Cavitation and bubble bursting as sources of oceanic ambient noise

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Cavitationlike bubble collapses and the bursting of floating bubbles have been proposed in the literature as sources of oceanic ambient noise at kilohertz frequencies. The first process is shown to be physically impossible in the oceanic environment. The noise produced by the second mechanism is estimated and shown to be too weak to be of any significance.

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INTRODUCTION

In a recent article, Shang and Anderson report very interesting data on high-frequency wind-dependent ambient noise in the ocean at low wind speed.[1] They also propose two mechanisms to explain these data, which we wish to consider in simple terms in this article.

The first mechanism invokes emission by collapsing cavitation bubbles. This notion appears to have originated with Furduev,[2] and was mentioned, more recently, by Kerman.[3] It seems to be rooted in a basic confusion between the *expansion* of a bubble and its *growth* by diffusion. Failure to recognize the fundamental difference between these two processes can lead to unphysical results, as we show in Sec. I. Even with this clarification, the question remains of whether, due to pressure fluctuations in the surrounding water, a bubble can expand sufficiently to undergo a violent collapse similar to those encountered in flow or acoustic cavitation. We examine this question in Sec. III and again come up with a negative answer.

The second mechanism implies that a significant amount of underwater noise can be produced by the bursting of bubbles that have risen to the ocean surface. We show that this process is a much more effective source of sound in air than in water. Indeed, we find that an unrealistically large number of bursting bubbles would be needed to account for even a small fraction of the observed noise. This result is confirmed qualitatively by observation. We therefore conclude that neither of the two mechanisms can possibly account for the reported data. As we have argued elsewhere,[4,5] bubbles appear to be important sources of oceanic ambient noise, but the mechanisms are quite different from those proposed in Ref. 1.

The precise description of the dynamics of a bubble is fairly complex,[6–10] but for the present purposes a very simple model is adequate. We take the bubble to be spherical, the liquid to be incompressible, and neglect the effect of viscosity. Furthermore, we also assume the gas contained in the bubble to behave polytropically with a polytropic index κ. With all these simplifications the Rayleigh–Plesset equation governing the dynamics of a bubble of radius R may be written[6,8,11]

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[p_{G0}\left(\frac{R_0}{R}\right)^{3\kappa} - \frac{2\sigma}{R} - p_0 - \Delta p(t)\right], \qquad (1)$$

where dots denote time differentiation, ρ is the liquid density, σ is the surface tension, p_0 is the static pressure, and $\Delta p(t)$ is the time-dependent part of the pressure. The vapor pressure is essentially constant and can be absorbed in p_0, even though it is a negligible correction at the temperature of oceanic waters. When $\Delta p = 0$, the bubble has an equilibrium radius R_0 and an equilibrium internal pressure p_{G0} that, from Eq. (1), are connected by

$$p_{G0} = p_0 + 2\sigma/R_0. \qquad (2)$$

The equilibrium radius can be obtained from this equation in terms of the number n of gas moles contained in the bubble and of the temperature T of the surrounding liquid by use of the perfect gas equation of state

$$3n\mathscr{R}T/4\pi R_0^3 = p_0 + 2\sigma/R_0, \qquad (3)$$

where $\mathscr{R}$ is the universal gas constant. The surface-tension term in this equation is negligible for R_0 greater than about 10 μm.

I. MASS DIFFUSION

The proposed mechanism of noise production by collapsing bubbles is described as follows in Ref. 1.

"The process of bubble cavitation in the surface turbulent layer is initiated when the turbulent pressure surrounding a vapor-air cavity . . . is reduced to the . . . threshold whereupon the bubble would grow to a larger radius by rectified diffusion. Some of the larger bubbles will be transformed into a transient cavity determined by the dynamic equation."

In the cavitation literature, the words *transient cavity* indicate a bubble which, having expanded substantially during a decrease of the liquid pressure, undergoes a violent collapse when the pressure recovers.[11] The violence of these collapses causes the substantial amount of medium and high-frequency noise characteristic of flow and acoustic cavitation.[12,13] It must be stressed that, during the expansion of these bubbles, a negligible amount of dissolved gas diffuses into them so that, at the beginning of the collapse, the internal pressure is essentially equal to the liquid vapor pressure, i.e., typically hundreds of times smaller than the external pressure. This enormous pressure imbalance causes the extremely energetic collapse and strong noise emission typical of this process. According to the model postulated in Refs. 1

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and 2, on the other hand, the increase of the bubble radius takes place as a consequence of air diffusion into the bubble, rather than because of a substantial drop of the water pressure. Therefore, the internal pressure is never very different from the external one and the very possibility of a violent collapse negated.

