A STUDY OF TOPICS IN COLLEGE PREPARATORY MATHEMATICS WITH EVALUATION OF THE USE OF A MANUAL IN MATRIX ALGEBRA FOR IN-SERVICE EDUCATION

A Dissertation

Presented to

the Faculty of the College of Education

The University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Education

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by

Oliver P. Monk

June 1966

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A STUDY OF TOPICS IN COLLEGE PREPARATORY MATHEMATICS WITH EVALUATION OF THE USE OF A MANUAL IN MATRIX ALGEBRA FOR IN-SERVICE EDUCATION

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ABSTRACT

Oliver P. Monk, A Study of Topics in College Preparatory Mathematics With Evaluation of the Use of a Manual in Matrix Algebra for In-Service Education (Unpublished doctoral dissertation, University of Houston, June, 1966).

Purpose of the study. It was the purpose of this study to: (1) identify the new topics in the eleventh and twelfth grade college preparatory mathematics programs; (2) develop a manual for one of these topics, matrix algebra, for in-service education of mathematics teachers; and (3) investigate experimentally the use of the manual in matrix algebra with high school students and with in-service mathematics teachers.

Methods and procedures. Descriptive research techniques of documentary analysis applied to survey studies, mathematics curriculum study programs, Texas state adopted textbooks and other textbooks in use in high schools within the Lamar Area School Study Council produced the identification of topics. Matrix algebra was selected from this identification, an intensive documentary analysis of the treatment of concepts conducted, and a manual for the introduction to matrix theory constructed.

In the experimental research for evaluation, student

samples consisted of eighty-four senior students from college preparatory and accelerated college preparatory classes in a high school within the Lamar Area School Study Council.

Students were pre-tested, presented the unit in matrix algebra, and given the post-test. Test scores were correlated with Scholastic Aptitude Test mathematical scores and high school grade point averages. Validity, reliability, and practicality of the test were determined. Teacher samples consisted of ninety-seven teachers from a 1965 in-service workshop, a 1965 summer graduate class, and a 1966 workshop. A total of one hundred and eleven teachers were enrolled in the program, and eighty-seven per cent voluntarily completed tests and questionnaires on teaching experience, semester hours in mathematics, degrees earned, previous experience with matrix algebra, teaching assignments, recommendations concerning workshops and other items. One group completed the post-test and two groups volunteered for the pre-test and the post-test. Data were tabulated and statistical analysis of scores of various sub-groups computed.

Teachers were classified into four groups by teaching experience and four groups by the number of semester hours in mathematics. An analysis of variance was applied to the test scores in a groups-within-treatment design.

<u>Summary of the findings</u>. Documentary analysis

revealed new topics to be analytic geometry, elementary functions, matrix algebra, probability and statistics, properties of number systems, and set theory. New approaches to other topics were identified. Ninety-four per cent of the teachers who listed the textbook in use in their schools reported using texts which included units in matrix algebra. Forty per cent reported experience with matrix algebra. The completed manual presumed no prior knowledge of matrix algebra and presented material beyond that included in the high school textbook.

Student performance on tests showed a gain in mean of 18.9 points on a thirty point test. Prior knowledge of matrix algebra had little or no influence on the post-test score. Validity and reliability coefficients were found to support the use of the test with group studies. Analysis of the student experiment supported including the study of matrix algebra in high school mathematics.

Two null hypotheses were formulated concerning significant differences in the test results of (1) four groups of different teaching experience and (2) the four groups of different semester hours in mathematics. F-ratio's computed did not allow rejection of the null hypothesis for either variable. No significant differences in groups were found by the analysis. A low correlation was found between the pre-test and post-test scores. Results indicated that the manual could be used satisfactorily with experienced and inexperienced groups and with teachers with wide variations in the number of semester hours in mathematics. Eighty-nine per cent of the teachers reported that such manuals and subject content workshops were necessary to improve mathematics education.

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THE PROBLEM AND ITS INVESTIGATION

Secondary school curricula in mathematics and science have received much attention in recent years. Mathematics curriculum study programs conducted in the past decade generated reforms in secondary school mathematics and led to what the National Council of Teachers of Mathematics called a revolution in school mathematics.[1] The nationwide effect of these programs was a general move toward curricular reform and the initiation of a drive to improve mathematics teaching. A Texas Curriculum Studies Commission, functioning in 1958 through 1960, led to the publication of new curriculum standards[2] and the eventual adoption of new textbooks for the statewide public school system. The National Science Foundation, along with several other institute and fellowship programs, was established to provide for education and re-education of high school teachers. Summer institutes, high school and college co-operative programs, fellowships, and academic year programs were offered for secondary school

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[1]G. Baley Price, "Progress in Mathematics and Its Implications for the Schools," The Revolution in School Mathematics, Washington, D. C., (September, 1961), p. 1.

[2]Mathematics 7-12: Bulletin 620, Texas Education Agency, Austin, Texas, (July, 1962), 189 pp.

mathematics teachers.

Curriculum revision and the new textbooks placed in the hands of high school students presented another problem. Many mathematics teachers completed their college programs before the new curriculum studies and were not attending institute programs. Their education for teaching mathematics was based upon the materials and content before the revolution in school mathematics. In-service education programs were necessary before the new materials and new approaches could be used effectively in the classroom. Questions concerning the feasibility of some topics for high school mathematics were raised, and additional problems developed concerning ways to fill the gaps in subject matter background of teachers already in service in the secondary schools.

I. THE PROBLEM

Statement of the problem. The problem was stated in the following divisions:

1. To identify the new topics and changes in content of the eleventh and twelfth grade college preparatory mathematics of the high schools within the Lamar Area School Study Council.
2. To select one topic, matrix algebra, from the identification and develop a manual for use

with in-service education programs.

3. To analyze experimentally the use of the manual with high school students and with in-service workshops in the Lamar Area School Study Council.

Significance of the problem. The Commission on Mathematics of the Texas Curriculum Studies recommended in 1959 that, as a result of curriculum reform, a planned program of professional improvement for teachers of mathematics should be in operation not later than 1965. The Commission report called on mathematics teachers to "attend summer institutes sponsored by the National Science Foundation," and to "attend local in-service education courses which place emphasis on subject content. . . ."[3] Emphasis on re-education of mathematics teachers has characterized much of the writing about the revolution in school mathematics. Teachers have been advised to determine the changes in objectives and content of the curriculum and new textbooks, and to upgrade themselves in subject matter understanding. Study groups have proposed various approaches to the problem. The following statements from leading and authoritative sources emphasize the significance of this problem.

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[3]Report of the Commission on Mathematics, Texas Curriculum Studies: Report No. 3, Texas Education Agency (July, 1959), p. 79.

In the summary of its report the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America stated that grade school teachers need a grasp of mathematics beyond the content of the courses they teach, and that a secondary teacher's understanding of mathematics must exceed the level at which he teaches and the level at which his college program has frequently ended. The report stated:

> Thus, within the next decade it is to be expected that teachers will be asked to teach material which many of our present teachers have never studied. The teacher's mastery of mathematical ideas must substantially exceed that represented by his textbook if he is to teach with a spirit of enthusiasm and inquiry which stimulates his students to explore both fundamental ideas and their applications.[4]

In 1960, Kenneth Brown, mathematics specialist for the United States Office of Education, reported that ninety per cent of all teachers need some re-education in mathematics. Brown stated that "In-service re-education of mathematics teachers is a must if the improved mathematics programs are to be successful."[5]

W. L. Duren, Jr., Dean of the College of Arts and

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[4]<u>Recommendations for the Training of Teachers of Mathematics: A Summary</u>, Committee on Undergraduate Program in Mathematics, Mathematical Association of America (January, 1961), p. 6.

[5]Kenneth E. Brown, "In-Service Re-education of Mathematics Teachers," <u>The American Mathematical Monthly</u>, LXVII (November, 1960), p. 918.

Sciences of the University of Virginia, made the following statement at a conference on in-service education of mathematics teachers held at the U. S. Office of Education:

> Despite their willingness to work, our school mathematics teachers do not have enough mathematical power to adapt to the demands of a new curriculum. . . . College mathematics can add to the competence of those who have already achieved some mastery of school mathematics, but it cannot recover many who failed to get a good high school mathematics education. Thus, the secondary school teacher is perhaps the key person in determining our national scientific and technical strength.[6]

In 1963, Frank B. Allen, then President of the National Council of Teachers of Mathematics, reported "When in-service training programs are considered, it is generally agreed that there are simply not enough people who are qualified to serve as instructors."[7] Allen called for development of media and materials for local programs. Again in his 1964 report he supported the Project Idaho in-service program and asked for more such state and local approaches to in-service education problems.[8]

The significance of the problem was stated by Kay and Woodby, directors of Project Idaho, as follows:

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[6] Ibid., pp. 918-919.

[7] Frank B. Allen, "The Council's Drive to Improve School Mathematics," The Mathematics Teacher, LVI (October, 1963), pp. 390-391.

[8] Frank B. Allen, "The Council's Drive to Improve School Mathematics--A Progress Report," The Mathematics Teacher, XVII (October, 1964), p. 373.

> Teachers of mathematics at all grade levels face the problem of upgrading their knowledge in mathematics because the content now being taught includes many ideas that are unfamiliar to many teachers. This upgrading problem exists for experienced as well as inexperienced teachers at all levels. . . .[9]

The National Science Foundation has made important contributions to the updating of mathematics education. These were not solutions to the total problem of in-service education. Edwina Deans has stated that the fact that many applications for the National Science Foundation institutes must be turned down for each one accepted "bears witness to the magnitude of reaching" all school personnel.[10] Examples of letters from 1965 mathematics institute directors include such statements as, "We wish to note that we could accommodate only about ten per cent of the applicants,"[11] and "The number of applications from deserving teachers was so much larger than the number of available stipends that it is necessary to disappoint many applicants."[12] In a

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[9]Richard Kay and Lauren G. Woodby, "Project Idaho," The Mathematics Teacher, LVIII (March, 1965), p. 241.

[10]Edwina Deans, Elementary School Mathematics, New Directions, U. S. Department of Health, Education and Welfare (Washington, D. C.: United States Government Printing Office, 1963), p. 93.

[11]Donald Solitar, Director of Mathematics Institute, Adelphi University (a letter dated March 10, 1965, from Garden City, New York).

[12]William T. Hale, Director of Mathematics Institute, University of Illinois (a letter dated March 9, 1965, from Urbana, Illinois).

March 15, 1965 letter from the National Science Foundation, it was reported that 287 of 1545 applicants, only 12.1 per cent, were awarded summer fellowships for science and mathematics study for the summer of 1965.[13]

Additional significance for this investigation comes from Jones and Coxford of the University of Michigan. They reported as follows:

> In-service education is a continuing need for all mathematics teachers who are faced with increasing pressure to use new methods and materials. This need is particularly striking for these teachers whose pre-service training was inadequate for their jobs. Re-training is vital for many teachers, especially since the majority of teachers in mathematics earned degrees before 1957.[14]

II. LIMITATIONS OF THE PROBLEM

The following limitations were observed:

1. Analyses of textbooks were limited: (a) to the state adopted textbooks furnished for eleventh and twelfth grade college preparatory courses, and (b) to the additional textbooks

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[13]Thomas D. Fontaine, Division Director of Graduate Education in Science, National Science Foundation (a letter dated March 15, 1965, from Washington, D. C.).

[14]Phillip S. Jones and Arthur F. Coxford, Jr., "Academic and Professional Preparation of Secondary School Mathematics Teachers," Review of Educational Research, Vol. XXXIV, No. 3 (National Educational Research Association, June, 1964), p. 326.

and materials in use in these courses in high schools within the Lamar Area School Study Council.

2. Identification of new topics was based upon documentary analysis of textbooks, survey studies, and curriculum study recommendations. More intensive analysis in matrix algebra and a pilot experience led to the development of the manual for matrix algebra.

3. In the voluntary structure of the in-service program there was no way to collect the specific data on teachers concerning mathematics course grades, previous test scores, or other criteria to allow statistical comparison. The structure prohibited setting up matched samples for a two-group experiment with variations in exposure to the manual. This limited the analysis with teachers to a one-group experimental method and use of the analysis of variance statistic.

III. DEFINITION OF TERMS USED

State adopted textbooks. A Texas state textbook committee selects five textbooks with the approval of the State Board of Education for each approved course, and the local school district may select one of these five to be furnished free to the local district as the state adopted book in that subject.

In-service education. In-service education has been defined as all the activities and programs after beginning to teach that represent efforts to improve teacher preparation and improve instructional programs. For this investigation it was defined as organized efforts for the education and re-education of secondary school mathematics teachers and the improvement of mathematics instruction.

New topics. This term was used to define topics in mathematics that were new to the current textbooks when compared with previous high school books. These were not "new" to mathematics, but "new" in the sense of inclusion in current high school curriculum materials.

IV. OVERVIEW OF THE STUDY

The study reports the application of descriptive research techniques to identify new topics in eleventh and twelfth grade college preparatory programs in high schools within the Lamar Area School Study Council. Matrix algebra was selected from this identification for reasons presented in the fourth chapter, and a manual for in-service education in matrix algebra was constructed. Documentary analysis produced the scope and sequence of the topic and the manual was constructed to prepare teachers to present the unit in matrix algebra to high school students.

Student samples were selected from a high school within the Lamar Area School Study Council. In-service teacher samples were: (1) a 1965 workshop of the Lamar Area School Study Council; (2) a 1965 summer graduate class at the University of Houston; and (3) a 1966 workshop of the Lamar Area School Study Council. Experimental analysis was based upon pre-test results and post-test results with students and teachers and upon data gathered by questionnaire from teachers. Copies of the manual, test, questionnaire, data for students, and data for teachers are included in the Appendix of the study.

A summary of the literature and of research related to the study is presented in the following chapter.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

Volumes of material were produced in the last decade concerning the curriculum reforms in mathematics education. A survey of the literature as reported in this chapter was necessary to the study for the following reasons:

1. To establish a background for understanding the curriculum reforms in mathematics at the secondary school level.
2. To identify the programs of in-service education in mathematics, both those in practice and those recommended.
3. To recognize the changes in the eleventh and twelfth grade programs and the resulting effects on students and teachers.
4. To gather material necessary for the selection of a topic in secondary mathematics and the development of a subject content unit in the topic.
5. To show the need for the study.
6. To review similar and related studies.

Many of the materials reviewed relating to this study were publications of various commissions, special committees, and curriculum reform programs. The publications of the mathematical societies, associations, and teachers' councils were important to the survey. Other materials came from the review of abstracts of theses and dissertations in the area of mathematics education. The following outline was followed in presenting the review of the literature:

1. Review of Curriculum Study Programs.
2. Historical Development of a Major Program.
3. Effects on Teacher Preparation Programs.
4. In-Service Education.
5. Problems of Curriculum Content.

I. REVIEW OF CURRICULUM STUDY PROGRAMS

The idea of major curriculum reform in mathematics education is not a new one. The President of the American Mathematical Society in 1902, E. H. Moore, called for "some basic reforms in the teaching of elementary and secondary school mathematics."[1] During the years of World War II, much was heard about the inadequacy of high school mathematics education, and many commissions and committees were organized to lead toward improvement of mathematics courses. In reporting this, Lynch also stated:

> During these years mathematics was one of the "top priority" courses in our high schools. Mathematics and mathematics educators, recognizing this, and determined

[1]Stanley J. Bezuska, "Mathematics," NEA Journal, LIII (May, 1964), p. 50.

> not to lose their advantage in the post-war years, set up a Post-War Commission on Mathematics. . . . The commission proposed that there be adequate mathematical training for all, each according to his needs, ...[2]

Among the early modern curriculum reform programs was the University of Illinois Committee on School Mathematics (UICSM). Work on the material of this program was begun in 1952 under the sponsorship of the Carnegie Corporation of New York and the University of Illinois.[3] The director of this program was Max Beberman and writing for a committee on the Analysis of Experimental Mathematics Programs, Beberman stated:

> In December, 1951, the Colleges of Education, Engineering and Liberal Arts and Sciences established the University of Illinois Committee on School Mathematics to investigate problems concerning the content and teaching of high school mathematics in grades 9-12. . . . We have introduced some new content, rearranged some of the traditional content, and developed many promising pedagogical techniques and approaches. . . .[4]

A number of programs located in various parts of the

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[2]Ina Lynch, "Forces That Have Influenced the School Mathematics Program," School Science and Mathematics, LXIV (April, 1964), pp. 255-57.

[3]Kenneth E. Brown, "The Drive to Improve School Mathematics," The Revolution in School Mathematics, (National Council of Teachers of Mathematics, Washington, D. C., 1961), p. 19.

[4]Max Beberman, "University of Illinois Committee on School Mathematics--Introductory Statement," An Analysis of New Mathematics Programs, (National Council of Teachers of Mathematics, Washington, D. C., 1963), pp. 57-58.

United States began working on curriculum revisions in the next five to six years. These programs and the new developments in secondary school mathematics were reviewed in a 1959 publication by the National Association of Secondary School Principals. The Director of Education for the American Association for the Advancement of Science, John R. Mayor, reported the mathematics curriculum studies of a ten year period, listing nine which ranged from complete studies to statements of policy. After supporting these as necessary improvements in mathematics education, Mayor stated:

> Various groups have expressed varying degrees of apprehension about "what is going on in mathematics." Those concerns may be categorized as follows:
>
> 1. That the new programs are too abstract and will not provide the most efficient preparation for the other sciences and engineering.
>
> 2. That the studies should have started at the elementary level, or at least be concurrent with elementary-level studies.
>
> 3. That teacher education programs even in 1958-59 are not preparing teachers to teach the new materials.
>
> All of these have some basis for support, but combined, these bases do not appear to be sufficient to detract from the apparent progress toward sound programs that is being made. Planning for the new School Mathematics Study Group already gives promise of directly alleviating any shortcomings due to these causes.[5]

In July of 1959, the Commission on Mathematics of the

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[5]John R. Mayor, "Efforts to Improve Programs and Teaching in Mathematics," The Bulletin of the National Association of Secondary School Principals, XXXXIII (May, 1959), p. 11.

Special Curriculum Study Program of the State of Texas published its report which reviewed some of the mathematics programs and presented a proposal for mathematics education from grades one through twelve.[6] This report was used as the basis for study programs in Texas school systems, and after a series of conferences and revisions, a bulletin was published in 1962.[7] Eleven curriculum study programs in mathematics were listed along with materials and current publications related to secondary school mathematics. A schedule for improving the mathematics curriculum in a school district was presented, and emphasis for the publication was given by J. W. Edgar, State Commissioner of Education, in the following statement:

> Throughout the nation the mathematics education of secondary school students is being strengthened. In step with this national trend, those responsible for mathematics education in Texas are upgrading the curriculum and instructional program.
>
> Responsibilities for developing guides and implementing the new state course descriptions in mathematics are with the local administrator and his staff. This bulletin, Mathematics 7 Through 12, is offered in the hope that it will be of assistance to local school personnel in strengthening mathematics offering and methodology.[8]

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[6] Commission on Mathematics, Texas Curriculum Studies, Report Number 3, Texas Education Agency (July, 1959).

[7] Mathematics 7 Through 12, Bulletin 620, Texas Education Agency, (Austin, Texas, July, 1962).

[8] J. W. Edgar, "Foreword," Mathematics 7 Through 12, Bulletin 620, Texas Education Agency, Austin, Texas. (July, 1962), p. iii.

Under the direction of the National Council of Teachers of Mathematics, and with the financial support of the National Science Foundation, eight conferences were held in late 1960. These were labeled Regional Orientation Conferences in Mathematics, and were planned to develop a concentrated effort toward rapid improvement of mathematics instruction. The consultants, speakers, and the resulting publication presented to mathematics educators the ideas behind the study programs, the experiences in classrooms where materials were used, and procedures for implementing new programs.[9] The director of these conferences, Frank B. Allen, in a summary stated:

> . . . . We must show the same appreciation for new mathematical ideas that we expect mathematicians to show for good teaching. We must, at all times, display the same positive attitude toward learning that we expect our pupils to display. We must remember that our decisions relative to teachability involve subjective judgements. Some concepts that seem at first to be unteachable are found on closer analysis to be appropriate for study by high school pupils. The teachability of a concept depends as much on the preparation and ability of pupils and teachers as on the inherent difficulty of the concept.[10]

In 1962, the Project on Instruction of the National Education Association presented a summary report on curriculum studies. The report covered studies in science,

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[9]*The Revolution in School Mathematics*, A Report of *Regional Orientation Conferences in Mathematics*, National Council of Teachers of Mathematics (Washingtor, D. C., 1961), 90 pp.

[10]*Ibid.*, p. 85.

mathematics, English, foreign languages, and social studies. In the chapter on mathematics, Fraser reported the purpose and plan for school mathematics from thirteen project sources.[11] Fraser presented a brief summary of the project and the status of the project as of January, 1962.

The number of projects and the variation in material produced by the curriculum study projects caused the appointment of a Committee on the Analysis of Experimental Mathematics Program. The National Council of Teachers of Mathematics established the committee and charged it with the responsibility of developing ways to "assist teachers and school systems in their consideration of program changes."[12] After organization and establishment of criteria for including an experimental program, the committee selected eight curriculum revision projects for the published analysis. Sub-committees were named to report on the content for the various levels. The committee identified eight issues as "crucial in the changing mathematics programs," and listed these as follows: "social applications, placement, structure, vocabulary methods, concepts vs.

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[11]Dorothy M. Fraser, Current Curriculum Studies in Academic Subjects, A Report for the Project on Instruction of the National Education Association, (Washington, D. C., June, 1962), pp. 27-42.

[12]Committee on Analysis of Experimental Programs, An Analysis of New Mathematics Programs, National Council of Teachers of Mathematics, (Washington, D. C., 1963), p. 1.

skills, proof, and evaluation."[13] Each of the programs was presented by the use of: (1) an introductory statement, usually by the project director, (2) the committee reports using the eight basic issues on topics and content material, and (3) concluding comments. Recommendations were presented on how the report can be used. Included was the statement that the report was not a "pre-digested evaluation," and that "each school must take the responsibility of making evaluations for its own needs."[14] The eight programs analyzed were:

1. The Boston College Mathematics Institute (BCMI).
2. The Greater Cleveland Mathematics Program (GCMP).
3. The Syracuse University-Webster College Madison Project.
4. The University of Maryland Mathematics Project (UMMaP).
5. The Ontario Mathematics Commission (OMC).
6. The School Mathematics Study Group (SMSG).
7. The Development Project in Secondary Mathematics at Southern Illinois University.

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[13] Ibid., pp. 2-4.

[14] Ibid., p. 4.

8. The University of Illinois Committee on School Mathematics (UICSM).[15]

A more complete understanding of the movement for curriculum revision represented by these study programs could come from an analytical review of the development of one of the major programs. Such a review will be presented in the following section of this chapter.

II. HISTORICAL DEVELOPMENT OF A MAJOR PROGRAM

A major program which combined the interests and forces of several associations and societies, and which involved people from many geographic, interest, and occupational areas, was the School Mathematics Study Group. The program, which will be identified in the remainder of this section by the abbreviation SMSG, exemplifies the movement for curriculum revision in mathematics at the secondary school level. A review of its development is presented in this section.

The beginning of SMSG was in the spring of 1958. The organization was a result of the efforts of the National Council of Teachers of Mathematics, the Mathematical Association of America, and the American Mathematical Society. After consultation, the president of the latter society

[15] Ibid., pp. 1-2.

appointed an organizing committee, which in turn appointed an advisory committee, and the SMSG was formed.[16] The three sponsoring organizations had the stated purpose "to organize a school mathematics study group whose objective would be the improvement of the teaching of mathematics in the schools."[17] With this united effort the support of the National Science Foundation was gained and financial support obtained for the project. Professor Edward G. Begle was appointed the director of SMSG, and headquarters were established on the campus of Yale University at New Haven, Connecticut. In the early days the study was sometimes referred to as the Yale Project.

In 1961, Professor Begle moved the headquarters of SMSG to Stanford University at Palo Alto, California, where it is now located.[18] The program is still basically financed by the National Science Foundation. The work of SMSG continues as it moves toward the fundamental aim stated by Professon Begle. The aims were: (1) to improve the teaching of mathematics in the schools, (2) to encourage more students to study more mathematics, and (3) to ensure that the

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[16]Committee on Analysis of Experimental Programs, _op. cit._, pp. 32-33.

[17]_Ibid._, p. 33.

[18]Brown, "The Drive to Improve School Mathematics," _op. cit._, p. 17.

mathematics studied by these students is "appropriate to the world of today."[19]

The development of SMSG was unique among the commissions and study groups in mathematics education. This uniqueness was stated by Brown to be a result of the inclusion in the study group of the combined thinking of "psychologists, testmakers, college and industrial mathematicians, biologists, and high school teachers."[20] A writing session was conducted in the summer of 1958 at Yale, and experimental use was made of the materials during the 1958-59 school year. These materials were described as "short units" by Begle when he listed the titles in a December, 1958, article.[21]

The need for more detailed curriculum outlines and improved content for school courses was accepted by the group. The outlines were started in the summer session of 1958, and plans were made for the completion of first drafts of text materials for courses of the ninth through the twelfth grade work by the summer of 1959. The participating teachers were anxious for early completion of these materials, and they

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[19] E. G. Begle, "The School Mathematics Study Group," The Bulletin of the National Association of Secondary School Principals, XXXXIII (May, 1959), p. 27.

[20] Brown, loc. cit.

[21] E. G. Begle, "The School Mathematics Study Group," The Mathematics Teacher, LI (December, 1958), pp. 616-18.

were interested in enlarging the area of their preparations to include seventh and eighth grade materials.

The writing teams were composed of high school and college mathematics teachers, and the close of the summer session in 1959 found the sample text materials ready for use in the 1959-60 school year. Brown reported that the textbooks and teachers' manuals for grades seven through twelve were tried in forty-five states.[22] There were more than four hundred teachers and over 42,000 pupils using these materials. Evaluations were made after each unit or chapter, and these evaluations were submitted by teachers, advisors, administrators, and some pupils to the SMSG headquarters. These evaluations and suggestions were used in the revision of the materials by the writing teams during the summer of 1960. Early recognition was given to the need to improve the mathematical background of teachers. This was stated as necessary "to give a teacher deeper insight into the mathematics as well as an understanding of the reasons for certain methods of presentation."[23]

During the 1960 summer evaluations and revisions SMSG

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[22] Brown, loc. cit.

[23] The Revolution in School Mathematics, "Questions and Answers" (A report of a panel's answers to questions submitted in writing at the Regional Orientation Conferences) National Council of Teachers of Mathematics, (Washington, D. C., 1961) p. 77.

personnel recognized that something further was necessary and that attention had to be focused on the grade school mathematics studies. SMSG adopted the basic philosophy that "a greater substance should be introduced earlier in the mathematics sequence."[24] With the accepted idea that no one can predict the need for mathematics in his future, either in concepts or in skills, then it becomes necessary that students be taught to understand and to apply the concepts and basic structure of mathematics as well as the basic skills in mathematics. Although SMSG started its program with students in the upper half of the class in mind, it was decided after experimental use of materials that the SMSG approach was necessary to all students early in their studies.[25] The program was increased in scope to include materials for grades four through twelve, and there were two levels of materials produced in grades seven, eight, and nine.

The materials produced by SMSG were sought by school systems, mathematics teachers, and commercial publishers across the nation. There were so many requests from publishers that SMSG adopted a statement of policy in 1961. The policy stated:

It is not intended that SMSG texts be considered as

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[24]Committee on Analysis of Experimental Mathematics Programs, op. cit., p. 33.

[25]Ibid., p. 33.

> the only suitable ones for such a curriculum. Indeed, a variety of texts of the same general nature is not only possible, but highly desirable.
>
> A major purpose of an SMSG text is to serve as a model and as a source of suggestions and ideas for the authors of this variety of texts. Textbook writers should feel free to use SMSG texts in this way and to adapt, expand, and improve them for their own purposes.[26]

The consideration of elementary mathematics programs by SMSG started with a conference called in 1959. Present were university professors of mathematics, high school and elementary teachers, supervisors and education specialists, representatives from scientific and government organizations with interests in mathematics, and psychologists with interest in learning theory. Deans reported on the conference and the development of the SMSG elementary program.[27] Following a critical study of elementary mathematics during 1960, a writing team produced experimental materials that were used by twenty-seven experimental centers in the 1960-61 school year. The centers were associated with colleges or universities for consultant services and involved 370 fourth, fifth, and sixth grade teachers, and over 12,000 students of the same levels.[28] Based on the evaluations and recommendations

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[26]"Questions and Answers," op. cit., p. 57.

[27]Edwina Deans, Elementary School Mathematics: New Directions, U. S. Department of Health, Education and Welfare, (U. S. Government Printing Office, 1963), p. 19.

[28]Ibid., p. 19.

from these uses, the materials were revised in the summer of 1961, and used in the 1961-62 school year.

The work of SMSG continues at Stanford University under the direction of Professor Begle, who has served continuously as the director of the program since its beginning. The influence of SMSG is reflected in the review of the literature and will also be seen in the topical analysis to be presented in the fourth chapter.

III. EFFECTS ON TEACHER PREPARATION PROGRAMS

Data from a survey of the trends of teacher education in mathematics reported by Schumaker indicated that changes in teacher education "came slowly and, unfortunately, tended to follow rather than to precede changes in the secondary school curriculum."[29] A study by Obourn and Brown revealed that of the nation's 118,298 mathematics teachers in the fall of 1961, twenty per cent were new to the school where they were teaching, and 12,652 (10.7 per cent) had had no previous experience in teaching.[30] The data showed that 37.3 per cent

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[29] John A. Schumaker, "Trends in the Education of Secondary School Mathematics Teachers," The Mathematics Teacher, LIV (October, 1961), pp. 413-22.

