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QUANTITATIVE TECHNIQUES OF PLANT LAYOUT

A Thesis

Presented to

the Faculty of the Department of Industrial Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

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by

Benito E. Flores-Sandoval

August 1964

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ABSTRACT

The purpose of this study is the development of quantitative techniques for the general plant layout problem, specifically for the development of the relative positions of a number of departments based upon a cross chart matrix that summarizes previous analysis of the materials handling between the department and which can include a correction for activity relationships between departments.

Two methods are studied in detail. One of them is the statistical technique of sampling at random and analyzing the results. The sample size is determined either parametrically or nonparametrically. The results obtained are as good as the ones found previously in the literature.

The other method studied is a variation of the dynamic programming technique. This method is referred to as an approximation in policy space. The results obtained through the application of both methods will always be local minimum at least. Occasionally a global minimum will be obtained. Presently there is no way to distinguish one from the other, but a reference point may be found to use a guidepost in the solution of the problem.

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

QUANTITATIVE TECHNIQUES OF PLANT LAYOUT

STATEMENT OF THE PROBLEM

The problem considered is the general plant layout problem, specifically the development of the relative positions of a number of departments (optimum layout) based upon a cross chart array that summarizes the materials handling between the departments and which can include a correction for activity relationships between departments. Two techniques are proposed. One is a statistical technique that will suboptimize the desired result. The other is an application of dynamic programming.

I. THE PROBLEM

Industrial Engineers are frequently faced with the problem of having to design a plant layout. Many authors have developed techniques and procedures that help in the design of a good plant layout. Richard Muther[1] for example, defines four phases in layout planning:

1. Location - Determination of the location of the area to be laid out.

2. General Overall Layout - Establishment of the general arrangement of the area to be laid out.

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[1]Richard Muther, Systematic Layout Planning (Industrial Education Indstitute, 1961)

3. Detailed Layout Plan - Location of each specific piece of machinery and equipment.

4. Installation - Measuring the physical moves of the equipment.

The second step is the one in which we are more interested. Muther states:

> Here the basic flow patterns and the areas allocated are brought together in such a way that the general size, relationship, and configuration of each major area is roughly established. This is sometimes termed block layout, area allocation or merely rough layout.

In an analysis of the general overall layout many things must be examined carefully. The products manufactured and the number of each that are going to be made must be studied, possibly through a forecast of sales. This study plus an analysis of the form of production will probably establish what kind of a layout we will have - whether it is a production line, or a job-shop, or a combination of both. In the general framework of our problem the job-shop will be the form assumed.

The next decision considers the flow of materials that will ensue because of the products manufactured. The routing of the products through the different departments must be analyzed in order to simplify and combine whenever possible to improve the flow of materials. Muther states:

> Flow-of-materials analysis is the heart of layout planning, whenever movement of materials is a major portion of the process-as when materials are large or heavy or many in quantity, or when transport or handling costs are high compared with costs of operation, storage, or inspection.[2]

One major part of an analysis of the flow-of-materials is intensity of flow. The intensity of flow is the magnitude of material movement. The magnitude of the movements over the various routings is the basic measure of relative importance of each route and therefore of relative closeness of operations or departments to each other.

The material can be measured, usually, in a common unit. These units can be pounds, tons, gallons, pallet-loads, tote-boxes, etc., per unit of time.

After all of the products manufactured have been studied and the flow of materials completed, groupings of the totals can be done in a cross chart, Fig. 1.

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[2] Ibid.

DEPARTMENTS

DEPARTMENTS		1	2	3	4	5	6
	1	0	3	4	6	6	5
	2		0	4	5	8	6
	3			0	7	5	5
	4				0	5	8
	5					0	6
	6						0

Figure 1

Cross-Chart

This chart summarizes the flow of materials. Numbers inside represent totals between departments in appropriate units. It should be noted that the numbers are totals and do not represent direction of flow.

Normally flow alone should not be the basis for a layout. In some industries, for example, there will be only a few pounds of material moved during the day, i.e. electronic industries. In other, even though the flow of materials indicate so, departments should not be close together for technical reasons - such as heat treating next to a department that uses inflammable substances.

