A TEST OF THE EQUIVALENT-RISK CLASS HYPOTHESIS AND A MULTIVARIATE ANALYSIS OF FIRMS' BUSINESS RISK DISCRIMINATORY CHARACTERISTICS

A Dissertation

Presented to

the Faculty of the Department of Finance

College of Business Administration

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

Charles Russell Idol

August, 1974

ACKNOWLEDGEMENTS

I would like to thank the members of my dissertation committee who extended to me their assistance during the preparation of this paper. Professor Steven Bolten offered a considerable amount of his time in aiding me research the literature and formulate my research objectives and methodology. Professors William Locander and William Sargent were particularly helpful to me with the quantitative aspects of the research. In addition, many useful suggestions were rendered throughout the development of the paper by Professors Francis Yeager and William Kretlow. My wife Carol is admired for the patience she evinced translating my atrocious handwriting and typing the numerous drafts. For her willingness to invest with me a considerable amount of prime time from our young happy years into the program which was culminated by this dissertation, a very special word of thanks is given.

A TEST OF THE EQUIVALENT-RISK CLASS HYPOTHESIS
AND A MULTIVARIATE ANALYSIS OF FIRMS'
BUSINESS RISK DISCRIMINATORY CHARACTERISTICS

An Abstract of a Dissertation
Presented to
the Faculty of the Department of Finance
College of Business Administration
University of Houston

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

---

by
Charles Russell Idol
August, 1974

ABSTRACT

The purpose of this paper was twofold: 1) to retest the assumptions of the equivalent-risk class hypothesis, and 2) to identify a set of business risk discriminatory characteristics.

The equivalent-risk class hypothesis assumes firms grouped according to industry classifications are both intragroup business risk homogeneous and intergroup business risk heterogeneous. The findings and implications of many significant research studies in the basic areas of corporate finance and investment theory are contingent on the validity of these two assumptions. Previous attempts to test the assumptions of the equivalent-risk class hypothesis have had conflicting conclusions. Further, the methodologies and business risk measures employed by these test efforts have been challenged. Hence, the lack of consistency in the results of these previous test attempts and the general acceptance in the literature of the assumptions of the equivalent-risk class hypothesis provided justification for a retest of the equivalent-risk class hypothesis' assumptions.

Before testing these assumptions, careful attention was given to both the development of a theoretically sound business risk measure and the appropriateness of the testing methodology. Neither assumption of the equivalent-risk class hypothesis was substantiated by the research findings. These findings strongly suggest industry classifications of firms are poor business risk discriminators for financial acamedicians and practitioners.

If the assumptions of the equivalent-risk class hypothesis are invalid, researchers must find new methods for business risk discrimination among firms. One such method was presented in this paper.

Thirty-five financial and operating variables (characteristics) were calculated for firms in two distinctly different business risk groups. A stepwise multiple discriminant analysis (MDA) program was applied to this data. The results of the MDA revealed discriminant functions containing only a small number of size and dividend policy related variables could correctly classify approximately 90% of the firms in the study into their respective business risk classes. Further, in the presence of size and dividend policy related variables, variables associated with long and short term capital turnover, profitability, and financial leverage were poor business risk discriminators among firms. Finally, the lower business risk firms were characterized by larger size (total assets) and more stable dividend policies with higher dividend payouts than the high business risk firms.

The results of the MDA should be encouraging to the financial community, for they imply firms can be categorized into their respective business risk classes by observing a small set of financial characteristics. Further research is needed in this area to develop the implications of these research findings into a more universal working model.

TABLE OF CONTENTS

LIST OF TABLES

INTRODUCTION

The primary emphasis of this research paper is on firms' business risk. Business risk is that aspect of a firm's total risk characteristics which is attributed to the firm's volatility in before tax operating profits. This volatility in the firm's operating profits can result from several characteristics of the firm such as stability of sales, changes in market and production technology, labor force behavior, quality of real production assets, quality of management, nature of the production process, nature of the raw materials market, governmental impacts of new legislation, and synchronization of operating profits with national and international economic conditions.

Previous research studies in the finance related areas have given too little attention to firms' business risk behavior. Typically, researchers have attempted to hold the effects of a firm's business risk constant by assuming firms grouped by industry classifications are both intragroup business risk homogeneous and intergroup business risk heterogeneous. These two assumptions have become known as the equivalent-risk class hypothesis. Thus, by selecting firms in only one industry, it has previously been assumed the interrelationships between other financial variables could be analyzed without bias from or need for specification of the firms' business behavior. Many classical financial research efforts on such basic topics as financial leverage and the cost of capital, valuation of the firm, dividend policy effects on the firm's value, and portfolio management have assumed business risk could be dealt with in this manner. Clearly, if the assumptions about industry classification of firms and firms' business risk behavior (the equivalent-risk class hypothesis) are invalid, the results of these previous research efforts may become somewhat suspect as to their validity.

Attempts have been made to test the validity of the equivalent-risk class hypothesis. However, little information has been gained from these attempts, since their results have been conflicting. Hence, the lack of consistency in the results of previous attempts to test the validity of the equivalent-risk class hypothesis and the preponderance in the literature of the validity of the equivalent-risk class hypothesis provide a justification for this research.

The first chapter presents a review of the literature of the equivalent-risk class hypothesis, research objectives, and research methodology. Several business risk measures are discussed and a proposed business risk measure is developed in Chapter 2. Chapter 3 contains tests of the two assumptions of the equivalent-risk class hypothesis. Then, an examination of key business risk discriminatory characteristics using multiple discriminant analysis is given in Chapter 4. The final chapter presents a summary of the research findings and a discussion of possible implications for future research.

CHAPTER 1

RESEARCH JUSTIFICATION, OBJECTIVES, AND METHODOLOGY

Section A - Research Justification

Empirical hypothesis testing which attempts to examine the relationships between a dependent variable and a set of explanatory variables is subject to erroneous conclusions resulting from problem misspecification and/or multicollinear relations among explanatory variables which may have been omitted. This problem is pertinent in the financial literature which attempts to study the effects of financial leverage, growth, and dividend policy on the firm's cost of capital. In such studies, attempts are usually made to circumvent these problems by grouping firms into homogeneous groups with respect to the omitted variables. Elton and Gruber [11] [12] discussed the need for homogeneous groups in economic hypothesis testing and the ill consequences which can occur if no attempt is made to hold constant the effects of omitted explanatory variables.

For example, consider a study which is attempting to examine the effect of financial leverage (as measured by a firm's debt-equity ratio) on the cost of equity capital (K_e). Further, suppose the firms in the sample were either very high or very low business risk firms. A scatter diagram of such firms is shown in Figure 1-1. Firms in the high business risk group (H) are denoted by a dot (·), while firms in the low business risk group (L) are denoted by a cross (x). If no attempt were made to account for the firms' different levels of business risk, a regression of K_e on financial risk would indicate a near zero regression coefficient as shown by the slope of line CC in Fig. 1-1. The resulting conclusion would be the independence of K_e and financial risk, implying a peculiar risk

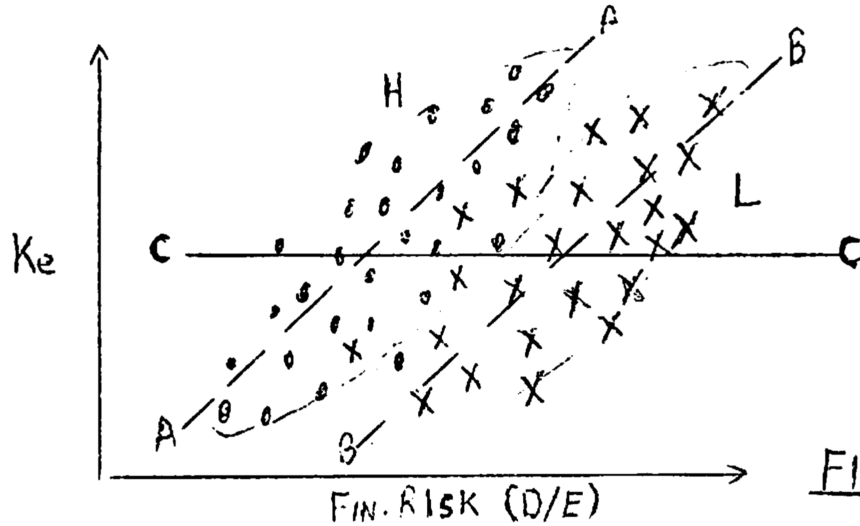

FIG. 1-1

indifference by investors. However, if the data were disaggregated into the high and low business risk groups and a regression of K_e on financial risk were run in each group, K_e would be shown to be quite related to financial risk as shown in Fig. 1-1 by the slopes of lines AA and BB in the high and low business risk groups, respectively.

A more striking example of the same problems can be seen in Fig. 1-2 where investors attach a high business risk premium on their

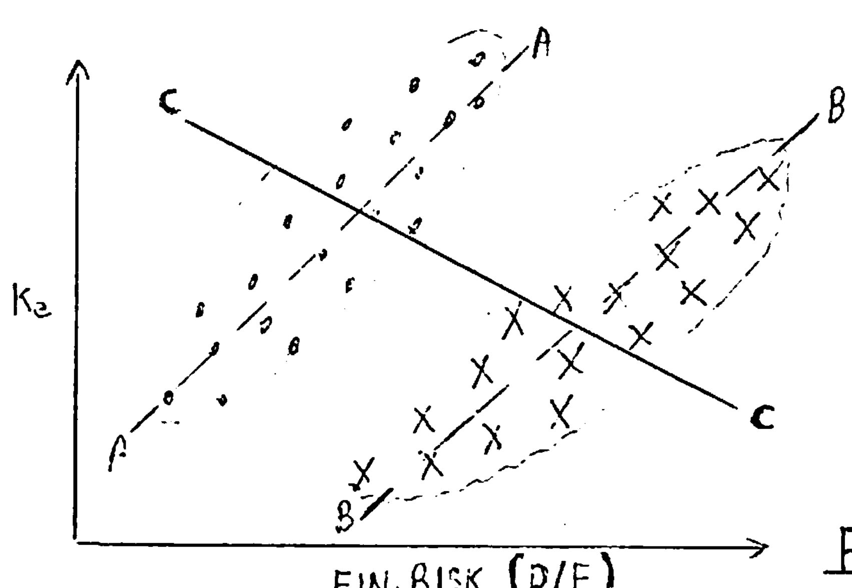

FIG. 1-2

required return on equity capital; hence, the firm's cost of equity capital (K_e) is very dependent on the firm's business risk. If no attempt were made to disaggregate the firms into their respective business risk groups, a regression of K_e on financial risk would imply firms could lower their cost of equity capital (K_e) while increasing their financial risk as depicted in Fig. 1-2 by the slope of line CC. Such risk seeking behavior inferred by the slope of CC is atypical. However, when the firms are disaggregated into their proper business risk groups and the regression of K_e on financial risk is analyzed, the traditional conclusions concerning the effect of financial risk on the firm's cost of equity capital are confirmed as shown in Fig. 1-2 by the slopes of lines AA and BB.

Since the late 1950's, considerable research in corporate finance has attempted to examine the relationship between a firm's cost of capital and its capital structure. Such studies have either included a business risk variable as an explanatory variable or analyzed firms in homogeneous business risk groups to hold constant the effect of the omitted business risk variable. In Wippern's [28] study on the effects of financial leverage on the firm's cost of capital, he uniquely attempted to select a financial risk explanatory variable which also incorporated the firm's business risk, thus allowing him to abstract from finding groups of firms which were business risk homogeneous. In a similar study, Brigham and Gordon [7] incorporated an explanatory business risk variable into their analysis. Modigliani and Miller's [19] [20] thought-stimulating works on the optimal capital structures of firms attempted to hold constant the effects of the omitted business risk variable by assuming firms in the same end-product industry classifications were homogeneous with respect to their business risk behavior and by examining only firms in the same industry classifications. Schwartz [23], in his attempts to analyze the effect of financial leverage on the firm's cost of capital, also assumed firms could be grouped into homogeneous business risk groups by the use of industry classification; however, he observed a correlation between a firm's business risk and its optimal financial leverage. Barges [4] not only examined firms in the same industry classification, but he also attempted to minimize business risk bias in his analysis with the use of book, rather than market, values in calculating his financial risk (debt-equity ratio) variable. Arditti [2] was shaken by his finding the cost of equity capital decreased with increased financial leverage. He concluded this phenomena was observed because he had omitted an explanatory business risk variable in his analysis which was positively correlated with the firm's cost of equity capital and negatively correlated with the firm's financial risk. Arditti's findings were identical to the relationships shown in Fig. 1-2 by the slope of line CC. Earlier research by Weston [26] had hinted in the same direction as Arditti's rationalization. Subsequent research by Baxter [5] and Aronson and Schwartz [3] supported Arditti's conclusions concerning the omitted business risk variable effects on the firm's cost of equity capital and its level of financial risk.

The appeal for finding groups of firms which are business risk homogeneous is not confined only to cost of capital studies. Recently, Melicher [18] attempted to identify operating and financial characteristics of firms which explained a firm's non-diversifiable market risk in Sharpe's [25] capital asset pricing model; however, he examined only electric utilities in an effort to hold constant the effects of an omitted business risk variable.

Section B - Objectives and Significance of This Research

The above mentioned segments of financial research stress the importance of both finding a theoretically sound business risk measure for firms and identifying groups of firms which evince homogeneous business risk characteristics. This paper not only adresses these two proglems, but also, an attempt will be made to identify a set of operating and financial characteristics of firms which will classify firms into their respective homogeneous business risk groups.

The objectives of this research are to:

1) Develop a theoretically sound measure which will cardinally measure a firm's business risk.

2) Retest the equivalent-risk class hypothesis that firms within the same end-product industry classification are business risk homogeneous, while industry groupings of firms are business risk heterogeneous.

3) Form groups of firms which are homogeneous according to the business risk measure.

4) Identify a set of operating and financial characteristics of firms which will classify firms into their respective homogeneous business risk groups.

Previous attempts have been made to quantify a firm's business risk by Wippern [27], Gonedes [14], Rao [22], and Bolten [6]. However, in these attempts, little emphasis was placed on providing the measures with a solid theoretical base. This paper, in Chapter 2, examines these previous attempts and presents a business risk measure which is developed from the basic concepts of total investor risk. The significance of developing a sound business risk measure cannot be overstated, since subsequent testing of the equivalent-risk class hypothesis and grouping firms into homogeneous business risk groups depend on such a measure.

If industry classifications of firms fail to satisfy the conditions of the equivalent-risk class hypothesis, studies which assume firms in the same industry arebusiness risk homogeneous may be subject to serious criticism for the reasons mentioned earlier by Elton and Gruber [11] [12]. The need for a thorough test of the equivalent-risk class hypothesis is justified, since considerable financial research has assumed its validity. Although previous researchers (Wippern [28], p. 617; Barges [4], p. 24) have doubted the equivalent-risk class hypothesis, attempts by Wippern [27], Gonedes [14], and Rao [22], to test the hypothesis have resulted in conflicting conclusions. These conflicting conclusions denote the significance of further research on the validity of the equivalent-risk class hypothesis.

Should the assumptions of the equivalent-risk class hypothesis be false, researchers must be capable of finding other means for classifying firms into homogeneous business risk groups. If a sound cardinal business risk measure could be developed, homogeneous groups of firms could be formed and reliance on industry classifications to provide homogeneous business risk groups could be circumvented. It will be interesting to note the similarity between the designed homogeneous business risk groups and the industry classifications of firms.

Not only will this research cluster firms into homogeneous business risk groups, but it will also attempt to find operating and financial variables that are significant in classifying firms into their respective homogeneous groups. If such variables exist, financial analysts and researchers could observe these key variables and obtain a priori information concerning the business riskiness of firms. Goodman and Williams [15] employed multiple discriminant analysis to identify a set of operating and financial variables which classified firms into standard industry classifications. Mingo and Pinches [21], also using multiple discriminate analysis, attempted to find if a firm's bond rating could be determined from a set of the firm's operating and financial characteristics. However, no investigation has been undertaken to identify firms business risk discriminatory variables. This aspect of the research could have an impact on the financial community in a similar manner as did Altman's [1] research pertaining to the identification of key financial and operating variables which predict corporate bankruptcy.

Section C - Previous Tests of the Equivalent-Risk Class Hypothesis

Wippern's Test:

Wippern [27] was the first to test the equivalent-risk class hypothesis which asserts that groups of firms classified by end-product industry classifications evince intraindustry homogeneity and interindustry heterogeneity according to business riskiness. Wippern selected sixty-one firms from the following eight industries:

Industry	Number of Firms
Baking	4
Cement	5
Electric Utility	17
Industrial Machinery	6
Domestic Oil	8
Paper	10
Rubber	5
Drug	6
Total	61

For each firm, he regressed the log (operating earnings before interest and taxes/share) on time over the ten year period 1954-1963. The antilog of the standard error of the estimate for each regression was Wippern's business risk measure for each of the sixty-one firms in his sample. Using parametric analysis of variance, he tested the null hypothesis that all eight groups of firms came from the same risk population; that is, no significant differences in business risk existed between industries. The null hypothesis was rejected at the 1% level of significance. The rejection of the null hypothesis favored the acceptance of the validity of the equivalent-risk class hypothesis; however, as Wippern noted:

> "This test, however, indicates nothing about the nature of the observed differences. The conventional analysis of variance gives no indication of whether the rejection of the

> hypothesis is attributable to only one or two of the classifications. In seeking to obtain more conclusive evidence it is desirable to determine whether the proxy uncertainty measure of each industry differs significantly from that of each other industry so that the existence of distinct risk classes may be demonstrated or refuted."[1]

To test the above mentioned possibility that the null hypothesis was rejected because of one or two highly heterogeneous industries, Wippern performed twenty-eight pairwise Scheffé interval tests to check for significant differences between each possible pairing of industry classifications.[2] The results of the Scheffé interval tests indicated only the following industries demonstrated any significant difference in business risk:

1) electric utility and machinery at the 1% level,
2) electric utility and oil at the 5% level,
3) electric utility and drug at the 5% level, and
4) electric utility and cement at the 25% level.

Wippern observed from these results:

> "The electric utility industry is one which is most frequently thought of as having a high degree of homogeneity among firms within the industry and one whose characteristics differ markedly from those of firms in other industries. Yet, on the basis of the proxy-uncertainty measure adopted, the industry risk-class assumption does not even differentiate between electric utilities and some industrials."[3]

Because of the above mentioned observations and the fact that not one Scheffé interval test between pairs of manufacturing industries indicated any significant difference in interindustry business risk, Wippern rejected the validity of the equivalent-risk class hypothesis.

---

1. Wippern [27], p. 17.

2. For a description of Scheffé interval tests, see Guenther [15A], pp. 360-366.

3. Wippern [27], p. 19.

Wippern's results are questionable for several reasons.

1) The business risk measure for each firm was the volatility of operating profits (operating earnings before interest and taxes) per share. Such a business risk measure is not an appropriate proxy of a firm's business risk since the volatility of this measure can occur from the firm's financing decision as well as its business riskiness. This argument is further pursued in Chapter 2 of this paper. Also, Davis, Dunn, and Williams [9] have shown the use of per share observations for interfirm comparisons is not appropriate. Wippern stated the validity of his findings was contingent on the appropriateness of his business risk measure:

> "Given the validity of the proxy-uncertainty variable employed, the statistical analysis provides clear evidence that industry groups do not provide an adequate basis on which to insure homogeneity of basic business uncertainty."[4]

2) The necessary conditions for a parametric analysis of variance test of intergroup homogeneity were not substantiated by Wippern. These necessary conditions are:

   a) the observations come from normally distributed populations, and

   b) the populations represented have uniform variances.[5]

---

4. Wippern [27], p. 19.

5. For more on these assumptions, see Freund [13], pp. 395-397, and Kirk [17], pp. 60-63.

Wippern could have tested for intragroup normality with "goodness of fit" tests and intergroup uniform variances with either pairwise group F ratio tests or one of several total group homogeneity tests.[6] Had a parametric analysis of variance test been inappropriate, Wippern should have used a nonparametric test.

3) Wippern's sample was too small, sixty-one firms. Two industries had six firms, two industries had five firms, and one industry had only four firms. Such small industry samples lowered the power of Wippern's test; that is, lowered the probability of rejecting a false null hypothesis.[7] Wippern was aware of this possible source of error.

> "There are two possible reasons why a greater number of significant differences amony industry variability measures were not found. The first is that the null hypothesis of no differences is, in fact, true. The second, however, is that the power of the test is not sufficient to avoid, in certain cases, accepting the null hypothesis when it is, in fact, false. This second situation may well affect the results of the test. The sample sizes of the baking, cement, and rubber industries are very small, and the power of the test to discriminate between these combinations is correspondingly low."[8]

6. Freund [13] explains the use of both "goodness of fit" tests (pp. 337-342), and F ratio tests (pp. 324-327). Kirk [17], pp. 61-62, discusses several total intergroup uniform variance tests.