[30] E. S. Obourn and R. E. Brown, Science and Mathematics Teachers in Public High Schools. U. S. Department of Health, Education, and Welfare, (U. S. Government Printing Office, 1963), p. 23-24.

were "part-time" mathematics teachers teaching three or fewer classes per day in mathematics. About fifteen per cent were teaching only one class in mathematics, and the summary stated:

> It may not always be possible to schedule classes under fully qualified full-time mathematics teachers; however, it is certainly desirable for a pupil to study mathematics under a teacher whose major interest is in that field.[31]

Fehr, the Head of the Department of the Teaching of Mathematics at Teacher's College, Columbia University, wrote in 1959 that lack of mathematical knowledge on the part of some mathematics teachers was "the greatest factor in the lack of student interest and in the poor quality of product of some of our schools. . . ."[32] Fehr reported further that a teacher must know his subject, must believe in what he is teaching, and must know how to communicate the subject so that his students "come into possession of treasured knowledge."[33] After an analysis of some of the problems in teacher education, Fehr proposed a program for the preparation of a mathematics teacher as follows:

The study of college mathematics should begin with

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[31] Ibid., p. 26.

[32] Howard F. Fehr, "The Education of Teachers of Mathematics," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), p. 169.

[33] Ibid., p. 169.

> a course in analytic geometry and the calculus running for one and one-half years. A teaching major can then be rounded off by the study of eighteen semester hours beyond the calculus. This work should include a study of modern algebra, both of the polynomical type, and the abstract approach through the study of matrices, groups, rings, fields and a study of geometry from the real number approach. . . . Another required year of study should be Probability and Statistics. Other courses from which selections can be made are Differential Equations, Theory, Symbolic Logic, and Finite Mathematics.[34]

In addition to this necessary subject-matter background, Fehr called for a course in methods of teaching mathematics, "which should be taught by a person competent in mathematics."[35]

Rourke reported the position of the Commission on Mathematics of the College Entrance Examining Board on teacher education in mathematics as necessarily including the objectives of philosophy, subject-matter, and methodology. For "prospective teachers of senior high school mathematics,"[36] Rourke listed a college major of thirty hours of college mathematics plus a mathematics education course covering the teaching of secondary school mathematics. The thirty hours had to include work in analytical geometry,

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[34] Ibid., p. 171.

[35] Ibid., p. 172.

[36] R. E K. Rourke, "The Commission on Mathematics of the CEEB and Teacher Education," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), p. 176.

calculus, analysis, abstract algebra, geometry, statistics and logic.[37]

The Committee on the Undergraduate Program in Mathematics (CUPM) was created by the Mathematical Association of America, and this committee reported its recommendation for the education of mathematics teachers in 1960. This report classified five levels of mathematics teaching from the elementary school through college teaching.[38] Level III was listed as teachers of mathematics in grades nine through twelve, and Level IV included teachers of advanced programs in high school and junior college teachers. The recommended minimum preparation for Level III teachers was thirty-three semester hours in a mathematics major with a minor field in which a "substantial amount of mathematics is used.[39] Topics listed, in addition to analytics and calculus, included abstract algebra, analysis, probability-statistics, number theory, and topology. Recommendations for Level IV were based on a master's degree with "at least two-thirds of the courses being in mathematics, and for which

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[37] Ibid., p. 178.

[38] Committee on Undergraduate Program in Mathematics, A Summary of Recommendations for the Training of Teachers of Mathematics, Mathematical Association of America, (January, 1961), p. 9.

[39] Ibid., p. 12.

an undergraduate program at least as strong as Level III training is a prerequisite."[40]

In 1959, the Commission on Mathematics of the Texas Curriculum Studies reported its recommendations on the pre-service education of mathematics teachers. The division of teacher certification of the Texas Education Agency had been requiring a minimum of eighteen semester hours of college mathematics for certification to teach high school mathematics, grades nine through twelve. The commission recommended:

> A teacher of mathematics in the senior high school will have completed a minimum of twenty-four hours of mathematics of which six hours of mathematics content are specifically designed to develop competencies for teaching senior high school mathematics.
>
> A teacher of calculus in the high school should have thirty hours of mathematics, none of which is below the calculus, and six hours of which are in advanced calculus.[41]

Joseph Landin of the University of Illinois compared the answers to the two questions, "What are the mathematical demands of the traditional curriculum upon the mathematics teacher," and, "What are the mathematical demands of the new mathematics curricula upon the mathematics teacher?"[42] The

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[40] Ibid., p. 13.

[41] Report of the Commission on Mathematics, Texas Curriculum Studies: Report No. 3, Texas Education Agency (July, 1959), p. 79.

[42] Joseph Landin, "The Secondary Mathematics Curriculum and the New Teacher," School Science and Mathematics, LXIII (May, 1963), p. 367.

traditional curriculum, according to Landin, called for a teacher preparation of eighteen to twenty college semester hours, including Algebra, Trigonometry, Analytic Geometry, Calculus, Advanced Geometry, and possibly Theory of Equations. Before listing the answer to the second question, Landin emphasized the concepts of mathematical structure presented in the new curricula and the necessity for specific courses in this area. Landin also declared as essential "a six-hour course devoted to the content and pedagogy of the new curricula."[43] The course requirements were listed as follows:

College Algebra	(3 semester hours)
Trigonometry	(2 semester hours)
Analytical Geometry	(3-4 semester hours)
Calculus	(6-8 semester hours)
Advanced Geometry	(3 semester hours)
Structure of Arithmetic	(3-5 semester hours)
Structure of Algebra	(6 semester hours)
Advanced Analytic Geometry	(3 semester hours)
Introduction to Higher Analysis	(6 semester hours)[44]

A review of the research dealing with recommended programs of teacher education and with certification

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[43] Ibid., p. 373.

[44] Ibid., p. 372-73.

requirements and actual qualifications of in-service mathematics teachers was reported by Estes in 1961. He summarized that the certification requirements and the actual qualifications were far below the minimums recommended for the preparation to teach the revised high school mathematics. Estes reported general agreement on the individual subjects to be included in the degree programs for mathematics teachers. The total number of semester hours in mathematics recommended by the different sources showed some variations but were reported by Estes as being in general agreement.

A study by Ford at the University of Missouri concerning the mathematics included in teacher education programs in colleges and universities of the Southern Association of Secondary Schools and Colleges was reported in 1962.[46] Ford found that the programs did not provide experience in the new secondary materials, and that the new teachers entering the profession through 1961 had not received the education in subject matter recommended by the national projects. Valsame reported similar findings from

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[45]Ronald V. Estes, "A Review of Research Dealing With Current Issues in Mathematics Education," School Science and Mathematics, LXI (November, 1961), p. 629-30.

[46]Patrick I. Ford, "The Mathematics Included in Programs for the Education of Secondary School Teachers in the Southern Association." (A doctor's dissertation, University of Missouri, Columbia, Missouri, 1962, 129 pp.) Abstract: Dissertation Abstracts, 23:543, No. 2, 1962.

a study of mathematics teacher education in the state of North Carolina.[47] Valsame's study indicated that new teachers were not receiving the amount nor the kind of subject matter training that had been recommended by the secondary school curriculum studies.

There have been a number of reports on alterations in programs or the instituting of new programs for teacher education. Among these were the co-operative undergraduate program in mathematics and education at the University of California,[48] the new master's degree program for mathematics teachers at the University of Chicago,[49] the University of Minnesota's co-operative program in teacher education,[50]

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[47]James Valsame, A Study of Selected Aspects of Mathematics Teacher Training in North Carolina as Related to Recent Trends in Mathematics Teaching. (A doctor's dissertation, University of North Carolina, Chapel Hill, North Carolina, 1961, 289 pp.) Abstract: Dissertation Abstracts, 23:549, No. 2, 1962.

[48]S. J. Bryant, J. L. Kelley, and Harley Flanders, "The Major in Mathematics for Teachers at the University of California," American Mathematical Monthly, LXX (The Mathematical Association of America, October, 1963), pp. 879-81.

[49]"New Master's Degree Program at the University of Chicago for the Training of Secondary Mathematics Teachers," American Mathematical Monthly, LXX (The Mathematical Association of America, October, 1963), pp. 884-85.

[50]W. H. Edson and J. W. Buchta, "A Co-operative Program for Teacher Education Leading to the B.A. and B.S. Degrees," American Mathematical Monthly, LXVIII (The Mathematical Association of America, March, 1961), pp. 290-91.

and the report of the teacher education requirements for the state of New York.[51]

The Minnesota program led to degrees in both education and mathematics, and the California program involved co-operative undergraduate sections, special classes, and a particular degree which completed the requirements for a teaching certificate and for graduate study in mathematics. The program at Chicago included courses in mathematics and education for a Master of Arts in the teaching of mathematics.

Jones and Coxford reported on "Mathematics Teacher Training Recommendations" and on "Certification and Pre-Service Programs," and predicted an increased stress on subject matter and on experience with new materials in the mathematics curriculum.[52] This report called for future efforts toward examination of the teaching-learning process for the purpose of evaluating "the effectiveness of the many modifications that are being introduced into the academic and professional training of teachers of mathematics."[53]

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[51] "Teacher Training Requirements in New York," American Mathematical Monthly, LXVIII (The Mathematical Association of America, June-July, 1961), p. 572.

[52] Phillip S. Jones and Arthur F. Coxford, Jr., "Academic and Professional Preparation of Secondary School Mathematics Teachers." Review of Educational Research, XXXIV (American Educational Research Association, June, 1964), pp. 322-25.

[53] Ibid., p. 331.

According to the studies reported, mathematics teacher preparation programs have been changed with further changes recommended. The majority of mathematics teachers now in service completed college programs before teacher preparation included the new or revised curriculum materials. This presented increased problems for in-service education. The next section will review the literature concerning the in-service education of mathematics teachers.

IV. IN-SERVICE EDUCATION

The in-service education, or re-education, of high school mathematics teachers was the subject of a conference in 1960 sponsored by the United States Office of Education and the National Council of Teachers of Mathematics. Brown and Snader prepared a publication that covered the highlights of the conference, the three main addresses made to the conference, the group work sessions on promising in-service practices, and the implications of the conference.[54] The great variation in the United States in the type of educational systems, the certification required of teachers, the assignment of teachers, and the content in mathematics

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[54]Kenneth E. Brown and Daniel W. Snader, *In-Service Education of High School Mathematics Teachers*, *U. S. Department of Health, Education, and Welfare*, Office of Education (U. S. Government Printing Office, Washington, D. C., 1961).

courses taught prompted a national survey by questionnaire. Syler reported the results of the survey to the conference.[55] The questionnaires on current problems and patterns of in-service education for mathematics teachers were mailed to colleges, state departments of education, and to individual school systems. The programs reported were grouped under the following headings:

A. Programs Reported by Colleges

1. Special courses for those already teaching regular courses attended by mathematics teachers.

2. National Science Foundation and other sponsored institutes.

3. Consultant services by college staffs.

B. Programs Reported by State Departments of Education

1. Reading by teachers (through provision of appropriate books and materials).

2. Study Groups--Local seminars or workshop type sessions--with or without outside help.

3. Formal classes--by extension services, correspondence, or locally sponsored classes.

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[55]Henry W. Syer, "Current Problems and Patterns of In-Service Education in High School Mathematics," _In-Service Education of High School Mathematics Teachers_, (compiled by Kenneth E. Brown and Daniel W. Snader), op. cit., p. 17.

4. Conferences and lectures and consultants.

5. Cooperative programs with nearby colleges.

6. Curriculum planning.

C. Programs Reported by School Systems.

1. Reading by teachers.

2. Study and discussion groups--with or without outside help.

3. Formal classes--locally sponsored, television lessons, correspondence, summer courses in school systems--with or without credit.

4. Conferences, lectures, and consultants.

5. Curriculum planning.

6. Cooperative programs with nearby colleges.[56]

The group work sessions on promising practices in in-service programs were divided by small, medium, and large school systems with state departments of education. All three sessions reported that, "the principal responsibility for providing an in-service education program rests with the local school system."[57] The program should begin at home and should be designed to take advantage of all institutes, special courses, special materials, and well-trained personnel

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[56]Ibid., pp. 18-35.

[57]Brown and Snader, op. cit., p. 56.

available. One of the promising procedures reported was "the use of seminars conducted by a subject matter specialist who deals with content related to a specific secondary school course."[58] Examples of seven city and county in-service programs and examples of six state-wide programs were cited in this report.[59]

Obourn and Brown presented data showing about thirty-seven per cent of the mathematics teachers teaching half time or less in mathematics, and about half a million students being taught by teachers who were poorly prepared and not interested in mathematics.[60]

From a study supported by The National Association of State Directors of Teacher Education and Certification, Viall reported data showing that a high percentage of mathematics classes were being taught by inadequately prepared teachers.[61] Conant cited this study by Viall and the study by Obourn and Brown as bases for his recommendation that teachers with less than nine hours of mathematics be re-assigned to other areas.[62]

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[58] Ibid., p. 57.

[59] Ibid., pp. 72-83.

[60] Obourn and Brown, op. cit., p. 25.

[61] William P. Viall, "Secondary Science and Mathematics Teachers Surveyed." Journal of Teacher Education, XIII (December, 1962), pp. 475-76.

[62] James Bryant Conant, The Education of American Teachers, (McGraw-Hill Book Company, New York, 1963), 275 pp.

Conant recommended that teachers with some experience, some scientific background, and with nine to seventeen hours of college mathematics be given intensive "training programs" to qualify them to remain in mathematics teaching.

A number of special programs of re-education of mathematics teachers have been in operation by various foundations and industries. Brown and Snader reported that these included "The National Science Foundation, The Ford Foundation, Shell Oil Company, General Electric Company, and the Camille and Henry Dreyfus Fund."[63] Rietz discussed the operation of the General Electric Fellowship Program, which began in 1945, with fifty high school science teachers attending a program at Union College in Schenectady, New York.[64] Over a fourteen year period, approximately 2,200 teachers in thirty-eight states participated as G. E. Fellows, and there were about one hundred and fifty secondary school mathematics teachers each summer in the fellowship program by 1957. Another in-service program was reported by Davis as a cooperative program of Syracuse University and area school districts.[65]

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[63]Brown and Snader, op. cit., p. 3.

[64]G. A. Rietz, "Industry Lends a Hand," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), pp. 181-86.

[65]Robert Davis, "A University Assists in an In-Service Program," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), pp. 187-89.

This program, called "The Syracuse Plan," was supported by a grant from the Alfred P. Sloan Foundation and involved mathematics teachers who taught in the high schools in the mornings and attended mathematics classes at Syracuse in the afternoons.

National Science Foundation Institutes. The institutes for high school mathematics and science teachers, sponsored by the National Science Foundation, were described as the "most massive efforts ever made in in-service education."[66] There were three types of programs initiated, the summer institute, the academic year institute, and the in-service institute. The objective of these programs was "to improve the mathematics education of students by upgrading the mathematical competence of their teachers."[67]

Mallinson reported a study of the development of the summer institute program of the National Science Foundation.[68] The first institutes were sponsored in 1953 with two programs for college teachers being offered. In the summer of 1954,

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[66]R. W. Burnett, "Academic and Professional Preparation of Science Teachers," Review of Educational Research, XXXIV (American Educational Research Association, June, 1964), p. 319.

[67]Jones and Coxford, op. cit., p. 326.

[68]George G. Mallinson, "The Summer Institute Program of the National Science Foundation." School Science and Mathematics, LXIII (February, 1963), pp. 95-104.

the first summer institute for high school teachers was sponsored at the University of Washington. Mallinson's study covered the development of the institute program from 1953 through 1961. The report showed that institute participants were selected from the more highly qualified mathematics and science teachers, and that the "vast numbers of unqualified teachers in science and mathematics classrooms throughout the United States" were not being offered institute participation.[69] Since 1959, the National Science Foundation has recognized the need for what has been called basic programs for the many unqualified teachers assigned to teach mathematics in our schools. The institute programs were reported to be likely to continue "throughout the forseeable future," and Mallinson reported the effect as follows:

> Projections prepared by the NSF and which appear in a table entitled, "Science and Mathematics Teacher Requirements, Turnover, Recruitment, and Training Opportunities Offered by the Institutes Section, National Science Foundation," indicate that in 1969 a secondary-school student is as likely to meet with an inadequately-trained science or mathematics teacher as he is at the present time. Thus, if the present effort of the National Science Foundation is sustained, we are not likely to sink any deeper in the "sea of incompetence."[70]

Orr and Young reported a study conducted by the

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[69] Ibid., p. 98.

[70] Ibid., p. 103.

American Institute for Research for the National Science Foundation.[71] Teacher questionnaires and teacher interviews were used with a sample of 491 public and private schools of various sizes and locations. The major findings stated that of the 169,000 science and mathematics teachers, about thirty-two per cent had attended National Science Foundation teacher training programs, thirteen per cent had applied and been rejected, but approximately fifty-five per cent had never made applications. Characteristics of the non-applicant were reported and their needs for re-education emphasized. Sixty-eight per cent of all teachers were depending on other means of in-service re-education in mathematics.

Wiersma evaluated results of National Science Foundation institutes through teachers' reactions to institute programs and through reported effects on the high school courses.[72] Using questionnaires to institute participants and to school personnel where participants taught, Wiersma found that teachers reacted favorably to the programs and

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[71]David B. Orr and Albert T. Young, Jr., "Who Attends NSF Institutes?" Science Teacher XXX (November, 1963), pp. 39-40.

[72]William Wiersma, Jr., "A Study of National Science Foundation Institutes: Mathematics Teachers' Reactions to Institute Programs and Effects of These Programs on High School Mathematics Courses." (A doctor's dissertation at the University of Wisconsin, 1962, 138 pp.). Dissertation Abstracts, XXIII (No. 4, 1962), pp. 1239-40.

reported that basic courses in mathematics were the most valuable part of institute experience. The effects on high school courses reported were: (1) an increase in the variety of topics, (2) less reliance on textbook, (3) more additional and related material presented, and (4) more stimulation of student initiative.

Another similar study was conducted by Whitaker in 1961.[73] This study supported the changes reported by Wiersma but also reported that institute participants with superior mathematics background did not report the use of experimental materials in class. More than one half of the summer institute participants who were less experienced and less adequately prepared reported using the experimental materials from the various curriculum reform studies. The study also emphasized the need for more institutes for the less adequately prepared mathematics teacher.

Participants in the academic year institute program at Oklahoma State University from 1956 through 1958 were the subject of a study by Ostlund.[74] He used personal interviews,

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[73]Mack L. Whitaker, "A Study of Participants in Summer Mathematics Institutes Sponsored by the National Science Foundation. (A doctor's dissertation at Florida State University, Tallahassee, Florida, 1961). Dissertation Abstracts, 22:2712, No. 8, 1962.

[74]Leonard A. Ostlund, "Retrospective on an N.S.F. Program," School Science and Mathematics, LXII (March, 1962), pp. 177-82.

tape recordings, and questionnaires to obtain data for his study. Ostlund reported the changes in economic factors, in teaching status, in activities and feelings, and in classroom procedures. Participants reported one hundred and eleven changes in classroom procedure. Ostlund stated that he was impressed by the variety and sincerity of these considerable changes in procedure.[75] Some of the changes listed were: (1) more activities, (2) broader understanding, (3) new concepts, (4) improved presentation, (5) encouragement of student initiative, (6) wider variety of topics and innovations, and (7) setting of higher goals.

Jones and Coxford reported that in 1960 the National Science Foundation sponsored two hundred and twelve summer institutes which involved mathematics teachers.[76] Mallinson listed a total of four hundred and eighty-one summer institutes sponsored by the National Science Foundation in 1962.[77] The 1965 listing of summer institutes included two hundred and four that listed mathematics, although some were multiple program offerings.[78] In another summary of the 1965

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[75] Ibid., p. 179.

[76] Philip S. Jones and Arthur F. Coxford, Jr., op. cit., p. 326.

[77] George G. Mallinson, op. cit., p. 97.

[78] "1965 Summer Institutes Sponsored by NSF," School Science and Mathematics, LXV (January, 1965), pp. 92-102.

institutes, it was stated that "more than 20,000 secondary school teachers of science and mathematics" would be participating in a total of four hundred and forty-nine institute programs.[79] Of this number, the summary reported one hundred and twenty-seven to be specifically for mathematics teachers.

Some other special in-service programs. A number of re-education programs other than institutes have been the subject of reports and studies. A 1963 report by DeVault, Houston, and Boyd concerned the effect of the use of consultants for in-service education.[80] This report was a part of a larger study concerning the use of television and the use of consultant services. The summary presented a positive value for consultant services and listed the need for identification of desired changes, the value of small group discussions, and appropriate selection of consultant services as necessary for best results.[81]

Kay and Woodby reported on the statewide in-service

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[79] "449 NSF Summer Institutes to Provide Study Opportunities," The Mathematics Teacher, LVIII (February, 1965), p. 155.

[80] M. Vere DeVault, W. Robert Houston, and Claude C. Boyd, "Do Consultant Services Make a Difference," School Science and Mathematics, LXIII (April, 1963), pp. 285-90.

[81] Ibid., p. 290.

program in Idaho. This program, labled "Project Idaho," was started with the training of local consultants through a summer seminar, and then these consultants conducted in-service programs in the local districts.[82] A number of different types of seminars, non-credit courses, and credit courses were reported in the first two years of the project. Although the program started in the elementary grade area, it is being expanded to all areas to "work toward the improvement of instruction in mathematics."[83]

Some different types of statewide programs to improve mathematics teaching were reported by Brown and Snader.[84] New York offered summer institute courses, part-time courses during the school year, and specialized personnel in mathematics along with a film developed for mathematics teachers. Minnesota developed a system of local and regional workshops, a special study of the use of School Mathematics Study Group materials by the Minnesota National Laboratory, and the development of the Minneapolis educational television station. Rhode Island reported a program which included those preparing to teach as well as those already teaching, and operated in

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[82]Richard Kay and Lauren Woodby, "Project Idaho," The Mathematics Teacher, LVIII (Number 3, March, 1965), pp. 241-43.

[83]Ibid., p. 243.

[84]Brown and Snader, op. cit., pp. 79-82.

the form of grants to eligible participants. Pennsylvania reported a system of workshops taught by selected instructors for two-hour sessions totaling about thirty hours of class instruction in mathematical concepts and the experimental programs. A number of large city school system programs were reported in this study also.[85]

A number of studies reported the development and use of films, television, and other audio-visual aids for in-service education. Mosteller presented a report on the Continental Classroom television course in mathematics, which was designed with teacher re-education as one of the objectives.[86] Davis reported on the development of tape recordings and films for in-service education uses,[87] and Rosenbloom presented the preparation of films for in-service education in the teachings of modern mathematics in the secondary school.[88] Evaluation of the use of the films has

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[85]Ibid., pp. 72-77.

[86]Frederick Mosteller, "Continental Classroom's TV Course in Probability and Statistics," The Mathematics Teacher, LVI (October, 1963), pp. 407-13.

[87]Robert B. Davis, "Syracuse University-Webster College Project," Science Course Improvement Projects: Courses, Written Materials, Films, Studies, (National Science Foundation Publication, Washington, D. C., 1962), p. 5.

[88]Paul C. Rosenbloom, "Films for Mathematics Teachers," Science Course Improvement Projects: Courses, Written Materials, Films, Studies, (National Science Foundation Publication, Washington, D. C., 1962), p. 52.

been recommended, but as yet has not been reported.

Another type of in-service education program was reported by the 1960 conference at the United States Office of Education.[89] Cooperative programs by a college and school systems in the area of that college were recommended. The Lamar Area School Study Council was organized in 1962 and reported twenty-eight school systems in membership in 1965.[90] The council sponsored conferences, summer workshops, short courses, educational television programs, night workshops, and special services programs. The purpose of the council was stated as "the improvement of instruction in the member schools." The list of sponsored activities for mathematics included a television series, short courses for credit, workshops, conferences, and publications. The need for in-service education and the position of a school district that falls behind in the re-education of its mathematics teachers was stated in the annual report as follows:

> Modern mathematics taught by a teacher who does not understand, and who, not understanding is not in sympathy with it, will only confuse the student, and will be less effective than with the traditional mathematics. School systems, . . . must make every effort to retrain their teachers by either having the teachers go back to

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[89]Brown and Snader, _op. cit._, p. 16.

[90]Lamar Area School Study Council, _Third Annual Report: LASSCO Yearbook 1964-65_, (compiled by Thomas T. Salter, Executive Secretary, Lamar State College, Beaumont, Texas, 1965), 33 pp.

college and take courses in modern mathematics, or by having them attend workshops, conducted by an instructor knowledgeable in modern mathematics. No longer will it suffice for the school system to require its teachers to spend an hour per week listening to the principal or supervisor read from a modern mathematics manual.[91]

V. PROBLEMS OF CURRICULUM CONTENT

As the previous sections of this chapter indicated the current curriculum reforms in mathematics changed school programs, textbooks, teacher preparation, and in-service education. Ferguson stated that the reform in mathematics meant at least four things to the teachers in secondary schools, and these were:

1. Change in the content of school mathematics;
2. Change in the approach to the familiar content;
3. Change in method of teaching;
4. Change in teacher preparation.[92]

Literature reviewing the changes in contents and the changes in the approaches to content for the eleventh and twelfth grades completes this section.

The questions and answers sessions of the Regional

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[91]John Creswell, "Modern Mathematics--The New Versus the Traditional," *Third Annual Yearbook*, (The Lamar Area Study Council, Beaumont, Texas, 1965), p. 32.

[92]W. Eugene Ferguson, "Current Reforms in the Mathematics Curricula--A Passing Phase or Progress?" *The Mathematics Teacher*, LVII (March, 1964), p. 143.

Orientation Conferences in Mathematics were summarized in the report published by the National Council of Teachers of Mathematics. The summary reported the topical outline of five of the curriculum projects[93] and the sequence of courses for grades nine through twelve as recommended by the panel of conference consultants.[94] Two programs were reported following a regular sequence in college preparatory mathematics, one with first year algebra in the eighth grade and the other starting algebra in the ninth grade. The eleventh grade program presented included algebra and introduction to trigonometry in the normal sequence, and covered advanced topics in algebra and elementary functions for those who started the program in the eighth grade. For this latter group the twelfth grade course specified a program of advanced topics which was presented as follows:

> We suggest that this be a one-year course containing such units as Polynomial Functions, Circular Functions, Exponential and Logarithmic Functions, Theory of Equations, Mathematical Induction, Postulational Systems, Matrix Algebra.[95]

The College Entrance Examination Board appointed a Commission on Mathematics, and the report of this commission was published in 1959. The commission stated, "The

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[93]*The Revolution in School Mathematics*, "Questions and Answers," pp. 65-68.

[94]*Ibid.*, p. 73.

[95]*Ibid.*, p. 73.

development of mathematics and the broadening of its applications have outrun the curriculum."[96] Recommendations for college preparatory mathematics topics and course sequences were made by the commission. Meder presented these proposals and the principles behind the revisions in the content.[97] The twelfth grade program proposed by the commission offered three possible semester programs. A course in Elementary Functions "should be included, no matter what other topics are chosen for the twelfth grade syllabus."[98] The other courses included Introduction to Probability and Statistical Inference, and Introduction to Abstract Algebra. The proposals emphasized that the program was secondary school mathematics and not collegiate or higher mathematics. On the subject of calculus in high school, the commission stated:

> The commission does not recommend the inclusion of a course in calculus as a part of the normal high school curriculum. The average student cannot be adequately prepared for such a course in three years, and anything less than a full-year course will ordinarily be time wasted, since it will not fit into any typical college program. Moreover, a course in

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[96]"Program for College Preparatory Mathematics," (College Entrance Examination Board, New York, 1959), p. 5.

[97]Albert E. Meder, Jr., "Proposals of the Commission on Mathematics of the College Entrance Examination Board," The Bulletin of the National Association of Secondary School Principals, XLIII (May, 1959), pp. 19-26.

[98]Ibid., p. 24.

> calculus deals with ideas that are mathematically quite sophisticated, and mathematical maturity is absolutely essential. There is no value whatever in a course in calculus that merely sets forth rules for calculation and formulas for solving certain types of problems without adequate attention to conceptual difficulties.[99]

Articles criticizing the curriculum proposals and the recommended revisions, and rebuttals to these articles, have appeared frequently. Morris Kline criticized the modern mathematics movement in the high schools with particular comments attacking the Commission on Mathematics of the College Entrance Examination Board.[100] The rebuttal to Kline's article appeared in the same publication and supported the proposals of the Commission.[101] R. J. Diamond delivered an address to Los Angeles City high school teachers which was later published, in which he attacked the proposals.[102] He questioned the position represented by the new programs, particularly in the introduction of abstractions, axiomatic systems such as Non-Euclidean geometry, mathematical logic.

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[99] Ibid., p. 24.

[100] Morris Kline, "The Ancients Versus the Moderns, A New Battle of the Books," The Mathematics Teacher, LI (October, 1958), pp. 418-27.

[101] Albert E. Meder, Jr., "The Ancients Versus the Moderns--a Reply," The Mathematics Teacher, LI (October, 1958), pp. 428-33.

