A STUDY OF THE BUCKLING OF BUILT-UP COLUMNS

A Thesis

Presented to

the Faculty of the Department of Civil Engineering

The University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

by

Chian-Li Chiu

June, 1968

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ABSTRACT

Many steel bridges, buildings, transmission and reception towers are constructed using built-up members. The buckling load of a built-up column is of interest to the designer because bending moments and stresses from the transverse loads are amplified when the compressive force approaches this critical load.

This thesis presents an analytical study to evaluate the critical buckling load for several types of built-up prismatic members. The effect of shear deformations on the buckling strength is discussed in detail. An experimental investigation on the measurement of buckling strength of built-up members was made on small scale test columns. Test results are compared with those predicted by the theoretical analysis. The tests verify the predictions of column buckling strengths made using the theoretical basis presented in this paper.

TABLE OF CONTENTS

1. INTRODUCTION

A built-up column is a compression member in which the components of the column are connected by a system of lacing bars or batten plates. The functions of the bracing system are to provide a mechanism for the transmission of shear by the column and to reduce the laterally unsupported length of the legs so that they may carry high compressive loads.

The earliest treatment of the problem on built-up columns was by Engesser[1] in 1889 in which he presented approximate formulas for the buckling load of latticed columns as well as that of columns with batten plates. In 1909 Engesser[2] published a refined analysis of the same problem, taking into account the secondary effect of the shearing forces. The failure of the Quebec Bridge[3] during construction in 1907 called attention to certain deficiencies of this type of column and gave rise to various theroretical and experimental investigations into the behavior of built-up columns.

The effect of the shearing forces which occur when a column deflects near the buckling load will be discussed in Chapter 2. Their influence on the critical load was found practically negligible in columns of solid cross section of the types conventional in structural design. This is entirely due to the fact that the shearing stresses

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[1]Engesser, F., Zentralblatt der Bauverwaltung, Vol. 11, p. 483, 1891.

[2]Engesser, F., Zentralblatt der Bauverwaltung, Vol. 29, 1909.

[3]"Royal Commission Quebec Bridge Inquiry Report," Ottawa, Canada, Vol. 1, 1908, p. 15050.

and the distortion caused by these stresses are very small, even in the worst case of an I-section buckling in the plane of the web. The conditions are different in built-up columns. The contribution of the shearing forces to the total deflection of the column is much greater in the case of built-up columns. The decrease in buckling strength due to the shear deflection is therefore much greater than in the case of column with solid cross section and depends upon the detailed arrangement and dimensions of the lacing elements.

The built-up column is in reality a three dimensional framework and the stability of which can be investigated more accurately by the methods of framework analysis. Many attempts have been made to approach the problems in this manner and to arrive at exact solutions of the column problem. However, all these studies, which considered the column as a framework, did not add much new knowledge concerning the performance of built-up columns beyond that already furnished by Engesser's work. They confirmed the results first derived by Engesser, and this is their importance. The exact solutions differ from the approximate formulas, in that the number n of panels in which the column is subdivided appears only in exact expressions for the critical load developed on the basis of the framework theory, but n affects the results only in cases where it is a small number. In practice n is usually greater than 4, and the exact and the approximate methods furnish nearly the same results. Therefore, in this paper the analysis is based on the approximate method, in which the built-up column is treated as a prismatic member. This approach is reasonable because in practice the column legs are considerably close to one another and the vertical length of a panel in the braced plane is usually small compared with the column height.

The purpose of this thesis is to present an analytical study to determine the elastic buckling load of built-up prismatic members. In order to confirm the theory derived, the small scale experimental column tests were performed.

In Chapter 2 the elastic buckling of prismatic bars will be discussed. Expressions for the critical load will be derived by means of solving the differential equation for the deflection curve of the fundamental column. The same expressions will be used for built-up prismatic members.

Laced columns and columns with batten plates will be discussed in Chapter 3. The effect of shear deformations on the buckling load is discussed in detail in each case. The three-legged column of prismatic cross section, which is often used as a guyed structure to support devices for the transmission and reception of radio and television signals, is also treated in detail in this chapter. Columns using perforated cover plates, instead of lacing bars or batten plates are often used in practice. In order to illustrate the overall theory the effect of shear deformations on the buckling strength for this kind of column is also discussed. However, for these columns the shear influence is not significant and may be neglected for practical purposes.

In order to determine how actual built-up columns behave, three test columns were designed, constructed, and tested in the structures laboratory. The test results are compared with those predicted by the theory and are discussed in detail in Chapter 4.

2. BUCKLING OF BARS

2.1. Euler's Column Formula

1. Column with Hinged Ends.

In a study of the strength of compression members it is useful to approach the subject by considering the behavior of an "ideal column", which is a solid bar assumed initially to be perfectly straight and compressed by a centrally applied load.

Consider first the case of a slender, ideal column with hinged ends, acted upon by a longitudinal force P applied along the centroidal axis of the ember (Fig. 2-1a). The column is assumed to be perfectly elastic, and the stresses do not exceed the proportional limit. If the load P is less than the critical value, the bar remains straight and undergoes only axial compression. This straight form of elastic equilibrium is "stable", which means that if a lateral force is applied and a small deflection produced, the deflection disappears when the lateral force is removed and the bar returns to its straight form. If P is gradually increased, a condition is reached in which the straight form of equilibrium becomes "unstable" and a small

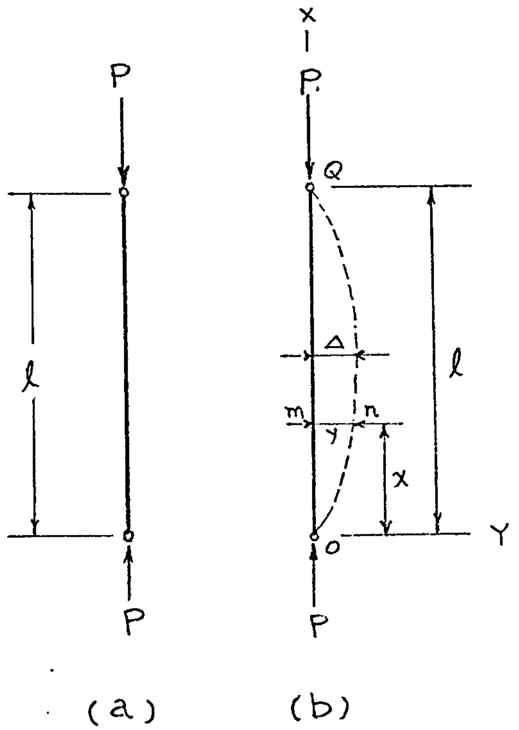

Fig. 2-1

lateral force will produce a deflection which does not disappear when the lateral force is removed. The "critical load (or Euler load)" is then defined as the axial force which is sufficient to keep the bar in such a slightly bent form (Fig. 2.1.b).

The critical load can be calculated by using the differential equation of the deflection curve. When the coordinate axes are taken as indicated in Fig. 2.1.b and also the column is assumed to be in a slightly deflected position, the bending moment at any cross section mn is

$$M = Py$$

and the differential equation becomes

$$EI \frac{d^2y}{dx^2} = -M = -Py$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = 0 \qquad (2\text{-}1)$$

The general solution of this equation is

$$y = A \sin kx + B \cos kx \qquad (a)$$

in which $k = \sqrt{\frac{P}{EI}}$, and A and B are constants of integration. For $x = 0$, $y = 0$, hence $B = 0$, and therefore

$$y = A \sin kx \qquad (b)$$

which is the general equation of the deflection curve. This is a sinusoidal curve, the value of y varying from zero for $kx = 0$, π, 2π, 3π, etc., to a maximum value of $\pm A$ for $kx = \frac{1}{2}\pi$, $3/2\,\pi$, etc. The value of A is therefore equal to Δ, the maximum deflection, and we have, in terms of Δ,

$$y = \Delta \sin kx = \Delta \sin\sqrt{\frac{P}{EI}}\, x \qquad (c)$$

For single curvature, as in Fig. 2-1.b, kx varies from 0 to π, hence for

$$x = l, \quad kl = \pi, \quad \text{or} \quad \sqrt{\frac{P}{EI}}\, l = \pi \qquad (d)$$

from which

$$P_{cr} = \frac{\pi^2 EI}{l^2} \qquad (2\text{-}2)$$

which is Euler's formula for long columns. This is the smallest critical load for the bar in Fig. 2-1 a.

Theoretically, the assumed conditions of equilibrium are satisfied for kl = 2π, 3π, etc., corresponding to curves with one, two or more nodes (Fig. 2-2). For double curvature, for example (Fig. 2-2a), we have kl = 2π and hence $P_{cr} = \frac{4\pi^2 EI}{l^2}$, which is four times the value given by Eq. (2-2). That is, the critical load for double curvature is four times that for single curvature. Multiple curvature cannot of course occur in practice with free, round-ended columns, but may readily be induced by intermediate lateral support. This

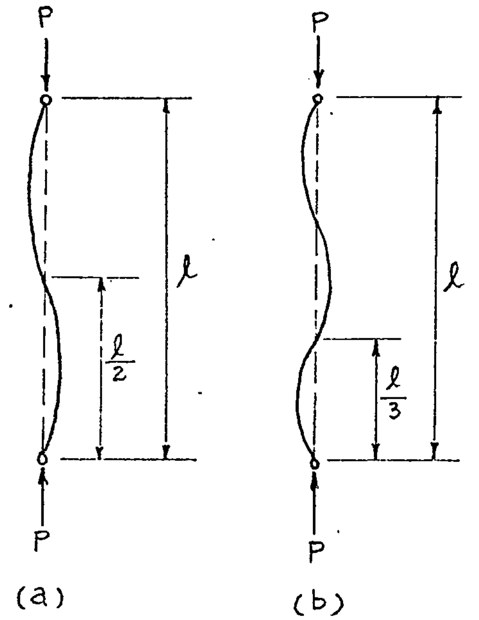

Fig. 2-2

analysis shows to some extent the strengthening value of such intermediate support for very long columns, or, a reduction of the unsupported length.

2. The Effect of End Conditions.

Eq. (2-2) has been derived for the case of freely pivoted ends, giving zero bending moments at these points. The critical loads for columns with some other end conditions can be obtained from the solution of the precedidng case by using for l a reduced length L, which is the actual length between points of inflection or of zero moment. In Fig. 2-3 are shown four cases of end condition with the length indicated between points of zero moment. Substituting these in Eq. (2-2) we have, for the four cases, the following theoretical formulas:

$$\left.\begin{array}{lll} \text{a) Hinged ends} & & P_{cr} = \dfrac{\pi^2 E I}{l^2} \\[2ex] \text{b) One end fixed, one end free} & & P_{cr} = \dfrac{\frac{1}{4}\pi^2 E I}{l^2} \\[2ex] \text{c) Both ends fixed} & & P_{cr} = \dfrac{4\pi^2 E I}{l^2} \\[2ex] \text{d) One end fixed, one end hinged} & & P_{cr} = \dfrac{\frac{9}{4}\pi^2 E I}{l^2} \end{array}\right\} \quad (2\text{-}3)$$

Eq. (2-3) are obtained from Eq. (2-2) by substituting in place of the length l of the bar a reduced length L. Thus we can write in general

$$P_{cr} = \frac{\pi^2 E I}{L^2} \qquad (2\text{-}4)$$

In b, the case of a prismatic bar with one end built-in and the other end free, the reduced length L is twice the actual length

(L = 2 l). In c, the case of a bar with both ends built-in, L is half the actual length. In d, with one end built-in and the other end hinged, L is two thirds of the actual length.

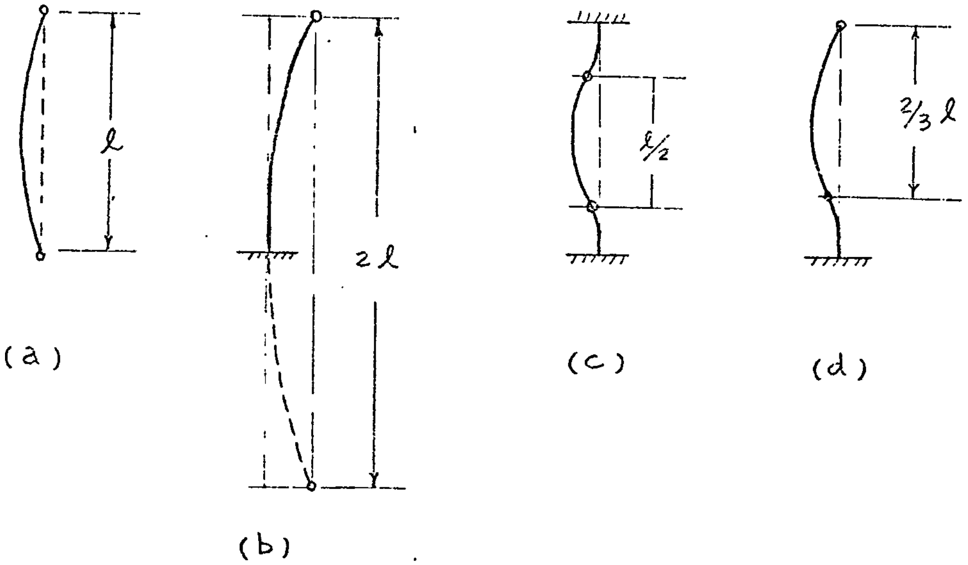

Fig. 2-3

3. The Effect of Shearing Force on the Critical Load.

In the preceding derivations of the equations for the critical loads, the effect of shearing force on the deflection was neglected. When buckling occurs, however, there will be shearing forces acting on the cross sections of the bar. The effect of these forces on the critical load will now be discussed for the hinged end column (Fig. 2-4). The change in angle of the deflection curve produced by the shearing force is

$$\frac{dy_s}{dx} = \gamma = \frac{nQ}{AG} = \frac{n}{AG} \cdot \frac{dM}{dx} \qquad \text{(e)}$$

where A is the total cross-sectional area of the column, G the

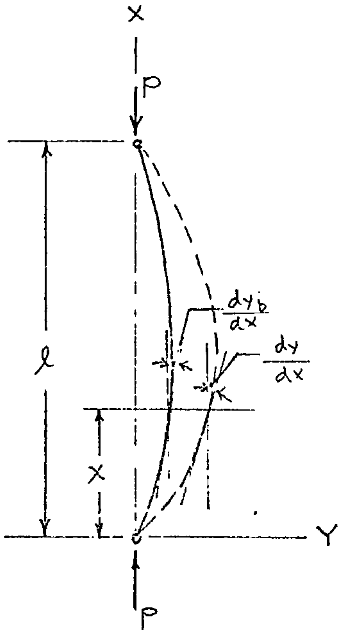

Fig. 2-4

modulus in shear, and n a numerical factor depending on the shape of the cross section.[1]

The rate of change of slope produced by the shearing force Q represents the additional curvature due to shear and is equal to

$$\frac{d^2y_s}{dx^2} = \frac{n}{AG} \cdot \frac{dQ}{dx} = \frac{n}{AG} \cdot \frac{d^2M}{dx^2} \qquad \text{(f)}$$

The total curvature of the deflection curve is now obtained by adding the curvature produced by the shearing force to the curvature produced by the bending moment. Then, for the column in Fig. 2-3, the differential equation of the deflection curve becomes

$$\frac{d^2y}{dx^2} = \frac{d^2y_b}{dx^2} + \frac{d^2y_s}{dx^2} = -\frac{M}{EI} + \frac{n}{AG} \cdot \frac{d^2M}{dx} \qquad \text{(g)}$$

---

[1]For a rectangular cross section the factor $n = 1.2$, and for a circular cross section $n = 1.11$. For an I beam bent about the minor axis of the cross section (i.e., bent in the plane of the flanges) the factor $n \approx 1.2\ A/A_f$, where A_f is the area of the two flanges. This value lies within the range 1.4 to 2.8 for the usual I beam and plate girder sections. If an I beam bends in the plane of the web (about the major axis) the factor $n \approx A/A_w$ where A_w is the area of the web. For this case values of n from 2 to 6 are typical for rolled steel sections.

since $M = P y$, thus we have

$$y'' = -\frac{Py}{EI} + \frac{nP}{AG} \cdot y'' \tag{h}$$

or

$$y''\left(1 - \frac{nP}{AG}\right) + \frac{P}{EI} y = 0 \tag{i}$$

Let $m = \sqrt{\dfrac{P}{EI\left(1-\frac{nP}{AG}\right)}}$, the general solution of this equation is

$$y = A \cos mx + B \sin mx \tag{j}$$

From the boundary condition: $x = 0$, $y = 0$;

$x = \ell$, $y = 0$;

we obtain the stability equation

$$\sin m\ell = 0 \tag{k}$$

from which the least critical value of the load is obtained

$$P_{cr} = \frac{P_e}{1 + \frac{nP_e}{AG}} \tag{2-5}$$

where $P_e = \dfrac{\pi^2 EI}{\ell^2}$ represents the Euler critical load for this case. Thus, owing to the action of shearing forces, the critical load is diminished in the ratio α . Thus $P_{cr} = \alpha \cdot P_e$;

$$\alpha = \frac{1}{1 + \frac{nP_e}{AG}} = \frac{1}{1 + \frac{\sigma_c}{G} n} \tag{l}$$

where $\sigma_{cr} = \dfrac{\pi^2 EI}{A\ell^2}$ represents critical stress.

From the fact that $\alpha < 1$,

therefore $P_{cr} < P_e$

Assume $\sigma_{cr} = \sigma_y = 33{,}000$ psi, $G = 12 \times 10^6$ psi, $n = 1.2$

then we have $\alpha = \dfrac{1}{1.003}$. It is seen that the ratio is very nearly equal to unity for solid columns, such as a column of rectangular cross section or a column with I cross section. Hence in these cases the effect of shearing force can usually be neglected. For built-up columns sonsisting of struts connected by lacing bars or batten plates, the shear effect may become of practical importance and will be the major part of this study. A graph of Eq. (2-5) is given in Fig. 2-5.

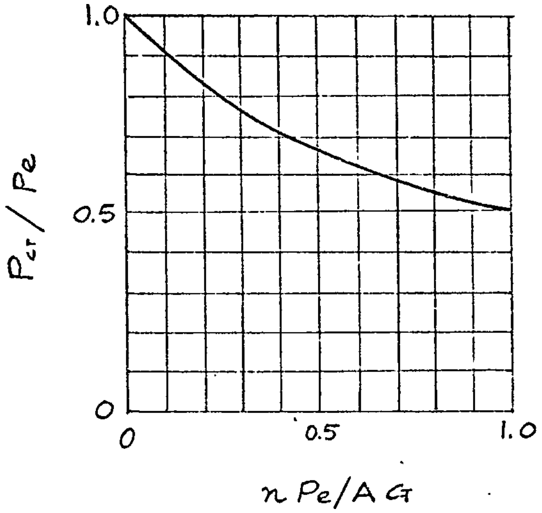

Fig. 2-5

4. Applicability of Euler's Column Formula

It was assumed in the previous discussion that the bar was very slender, so that the maximum compressive stresses which occurred during buckling remained within the proportional limit of the material. Only under these conditions will the preceding equations for the critical loads be valid. To establish the limit of applicability of these formulas the fundamental case of hinged ends (Fig. 2-1) will be illustrated.

