STATISTICAL DC NODE VOLTAGE CHARACTERISTICS
BY MONTE-CARLO TECHNIQUES

A Thesis

Presented to

the Faculty of the Cullen College of Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science

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by

Min-Chang Tseng

May, 1971

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ABSTRACT

The calculation of the standard deviations and mean values of the node voltages of a given network by ECAP are based upon the assumptions that a node voltage e_k is a linear function of the circuit variables P_j such that $e_k = \sum_j K_j P_j$, and that all variable parameters are normally distributed. Regarding the first assumption, e_k realistically cannot be expected to be linear with respect to all its variable parameters. The second assumption does not hold in some cases where, for example, the variable parameters may be assummed to have uniform distribution.

In either case, ECAP analysis will generally not yield the actual mean and standard deviation of node voltages. Therefore it was desirable to modify the ECAP program so that the user may get a more accurate estimate of the mean values and standard deviations.

To fulfill this requirement, a computer program using the Monte-Carlo method has been developed. This method generates normally distributed random variable parameters of the network and then calculates the node voltages. From these, in turn, the mean values and standard deviation of node voltages are calculated.

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CHAPTER 1 INTRODUCTION

The Electronic Circuit Analysis Program(ECAP) is an integrated system of programs developed primarily to aid the electrical engineer in the design and analysis of electronic circuits. There are four closely related programs in the system.

INPUT LANGUAGE. This program acts as the communication link between the user and the three analysis programs. The language is user-oriented and allows complex circuits to be simply described to the computer from easily developed equivalent circuits. Six basic statements used in the program completly define the topology of the circuit, the circuit element values, the type of analysis to be performed, the driving functions, and the output required. This simple scheme makes it possible to learn to use the program in a short time.

DC ANALYSIS. The DC analysis program obtains the DC steady state solution of linear electrical networks and provides worst case analysis, standard deviation(statistical) analysis, and sensitivity coefficient calculations if requested. This program also provides an automatic parameter-modification capability.

AC ANALYSIS. The AC analysis program obtains the steady state solution of linear electrical networks to sine-wave excitation at a fixed frequency. Since this program also contains the automatic parameter-modification capability, it is easy to obtain frequency and phase response solution. TRANSIENT ANALYSIS. The transient analysis program provides the time response solution of linear or nonlinear electrical networks subject to arbitrary, user-specified driving functions. Nonlinear elements are modeled by using combinations of linear elements and switches to provide piecewise linear approximations to the nonlinear characteristics.

The ideal DC network analysis program should be capable of sensitivity, tolerance and worst case studies. In the ECAP program, calculation of the standard deviations and the means of the node voltages is based upon the assumptions that all random variable parameters are normally distributed and that node voltages are linear with respect to circuit variable parameters. This thesis has two purposes: First to show that the ECAP program does not compute the true standard deviations and means of node voltages; Second to apply the Monte-Carlo technique in the program so that it will be capable of computing more accurate standard deviations and means of node voltages in a complex network.

CHAPTER 2 CALCULATION OF STANDARD DEVIATION

2-1 Standard Deviation Calculation by ECAP

From basic probability theory the variance of the variable y is

$$\sigma^2(y) = \sum_i a_i^2 \sigma^2(x_i)$$

where $\sigma(y)$ is defined as the standard deviation of y when

$$y = \sum_i a_i x_i$$

and where the x_i are statistically independent random variables with standard deviation $\sigma(x_i)$ and the a_i are constants. If it is assumed that a node voltage e_k is a linear function of the circuit variables P_j such that

$$e_k = \sum_j K_j P_j$$

where the P_j are statistically independent random variables with standard deviations $\sigma(P_j)$ and the K_j are constants, then

$$\sigma^2(e_k) = \sum_j K_j^2 \sigma^2(P_j)$$

Realizing that

$$K_j = \frac{\partial}{\partial P_j} (\sum_j K_j P_j) = \frac{\partial e_k}{\partial P_j}$$

then

$$\sigma^2(e_k) = \sum_j (\frac{\partial e_k}{\partial P_j})^2 \sigma^2(P_j)$$

It is further assumed that

$$\sigma(P_j) = \frac{P_{jmax} - P_{jmin}}{6} \qquad (1)$$

For normally distributed P_j

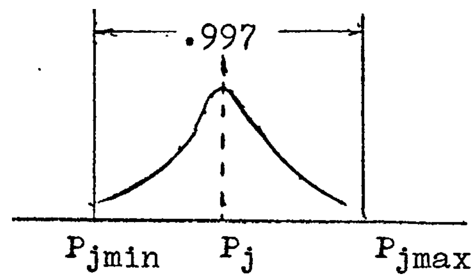
$$P_{jmax} = P_{jm} + 3(P_j)$$

and $$P_{jmin} = P_{jm} - 3(P_j)$$

and $P_{jmax}-P_{jmin}$ includes 99.7% of the area under the distribution curve as indicated in Fig. 1. P_{jmax} and P_{jmin} are, respectively, the specified maximum and minimum values for P_j and P_{jm} is the mean of P_j. This gives an expression for the standard deviation of e_k which can be written exclusively as a function of terms available in ECAP. That is, the standard deviation of node voltage e_k is calculated by ECAP as

$$\sigma(e_k)=\left\{\sum_j \left(\frac{\partial e_k}{\partial P_j}\right)^2 \left(\frac{P_{jmax} - P_{jmin}}{6}\right)^2\right\}^{\frac{1}{2}} \qquad (2)$$

Fig. 1 Normal Distribution and Probabilities

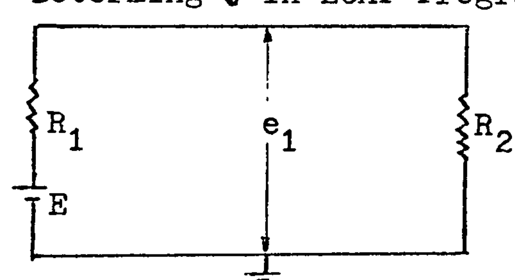


As an example of standard deviation calculation, the following equation is used by ECAP in determing $\sigma(e_k)$ for the circuit of Fig. 2.

$$\sigma(e_1) = \left\{\left(\frac{\partial e_1}{\partial R_1}\right)^2 \left(\frac{R_{1max} - R_{1min}}{6}\right)^2 + \left(\frac{\partial e_1}{\partial R_2}\right)^2 \left(\frac{R_{2max} - R_{2min}}{6}\right)^2 + \left(\frac{\partial e_1}{\partial E}\right)^2 \left(\frac{E_{max} - E_{min}}{6}\right)^2\right\}^{\frac{1}{2}}$$

Fig. 2 Illustrative DC Network for

Determing σ in ECAP Program



2-2 Calculation of the Partial Derivatives

From Eq.(2), it is very clear that we have to compute the partial derivatives of the node voltages with respect to each of the circuit parameters so that we are able to get the standard deviations of node voltages. The development of the calculations is as follows.

The topological properties of an electrical circuit can be represented by a graph, and the graph can be conveniently represented by a matrix $\bar{A}$ containing only the element +1, -1, and 0. Each element a_{ij} of matrix $\bar{A}$ is determined by the rules given in Table 1,

Table 1

Connectivity of Values of a_{ij}

a_{ij}	Connectivity: branch i, node j
+1	Node j is the initial node of branch i
-1	Node j is the final node of branch i
0	Branch i is not connected to node j

the $\bar{A}$ matrix is the branch to node incidence matrix. If we define circuit parameter matrices as

$\bar{E}$ = voltage source

$\bar{I}$ = current source

$\bar{Y}$ = conductance matrix

$\bar{A}_t$ = transpose of $\bar{A}$

then we obtain the nodal equation

$$\bar{e} = (\bar{A}_t\overline{YA})^{-1}\ \bar{A}_t(\bar{I}-\overline{YE}) \tag{3}$$

where $\bar{e}$ = node voltage vector

$(\bar{A}_t\overline{YA})$ = nodal conductance matrix

$\bar{A}_t(\bar{I}-\overline{YE})$ = equivalent current vector

$(\bar{A}_t\overline{YA})^{-1}$ = nodal impedance matrix

The derivative of the node voltages $\bar{e}$, as presented in Eq.(3), with respect to the resistance in the (i)th branch is

$$\frac{\partial\bar{e}}{\partial R_i} = \frac{\partial(\bar{A}_t\overline{YA})^{-1}}{\partial R_i}\bar{A}_t(\bar{I}-\overline{YE}) + (\bar{A}_t\overline{YA})^{-1}\ \bar{A}_t\ \frac{\partial(\bar{I}-\overline{YE})}{\partial R_i}$$

Realizing that

$$\frac{\partial(\bar{A}_t\overline{YA})^{-1}}{\partial R_i} = -(\bar{A}_t\overline{YA})^{-1}\ \bar{A}_t\ \frac{\partial\bar{Y}}{\partial R_i}\bar{A}(\bar{A}_t\overline{YA})^{-1}$$

and $$\frac{\partial\bar{E}}{\partial R_i} = \frac{\partial\bar{I}}{\partial R_i} = 0$$

$\because$ $\bar{e}' = \overline{Ae}$ .... branch voltage

$$\therefore\ \frac{\partial\bar{e}}{\partial R_i} = -(\bar{A}_t\overline{YA})^{-1}\ \bar{A}_t\ \frac{\partial\bar{Y}}{\partial R_i}(\bar{E} + \bar{e}') \tag{4}$$

In a similar manner, the differentiation of Eq.(3) with respect to the other circuit parameters yields

$$\frac{\partial \overline{e}}{\partial \beta_{ji}} = -(\overline{A}_t \overline{YA})^{-1} \; \overline{A}_t \; \frac{\partial \overline{Y}}{\partial \beta_{ji}} \; (\overline{E} + \overline{e}') \qquad (5)$$

$$\frac{\partial \overline{e}}{\partial E_i} = -(\overline{A}_t \overline{YA})^{-1} \; \overline{A}_t \overline{Y} \; \frac{\partial \overline{E}}{\partial E_i} \qquad (6)$$

$$\frac{\partial \overline{e}}{\partial I_i} = (\overline{A}_t \overline{YA})^{-1} \; \overline{A}_t \; \frac{\partial \overline{I}}{\partial I_i} \qquad (7)$$

where β_{ji} is current gain from the (i)th branch to (j)th branch. Next, ϵ_i and $\overline{\epsilon}_j$ are defined, respectively, as column and row vectors of dimension m(where m is the number of branches in the graph).

$$\epsilon_i = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix} ; \qquad \overline{\epsilon}_j = \begin{bmatrix} 0 & . & . & 0 & 0 & 1 & . & . & 0 \end{bmatrix}$$