7. For more on the power of a test, refer to Siegel [24], pp. 8-10.

8. Wippern [27], p. 19.

Gonedes' Test:

Gonedes' [14] sample consisted on eight industry classifications (paper, steel, cosmetics, department stores, air transport, special machinery, textiles, and publishing houses) with ten firms in each industry. He justified the significance of his research by challenging Wippern's [27] work. Gonedes, contending Wippern had not substantiated the appropriateness of a parametric analysis of variance test in his paper, employed a Kruskal —Wallis nonparametric one way analysis of variance by ranks test.[9] He calculated for each firm the relative deviation between the annual growth rate in the firm's EBIT for each year between 1957 and 1968 and the firm's compound growth rate of EBIT over the 1957-1968 time period. Thus, Gonedes determined ten business risk measures for each of his eighty sample firms. Measuring business risk in such a manner and employing the Kruskal-Wallis test, he tested both for intraindustry homogeneity and interindustry heterogeneity. With his eight intraindustry tests, Gonedes found only two industries (textiles and department stores) intraindustry homogeneous at the 1% level. Aggregating industry data and testing for interindustry heterogeneity, he found the eight industry groupings of firms were heterogeneous at the 1% level of significance; a finding which substantiated partial assumptions of the equivalent-risk class hypothesis.[10]

9. A thorough presentation and description of the Kruskal-Wallis test is given in Siegel [24], pp. 184-193.

10. Rao [22], p. 1763, noted if the steel industry had been deleted from Gonedes' industry classifications, interindustry heterogeneity would not have been substantiated by a Kruskal-Wallis test on the remaining seven industries. Rao reported no test results on this point.

However, since intraindustry homogeneity could not be established, Gonedes concluded:

> "The empirical results we presented are not wholly consistent with the equivalent-risk class hypothesis."[11]

Although Gonedes based the significance of his research on the appropriateness of Wippern's methodology, he made no attempt to test the appropriateness of his nonparametric methodology. Had Gonedes shown the basic assumptions of a parametric analysis of variance test (observations from normal populations with uniform variances) were seriously violated, his use of the less powerful nonparametric test would have been justified. So one question is, was a nonparametric test appropriate?[12] Also, as discussed in Chapter 2, it is doubtful if the Gonedes risk measure is a legitimate proxy for business risk.

Rao's Test:

Rao [22] replicated Gonedes' [14] work using ninety-seven Indian firms from the following five industries:

---

11. Gonedes [14], p. 167.

12. In all of Gonedes' analysis of variance tests, he was working with populations of equal observations. Under these conditions, failure of the populations to be normally distributed or to have equal variances does not seriously affect the validity of a parametric analysis of variance test (Guenther [16], p. 63; Kirk [17], pp. 60-63).

Industry	Number of Firms
Paper	11
Cement	11
Textiles	8
Electric Utility	35
Sugar	32
Total . . .	97

He randomly selected his five industries, but he non-randomly selected his firms in favor of larger firms. Rao's study covered the six year period 1958-1964. Interestingly, his results were exactly opposite to Gonedes' results; that is, Rao concluded intra-industry homogeneity existed, but not interindustry heterogeneity (both tests were to the 1% level of significance).

Like Gonedes, Rao made no attempt to justify the appropriateness of the use of a less powerful nonparametric test. Further, he accepted, without any theoretical support, the validity of Gonedes' business risk proxy. Strong general conclusions from Rao's work are difficult to make since he non-randomly biased his sample in favor of larger firms.

A Comment on These Tests:

It is apparent from the research by Wippern [27], Gonedes [14], and Rao [22] that no definite conclusions on the validity of the equivalent-risk class hypothesis can be made based on these conflicting research findings. As will be discussed in Chapter 2, the business risk measures employed by these earlier researchers were questionable. In no study was an effort made to justify the appropriateness of the methodology (parametric or nonparametric) employed. These facts intensify the need for thorough re-examination of the equivalent-risk class hypothesis. It is hoped this research will make a significant contribution to the body of knowledge in this area.

Section D - Methodology

The study sample of this research will contain one hundred forty-four industrial firms. Twelve industry classifications, which contain at least twelve firms, will be randomly selected from all possible industrial classifications on the COMPUSTAT TAPES [8].[13] From each of the twelve classifications, twelve firms will be randomly selected. The study sample will be presented in Chapter 3.

A theoretical development of the business risk measure will be given in Chapter 2. Each firm's before tax operating return on total assets (K_a) will be calculated for each of the ten years 1963-1972. Then, K_a will be linearly regressed on time over the 1963-1972 time period for each firm. The standard error of the estimate of this regression equation will be each firm's business risk measure.

Having quantified each firm's business risk, the assumptions of the equivalent-risk class hypothesis will be tested in Chapter 3. An attempt will be made to use a parametric one-way analysis of variance technique to test the null hypothesis that industrial firms grouped by industry classifications are intergroup business risk homogeneous.[14]

---

13. Industry classifications on the COMPUSTAT TAPES [8] roughly correspond to the Bureau of Management & Budget's Standard Industry Classification (SIC) Codes [25A].

14. For a discussion of parametric analysis of variance tests, see Guenther [16], Chapter 2, or Freund [13], Chapter 14.

A rejection of this null hypothesis would imply intergroup heterogeneity, thus supporting this assumption of the equivalent-risk class hypothesis. The null hypothesis will be tested at the 5% level of significance. Power tests will also be performed at all significant levels to determine the probability of rejecting a false null hypothesis. If the null hypothesis is rejected at the 5% level, pairwise F tests will be performed on all possible pairwise combinations of the twelve groups of firms to indicate which group/groups was/were business risk heterogeneous.

Then, the one hundred forty-four firms will be rank ordered according to their measured business risk from the highest to the lowest business risk firm. If intragroup business risk homogeneity exists, as purported by the assumptions of the equivalent-risk class hypothesis, one would expect firms in a particular industry group to be clustered within a narrow interval in the rank ordered scale (1 - 144). These industry group clusterings will be presented and the relevance of these group clusterings to the intragroup homogeneity assumption of the equivalent-risk class hypothesis will be discussed.

In order to justify the use of parametric techniques, the assumptions of group normality and uniform variances must be performed. Since the twelve populations sampled will have equal observations (twelve), these assumptions must be seriously violated for the use of parametric techniques to be abandoned.[15] However, if nonparametric methods must be employed, a Kruskal-Wallis one way analysis of variance by ranks will be

15. See Footnote 12 for support of this argument.

performed to test the null hypothesis of intragroup and intergroup business risk homogeneity.[16]

In Chapter 4, two groups of firms will be formed. One group will consist of forty high risk firms; the other group will consist of forty low risk firms. Each group will be subdivided into two samples: an original sample of twenty-five firms and a holdout sample of fifteen firms. The appropriate parametric or nonparametric analysis of variance test will be used to verify the business risk distinctness of these two groups. Then, a set of thirty-five financial and operating variables will be calculated for each of the eighty firms in the two business risk groups. A stepwise multiple discriminant analysis (MDA) program will be applied to the thirty-five variables of the fifty firms in the original samples from each risk group to:

1) develop discriminant functions (comprised of linear combinations of the thirty-five financial and operating variables) that will discriminate between high and low business risk firms, and
2) determine a set of key business risk discriminatory variables among firms.[17]

Such a MDA program will begin by selecting the best discriminating variable and incorporating this variable in two linear discriminant functions (one function per risk group). This process will be reiterated and additional variables added to the discriminant functions until the discriminatory powers of an additional variable in the discriminant functions

16. Reference to this test is given in Footnote 9.

17. Refer to BMD Biomedical Computer Programs [10], program #BMD 07M, for a presentation of this technique.

becomes marginal. At this poing, the program will terminate, and both the discriminate functions and their ability to classify firms into their respective risk groups will be read out. In order to test the ability of this set of financial and operating variables as business risk discriminators, the discriminant functions will be applied to the holdout samples of fifteen firms from each of the two risk groups.

After the above research has been performed, a summary of the research findings, the significance of the findings, and the implications for future research will be discussed in Chapter 5.

REFERENCES

1. Altman, Edward I., "Financial Ratios, Discriminant Analysis, and the Prediction of Corporate Bankruptcy," Journal of Finance, Sept., 1968.

2. Arditti, Fred D., "Risk and the Required Return on Equity," Journal of Finance, March, 1967.

3. Aronson, J. Richard and Eli Schwartz, "Some Surrogate Evidence in Support of the Concept of Optimal Financial Structure," Journal of Finance, March, 1967.

4. Barges, Alexander, The Effect of Capital Structure on the Cost of Capital. Prentice-Hall, Inc., Englewood Cliffs, J. J., 1963.

5. Baxter, Nevins D., "Leverage, Risk of Ruin and the Cost of Capital," Journal of Finance, Sept., 1967.

6. Bolten, Steven E., Security Analysis and Portfolio Management, An Analytical Approach to Investments. Holt, Rinehart, and Winston, Inc., New York, 1970.

7. Brigham, Eugene F. and Myron Gordon, "Leverage, Dividend Policy and the Cost of Capital," Journal of Finance, March, 1968.

8. COMPUSTAT TAPES, Investors Management Science, Inc., Denver, 1972.

9. Davis, E. G., D. M. Dunn, and W. H. Williams, "Ambiguities in the Cross-Section Analysis of Per Share Financial Data," Journal of Finance, December, 1973.

10. Dixon, W. J. (Editor), BMD Biomedical Computer Programs. University of California Press, Berkeley, 1973.

11. Elton, Edwin J. and Martin J. Gruber, "Homogeneous Groups and the Testing of Economic Hypothesis," Journal of Financial and Quantitative Analysis, January, 1970.

12. ______________________, "Improved Forecasting Through the Design of Homogeneous Groups," Journal of Business, October, 1971.

13. Freund, John E., Mathematical Statistics, Second Edition. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971.

14. Gonedes, Nicholas J., "A Test of the Equivalent-Risk Class Hypothesis," Journal of Financial and Quantitative Analysis, June, 1969.

15. Goodman, M. L. and W. H. Williams, "A Statistical Grouping of Corporations by their Financial Characteristics," Journal of Financial and Quantitative Analysis, September, 1971.

15A. Guenther, William C. Concepts of Statistical Inference, Second Edition. McGraw-Hill, New York, 1973.

16. Guenther, William C., Analysis of Variance. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964.

17. Kirk, Roger E., Experimental Design: Procedures for the Behavioral Sciences. Brooks/Cole Publishing Co., Belmont, California, 1968.

18. Melicher, Ronald W., "Financial Factors Which Influence Beta Variations Within a Homogeneous Industry Environment," Journal of Financial and Quantitative Analysis, March, 1974.

19. Modigliani, Franco and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, June, 1958.

20. ____________________, "The Cost of Capital, Corporation Finance, and the Theory of Investment. Reply," American Economic Review, September, 1958.

21. Mingo, K. A. and G. E. Pinches, "A Multivariate Analysis of Industrial Bond Ratings," Journal of Finance, March, 1973.

22. Rao, N. Krishna, "Equivalent-Risk Class Hypothesis: An Empirical Study," Journal of Financial and Quantitative Analysis, June, 1972.

23. Schwartz, Eli, "Theory of the Capital Structure of the Firm," Journal of Finance, March, 1959.

24. Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences. McGraw-Hill, New York, 1956.

25. Scharpe, William F., "A Simplified Model for Portfolio Analysis," Management Science, January, 1963.

25A. Standard Industry Classification Manuel, Office of Management and Budget, Washington, D. C., 1972.

26. Weston, J. Fred, "A Test of Cost of Capital Propositions," Southern Economic Journal, October, 1963.

27. Wippern, R. F., "A Note on the Equivalent-Risk Class Assumption," Engineering Economist, Spring, 1966.

28. ____________________, "Financial Structure and the Value of the Firm," Journal of Finance, December, 1966.

CHAPTER 2

THE BUSINESS RISK MEASURE

The emphasis of this paper is on testing the assumptions of the equivalent-risk class hypothesis and on finding a set of operating and financial ratios that will help classify firms into homogeneous business risk groups. Hence, the selection of a viable business risk measure for each firm is fundamental to the purpose of this research.

This chapter examines business risk measures which have been used in previous studies relative to the equivalent-risk class hypothesis in Sections A and B. Another possible business risk measure is discussed in Section C. Finally, the business risk measure used in this study is presented in Section D.

Section A - Wippern's Measure

Wippern [13] selected a sample of sixty-one firms from eight industries. He measured business risk for each firm as follows:

> "The variability of the operating earnings per share for each firm is measured by the antilog of the standard error around the logarithmic regression of annual earnings observations over the ten-year period 1954-1963. The antilog of the standard error of a logarithmic regression measures percentage variations around the line. The measure of dispersion is taken around the regression of income on time to avoid the influence of earnings growth or decline on variability."[1]

Hence, Wippern regressed the log of each firm's earnings before interest and taxes (EBIT) per share on time over the ten year period 1954-1963 as shown below:

---

1. Wippern [13], p. 16.

$$\log(\text{EBIT/Shr.})_i = a_i + b_i t_j + e_{ij} \qquad (2\text{-}1)$$

for the ith firm, where j = 1954, ..., 1963. The antilog of the standard error of $\log(\text{EBIT/Shr.})_i$ in Equation 2-1 was Wippern's measure for each firm's business risk.

Wippern's use of the variability of EBIT/Shr. as a measure for a firm's business risk is somewhat questionable. A firm's EBIT can vary from year to year as a result of the firm's total business risk complexion such as variations in market conditions affecting the firm, economic environment, competitor's behavior, operating effectiveness of the firm's assets, quality of the firm's assets, government regulations, quantity of sales, or technology pertinent to the firm's production process. Assuming a firm's total assets and shares outstanding remain fixed, the variability in EBIT/Shr. which results from these variations would be a legitimate proxy for business risk. However, a firm's EBIT may vary over time as a result of changes in the magnitude of the firm's total assets. If such changes in total assets were financed by identical changes in the firm's total liabilities, the accompanying variability in EBIT/Shr. would not be a legitimate business risk proxy; instead, the variability of EBIT/Shr. would have occurred because of the financing decision of the firm. Hence, since the variability of EBIT/Shr. could result from both the firm's business risk characteristics and the firm's financing decision, this measure is not a suitable proxy for interfirm business risk comparisons.

Further, a recent study by Davis, Dunn, and Williams [5] argued the use of per share observations for interfirm comparisons is not appropriate. They noted:

> "For example, one could study the data available for the oil industry in 1967 on the basis of the number of shares outstanding in

either 1967 or 1972. Since both are studies of 1967, one might intuitively expect the same results in either case. Unfortunately, this is not so. Not only do the obvious statistical characteristics such as univariate means and variances of the per share variables change, but so also do regression and correlation coefficients between these variables. The mean and variance changes may not always be critical, but changes in the relationships among variables are certainly very important. Furthermore there is no simple way to compare correlations computed on the basis of two different years. Such a correlation is influenced by the number of shares which each company has outstanding at the time of the analysis."[2]

For the above mentioned reasons, Wippern's business risk measure was not employed in this research.

Section B - Gonedes' Measure

Gonedes [6] randomly selected eight industry classifications, taking at random ten firms from each classification to make his total study sample of eighty firms. Then, he measured for each firm the relative deviation between the annual growth rate in the firm's EBIT for each year between 1957 and 1968 and the firm's compound growth rate of EBIT over the 1957-1968 time period. Equation 2-2 depicts this risk measure.

$$BR_{it} = \frac{R_{it} - K_i}{K_i} , \qquad (2\text{-}2)$$

where R_{it} = the growth rate in the ith firm's EBIT in time period t, for t = 1957, ..., 1968, and

K_i = the compound growth rate in the ith firm's EBIT over the ten year period 1957-1968.

Thus, Gonedes calculated ten business risk measures for each of the eighty firms in his sample.

Gonedes' business risk measure avoided the suggested shortcomings of Wippern's per share risk measure. However, Gonedes' measure (which was

2. Davis, Dunn, and Williams [5], pp. 1241-1242.

heavily based on a compound growth rate in EBIT over a specified time period) may be a poor proxy to use for interfirm business risk comparisons because such a measure is arbitrarily dependent on the beginning and the end of period EBITs from which each firm's compound growth rate in EBIT is calculated. That is, the denominator in Eqn. 2-2 for each firm in dependent on the end years of the study (1957 and 1968 in the Gonedes study). Had Gonedes selected another time period, say 1956-1967 or 1958-1969, then the denominator of each firm's annual business risk measure would probably have changed. Since any B_{it} in Eqn. 2-2 is very sensitive to changes in K_i, such a change in the study end years over which K_i is calculated would vastly alter each year's assessment of a firm's business risk. Hence, Gonedes' conclusions lacked the powers of external validity since they were very contingent on the beginning and ending years of the study from which his business risk measure was calculated. Apparently Gonedes felt somewhat ill at ease with his risk measure since he noted:

> "It should be emphasized that the surrogate of business uncertainty which we chose to employ is one of several surrogates which may be utilized with respect to business risk. We recognize that different measurement methodologies may provide foundations for implications which are diametrically opposed to the implications presented below."[3]

Further, using the Gonedes business risk measure, how could interfirm business risk assessments be made if one or more of the denominators in Eqn. 2-2 were either very near (or equal to) zero or negative? Gonedes did not elaborate on these possibilities. Because of these objections, the Gonedes business risk measure was not used in this research.[4]

---

3. From Gonedes [6], p. 165.

4. Rao [11] used Gonedes' business risk measure for a different sample of firms over a different time period, so the same arguments apply to Rao's risk measure as apply to the Gonedes risk measure.

Section C - Bolten's Proposed Business Risk Measure

Bolten [3] has suggested a business risk measure similar to a coefficient of variation. The business risk (BR_i) for the ith firm is shown in Eqn. 2-3.

$$BR_i = \frac{\sum_{t=1}^{n} (X_{it} - \overline{X_i})^2/n}{\overline{X_i}} , \qquad (2\text{-}3)$$

where $\overline{X_i}$ = average expected EBIT for n years, and

$(X_{it} - \overline{X_i})^2$ = squared deviation of the ith firm's actual EBIT in year t from $\overline{X_i}$, and n = number of years for which BR_i is calculated.

Thus, Bolten's risk assessment is a measure of a firm's dispersion of actual EBITs about the average expected EBIT relative to the average expected EBIT. Since this measure relies on the researcher's subjective estimate of the expected EBIT for every firm in each year of the study (in order to calculate $\overline{X_i}$ for each ith firm), independent researchers analyzing the same firms over identical time periods could conclude differing business risk measures for each firm. Such differences among researchers exposed to the same historical data would make research findings concerning the equivalent-risk class hypothesis using Bolten's risk measure contingent on the researcher's subjective estimate of the expected EBIT for each firm. For this reason, Bolten's business risk measure was not chosen for this study.

Section D - The Business Risk Measure Used in This Research

As mentioned earlier in this chapter, any research pertaining to the equivalent-risk class hypothesis is very contingent upon the theoretical soundness of the business risk measure employed. A business risk measure should have several attributes.

1) The measure should be a legitimate proxy of the firm's performance variability over time attributable only to the firm's business risk environment.[5]

2) The measure should allow firms to be ranked (at least ordinally). Such rankings would allow for the classification of firms into homogeneous business risk groups. Then, the financial and operating characteristics of firms in these homogeneous risk groups could be examined for structural relationships which may help explain firms' business riskiness.

3) The measure should be objectively measured. That is, it should not be dependent on subjective estimates which can vary among different researchers.

Previous attempts to quantify a firm's business risk (Gonedes [6], Rao [11], Wippern [13], Bolten [3]) have used some measure of the volatility of the firm's operating earnings before interest and taxes (EBIT) as the business risk proxy.[6] Since EBIT is the firm's operating income before interest charges, its volatility (if properly scaled for the firm's size) seems to be a legitimate business risk measure. The volatility of EBIT must be scaled for the firm's size to allow for interfirm business risk comparisons.[7]

---

5. For a thorough discussion of "the firm's business risk environment" see Bolten [3], pp. 241-247.

6. Also, most classical finance textbooks refer to measures of business risk in this manner. See Quirin [10], p. 299, VanHorne [12], p. 46, and Weston and Brigham [14], pp. 311-313.

7. For more on the scaling of measures to firm size for interfirm comparisons, see Davis, Dunn, and Williams [5], p. 1247, and Altman and Schwartz [1], p. 605.

The volatility of a firm's before tax operating return on total assets (EBIT/total assets) is a measure which accounts for both a firm's variability in EBIT and size (total assets). Thus, if the volatility of firms' before tax operating returns on total assets could be quantified, interfirm business risk comparisons and rankings could be made.

Further, the volatility of EBIT/TA has theoretical appeal as a business risk measure from the firm's owners' (equity shareholders) viewpoint.. The firm's after tax return on equity (K_e) can be shown to be a function of the firm's before tax return on total assets. The following financial definitions and symbols are given:

K_e^* = after tax return on the firm's common owners' equity.

NI = after tax return available to common equity shareholders.

NW = net worth of the firm (common equity at par, surplus on common equity, and retained earnings).

TA = total assets of the firm.

D = total debt of the firm.