Let A = sectional area of column

r = least radius of gyration

σ_{cr} = critical value of the compressive stress

Dividing the critical load from Eq. (2-2) by A, and letting $r = \sqrt{\frac{I}{A}}$, we have

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2} \tag{2-6}$$

This stress depends only on the modulus of leasticity E of the material and on the slenderness ratio l/r. The expression is valid as long as the stress σ_{cr} remains within the proportional limit. When the proportional limit and the modulus E are known for a particular material, the limiting value of the slenderness ratio l/r can be found readily from Eq. (2-6). For example, for a structural steel with a proportional limit of 30,000 psi and E = 30,000,000 psi, we find the minimum l/r from Eq. (2-6) to be about 100. Consequently, the critical load for a bar of this material, having hinged ends, can be calculated from Eq. (2-2) if l/r is greater than 100. If l/r is less than 100, the compressive stress reaches the proportional limit before buckling can occur and Eq. (2-2) cannot be used.[1]

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[1]For the buckling of bars compressed beyond the proportional limit see Timoshenko, "Theory of Elastic Stability" 2nd Ed., Chap. 3, p. 163-184, McGraw-Hill Book Company, Inc., 1961.

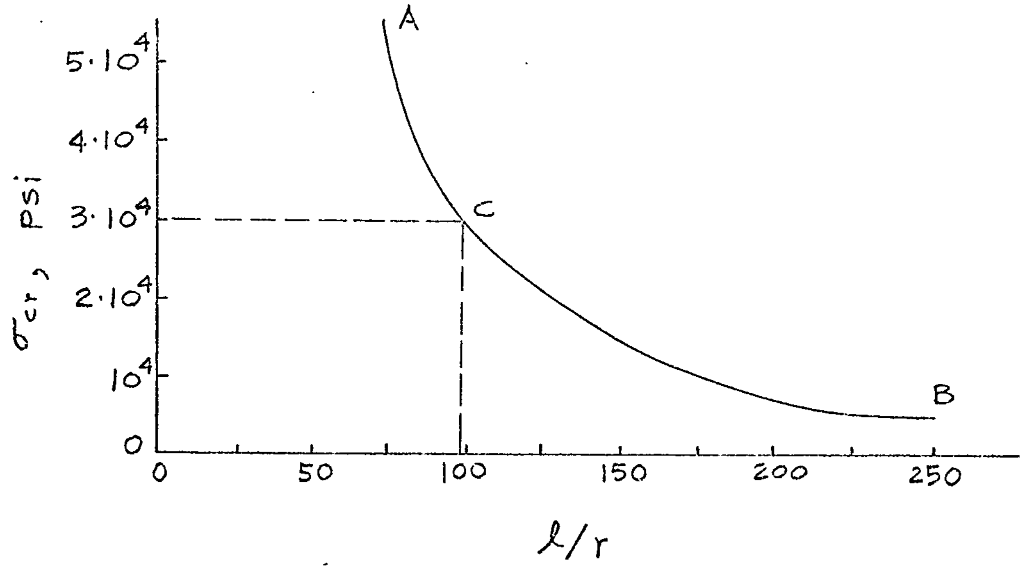


Fig. 2-6

Eq. (2-6) can be represented graphically by the curve A C B in Fig. 2-6, where the critical stress is plotted as a function of ℓ/r. The curve approaches the horizontal axis asymptotically, and the critical stress approaches zero as the slenderness ratio increases. The curve is also asymptotic to the vertical axis, but is applicable in this region only as long as the stress σ_{cr} remains below the proportional limit of the material. The curve in Fig. 2-6 is plotted for the structural steel mentioned above, and point C corresponds to a proportional limit of 30,000 psi. Thus only the portion B C of the curve can be used.

Proceeding as for a bar with hinged ends, we can find the same expressions for the critical stresses analogous to Eq. (2-6) for the other cases shown in Fig. 2-3.

These equations are written in general

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \tag{2-7}$$

in which L is a reduced length for each case. Thus the results obtained for the fundamental case can be used for other cases of buckling of bars by using the reduced length instead of the actual length of the bar.

5. Revisions to Euler Formula in the Inelastic Range

As stated in the preceding section, the Euler formula is satisfactory only for cases where stresses are proportional to strains. Outside of the elastic range E is not constant and the Euler formula must be revised in order for it to be applicable in the inelastic range.

In 1889 Friedrich Engesser proposed the so-called tangent-modulus theory[1] in which the moduli of elasticity for stresses above the proportional limit are determined from the slope of the stress strain curve. Engesser assumed that the column remained straight until failure and that the tangent modulus was constant for the entire cross section of a column. To plot a curve of the Euler equation above the proportional limit with the Engesser proposal it is necessary to assume certain values of P/A , find the tangent modulus for each from a stress-strain curve, and then use the Euler

[1] Zeitschrift fur Architekur und Ingenieurwesen, 1889.

equation to determine the corresponding L/r values for plotting against P/A. The application of this method results in curves which come much closer to test-result curves than does the use of the regular Euler expression.

During the last seventy years applications of the tangent-modulus theory by the engineering profession have made a full circle from considerable respect to little respect and back to considerable respect. After its introduction many engineers claimed that the theory was poor because a very important fact was not considered in its development. The fact supposedly neglected was that the strains on one side of the column were decreasing as were the stresses on that side, and that these changes were made in the range of the elastic modulus.

Based on this criticism Engesser revised his old theory and introduced the reduced-modulus or double-modulus theory in 1895. A reduced modulus somewhat greater than the tangent modulus is used and the estimated load which a column can support is larger than that given by the original Engesser theory. For a good many years the double-modulus theory was accepted as being the correct theory of column action in the inelastic range, but in recent years many doubts have been voiced about the double-modulus theory. Actual test results fall in between the values given by the two theories and in fact they tend to be closer to the tangent-modulus values than to the reduced-modulus values. Furthermore, the tangent-modulus values are on the safe side while the double-modulus values are on the unsafe side.

F. R. Shanley presented a paper in 1947[1] which discussed the shortcomings of the double-modulus theory and showed that the original tangent modulus theory was the better of the two. Curves are presented in Fig. 2-7 showing the comparison of the results obtained by using these two formulas and also the Euler formula.

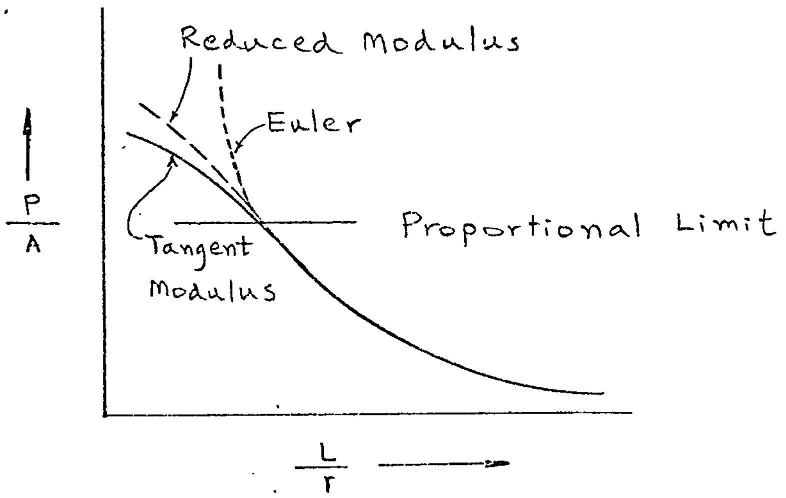


Fig. 2-7
(after Shanley)

[1] F. R. Shanley, "The Column Paradox," Journal of the Aeronautical Sciences, (May 1947), p. 26.

3. BUCKLING OF BUILT-UP COLUMNS

3.1. Description

Built-up compression members are often used for very large structures where the members are long and support very heavy loads.

When built-up sections are used they are connected on their open sides with a system of lacing bars or batten plates. The functions of the bracing system are to provide a mechanism for the transmission of shear and to reduce the laterally unsupported length of the column components so that they may carry high compressive loads.

The built-up sections shown in Fig. 3-1 are often used for structural compression members. Four angles are sometimes arranged as shown in (a) to produce larger r values. This type of member may often be seen in towers and in crane booms. A pair of channels (b) is often used as a building column or as a web member in a large truss. Sometimes the channels may be turned out as shown in (c). A built-up section (d) consisting of a pair of channels with perforated cover plates is also frequently used for the compression members of buildings and bridge trusses.

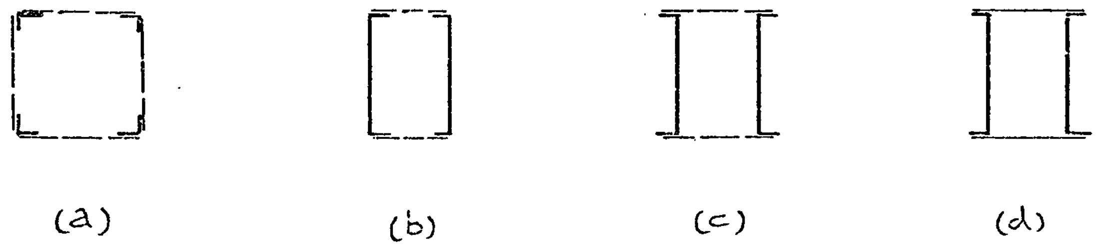


Fig. 3-1

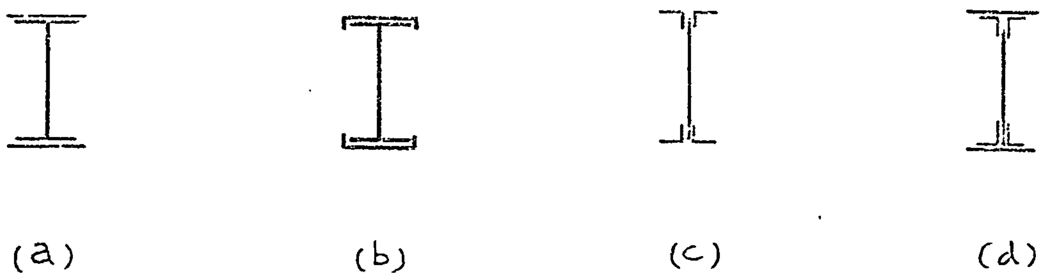


Fig. 3-2

The built-up sections shown in Fig. 3-2 are used when the rolled shapes do not have sufficient strength to resist the column loads; the areas in these sections are increased by adding plates to the flanges. In the evaluation of the critical buckling load for these non-latticed built-up columns, the lateral shearing forces are small and can be neglected. Eq. (2-4) for a single column is used to calculate the critical load for the non-latticed built-up columns.

The critical load for the built-up latticed column is always less than for a solid column having the same cross-sectional area and the same slenderness ratio ℓ/r . This decrease in the critical load is due primarily to the fact that the effect of shear on deflections is much greater for a built-up latticed column than for a solid bar. The shearing forces are therefore definitely not negligible for the built-up latticed columns. The actual values of the critical loads depend upon the detailed arrangement and the dimensions of the bracing systems.

If the built-up column has a large number of panels, Eq. (2-5), derived for a solid bar, can be adapted to the calculation of the critical load. We can write Eq. (2-5) in the form

$$P_{cr} = \frac{P_e}{1 + \frac{P_e}{K}} \qquad (3\text{-}1)$$

where P_e is the Euler critical load and the quantity $1/K$ for a built-up column corresponds to n/AG for a solid bar. Thus the factor $1/K$ is the quantity by which the shearing force Q is multiplied in order to obtain the additional slope γ of the deflection curve due to shear. Thus we have

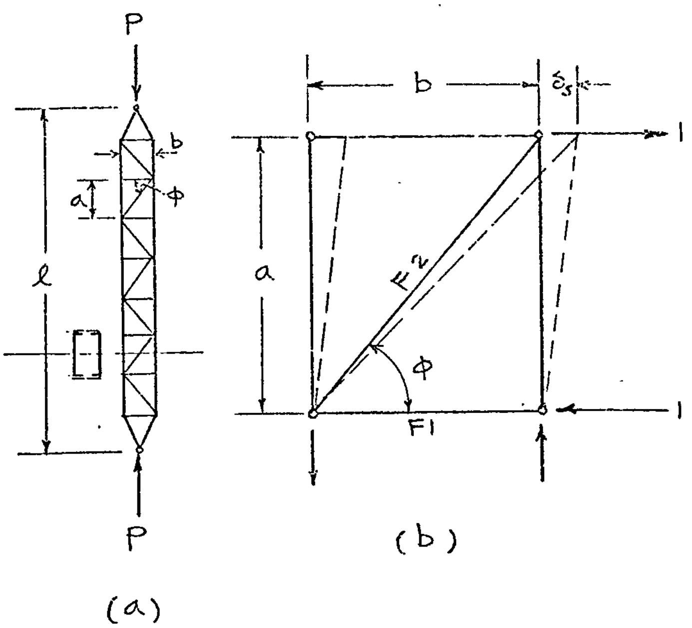
$$\gamma = \frac{Q}{K} \qquad (a)$$

and to determine the quantity $1/K$ in any particular case we must investigate the lateral displacements produced by the shearing force.

3.2. Laced Column

It was pointed out in the preceding section that the additional change in slope of the deflection curve due to the shearing force is expressed by $\frac{1}{K} \cdot Q$, and the effect of shear on the critical load in Eq. (3-1) is represented by $\frac{1}{K}$ which represents the change in slope of the deflection curve due to the unit shearing force.

Fig. 3-3 shows the deformation of one panel under the unit shearing force. Assuming hinges at the joints, the angular displacement produced by the unit shearing force can be taken as $\frac{\delta_s}{a}$ when the deformation is small. By using the principle of virtual work we have $\delta_s = \sum \frac{N^2 \ell}{AE}$, in which N, A and



ℓ are an axial load, cross-sectional area and the length of the member, respectively. In the members F1, $N = 1$, and in F2, $N = \frac{1}{\cos\phi}$, thus we have

Fig. 3-3

$$\delta_s = \frac{1 \times b}{E \cdot A_b} + \frac{1 \times a}{\cos^2\phi \cdot \sin\phi \cdot E \cdot A_d}$$

$$= \frac{a}{E}\left(\frac{1}{\sin\phi \cdot \cos^2\phi \cdot A_d} + \frac{1}{\tan\phi \cdot A_b}\right) \qquad \text{(b)}$$

$$\frac{1}{K} = \frac{\delta_s}{a} = \frac{1}{E}\left(\frac{1}{\sin\phi \cdot \cos^2\phi \, A_d} + \frac{1}{\tan\phi \cdot A_b}\right) \qquad \text{(3-2)}$$

Substitute in Eq. (3-1) the critical load for a strut with hinged ends (Fig. 3-3a) is

$$P_{cr} = \frac{\pi^2 E I}{\ell^2} \frac{1}{1 + \frac{\pi^2 E I}{\ell^2}\left(\frac{1}{E A_d \sin\phi \cdot \cos^2\phi} + \frac{1}{E A_b \tan\phi}\right)} \qquad \text{(3-3)}$$

In this expression I is the moment of intertia of the cross section of the strut, that is, $I = 2\,I_c + \frac{A_c\, b^2}{2}$, in which A_c and I_c are the area and the moment of inertia of the channel, while A_d and A_b are the areas of two diagonals and two battens, respectively. If A_b and A_d are small in comparison with the area of the channels (Fig. 3-3a) or other main members, the critical load from Eq. (3-3) may be considerably lower than the Euler value. The effect of A_d on critical load is larger than that of A_b. Thus the laced column may be considerably weaker than a solid strut with the same EI, but since the amount of material used is less, the laced column may be more economical. When Eq. (3-3) is used, the actual built-up column is replaced by an equivalent column of a reduced length L which is to be determined from the equation

$$L = \ell \sqrt{1 + \frac{\pi^2 EI}{\ell^2}\left(\frac{1}{E A_d \sin\phi \cos^2\phi} + \frac{1}{E A_b \tan\phi}\right)} \quad \text{(c)}$$

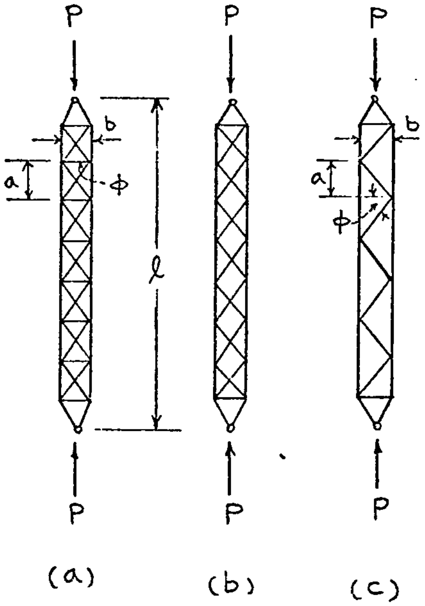


Fig. 3-4

When there are two diagonal lacing bars in each panel (Fig. 3-4a) the shearing force will stress one diagonal in tension and the other in compression. The battens do not take part in the transmission of shearing force, and the system is equivalent to that shown in Fig. 3-4b. The critical load in this case can be obtained from Eq. (3-3) by omitting the term containing A_b and doubling the cross-sectional area A_d. Thus we have

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \frac{1}{1 + \frac{\pi^2 EI}{\ell^2} \frac{1}{E A_d \sin\phi \cos^2\phi}} \quad (3\text{-}4)$$

and a reduced length for an equivalent column is

$$L = \ell \sqrt{1 + \frac{\pi^2 E I}{\ell^2}\left(\frac{1}{E A_d \sin\phi \cos^2\phi}\right)} \qquad \text{(d)}$$

In the above equations A_d denotes the cross-sectional area of four diagonals, two on each side of the column in the same panel.

Eq. (3-4) can be used also in the case of a single system of diagonal bars (Fig. 3-4c) provided A_d is the area of two diagonals and ϕ is measured as shown.