The rectified mass diffusion process mentioned in the above quotation and in Ref. 2 can be explained in the following terms.[6,8,14–16] Consider a bubble executing volume pulsations. If the bubble expands, the partial pressure of the gas contained in it decreases and, therefore, by Henry's law, the concentration of gas dissolved in the neighborhood of the bubble surface also falls. A concentration gradient is set up in the liquid that drives some of the dissolved gas into the bubble. Upon compression the reverse happens but, due to the reduced area and thicker boundary layer, it turns out that the amount of gas lost during compression is less than that gained during expansion so that there is a net increase in the amount of gas contained in the bubble. It is essential to note that this process does not require the bubble to execute oscillations of large amplitude. As a matter of fact, a theory, based on the assumption that for such low-amplitude oscillations linearization is applicable, is sufficient to explain many of the experimental data.[15,16] Acoustically, a bubble growing by rectified diffusion would behave as a weak monopole source with the same time dependence as the liquid pressure causing the oscillations (with the possible addition of a few harmonics) and could in no way be assimilated to a cavitation bubble.

This mass transfer can change the amount of gas contained in the bubble by orders of magnitude, but occurs on a time scale thousands of times greater than the period of the volume pulsations. Therefore, the effect of mass diffusion can be incorporated into the description of the radial dynamics of a bubble simply by allowing the equilibrium radius R_0 given by Eq. (3) to change very slowly with time in response to the changing number n of gas moles contained in the bubble.

The misunderstanding at the root of the proposed mechanism can be illustrated by the following analogy. Consider a mechanical oscillator. If the amplitude of the oscillations remains limited, the entire system can be (slowly) translated by arbitrarily large amounts without affecting their energy. A confusion between the oscillation amplitude and the amount of translation (analogous to the radius increase due to diffusion) can only lead to physically absurd conclusions.

Having thus clarified the proper role of mass diffusion in the postulated mechanism, the question still remains of whether in the oceanic environment pressure fluctuations (due, e.g., to turbulence) can occur of sufficient magnitude as to cause the expansion of a bubble to a degree sufficient for a violent collapse to take place. We address this question in the next section.

II. BUBBLE COLLAPSE

The degree of expansion necessary to promote a violent collapse, i.e., to turn a bubble into a transient cavity, is not a precisely defined quantity but is normally taken to be at least a factor of 2 in radius, which is nearly an order of magnitude in volume.[11] We now consider a number of ways in which such an expansion can take place, and show that they are impossible in the oceanic environment.

Consider first a decrease of the liquid pressure acting on the bubble so slow that the inertia terms in the left-hand side of (1) are negligible and the process is isothermal, $\kappa = 1$. Equation (1) then reduces to

$$p_{G0}(R_0/R)^3 - 2\sigma/R - p_0 - \Delta p = 0\,. \tag{4}$$

This relation determines the radius of a bubble when the ambient pressure is $p_0 + \Delta p$ given that the radius, when the pressure is p_0, is R_0. It is readily shown that it possesses a stable solution as long as $-\Delta p < p_0$ or, more precisely, as long as $-\Delta p < p_0 - p_v$, where p_v, is the vapor pressure. (Recall that $\Delta p < 0$ in this argument.) In this case, the bubble expands gradually as the pressure falls remaining at every instant in a state of quasiequilibrium with the external pressure. On the other hand, if $-\Delta p > p_0 - p_v$, all bubbles greater than a certain critical radius R_{cr} will grow explosively. This critical radius is given by