I = total interest expense of debt to the firm.

K_d = average before tax cost of debt to the firm.

EBIT = firm's operating earnings before interest and taxes.

K_a = firm's before tax operating return on total assets.

X = firm's corporate income tax rate on total taxable income.

From these definitions, it follows that:

$$TA = NW + D, \tag{2-4}$$

and $$K_e^* = NI/NW, \tag{2-5}$$

and $$NI = (EBIT - I)(1 - X), \tag{2-6}$$

and $$K_d = I/D, \tag{2-7}$$

and $$K_a = EBIT/TA. \tag{2-8}$$

Substituting Eqn. 2-6 into Eqn. 2-5 yields

$$K_e^* = \left[(EBIT - I)(1 - X)\right] / NW \,. \tag{2-9}$$

However, solving for EBIT in Eqn. 2-8 and for I in Eqn. 2-7 and substituting these values into Eqn. 2-9 results in

$$K_e^* = \left(\left[K_a(TA) - K_d(D)\right]\left[1 - X\right]\right)/NW \,. \tag{2-10}$$

Replacing TA with NW + D (from Eqn. 2-4) in Eqn. 2-10 produces an expression

$$K_e^* = \left(\left[K_a(NW + D) - K_d(D)\right]\left[1 - X\right]\right)/NW. \tag{2-11}$$

Reducing Eqn. 1-11,

$$K_e^* = \left[K_a + (D/NW)(K_a - K_d)\right]\left[1 - X\right] . \tag{2-12}$$

The net worth of the firm (NW) is commonly referred to in the literature as the firm's total equity (E). Thus, Eqn. 2-12 is the familiar expression

$$K_e^* = \left[K_a + (D/E)(K_a - K_d)\right]\left[1 - X\right]. \tag{2-13}$$

Eqn. 2-13 depicts the important functional relationship between the firm's after tax return on owners' equity (K_e^*) and the before tax operating return on total assets (K_a), the debt-equity ratio (D/E), the average cost of debt (K_d), and the income tax rate (X) applicable to taxable income (EBIT - I). Holding the firm's debt-equity ratio and tax rate constant and then differentiating Eqn. 2-13 with respect to K_a yields

$$\frac{\Delta K_e^*}{\Delta K_a} = (1 + D/E)(1 - X). \tag{2-14}$$

Or, the change in K_e^* which accompanies a change in K_a is

$$\Delta K_e^* = (1 + D/E)(1 - X)(\Delta K_a). \tag{2-15}$$

Thus, given a level of financial risk for the firm (D/E) and tax rate (X), then Eqn. 2-15 shows the sensitivity of a change in K_e^* to a change in K_a.

An investor's total risk for assuming an equity position in a firm may be considered the volatility of K_e^*. Thus, the more volatile K_e^*, the more uncertainty, or total risk to the investor. The volatility in K_e^*

results from both the firm's financial and business risk. Then, given the firm's financial risk (D/E) and tax rate (x), the volatility of K_e^* results from the firm's business risk. From Eqn. 2-15, the more volatile K_a, the more volatile K_e^*; therefore, the volatility of K_a appears to be a legitimate proxy for a firm's business risk.

Hence, for this research, a quantification of the volatility of K_a will be the business risk measure employed. Since K_a is scaled for the firm's size, such a business risk measure would be suitable for comparing the business risk among firms.

Borrowing from the literature in investments, several volatility measures applicable to common stock prices could be applied to measuring the volatility of K_a.[8] The standard deviation of K_a with respect to its average value over a specified time period is an appealing measure of volatility; however, such a measure would be biased against firms which have experienced a rapid growth rate.[9] Thus, the volatility measure should be insensitive to the time trend (or growth rate) of K_a. To overcome these growth effects on the volatility measure of a firm's K_a, this study will quantify each firm's business risk as the standard error of the estimate (SE_i) of the regression equation,

$$K_{ait} = a_i + b_i t + e_{it}, \quad t = 1, \ldots, 10 \qquad (2\text{-}16)$$

where t = time period,

K_{ait} = the ith firm's K_a in time period t,

e_{it} = error term in time period t for the ith firm.

---

8. Refer to Norgaard [8], Altman and Schwartz [1], [2], and Pinches and Kenney [9] for a review of such volatility measures.

9. See Norgaard [8], pp. 1073-1074, Footnote 6.

The ten-year period 1963-1972 will be examined. For each firm, K_{ait} will be regressed on time over the ten-year period and SE_i (the standard error of the estimate) will be calculated as shown in Eqn. 2-17.

$$SE_i = \left[\frac{\sum_{t=1}^{n} (K_{ait} - \hat{K}_{ait})^2}{n-2} \right]^{\frac{1}{2}}, \quad t=1,\ldots,n \; ; \quad n=10, \qquad (2\text{-}17)$$

where $\hat{K}_{ait}$ = the estimate of K_{ait} in time period t.

A firm's business risk measured in this manner appears to satisfy the three attributes mentioned earlier of an ideal business risk measure. That is, the proposed business risk measure (SE_i):

1) has a solid theoretical base evincing its legitimacy as a proxy for a firm's business risk,
2) would allow firms to be ranked both cardinally and ordinally according to their business riskiness, and
3) is objectively determined.

REFERENCES

1. Altman, Edward I. and Robert A. Schwartz, "Common Stock Price Volatility Measures and Patterns," Journal of Financial and Quantitative Analysis, January, 1970.

2. ________________, "Behavior of Industrial Stock Price Indices," Journal of Finance, September, 1973.

3. Bolten, Steven E., Security Analysis and Portfolio Management, An Analytical Approach to Investments. Holt, Rinehart, and Winston, Inc., New York, 1970.

4. Brigham, Eugene F. and J. Fred Weston, Managerial Finance, Fourth Edition. Holt, Rinehart, and Winston, Inc., New York, 1972.

5. Davis, E. G., D. M. Dunn and W. H. Williams, "Ambiguities in the Cross-Section Analysis of Per Share Financial Data," Journal of Finance, December, 1973.

6. Gonedes, Nicholas J., "A Test of the Equivalent-Risk Class Hypothesis," Journal of Financial and Quantitative Analysis, June, 1969.

7. Klemkosky, R. C., "The Bias in Performance Measures," Journal of Financial and Quantitative Analysis, June, 1973.

8. Norgaard, Richard L., "Evaluating Risk, Returns, and Trends," Journal of Financial and Quantitative Analysis, September, 1971.

9. Pinches, George E. and William R. Kenney, Jr., "The Measurement of the Volatility of Common Stock Prices," Journal of Finance, March, 1971.

10. Quirin, G. David, The Capital Expenditure Decision. Richard D. Irwin, Inc., Homewood, Ill., 1967.

11. Rao, N. Krishna, "Equivalent-Risk Class Hypothesis: An Empirical Study," Journal of Financial and Quantitative Analysis, June, 1972.

12. VanHorne, James C., Financial Management and Policy, Second Edition. Prentice Hall, Englewood Cliffs, N. J., 1971.

13. Wippern, R. F., "A Note on the Equivalent Risk Class Assumption," Engineering Economist, Spring, 1966.

CHAPTER 3

THE TESTS OF THE EQUIVALENT-RISK CLASS HYPOTHESIS

The purpose of this chapter is to thoroughly test the classical equivalent-risk class hypothesis which asserts that firms grouped by industry classifications are intragroup homogeneous and intergroup heterogeneous with respect to their business riskiness. The significance of the validity of this hypothesis was discussed in Chapter 1. Previous research in this area (Wippern [11], Gonedes [6], Rao [10]) was also discussed in Chapter 1.

The study sample of one hundred forty-four industrial firms from twelve industries is presented in Section A. Using the business risk measurement developed in Chapter 2, the one hundred forty-four firms' business risk measures are then presented in Section B. The parametric analysis of variance model used to test the intergroup heterogeneity assumption of the equivalent-risk class hypothesis is discussed in Section C, and tests of the model's assumptions are analyzed in Section D. The results of the tests for intergroup heterogeneity are presented in Section E. In Section F, the intragroup business risk homogeneity of twelve industry groups was investigated. The research findings of Sections E and F are summarized and compared to the findings of previous research studies on the equivalent-risk class hypothesis in Section G.

Section A - The Study Sample

Twelve industry classifications from the COMPUSTAT TAPES [2] were randomly selected. These industry classifications closely corresponded to those classifications of the Bureau of Management and Budget [1]. For an industry classification to be included in the study it had to contain a minimum of twelve firms with complete financial data reported on the COMPUSTAT TAPES from 1963 through 1972. The twelve industries of this study are shown below.

Industry	COMPUSTAT TAPE Industry Code No.
Metals - Misc.	1000
Food - Meat Packers	2010
Textile Apparel Mfg.	2300
Paper	2600
Chemical & Chem. Preparations	2899
Oil - Integrated Domestic	2912
Tire & Rubber	3000
Steel - Minor	3311
Electronics	3670
Auto Parts	3714
Air Transport	4511
Conglomerates	9997

From each of the above twelve industry groupings, twelve firms were randomly selected, provided each firm selected had complete financial data reported for the ten year time span 1963-1972. The total study sample was one hundred forty-four firms. The study sample is given in Appendix A-1.

Section B - Quantification of the Firms' Business Risks[1]

Before quantifying each firm's business risk, the before tax operating return on total assets had to be calculated from COMPUSTAT TAPES [2] data for each of the ten years (1963-1972) as shown in Eqn. 3-1.

$$K_{ait} = \frac{EBIT_{it}}{TA_{it}} \ , \quad t = 1963, \ldots, 1972 \ , \qquad (3\text{-}1)$$

where K_{ait} = the ith firm's before tax operating return on total assets in year t,

$EBIT_{it}$ = the ith firm's earnings before interest and taxes in year t, and

TA_{it} = the ith firm's total assets in year t.

Then, K_{ait} was linearly regressed on time over the ten year (1963-1972) period for each of the one hundred forty-four firms in the study according to Eqn. 3-2.[2]

$$\hat{K}_{ait} = a_i + b_i t, \quad t = 1, \ldots, 10 \ , \qquad (3\text{-}2)$$

where $\hat{K}_{ait}$ = an estimate of the ith firm's before tax operating return on total assets in year t,

a_i = the ith firm's regression constant term, and

b_i = the ith firm's regression coefficient term.

Finally, each firm's business risk was measured by the standard error of the estimate (SE_i) in Eqn. 3-3.

---

1. The theoretical development of the business risk measure used in this study was presented in Chapter 2, Section D.

2. The regression program used was the ECON, OLS ECONOMETRIC ESTIMATION PROGRAM [3].

$$SE_i = \left[\frac{\sum_{t=1}^{n} (\hat{K}_{ait} - K_{ait})^2}{n - 2} \right]^{1/2}, \quad t = 1,\ldots,n; \quad n = 10 \text{ observations (years)}, \tag{3-3}$$

where $\hat{K}_{ait}$ = the estimate (from Eqn. 3-2) of the actual K_{ait} (from Eqn. 3-1) for the ith firm in year t.

The results of this regression analysis of the one hundred forty-four firms in the study are given in Appendix A-2, Tables 1 - 12.

From Appendix A-2, it should be noted that no significant linear correlation between K_a and time existed for approximately sixty percent of the firms in the study. This result was not surprising since one would expect a firm's K_a to either 1) fluctuate about some average value, or 2) secularly increase in boom years and secularly decrease in recessionary years. Either of these two situations would result in a low linear correlation between K_a and time; hence, a statistically insignificant F statistic. Also, if secular trends in K_a exist over the ten-year time period, one would expect the error terms to be autocorrelated. Thus, a good number of firms (approximately 14 percent) in the study had Durbin-Watson statistics which at least indicated autocorrelation may have existed.[3] Regardless of either the statistical significance of the F statistic or the presence of autocorrelation, the standard error of the estimate (SE) remains a legitimate proxy for a firm's business risk (volatility in K_a over time).

---

3. A firm statement on autocorrelation in each regression equation cannot be made since ten is such a small number of observations. However, either a very high or a very low Dubin-Watson statistic indicated autocorrelation may be suspected. For more on this discussion, see Yamane [12], pp. 809-813.

Section C - The Parametric Analysis of Variance Model

This study contained twelve groups (industry classifications) of firms, each group containing twelve firms (see Appendix A-1). If the assumptions of the equivalent-risk class hypothesis are true, each group should evince a mean business risk distinct from the mean business risk of any other group. For example, the metal group's mean business risk should be different from each of the other eleven groups' mean business risks. The same should hold true for the other eleven groups. Also, within any group, business risk homogeneity should exist. If either of these two conditions (intergroup heterogeneity and intragroup homogeneity) do not hold, the assumptions of the equivalent-risk class hypothesis are not validated.

The F test:

An ideal test of the intergroup heterogeneity assumption of the equivalent-risk class hypothesis can be made using parametric analysis of variance (ANOVA) if the assumptions of the parametric ANOVA model can be substantiated. For example, suppose k independent groups of firms are taken with n firms per group. The total number of firms in the analysis is N firms, where $N = nk$. Further, suppose the business risk for each ith firm in the jth group is given as X_{ij}, for $i = 1,\ldots,n$ and $j = 1,\ldots,k$. A parametric ANOVA of the validity of the equivalent-risk class hypothesis could be employed if the following assumptions are true:

1) the X_{ij}'s are independent,
2) the X_{ij}'s are continuous random variables,
3) the X_{ij}'s in each of the k groups are normally distributed, and
4) the variances in each of the k groups are the same.

Assuming these assumptions hold, a parametric ANOVA would proceed in the following manner. The grand mean ($\overline{X_T}$) of the N firms' business risks is

$$\overline{X_T} = \sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij} \Big/ N. \tag{3-4}$$

The total variation of all N observations about the grand mean ($\overline{X_T}$) is the total sum of squares (SST), where,

$$SST = \sum_{j=1}^{k} \sum_{i=1}^{n} (X_{ij} - \overline{X_T})^2 . \tag{3-5}$$

It can be shown that the total sum of squares is comprised of two components: the between groups sum of squares (SSB) and the within groups sum of squares (SSW).[4] The between groups sum of squares measures the portion of total variation attributable to variations between the k group means and the grand mean as shown in Eqn. 3-6.

$$SSB = n \sum_{j=1}^{k} (\overline{X}_j - \overline{X_T})^2, \tag{3-6}$$

where n (the number of observations in each jth group) is the same for all k groups, and

$$\overline{X_j} = \sum_{i=1}^{n} X_{ij} \Big/ n. \tag{3-7}$$

$\overline{X_j}$ in Eqn. 3-7 represents the group mean business risk of the jth group. That portion of the total variation (SST) attributable to variations within the groups (SSW) about the groups' means is expressed in Eqn. 3-8.

---

4. For a proof of this statement, refer to Edwards[4], pp. 112-115.

$$SSW = \sum_{j=1}^{k} \sum_{i=1}^{n} (X_{ij} - \overline{X_j})^2 . \qquad (3\text{-}8)$$

The degrees of freedom (df) associated with the total sum of squares (SST) in Eqn. 3-5 is N - 1. The between groups sum of squares (SSB) in Eqn. 3-6 has k - 1 degrees of freedom and the within groups sum of squares (SSW) in Eqn. 3-8 has N - k degrees of freedom. The between groups mean square (MSB) and the within groups mean square (MSW) are estimates of the between groups and within groups variances, respectively. These variance estimates are shown in Eqns. 3-9 and 3-10.

$$MSB = SSB \,/\, (k-1) . \qquad (3\text{-}9)$$

$$MSW = SSW \,/\, (N-k) . \qquad (3\text{-}10)$$

F_c is defined as the ratio of the between groups mean square to the within groups mean square. Algebraically,

$$F_c = MSB \,/\, MSW . \qquad (3\text{-}11)$$

F_c has an F distribution with k - 1 and N - k degrees of freedom.

Parametric ANOVA employs a one-sided F test to check the null hypothesis (H_o) that no significant differences exist between the k groups' mean business risks. Once the α level of the test has been specified, the criterion for accepting the null hypothesis (H_o) is to accept H_o if $F_c \leq F_{\alpha,\ k-1,\ N-k}$. If $F_c > F_{\alpha,\ k-1,\ N-k}$, then H_o must be rejected and the alternative hypothesis (H_1) that significant differences do exist between the k groups' mean business risks must be accepted. The α level of an F test is the minimum probability of rejecting H_o when H_o is true. A table of the F distribution is given in Guenther [7], pp. 486-499.

---

5. For a more thorough discussion of parametric ANOVA and the associated F test, refer to Freund[5], pp. 395-404; Guenther[8], Chapter 2; Guenther[7], Chapter 7; Kirk[9], Chapter 2; and Edwards[4], Chapter 7.

The α Level and the Power of an F Test:

When engaged in statistical hypothesis testing, the researcher must be cognizant of either rejecting a true H_o or accepting a false H_o. If a true H_o is rejected, a Type I error is committed. If a false H_o is accepted, a Type II error is committed. α is defined as the probability of committing a Type I error, and β is the probability of committing a Type II error. The probability of rejecting H_o when H_o is false equals $(1 - \beta)$ and is called the power of the test.

When an ANOVA F test is used to test the intergroup business risk heterogeneity assumption of the equivalent-risk class hypothesis, the null hypothesis (H_o) of the F test is that the industry groups are intergroup business risk homogeneous. Only an acceptance of H_o refutes this contention of the equivalent-risk class hypothesis. Therefore, an F test α level of 5% would allow for a more conservative rejection of the intergroup business risk heterogeneity assumption than would an F test α level of 1%. For this reason, the 5% level of α was chosen for this research.

A second reason for using a 5% level in this research was the power of the F test. As α is decreased, the probability (β) of committing a Type II error is increased and the power of the test $(1 - \beta)$ is decreased. Therefore, at the 5% level, the F test is more powerful than at the 1% level.

Section D - Testing the Parametric ANOVA Assumptions

In the previous section, the four assumptions of the parametric ANOVA model were enumerated. Each of the one hundred forty-four business risk measures was calculated independently; therefore, the assumption of observation independence was met. Also, since the risk measures were continuous random variables, assumption number two was also met. This section deals with the normality and equal variance assumptions.

The Normality Assumption:

The normality assumption assumes the business risk measures of the twelve firms in each of the twelve industry groups are normally distributed about their respective group means. To test this assumption, each group was subjected to a chi-square goodness of fit test.[6] The mean business risk $\overline{X_j}$ and estimated standard deviation (S_j) of each jth group of the twelve industry groups were calculated according to Eqns. 3-12 and 3-13, respectively.

$$\overline{X_j} = \sum_{i=1}^{n} X_{ij}/n \text{ , for } n = 12 \text{ and all } j = 1,\ldots,12. \qquad (3\text{-}12)$$

$$S_j = \left[\sum_{i=1}^{n} (X_{ij} - \overline{X_j})^2 / (n-1)\right]^{1/2}, \text{ for } n=12 \text{ and all } j=1,\ldots,12. \qquad (3\text{-}13)$$

These means and estimated standard deviations are shown in Appendix A-3. Then, for each ith firm in each jth industry, the unit standard deviations (Y_{ij}) of X_{ij} from $\overline{X_j}$ was calculated as shown in Eqn. 3-14.[7]

$$Y_{ij} = \frac{X_{ij} - \overline{X_j}}{S_j} \quad . \qquad (3\text{-}14)$$

These calculations are given in Appendix A-4.

---

6. Freund[5], pp. 334-338 and Guenther[7], pp. 316-321 discuss this test in detail.

7. Refer to Appendix A-2 for the business risk measure X_{ij} of each of the firms in the study sample.

The goodness of fit tests of the twelve industry groups are contained in Tables 3-1 through 3-12. Five intervals were chosen as shown in column one of the test tables. Column two is the probability (p_ℓ) of a unit standard deviation occuring within the ℓth interval if the business risk measures for the industry group are normally distributed with group mean $\overline{X_j}$ and standard deviation S_j. (These probabilities were obtained from Guenther [7], pp. 480-481.) Column three is the expected number (e_ℓ) of unit standard deviations in the ℓth interval where

$$e_\ell = (p_\ell)(n), \; n=12. \tag{3-15}$$

The actual number (f_ℓ) of unit standard deviations within each of the ℓ intervals is given in Column four. The difference ($f_\ell - e_\ell$) between the actual and expected unit standard deviation in each ℓth interval and this difference squared $(f_\ell - e_\ell)^2$ divided by the expected number (e_ℓ) of unit standard deviations in each ℓth interval are given in Columns five and six, respectively.

The calculated chi-square statistic (X_c) for each industry group was determined by Eqn. 3-16.

$$X_c^{\,2} = \sum_{\ell=1}^{m} (f_\ell - e_\ell)^2 / e_\ell \, , \tag{3-16}$$

Where ℓ = interval number and m = 5 intervals.

Given an α level, the null hypothesis (H_o) that the twelve observations within a jth industry group are normally distributed with mean $\overline{X_j}$ and standard deviation S_j was accepted only if $X_c^{\,2} \leq X_{\alpha,\, m-3}^{2}$. Any significant $X_c^{\,2}$ at either the 5% or 1% levels are noted in Tables 3-1 through 3-12. (These significant points in the chi-square distribution were obtained from Guenther [7] p. 483.)