This difficulty can be overcome by ranking the departments with respect to each other by artificial means.[3] A

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[3] *Ibid*.

possible ranking could be

Relationship of Departments	Weighting Factor
Closeness Necessary	-3
Closeness Very Important	-2
Closeness Important	-1
Ordinary Closeness	0
Closeness Unimportant	1
Closeness Not Desirable	3

As example of this weighting the cross chart presented above can be transformed to the following:

0	3	4	6	6	5		0	4	3	7	9	2
	0	4	5	8	6			0	1	4	7	3
		0	7	5	5				0	6	4	8
			0	5	8					0	2	6
				0	6						0	5
					0							0

Figure 2

Cross charts as originally prepared and corrected for activity relationships.

In this problem the assumption is that such an analysis has been performed and the cross chart completed. It should be noted that the values are treated as deterministic, a valid assumption on a short range basis. However, it should be realized that the values in reality are not deterministic, but probablistic. The removal of the deterministic assumption changes the nature of the problem and increases its complexity. The solution of the problem on a probablistic basis is not a part of this study. It is assumed that the error introduced in treating the values as deterministic is negligible.

Suppose then, that there are six departments or work centers X_i (i = 1, 2, . . ., 6) that are to be assigned to a large rectangular area. We have, as shown before, compiled a cross chart that indicates the future expected flow of materials between departments (Fig. 2).

In figure 3 we show in lattice form the "centers of gravity" of the areas where the departments are to be allocated.

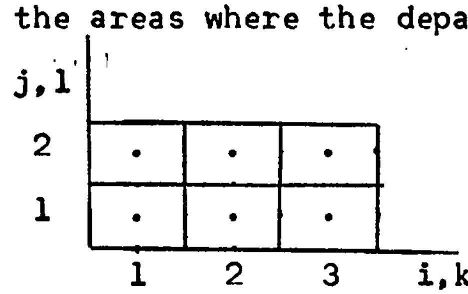

Figure 3

Lattice form of centers of gravity of a (2x3) six department layout.

All of the material moved between the department where point (1,1) lies and the department where point (1,2) is, will travel one unit of distance, where one unit of distance could equal 50, 100, 200, or other appropriate distance.

The travel in these types of layouts will be done horizontally and/or vertically, usually coinciding with the aisle distribution in a factory.

Let us now state the problem completely.

First, an area is available or planned where future activities can be performed. Whatever the shape of building discussed (rectangular, "L" shaped, "U" shaped) we will allocate the departments that will constitute the plant. These departments have equal rectangular area (see Chapter V for explanation). In previous analysis we have compiled information concerning the flow of materials in the appropriate units, and the effect of their relationships was duly noted.

We are now trying to allocate the departments to the areas in such a form that the evaluation of the layout — multiplying all of the weights of the cross chart times the distance between departments — will result in a minimum (optimum) number of units. An area is then selected to which a group of six departments are to be allocated. All of the possibilities can be evaluated by means of the following model:

$$\text{Evaluation}=\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} C(A(i,j):A(k,l)) \cdot \left(\left| k-i \right| - \left| l-j \right| \right)$$

where C is the cross chart array and the A array represents the department numbers. If i=1 and j=1 A(1,1)=2 represents the number of the department in cell (1,1), in this case department 2. If A(1,2)=4, the value of C(2,4)=5 (see Fig. 1).

The model can be used then for the evaluation of feasible solutions.

CHAPTER II

PREVIOUS WORK

There have been several attempts towards the solution of the problem.

Before we begin, it should be noted that the problem has an inherent difficulty. If there are twelve departments to be allocated to twelve areas, there are twelve factorial possible ways of allocating the departments. The difficulty then consists of comparing all alternative plant layouts with each other. Another difficulty is the selection of a suitable measure of effectiveness. Materials handling is the measure of layout effectiveness used in this study.

One of the first attempts was made by Peter C. Noy[4] when he designed an evaluation technique that considers the sequence of operations of a number of parts in a process-type (job-shop) layout. The technique assumes that the various production centers are located in a straight line, but not necessarily that the centers lie physically in a straight line. The products cannot criss cross from one machine to another. Product flow must follow along a given line until it comes to another operation. This technique obviously is limited in scope.

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[4]Peter C. Noy, "Make the Right Plant Layout, Mathematically," American Machinist, Vol. 101, No. 6, p. 121.

A technique similar to the one just described was developed by R. J. Wimmert.[5] Wimmert considers volume demand in evaluating production center locations. This approach is more general since it does not restrict flow to the straight-line-type of process layout. However, his method rapidly becomes complex since it tries to analyze all possible combinations.

In this method Wimmert analyzes machine rather than departments and develops a load-path (units of weight x units of distance) matrix, combining pairs of machines. After all of the possible combinations of pairs of machines are established, a systematic analysis will eliminate undesirable alternatives.