$$R_{cr} = \tfrac{4}{3}\{\sigma/[-\Delta p - (p_0 - p_v)]\}\,. \tag{5}$$

It is obvious that R_{cr} as given by this equation becomes positive, and therefore physically meaningful, only when the ambient pressure $p_0 + \Delta p$ falls below the vapor pressure. This relation only becomes applicable therefore when the ambient pressure has fallen *essentially by more than one atmosphere*. For this to happen due, for instance, to a Bernoulli effect, the bubble should move at a speed in excess of 14 m/s with respect to the water, which is clearly impossible. Furduev[2] used Eq. (5) but reversed the sign of the pressures in the denominator and therefore obtained erroneous results. If, rather than reaching the limit of unstable growth, we only consider the pressure reduction necessary to double the radius so that a violent collapse would take place when the pressure recovers to the value p_0, we find

$$-\Delta p = \tfrac{7}{8}p_0 + \tfrac{3}{4}(\sigma/R_0)\,, \tag{6}$$

which gives a pressure reduction ranging from 1.2 atm when $R_0 = 1\,\mu$m to 0.88 atm when R_0 is so large that the last term is negligible. Again, such enormous pressure *decreases* appear most unlikely in the ocean.

If we now wish to include inertial effects in the analysis, it is useful to make use of the following first integral of Eq. (1) that can easily be derived if Δp is considered to be a constant:

$$\begin{aligned} R^3\dot{R}^2 - R_i^3\dot{R}_i^2 &= \frac{2}{3\rho}\left[\frac{p_{G0}R_0^{3\kappa}}{\kappa-1}\left(\frac{1}{R_i^{3(\kappa-1)}} - \frac{1}{R^{3(\kappa-1)}}\right)\right. \\ &\quad \left. - 3\sigma(R^2 - R_i^2) - (p_0 + \Delta p)(R^3 - R_i^3)\right]. \end{aligned} \tag{7}$$

Here the index i denotes initial values. To study the conditions that would be necessary for an expansion of the bubble, we idealize the process by assuming the bubble to be quiescent and in equilibrium with a radius R_0 at the instant at which the variable part of the pressure field takes on the value $\Delta p = -P$, with $P > 0$. We want to estimate the value of P necessary to cause an expansion up to twice R_0. To apply (7) to this situation, we let $R_i = R_0$, $R = 2R_0$, $\dot{R}_i = 0$, and $\dot{R} = 0$, as is appropriate at the end of the growth, to find

$$\frac{P}{p_0} = 1 + \frac{1}{7}\frac{1}{\kappa - 1}\left(1 - 2^{-3(\kappa - 1)}\right). \quad (8)$$

Here we have dropped surface tension terms that are important only for micron-size bubbles. Since surface tension tends to oppose the expansion of the bubble, its effect would be in the direction of increasing the estimate of P given by (8), as can readily be shown. The right-hand side of (8) ranges from 1.3 for $\kappa = 1$ to 1.2 for $\kappa = 7/5$. Again we conclude that the postulated expansion of the bubble would require a reduction of the static pressure of the order of 1 atm. The difference with the previous quasistatic estimate is due to the inclusion of inertia, which resists motion, and is, therefore, in the direction that could be anticipated on intuitive grounds. The two processes considered in which the pressure is decreased so slowly that the equilibrium is maintained during the expansion, or is abruptly decreased to its final value, bracket any more realistic situation in which the decrease would be at a finite (as opposed to infinitesimal or infinite) rate. Clearly, in the oceanic environment, a bubble cannot possibly expand sufficiently to become a transient cavity, which is the only way in which noise similar to cavitation noise could be generated.

Another possibility that may perhaps be envisaged is that of the compression of a bubble in equilibrium due to the rapid rise of the surrounding pressure, e.g., because of the impact of a large breaking wave. This process of course cannot take place at low wind speeds such as those considered in Ref. 1 but we include it for completeness. It is obvious that, in order to radiate noise, the compression must occur so rapidly that pressure gradients are set up in the neighborhood of the bubble. In this case, we can therefore again use Eq. (7) considering the bubble to be initially overexpanded with respect to its final radius at the end of the compression caused by a sudden pressure increase to the level $p_0 + P$. In the initial state, we have $R_i = R_0$, $\dot{R}_i = 0$, while in the final state $R = \frac{1}{2}R_0$, $\dot{R} = 0$. A simple calculation now gives

$$\frac{P}{p_0} = \frac{8}{7}\frac{2^{3(\kappa - 1)} - 1}{\kappa - 1} - 1 + \frac{2}{7}\left(8\,\frac{2^{3(\kappa - 1)} - 1}{\kappa - 1} - 9\right)\frac{\sigma}{R_0 p_0}. \quad (9)$$

The two numerical constants in the right-hand side have the values 1.4 and 2.2 for $\kappa = 1$, and 2.7 and 4.8 for $\kappa = 7/5$. If converted to stagnation pressures by means of Bernoulli's equation, for $p_0 = 1$ atm, these results would correspond to flows with velocities in excess of 17 m/s instantaneously brought to rest. We obtain this estimate for the most favorable case of $\kappa = 1$ and negligible surface tension effects. Again, this is all but impossible in the oceanic environment.