TABLE 3-1 - Goodness of Fit Test for Metals - Misc.

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_l(a \le X_{ij} \le b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
l=2, -1.5 to - .5	.24	2.88	4	1.12	.44
l=3, - .5 to .5	.38	4.56	3	-1.56	.53
l=4, .5 to 1.5	.24	2.88	5	2.12	1.56
l=5, 1.5 to ∞	.07	.84	0	.84	.84

$X^2_{.01,2} = 5.9$

$X^2_{.05,2} = 9.2$

$X^2_c = 4.21$

TABLE 3-2 - Goodness of Fit Test for Meat Packers

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_l(a \le X_{ij} \le b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
l-2, -1.5 to - .5	.24	2.88	5	2.12	1.56
l=3, - .5 to .5	.38	4.56	4	- .56	.07
l=4, .5 to 1.5	.24	2.88	2	- .88	.27
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 2.77$

TABLE 3-3 - Goodness of Fit Test for Textiles

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_l(a \leq X_{ij} \leq b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, - ∞ to -1.5	.07	.84	0	- .84	.84
l=2, -1.5 to - .5	.24	2.88	5	2.12	1.56
l=3, - .5 to .5	.38	4.56	3	-1.56	.53
l=4, .5 to 1.5	.24	2.88	3	.12	.01
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 2.97$

TABLE 3-4 - Goodness of Fit Test for Paper

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_l(a \leq X_{ij} \leq b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, - ∞ to -1.5	.07	.84	0	- .84	.84
l-2, -1.5 to - .5	.24	2.88	2	- .88	.27
l=3, - .5 to .5	.38	4.56	9	4.44	4.32
l=4, .5 to 1.5	.24	2.88	0	-2.88	2.88
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 8.34^*$, significant at 5% level

TABLE 3-5 - Goodness of Fit Test for Chem. & Chem. Preparations

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_1(a \le X_{ij} \le b)$	e_1	f_1	$f_1 - e_1$	$\frac{(f_1 - e_1)^2}{e_1}$
1=1, - ∞ to -1.5	.07	.84	0	- .84	.84
1=2, -1.5 to - .5	.24	2.88	3	.12	.01
1=3, - .5 to .5	.38	4.56	8	3.44	2.60
1=4, .5 to 1.5	.24	2.88	0	-2.88	2.88
1=5, 1.5 to ∞	.07	.84	1	- .1	.03

$X^2_{.01,2} = 5.9$

$X^2_{.05,2} = 9.2$

$X^2_c = 6.36$*, significant at 5% level

TABLE 3-6 - Goodness of Fit Test for Oils

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_1(a \le X_{ij} \le b)$	e_1	f_1	$f_1 - e_1$	$\frac{(f_1 - e_1)^2}{e_1}$
1=1, - ∞ to -1.5	.07	.84	0	- .84	.84
1-2, -1.5 to - .5	.24	2.88	4	2.12	1.56
1=3, - .5 to .5	.38	4.56	5	.44	.04
1=4, .5 to 1.5	.24	2.88	2	- .88	.27
1=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 2.74$

TABLE 3-7 - Goodness of Fit Test for Tire & Rubber

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_l(a \leq X_{ij} \leq b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
l=2, -1.5 to - .5	.24	2.88	4	1.12	.44
l=3, - .5 to .5	.38	4.56	6	1.44	.45
l=4, .5 to 1.5	.24	2.88	1	-1.88	1.23
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.9$

$X^2_{.05,2} = 9.2$

$X^2_c = 2.99$

TABLE 3-8 - Goodness of Fit Test for Steel

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_l(a \leq X_{ij} \leq b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
l-2, -1.5 to - .5	.24	2.88	4	1.12	.44
l=3, - .5 to .5	.38	4.56	3	-1.56	.53
l=4, .5 to 1.5	.24	2.88	4	1.12	.44
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 2.28$

TABLE 3-9 - Goodness of Fit Test for Electronics

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_1(a \leq X_{ij} \leq b)$	e_1	f_1	$f_1 - e_1$	$\frac{(f_1 - e_1)^2}{e_1}$
1=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
1=2, -1.5 to - .5	.24	2.88	6	3.12	3.38
1=3, - .5 to .5	.38	4.56	2	-2.56	1.44
1=4, .5 to 1.5	.24	2.88	3	.12	.01
1=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.$

$X^2_{.05,2} = 9.$

$X^2_c = 5.70$

TABLE 3-10 - Goodness of Fit Test for Auto Parts

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_1(a \leq X_{ij} \leq b)$	e_1	f_1	$f_1 - e_1$	$\frac{(f_1 - e_1)^2}{e_1}$
1=1, $-\infty$ to -1.5	.07	.84	0	- .84	.84
1-2, -1.5 to - .5	.24	2.88	3	.12	.01
1=3, - .5 to .5	.38	4.56	6	1.44	.45
1=4, .5 to 1.5	.24	2.88	2	- .88	.27
1=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 1.60$

TABLE 3-11 - Goodness of Fit Test for Air Transport

Interval (a,b) in Unit Std. Deviations	Interval Probability $P_l(a \le X_{ij} \le b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, - ∞ to -1.5	.07	.84	0	- .84	.84
l=2, -1.5 to - .5	.24	2.88	4	1.12	.43
l=3, - .5 to .5	.38	4.56	2	-2.56	1.44
l=4, .5 to 1.5	.24	2.88	6	3.12	3.38
l=5, 1.5 to ∞	.07	.84	0	- .84	- .84

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 6.93^*$, significant at 5% level

TABLE 3-12 - Goodness of Fit Test for Conglomerates

Interval (a,b) in Unit Std. Deviations	Internal Probability $P_l(a \le X_{ij} \le b)$	e_l	f_l	$f_l - e_l$	$\frac{(f_l - e_l)^2}{e_l}$
l=1, - ∞ to -1.5	.07	.84	0	- .84	.84
l-2, -1.5 to - .5	.24	2.88	3	.12	.01
l=3, - .5 to .5	.38	4.56	6	1.44	.45
l=4, .5 to 1.5	.24	2.88	2	- .88	.27
l=5, 1.5 to ∞	.07	.84	1	.16	.03

$X^2_{.01,2} = 5.99$

$X^2_{.05,2} = 9.21$

$X^2_c = 1.60$

Unfortunately, the powers of the chi-square goodness of fit test were weak since the number of observations (n=12) in each industry group was small. Therefore, only conclusions about approximate normality could be made. The tests revealed a significant X_c^2 at the 5% level in only three industries (paper, chemical, air) and no significant X_c^2 at the 1% level. Therefore, it was concluded the business risk measures in each of the twelve industry groups were "approximately" normally distributed and the normality assumption was substantiated.

The Equal Variance Assumption:

The equal variance assumption assumes the variances of the twelve industry groups are the same. To test this assumption, the variance (S_j^2) of each of the twelve industry groups was calculated by squaring each standard deviation (S_j). These group variances are listed in Appendix A-3. A Cochran test of equal variances was performed on the twelve group variances to test for homogeneity. This test employes the use of a Cochran statistic (C) calculated according to Eqn. 3-17.

$$C = S_j^2(\text{largest}) \Big/ \sum_{j=1}^{k} S_j^2 , \qquad (3\text{-}17)$$

where k = the number of group variances tested with n observations per group. Given an α level, if $C \leq C_{\alpha,k,n-1}$, then the null hypothesis (H_o) of equal k group variance is accepted. Otherwise, the k group variances are not homogeneous. (For a distribution of Cochran's statistic, see Guenther [7], pp. 524-525.)

The twelve group variance Cochran statistic (C_T) was .3662. With degrees of freedom k(12) and n-1(11), C_T was significant at the 1%

level; therefore, H_0 was rejected. This result was not startling since the largest group variance (textile) was 23.67 and the smallest group variance (oil) was .78. However, since the twelve industry group variances were heterogeneous (violating the equal variance assumption of the parametric ANOVA model) the need existed to determine all homogeneous and heterogeneous variances. To accomplish this task, sixty-six paiwise F tests of equal variances were performed.

With such a pairwise F test, the homogeneity of two variances can be tested by calculating an F statistic (F_{ij}) as shown in Eqn. 3-18.

$$F_{ij} = \frac{S_i^2}{S_j^2} . \tag{3-18}$$

The numerator of Eqn. 3-18 is the larger variance of the two variances. F_{ij} has an F distribution (Appendix A-1) with $n_i - 1$ and $n_j - 1$ degrees of freedom where n_i and n_j are the number of observations from which S_i^2 and S_j^2, respectively, were calculated. With an α level of significance specified, if $F_{ij} \leq F_{\alpha/2, n_i-1, n_j-1}$, then accept the null hypothesis of equal variances; otherwise, reject H_0 and conclude the two variances are not the same.[9] Unlike the one-sided F test used to test the null hypothesis of euqal group means in the parametric ANOVA model, the F test for equal variances is a two-sided model.

Using pairwise F tests for equal variances as a grouping criterion, the twelve industry groups were clustered into homogeneous variance subgroups. In Table 3-13, the industry groups were both row

9. A thorough explanation of the pairwise F test for equal variances is given by Guenther[7], pp. 233-241.

Variance (s^2) Industry	.78 Oil	1.28 Chem.	1.53 Metal	1.54 Steel	1.62 Paper	1.89 Congl.	2.49 Air	6.06 Meat	6.71 Auto P.	8.38 Elect.	8.67 Rubber	23.67 Textile
Oil		1.65	1.96	1.98	2.07	2.43	3.20	7.77**	8.61**	10.75**	11.12**	30.36**
Chemicals			1.19	1.20	1.26	1.47	1.94	4.72*	5.23*	6.53**	6.75**	18.43**
Metal - Misc.				1.01	1.06	1.24	1.63	3.97*	4.39*	5.48**	5.67**	15.47**
Steel					1.05	1.22	1.61	3.92*	4.34*	5.43**	5.62**	15.32**
Paper	Subgrp. A	Subgrp. B	Subgrp. C	Subgrp. D		1.17	1.54	3.75*	4.15*	5.19*	5.37**	14.65**
Conglomerates	Elect.	Air	Congl.	Oil			1.32	3.20	3.55*	4.43*	4.59*	12.52**
Air Transport	Rubber	Meat	Air	Chem.				2.43	2.69	3.36	3.48	9.49**
Meat	Textiles	Auto	Meat	Metal					1.11	1.38	1.43	3.91*
Auto Parts		Elect.		Steel						1.25	1.29	3.53*
Electronics		Rubber		Paper							1.03	2.82
Rubber & Tire				Congl.								2.73
Textile Apparel				Air								

$F_{.01,11,11} = 5.33$

$F_{.05,11,11} = 3.48$

** Significant at 1% level
* Significant at 5% level

TABLE 3-13 - Pairwise F Tests for Variance Homogeneity and column rank ordered from the lowest variance group to the highest variance group. The entries in Table 3-13 are all sixty-six possible pairwise F ratios. With a 5% α level, only four equal variance subgroups (A, B, C, D) existed. These four subgroups are shown at the bottom of Table 3-13. To confirm the variance homogeneity of these four subgroups, the Cochran statistic for each subgroup was determined and tested for significance at the 5% level in Table 3-14.

TABLE 3-14 - Cochran Tests of the Equal Variance Subgroups

Subgroup A	Subgroup B	Subgroup C	Subgroup D
Electronics	Air	Conglomerates	Oil
Rubber	Meat	Air	Chemical
Textile	Auto	Meat	Metal
	Electronics		Steel
	Rubber		Paper
			Conglomerates
			Air
$C_A = .5812$	$C_B = .2684$	$C_C = .5801$	$C_D = .2239$
$C_{.05,3,11} = .6025$	$C_{.05,5,11} = .4118$	$C_{.05,3,11} = .6025$	$C_{.05,7,11} = .3154$

The Cochran tests in Table 3-14 confirmed the intragroup variance homogeneity of the four subgroups A, B, C, and D. Hence, for these four subgroups, the uniform variance assumption was substantiated.

The Appropriateness of the Parametric ANOVA Model:

The test results of this section have shown both the business risk measures in each of the twelve industry groups were approximately normally distributed about their respective group's mean business risk and each of the four subgroups (A, B, C, D) was intragroup variance homogeneous. Therefore, the use of a parametric ANOVA model to test the intergroup business heterogeneity assumptions of the equivalent-risk class hypothesis in the four subgroups was statistically justified.

Had not the appropriateness of the use of the parametric ANOVA model been substantiated, the subsequent ANOVA F tests and conclusions could be questioned in a manner similar to Gonedes' [6] criticisms of Wippern's [11] work.[10]

Section E - The ANOVA F Tests for Intergroup Heterogeneity

A parametric ANOVA F test was applied to each of the four equal variance subgroups from Section D. With an ANOVA F test for each subgroup, the null hypothesis (H_o) that no significant differences between the subgroup's members' mean business risks existed was tested.

---

10. It should be noted, even when the normality and equal variances are violated, the parametric ANOVA model may still be appropriate if the k groups all contain the same number (n) of observations. That is, the F test associated with the ANOVA model is very robust with respect to these two key assumptions given equal group sizes. (Kirk [9], pp. 60-63; Edwards [4], pp. 152-153; and Guenther [8], p. 63).

The F tests were conducted at the 5% level of significance for the reasons cited in Section C of this chapter. Then all possible pairwise F tests were calculated within each subgroup. If the intergroup heterogeneity assumptions of the equivalent-risk class hypothesis hold, the null hypothesis of each pairwise F test should be rejected at the test's α level (5%); otherwise, it must be concluded no significant difference in the mean business risk between the two industry groups in the F test existed.

Minimum Powers of the F Tests:

As discussed earlier in Section C, the power of a test is the probability of rejecting a false null hypothesis. A high power of the test implies greater test reliability. To determine the power of an F test employed in the parametric ANOVA model, the statistic ϕ is calculated in the following manner:[11]

$$\phi = \left[(n/k) \sum_{j=1}^{k} B_j^2\right]^{\frac{1}{2}} / \sigma \tag{3-19}$$

where $B_j = \overline{X_j} - \overline{X_T}$. (3-20)

In Eqn. 3-19, n is the number of observations in each jth group (assume $n_1 = n_2 = \ldots = n_k$), k is the number of groups in the ANOVA F test, and σ is the standard deviation of the population from which the k observations were taken. $\overline{X_j}$ and $\overline{X_T}$ in Eqn. 3-20 are the jth

11. Refer to Guenther [8], pp. 47-50 and Kirk [9], pp. 107-109 for a more detailed discussion of the power of an ANOVA F test.

group and total k group means defined in Eqn. 3-7 and Eqn. 3-4, respectively. The ϕ statistic has a non-central F distribution with (k-1) and (nk-k) degrees of freedom. Given the α level of the ANOVA F test, ϕ , and degrees of freedom, the power of the F test is found from a table of the non-central F distribution.[12]

The larger the value of ϕ , given degrees of freedom and α , the more powerful the F test. This is logical since as ϕ increases, the B_j terms are getting larger implying greater differences between the k group means. The larger the differences between the k group means, the higher the probability H_0 (the null hypothesis of equal k group means) will be rejected in the ANOVA F test.

The power of the ANOVA F test is very sensitive to the population's parameter σ . Since σ is usually unknown, using the nk observations' standard deviation (S_T) about $\overline{X_T}$ as an estimate for σ can produce an erroneous F test power if $\sigma \neq S_T$. To circumvent this problem, the B_j terms in Eqn. 3-20 are usually defined as multiples of σ . Then, ϕ becomes a pure arithmetic number since the σ term in both the numerator and denominator of Eqn. 3-19 can be factored out.

If the maximum difference between any two of the k group means is $a\sigma$ (where a is any real number), then the minimum power of the ANOVA F test can be determined if the B_j terms are chosen such that ϕ is calculated according to Eqn. 3-21.[13]

---

12. These tables are quite voluminous, since a distince non-central F distribution exists for each different k-1 degree of freedom. A set of these tables is given in Guenther [8], Appendix Table 6 for (k-1)=1 through (k-1)=8 degrees of freedom.

13. Refer to Guenther [8], pp. 48-49 for a derivation of Eqn. 3-21.

$$\phi = a \left[\frac{n}{2k}\right]^{1/2} . \qquad (3\text{-}21)$$

Table 3-15 shows the minimum powers of all ANOVA F tests used in this study for three possible maximum differences (1σ, 1.25σ, 1.5σ) between any two of the k group means with $\alpha = 5\%$.

TABLE 3-15 - Power of the ANOVA F Tests

Maximum a	1σ		1.25σ		1.5σ	
k Group in F test	ϕ	Min. Power	ϕ	Min. Power	ϕ	Min. Power
2	1.732	.65	2.165	.83	2.598	.94
3	1.414	.55	1.768	.74	2.12	.88
4	1.225	.48	1.531	.66	1.85	.85
5	1.095	.42	1.369	.63	1.65	.83
6	1.000	.40	1.250	.61	1.50	.79
7	.923	.38	1.157	.57	1.38	.77

$S_T = 2.60$ (nk estimate of σ) $\alpha = 5\%$

The powers of all F tests in Table 3-15 could have been increased had the number of firms from each of the twelve industry groups used in the study exceeded twelve. However, to obtain industry group samples larger than twelve firms would have been quite difficult without decreasing the number of industry groups in the study since most industry groupings on the COMPUSTAT TAPES [2] have twelve or fewer firms with comparable data reported for the 1963-1972 time span.

F Tests' Results:

After an ANOVA F test was applied to each of the four uniform variance subgroups, all possible pairwise F tests were conducted within each subgroup. A table of the F tests' results was then constructed for each subgroup similar to Table 3-14. In this table, the industry groups in the subgroup were rank ordered in both rows and columns from lowest to highest industry group mean business risk. The entries in the upper right diagonal side of the table are the calculated F ratios of the ANOVA pairwise F tests between the industry groups. Significant F ratios at the 5% test level are noted and industry groups which did not evince intergroup business risk heterogeneity at the 5% level are clustered at the bottom of each table in the red brackets. From these tables, it was clear which industry groups did not conform to the equivalent-risk class hypothesis assumption of intergroup business risk heterogeneity.

From the ANOVA chart in Table 3-16, the null hypothesis (H_0) of equal industry group mean business risk was accepted when the F test was applied to the entire subgroup A. The pairwise F tests' results in Table 3-17 indicated no significant difference in the mean industry group business risk existed either between textile apparels and electronics or between tire and rubber and textile apparels; however, tire and rubber and electronics were distinct. The pairwise F tests' results were more reliable than the F test conclusion in Table 3-16, since from Table 3-15, the power of any pairwise (k=2) ANOVA F test is greater than the power of any ANOVA F test for k> 2. Therefore, each industry group in Subgroup A was not business risk heterogeneous.

TABLE 3-16 - Subgroup A ANOVA F Test

Source	SS	dF	MS	F	$F_{.05,2,33}$
SSB	63.32	2	31.66	2.33	3.29
SSW	448.03	33	13.58		
SST	511.35	35			

TABLE 3-17 - Pairwise F Tests for Subgroup A

$\overline{X}_j$	2.94	4.74	6.18
Industry Group	Rubber	Textile	Elect.
Tire & Rubber		1.04	7.44*
Textile Apparel			.77

$F_{.05,1,22} = 4.30$

* Significant at 5% level

For Subgroup B, the five industry groups did not exhibit homogeneous business risk (Table 3-18). However, within B, two business risk homogeneous sets of industry groups existed as shown in Table 3-19. One homogeneous set consisted of tire and rubber, auto parts, air transport, meat packers, and electronics. The other homogeneous set was meat packers and electronics. Thus, the intergroup heterogeneity assumption of the equivalent-risk class hypothesis was not validated in Subgroup B.

TABLE 3-18 - Subgroup B ANOVA F Test

Source	SS	dF	MS	F	$F_{.05,4,55}$
SSB	80.29	4	20.07	3.10*	2.54
SSW	355.52	55	6.46		
SST	435.81	59			

* Significant at 5% level

TABLE 3-19 - Pairwise F Tests for Subgroup B

$\overline{X}_j$	2.94	3.12	4.12	4.36	6.18
Industry Group	Rubber	Auto	Air	Meat	Elect.
Tire & Rubber		.03	1.52	1.65	7.39*
Auto Parts			1.32	1.44	7.44*
Air Transport				.08	4.65*
Meat Packers					2.74

$F_{.05,1,22} = 4.30$

* Significant at 5% level

As in Subgroup B, the F test shown in Table 3-20 for Subgroup C supported the intergroup heterogeneity assumption of the equivalent-risk class hypothesis. However, when C was examined in Table 3-21, meat packers and air transport were business risk homogeneous.