[102] R. J. Diamond, "A Commentary Inspired by the New Mathematics Programs," School Science and Mathematics, LXIII (November, 1963), pp. 658-64.

and an inquiry into the "foundations" of mathematics. Diamond said, "The important thing in teaching high school mathematics is to get on with the mathematics and avoid any inquiry into the nature of number."[103] The rebuttal to this commentary was presented by Gail Young, who pointed out that most criticisms expressed applied to some mathematics programs, but not to all of the curriculum revision programs.[104] Young agreed with Diamond in the statement that school mathematics must be useful mathematics, and that the usefulness must be made clear in the classroom. Young stated that mathematical needs have changed with the birth of new areas of application and the essential mathematical tools for today make the new curriculum content even more necessary.[105]

Surveys of college mathematics departments and of high school mathematics teachers concerning the content of the college preparatory fourth and fifth year programs have been conducted. Brown reported such a survey of the heads of mathematics departments in Ohio Colleges.[106] Using a

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[103] Ibid., pp. 661-62.

[104] Gail S. Young, "Comments on a 'Commentary,'" School Science and Mathematics, LXIV (June, 1964), pp. 545-49.

[105] Ibid., p. 547.

[106] Robert S. Brown, "Survey of Ohio College Opinions With Reference to High School Mathematics Programs." The Mathematics Teacher, LVI (April, 1963), pp. 245-47.

questionnaire where each question called for a statement explaining the reason for the answer, Brown tabulated the results and comments for the questions. Sixty-nine per cent of the responses said that calculus should not be included in high school content, and twenty-five per cent said it should with the qualifications that the course "be taught for a full year as a college-level course by a college caliber-teacher to highly-select students."[107] Brown's table of topics that should be included in the preparation of college bound mathematics students listed twelve topics that ranged in placement from the ninth through the twelfth grades.[108]

Carl B. Allendoerfer, Head of the Department of Mathematics at the University of Washington, presented a position concerning the teaching of calculus in high school organized around the following three points:

1. Although calculus is an essential part of a mathematical education, its importance relative to algebra, geometry, and other subjects has been overemphasized.

2. Calculus is now being taught too early to students who would be far better off studying other mathematical subjects.

3. When it is taught, calculus is presented in the wrong way.[109]

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[107] Ibid., p. 246.

[108] Ibid., p. 247.

[109] Carl B. Allendoerfer, "The Case Against Calculus," The Mathematics Teacher, LVI (November, 1963), p. 482.

Allendorfer stressed a need for a full treatment of analytical geometry, and then topics of probability, matrix algebra or finite mathematics before the teaching of calculus in high school.[110]

Another survey of two hundred and five colleges was made by sending questionnaires to the directors of admission and to the chairmen of mathematics departments of these colleges. Blank reported the results of his survey of more than four hundred individuals when he received responses from sixty-three per cent of the individuals and ninety per cent of the institutions.[111] Conclusions were that offering a fifth year of high school college preparatory mathematics was highly advisable, and that the most important courses are analytic geometry and probability and statistics. A high school survey also conducted by Blank revealed that the most widely offered course was a year's program in analytic geometry and calculus. The second most popular was elementary functions and matrix algebra and the third was probability and statistics.[112]

During the 1963-64 school year, Buchanan conducted a

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[110]Ibid., p. 483.

[111]William R. Blank, "A Survey Concerning Advanced Mathematics Curriculum," The Mathematics Teacher, LVII April, 1964), pp. 209-11.

[112]Ibid., p. 209.

study of two hundred and thirty-three departments of mathematics in United States colleges, which offered a graduate program in either mathematics, applied mathematics, or statistics. The study concerned the content of the twelfth year high school mathematics course.[113] The data showed analytic geometry as the most frequent first choice selection with matrix algebra and additional elementary functions tied as second most popular first choice. When first and second choices were combined, then analytic geometry, elementary functions, probability and statistics, and matrix algebra were the four topics selected.[114]

SUMMARY

The literature revealed that there were many similarities and yet many variations in the proposals and recommendations of the curriculum revision programs. The total activity and size of the movements indicated that these programs were not passing phases in the mathematics curriculum. The rate of growth and expansion of the School Mathematics Study Group exemplified the force for change. The effect on textbooks and curriculum material revealed in

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[113] O. Lexton Buchanan, Jr., "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth-Year Course in Mathematics," _The Mathematics Teacher_, LVIII (March, 1965), pp. 223-25.

[114] _Ibid._, p. 225.

the literature evidenced the permanent effect of the movement for curriculum revision. The existence of the many different programs and the proliferation of articles, opinions, surveys, and studies showed the continuing need for the concerted effort emphasized throughout the literature.

Data presented indicated that a major problem is teacher preparation. The changes in certification requirements and the additions to college degree requirements showed an effect on pre-service education by these programs. The number of mathematics teachers who had a poor background for teaching secondary school mathematics in high school classrooms across the nation documented the need for in-service education programs. It was evident that the National Science Foundation and other sponsors of institutes for mathematics teachers were making a tremendous effort toward re-education of teachers and improvement of instruction in mathematics. The National Science Foundation programs have proved to be successful even though there has been criticism of certain institute practices. Evidence in the literature indicated these institutes were effective programs for in-service education and that such programs must be continued.

Evidence was also presented that in-service education must begin at the local level and must include much more than the institute programs. State and local programs built around well qualified instructors and consultants reported good advances toward solutions of in-service problems. Studies reviewed recommended well planned workshops and short courses under qualified instructors as essential parts of in-service programs.

Content studies and surveys of opinions concerning topics for eleventh and twelfth grade courses revealed the problems in the area of content. The major problems faced were identified as the selection of topics to be studied and the proper preparation of teachers for presentation of the topics.

From the review it was evident that much effort and financial support has been invested in the mathematics curriculum programs. The studies presented show also that continued effort is necessary. The third chapter here will present the setting, development of materials, and procedure of this study.

CHAPTER III

DESIGN AND PROCEDURE FOR THE STUDY

To fulfill the purpose of the study an identification of topics new to secondary school mathematics courses was necessary. From this identification of topics, one was selected by procedures to be outlined and a manual presenting an introduction of matrix algebra was developed. The next step in the study involved the use of the manual and an evaluation of the results of this use. The multiple nature of the problem involved the use of the following research designs.

I. DESIGN

Descriptive research design. A descriptive research design was necessary to identify topics and develop the manual. The survey studies design[1] applied to curriculum study programs, surveys of mathematics programs, and textbooks and materials in use in the eleventh and twelfth grades of the high schools of the Lamar Area School Study Council produced the identification of topics. More intensive documentary analysis of the treatment of matrix algebra by these programs and textbooks was designed for development of the manual.

[1]Deobold B. Van Dalen and William J. Meyer, Understanding Educational Research, (McGraw-Hill Book Company, New York, 1962), pp. 187-194.

Experimental research design. The design commonly called a one-group method was applied to the evaluation of the use of the manual with a selected sample of high school seniors.[2] Students were to be tested, exposed to the variable represented by the manual in matrix algebra, and tested again. Performances were to be evaluated in relation to other scores and data collected for student samples. An instrument for teaching the measurements was developed. The one-group design was used with teacher groups also, and the analysis included application of both descriptive and inferential statistics. The groups-within-treatment design for evaluation was constructed with an analysis of variance statistic applied on test performance results.

Procedure for the study. The procedures followed in this study will be presented under the following outline:

1. Setting for the study.
2. Procedure for identification of topics.
3. Procedure for development of the manual.
4. Procedure for evaluation.
5. Procedure for developing survey instruments.
6. Treatment of the data.
7. Summary.

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[2] *Ibid.*, pp. 230-232.

II. SETTING FOR THE STUDY

Textbooks and materials. In September, 1963, the new set of state adopted textbooks in mathematics was issued to the public high schools in Texas. These books covered material through the eleventh grade, with some books going into one semester of the twelfth grade. The Texas Education Agency had previously sent to each high school the bulletin on mathematics in grades seven through twelve.[3] School districts were offering courses at the twelfth grade level for which no state adopted textbooks were available. In the review of the literature in the second chapter a number of surveys reported on recommended units for these twelfth grade courses. The fourth chapter will present topical tabulations of these materials and surveys. Textbooks and materials examined were limited to the Texas state adopted textbooks and those additional books in use in high schools of the Lamar Area School Study Council. The new materials are presented in Table I.

Lamar Area School Study Council. The organization in February, 1962, of a council of schools to promote the common educational interests of the member schools marked

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[3] *Mathematics 7 Through 12*, Texas Education Agency (Austin, Texas, July, 1962), 189 pp.

TABLE I

LISTING OF NEW MATHEMATICS TEXTBOOKS AND MATERIALS

	Author and Title	Publisher	Date of Publication	State Adopted
1.	Mary P. Dolciani, S. L. Berman, W. Wooten Modern Algebra and Trigonometry	Houghton Mifflin Company	1963	Yes
2.	Alice L. Griswold, M. L. Keedy, J. F. Schacht Contemporary Algebra and Trigonometry	Holt, Rinehart, and Winston	1963	Yes
3.	R. E. Johnson, L. L. Lendsey, W. E. Slesnick, G. E. Bates Modern Algebra, Second Course	Addison-Wesley Company	1962	Yes
4.	Glen D. Vannatta, A. W. Goodwin, H. P. Fawcett Algebra Two, A Modern Course	Charles E. Merrill Company	1962	Yes
5.	A. M. Welchons, W. R. Krickenberger, H. R. Pearson Book 2, Algebra	Ginn and Company	1962	Yes
6.	School Mathematics Study Group Introduction to Matrix Algebra	Yale University Press	1960	No
7.	School Mathematics Study Group Intermediate Mathematics, Part I, II, III	Yale University Press	1960	No
8.	Robert C. Fisher and Allen D. Ziebur Integrated Algebra and Trigonometry	Prentice-Hall, Inc.	1958	No
9.	Carl B. Allendoerfer, Cletus O. Oakley Principles of Mathematics, Second Edition	McGraw-Hill Book Company	1963	No
10.	Theodore Herberg, James D. Bristol Elementary Mathematical Analysis	D. C. Heath Company	1962	No
11.	M. P. Dolciani, E. F. Beckenbach, A. J. Donnelly, R. C. Jurgensen, William Wooten Modern Introductory Analysis	Houghton Mifflin Company	1964	No
12.	Glen D. Vannatta, W. H. Carnahan, H. P. Fawcett Advanced High School Mathematics, Expanded Edition	Charles E. Merrill Company	1965	No

the official beginning of the Lamar Area School Study Council. Since the organization of the council, twelve additional school districts have joined, making a total of twenty-eight school districts in membership for 1965-66. Financial support for the study council was provided by the member school systems and by Lamar State College. Details are included to show the organization for in-service education. The list of school districts and other information including the number of students in average daily attendance, each district's share of the budget, and the teacher quota assignment are presented in Table II.[5] Local Industry demonstrated interest in Lamar Area School Study Council through contributions to be used at the discretion of the directors. Some workshops and conferences received joint financial support from other sources including the Texas Education Agency, Science Research Associates, and several textbook publishing companies.

The activities of the Lamar Area School Study Council in the area of mathemtaics curriculum and the improvement of instruction in mathematics were started during the 1962-63

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[4]History of the Lamar Area School Study Council, Mimeographed Report Prepared by the Staff of the Lamar Area School Study Council, 1964, p. 1.

[5]Third Annual Yearbook, Lamar Area School Study Council, (Lamar State College of Technology, Beaumont, Texas, 1965), p. 6.

TABLE II

LAMAR AREA SCHOOL STUDY COUNCIL MEMBERSHIP, COST, AND TEACHER QUOTA 1965-66

District	1965-66 ADA	District Share of Budget	Teacher Quota
Port Arthur	14,622	2,558.85	162
Beaumont	13,873	2,427.78	157
South Park	11,272	1,972.60	121
Port Neches	6,007	1,051.23	63
Orange	4,849	848.58	53
Nederland	4,606	806.05	48
Vidor	4,308	753.90	45
Silsbee	3,476	608.30	37
Jasper	2,816	492.80	31
Bridge City	2,254	394.45	25
Liberty	2,237	391.48	25
West Orange	1,673	292.78	19
Hardin-Jefferson	1,599	279.83	18
Kirbyville	1,501	262.68	17
Woodville	1,431	250.43	17
Little Cypress	1,412	247.10	17
Buna	1,149	201.08	14
Anahuac	1,079	188.83	13
Kountze	1,078	188.65	13
East Chambers	1,000	175.00	12
Hamshire-Fannett	915	160.13	12
Orangefield	793	138.78	10
Hull-Daisetta	734	128.45	10
Chance-Loeb	693	115.00	9
Warren	609	115.00	9
Deweyville	433	115.00	9
Evadale	311	115.00	9
Devers	169	115.00	9
Totals	86,899	$15,394.76	984

school year. Night workshops were sponsored with mathematics teachers from member high schools meeting for three hour workshops on four nights, two in the fall semester and two in the spring semester.[6] A summary of the activities of the council in mathematics is presented in Table III. The short courses listed were of two to three weeks duration and were offered for teachers of the primary through the eighth grades. Several of the conferences listed involved the authors of the textbooks being introduced in the Texas public schools. The television series was presented on Sunday afternoons over the television stations that were local to the area of the Lamar Area School Study Council. The workshops, held at night during the school year, covered three hours each meeting for a total of twelve hours of instruction.[7]

The development of the manual was aided by a trial presentation to a workshop meeting at Lamar College. The evaluations were conducted within member schools of the council and at workshops of the council. The one exception to this setting of the study was an evaluation of the manual conducted during class meetings of a graduate class in methods in secondary school mathematics at the University of Houston in the summer of 1965.

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[6]Third Annual Yearbook, Lamar Area School Study Council, p. 7.

[7]Ibid.

TABLE III

A SUMMARY OF ACTIVITIES OF THE LAMAR AREA SCHOOL STUDY COUNCIL IN MATHEMATICS 1962-65

Activity	Description	Date
Night Workshops	Three hour sessions with workshop director for elementary and secondary areas.	1962-63
Spring Conference	One day conference on Modern Mathematics (met on a Saturday 8 A. M. to 4 P. M.).	Spring 1963
Short Course	Modern Mathematics Short Course (non credit), elementary level.	Fall 1963
Television Series	"Modern Math for Parents," Instructor, Dr. John Creswell of the University of Houston (appeared on Sundays throughout the year).	1963-1964
Spring Conference	One day conference on Modern Mathematics offered primary section, elementary section, junior high section, and senior high section (Saturday, 8 A. M. to 3 P. M.).	Spring 1964
Summer Workshop	Modern Mathematics through the 8th grade (non credit).	Summer 1964
Summer Workshop	Modern Mathematics, elementary level (college credit given).	Summer 1964
Night Workshops	Mathematics for the Elementary grades met for four sessions, junior high mathematics for four sessions, and senior high mathematics for four sessions.	1963-1964
Night Workshops (Four sections)	1. Modern Mathematics in the Elementary School. 2. Modern Mathematics in the Junior High School 3. Advanced Junior High Mathematics. 4. Modern Senior High Mathematics.	1964-1965
Short Course	Modern Mathematics, Elementary Level, (college credit given).	Summer 1965

III. PROCEDURE FOR IDENTIFICATION OF TOPICS

Three basic sources were used to identify topics that met the criteria of being new to secondary school mathematics courses or being new or unfamiliar to secondary school mathematics teachers who must present the topics to students. The first source was a documentary analysis of textbooks and materials now in use in comparison with the state-adopted books furnished prior to the current books. The object of the comparison was to identify material in current texts that did not appear in previous texts. The second source was a topical analysis of recommendations by the various curriculum revision groups, such as the School Mathematics Study Group and the Texas Curriculum Studies. Studies and surveys of opinions of college and high school teachers about topics for eleventh and twelfth grade courses served as the third source for the identification. This procedure is reported fully in the fourth chapter of this study.

IV. PROCEDURE FOR DEVELOPMENT OF THE MANUAL

Reasons for election of matrix algebra from topics identified are presented in the fourth chapter. A manual was developed for introduction to the theory of matrices. (See Appendix for complete manual). A more intensive documentary analysis was conducted to determine the scope and sequence of a high school unit. A pilot presentation to seniors was conducted in 1964. The class had completed four years of college preparatory mathematics through the eleventh grade, and the students were taking a fifth year program in their senior year of high school. Based on that experience and the reactions of the regular teacher of the class, the manual was revised. In the spring of 1964, arrangements were made with the director of the mathematics workshop of the Lamar Area School Study Council to present the manual to one session to get the reactions from a group of teachers and to determine needed revisions of the manual. After further revision the manual was presented to workshop programs in 1965 and 1966 and to a graduate class in methods in 1965.

V. PROCEDURE FOR EVALUATION

Procedures to analyze the use of the manual involved use with teachers and students. Initial evaluation was with the use of the manual as a teaching guide for a unit with a group of twelfth grade students in a member high school of the Lamar Area School Study Council. In this evaluation a pre-test and a post-test were used in what Barnes has called a "before-after study with a single group."[8] Based upon the results of analysis reported in the fifth chapter a two-weeks

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[8]Fred P. Barnes, Research for the Practioner in Education (Department of Elementary School Principals, National Education Association, 1964), p. 61.

unit was presented. Four mathematics classes were selected for the experiment. Two groups were enrolled in fifth year or accelerated programs in mathematics, and two classes were regular fourth year college preparatory mathematics students. The classes were arranged so that one accelerated class and one regular class were combined to form a morning group for instruction, and the remaining two classes were combined to form an afternoon group. Eighty-four students were included in the two groups.

The second evaluation of the use of the manual involved three groups of mathematics teachers. Two in-service workshop programs of the Lamar Area School Study Council co-operated with the study. In a 1965 workshop each participant received a copy of the manual and two discussion or lecture sessions were conducted on the concepts in the manual. Six weeks after the close of the workshop participants were asked to complete a short questionnaire and a content survey test. By arrangement with the instructor, Dr. John Creswell of the University of Houston, the manual was presented to graduate students enrolled in a course of methods in secondary school mathematics. Pre-tests, post-tests, and questionnaires were used. A 1966 workshop presentation completed the sample of one hundred and eleven in-service teachers. Participants volunteered to complete the pre-test and the questionnaire at the first meeting, spend the three hours workshop time with the manual, and take the post-test. This completed the collection of data for the evaluation.

Student test scores were tabulated and descriptive statistics computed. Correlations with high school grade point averages and other mathematics scores were used to determine the validity and reliability of the test. Analysis of significant differences, gain on the tests, and an item analysis were used in this evaluation of the use of the manual. A number of statistical comparisons were made of subgroups based on variables revealed by the questionnaire. An analysis of variance technique was used to test hypotheses concerning the results with the teachers participating.

VI. DEVELOPMENT OF THE EVALUATION INSTRUMENTS

A descriptive survey instrument and a content survey instrument were both necessary for the collection of data in the study. The construction of a short questionnaire to gather background data on the teacher participants was necessary. In this questionnaire the teacher was encouraged to present opinions and recommendations concerning the manual. Since the questionnaire and the content test were given to participants at the same time, it was necessary to control the total amount of time for completing both instruments. Books on research in education were reviewed for points of consideration in the construction and administering of questionnaires. The questionnaires used in other studies were reviewed.[11]

From examination of the high school texts and materials covering matrix algebra, some ideas about the nature of questions on the content test for students were gathered. In the trial presentation of the unit to students in 1964 a ten problem test was given, but the subjective nature of scoring made it evident that an objective form of multiple choice items was necessary for data collection. Thorndike and Hagan discussed the theory and practice of construction of objective tests and included instructions for writing good multiple choice items.[12] After a review of the manual and preparation of a supply of items, a test of thirty responses with five choices on each was constructed.

The two instructors whose high school classes were selected for the experiment then screened and edited each item of the test. Recommendations were made that the pretest be given in two parts. The first part was to consist of eighteen questions concerning definitions, operations,

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[11]Oran Bright Bailey, "An Analysis of the Teaching Problems of Beginning Junior High School Science Teachers Within the Lamar Area School Study Council," (unpublished Doctor's dissertation, University of Houston, Houston, Texas, 1965).

[12]R. L. Thorndike and Elizabeth Hagen, Measurement and Evaluation in Psychology and Education, Second Edition, John Wiley and Sons, (New York, 1961), pp. 60-95.

and understanding of laws. The second part consisted of twelve questions and problems in matrix operations. By handing out the second part after the student had completed the first part, the items of the second section of the test could be written in problem notation and form that would test for understanding and application without giving away information that could affect the ability to select the correct response on the first part of the test.

In addition to the two regular instructors of the student classes, interviews were conducted with three people working in mathematics education. These individuals were experienced secondary school teachers and each had completed at least two National Science Foundation Summer Institutes in mathematics. They represented college mathematics work, a supervisor and department head, and a respected high school teacher. Each had participated in Lamar Area School Study Council workshops. Questionnaires and tests were studied and several revisions made to include suggestions agreed upon. The final form of the questionnaire included nine factual or personal data requests and two open end items. The test on content consisted of thirty multiple choice items. Means and standard deviations were calculated for pre-tests and post-tests. These are reported along with correlations, standard errors, and confidence limits in the fifth and sixth chapters. Validity and reliability coefficients are reported also.

VII. TREATMENT OF THE DATA

Data obtained by the experiment with students were analyzed and a number of statistical considerations determined. The test scores were collected, tabulated, and the means and standard deviations calculated. Tests for significance were made between the regular and accelerated student's results. The purpose of the evaluation of the use of the manual with students was to determine the feasibility of the topic for high school students, and to test the validity, reliability, and practicality of the testing instrument. Students' scores on the Scholastic Aptitude Test mathematical section were tabulated and product-moment correlations calculated. High school grade point averages for the students were collected and a correlation with matrix algebra test results determined. These correlations along with content analysis and item analysis were used to establish validity. Reliability of the test was calculated by the Spearman-Brown split halves method and the Kuder-Richardson Formula twenty-one. These and other results for students are reported in the sixth chapter.

Data obtained by the questionnaire were tabulated and teachers grouped according to the variables represented. Test results were collected and the means and standard deviations for various groupings calculated. The "t" test for significance in the difference in means was used on the pre-test for some variables within the sample of teachers. Correlations in the pre-test and post-test results were determined. The factors of teaching experience and preparation for mathematics teaching were selected as variables for further analysis. Data from questionnaires were used to classify teachers into the groups-within-treatment design. A null hypothesis was formulated, and the analysis of variance statistic used to test the hypothesis. Teachers were then re-classified into four groupings based upon the preparation for teaching as measured by the number of semester hours earned in mathematics. The analysis variance technique was applied to this groups-within-treatment design.

A purpose of the evaluation of the use of the manual in a teacher workshop was to determine the values placed on such workshops and such materials as represented by the manual by the participants. Open end items on the questionnaires were designed to collect responses for analyzing this purpose. The seventh chapter of this study reports the analysis of the experiment with mathematics teachers.

VIII. SUMMARY

This chapter has presented a description of the setting of the study, the instruments for the study, and the research procedures that were used in the collecting, processing, and reporting of the data. The setting involved the students and teachers of the high schools of the Lamar Area School Study Council and the textbooks and materials in use in eleventh and twelfth grade mathematics courses. Both descriptive and experimental research designs were employed in the study. Documentary analysis was applied to the problem of identifying topics and of developing the topic of matrix algebra for the study.

Necessary research instruments for data collection were constructed. A questionnaire and a content survey were developed. The experiment with the student group was used to develop and validate the content survey. Questionnaires collected data on a number of variables for each participating teacher. Treatment of the data involved both descriptive and inferential statistics. The experimental research design employed the one-group method of testing, exposure to variable and re-testing. This applied to the student groups and the teacher groups. The formation of a null hypothesis and the testing of the hypothesis by the analysis of variance produced an evaluation of the use of the manual.

The fourth chapter will present the analysis of textbooks and materials, surveys, and curriculum programs to identify the new topics.

CHAPTER IV

THE IDENTIFICATION OF NEW TOPICS AND THE SELECTION OF A TOPIC FOR FURTHER DEVELOPMENT

Three sources were examined to identify the topics in mathematics being treated in eleventh and twelfth grade programs. Surveys made of opinions of high school and college mathematics teachers concerning what topics should be included in high school programs were analyzed. The curriculum study programs that presented topical outlines for the last two years of high school mathematics were compared. Textbooks and materials were analyzed including the new state adopted texts and some materials not furnished by the state in use by the high schools in the Lamar Area School Study Council.

The suggestions for the type of documentary analysis recommended by Van Dalen[1] and by Rummel[2] are followed in this chapter. The purpose of the documentary analysis presented in the first three sections was to identify the units, both proposed and in practice, in the final two years

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[1]Deobold B. Van Dalen and William J. Meyer, Understanding Educational Research, (McGraw-Hill Book Company, York, Pennsylvania, 1962), pp. 192-193.

[2]J. Francis Rummel, An Introduction to Research Procedure in Education, Second Edition, (Harper and Row, New York, New York, 1958), p. 164.

of high school mathematics. The analysis was primarily concerned with the presence or absence of content material. The quantitative analysis conducted was not sufficient to determine the degree of treatment of concepts by each program or textbook. Synthesis of the three analyses led to identification of topics new to the high schools involved and necessary for in-service programs. The basis for selection of one of these topics for the development of the manual is reported in section four. A more detailed and qualitative analysis was necessary for the development of the manual for in-service education, and this is reported in the succeeding chapter.

I. IDENTIFICATION OF TOPICS FROM SURVEYS

Recent literature in mathematics education was found to be full of the opinions of greater and lesser authorities concerning what topics to present in high school. Individual opinions and articles by authorities were discarded and only surveys which reported the procedure of the survey, the group questioned, and the treatment of responses were included in this analysis. Four surveys were included and three are reported in a tabulation summarizing 1963, 1964, and 1965 surveys. (See Table IV).

Heads of the Departments of Mathematics of the colleges and universities in the state of Ohio were surveyed

TABLE IV

A SUMMARY OF THREE SURVEYS FOR IDENTIFYING TOPICS IN SENIOR HIGH SCHOOL MATHEMATICS

	1963, Brown[a]	1964, Hackett[b]	1965, Buchanan[c]
Question Asked in Survey	What topics should be included in the preparation of college-bound math students?	Please rate on a check-list the order of topics for a fifth year of high school mathematics.	Choose from the courses listed on the post card by order of choice the one you most prefer to follow one semester of elementary functions.
Responses	Thirty-Two Colleges in the State of Ohio	Forty Selected Colleges in the United States	One Hundred and Sixty-Nine Colleges in the United States
Topics Listed by Rank or Order of Highest Per Cent of Responses	1. Inequalities . . . 91% 2. Number Systems . . 79% 3. Analytic Geometry . . . . . 75% 4. Algebra of Sets . 75% 5. Elementary Functions . . . . 66% 6. Probability . . . 63% 7. Matrix Algebra and Determinants . . . 56% 8. Statistics . . . . 50% 9. Limit Concepts . . 22% 10. Vector Analysis . 19% 11. Integral Calculus 3% 12. Differential Calculus 3%	1. Absolute Value 2. Analytic Trigonometry 3. Trigonometric Functions 4. Inequalities 5. Algebraic Functions 6. Analytic Geometry 7. Exponential Functions 8. Logarithmic Functions 9. Mathematical Induction 10. Polar Coordinates 11. Determinants 12. Parametric Equations 13. Sets, Algebra of Sets 14. Partial Fractions 15. Probability with Statistical Inference 16. Sequences and Series 17. Limit Concept 18. Number Bases 19. Groups 20. Matrices 21. Non-Euclidean Geometry	FIRST CHOICES 1. Analytic Geometry 2. More Elementary Functions 3. Matrix Algebra 4. Probability and Statistics 5. Calculus 6. Modern Algebra 7. Combination FIRST AND SECOND CHOICES COMBINED 1. Analytic Geometry 2. More Elementary Functions 3. Probability and Statistics 4. Matrix Algebra 5. Modern Algebra 6. Calculus 7. Combination

a. Summary from article in The Mathematics Teacher, April, 1963.
b. Summary from article in Newsletter of the Sabine Area Council of Teachers of Mathematics, Port Arthur, Texas, March 23, 1965.
c. Summary from article in The Mathematics Teacher, March, 1965.

by Brown in 1963.[3] The stated purpose was to determine what mathematics should be taught to seniors in high schools following the completion of a course in trigonometry. Forty-nine colleges were mailed questionnaires, and thirty-two responded. Answers to four questions were tabulated and presented in Brown's report. In answer to a question asking if calculus should be included in a high school college preparatory program, sixty-nine per cent responded "no," six per cent said "maybe," and twenty-five per cent said "yes." Of those not reporting in the negative, Brown stated:

> Most of those saying "yes" or "maybe" wrote strong qualifying statements to support their answer. Most of these were similar to the one that stated, "Only if it is taught as a full-year, college level course by a college-caliber teacher, to a very select group for advanced placement purposes."[4]

In order to find which of the newer topics college mathematics teachers recommended, Brown asked the question, "What topics should be included in the preparation of our college-bound mathematics students?" Responses were in the form of lists of topics, and the tabulations of these lists are presented in Table IV. The topics selected by at least fifty per cent of the total who responded were as follows: inequalities, ninety-one per cent; number systems, seventy-

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[3]Robert S. Brown, "Survey of Ohio College Opinions With Reference to High School Mathematics Programs," The Mathematics Teacher, LVI (April, 1963), pp. 245-247.

[4]Ibid., p. 246.

nine per cent; analytic geometry, seventy-five per cent; algebra of sets, seventy-five per cent; elementary functions, sixty-six percent; probability, sixty-three per cent; matrix algebra and determinants, fifty-six per cent; and statistics, fifty per cent. Brown reported that the topics of inequalities, number systems, and algebra of sets were general topics introduced as early as the eighth grade or ninth grade of college preparatory mathematics.[5] Omitting these three then the remaining topics recommended by fifty per cent or more of the responses to Brown's survey were analytic geometry, elementary functions, matrix algebra and determinants, probability, and statistics.