The vector ϵ_i has a 1 in the (i)th location and zeros elsewhere. The (i)th location represents the (i)th row of a matrix. Similarly, $\overline{\epsilon}_j$ has a 1 in the (j)th location(representing the (j)th column of a matrix) and zeros elsewhere. Then the product $\epsilon_i \overline{\epsilon}_j$ is a matrix with a 1 in the (ij)th location and zeros elsewhere. With this notation

$$\frac{\partial \overline{Y}}{\partial R_i} = -\frac{1}{R_i^2} \epsilon_i \overline{\epsilon}_i - \sum_{j \neq i} \frac{\beta_{ji}}{R_i^2} \epsilon_j \overline{\epsilon}_i$$

or $$\frac{\partial \overline{Y}}{\partial R_i} = -\frac{1}{R_i^2}\sum_{j=1}^{m} \beta_{ji}\epsilon_j\overline{\epsilon}_i \qquad \text{where } \beta_{ii} = 1$$

where i is the column in the $\overline{Y}$ matrix corresponding to the (i)th branch, and j is the row corresponding to the (j)th branch. The term

$$\frac{\beta_{ji}}{R_i^2}\epsilon_j\overline{\epsilon}_i$$

is present only if the (i)th branch is the controlling (or <u>from</u>) branch for a dependent current sorce assigned to the (j)th branch. If the (i)th branch is the <u>from</u> branch for dependent current source assigned to other branches as well, then additional similar terms must appear in the above expression. In a similar manner

$$\frac{\partial \overline{Y}}{\partial \beta_{ji}} = \frac{1}{R_j}\epsilon_i\overline{\epsilon}_j$$

$$\frac{\partial \overline{E}}{\partial E_i} = \epsilon_i$$

$$\frac{\partial \overline{I}}{\partial I_i} = \epsilon_i$$

Thus, the formulas for the partial derivatives become

$$\frac{\partial \bar{e}}{\partial R_i} = (\bar{A}_t \overline{YA})^{-1} \, \bar{A}_t(\epsilon_i + \sum_{j \neq i} \epsilon_j \beta_{ji}) \, \frac{1}{R_i^2} \, (E_i + e_i') \tag{8}$$

$$\frac{\partial \bar{e}}{\partial \beta_{ji}} = -(\bar{A}_t \overline{YA})^{-1} \, \bar{A}_t \epsilon_i \, \frac{1}{R_j} \, (E_j + e_j') \tag{9}$$

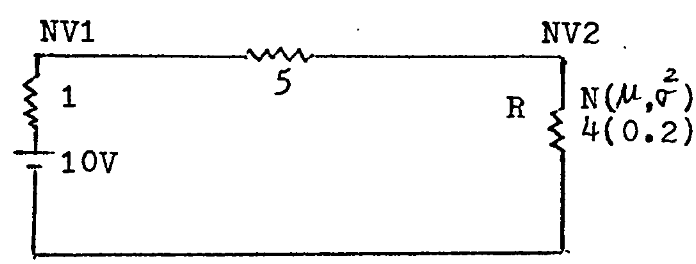
$$\frac{\partial \bar{e}}{\partial E_i} = -(\bar{A}_t \overline{YA})^{-1} \, \bar{A}_t \bar{Y} \epsilon_i \tag{10}$$

$$\frac{\partial \bar{e}}{\partial I_i} = (\bar{A}_t \overline{YA})^{-1} \, \bar{A}_t \epsilon_i \tag{11}$$

2-3 Actual Standard Deviation

Actually, the mean of node voltage e_k does not depend linearly on the nominal values of the circuit variables, i.e., the mean node voltage not only depends upon the nominal values of circuit parameters but it also depends on the variances of circuit parameters. For example, in a simple circuit as Fig. 3,

Fig. 3 DC Network For Explanation of Actual Standard Deviation



the node voltage of NV2 is represented by

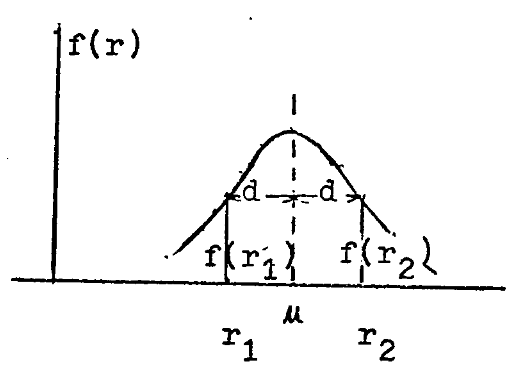
$$y = NV2 = \frac{10R}{6 + R} \tag{12}$$

where R has a distribution $N(\mu,\sigma^2)$. The notation $N(\mu,\sigma^2)$ represents a normal distribution and characterizes two parameters μ(the expectation) and σ^2(variance) of R. In this case, the nominal value μ of R is 4Ω and its per cent tolerance is 20%. If R is normally distributed, the graph of pdf(probability density function) of a normally distributed random variable is symmetric about μ as shown in Fig. 4, i.e.,

$$f(r_1) = f(r_2)$$

where f(r) is probability density function of random variable R.

Fig. 4 Normal Distribution



Then, we may pick up two points r_1 and r_2 symmetrical to the expected value and from Eq.(12) the y values corresponding to these two points will be

$$y_1 = \frac{10r_1}{6+R} = \frac{10(\mu - d)}{6+\mu-d}$$

$$y_2 = \frac{10r_2}{6+R} = \frac{10(\mu + d)}{6+\mu+d}$$

Then the average value of y_1 and y_2 will be

$$\frac{y_1 + y_2}{2} = \frac{10(12\mu + 2\mu^2 - 2d^2)}{2\left\{(6+\mu)^2 - d^2\right\}}$$

and because $d \geq 0$

$$\frac{y_1 + y_2}{2} = \frac{10(12\mu + 2\mu^2 - 2d^2)}{2\left\{(6+\mu)^2 - d^2\right\}} \leq \frac{10(12\mu + 2\mu^2)}{2(6+\mu)^2}$$

$$= \frac{10}{6+\mu} \qquad (13)$$

and the mean of NV2

$$E(NV2) = \int_{-\infty}^{\mu} \frac{(y_1 + y_2)}{2} f(r) \leq \int_{-\infty}^{\mu} \frac{10}{6+\mu} f(r)$$

The leftmost term is the mean computed by ECAP. Therefore the actual mean is less than that obtained from ECAP.

Since the node voltage e_k is not a linear function of the circuit variables P_j, we are unable to get the true standard deviation from ECAP.

CHAPTER 3 MONTE-CARLO TECHNIQUES

As mentioned before, the variance calculated by ECAP can not adequately represent variation in network behavior. For a more accurate distribution of node voltages, we can apply Monte-Carlo techniques. The procedure consists of solving several thousand cases of a given network problem with parameter values(such as resistances and sources) chosen at random over certain specified tolerance ranges. Then, means and standard deviations for these results are calculated.

3-1 Generation of Random Numbers

The uniform distribution may be used to generate nearly all other distributions. To obtain a long sequence of uniformly distributed random numbers, the power residue method is most often used. The following Fortran program from the IBM System/360 Scientific Subroutine Package is based on the power residue method for generation of uniformly distributed random numbers.

```
     SUBROUTINE RANDU(IX,IY,YFL)
     IX=IX*65339
     IF (IY) 5,6,6
   5 IY=IY+2147483647+1
   6 YFL=IY
```

```
YFL=YFL*0.45656613E-9
RETURN
END
```

where

IX: For the first entry this must contain any odd integer number with nine or fewer digits. After the first entry IX will be the previous value of IX computed by this subroutine.

IY: A resultant integer random number requied for the next entry to this subroutine. The range of this number is between zero and 2**31.

3-2 Normally Distributed Random Numbers

Then from the Central Limit Theorem, the distribution of the random variable formed from the sum of n uniformly distributed independent variables tends to approach the normal distribution as $n \to \infty$. That is if $X_1, X_2, \ldots X_n$ is a sequence of an independent random variable with expected value $E(X_i)=\mu_i$ and variance $V(X_i)=\sigma_i^2$ $(i=1, 2, \ldots)$ then

$$Z_n = \frac{X - \sum_{i=1}^{n} \mu_i}{\left(\sum_{i=1}^{n} \sigma_i^2\right)^{\frac{1}{2}}}$$

approximates the distribution of N(0,1) (this expression is defined in 2-3) as n becomes large. Since it is a uniform distribution, E(X)=0.5, and V(X)=1/12. Therefore if we select n=12, then

$$Z_n = \frac{X - 12*0.5}{(12*1/12)^{\frac{1}{2}}} = X - 6 \tag{14}$$

Again by statistical analysis, if Z_n has a distribution of $N(\mu,\sigma^2)$ and if $Y=aZ_n+b$, then Y has a distribution of $N(a\mu+b,\ a^2\sigma^2)$. Since Z_n of equation (14) has the distribution of N(0,1), the expected value and variance of Z_n will be 0 and 1 respectively. Therefore to get an expected value (b) and standard deviation (a) for a normal distribution $N(b,a^2)$ from a standarized normal distribution N(0,1), we merely apply the formula

$$Y = aZ_n + b = a(X - 6) + b$$

3-3 The Fortran Program for Generation of Normally Distributed Random Numbers

The following program is a routine based on the Central Limit Theorem, and reprinted from "IBM System/360 Scientific Subroutine Package".

```
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0
      DO 50 I = 1, 12
      CALL RANDU(IX,IY,Y)
      IX = IY
   50 A = A + YFL
      V = (A - 6.0)*S + AM
      RETURN
      END
```

where

IX: IX must contain an odd integer number with nine or fewer digits on the first entry to GAUSS.

AM: The desired mean of the normal distribution.

V: The value of the computed normal random variable.

3-4 The Output of Normally Distributed Random Numbers

R1 of the Table 2 are normally distributed random numbers generated by the subroutine GAUSS, and NV1, NV2, NV3 are node voltages of a network of Fig. 5 corresponding to random variable parameters R1 and R2.