The size of the matrix will be given by the well known combinational formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where k = 2 (pair of machines)

and n = number of machines.

In a layout of six machines, the size of the matrix would be 15 x 15 and in one of twelve machines it would

[5] R. J. Wimmert, "A Mathematical Method of Equipment Location," Journal of Industrial Engineering, Vol. IX, No. 6, (1958), p. 498.

be a 66 x 66 matrix. In a situation where fifty machines are involved (not uncommon) the size of the matrix would be so large that analysis would be prohibitively expensive

A parallelism can be drawn between this analysis and our problem. It can be assumed that departments can be used rather than machines and in this case we could analyze the matrix. Nevertheless, the number of combinations is so large that costs prevent complete analysis. A further, and more difficult, drawback is that there is no positive check as to when the best feasible solution is found, except by ennumeration.

Specialized forms of the linear programming method can be used in an attempt to solve the problem. Again the solutions found do not solve the problem, but they can be used as reference points towards which to guide the problem.

To exemplify the preceeding we will show the methods used by Frederich S. Hillier.[6] Hillier shows two forms of analysis. The first one starts by selecting a random layout. In our problem this would be equivalent to selecting an arrangement of six departments to six locations. He then evaluates what would happen if a department were moved to the right, left, up or down from its assumed relative position. The evaluation consists of the change in the sum of products

[6]Frederich S. Hillier, "Quantitative Tools for Plant Layout Analysis", Journal of Industrial Engineering, (Jan.-Feb. 1963), pp. 33-40.

of the cross chart values and the distance moved compared to the original value. The change can either be positive, negative or zero, depending on whether there is no improvement, improvement or indifference in the new layout. In this manner a better layout is formed. The author concedes that it is possible for the optimum not to be reached.

To exemplify the second approach, let us examine a 3 x 4 arrangement where twelve departments are to be allocated. These twelve departments can also occupy four different types of locations:

Center (Cr) = 2
Top-Bottom Middle (TM, BM) = 4
Side-Middle (SM) = 2
Corner(C) = 4

C	TM	TM	C
SM	CR	CR	SM
C	BM	BM	C

All of the departments can occupy any of the positions. Placing department 1 in a corner and closest to it the one department with the highest value (units in tons/week or thousands of units/month or any appropriate unit.) in the cross chart (Fig. 4), and next to it the next highest value in the cross chart an optimum allocation can be done, but only for department 1 in a corner position.

DEPARTMENTS

DEPARTMENTS		1	2	3	4	5	6	7	8	9	10	11	12
	1	0	5	2	4	1	0	0	6	2	1	1	1
	2		0	3	0	2	2	2	0	4	5	0	0
	3			0	0	0	0	0	5	5	2	2	2
	4				0	5	2	2	10	0	0	5	5
	5					0	10	0	0	0	5	1	1
	6						0	5	1	1	5	4	0
	7							0	10	5	2	3	3
	8								0	0	0	5	0
	9									0	0	10	10
	10										0	5	0
	11											0	2
	12												0

FIGURE 4

Cross Chart from Frederich S. Hillier article.

Illustrating further this example the values of the row of department 1 can be arranged in descending order, they are:

Department number 8,2,3,4,9,5,10,11,12,6,7

Value (in descending order) 6,5,4,2,2,1,1,1,1,0,0.

If department 1 has been allocated to a corner position then all of the others should follow the descending value pattern illustrated above as follows:

1	8	9	5
2	4	12	7
3	11	10	6

The evaluation of this arrangement with respect to department #1 in a corner position would be

$$S = 6x1+5x1+4x2+2x2+2x2+1x3+1x3+1x3+1x4+0x4+0x5$$

$$= 6+5+8+4+4+3+3+3+4 = 40$$

If we proceed to evaluate department #1 in all of the other three positions - top middle, side middle and center - evaluate, and proceed for all departments, we could form a table such as Figure 5.

Type of Location

Work Center	Corner	Top or Bottom Middle	Center	Side Middle
1	40	33	29	34
2	40	32	29	34
3	36	29	27	31
4	55	46	41	48
5	37	30	28	31
6	49	41	36	42
7	55	46	41	48
8	55	48	43	48
9	57	49	44	50
10	43	35	30	36
11	72	60	52	63
12	35	30	28	31

Figure 5

Minimum total distance loads to or from work center must travel.

From this table it is possible, by means of the Hungarian method,[7] to find the optimum solution, that is, where each department should be allocated in order to minimize materials handling (optimum layout).