Buchanan surveyed the colleges and universities in the United States which offered graduate programs in departments of mathematics, applied mathematics, or statistics.[6] Questionnaires were mailed to two hundred and thirty-three department chairmen, and responses were received from one hundred and sixty-nine, or seventy-three per cent. The original purpose of the survey was to collect data in conjunction with a study of the teaching of a unit on limits for the twelfth year course. The findings were reported as

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[5] Ibid., p. 247.

[6] O. Lexton Buchanan, Jr., "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth Year Course in Mathematics." The Mathematics Teacher, LVIII (March, 1965), pp. 223-225.

a doctoral dissertation by Buchanan at the University of Kansas.[7] Included with the questionnaire was a post card and instructions for filling in the card. Listed on the survey card were seven choices of topics with instructions to mark the first choice and the second choice for semester units in the twelfth grade following the completion of a semester course in elementary functions as recommended by the College Entrance Examination Board's mathematics program. First choices tabulated revealed the preference of analytic geometry first, matrix algebra and additional elementary functions received an equal number of selections for second, and probability and statistics for fourth. Combining the selections of first and second choices, the same four topics were preferred in the order of analytic geometry, elementary functions, probability and statistics, and matrix algebra.[8] (See Table IV).

Blank reported a survey of high schools offering advanced programs in mathematics and a survey of college mathematics department chairmen and directors of admission.[9]

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[7] O. Lexton Buchanan, Jr., "A Unit on Limits for the Twelfth Year Course in Mathematics." (unpublished Doctor's Dissertation, University of Kansas, Lawrence, 1964).

[8] Buchanan, "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth Year Course in Mathematics," op. cit., p. 225.

[9] William R. Blank, "A Survey Concerning Advanced Mathematics Curriculum," The Mathematics Teacher, LVII (April, 1964), pp. 208-211.

Of the 163 high schools mailed questionnaires, ninety-nine completed and returned them. Eighty-one schools reported offering the advanced mathematics courses. Twenty-two states were represented by the eighty-one schools that reported. The most popular course being offered in the high schools, according to Blank's summary of responses, was analytic geometry and calculus. The second was probability and statistics, and the third was elementary functions and matrix algebra.[10]

For the second part of the survey by Blank, more than four hundred questionnaires were mailed to chairmen of mathematics departments and to directors of admission for 205 colleges with at least one college from each state. Responses were received from sixty-three per cent of the individuals, but these represented over ninety per cent of the colleges with a total of 224 usable questionnaires analyzed for the study. Blank stated that his results agreed with those reported in the earlier study by Brown of opinions of Ohio colleges.[11] Blank summarized from his study that accelerated mathematics students in their senior year of high school should be offered a one semester course in analytic geometry and a one semester course in probability

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[10] _Ibid._, p. 209.

[11] Robert S. Brown, _op. cit._

and statistics. Blank did not report other topics beyond the first and second choices.[12]

Mathematics department chairmen of forty selected colleges were the subject of a survey by Harold Hackett, a National Science Foundation Institute participant.[13] While attending an institute at Montclair State College in New Jersey, Hackett mailed a checklist asking mathematics chairmen to rate a list of topics for a fifth year course in high school mathematics. A total of forty topics were listed, ranging from mathematics of finance and slide rule through topology and Boolean algebra. Ratings of the first twenty-one are presented in Table IV. Grouping these into similar course headings, the choices in the order of ratings were analytic geometry and analytic trigonometry, the study of functions, determinants and matrices, and probability and statistics.

The surveys presented showed the occurrence of a topic in a listing, and to a degree the relative importance of a topic was shown by the ratings, percentages, and choices. One survey offered seven choices of pre-selected topics, another offered forty choices of pre-selected topics, and

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[12] William R. Blank, *op. cit.*, p. 211.

[13] Harold Hackett, "Colleges Rank Order of Topics for a Fifth Year of High School Math." *Newsletter of the Sabine Area Council of Teachers of Mathematics.* (March 23, 1965). p. 3.

two called on the respondent to list topics. The tabulation presented amounts to a comparison of the 1963, 1964 and 1965 recommendations of college mathematics department chairmen concerning the topics needed by the college preparatory mathematics student in the last two years of high school.

II. IDENTIFICATION OF TOPICS FROM CURRICULUM STUDIES

Fraser reported summaries of thirteen mathematics curriculum study programs covering the elementary, junior high, and high school programs.[14] Statements and summaries of the work of each of the commissions, study groups, or curriculum projects were presented. Five of the programs listed general course titles or descriptions for grades eleven and twelve. Another publication presented outlines of some major programs as they existed in 1961.[15] One program reported materials for the eleventh and twelfth grades, and one stated that such material would be available in 1962. A third publication presented an analysis of eight curriculum revision projects. The committee conducting the analysis emphasized that it was not evaluating the various

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[14]Dorothy M. Fraser, Current Curriculum Studies in Academic Subjects, A Report Prepared for the Project on Instruction, (National Education Association, March, 1963), pp. 28-42.

[15]"Questions and Answers," The Revolution in School Mathematics, (National Council of Teachers of Mathematics, 1961), pp. 65-68.

experimental programs, but conducting an analysis within an established frame of reference.[16]

Examination of the three publications revealed the following studies with programs listed for the eleventh and twelfth grades: Commission on Mathematics of the College Entrance Examination Board, Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, University of Illinois Committee on School Mathematics, School Mathematics Study Group, and the Ball State Teachers College Experimental Program. The programs for these five groups are summarized in Table V. Comparison of grade eleven reports showed the course of intermediate mathematics, including the topics of advanced algebra and an introduction to trigonometry, recommended by four of the five studies. Grade twelve revealed a wider variation, but the topics of elementary functions, analytic geometry, matrix algebra, and probability and statistics were identified.

The Commission on Mathematics for the state of Texas presented a proposed program in 1959. One of the purposes of the study commission was to prepare outlines to indicate subject matter content desired by Texas Schools. These

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[16] Committee on the Analysis of Experimental Mathematics Programs, "An Analysis of New Mathematics Programs," (National Council of Teachers of Mathematics, Washington, D. C., 1963), p. 1.

TABLE V

SUMMARY OF COLLEGE PREPARATORY MATHEMATICS CURRICULUM STUDY PROGRAMS
FOR GRADES ELEVEN AND TWELVE

STUDY GROUP	PROGRAM FOR GRADE ELEVEN	PROGRAM FOR GRADE TWELVE
Commission on Mathematics of the College Entrance Examination Board (1959 Report)	Intermediate Mathematics - Real and complex numbers, algebra and elementary trigonometry centered around coordinates, vectors, complex numbers.	Three Alternative Programs: 1. Elementary functions and introductory probability. 2. Elementary functions and modern algebra. 3. Elementary functions expanded to two semesters.
Secondary School Curriculum Committee of the National Council of Teachers of Math. (1959 Report)	Algebra and Trigonometry.	Two one-semester courses selected from Probability and Statistics, Analytic Geometry, Mathematical Analysis, Study of Functions.
School Mathematics Study Group (SMSG) (1961 Report)	Intermediate Mathematics - Study of algebra, trigonometry, functions, vectors developed as mathematical system, coordinate geometry.	Two one-semester courses (After completion of topics from intermediate mathematics): 1. Elementary functions. 2. Introduction to Matrix Algebra.
University of Illinois Committee on School Mathematics (UICSM) (1962 Report)	(The Committee stresses the sequential nature of all of its units on high school mathematics. Last five generally covered in last two years). Unit 7: Mathematical Induction. Unit 8: Sequences. Unit 9: Exponential and Logarithmic Functions. Unit 10: Circular Functions and Trigonometry. Unit 11: Polynomial Functions and Complex Numbers.	
Ball State Teachers College Experimental Program (1962 Report)	Algebra II - Treating topics conventionally covered in intermediate and advanced algebra.	Pre-Calculus Mathematics - With emphasis on Analytic Geometry and Trigonometry.

outlines were to be presented to textbook committee personnel and to textbook publishers.[17] A comparison of the detailed outline of topics for grades eleven and twelve for the School Mathematics Study Group[18] and the Texas Curriculum Studies[19] revealed the strong influence of the national curriculum program on the mathematics curriculum outlines for Texas schools. (See Table VI).

Further analysis was necessary to identify clearly the placement of topics. Some variations were evident, and the ninth and tenth grade outlines for these two programs were compared. These showed a variation in the placement and sequence of some topics creating the variations in the eleventh and twelfth grades. The most obvious and influential of these was the coverage given topics in trigonometry during the ninth through the eleventh grades by the School Mathematics Study Group. This reduced the coverage needed in trigonometry in the twelfth grade program as shown in Table VI. The Texas program introduced logarithms and numerical trigonometry earlier than the twelfth grade but

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[17]Commission on Mathematics, "Texas Curriculum Studies, Report Number 3," (Texas Education Agency, July, 1959), p. 1.

[18]"Questions and Answers," _The Revolution in School Mathematics_, _op_. _cit_., pp. 66-67.

[19]Commission on Mathematics, "Texas Curriculum Studies Report Number 3," _op_. _cit_., pp. 65-69.

TABLE VI

COMPARISON OF SCHOOL MATHEMATICS STUDY GROUP AND TEXAS CURRICULUM STUDIES PROGRAMS FOR GRADES ELEVEN AND TWELVE

	SCHOOL MATHEMATICS STUDY GROUP	TEXAS CURRICULUM STUDIES
GRADE ELEVEN	<u>Intermediate Mathematics</u> 1. Number Systems, 2. Introduction to Coordinate Geometry, 3. Function Concept and the Linear Function, 4. Quadratic Functions and Equations, 5. Complex Number Systems, 6. Equations of First and Second Degree in Two Variables, 7. Systems of Equations in Two Variables, 8. Systems of First Degree Equations in Three Variables, 9. Logarithms and Exponents, 10. Introduction to Trigonometry, 11. The System of Vectors, 12. Polar Form of Complex Numbers, 13. Sequences and Series, 14. Permutations, Combinations and the Bionomial Theorem, 15. Algebraic Structures.	1. Structures of Number System, 2. Systems of Linear Equations and Inequalities, 3. Special Products and Factoring, 4. Functions and Fractional Equations, 5. Graphs, 6. Variation, 7. Powers, Roots, Irrational and Complex Numbers, 8. Logarithms and Numerical Trigonometry, 9. Structure of Algebra-Postulational, 10. Sets, Relations, Functions, 11. Linear and Quadratic Functions, 12. Progressions and Bionomial Expansion, 13. Permutations, Combinations, 14. Probability and Statistics.
GRADE TWELVE	<u>Elementary Functions</u> 1. Functions, 2. Polynomial Fractions, 3. Tangents to Graphs of Polynomial Functions, 4. Exponential and Logarithmic Functions, 5. Circular Functions. <u>Introduction to Matrix Algebra</u> 1. Matrix Operations, 2. The Algebra of 2x2 Matrices, 3. Matrices and Linear Systems, 4. Representation of Column Matrices as Geometric Vectors, 5. Transformations of the Plane	<u>Trigonometry</u> - Definitions, Graphing, Identities Equations and Inequalities, Relations and Tables, Triangle Solving, Complex Numbers, Applications, Logarithms. <u>Elementary Analysis</u> - Mathematical Proof, Determinants, Vectors, Complex Numbers, Series, Algebra of Polynomials, Analytic Geometry (st. line), Circles and Systems of Circle, Order Relations, Equations and Lucus, Elementary Functions, Introduction to Limits, Numerical Analysis, Matrices, Logic, Theory of Numbers, Modulo Arithmetic, Groups and Fields.

the major coverage of trigonometry remained in the twelfth grade program as shown. (See Table VI). The topics presented in the last year of the proposed Texas program for the eleventh grade and the twelfth grade elementary analysis course matched generally with the topics from the last part of the School Mathematics Study Group's eleventh grade and full twelfth grade program.

III. IDENTIFICATION OF TOPICS FROM TEXTBOOKS

The current and previous lists of Texas state adopted textbooks were examined. The textbooks currently in use were furnished to the state's public schools beginning in September of 1963. The previous contract was in force from September, 1955, until August, 1963. From the Catalogue of Current Adoption Textbooks, 1962-63 a list of previous adoption books for the second course in algebra was recorded.[20] (See Table VII). The 1964-65 catalogue provided the listing of current adoption textbooks for the eleventh grade course in algebra.[21] Information in the catalogues revealed that twelfth grade textbooks had not been revised, with the same textbooks being furnished in solid geometry

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[20] Catalogue of Current Adoption Textbooks, 1962-63, Bulletin 623, (Texas Education Agency, Austin, 1962), p. 31.

[21] Catalogue of Current Adoption Textbooks, 1964-65, Bulletin 650, (Texas Education Agency, Austin, 1964), p. 31.

TABLE VII

PREVIOUS AND CURRENT ADOPTION ALGEBRA II TEXTBOOKS AND THEIR USE IN THE LAMAR AREA SCHOOL STUDY COUNCIL HIGH SCHOOLS

Contract Period	Number of High Schools Using Book	Author and Title	Copyright Date	Publisher
1955	No responses	1. Aiken, *Algebra, Its Big Ideas and Basic Skills, Book 2*	1954	McGraw-Hill Company
	No responses	2. Fehr, *Algebra - Course 2*	1955	D. C. Heath Company
	No responses	3. Freilich, *Algebra for Problem Solving, Book 2*	1953	Houghton Mifflin
1963	4	4. Mallory, *Second Algebra*	1952	Singer Book Company
	15	5. Smith, *Algebra 2*, Texas Edition	1954	Row, Peterson Company
	(Total - 19)	(List taken from *Catalogue of Current Adoption Textbooks, 1962-63* published by The Texas Education Agency, Bulletin 623, page 31)		
1963	23	1. Dolciani, *Modern Algebra and Trigonometry*	1963	Houghton Mifflin
	No responses	2. Griswold, *Contemporary Algebra and Trigonometry*	1963	Holt, Rinehart, Winston
	No responses	3. Johnson, *Modern Algebra, Second Course*	1962	Addison-Wesley
1969	No responses	4. Vannatta, *Algebra Two, A Modern Course*	1962	C. E. Merrill
	2	5. Welchons, *Book 2 Algebra*	1962	Ginn and Company
	(Total - 25)	(List taken from *Catalogue of Current Adoption Textbooks, 1964-65* published by The Texas Education Agency, Bulletin 650, page 31)		

since 1955, and the textbooks in trigonometry since 1960.[22] The textbook division of the Texas Education Agency reported the adoption of a new trigonometry textbook scheduled for February of 1966 to be made available to schools in September, 1966. No textbooks were furnished or adopted for the elementary analysis course for the 1966-67 school year.

A survey of the high schools of the Lamar Area School Study Council was made to determine the textbooks in use for the second course in algebra. Mathematics teachers from some high schools reported by listing the books in use in their schools on a form provided them, and other responses were collected by telephone interview with school personnel. All schools reporting except one followed the college preparatory sequence of first year algebra, geometry, and second year algebra; so this placed the second year of algebra as the regular eleventh grade course. There are thirty-four high schools in the school districts which are members of the Lamar Area School Study Council. Twenty-five responses were tabulated in the survey of current textbooks, and twenty-three of the schools reported the textbook published by Houghton Mifflin Company as the basic text.[23] The other

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[22] Ibid., pp. 31-32.

[23] Mary P. Dolciani, Simon L. Berman, and William Wooten, Modern Algebra and Trigonometry, (Houghton Mifflin Company, Boston, 1963), 658 pp.

two schools were using the publication by Ginn and Company.[24] An effort was made to identify the previous textbook for the second year algebra course. A total of nineteen responses was recorded and the majority of these were from memory of previous texts, and the accuracy was noted as questionable. Of the nineteen responding, fifteen reported using the same textbook, and the other four were using a second book. The lists of second year algebra textbooks and the results of the survey were presented in Table VII.

Comparisons of the content by unit titles of the most frequently used textbook from the current and the previous lists were made to determine new topics in the textbook issued to the student beginning in September, 1963. The differences in terminology presented a difficulty in comparison by chapter titles only. (See Table VIII). The relative importance of a topic and the depth of treatment of a selected topic were not revealed by the unit outlines. The authors presented a plan for using the out-of-adoption textbook in the following statements:

> Chapters I-VI review the fundamentals of algebra with certain additional topics to enlarge the student's mathematical concepts, . . .

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[24] A. M. Welchons, W. R. Krickenberger, Helen R. Pearson, Algebra, Book Two, (Ginn and Company, Boston, 1962), 591 pp.

> The following plan illustrates possible selections of material for study in the time periods designated.
>
> First half of the year: Chapters I-VIII. (For a minimum course, starred topics and the exercises beyond those designated as basic may be omitted.)
>
> Second half of the year: Chapters IX-XII, and either Chapter XIII or Chapter XIV. (Starred topics and special exercises may be omitted in a minimum course.)[25]

The first half of the year program presented was a review except for the topics of the unit on powers and roots, and the unit on logarithms and the slide rule. Some additional concepts were added to the six chapters reviewing elementary algebra. The second half presented the topics of numerical trigonometry, quadratic functions and equations, systems of quadratic equations, series, and then a choice of either rates of change or permutations, combinations, and probabilities.

The teachers' manual for the Houghton Mifflin textbook presented four recommendations for use of the text in 180 day courses. Each recommendation covered 170 days of treatment of the topics plus a ten day review at the close of the year. In each course some chapters were omitted, and these were recommended for coverage in the twelfth grade. The four recommendations were as follows:

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[25]C. A. Smith, W. F. Totten, H. R. Douglass, Algebra 2 (Row, Peterson and Company, Evanston, Illinois, 1954), p. v.

SUGGESTED 180 DAY TIME SCHEDULE (INCLUDING TESTING)[26] *

Chapter	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Option I	15	15	15	15	18	18	18	18	18	20	0	0	0	0	0	0
Option II	12	12	14	12	15	12	15	15	15	18	15	15	0	0	0	0
Option III	10	10	13	10	15	12	15	15	15	18	0	0	9	9	9	10
Option IV	10	10	12	10	13	12	15	15	15	18	15	15	10	0	0	0

*A final ten day review included on each course.

Option one in the table was a minimal course with option two being an extended course covering trigonometry. The third option presented an enriched course in algebra, and the final option recommended was an enriched course in algebra and trigonometry.

A comparison of minimal courses from the two books identified the new topics of sets, number systems, study of functions, trigonometry, and complex numbers. A program which presented an enriched course in algebra identified the additional new topics of binomial expansions, polynomial functions, matrices and determinants, and a much stronger treatment of probability. (See Table VIII).

Five state adopted textbooks were provided for the eleventh grade mathematics program. Analysis of one of these with the previous state adopted textbook identified

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[26] "Teachers' Manual," Modern Algebra and Trigonometry, (Houghton Mifflin Company, Boston, 1963), p. 6.

TABLE VIII

COMPARISON OF CHAPTER TOPICS OF PREVIOUS AND CURRENT SECOND YEAR ALGEBRA TEXTBOOK

Current Textbook (Dolciani, Berman, Wooten, *Modern Algebra and Trigonometry*, Houghton Mifflin Co.)	Previous Textbook (Smith, Totten, Douglass, *Algebra 2*, Row, Peterson and Company)
Chapter Titles	Chapter Titles
1. Sets of Numbers; Axioms	1. Fundamental Operations--Review
2. Open Sentences in One Variable	2. First Degree Equations and Problems
3. Systems of Linear Open Sentences	3. Special Products and Factoring
4. Polynomials and Factoring	4. Fractions
5. Rational Numbers and Expressions	5. Functional Relationships
6. Relations and Functions	6. Systems of Linear Equations
7. Irrational Numbers and Quadratic Equations	7. Powers and Roots
8. Quadratic Relations and Systems	8. Logarithms and the Slide Rule
9. Exponential Functions and Logarithms	9. Numerical Trigonometry
10. Trigonometric Functions and Complex Numbers	10. Quadratic Functions and Equations
11. Trigonometric Identities and Formulas	11. Systems of Quadratic Equations
12. The Circular Functions and Their Inverses	12. Series
13. Progressions and Bionomial Expansions	13. Permutations, Combinations, and Probability
14. Polynomial Functions	14. Rates of Change
15. Matrices and Determinants	
16. Permutations, Combinations, and Probability	

some new topics. An examination of the other four textbooks was necessary to determine that the differences found were consistent with other current textbooks. Using the content of the book by Dolciani, Berman, and Wooten[27] as the reference list for topics and units included, the analysis was made and the results are reported in Table IX. From the analysis the emphasis on the study of functions, complex numbers, matrices and determinants, probabilities, and systems of numbers and equations was identified. Variations were found in the content and approach of the five books. Several books varied as to the amount of detail and depth of coverage of specific topics, but the tabulation showed general agreement on the topics covered. Two textbooks in particular were almost identical in the material presented. A further analysis of the teachers' manuals revealed very similar time schedules for the presentation of the topics, both in average classes and for enriched programs.[28,29]

A survey of the high schools of the Lamar Area School Study Council was made to identify the textbooks in use for twelfth grade college preparatory mathematics. Some reported

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[27]Mary P. Dolciani, et. al., _op. cit._

[28]"Teachers' Manual," _Modern Algebra and Trigonometry_, _op. cit._

[29]Glen D. Vannatta, A. W. Goodwin, H. P. Fawcett, "Teachers' Guide," _Algebra Two, A Modern Course_, (Charles E. Merrill Books, Columbus, Ohio, 1962), pp. 4-5.

TABLE IX

TOPICAL ANALYSIS OF TEXAS STATE ADOPTED TEXTBOOKS FOR SECOND COURSE IN ALGEBRA

Dolciani, Berman, Wooten, Modern Algebra and Trigonometry, (Houghton Mifflin Company).	Welchons, Krickenberger, Pearson, Algebra, Book 2, (Ginn Company).	Griswold, Keedy, Schacht, Contemporary Algebra and Trigonometry, (Holt, Rinehart, Winston).	Vannatta, Goodwin, Fawcett, Algebra 2, (C. E. Merrill).	Johnson, Lindsey, Slesnick, Bates, Modern Algebra, Second Course, (Addison-Wesley).
Sets of Numbers, Axioms	No set notation	x	x	x
Open Sentences in One Variable	x	x	x	x
Systems of Linear Open Sentences	x	x	x	x
Polynomials and Factoring		x	x	x
Rational Numbers and Expressions	"Fractions"	x	x	x
Relations and Functions	x	x	x	x
Irrational Numbers	"Complex Fraction"	x	x	x
Quadratic Equations	x	x	x	x
Quadratic Relations	x	x	x	x
Exponential Functions and Logarithms	Computational	x	x	x
Trigonometric Functions	x	x	x	x
Complex Numbers	x	x	x	x
Trigonometric Identities		x	Very light	
The Circular Functions and Inverses		x	x	
Progressions and Binomial Expansion	x	x	x	x
Polynomial Functions	Higher Degree Equations	x	x	x
Matrices and Determinants	Determinants Only		x	
Permutations, Combinations and Probability	x		x	x
Additional Topics Treated in Other Texts Comments	Rates of Change, Statistics	Vectors, Heavy on Trigonometry	Vectors	Vectors

the use of the enrichment topics for the eleventh grade state adopted texts. From this survey the seven textbooks reported in the previous chapter were identified. (See Table I). All the texts for courses of elementary analysis, probability and statistics, advanced mathematics, and similar titles for the last one or two semesters of the twelfth grade program involved one or more of these textbooks. They were listed as follows:

1. Mary P. Dolciani, E. F. Beckenbach, A. J. Donnelly, R. C. Jurgensen, and W. Wooten, Modern Introductory Analysis, (Houghton Mifflin Company, Boston, 1964).

2. Glen D. Vannatta, W. H. Carnahan, H. P. Fawcett, Advanced High School Mathematics, (Charles E. Merrill Books, Columbus, Ohio, 1965).

3. Robert C. Fisher and Allen D. Ziebur, Integrated Algebra and Trigonometry, (Prentice-Hall Company, Englewood Cliffs, New Jersey, 1958).

4. Carl B. Allendoerfer and Cletus O. Oakley, Principles of Mathematics, Second Edition, (McGraw-Hill Book Company, New York, 1963).

5. School Mathematics Study Group, Elementary Functions, (Part 1) and (Part 2), Yale University Press, New Haven, Connecticut, 1960).

6. School Mathematics Study Group, Introduction to Matrix Algebra, (Yale University Press, New Haven, Connecticut, 1960).

7. Theodore Herberg and J. D. Bristol, Elementary Mathematical Analysis, (D. C. Heath Company, Boston, 1962).

Topical comparisons of these materials are presented in Table X. The first column listed general headings for units as found in the earlier analysis of surveys, and the

TABLE X

TOPICAL ANALYSIS OF NON-STATE ADOPTED TWELFTH GRADE TEXTS IN USE IN HIGH SCHOOLS OF THE LAMAR AREA SCHOOL STUDY COUNCIL

General Topics	Detailed Lists Within General Topics	Modern Introductory Analysis Houghton Mifflin	Advanced High School Mathematics C. E. Merrill	Integrated Algebra and Trigonometry Prentice-Hall	Principles of Mathematics McGraw-Hill	S.M.S.G. 1. Elementary Functions 2. Matrix Algebra	Elementary Mathematical Analysis D. C. Heath
Analytic Geometry	Straight Line (Plane)	x	x		x		x
	Second Degree (Circle, Ellipse, etc.)	x	x		x		
	Transformations	x	x		x		
Elementary Functions	Linear Relations	x	x	x	x	x	x
	Circular, Trigonometric Exponential, Logarithmic	x	x	x	x	x	x
	Polynomial	x	x	x		x	x
Matrix Algebra	Matrices	x	x	x	2x2 only	x	
	Determinants	x	x	x		x	
	Vector Approach		x			x	
Probability and Statistics	Permutations	x	x	x	x		x
	Combinations	x	x	x	x		x
	Probabilities	x	x	x	x		x
	Statistics		x				
Other Topics Treated (Some are included in above grouping in some texts but checked again here)	Sets Algebra of Sets	x	x		x		x
	Number Systems Complex Numbers	x	x	x	x		x
	Limit Concepts Sequences, Series	x	x	x	x		
	Modern Algebra Fields, Groups	x			x		x
	Mathematical Induction	x		x			x
	Vectors	x	x				x
	Inequalities	x	x	x			x
	Systems of Equations			x	x		x

second column presented detailed listings within the general headings. The next six columns are used to compare the topics offered in each text. The School Mathematics Study Group texts are combined into one column. The type of treatment or emphasis placed on the topic was not identified by this comparison. Van Dalen stated that this type of documentary analysis "may not accurately reveal the cruciality or importance of the items being analyzed."[30] The topics presented by materials in use in the twelfth grade programs of the high schools of the Lamar Area School Study Council were identified by this analysis.

Topics revealed in Table X were compared with the previous textbooks for the twelfth grade. The fusion of solid geometry and plane geometry courses and the placement of this fused geometry at tenth grade level caused the 1949 edition solid geometry textbook[31] to be dropped from the twelfth grade program. The state-adopted textbooks providing for the one semester course in trigonometry since 1960 were exemplified by the book authored by Welchons and Krickenberger.[32] Content of the book, first copyrighted in

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[30]Deobold Van Dalen, op. cit. pp. 193-194.

[31]Raleigh Schorling, Rolland Smith, and John Clark, Modern School Solid Geometry, (World Book Company, Yonkers-on-Hudson, New York, 1949). 256 pp.

[32]Catalogue of Current Adoption Textbooks, 1964-65, op. cit., p. 32.

1954, was presented by the following list of units:

> 1. Functions of an Acute Angle, Solution of Right Triangles, 2. Approximate Numbers, Logarithms, 3. The Slide Rule, 4. Solutions of Right Triangles by Logarithms, 5. Trigonometric Functions of Any Angle, 6. Radian Measure Mil Measure, 7. Line Values and Graphs of Functions, 8. Fundamental Relations, 9. Functions of Two Angles, 10. Oblique Triangles, Forces and Vectors, 11. Inverse Functions, 12. Complex Numbers and Hyperbolic Functions, 13-15. Brief Presentations of the Right Spherical, General Spherical, and Terrestrial Triangles.[33]

Emphasis in the content was on numerical trigonometry. Comparison of the materials on the list with those presented in Table X revealed the new topics for the twelfth grade program. Generally, the seven texts showed the new topics to be analytic geometry, matrix algebra, probability and statistics, and sets. Much more thorough treatment was given to the units and topics that covered complex numbers and the study of functions.

Since new textbooks for twelfth grade trigonometry were in the process of adoption by the state of Texas, a sample copy of the textbook in the Houghton Mifflin series was examined. The state textbook committee unanimously selected this as one of the five textbooks during the fall of 1965.[34]

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[33] A. M. Welchons and W. R. Krickenberger, Trigonometry With Tables, (Ginn and Company, Boston, 1957), p. vii.

[34] "Official Report on Books and Teaching Aids Listed in the Subjects and Grades Included in Proclamation No. 41 of the State Board of Education," (a letter of October 22, 1965, from State Textbook Committee member Miss Lera McFarland, Beaumont, Texas).

Since ninety-two per cent of the high schools of the Lamar Area Study Council were using the eleventh grade textbook from this series (twenty-three out of twenty-five reported in Table VI), this book by Wooten, Beckenbach, and Dolciani was included.[35] Topics presented in the book were listed as follows:

> 1. Sets, Relations, and Functions; 2. Circular Functions, Identities; 3. Graphs of Circular Functions, Applications; 4. Inverse of Circular Functions; 5. Trigonometric Functions, Solution of Triangles; 6. Vectors, Applications, Polar Coordiantes; 7. Complex Numbers, Polar Representation of Complex Numbers and Vectors; 9. Infinite Series and Trigonometric Functions; Appendix A. Using Logarithms; Appendix B. Spherical Trigonometry.[36]

The approach and content of the book agreed with the materials of Table X. The comparison with Table X and with the previous state adopted textbooks verified the identification of the new topics.

