in solving the problems. The better problem solvers were more proficient in starting the attack on the problem and were able to apply relevant knowledge to the solution. The good problem solvers had a definite feeling of confidence in their ability to solve problems.

Individual Differences

The literature on problem solving as related to individual differences was limited. Clark and Eads listed the following guideposts for the teacher's use in determining a pupil's readiness for certain problems:

1. They can find the solution readily.
2. They can change the numbers in the problem to other reasonable numbers.
3. They are not distracted by extraneous data.
4. They can change items in the problem to other reasonable items.
5. Often they can solve the problem in more than one way.
6. They can devise other problems using the same situation.
7. They can talk about the problem, tell things about it that were not stated, invent circumstances that created the problem, etc.
8. They can explain why they used the method they used in solving the problem. (5:264)

Jones and Pingry (26) stated that the school's goal should not be to reduce differences among children but to consider them in teaching. They advocated the use of more materials in mathematics that were suitable for different levels of ability. They also stated that research was needed to aid in decisions on providing for individual differences.

Weaver suggested planning that permitted flexibility in the assignment of pupils to classes. He stated, "Individualism does not mean that each child must be taught separately." (50:75) Variation in instruction for individual differences may be classified as (1) variation in instruction time, (2) variation in methods and materials, and (3) variation in the kind and level of mathematical content.

In his experiment Riedesel (38) provided for two levels of achievement in his assignments for certain sixth graders. He prepared problem-solving practice sheets for pupils above average in achievement. Another set of practice sheets were prepared for the below-average sixth graders. These differentiated assignments were not a significant phase of Riedesel's experiment.

Summary

An attempt to summarize the research and literature related to the improvement of problem-solving ability should be approached with caution since the findings are sometimes contradictory.

Some generalizations arising from a survey of the literature follow:

1. Certain activities involving pupils in problem solving have resulted in improvement in problem-solving achievement.

2. There was no one method better than others in developing problem-solving ability, although some systematic procedure was better than none.

3. Research indicated that pupils should not be restricted to a single procedure in their problem-solving efforts.

4. Certain behavioral traits such as reasoning and memory had high correlation with success in problem solving.

5. Reading ability had considerable positive correlation with problem-solving ability according to findings of some research but not according to other research.

CHAPTER II

DESIGN OF THE STUDY

The purpose of this study was to determine the effect of individualized assignments of verbal problems in mathematics on the achievement of fifth-grade pupils in mathematical concepts and problem solving.

Null Hypotheses To Be Tested

Pupils involved in the study were separated into categories according to sex, mental ability, and reading ability. These categories were established to provide additional information concerning the effect of the experimental treatment on pupils' achievement. Comparisons between subgroups of the experimental and control groups that involved null hypotheses to be tested are illustrated in Figures 1--8. The score gains on the arithmetic concepts section of the *Iowa Tests of Basic Skills* (31) were used as criterion measures for one phase of the statistical analysis. The score gains on the problem-solving section of the *Iowa Tests of Basic Skills* (31) were used as criterion measures of the other phase of the statistical analysis. The raw score of the final test minus the raw score of the pretest was considered the score gain.