The method has several draw backs. For example, even though it shows that a department should be allocated to a corner, it does not say which corner. Consequently, the possible combinations are given by: (No. of corners)! x (No. of centers)! x (No. of side middle)! x (No. of top and bottom middle)! = $2! \cdot 4! \cdot 4! \cdot 2! = 2304$.

An even more important limitation is the following. When an arrangement is examined, for example, when department 1 is evaluated in a corner position, all of the other departments are distributed so as to minimize the value of having department 1 in a corner position.

When we have the minimum solution, as given by the Hungarian method, further analysis is necessary. If we overlap all of the evaluations used to find the minimum values that are to be used from figure 5 (chosen by the Hungarian method) and they coincide, we have a feasible solution. The solution otherwise is not a plausible one. This type of analysis is complex and the number of computations required

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[7] M. Sasieni, A. Yaspan, L. Friedman, Operations Research Methods and Problems (John Wiley, 1959).

make of it an awkward method with no guarantee of a solution, least of all a minimum feasible solution.

Attempts have been made, then, in the direction of solving the aforementioned problem. Some of the previous work has pointed towards the solution or perhaps towards a guideline for the solution but not quite reaching an optimum point. This study goes one step further.

CHAPTER III

THE PROPOSED METHODS

A. STATISTICAL RANDOM SAMPLING

The first method proposed for a solution of a problem of this nature is as follows:

There are many possibilities as to how an area is to be laid out, whether it is rectangular, "L" shaped, or "U" shaped, or practically in any symmetrical form. A rectangular area and twelve departments that are to be allocated to the area make up for twelve factorial combinations.

Utilizing models such as the one mentioned in the first chapter, all of the possible layouts can be evaluated. These values constitute a set ranging from the best possible to the worst possible, that is from the smallest to the largest values (see page 54 Appendix E for an example). This set of values can be assumed to be the possibility space and K as a random variable that will assume those values. K is a function defined in the sample space[8] even though the term random function is more appropriate.

Then,

$K =$ random variable $K_1, K_2, K_3, \ldots, K_n$ as the values it assumes and $P\left\{K = K_i\right\} =$ probability that K assumes the value K_i.

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[8]William Feller, An Introduction to Probability Theory and its Applications (John Wiley, 1950), p. 199.

A function $P\left\{K = K_i\right\} = f(K_i)$ $i = 1, 2, \ldots, n$ is called the probability distribution of the random variable K which is defined on the aggregate of values K_i assumed by K.

$$f(K_i) \geqslant 0 \text{ and } \sum_i f(K_i) = 1$$

Since $f(K_i)$ will have values only as a particular arrangement is formed and has no value at any other point, meaning that the number of evaluations between n and m is a finite set, the distribution function is referred as a discontinuous distribution function.

Strictly speaking though, all random variables are discontinuous because in practice only multiples of the smallest units which the apparatus can measure will be measured. If the unit is small compared to the variations observed, an abstraction can be made from this fact and K can be treated as a continuous[9] variable[10]. Any discontinuous distribution function can be handled by means of the Dirac δ function[11] and operated upon as a continuous distribution function[12].

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[9]Defined as being continuous everywhere and piecewise differentiable with a continuous derivative (meaning that it exists and is continuous except possibly at certain points which occur at most a finite number of times in any finite interval).

[10]Niels Arley, K. Rauder Buch, Introduction to the Theory of Probability and Statistics (John Wiley, 1950), pp. 33, 36.

[11]P.A.M. Dirac, Principles of Quantum Mechanics (1930).

[12]Niels Arley, K. Rauder Buch, Introduction to the Theory of Probability and Statistics (John Wiley, 1950) pp. 33, 36.

The Dirac δ function is defined as

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} f(u)du = 1$$

Another means by which the discontinuous function could be handled would be by means of the Stieltjes integral, which can handle continuous and discontinuous functions. Neither one of the methods mentioned were utilized, they are mentioned to illustrate the fact that they could be used.

Now that the assumption of continuity can be made, or if the function is discontinuous it still could be handled, the analysis can proceed.

A set of values does exist which correspond to the different types of arrangements which can possibly be formed. These values are not known unless a complete ennumeration is made. Nevertheless statistics provides means to analyze the values of the distribution function.