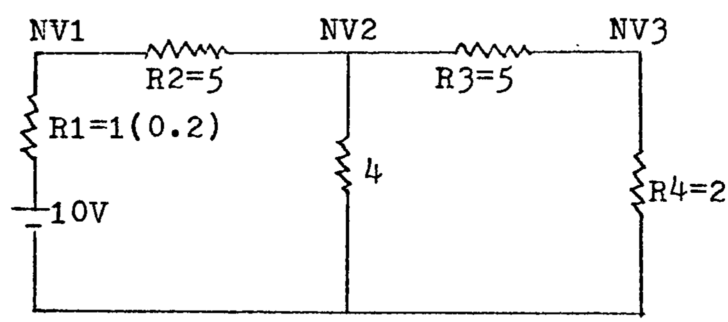
Table 2 Normally Distributed Random Node Voltages of Fig. 5

R1	NV1	NV2	NV3
0.91688	8.87349	2.73030	1.09212
1.21688	8.55805	2.63324	1.05330

0.67658	9.14344	2.81337	1.12535
1.09727	8.68108	2.67110	1.06844
1.15638	8.61984	2.65226	1.06090
0.83948	8.95868	2.75652	1.10261
0.95982	8.82692	2.71597	1.08639
1.36539	8.41005	2.58771	1.03508
0.83341	8.96543	2.75859	1.10344
1.25238	8.52220	2.62221	1.04889
0.89162	8.90111	2.73880	1.09552
0.95828	8.82858	2.71648	1.08659
1.24190	8.53275	2.62546	1.05018
0.84641	8.95099	2.75415	1.10166
1.20967	8.56536	2.63550	1.05420
1.01053	8.77255	2.69925	1.07970
0.96331	8.82316	2.71482	1.08593
0.89988	8.89206	2.73602	1.09441
1.13917	8.63758	2.65772	1.06309
1.34419	8.43086	2.59411	1.03764
0.86657	8.92868	2.74728	1.09891
1.17862	8.59702	2.64524	1.05810
0.99281	8.79147	2.70507	1.08203
0.86883	8.92618	2.74652	1.09861
1.30814	8.46649	2.60507	1.04203
1.13594	8.64092	2.65874	1.06350
0.77079	9.03567	2.78021	1.11208

1.18154	8.59403	2.64432	1.05773
0.73195	9.07979	2.79378	1.11751
.	.	.	.
.	.	.	.
Expectation	9.78245	2.70229	1.08092
Std. Dev.	0.21398	0.06584	0.02634

Fig. 5 DC Network For Generation of Normally Distributed Random Node Voltages



3-5 The Number of Trials

Since the Monte-Carlo technique is a statistical method of analyzing a given network by generation of random variations within the tolerance limits, the final results are expected to fluctuate accordingly for a small number of trials, but to converge as the number of trials increases. More accurate results require a larger number of trials and therefore more computation time. If we assume that the random numbers are perfect, the only errors we can obtain are due to the basic law of statistics. We know that these errors (deviations from the true value) decrease with $1/\sqrt{n}$, where n is the number of trials.[1] For this thesis the appropriate number of trials was determined by experiment. The results of the experiment are shown in Table 3, and are based on the DC network of Fig. 6. These data are plotted in Fig. 7. On the basis of these results, the number of trials chosen was 2000.

Fig. 6 DC Network for Deciding Number of Trials

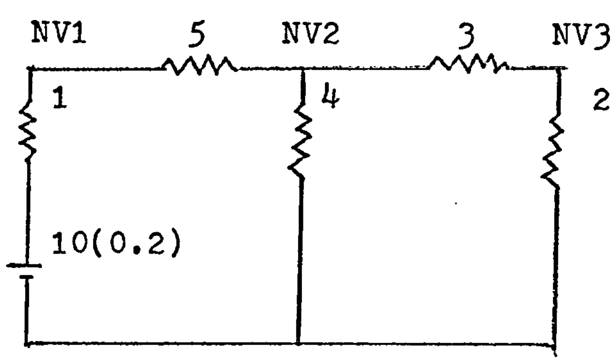


Table 3 Mean Values And Standard Deviations By Analytical Solution, ECAP, And Monte-Carlo

	node	NV1	NV2	NV3
MEAN VALUE				
analytical sol.		8.7837837	2.7027027	1.0810811
ECAP		8.7837829	2.7027018	1.0810806
	250 trials	8.7990176	2.7073900	1.0829560
	500 trials	8.8016720	2.7082068	1.0832827
Monte-	750 trials	8.8097533	2.7106933	1.0842773
Carlo	1000 trials	8.7826153	2.7023432	1.0809373
	1400 trials	8.7727443	2.6993059	1.0797224
	1500 trials	8.7769598	2.7006030	1.0802412
	2000 trials	8.7763895	2.7004275	1.0801710
	5000 trials	8.7688098	2.6980953	1.0792381
STANDARD DEVIATION				
	node	NV1	NV2	NV3
analytical sol.		0.58558530	0.18018018	0.072072072
ECAP		0.58558530	0.18018001	0.072072029
	250 trails	0.59462706	0.18296217	0.073184869
	500 trails	0.59527872	0.18008576	0.072034304
Monte-	750 trails	0.59403679	0.18278055	0.073112220
Carlo	1000 trails	0.58916875	0.18128269	0.072513077
	1400 trials	0.59079806	0.19178402	0.072136080
	1500 trials	0.59186956	0.18211371	0.072845484
	2000 trials	0.58534149	0.18215070	0.072042030
	5000 trials	0.58527213	0.18008373	0.072033493

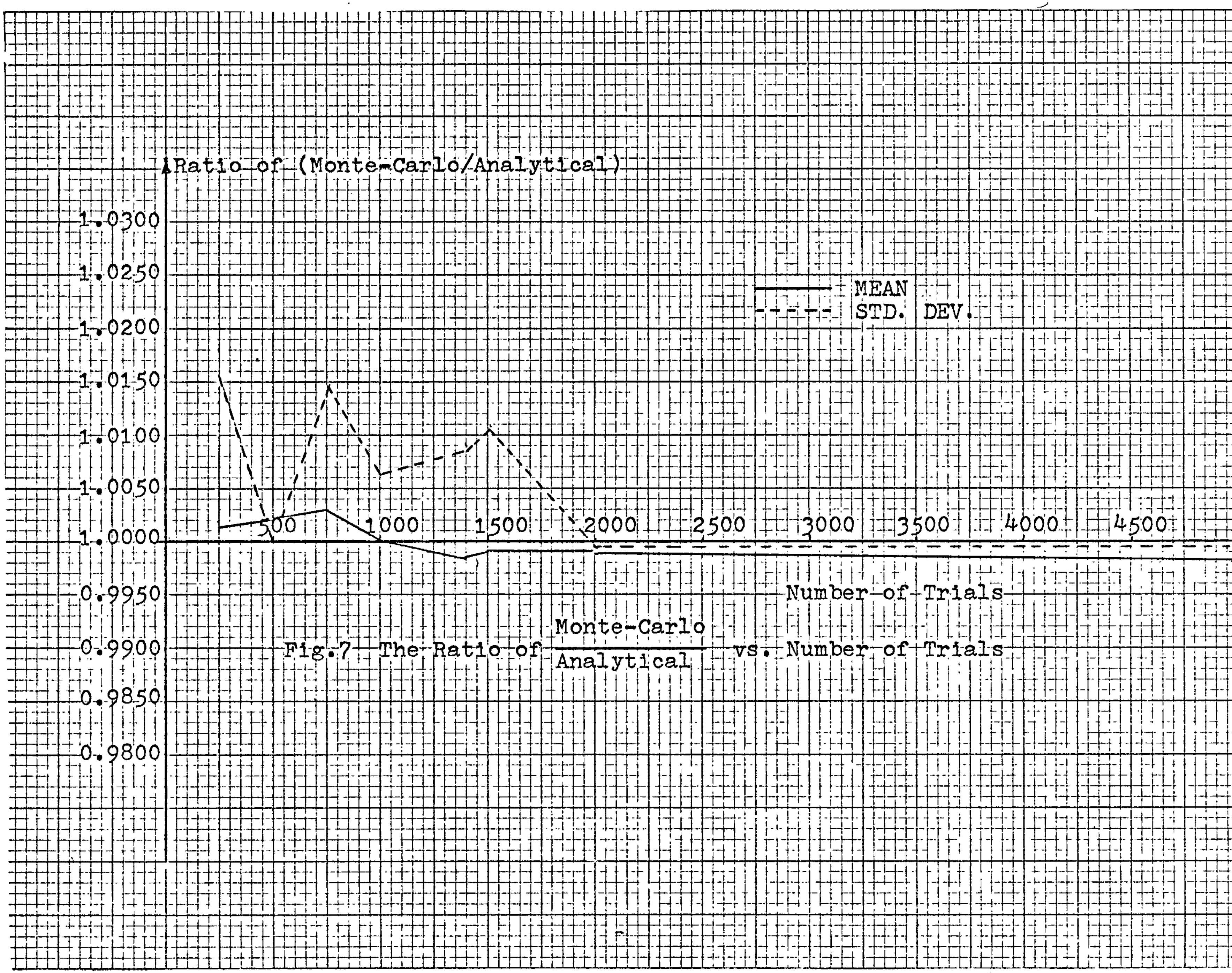


Fig.7 The Ratio of $\frac{\text{Monte-Carlo}}{\text{Analytical}}$ vs. Number of Trials

3-6 Calculation of Mean Values and Standard Deviation

If the value of a discrete random variable X are X_1, X_2, . . X_n, then the mean $\bar{X}$ and the standard deviation σ are defined as

$$\bar{X} = (X_1 + X_2 + \ldots + X_n)/n \tag{15}$$

$$\sigma = \left\{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})\right\}^{\frac{1}{2}} \tag{16}$$

If we apply the above Eqs. directly to a computer program for computation of the standard deviation σ, then we are able to get the standard deviation only after a solution of the mean $\bar{X}$ is obtained. In other words, we have to store each result of the random node voltage X_i in each iterative cycle for applying Eq.(16). This procedure would require more core memory than necessary. In order to save core memory, the computation procedure was changed as follows. At the first iterative cycle, we generate the node voltages $\tilde{X}$ by using nominal values of circuit parameters. For subsequent iterations, which generate the random node voltages, we sum up the difference and the square of the difference, i.e.,

$$SD = \sum_{i=1}^{n} \Delta X_i = \sum_{i=1}^{n} (X_i - \tilde{X})$$

$$SDD = \sum_{i=1}^{n} (\Delta X_i)^2$$

After completion of a calculation cycle, DEF(the difference of $\overline{X}$ and $\tilde{X}$) may be calculated as follows:

$$\overline{X} = DEF + \tilde{X} \qquad \text{where} \quad DEF = \frac{SD}{n}$$

The standard deviation will be

$$= \left\{ \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{1}{n-1} \sum_{i=1}^{n} (X_i - (SD/n + \tilde{X}))^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{1}{n-1} \sum_{i=1}^{n} (\Delta X_i - SD/n)^2 \right\}^{\frac{1}{2}}$$

$$= \left[\frac{1}{n-1} \sum_{i=1}^{n} \left\{ (\Delta X_i)^2 + (SD/n)^2 - 2\Delta X_i . SD/n \right\} \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{n-1} \left\{ SDD + (SD)^2/n \right\} \right]^{\frac{1}{2}}$$

$$= \left\{ \frac{1}{n - 1} \left(SDD - SD.\ DEF \right) \right\}^{\frac{1}{2}} \qquad (17)$$

Eqs.(16) and (17) were used to calculate the mean and standard deviation. This avoids the need for setting up a large number of arrays for storing the random node voltages resulting in each iterative cycle.