IV. SELECTION OF A TOPIC

Previous sections reported the analysis of surveys, curriculum studies, and textbooks. A number of topics in mathematics were revealed as receiving a new or altered emphasis in high school courses. From the surveys the general

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[35]William Wooten, Edwin F. Beckenbach, and Mary P. Dolciani, Modern Trigonometry, (Houghton Mifflin Company, Boston, 1966), 415 pp.

[36]Ibid., pp. v-vii.

headings of analytic geometry, elementary functions, matrix algebra, and probability and statistics were recommended. From the eleventh and twelfth grade curriculum studies, the same recommendations were emphasized. Analysis of the textbook materials revealed these topics in the eleventh and twelfth grade textbooks; and, in addition, new treatments or new approaches to the study of complex numbers, sets, and number systems were presented. To satisfy the purpose of developing and evaluating a manual to be used for in-service education, a topic was selected from those identified above.

The School Mathematics Study Group so strongly recommended the areas of elementary functions and matrix algebra that writing teams were instructed to prepare a semester unit in each topic for their twelfth grade course. Participation in a National Science Foundation Institute for high school mathematics at Washington State University, Pullman, Washington,[37] during the summer of 1960 and the summer of 1961 emphasized the need for the education of high school mathematics teachers in the theory of numbers, systems of equations and numerical analysis, higher geometry, and linear algebra. The study of matrix algebra, which included

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[37]Dr. S. G. Hacker, Director, Summer Institute for High School Mathematics Teachers, Washington State University, Pullman, Washington (an eight weeks program sponsored by the National Science Foundation).

the properties and operations of matrices and determinants, was included in the system of equations and numerical analysis, and also in the study of linear algebra which emphasized the treatment of vectors and vector spaces.

A third influence on the selection of the topic was an experience meeting with one session of a workshop for in-service education of mathematics teachers in the spring of 1964. The workshops, sponsored by the Lamar Area School Study Council for teachers of member high schools, met for four sessions and at the fourth session some ideas about matrix algebra were presented. In a short evaluation discussion with the twenty-six teachers present, it was learned that only one had received any instruction in matrix algebra or had presented a unit in matrix algebra to high school students, although all twenty-six recognized the presence of the topic in the eleventh grade materials of the current state adopted textbooks which they were using with their high school classes. (See Table VIII).

Following this experience, a unit in matrix algebra was presented to one class of advanced mathematics students in a member high school of the Lamar Area School Study Council. The results confirmed the interest in the use of matrix algebra as the selected topic. The committee on Analysis of Experimental Programs made the following statement:

> At the twelfth grade level, the topic of matrices is an excellent choice for starting the student in an easy way on the subject of linear algebra. He should at least be introduced to the subject before going to college.[38]

Fulfillment of this statement required that in-service teachers be given some preparation and sufficient encouragement and confidence to cause the recommended unit in matrix algebra to be presented to high school students.

V. SUMMARY

A documentary analysis for the purpose of identifying the new topics in mathematics and selecting a topic for further development was presented in this chapter. A number of new topics and new approaches to topics for high school mathematics were identified from surveys, experimental curriculum study programs, and the textbooks examined. Chief among these were analytic geometry, probability and statistics, elementary functions, and matrix algebra for the twelfth grade program. The eleventh grade program analysis identified systems of equations, matrix algebra, sets, number systems, and an increased emphasis on the study of functions, complex numbers, and trigonometry.

The topic of matrix algebra was selected for the development of a manual for presentation of a topic for in-service

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[38] Committee on Analysis of Experimental Programs, *op. cit.*, p. 48.

education. A more detailed analysis of content and treatment of concepts was necessary for developing the manual. The following chapter reports this continued documentary analysis and the development of the manual and the survey forms used in the evaluation of the manual.

CHAPTER V

THE DEVELOPMENT OF A MANUAL FOR MATRIX ALGEBRA AND DEVELOPMENT OF SURVEY INSTRUMENTS

In the preceeding chapter nine textbooks were found to present some concepts of matrix algebra, which included matrices and determinants. Results of further documentary analysis to determine treatment of concepts within the topic are reported in the succeeding section. Procedures for development of a manual introducing matrix theory are reported, followed by presentation of some procedures for evaluating the use of the manual.

I. ANALYSIS OF TREATMENT OF MATRIX ALGEBRA IN HIGH SCHOOL TEXTS

The analysis reported here was designed to determine the specific development of the topic of matrix algebra as presented in the high school student materials. The recommended time schedule for presenting the topic and the specific list of concepts covered indicated the limits for the coverage of matrix algebra. Textbooks were not evaluated by the analysis, but the quantitative and qualitative treatment of the topic was analyzed.

Recommended time schedule. Comparisons of some recommended presentation times are reported in Table XI. Eight different treatments were summarized in the time schedule and the recommended time ranged from three class days to fifteen class days. The majority showed nine to ten class days or approximately two weeks for a unit in matrix algebra at either the eleventh or the twelfth grade. The School Mathematics

TABLE XI

COMPARISON OF PROPOSED TIME SCHEDULE FOR UNITS ON MATRIX ALGEBRA

Number, Author, Title, Publisher	Title of Unit	Grade Placement	Time Proposed
1. Vannatta, Goodwin, Fawcett, Algebra Two, A Modern Course, Charles E. Merrill Books, Inc. Columbus, Ohio, 1962	Chapter 13: Matrices and Determinants	11 Second Semester	Two weeks
2. Dolciani, Berman, Wooten, Modern Algebra and Trigonometry, Houghton Mifflin Company, Boston, 1963	Chapter 15: Matrices and Determinants	11 Second Semester	Nine class days
3. Welchons, Krickenberger, Pearson, Algebra, Book Two, Ginn and Company, Boston, 1962	Chapter 16: Determinants	11 Second Semester	No proposed time
4. Vannatta, Carnahan, Fawcett, Advanced High School Mathematics, Expanded Edition, Charles E. Merrill Books, Inc., Columbus, Ohio, 1965	Chapter 16: Matrices and Vectors	11 First or Second Semester	Approximately two weeks
5. Dolciani, Beckenbach, Jurgensen, Donnelly, Wooten, Modern Introductory Analysis, Houghton Mifflin Company, Boston, 1964	Chapter 13: Analytic Geometry and Matrices	12 Second Semester	Fifteen days on total chapter--8-10 on matrices
6. Fisher, Ziebur, Integrated Algebra and Trigonometry, Prentice-Hall, Englewood Cliffs, New Jersey, 1958	Chapter 7: Systems of Equations	12 First or Second Semester	No proposed time about 70% on matrices
7. Allendoerfer, Oakley, Principles of Mathematics, Second Edition, McGraw-Hill Book Company, New York, 1963	Unit 3.8: 2x2 Matrices	12 First Semester	Brief introduction (3 days)
8. Wooten, Beckenbach, Dolciani, Modern Trigonometry, Houghton Mifflin Company, Boston, 1966	Chapter 8 Matrices	12 First Semester	Seven class days

Study Group semester course was not included in the table. The recommended time schedule for this course was as follows:

Chapter I	Matrix Operations--two weeks
Chapter II	The Algebra of 2x2 Matrices--four weeks
Chapter III	Matrices and Linear Systems--two weeks
Chapter IV	Representation of Column Matrices as Geometric Vectors--three weeks
Chapter V	Transformations of the Plane--three weeks
Appendix	Research Exercises--four weeks.[1]

The material presented in this course was exhaustively detailed and involved such proofs as the ring and group properties of sub-sets of matrix algebra. An eighteen weeks' program well afforded this type of approach, but the materials of the eight textbooks recommended the inclusion of a unit of approximately two weeks only in the high school courses.

Analysis of content. A qualitative analysis of the content of the eight textbooks revealed the concepts covered by the units. (See Table XII). The greatest variations in approach were a unit on determinants[2] and a unit on two by two matrices examined as a non-commutative ring presented in another textbook.[3] Seven of the eight textbooks

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[1]School Mathematics Study Group, Introduction to Matrix Algebra, Commentary for Teachers, (Yale University Press, 1962), pp. iv-v.

[2]A. M. Welchons, W. R. Krickenberger, Helen Pearson, Algebra, Book Two, (Ginn and Company, Boston, 1962), pp. 418-435.

[3]C. B. Allendoerfer, C. O. Oakley, Principles of Mathematics, Second Edition, (McGraw-Hill Book Company, New York, 1963), pp. 101-104.

TABLE XII

ANALYSIS OF CONCEPTS PRESENTED IN EIGHT TEXTBOOKS OF TABLE XI IN LIMITS OF MATRIX ALGEBRA

Book Number (From Table XI)	1	2	3	4	5	6	7	8
Matrices								
Terminology, Definitions	x	x		x	x	x	x	x
Equality of Matrices	x	x		x	x	x		x
Addition, Additive Identity	x	x		x	x	x		x
Scalar Multiplication	x	x		x	x	x		x
2x2 Matrices	x	x		x	x	x	x	x
Basic Properties, Operations	x	x		x	x	x	x	x
Multiplication	x	x		x	x	x	x	x
Identity Matrix	x	x		x	x	x	x	x
The Inverse of a Matrix		x		x	x	x	x	x
Determinant of 2x2	x	x	x	R	x	x	x	x
Invertible Matrices		x		x	x		x	x
Higher Matrices		x		x	x	x		
Properties		x		x	x			
Transformations		x		x	x			
Multiplication		x		x	x			
Determinants	x	x	x	R	A	x		
Third Order of Calculation	x	x	x	R		x		
Expansion by Minors	x	x	x	R		x		
Properties	x	x	x	R		x		
Cramer's Rule	x	x	x	R		x		
Special Treatments				x	x			x
Complex Numbers				x	x			x
Rotations				x	x			x
Matrix Solution of Systems of Linear Equations		x		x	x	x		x
Vectors and Matrices				x	x			x

x Indicates treatment of the concept in the textbook.

R Indicates brief review only, presuming prior knowledge.

A No review presented, but background knowledge assumed.

presented complete treatment of the algebra of two by two matrices. The concepts of properties and relations were presented before introducing the operations of addition, scalar multiplication, and multiplication. Additive and multiplicative identities were presented and the determinant function defined. Most texts presented a proof of the multiplicative inverse for two by two matrices, and then presented some applications with solutions of linear equations. Four of the textbooks presented thorough treatments of determinants and developed the properties and the process of expansion by minors. Two of the texts presumed a prior knowledge of determinant operations. The application of Cramer's Rule with systems of equations was presented in five of the textbooks. (See Table XII).

Three of the textbooks listed in Table XI and Table XII branched into some special treatments of problems in trigonometry and analytic geometry using matrix applications. Units of complex numbers, trigonometric functions and rotations, and problems with vectors were treated in matrix theory. Matrices of higher than the second order were introduced by four of these high school textbooks, although none carried the concept of inverses of matrices beyond work with second order matrices. The analysis of Table XII indicated that seven texts generally agreed on the treatment of concepts through the algebra of two by two matrices. Beyond that there were several approaches to more advanced mathematical applications of matrix theory.

II. DEVELOPMENT OF THE MANUAL

The Mathematical Association of America stated that a mathematics teacher at a particular level must have an understanding of the mathematics that will be covered in succeeding courses by the students. In recognizing that new topics such as matrix theory were being included at the senior high school level, a publication recommended that a teacher have a "grasp of mathematics beyond the nominal content" of the course or level being taught.[4] Creswell summed up this idea of the need for the teacher to understand both concepts and processes thoroughly when he stated, "Teachers cannot teach what they themselves do not understand."[5] With these thoughts applied in conjunction with the analysis of section one of this chapter, the content of the manual designed to be used for in-service education for high school mathematics teachers was planned.

Following the pilot presentation in a workshop situation in 1964, the decision was made that the manual would have to be so written as to presume no advance knowledge of matrices or determinants. In the unit taught with the class of advanced mathematics students in 1964, it was evident that some explanations concerning applications and reasons for including matrix algebra in high school mathematics were needed. The manual started with an introduction that included some history and applications for the topic. (See Appendix for manual). The second section presented some literature and textbooks for teachers to use as

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[4]Committee on the Undergraduate Program in Mathematics, "A Summary of Recommendations for the Training of Teachers in Mathematics," (Mathematical Association of America, January, 1961), pp. 5-7.

[5]John L. Creswell, "The Competence in Arithmetic of Prospective Georgia Elementary Teachers," The Arithmetic Teacher, (April, 1964), p. 248.

reference and supplementary materials. The first two sections were designed to show the teacher in an in-service program some importance of the topic, and also to help the teacher with background material in matrix theory during the introduction of matrix algebra to high school students. Applications in the social and industrial sciences as well as in mathematics and engineering were presented.

Terminology and basic definitions. Symbols and basic definitions used in the manual were determined to be consistent with those used in the high school textbooks analyzed. Two textbooks used in college courses in National Science Foundation Summer Institutes in 1960 and 1961 were reviewed for consistency here also.[6,7] These two books were used for determining some of the more advanced concepts included in the manual. Two additional recent books were reviewed to check further the consistency of terms and definitions. A book by Meserve, Pettofrezzo, and Meserve designed to prepare students for a course combining analytic geometry and calculus included a chapter on the algebra of matrices.[8] The second book was a 1965 publication designed for advanced mathematics students in the senior year, or for college algebra or a unified college freshman mathematics course. This book also included a chapter on

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[6] D. C. Murdoch, Linear Algebra for Undergraduates, (John Wiley and Sons, Inc., New York, 1957), 239 pp.

[7] Kaiser S. Kunz, Numerical Analysis, (McGraw-Hill Book Company, Inc., New York, 1957), 381 pp.

[8] Bruce E. Meserve, A. J. Pettofrezzo, Dorothy T. Meserve, Principles of Advanced Mathematics, (L. W. Singer Company, Syracuse, New York, 1964), pp. 661-729.

vectors and matrices, and both books agreed with the terminology and definitions for the manual.[9]

Concepts presented in manual. Presentation of the algebra of matrices started with the equality relation, addition operation, and scalar multiplication. Ideas and examples were presented for matrices of second and third order, and also explained for a matrix of any order. (See Appendix for manual). After introduction of matrix multiplication, review and comparison with the laws and properties of our number system, such as the commutative, associative, and distributive properties for addition and multiplication were presented for matrix operations.

The matrix solution of systems of linear equations, using the properties of row transformations, was presented. Since the determinant function was a part of the understanding necessary for working with inverses of matrices, the section on determinants was presented next. The schematic method of calculation of a third order determinant was shown to be consistent with the expansion by minors concept for calculating the value of a determinant. The direct method of solution of unknowns in a system of equations by the method of "Cramer's Rule" was demonstrated. (See manual in Appendix). Inverses of matrices were then presented, first for the two by two matrix. Using a method called the "elimination method" by Kunz, the inverse of a higher matrix was explained.[10] The

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[9]C. B. Allendoerfer and C. O. Oakley, Fundamentals of Freshman Mathematics, (McGraw-Hill Book Company, New York, 1965), pp. 159-186.

[10]Kaiser S. Kunz, op. cit., pp. 234-235.

remaining parts of the manual presented examples and exercises for applying the elementary theory already introduced in solutions of problems. The solution of systems of linear equations by the inversion of the matrix of the coefficients illustrated many of the concepts introduced in the manual.

The manual was designed as an introduction to the theory of matrix algebra and was intended to cover the materials necessary for preparation to teach the two weeks' units outlined in the high school textbooks. Teachers in an in-service program were urged by the manual to go beyond the material which can be presented in a short workshop situation. The manual emphasized the need for going beyond the content presented for better understanding and more effective presentation of the material to high school students.

III. DEVELOPMENT FOR EVALUATION OF THE MANUAL

Following the pilot approach to the topic in 1964 and the preparation of the manual for the introduction to matrix theory, a meeting was held with the research committee and recommendations were made to conduct evaluations of the manual. Members of the committee made various suggestions which crystallized into the idea of two evaluations. One of these was to be with teachers in a workshop situation in 1965. The other evaluation was scheduled for the spring of 1965 with some senior mathematics classes in a member high school of the Lamar Area School Study Council. The first step necessitated by the evaluation was the planning and construction of the data gathering instruments. Before these tools could be developed, it was necessary to determine what questions were to be answered. The survey materials were to be constructed to collect data for analysis of these questions.

Questions for evaluation with teachers. Two types of data were needed where the teachers were involved. The first type concerned the background, education, and experience of the teachers in the workshop. The second type concerned the understanding of content of matrix algebra following an in-service program with the manual. Questions to be answered were identified as:

1. Was the manual a satisfactory approach, as measured by the survey materials, to preparing a teacher for teaching a prescribed unit in matrix algebra?
2. Was there a relationship between the teaching experience, number of hours in mathematics, or other areas of background, and scores made by teachers on the content survey?
3. Was there a relationship between solicited responses from teachers and other factors, such as educational background or test scores?

The second and third questions called for a data gathering questionnaire. Following the suggestions and precautions set forth by Van Dalen in his chapter on "Tools of Research"[11] a short instrument meeting some of the characteristics of the closed form and the open form

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[11]Deobold B. Van Dalen and William J. Meyer, Understanding Educational Research, McGraw-Hill Book Company, (New York, 1962), pp. 255-257.

questionnaire was developed. The first question called for a test or a content survey instrument to attempt to measure the understanding of the content the teacher would be called upon to impart to his students. Since collection of data depended upon voluntary participation by practicing teachers, it was necessary to keep the total amount of participation time within reason and still collect sufficient information to produce reliable results.

The concept that the right question must be asked to get the right answer was applied in developing the questionnaire. Following the original test construction the instrument was revised several times to correct weaknesses discovered when co-operating teachers checked the questions. Several items were rewritten or placed in different sequence to generate a more complete response. The completed questionnaire contained nine quantitative response type items and two open end questions where comments were solicited. (See Appendix for copy). Results of the use of the questionnaire are presented in the sixth chapter of this report.

Testing the content knowledge of the participating teachers presented a more difficult problem. There were no standardized tests available which measured an understanding of matrix algebra. It was necessary that a test be developed for this purpose. A test was constructed and used with the experimental presentation of the unit in matrix algebra to one class of students in 1964. This test was a ten item problem solving approach. Examination of this test made it clear that an objective item test was necessary to afford good collection and comparison of test scores. Objective test items properly constructed were found to remove subjectivity in scoring which occurred when the ten problem test was scored by different individuals. The necessity for objectivity in scoring was noted by Thorndike and Hagan[12] and by Van Dalen.[13]

When constructing an appraisal instrument, Van Dalen advised that questions of validity, reliability, and practicality must be answered.[14] One of the major limitations of the investigation developed when no available criteria were found for analyzing the validity of such a test with a teacher group. There was no "captive audience" or known group with which to compare the tested group. There were no ways or means to insist on another test or to obtain specific quantitative data on grades or other test scores of these teachers for comparison. For purposes of this study, it was decided to develop the test for use with a student group and then use the test with the teacher group as a content survey. The procedures for development of the test are reported in the succeeding section.

Questions for evaluation with students. The basic question for this evaluation was, "Can high school students grasp the fundamental concepts of elementary matrix theory when they are taught by the method

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[12] R. L. Thorndike and Elizabeth Hagan, Measurement and Evaluation in Psychology and Education, (John Wiley and Sons, New York, 1961), p. 47.

[13] Deobold B. Van Dalen and William J. Meyer, op. cit., p. 264.

[14] Ibid., pp. 264-266.

and approach set forth in the manual?" To answer this question, it was necessary to plan a situation where students could be pre-tested, exposed to the teaching of the unit on matrix algebra, and then tested again.

This procedure represented the type of experimental design referred to by Van Dalen as the "one group method."[15] Barnes called this a "before-after study with a single group."[16] This design met again with the problem that there were no standardized tests available on the subject of matrix algebra, which necessitated the development of a measuring instrument. Standardizing a test would require many testings with many populations complete with the development of norms and the appropriate tables for comparison of scores. The job at hand was to develop an appraisal instrument which would be sufficiently objective, valid, reliable, and practical to accomplish the desired evaluation. The results of this one group method experiment are analyzed in the seventh chapter. Some points of statistical consideration necessary for development of the test are reported in the next section.

Development of the test. As mentioned earlier, the test was constructed to consist of objective type items. Assistance was gained from a publication by the Educational Testing Service,[17] and from a

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[15] Ibid., p. 230.

[16] Fred P. Barnes, Research for the Practitioner in Education, (Department of Elementary School Principals, National Education Association, 1964), p. 61.

[17] Test Development Division of Educational Testing Service, "Multiple-Choice Questions: A Close Look," (Educational Testing Service, Princeton, New Jersey, 1963), 43 pp.

review of recommendations for constructing objective tests presented by Thorndike and Hagan.[18] A supply of items was prepared and each question was provided with five possible choices for the correct answer. The classes were selected for the experiment, and the aid of the mathematics teachers involved obtained. The test was planned to be administered in a normal class time which made the time limit for testing approximately fifty minutes. Test items were checked by the teachers and a selection of thirty items agreed upon. The first eighteen items were placed in a sequence designed to test an understanding of terminology, definitions, and operations with matrices and determinants. The final questions were arranged to test applications of matrix algebra including solutions of problems. The thirty items selected were compared with chapter and unit tests in the textbooks analyzed, and with the concepts presented in the manual to verify content validity.

To determine further validity of the test, it was necessary to select criteria with which to compare the test scores. Since there were no standardized instruments available measuring matrix algebra only, two other criteria were selected. Scores on the mathematical section of the Scholastic Aptitude Test of the College Entrance Examination Board were correlated with the obtained test scores. As a further check the high school grade point averages were correlated with the obtained test scores. Using the Pearson product-moment correlation method, a co-efficient of $r = .768$ was found with the matrix test scores and the

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[18] R. L. Thorndike and Elizabeth Hagan, op. cit., pp. 567-570.

mathematics section of the college entrance test. Scores for eighty students were included in the correlation. Complete scores for all students are listed in the Appendix, and results of the correlation are presented in Table XV in the sixth chapter. The product-moment correlation with the high school grade point average produced a coefficient of r = .772. All semester grades earned for the previous semesters of high school work were included in the average, which was determined by assigning four points for a grade of "A," three points for a "B," two for a "C," and one for a "D."

The standard error of the mean and the standard error of the product-moment correlation were explained by Meyer.[19] Computing the standard error of the mean produced a value of $S_m = \pm .342$. (See Table XV). Limits for the ninety-five per cent confidence interval were calculated to be 26.14 ± 0.67. Statistically, this indicated that ninety-five per cent of the sample populations of the same size drawn from twelfth grade students of similar mathematical background would earn scores on this test with a computed mean within the limits of 25.47 and 26.81. The ninety-five per cent confidence interval for the product-moment correlation coefficient was within the limits of r = .666 and r = .844. (See Table XV).

The reliability of the appraisal instrument was tested by the split-half method. Dividing the test into odd and even halves and applying the Kuder Richardson "Formula 21," a coefficient of r = .72

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[19]Deobold B. Van Dalen and William J. Meyer, op. cit., pp. 304-312.

was determined.[20] Applying the Spearman-Brown formula, a coefficient for the full length test of $r_{11} = .73$ was derived.[21] (See Table XV, Chapter Six). Using the prophecy formula as a correction for the thirty item time limit, a test three times as long, with the ninety items matching the thirty in construction, would produce a coefficient of $r = .89$. Indicated here was the fact that the reliability of the test could be increased if necessary by increasing its length. Using the $r_{11} = .73$, the standard error of measurement was calculated to be $SE_M = 1.63$. (See Table XV). The obtained value indicated that there was about one chance in three that an individual's obtained score differed from his "true" score by as much as 1.63 points. There was only one chance in twenty that the score would differ by as much as 3.26 score points for an individual. Application of these statistical concepts indicated satisfactory reliability. Considering the values for group study explained by Thorndike and Hagan, the reliability coefficient of $r = .73$ permitted the use of such an instrument to "make useful studies and draw accurate conclusions about groups."[23]

More complete discussions of the analysis of test results are presented in the sixth and seventh chapters. Only the results necessary to consider the validity, reliability, and practicality were presented

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[20]R. L. Thorndike and Elizabeth Hagan, _op. cit._, p. 179.

[21]_Ibid._, p. 181.

[22]_Ibid._, p. 183.

[23]_Ibid._, p. 190.

at this point. Complete lists of test scores and further calculations are reported in the succeeding chapters and in the Appendix.

IV. SUMMARY

Descriptive research in the form of documentary analysis identified the treatment of concepts within the selected topic of matrix algebra. The procedure for development of the manual was reported and some explanations were presented, showing why the manual started where it did, included the concepts selected, and why the material beyond high school level was presented. Recommendations were made for teachers in subject area to be prepared in content that comes both before and beyond the area or level at which they instruct. The manual was designed to meet this need for in-service education of mathematics teachers in the topic of matrix algebra.

Evaluation of the use of the manual was carried out with a 1965 workshop for teachers in the Lamar Area School Study Council, with a 1965 experimental use with four selected classes of high school seniors, with a small class of mathematics teachers enrolled in a summer graduate course at the University of Houston, and with a 1966 workshop with teachers. An appraisal instrument was developed, and tests for validity and reliability were conducted. Objectivity and practicality were verified, and statistics and factors examined supported the test as a suitable instrument for use in the evaluation. The results of the evaluation with students are reported in the succeeding chapter.

CHAPTER VI

ANALYSIS OF PRESENTATION OF A UNIT IN MATRIX ALGEBRA TO SELECTED HIGH SCHOOL SENIORS

To implement new topics in mathematics materials must be provided and teachers prepared to make the best use of the materials. Research reports recommended that experiments on suitability of new topics should be conducted. Some studies concerning the evaluation and effectiveness of topics in mathematics were reported by Dessart, and a common problem was reported by those attempting to evaluate the effectiveness of the new topics.[1] Appraisal instruments available were inadequate, and often standardized tests were used even though the test did not include questions covering the new topic. Dessart recommended additional basic research on the design of appraisal instruments "to tap adequately the effectiveness of experimental topics . . . ."[2] The experiment with selected high school seniors was to serve two purposes. The suitability of the topic of matrix algebra was examined and an appraisal instrument suitable for use with teachers in a workshop program was developed.

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[1]Donald J. Dessart, "Mathematics in the Secondary School," Chapter IV, "Natural Sciences and Mathematics," Review of Educational Research, XXXIV (American Educational Research Association, June, 1964), pp. 298-312.

[2]*Ibid.*, p. 302-303.

I. DESIGN OF THE EXPERIMENT

The design selected for the experimental use of the manual consisted of a pre-test, the presentation of a unit in matrix algebra, and a post-test. Van Dalen and Meyer called this a one group method experimental design with the testing, exposure to variable, and re-testing of a single group.[3] Scandura presented the "teach-test" method as a design of "naturalistic" research, a division of action research, being carried on in mathematics education at Florida State University.[4] A test was prepared and the test and the manual used in this one group method design.

To carry out the design student groups were selected, the testing and instruction planned with the cooperating school and teachers, and the experiment conducted.

II. THE STUDENT GROUPS

Selecting the students. Two programs of college preparatory mathematics were offered at the twelfth grade level of this high school of the Lamar Area School Study Council.

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[3]Deobold E. Van Dalen and William J. Meyer, Understanding Educational Research, McGraw-Hill Book Company, New York, 1962), pp. 230-231.

[4]J. M. Scandura, "Educational Research and the Mathematics Educator," The Mathematics Teacher, LVII (The National Council of Teachers of Mathematics, February, 1965), p. 135.

Two classes of advanced high school mathematics were conducted for students who had been accelerated in mathematics and completed the normal four year program by the end of the eleventh grade. There were seven classes of the normal twelfth grade program which consisted of a first semester study of trigonometry and a second semester study of elementary analysis. The school reported that students were randomly assigned to the regular mathematics classes. The two trigonometry analysis classes which met at the same school periods as the two accelerated classes were selected. These were placed with the accelerated classes to form two experimental groups, each made up of a class of regular twelfth grade mathematics students and a class of accelerated students. This assignment formed a group of forty-two students in one section and a group of forty-three in the other.

Analysis of descriptive information on students. Of the eighty-five students one was absent from the pre-test and was not included in the test analysis. Of the eighty-four students included in the experiment thirty-four had completed first year algebra in the eighth grade, second year algebra in the ninth grade, geometry in the tenth grade, trigonometry and analysis in the eleventh grade, and were enrolled in analytic geometry and analysis in the twelfth grade. The remaining fifty students had completed these courses through trigonometry and as seniors were enrolled in analysis as their eighth semester college preparatory course.

Descriptive data for these students are presented in Table XIII. High school grade point averages, recorded scores on the Otis Intelligence Test, and the recorded scores on the mathematical section of the Scholastic Aptitude Test of the College Entrance Examination Board are reported in the table. Analysis of the data presented a picture of the differences between the accelerated students and the regular students. Using the "t" test for significance of difference in means of the accelerated and regular groups, the null hypothesis that there existed no significant difference when measured by the Scholastic Aptitude Test could be rejected at the .001 level of significance since t=8.845. This indicated a significant difference in mathematical background and ability of the two groups of students involved in experiment.

<u>Background in matrix algebra</u>. After the administering of the pre-test a discussion was held with each group of students to determine their recall of any previous experience with matrix algebra. Two students reported having studied the topic briefly in summer institutes for high school students. The accelerated students reported having examined two by two matrices as an example of a non-commutative ring.