It is seen that the value of critical load for the laced column is determined from the size of column components, its configuration, the length of the column and also depends upon the arrangement and dimensions of bracing systems. The effects of these factors on the buckling strength are further illustrated by the following numerical examples:

(a) Column (Fig. 3-3a)

No.	a (in)	b (in)	l (ft)	Channel Section (in)X(in)X(lb/ft)	Bracing Diagonal (in)X(in)	Bracing Horizontal (in)X(in)	P_{cr} (K) Eq.(3-3)	$\frac{L}{r}$
1	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	41.0	148
2	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/8	1/8 X 3/8	43.8	143
3	6	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	18.3	221
4	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/16	1/8 X 3/16	31.8	147
5	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/8	1/8 X 3/8	33.5	144
6	6	3	28	3 X 1 1/2 X 4.1	3/16 X 3/16	1/8 X 3/16	14.0	220
7	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/8	1/8 X 3/16	43.4	143
8	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/8	41.3	147
9	8	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	38.8	151
10	8	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	17.8	224
11	6	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	30.8	170
12	8	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	30.1	172

(b) Column 2. (Fig. 3-4a)

No.	a (in)	b (in)	l (ft)	Column Components: Channel (in)X(in)X(lb/ft)	Bracing: Diagonal (in)X(in)	Bracing: Horizontal (in)X(in)	P_{cr} (K) Eq.(3-4)	$\frac{L}{r}$
1	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	44.1	142
2	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/8	1/8 X 3/8	45.5	140
3	6	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	18.9	217
4	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/16	1/8 X 3/16	33.7	143
5	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/8	1/8 X 3/8	34.5	141
6	6	3	28	3 X 1 1/2 X 4.1	3/16 X 3/16	1/8 X 3/16	14.3	219
7	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/8	1/8 X 3/16	45.5	140
8	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/8	44.1	142
9	8	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	42.7	145
10	8	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	18.7	220
11	6	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	32.2	166
12	8	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	1/8 X 3/16	31.8	167

(c) Column 3. (Fig. 3-4c)

No.	a (in)	b (in)	l (ft)	Column Components Channel (in)X(in)X(lb/ft)	Bracing Diagonal (in)X(in)	P_{cr} (K) Eq. (3-4)	$\frac{L}{r}$
1	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	41.6	146
2	6	3	18	4 X 1 5/8 X 5.4	3/16 X 3/8	44.1	142
3	6	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	18.4	220
4	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/16	32.2	146
5	6	3	18	3 X 1 1/2 X 4.1	3/16 X 3/8	33.7	143
6	6	3	28	3 X 1 1/2 X 4.1	3/16 X 3/16	14.0	220
7	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-
9	8	3	18	4 X 1 5/8 X 5.4	3/16 X 3/16	39.2	150
10	8	3	28	4 X 1 5/8 X 5.4	3/16 X 3/16	17.9	222
11	6	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	31.3	224
12	8	4	28	4 X 1 5/8 X 5.4	3/16 X 3/16	30.5	171

3.3. Column with Batten Plates

The battened column, subdivided by the battens into panels, is a highly redundant structure and its exact analysis would be rather laborious. However, in studying the stability problem of this system simplifying assumptions have been found concerning the local deformations of the framework which can be applied to determine the internal forces of the system. From the theory of rectangular frameworks it is known that the distorted form of such a framework is characterized by points of inflection at the center of the transverse members, and approximately at the mid-point of each chord member. Therefore the redundant system can be replaced by a statically determinate framework having articulated links at the midpoint of each member as shown in Fig. 3-5.

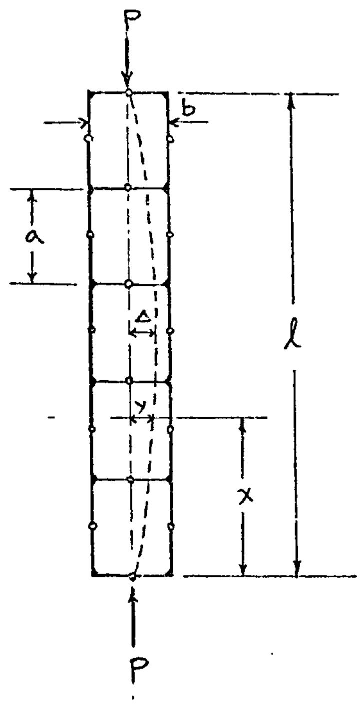


Fig. 3-5

3.3.1. Two-legged Column with Batten Bracing

In the case of a two-legged column made with battens only, as shown in Fig. 3-6a, the lateral displacement produced by the unit shearing force can be obtained by considering the deformations of an element of the strut between the points of inflection which are assumed to be at the mid-points of the panels mn and m_1n_1. The bending of the element will be as shown in

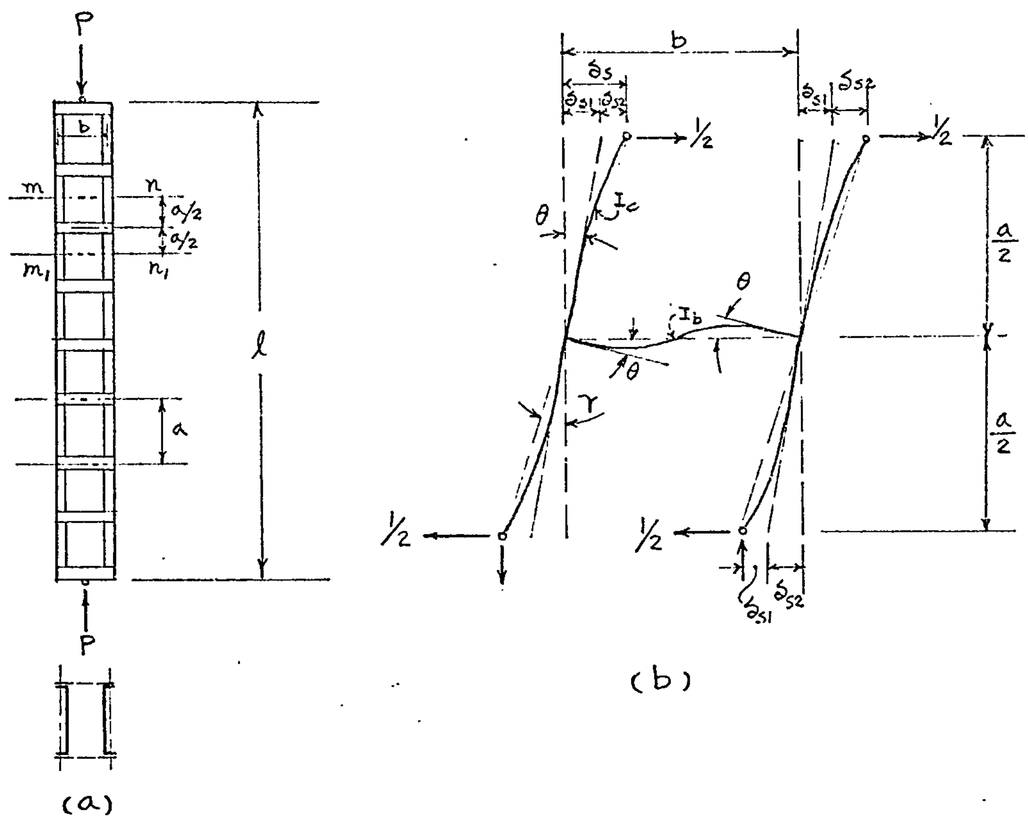


Fig. 3-6

Fig. 3-6b. The lateral deflection consists of the sum of the displacements δ_{s1} , due to bending of the batten, and δ_{s2} , due to bending of the channels. Fig. 3-7 shows in the bending of the batten that there are couples acting at the ends, and the angle of rotation at each end of the batten is obtained by the conjugate-beam method,

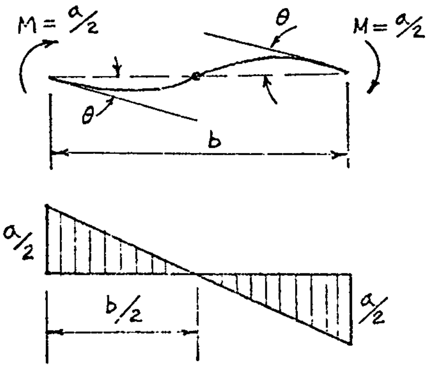


Fig. 3-7

$$\theta = \frac{M \cdot b}{3EI_b} - \frac{M \cdot b}{6EI_b} = \frac{ab}{12\,EI_b}$$

The total lateral displacement is

$$\delta_s = \delta_{s1} + \delta_{s2} \quad ;$$

$$\delta_{s1} = \frac{\theta a}{2} = \frac{a^2 b}{24\,EI_b} \qquad \text{(e)}$$

$$\delta_{s2} = \frac{1}{2}\left(\frac{a}{2}\right)^3 \frac{1}{3EI_c} = \frac{a^3}{48EI_c} \qquad \text{(f)}$$

where b is the length of the batten, $E\,I_b$ is its flexural rigidity and $E\,I_c$ is the flexural rigidity of one of the vertical channels. The total angular displacement produced by the unit shearing force is

$$\gamma = \frac{1}{K} = \frac{\delta_s}{\frac{a}{2}} = \frac{ab}{12EI_b} + \frac{a^2}{24EI_c} \qquad \text{(g)}$$

Substituting into Eq. (3-1) yields

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \; \frac{1}{1 + \frac{\pi^2 EI}{\ell^2}\left(\frac{ab}{12EI_b} + \frac{a^2}{24EI_c}\right)} \qquad \text{(3-5)}$$

and a reduced length for an equivalent column is

$$L = \ell\sqrt{1 + \frac{\pi^2 EI}{\ell^2}\left(\frac{ab}{12EI_b} + \frac{a^2}{24EI_c}\right)} \qquad \text{(h)}$$

In the above equations the factor $\frac{\pi^2 EI}{\ell^2}$ represents the critical load for the entire column calculated as for a solid column. It is seen that when b is large or when flexural rigidity $(E\,I_b)$ of the batten is small, the critical load is much lower than that given by Euler's formula.

If the flexural rigidity of the batten is very large, assume $E\,I_b = \infty$; Eq. (3-5) produces in this case

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \; \frac{1}{1 + \frac{\pi^2 a^2}{24\ell^2} \cdot \frac{I}{I_c}} \qquad \text{(3-6)}$$

I and I_c in the above formula can be represented by $2\,A_c\,r^2$ and $A_c\,r_a^2$, respectively, therefore Eq. (3-6) can be written in the form of

$$P_{cr} = P_e \frac{1}{1 + \frac{\pi^2 a^2}{24} \frac{2\,A_c\,r^2}{\ell^2 A_c\,r_a^2}}$$

$$= P_e \frac{1}{1 + 0.83 \frac{(a/r_a)^2}{(\ell/r)^2}} \qquad (j)$$

For safety purposes we may use 1.0 in lieu of the fraction of 0.83 in the above expression, thus Eq. (3-6) becomes

$$P_{cr} = P_e \frac{\lambda^2}{\lambda^2 + \lambda_a^2} \qquad (3\text{-}6a)$$

In this equation $\lambda = \frac{\ell}{r}$ represents the slenderness ratio of the built-up column, and $\lambda_a = \frac{a}{r_a}$ represents the slenderness ratio of one channel between the battens of the panel.

In the calculation of the angular displacement γ, the shear in the batten must be taken into consideration also. From Eq. (3-7) it can be seen that the shearing force in the batten is $\frac{a}{b}$, and the corresponding shearing strain in

$$\left(\frac{a}{b}\right) \frac{n}{A_b\,G} = \frac{n\,a}{b\,A_b\,G} \qquad (k)$$

For this case A_b is the cross-sectional area of two battens and n equals 1.2 for the rectangular section. Adding this expression to the previous Eq. (g), we obtain

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \frac{1}{1 + \frac{\pi^2 EI}{\ell^2}\left(\frac{ab}{12EI_b} + \frac{a^2}{24EI_c} + \frac{na}{bA_bG}\right)} \qquad (3\text{-}7)$$

instead of Eq. (3-5).

3.3.2. Effect of Local Failure

A built-up column represents a framework which will collapse if any member of the structure begins to yield locally before the critical load P_{cr} is reached for which the column was designed. It is therefore necessary to take the possibility of local failure into account in the evaluation of the critical load.

If the vertical channels of the built-up column represented in Fig. 3-8a are very flexible, or if the distance between battens is large, collapse of the column may occur as a result of local buckling of the channels between two consecutive battens.

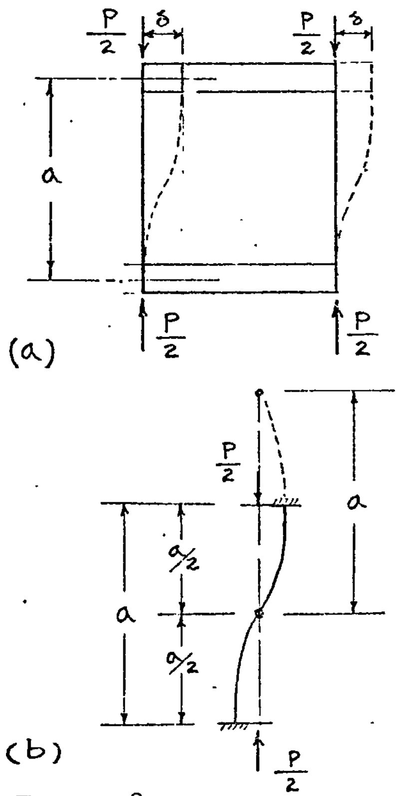


Fig. 3-8

Assuming that the rigidity of the battens is very large, the critical value of the compressive force at which the assumed buckling will occur is found from Fig. 3-8b

$$\frac{P}{2} = \frac{\pi^2 E I_c}{a^2}$$

or

$$P = \frac{2\pi^2 E I_c}{a^2} \qquad (1)$$

In the analysis of the bending of a beam-column with hinged ends[1], the maximum deflection δ is

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[1] See Timoshenko, S. and Gere, J., "The Theory of Elastic Stability", 2nd Edition, McGraw Hill Book Co., Inc., New York, 1961, p. 29, Eq.(a).

given in the form

$$\delta = \delta_o \left(\frac{1}{1-\alpha} \right) \qquad \text{(m)}$$

in which δ_o is the maximum deflection due to a lateral load only and α represents the ratio of P/P_{cr} . Eq. (m) expresses the effect of axial load on the deflection; the δ_o due to the lateral load only is increased by the amplification factor $\frac{1}{1-\alpha}$ when an axial load is also present.

The effect of the axial load $P/2$ on the bending of the vertical channels can now be taken into account by writing Eq. (f) in the form of Eq. (m),

$$\delta_{s2} = \frac{a^3}{48EI_c} \frac{1}{1-\alpha} \qquad \text{(n)}$$

where

$$\alpha = \frac{P_{cr}}{\frac{2\pi^2 EI_c}{a^2}} \qquad \text{(o)}$$

Using expression (n) for δ_{s2} , the critical load P_{cr} for the strut shown in Fig. 3-6b can be written in the form

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \frac{1}{1 + \frac{\pi^2 EI}{\ell^2}\left[\frac{ab}{12EI_b} + \frac{a^2}{24EI_c}\frac{1}{(1-\alpha)} + \frac{na}{bA_bG}\right]} \qquad \text{(3-8)}$$

Since α depends upon P_{cr} , this equation can be solved best by trial and error.[1] It should be noticed also that the critical load

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[1]See Appendix for the computer solution written in the FORTRAN language.

for the column between battens (Fig. 3-8a) is always less than that given by Eq. (1), inasmuch as the battens are not rigid. This means that the true value of α is larger than that given by Eq. (o) and hence the true critical load is less than that obtained from Eq. (3-8). However, these differences are not of practical significance, since the term in the denominator of Eq. (3-8) containing I_c is usually small compared with the term containing I_b.

The critical load for the column with batten bracing is determined from the configuration and size of the column components, and also depends upon the arrangement and dimensions of the particular batten system. The effects of these factors on the buckling strength are further illustrated by the following numerical examples: (See Table 3-1).

It is interesting to note that column 10 in the Table 3-1 has a larger value of P_{cr} than that of column 1 although the latter has a larger b value.

Table 3-1

No.	a (in)	b (in)	l (ft)	Channel (in)X(in)X(lb/ft)	Batten (in)X(in)	P_{cr} (K) Eq.(3-5)	*Eq.(3-7)	Eq.3-8)	*L/r
1	6	4	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	29.34	28.34	28.34	155
2	6	4	12	3 X $1\frac{1}{2}$ X 4.1	$\frac{1}{4}$ X $\frac{1}{2}$	48.02	46.67	46.67	121
3	6	4	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	21.25	20.72	20.72	182
4	8	4	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	23.18	22.36	22.36	180
5	8	4	12	3 X $1\frac{1}{2}$ X 4.1	$\frac{1}{4}$ X $\frac{1}{2}$	39.30	38.10	38.10	134
6	8	4	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	17.82	17.33	17.33	198
7	12	4	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	16.30	15.68	15.67	210
8	12	4	12	3 X $1\frac{1}{2}$ X 4.1	$\frac{1}{4}$ X $\frac{1}{2}$	28.70	27.73	27.70	157
9	12	4	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	13.45	13.03	13.02	230
10	6	3	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	30.54	29.12	29.12	117
11	8	3	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	25.27	23.97	23.97	129
12	12	3	12	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	12.48	17.66	17.66	150
13	6	5	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	-	21.03	-	180
14	8	5	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	-	16.91	-	200
15	12	5	20	3 X $1\frac{1}{2}$ X 4.1	1/8 X $\frac{1}{2}$	-	12.13	-	236
16	24	4	12	3 X $1\frac{1}{2}$ X 4.1	3/16 X 1	-	45.76	43.55	122
17	36	4	12	3 X $1\frac{1}{2}$ X 4.1	3/16 X 1	-	31.02	27.45	148
18	24	4	18	3 X $1\frac{1}{2}$ X 4.1	3/16 X 1	-	32.27	31.48	147
19	36	4	18	3 X $1\frac{1}{2}$ X 4.1	3/16 X 1	-	24.18	22.45	174

3.4. Three-legged Column

3.4.1. Description

The three-legged laced tower of prismatic cross section is often used as a guyed structure to support devices for the transmission and reception of radio and television signals. The legs almost invariably are arranged to occupy the vertices of an equilateral triangle in cross section. Bracing connects the legs to form three braced planes, each parallel to the axis of the tower and each making a dihedral angle of 60° with the others. The bracing system may consist of diagonals, battens, or a combination of both. The three braced planes of a tower are subdivided into vertically stacked panels by the bracing members and are usually identical.

The buckling loads for the equilateral triangle tower, (1) with triangulated web systems of various configurations, and (2) with nontriangulated web systems (battens only), are treated in this article.