CHAPTER 4 THE MODIFIED ECAP PROGRAM

4-1 The Features of ECAP

To modify the present ECAP program, we have to understand the structure of it first. The ECAP system consists of four major programs;

(A) ECAP Input Language

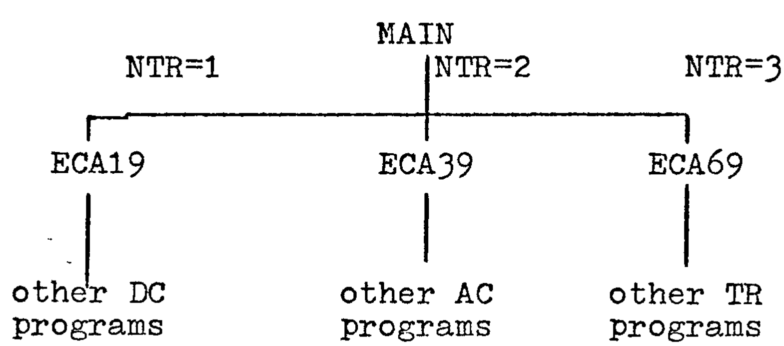
(B) DC Analysis

(C) AC Analysis

(D) Transient Analysis

The structure is shown in Fig.8. Modification of the program affects only the ECAP input language and DC analysis programs. The functions of these two major programs are explained below.

Fig. 8 ECAP Block Diagram



The ECAP input language program interprets the ECAP problem-oriented language statements and stores the input data into an appropriate set of fiags and arrays for the specified analysis section of the program.

ECA: The initialization link for Language Program.

ECA01: Pre-sets information in the program for use throughout the read in cycle.

ECA02: Keeps track of the number of branches and type of data i.e. branch(B), node(N), component(R,L,C,G,) etc..

ECA03: Keeps track of the number of nodes and sets up the arrays NINIT(K) and NFIN(K) where the value of K is the branch numbers. The integer value stored in NINT(K) is the originating node number for branch K and the terminating node is stored in NFIN(K).

ECA04: Used only for time varying sources(E or I).

ECA05: Stores the maximum, and initial value of an element in branch K in the Kth element of the arrays YMAX, YMIN, and Y(the values for G and C are stored as on the input data statement while the values for R and L are stored as 1./R and 1./L). The source arrays EMAX, EMIN, E, AMPMAX, AMPMIN, and AMP are also set up in this subroutine.

ECA06: Interprets Command, Solution Control, and Output Specification cards i.e., cards with nothing in columns 1-5). It also provides a final check for consistency in the output variable specifications(modifying some of them as required).

ECA07: Used if a system control card is read in (trace, dump

switch) or if an error is found in the input statements.
ECA08: This subroutine used to set the initial value of numerous element in the program(i.e. all array elements to zero, initial values for SHORT, OPEN, START, FINISH, DELTA, etc.) and is called in during the read in cycle for an AC, DC, or TR input statement.
ECA09: Used to interpret the numerical information in an input statement(called in each time a numerical value is encountered on an input statement).[2]

The DC analysis section of program is entered by a call from MAIN to ECA19. All of the calls to the other DC subprograms are made via ECA19. Very brief descriptions of the subroutines are outlined as follows.
ECA19: DC analysis link.
ECA20: After setting the initial value of certain parameter, ECA22 through ECA30 are called to compute and print the initial solutions.
ECA22: Forms the nodal conductance matrix ZPRL(NNODE, NNODE).
ECA23: Computes the equivalent current vector EQUCUR(NNODE).
ECA24: Prints the nodal conductance matrix and current vector.
ECA25: Prints the nodal, branch, element, branch currents, the branch power loses, and also checks unbalance.
ECA26: Inverts the conductance matrix ZPRL and puts the inverse. in ZPRL. Computes the node voltage SMLEP(NNODE) by

multiplying the impedances matrix by current vector.

ECA27: Sets values for W.C. and prints its results.

ECA28: Computes the partial derivatives of the node voltages with respect to each circuit parameter. Also computes and prints the node voltage sensitivities, if they are required. Computation of standard deviation and comparing sign of partial are done by this subroutine.

ECA29: Prints the estimated standard deviation.

ECA30: A routine for parameter modofication solution.

ECA31: A routine for parameter iteration.

4-2 The Features of Modified ECAP

Incorporating the Monte-Carlo techniques for standard deviation calculations, requires changes in or additions to some parts of ECAP. In the ECAP program there are no provisions for defining the standard deviation of circuit parameters directly. Only maximum and minimum values may be specified, and in the program standard deviations are computed as

$$\sigma(P_j) = \frac{P_{jmax} - P_{jmin}}{6}$$

where P_{jmax} and P_{jmin} are, respectively, the specified maximum and minimum values for P_j. The algebraic minimum and maximum are explicitly specified following the nominal value, such as R=3(2,10) or E=-12(-13,-9). The minimum value is always the least positive, or most negative value in the parentheses following the nominal value. Alternatively a percentage tolerance may be given following the nominal value such as E=34(0.35) to specify a 25% tolerance on a 24V nominal voltage. The program then calculates the maximum and minimum values of E as 18 and 30, respectively. Therefore it is necessary to set up new variables for the modified ECAP program to define a standard deviation for DC network parameters such as

SIGY(branch): The standard deviation for circuit Y parameter.

SIGE(branch): The standard deviation for circuit E parameter.

SIGA(branch): The standard deviation for circuit I(current source) parameter.

SIGM(branch): The standard deviation for transconductance parameter.

Two new flags, MONTE and NWO are also added. If a user wants to use the Monte-Carlo technique for standard deviation analysis, an SR control card is used instead of the ST card. In this case, MONTE is set to be one, otherwise it is set to be zero. After completing the Monte-Carlo technique, NWO will be reset to be one. The following are programs to be modified for the new ECAP.

(A) Zeroing SIGY, SIGE, SIGA, and SIGM arrays.

```
      YTERM(I)=0.0                                            LA080530
      YTERML(I)=0.0                                           LA080540
 1011 YTERMH(I)=0.0                                           LA080550
      DO 1025 I=1,25                                          LA080551
      SIGY(I)=0.0                                             LA080552
      SIGA(I)=0.0                                             LA080553
      SIGM(I)=0.0                                             LA080554
 1025 SIGE(I)=0.0                                             LA080555
      DO 1015 I=1,50                                          LA080560
```

(B) Computing the Standard Deviation of Variable Parameters from the Given Maximum and Minimum Values.

```
 3895 YTERML(NUM)=YTERM(NUM)-TNUM*ABS(YTERM(NUM))             LA031010
      YTERMH(NUM)=YTERM(NUM)+TNUM*ABS(YTERM(NUM))             LA031020
      SIGM(NUM)=(YTERMH(NUM)-YTERML(NUM))/6.                  LA031021
      GO TO 19                                                LA031030
 3893 YTERMH(NUM)=TNUM                                        LA031040
      SIGM(NUM)=(YTERMH(NUM)-YTERML(NUM))/6.                  LA031041
      .                                                       .
      .                                                       .
      GO TO 2310                                              LA031200
   85 YTERMH(NUM)=TNUM*Y(NBRN)                                LA031210
      SIGM(NUM)=(YTERMH(NUM)-YTERML(NUM))/6.                  LA031211
   19 ITRANS=2                                                LA031220
      .                                                       .
      .                                                       .
```

```
3855 YMIN(NUM)=Y(NUM)/(1.0+TNUM)                  LA050530
     YMAX(NUM)=Y(NUM)/(1.0-TNUM)                  LA050540
     SIGY(NUM)=(YMAX(NUM)-YMIN(NUM))/6.           LA050541
     GO TO 19                                     LA050550
3853 YMIN(NUM)=1.0/TNUM                           LA050560
     SIGY(NUM)=(YMAX(NUM)-YMIN(NUM))/6.           LA050561
     GO TO 19                                     LA050570
     .                                            .
3748 YMIN(NUM)=Y(NUM)*(1.0-TNUM)                  LA050710
     YMAX(NUM)=Y(NUM)*(1.0+TNUM)                  LA050720
     SIGY(NUM)=(YMAX(NUM)-YMIN(NUM))/6.           LA050721
     GO TO 19                                     LA050730
3746 YMAX(NUM)=TNUM                               LA050740
     SIGY(NUM)=(YMAX(NUM)-YMIN(NUM))/6.           LA050741
     GO TO 19                                     LA050750
     .                                            .
     .                                            .
3915 EMIN(NUM)=E(NUM)-TNUM*ABS(E(NUM))            LA050850
     EMAX(NUM)=E(NUM)+TNUM*ABS(E(NUM))            LA050860
     SIGE(NUM)=(EMAX(NUM)-EMIN(NUM))/6.           LA050861
     GO TO 2310                                   LA050870
3913 EMAX(NUM)=TNUM                               LA050880
     SIGE(NUM)=(EMAX(NUM)-EMIN(NUM))/6.           LA050881
     GO TO 2310                                   LA050890
     .                                            .
```

```
      AMPMAX(NUM)=AMP(NUM)+TNUM*ABS(AMP(NUM))           LA051020
      SIGA(NUM)=(AMPMAX(NUM)-AMPMIN(NUM))/6.            LA051021
      GO TO 2310                                        LA051030
 3923 AMPMAX(NUM)=TNUM                                  LA051040
      SIGA(NUM)=(AMPMAX(NUM)-AMPMIN(NUM))/6.            LA051041
      .                                                 .
  501 YMAX(N)=1.0/VSECND(M)                             DC300410
      YMIN(N)=1.0/VLAST(M)                              DC300420
      SIGY(N)=(YMAX(N)-YMIN(N))/6.                      DC300421
  503 Y(N)=1.0/VFIRST(M)                                DC300430
      .                                                 .
      .                                                 .
      SIGY(N)=(YMAX(N)-YMIN(N))/6.                      DC300481
  508 Y(N)=VFIRST(M)                                    DC300490
      .                                                 .
      .                                                 .
  601 YTERML(N)=VSECND(M)                               DC300570
      YTERMH(N)=VLAST(M)                                DC300580
      SIGM(N)=(YTERMH(N)-YTERML(N))/6.                  DC300581
  602 YTERM(N)=VFIRST(M)                                DC300590
      .                                                 .
      .                                                 .
  604 YTERML(N)=VSECND(M)*Y(II)                         DC300640
      YTERMH(N)=VLAST(M)*Y(II)                          DC300650
      SIGM(N)=(YTERMH(N)-YTERML(N))/6.                  DC300651
```

```
606 YTER(N)=VLAST(M)*Y(II)                          DC300660
    .                                               .
    .                                               .
701 EMIN(N)=VSECND(M)                               DC300710
    EMAX(N)=VLAST(M)                                DC300720
    SIGE(N)=(EMAX(N)-EMIN(N))/6.                    DC300721
703 E(N)=VFIRST(M)                                  DC300730
    .                                               .
    .                                               .
801 AMPMIN(N)=VSECND(M)                             DC300780
    AMPMAX(N)=VLAST(M)                              DC300790
    SIGA(N)=(AMPMAX(N)-AMPMIN(N))/6.                DC300791
803 AMP(N)=VFIRST(M)                                DC300800
    .                                               .
    .                                               .
```

(C) Interpreting SR Card for Standard Deviation Calculation by Monte-Carlo Technique.

```
121 DO 10 K=1,11                                    LA060400
    IF (NWORDS(ICOL)-NMCD(1,K)) 10,6,10             LA060410
  6 IF (NWORDS(ICOL+1)-NMCD(2,K)) 10,7,10           LA060420
 10 CONTINUE                                        LA060430
    GO TO 104                                       LA060440
  7 IF (K.EQ.1.OR.K.EQ.4) MONTE=1                   LA060441
    MAC=MAC+1                                       LA060450
    IF (MAC-6) 11,135,500                           LA060460
```

```
104 DO 16 MTYPE=1,20                                    LA060640
    IF(NWORDS(ICOL)-INDC(1,MTYPE)) 16,12,16             LA060650
 12 IF(NWORDS(ICOL+1)-INDC(2,MTYPE)) 16,12,16           LA060660
 16 CONTINUE                                            LA060670
    IF(NWORDS(ICOL)-INDC(1,8)) 4047,15,4047             LA060671
 15 IF(NWORDS(ICOL+1)-INDC(2,1)) 4047,18,4047           LA060672
 18 MONTE=1                                             LA060673
    MTYPE=8                                             LA060674
    GO TO 13                                            LA060675
4047 M3=10                                              LA060680
    GO TO 805                                           LA060690
135 M3=30                                               LA060700
    GO TO 805                                           LA060710
 13 ICOL=ICOL+1                                         LA060720
    .                                                   .
```

(D) Computation of Random Node Voltages, Mean Values, and Standard Deviations.