TABLE XIII

DESCRIPTIVE INFORMATION FOR HIGH SCHOOL SENIORS

Group and Number	Accelerated Group N = 34	Regular Group N = 50	Total Group N = 84
Mean for IQ Scores	125.94	116.76	120.52
Range for IQ Scores	24 112 to 136	37 95 to 132	41 95 to 136
Per Cent of Group Above Total Mean IQ	73.5%	34.0%	50.0%
Mean for SAT-Math Scores	608.61	484.96	535.33
Range for SAT-Math	295 452 to 747	365 353 to 718	394 353 to 747
Per Cent of Group Above Total Mean SAT-Math	90.9%	42.6%	61.7%
Mean for High School Grade Point Averages	3.697	3.030	3.290
Range for High School Grade Point Averages	0.833 3.167 to 0.833	2.167 1.833 to 4.000	2.167 1.833 to 4.000
Per Cent of Group Above Total Mean High School Grade Point Averages	88.2%	38%	58.3%

From the regular teacher's lesson plan book this was found to have covered two class days seven months before the pre-test was administered. A total of seven of the regular college preparatory students indicated having heard "about one lecture on the existence of determinants and matrices" when they were in a second year algebra course two years earlier.

III. PRESENTING THE UNIT AND TESTING

The pre-test. Without advance announcement concerning the nature of the topic or the test the students were grouped and the test administered. The oral instructions given recommended that there be no "wild guessing" since a pre-test was designed to determine what a student knew and understood about a subject before instruction was given. Students were asked to read and attempt all questions if they knew the answer or could reason to the answer from prior knowledge. After completion of the pre-test papers were collected and put away until the completion of the post-test. The test was not graded or discussed in the class until after the post-test and then both were discussed. This was done to minimize teaching for the test and to minimize test question recall by the students.

Analysis of the pre-test results revealed a mean for the total group of 7.25 and a standard deviation of $\sigma = 5.15$

on the thirty item test. The accelerated group had a mean of 11.24 and a $\sigma = 4.47$. The regular group had a mean of 4.54 and a $\sigma = 2.57$. Other data are presented in Table XIV.

An item analysis was conducted and examined with the cooperating teachers. There were six questions responded to correctly by as many as twenty-five per cent of the regular group. Examination of these revealed that a student who had encountered and remembered the definition of a matrix and who knew the meaning of the associative, commutative, and distributive laws could have responded correctly to these questions. For the accelerated group there was a total of 381 correct responses out of a possible 1020. For the regular group there were 205 correct responses out of a possible 1500. This again emphasized the difference in the background in matrix algebra between the accelerated and regular groups. There was a total of 24.25 per cent correct responses from the total group on the pre-test.

<u>Teaching the unit</u>. The time schedule recommended by the Houghton Mifflin series of textbooks was followed. (See Table XI, Chapter V). A total of ten class days was used including the testing time. Students were not given copies of the manual but material presented followed the sequence, procedure and examples used in the manual. The pre-tests were turned in within a forty minute period of time and students were allowed forty-five minutes on the post-test. The

TABLE XIV

COMPARISON OF PRE-TEST AND POST-TEST SCORES ON MATRIX ALGEBRA TEST

		Accel- erated Group	Reg- ular Group	Total Group
PRE-TEST	Mean	11.24	4.54	7.25
	Standard Deviation	4.47	2.57	5.15
	Low Score	4	0	0
	High Score	22	11	22
	Range	18	11	22
POST-TEST	Mean	28.68	24.54	26.14
	Standard Deviation	1.62	2.77	3.13
	Low Score	23	17	17
	High Score	30	30	30
	Range	7	13	13
GAIN	Lowest	6	11	6
	Highest	26	26	26
	Mean	17.44	20.26	19.12

Product-Moment correlation of Pre-test with Post-test for accelerated group	$r = + .094$
Product-Moment correlation of Pre-test with Post-test for regular group	$r = + .2$

net time occupied eight clock hours of instruction with no more than forty-five minutes allowed per day. The pre-test was administered on Monday of one week and the post-test on Friday of the succeeding week, or on the twelfth calendar day after the pre-test.

Efforts were made to control all variables of instruction possible. Questions asked or group interactions in one class period were repeated as nearly as possible with the other group.

The post-test. Analysis of the post-test revealed a mean for the total group of 26.14 and a standard deviation of $\sigma = 3.13$. The regular group earned a mean of 24.54 and a $\sigma = 2.77$. The accelerated group scored a mean of 28.68 and $\sigma = 1.62$. (See Table XIV). The following tabulation reported comparisons of the means, gains in the means, and

Group	Mean of Pre-Test	Mean of Post Test	Gain of Mean	Mean of Gain
Accelerated	11.24	28.68	17.44	17.44
Regular	4.54	24.54	20.00	20.26
Total	7.25	26.14	18.89	19.12

the means of the gains from the pre-test to the post-test by student groups. The null hypothesis of no significant difference in the means of the accelerated group and the regular group was formulated. The "t" test for significance produced a value of t = 7.81 and the null hypothesis was rejected at the .01 level of confidence. There was a significant difference in the scores on the post-test.

An item analysis of the post-test revealed 87.38 per cent correct responses out of the 2520 possible responses. The most frequently missed items were those requiring applications of theory in problem solving and requiring some computations where errors in arithmetic could have affected the result. These questions were the type of application requiring drill and repeated use of procedures to develop skills for solving quickly and accurately.

Correlations and standard errors. The product-moment correlation of the pre-test with the post-test for the accelerated group yielded a value of r = + .09. The correlation coefficient for the regular group was r = + .20. (See Table XIV). These are significant in that they establish that a student did not have to do well on the pre-test in order to do well on the post-test. This indicated that the instruction by use of the manual started at the right place and did not presume a prior knowledge of the tested material. The value of the coefficient for the accelerated group was minimized due to the heavy cluster of scores in the top interval where rankings or correlations would be heavily influenced by the lack of dispersion or spread of scores.

Product-moment correlations of post-test results with SAT-Mathematics scores and grade point averages were computed. (See Table XV). These were reported in the previous chapter when external validity was presented. The coefficient of r = + .77 found with both external criteria supported the validity of the matrix algebra test. To examine reliability the post-test scores on odd and even numbered problems were correlated using the product-moment technique. A coefficient of reliability of r = + .73 could be increased according to the Spearman Brown Prophecy Formula by lengthening the test.[5] (See Table XV). The standard error for the mean, the correlation, and the standard error of measurement were calculated and are presented in Table XV.

Test scores are reported in the Appendix along with other data for the student group. The test on matrix algebra was determined to be a sufficient appraisal instrument for analysis of groups using the manual. The designed test had too much ceiling for the accelerated group and a longer and more difficult selection of items would be superior for working with the accelerated students.

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[5]R. L. Thorndike and Elizabeth Hagan, Measurement and Evaluation in Psychology and Education, (John Wiley and Sons, New York, 1962), p. 187.

TABLE XV

REPORT OF CORRELATIONS AND STANDARD ERRORS OF STATISTICS ON MATRIX ALGEBRA TEST

Statistic	Procedure or Criteria	Value of Statistic
Standard Error of the Mean	Matrix Algebra Post-Test	$S_m = \pm .342$
95% Confidence Limit for Mean	Matrix Algebra Post-Test	$26.14 \pm .67$
Product-Moment Correlation "r"	Matrix Post-Test With SAT-Math Score of CEEB.	$r = + .77$
Product-Moment Correlation "r"	Matrix Post-Test With High School Grade Point Average	$r = + .77$
Standard Error of Correlation	Convert r = + .77 Into Standard or Z Terms	$S_z = \pm .11$
95% Confidence Limit for Correlation	Determine Z_r to be $1.020 \pm .216$, Then Convert to "r"	$r = + .666$ to $r = + .844$
Product-Moment Correlation "r"	Using Spearman-Brown Formula on Split-halves of Matrix Algebra Test	$r_{11} = + .73$
Kuder-Richardson Formula "21"	Determining Test Reliability for Matrix Algebra Test	$r_{11} = + .72$
Standard Error of Measurement	For $r_{11} = + .73$ Then SE Given Matrix Algebra Score Points by SE = 3.13 1 - .73	$SE = \pm 1.6$
Correction Due to Length of Test	Spearman-Brown Prophecy Formula Increasing Length if Desire to Increase Reliability	$r_{33} = + .89$

IV. SUMMARY

Students were selected and the experiment arranged. A pre-test was administered, the unit on matrix algebra using the manual was presented to the students, and a post-test was administered. Test results indicated that the material presented in the manual can be understood and applied by high school college preparatory mathematics students. Both the regular college preparatory students and the accelerated students demonstrated this ability to understand, interpret, and apply the material. The item analysis further supported this summary since questions of both interpretation and application had high percentages of successful responses. The implication presented was that a mathematics teacher could prepare in the areas covered by the manual, follow the procedure of presentation outlined in the manual, and expect success, as measured by this test, in understanding concepts, operations, and elementary applications of matrices and determinants.

Summarizing these results required attention to the possible operation of the "Hawthorne effect."[6] Using a pre-test established the students' awareness of the experiment, and placing the two groups of students together to receive

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[6]Desmond L. Cook, "The Hawthorne Effect in Educational Research," Phi Delta Kappan, XLIV (December, 1962), pp. 116-122.

instruction from someone other than the regular instructor increased the possibility of the effect. However, instruction and testing presented by an instructor other than the grade-giver possibly minimized the Hawthorne Halo. This must be recognized as one of the limitations or problems of the test, re-test designs when the participants are high school students aware of the research or experiment.

The succeeding chapter, the seventh, will report on the use of the appraisal instrument and the manual with groups of teachers in workshop situations.

CHAPTER VII

ANALYSIS OF THE USE OF THE MATRIX THEORY MANUAL WITH MATHEMATICS TEACHERS

The manual was used in the presentation of an introduction to matrix theory to three groups of teachers. Two of these groups were made up of in-service mathematics teachers participating in a workshop sponsored by the Lamar Area School Study Council. One group met in February of 1965 and the second met in February of 1966. The third group of participants was a graduate class in methods in secondary school mathematics at the University of Houston in the summer of 1965.

I. DESIGN OF THE EXPERIMENT

The workshop program represented a voluntary participation by a group of teachers meeting for an in-service program in modern senior high school mathematics. Arrangements for voluntary participation by teachers in an experimental design involving testing was difficult. The classical design of setting up two matched groups for testing, exposure to variable, and re-testing was not suitable due to this voluntary structure. A selective or controlled experimental situation similar to the National Science Foundation summer institutes was necessary to produce material samples, and this was not possible in the in-service program available for the experiment. The many variables and the range in variable represented by the teaching experience, semester hours in mathematics, background in matrix algebra, time since last college work, mathematics institute participation, and age and sex reduced the possibility of creating two matched samples and varying the treatment between testings. This limitation suggested the need for an analysis of variance design. Meyer presented an explanation of this technique in a unit on inferential statistics,[1] and Guilford outlined the procedure in a chapter titled "Introduction to Analysis of Variance."[2]

Development of the procedure followed involved the use of a questionnaire, presentation of the manual, and testing. Test scores and data from questionnaires were tabulated. Participants were classified into four groups based upon the variable of teaching experience and an analysis of variance computed. Teachers were then re-classified into four groups based upon the number of semester hours in mathematics and an analysis of variance applied. This procedure

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[1]Deobold B. Van Dalen and William J. Meyer, Under standing Educational Research, (McGraw-Hill Book Company, New York, 1962), pp. 322-329.

[2]J. P. Guilford, Fundamental Statistics in Psychology and Education, Fourth Edition, (McGraw-Hill Book Company, New York, 1965), pp. 268-303.

was followed to determine if significant differences existed among the groups in the effects of the use of the manual in an in-service program as measured by the content survey.

II. THE PARTICIPATING TEACHERS

Thirty-seven teachers representing fourteen school districts of the Lamar Area School Study Council participated in the 1965 workshop. There were twenty-nine who returned the questionnaire and completed the content survey. The summer graduate class enrollment was fourteen, and twelve complete sets of data were collected. Sixty-one teachers representing nineteen school districts and twenty-six high schools were enrolled in the 1966 workshop and fifty-six of these completed the questionnaire and the post-test. A total of one hundred and eleven teachers were enrolled in the three groups, and of these ninety-seven, or just over eighty-seven per cent, voluntarily completed the experiment. Descriptive information for the ninety-seven teachers was tabulated from the questionnaires and produced the following analysis. (See Table XVI).

Analysis of descriptive data. Teaching experience among the participants ranged from two members of the summer class four weeks away from starting their first teaching to two members of a workshop in their forty-first year of teaching mathematics. Workshop participants included four

TABLE XVI

DESCRIPTIVE DATA FOR TEACHER PARTICIPANTS COMPLETING USE OF MATRIX ALGEBRA MANUAL AND SURVEYS

	Workshop 1965	Graduate Class 1965	Workshop 1966	Total	Per Cent of Group
Male	19	5	31	55	56.7%
Female	10	8	24	42	43.3%
Master's Degrees	14	1	18	33	34%
Bachelor's Degrees	15	11	38	64	66%
First Degree Earned 1963 or Later	8	5	22	35	36.1%
Reported Experience in Matrix Algebra	14	4	22	40	41.2%
No Previous Matrix Algebra	15	8	33	57	58.8%
NSF Institute Participants	9	1	15	25	25.8%
Under Five Years Teaching Experience	12	10	26	51	52.6%
Five Years or More Teaching Experience	17	2	30	46	47.4%
33 Semester Hours Mathematics or Less	14	10	22	46	47.4%
34 Semester Hours Mathematics or More	15	2	34	51	52.6%
Reported Teaching Mathematics	29	9	54	92	94.8%
Total With Test Results	29	12	56	97	100%
Per Cent of Total Group	29.9%	12.4%	57.7%	100%	

teachers who were in their first semester of teaching and five who had completed more than thirty years of teaching. Table XVI shows a dividing point of below five years experience contrasted with five years and above. Fifty-two and six-tenths per cent of all participants had less than five years experience and the remaining forty-seven plus per cent had been teaching five years or more.

There were twenty-five teachers in the experiment who had attended at least one National Science Foundation Institute. These included thirteen men and twelve women. Twenty of these teachers had thirty semester hours or more in college mathematics courses when they were selected for the institutes, while only two had fewer than the twenty-four semester hours required for certification in Texas when they were selected. Ten of the twenty-five had participated in more than one institute, and seventeen reported holding master's degrees. Mathematics institute participants represented twenty-five and eight-tenths per cent of the total group of ninety-seven teachers.

Every teacher enrolled held at least a bachelor's degree. Thirty-four per cent had completed master's degrees and sixty-six per cent held bachelor's degrees. Dates that degrees were earned ranged from 1925 until January, 1966 for bachelor's degrees, and from 1940 through 1965 for master's degrees. Twenty-eight teachers held bachelor's degrees with twelfth grade work in analysis and analytic geometry. (See Table XVII). Courses being taught were not listed by two workshop participants and three of the summer graduate class members were not assigned at that time to teach mathematics.

The questionnaire included an item that called for a reaction concerning whether subject matter content workshops and manuals such as that on matrix algebra were needed for in-service education of mathematics teachers. Answers were not given for six per cent, five per cent said no, and eighty-nine per cent answered yes. (See Table XVII). Four of the five teachers who answered no wrote in statements concerning the need to emphasize methods of presenting topics to high school students.

III. PRESENTING THE MANUAL AND TESTING

Two of the experimental groups agreed to take a pre-test on the content survey. This was given with the instruction to avoid wild guessing but to attempt any item that mathematical knowledge and reasoning could be followed to produce the correct choice. A copy of the manual on the introduction to matrix algebra was presented to each participant. A maximum of three hours exclusive of the final testing time was used to discuss the manual. The intensive study of the manual was left to the individual teacher.

The two groups who took the pre-test were administered

TABLE XVII

DATA FROM OPEN ITEMS ON QUESTIONNAIRE FOR TEACHERS PARTICIPATING IN UNIT OF MATRIX ALGEBRA

QUESTION	TABULATION OF ANSWERS
What subjects are you assigned to teach?	1. Algebra I . . . 40 teachers - 41.2% 2. Algebra II . . 40 teachers - 41.2% 3. Geometry . . . 42 teachers - 43.3% 4. Trigonometry . 20 teachers - 20.6% 5. Consumer Math . 14 teachers - 14.4% 6. Analysis I or II . . . . . 10 teachers - 10.3% 7. Others: Related Math - 8 Analytic Geometry - 3 General Math - 3 Seventh and eighth grade - 3 Probability and Statistics - 2 8. Not assigned to teach Math - 3 9. Unanswered - 2
Do you believe that workshops in subject content and manuals such as this one in high school mathematics are needed?	1. Yes . . . . 86 teachers - 88.7% 2. No . . . . 5 teachers - 5.1% 3. Unanswered . . 6 teachers - 6.2%
What textbooks are being used by your school for the following courses: Algebra II, Trigonometry, Analysis, Advanced Math?	1. Answered with textbook that included matrix algebra 68 teachers - 70.1% 2. Answered with textbook that included determinants only 3 teachers - 3.1% 3. No answer - 26 teachers - 26.8%

the post-test after completing study of the manual. The first group of teachers took the test individually as a follow-up test six weeks after the close of the workshop. The cover letter, questionnaire, and survey test are presented in the Appendix. No materials were allowed to be used during the content survey in any of the testings. The following section will present some analysis of the test results.

IV. ANALYSIS OF TEST SCORES

The pre-test. There were sixty-seven sets of scores included in the analysis of the pre-test results. (See Table XVIII). The pre-test was completed by the graduate class in 1965 and the workshop group in 1966. The standard deviation for the pre-test group was $\sigma = 8.56$. The means for a number of classifications of variable are presented in Table XVIII. Using the statistical test of significance developed by Fisher and described by Van Dalen and Meyer,[4] some significant differences were found. One example reported the "t" test between two groups of teachers, one with thirty-three semester hours or less in mathematics, and the other thirty-four semester hours or more. A value $t = 69.92$ with sixty-five degrees of freedom was found, allowing the null hypothesis of no significant difference to be rejected

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[4]Deobold B. Van Dalen and William J. Meyer, op. cit., p. 318.

TABLE XVIII

ANALYSIS OF PRE-TEST SCORES BY TEACHERS ON UNIT IN MATRIX ALGEBRA

Group	N	Low Score	Range	Mean
Graduate Class	13	0	27	7.69
1966 Workshop	54	0	27	12.03
TOTAL GROUP	67	0	27	11.1
Reporting Previous Matrix Algebra	39	2	25	17.72
Reporting Degrees Earned Since January, 1963	26	0	27	11.23
NSF Institute Participants	16	6	27	15.88
Reporting 33 Semester Hours or Less in Mathematics	31	0	27	5.55
Reporting 34 Semester Hours or More in Mathematics	36	0	27	15.55

The results of the pre-test analysis indicated the differences in the knowledge of matrix algebra as measured by the testing instrument before the use of the manual in an in-service program.

The post-test. Ninety-six complete sets of test data were collected by the use of the post-test. The analysis of these results produced a standard deviation for the total group of $\sigma = 4.34$. The standard deviation on the post-test was about one-half the standard deviation of the pre-test scores. The mean for the post-test was 26.12, and this represented a gain in the mean of 15.02 from the pre-test mean. The range, mean, and standard deviation for various groupings determined by variables within the participants were computed. (See Table XIX). The standard error of the mean for the total group was computed to be $S.E._{M.} = \pm .44$. The ninety-five per cent confidence limit for the mean was determined at $26.12 \pm .86$. This statistic presented the relation that if any other groups of teachers were drawn from a similar population with similar backgrounds to obtain means on the test, the means would fall ninety-five per cent of the time between $M = 25.26$ and $M = 26.98$. (See Table XIX).

Other statistical characteristics were studied. A product-moment correlation between pre-test and post-test scores was found to be $r = .4$. This indicated that a high

TABLE XIX

ANALYSIS OF POST-TEST SCORES BY TEACHERS
ON UNIT OF MATRIX ALGEBRA

Group	N	Range	Mean	Standard Deviation
Graduate Class, 1965	12	19	24.83	4.94
1965 Workshop	29	14	25.82	4.46
1966 Workshop	55	18	26.54	4.16
TOTAL GROUP	96	19	26.12	4.34
Previous NSF Participants	25	18	26.90	3.20
With 34 Semester Hours or More in Mathematics	50	18	26.79	3.82
With 33 Semester Hours or Less in Mathematics	46	19	25.80	4.44
With Four Years or Less Teaching Experience	51	17	26.55	3.62
With Five Years or More Teaching Experience	45	19	25.55	4.88
Reporting Previous Matrix Algebra Work	39	14	27.30	3.23
Reporting No Previous Matrix Algebra Work	57	19	25.33	4.86
Reporting Degrees Earned Since January, 1963	35	15	27.29	3.22

score on the pre-test could not be used to predict a high score on the post-test. An analysis of variance statistic was applied and reported in the next section.

V. ANALYSIS OF VARIANCE

Application of the analysis of variance technique necessitated classifying the participants into three or more groups based upon a selected variable.[5] Since this study was concerned with the use of the manual in an in-service program for mathematics teachers two variables were selected and an analysis of variance on the matrix algebra post-test results computed for each of the variables.

Variable of teaching experience. The number of years of teaching experience was selected as a logical variable for classifying the participants. The question formulated was, "Will there be significant differences in matrix algebra test results for groups of teachers with different teaching experience following study of the manual?" An analysis of variance would test for any significant differences between the groups based upon different teaching experience. Four classifications of the variable of teaching experience were determined. One group was based upon the beginning teacher as defined by certain classification requirements of the

[5] *Ibid.*, pp. 322-324.

Texas Education Agency and included teachers with less than three years of experience.[6] The second group was determined after a study of the beginning teacher as classified in a recent doctoral dissertation,[7] and included those teachers with at least three and less than five years teaching experience. The third classification was somewhat arbitrarily defined as the category of five to ten years of teaching experience. The only justification for breaking at the ten year mark was that this level represented the point at which most school districts begin to present service awards and "ten-year pins" and represented a logical dividing point in the teaching experience. The fourth group covered all teachers who had been teaching for eleven years or more. The null hypothesis for this group stated that there were no significant differences in the achievement on the matrix algebra test by the different experience groups after studying the manual.

Thirty-one participants with post-test scores were reported with two years or less teaching experience, twenty had three to four years of teaching experience, twenty had

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[6] Handbook for Secondary School Principals, Bulletin 639, (Texas Education Agency, Austin, Texas, December, 1963), pp. 67-70.

[7] Oran Bright Bailey, "An Analysis of the Teaching Problems of Beginning Junior High School Science Teachers Within the Lamar Area School Study Council," (unpublished Doctor's Dissertation, University of Houston, Houston, 1965).

completed from five to ten years of teaching, and twenty-five had been in-service as teachers eleven years or more. The F-ratio computed by the analysis of variance with the four groups was F = 0.79. (See Table XX).

The null hypothesis cannot be rejected with an F < 1.0.

TABLE XX

TEACHING EXPERIENCE--ANALYSIS OF VARIANCE

Source of Variance	Sum of Squares	df	Mean Square	F
Between Treatments	47.00	3	15.67	0.79
Within Groups	1827.74	92	19.87	
Total	1874.74	95		

With the degrees of freedom in this distribution, rejection of the null hypothesis required an F = 2.71 for the .05 level and an F = 3.99 for the .01 level of confidence.[8]

Variable of semester hours in mathematics. The number of semester hours in mathematics earned by the teachers was selected as the second variable. The question stated, "Will there be significant differences in achievement on the matrix algebra test by groups of teachers with different backgrounds in mathematics following the use of the manual?" The null hypothesis stated that there were no significant differences

[8] J. P. Guilford, op. cit., p. 586.

in the test results of the four groups with different mathematical backgrounds. It must be accepted that the number of semester hours in mathematics does not necessarily establish an equal background since two teachers with thirty semester hours may have taken different courses to arrive at the same total. For purposes of this study the number of hours represented a logical criterion since the point to be established was to what degree teachers of different training and background could profit from such an in-service experience.

The first dividing point determined was taken from the recommendations of the Mathematical Association of America.[9] The minimum preparation for a high school teacher of college preparatory mathematics was set at eleven courses, or thirty-three semester hours. Thirty-three hours or less and thirty-four hours or more provided the middle division of participants. A space of eight semester hours above and below this point provided clear dividing lines. There were twenty-one teachers with twenty-five semester hours or less participating in the use of the manual. Twenty-five teachers had earned from twenty-six to thirty-three semester hours, and twenty-two teachers were in the third classification of thirty-four through forty-one semester hours. The fourth group of twenty-eight teachers reported a total of forty-two

[9]Committee on Undergraduate Program in Mathematics, _op_. _cit_., pp. 12-13.

or more semester hours in mathematics.

The F-ratio computed in the analysis with these four groups was F = 1.63. (See Table XXI). With the degrees of freedom in the distribution it was reported earlier that an

TABLE XXI

SEMESTER HOURS IN MATHEMATICS ANALYSIS OF VARIANCE

Source of Variance	Sum of Squares	df	Mean Square	F
Between Treatments	65.65	3	21.85	1.63
Within Groups	1605.31	92	13.38	
Total	1670.96	95		

F = 3.99 for the .01 level and an F = 2.71 for the .05 level of confidence were necessary to reject the null hypothesis. Rejection was not possible with F = 1.63. There were no significant differences found by the analysis of variance with groups of different mathematical education.

VI. SUMMARY

Two groups of teachers were pre-tested, presented a manual on the introduction to matrix theory, and administered a post-test. A third group was presented the manual and administered the content survey instrument as a post or follow up test. Questionnaires were completed by the participants. Ninety-six teachers representing eighty-seven per cent of the total enrollment of the three groups completed the test and the questionnaire.

The questionnaire reported wide variations in the teaching experience, education in mathematics, total educational background, and time since college work by the participants. Pre-test results revealed significant differences in knowledge of matrix algebra by certain groups as indicated by the "t" test for significance between means. The questionnaire reported that more than ninety-four per cent of the teachers were currently teaching mathematics and sixty per cent were teaching some work in the last two years of college preparatory mathematics. Eighty-eight and seven-tenths per cent of the teachers stated that subject content workshops such as the one on matrix algebra were needed while only five per cent stated that methods were needed instead of subject content.

Only twenty-five per cent of the teachers reported participation in a National Science Foundation or other institute for improving the education of mathematics teachers. This was evenly divided with thirteen men and twelve women. This group of institute participants made the highest mean score on the pre-test. (See Table XVIII).

Participating teachers were classified into four groups based upon their teaching experience, and an analysis of variance computed. The null hypothesis was not rejected.

Participants were reclassified into four groups on the variable of preparation for teaching as measured by the number of semester hours in mathematics earned by each teacher. An analysis of variance was computed, and again the null hypothesis could not be rejected. Failure to reject the hypotheses suggested that there were no significant differences revealed by the post-test results as a result of the variables of teaching experience and number of semester hours in mathematics. This supported the use of the manual with wide ranges of these variables in an in-service program.

Pre-test results suggested that only a few teachers had been exposed to matrix algebra in either pre-service or in-service education. Only one out of three graduates from college since January of 1963 reported previous work with matrix algebra. Eliminating the lowest experience group, sixty-three per cent of the teachers had not participated in National Science Foundation programs.

No attempt was made in the study to analyze the sex variable. Attitudes towards mathematics were treated only as reflected by questions concerning the need for workshops and manuals in subject content. Applications with students were treated in the sixth chapter.

Chapter VIII, the concluding chapter, will summarize the results of the study and present their implications as conclusions and recommendations.

CHAPTER VIII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

I. SUMMARY

The purposes of this study were as follows: (1) to identify the topics new to senior high school college preparatory mathematics; (2) to select a topic and develop a manual for in-service education in the topic; (3) to analyze the experimental use of the manual with high school seniors and in an in-service program with mathematics teachers. The first purpose was carried out through a documentary analysis of surveys, study programs, previous and current state adopted textbooks for the eleventh and twelfth grades, and by an analysis of the additional textbooks not furnished by the state in use in the high schools of the Lamar Area School Study Council. A more intensive documentary analysis of the materials which presented some treatment of the topic of matrix algebra was necessary to fulfill the second purpose. Presentation of a unit in matrix algebra to student samples was first involved in the third area, and then the manual was presented to in-service teachers of mathematics. Data collected from the student and teacher samples by tests and questionnaires were then analyzed.

The study was conducted from February, 1965 to

February, 1966. The student sample was selected from senior level students in a high school of the Lamar Area School Study Council. Two of the teacher samples were workshop groups of the Lamar Area School Study Council and a third sample included was a graduate class in methods in secondary school mathematics at the University of Houston.

Summary of the related literature. A number of curriculum reform programs in mathematics developed around the nation. Recommendations for change in content, placement, and emphasis on the many topics in mathematics were made. Texas, as well as other states, named a curriculum studies commission in mathematics to move the high schools of the state to meet the new curriculum proposals. New mathematics textbooks were adopted for the second year course in algebra at the eleventh grade. Plans were made for other adoptions in trigonometry and mathematical analysis for the twelfth grade.

Programs for re-education of mathematics teachers and recommendations for revised preparation requirements came from many sources. The National Science Foundation came into existence and sponsored summer institutes and academic year institutes for the in-service education of mathematics teachers. These institutes were highly successful for those who attended. Literature revealed that the majority of mathematics teachers were either not applying or not being accepted for institutes, and that in-service education was still a major local problem. A number of state and local programs in the elementary and the secondary fields were reported. Workshops and short courses in the new topics were recommended.

Reviews of research in mathematics education recommended in-service programs to determine what topics were to be studied and what would be their scope and sequence. Further recommendations included programs to improve the preparation of teachers for presentation of the new topics. Further research was recommended on the suitability or feasibility of including new topics in the high school curriculum, and upon development of suitable materials for measuring and studying these new programs.