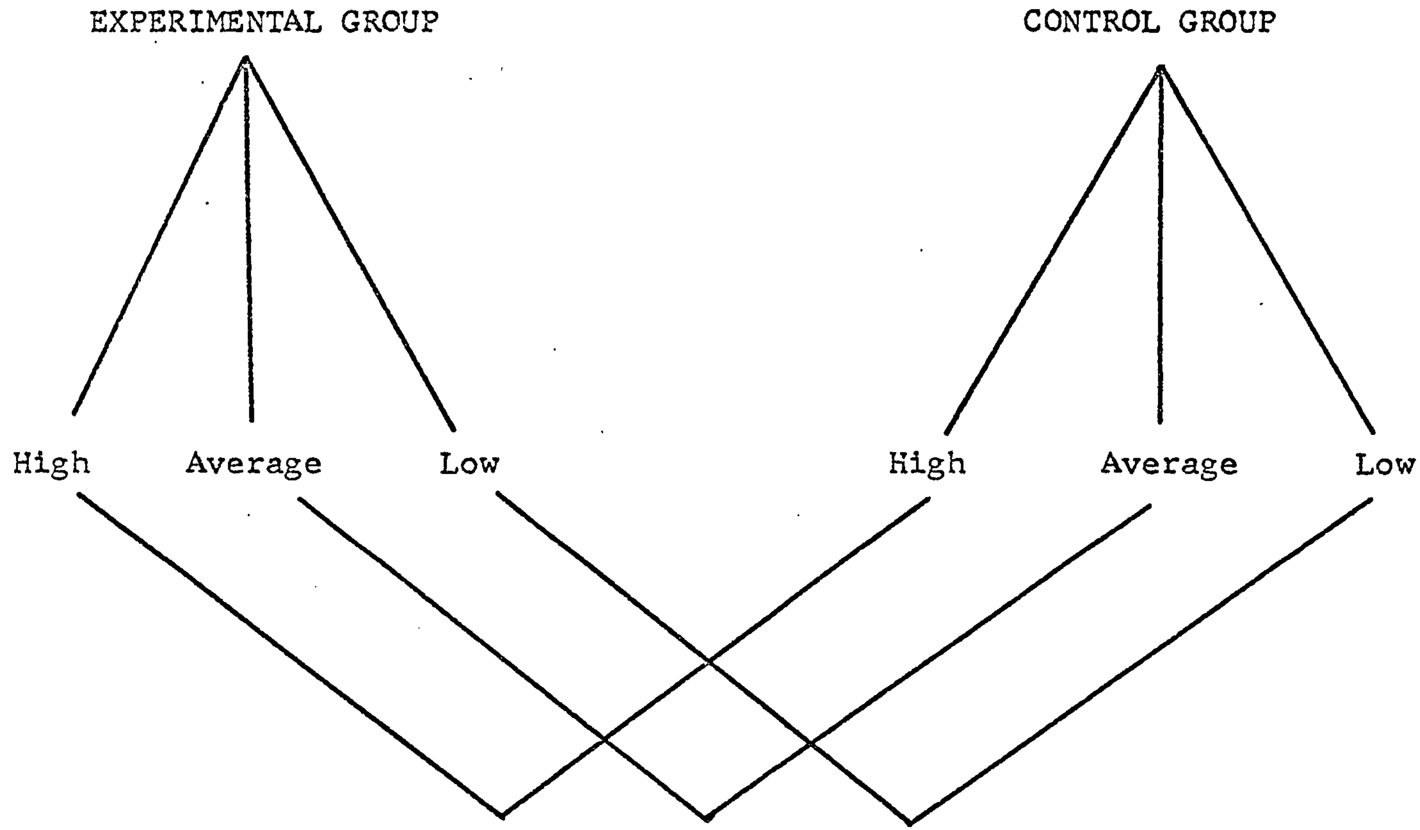

Figure 1

COMPARISON OF SCORE GAINS ON ARITHMETIC
CONCEPTS TEST FOR READING LEVELS

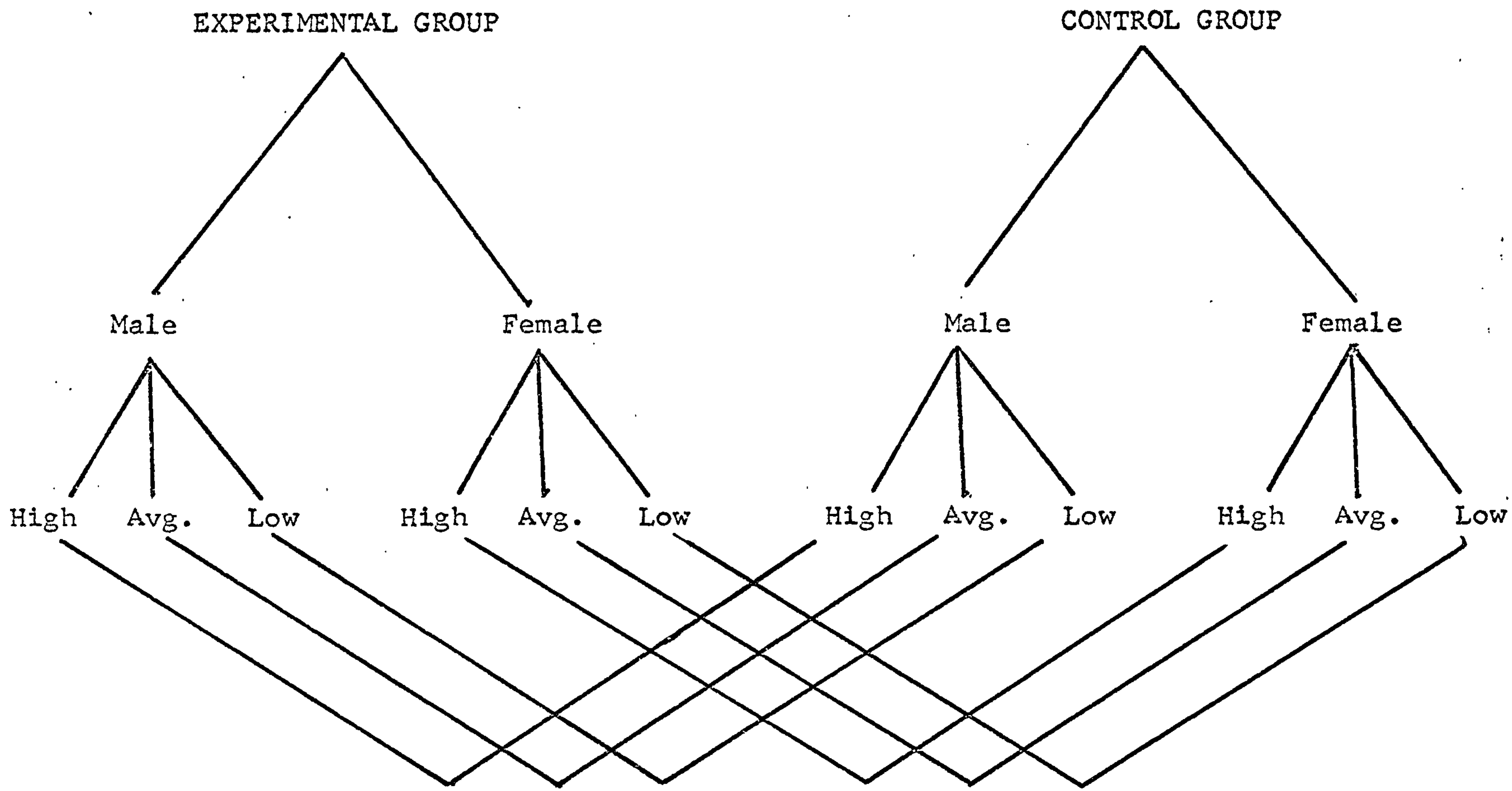

Figure 2

COMPARISON OF SCORE GAINS ON ARITHMETIC CONCEPTS
TEST FOR SEX AND READING LEVELS

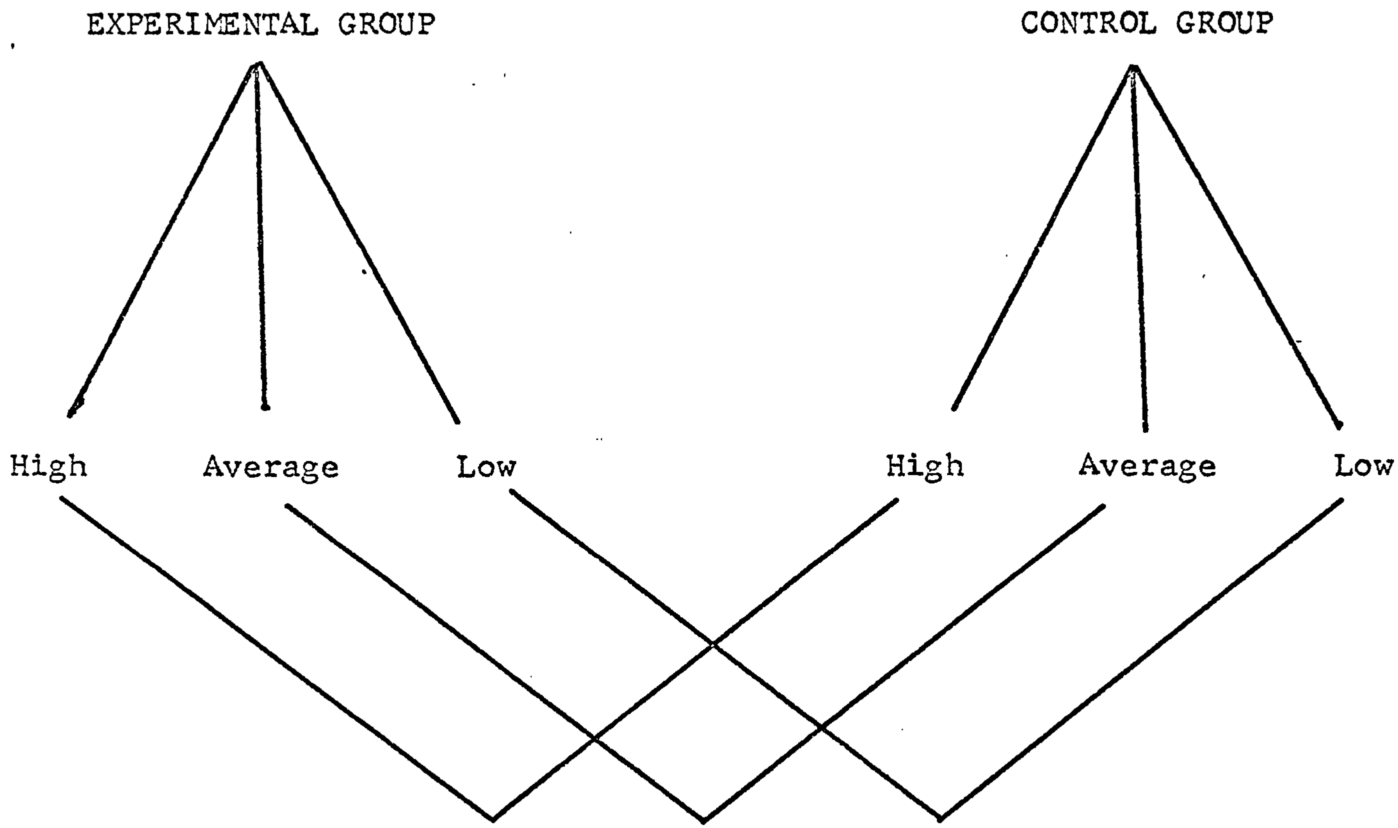

Figure 3

COMPARISON OF SCORE GAINS ON ARITHMETIC
CONCEPTS TEST FOR INTELLIGENCE LEVELS

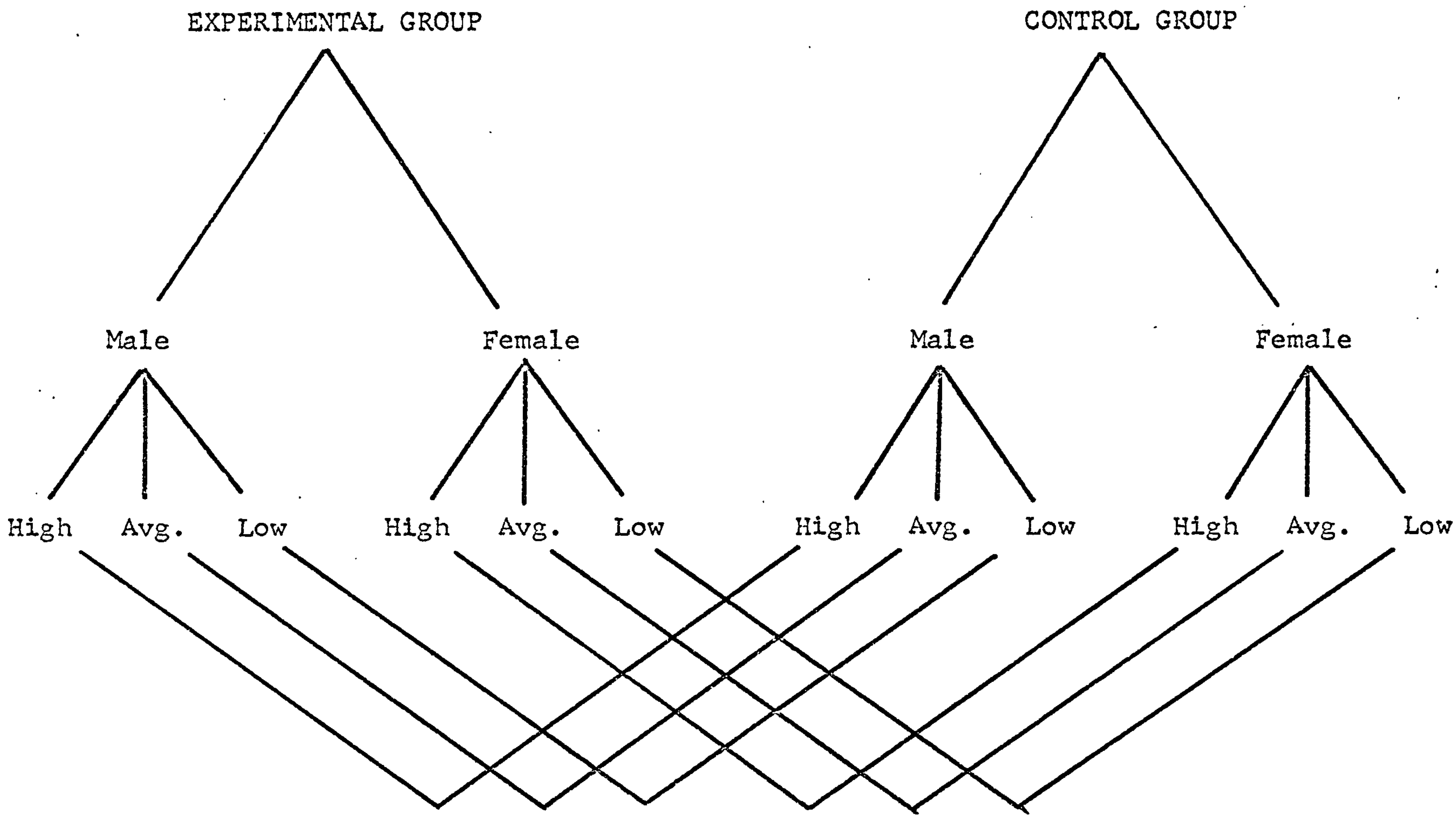

Figure 4

COMPARISON OF SCORE GAINS ON ARITHMETIC CONCEPTS TEST FOR SEX AND INTELLIGENCE LEVELS

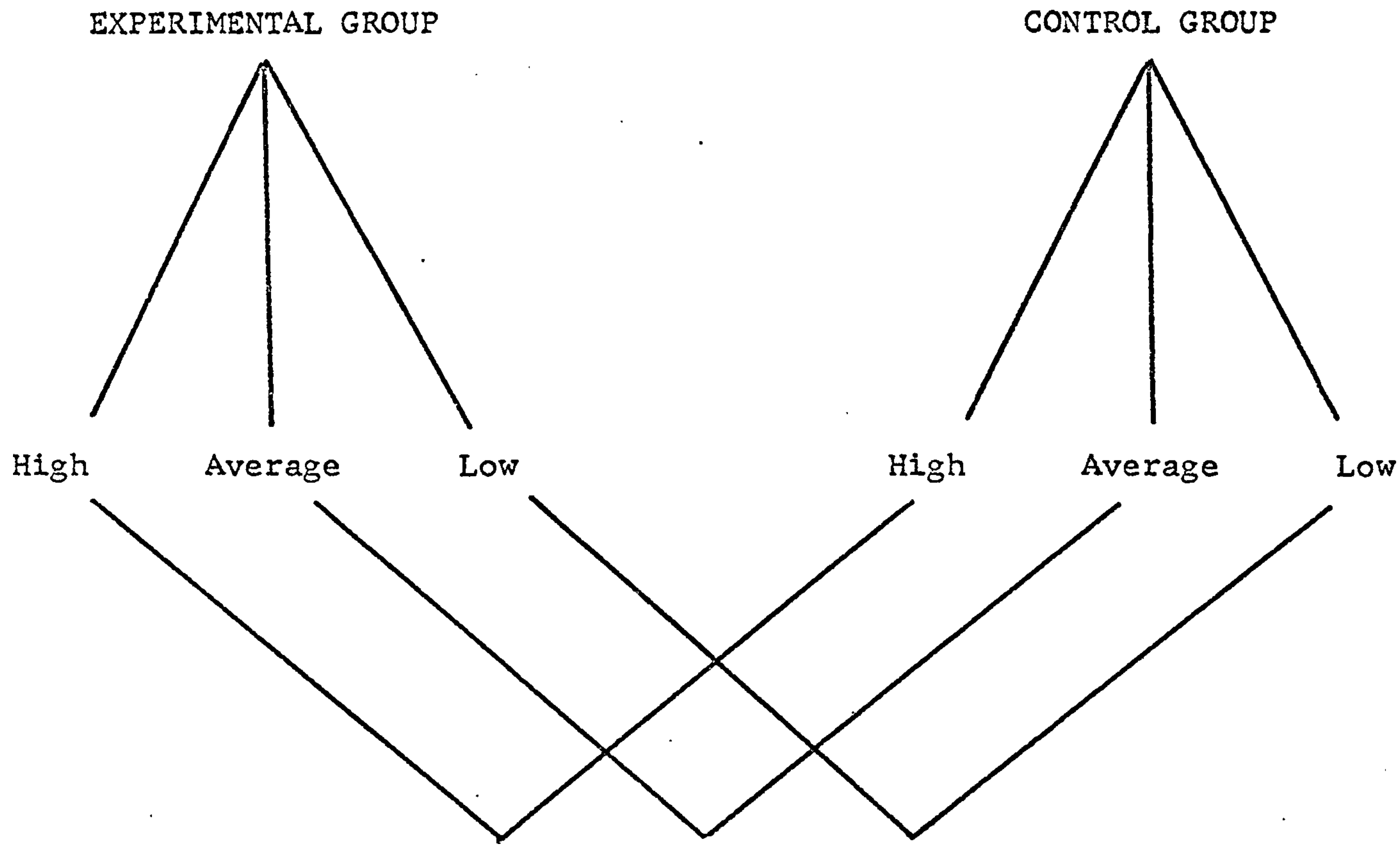

Figure 5

COMPARISON OF SCORE GAINS ON PROBLEM-SOLVING TEST FOR READING LEVEL

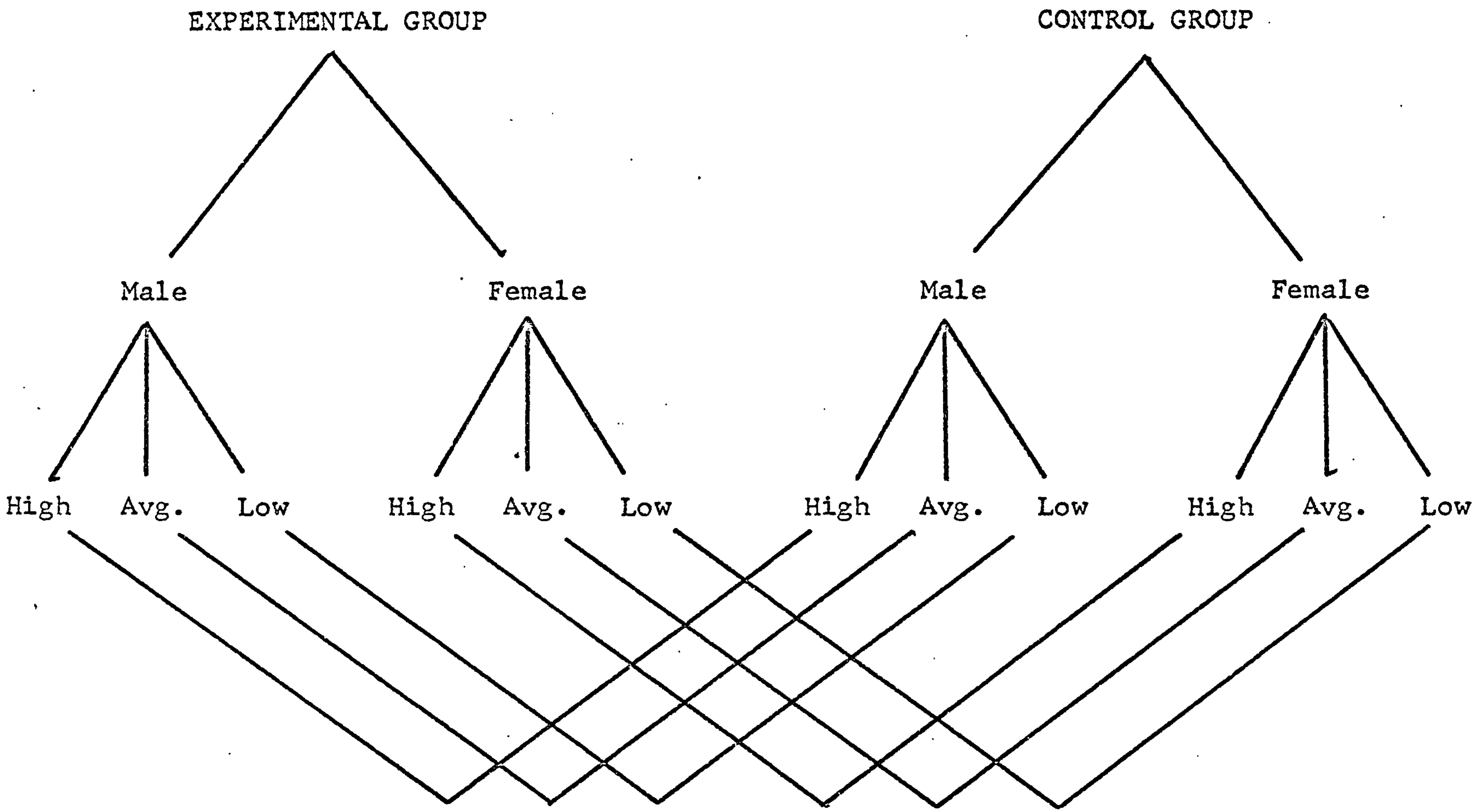

Figure 6

COMPARISON OF SCORE GAINS ON PROBLEM-SOLVING
TEST FOR SEX AND READING LEVELS

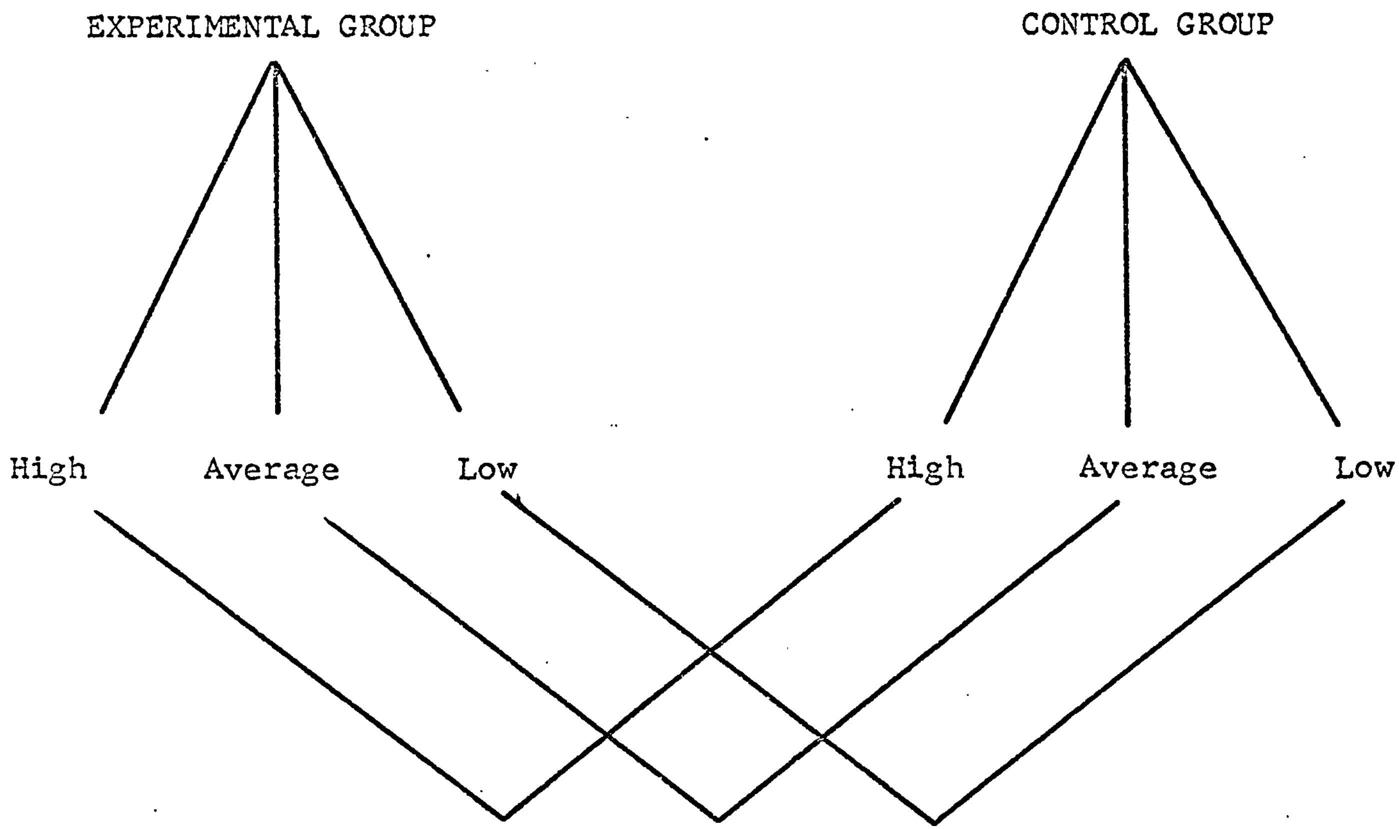

Figure 7

COMPARISON OF SCORE GAINS ON PROBLEM-SOLVING
TEST FOR INTELLIGENCE LEVELS

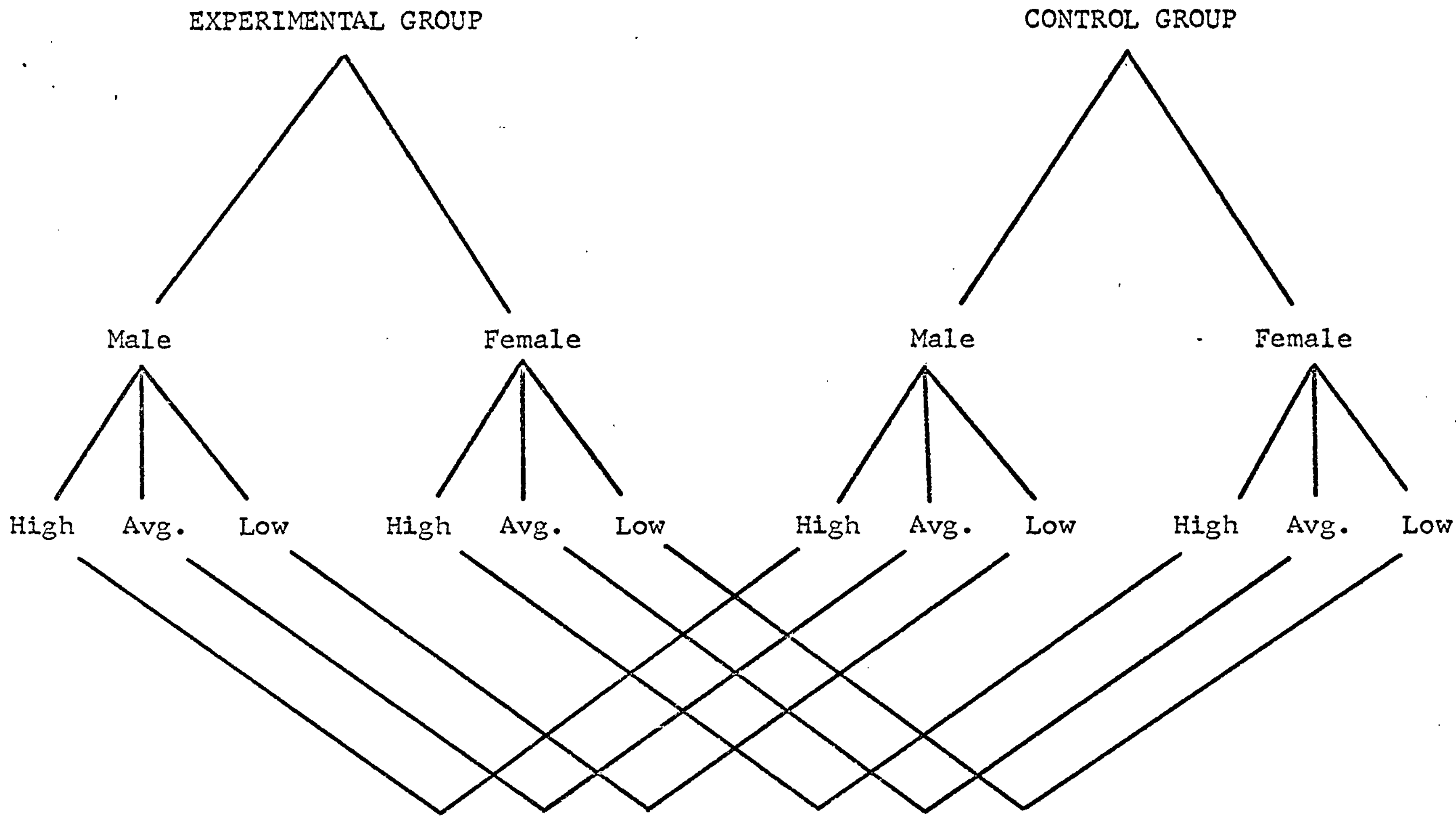

Figure 8

COMPARISON OF SCORE GAINS ON PROBLEM-SOLVING
TEST FOR SEX AND INTELLIGENCE LEVELS

Using the score gains of the arithmetic concepts section of the standardized test as a dependent variable, the following null hypotheses were tested:

1. There is no difference in score gains by pupils using individualized problem-solving assignments and pupils using regular textbook materials.

2. There is no difference in score gains by boys using individualized problem-solving assignments and boys using regular textbook materials.

3. There is no difference in score gains by girls using individualized problem-solving assignments and girls using regular textbook materials.

4. There is no difference in score gains by pupils of high reading level using individualized problem-solving assignments and pupils of high reading level using regular textbook materials.

5. There is no difference in score gains by pupils of average reading level using individualized problem-solving assignments and pupils of average reading level using regular textbook materials.

6. There is no difference in score gains by pupils of low reading level using individualized problem-solving assignments and pupils of low reading level using regular textbook materials.

7. There is no difference in score gains by boys of high reading level using individualized problem-solving assignments and boys of high reading level using regular textbook materials.