By calling the subroutine ECA22B from ECA20, the modified ECAP generates normally distributed parameters according to the given standard deviation and the mean of the circuit parameters. Subroutines ECA22, ECA23, and ECA26 are still used for computation of the node voltages from random variable parameters via ECA20. The following is the modified ECA20.

```
      SUBROUTINE ECA20(ZPRL)                                            DC200000
C                                                                       DC200010
      DOUBLE PRECISION ZPRL(50,50)                                      DC200020
C                                                                       DC200030
      COMMON NMAX,NNODE,NTERMS,NUMBL,NUMBR,NUMBC,IRTN,NTRACE,NSWTCH,KTO,DC200040
     1 NPRINT(10)                                                       DC200050
      COMMON E(200),EMIN(200),EMAX(200),AMP(200),AMPMIN(200),AMPMAX(200)DC200060
      COMMON Y(200),YMIN(200),YMAX(200),NINIT(200),NFIN(200),MODE1(200) DC200070
      COMMON YTERM(200),YTERMH(200),YTERML(200),IROWT(200),ICOLT(200)   DC200080
      COMMON ERROR1,ISEQ,MSEQ,MO,NUMMO,VFIRST(50),VSECND(50),VLAST(50)  DC200090
      COMMON MOBRN(50),MOPARM(50),MOSTEP(50),IWCOUT(4)                  DC200100
      COMMON /TSENG/ SIGY(25),SIGE(25),SIGA(25),SIGM(25),MONTE,NWO
C                                                                       DC200110
C     THE FOLLOWING VARIABLES ARE USED ONLY IN THE ECAP D.C. ANALYSIS   DC200120
C                                                                       DC200130
      COMMON AX1,SMLEP(50),CURR(200),SMLE(200),EQUCUR(50),EX(200)       DC200140
      COMMON EB(200),AMPX(200),AMPB(200),VNOM(50),STDSQ(50),L,M,ITOL    DC200150
      COMMON JX1,JX4,JX5,DELTA,DUM1(28)                                 DC200160
C                                                                       DC200170
      COMMON LANG(265)                                                  DC200180
C                                                                       DC200190
      COMMON MATA(200,4,3),YX(200),YB(200),YTERMX(200),YTERMB(200)      DC200200
      COMMON WCMAX(50),WCMIN(50)                                        DC200210
      DIMENSION VM(25),SD(25),SDD(25)
      DOUBLE PRECISION VM,SD,SDD,D,DEL,RITER,RITERM
C                                                                       DC200220
      DOUBLE PRECISION SMLEP,CURR,SMLE,EQUCUR                           DC200230
C                                                                       DC200240
C                                                                       DC200250
      IF( NTRACE )41,41,40                                              DC200260
 2    FORMAT(' DC MAINLINE-ECA20 ENTERED. IRTN=',I3 )                   DC200270
 40   WRITE(6, 2 )  IRTN                                                DC200280
 41   CONTINUE                                                          DC200290
      NWO=0
      GO TO (1,420,400),IRTN                                            DC200300
    1 JX4=0                                                             DC200310
      DO 10 I=1,NMAX                                                    DC200320
      YX(I)=Y(I)                                                        DC200330
      EX(I)=E(I)                                                        DC200340
   10 AMPX(I)=AMP(I)                                                    DC200350
      IF( NTERMS ) 30, 5, 30                                            DC200360
   30 DO 11 I=1,NTERMS                                                  DC200370
   11 YTERMX(I)=YTERM(I)                                                DC200380
 5    CALL ECA22(ZPRL)                                                  DC200390
      CALL ECA23                                                        DC200400
      IF(NPRINT(10))7,7,8                                               DC200410
    8 JX1=1                                                             DC200420
      CALL ECA24(ZPRL)                                                  DC200430
 7    CALL ECA26(ZPRL)                                                  DC200440
      IF(NPRINT(10))3,3,4                                               DC200450
    4 JX1=2                                                             DC200460
      CALL ECA24(ZPRL)                                                  DC200470
    3 CALL ECA25                                                        DC200480
  420 IF(JX4)419,419,20                                                 DC200490
```

```
  419 I=ISEQ+1
      GO TO (302,16,17,18,18),I
   18 DO 19 I=1,NNODE
   19 STDSQ(I)=0.0
      IF(ISEQ-3)16,16,17
   17 DO 25 K=1,3
      DO 25 I=1,4
      NUM=NMAX
      IF(I-2)22,23,22
   23 IF(NTERMS)25,25,21
   21 NUM=NTERMS
   22 DO 25 J=1,NUM
      MATA(J,I,K)=0
   25 CONTINUE
      DO 24 I=1,NNODE
 24   VNOM(I)=SMLEP(I)
 16   L=1
      M=NNODE
      GO TO 850
   20 L=JX5
      M=JX5
 850  CALL ECA28(ZPRL)
      IF(MONTE.EQ.0) GO TO 848
      IF(NWO) 848,451,848
 451  DO 15 I=1,NNODE
      SD(I)=0.
   15 SDD(I) = 0.
      IX=4975331
      NITER=2001
      RITER=2000.
      RITERM=1999.
      DO 32 NNN=1, NITER
      IF(NNN.EQ.1) GO TO 415
      DO 28 NN=1,NMAX
      IF(SIGY(NN)) 29,28,29
 29   S=SIGY(NN)
      AM=Y(NN)
      CALL ECA22B(IX,S,AM,VV)
      YX(NN)=VV
 28   CONTINUE
      DO 56 NN=1,NMAX
      IF(SIGE(NN)) 57,56,57
 57   S=SIGE(NN)
      AM=E(NN)
      CALL ECA22B(IX,S,AM,VV)
      EX(NN)=VV
 56   CONTINUE
      DO 38 NN=1,NMAX
      IF(SIGA(NN)) 36,38,36
 36   S=SIGA(NN)
      AM=AMP(NN)
      CALL ECA22B(IX,S,AM,VV)
      AMPX(NN)=VV
 38   CONTINUE
```

```
      DO 39 NN=1,NTERMS
      IF(SIGM(NN)) 37,39,37
 37   S=SIGM(NN)
      AM=YTERM(NN)
      CALL ECA22B(IX,S,AM,VV)
      YTERMX(NN)=VV
 39   CONTINUE
 415  CALL ECA22(ZPRL)
      CALL ECA23
      CALL ECA26(ZPRL)
      IF(NNN.NE.1) GO TO 48
      DO 44 I=1,NNODE
 44   VM(I)=SMLEP(I)
      GO TO 32
 48   DO 79 I=1,NNODE
      D        = SMLEP(I) - VM(I)
      SD(I)    = SD(I) + D
      SDD(I)   = SDD(I)  +  D * D
 79   CONTINUE
 32   CONTINUE
      WRITE(6,300)
      WRITE(6,353)
      DO 35 I=1,NNODE
      DEL  = SD(I) / RITER
      VM(I) = VM(I) + DEL
      SD(I) = DSQRT(DABS((SDD(I)-DEL*SD(I)) / RITERM ))
      WRITE(6,303) I,VM(I),SD(I)
 35   CONTINUE
 300  FORMAT(///' STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO'/)
 353  FORMAT(' NODE',4X,'MEAN',15X,'STD. DEV.')
 303  FORMAT(1X,I2,1X,2(4X,E15.8))
      NWO=1
 848  IF(JX4    )  9995,880, 820
  880 IF(ISEQ-2)301,800,801                                             DC200730
 801  IF(MONTE) 802,802,805
 802  CALL ECA29
 805  IF(ISEQ-3) 301,301,800
  800 JX5=0                                                             DC200760
      ISEQ=0                                                            DC200770
      WRITE(6,803)                                                      DC200780
  803 FORMAT(/// 39H WORST CASE SOLUTIONS FOR NODE VOLTAGES///,         DC200790
     1 5H NODE,5X,5HWCMIN,14X,7HNOMINAL,12X,5HWCMAX/)                   DC200800
  820 CALL ECA27                                                        DC200810
      IF(JX4-1)302,5,5                                                  DC200820
  301 ISEQ=0                                                            DC200830
  302 IF(MO)9995,9995,400                                               DC200840
  400 IF(NUMMO)402,119,402                                              DC200850
  402 CALL ECA30                                                        DC200860
      CALL ECA31                                                        DC200870
      NWO=0
      GO TO 5                                                           DC200880
  119 ISEQ=MSEQ                                                         DC200890
      IWCOUT(1)=IWCOUT(3)                                               DC200900
      IWCOUT(2)=IWCOUT(4)                                               DC200910
```

```
      IWCOUT(3)=0                                                       DC200920
      IWCOUT(4)=0                                                       DC200930
      MO=0                                                              DC200940
      IF( ITOL )419,419,900                                             DC200950
  900 MOSTEP(ITOL)=0                                                    DC200960
      GO TO 419                                                         DC200970
 9995 IRTN=1                                                            DC200980
 9996 IF( NTRACE )9999,9999,9998                                        DC200990
 9997 FORMAT(' DC MAINLINE-ECA20 EXIT. IRTN=',I3 )                      DC201000
 9998 WRITE(6, 9997) IRTN                                               DC201010
 9999 RETURN                                                            DC201020
      END                                                               DC201030
```

(E) The Subroutine for Generation of Normally Distributed Random Numbers.

EXA22B is a subroutine which combines GAUSS and RANDU which are listed in Chapter 3. ECA22B is used for generation of normally distributed random numbers and is called from ECA20. The following is the listing of ECA22B.

```
      SUBROUTINE ECA22B(IX,S,AM,V)
      A=0.
      DO 50 I=1,12
      IX=IX*65539
      IF (IY) 5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*0.4656613E-9
      IX=IY
   50 A=A+YFL
      V=(A-6.0)*S+AM
      RETURN
      END
```

CHAPTER 5 THE OUTPUT OF THE MODIFIED ECAP

In DC network analysis there are three solution control cards. The use of these cards is illustrated in Table 4.