The statistical technique is sampling. Sampling from a population, whether finite or infinite, will reveal facts concerning the parameters of the distribution. The sample values can be used to estimate the true values of the mean, variance, skewness, kurtosis. These parameters indicate the shape of the distribution from which sampling is being done. In the plant layout problem being analyzed the smallest possible values, given by the evaluating model, are the ones of interest and the parameters of the population can indicate possible ranges where the smallest values will be found.

Assume that a random sample has been drawn from a population. From the sample, the sample mean $\bar{X}$, the sample variance S^2 can be found. These values can be used to estimate the true population values. This is true when something else is known about the distribution, for instance, that it is approximately normally distributed.

If the population is truly - or close enough to - Normal, then the solution to the problem can be obtained.

It is known that the Normal distribution has the following shape and formula.

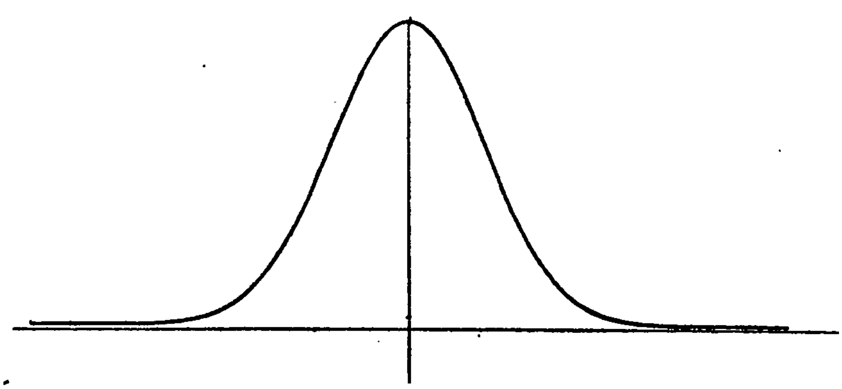

Figure 6

The Normal Distribution. $f(x) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-(x-\mu)^2/2\sigma^2}$

The areas under the Normal curve represent probabilities. If the estimates of the parameters can be found by means of sampling, then, the value of the standard deviation can be estimated. Looking up in a table of probabilities based on the normal distribution the value of $\bar{X} \pm 3\sigma$ has a probability of occuring of only 0.0013.

An evaluation, by means of a model, of a layout that yields a value smaller than $\bar{X} - 3\sigma$ will be a very good one, possibly not the best, but better in numerical value than 99.87% of the possible values that the distribution has.

Continuing with the same assumption, the only real problem is how to sample and what determines the size of the sample.

In order to sample, a computer[13] was used which would develop a random layout and evaluate it. This operation was repeated the number of times required by sample size.

The sample size is determined by the desired precision of the sample and can be determined by[14]

$$n = \frac{\left(\frac{tS}{d}\right)^2}{1 + \frac{1}{N}\left(\frac{tS}{d}\right)^2}$$

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[13]IBM 709 of Texas A & M University and the IBM 7094 located in the NASA facilities in Houston, Texas.

[14]W. G. Cochran, <u>Sampling Techniques</u> (John Wiley & Sons, 1953), p. 55.

where

n is the sample size,

N is the population size,

d is the permissible error,

S is an estimate of the standard deviation,

and t is the abscissa of the normal distribution at some desired confidence level (1.96 = 95%).

Suppose the distribution is not normal? In this case a transformation of the values found from the evaluation of the layouts can be used to see whether the distribution can be normalized.

For example, the values of the evaluation are part of a set X (X_i = values i = 1, 2, . . . , n) composed of the X_i evaluations.

A transformation of this set to another one called Y composed of Y_i evaluation where

$$Y_i = \left(\frac{X_i}{2}\right)^{1/3} \quad \text{for } i = 1, 2, \ldots, n$$

can be performed. This new set can be considered to be a sample whose mean, variance, skewness and kurtosis can be evaluated. If the skewness is close to zero and the kurtosis close to three, this would indicate normality (see page 64 Appendix E).

If this evaluation does not reveal normality another transformation can be tried to examine the new parameters of the sample. This procedure can be repeated until normality is approximated. There is no guarantee that this will be the case.

To circumvent this problem a non-parametric formula was utilized that would not restrict the analysis to normal distributions or suitable transformations of the values.

Assume that ϵ is a predetermined confidence level given in percent and that 100β is a percent of the total population of a distribution.

If an assurance of ϵ is desired that at least 100β percent of the output of a random process will be included between the largest and smallest values, a random sample of size N must be obtained where N is given by the equation found by S. S. Wilks[15]

$$N\beta^{N-1} - (N-1)\beta^{N} = 1 - \epsilon .$$

For example, if we want an assurance of 99% (ϵ) that at least 99% (100β) of the values of the population should be included, what should be the sample size?