8. There is no difference in score gains by boys of average reading level using individualized problem-solving assignments and boys of average reading level using regular textbook materials.

9. There is no difference in score gains by boys of low reading level using individualized problem-solving assignments and boys of low reading level using regular textbook materials.

10. There is no difference in score gains by girls of high reading level using individualized problem-solving assignments and girls of high reading level using regular textbook materials.

11. There is no difference in score gains by girls of average reading level using individualized problem-solving assignments and girls of average reading level using regular textbook materials.

12. There is no difference in score gains by girls of low reading level using individualized problem-solving assignments and girls of low reading level using regular textbook materials.

13. There is no difference in score gains by pupils of high intelligence using individualized problem-solving assignments and pupils of high intelligence using regular textbook materials.

14. There is no difference in score gains by pupils of average intelligence using individualized problem-solving assignments and pupils of average intelligence using regular textbook materials.

15. There is no difference in score gains by pupils of low intelligence using individualized problem-solving assignments and pupils of low intelligence using regular textbook materials.

16. There is no difference in score gains by boys of high intelligence using individualized problem-solving assignments and boys of high intelligence using regular textbook materials.

17. There is no difference in score gains by boys of average intelligence using individualized problem-solving assignments and boys of average intelligence using regular textbook materials.

18. There is no difference in score gains by boys of low intelligence using individualized problem-solving assignments and boys of low intelligence using regular textbook materials.

19. There is no difference in score gains by girls of high intelligence using individualized problem-solving assignments and girls of high intelligence using regular textbook materials.

20. There is no difference in score gains by girls of average intelligence using individualized problem-solving assignments and girls of average intelligence using regular textbook materials.

21. There is no difference in score gains by girls of low intelligence using individualized problem-solving assignments and girls of low intelligence using regular textbook materials.

Using the score gains of the problem-solving section of the standardized test as a dependent variable, the following null hypotheses were tested:

22. There is no difference in score gains by pupils using individualized problem-solving assignments and pupils using regular textbook materials.

23. There is no difference in score gains by boys using individualized problem-solving assignments and boys using regular textbook materials.

24. There is no difference in score gains by girls using individualized problem-solving assignments and girls using regular textbook materials.

25. There is no difference in score gains by pupils of high reading level using individualized problem-solving assignments and pupils of high reading level using regular textbook materials.

26. There is no difference in score gains by pupils of average reading level using individualized problem-solving assignments and pupils of average reading level using regular textbook materials.

27. There is no difference in score gains by pupils of low reading level using individualized problem-solving assignments and pupils of low reading level using regular textbook materials.

28. There is no difference in score gains by boys of high reading level using individualized problem-solving assignments and boys of high reading level using regular textbook materials.

29. There is no difference in score gains by boys of average reading level using individualized problem-solving assignments and boys of average reading level using regular textbook materials.

30. There is no difference in score gains by boys of low reading level using individualized problem-solving assignments and boys of low reading level using regular textbook materials.

31. There is no difference in score gains by girls of high reading level using individualized problem-solving assignments and girls of high reading level using regular textbook materials.

32. There is no difference in score gains by girls of average reading level using individualized problem-solving assignments and girls of average reading level using regular textbook materials.

33. There is no difference in score gains by girls of low reading level using individualized problem-solving assignments and girls of low reading level using regular textbook materials.

34. There is no difference in score gains by pupils of high intelligence using individualized problem-solving assignments and pupils of high intelligence using regular textbook materials.

35. There is no difference in score gains by pupils of average intelligence using individualized problem-solving assignments and pupils of average intelligence using regular textbook materials.

36. There is no difference in score gains by pupils of low intelligence using individualized problem-solving assignments and pupils of low intelligence using regular textbook materials.

37. There is no difference in score gains by boys of high intelligence using individualized problem-solving assignments and boys of high intelligence using regular textbook materials.

38. There is no difference in score gains by boys of average intelligence using individualized problem-solving assignments and boys of average intelligence using regular textbook materials.

39. There is no difference in score gains by boys of low intelligence using individualized problem-solving assignments and boys of low intelligence using regular textbook materials.

40. There is no difference in score gains by girls of high intelligence using individualized problem-solving assignments and girls of high intelligence using regular textbook materials.

41. There is no difference in score gains by girls of average intelligence using individualized problem-solving assignments and girls of average intelligence using regular textbook materials.

42. There is no difference in score gains by girls of low intelligence using individualized problem-solving assignments and girls of low intelligence using regular textbook materials.

Sample

Twelve sections of fifth-grade pupils in five different schools in the Bryan Independent School District were selected to participate in the study. Two classes at Bonham Elementary School, two classes at Bowie Elementary School, one class at Carver Elementary School, three classes at Milam Elementary School, and four classes at Ross Elementary School constituted the twelve classes involved in the study. Table I, page 39, gives a summary of the size of the classes involved.

The classes involved in the study represented a wide variety of pupil ability and pupil achievement. Table II, page 40, provides a summary of the mean values of intelligence quotients and mean values of reading grade equivalent scores (31) of the twelve classes.

One of the classes was self-contained, but the other classes were taught mathematics by a teacher who taught all the fifth-grade mathematics for that particular school. The five teachers were selected because of their interest in mathematics and their willingness to participate in the study. Approval of the study was previously obtained from the superintendent and director of instruction for the school district and each school principal.

For the purposes of this study the boys of each class and the girls of each class were separated into two groups of

TABLE I

NUMBER OF PUPILS IN EACH FIFTH-GRADE CLASS INVOLVED IN THE STUDY

Class	Number of boys	Number of girls	Total pupils
1	12	12	24
2	10	13	23
3	15	17	32
4	16	16	32
5	11	17	28
6	15	13	28
7	12	16	28
8	13	14	27
9	13	17	30
10	15	16	31
11	16	12	28
12	15	14	29
	163	177	340

TABLE II

AVERAGE INTELLIGENCE QUOTIENT AND AVERAGE READING GRADE EQUIVALENT OF THE CLASSES INVOLVED IN THE STUDY

Class	Mean intelligence quotient	Mean reading grade equivalent
1	101.8	54.6
2	101.3	53.4
3	87.7	45.7
4	86.3	47.4
5	84.7	43.3
6	90.8	43.3
7	95.4	47.8
8	92.9	51.1
9	110.5	64.0
10	106.4	59.2
11	102.1	55.4
12	107.9	61.8

equal size. One-half of the members of each class were treated as the experimental group and the others as the control group. The investigator used a table of random numbers (11) in selecting the members of each class to participate as the experimental group. Lists of the selected pupils were given to each teacher who treated these pupils as the experimental group and the other pupils in each class as the control group.

The random selection of one-half of each mathematics class to serve as the experimental group was done to control certain variables--(1) initial differences in pupil ability, (2) teacher effectiveness, and (3) class situations. The situation of having both experimental and control groups working in the same classroom may have caused contamination of one group by the other.

There were 340 pupils in the twelve classes; of these 340, complete data were collected for 316. Some pupils moved away during the study, and some missed the necessary tests. If a pupil missed one phase of the testing, his record was not included in the final analysis.

Materials and Procedures

Materials used by the experimental group of children were verbal problems selected from the elementary mathematics textbooks used in the public schools of the state of Texas (see Appendix C). In order to identify the level of difficulty, verbal problems taken from the first part of second-grade books were labeled twenty-one, and the problems taken from the last part of the second-grade books were labeled twenty-two. Problems from the first part of the third-grade books were labeled thirty-one, and those from the second part of third-grade books were labeled thirty-two.

In a similar manner problem-solving assignments were prepared and labeled for two levels at the fourth, fifth, and sixth grades, and one level at the seventh grade. The investigator assumed that the problems included in a textbook had the level of difficulty represented by the grade level of the textbook. The wording of the verbal problems was not altered except for changes of proper names so that the reading level was not changed. In some cases the numbers used in the problems were changed slightly. Five problems were included in each assignment. Fifteen sets of five problems each were prepared for both levels of third, fourth, fifth, and sixth grades, and ten sets were prepared for the other levels. Enough copies were prepared so that each child used his assignment sheet for his computation. Enough answer sheets were prepared for each level so that each class received several copies. Appendix A contains sample copies of assignment sheets, answer sheets, and a copy of pupil's personal record sheet.

Some details of the materials used in the study were formulated and tested in a pilot study conducted during the fall of 1965.

The readability level of each verbal problem level was checked by use of the Spache Readability Formula (10), and the Dale-Chall Readability Formula (10). The readability level matched the level designated as the problem level in ninety per cent of the problem sets. The readability level determined by the readability formulae was the instructional level which indicated appropriate reading material for reading instruction. The reading level of verbal problems should be lower than material used for reading instruction. Pupils should not be limited in their efforts to solve verbal problems because they can not read the conditions stated in the problem. Most of the materials used in this study were more difficult to read than the labeled grade level indicated.

Using results of the pilot study, it was determined that the low level problems were too easy for pupils having the corresponding grade equivalent score on the _Iowa Tests of Basic Skills_ (31), and the higher level problems were too hard for pupils with the corresponding grade equivalent score on the standardized test. If a pupil had a grade equivalent score in problem solving of third grade or below, raising this score by one grade provided the appropriate level for problem-solving assignments. If the pupil had a grade equivalent score of seventh grade or above, a sixth-grade assignment was appropriate.

The proposed study was discussed with principals in each school and individual conferences were held with the fifth-grade teachers agreeing to participate in the study. Materials and records of the pilot study were shown the teacher. Further discussion of procedures to be used in the present study followed the demonstration of materials to be used. Instruction sheets given to each teacher and principal are included in Appendix B.

Each teacher administered the arithmetic sections of the _Iowa Tests of Basic Skills_: _Form 2_ (31) to their mathematics classes during the week ending February 17, 1967. The tests were scored and the grade equivalent scores determined. The grade equivalent score on problem solving aided the teacher in deciding individual level assignments in the experimental group. Work began with the experimental materials on February 27, 1967.

After each child had completed two of the assignments, an analysis of the results was made and a decision made as to the necessity of changing his assignment to easier or more difficult problems. An effort was made to provide problem-solving assignments that were not too difficult nor too easy. Each pupil was allowed fifteen minutes to work independently on his problem-solving assignment. After finishing the assignment, the pupil checked his work with the appropriate answer sheet. The pupil made a record of his correct answers on his personal record sheet and used the answer sheet in his study of the problems that he failed to solve. The pupils of the experimental group worked the individualized problem-solving assignments three days a week for a period of ten weeks. Fifteen sets of problems were selected for most of the levels. Most of the pupils worked at one level for five weeks and then worked at a higher level for the last five weeks of the experimental period. The teacher in some cases varied this procedure for certain individuals.

During the ten weeks the experimental group worked problems at their ability level, the others worked with regular fifth-grade textbook materials. Each teacher kept a log of the time each group spent on verbal problems and planned pupil assignments so that the experimental group did not have the experimental problem-solving assignments in addition to assignments given the whole class.

The teachers involved in the study differed in their control of the time each group spent on verbal problems. Some were able to balance the time, but other teachers did not provide as much time on verbal problems for the control group as the experimental group used. Using the log of time spent on verbal problems kept by each teacher, the total time spent on verbal problems by the two groups in each class was determined. The control group as a whole averaged approximately one-half as much time on verbal problems as did the experimental group.

Since the teachers had pupils of the control group and the experimental group together in the same classroom, contamination of one group by the other was likely. The investigator discussed with each teacher ways that each might explain the experimental program to pupils to avoid excessive reaction of one group to the other. The teachers were encouraged to use their own judgment as to the management of the two groups to avoid possible contamination. Thus each teacher attempted to control this contamination in various ways. There was little evidence to indicate how well this factor was controlled. The uncertain control of contamination of the groups may have affected the results of the experimental study.