For a prismatic member of Fig. 2-4, the least critical value of the load P, considering the effect of shearing force, is given by Eq. (3-1)

$$P_{cr} = \frac{P_e}{1 + \frac{P_e}{K}}$$

in which $K = \dfrac{1}{n/AG} = \dfrac{1}{\gamma}$ is defined as the shearing stiffness. The expression K for use with the three-legged tower will now be derived:

Let Q_P be the shearing force in one of the braced planes and K_P the shearing stiffness of a braced plane. The angle between a braced plane and the direction in which the buckled tower cross section is displaced is designated by β (Fig. 3-10). The moment of

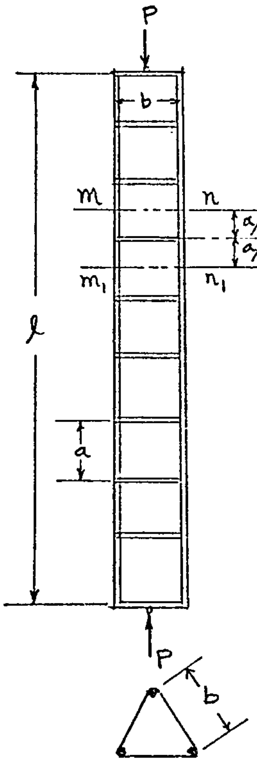

Fig. 3-9

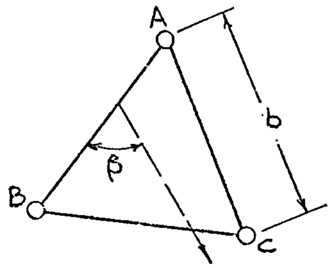

Fig. 3-10

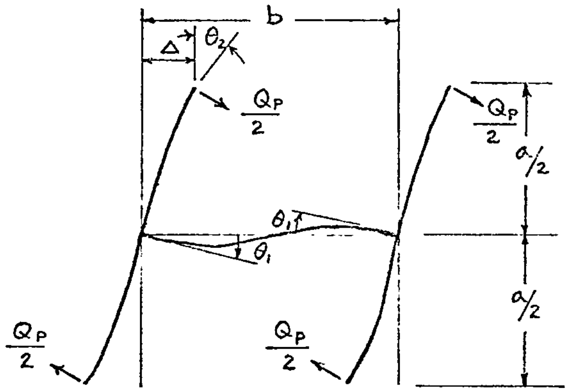

Fig. 3-11

inertia about any axis of the section, I , is invariant. The displacement of a braced plane such as A B in Fig. 3-10 consists of a component along direction A B , y_P , and a component perpendicular to plane A B . The former results in a shear resistance directed from B to A. Designating the total displacement of the cross section by y , then

$$y_P = y \cos \beta \tag{3-9}$$

The curvature in plane A B associated with axial deformations of legs A and B is designated y''_{bp} , and from Fig. 3-12

$$\epsilon_B = \frac{\overline{st}}{\overline{nq}} = -\frac{\frac{b}{2}}{-\rho} = \frac{\frac{b}{2}}{\rho}$$

$$\epsilon_A = -\frac{\overline{rq}}{\overline{nq}} = -\frac{-\frac{b}{2}}{-\rho} = -\frac{\frac{b}{2}}{\rho}$$

Thus, y''_{bp} may be expressed

$$y''_{bp} = \frac{\epsilon_B - \epsilon_A}{b} = \frac{1}{\rho} = -\frac{P}{EI}\, y \cos\beta \qquad (3\text{-}10)$$

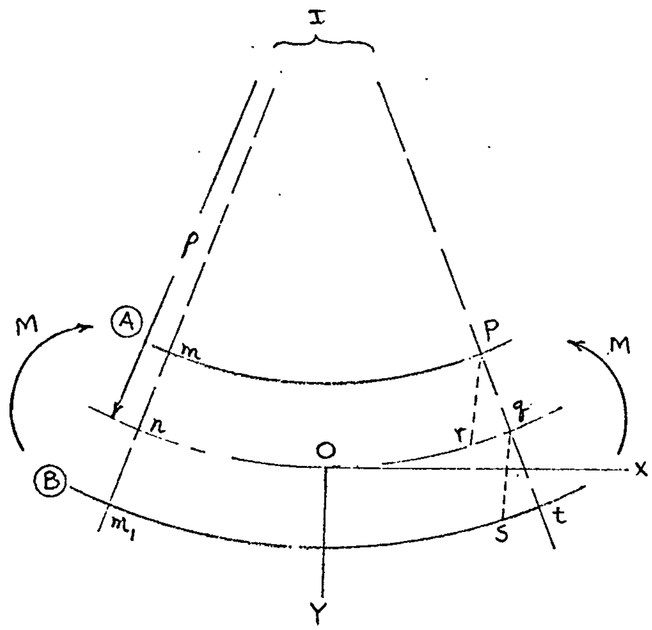

Fig. 3-12

Assume the shear in the leg of a column is taken as $\frac{Py'}{3}$, and the shear in each leg is resolved into components in the braced planes at each vortex of the equilateral triangle A B C . The shearing force Q_p in each plane is

$$Q_p = \frac{2}{3} P y' \cos\beta \qquad (3\text{-}11)$$

Thus the relation for the panel distortions due to shear is obtained:

$$y'_{sp} = \frac{Q_p}{K_p} = \frac{\frac{2}{3} P \cdot y' \cos\beta}{K_p} \qquad (3\text{-}12)$$

$$y''_{sp} = \frac{P y'' \cos\beta}{\frac{3}{2} K_p} \tag{3-13}$$

Eqs. (3-9), (3-10) and (3-13) yield

$$\begin{aligned} y'' \cos\beta &= y_p'' = y''_{bp} + y''_{sp} \\ &= \left(-\frac{P}{EI} y + \frac{P y''}{\frac{3}{2} K_p} \right) \cos\beta \end{aligned} \tag{3-14}$$

from which

$$y'' + \frac{P}{EI} \left(\frac{1}{1 - \frac{2P}{3K_p}} \right) y = 0 \tag{3-15}$$

It is noted that Eq.(i) in Chapter 2 can be written in the form

$$y'' + \frac{P}{EI} \left(\frac{1}{1 - \frac{P}{K}} \right) y = 0$$

which is identical to Eq. (3-15), with the exception that $\frac{3}{2}$ (K_P) is found in the latter, while K appears in the former. The foregoing implies that Eq. (3-1) may be used to determine the buckling load of three-legged towers provided $\frac{3}{2}$ K_P is substituted for K. The formula corresponding to Eq. (3-1) for the three-legged tower is therefore

$$P_{cr} = \frac{P_e}{1 + \frac{P_e}{\frac{3}{2} K_p}} \tag{3-1a}$$

Eq. (3-1a) is valid for both web systems, namely (1) with triangulated bracing, and (2) with non-triangulated bracing. The stiffness K_P

for each particular case is evaluated according to the methods shown in the previous section.

3.4.2. Column with Nontriangulated Web System[1]

For the case of a column with batten bracing only (Fig. 3-11),

$$\frac{1}{K_p} = \frac{\gamma_p}{Q_p} = \frac{2\Delta}{a Q_p} = \frac{1}{24E}\left(\frac{2ab}{I_b} + \frac{a^2}{I_c}\right) \quad (3\text{-}16)$$

In this expression I_b is the moment of inertia of the batten cross section and I_c is the moment of inertia of a leg of the tower about its centroidal axis. It is here assumed that the column legs are circular or tubular so that any axis of the cross section is a principal axis, and I_c is invariant. The vertical and horizontal dimensions of the panel in the braced planes are a and b, respectively. Eq. (3-16) ignores the possibility of additional effects due to shear distortion in the batten.

The formula corresponding to Eq. (3-1) for three-legged towers with batten bracing are, therefore

$$P_{cr} = \frac{\dfrac{\pi^2 EI}{\ell^2}}{1 + \dfrac{\pi^2 I}{\ell^2}\,\dfrac{1}{36}\left(\dfrac{2ab}{I_b} + \dfrac{a^2}{I_c}\right)} \quad (3\text{-}17)$$

The rotation of a tower leg, θ_2, at a point midway between two battens is

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[1]Tomayo, J. Y., and Ojalud, M., "Buckling of Three-legged Columns with Batten Bracing," Proceedings of the A. S. C. E., Vol. 91, No. ST 1, February, 1965.

$$\theta_2 = \frac{\Delta}{\frac{a}{2}} + \frac{\frac{a}{2}}{3EI_c}\left(0 - \frac{-\frac{Q_p a}{4}}{2}\right)$$

$$= \frac{Q_p}{24E}\left(\frac{2ab}{I_b} + \frac{a^2}{I_c}\right) + \frac{Q_p a^2}{48EI_c}$$

$$= \frac{Q_p}{E}\left(\frac{ab}{12I_b} + \frac{a^2}{16I_c}\right) \qquad (3\text{-}18)$$

To simplify subsequent expressions, the following substitutions are introduced:

$$N = \frac{ab}{12I_b} + \frac{a^2}{16I_c} \qquad (3\text{-}19)$$

and . .

$$P_e = \frac{\pi^2 EI}{\ell^2}$$

The component of shear in one leg along the direction of the braced plane due to θ_2 is $\frac{P\theta_2}{3}$. The total panel shear is

$$Q_p = \frac{2}{3}P\theta_2 + \frac{2}{3}Py_b' \cos\beta \qquad (3\text{-}20)$$

Substituting from Eq. (3-18),

$$Q_p = \frac{\frac{2}{3}Py_b' \cos\beta}{1 - \frac{2}{3}\frac{P}{E}N} \qquad (3\text{-}21)$$

The curvature associated with shear distortion is designated y''_{SP}, then

$$y'_{sp} = \gamma = \frac{Q_p}{K_p} = \frac{\frac{2}{3}\frac{P}{K_p}y_b' \cos\beta}{1 - \frac{2}{3}\frac{P}{E}N}$$

and

$$y''_{sp} = \frac{\frac{2}{3}\frac{P}{K_p} y_b'' \cos\beta}{1 - \frac{2}{3}\frac{P}{E} N} \tag{3-22}$$

The curvature associated with axial changes in the length of the legs is given by Eq. (3-10). Thus, the governing differential equation is obtained from the following:

$$y_p'' = y'' \cos\beta = y''_{bp} + y''_{sp}$$

$$y'' \cos\beta = \left(-\frac{P}{EI} y + \frac{\frac{2}{3}\frac{P}{K_p} y_b''}{1 - \frac{2}{3}\frac{P}{E} N} \right) \cos\beta$$

Substitute $\quad y_b'' = -\frac{Py}{EI}$

$$y'' + \left(\frac{P}{EI} + \frac{\frac{2}{3}\frac{P}{K_p}\frac{P}{EI}}{1 - \frac{2}{3}\frac{P}{E} N} \right) y = 0$$

or

$$y'' + \frac{P}{EI}\left(1 + \frac{\frac{2}{3}\frac{P}{K_p}}{1 - \frac{2}{3}\frac{P}{E} N} \right) y = 0 \tag{3-23}$$

Solving for the pin-ended condition yields the following solution for the buckling load P in implicit form:

$$P\left(1 + \frac{\frac{2}{3}\frac{P}{K_p}}{1 - \frac{2}{3}\frac{P}{E} N} \right) = P_e$$

Therefore

$$\frac{2}{3}\left(\frac{1}{K_p} - \frac{N}{E} \right) P^2 + \left(1 + \frac{2}{3}\frac{N}{E} P_e \right) P - P_e = 0$$

or

$$P^2 + \frac{\frac{3}{2} + P_e \frac{N}{E}}{\frac{1}{K_p} - \frac{N}{E}} P - \frac{\frac{3}{2} P_e}{\frac{1}{K_p} - \frac{N}{E}} = 0$$

Solving for P ,

$$P_{cr} = \frac{-T - \sqrt{T^2 + R}}{2} \qquad (3\text{-}24)$$

in which

$$T = \frac{\frac{3}{2} + \frac{N}{E} P_e}{\frac{1}{K_p} - \frac{N}{E}} \qquad (3\text{-}25)$$

and

$$R = \frac{b\, P_e}{\frac{1}{K_p} - \frac{N}{E}} \qquad (3\text{-}26)$$

Eq. (3-24) may be regarded as a refinement of Eq. (3-17). Whereas the latter considers the change in slope associated with shear type deformation to be $\frac{2\Delta}{a}$ (Fig. 3-11) for purposes of computing the shearing force, the full value θ_2 is used in the derivation of Eq. (3-24). The buckling loads calculated from Eq. (3-24) and (3-17) for different column dimensions are shown in Table 3-2.

Table 3-2. (See Fig. 3-9)

No.	a (in)	b (in)	l (ft-in)	leg (in. Dia.)	Batten (in. Dia.)	P_{cr} (K) Eq (3-17)	*Eq. (3-24)
1	4	6	4 - $2\frac{1}{2}$	3/8	$\frac{1}{4}$	3.805	3.816
2	6	6	4 - $2\frac{1}{2}$	3/8	3/8	9.12	7.96
3	5	6	5 - $2\frac{1}{2}$	3/8	$\frac{1}{4}$	2.99	2.98
4	6	10	8 - $2\frac{1}{2}$	$\frac{1}{2}$	3/8	7.80	7.36

3.4.3. Column with Triangulated Web System.

For the case of a column with diagonal and horizontal bracing arranged symmetrically about the longitudinal axis, the stiffness K_p can be evaluated by the same analysis as before.

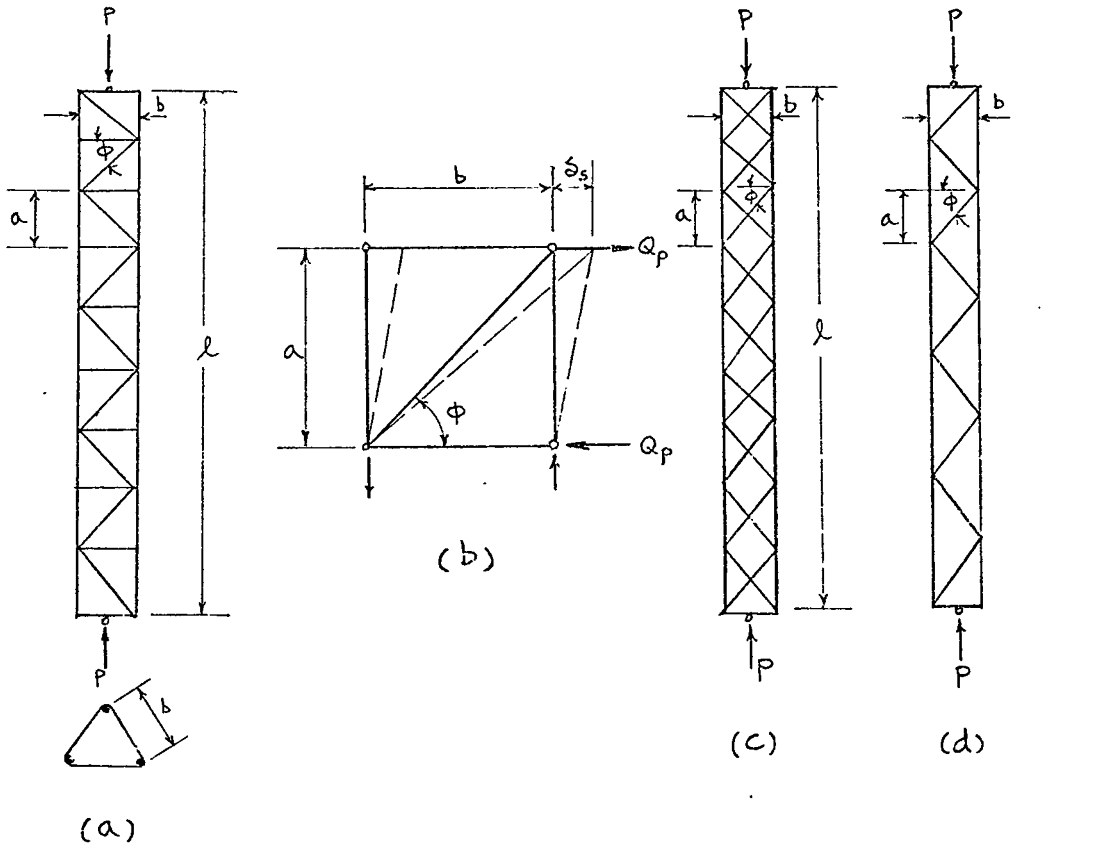

Fig. 3-13

For the case of the column in Fig. 3-13a,

$$\frac{1}{K_p} = \frac{\gamma_p}{Q_p} = \frac{\delta_s}{a} = \frac{1}{E}\left(\frac{1}{\sin\phi \cdot \cos^2\phi \, A_d} + \frac{1}{\tan\phi \cdot A_b}\right) \quad (3\text{-}27)$$

In this equation A_d is the cross sectional area of the diagonal bracing, and A_b is the cross sectional area of the horizontal bracing. It is again assumed that the column legs are circular or tubular and that I_c is invariant.

Inserting Eq. (3-27) in Eq. (3-1a), the critical load for the triangulated lacing column (Fig. 3-13a) is determined to be

$$P_{cr} = \frac{\dfrac{\pi^2 EI}{\ell^2}}{1 + \dfrac{2}{3}\dfrac{\pi^2 I}{\ell^2}\left(\dfrac{1}{A_d \cdot \sin\phi \cdot \cos^2\phi} + \dfrac{1}{A_b \cdot \tan\phi}\right)} \qquad (3\text{-}28)$$

and a reduced length for the equivalent column is

$$L = \ell\sqrt{1 + \frac{2}{3}\frac{\pi^2 I}{\ell^2}\left(\frac{1}{A_d \cdot \sin\phi \cdot \cos^2\phi} + \frac{1}{A_b \cdot \tan\phi}\right)} \qquad (3\text{-}29)$$

In the above equations, A_d is the cross-sectional area of the diagonal web, and A_b is that for the horizontal bracing.

The expressions for the buckling load of triangulated web systems of the configurations shown in Figs. 3-13c and d are also shown in Art. 3.2. The buckling load for the web system of two diagonals (Fig. 3-13c) for an equilateral triangular column is therefore

$$P_{cr} = \frac{\dfrac{\pi^2 EI}{\ell^2}}{1 + \dfrac{2}{3}\dfrac{\pi^2 I}{\ell^2}\left(\dfrac{1}{\sin\phi \cdot \cos^2\phi \cdot A_d}\right)} \qquad (3\text{-}30)$$

and a reduced length for the equivalent column is

$$L = \ell\sqrt{1 + \frac{2}{3}\frac{\pi^2 I}{\ell^2}\left(\frac{1}{\sin\phi \cdot \cos^2\phi \cdot A_d}\right)} \qquad (3\text{-}31)$$

In the above equations, A_d denotes the cross-sectional area of the two diagonals in the web panel.

Eq. (3-30) can be used also in the case of a web system composed of a single diagonal (Fig. 3-13d) provided A_d is the area of one diagonal and ϕ is measured as shown.

3.5. Column with Perforated Cover Plates

Columns with perforated cover plates[1] instead of lacing bars or batten plates are often used in practice. The effect of shear deformation on the buckling strength of these columns will be illustrated in this article.

The cross section of a typical column with perforated cover plates is shown in Fig. 3-14a. In the calculation of the cross-sectional area and moment of inertia of the column, the properties of the net area (section n n) can be used with sufficient accuracy for most practical purposes. In determining the lateral displacement due to unit shearing force, consider as before a typical element from the column (Fig. 3-14b). This element is similar to the element from the column with batten plates (Fig. 3-6b), except that instead of a narrow batten plate the portion of the cover plate between perforations is used. Thus, finally the idealized element of Fig. 3-14c is obtained where the horizontal cross member can be considered as infinitely rigid. The lengths of the vertical projections, which are treated as cantilever beams, will be somewhere between $c/2$ and $a/2$, where c is the length of a perforation. The value $3\,c/4$ is reasonable and gives results which agree with experiments[2].

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[1]M. W. White and B. Thurlimann, "Study of Columns with Perforated Cover Plates", AREA Bull. 531, 1956.

[2]Ibid.