TABLE 3-20 - Subgroup C ANOVA F Test

Source	SS	dF	MS	F	$F_{.05,2,33}$
SSB	36.60	2	18.30	5.26*	3.29
SSW	114.86	33	3.48		
SST	151.46	35			

* Significant at 5% level

TABLE 3-21 - Pairwise F Tests for Subgroup C

$\overline{X_j}$	2.11	4.12	4.36
Industry Group	Congl.	Air	Meat
Conglomerates		11.10*	7.61*
Air Transport			.08

$F_{.05,1,22} = 4.30$

* Significant at 5% lev

Although the seven industry groups in Subgroup D were business risk heterogeneous (Table 3-22), four business risk homogeneous sets of industry groups were found from the pairwise F tests in Table 3-23. The existence of each of these four homogeneous sets strongly indicated firms grouped by industry classifications within these sets were not intergroup business risk heterogeneous as purported by the equivalent-risk class hypothesis.

TABLE 3-22 - Subgroup D ANOVA F Test

Source	SS	dF	MS	F	$F_{.05,6,77}$
SSB	73.20	6	12.20	7.67^{**}	2.22
SSW	122.52	77	1.59		
SST	195.72	83			

** Significant at 1% level

TABLE 3-23 - Pairwise F Tests for Subgroup D

$\overline{X}_j$	1.22	1.85	2.11	2.60	2.65	3.63	4.12
Industry Group	Oil	Paper	Congl.	Chem.	Metal	Steel	Air
Oil		1.95	3.57	11.01*	10.65*	29.96*	30.93*
Paper			.25	2.34	2.49	12.10*	15.17*
Conglomerates				.89	1.02	8.05*	11.10*
Chemical					.01	4.52*	7.41*
Metal						3.73	6.47*
Steel							.73

$F_{.05,1,22} = 4.30$

* Significant at 5% level

Section F - An Examination of Intragroup Homogeneity

In the previous section, the equivalent-risk class hypothesis' assumption of intergroup business risk heterogeneity was not substantiated for the twelve industry groups used in the study. This section examines the assumption of intragroup homogeneity.

The one hundred forty-four firms from the twelve industry groups were rank ordered according to their measured business risk (SE_i) from highest to lowest risk. The highest risk firm (Aileen) was assigned a rank of one and the lowest risk firm (ITT) was assigned a rank of one hundred forty-four. This rank ordering is given in Appendix A-6. The rank ordering was then divided into three risk groups (high, medium, low). Each of the three risk groups contained one third of the firms in the study sample (forty-eight firms per group). The high risk group consisted of firms ranked one through forty-eight; the medium risk group consisted of firms ranked forty-nine through ninety-six; the high risk group consisted of firms ranked ninety-seven through one hundred forty-four.

The number of firms from each of the twelve industry groups in each of the three risk groups is shown in Table 3-24. The industry groups were rank ordered in Table 3-24 from highest to lowest mean group business risk ($\overline{X_j}$). The mean group business risk ($\overline{X_j}$) and the standard deviation (S_j) of the twelve firms in each group about $\overline{X_j}$ is also shown in Table 3-24 for the twelve industry groups.[14] Eight of the twelve industry groups evinced intragroup business risk homogeneity and these groups could be categorized as being either a high, medium, or low business risk group. The metal group was marginally intragroup business risk homogeneous. However, three industry groups (textile apparel, auto parts, and tire & rubber) were definitely not intragroup business risk homogeneous.

These findings suggest firms grouped by industry classifications are not necessarily intragroup business risk homogeneous. Thus, the

---

14. The $\overline{X_j}$ and S_j terms for each industry group are presented in Appendix A-3.

intragroup business risk homogeneity assumption of the equivalent-risk class hypothesis was not thoroughly substantiated by these research findings.

TABLE 3-24 - Number of Industry Occurrences in the Three Risk Groups

j	Industry (j)	$\overline{X}_j$	S_j	Risk Group High	Risk Group Med.	Risk Group Low	Intragroup Homogeneous	Risk Group
1	Electronics	6.18	2.89	10	2	0	Yes	High
2	Textile Apparel	4.74	4.87	5	4	3	No	None
3	Meat Packers	4.36	2.46	7	4	1	Yes	High
4	Air Transport	4.12	1.58	7	5	0	Yes	High
5	Steel	3.63	1.24	5	7	0	Yes	Med-High
6	Auto Parts	3.12	2.59	3	4	5	No	None
7	Tire & Rubber	2.94	2.95	3	2	7	No	None
8	Metals	2.65	1.24	4	4	4	Somewhat	Med-Low
9	Chemicals	2.60	1.13	1	9	2	Yes	Med.
10	Conglomerates	2.11	1.37	1	5	6	Yes	Med-Low
11	Paper	1.85	1.27	1	1	10	Yes	Low
12	Oils	1.22	.88	1	1	10	Yes	Low

High Risk Group ($\overline{X}_j$) Range - 3.51-18.09

Medium Risk Group ($\overline{X}_j$) Range - 1.90-3.51

Low Risk Group ($\overline{X}_j$) Range - .35-1.89

Section G - Summary of Chapter 3

The research findings clearly indicated for the twelve randomly selected industry groupings of firms used in this study, the assumptions of the equivalent-risk class hypothesis did not hold.

The following sets of industry groups were found to be business risk homogeneous:

1) tire & rubber and textiles;
2) textiles and electronics;
3) tire & rubber, auto parts, air transport, and meat packers;
4) meat packers and electronics;
5) air transport and meat packers;
6) domestic oils and paper;
7) paper, conglomerates, chemical & chemical preparations, and metals;
8) metals and steel; and
9) steel and air transport.

In each of the above nine homogeneous sets, as much business risk variation existed among the industry groups in the set as between the industry groups in the set. Had the assumptions of the equivalent-risk class hypothesis been supported by the research findings, no homogeneous sets would have existed since all twelve groups of firms would have been intergroup business risk heterogeneous.

Further, three industry groups of firms (textile apparel, auto parts, and tire & rubber) were found not to be intragroup business risk homogeneous. This finding refuted the intragroup business risk homogeneity aspect of the equivalent-risk class hypothesis.

These findings supported the conclusions of Wippern's [11] work. Although both this research and Wippern's employed a parametric ANOVA model to test the equivalent-risk class hypothesis, several differences in the two works should be noted. First, Wippern did not give a theoretical justification of his business risk measure. A theoretical framework was developed to justify the business risk measure used in this research (Chapter 2, Section D). Also, as discussed in Chapter 2, Wippern's risk measure has been questioned as not being a legitimate business risk measure. Second, as noted in Chapter 1, Wippern selected industry groups with an uneven number of firms in each group, making the power calculation of Wippern's F test quite difficult. Both by designing his test in such a manner (uneven group sample sizes) and by not testing the assumptions of the parametric ANOVA model, Wippern's work became subject to skepticism. Thus, both by selecting equal sample sizes from each of the twelve industry groups used in the study and by confirming the assumptions of the parametric ANOVA model, the efforts of this research cannot be subjected to the same criticisms as was Wippern's. Further, Wippern did not employ pairwise F tests to find homogeneous business risk clusters of industry groups of firms. Had he used such pairwise F tests, his groupings would have been shaky since the powers of his F tests in some cases would have been quite poor due to such small group sizes.

Gonedes [6] and Rao [10] justified their works on the possible inappropriateness of Wippern's parametric ANOVA model, although neither justified the use of their less powerful nonparametric techniques by invalidating the parametric ANOVA model assumptions. Since the use of the parametric ANOVA model was shown in Section D to be appropriate, the Gonedes and Rao method was not applicable. Thus, stronger conclusions can be made from this research than from either Gonedes' or Rao's work since parametric techniques are more powerful than nonparametric techniques.

The invalidity of the equivalent-risk class hypothesis indicated by these research findings suggests financial researchers must both find means of grouping firms into homogeneous business risk groups not contingent on industry classifications and develop new methods for business risk discrimination among firms. These topics will be further discussed in Chapter 4.

REFERENCES

1. Bureau of Management & Budget, Standard Industry Classification Manual. U. S. Government Printing Office, Washington, D. C., 1972.

2. COMPUSTAT TAPES, Investors Management Service, Inc., Denver, 1972.

3. ECON, OLS Econometric Estimation Program, by Morris Normal, adapted for Univac 1108 by R. William Thomas, Economics Department, University of Houston, Houston, Texas.

4. Edwards, Allen L., Experimental Design in Psychological Research, Fourth Edition. Holt, Rinehart, and Winston, Inc., New York, 1972.

5. Freund, John E., Mathematical Statistics, Second Edition. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971.

6. Gonedes, Nicholas J., "A Test of the Equivalent-Risk Class Hypothesis," Journal of Financial and Quantitative Analysis, June, 1969.

7. Guenther, William C., Concepts of Statistical Inference, Second Edition. McGraw-Hill, New York, 1973.

8. ______________, Analysis of Variance. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964.

9. Kirk, Roger E., Experimental Design: Procedures for the Behavioral Sciences. Brooks/Cole Publishing Co., Belmont, California, 1968.

10. Rao, N. Krishna, "Equivalent-Risk Class Hypothesis: An Empirical Study," Journal of Financial and Quantitative Analysis, June, 1972.

11. Wippern, R. F., "A Note on the Equivalent-Risk Class Assumption," Engineering Economist, Spring, 1966.

12. Yamane, Taro, Statistics: An Introductory Analysis, Second Edition. Harper & Row, New York, 1969.

CHAPTER 4

AN EXAMINATION OF BUSINESS RISK CLASSIFIER VARIABLES USING MULTIPLE DISCRIMINANT ANALYSIS

From Chapter 3, it was shown industry classifications do not appear to be reliable business risk classifiers of firms since the assumptions of the equivalent-risk class hypothesis were not substantiated. These findings suggest financial researchers and practitioners cannot be certain a group of firms from any particular industry classification will be either intragroup business risk homogeneous or intergroup business risk heterogeneous vis-a-vis a group of firms from any other industry classification.

Then, if industry classifications are not valid business risk discriminators among firms, does a criterion exist which will allow firms to be identified according to their business riskiness? It is the purpose of this chapter to investigate such a business risk discriminatory criterion. Explicitly, this chapter presents a method for identifying a set of financial and operating ratios that may be useful for business risk discrimination among firms.

Multiple discriminant analysis (MDA) was the quantitative technique employed to identify these discriminatory ratios. A brief description of the MDA model and the previous applications of MDA in financial research are discussed in Section A.

In Section B, the one hundred forty-four firms of the study were rank ordered from highest to lowest according to their measured business risk. Then, two groups of firms were formed. One group consisted of forty high business risk firms, while the other consisted of forty low business risk firms. The business risk heterogeneity of these two risk groups was confirmed by the use of a Kruskal-Wallis nonparametric analysis of variance test.

Next, a set of thirty-five financial and operating ratios is presented in Section C which were calculated for each of the eighty firms in the two distinct business risk groups of Section B. The rationale for selecting these variables and a factor analysis of the interrelationships among these variables are also discussed in Section C.

In Section D the forty firms in each risk group were divided into an original sample of twenty-five firms and a holdout sample of fifteen firms. A stepwise MDA program was applied to the fifty firms in the original samples from the high and low business risk groups to develop linear combinations (discriminant functions) of the thirty-five variables which maximize discrimination between the two risk groups. The discriminatory powers of the discriminant functions were tested by classification matrices for both the original and holdout samples of each risk group. The significance of the financial and operating ratios which entered into the discriminant functions, the functions' discriminatory powers, and the firms misclassifications by the discriminant function are discussed.

Section E summarizes the results of the stepwise MDA program and notes the significant research findings of the chapter.

Section A - The MDA Model and Previous Applications of MDA in Financial Research

The MDA Model:

MDA is a multivariate technique used to classify an observation into one of g mutually exclusive groups based on a set of m independent measured characteristics or variables. The MDA technique assumes:

1) the g groups are mutually exclusive and known,
2) the set of m variables is known for each observation in each group,
3) the m variables have a multivariate normal population within each of the g groups, and
4) the variance-covariance matrices for the m variables within each of the g groups are the same.

Given these assumptions, MDA attempts to develop linear combinations of the m variables which maximize discrimination between the g firms.

For example, consider two groups composed of n observations per group with each observation having a set of m variables. Assuming the four MDA assumptions hold, then an MDA routine would develop a general discriminant function (y) as shown in Eqn. 4-1.

$$y = a_1 x_1 + a_2 x_2 + \dots + a_m x_m, \tag{4-1}$$

where a_i is the discriminant coefficient of the ith independent variable. Then, a discriminant score could be calculated for each jth observation according to Eqn. 4-2.

$$y_j = a_1 x_{1j} + a_2 x_{2j} + \dots + a_m x_{mj}, \tag{4-2}$$

where x_{ij} is the jth observation's value of the ith independent variable. The general discriminant function in Eqn. 4-1 is derived in such a manner as to maximize the ratio λ of the between groups sum of squares of the 2n discriminant scores to the within groups sum of squares of the 2n discriminant scores.

Notice, the general discriminant function (y) in this example has reduced the dimensionability of the classification problem from an m dimensional plane to a one dimensional plane. In general, when the number of groups is g (for $g \geq 2$), then to achieve observation classification, the dimensionality of the problem could be reduced from an m dimensional plane to a (g - 1) dimensional plane with the use of MDA. The coefficients of the first of the (g - 1) discriminant functions would be derived exactly as in the two group case (maximize λ). The coefficients of the (g - 1)th discriminant function would be derived from maximizing λ given the discriminatory powers of the first, second, ..., and (g - 2)nd discriminant functions. Hence, the discriminatory powers of the discriminant functions are monotonic decreasing from the first through the (g - 1)th discriminant function.[1]

Previous Applications of MDA in Financial Research:

Smith [13] was one of the first to use MDA in investment analysis to classify securities into investment groupings. He used a Merrill Lynch classification of securities into either investment, trading, or speculative groups. Then, he selected thirty-three securities (eleven per group) to be classified according to a MDA program. From five groups of financial data, seven financial variables were selected to include into the two discriminant functions. These seven variables were as follows: dividend yield, dividend payout ratio, five year earnings per share growth, current ratio, price earnings ratio, annual sales to shares outstanding ratio, and five year total asset growth. Smith's analysis correctly classified 88% of the thirty-three firms. He used no holdout sample.

1. For a more thorough discussion of the MDA methodoloty, refer to Cooley & Lohnes [4], Chapter 9; Tatsuoka [14], Chapter 6; Morrison [11], pp. 130-133; Smith [13], pp. 8-16, and Anderson [2], Chapter .

Altman [1] used MDA to determine financial and operating ratios that would predict corporate bankruptcy. He selected thirty-three firms which had applied for bankruptcy under Chapter X of the Bankruptcy Act during the 1946-1965 time period and thirty-three non-bankrupt firms. From twenty-two financial and operating ratios, Altman found a discriminant function which incorporated the following five ratios: working capital to total assets, retained earnings to total assets, market value of equity to book value of total debt, sales to total assets, and earnings before interest and taxes (EBIT) to total assets. Then, applying this discriminant function to a holdout sample of sixty-six equally divided bankrupt and non-bankrupt firms, his model was 78% accurate in bankruptcy discriminations.

Whereas Altman's work applied primarily to larger firms, Edmister [6] used stepwise MDA to successfully determine failure of smaller firms. Using three year averages of financial and operating ratios and ratio trend analysis, he developed a discriminant function incorporating seven ratio and trend variables which were 93% accurate in discriminating between failure and non-failure small firms when applied to a sample of forty-two firms.

Goodman and Williams [7] attempted to investigate whether groupings of firms according to financial characteristics were similar to predetermined industrial groupings. First, they selected eleven of fifty-seven financial variables from COMPUSTAT TAPES [3] in both 1966 and 1967 to develop a discriminant function in each year to classify firms as either industrial or utility firms. A discriminant function was developed for each year, each being roughly 98% accurate in its classification. Later, they selected fourteen financial variables to develop four discriminant functions to classify industrial firms as either chemical, drug, domestic oil, steel, or electronics firms. For the years 1946-1967, twenty-one sets of discriminant functions (one set per year) were developed using the fourteen variables. For this twenty-one year span, the MDA classifications were 73% correct with the most common misclassifications occuring between chemical and drug firms. Goodman and Williams did not apply their discriminant functions to holdout samples in any of the above cases.

Mingo and Pinches [9] used MDA to classify one hundred and eighty new higher grade corporate bond issues from 1967-1968 into the five Moody bond ratings (Aa, A, Baa, Ba, B). They held a holdout sample of forty-eight issues. They first factor analyzed thirty-five financial variables for each new issue and identified seven factors; however, they failed to report the eigenvalues and communalities for the factor analysis. Only five variables were selected from the seven factors to include in the MDA model.(No rationale was given for selection of those five variables.) A dummy sixth variable, subordination of the issue to other debt issues, was also included. Their model classified 70% of the model observations correctly and 65% of the holdout sample observations correctly. The dummy sixth variable, subordination, alone classified 80% correctly.

Recently, Latané and Reinhart [8] have attempted to use MDA to prove security price-earnings ratios (P/E) do not come from a homogeneous universe. That is, they showed MDA was a better classifier of securities according to four groups of P/E ratio annual changes than was a single universal multiple regression model (REG). Latané and Reinhart collected eighteen financial variables for each security. Data was collected for two hundred and seventy-four securities in 1961 and three hundred and twenty-one securities in 1971. They selected one-third of their observations for a holdout sample in each time period. Their regression models for 1961 and 1971 made the yearly change in the P/E ratio the dependent variable and the eighteen financial characteristics of each security the independent variables. The regression R^2 for 1961 and 1971 were .53 and .39, respectively (both significant to the .001 level). Their 1961 and 1971 MDA models each had only three variables in the discriminant functions and the authors did not elaborate on other possible functions. The percent correct classifications using both REG and MDA models in each year are given below:

	MDA	REG
1961	52%	45%
1971	50%	32%

Although the MDA models were better classifiers than the REG models, the results are not overly impressive.

It is evident from the above mentioned studies that MDA has been applied to a wide assortment of financial research in recent years. However, its application to the investigation of discrimination among firms in different business risk classes has not previously been attempted.

Section B - The Design of Homogeneous Business Risk Groups

In order to apply MDA techniques, there must exist g (where $g \geq 2$) groups of observations which are both identifiable and distinctly different. To meet this requirement, the one hundred forty-four firms were rank ordered from highest to lowest according to their measured business risk. Each firm's business risk was calculated by Eqn. 3-3. The rank ordering of the firms and each firm's measured business risk (SE_i) are shown in Appendix A-5.

Two groups of firms were formed for the MDA program. One group consisted of forty high business risk firms; the other group consisted of forty low business risk firms. The forty firms in each risk group were further divided into an original sample of twenty-five firms and a holdout sample of fifteen firms. The firms included in the original and holdout samples of each risk group are noted by an (X) in either the (O) or (H) columns in Appendix A-5. The remaining sixty-four unchecked firms in Appendix A-5 were not used in the MDA.

To confirm the business risk heterogeneity of the two groups, a Kruskal-Wallis (K-W) nonparametric one-way ANOVA test was performed.[2] A parametric ANOVA test of equal group mean business risk was not applicable since both the normality and equal variance assumptions of the parametric ANOVA model appeared to be seriously violated in this case. To employ the K-W test, the eighty firms in the two groups were ranked from 1 - 80 with the lowest risk firm in the two groups (Atlantic-

2. A complete presentation of the K-W ANOVA test assumptions and methods is presented in Siegel [12], pp. 184-194.

Richfield) ranked 1 and the highest risk firm in the two groups (Aileen) ranked 80. Then, the K-W test statistic H_c was calculated according to Eqn. 4-3.

$$H_c = \frac{12}{N(N+1)} \sum_{j=1}^{k} (R_j^2 / N_j) - 3(N+1) = 59.26 , \qquad (4\text{-}3)$$

where, k = 2 = number of groups,

N_j = 40 = number of firms in each jth group,

N = 80 = total firms in two groups, and

R_j = sum of ranks in the jth group.

In general, the K-W test statistic has a chi-square distribution with (k-1) degrees of freedom. Given the α level of the test, if $H_c \leq \chi^2_{\alpha,\ k-1}$, then accept the null hypothesis that the N observations come from the same population. Since H_c as calculated in Eqn. 4-3 vastly exceeded $\chi^2_{.001,1}$, the K-W null hypothesis of intergroup business risk homogeneity was rejected at the .1% level. The power of the K-W test was not calculated; however, the K-W test has a very high power efficiency rating (95%) relative to the F test used in the parametric ANOVA model.[3]

The results of the K-W test indicated the two groups of firms were both identifiable and distinct as high and low business risk groups. Thus, the two groups were suitable for the MDA.

Section C - Financial and Operating Variables Used in the MDA

For each of the forty firms in both the high and low business risk groups used in the MDA, thirty-five financial and operating variables (ratios) were calculated from 1972 data on the COMPUSTAT TAPES [3]. These variables measured such firm characteristics as liquidity, financial

3. Siegel [12], pp. 192-193.

leverage, cash flow and debt coverage, profitability, asset turnover, size, secondary equity market behavior, and dividend policy.