Table 4 The Function of Solution Control Card

Solution control cards	Operation requested
SE	Requests sensitivity and partial derivative calculation
WO	Requests worst case calculation as well as sensitivity
ST	Requests standard deviation calculation as well as sensitivity

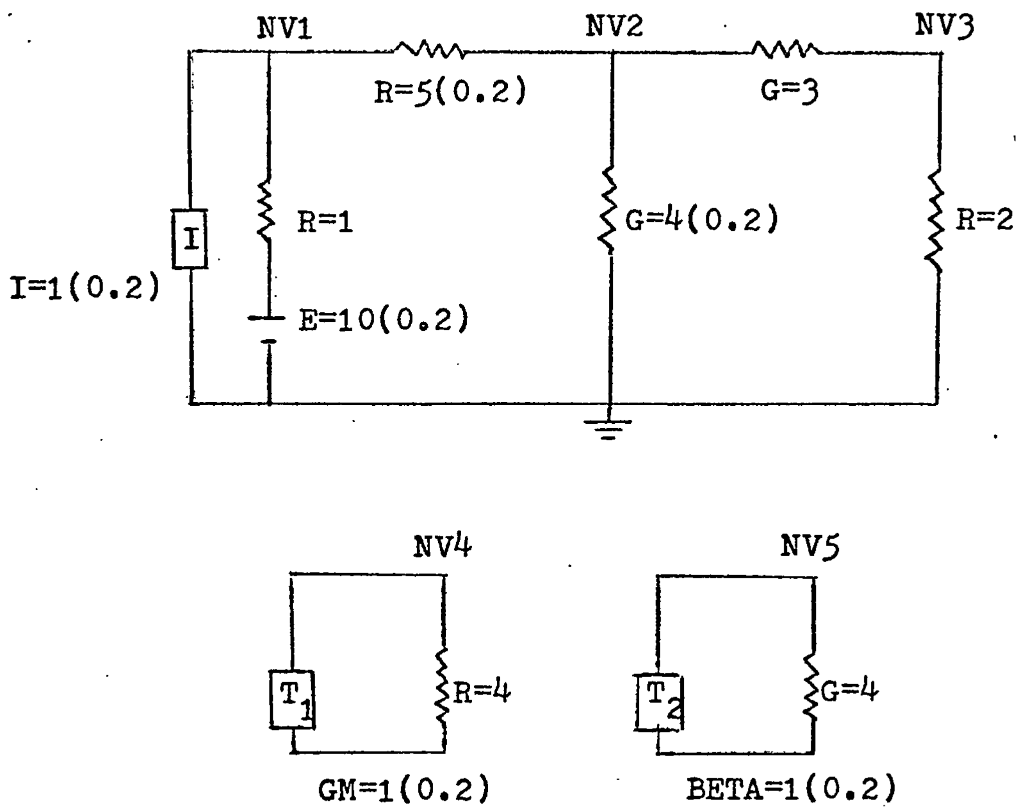
Worst case analysis and standard deviation will be calculated and printed for any DC problem in which the appropriate WO and ST cards are included. They need not be requested on the PRINT card. The same is true for the output request by an SE card, except when a WO or ST card or both are included, in which case SE must be requested via the PRINT card.

In the modified ECAP program a user may use either the SR or ST card optionally for the standard deviation calculation. A user may still use the other solution control cards along with SR or ST card.

The outputs resulting from use of ST or SR solution control cards along with the other solution control cards for DC circuit parameters R, E, G, I, GM, and BETA are listing in the APPENDIX.

A modify routine, defined as a set of ECAP statement starting with MO and ending with EX is still available in the modified ECAP program. Therefore a user may leave all parameter values used in a modify solution unchanged and only substitute an ST for the SR card. Thus the user is able to compare the results obtained via the ST and SR routine by the use of an MO routine. The purpose of the output of the modified ECAP listing on APPENDIX is to check the capabilities of the SR card along with the other solution control cards in a program. Only 25 trials are taken, and its input data is based on the network of Fig. 9.

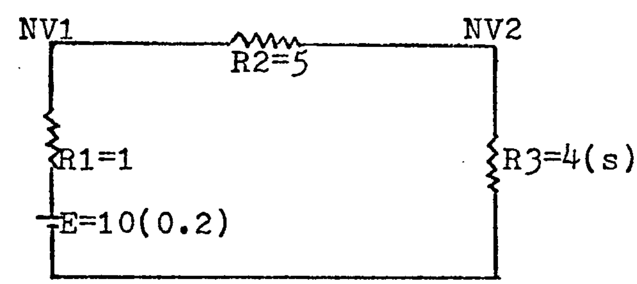
Fig. 9 DC Network For Checking Capability of The Modified ECAp



CHAPTER 6 DISCUSSION

As mentioned before, the actual mean values of node voltages are usually nonlinear and therefore analytical solutions are impossible. This means that a direct comparison of the accuracy of ST and SR calculation cannot be made.

Fig. 10 DC Network of Variable Tolerance in R3



Changing the percentage tolerance, s, of R3 in Fig. 10 and using SR(2000 trials) and ST calculations produces different solutions for standard deviations and means as a function of s shown in Table 5 and 6. When s=0, it is a linear problem and we have its analytical solution, i.e.,

NV1=9.00

NV2=4.00

σ(NV1)=0.60000

σ(NV2)=0.2666667

Only in this case is the ECAP ST solution identical with the analytical solution.

From Tables 5 and 6, the ratio of the ECAP to the Monte-Carlo solution is formed as shown in Table 7. Consider the ratio ECAP/Monte-Carlo at s=0. As shown in Fig. 11 and Table 7, the mean is 1.0008, the standard deviation is 1.0004. This means the errors from the Monte-Carlo technique are negligible. We may therefore conclude that values obtained by Monte-Carlo techniques are sufficiently close to those obtained by analytical solution. From s=0.01 on, the problem becomes nonlinear and we can no longer get true values from ECAP. This is why the curves of Fig. 11 jump abruptly between s=0 and s=0.01. Then, as s increases, the ratios tend to gradually decrease. Since we may assume that the solutions by the Monte-Carlo technique are almost identical with analytical solutions, we may conclude again that the mean values and the standard deviations of node voltages of Fig. 10 by ECAP will become smaller by comparison with the true value as s increases.

Table 5

Mean Values of Node Voltages

	By ECAP		By Monte-Carlo	
	NV1	NV2	NV1	NV2
E=10(0.2) R3=4	9.0000000	3.9999999	8.9924238	3.9966327
E=10(0.2) R3=4(0.01)	9.0000000	3.9999999	9.0052612	4.0023833
E=10(0.2) R3=4(0.05)	9.0000000	3.9999999	9.0046685	4.0048273
E=10(0.2) R3=4(0.1)	9.0000000	3.9999999	9.0053100	4.0086760
E=10(0.2) R3=4(0.2)	9.0000000	3.9999999	9.0071282	4.0195852
E=10(0.2) R3=4(0.3)	9.0000000	3.9999999	9.0099692	4.0366313

Table 6

Standard Deviations of Node Voltages

	By ECAP		By Monte-Carlo	
	NV1	NV2	NV1	NV2
E=10(0.2) R3=4	0.59999967	0.26666653	0.59974969	0.26655541
E=10(0.2) R3=4(0.01)	0.60000116	0.26678640	0.59770202	0.27576638
E=10(0.2) R3=4(0.05)	0.60003674	0.26964974	0.59774204	0.26869938
E=10(0.2) R3=4(0.1)	0.60014784	0.27840799	0.59787059	0.27790225
E=10(0.2) R3=4(0.2)	0.60059202	0.31098390	0.59843488	0.31520160
E=10(0.2) R3=4(0.3)	0.60133153	0.35876316	0.59958200	0.38017969

Table 7

The Ratio of ECAP/Monte-Carlo

	Standard Deviation		Mean	
	NV1	NV2	NV1	NV2
E=10(0.2) R3=4	1.0004	1.0004	1.0008	1.0008
E=10(0.2) R3=4(0.01)	1.0038	1.0038	0.9995	0.9994
E=10(0.2) R3=4(0.05)	1.0038	1.0035	0.9995	0.9988
E=10(0.2) R3=4(0.1)	1.0038	1.0018	0.9994	0.9979
E=10(0.2) R3=4(0.2)	1.0036	0.9866	0.9992	0.9951
E=10(0.2) R3=4(0.3)	1.0029	0.9437	0.9989	0.9909