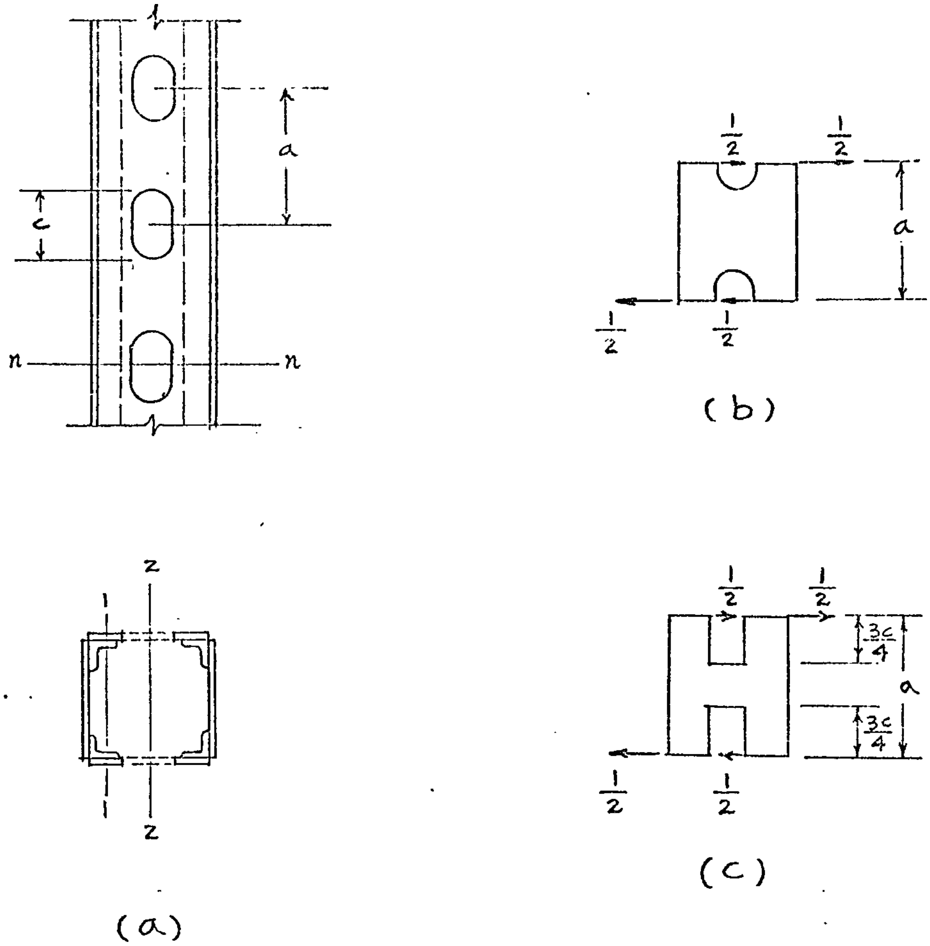

Fig. 3-14

The equations for a column with batten plates can now be modified for this case. Assume the cross member (analogous to a batten) is infinitely rigid, $I_b = \infty$ may be substituted in Eq. (e) and $\delta_{s1} = 0$ is obtained. The displacement δ_{s2} is determined as the deflection of a cantilever (Eq. (f)) , and δ_{s2} is obtained as

$$\delta_{s2} = \frac{1}{2}\left(\frac{3c}{4}\right)^3 \frac{1}{3EI_f} = \frac{9\,c^3}{128\,EI_f} \qquad \text{(P)}$$

In this expression I_f represents the moment of inertia of the "flange" of the column, that is, the entire effective area of the column on one side of the z axis, taken about the centroid of the flange (axis 1-1). The angular displacement due to unit shearing force is

$$\gamma = \frac{1}{K} = \frac{\delta_{s1} + \delta_{s2}}{\frac{a}{2}} = \frac{9c^3}{64\, a E I_f}$$

from which is obtained

$$P_{cr} = \frac{\pi^2 E I}{\ell^2} \frac{1}{1 + \frac{\pi^2 E I}{\ell^2}\left(\frac{9c^3}{64\, a E I_f}\right)} \qquad (3\text{-}32)$$

as the critical load for a column with perforated cover plates.

The tests made on such columns have not indicated any weakness due to shear deformation[1]. Therefore, for these columns the shear effect may be neglected for practical purposes. The design of such columns hinges on the effective area of the perforated plates.

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[1]Stang, A. H., and Greenspan, M., "Perforated Cover Plates for Steel Columns, National Bureau of Standards (U.S.), Journal of Research, Vol. 28, 1942, Research Papers RP 1473 and RP 1474, and Vol. 40, 1948, Research Papers RP 1861 and RP 1880.

4. BUILT-UP COLUMN TESTS

In order to obtain some experimental confirmation on the buckling strength of built-up columns, several column tests were performed for columns with diagonal bracing as well as for columns with batten systems. Three test columns were designed and tested as pinned-end columns using the universal testing machine in the Civil Engineering structures laboratory. The tests confirmed the predictions of column buckling strengths made on the theoretical basis presented in this paper.

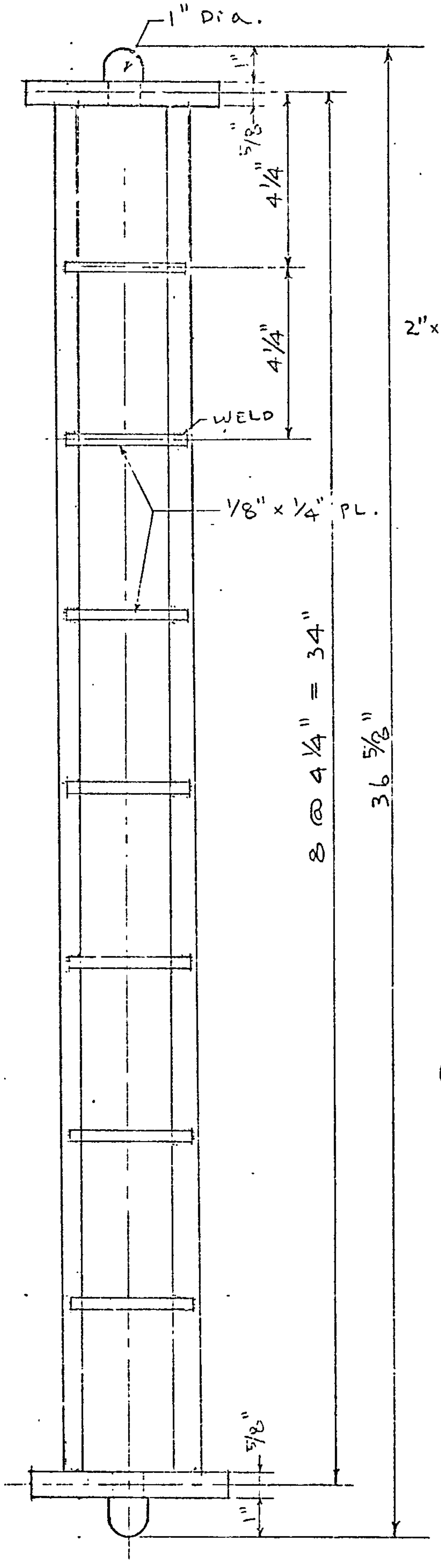
4.1. Test Specimens

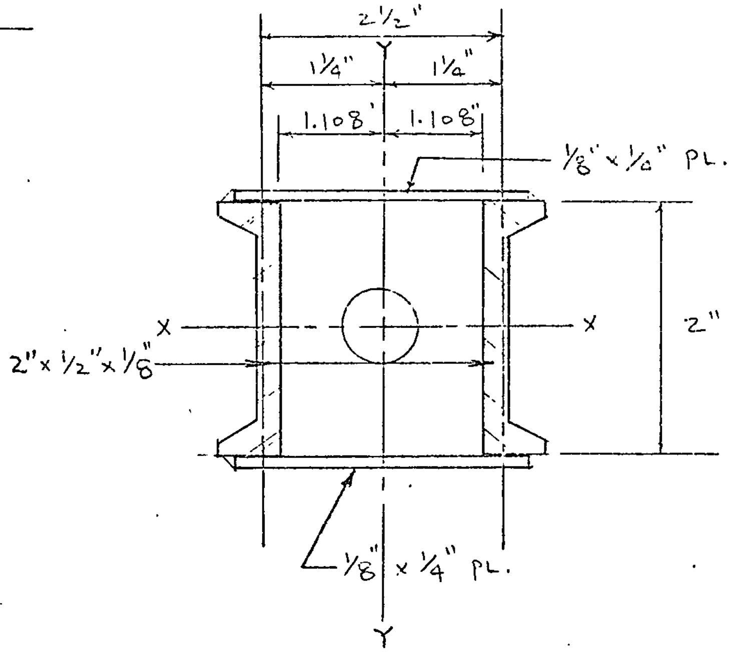
The dimensions of the column specimens were designed to fit within the existing testing machine in the structures laboratory.

Test Column No. 1 was fabricated of steel channel of 2 x 2 x 1/8 inches with 1/8 x $\frac{1}{4}$ inch bars for the battens. The 5/8 inch rectangular end plates used at the top and bottom had holes bored in them to accommodate hemispherical bearing heads of hardened steel. Details of fabrication are shown in Fig. 4-1 and 4-2.

Test Column No. 2 was fabricated of steel with 3/8 inch diameter bars for the triangle legs and $\frac{1}{4}$ inch bars for the battens. Again the 5/8 inch thick triangular plates used at the top and bottom had holes bored in them to accommodate the hemispherical bearing heads. Details of fabrication are shown in Fig. 4-3 and 4-4.

Test Column No. 3 was fabricated of steel with 3/8 inch



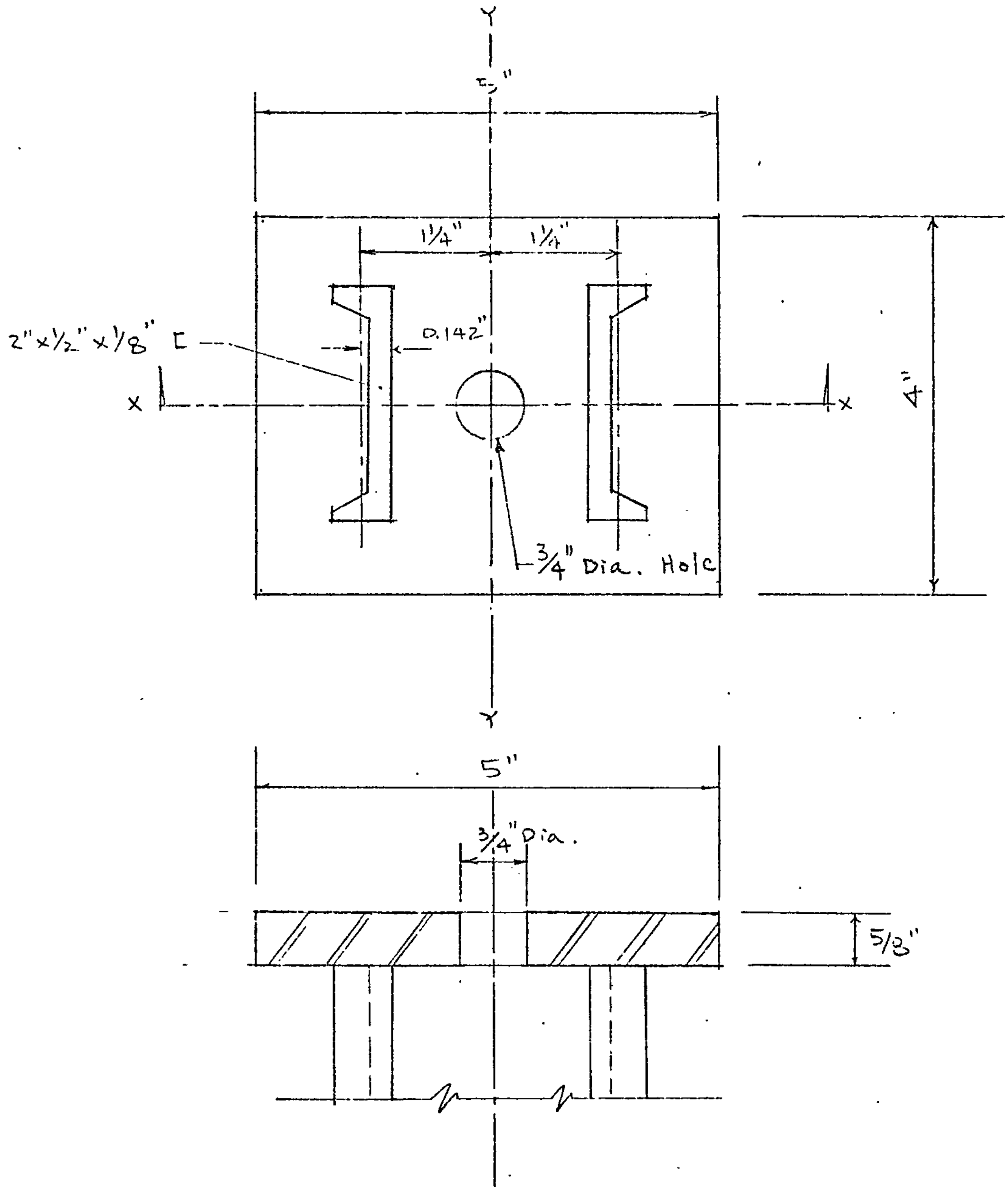


(b) Cross Section

(a) Elevation

Fig. 4-1

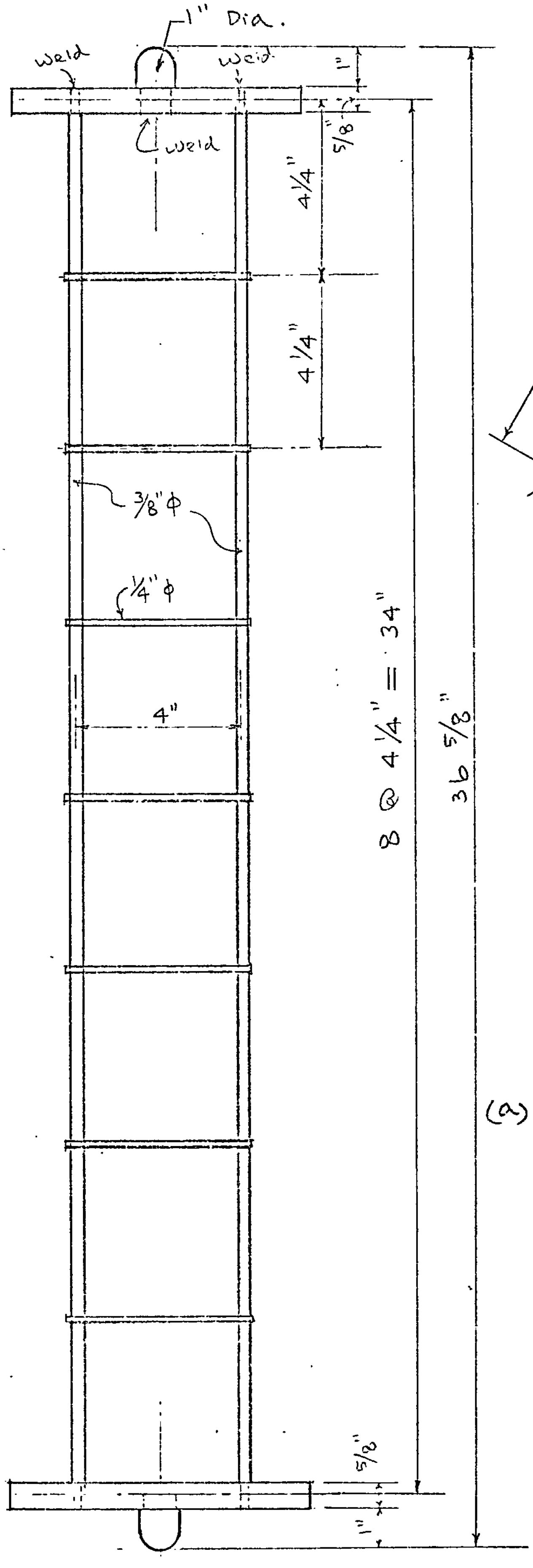
Test Column No. 1

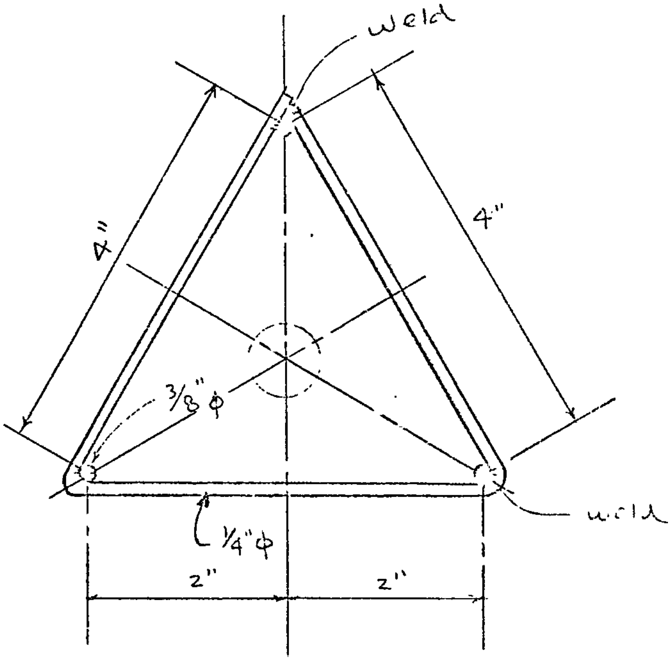


Column End Plate

Fig. 4-2

Test Column No. 1



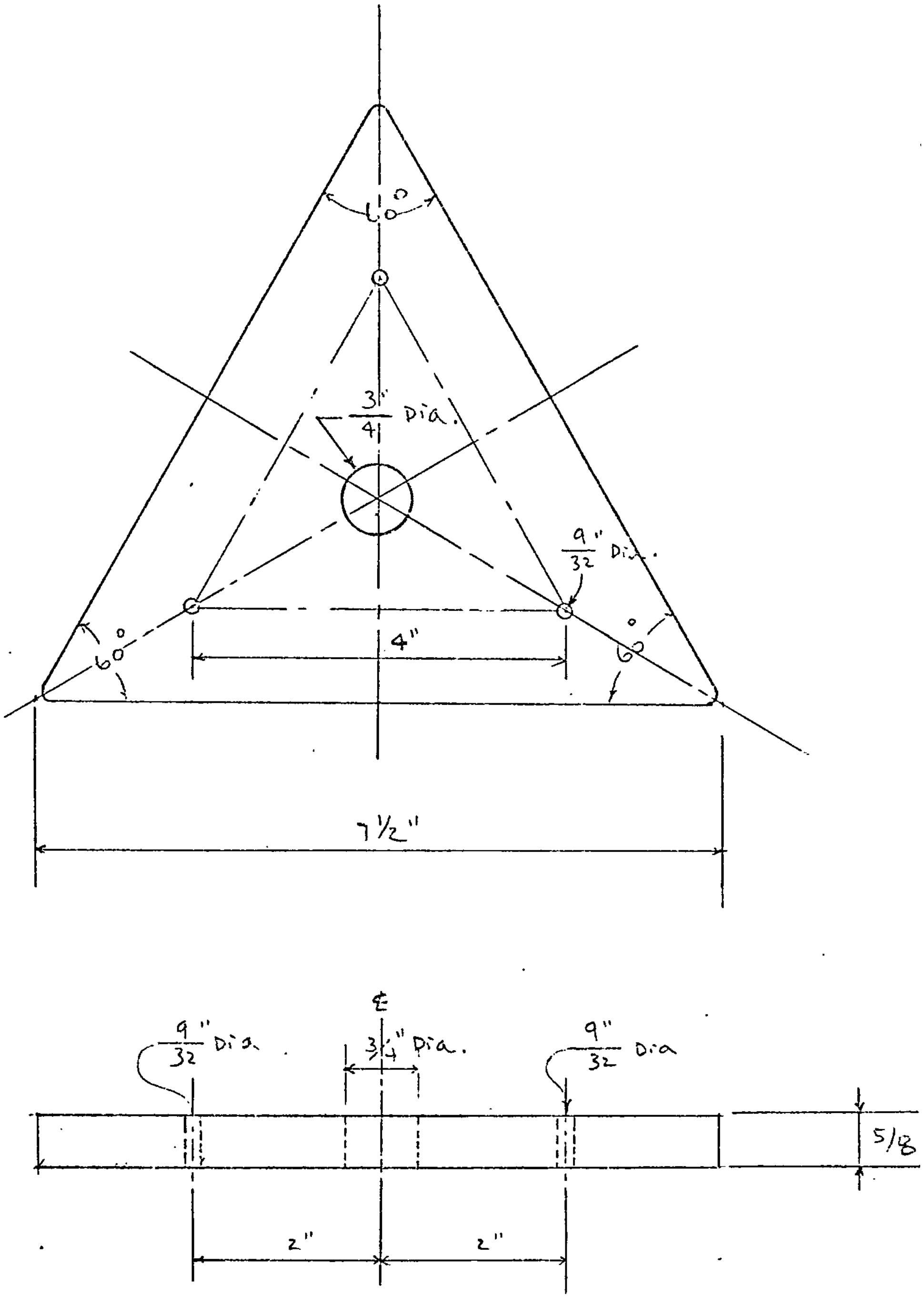


(b) Cross Section

(a) Elevation

Fig. 4-3

Test Column No. 2



Column End Plate

Fig. 4-4

Test Column No. 2

diameter bars for the trianglelegs and $\frac{1}{4}$ inch diameter bars for the diagonal bracing. The same diagonal pattern was used for each of the three braced panels of the column. Details of fabrication are shown in Fig. 4-5 and 4-6.