During the week ending May 12, 1967, the final standardized test was administered to all pupils in the twelve classes involved in the study. The arithmetic concepts section A-1 and the problem-solving section A-2 of the *Iowa Tests of Basic Skills: Form 3* (31) were given and scored.

Selection and Administration of Tests

To measure possible gains in achievement in the area of mathematics, the *Iowa Tests of Basic Skills: Forms 2 and 3*

(31) were selected by the investigator. The two arithmetic sections of the test provided adequate measurement of performance in the two areas being investigated. The coefficient of reliability for the arithmetic concepts test was .86. For the problem-solving section the coefficient of reliability was .82. The arithmetic concepts section A-1 for the fifth grade consisted of forty-two items. There were twenty-nine items on the problem-solving section A-2 of the standardized test. The pretest and the final test were administered by the teachers and scored by the investigator.

Each pupil's intelligence was measured by the *Otis Quick-Scoring Mental Ability Test, Beta Test, Form Em* (35). Most of the pupils were tested in the spring of 1966 by their teachers. The other students were tested in the spring of 1967 by school counselors. The median intelligence quotient for the twelve classes involved in the study was ninety-five. By using a range of median plus seven and a median minus seven for a measure of average pupils, three numerically balanced subgroups were formed. (See Table III, page 49.)

To separate the pupils into three numerically balanced subgroups based on reading ability, reading test scores (grade equivalent) on the *Iowa Tests of Basic Skills: Form 1* (31) were utilized. This form of the standardized test was administered by the teachers to all fifth graders in the school district in the fall of 1966. The median reading grade equivalent score for the twelve classes was fifty-one. By using a range of the median grade equivalent score plus five and the median grade equivalent score minus five as a measure of average reading ability, three numerically balanced subgroups based on reading ability were formed. (See Table IV, page 49.)

Statistical Analysis of Data

The results of the experimental use of individualized problem-solving assignments were determined by two criterion measures: (1) score gains in arithmetic concept achievement, and (2) score gains in problem-solving achievement. Using arithmetic concepts score gains as the dependent variable, analysis of variance technique was used to estimate variance due to sources such as sex, reading level, intelligence level, and the use of experimental materials. It was determined by the use of an F test whether the variance due to each source was significant at the five per cent level of confidence.

Analysis of variance, one-way classification, was used in testing each null hypothesis. In this analysis, the use of individualized problem-solving assignments and the use of regular fifth-grade textbook materials were considered the source of variation in one of the following categories of pupils: (1) all pupils, (2) boys, (3) girls, (4) high level readers, (5) average level readers, (6) low level readers,

TABLE III

NUMBER OF PUPILS IN SUBGROUPS BASED ON SEX AND INTELLIGENCE

	Male	Female	Total
EXPERIMENTAL GROUP			
High Intelligence	25	32	57
Average Intelligence	28	23	51
Low Intelligence	22	25	47
CONTROL GROUP			
High Intelligence	24	31	55
Average Intelligence	31	24	55
Low Intelligence	27	24	51

TABLE IV

NUMBER OF PUPILS IN SUBGROUPS BASED ON SEX AND READING LEVEL

	Male	Female	Total
EXPERIMENTAL GROUP			
High Reading	22	31	53
Average Reading	28	23	51
Low Reading	26	26	52
CONTROL GROUP			
High Reading	23	40	63
Average Reading	26	21	47
Low Reading	30	20	50

(7) high intelligence level, (8) average intelligence level, (9) low intelligence level, (10) boys of high reading level, (11) boys of average reading level, (12) boys of low reading level, (13) girls of high reading level, (14) girls of average reading level, (15) girls of low reading level, (16) boys of high intelligence, (17) boys of average intelligence, (18) boys of low intelligence, (19) girls of high intelligence, (20) girls of average intelligence, and (21) girls of low intelligence. The five per cent level of confidence was used in deciding whether to reject the null hypothesis.

The analysis described above was repeated with the other criterion measure--the score gains on the problem-solving section of the standardized test.

In order to balance the number of score gains in each cell, a table of random numbers was used to select twenty cases in each category. The actual number of cases in each category of both the experimental and the control group ranged from twenty to forty.

Assumptions and Limitations

A major function of this study involved evaluation of materials used in problem solving. It was assumed that the standardized test used in the study measured pupil achievement in mathematics. It was also assumed that the procedures actually used in connection with the experimental and control groups were those that were planned and described.

The study was limited in the following ways:

1. The subjects of the study were limited to twelve sections of fifth graders from one school district.

2. The duration of the study was limited to a period of ten weeks.

3. Materials used were limited to selections from mathematics textbooks on the Texas state-adopted list (see Appendix C) for grades two through seven.

4. The grade level of the materials selected was determined largely by the level of the textbook from which the problems were selected.

5. Activities involving the use of the special mathematics material by the teachers may not have been uniform from group to group.

6. The time variable was not completely controlled. The control group averaged working approximately one-half as long as the experimental group worked on verbal problems.

7. Procedures for preventing contamination of one group by the other were limited.

Explanation of Terms

Verbal problem solving. This activity involves the answering of a question or the finding of an element that satisfies a condition which is represented by words and other symbols in sentence form.

Achievement in mathematics. The score made on a standardized achievement test in mathematics was considered an indication of the pupil's achievement in mathematics.

Individualized assignments. Assignments of practice sets of verbal problems labeled as having a certain grade level of difficulty to a pupil having the ability to work problems of that level were considered individualized assignments.

Experimental group. Pupils who were given practice sheets in problem solving that matched their level of achievement in problem solving were considered the experimental group.

Methods. Methods referred to preliminary problem-solving activities of a pupil in his effort to simplify the stated conditions.

Problem-solving ability. Pupils who were able to find a solution to verbal problems without assistance were considered to have problem-solving ability. A score on the problem-solving section of a standardized achievement test was considered as a measure of this ability.

High reading level. Pupils making a grade equivalent score above fifty-six on the reading section of the Iowa Tests of *Basic Skills*: *Form 1* (31) were considered as a group having a high level of reading ability.

Average reading level. Pupils making a grade equivalent score forty-six to fifty-six inclusive on the reading section of the *Iowa Tests of Basic Skills*: *Form 1* (31) were considered as a group having average level of reading ability.

Low reading level. Pupils making a grade equivalent score below forty-six on the reading section of the *Iowa Tests of Basic Skills*: *Form 1* (31) were considered as a group having a low level of reading ability.

High intelligence level. Pupils having an intelligence quotient above 102 on the *Otis Mental Ability Test*: *Beta Test*, *Form Em* (35) were considered as a group having high intelligence.

Average intelligence level. Pupils having an intelligence quotient from 88 to 102 inclusive on the *Otis Mental Ability Test*: *Beta Test*, *Form Em* (35) were considered as a group having average intelligence.

Low intelligence level. Pupils having an intelligence quotient below eighty-eight on the *Otis Mental Ability Test*: *Beta Test*, *Form Em* (35) were considered as a group having low intelligence.

Summary

The study was designed to determine the effect of individualized assignments of verbal problems on the achievement of fifth-grade pupils in mathematics.

Forty-two null hypotheses involving various subgroups of the experimental and control groups were tested. Pupils for the experimental group were randomly selected from each of twelve sections of fifth-grade pupils in the same public school district. The other pupils in each section were used as the control group. For purposes of analysis, subgroups based on sex, reading ability, and intelligence were formed from the experimental and control groups.

The experimental group was given individualized verbal problems-solving assignments for a period of ten weeks. The control group worked with regular fifth-grade textbook materials.

A comparison of achievement in mathematics was made between the experimental group and the control group. A pretest and a final test were used to provide a raw score gain for each pupil. Analysis of variance was used to determine whether differences in score gains of the experimental group and the control group were due to some factor other than chance. A five per cent level of confidence was used in indicating the significance of score gain differences.

Limitations of the study included (1) the size of the sample, (2) length of the experimental period, (3) control of time both groups spent on problem solving, (4) control of contamination of one group by the other, and (5) the quality of problem-solving assignments.

CHAPTER III

PRESENTATION AND ANALYSIS OF DATA

The purpose of this study was to determine the effect of individualized assignments of verbal problems in mathematics on the achievement of fifth-grade pupils in mathematical concepts and problem solving.

Presentation of Data

The arithmetic sections of the *Iowa Tests of Basic Skills: Forms 2 and 3* (31) were administered to all pupils as a pretest and a final test. The individual pupil's raw score on the pretest was subtracted from his raw score on the final test to obtain a figure referred to as his score gain. Each pupil had a score gain on the arithmetic concepts section and a score gain on the problem-solving section.

The experimental and control groups were divided into twenty subgroups based on sex, reading ability, and intelligence. Twenty score gains were randomly selected from the basic subgroups to form equal cells for analysis. Combining the score gains of boys and girls for each level of reading and intelligence created cells having forty score gains.

The mean of the score gains in each category are presented in Table V.

TABLE V

THE MEAN SCORE GAINS BY GROUPS ON ARITHMETIC CONCEPTS TEST AND PROBLEM-SOLVING TEST

	Arithmetic Concepts		Problem-Solving	
	Experimental Group	Control Group	Experimental Group	Control Group
All pupils	1.25	.40	.84	1.16
Boys	1.41	.77	.67	.70
Girls	1.02	.03	1.01	1.55
High reading	.78	.28	.88	1.85
Avg. reading	1.45	-.25	1.50	.50
Low reading	1.53	1.18	.15	1.03
Boys, high reading	.35	.90	.60	.85
Boys, avg. reading	2.20	.05	1.70	-.55
Boys, low reading	1.90	1.40	-.30	1.80
Girls, high reading	1.20	-.35	1.15	2.85
Girls, avg. reading	.70	-.50	1.30	1.55
Girls, low reading	1.15	.95	.60	.25
High I. Q.	1.33	1.48	.30	1.80
Avg. I. Q.	1.13	.50	2.57	.80
Low I. Q.	1.05	1.33	.55	1.15
Boys, high I. Q.	2.70	2.45	-1.25	.95
Boys, avg. I. Q.	.40	-.20	2.95	-.05
Boys, low I. Q.	.45	2.25	.00	1.60
Girls, high I. Q.	-.05	.50	1.85	2.65
Girls, avg. I. Q.	1.85	1.20	2.40	1.65
Girls, low I. Q.	1.65	.40	1.10	.70

Analysis of Data

Score gains on the arithmetic concepts section and score gains on the problem-solving section of the Iowa Tests of Basic Skills were used as dependent variables with the experimental treatment and various subgroups as independent variables in two-way analysis of variance (19). To balance the cells, twenty measures were selected from each basic subgroup by using a table of random numbers. In the various groupings there was no significant variance at the five per cent level of confidence. The results of this analysis of variance is summarized in Tables VI through XI.

In analysis of variance with arithmetic concepts test score gains as the criterion variable and the experimental treatment as a source of variance (see Table VI, page 60), the obtained F value was 1.99 which was not significant at the five per cent level of confidence. The variance due to grouping based on sex of the subjects resulted in an F value of .99 which was not significant at the five per cent level of confidence.

With arithmetic concepts test score gains as the criterion variable and two treatments along with three subgroups based on reading ability as independent variables, an F value of .77 indicated no significant difference at the five per cent level of confidence due to reading ability. Variance due to the two treatments resulted in an F value of 1.99 which was not significant at the five per cent level of confidence. These results are summarized in Table VII, page 60.

Using the two treatments and three subgroups based on intelligence as sources of variance with the arithmetic concepts test score gains as the criterion variable, F values of .01 and .48 were obtained for the experimental treatment and intelligence levels respectively. Neither of these values was significant at the five per cent level of confidence (see Table VIII, page 61).

The analysis of variance with the criterion variable of problem-solving test score gains and the independent variables consisting of two treatments with two subgroups based on sex (see Table IX, page 61) resulted in F values of .25 and 1.14. Neither of these values was significant at the five per cent level of confidence.