Arguments similar to the ones given above also show that turbulent fluctuations cannot cause "nonlinear, large-amplitude excursions of the bubble wall" as postulated in the model of Ref. 3. If we simply use Eq. (4) for an estimate disregarding surface tension, we see that a 10% volume change (which is a 3% radius change, and can hardly qualify as a large-amplitude excursion) requires a 10% pressure fluctuation Δp. If this is converted to a velocity fluctuation by the order-of-magnitude estimate $\Delta p \sim \rho\,\overline{u'^2}$, we find a result of the order of 3 m/s. Even if at all possible, such a large turbulence intensity cannot be expected to be a frequent occurrence in the ocean.

III. THE NOISE PRODUCED BY BURSTING BUBBLES

The second mechanism mentioned in Ref. 1 as a possible source of kHz frequency noise is the bursting of bubbles at the ocean's surface. The physical process may be described in the following terms.

Consider a floating bubble just before it bursts. Its cap is bounded by two nearly spherical surfaces of nearly equal radius R across each one of which the pressure jumps by $2\sigma/R$. The bubble internal pressure will therefore exceed by $4\sigma/R$ the atmospheric pressure p_0. When the cap ruptures the bubble gas at a pressure $p_0 + 4\sigma/R$ is brought into contact with air at a pressure p_0, and therefore a strong compression wave is radiated in the air. This is the origin of the noise familiar to anyone who has put a glass of carbonated beverage close to his ear. As the compressed air is ejected from the bubble, and before the liquid has had much of a chance to move, standing waves are set up in the cavity but the impedance mismatch between air and water is so large that virtually none of the associated acoustic energy penetrates into the water.

The process looks different from the water side. Here the bubble boundary consists only of one surface and therefore the pressure is always an amount $2\sigma/R$ smaller than the pressure inside the bubble. When the pressure in the bubble is brought to p_0 by the bursting of the cap, the pressure on the liquid side of the bubble surface is therefore smaller than p_0. The resulting pressure gradient, combined with the hydrostatic head, promotes the motion of the liquid and the filling of the cavity. The pressure field associated with this motion evolves on a relatively long time scale, is essentially hydrodynamic, and does not propagate. The only significant source of a propagating pressure disturbance in the water is the rapid, almost steplike, change of the pressure along the bubble surface from the value $p_0 + \rho g h$, where h is the depth of the particular point considered, to the value $p_0 - 2\sigma/R$ that prevails after the rupture of the cap. Averaging over h we get a resultant pressure drop of the order of σ/R. Before proceeding to obtain an estimate of the spectrum and intensity radiated in the water, we wish to stress that the mechanism that radiates noise in the air, the compression wave, is totally absent in the water.

Let the pressure on the water side of the immersed part of the bubble vary with time as $p_S(t)$ when the cap ruptures. We neglect any spatial variation of this quantity, which is adequate for the calculation of the dominant part of the radiated sound. In an unbounded medium, at a distance r from the center of the bubble, this would give rise to the monopole pressure field $(R/r)p_s(t)$. We can account for the pressure-release nature of the water surface by introducing an image source that interferes destructively with the real one and therefore results in a dipole emission. The radiated pressure field $p(\mathbf{x},t)$ at a distance much greater than R will then have the form

$$p(\mathbf{x},t) = -d\cos\theta\frac{\partial}{\partial r}\left[\frac{R}{r}p_S\left(t-\frac{r}{c}\right)\right], \tag{10}$$

whre d is the effective separation of the real and image monopole sources, c is the speed of sound in the water, and θ is the angle between the direction of observation and the normal to the free surface. At a distance greater than a few wavelengths, the previous expression can be simplified by omitting the term proportional to r^{-2},