At each end of the column specimens $\frac{1}{2}$ inch radius hemispherical bearing heads and brackets of hardened steel were designed to serve as pinned-end supports. These bearing pieces were heat-treated for one hour at 1,330° F temperature and were then oil quenched. The details of construction are shown in Figs. 4-7 and 4-8.

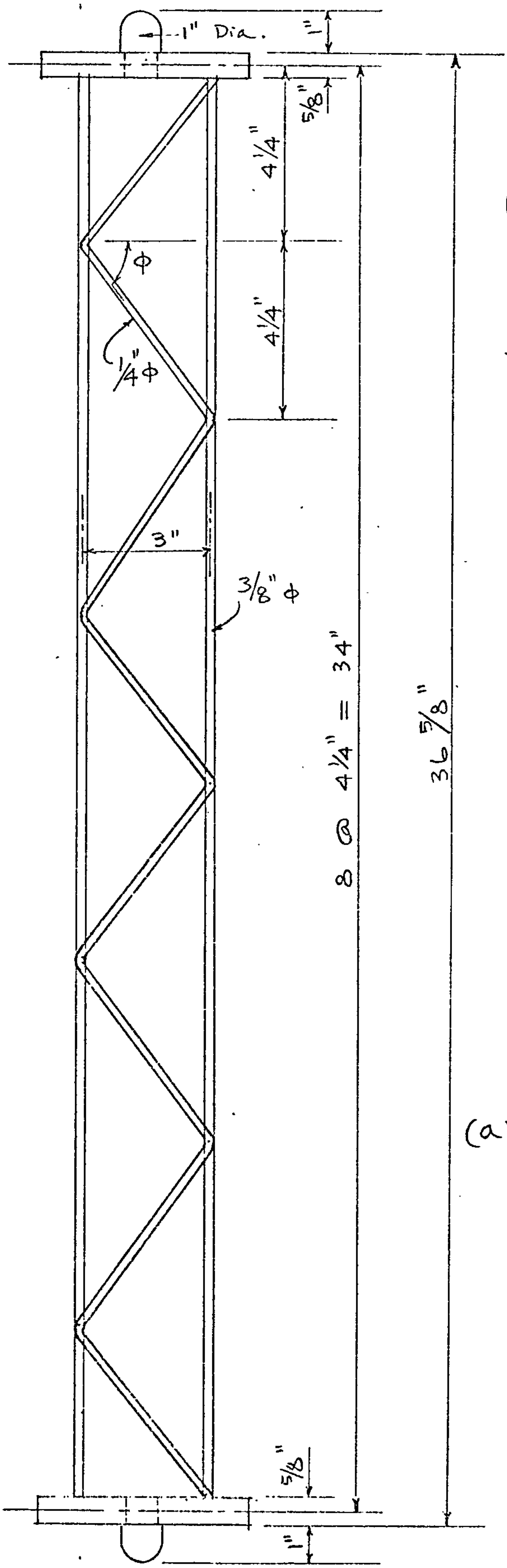
The test column properties are as follows:

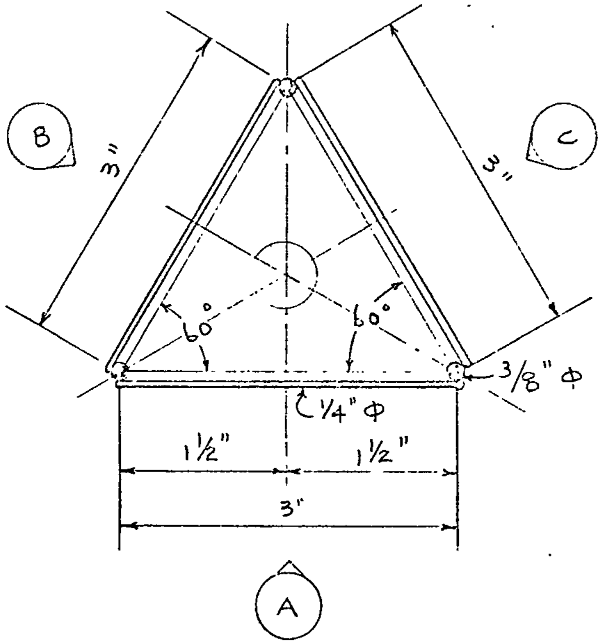
<u>Test Column No. 1</u>

$a = 4.25$ in.	$b = 2.5$ in.
$A_c = 0.379$ in.^{2}	$I_c = 0.0064$ in.^{4}
$A_b = 0.0624$ in.^{2}	$I_b = 0.000326$ in.^{4}
$l = 36.625$ in.	$E = 29.6 \times 10^3$ Ksi
$N = 1.2$	$G = 12 \times 10^3$ Ksi
$P_e = 260.73$ Kips	$\frac{1}{K} = 0.04403$ Kip^{-1}
$L = 189.14$ in.	

<u>Test Column No. 2</u>

$A_c = 0.1105$ in.^{2}	$a = 4.25$ in.
$b = 4.0$ in.	$l = 36.625$ in.
$I = 0.887$ in.^{4}	$I_b = 0.000192$ in.^{4}
$I_c = 0.000971$ in.^{4}	$E = 29.6 \times 10^3$ Ksi.
$P_e = 193.18$ Kips	$N = 8541.10$ in.$^{-2}$
$\frac{1}{K_P} = 0.27546$ Kips^{-1}	$T = -4372.10$ Kips
$R = -88,529.00$ Kips^2	



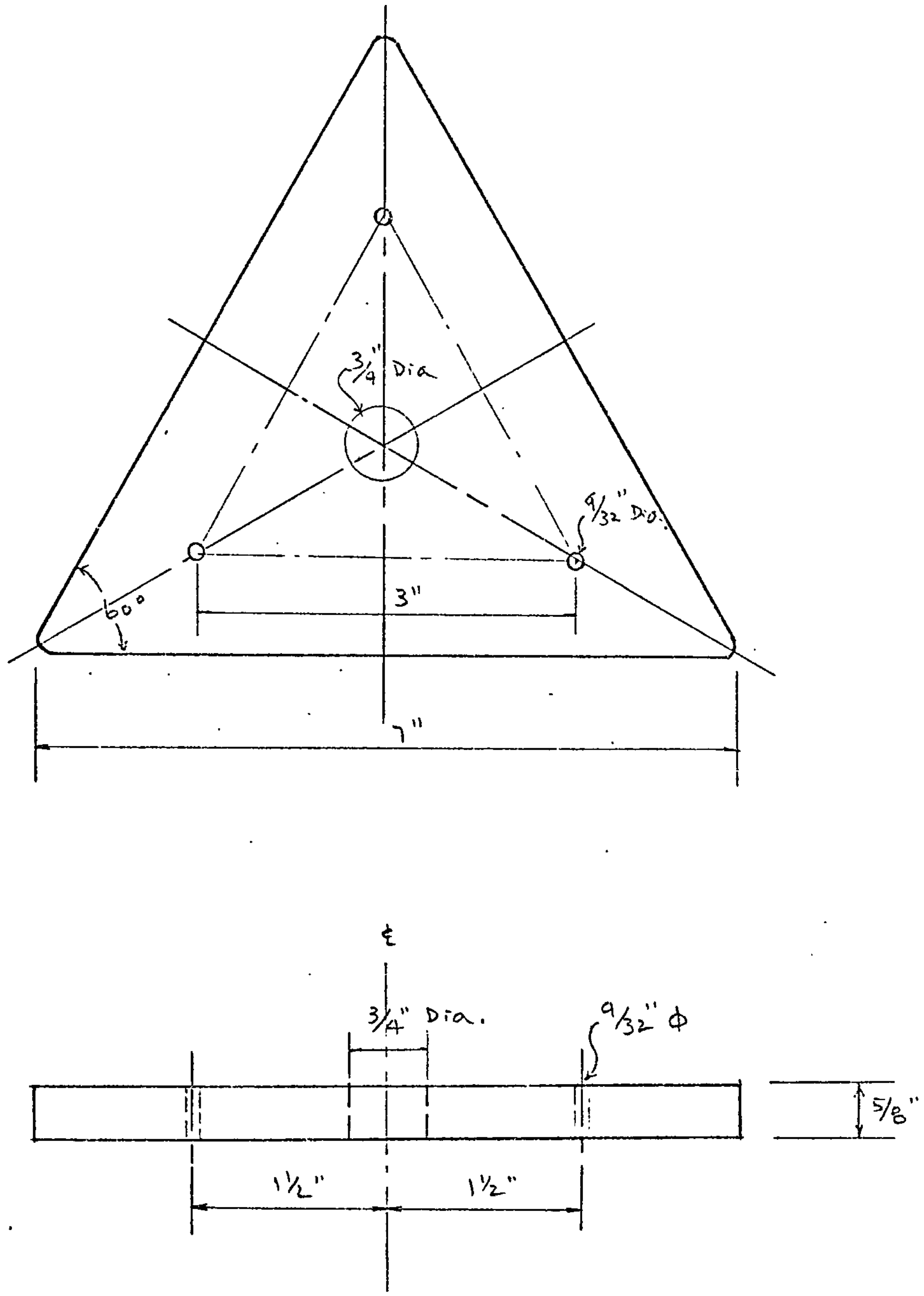


(b) Cross Section

(a) Elevation (A) (B) & (C)

Fig. 4 – 5

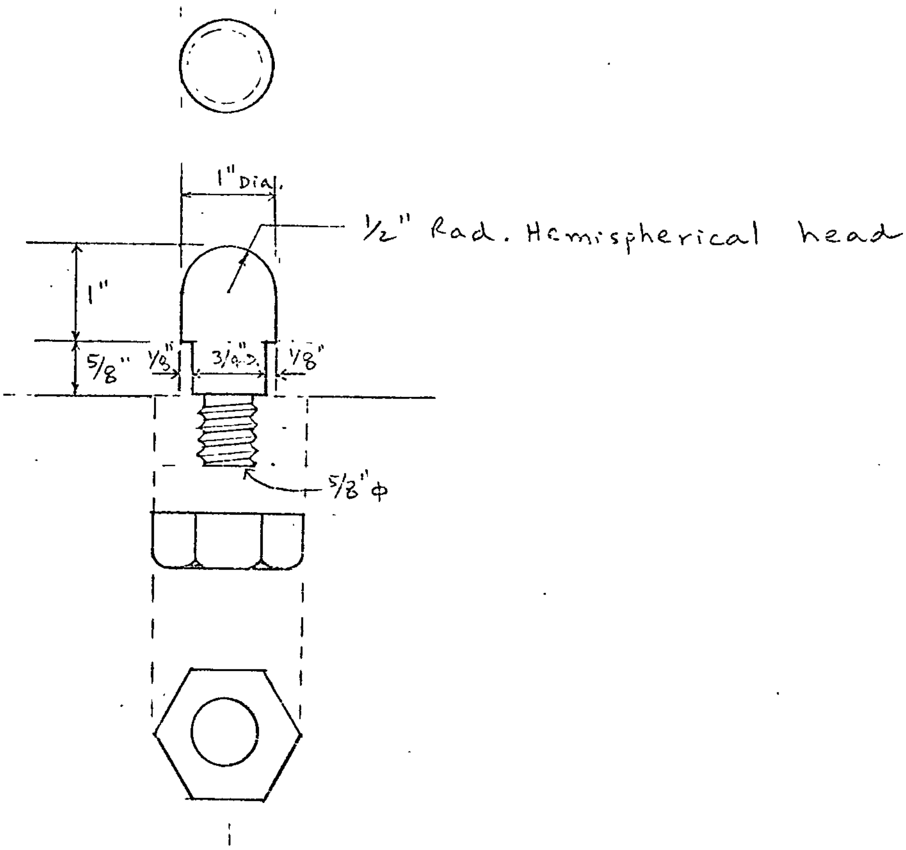
Test Column No. 3



Column End Plate

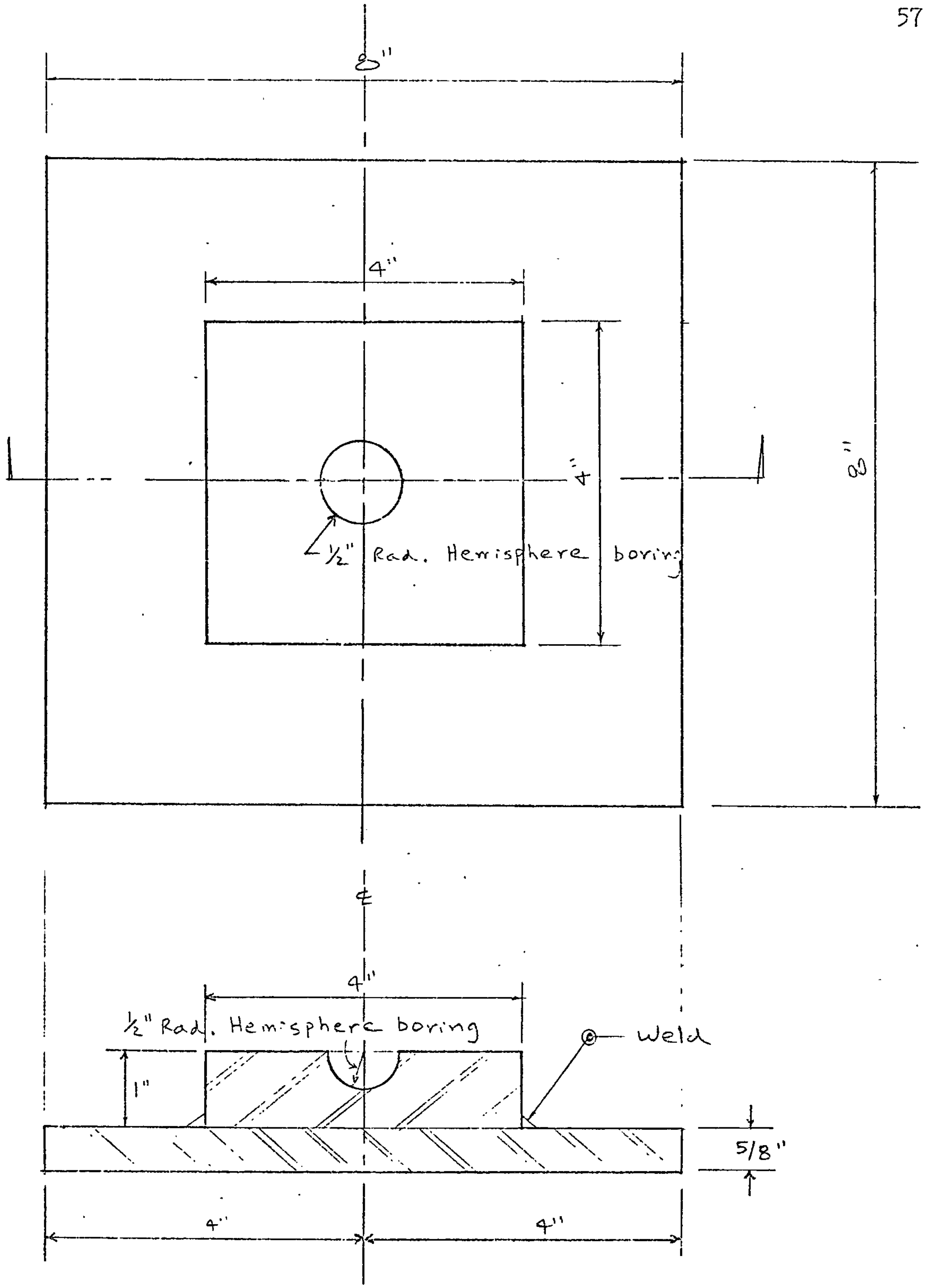
Fig. 4-6

Test Column No. 3



Hemispherical Bearing Head for Test Columns

Fig. 4 - 7



Hemispherical Bearing Brackets for Test Columns

Fig. 4-8

Test Column No. 3

A_c = 0.1105 in.^{2}

b = 3.0 in.

I = 0.501 in.^{4}

I_c = 0.000971 in.^{4}

ϕ = 46.667°

P_e = 109.11 Kips

L = 39.12 in.

a = 4.25 in.

l = 36.625 in.