As shown in Table X, page 63, variances due to three subgroups based on reading ability and the two treatments with problem-solving test score gains as the criterion variable were not significant at the five per cent level of confidence. The obtained values of F were .63 and .25.

With the criterion variable of problem-solving test score gains and independent variables based on two treatments and three subgroups based on intelligence, the F values obtained were .02 and .97. As shown in Table XI, page 63,

TABLE VI

ANALYSIS OF VARIANCE OF ARITHMETIC CONCEPTS TEST SCORE GAINS OF PUPILS IN EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO SUBGROUPS BASED ON SEX

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	43.3	1	43.3	1.99	n.s.
Sex	21.6	1	21.6	.99	n.s.
Within sets	5144.6	236	21.8		

TABLE VII

ANALYSIS OF VARIANCE OF ARITHMETIC CONCEPTS TEST SCORE GAINS BY EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO SUBGROUPS BASED ON THREE READING LEVELS

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	43.3	1	43.3	1.99	n.s.
Reading level	33.3	2	16.7	.77	n.s.
Within sets	5112.1	234	21.8		

TABLE VIII

ANALYSIS OF VARIANCE OF ARITHMETIC CONCEPTS TEST SCORE GAINS BY EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO SUBGROUPS BASED ON THREE LEVELS OF INTELLIGENCE

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	0.2	1	0.2	.01	n.s.
Intelligence	14.1	2	7.05	.48	n.s.
Within sets	3463.8	234	14.8		

TABLE IX

ANALYSIS OF VARIANCE OF PROBLEM-SOLVING TEST SCORE GAINS BY EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO SUBGROUPS BASED ON SEX

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	4.8	1	4.8	.25	n.s.
Sex	21.6	1	21.6	1.14	n.s.
Within sets	4473.8	236	19.0		

neither of these values was significant at the five per cent level of confidence.

Analysis of variance, one-way classification, was used in testing a number of null hypotheses. For the purpose of this study it was decided that an F value large enough to be significant at the five per cent level of confidence was needed to reject a null hypothesis.

The null hypothesis that the experimental group score gains in arithmetic concepts are no different from control group score gains was not rejected (see Table XII, page 64). With 1 and 400 degrees of freedom, an F value of 3.86 is significant at the five per cent level of confidence (19). Thus, the obtained F value of 2.00 with 1 and 238 degrees of freedom was not significant.

The F values for the variance in different subgroups were not large enough to be significant at the five per cent level of confidence. Consequently, none of the null hypotheses involving the arithmetic concepts test scores were rejected.

The null hypothesis that the experimental group score gains in problem solving are no different from the control group score gains in problem solving was not rejected (see Table XIII, page 65). With 1 and 400 degrees of freedom, an F value of 3.86 is significant at the five per cent level of confidence. Hence, the obtained F value of .27 with 1 and 238 degrees of freedom was not significant.

TABLE X

ANALYSIS OF VARIANCE OF PROBLEM-SOLVING TEST SCORE GAINS BY EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO THREE SUBGROUPS BASED ON READING LEVEL

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	4.8	1	4.8	.25	n.s.
Reading level	24	2	12	.63	n.s.
Within sets	4425.5	234	18.9		

TABLE XI

ANALYSIS OF VARIANCE OF PROBLEM-SOLVING TEST SCORE GAINS BY EXPERIMENTAL GROUP AND CONTROL GROUP SEPARATED INTO THREE SUBGROUPS BASED ON INTELLIGENCE

Source of variance	Sum of squares	Degrees of freedom	Estimate of variance	F	Significance at 5% level
Experimental group and control group	0.4	1	0.4	.02	n.s.
Intelligence	34.7	2	17.4	.97	n.s.
Within sets	4221	234	18		

TABLE XII

ANALYSIS OF VARIANCE OF ARITHMETIC CONCEPTS TEST SCORE GAINS OF PUPILS IN EXPERIMENTAL AND CONTROL GROUPS

Group	F	d. f. Between	d. f. Within	Test of null hypothesis at 5% level
All pupils	2.00	1	238	failed to reject
Boys	.74	1	118	failed to reject
Girls	1.17	1	118	failed to reject
High reading level	.25	1	78	failed to reject
Avg. reading level	2.35	1	78	failed to reject
Low reading level	.12	1	78	failed to reject
Boys, high reading	.10	1	38	failed to reject
Boys, avg. reading	3.08	1	38	failed to reject
Boys, low reading	.15	1	38	failed to reject
Girls, high reading	2.23	1	38	failed to reject
Girls, avg. reading	.43	1	38	failed to reject
Girls, low reading	.02	1	38	failed to reject
High I. Q. level	.04	1	78	failed to reject
Avg. I. Q. level	.60	1	78	failed to reject
Low I. Q. level	.08	1	78	failed to reject
Boys, high I. Q.	.05	1	38	failed to reject
Boys, avg. I. Q.	.34	1	38	failed to reject
Boys, low I. Q.	1.96	1	38	failed to reject
Girls, high I. Q.	.32	1	38	failed to reject
Girls, avg. I. Q.	.29	1	38	failed to reject
Girls, low I. Q.	.74	1	38	failed to reject

TABLE XIII

ANALYSIS OF VARIANCE OF ARITHMETIC PROBLEM-SOLVING TEST SCORE GAINS OF THE PUPILS IN EXPERIMENTAL AND CONTROL GROUPS

Group	F	d. f. Between	d. f. Within	Test of null hypothesis at 5% level
All pupils	.27	1	238	failed to reject
Boys	.002	1	118	failed to reject
Girls	.48	1	118	failed to reject
High reading level	.98	1	78	failed to reject
Avg. reading level	1.04	1	78	failed to reject
Low reading level	.85	1	78	failed to reject
Boys, high reading	.03	1	38	failed to reject
Boys, avg. reading	2.64	1	38	failed to reject
Boys, low reading	2.28	1	38	failed to reject
Girls, high reading	1.65	1	38	failed to reject
Girls, avg. reading	.04	1	38	failed to reject
Girls, low reading	.08	1	38	failed to reject
High I. Q. level	2.32	1	78	failed to reject
Avg. I. Q. level	4.29	1	78	rejected
Low I. Q. level	.39	1	78	failed to reject
Boys, high I. Q.	2.19	1	38	failed to reject
Boys, avg. I. Q.	4.95	1	38	rejected
Boys, low I. Q.	1.29	1	38	failed to reject
Girls, high I. Q.	.44	1	38	failed to reject
Girls, avg. I. Q.	.39	1	38	failed to reject
Girls, low I. Q.	.09	1	38	failed to reject

The subgroup consisting of pupils with average intelligence had test scores in problem solving that resulted in the rejection of the null hypothesis that there is no difference between problem-solving score gains of the experimental subgroup and control subgroup. With 1 and 78 degrees of freedom, an F value of 3.96 is significant at the five per cent level of confidence. Thus, the obtained F value of 4.29 was significant.

The subgroup consisting of boys with average intelligence had test scores in problem solving that resulted in the rejection of the null hypothesis that there is no difference between problem-solving score gains of the experimental subgroup and the control subgroup. With 1 and 38 degrees of freedom, an F value of 4.10 is significant at the five per cent level of confidence. Therefore, the obtained F value of 4.95 was significant.

None of the other null hypotheses concerning problem-solving scores and the subgroups other than the ones above were rejected.

Summary

The score gains of the pupils in this study were considered to be the difference in raw scores made on the pretest and the final test. Data were separated into various categories based on sex, reading ability, and intelligence.

With the score gains on the arithmetic concepts test as the criterion measure, no significant difference was found between the experimental groups and the corresponding control groups.

Pupils of the experimental subgroup having average intelligence had greater score gains in problem solving significant at the five per cent level of confidence than the corresponding control subgroup. The experimental subgroup consisting of boys with average intelligence had greater gains in problem-solving at the five per cent level of confidence than the corresponding control subgroup. With the score gains on the problem-solving test as the criterion measure, no significant difference was found between other experimental groups and corresponding control groups.

CHAPTER IV

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

Summary

What effect did individualized assignments of verbal problems have on the achievement of fifth-grade pupils in mathematical concepts and problem solving? The study provided an answer through the testing of the following null hypotheses:

1. There is no difference in the score gains on the arithmetic concepts test by pupils using individualized problem-solving assignments and pupils using regular textbook materials.

2. There is no difference in the score gains on the problem-solving test by pupils using individualized problem-solving assignments and pupils using regular textbook materials.

Additional information was sought by testing these hypotheses for subgroups: (1) boys, (2) girls, (3) high reading level, (4) average reading level, (5) low reading level, (6) boys, high reading, (7) boys, average reading, (8) boys, low reading, (9) girls, high reading, (10) girls, average reading, (11) girls, low reading, (12) high intelligence level, (13) average intelligence level, (14) low intelligence level,

(15) boys, high intelligence, (16) boys, average intelligence, (17) boys, low intelligence, (18) girls, high intelligence, (19) girls, average intelligence, and (20) girls, low intelligence.

Twelve classes of fifth graders in five different schools participated in the study. One-half of the members of each class were randomly selected to use the specially prepared individualized problem-solving assignments. The other members of each class worked on assignments from the regular fifth-grade mathematics textbooks. The mathematics teachers gave their classes the arithmetic sections of the Iowa Tests of Basic Skills: Form 2 before using the specially prepared materials. The investigator scored the tests and reported the grade equivalent scores to the teachers. The teachers used these scores on the problem-solving test in deciding what level of problems each member of the experimental group should be assigned. The experimental group worked on these individualized problem-solving assignments three days a week for ten weeks.

In order to prevent contamination of one group by the other, each teacher planned activities involving the control group while the experimental group worked on the individualized assignments. The control group used regular fifth-grade textbook materials for work with verbal problems. Efforts were made to explain the use of the special materials by selected pupils so that all understood that a pupil's status in the class did not depend on the use of the special materials.

The arithmetic concepts and problem-solving sections of the Iowa Tests of Basic Skills: Form 3 were given to all class members by their teachers at the end of the ten weeks. The investigator scored the tests and tabulated the scores along with the pretest scores, intelligence quotients, and reading grade equivalent scores of the twelve classes. Complete data were obtained for 316 children.

With the raw score gains on the arithmetic concepts section of the standardized test as criterion measures, analysis of variance was used in testing the null hypotheses concerning the experimental treatment and the various subgroups subject to the treatment. Analysis of variance using the raw score gains on the problem-solving section of the standardized test as criterion measures was utilized in testing the null hypotheses concerning the experimental treatment and the various subgroups subject to the treatment.

The following is a summary of the findings of this study:

1. The difference in score gains on problem solving was significant at the five per cent level of confidence between pupils of average intelligence in the experimental group and pupils of average intelligence in the control group, with the experimental subgroup having the greater gains.

2. There was a difference significant at the five per cent level in problem-solving score gains of boys with average intelligence in the experimental group and problem-solving score gains of boys with average intelligence in the control group with the experimental subgroup having the greater gains.

3. There was no significant difference in the score gains on arithmetic concepts by the experimental group or any of its subgroups and the control group or corresponding subgroups.

4. There was no significant difference in the problem-solving score gains of the experimental group as a whole and the control group as a whole.

5. There was no significant difference in the problem-solving score gains of the following experimental subgroups and corresponding control subgroups: (1) boys, (2) girls, (3) high reading, (4) average reading, (5) low reading, (6) boys, high reading, (7) boys, average reading, (8) boys, low reading, (9) girls, high reading, (10) girls, average reading, (11) girls, low reading, (12) high intelligence (13) low intelligence, (14) boys, high intelligence, (15) boys, low intelligence, (16) girls, high intelligence, (17) girls, average intelligence, and (18) girls, low intelligence.

Conclusions

With the limitations of the study in mind, the following conclusions were drawn from the findings:

1. Fifth-grade pupils using individualized problem-solving assignments do not perform better in the area of mathematical concepts than pupils using regular fifth-grade textbook materials.