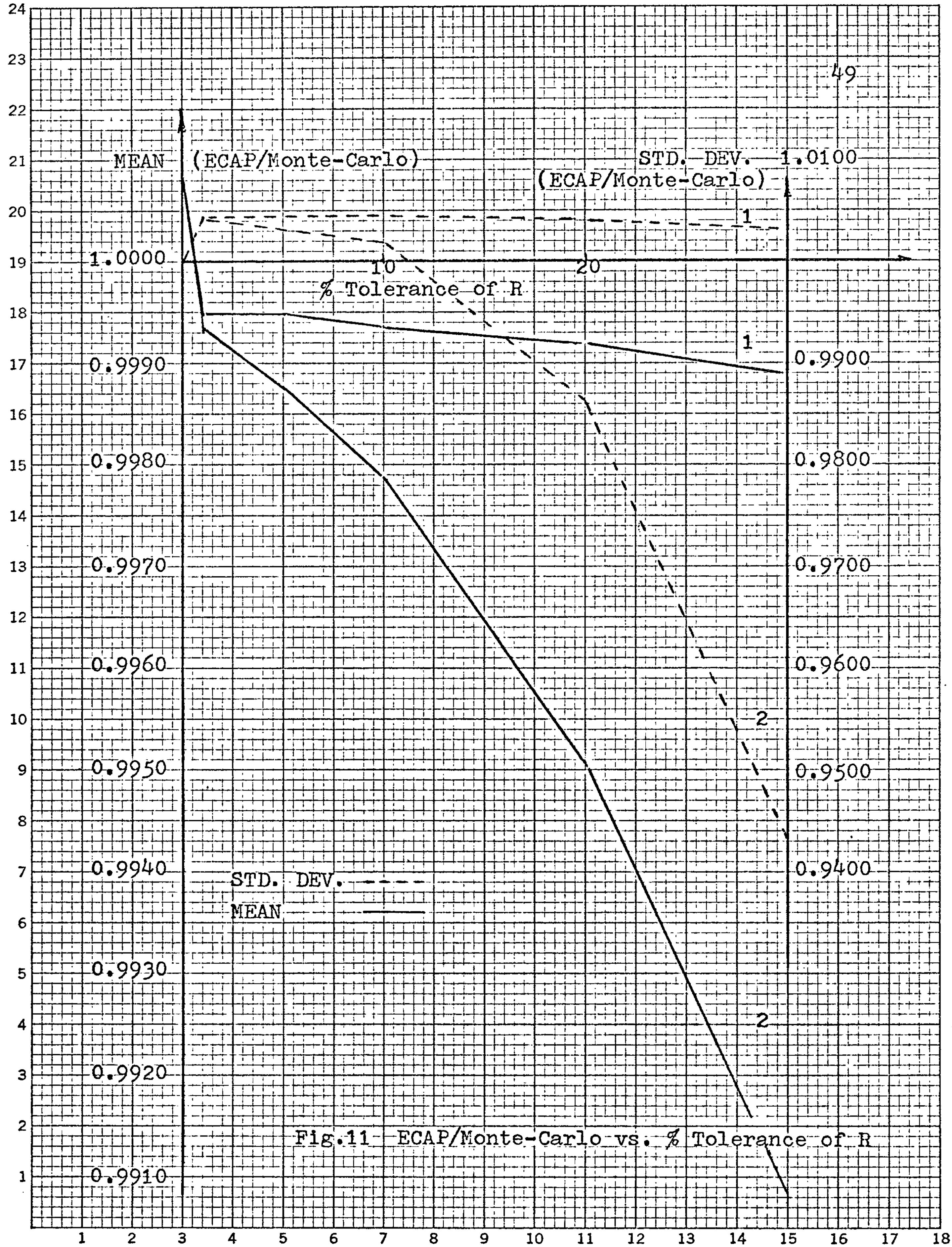


Fig.11 ECAP/Monte-Carlo vs. % Tolerance of R

APPENDIX

Tests of Compatibility Between Original and Modified ECAP

ECAP

ST Solution Control Card

```
     DC
B1   N(0,1),R=1,E=10(0.2),I=1(0.2)
B2   N(1,2),R=5(0.2)
B3   N(2,3),G=3
B4   N(3,0),R=2
B5   N(2,0),G=4(0.2)
B6   N(4,0),R=4
 T1  B(4,6),GM=1(0.2)
B7   N(5,0),R=4
T2   B(4,7),BETA=1(0.2)
     ST
     PR,CA,NV
     EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	-0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

STANDARD DEVIATIONS OF NODE VOLTAGES

NODE	STD. DEV.	NOM.-STD. DEV.	NOMINAL	NOM.+STD. DEV.
1	0.56768668E 00	0.69867172E 01	0.75544042D 01	0.81220903E 01
2	0.35851061E-01	0.29057378E 00	0.32642485D 00	0.36227590E 00
3	0.30729476E-01	0.24906325E 00	0.27979273D 00	0.31052220E 00
4	0.14379036E 00	-0.12629604E 01	-0.11191709D 01	-0.97538054E 00
5	0.71895182E-01	-0.63148063E 00	-0.55958547D 00	-0.48769027E 00

MODIFIED ECAP

SR Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(0.2),I=1(0.2)
B2    N(1,2),R=5(0.2)
B3    N(2,3),G=3
B4    N(3,0),R=2
B5    N(2,0),G=4(0.2)
B6    N(4,0),R=4
 T1   B(4,6),GM=1(0.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.2)
      SR
      PR,CA,NV
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	-0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.73749999D 01	0.53648947D 00
2	0.32392165D 00	0.47545375D-01
3	0.27764713D 00	C.40753178D-01
4	-0.11207355D 01	0.18139647D 00
5	-0.55286349D 00	0.89735855D-01

ECAP

ST,WO, and SE Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(0.2),I=1(0.2)
B2    N(1,2),R=5(0.2)
B3    N(2,3),G=3
B4    N(3,0),R=2
B5    N(2,0),G=4(0.2)
B6    N(4,0),R=4
 T1   B(4,6),GM=1(0.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.2)
      ST
      WO
      SE
      PR,CA,NV
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	-0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

STANDARD DEVIATIONS OF NODE VOLTAGES

NODE	STD. DEV.	NOM.-STD. DEV.	NOMINAL	NOM.+STD. DEV.
1	0.56768668E 00	0.69867172E 01	0.75544042D 01	0.81220903E 01
2	0.35851061E-01	0.29057378E 00	0.32642485D 00	0.36227590E 00
3	0.30729476E-01	0.24906325E 00	0.27979273D 00	0.31052220E 00
4	0.14379036E 00	-0.12629604E 01	-0.11191709D 01	-0.97538054E 00
5	0.71895182E-01	-0.63148063E 00	-0.55958547D 00	-0.48769027E 00

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901056E 01	0.75544033E 01	0.96606054D 01
2	0.18085104E 00	0.32642484E 00	0.58507449D 00
3	0.15501517E 00	0.27979273E 00	0.50149242D 00
4	-0.24071627E 01	-0.11191702E 01	-0.49604866D 00
5	-0.12035809E 01	-0.55958545E 00	-0.24802433D 00

MODIFIED ECAP

SR, WO, and SE Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(0.2),I=1(0.2)
B2    N(1,2),R=5(0.2)
B3    N(2,3),G=3
B4    N(3,0),R=2
B5    N(2,0),G=4(0.2)
B6    N(4,0),R=4
 T1   B(4,6),GM=1(0.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.2)
      SR
      WO
      SE
      PR,CA,NV
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	-0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.73749999D 01	0.53648947D 00
2	0.32392165D 00	0.47545375D-01
3	0.27764713D 00	0.40753178D-01
4	-0.11207355D 01	0.18139647D 00
5	-0.55286349D 00	0.89735855D-01

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901056E 01	0.75544033E 01	0.96606054D 01
2	0.18085104E 00	0.32642484E 00	0.58507449D 00
3	0.15501517E 00	0.27979273E 00	0.50149242D 00
4	-0.24071627E 01	-0.11191702E 01	-0.49604866D 00
5	-0.12035809E 01	-0.55958545E 00	-0.24802433D 00

MODIFIED ECAP

SR, SE, and WO Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(8,12),I=1(0.8,1.2)
B2    N(1,2),R=5.(4,6)
B3    N(2,3),G=3.
B4    N(3,0),R=2.
B5    N(2,0),G=4.(3.2,4.8)
B6    N(0,4),R=4
T1    B(4,6),GM=1(0.8,1.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.8,1.2)
      SR
      SE
      WO
      PR,CA,NV
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.139896370 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.73749998D 01	0.53648967D 00
2	0.323921650 00	0.47545387D-01
3	0.27764712D 00	0.40753189D-01
4	0.11207355D 01	0.18139651D 00
5	-0.55286348D 00	0.89735871D-01

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901047E 01	0.75544033E 01	0.96606062D 01
2	0.18085104E 00	0.32642484E 00	0.58507465D 00
3	0.15501517E 00	0.27979273E 00	0.50149255D 00
4	0.49604857E 00	0.11191702E 01	0.24071639D 01
5	-0.12035818E 01	-0.55958545E 00	-0.24802429D 00

MODIFIED ECAP

ST, SE, and WO Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(8,12),I=1(0.8,1.2)
B2    N(1,2),R=5.(4,6)
B3    N(2,3),G=3.
B4    N(3,0),R=2.
B5    N(2,0),G=4.(3.2,4.8)
B6    N(0,4),R=4
T1    B(4,6),GM=1(0.8,1.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.8,1.2)
      ST
      SE
      WO
      PR,CA,NV,SE
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

PARTIAL DERIVATIVES AND SENSITIVITIES OF NODE VOLTAGES

WITH RESPECT TO RESISTANCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.20527799D 01	-0.20527791E-01
1	2	-0.88700359D-01	-0.88700326E-03
1	3	-0.76028880D-01	-0.76028844E-03
1	4	-0.30411552D 00	-0.30411545E-02
1	5	0.15205776D 00	0.15205771E-02
2	1	0.23219407D 00	0.11609700E-01
2	2	-0.52430919D-01	-0.26215452E-02
2	3	-0.44940788D-01	-0.22470388E-02
2	4	-0.17976315D 00	-0.89881532E-02
2	5	0.89881576D-01	0.44940747E-02
3	1	0.21745546D-02	0.72485127E-05
3	2	0.13047328D-01	0.43491076E-04
3	3	-0.10872774D 00	-0.36242558E-03
3	4	-0.43491094D 00	-0.14497025E-02
3	5	0.21745547D 00	0.72485115E-03
4	1	0.21745534D-02	0.43491047E-04
4	2	0.13047321D-01	0.26094634E-03
4	3	0.31168600D-01	0.62337168E-03
4	4	-0.43491093D 00	-0.86982138E-02

4	5	0.21745535D 00	0.43491051E-02
5	1	0.18942788D 00	0.47356938E-03
5	2	0.11365673D 01	0.28414177E-02
5	3	0.97420055D 00	0.24355007E-02
5	4	0.38968022D 01	0.97420029E-02
5	5	-0.19484011D 01	-0.48709996E-02
6	1	0.0	0.0
6	2	0.0	0.0
6	3	0.0	0.0
6	4	0.27979255D 00	0.11191696E-01
6	5	0.0	0.0
7	1	0.0	0.0
7	2	0.0	0.0
7	3	0.0	0.0
7	4	0.0	0.0
7	5	-0.13989636D 00	-0.55958517E-02

WITH RESPECT TO BETAS

BETA	NODE	PARTIALS	SENSITIVITIES
1	1	0.0	0.0
1	2	0.0	0.0
1	3	0.0	0.0
1	4	0.55958533D 00	0.11191703E-01
1	5	0.0	0.0
2	1	0.0	0.0
2	2	0.0	0.0
2	3	0.0	0.0
2	4	0.0	0.0
2	5	-0.55958533D 00	-0.55958517E-02

WITH RESPECT TO VOLTAGE SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	0.83937825D 00	0.83937764E-01
1	2	0.36269428D-01	0.36269415E-02
1	3	0.31088081D-01	0.31088069E-02
1	4	0.12435233D 00	0.12435228E-01
1	5	-0.62176163D-01	-0.62176138E-02

WITH RESPECT TO CURRENT SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.83937825D 00	-0.83937794E-02
1	2	-0.36269428D-01	-0.36269403E-03
1	3	-0.31088081D-01	-0.31088060E-03
1	4	-0.12435233D 00	-0.12435229E-02
1	5	0.62176163D-01	0.62176143E-03

STANDARD DEVIATIONS OF NODE VOLTAGES

NODE	STD. DEV.	NOM.-STD. DEV.	NOMINAL	NOM.+STD. DEV.
1	0.56768698E 00	0.69867172E 01	0.75544042D 01	0.81220903E 01
2	0.35851073E-01	0.29057378E 00	0.32642485D 00	0.36227590E 00
3	0.30729488E-01	0.24906319E 00	0.27979273D 00	0.31052220E 00
4	0.14379042E 00	0.97538048E 00	0.11191709D 01	0.12629604E 01
5	0.71895182E-01	-0.63148063E 00	-0.55958547D 00	-0.48769027E 00

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901047E 01	0.75544033E 01	0.966060620D 01
2	0.18085104E 00	0.32642484E 00	0.585074650D 00
3	0.15501517E 00	0.27979273E 00	0.501492550D 00
4	0.49604857E 00	0.11191702E 01	0.240716390D 01
5	-0.12035818E 01	-0.55958545E 00	-0.248024290D 00

ECAP

ST, SE, and WO Solution Control Card