A_d = 0.0491 in.^{2}

E = 29.6 X 10^3 Ksi

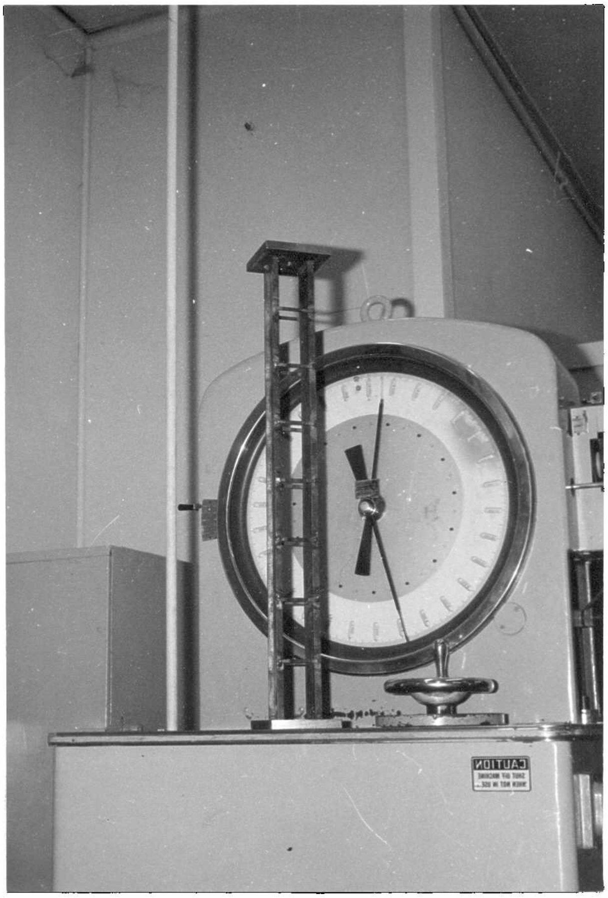
$\frac{1}{K_p}$ = 0.0020037 Kips^{-1}

The predicted loads were as follows:

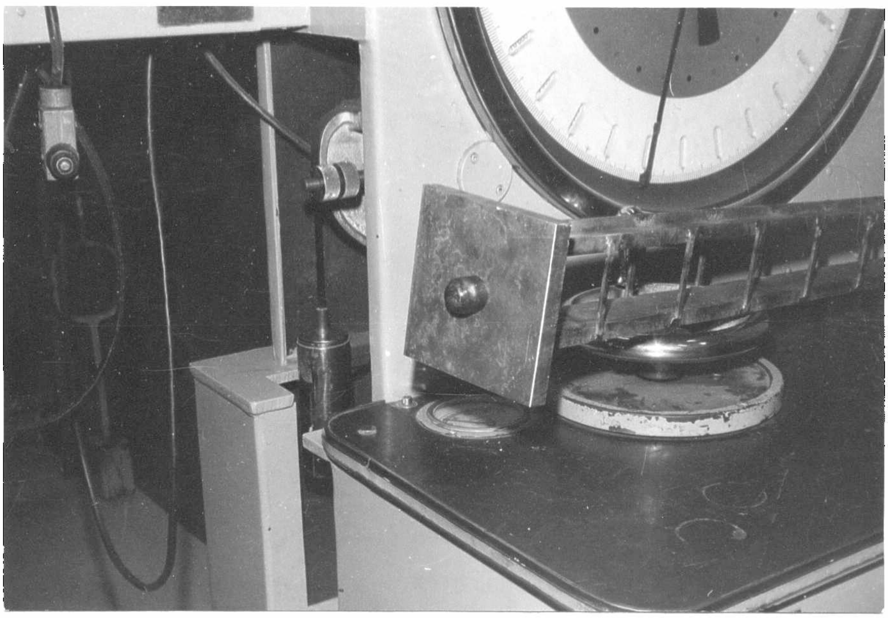
Description / Test Column	Predicted load in Kips				$\frac{L}{r}$
	Two-legged Column	Three-legged Column			
	Eq. (3-7)	Eq. (3-17)	Eq. (3-24)	Eq. (3-30)	
No. 1	9.78	-	-	-	151
No. 2	-	4.89	5.07	-	138
No. 3	-	-	-	95.2	105

4.2. Test Set-up

General views of the test set-up and the test arrangements are shown in Figs. 4-9 through 4-17. The three specimens were tested in the 200,000 pound mechanical beam and weight testing machine. A transit was used to aid the alignment of the column in a true vertical position so that the load would be applied exactly along the central

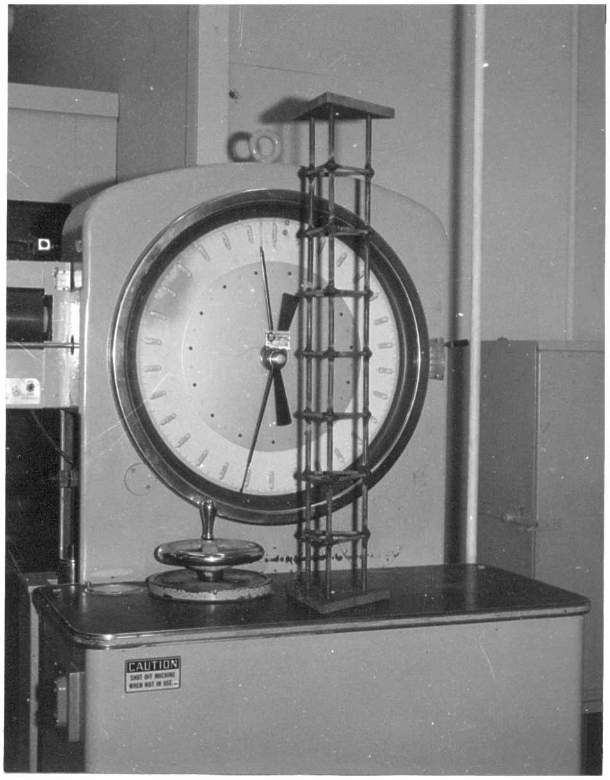


(a) Column No. 1

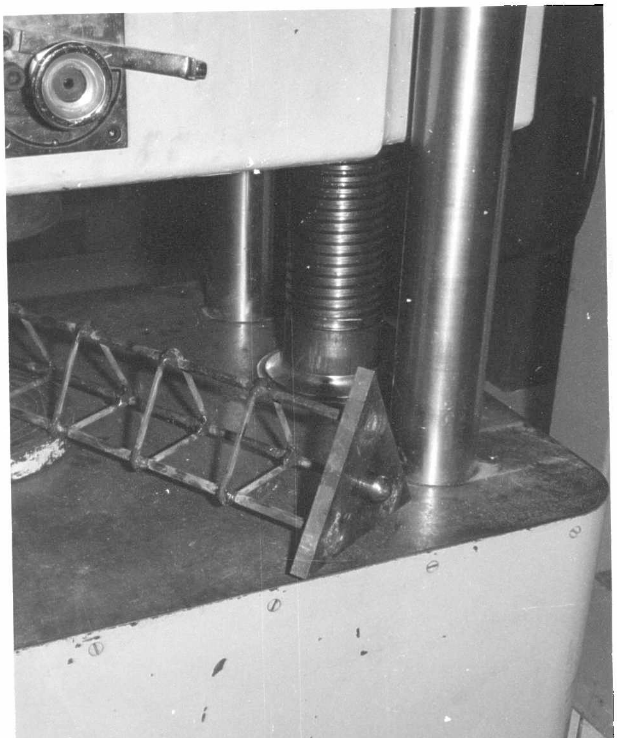


(b) Column Head

Fig. 4-9

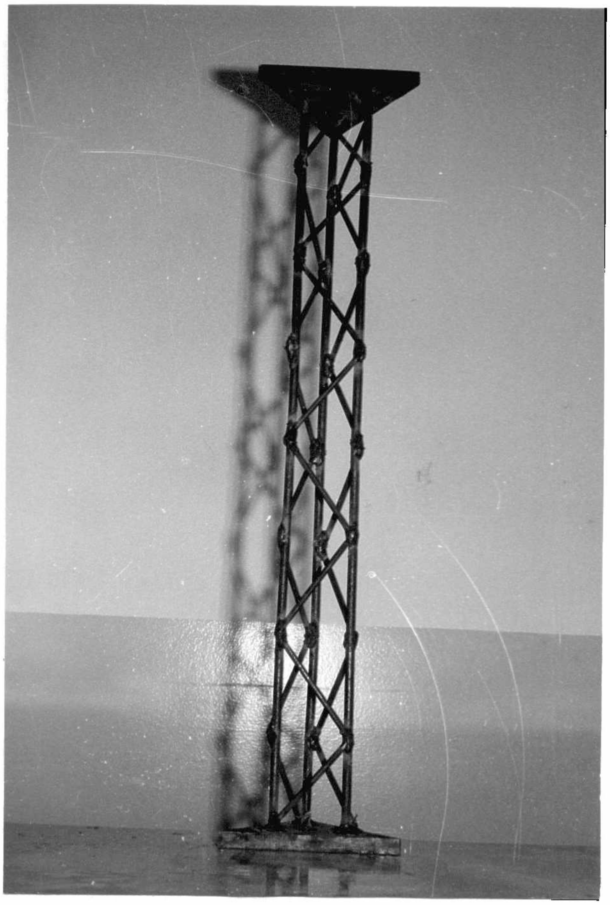


(a) Column No. 2

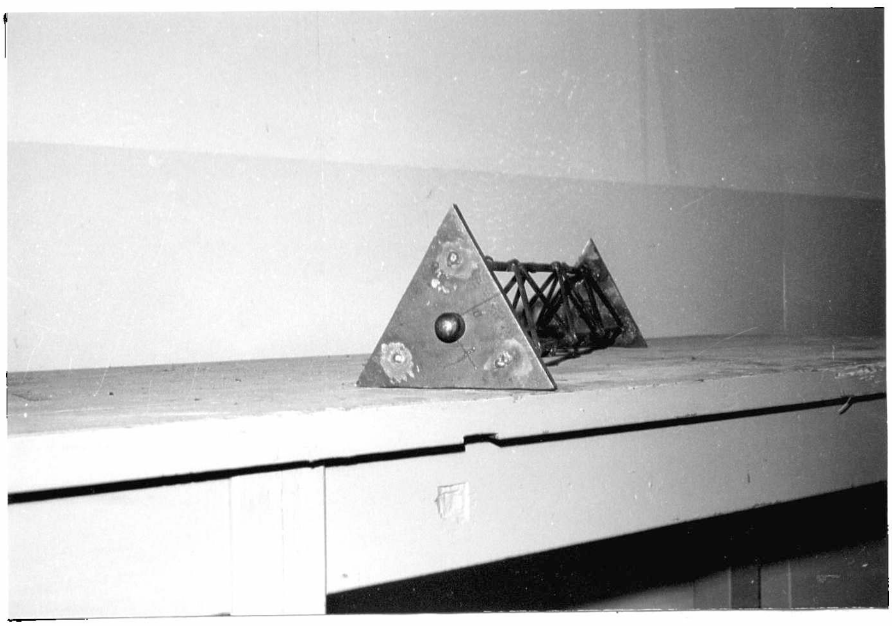


(b) Column Head

Fig. 4-10



(a) Column No. 3



(b) Column Head

Fig. 4-11

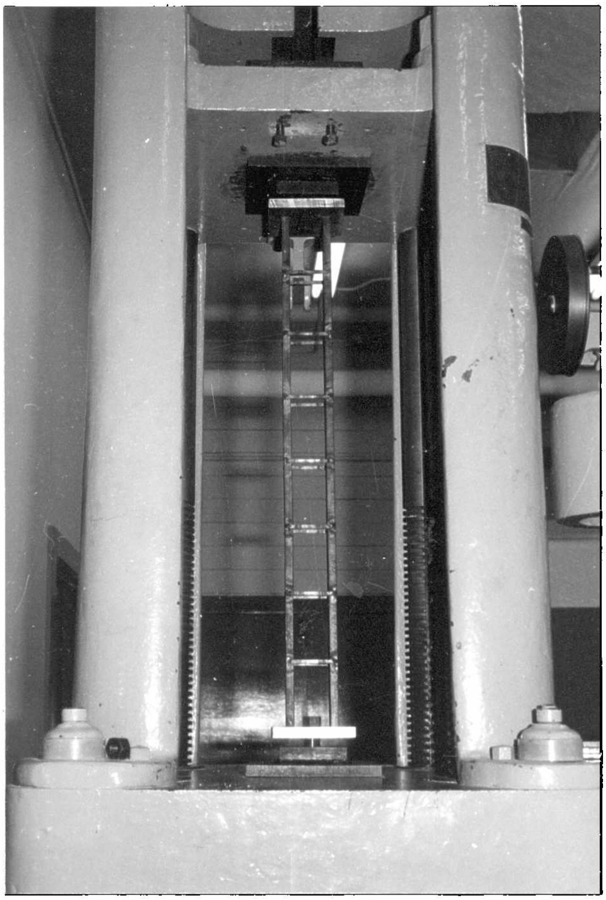


Fig. 4-12

Subcritical Load

(Col. No. 1)

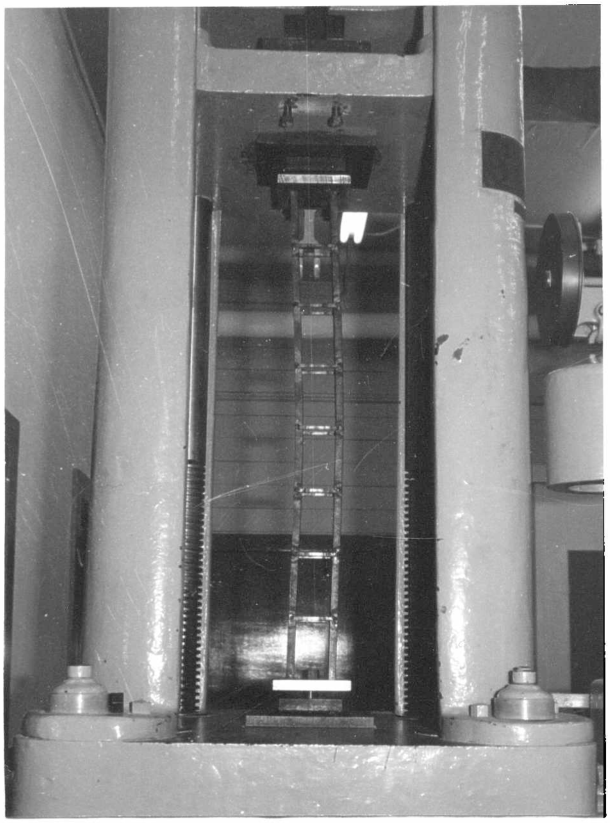


Fig. 4-13

Column at Buckling Load

(Col. No. 1)

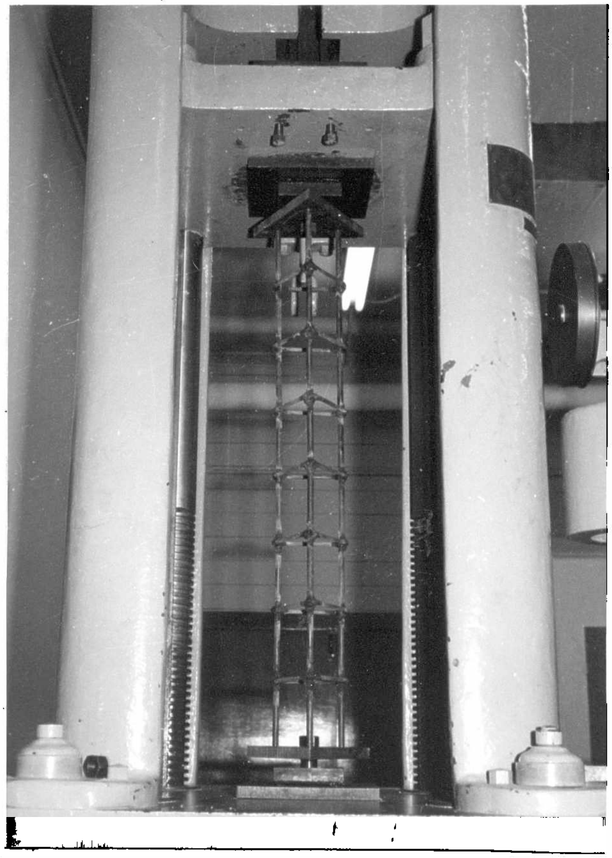


Fig. 4-14

Subcritical Load

(Col. No. 2)

Fig. 4-15

Column at Buckling Load

(Col. No. 2)

Fig. 4-16

Subcritical Load

(Col. No. 3)

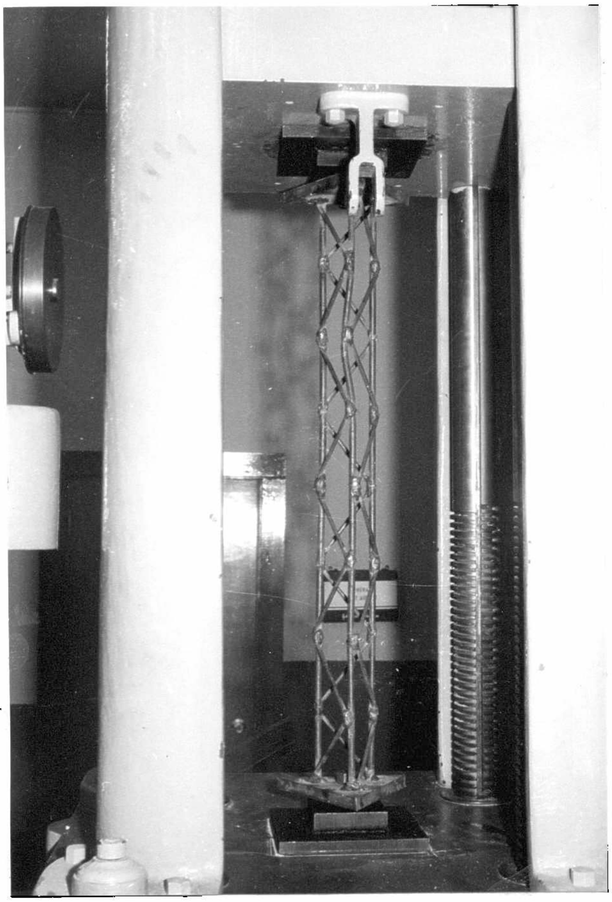


Fig. 4-17

Column Failed Locally

(Col. No. 3)

axis at mid-height and as close as possible to the central axis at the ends. The load was applied in small increments and the lateral deflection at mid-height was observed using the transit. The critical load was read directly from the test machine.

4.3. Test Results

The column buckling loads, as determined from the above tests, were found to be 9.50 Kips for Column No. 1 and 4.80 Kips for Column No. 2. Column No. 3 failed locally at 14.8 Kips.

Figures 4-12, 4-14 and 4-16 show the columns in the testing machine with a small initial compressive load. In Figures 4-13 and 4-15 the compressive loads have practically reached the maximum observed values, and transverse deflections can be seen. After the ultimate load was reached the columns continued to deflect with no change in the load. In Figure 4-17 the local buckling may be seen in one of the triangular legs.

Figures 4-18 and 4-19 show some of the other test columns that failed locally during testing; such failures were believed to be due to the small size of components and inadequate workmanship in fabrication.

4.4. Discussion and Remarks

The test result was 2.5 per cent below the predicted value for Column No. 1. For Column No. 2 the test value was 5 per cent below the predicted value based on Eq. (3-24) and 1.75 per cent below the predicted value based on Eq. (3-17). The small discrepancy

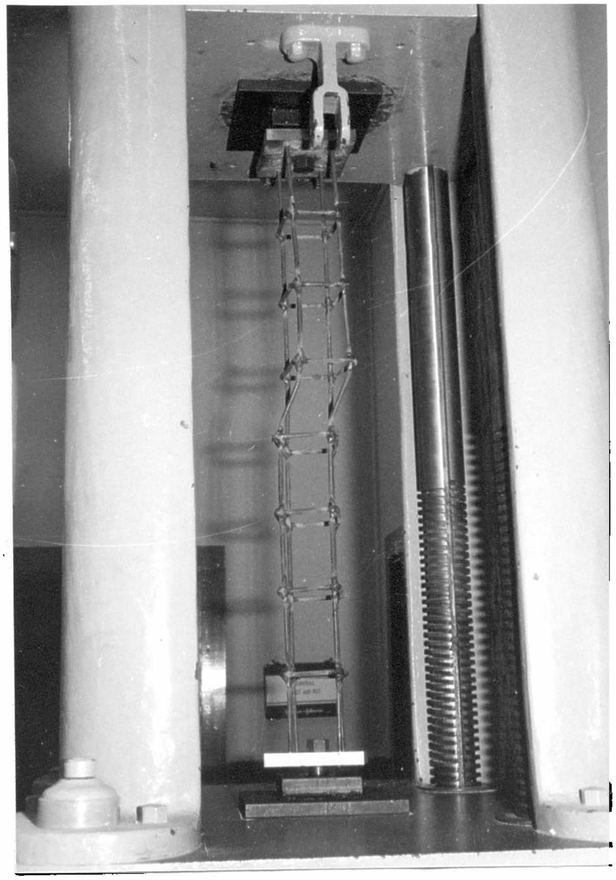


Fig. 4-18

Typical Local Failure

of a Batten Column

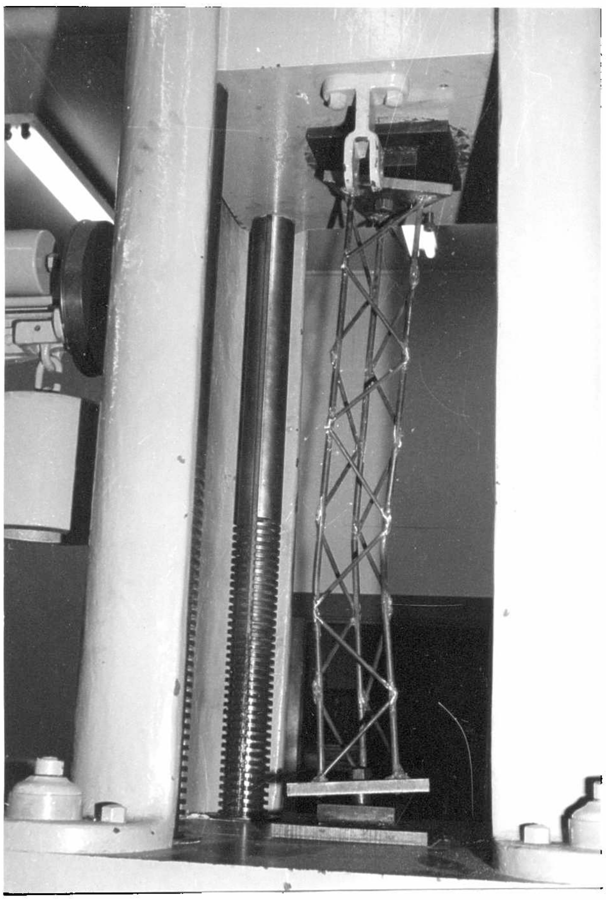


Fig. 4-19

Typical Local Failure

of a Triangular Laced Column

between the theoretical values and the experimental results are considered to be due to the slight imperfections in alignment, initial out-of-straightness, and unsymmetrical residual stress distribution in the test columns.