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[15] S. S. Wilks, Mathematical Statistics (Princeton University Press, 1947), pp. 90-93.

The equation can be solved for N in several ways, trial and error, graphically, etc. In this example the answer is $N = 660$.[16]

Considering only the upper or lower tolerance limit- in our problem it would be the lower limit - the sample size N is given by[17]

$$N = \frac{\ln(1-\epsilon)}{\ln \beta}$$

Statistics is not a deductive science. It is a guide to induction, it is the science that explains the behavior of data phenomena according to repeatable patterns.[18] Having an ϵ of 0.999 and a 100β of 99.9% does not give 100% assurance that the lowest and highest values of a random sample will include 99.9 percent of the population values.

To add a certain amount of security a variation to Hillier's method[19] was devised. This method is far more general and can indicate whether a solution is feasible or not.

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[16]Acheson J. Duncan, Quality Control and Industrial Statistics (Richard D. Irwin, Inc., 1959), pp. 113-115.

[17]S. S. Wilks, "Statistical Production With Special Reference to the Problem of Tolerance Limits", Annuals of Mathematical Statistics, Vol. 13 (1942), pp. 400-409.

[18]Norbert Wiener, The Human Use of Human Beings, Doubleday and Co., New York, 1954.

[19]Frederich S. Hillier, "Quantitative Tools for Plant Layout Analysis", Journal of Industrial Engineering, Jan.-Feb. 1963, pp. 33-40.

Suppose that, as Hillier states, a cross chart and future layout of twelve departments in a 3 x 4 arrangement (figure 4), has been developed.

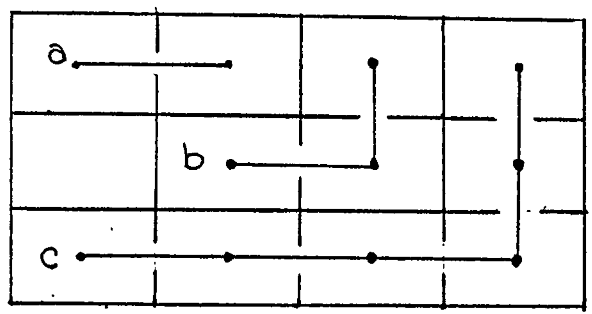

Figure 7

Example of typical 3 x 4 arrangement with 1(a), 2(b), and 5(c), unit distance relationships drawn in. A unit is the distance between the centers of two areas.

Counting the possible number of distance relationships of 1, 2, 3, 4, and 5 units between the areas where the departments are to be allocated and find them as follows:

Distance of	Number of such relationships
1 units	17
2 units	22
3 units	17
4 units	8
5 units	2

The cross chart (figure 4) has 66 values[20] or relationships between the departments. Arranging these values

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[20] The number 66 is found by adding the number of values on the upper triangular half of the cross chart matrix without including the main diagonal. 144/2 - 12/2 = 66

in descending order, multiplying the first 17 by 1 (see table above); the next 22 by 2 (see table above) etc., and adding all of the products will yield the minimum value of a layout arrangement.

Placing, in an array of the same size of the cross chart matrix, the corresponding unit distance values found above and duplicating them in the lower triangular half, a test matrix is created. Adding the rows (or columns) of this matrix will yield the values 20, 20, 26, 26, 24, 24, 24, 24, 30, 30, 30, 30 not necessarily in any order.[21]

If any of these sums is violated, the solution is not feasible, but can be used as a reference point. Unfortunately, at the present, no device or test is known to find how far off we are from the smallest feasible solution, except through ennumeration.

Summing up, a finite universe of values exists. These values are found by evaluating different layouts. To these values the characteristic of continuity has been attached. Assuming this, sampling techniques are utilized to obtain random samples that will yield parameters that can be used to find probabilistic estimates[22] showing how small a value is compared to the total population.

[21]For a full explanation see Appendix A.

[22]See Appendix D.

The sample size can be determined either parametrically[23] or non-parametrically and the layouts designed and evaluated by means of a computer.

The higher the precision desired the larger our sample must be (see Appendix D). This indicates more computer time and, of course, higher cost.

Cost of finding a better solution will be the restriction as to the magnitude of sample size.