2. Considering all pupils without regard to subgroups, fifth-grade pupils using individualized problem-solving assignments do not solve problems better than pupils using regular fifth-grade textbook materials.

3. Fifth-grade pupils of average intelligence perform better in problem solving after using individualized problem-solving assignments than children of average intelligence using regular fifth-grade textbook materials.

4. As a subgroup, boys of average intelligence perform better in problem solving after using individualized problem-solving assignments than boys of average intelligence using regular fifth-grade textbook materials.

Implications

The following implications are a result of the findings of this study:

1. Preparing special below grade level and above grade level assignments in problem solving of the type used in this

study is not essential in the teaching of mathematics to fifth-grade children.

2. Intensive practice on problem solving is not sufficient in itself for causing significant gains in problem-solving achievement.

Recommendations

In the light of the findings of this study the following recommendations are made:

1. Similar experiments should be executed in which the same amount of material for individualized problem-solving assignments is used over a longer period of time.

2. Studies similar to the present one should be utilized for study of provision for individual differences in reading and other subject areas.

3. Studies should be implemented to determine the relationship of various personality traits to success in problem solving and ways that a teacher can cause children to develop the traits needed for successful problem solving.

4. More study should be undertaken to determine the objectives for the use of problem-solving materials and ways of measuring the achievement of these objectives.

5. A similar study should be undertaken that would include another subgroup classification such as three levels of problem-solving ability.

6. A study similar to the present investigation should be implemented that would include an analysis of pupil attitudes toward verbal problem solving before the use of the special materials and pupil attitudes toward problem solving after using the materials for the experimental period.

7. A study should be undertaken to determine the effect that the use of individualized problem-solving assignments along with instruction in two or more methods of problem solving would have on children's achievement in problem solving.

8. A study similar to the present one should be implemented with a larger number of pupils and the use of separate sections of fifth graders for experimental and control groups. Contamination safeguards should be carefully planned and executed.

9. Replication of present study should be completed using more instruments for testing pupil achievement. Special consideration should be given to measuring achievement of pupils above and below average in ability.

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APPENDIX

APPENDIX A

Sample of Materials Used by Experimental Group

Problem Solving

21 Set 1

1. In the garden 54 bees buzzed around the flowers. In the hives 42 worked to make honey. What a great hum all _______ of these bees made!

Answer _________

2. Alice is having a birthday party. She is 8 years old. In how many years will she be 15 years old?

Answer _________

3. Father took Kathy and Gary to the amusement park. He bought each of them 8 tickets. How many tickets did he buy?

Answer _________

4. Bill misspelled 7 of 25 words. How many words did he spell correctly?

Answer _________

5. Bob walks 5 blocks to school. Tom walks 4 blocks to school. How many are 5 blocks and 4 blocks?

Answer _________

CN

Problem Solving

21 Set 2

1. Robert collects baseball pictures. He had 78 baseball pictures. He gave 25 to his brother. How many does Robert have now?

Answer __________

2. There were 7 books on the shelf. Then 2 more books were placed on the shelf. How many books were on the shelf?

Answer __________

3. Andy pasted 12 pictures of airplanes and 17 pictures of boats in his scrapbook. He pasted __________ more pictures of boats than of airplanes.

Answer __________

4. Joe bought 2 pencils for 8 cents each. How much did he spend?

Answer __________

5. Jane is having a birthday party. She is 8 years old. In how many years will she be 15 years old?

Answer __________

CN

Problem Solving

22 Set 1

1. George has 96 marbles. He bought 30 of them this year. How many marbles did he have last year?

Answer ________

2. At the dairy the class saw 46 cans of milk placed on a truck. A second truck took on 37 cans of milk. ________ cans of milk were loaded on the two trucks.

3. Lou's grandmother is 57 years old. Lou's mother is 29 years old. How many years older than her mother is her grandmother?

Answer ________

4. Jack has 38 marbles. Steve has 19 marbles. How many marbles do both boys have?

Answer ________

5. Jean had 16 stamps. She gave Sue 7 stamps. Jean had ________ stamps left.

Problem Solving

22 Set 2

1. Tom had 12 pencils. He lost 5 of them. How many are left?

Answer __________

2. Sue had 20¢. She wanted to buy a book for 30¢. How much more money does she need?

Answer __________

3. John has four tops. He gives them all to Sam. How many does John have left?

Answer __________

4. Bill has a nickel. Joe has a nickel and a penny. How many cents do they have together?

Answer __________

5. Sam has fourteen apples, and Bill has nine less. How many does Bill have?

Answer __________

CN

Problem Solving

31 Set 1

1. Joe has 6 new books and 5 old books. How many books has he?

Answer ________

2. A farmer had 28 turkeys. He sold 9 turkeys. How many turkeys has he left?

Answer ________

3. Wool from one sheep will make four coats for children. Wool from 3 sheep will make how many coats for children?

Answer ________

4. Jane had a piece of cloth 63 inches long. She cut off and used a piece 28 inches long. How long was the piece of cloth that was left?

Answer ________

5. James found a board 6 feet long to mend the back steps. He divided the board into 2 equal pieces. How long was each piece?

Answer ________

CN

Problem Solving

31 Set 2

1. Tom caught 5 fish. Joe caught 4 fish and Jim caught 7 fish. How many fish did the three boys catch?

Answer ________

2. There are 4 windows in the room. The children put 4 plants on each window sill. How many plants are there altogether?

Answer ________

3. Sue had 25 cents. Her mother gave her 15 cents. How many cents did Sue have then in all?

Answer ________

4. The girls made 14 cookies. They put pink sugar on one half of them. What is one half of 14?

Answer ________

5. A new pencil is 7 inches long. How many pencils put end to end would equal 35 inches?

Answer ________

CN

Problem Solving

32 Set 1

1. A case of eggs at the hatchery held 144 eggs. The man threw away 26 cracked eggs. How many of the eggs did he have left?

Answer ________

2. Jack's father bought Jack a pair of shoes for $6.49 and a shirt for $2.19. How much money did he spend?

Answer ________

3. The United States opened the Panama Canal in 1914. How old was the canal in 1963?

Answer ________

4. Mrs. Jones gave 4 apples each to 6 boys. How many apples in all did she give to the boys?

Answer ________

5. For the book fair, the girls bought colored paper to cover the tables. The paper was 8¢ a yard. They needed 24 yards. What did they pay for the paper?

Answer ________

CN

Problem Solving

32 Set 2

1. Sally's mother had $5.00. She spent $3.65 for meat. How much money did she have left?

Answer ________

2. At the school play there were 237 parents and 126 children. How many people were at the school play?

Answer ________

3. Tommy cut some pieces of string 6 feet long to tie bundles of paper. How many 6-foot pieces could he cut from 36 feet of string?

Answer ________

4. Carl bought 6 packages of seed to plant in the garden. The seed cost $ .15 a package. How much did the six packages cost?

Answer ________

5. 540 pieces of candy. 6 pieces in each bag. How many bags?

Answer ________

CN

Problem Solving

41 Set 1

1. Kay and Jane put their money together so they could buy their father a tie. Kay had $1.15 and Jane had $ .75. How much could they spend for the tie?

Answer ________

2. Joe helps at his father's grocery store after school. He put 32 cans of peaches on the shelf in 8 rows. How many were in each row?

Answer ________

3. There were 24 clowns in the clown parade. Eight clowns were riding. The others were walking. How many clowns were walking?

Answer ________

4. Jane weighs 63 pounds. Sue weighs 59 pounds. How many pounds less than Jane does Sue weigh?

Answer ________

5. A gas tank on a sports car holds 17 gallons when full. 9 gallons have been put in. How many more are needed to fill the tank?

Answer ________

CN

Problem Solving

41 Set 2

1. Joe's father bought 14 gallons of gasoline at one station and 9 at another. How much did he buy altogether?

Answer ________

2. Bob found 203 pecans and 185 hickory nuts. How many nuts did he find?

Answer ________

3. Kay read four stories. The stories were 46 pages long, 54 pages long, 38 pages long and 43 pages long. How many pages did Kay read in all?

Answer ________

4. Mrs. Jones paid $1.65 each for 3 rosebushes. How much did all of the bushes cost?

Answer ________

5. Jane needs 45¢ to buy a ring. She has saved 9 pennies. The money that she needs is how many times more than the money she has?

Answer ________

CN

Problem Solving

42 Set 1

1. Mrs. Yeager spent $3.35 for lunch during the week. She bought 5 lunches and paid the same amount for each of them. How much did each lunch cost?

Answer __________

2. Mr. Bounds planted 9 rows of seedlings and put 26 seedlings in each row. How many seedlings did he plant?

Answer __________

3. How many yards are there in 33 feet? (1 yard = 3 feet)

Answer __________

4. Nan said it was 42 days until her birthday. How many weeks was this?

Answer __________

5. Driving. 47 miles each hour. 25 hours. How far?

Answer __________

CN

Problem Solving

42 Set 2

1. Judy sold 7 children's tickets to the school play at 35¢ each, and a ticket to her mother for 75¢. How much money should Judy have received for the tickets?

Answer ________

2. John had 24 rabbits to sell. His uncle said that he would take 3/4 of them. How many rabbits can John sell to his uncle?

Answer ________

3. There are 1760 yards in a mile. What is the distance in yards that a car travels on a road that is 3 miles long?

Answer ________

4. Two different recipes for cookies call for 3/4 pound and 1/2 pound of butter. How much butter will be needed to make the cookies?

Answer ________

5. What number is 5 greater than 8 times 9?

Answer ________

CN

Problem Solving

51 Set 1

1. There are 144 square feet in a rectangular rug. The rug is 9 feet wide. How long is it?

Answer ________

2. Eighteen boys are divided into two equal teams. How many are on each team?

Answer ________

3. The winner of a bicycle race traveled 160 miles in 8 hours. What was his average speed?

Answer ________

4. Larry had $4.00 to spend. He went to the hobby shop and bought a model airplane costing $1.50, a tube of glue for 19¢, and a small bottle of paint for 15¢. How much did he spend? How much money did he have left?

Answer ________

5. How much change will you get from a $2 bill if you buy 6 cans of corn at 24¢ a can?

Answer ________

CN

Problem Solving

51 Set 2

1. Bob had 72 pennies in a collection. He gave 29 of them to Andy. How many pennies did he have left?

Answer ________

2. The Jones family uses 3 pounds of butter a week. Find the cost of butter used in 4 weeks at $ .76 a pound.

Answer ________

3. How much does 1/4 yard of ribbon cost at 80 cents a yard?

Answer ________

4. The distance from Carter to Dover is 27 7/10 miles. How many miles in the round trip?

Answer ________

5. A jet airliner traveled 3250 miles in 5 hours. How many miles did it average per hour?

Answer ________

CN

Problem Solving

52 Set 1

1. A water tank contained $5\frac{1}{2}$ gallons of water; 2 1/8 gallons were used. How much water was left in the tank?

Answer _________

2. There are 5,100 pounds of apples to be shipped in boxes containing 30 pounds each. How many boxes will be filled with apples?

Answer _________

3. Harold walked 3 3/4 miles in the morning and 2 3/5 miles in the afternoon. How many miles did he walk that day?

Answer _________

4. A rectangular piece of metal is 8 feet by 13 feet. How much does it weigh if each square foot weighs 2 pounds?

Answer _________

5. If the number 34 is increased by one half of itself, what is the resulting number?

Answer _________

CN

Problem Solving

52 Set 2

1. One day Larry gathered 136 eggs. How many dozen did he gather?

Answer ________

2. In the mountains the family car averaged 11.8 miles per gallon of gasoline, while in level country the car averaged 15.6 miles per gallon. How many more miles per gallon did the car average in level country than in the mountains?