The test results of the above two cases show reasonable agreement with the theoretical calculations. Therefore, it is concluded that the theoretical formulas derived from the analysis based on the approximate method can be used to predict the buckling load for built-up columns of batten type construction.

For the three-legged column with batten bracing the test results indicate either Eq. (3-17) or Eq. (3-24) may be used to predict the buckling load. However, this may not always be the case. The effect can be seen by comparing the loads predicted from the two equations for a column of different dimensions. Assume Column No. 2 in Table 3-2 for example, the predicted buckling loads are: according to Eq. (3-17), $P_{cr} = 9.12$ Kips, and according to Eq. (3-24), $P_{cr} = 7.96$ Kips. It is the writer's considered opinion that inasmuch as Eq. (3-24) has a more rational basis, it is the more reliable. Further tests are needed in which the parameters are systematically varied before one can establish the relative validity of these two equations.

For Test Column No. 3, the three-legged column with diagonal bracing, the experimental result shows that the column failed locally in one of the column legs at 14.8 Kips. The deficiency was considered to be due to the imperfection in workmanship, initial crookedness of the column legs and possibly due to the inaccurate sizing of the leg members between two consecutive bracings of the web panel. In order to prevent failure of the leg members prior to primary failure of the column, the column length would have to be increased so that a truly long column, by L/r terminology, was used. However, the present testing machine limits the useful column height to that employed in the present tests.

In Column No. 2, if the $\frac{1}{4}$ inch diameter bars used for the battens were replaced with bars of rectangular section of equal cross sectional area (1/8"X 7/16") and the other dimensions remained unchanged, then according to Eq. (3-24) $P_{cr} = 15$ Kips would be obtained. In this case the buckling strength of the column is increased from 5 Kips to 15 Kips due to the change of cross sectional shape of the battens, although the cross sectional area remains the same. The web shear rigidity of the column is thus greatly increased. If the web system of Column No. 2 was changed from non-triangulated web (battens only) to a triangulated web system (lacing pattern) as in the case of Column No. 3, and the other dimensions remained unchanged, then it would be found from the predicted buckling loads of these two columns that the web rigidity would be greatly increased in this case too.

Robert A. Williamson[1] introduces the effective thickness (t_e) as a measure of the potential effect of web rigidity in built-up columns. He expresses the critical buckling load in terms of the

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[1] See discussion by Williamson, Robert A., A.S.C.E. Vol. 91, No. ST 5, October 1965.

effective thickness. For three-legged columns with triangulated web systems (systems with both diagonals and battens) he expresses the critical buckling load by an equation of the following form:

$$P_{cr} = \frac{P_e}{1 + \dfrac{2 P_e}{3 G b t_e}} \tag{4-1}$$

For the case of triangulated systems, by replacing each triangulated panel with a fictitious solid panel of equivalent shear rigidity, the effective thickness, t_e , is given by

$$t_e = \frac{a b}{\dfrac{G}{E} \sum \dfrac{l_w^3}{A_w}} \tag{4-2}$$

In this expression l_w = the length of each acting web member in the panel, and A_w = its cross-sectional area. The summation extends over the length of the panel a . By evaluating t_e from Eq. (4-2) and inserting this value in Eq. (4-1), equations identical with Eqs. (3-28) and (3-30) for evaluation of buckling loads are derived. A similar approach is applicable to nontriangulated web systems (batten only). For the case of Eq. (3-17) in which the buckling shear is regarded as being dependent on the rotation of the line adjoining the inflection points in adjacent panels, the corresponding value of t_e is given by

$$t_e = \frac{24 E I_c}{G a^2 b} \left[\frac{1}{1 + \left(\frac{2b}{a}\right)\left(\frac{I_c}{I_b}\right)} \right] \tag{4-3}$$

obtained by equating the shear resistance of the solid panel to that of the nontriangulated panel which it replaces. Inserting Eq. (4-3) in Eq. (4-1) a result which coincides with Eq. (3-17) is obtained. Eq. (3-24) considers the buckling shear to be a function of the rotation of the leg at the inflection point. Based on this assumption, equating the shear distortion yields

$$t_e = \frac{24 E I_c}{G a^2 b} \left[\frac{1}{\frac{3}{2} + \left(\frac{2b}{a}\right)\left(\frac{I_c}{I_b}\right)} \right] \qquad (4\text{-}4)$$

Substitution in Eq. (4-1) gives an expression which is essentially equivalent to Eq. (3-24).

The concept of effective thickness provides a convenient measure of the potential effect of web flexibility. It is of interest to note that t_e, in the case of tall guyed towers with a conventional triangulated web system, is usually more than 0.001 in., whereas t_e for the author's test column No. 2, with its nontriangulated web system, is several orders of magnitude less -- approximately 0.000038 in. The low critical buckling strength of Test Column No. 2 in this investigation demonstrates the importance of the influence of low web rigidity in degrading the buckling strength.

For a critical load beyond the proportional limit of the material the equations derived in this paper can be used also by introducing E_t instead of E. Take the case shown in Fig. 3-6. Under the action of an axial load the chords of the column will be uniformly compressed while the battens are unstressed. Hence in calculating the critical load which is beyond the proportional limit, Eq. (3-5) can be used. The substitution of E_t for E is necessary only in those terms relating to the chords while E and G are retained in those terms relating to the battens. Thus, if compression of the column exceeds the proportional limit, the battens become relatively more rigid and the properties of the built-up column approach those of a solid column.

```
C     BUCKLING LOAD FOR LACED COLUMN NO. 1 (FIG. 3-3 )
      REAL I, IC, IK, L
    1 READ(5,100) A, B, AC, IC, E, L, PHI, AD, AB
  100 FORMAT(8F10.5)
      I=2.*IC+AC*B**2/2.
      WRITE(6,101) A, B, AC, IC, E, L, PHI, AD, AB,I
  101 FORMAT(8X, 'A=',E12.5,5X,'B=',E12.5,5X,'AC=',E12.5,5X,'IC=',E12.5/
     18X,'E=',E12.5,5X,'L=',E12.5,5X,'PHI=',E12.5,5X,'AD=',E12.5,5X,'AB=
     2',E12.5,5X,'I=',E12.5//)
      X=1.74533E-2*PHI
      PE=3.1416**2*E*I/L**2
      IK=1./E*(1./(SIN(X)*COS(X)**2*AD)+COS(X)/(SIN(X)*AB))
      REDL=L*SQRT(1.+PE*IK)
      PCR=3.1416**2*E*I/REDL**2
      SLR=REDL/SQRT(I/(2.*AC))
      WRITE(6,102) PE, IK, REDL, SLR, PCR
  102 FORMAT(8X,'PE=',E12.5,5X,'1/K=',E12.5,5X,'REDL=',E12.5,5X,'L/R=',
     1E12.5/8X,'PCR=',E12.5//)
      GO TO 1
      END

        000020
```

```
C     BUCKLING LOAD FOR LACED COLUMN NO. 2 AND NO. 3 ( FIG. 3-4 )
      REAL I, IC, IK, L
    1 READ(5,100) A, B, AC, IC, E, L, PHI,AD
  100 FORMAT(8F10.5)
      WRITE(6,101) A, B, AC, IC, E, L, PHI, AD
  101 FORMAT(8X, 'A=',E12.5,5X,'B=',E12.5,5X,'AC=',E12.5,5X,'IC=',E12.5/
     18X,'E=',E12.5,5X,'L=',E12.5,5X,'PHI=',E12.5,5X,'AD=',E12.5//)
      I=2.*IC+AC*B**2/2.
      X=1.74533E-2*PHI
      PE=3.1416**2*E*I/L**2
      IK=1./E*(1./(SIN(X)*COS(X)**2*AD))
      REDL=L*SQRT(1.+PE*IK)
      PCR=3.1416**2*E*I/REDL**2
      SLR=REDL/SQRT(I/(2.*AC))
      WRITE(6,102) PE, IK, REDL, SLR, PCR
  102 FORMAT(8X,'PE=',E12.5,5X,'1/K=',E12.5,5X,'REDL=',E12.5,5X,'L/R=',
     1E12.5/8X,'PCR=',E12.5//)
      GO TO 1
      END

        000019
```

```
C     BUCKLING LOAD FOR BUILT-UP COLUMN WITH BATTEN BRACING
      REAL I, IC, IB, L, IK, N
    1 READ(5, 100) A, B, AC, IC, AB, IB, L, E, N, G
  100 FORMAT(8F10.5)
      I=2.*IC+AC*B**2/2.
      WRITE(6, 101) A, B, AC, I, IC, AB, IB, L, E, N, G
  101 FORMAT(8X,'A=',E12.5,5X,'B=',E12.5,5X,'AC=',E12.5,5X,'I=',E12.5,5X
     1,'IC=',E12.5/8X,'AB=',E12.5,5X,'IB=',E12.5,5X,'L=',E12.5,5X,'E=',E
     212.5,5X,'N=',E12.5,5X,'G=',E12.5//)
      PE=3.1416**2*E*I/L**2
      IK=A*B/(12.*E*IB)+A**2/(24.*E*IC)+N*A/(B*AB*G)
      REDL=L*SQRT(1.+PE*IK)
      PCR=3.1416**2*E*I/REDL**2
      SLR=REDL/SQRT(I/(2.*AC))
      WRITE(6,102) PE, IK, REDL, SLR, PCR
  102 FORMAT(8X,'PE=',E12.5,5X,'1/K=',E12.5,5X,'REDL=',E12.5,5X,'L/R=',
     1E12.5/8X,'PCR=',E12.5//)
      GO TO 1
      END

         000019
```

```
C      BUCKLING LOAD FOR BUILT-UP COLUMN WITH BATTEN BRACING
C      EFFECT OF LOCAL BUCKLING IS CONSIDERED
       REAL I, IC, IB, L, IK, N
     1 READ(5, 100) A, B, AC, IC, AB, IB, L, E, N, G, PCR1
   100 FORMAT(8F10.5)
       I=2.*IC+AC*B**2/2.
       WRITE(6, 101) A, B, AC, I, IC, AB, IB, L, E, N, G, PCR1
   101 FORMAT(8X, 'A=',E12.5,5X, 'B=',E12.5,5X, 'AC=',E12.5,5X,'I=',E12.5
      1,5X, 'IC=',E12.5,5X, 'AB=',E12.5/8X, 'IB=',E12.5,5X, 'L=',E12.5,5X
      2, 'E=',E12.5,5X,'N=',E12.5,5X,'G=',E12.5/8X, 'PCR1=',E12.5//)
       PE=3.1416**2*E*I/L**2
       DO 10 K=1, 100000
       ALPHA=PCR1/(2.*3.1416**2*E*IC/A**2)
       IK=A*B/(12.*E*IB)+A**2/(24.*E*IC*(1.-ALPHA))+N*A/(B*AB*G)
       REDL=L*SQRT(1.+PE*IK)
       PCR=3.1416**2*E*I/REDL**2
       SLR=REDL/SQRT(I/(2.*AC))
       IF (ABS(PCR-PCR1) .LE. 1.E-5) GO TO 20
       IF (PCR .LT. PCR1) GO TO 30
       PCR1=PCR1+1.E-2
       GO TO 10
    30 PCR1=PCR1-1.E-2
    10 CONTINUE
       WRITE(6, 103)
   103 FORMAT(8X, 'ITERATION TO BE FURTHER INCREASED'///)
       GO TO 1
    20 WRITE(6,102) ALPHA, IK, REDL, SLR, PCR
   102 FORMAT(8X, 'ALPHA=',E12.5,5X, '1/K=',E12.5,5X, 'REDL=',E12.5,5X,
      1'L/R=',E12.5/8X, 'PCR=',E12.5//)
       GO TO 1
       END
```

C00021

```
C     BUCKLING LOAD FOR THREE-LEGGED COLUMN WITH BATTEN BRACING
      REAL  I, IB, IC, IKP, L, N
    1 READ(5,100) AC, A, B, L, I, IB, IC, E
  100 FORMAT(8F10.5)
      WRITE(6,101) AC, A, B, L, I, IB, IC, E
  101 FORMAT(8X, 'AC=',E12.5,5X, 'A=',E12.5,5X,'B=',E12.5,5X,'L=',E12.5/
     18X,'I=',E12.5,5X,'IB=',E12.5, 5X,'IC=',E12.5,5X,'E=',E12.5//)
      N=A*B/(12.*IB)+A*A/(16.*IC)
      PE=3.1416**2*E*I/L**2
      IKP=(2.*A*B/IB+A*A/IC)/(24.*E)
      R=6.*PE/(IKP-N/E)
      T=(1.5+PE*N/E)/(IKP-N/E)
      PCR=(-T-SQRT(T**2+R))/2.
      PCRA=PE/((1.+3.1416**2*I)/L**2/36.*(2.*A*B/IB+A*A/IC))
      WRITE(6,102) PE, N, IKP, T, R, PCR, PCRA
  102 FORMAT(8X,'PE=',E12.5,5X,'N=',E12.5,5X,'1/KP=',E12.5,5X,'T=',E12.5
     1,5X,'R=',E12.5/8X,'PCR(EQ.3-24)=',E12.5/8X,'PCR(EQ.3-17)=',E12.5//
     2/)
      GO TO 1
      END

        000020
```

```
C       BUCKLING LOAD FOR THREE-LEGGED COLUMN WITH LACED BRACING(FIG3-13A)
        REAL I, IC, IKP, L
      1 READ(5, 100) AC, A, B, L, I, AD, IC, E, PHI, AB
    100 FORMAT(8F10.5)
        WRITE(6,101) AC, A, B, L, I, AD, IC, E, PHI, AB
    101 FORMAT(8X, 'AC=',E12.5,5X, 'A=',E12.5,5X,'B=',E12.5,5X,'L=',E12.5/
       18X,'I=',E12.5,5X,'AD=',E12.5, 5X,'IC=',E12.5,5X,'E=',E12.5,5X,'PHI
       2=',E12.5,5X,'AB=',E12.5//)
        X=1.74533E-2*PHI
        PE=3.1416**2*E*I/L**2
        IKP=1./E*(1./(SIN(X)*COS(X)**2*AD)+COS(X)/(SIN(X)*AB))
        REDL=L*SQRT(1.+PE*2./3.*IKP)
        PCR=3.1416**2*E*I/REDL**2
        SLP=REDL/SQRT(I/(3.*AC))
        WRITE(6,102) PE,IKP,REDL,SLP,PCR
    102 FORMAT(8X,'PE=',E12.5,5X,'1/KP=',E12.5,5X,'REDL=',E12.5,5X,'L/R=',
       1E12.5/8X,'PCR=',E12.5//)
        GO TO 1
        END

          000019
```

```
C      BUCKLING LOAD FOR THREE-LEGGED COLUMN WITH LACED BRACING(FIGS3-13C AND D)
       REAL I, IC, IKP, L
     1 READ(5, 100) AC, A, B, L, I, AD, IC, E, PHI
   100 FORMAT(8F10.5)
       WRITE(6,101) AC, A, B, L, I, AD, IC, E, PHI
   101 FORMAT(8X, 'AC=',E12.5,5X, 'A=',E12.5,5X,'B=',E12.5,5X,'L=',E12.5/
      18X,'I=',E12.5,5X,'AD=',E12.5, 5X,'IC=',E12.5,5X,'E=',E12.5,5X,'PHI
      2=',E12.5//)
       Y=1.74533E-2*PHI
       PE=3.1416**2*E*I/L**2
       IKP=1./E*(1./(SIN(Y)*COS(Y)**2*AD))
       REDL=L*SQRT(1.+PE*2./3.*IKP)
       PCR=3.1416**2*E*I/REDL**2
       SLR=REDL/SQRT(I/(3.*AC))
       WRITE(6,102) PE,IKP,REDL,SLR,PCR
   102 FORMAT(8X,'PE=',E12.5,5X,'1/KP=',E12.5,5X,'REDL=',E12.5,5X,'L/R=',
      1E12.5/8X,'PCR=',E12.5//)
       GO TO 1
       END

        000019
```

6. NOTATIONS

The following symbols were adopted for use in this paper:

a, b, c, d	Numerical coefficients, distances
A_c	Area of one leg of built-up column
E	Modulus of elasticity
G	Modulus of leasticity in shear
I	Centroidal moment of inertia of column cross section
I_b	Centroidal moment of inertia of batten cross section
I_c	Centroidal moment of inertia of a leg of the column
K	Shearing stiffness
K_p	Shearing stiffness of one panel
l	Height of column or tower
L	Reduced length
M	Moment at a section
N	A term equal to $\frac{a\,b}{12\,I_b} + \frac{a^2}{16\,I_c}$ in three-legged tower
P	Axial force in column
P_e	Euler critical load $= \frac{\pi^2 E I}{\ell^2}$
P_{cr}	Critical buckling load $= \frac{\pi^2 E I}{L^2}$
Q	Shearing force on a column cross section
Q_p	Shearing force in a panel
r	Radius of gyration
R	A term equal to $\frac{6\,P_e}{\frac{1}{K_p} - \frac{N}{E}}$ in three-legged tower

$$T = \frac{\frac{3}{2} + P_e \frac{N}{E}}{\frac{1}{K_p} - \frac{N}{E}}$$ in three-legged tower.

X, Y	Rectangular coordinates
y	Deflection of the buckled column
y_{sp}	Deflection due to shear in a braced plane
y_{bp}	Deflection due to flexure in a braced plane
Z	Section modulus
α, β	Angles, ratios
γ	Shearing unit strain
δ	Deflection
θ	Angle
λ	Numerical factor, slenderness ratio
ρ	Radius of curvature
σ	Unit normal stress
σ_{cr}	Compressive unit stress at critical load
ϕ	Angle

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