The means and standard deviations of the thirty-five variables in each sample (original and holdout) of both risk groups were calculated. Careful attention was given to the variances of all variables in both original samples from which the discriminant functions were derived. (The data of the firms in both holdout samples was not used to develop the discriminant functions since the purpose of the holdout samples was to serve as an unbiased check on the discriminatory powers of the discriminant function.) An F test for equal variances was applied to each of the thirty-five variables in the two original samples. If either the results of the F test indicated a significant difference in variance at the 5% level or the variances were of a high magnitude, that variable was transformed for the eighty firms to improve intersample variance homogeneity. Prior to these transformations, the F tests indicated the variances of four variables in the two original samples were significantly different at the 1% level and the variance of one variable was significantly different at the 5% level. After transformations, only the variance of one variable ($\log_{10}$(sales/current assets)) in the two original samples was significantly different at the 5% level. These results indicated the transformations did improve variance homogeneity. The thirty-five variables used in the MDA after transformations are shown in Table 4-1.

---

4. Refer to Weston & Brigham [16], Chapter 2 and Van Horne [15], Chapter 25 for more discussion of the financial and operating variables presented in Table 4-2.

TABLE 4-1 - Variables (Ratios) Used in MDA

Variable Number	Variable (Ratio)	Transformation
1	Current Ratio (CA/CL)	
2	Quick Ratio (CA-INV.)/(CL)	
3	Working Capital/Total Assets	
4	Total Debt/Total Assets	
5	Long Term Debt/Total Assets	
6	Cash Flow/Net Worth	For variables 6-9, negative ratios replaced with 0 and all ratios > 1 replaced with 1.
7	Cash Flow/Total Assets	
8	Cash Flow/Total Debt	
9	Cash Flow/Long Term Debt	
10	Cash Flow/Sales	
11	Inventory/Total Assets	
12	EBIT/Sales	For variables 12-15, negative numbers replaced with 0.
13	EBIT/Total Assets	
14	Net Income/Net Worth	
15	Net Income/Total Assets	
16	(Cost of Goods Sold)/Sales	
17	Mean Mkt. Price per Shr. Common Stock	
18	Price Range per Shr. Common Stock	
19	Total Assets	Log_{10}
20	Shares Common Stock Traded	Log_{10}
21	Common Stock Dividend Payout	
22	Common Stock Mean Dividend Yield	
23	(Annual Capital Expend.)/Total Assets	
24	Stock Exchange Listing	1=NYSE; 0=AMSE
25	No. of Yrs. of Consec. Non-dec. Dividends	
26	Average Collection Period	
27	Total Debt/Net Worth	Add 1; Log_{10}
28	Long Term Debt/Net Worth	Add 1; Log_{10}
29	EBIT/Fixed Financial Charges	For variables 29 & 30, neg. numbers replaced with 0 and ratios $\geq$ 50 replaced with 50; Log_{10}
30	(EBIT + Depreciation)/Fixed Financial Chgs.	
31	Sales/Inventory	Log_{10}
32	Sales/Net Plant	Log_{10}
33	Sales/Current Assets	Log_{10}
34	Sales/Total Assets	Add 1; Log_{10}
35	(Cost of Goods Sold)/Inventory	Log_{10}

As discussed in Section A of this chapter, MDA assumes the variance-covariance matrices of the variables within each of the distinct groups are the same. The test for equality of these matrices of the high risk original sample and the low risk original sample was not conducted; however, the transformations should have improved the homogeneity of these two matrices. Further, the multivariant ANOVA F tests employed in MDA are robust with respect to this assumption if the original sample sizes are equal and the total original sample (the sum of the number of observations in each distinct group's original sample over all distinct groups) is large.[5] The original samples in each risk group were chosen such that these two conditions were satisfied.

Since the discriminant function developed in the MDA were derived from only one year's data (1972), the question of the stability (or representativeness) of the 1972 data could be raised. In order to test for this stability, the thirty-five variables of all eighty firms (this included both the holdout and original samples of each risk group) were factor analyzed using a BMD03M R-type factor analysis program.[6] If the same common factors emerged from the 1972 data as were present in previous independent research studies relevant to the identification of common factors in financial data, then the stability of the 1972 data

---

5. See Morrison [11], Section 4.9 for a discussion of the consequences of unequal variance-covariance matrices when either the original samples vary in size or the total original sample is not sufficiently large.

6. A description of the BMD03M program is presented in Dixon [5], BMD03M Section. For a discussion on general factor analysis multivariant techniques, refer to Morrison [11], Chapter 8, and Cooley & Lohnes [4], Chapters 4 and 5.

used in the study would be strengthened. Table 4-3 shows significant factor loadings of the thirty-five variables on six common factors which evolved after factor rotation.[7] The six common factors explained 79% of the total variation of all thirty-five variables. An interpretation of these six factors is given in Table 4-2. Although Factor 6 had only one variable (price range per share of common stock) with a factor loading $\geq .7$, the mean price per share of common stock loaded high (-.55) on Factor 6. In addition, two firm size variables ($\log_{10}$(total assets) and $\log_{10}$(shares traded)) loaded high on Factor 6; hence, Factor 6 was designated a market behavior and size factor.

TABLE 4-2 - Factor Interpretation

Factor Number	Interpretation
1	Return on Investment
2	Long Term Capital Turnover
3	Short Term Capital Turnover
4	Financial Leverage
5	Dividend Policy
6	Market Behavior & Size

7. Only variable factor loadings $\geq .7$ were reported in Table 4-3, since such a loading implies the factor accounted for approximately 50% or better of the total variation of the variable.

TABLE 4-3 - Variable Factor Loadings

Variable #	Variable	Factor Number 1	2	3	4	5	6
14	Net Income/Net Worth	.90					
15	Net Income/Total Assets	.88					
13	EBIT/Total Assets	.84					
29	EBIT/Fixed Financial Charges	.81					
30	(EBIT + Depr.)/Fixed Fin.Chgs.	.72					
32	Sales/Net Plant		-.88				
10	Cash Flow/Sales		.80				
11	Inventory/Total Assets		-.79				
34	Sales/Total Assets		-.72				
35	Cost of Goods Sold/Inventory			-.93			
31	Sales/Inventory			-.90			
33	Sales/Current Assets			-.88			
5	Long Term Debt/Total Assets				-.93		
4	Total Debt/Total Assets				-.92		
28	Long Term Debt/Net Worth				-.91		
27	Total Debt/Net Worth				-.90		
22	Mean Common Stock Dividend Yield					.90	
21	Common Stock Dividend Payout					.86	
25	Cons.Years Non-dec. Common Stk.Div.					.75	
18	Price Range per Shr. of Common Stk.						-.71

Note: Only factor loadings $\geq$ .7 reported

The variables associated with the six rotated factors which resulted from the factor analysis shown in Table 4-3 closely resembled those variables associated with six of the seven common factors of a bond classification study conducted by Mingo and Pinches [9] . Also, the factor identifications of several of the variables of this study were very similar to the results of another recent study by Mingo and Pinches [10] dealing with the stability of variable-factor associations over time. Thus, the factor analysis supported and strengthened the stability question about the use of the 1972 data in the MDA.

Section D - MDA Results

The forty firms in each of the high and the low business risk groups were subdivided into an original sample of twenty-five firms and a holdout sample of fifteen firms. The original samples were used to develop discriminant functions, while the holdout samples were used to serve as an unbiased test of the discriminatory powers of the discriminant functions. The firms in the original and holdout samples of each risk group used in the MDA are denoted in Appendix A-5. The thirty-five variables (Table 4-1) for each of the eighty firms used in the MDA were then calculated.

A stepwise MDA program was applied to the thirty-five variables of the fifty firms composing the two original samples to develop sets of two discriminant functions to be used for the classification of firms into either the high or low business risk groups.[8] The discriminatory

8. The BMD07M stepwise MDA program was used in this study. A description of this program is given in Dixon [5] , Section BMD07M, "Stepwise Discriminant Analysis."

powers of these discriminant functions were tested at each iteration of the program via classification matrices for firms in both the original and holdout samples.

The initial phase of the stepwise MDA program was to calculate the sample means and sample standard deviations of the thirty-five operating and financial variables (ratios) in the original and holdout samples. Then, at each iteration, the program selected as the variable to enter into the two discriminant functions that variable with the largest F ratio (F to enter) of between to within original samples' sum of squares both given the set of variables already included in the discriminant functions and providing the F to enter ratio was ≥ 1.2. Also, at each iteration, any variable was deleted from the two functions if its F ratio (F to exit) of between to within original samples' sum of squares (given the set of variables already included in the discriminant functions) was less than 1.0. The multivariant ANOVA F ratio of between to within original samples' sum of squares was calculated after each iteration (the addition or the removal of a variable from the discriminant functions). This F ratio was used to test the null hypothesis of equal original samples' multivariant centroids in the k dimensional space, where k was equal to the number of variables in the discriminant functions after the iteration. The two discriminant functions and a classification matrix for both the fifty firms in the two original samples and the thirty firms in the two holdout samples were reported after each iteration. The program terminated when both the following conditions existed:

1) the F to enter ratio of each variable not included in the discriminant functions was less than 1.2, and

2) the F to exit ratio of each variable included in the discriminant functions was greater than 1.0.

Upon termination, the program reported the classification of each of the eighty firms into either the high or low business risk groups. The classification of a firm was based on the minimum Mahalanobis distance in the two discriminant scores' dimensional space from the firm to the high and low risk original sample centroid.

Table 4-4 shows the results of the stepwise MDA after the final iteration. Only one variable ($\log_{10}$(sales/inventory)) was entered and later removed from the discriminant functions. After the final iteration (#15), thirteen variables were in the discriminant functions. These functions correctly classified all fifty firms (100%) in the original samples and twenty-two firms (73%) in the holdout samples. This classification was considerably better than the 50-50 chance classification in each set of samples.

These results were significant. They suggest MDA can be useful in identifying a set of business risk discriminatory variables.

Optimal classification in the holdout samples occurred in the fourth iteration with twenty-six of the thirty firms (87%) in the high and low business risk groups' holdout samples being correctly classified. With the first four entered variables in the discriminant functions, 94% of the original samples were correctly classified and 91% of both original and holdout samples were correctly classified. As seen in Table 4-4, after the third iteration, the increase in the discriminatory

Iteration #	Variable	Action on Variable	Factor #	#(k) of Variables in Discriminant Fns.	% Firms Correctly Classified: Original Sample	Holdout Sample	Total Sample	F Ratio to Test For Equal Group Centroids in k Dimensional Space	Degrees of Freedom
1	Log_{10}(Total Assets)	Entered	6	1	84	77	81	47.97	1,48
2	Yrs. Cons. Div.	Entered	5	2	90	80	86	38.93	2,47
3.	Dividend Payout	Entered	5	3	94	83	90	30.15	3,46
4	Log_{10}(Sales/Inventory) *	Entered	3	4	94	87	91	25.38	4,45
5	Cash Flow/Long Term Debt	Entered	4	5	96	77	89	21.93	5,44
6	Mean Per Shr.Com.Stk.Price	Entered	6	6	96	77	89	20.86	6,43
7	Cash Flow/Total Debt	Entered	4	7	94	77	88	19.27	7,42
8	Log_{10}(Long Term Debt/Net Worth)	Entered	4	8	94	77	88	17.93	8,41
9	Long Term Debt/Total Assets	Entered	4	9	96	73	88	16.47	9,40
10	Log_{10}(Sales/Net Plant)	Entered	2	10	96	77	89	15.09	10,39
11	Cash Flow/Long Term Debt	Removed	—	9	96	77	89	16.68	9,40
12	Log_{10}(Shares Traded)	Entered	6	10	96	73	88	15.57	10,39
13	Cost of Goods Sold/Sales	Entered	1	11	100	77	91	14.98	11,38
14	Working Capital/Total Assets	Entered	2	12	100	73	90	14.58	12,37
15	Total Debt/Total Assets	Entered	4	13	100	73	90	13.62	13,36

Note: All F ratios sign. at 1% level

* Removed in Iteration # 11

TABLE 4-4 - Summary of MDA Program Results powers of the discriminant functions became marginal with additional variables included in the functions.

The above finding implies firms can be categorized as being either high or low business risk firms by observing a small set of financial and operating variables.

An examination of the first three variables which entered in the discriminant functions revealed firm size (total assets) and dividend policy related variables were the most significant business risk classifier characteristics of firms. The univariate means of the first three entered variables were distinctly different for the high and low business risk groups (Table 4-5).

TABLE 4-5 - Univariate Means of Key Classifier Variables

Variable/Sample	High Original	Low Original	High Holdout	Low Holdout
Log_{10}(Total Assets)	1.77	2.79	2.03	2.79
Yrs. Consecutive Non-Decreasing Div.	1.76 Yrs.	10.96 Yrs.	2.73 Yrs.	13.0 Yrs.
Dividend Payout	10%	37%	15%	45%

Table 4-6 indicates firms which were misclassified with the fourth iteration discriminant functions that had optimal discriminatory powers in the holdout samples. The probable reason for misclassification in each case was associated with either size and/or dividend policy related variables.

TABLE 4-6 - Iteration Four Misclassifications

Firm	Group	Sample	$Log_{10}(TA)$	Yrs.Div.	Div.Payout	Probable Reason For Misclassification
Whittaker	High	Original	2.76	0	0	Size
N.W. Airlines	High	Holdout	2.96	18 yrs.	54%	Size & Dividend Policy
Adams Mills	High	Holdout	1.70	4 yrs.	91%	Dividend Policy
Maremont	Low	Original	2.21	0	0	Size & Dividend Policy
Kaiser Ind.	Low	Holdout	2.79	0	0	Dividend Policy
Reserve Oil	Low	Holdout	1.94	0	0	Size & Dividend Policy

Thus, the results of the MDA and an examination of univariate means revealed relative to high business risk firms, low business risk firms:

1) are larger in size (total assets),

2) have a more stable dividend policy, and

3) have a larger dividend payout.

These findings are consistent with the traditional financial theory on firm development. New firms are typically considered to have considerable business risk because they are usually either trying to break into untested product markets, competing with the older, more established firms, or developing a new high risk technology. Their initial problem is capitalization due to their poor access to capital markets. Equity capital is difficult to raise since the newer firms do not have wide investor exposure resulting in weak primary and secondary markets for their securities. Without an adequate equity base, investment bankers are somewhat skeptical of underwriting large long term debt issues of the newer firms. Thus, the lack of access to capital markets by the newer, more business risky firms results in small initial size (total assets) and dependence on internal financing. Dependence on internal financing with retained earnings implies low dividend payouts and unstable dividend policies.

On the other hand, older firms should have proven (less risky) product markets, a maturely developed technology, and experienced management. These factors should generally cause older firms to be less business risky than new firms. These older and lower risk firms should have acquired a ready access to capital markets resulting in larger size (total assets) and less dependence on internal financing (which implies higher dividend payouts and stable dividend policies).

Finally, of the thirteen variables in the discriminant functions after the fifteenth (final) iteration shown in Table 4-4, the following factor representations were evident:

1) Only one variable associated with the return on investment factor (Factor #1) was present and that variable did not enter into the functions until the thirteenth iteration.
2) Two variables associated with the long term capital turnover factor (Factor #2) were present. They did not enter the functions until the tenth and fourteenth iterations.
3) No variables associated with the short term capital turnover factor (Factor #3) were present. Log_{10}(sales/inventory) entered into the functions in the fourth iteration, but it was removed in the eleventh iteration.

4) Five financial leverage factor (Factor #4) variables were present; however, none of these variables entered into the functions early. Their entries occurred in the fifth, seventh, eighth, ninth, and fifteenth iterations.

5) Two of the three dividend policy factor (Factor #5) variables entered into functions in early iterations (the second and third iterations).

6) Three market behavior and size factor (Factor #6) variables were present in the functions. They entered in the first, sixth, and twelth iterations.

These findings have important implications for future researchers and practitioners. They imply:

1) Variables associated with profitability and capital turnover are relatively insignificant as business risk classifiers.

2) Variables associated with dividend policy, size and market behavior, and financial leverage may be very good business risk discriminators among firms.

Section E - Summary of Chapter 4

The purpose of the research presented in this chapter was to identify a set of operating and financial characteristics of firms which can be used to classify firms into their proper business risk groups. The quantitative technique employed was multiple discriminant analysis (MDA).

In Section A, a brief description of MDA techniques and previous application of MDA in financial research were discussed. It was noted no previous attempts have been made to apply MDA to the identification of business risk discriminating characteristics of firms.

The one hundred forty-four firms in the total study sample were rank ordered from highest to lowest measured business risk in Section B. Then, a high risk group and a low risk group of forty firms per group were designed. The business risk distinctness of these two groups was confirmed using a Kruskal-Wallis nonparametric analysis of variance test. Within each group, an original sample of twenty-five firms and a holdout sample of fifteen firms were designated for the MDA program.

Section C discussed the thirty-five variables calculated for the eighty firms in the two risk groups. The rationale for transformations applied to certain variables was presented. Since only 1972 data was used in the MDA, the thirty-five variables were factor analyzed to test if the 1972 data was representative of other years' data. Since variable-factor relationships similar to those relationships found in previous research studies were found, the representativeness of the 1972 data was supported.

A stepwise MDA program was applied to the thirty-five variables of the eighty firms in the two risk groups in Section D. Thirteen variables were present in the final set of discriminant functions. These functions classified 100% of the firms in the original samples correctly and 73% of the firms in the holdout samples correctly. These classifications were significantly superior to 50-50 chance classifications. Optimal classification (87%) in the holdout samples occurred in the program's fourth iteration. After the third iteration, the discriminant functions contained a size and two dividend policy related variables; these functions classified 94% of the original samples and 83% of the holdout samples correctly. Only marginal increases in the discriminatory powers of the business risk discriminant functions were present after the third iteration. In fact, the third iteration discriminant functions had better discriminatory powers than did functions of most later iterations containing more variables.

The MDA results indicated variables associated with profitability and capital turnover were relatively unimportant business risk discriminators among firms. However, dividend policy, size, and market behavior related variables were key business risk discriminators. Financial leverage variables may be important discriminatory variables in the absence of the key discriminators.

Further, the lower business risk firms were characterized as being large (size in total assets) and having stable dividend policies with high dividend payouts relative to the higher business risk firms.

These findings are highly significant to financial researchers and practitioners. They not only imply financial and operating characteristics of firms may be used to classify firms into the respective risk classes, but also, only a small set of key discriminatory characteristics may be necessary for this purpose. Perhaps, future research will indicate firms grouped into equivalent business risk groups according to their financial and operating characteristics are both more intragroup business risk homogeneous and more intergroup business risk heterogeneous than firms grouped into equivalent business risk groups by industry classifications.

REFERENCES

1. Altman, Edward I., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance, September, 1968.

2. Anderson, T. W., An Introduction to Multivariate Statistical Analysis. John Wiley & Sons, New York, 1958.

3. COMPUSTAT TAPES, Investors Management Science, Inc., Denver, 1972.

4. Cooley, William W. and Paul R. Lohnes, Multivariate Data Analysis. John Wiley & Sons, New York, 1971.

5. Dixon, W. J. (ed.), BMD: Biomedical Computer Programs. University of California Press, Berkeley, 1971.

6. Edmister, Robert O., "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction," Journal of Financial and Quantitative Analysis, March, 1972.

7. Goodman, M. L. and W. H. Williams, "A Statistical Grouping of Corporations by Their Financial Characteristics," Journal of Financial and Quantitative Analysis, September, 1971.

8. Latane, H. A. and W. J. Reinhart, "A Multivariate Analysis of Earnings-Price Ratios." Unpublished paper given at the Southern Finance Association meeting in Houston, Texas, November, 1973.

9. Mingo, K. A. and G. E. Pinches, "A Multivariate Analysis of Industrial Bond Ratings," Journal of Finance, March, 1973.

10. Mingo, K. A., G. E. Pinches and J. K. Caruthers, "The Stability of Financial Patterns in Industrial Organizations," Journal of Finance, May, 1973.

11. Morrison, Donald F., Multivariate Statistical Methods. McGraw-Hill, New York, 1967.

12. Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences. McGraw-Hill, New York, 1956.

13. Smith, K. V., "Classification of Investment Securities Using MDA." Paper #101, Herman C. Krannert Graduate School of Industrial Administration, Purdue University, Lafayette, Indiana, 1965.

14. Tatsuoka, Maurice M., Multivariate Analysis: Techniques for Educational and Psychological Research. John Wiley & Sons, New York, 1971.

REFERENCES

15. VanHorne, James C., Financial Management and Policy, Third Edition. Prentice Hall, Englewood Cliffs, N. J., 1974.

16. Weston, J. Fred and Eugene F. Brigham, Managerial Finance, Fourth Edition. Holt, Rinehart and Winston, Inc., New York, 1972.

CHAPTER 5

SUMMARY AND CLOSING COMMENTS

The primary emphasis of this research paper was on firms' business risk. Business risk is that aspect of a firm's total risk characteristics which is attributed to the firm's volatility in before tax operating profits. This volatility in the firm's operating profits can result from several characteristics of the firm such as:

1) stability of sales,
2) changes in market and production technology,
3) labor force behavior,
4) quality of real production assets,
5) quality of management,
6) nature of the production process,
7) nature of the raw materials market,
8) governmental impacts of new legislation, and
9) synchronization of operating profits with national and international economic conditions.