```
        DC
  B1    N(0,1),R=1,E=10(8,12),I=1(0.8,1.2)
  B2    N(1,2),R=5.(4,6)
  B3    N(2,3),G=3.
  B4    N(3,0),R=2.
  B5    N(2,0),G=4.(3.2,4.8)
  B6    N(0,4),R=4
  T1    B(4,6),GM=1(0.8,1.2)
  B7    N(5,0),R=4
  T2    B(4,7),BETA=1(0.8,1.2)
        ST
        SE
        WO
        PR,CA,NV,SE
        EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

PARTIAL DERIVATIVES AND SENSITIVITIES OF NODE VOLTAGES

WITH RESPECT TO RESISTANCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.20527799D 01	-0.20527791E-01
1	2	-0.88700359D-01	-0.88700326E-03
1	3	-0.76028880D-01	-0.76028844E-03
1	4	-0.30411552D 00	-0.30411545E-02
1	5	0.15205776D 00	0.15205771E-02
2	1	0.23219407D 00	0.11609700E-01
2	2	-0.52430919D-01	-0.26215452E-02
2	3	-0.44940788D-01	-0.22470388E-02
2	4	-0.17976315D 00	-0.89881532E-02
2	5	0.89881576D-01	0.44940747E-02
3	1	0.21745546D-02	0.72485127E-05
3	2	0.13047328D-01	0.43491076E-04
3	3	-0.10872774D 00	-0.36242558E-03
3	4	-0.43491094D 00	-0.14497025E-02
3	5	0.21745547D 00	0.72485115E-03
4	1	0.21745534D-02	0.43491047E-04
4	2	0.13047321D-01	0.26094634E-03
4	3	0.31168600D-01	0.62337168E-03
4	4	-0.43491093D 00	-0.86982138E-02

4	5	0.21745535D 00	0.43491051E-02
5	1	0.18942788D 00	0.47356938E-03
5	2	0.11365673D 01	0.28414177E-02
5	3	0.97420055D 00	0.24355007E-02
5	4	0.38968022D 01	0.97420029E-02
5	5	-0.19484011D 01	-0.48709996E-02
6	1	0.0	0.0
6	2	0.0	0.0
6	3	0.0	0.0
6	4	0.27979255D 00	0.11191696E-01
6	5	0.0	0.0
7	1	0.0	0.0
7	2	0.0	0.0
7	3	0.0	0.0
7	4	0.0	0.0
7	5	-0.13989636D 00	-0.55958517E-02

WITH RESPECT TO BETAS

BETA	NODE	PARTIALS	SENSITIVITIES
1	1	0.0	0.0
1	2	0.0	0.0
1	3	0.0	0.0
1	4	0.55958533D 00	0.11191703E-01
1	5	0.0	0.0
2	1	0.0	0.0
2	2	0.0	0.0
2	3	0.0	0.0
2	4	0.0	0.0
2	5	-0.55958533D 00	-0.55958517E-02

WITH RESPECT TO VOLTAGE SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	0.839378250 00	0.83937764E-01
1	2	0.36269428D-01	0.36269415E-02
1	3	0.31088081D-01	0.31088069E-02
1	4	0.12435233D 00	0.12435228E-01
1	5	-0.62176163D-01	-0.62176138E-02

WITH RESPECT TO CURRENT SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.83937825D 00	-0.83937794E-02
1	2	-0.36269428D-01	-0.36269403E-03
1	3	-0.31088081D-01	-0.31088060E-03
1	4	-0.12435233D 00	-0.12435229E-02
1	5	0.62176163D-01	0.62176143E-03

STANDARD DEVIATIONS OF NODE VOLTAGES

NODE	STD. DEV.	NOM.-STD. DEV.	NOMINAL	NOM.+STD. DEV.
1	0.56768698E 00	0.69867172E 01	0.75544042D 01	0.81220903E 01
2	0.35851073E-01	0.29057378E 00	0.32642485D 00	0.36227590E 00
3	0.30729488E-01	0.24906319E 00	0.27979273D 00	0.31052220E 00
4	0.14379042E 00	0.97538048E 00	0.11191709D 01	0.12629604E 01
5	0.71895182E-01	-0.63148063E 00	-0.55958547D 00	-0.48769027E 00

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901047E 01	0.75544033E 01	0.966060620D 01
2	0.18085104E 00	0.32642484E 00	0.585074650D 00
3	0.15501517E 00	0.27979273E 00	0.501492550D 00
4	0.49604857E 00	0.11191702E 01	0.240716390D 01
5	-0.12035818E 01	-0.55958545E 00	-0.248024290D 00

MODIFIED ECAP

SR, SE, and WO Solution Control Card

```
      DC
B1    N(0,1),R=1,E=10(8,12),I=1(0.8,1.2)
B2    N(1,2),R=5.(4,6)
B3    N(2,3),G=3.
B4    N(3,0),R=2.
B5    N(2,0),G=4.(3.2,4.8)
B6    N(0,4),R=4
T1    B(4,6),GM=1(0.8,1.2)
B7    N(5,0),R=4
T2    B(4,7),BETA=1(0.8,1.2)
      SR
      SE
      WO
      PR,CA,NV,SE
      EX
```

NODE VOLTAGES

NODES	VOLTAGES			
1- 4	0.75544042D 01	0.32642485D 00	0.27979273D 00	0.11191709D 01
5- 5	-0.55958547D 00			

ELEMENT CURRENTS

BRANCHES	CURRENTS			
1- 4	0.24455958D 01	0.14455958D 01	0.13989637D 00	0.13989637D 00
5- 7	0.13056994D 01	0.27755576D-16	0.0	

PARTIAL DERIVATIVES AND SENSITIVITIES OF NODE VOLTAGES

WITH RESPECT TO RESISTANCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.20527799D 01	-0.20527791E-01
1	2	-0.88700359D-01	-0.88700326E-03
1	3	-0.76028880D-01	-0.76028844E-03
1	4	-0.30411552D 00	-0.30411545E-02
1	5	0.15205776D 00	0.15205771E-02
2	1	0.23219407D 00	0.11609700E-01
2	2	-0.52430919D-01	-0.26215452E-02
2	3	-0.44940788D-01	-0.22470388E-02
2	4	-0.17976315D 00	-0.89881532E-02
2	5	0.89881576D-01	0.44940747E-02
3	1	0.21745546D-02	0.72485127E-05
3	2	0.13047328D-01	0.43491076E-04
3	3	-0.10872774D 00	-0.36242558E-03
3	4	-0.43491094D 00	-0.14497025E-02
3	5	0.21745547D 00	0.72485115E-03
4	1	0.21745534D-02	0.43491047E-04
4	2	0.13047321D-01	0.26094634E-03
4	3	0.31168600D-01	0.62337168E-03
4	4	-0.43491093D 00	-0.86982138E-02

4	5	0.21745535D 00	0.43491051E-02
5	1	0.18942788D 00	0.47356938E-03
5	2	0.11365673D 01	0.28414177E-02
5	3	0.97420055D 00	0.24355007E-02
5	4	0.38968022D 01	0.97420029E-02
5	5	-0.19484011D 01	-0.48709996E-02
6	1	0.0	0.0
6	2	0.0	0.0
6	3	0.0	0.0
6	4	0.27979255D 00	0.11191696E-01
6	5	0.0	0.0
7	1	0.0	0.0
7	2	0.0	0.0
7	3	0.0	0.0
7	4	0.0	0.0
7	5	-0.13989636D 00	-0.55958517E-02

WITH RESPECT TO BETAS

BETA	NODE	PARTIALS	SENSITIVITIES
1	1	0.0	0.0
1	2	0.0	0.0
1	3	0.0	0.0
1	4	0.55958533D 00	0.11191703E-01
1	5	0.0	0.0
2	1	0.0	0.0
2	2	0.0	0.0
2	3	0.0	0.0
2	4	0.0	0.0
2	5	-0.55958533D 00	-0.55958517E-02

WITH RESPECT TO VOLTAGE SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	0.83937825D 00	0.83937764E-01
1	2	0.36269428D-01	0.36269415E-02
1	3	0.31088081D-01	0.31088069E-02
1	4	0.12435233D 00	0.12435228E-01
1	5	-0.62176163D-01	-0.62176138E-02

WITH RESPECT TO CURRENT SOURCES

BRANCH	NODE	PARTIALS	SENSITIVITIES
1	1	-0.83937825D 00	-0.83937794E-02
1	2	-0.36269428D-01	-0.36269403E-03
1	3	-0.31088081D-01	-0.31088060E-03
1	4	-0.12435233D 00	-0.12435229E-02
1	5	0.62176163D-01	0.62176143E-03

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.73749998D 01	0.53648967D 00
2	0.32392165D 00	0.47545387D-01
3	0.27764712D 00	0.40753189D-01
4	0.11207355D 01	0.18139651D 00
5	-0.55286348D 00	0.89735871D-01

WORST CASE SOLUTIONS FOR NODE VOLTAGES

NODE	WCMIN	NOMINAL	WCMAX
1	0.54901047E 01	0.75544033E 01	0.96606062D 01
2	0.18085104E 00	0.32642484E 00	0.58507465D 00
3	0.15501517E 00	0.27979273E 00	0.50149255D 00
4	0.49604857E 00	0.11191702E 01	0.24071639D 01
5	-0.12035818E 01	-0.55958545E 00	-0.24802429D 00

MODIFIED ECAP

Using MO Card to Compare ST and SR Solutions

```
      DC
B1    N(0,1),E=10(0.2),R=1
B2    N(1,2),R=5
B3    N(2,0),R=4
      SR
      EX
```

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.89924238D 01	0.59974969D 00
2	0.39966327D 01	0.26655541D 00

```
      MO
B1    N(0,1),E=10(0.2),R=1
B2    N(1,2),R=5
B3    N(2,0),R=4(0.05)
      SR
      EX
```

STANDARD DEVIATIONS OF NODE VOLTAGES BY MONTE-CARLO

NODE	MEAN	STD. DEV.
1	0.90046685D 01	0.59774204D 00
2	0.40048273D 01	0.26869938D 00

```
      MO
B1    N(0,1),E=10(0.2),R=1
B2    N(1,2),R=5
B3    N(2,0),R=4(0.01)
      ST
      EX
```

STANDARD DEVIATIONS OF NODE VOLTAGES

NODE	STD. DEV.	NOM.-STD. DEV.	NOMINAL	NOM.+STD. DEV.
1	0.60000116E 00	0.83999987E 01	0.90000000D 01	0.96000004E 01
2	0.26678640E 00	0.37332134E 01	0.39999999D 01	0.42667856E 01

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