2. DYNAMIC PROGRAMMING VARIATION APPROACH

In this second method a variation of an application of dynamic programming is utilized.[24]

Let us define the terms:

g_j = the smallest cost of the jth configuration, given some initial configuration

$\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}$ = the sets of permissible one-two-three-four-and five unit distance interchanges among the members of the allocation matrix.

$\{C_1(P_1)\}, \{C_2(P_2)\}, \{C_3(P_3)\}, \{C_4(P_4)\}, \{C_5(P_5)\}$ = the sets of evaluations associated with the interchanges above mentioned.

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[23] W. G. Cochran, Sampling Techniques (John Wiley and Sons, 1953), p. 55.

[24] Richard Bellman, Adaptive Control Processes, (Princeton University Press, 1961), pp. 232-237.

Using the definitions a functional equation can be defined

$$g_{j+1} = \underset{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\}}{\text{Minimum}} \begin{bmatrix} C_1(P_1) \\ C_2(P_2) \\ C_3(P_3) \\ C_4(P_4) \\ C_5(P_5) \end{bmatrix} \leqslant g_j$$

where g_o = the evaluation associated with the initial configuration. The functional equation shows the relationship between the evaluation of the jth and j + 1th configuration. Knowing the value for the jth configuration, the functional equation shows that the choice between all the possible interchanges - whether 1, 2, 3, 4, or 5 unit distance - will be the one that will yield the lowest evaluation.

After having found the least value for the j + 1 configuration, one more application of the recursive equation will yield the smallest cost configuration for the j + 2 configuration and so on.

To start the calculations, the 0^{th} evaluation is obtained by choosing arbitrarily an initial configuration and computing its value. These calculations will yield g_o.

If we have a 3 x 4 configuration there are 66 possible 2 cell interchanges. We can analyze all of them and find the lowest evaluation. If interchange of the department that occupies, i.e., cell (1,1) and the one that occupies cell (3,4) is the one that yields the minimum cost (of the set C_5 (P_5)) then the functional equation is solved. After the interchange that gave the minimum value is executed the new layout (configuration) will be used to repeat the process.

The method of solution is an example of iteration in policy space used in dynamic programming. This technique consists in choosing a policy, (here the first configuration) and then improving upon it. That is, finding another configuration which yields a smaller value.

The larger advantage of this technique is that the new arrangements will only be made if they yield a lower value in the evaluation of the layout by means of the model.

This algorithm will always yield at least the local minimum and possibly even an absolute minimum. The method is a variant of the gradient optimization technique which finds the local minimum associated with the initial point of departure.

The results found in the previous section concerning a lower bound as a check of separation from an absolute minimum can be used here too.

To further investigate minimum configuration the same type of functional equation can be constructed that will yield improvements for 3 cell, 4 cell, etc. interchanges.

The complexity of the analysis is naturally increased. Programming and execution time in a computer are also increased and the upper bound on analysis is cost of improving a layout vs the savings gained by it. The quantification of this criterion is considered outside of the scope of this thesis.

CHAPTER IV

METHODOLOGY

A. RANDOM SAMPLING

The procedure followed throughout the study can be described in the following manner:

1. Cross-Chart. The cross charts used in all of the examples of this thesis are developed or designed to give information concerning specific cases or applications.

2. Minimum Theoretical Value. Utilizing the method described in detail in Appendix A the minimum theoretical value was found. This value can be used as a guidepost. The value found for a specific layout may not be a feasible solution, nevertheless its function as a guidepost is not disturbed by this fact.

3. Development of Layouts. The development or design of a layout by a computer was obtained by the specification of a matrix of n x m size. This matrix represents a rectangular area where n x m departments are to be allocated. In the matrix each of the locations is filled initially with a number from 1 to n x m representing in this form the department number.

Afterwards different departments, or values in the cells, are interchanged randomly forming a new arrangement or layout. It can be seen that an arrangement can repeat itself but the possibilities of this happening are rather small. The computer program flow chart that accomplishes the above can be found in Appendix C.

4. <u>Random Layouts</u>. In previous chapter the idea of a sample drawn randomly has been illustrated. In the problem a sample value is the evaluation of a particular layout. The sample must be random, though, and some means of checking this out was developed.

The check is a simple X^2 test. In the previous paragraph the description of the development of layouts was presented. Assume that every time that a new layout is developed the value in the cell $A_{i,j}$ is recognized. If the layout is a 3 x 4, then, the value of $A_{i,j}$ in any particular arrangement can be from 1 to 12. If we develop N layouts, tally counts can be made as to the number of times the number 1 appears, the number of times the number 2 appears, etc. in location $A_{i,j}$. The comparison of these values with the theoretical value N / 12 will give the necessary statistic used in a χ^2 test of hypotheses. For examples of this discussion see pages 62 and 63 of Appendix E.