$$p = d\cos\theta(R/rc)p_s'(t-r/c). \tag{11}$$

If there are N bubbles bursting per unit area and unit time, the resulting average intensity spectrum $\hat{I}$ below the surface is

$$\hat{I}(\omega) = \pi(Nd^2R^2/\rho c^3)\omega^2|\hat{p}_S(\omega)|^2, \tag{12}$$

where $\hat{p}_S$ is the Fourier transform of $p_S(t)$. A very precise knowledge of this quantity is not required for a rough estimate of the effect, since we already know that the magnitude of p_S is of the order of σ/R. If we let

$$p_S = (\sigma/R)F(t/\tau), \tag{13}$$

where τ is a typical time scale, the previous result (12) becomes

$$\hat{I} = (\pi Nd^2\sigma^2/\rho c^3)\omega^2\tau^2|\hat{F}(\omega\tau)|^2. \tag{14}$$

We may expect τ to be of the order of the time R/c_g, where c_g is the speed of sound in air, required for pressure pulses to propagate through the bubble. If this estimate is correct, for ω in the kHz range, we have $\omega\tau \ll 1$. The Fourier transform of F can therefore be estimated by breaking up the integral over time into a part from 0 to τ, and a part from τ to infinity. In the first contribution the kernel $\exp(i\omega t)$ can be substituted by 1, while the second contribution is essentially zero since $F \simeq 0$ for $t > \tau$. One therefore finds, by use of the theorem of the mean, that

$$\hat{F} \simeq (2\pi)^{-1/2}\int_0^\tau F(t)dt = (2\pi)^{-1/2}\tau\bar{F}, \tag{15}$$

where $\bar{F}$ is a constant of order 1 that will be omitted in the following. This approximate evaluation can be explicitly checked, for example, if the form $F = -H(t)\exp(-t/\tau)$ used in Ref. 1 is assumed. In this case, $\bar{F} = 1$ exactly. The order of magnitude of $\hat{I}$ for frequencies much smaller than τ^{-1} is then

$$\hat{I} \simeq NR^4\sigma^2\omega^2/2\rho c^3c_g^2, \tag{16}$$

where we have set $\tau = R/c_g$ and have also taken the distance d of the real from the image source to be of order R. Setting in this expression $\omega = 2\pi\times 1$ kHz, $R = 1$ mm, $\sigma = 50$ erg/cm^2, $\rho = 1$ g/cm^3, $c = 1500$ m/s, $c_g = 330$ m/s, we can calculate the number of bursting bubbles necessary to produce the reference intensity *re*: 1 μPa/Hz$^{1/2}$, i.e., a level of 10 dB. This number is found to be about 480 events per cm^2 per second. A more realistic, although still very low, intensity of 50 dB would require 48 000 000 bubbles bursting per second per cm^2. Clearly the contribution due to this process is quite irrelevant to the ambient noise problem.

We may add that this conclusion is borne out by some preliminary experimental results obtained by H. C. Pumphrey at the University of Mississippi,[17] who found that a bursting bubble produces a significant amount of noise in the air, but very little in the water. The peak pressure is found to be at least two orders of magnitude smaller than that radiated by the oscillations of the bubble entrained by a drop impacting the water surface.[18,19] As was argued in Refs. 4 and 5, bubbles do contribute significantly to oceanic ambient noise, but through volume pulsations rather than by collapsing or bursting.

IV. CONCLUSIONS

We have shown that, in the oceanic environment, air bubbles present in the water can in no way act as noise sources in the same way as bubbles in a cavitating system. In the first place, mass diffusion phenomena are irrelevant in judging the degree of expansion of a bubble. Second, the pressure fluctuations to which a bubble may reasonably be subject in the ocean are far smaller than those that would be necessary to cause the rapid expansions and violent collapses typical of cavitation noise. We have also tried to estimate the amount of noise radiated in the water by the bursting of a floating bubble, and we have concluded that tens of millions of bubbles bursting per cm^2 per second would be necessary to come anywhere close to the observed levels. We conclude that this mechanism is an unimportant source of noise as well.

We do believe that bubbles are significant acoustic sources in the ocean but, as was argued elsewhere,[4,5] their main role is associated with volume pulsations driven by the surrounding turbulence or arising due to other perturbations of the equilibrium state.

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