Previous research studies in the finance related areas have given too little attention to firms' business risk behavior. Typically, researchers have attempted to hold the effects of a firm's business risk constant by assuming firms grouped by industry classifications are both intragroup business risk homogeneous and intergroup business risk heterogeneous. (These two assumptions have become known as the equivalent-risk class hypothesis.) Thus, by selecting firms in only one industry, it has previously been assumed the interrelationships between other financial variables could be analyzed without bias from or need for specification of the firms' business risk benavior. Many classical financial research efforts on such basic topics as financial leverage and the cost of capital, valuation of the firm, dividend policy effects on the firm's value, and portfolio management have assumed business risk could be dealt with in this manner. Clearly, if the assumptions about industry classification of firms and firms' business risk behavior (the equivalent-risk class hypothesis) are invalid, the results of these previous research efforts may become somewhat suspect as to their validity.

Attempts have been made to test the validity of the equivalent-risk class hypothesis. However, little information has been gained from these attempts, since their results have been conflicting. Hence, the lack of consistency in the results of previous attempts to test the validity of the equivalent-risk class hypothesis and the preponderance in the literature of the validity of the equivalent-risk class hypothesis provided the justification for this research.

Section A - Research Objectives

The objectives of this research were to:

1) develop a theoretically sound measure which will cardinally measure a firm's business risk,
2) retest the assumptions of the equivalent-risk class hypothesis which asserts firms grouped by industry classification are both intragroup business risk homogeneous and intergroup business risk heterogeneous,

3) form groups of firms which are homogeneous and unique according to the business risk measure, and

4) identify a set of operating and financial characteristics of firms which can be used as business risk discriminatory characteristics.

Section B - The Business Risk Measure

The theoretical justification for the use of the volatility in the ratio of earnings before interest and taxes to total assets as a firm's cardinal business risk measure was presented in Chapter 2. A measurement of the volatility of this ratio was also given in Chapter 2. The development of a theoretically sound business risk measure was significant since previous researchers provided little justification for their business risk measures. This finding was surprising. Perhaps this paper has made a contribution in the area of presenting a theoretically sound business risk measure which has universal appeal.

Section C - The Tests of the Assumptions of the Equivalent-Risk Class Hypothesis

Twelve firms from each of twelve industry classifications comprised the study sample. Each firm's business risk was measured. It was shown the business risk measures were approximately normally distributed in each industry group and four equal business risk variance subgroups of industry classifications of firms existed. Appropriately, parametric analysis of variance pairwise F tests were applied to the four subgroups of industry classifications to test the validity of the intergroup heterogeneity assumption of the equivalent-risk class hypothesis. The results of the tests indicated the following groups of firms were not intergroup business risk heterogeneous:

1) tire & rubber and textiles;
2) textiles and electronics;
3) tire & rubber, auto parts, air transport, and meat packers;
4) meat packers and electronics;
5) air transport and meat packers;
6) domestic oils and paper;
7) paper, conglomerates, chemical & chemical preparations, and metals;
8) metals and steel;
9) steel and air transport.

These findings strongly suggest this assumption of the equivalent-risk class hypothesis is invalid.

The firms in the study were then rank ordered from the highest to lowest business risk firms. The forty-eight highest business risk firms were designated as the high risk group, the next forty-eight firms were designated as the medium risk group, and the lowest forty-eight firms were designated as the low risk group. The number of firms from each industry classification in each risk class was noted. This procedure indicated three groups of firms:

1) textile apparel,
2) auto parts, and
3) tire & rubber

were not intragroup business risk homogeneous, as purported by the equivalent-risk class hypothesis.

Thus, neither the intergroup business risk heterogeneity assumption nor the intragroup business risk homogeneity assumption of the equivalent-risk class hypothesis was substantiated by this research.

Section D - The Identification of a Set of Business Risk Discriminatory Characteristics of Firms

Two business risk distinct groups of firms were formed (a high and a low business risk group). The distinctness of these two groups was confirmed using a Kruskal-Wallis test. Each group (composed of forty firms) was divided into an original sample of twenty-five firms and a holdout sample of fifteen firms. Thirty-five financial and operating characteristics (variables) were calculated for each firm in the two groups for the year 1972. These variables measured such firm characteristics as profitability, short term capital turnover, long term capital turnover, financial leverage, dividend policy, and size & market behavior. The thirty-five variables of each of the eighty firms in the two risk groups was then factor analyzed. The stability (representativeness) of this 1972 data was supported by the similarities in the variable-factor relationships of the 1972 data used in the study and those variable-factor relationships in previous independent research studies.

A stepwise multiple discriminant analysis (MDA) program was employed to identify business risk discriminatory variables. The MDA results were as follows:

1) Discriminant functions containing only three variables ($\log_{10}$(total assets), consecutive years of non-decreasing dividends per share, dividend payout ratio) correctly classified 94% of the original samples and 83% of the holdout samples. Further, only marginal increases occurred in the discriminatory powers of discriminant functions containing variables additional to the three above mentioned variables.
2) Variables associated with profitability, short term capital turnover, long term capital turnover, and financial leverage were relatively unimportant business risk discriminatory variables; while variables associated with dividend policy and size were key business risk discriminators.
3) Relative to the high business risk firms, the low business risk firms were large (size in total assets) and had stable dividend policies with high dividend payouts.

These results are very significant since they imply a small set of characteristics exist which can serve as business risk discriminators among firms. No prior published research has investigated this area; a fact which places even greater emphasis on these research findings.

Section E - Implications of the Research Findings & Suggestions for Future Researchers

The significant findings of this research were:

1) the invalidity of the assumptions of the equivalent-risk class hypothesis, and
2) the identification of a small set of business risk discriminatory characteristics (variables).

If the assumptions of the equivalent-risk class hypothesis are in fact invalid, acamedicians need to re-examine the classical works on the cost of capital controversies, firm valuation, and optimal dividend policy which abstracted from the business risk effects by assuming the validity of the equivalent-risk class hypothesis. Further, portfolio managers should be aware that attempts to diversify portfolio business risk by selecting securities of firms in different industries may be futile. The same applies to diversification oriented firms seeking to acquire firms from different industry classifications to stabilize their operating profits. Likewise, financial intermediaries such as commercial banks, governmental insuring agencies, insurance companies, and investment bankers should be ill advised to attach uniform business risk premiums to all firms in any industry classification.

The identification of a small set of business risk discriminatory variables should be encouraging to the financial community, since the validity of the equivalent-risk class hypothesis has been strongly challenged. It must be noted, however, this finding cannot be universalized until future research can thoroughly substantiate its validity. Given the methodology and findings of this research as a starting point, future researchers need to investigate the following issues:

1) Do the same key business risk discriminatory variables appear with other samples of firms over different time periods?
2) Only one year's data (1972) was used in this study's MDA analysis. Should the data be three year averages to

sooth out any transient effects in the variables?

3) How rapidly does the discriminatory powers of the set of key discriminator variables deteriorate when the number of distinct business risk groups is increased, with each group less business risk distinct than the two groups used in this study?

4) Should a firm's total business risk behavior in operating earnings before interest and taxes to total assets (EBIT/TA) be characterized by three parameters such as level, trend, and volatility rather than only a volatility measure? If so, perhaps the use of cluster analysis on the three parameters of firms would produce genuinely business risk unique groups of firms independent from industry classifications.

5) Can firms be grouped into equivalent "total" risk groups according to their business risk and financial risk characteristics which are intragroup total risk homogeneous and intergroup total risk heterogeneous? Then, given the total risk groups, what are implications for portfolio managers, professional lenders, diversification oriented firms, and private investors?

With the increased awareness and popularity of multivariate techniques in the financial literature, these issues are sure to be addressed in the near future. It is hoped these research efforts will be helpful to future researchers engaged in these endeavors.

TABLE OF APPENDICES

APPENDIX A-1 - The Study Sample

Metals - Misc.

Amer. Metal Climax
Amer. Smelting & Ref.
Brush-Wellman
Cerro
Cleve-Cliffs Iron
Cyprus Mines
Driver Harris
Fansteel
Hanna Mining
Intl. Nickel Canada
Molyb-Denum
Utah Intl.

Food - Meat Packers

Bluebird
General Host
Greyhound
Hormel
Hygrade
Iowa Beef
Kane-Miller
Mayer
Tobin
United Bros.
Zion
Rath

Textile Apparel Mfg.

Adams Mills
Aileen
Blue Bell
Chadbourn
Cluett Products
Genesco
Hart, Schaffner & Marx
Jonathan Logan
Munsingwear
Originala
Pioneer
Warnaco

Paper

Crown Zellerbach
Great Northern
Hammermill
Intl. Paper
Kimberly Clark
Mead
Scott
Sorg
St. Regis
Union Camp
Westvaco
Whippany

APPENDIX A-1 - The Study Sample

Chemical & Chem. Preparations

Ansul
Conwood
Ethyl
Fairmont
Grow Chem.
Lubrizol
Diversity
Nalco
Oakite
Purex
Sun Chem.
West Chem.

Oil - Integrated Domestics

Ashland
Atlantic-Richfield
Cities Service
Marathon
Quaker State
Conoco
Phillips
Reserve
Shell
Skelly
Sun
Union

Tire & Rubber

A O Industries
Armstrong
Carlisle
Cooper Tire & Rubber
Dayco
Firestone
General
Goodrich
Goodyear
Mansfield
Mohawk
Uniroyal

Steel - Minor

Alanwood
Allegheny Ludlum
Ampco Pittsburg
Carpenter
Copperweld
Dominion Fndrs.
Florida
Kaiser
Latrobe
McLouth
Pheonix
Standard Alliance

APPENDIX A-1 - The Study Sample

Electronics

Ambac
Collins
Conrac
Edo Corp.
Fairchild
Hazeltine
High Voltage
Ratheon
Sanders Assoc.
Servco
Sparton
VLN

Auto Parts

Aspro
Amer. Safety
Bearings, Inc.
Bendix
Borg-Warner
Champion
Dana
Eaton
Gould
Howell
Maremont Corp.
Napco

Air Transport

Allegheny
American
Braniff
Continental
Delta
Eastern
Flying Tiger
Frontier
National
Northwest
UAL
Western

Conglomerates

Avco
Gulf-Western
Indian Head
I T & T
Kaiser Inds.
Kidde
Litton
Signal
Teledyne
Tenneco
Textron
Whittaker

APPENDIX A-2 - Regression Results (Notes)

a_i = the ith firm's regression constant.

b_i = the ith firm's regression coefficient.

r^2 = the coefficient of determination (percent of total variation explained by the linear regression).

d = Durbin-Watson statistic.

SE_i = ith firm's business risk (standard error of the estimate).

F = regression F statistic.

APPENDIX A-2 - Regression Results

TABLE 1 - Metals - Misc.

Firm	a_i	b_i	r^2	F	d	SE
Amer. Metal Climax	10.0	- .43	.65	17.57**	2.86	.94
Amer. Smelting & Ref.	13.5	-1.20	.65	17.37**	1.51	2.60
Brush-Wellman	1.3	1.40	.45	8.66*	1.63	4.33
Cerro	16.0	-1.32	.51	10.29*	1.29	3.75
Cleve-Cliffs Iron	6.1	.14	.04	.71	1.96	1.50
Cyprus Mines	3.73	.63	.37	6.18*	1.84	2.31
Driver Harris	4.4	.15	.11	.12	1.79	3.81
Fansteel	5.8	.02	.12	.00	1.89	4.17
Hanna Mining	4.8	.16	.09	1.89	1.87	1.06
Intl.Nickel Canada	24.1	-1.45	.59	14.03**	2.68	3.51
Molyb-Denum	.07	4.5	.12	.07	1.51	2.39
Utah Intl.	2.7	.48	.48	9.15*	1.99	1.45

*** Autocorrelation suggested by Durbin-Watson statistic (d)

** Significant at 1% level

* Significant at 5% level

TABLE 2 - Meat Packers

Firm	a_i	b_i	r^2	F	d	SE
Bluebird	.74	1.63	.38	6.43*	2.29	5.84
General Host	4.05	- .05	.12	.03	1.88	2.87
Greyhound	22.91	-1.65	.92	109**	1.39	1.43
Hormel	9.16	.82	.25	3.62	1.97	3.91
Hygrade	-1.26	1.26	.21	3.34	1.93	6.28
Iowa Beef	22.69	-1.54	.39	6.68*	2.15	5.40
Kane-Miller	8.1	- .04	.12	.03	1.39	2.16
Mayer	17.2	.12	.10	.16	2.16	2.82
Tobin	19.6	-1.71	.83	46.03**	1.93	2.29
United Bros.	5.6	.10	.12	.03	.68***	4.97
Zion	8.4	-1.11	.38	6.60*	1.26	3.91
Rath	-2.2	.56	.09	.24	1.77	10.41

APPENDIX A-2 - Regression Results

TABLE 3 - Textiles

Firm	a_i	b_i	r^2	F	d	SE
Adams Mills	13.0	- .28	.06	.45	1.19	3.82
Aileen	12.9	1.81	.02	.83	1.13	18.09
Blue Bell	11.1	.19	.01	.92	1.28	1.85
Chadbourn	8.3	-.86	.15	2.63	1.49	4.81
Cluett Products	15.6	-.77	.63	16.51**	1.57	1.72
Genesco	12.6	-.11	.09	.23	.95***	2.06
Hart,Schaffner& Marx	15.3	-.47	.11	2.09	.74***	2.94
Jonathan Logan	13.1	.05	.10	.18	1.64	1.06
Munsingwear	16.3	-.22	.01	.98	2.34	2.05
Originala	52.1	-4.33	.67	19.27**	1.92	8.96
Pioneer	19.5	-1.23	.12	2.23	.90***	7.48
Warnaco	14.6	- .64	.44	8.16*	1.85	2.03

*** Autocorrelation suggested by Durbin-Watson statistic(d)

** Significant at 1% level

* Significant at 5% level

TABLE 4 - Paper

Firm	a_i	b_i	r^2	F	d	SE
Crown Zellerbach	12.5	- .58	.85	50.38**	2.62	.74
Great Northern	7.3	.22	.09	1.87	1.19	1.45
Hammermill	12.3	- .79	.71	22.86**	1.23	1.49
Intl. Paper	12.7	- .55	.50	9.88*	2.22	1.58
Kimberly Clark	12.8	- .41	.36	6.40*	1.90	1.47
Mead	10.4	- .53	.44	8.18*	1.12	1.69
Scott	18.3	-1.10	.81	40.21**	1.58	1.58
Sorg	11.0	-1.17	.22	3.48	1.03	5.69
St. Regis	6.6	- .10	.04	.65	1.78	1.15
Union Camp	11.7	- .29	.20	3.30	2.10	1.44
Westvaco	10.9	- .75	.43	7.75*	1.04	2.44
Whippany	10.1	- .94	.80	36.45**	3.36***	1.42

APPENDIX A-2 - Regression Results

TABLE 5 - Chem. & Chem. Preparations

Firm	a_i	b_i	r^2	F	d	SE
Ansul	13.0	- .28	.04	.66	***.75	3.16
Conwood	19.6	.16	.07	.45	***.63	2.23
Ethyl	14.0	- .10	.07	.41	1.54	1.39
Fairmont	18.8	-2.04	.85	**53.57	2.25	2.54
Grow Chem.	20.6	-1.45	.67	**19.34	2.89	2.99
Lubrizol	23.5	.16	.08	.32	1.23	2.52
Diversity	12.9	-.32	.04	1.40	1.29	2.46
Nalco	23.5	.32	.01	1.08	1.17	2.79
Oakite	28.6	-.20	.11	.10	***.69	5.70
Purex	19.5	-.68	.50	*10.13	2.21	1.95
Sun Chem.	8.6	.01	.12	.00	1.32	1.27
West Chem.	16.0	.08	.11	.11	1.94	2.16

*** Autocorrelation suggested by Durbin-Watson statistic (d)

** Significant at 1% level

* Significant at 5% level

TABLE 6 - Oils

Firm	a_i	b_i	r^2	F	d	SE
Ashland	12.4	- .33	.10	1.96	***.87	2.12
Atlantic-Richfield	6.93	- .03	.10	.17	1.46	.70
Cities Service	7.5	- .02	.12	.03	***.68	1.02
Marathon	7.9	1.07	.82	**43.16	1.33	1.48
Quaker State	19.7	.24	.07	.39	1.14	3.55
Conoco	6.4	.60	.73	**25.79	1.10	1.06
Phillips	8.9	- .28	.67	**19.49	1.96	.58
Reserve	5.6	- .27	.15	2.59	1.27	1.50
Shell	11.1	- .41	.64	**17.27	***.80	.89
Skelly	6.8	.00	.13	.00	1.83	.84
Sun	8.0	.08	.23	3.76	1.34	.39
Union	9.5	- .29	.74	**26.79	1.36	.51

APPENDIX A-2 - Regression Results

TABLE 7 - Tire & Rubber

Firm	a_i	b_i	r^2	F	d	SE
A O Industries	14.5	.24	.12	.04	2.78	11.38
Armstrong	11.7	- .54	.53	11.25**	1.77	1.46
Carlisle	27.1	-1.17	.24	3.82	1.06	5.44
Cooper Tire & Rubber	14.4	- .60	.25	4.05	1.26	2.71
Dayco	11.6	- .20	.11	2.14	.69***	1.27
Firestone	14.9	- .47	.62	15.67**	1.53	1.08
General	10.8	- .14	.09	.25	2.22	2.46
Goodrich	8.8	- .15	.04	.64	1.98	1.73
Goodyear	13.9	- .30	.42	7.45*	2.28	.99
Mansfield	.30	.67	.18	3.00	1.38	3.51
Mohawk	11.2	- .30	.18	2.96	1.32	1.59
Uniroyal	8.9	- .21	.04	1.36	2.16	1.60

*** Autocorrelation suggested by Durbin-Watson statistic (d)

** Significant at 1% level

* Significant at 5% level

TABLE 8 - Steel

Firm	a_i	b_i	r^2	F	d	SE
Alanwood	5.1	.02	.12	.01	1.63	2.15
Allegheny Ludlum	16.7	-1.05	.52	10.69*	1.32	2.91
Ampco Pittsburg	3.4	.58	.32	5.27	1.81	2.31
Carpenter	24.5	-1.1	.16	2.73	.80***	6.06
Copperweld	11.6	- .01	.13	.00	1.38	3.24
Dominion Fndrs.	17.6	- .90	.64	17.25**	1.35	1.96
Florida	13.0	.99	.25	3.93	1.67	4.56
Kaiser	9.5	- .76	.29	4.60	.60***	3.22
Latrobe	16.8	-1.84	.59	14.07**	1.26	4.46
McLouth	17.6	-2.20	.67	19.09**	1.55	4.57
Pheonix	6.0	-1.00	.24	3.80	.99***	4.66
Standard Alliance	6.8	- .07	.12	.03	2.51	3.46

APPENDIX A-2 - Regression Results

TABLE 9 - Electronics

Firm	a_i	b_i	r^2	F	d	SE
Ambac	12.5	- .42	.09	1.88	2.10	2.77
Collins	15.7	-2.32	.43	* 7.72	*** .51	7.60
Conrac	18.2	- .50	.00	1.03	1.32	4.50
Edo Corp.	11.4	- .79	.19	3.08	1.51	4.09
Fairchild	10.9	-1.68	.22	3.55	1.41	8.08
Hazeltine	14.3	-1.63	.22	3.56	1.04	7.85
High Voltage	12.8	-1.57	.54	** 11.51	2.04	4.19
Ratheon	8.8	.45	.20	3.28	*** .59	2.27
Sanders Assoc.	31.1	-4.11	.57	** 12.80	*** 2.97	10.43
Servco	7.2	.68	.08	.33	1.72	10.84
Sparton	10.6	.18	.12	.05	2.19	7.42
VLN	6.6	.17	.11	.14	2.35	4.08

*** Autocorrelation suggested by Durbin-Watson statistic (d)

** Significant at 1% level

* Significant at 5% level

TABLE 10 - Auto Parts

Firm	a_i	b_i	r^2	F	d	SE
Aspro	22.4	-1.47	.43	* 7.84	1.07	4.78
Amer. Safety	21.4	- .96	.03	.74	1.24	10.15
Bearings, Inc.	21.8	- .07	.09	.26	1.92	1.19
Bendix	11.1	- .27	.10	2.05	1.69	1.68
Borg-Warner	14.6	- .49	.65	** 17.91	1.34	1.05
Champion	30.2	- .45	.29	4.67	2.68	1.89
Dana	24.1	- .99	.66	** 18.21	*** 3.06	2.11
Eaton	19.2	- .55	.23	3.73	1.92	2.59
Gould	11.3	- .18	.05	.56	2.49	2.17
Howell	24.1	-2.53	.64	** 17.31	1.93	5.53
Maremont Corp.	6.9	.37	.19	3.17	1.32	1.89
Napco	4.6	.22	.03	.67	1.93	2.39