Summary of the research procedure. The survey study type of descriptive research was employed in the identification of topics, the selection of a topic, and as a guide to the development of the manual. The data obtained from the documentary analysis of textbooks and materials revealed the topics in mathematics in those textbooks that had been placed in the students' hands. The analysis was used to identify clearly the recommendations and proposed program, and also the actual effect in the classroom as measured by the treatment of topics in the materials issued to students. The study of surveys, analysis of past and current state adopted books, and of new textbooks being placed in use in the high schools identified the new topics and how they were treated in student materials.

A pilot approach to presenting a unit in matrix algebra was made in 1964. A student group was presented a two weeks unit and one meeting of an in-service workshop was used in a lecture presentation of an introduction to matrix theory. From these two experiences the manual was revised and studied further. The documentary analysis caused further revision of the manual and a preliminary development of a measuring instrument. A student sample was selected in 1965 and the measuring tool refined in conjunction with the cooperating teachers of the student group. A one-group method experimental design was used with the presentation of the unit in matrix algebra following the procedure of the manual. Test results were used to further refine the test and the manual in preparation for the use with teacher groups. Validity and reliability of the test for the study of groups was established.

A questionnaire was developed to collect data from the participating teachers. The questions produced responses in the following areas:

1. Years of teaching experience.
2. Preparation for teaching mathematics.
3. National Science Foundation Institute experience.

4. Degrees earned and dates earned.

5. Previous experience in matrix algebra.

6. Recommendations concerning workshops in mathematics.

7. Sex of participant.

8. Teaching assignment.

The content survey developed was a thirty item test measuring concepts and applications of matrix theory. The tests were administered personally in a pre-test and post-test experiment with high school seniors. These were scored and a statistical analysis conducted. The content surveys and questionnaires were administered in a pre-test and post-test experiment with two groups of teachers. The test and questionnaire were administered in a follow-up program to one teacher workshop group. Tests were scored and data from questionnaires tabulated. Teachers participating were classified on the basis of the variables revealed by the questionnaires and test results statistically analyzed. Ranges, means and standard deviations were computed for the various groups. The variable of teaching experience was selected, teachers classified by this variable, and an analysis of variance computed. The variable of preparation for teaching mathematics as measured by the semester hours of college mathematics was selected, teachers classified by this variable, and an analysis of variance computed on the test scores.

Results of the study were determined by tabulation of number and per cent of responses to items of the questionnaire. Means on test scores, tests for significance of difference in means, correlations, and analysis of variance techniques were utilized to find other results.

Summary of findings of documentary analysis. The survey studies technique applied to surveys of mathematics education, experimental curriculum study programs, and textbooks and classroom materials revealed the agreements and variations concerning new topics and new approaches and placement of topics. New topics identified were analytic geometry, properties of number systems, matrix algebra, probability and statistics, and sets and set theory. New approaches and new treatments were identified for the topics of complex numbers, inequalities, elementary functions, and polynomials. High schools of the Lamar Area School Study Council were found to be using state adopted textbooks which included these same topics. Textbooks other than those furnished by the state were found to be in use in new courses in elementary analysis and senior level advanced mathematics courses. Analysis of these textbooks further identified the topics of analytic geometry, matrix algebra, probability and statistics, elementary functions, algebra of sets, and number systems.

From the identified topics matrix algebra was chosen for further development. The selection was suggested by the placement and emphasis given the topic by the study program, surveys, and the textbooks analyzed. Other influences were two mathematics institutes attended in 1960 and 1961, a pilot use of the topic with students and with teachers in 1964, and the fact that ninety-two per cent of the high schools surveyed were using a second year algebra textbook in the eleventh grade which included a unit on matrix algebra. A more intensive documentary analysis was necessary to determine the treatment of concepts in matrix algebra in texts and materials. This analysis produced the basis for development of the manuals used in the study.

Summary of findings with student experiment. Statistical analysis of the pre-test and post-test results for the eighty-four senior students revealed a number of facts related to the objectives of the study. Test results produced product-moment correlations of $r = .77$ with Scholastic Aptitude Test mathematical scores and with high school grade point averages. The regular college preparatory mathematics senior students and advanced mathematics senior students showed a total mean gain of 19.12 over the pre-test scores. A post-test mean of 26.14 was found. The ninety-five per cent confidence interval for the mean on the post-test was 26.14 ± 0.67. The ninety-five per cent confidence interval for the correlation was from $r = .67$ to $r = .84$.

Student scores produced a reliability coefficient of r = .73 when the Spearman-Brown formula was applied to an odd and even split-halves technique. Satisfactory reliability for group studies was found for the content survey. Both regular and advanced mathematics students at the senior high level demonstrated the ability to understand, interpret, and apply the material after the presentation of the unit on matrix algebra by the procedure outlined in the manual. A product-moment correlation between pre-test and post-test scores produced a coefficient of r = + .09 for the advanced group and r = + .2 for the regular group. An item analysis on the pre-test and the post-test revealed the student's understanding of the concept and applications presented in the test.

Summary of the findings with teacher groups. The tabulation of responses to questionnaire items revealed that just under eighty-nine per cent of the teachers believed that such subject content workshops were necessary. Six per cent did not respond to the question and five per cent thought that methods were needed more than subject content. Ninety-five per cent of the teachers completing the tests and questionnaires were assigned to teach mathematics courses from the seventh through the twelfth grades. Although twenty-six per cent did not report the textbooks in use in second year algebra and above, seventy per cent reported using a series which included a two-weeks unit in matrix algebra at the eleventh grade and another unit which included matrix algebra at the twelfth grade.

Twenty-five teachers who participated had attended National Science Foundation institutes and ten of these had attended more than one institute. These teachers reported a mean of 12.64 years of teaching experience. The mean on the pre-test for this group was 15.8 and the mean on the post-test was 26.9. Seventeen of these teachers had earned forty or more semester hours in mathematics and the group of twenty-five showed a mean of 48.8 semester hours.

Thirty-four of the ninety-seven teachers had earned master's degrees in some field. Thirty-five participants had earned a bachelor's degree since 1963, but every participant held at least a bachelor's degree. There were fifty-one teachers with less than five years of teaching experience, and this group had a mean of two and three-tenths years of experience. The remaining forty-six teachers had a mean of 15.5 years of teaching experience and the total group had a mean teaching experience of eight and four-tenths years. The group of teachers with less than five years experience earned a mean score of 26.55 on the post-test and those with five years or more earned a mean score of 25.55 on the post-test.

There were fifty-five men and forty-two women. Of these thirty-nine had previous experience in matrix algebra and fifty-eight did not. Those with matrix algebra experience had a pre-test mean of 17.72 and a post-test mean of 27.13. Those who reported no previous matrix algebra had a pre-test mean of 5.93 and a post-test mean of 25.33. The total group pre-test mean was 11.1 and the post-test mean for all participants was 26.12. The standard error of the mean was $S.E._{M.} = .44$ and the ninety-five per cent confidence limit was found to be 26.12 ± .86.

Forty-six teachers with thirty-three semester hours or less in mathematics earned a pre-test mean of 5.55, and the fifty participants with thirty-four semester hours or more in mathematics earned a pre-test mean of 15.55. The "t" test for significance revealed that the null hypothesis of no significant difference was rejected at the .01 level of confidence. Post-test means were 25.6 and 26.9 respectively.

Teachers were classified into four groups based upon the years of teaching experience reported. A null hypothesis stating that there were no significant differences in the post-test means earned by these four groups was formulated. Application of the analysis of variance produced an F-ratio of F = 0.79, and the null hypothesis could not be rejected. Failure to reject the hypothesis suggested the interpretation that there were no significant differences in the post-test results attributable to the differences in teaching experience.

Another classification of four groups was determined based upon the variable of the number of semester hours in mathematics earned by each teacher. A null hypothesis stating that no significant differences existed with these four groups on the post-test results was formulated. An F-ratio of F = 1.63 was computed and again the null hypothesis could not be rejected. This was interpreted that no significant differences in the post-test results could be attributed to the variations in the number of semester hours in mathematics earned by the participants.

Failure to reject both null hypotheses was interpreted to mean that post-test results did not reveal any differences in achievement after use of the manual in matrix algebra because of teaching experience or mathematical background. This supported the finding that the manual was satisfactory for use with both experienced and inexperienced teachers, and with teachers having varied numbers of semester hours in college mathematics.

Although ninety-four per cent of the teachers who listed the textbooks in use in their schools reported textbooks which included a unit in matrices, there were sixty per cent who reported no previous experience with matrix algebra. Excluding all teachers with less than two years of teaching experience, sixty-three per cent of the mathematics teachers had either not applied or not been accepted to National Science Foundation institutes or similar programs. Sixty-five per cent of the teachers who had earned bachelor's degrees since January, 1963, reported no previous experience with matrix algebra. These figures indicated that two out of three teachers, including recent graduates of mathematics teacher education programs, had no experience with matrix algebra; however, over ninety per cent of the high schools were using a book with the unit included. Only one out of three teachers with two years or more of experience had attended an institute for mathematics teacher education.

II. CONCLUSIONS

1. A majority of the secondary school mathematics teachers in the area represented by this study were not adequately prepared to teach the new topics in the mathematics curriculum.
2. Current state adopted textbooks for the eleventh grade and new texts for the twelfth grade in use in the Lamar Area School Study Council meet the recommendations of curriculum study programs such as the School Mathematics Study Group and the Committee on the Undergraduate Program in Mathematics.
3. New curriculum materials and recommendations by

commissions and study groups for mathematics in the secondary schools are less different than some sources have suggested. The major differences in many of the suggested programs are concerned with placement, sequence of topics, and degree of emphasis rather than with what mathematics should be taught in high school.

4. Inadequate measuring instruments will hinder research in mathematics education in secondary schools until standardized tests including treatments of new topics are developed.
5. As represented by this sample, college preparatory mathematics students at the senior level are interested in the new topics in mathematics and learn quickly to work with them.
6. The majority of the mathematics teachers in the high schools of the Lamar Area School Study Council either have not applied or have not been accepted for National Science Foundation institutes or similar programs. Teachers who have attended National Science Foundation institutes were already better prepared to teach mathematics than those who have not applied or not been accepted.
7. Based upon the teachers who had earned bachelor's

degrees since January, 1963, the content of the majority of college programs preparing mathematics teachers for high schools has not yet been altered to include recommendations of study groups such as the Committee on the Undergraduate Program in Mathematics. Some courses covering new mathematics content were being offered but were not included in the required preparation programs for mathematics teachers.

8. In-service workshop programs are successful means of subject content re-education in mathematics even with wide ranges in teaching experience and mathematical background.

9. The manual "An Introduction to the Theory of Matrices" is a satisfactory guide or text for an in-service program in matrix algebra. More time with teachers for discussion of the manual should be allowed than was provided in this experiment.

10. High school mathematics teachers are interested in the improvement of mathematics education. With the proper approach and motivation they will participate in research and study programs, including testing, such as this study represents.

III. RECOMMENDATIONS

1. It is recommended that matrix algebra topics be included in the eleventh or the twelfth grade for the regular college preparatory program and that accelerated students be required to study the topic a second time in its applications to geometry and vectors.
2. It is recommended that the results of surveys of college mathematics departments, curriculum revision programs, and content study groups be followed to the extent that topics identified by the analysis of these recommendations be included in high school programs. Calculus should be omitted from the high school mathematics curriculum except in the case of extremely accelerated students who have completed the study of such topics as analytic geometry, linear algebra, theory of functions, statistics, and analysis.
3. It is recommended that the Lamar Area School Study Council strengthen the in-service mathematics education program by enlarging its program to offer in-service institutes and summer institutes in subject matter and methods courses for mathematics teachers.

4. It is recommended that content manuals for in-service education be prepared in the new high school topics, and that they be constructed with the in-service participant in mind. That is, the manual must presume no prior knowledge of the topic, and must take the teacher beyond the material to be covered in the high school classroom.
5. It is recommended that efforts be made to encourage school districts to require re-education or in-service work in mathematics education with those currently teaching mathematics. There are many teachers not attending, indeed not even applying, for summer institutes or other programs, and students continue to be assigned to these teachers annually for their mathematics instruction and inspiration.
6. It is recommended that research in mathematics education be intensified, both in the secondary school classroom and with the many problems in teacher education and re-education. Teachers need to be encouraged to participate more in research, and arrangements made for participation so that more accurate evaluation procedures and experimental designs may be followed.

7. It is recommended that colleges examine present programs of teacher education in mathematics considering the current recommendations.

IV. RECOMMENDATIONS FOR FURTHER STUDY

1. Teaching assignments compared to the number of semester hours suggested a need for the study of assignments of teachers to determine what criteria for assignment, if any, are being followed. Are assignments being made based upon the best preparation to teach the given subject or upon other criteria? This indicated a need for a study of the procedures being used and the effects on teacher initiative and student achievement.
2. Absence of standardized measuring instruments for students and for teachers in the new topics in mathematics indicated a need for research to develop such instruments. The voluntary nature of teacher participation in this study would have to be changed before standardization and evaluation techniques could be formulated to develop such instruments accurately. The need for development of such yardsticks for comparison in mathematics education is strongly indicated.

3. A study of the content of college mathematics courses required for completion of certification to teach high school mathematics should be conducted. The recommendations that teachers be thoroughly familiar, not only with the subject being taught, but also with the material which comes before and which follows that subject are most valid. This indicates a study to determine how well teacher preparation programs are accomplishing this objective.
4. Since improvement in education takes place only if change reaches the classroom and the student, a study to determine what topics and approaches are actually being presented in the mathematics classroom should be conducted.
5. National Science Foundation institutes represent an opportunity to conduct research on mathematics education on a national level. Recognizing that programs for the institutes are quite complete, effort should be made to take advantage of this "captive" group to conduct research of significance to improvement of mathematics education.

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APPENDIXES

APPENDIX A

AN INTRODUCTION TO THE THEORY OF MATRICES

I. INTRODUCTION

The theory of matrices is one of the topics becoming more prominent in secondary school mathematics. The algebra of matrices represents an opportunity for the study and application of the laws and structure of mathematics with new and interesting material for the high school mathematics student. This study will serve to emphasize the relation and significance in mathematics of structural definitions and laws of operations. We have worked and drilled so much with the definitions and operations of our number system that we often do not realize their full significance. The properties and operations of matrix algebra are such that they emphasize the properties and operations we work with in all algebra.

The study of the theory of matrices is important to the student of mathematics, science, and engineering. Our modern computers do many of their computational operations with matrices. The area of atomic physics expresses many of its problems in the algebra of matrices. Solutions of vectors, vector spaces, systems of linear equations and of the mathematics of quantum mechanics may all be expressed in matrix algebra. Other applications such as inventory control in industrial and business operations, cost analysis studies, deployment problems in military tactics, and data analysis work in statistics employ matrix theory.

The term matrix (singular) was first used around 1850 by J. J. Sylvester; and the foundations of the theory of matrices were formulated by Arthur Cayley, an Englishman, in 1858, during the reign of Queen Victoria. The development of the areas of quantum mechanics and mass data analysis, along with electronic computers, has only served to increase the significance of this topic of mathematics. A unit in matrix algebra in the latter part of a second year algebra course and a more complete unit in senior level courses for advanced mathematics students seem highly desirable.

II. LITERATURE AND TEXTBOOKS

There are five state adopted textbooks for the second course in algebra in Texas public schools. Three of these textbooks have units on matrix algebra and determinants and one of the books has a unit on determinants. Only one of the texts does not present an approach to working with matrices and determinants. The Charles E. Merrill Company publishes the book Algebra Two, A Modern Course by Vannatta, Goodwin, and Fawcett, which includes a chapter on the theory of matrices. The book Modern Algebra, Book Two, by Dolciani, Berman and Wooten and published by Houghton Mifflin Company, has an excellent introduction to the theory of matrices and operations with determinants. Modern Algebra, Second Course by

Johnson is published by Addison-Wesley and presents topics on matrices and determinants. Welchons, Krickenberger, and Pearson are the authors of Book Two Algebra, published by the Ginn Company, which presents a unit on determinants and operations with determinants. The book published by the Holt, Rinehart and Winston Company entitled Contemporary Algebra and Trigonometry by Griswold, Keedy, and Schacht is the only one of the state adopted books which does not give time and attention to this topic. The book selected for use in second year algebra by the greatest number of school districts in Texas was the Houghton Mifflin Company textbook which presents a fairly complete introduction to matrix theory.

The School Mathematics Study Group writing team produced a manual for students and a teacher's commentary on Introduction to Matrix Theory. There is now available a revised edition of the manual, and SMSG still includes this among their recommended topics for high school mathematics. An excellent study for teachers on the theory of matrices may be found in the book Linear Algebra for Undergraduates by D. C. Murdock and published by John Wiley & Sons of New York. Another excellent treatment of matrix algebra may be found in Higher Algebra for Undergraduates by Marie Weiss and revised by Roy Dubisch. This revised edition was published by John Wiley & Sons in 1962. Possibly the best illustration of the techniques and the operations of matrices and determinants may be found in the book Numerical Analysis by K. S. Kunz, a research physicist for the Schlumberger Well Surveying Corporation. Mr. Kunz's book grew out of a set of lecture notes presented at the Computation Laboratory of Harvard University and gives many interesting operations and uses of the various methods of matrices and determinants.

III. TERMINOLOGY AND BASIC DEFINITIONS

A matrix is a rectangular array of numbers arranged in rows and columns and enclosed in brackets or large parentheses as shown:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \quad \begin{pmatrix} 1 & 2 & 7 & 5 \\ 3 & 6 & 2 & 2 \\ 1 & 0 & 2 & 3 \end{pmatrix}.$$

The individual numbers of the array are called entries or elements of the matrix. The matrix has a dimension that is determined by the number of rows (horizontal) and the number of columns (vertical) of the matrix. Of the matrices shown above, the first has three rows and three columns, so it is a three by three (3x3) matrix. The second would be a 3x4 matrix. The dimension is always given with the number of rows preceding the number of columns. A square matrix is a matrix with the same number of rows and columns, such as 2x2, 3x3, or nxn. A matrix with only one row is called a row matrix or row vector, and similarly a column matrix or column vector is the matrix with only one column.

Matrices are usually designated by capital letters and the elements or entries by small letters. Examples are as follows:

$$A_{3x2} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} , \qquad B = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} ,$$

$$C_{mxn} = \begin{pmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{pmatrix} .$$

From the examples matrix C has m rows and n columns, therefore the dimension of mxn. Matrix B is a column matrix with the dimension 4x1.

For every matrix A, there exists a corresponding matrix, A^t, called the transpose of A. The transpose of a matrix is formed by interchanging the rows and columns as shown with the first row becoming the first column of the transpose, the second row becoming the second column of the transpose, and the third row becoming the third column.

$$\text{If} \quad A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \text{, then } A^t = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} .$$

If matrix B has a dimension of 3x4, then we can readily see that B^t must have the dimension 4x3. If all the entries of a matrix are zeros, then the matrix is called a zero matrix. Examples below show that we can have zero matrices of different dimensions:

$$0_{2x3} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} , \quad 0_{2x2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} .$$

IV. THE ALGEBRA OF MATRICES

Equality relation

The relation of equals has the following definition. Two matrices are equal <u>if and only if</u> they have the <u>same dimension and the same corresponding entries</u>.

$$\text{If} \quad A = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} , \quad B = \begin{pmatrix} c_1 & c_2 \\ d_1 & d_2 \end{pmatrix}$$

for these two matrices, and from this definition, if A = B, then $a_1 = c_1$, $a_2 = c_2$, $b_1 = d_1$, and $b_2 = d_2$. At this point, it would be meaningful to cover the properties of the equality relation. Demonstrate to a class why this relation satisfies the reflective, symetric, and transitive properties.

Addition Operation

The sum of two matrices of the same dimension is a matrix whose entries are the sums of the corresponding entries of the matrices being added. This means that matrix addition is defined only for matrices of the same dimension. The sum of two matrices then will be a unique matrix of the same dimension as those being added. We can show here that the commutative and the associative laws hold for matrix addition:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} a+1 & b+2 \\ c+1 & d+3 \end{pmatrix}.$$

From the definition of a zero matrix and the definition of addition, we see that the additive identity in matrix algebra will be a zero matrix of the same dimension, and

$$A_{mxn} + 0_{mxn} = A_{mxn}.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} o & o \\ o & o \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The additive inverse of a matrix will be the matrix each of whose elements are the negative of the corresponding elements of the original matrix. If $A + B = \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$, then we know that A is a 2x2 matrix with each element of B being the negative of the corresponding element of A. The additive identity and the additive inverse both exist in the operation of matrix addition.

Scalar Multiplication

In matrix algebra, any real number or constant is called a scalar. We can multiply a scalar times a matrix, and the result of this operation will be a matrix of the same dimension as the original matrix. The product of a scalar and a matrix is a matrix each of whose elements is the product of the scalar and its corresponding element from the first matrix. If k is a scalar, then by definition

$$k \cdot \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} ka & kd \\ kb & ke \\ kc & kf \end{pmatrix}.$$

We can show that scalar multiplication will be commutative and associative. We now have sufficient affluence in working with matrices that we can solve such problems as finding the value of a, b, c, and d in the relation:

$$3 \begin{pmatrix} a & b \\ c & d \end{pmatrix} - 2 \begin{pmatrix} 2 & 8 \\ 3 & 0 \end{pmatrix} = 4 \begin{pmatrix} -1 & 2 \\ 3 & 3 \end{pmatrix} .$$

Performing scalar multiplication, we will have

$$\begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix} + \begin{pmatrix} -4 & -16 \\ -6 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 12 & 12 \end{pmatrix} .$$

Then performing the addition operation, we will have

$$\begin{pmatrix} 3a - 4 & 3b - 16 \\ 3c - 6 & 3d + 0 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 12 & 12 \end{pmatrix} .$$

From the definition of the equals relation, we then have the following solution:

$$\begin{array}{cccc} 3a - 4 = -4 & 3c - 6 = 12 & 3b - 16 = 8 & 3d + 0 = 12 \\ 3a = 0 \;, & 3c = 18 \;, & 3b = 24 \;, & 3d = 12 \\ a = 0 & c = 6 & b = 8 & d = 4 \end{array} .$$

We are given the three matrices:

$$X = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} , \quad Y = \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} , \quad Z = \begin{pmatrix} a_3 & b_3 \\ c_3 & d_3 \end{pmatrix}$$

We should be able to establish the proof of the following relations:

1. $(X + Y)^t = X^t + Y^t$, 4. $(X + Y) + Z = X + (Y + Z)$,

2. $k(X + Y) = kX + kY$, 5. $(X^t)^t = X$,

3. $(k + 1) X = kX + 1X$, 6. $(-1) X = -X$.

Matrix Multiplication

The multiplication of two matrices will give a third matrix whose elements are found by adding the products obtained by multiplying the elements of a row of one matrix with the corresponding elements of a column of the other matrix. We must notice that two matrices to be multiplied must have the condition that the number of elements of the rows of the first is the same as the number of elements of the columns of the second. This is a necessary condition for matrix multiplication, and the operation is not defined unless this condition is fulfilled. Some examples of matrix multiplication are:

1. $$(a\ b\ c) \cdot \begin{pmatrix} x \\ y \\ x \end{pmatrix} = (ax + by + cz),$$

2. $$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \circ \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} ax + bz & ay + bw \\ cx + dz & cy + dw \end{pmatrix},$$

3. $$\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \circ \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1\circ 1 + 0 \circ -1 \\ 2\circ 1 + -1 \circ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

From these examples we can see then that A_{2x2} times B_{2x1} will give a C_{2x1}, and generally, that A_{mxn} times B_{nxp} will give C_{mxp}.

We are now able to show that matrix multiplication is not generally commutative. AB means the <u>left multiplication</u> of A and B and BA means the <u>right multiplication</u> of B and A. If

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} ,$$

then we can do the indicated multiplication as follows:

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} \quad , \quad BA = \begin{pmatrix} 5 & -2 \\ 5 & -3 \end{pmatrix} .$$

Since AB $\neq$ BA, then we have learned that the commutative law does not hold for matrix multiplication generally.

From our definition of a <u>square matrix</u> and from matrix multiplication we can show the square matrix whose <u>main diagonal</u> from upper left to lower right consists of elements of one, while all other elements are zero, will have a unique property. Multiplication of any square matrix A by this special square matrix we will call I will give the matrix A, or

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \circ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a + 0 & b + 0 \\ c + 0 & d + 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix};$$

and

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}.$$

This gives us our multiplicative identity for matrices. It is called the Identity Matrix. We must recognize that the Identity matrix is defined only for the set of square matrices. I_{3x3} is the identity element in matrix multiplication for the set of 3x3 matrices. We can show that $A \circ I = A$ and that $I \circ A = A$.

From the study of our number system we know that if $a \cdot b = 0$, then either $a = 0$ or $b = 0$. Consider the product of the given matrices:

$$\begin{pmatrix} -2 & 1 \\ -4 & 2 \end{pmatrix} \circ \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

This tells us that in matrix multiplication that $AB = 0$ does not imply that either A or B must be a zero matrix. We can now show that the associative law (AB)C = A(BC) holds for matrix multiplication. We will have <u>two</u> distributive laws due to left hand and right hand multiplication. We can show that these are valid. We are now ready to solve the following types of problems:

1. Show why $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is not an "identity matrix;"

2. $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{pmatrix}$, find A^2, a^3, $(-A)^2$;

3. Let A and B be 2x2 matrices. Show that

$$(A \cdot B)^t = B^t \cdot A^t;$$

4. $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -5 \\ -2 & 2 \end{pmatrix}$,

   show that $AB = BA = I$.

V. A MATRIX SOLUTION OF SYSTEMS OF LINEAR EQUATIONS

It is possible with what we know of matrices to operate on a system of linear equations to find the common solution. Given the system of linear equations:

$$x - y = 2,$$
$$2x + 5y = 11.$$

We can write these in matrice form as:

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 5 & 11 \end{pmatrix} .$$

Since each is one equation, we can perform transformations on these rows. Any operation which will result in an equivalent system of equations may be performed on the matrix. We remember from early algebra that we can do certain operations without changing the value of the equations. Let us multiply the elements of the first row by 2 and subtract the results, element by element, from the second row. This would give:

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 7 & 7 \end{pmatrix} .$$

Dividing the second row by 7, we have:

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \end{pmatrix} .$$

By adding the elements of the second row now to the first row we will get the matrix:

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{pmatrix} .$$

From this we see that $x = 3$ and $y = 1$.

A second example using three unknowns is as follows:

$$\begin{aligned} x - 2y + z &= 7, \\ 3x + y - z &= 2, \\ 2x + 3y + 2z &= 7. \end{aligned}$$

The matrix for this system would be:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{pmatrix}.$$

Multiplying row one by 3 and subtracting from row two, and multiplying row one by 2 and subtracting from row three, we have the matrix:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 7 & 0 & -7 \end{pmatrix}.$$

Subtract row two from row three:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 0 & 4 & 12 \end{pmatrix}.$$

Divide row three by 4:

From this we see that $x = 3$ and $y = 1$.

A second example using three unknowns is as follows:

$$\begin{aligned} x - 2y + z &= 7, \\ 3x + y - z &= 2, \\ 2x + 3y + 2z &= 7. \end{aligned}$$

The matrix for this system would be:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{pmatrix}.$$

Multiplying row one by 3 and subtracting from row two, and multiplying row one by 2 and subtracting from row three, we have the matrix:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 7 & 0 & -7 \end{pmatrix}.$$

Subtract row two from row three:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 0 & 4 & 12 \end{pmatrix}.$$

Divide row three by 4:

$$\begin{pmatrix} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

Multiply row three by 4 and add the result to row two. Subtract row three from row one:

$$\begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 0 & -7 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

Divide row two by 7:

$$\begin{pmatrix} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

Multiply row two by 2 and add to row one:

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

Therefore the common solution is $x = 2$, $y = -1$, and $z = 3$.

IV. THE DETERMINANT

We know from our number system that if $a \cdot b = 1$, then b is the <u>inverse</u> of a. In matrix multiplication we will have a similar relation that if $A \cdot B = I$, then B must be the inverse of A. In order to work with the inverse of matrices we must first study the property of determinants. Associated with every square matrix of any dimension is a real number called the determinant of the matrix. For 2x2 matrices the determinant is found by adding the product of the elements of the upper left to lower right diagonal to the negative of the product of the elements of the other diagonal.

Determinants of Second and Third Order Matrices

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then the determinant of A is ad-bc.

The determinant is designated usually by the symbol delta,

If $A = \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$, then $\delta(A) = 5$.

Let us now look at 3x3 matrices. The determinant is shown usually as the elements of the matrix set up in verticle bars. We define the determinant of order three as follows:

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}, \qquad \delta(A) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} .$$

$$\delta(A) = a_1 \circ b_2 \circ c_3 + a_2 \circ b_3 \circ c_1 + a_3 \circ b_1 \circ c_2 - a_2 \circ b_1 \circ c_3 - a_1 \circ b_3 \circ c_2 - a_3 \circ b_2 \circ c_1 .$$

This is found by adding the products of elements on left to right diagonals and then adding the negative of the products of elements on right to left diagonals in this scheme:

$$\left| \begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array} \right. .$$

<u>Expansion by Minors</u>

The scheme just shown is good only for the <u>third order determinant</u>. Emphasize to students that the determinant is a real number, not an array of numbers, even though notation seems very similar to the matrix notation. A matrix is an array of numbers while a determinant is a particular real number found by the "expansion" of the array of numbers defining the determinant. Expansion of any determinant of any order may be done by a process called "<u>expanding by minors</u>." The minor of any element in a determinant is the determinant resulting from the deletion of the row and column containing that element. The minor of the element b_1 in the following determinant is as shown:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad , \quad \text{minor of } b_1 = \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} \; .$$

The expansion of δ about the elements of the first row will be given by:

$$\delta = a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \quad - b_1 \cdot \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} \quad + c_1 \cdot \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \; .$$

Notice that we have +1 times a_1 and c_1 and -1 times b_1 in this expansion. We multiply the resulting product of the element and its minor by either +1 or -1 according as the sum of the number of the row plus the number of the column is an even or odd integer. The element b_1 is in the second column and the first row and the sum is three, an odd integer; therefore, we multiply by the negative one. A fourth order determinant may be expanded first to an expression involving third order determinants and then each third order determinant expanded. In this way we can arrive at the real number which is the value of the determinant. Further examination of expanses, such as Laplace's Expansion, is encouraged to aid the teacher in understanding determinants.