Answer ________

3. A rectangle is 3 feet long and 2 feet wide. What is the ratio of its length to its width?

Answer ________

4. Three boys on a camping trip went to a spring to get drinking water for their camp. Each boy carried one gallon of water. What was the total weight of the water if one gallon weighs 8 1/3 pounds?

Answer ________

5. Mary has 2 dollar bills, 2 fifty-cent pieces, 3 quarters, 7 dimes, 3 nickels, and 18 pennies in her purse. How much money is this?

Answer ________

CN

Problem Solving

61 Set 1

1. It is 2525 miles from Los Angeles to New York by air. How many miles an hour must a jet plane fly to make the flight in 5 hours?

Answer _________

2. If James earns \$ .50 per hour, how long will it take him to earn \$25?

Answer _________

3. A truck and its load of coal weighed 14,870 pounds. The empty truck weighed 5996 pounds. Find the weight of the coal.

Answer _________

4. Bill withdrew 2/3 of his \$15 savings from the bank. He spent 1/5 of the amount he withdrew. How much did he spend?

Answer _________

5. Joe bought 5 cans of pears. Each can cost \$ .34. He gave the clerk a five dollar bill. How much change should he receive?

Answer _________

CN

Problem Solving

61 Set 2

1. The cost of repairing the foundation of a building is \$6495. \$2480 of this amount was the cost of labor. How much was spent for other things?

Answer ________

2. How many pounds of potatoes can you buy for \$0.50 if the potatoes cost 6¢ per pound?

Answer ________

3. A mile contains 5280 feet. How many feet are there in a quarter of a mile?

Answer ________

4. On four tests Joe made scores of 85, 90, 67, 78. To the nearest tenth, what was his average score on these four tests?

Answer ________

5. At 36 cents a dozen how much does 4 oranges cost?

Answer ________

CN

Problem Solving

62 Set 1

1. Mr. Rogers used 3 quarts of paint to cover 225 square feet of surface. He has another 6 quarts of paint of the same kind. How many square feet might he expect to cover with it?

Answer ________

2. Mr. Johnson drove 1125.9 miles in March, 987.6 miles in April, and 1417.8 miles in May. On the average, how far did he drive each month?

Answer ________

3. Mrs. Jones invited 15 people to a party. She planned to buy 3 pounds of meat for each 8 guests. How many pounds of meat should she buy for the party?

Answer ________

4. A pitcher won 19 of the 26 games that he pitched. What per cent of his games did he win?

Answer ________

5. Betty sold 17 tickets for a play. This was 10% of all tickets sold. How many tickets were sold?

Answer ________

CN

Problem Solving

62 Set 2

1. Virgil gained 5.25 pounds in 6 months. To the nearest hundredth pound, what was his average gain in each of the months?

Answer ________

2. Mary wanted to earn money to buy a $24.50 bicycle. At $ .75 an hour, how many hours would she have to work to earn the money?

Answer ________

3. Arthur earned $1.80 in 6 hours. At this rate, how much did he earn in 2 hours?

Answer ________

4. Joe spelled correctly 85% of the words on a 50-word test. How many words were spelled correctly?

Answer ________

5. If a number is increased by 16, it will be 3 times as large as 15. What is the number (original)?

Answer ________

CN

Problem Solving

71 Set 1

1. Don had 18 baseball cards. After he bought 24 more, he had how many baseball cards?

Answer ________

2. The Cranes have a large garden 50 feet by 124 feet. How many square feet are available for planting?

Answer ________

3. A manufacturing company's actual sales were 110% of the estimated sales of $200,000. What were the actual sales?

Answer ________

4. Jim bought 2 pencils priced at 3 for 15¢ and 3 erasers priced at 4¢ each. How many cents in all did he spend for these articles?

Answer ________

5. Ken had more than $9.10. He then earned $2.75, but he still had less than $14.00. How much money did Ken have before he earned the $2.75?

Answer ________

CN

Problem Solving

71 Set 2

1. Lois worked in a bakery 4 days last week. She worked an average of 1 1/2 hrs. per day. How many hours in all did she work during the 4 days?

Answer ________

2. Alice saves $4 out of each $9 that she earns. If she continues to save at this rate, how many dollars will she have saved when she has earned $216?

Answer ________

3. In a town of ten thousand population, twenty-two hundred attend school. What part of the population is in school?

Answer ________

4. A meat roast that weighed 6 pounds decreased in weight while cooking to 3.9 pounds. What was the per cent of decrease?

Answer ________

5. Joe has a piece of plywood in the shape of a parallelogram with sides 9 ft. and 17 1/2 ft. What is the perimeter of the piece of plywood in inches?

Answer ________

CN

21 Answer Sheet 1

21
Set 1

1. Answer 96
2. Answer 7 years
3. Answer 16
4. Answer 18
5. Answer 9

21
Set 2

1. Answer 53
2. Answer 9
3. Answer 5
4. Answer 16 cents
5. Answer 7 years

21
Set 3

1. Answer 17
2. Answer 57
3. Answer 1 year
4. Answer 365 days
5. Answer 16

21
Set 4

1. Answer 4
2. Answer 7 cents
3. Answer 8
4. Answer 9¢
5. Answer 8

21
Set 5

1. Answer 4¢
2. Answer 14 minutes
3. Answer 26
4. Answer 10
5. Answer 14

21
Set 6

1. Answer 13
2. Answer 16 cents
3. Answer 9 cents
4. Answer 12
5. Answer 9

42 Answer Sheet 1

42
Set 1

1. Answer $ .67
2. Answer 234
3. Answer 11 yards
4. Answer 6 weeks
5. Answer 1175 miles

42
Set 2

1. Answer $ 3.20
2. Answer 18
3. Answer 5280 yards
4. Answer 1 1/4 pounds
5. Answer 77

42
Set 3

1. Answer $ .45
2. Answer 216 miles
3. Answer 74 yards
4. Answer 495 mph
5. Answer 2

42
Set 4

1. Answer $12.30
2. Answer 1 1/2 quarts
3. Answer $ 6.25
4. Answer 1344 miles
5. Answer 6 feet

42
Set 5

1. Answer 450,000 pounds
2. Answer 4 o'clock
3. Answer 31
4. Answer 1 3/4 pounds
5. Answer 15

42
Set 6

1. Answer 128 miles
2. Answer 150 days
3. Answer 12 years
4. Answer 18
5. Answer $ .21

NAME ____________________ School ____________
Section ____________

PROBLEM SOLVING RECORD

Level	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10	Set 11	Set 12	Set 13	Set 14	Set 15
7 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
6 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
6 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
5 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
5 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
4 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
4 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
3 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
3 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
2 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
2 1	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o
1 2	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o	o o o o o

APPENDIX B

Instructions for Teacher

RESEARCH IN THE USE OF INDIVIDUALIZED ASSIGNMENTS OF VERBAL PROBLEMS

Will independent practice with verbal problems matched in difficulty to pupil's ability result in greater achievement in arithmetic by certain fifth grade pupils? The hypothesis of the investigator is that a pupil's experience with verbal problems will result in more learning if the problems are hard but not too hard for the individual pupil.

One half of each section of fifth graders will use the special materials and the other half of the class will practice on material in text or on material provided by the teacher. Pupils in each class that are to use the special materials will be randomly selected by the investigator. They will be called the experimental group,and the others will be called the control group.

All will be given the arithmetic section of the Iowa Tests of Basic Skills at the beginning of the study. At the end of ten weeks they will be given another form of the same test. The teacher will use results of first test to aid her in deciding on the problem solving level of each pupil in the experimental group. Each member of this group will be given a set of problems of his level to solve independently. Each pupil will be allowed fifteen minutes to try the five problems. At the end of this time, he will check his paper with aid of an answer sheet. He will record the results on a personal record sheet. He will be allowed about five minutes to attempt any that he missed by using the answer sheet for help. This work will be done on another sheet of paper. The original work on problem set will be collected by the teacher. The other members of the class will practice on material in textbook at the time the experimental group is working with the special material.

When the teacher notices that the problem sets are too easy or too hard for a pupil, she will instruct the pupil to try a different level set the next time. The teacher may discuss methods of problem solving at another time. The special materials are not to be solved by any particular method. The emphasis should be on choosing a method to suit the problem or the individual's preference. The form of the answer is not critical in this activity except that measurements should include the unit of measure as part of answer.

There will be thirty sets of problems to solve in a ten week period. This means that three per week would be desirable. A preliminary study using these materials seemed to indicate that the pupils made large gains in areas of arithmetic other than problem solving. This means that in all probability their time will be well spent.

RESEARCH ON PROBLEM SOLVING

Suggestions for teachers participating in research:

Administer the arithmetic part of Iowa Tests of Basic Skills to all pupils as early as possible. Test booklets and answer sheets will be available soon. Use Form 2 of the test. It may be given at one period or at two periods on successive days. It requires 65 minutes. The test results and names of pupils in the experimental group will be given each teacher as soon as possible.

It is hoped that use of materials can begin on February 27. They will be used for ten weeks and then another form of the test will be given all pupils of each participating teacher.

The teacher may manage the materials as she likes. A monitor system may help. Pupils should not receive help while trying problems. When pupil completes the practice sheet, he scores his paper and marks the results on his own record sheet. The teacher should notice personal record sheet of each. She will decide whether a pupil is to try a higher level or a lower level. No pupil will stay at one level for more than 15 sets (10 sets at 21 or 22 level). If pupil gets all correct or none correct for two successive sets, the teacher should consider moving pupil to another level.

Each teacher should keep up with the amount of time that the control group are working verbal problems during the time the experimental group are using their verbal problem worksheets. This information can be used in estimating the ratio of time spent on problem solving by the two groups. This information is to be included in the written report of the study.

It may be that the participating teacher will want to assign the whole class some practice exercises and allow the practice sets of verbal problems to substitute for a certain number of the regular exercises. It is hoped that the experimental group do not feel that the special materials are just added on to regular practice by whole class.

Pupils should understand that word problems are supposed to be hard to do. The pupil is not expected to know what to do immediately. He is expected to figure out what to do and then do it. These materials include problems of the proper difficulty for all pupils.

APPENDIX C

State-Adopted Mathematics Textbooks

STATE-ADOPTED MATHEMATICS TEXTBOOKS

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Deans, Edwina, Robert B. Kane, and Robert A. Oesterle. Modern Mathematics Labtext 2. New York: American Book Company, 1964.

_______. Developing Mathematics. New York: American Book Company, 1963.

Deans, Edwina and others. Understanding Mathematics. New York: American Book Company, 1963.

_______. Learning Mathematics. New York: American Book Company, 1963.

_______. Unifying Mathematics. New York: American Book Company, 1963.

Educational Research Council of Greater Cleveland. Greater Cleveland Mathematics Program 2. Chicago, Illinois: Science Research Associates, Inc., 1961, 1962.

Eicholz, Robert E. and others. Elementary School Mathematics, Book 2, Book 3, Book 4, Book 5, and Book 6. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1963.

Gundlach, Bernard H., Ronald C. Welch, and Edward G. Buffie. Sets, Numbers, Numerals 2. River Forest, Illinois: Laidlaw Brothers, 1965.

McSwain, E. T. and others. Arithmetic 3. River Forest, Illinois: Laidlaw Brothers, 1965.

_______. Arithmetic 4. River Forest, Illinois: Laidlaw Brothers, 1965.

_______. Arithmetic 5. River Forest, Illinois: Laidlaw Brothers, 1965.

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Morton, R. L. and others. Modern Arithmetic Through Discovery, Book 3, Book 4, Book 5, and Book 6. Morristown, New Jersey: Silver Burdett Company, 1963.

Suppes, Patrick. Sets and Numbers - Book 2. Syracuse, New York: The L. W. Singer Company, Inc., 1965.

Van Engen, Henry and others. Seeing Through Mathematics, Book 1. Chicago, Illinois: Scott, Foresman and Company, 1962, 1961.