APPENDIX A-2 - Regression Results

TABLE 11 - Air Transport

Firm	a_i	b_i	r^2	F	d	SE
Allegheny	4.8	- .10	.11	.14	1.11	2.30
American	10.3	- .94	.68	20.11**	1.50	1.90
Braniff	8.1	- .22	.06	.50	1.82	2.88
Continental	15.4	-1.13	.27	4.26	.99***	4.99
Delta	22.8	-1.28	.50	10.03*	1.37	3.68
Eastern	1.27	.32	.03	.71	1.41	3.46
Flying Tiger	6.4	.31	.09	.26	1.56	5.47
Frontier	13.1	-1.17	.17	2.79	.86***	6.35
National	21.2	-1.64	.44	8.07*	1.34	5.25
Northwest	28.0	-2.44	.59	13.95**	.72***	5.94
UAL	8.8	- .66	.42	7.65*	2.08	2.15
Western	22.5	-2.21	.61	15.33**	1.20	5.12

*** Autocorrelation suggested by Durbin-Watson statistic (d)

** Significant at 1% level

* Significant at 5% level

TABLE 12 - Conglomerates

Firm	a_i	b_i	r^2	F	d	SE
Avco	19.4	-1.88	.89	75.46**	2.26	1.96
Gulf-Western	13.1	-1.06	.74	26.32**	1.21	1.88
Indian Head	8.9	.40	.43	7.90*	1.86	1.30
I T & T	8.0	.05	.10	2.00	.60***	.35
Kaiser Inds.	- .7	.16	.03	.74	1.29	1.73
Kidde	5.7	.85	.42	7.62*	.78***	2.81
Litton	17.1	-1.19	.79	35.02**	1.36	1.82
Signal	8.1	- .40	.18	2.96	1.68	2.10
Teledyne	13.1	- .43	.08	1.80	.61***	2.93
Tenneco	5.3	.17	.67	18.90**	1.41	.36
Textron	15.2	.08	.11	.10	.65***	2.47
Whittaker	8.5	.18	.11	.08	1.15	5.63

APPENDIX A-3 - Industry Statistics

		Metal	Meat	Textiles	Paper	Chem.	Oil	Rubber	Steel	Elect.	Auto	Air	Congl.
						Industry Groups							
Statistics	$\sum_{i=1}^{n} X_{ij}$	31.82	52.29	56.87	22.14	31.16	14.64	35.22	43.56	74.12	37.42	49.49	25.34
	$\overline{X_j}$	2.65	4.36	4.74	1.85	2.60	1.22	2.94	3.63	6.18	3.12	4.12	2.11
	$\left(\sum_{i=1}^{n} X_{ij}\right)^2/n$	84.38	227.85	269.52	40.85	80.91	17.86	103.37	158.12	457.81	116.69	204.11	53.51
	$\sum_{i=1}^{n} (X_{ij})^2$	101.20	294.49	529.92	58.62	95.04	26.44	198.79	175.12	550.02	190.52	231.53	74.31
	S_j^2	1.53	6.06	23.67	1.62	1.28	.78	8.67	1.54	8.38	6.71	2.49	1.89
	S_j	1.24	2.46	4.87	1.27	1.13	.88	2.95	1.24	2.89	2.59	1.58	1.37

Notes:

X_{ij} = measured business risk of the ith firm in the jth industry.

$\overline{X_j}$ = mean business risk in the jth industry.

S_j = the standard deviation of firms' measured business risk in the jth industry.

S_j^2 = the variance of firm's measured business risk in the jth industry.

Number of firms per industry group (n) = 12.

Number of industry groups = 12.

APPENDIX A-4 - Unit Standard Deviations Within Industries (Notes)

The unit standard deviation (Y_{ij}) for each firm was calculated according to the following equation:

$$Y_{ij} = \frac{X_{ij} - \overline{X_j}}{S_j} ,$$

where

X_{ij} = the ith firm's (in the jth industry) business risk, and

$\overline{X_j}$ = the jth industry's mean group business risk, and

S_j = the standard deviation of firms' measured business risk in the jth industry.

APPENDIX A-4 - Unit Standard Deviations Within Industries

Metals	Meat Packers	Textiles
-1.38	.60	- .19
- .04	- .61	2.74
1.35	-1.19	- .59
.89	- .18	.01
- .92	.78	- .62
- .27	.42	- .55
.94	- .89	- .37
1.23	- .62	.76
-1.28	2.46	- .55
.69	- .84	.87
- .21	.25	.56
- .97	- .18	- .56

Paper	Chemicals	Oils
- .87	.50	1.02
- .31	.33	- .59
- .28	-1.07	- .23
- .21	- .05	.66
- .30	.35	2.65
- .13	- .07	- .18
- .21	- .12	- .73
3.02	.17	.32
- .55	2.74	- .38
- .32	- .58	- .43
.46	-1.18	- .94
.34	- .39	- .81

APPENDIX A-4 - Unit Standard Deviations Within Industries

Tire & Rubber	Steel	Electronics
2.86	- 1.19	-1.18
- .54	- .58	.49
.85	- 1.06	- .58
- .08	1.96	- .72
- .57	- .31	.66
- .63	- 1.35	.58
- .16	.75	- .69
- .41	- .33	-1.35
- .66	.67	1.47
.19	.76	1.61
- .46	.83	.43
- .45	- .14	- .73

Auto Parts	Air Transport	Conglomerates
.64	-1.15	- .11
2.71	-1.41	- .17
- .75	- .78	- .59
- .56	.55	-1.28
- .80	- .28	- .28
- .47	- .42	.51
- .39	.85	- .21
- .20	1.41	- .01
- .37	.72	.60
.93	1.15	-1.28
- .47	-1.25	.26
- .28	.63	2.57

APPENDIX A-5 - Rank Ordering of Study Sample

Rank	SE_i	MDA Group	Sample O	Sample H	Firm	Industry
1	18.09	High		X	Aileen	Textile Apparel
2	11.38	High	X		A O Industries	Tire & Rubber
3	10.84	High	X		Servco	Electronics
4	10.43	High		X	Sanders Assoc.	Electronics
5	10.41	High	X		Rath	Meat Packers
6	10.15	High	X		Amer. Safety	Auto Parts
7	8.96	High		X	Originala	Textile Apparel
8	8.08	High	X		Fairchild	Electronics
9	7.85	High	X		Hazeltine	Electronics
10	7.60	High		X	Collins Radio	Electronics
11	7.48	High	X		Pioneer	Textile Apparel
12	7.42	High	X		Sparton	Electronics
13	6.35	High		X	Frontier	Air Transport
14	6.28	High	X		Hygrade	Meat Packers
15	6.06	High	X		Carpenter	Steel
16	5.94	High		X	Northwest	Air Transport
17	5.84	High	X		Bluebird	Meat Packers
18	5.70	High	X		Oakite	Chemicals
19	5.68	High		X	Sorg	Paper
20	5.63	High	X		Whittaker	Conglomerate
21	5.53	High	X		Howell	Auto Parts
22	5.47	High		X	Flying Tiger	Air Transport
23	5.44	High	X		Carlisle	Tire & Rubber
24	5.40	High	X		Iowa Beef	Meat Packers
25	5.25	High		X	National	Air Transport
26	5.12	High	X		Western	Air Transport
27	4.99	High	X		Continental	Air Transport
28	4.97	High		X	United Brands	Meat Packers
29	4.81	High	X		Chadbourn	Textile Apparel
30	4.78	High	X		Aspro	Auto Parts
31	4.66	High		X	Pheon'x	Steel
32	4.57	High	X		McLouth	Steel
33	4.55	High	X		Florida Steel	Steel
34	4.50	High		X	Conrac	Electronics
35	4.46	High	X		Latrobe	Steel
36	4.33	High	X		Brush-Wellman	Metals
37	4.19	High		X	High Voltage	Electronics
38	4.17	High	X		Fansteel	Metals
39	4.08				Edo Corp.	Electronics
40	4.08	High		X	VLN	Electronics
41	3.91				Zion	Meat Packers
42	3.91				Hormel	Meat Packers
43	3.82	High		X	Adams Mills	Textile Apparel
44	3.81				Driver Harris	Metals

APPENDIX A-5 (Continued)

Rank	SE_i	MDA Group	Sample O	Sample H	Firm	Industry
45	3.75				Cerro	Metals
46	3.68				Delta	Air Transport
47	3.55				Quaker State	Oils
48	3.51				Mansfield	Tire & Rubber
49	3.51				Intl.Nickel Canada	Metals
50	3.45				Eastern	Air Transport
51	3.45				Standard Alliance	Steel
52	3.24				Copperweld	Steel
53	3.22				Kaiser Steel	Steel
54	3.16				Ansul	Chemicals
55	2.99				Grow Chem.	Chemicals
56	2.94				Hart,Schaffer & Marx	Textile Apparel
57	2.93				Teledyne	Conglomerates
58	2.91				Allegheny Ludlow	Steel
59	2.87				Braniff	Air Transport
60	2.87				General Host	Meat Packers
61	2.81				Mayer	Meat Packers
62	2.80				Kiddie	Conglomerates
63	2.79				Nalco	Chemicals
64	2.77				Ambac	Electronics
65	2.71				Cooper	Tire & Rubber
66	2.60				Amer. Smelting	Metals
67	2.58				Eaton	Auto Parts
68	2.54				Fairmont	Chemicals
69	2.52				Lubrizol	Chemicals
70	2.47				Textron	Conglomerates
71	2.46				Diversity	Chemicals
71	2.46				General	Tire & Rubber
73	2.44				Westvaco	Paper
74	2.39				Molybdenum	Metals
75	2.39				Napco	Auto Parts
76	2.31				Ampco Pittsburg	Steel
77	2.30				Cyprus Mines	Metals
78	2.30				Allegheny	Air Transport
79	2.29				Tobin	Meat Packers
80	2.27				Ratheon	Electronics
81	2.23				Conwood	Chemicals
82	2.17				Gould	Auto Parts
83	2.16				West Chemicals	Chemicals
84	2.16				Kane-Miller	Meat Packers
85	2.15				Alanwood	Steel
86	2.15				UAL	Air Transport
87	2.11				Ashland	Oils
88	2.10				Dana	Auto Parts
89	2.10				Signal	Conglomerates
90	2.06				Genesco	Textile Apparel

APPENDIX A-5 (Continued)

Rank	SE_i	MDA Group	Sample O	Sample H	Firm	Industry
91	2.05				Munsingwear	Textile Apparel
92	2.03				Warnaco	Textile Apparel
93	1.96				Avco	Conglomerates
94	1.95				Dominion Fndrs.	Steel
95	1.95				Purex	Chemicals
96	1.90				American	Air Transport
97	1.89	Low		X	Champion	Auto Parts
98	1.88	Low	X		Maremont	Auto Parts
99	1.87	Low	X		Gulf-Western	Conglomerates
100	1.85	Low		X	Blue Bell	Textile Apparel
101	1.82	Low	X		Litton	Conglomerates
102	1.73	Low	X		Goodrich	Tire & Rubber
103	1.72	Low		X	Kaiser Inds.	Conglomerates
104	1.72	Low	X		Cluett Products	Textile Apparel
105	1.69	Low	X		Mead	Paper
106	1.68	Low		X	Bendix	Auto Parts
107	1.60	Low	X		Uniroyal	Tire & Rubber
108	1.58	Low	X		Mohawk	Tire & Rubber
109	1.58	Low		X	International	Paper
110	1.57	Low	X		Scott	Paper
111	1.50	Low	X		Cleveland-Cliffs	Metals
112	1.50	Low		X	Reserve	Oils
113	1.49	Low	X		Hammermill	Paper
114	1.47	Low	X		Marathon	Oils
115	1.47	Low		X	Kimberly-Clark	Paper
116	1.45	Low	X		Armstrong	Tire & Rubber
117	1.44	Low	X		Utah Intl.	Metals
118	1.44	Low		X	Great Northern	Paper
119	1.43	Low	X		Union Camp	Paper
120	1.42	Low	X		Greyhound	Meat Packers
121	1.41	Low		X	Whippany	Paper
122	1.39	Low	X		Ethyl	Chemicals
123	1.30	Low	X		Indian Head	Conglomerates
124	1.26	Low		X	Dayco	Tire & Rubber
125	1.26	Low	X		Sun Chemical	Chemicals
126	1.19	Low	X		Bearings, Inc.	Auto Parts
127	1.15	Low		X	St. Regis	Paper
128	1.08	Low	X		Firestone	Tire & Rubber
129	1.06	Low	X		Continental Oil	Oils
130	1.05	Low		X	Jonathan Logan	Textile Apparel
131	1.05	Low	X		Hanna Mining	Metals
132	1.05	Low	X		Borg Warner	Auto Parts
133	1.02	Low		X	Cities Service	Oils
134	.99	Low	X		Goodyear	Tire & Rubber
135	.94				Amer. Metal Climax	Metals

APPENDIX A-5 (Continued)

Rank	SE_i	MDA Group	Sample O	Sample H	Firm	Industry
136	.89	Low		X	Shell	Oils
137	.84				Skelly	Oils
138	.74				Crown Zellerbach	Paper
139	.70	Low		X	Atlantic-Richfield	Oils
140	.58				Phillips	Oils
141	.51				Union	Oils
142	.39				Sun	Oils
143	.36				Tenneco	Conglomerates
144	.34				ITT	Conglomerates

BIBLIOGRAPHY

Books

Aaker, David A. Multivariate Analysis in Marketing: Theory & Application. Belmont: Wadsworth Publishing Co., 1971.

Anderson, T. W. An Introduction to Multivariate Statistical Analysis. New York: John Wiley & Sons, 1958.

Barges, Alexander. The Effect of Capital Structure on the Cost of Capital. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963.

Bolten, Steven E. Security Analysis and Portfolio Management, An Analytical Approach to Investments. New York: Holt, Rinehart, and Winston, Inc., 1970.

Brigham, Eugene F. and J. Fred Weston. Managerial Finance. 4th ed. New York: Holt, Rinehart, and Winston, Inc., 1972.

Buchtel, Foster S. A Basic Guide for Preparation and Submission of Proposals for Research Support. Kalamazoo: Wolverine Press, 1970.

Cooley, William W. and Paul R. Lohnes. Multivariate Data Analysis. New York: John Wiley & Sons, 1971.

Dixon, W. J. (ed.) BMD Biomedical Computer Programs. Berkeley: University of California Press, 1973.

Edwards, Allen L. Experimental Design in Psychological Research. 4th ed. New York: Holt, Rinehart, and Winston, Inc., 1972.

Guenther, William C. Concepts of Statistical Inference. 2nd ed. New York: McGraw-Hill, 1973.

____________________. Analysis of Variance. Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1964.

Kirk, Roger E. Experimental Design: Procedures for the Behavioral Sciences. Belmont, California: Brooks/Cole Publishing Co., 1968.

Morrison, Donald F. Multivariate Statistical Methods. New York: McGraw-Hill, 1956.

Quirin, G. David. The Capital Expenditure Decision. Homewood, Ill.: Richard D. Irwin, Inc., 1967.

Siegel, Sidney. Nonparametric Statistics for the Behavioral Sciences. New York: McGraw-Hill, 1956.

Turabian, K. L. A Manual for Writers of Term Papers, Theses, and Dissertations. Revised ed. Chicago: University of Chicago Press, 1955.

Tatsuoka, Maurice M. Multivariate Analysis: Techniques for Educational and Psychological Research. New York: John Wiley & Sons, 1971.

VanHorne, James C. Financial Management and Policy. 2nd ed. Englewood Cliffs, N. J.: Prentice-Hall, 1971.

__________________. Financial Management and Policy. 3rd ed. Englewood Cliffs, N. J.: Prentice-Hall, 1974.

__________________. (ed.) Foundations for Financial Management. Homewood: Richard D. Irwin, Inc., 1966.

Yamane, Taro. Statistics: An Introductory Analysis. 2nd ed. New York: Harper & Row, 1969.

Periodicals

Altman, Edward I. "Financial Ratios, Discriminant Analysis,and the Prediction of Corporate Bankruptcy." Journal of Finance, XXIII (Sept., 1968), 589-609.

Altman, Edward I. and Robert A. Schwartz. "Common Stock Price Volatility Measures and Patterns." Journal of Financial and Quantitative Analysis, IV (January, 1970), 603-626.

__________________. "Behavior of Industrial Stock Price Indices." Journal of Finance, XXVIII (September, 1973), 957-972.

Arditti, Fred D. "Risk and the Required Return on Equity." Journal of Finance, XXII (March, 1967), 19-36.

Aronson, J. Richard and Eli Schwartz. "Some Surrogate Evidence in Support of the Concept of Optimal Financial Structure." Journal of Finance, XXII (March, 1967), 10-18.

Baxter, Nevins D. "Leverage, Risk of Ruin and the Cost of Capital." Journal of Finance, XXII (September, 1967), 395-404.

Brigham, Eugene F. and Myron Gordon. "Leverage, Dividend Policy and the Cost of Capital." Journal of Finance, XXIII (March, 1968), 85-104.

Davis, E. G., D. M. Dunn and W. H. Williams. "Ambiguities in the Cross-Section Analysis of Per Share Financial Data." Journal of Finance, XXVIII (December, 1973), 1241-1248.

Edmister, Robert O. "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction." Journal of Financial and Quantitative Analysis, VII (March, 1972), 1477-1494.

Elton, Edwin J. and Martin J. Gruber. "Homogeneous Groups and the Testing of Economic Hypothesis." Journal of Financial and Quantitative Analysis, IV (January, 1970), 581-602.

____________________. "Improved Forecasting Through the Design of Homogeneous Groups." Journal of Business, XXXXIV (October, 1971), 432-450.

Gonedes, Nicholas J. "A Test of the Equivalent-Risk Class Hypothesis." Journal of Financial and Quantitative Analysis, IV (June, 1969), 159-178.

Goodman, M. L. and W. H. Williams. "A Statistical Grouping of Corporations by Their Financial Characteristics." Journal of Financial and Quantitative Analysis, VI (September, 1971), 1095-1104.

Klemkosky, R. C. "The Bias in Performance Measures." Journal of Financial and Quantitative Analysis, VIII (June, 1973), 505-516.

Melicher, Ronald W. "Financial Factors Which Influence Beta Variations Within a Homogeneous Industry Environment." Journal of Financial and Quantitative Analysis, IX (March, 1974), 231-242.

Mingo, K. A. and G. E. Pinches. "A Multivariate Analysis of Industrial Bond Ratings." Journal of Finance, XXVIII (March, 1973), 1-18.

Mingo, K. A., G. E. Pinches and J. K. Caruthers. "The Stability of Financial Patterns in Industrial Organizations." Journal of Finance, XXVIII, (May, 1973), 389-396.

Modigliani, Franco and M. H. Miller. "The Cost of Capital, Corporation Finance, and the Theory of Investment." American Economic Review, XLVIII (June, 1958), 261-297.

____________________. "The Cost of Capital, Corporation Finance and the Theory of Investment. Reply." American Economic Review, IL (September, 1958), 655-659.

Norgaard, Richard L. "Evaluating Risk, Returns, and Trends." Journal of Financial and Quantitative Analysis, VI (September, 1971), 1069-1082.

Pinches, George E. and William R. Kenney, Jr. "The Measurement of the Volatility of Common Stock Prices." Journal of Finance, XXVI (March, 1971), 119-126.

Rao, N. Krishna. "Equivalent-Risk Class Hypothesis: An Empirical Study." Journal of Financial and Quantitative Analysis, VII (June, 1972), 1763-1772.

Schwartz, Eli. "Theory of the Capital Structure of the Firm." Journal of Finance, XIV (March, 1959), 18-39.

Scharpe, William F. "A Simplified Model for Portfolio Analysis." Management Science, Vol. 1, No. 2 (January, 1963), 277-293.

Tysseland, Milford S. "Further Tests of the Validity of the Industry Approach to Investment Analysis." Journal of Financial and Quantitative Analysis, VI (March, 1971), 835-848.

Weston, J. Fred. "A Test of Cost of Capital Propositions." Southern Economic Journal, XXX (October, 1963), 105-112.

Wippern, R. F. "A Note on the Equivalent-Risk Class Assumption." Engineering Economist, Vol. II, No. 3 (Spring, 1966), 13-22.

__________. "Financial Structure and the Value of the Firm." Journal of Finance, XXI (December, 1966), 615-634.

Other Sources

COMPUSTAT TAPES. Investors Management Science, Inc., Denver, 1972.

Norman, Morris. ECON, OLS Econometric Estimation Program. (Adapted for Univac 1108 by R. William Thomas) Houston: University of Houston Economics Department.

Smith, K. V. "Classification of Investment Securities Using MDA." Paper #101. Lafayette: Herman C. Krannert Graduate School of Industrial Administration, Purdue University, 1965.

U. S. Executive Office of the President, Bureau of Management & Budget. Standard Industry Classification Manual. Washington, D. C., 1972.

Unpublished Material

Latané, H. A. and W. J. Reinhart. "A Multivariate Analysis of Earnings-Price Ratios." Presented at the Southern Finance Association meeting in Houston, Texas, November, 1973.