Properties of Determinants

There are some special properties of determinants which may be used to simplify expansions. These are general and apply to any order determinant.

1. If two rows or two columns of a determinant are identical, then the value of the determinant is zero.
2. If any two rows or any two columns of a determinant are interchanged, the resulting determinant is the negative of the original determinant.
3. If all rows and columns of a determinant are interchanged in order, the resulting determinant equals the original one.
4. If the elements of one row or one column are multiplied by the real number k, the resulting determinant is k times the original one.
5. If one row or one column has zero for every element, the determinant is zero.
6. If each element of one row or one column is multiplied by a real number k and if the resulting products are then added to the corresponding elements of another row or another column, respectively, the resulting determinant equals the original one.

VII. THE INVERSE OF A MATRIX

The <u>inverse</u> of a matrix always refers to the multiplicative inverse. We refer to the additive inverse of a matrix as the negative of the matrix. Thus we have the matrix A, the additive inverse $-A$, and the multiplicative inverse A^{-1}. We have discussed the identity matrix I. We shall define the A^{-1} as the matrix which multiplies with A to give I. In the expression

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \circ A^{-1} = I,$$

we want to find the matrix A^{-1} in terms of the elements of the given matrix. We can see that

$$\begin{array}{ccccc} A & \circ & B & = & I, \\ \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} & \circ & \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} & = & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{array}$$

Therefore, we see that B is the inverse of A, since the product is the identity matrix. <u>Does every 2x2 matrix have an inverse</u>? Let us consider the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and let $A^{-1} = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$, then we have $A \circ A^{-1} = I$, or

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \circ \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

This will give us the relation

$$\begin{pmatrix} ax + bz & zy + bw \\ cx + dz & cy + dw \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

From the equality relation, this will be true if and only if

$$\begin{aligned} ax + bz &= 1, & ay + bw &= 0, \\ cx + dz &= 0, & cy + dw &= 1. \end{aligned}$$

Solving these two pairs of equations, we find:

$$x = \frac{d}{ad-bc}, \quad y = \frac{-b}{ad-bc}, \quad z = \frac{-c}{ad-bc}, \quad w = \frac{a}{ad-bc}.$$

Substituting in the previous relation for A^{-1} we will have

$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}.$$

Using our knowledge of scalar multiplication this becomes

$$A^{-1} = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

We can see that this will be true if and only if $ad-bc \neq 0$, or A will have an inverse if the determinant of A does not equal zero. This relation enables us to find the inverse for any second order matrix if the determinant of the matrix is not zero. An invertible matrix is said to be non-singular. The inverse of a matrix of order greater than two can be found although it is somewhat more difficult.

There are a number of methods for computing inversions of matrices. The simplest and most direct is called the "Elimination Method." Any non-singular matrix may be "Augmented" and then the inverse found by the elimination method. Since we can solve systems of linear equations readily in matrix algebra, the procedures of inversion of matrices of higher order than two will be most interesting to the better student or to the advanced classes at the senior level. An example of this is as follows:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \end{pmatrix}$$

$$\text{Augmented } A = \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 7 & 0 & 0 & 1 \end{pmatrix}.$$

We have augmented matrix A with the elements of the identity matrix of the same dimension. We now perform row and column operations to reduce certain elements to zero and others to one to get the elements of the identity matrix on the left side. Multiplying row one by four and subtracting from corresponding elements of row two, then multiplying row one elements by seven and subtracting from row three, we will have:

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -14 & -7 & 0 & 1 \end{pmatrix}.$$

Now if we multiply the second row by two and subtract from the third, we will get:

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & 0 & -2 & 1 & -2 & 1 \end{pmatrix}.$$

Dividing row two by -3 and row three by -2, we will get:

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & 1/3 & 0 \\ 0 & 0 & 1 & -1/2 & 1 & -1/2 \end{pmatrix}.$$

Proceeding in this same manner, we will come to the result:

$$\begin{pmatrix} 1 & 0 & 0 & -13/6 & 5/3 & -1/2 \\ 0 & 1 & 0 & 7/3 & -7/3 & 1 \\ 0 & 0 & 1 & -1/2 & 1 & -1/2 \end{pmatrix}.$$

Therefore, the inverse of matrix A will be:

$$A^{-1} = \begin{pmatrix} -13/6 & 5/3 & -1/2 \\ 7/3 & -7/3 & 1 \\ -1/2 & 1 & -1/2 \end{pmatrix}.$$

We have now reached the point where we can solve problems requiring the multiplying by inverses of matrices. Some examples are:

1. $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -2 & 3 \end{pmatrix}$,

2. $\begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$,

3. $\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$.

VIII. CRAMER'S RULE

No attempt will be made to write this out, but reference is made to any good approach to solutions of systems of linear equations using matrices and determinants. Cramer's Rule states that for the system of equations:

$$a_1x + b_1y + c_1z = d_1,$$
$$a_2x + b_2y + c_2z = d_2,$$
$$a_3x + b_3y + c_3z = d_3,$$

we can write the matrix equation:

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \circ \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} .$$

If D denotes the determinant of the coefficient matrix, and $D \neq 0$, we can find the values for the unknowns directly by the following relation:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D} , \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D} , \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D} .$$

If $D \neq 0$, we can thus find expressions for x, y, and z. The determinant in the numerator is the same as the coefficient matrix determinant with the constant term replacing the coefficients of the variable being solved for in that relation.

Cramer's Rule is applicable to any number of equations and can be used to solve "n" equations in "n" unknowns. The value may be set up directly using the determinants. A computer can easily be programmed to solve the expansion of the determinant, so Cramer's Rule is used in solving large systems of equations by arranging data in rectangular form.

IX. SUMMARY STATEMENT

There are many applications to matrix algebra, and as illustrated one can go into as much depth as time, interest, and need will dictate. We must study our content thoroughly and be prepared to present it clearly and concisely. It is hoped that this introductory unit in matrix algebra will challenge teachers to study all new topics in secondary mathematics. We must organize and present them in an interesting and informative way for the benefit of our student.

APPENDIX B

SURVEY OF WORKSHOP PARTICIPANTS MODERN SENIOR HIGH SCHOOL MATHEMATICS - 1965 LAMAR AREA SCHOOL STUDY COUNCIL

1. What is your sex? Male ______ Female ______

2. What kind of degree or degrees do you have and what are the majors, minors, and dates of completion?

   Bachelor
   Major ______________________

   Minor ______________________

   Date completed ______________________

   Masters
   Major ______________________

   Minor ______________________

   Date of completion ______________________

3. How many years of teaching experience do you have? (Count the present year.)

   Total years ______________________

   Years in mathematics ______________________

4. How many semester hours do you have in mathematics?

   ____

5. Have you ever attended a summer institute in mathematics sponsored by the National Science Foundation? Yes ______ No ______

If your answer is yes, list the year and number of weeks

Year______ Number of weeks______ Semester hours_______

Year______ Number of weeks______ Semester hours_______

Year______ Number of weeks______ Semester hours_______

6. Were you present for the presentation of the introduction to matrix theory in this workshop? Yes _____ No _____

7. Have you had instruction in matrix theory before this workshop presentation? Yes _____ No _____

If yes, give the following:

In college course ________ Date ____________________

In other workshops ________ Date ____________________

8. What courses in mathematics are you teaching?

Algebra I __________ Trigonometry __________

Algebra II __________ Analysis I or II __________

Geometry __________ Analytic Geometry __________

Consumer Math __________ Other __________

9. What textbooks are being used by your school for the following?

Algebra II ______________________________

Trigonometry ______________________________

Analysis I ______________________________

Advanced Math ______________________________

10. Please make any comment or evaluation you would care to about the hand-out unit on matrices. Was it helpful, clear, did it begin at the best place, etc.?

Comments:

11. Do you feel that manuals and presentations such as this in other units of high school mathematics are needed?

Yes ____

No ____

Comments:

APPENDIX C

MATRIX ALGEBRA CONTENT SURVEY

PRE-TEST: PART A

Place a circle around the letter in front of the correct answer:

1. A matrix is:

a. a system of linear equations.
b. an array of numbers.
c. a system of quadratic equations.
d. a rectangular array of numbers.
e. a 2x2 set of numbers.

2. The dimension of a matrix is:

a. the product of its entries.
b. the rows divided by the columns.
c. the length times the width.
d. found by using determinants.
e. the number of rows and columns.

3. A square matrix is:

a. one whose length equals its width.
b. one whose entries give a perfect square.
c. one that has the same number of rows and columns.
d. one that has a rational root.
e. found by multiplying a matrix times itself.

4. In matrix addition the operation requires:

a. adding a real number to a matrix.
b. adding a matrix to a complex number.
c. adding a determinant and a matrix.
d. adding a matrix to a matrix.
e. finding the determinants, then adding them.

5. The result of matrix addition is:

a. another matrix.
b. a real number.
c. a complex number.
d. no defined answer.
e. a real value if the determinant is not zero.

6. Matrix multiplication is:

   a. defined for all matrices.
   b. defined for two matrices with equal number of entries.
   c. defined if the number of rows of one equals the number of columns of the other.
   d. defined for matrices whose entries are real numbers.
   e. defined for square matrices only.

7. In matrix theory, scalar multiplication is:

   a. the product of all the elements.
   b. the product of the determinants.
   c. the product of a constant times a matrix.
   d. the result of multiplying two matrices.
   e. finding the proper units of measure.

8. The result of matrix multiplication is:

   a. a scalar product.
   b. a determinant.
   c. another matrix.
   d. a square matrix.
   e. a minor of the original matrix.

9. Two matrices are equal if and only if:

   a. the sum of the elements is equal.
   b. they have the same dimension and the sums of the elements are equal.
   c. they are both square matrices.
   d. the determinants can be added or multiplied.
   e. they have the same dimension and every element of one is identical to every element of the other.

10. In matrix algebra there exists:

    a. an additive inverse, but no additive identity.
    b. an additive identity, but no multiplicative identity.
    c. a multiplicative inverse, but no additive identity.
    d. both additive and multiplicative identities.
    e. no inverses.

11. Matrix multiplication is:

    a. commutative.
    b. associative, but not commutative.
    c. not associative.
    d. commutative and associative.
    e. neither commutative nor associative.

12. Associated with every square matrix of any dimension is a real number called:

    a. a scalar of the matrix.
    b. the determinant of the matrix.
    c. the square root of the matrix.
    d. another matrix.
    e. a column vector.

13. The additive identity in matrix algebra is:

    a. a scalar of zero.
    b. a zero matrix of any dimension.
    c. a zero matrix of the same dimension.
    d. the number zero.
    e. a determinant of the matrix.

14. The multiplicative inverse of a square matrix is defined and can be found only if:

    a. it is factorable.
    b. a non-zero scalar exists.
    c. it is a non-singular matrix.
    d. the determinant equals zero.
    e. the value of the matrix is unity.

15. The value of a determinant can always be found by:

    a. adding all the elements.
    b. expanding by minors.
    c. inverting the matrix.
    d. factoring out scalars.
    e. subtracting row elements from column elements.

16. Any non-singular matrix may be augmented and the inverse found through a process called the elimination method. This is done by:

    a. multiplying the elements of the matrix.
    b. using the elements of the identity matrix of the same dimension.
    c. finding the determinants of the matrix.
    d. replacing the entries of the matrix.
    e. using both matrix addition and multiplication.

17. Only one of the following is not always true. Select it as the answer:

a. if any two rows of a determinant are identical, the determinant is 0.
b. if any two columns of a determinant are identical, the determinant is 0.
c. if one row has 0 for every element, the determinant is 0.
d. if one column has 0 for every element, the determinant is 0.
e. if you interchange two rows of a determinant, the determinant will be 0.

18. The transpose of a matrix is found by:

a. evaluating the determinant.
b. interchanging the rows and columns.
c. multiplying by the identity matrix.
d. inverting the matrix.
e. factoring out all scalars.

MATRIX ALGEBRA CONTENT SURVEY

PRE-TEST: PART B

Place a circle around the letter in front of the correct answer:

19. Only one of the following is not always true. Pick it as the answer:

    a. $A + B = B + A$

    b. $A \circ A^{-1} = I$

    c. $A \circ (B + C) - A \circ B + A \circ C$

    d. $A \circ I - A$

    e. $A \circ B = B \circ A$

20. One of these statements is not always true. Select it as the answer:

    a. $(A + B)^t = A^t + B^t$

    b. $(A^t)^t = A$

    c. If $A = B$, then $A^t = B^t$

    d. If $A = B$, then $A^{-1} = B^t$

    e. $A^t + I = (A + I)^t$

21. Pick the incorrect statement as the answer:

    a. $I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

    c. $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

    b. $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

    d. $A \circ I = A$

    e. $I \circ A = A$

22. Pick the <u>incorrect</u> statement as the answer:

a. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 4 & 3 \end{pmatrix}$

b. $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

c. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 1 & 6 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

e. $\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ -5 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

23. The addition below gives:

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 4 \\ 0 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 5 \\ 0 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix} = :$$

a. $\begin{pmatrix} 3 & 7 & 5 \\ 8 & 4 & 3 \\ 3 & 7 & 5 \end{pmatrix}$ b. $\begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix}$ c. $\begin{pmatrix} 1 & 5 \\ 1 & 5 \\ 1 & 5 \end{pmatrix}$

d. $\begin{pmatrix} 4 & 5 \end{pmatrix}$ e. $\begin{pmatrix} 2 & 3 & 5 \\ 3 & 3 & 4 \\ 3 & 2 & 2 \end{pmatrix}$

24. Find A·B when $A = \begin{pmatrix} -1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{pmatrix}$:

a. $\begin{pmatrix} -1 & 0 & -2 \\ 4 & 0 & 1 \\ 1 & 1 & 6 \end{pmatrix}$ b. $\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$ c. $\begin{pmatrix} 3 & 2 & 8 \\ 5 & 1 & 2 \\ 5 & 2 & 6 \end{pmatrix}$

d. $\begin{pmatrix} 8 & 0 \end{pmatrix}$

e. $\begin{pmatrix} -1 & 4 & 1 \\ 0 & 0 & 1 \\ -2 & 1 & 6 \end{pmatrix}$

25. Given $A = \begin{pmatrix} 7 & 2 & 1 \\ 4 & 1 & 3 \\ 0 & 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, Find A·B:

a. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ b. 8 7 c. $\begin{pmatrix} 7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

d. $\begin{pmatrix} 7 & 2 & 1 \\ 4 & 1 & 3 \\ 0 & 5 & 6 \end{pmatrix}$ e. $\begin{pmatrix} 10 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 11 \end{pmatrix}$

26 If $M = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$, find M^t.

a. $\begin{pmatrix} y_2 & x_2 \\ y_1 & x_1 \end{pmatrix}$

b. $\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$

c. $\begin{pmatrix} x_2 & y_2 \\ x_1 & y_1 \end{pmatrix}$

d. $\begin{pmatrix} y_1 & x_1 \\ y_2 & x_2 \end{pmatrix}$

e. $x_1y_2 - y_1x_2$

27. If $M = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$ find M^{-1}.

a. $\begin{pmatrix} x_1 & -x_2 \\ -y_1 & y_2 \end{pmatrix}$

b. $\dfrac{1}{x_1y_2 - x_2y_1} \begin{pmatrix} y_2 & -y_1 \\ -x_2 & x_1 \end{pmatrix}$

c. $\begin{pmatrix} \frac{1}{x_1} & \frac{1}{y_1} \\ \frac{1}{x_2} & \frac{1}{y_2} \end{pmatrix}$

d. $\dfrac{x_2 \quad -x_1}{y_2 \quad -y_1} \begin{pmatrix} x_2 & y_2 \\ x_1 & y_1 \end{pmatrix}$

e. $x_1y_2 - y_1x_2$

28. Expanding the determinant given at the right about the second column, the minor of the element B_3 will be:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

a. $\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ b. $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ c. $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

d. $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ e. $b_1b_2 - a_3c_3$

29. For the given system of equations, the value of the determinant of the coefficient matrix will be:

$$\begin{aligned} 2x - y - z &= 1 \\ 2x - 3y - 4z &= 0 \\ x + y - z &= 4 \end{aligned}$$

a. -6 b. 11 c. -4 d. 25 e. 5

30. Given $\begin{matrix} 2 & -3 \\ 1 & 4 \end{matrix} \cdot \begin{matrix} x \\ y \end{matrix} = \begin{matrix} 1 \\ 6 \end{matrix}$ find x and y:

a. $x = 22$
$y = 11$

b. $x = 1$
$y = 6/5$

c. $x = 2$
$y = 1$

d. $x = 1/3$
$y = 6$

e. $x = -1$
$y = -2$

APPENDIX D

Dear Mathematics Teacher:

By way of the enclosed material I am collecting data concerning the retention of the content we covered in the workshop in senior high school mathematics. The material consists of a short questionnaire and a content survey.

Permission has been granted for me to contact you and ask your cooperation in this project. Only the workshop participants can assist in this data collection. Please fill in the two sets of material and mail to me. You are asked not to consult with anyone or refer to any material as you answer the survey. If you want a copy of the survey, put a note in the envelope to this effect and I will see that you receive it.

I know I am asking much at this busy time, and I appreciate your help. I can accept only those which are answered and returned immediately without reference to the manual or other materials.

Sincerely,

Oliver P. Monk

APPENDIX E

DATA FOR STUDENT GROUP

Number	Pre-Test Score	Post-Test Score	Gain	SAT-Math	H. S. Grade Point Average	Otis I. Q.
A1	10	29	19	567	3.833	130
A2	16	28	12	666	3.167	127
A3	5	24	19	527	3.262	115
A4	22	28	6	738	3.786	135
A5	19	28	9	---	4.000	132
A6	11	27	16	633	3.286	132
A7	6	30	24	543	3.762	132
A8	16	28	12	685	3.952	129
A9	8	27	19	589	3.500	126
A10	13	30	17	571	3.952	132
A11	11	26	15	460	3.524	117
A12	4	30	26	584	3.833	128
A13	16	30	14	707	3.833	129
A14	5	23	18	452	3.286	120
A15	18	30	12	650	4.000	135
A16	16	29	13	642	3.952	132
A17	13	30	17	661	4.000	128
A18	14	29	15	633	3.429	117
A19	13	29	16	567	3.524	126
A20	9	28	19	584	3.786	117
A21	5	30	25	600	3.810	124
A22	12	30	18	608	3.667	127

DATA FOR STUDENT GROUP

Number	Pre-Test Score	Post-Test Score	Gain	SAT-Math	H. S. Grade Point Average	Otis I. Q.
A23	5	29	24	565	3.452	112
A24	8	30	22	606	3.571	127
A25	14	29	15	703	3.881	133
A26	7	29	22	580	3.786	129
A27	13	30	17	518	3.810	122
A28	9	30	21	694	3.976	136
A29	4	29	25	606	3.690	112
A30	13	29	16	625	3.976	129
A31	8	29	21	608	3.690	134
A32	13	29	16	747	3.452	131
A33	12	30	18	580	3.571	114
A34	14	29	15	625	3.714	113
R1	2	19	17	444	2.595	118
R2	10	29	19	518	3.286	117
R3	3	24	21	518	3.119	123
R4	4	26	22	518	2.976	110
R5	4	22	18	444	2.952	108
R6	4	30	26	718	3.619	116
R7	5	23	18	427	3.050	113
R8	2	22	20	518	1.929	120
R9	4	29	25	567	4.000	125
R10	5	28	23	584	3.976	121

DATA FOR STUDENT GROUP

Number	Pre-Test Score	Post-Test Score	Gain	SAT-Math	H. S. Grade Point Average	Otis I. Q.
R11	4	24	20	492	2.976	117
R12	3	24	21	567	2.762	132
R13	1	26	25	430	2.976	110
R14	11	26	15	559	3.286	128
R15	4	22	18	427	2.048	105
R16	8	25	17	589	2.905	125
R17	1	26	25	---	3.762	119
R18	10	21	11	353	2.286	101
R19	1	26	25	427	3.452	115
R20	4	25	21	613	3.905	131
R21	3	26	23	546	2.816	112
R22	0	26	26	543	3.190	104
R23	3	25	22	510	3.286	131
R24	7	26	19	497	3.571	119
R25	2	24	22	576	2.762	106
R26	1	25	24	559	3.214	124
R27	--	27	--	---	3.929	---
R28	4	18	14	370	2.119	109
R29	10	23	13	452	2.357	111
R30	6	27	21	551	3.524	116
R31	2	17	15	382	1.929	115
R32	4	26	22	477	3.833	121

DATA FOR STUDENT GROUP

Number	Pre-Test Score	Post-Test Score	Gain	SAT-Math	H. S. Grade Point Average	Otis I. Q.
R33	1	24	23	576	3.500	126
R34	2	26	24	440	3.667	121
R35	1	27	26	576	3.190	107
R36	2	22	20	477	3.500	109
R37	1	22	21	555	3.357	127
R38	4	25	21	---	3.452	123
R39	3	26	23	536	3.190	128
R40	3	20	17	378	2.405	107
R41	3	23	20	545	1.833	109
R42	3	23	20	439	2.357	113
R43	10	30	20	603	3.300	---
R44	7	25	18	526	3.310	120
R45	5	21	16	362	2.175	112
R46	2	25	23	---	2.667	95
R47	5	24	19	554	1.929	119
R48	6	27	21	427	3.071	122
R49	6	26	20	590	3.405	123
R50	5	24	19	518	3.429	120
R51	3	27	24	518	3.310	118

APPENDIX F

DATA FOR TEACHER PARTICIPATION

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
WS1	M	6	28	22	2	27	B-1965	0
WS2	F	2	27	25	1	24	B-1966	0
WS3	F	27	30	3	19	60	B-1947 M-1956	4
WS4	F	6	30	24	8	24	B-1943	0
WS5	F	13	30	17	10	39	B-1942 M-1947	0
WS6	M	16	26	10	7	42	B-1959 M-1964	1
WS7	F	13	30	17	2	37	B-1964	0
WS8	M	2	22	20	17	30	B-1949 B-1957	0
WS9	F	20	28	8	3	42	B-1963	0
WS10	F	27	29	2	2	56	B-1963 M-1965	1
WS11	F	5	28	23	2	18	B-1964	0
WS12	M	19	29	10	41	50	B-1928 M-1940	0
WS13	M	20	30	10	6	36	B-1957	1
WS14	M	10	26	16	5	46	B-1961	1
WS15	M	14	29	15	4	36	B-1963	0
WS16	F	3	23	20	18	24	B-1950 M-1953	0
WS17	M	5	29	24	2	33	B-1964	0
WS18	F	--	30	--	2	30	B-1964	0
WS19	F	14	27	13	41	35	B-1925	0

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
WS20	F	24	30	6	14	34	B-1952 M-1960	1
WS21	F	26	25	--	4	61	B-1960	0
WS22	F	13	27	14	27	45	B-1928 M-1952	1
WS23	F	27	30	3	32	62	B-1930 M-1943	4
WS24	M	19	23	4	4	36	B-1962	1
WS25	M	23	30	7	4	48	B-1961	0
WS26	M	6	25	19	14	45	B-1950 M-1954	2
WS27	M	1	20	19	1	18	B-1965	0
WS28	M	8	26	18	5	35	B-1961 M-1965	0
WS29	M	1	27	26	9	21	B-1956 M-1964	0
WS30	M	12	28	16	1	36	B-1966	0
WS31	M	0	26	26	26	12	B-1933 M-1952	0
WS32	M	0	--	--	2	31	B-1964	0
WS33	M	5	23	18	9	21	B-1955	0
WS34	F	20	27	7	3	39	B-1963	1
WS35	M	17	29	12	8	30	B-1958	2
WS36	M	16	29	13	3	35	B-1963	0
WS37	M	25	30	5	6	40	B-1961 M-1965	0
WS38	M	3	26	23	7	24	B-1957	0

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
WS39	M	17	27	10	8	52	B-1956	0
WS40	M	9	24	15	3	30	B-1963	0
WS41	F	18	28	10	2	30	B-1964	0
WS42	M	6	12	6	17	62	B-1948	1
WS43	F	10	27	17	3	39	B-1963	0
WS44	F	15	29	14	4	36	B-1961	0
WS45	F	16	30	14	2	42	B-1964	0
WS46	M	11	30	19	2	40	B-1964	0
WS47	M	7	29	22	1	25	B-1966	0
WS48	-	0	--	--	7	36	B-	0
WS49	M	2	21	19	3	18	B-1939	0
WS50	F	13	27	14	1	36	B-1965	0
WS51	F	6	15	9	1	30	B-1965	0
WS52	M	22	--	--	17	70	B- M-1953	1
WS53	M	20	29	9	2	30	B-1961 M-1963	0
WS54	F	0	24	24	25	21	B-1937	0
WS55	F	10	28	18	5	42	B-1961	2
WS56	M	--	29	--	11	57	B-1953 M-1960	2
WS57	-	--	13	--	--	--	------	-

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
WS39	M	17	27	10	8	52	B-1956	0
WS40	M	9	24	15	3	30	B-1963	0
WS41	F	18	28	10	2	30	B-1964	0
WS42	M	6	12	6	17	62	B-1948	1
WS43	F	10	27	17	3	39	B-1963	0
WS44	F	15	29	14	4	36	B-1961	0
WS45	F	16	30	14	2	42	B-1964	0
WS46	M	11	30	19	2	40	B-1964	0
WS47	M	7	29	22	1	25	B-1966	0
WS48	-	0	--	--	7	36	B-	0
WS49	M	2	21	19	3	18	B-1939	0
WS50	F	13	27	14	1	36	B-1965	0
WS51	F	6	15	9	1	30	B-1965	0
WS52	M	22	--	--	17	70	B- M-1953	1
WS53	M	20	29	9	2	30	B-1961 M-1963	0
WS54	F	0	24	24	25	21	B-1937	0
WS55	F	10	28	18	5	42	B-1961	2
WS56	M	--	29	--	11	57	B-1953 M-1960	2
WS57	-	--	13	--	--	--	------	-

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
WS58	F	--	30	--	--	--	------	-
WS59		--	23	--	--	--	------	-
SS1	M	1	26	25	6	15	B-1949 M-1963	0
SS2	F	27	30	3	--	42	B-1964	0
SS3	F	10	28	18	4	33	B-1960	1
SS4	F	1	25	24	1	12	B-1964	0
SS5	M	4	22	18	4	18	B-1961	0
SS6	F	--	24	24	--	30	B-1936	0
SS7	F	27	29	2	2	30	B-1963	0
SS8	M	1	27	26	1	35	B-1965	0
SS9	F	5	30	25	2	30	B-1963	0
SS10	F	17	--	--	--	--	------	-
SS11	M	4	11	7	13	15	B-1952	0
SS12	F	1	22	21	3	32	B-1957	0
SS13	M	2	22	--	2	15	B-1939	0
W1	M	--	26	--	6	30	B-1959 M-1962	1
W2	M	--	29	--	16	70	B-1949 M-1953	1
W3	F	--	29	--	1	25	B-1964	0
W4	M	--	20	--	1	23	B-1964	0

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
W5	F	--	24	--	40	36	B-1925	0
W6	M	--	25	--	6	32	B-1959 M-1963	0
W7	F	--	30	--	12	57	B-1953 M-1958	1
W8	M	--	30	--	2	27	B-1963	0
W9	M	--	15	--	29	--	B-1925 M-1941	0
W10	F	--	18	--	27	42	B-1929 M-1952	1
W11	M	--	25	--	2	33	B-1963	0
W12	F	--	27	--	31	23	B-1948 M-1954	0
W13	M	--	16	--	19	29	B-1947 M-1947	0
W14	M	--	27	--	2	42	B-1963	0
W-15	F	--	16	--	8	27	B-1957	0
W-16	M	--	28	--	4	60	B-1962 M-1964	3
W17	M	--	27	--	2	44	B-1963	0
W18	F	--	30	--	1	33	B-1962	0
W19	M	--	27	--	40	44	B-1928 M-1940	0
W-20	F	--	29	--	13	34	B-1952 M-1960	1
W21	M	--	27	--	1	15	B-1960	0
W22	F	--	28	--	1	30	B-1965	0
W23	M	--	28	--	15	62	B-1950 M-1962	2

DATA FOR TEACHER PARTICIPANTS

Number	Sex	Pre-Test Score	Post-Test Score	Gain	Teaching Experience (Years)	Semester Hours in Math	Degree - Date Bachelor's - B Master's - M	NSF Number
W24	M	--	29	--	1	33	B-1964	0
W25	F	--	30	--	10	30	B-1933	2
W26	M	--	29	--	5	36	B-1961	0
W27	M	--	28	--	8	60	B-1956 M-1961	0
W28	M	--	26	--	19		B-1946 M-1953	5
W29	M	--	26	--	3	39	B-1962	0