5. <u>Model Design</u>. A model used to evaluate the layout depends on the shape of the layout. For rectangular shaped areas the model presented in Chapter I can be utilized. The same model can be used also for "L" shaped layouts. This can be accomplished by placing zeroes in the first quadrant where there would be no construction.

In a "U" shaped layout a special model was designed that would take care of a possible problem. A factory that has this shape will probably have a courtyard. Traffic will utilize this courtyard short cut and change the distance relationship between two departments.

6. <u>Sample Size</u>. The sample size, or number of random layouts developed by the computer, was predetermined by the precision and reliability desired in our sample. Tabulated values can be found in Appendix D.

7. <u>Inspection of Results</u>. After N (sample size) layouts were designed and evaluated by the computer the results were inspected. The inspection of the results was used to find the minimum and maximum values of the sample and the layout that produced the results (see examples Appendix E). This minimum value and the decision was made whether to increase the size of our sample (or not) depending on the closeness of the two values.

8. <u>Further Inspection</u>. Affecting the closeness of the values mentioned in the previous paragraph is a particularity of these layouts. Regardless of the values of the cross chart for every rectangular arrangement, there are three others that will produce the same result when evaluated. For every square arrangement there are seven others that will produce the same result (see Appendix B). This

result affects the values found in the following form. If the values found are R_1 and R_2 there are, assuming, M layouts whose evaluation will be within the range $R_1 - R_2$. The true number of values is only M/4 or M/8 because of the particularity of the layouts.

This result decreases the number of possibilities that need to be examined.

B. DYNAMIC PROGRAMMING

1. Cross-Chart. The cross charts used in all of the examples of this thesis are developed or designed to give information concerning specific cases or applications.

2. Minimum Theoretical Value. Utilizing the method described in detail in Appendix A the minimum theoretical value was found. This value can be used as a guidepost. The value found for a specific layout may not be a feasible solution, nevertheless its function as a guidepost is not disturbed by this fact.

3. Development of Layouts. The development or design of a layout by a computer was obtained by the specification of a matrix of n x m size. This matrix represents a rectangular area where n x m departments are to be allocated. In the matrix each of the locations is filled initially with a number from 1 to n x m representing in this form the department number.

Afterwards an examination is performed in the following form. In a methodic manner evaluate all of the possible layouts that can be formed by the interchange of two departments.

Select the layout which will yield the minimum value in the evaluation. This layout will serve as a basis for the next interchange in the same manner as the initial layout was the basis for this result, (see pages 76-85 of Appendix E). Perform as many improvements as possible. Whenever the two cell interchanges will not yield improved solutions, a systematic evaluation is made of all the possible three cell interchanges to see if improved solutions are found. If there are none, possibly four cell interchanges could improve on our best solution. This was not tried in this study. If there was an improvement with three cell interchanges, the procedure was repeated again until no further improvements were found. Upon this event two cell interchanges were repeated trying to improve the solutions found. The alternate procedure was followed until no further improvements were found.

4. Model Design. A model used to evaluate the layout depends on the shape of the layout. For rectangular shaped areas the model presented in Chapter I can be utilized. The same model can be used also for "L" shaped layouts. This can be accomplished by placing zeroes in the first quadrant where there would be no construction.

In a "U" shaped layout a special model was designed that would take care of a possible problem. A factory that has this shape will probably have a courtyard. Traffic will utilize this courtyard short cut and change the distance relationship between two departments. The same problem would occur if the U shape form is in a multi-story building.

CHAPTER V

RESULTS OBTAINED

A. SAMPLING

The computer programs executed for the sampling procedure produced a large amount of output. In order to decrease the time spent in the examination of all of the sample values only condensed results appear in this study.

In Appendix E pages 65 through 75 the condensed results of the program runs are shown.

The evaluation of the programs can be presented in Table I . In this table the sample size determined by the precision required (Appendix D) yielded the number of layouts evaluated. From all of these layouts only the minimum and maximum values are shown. The important value is the smallest since this layout will have the smallest material handling cost. It can be noted that with a sample of 720 layouts a minimum value of 319 was found and even though the sample size in another run was increased to 3000, the minimum value found in this sample was 321. This is a characteristic of a true random sample.

Sample Size N	Values Found Minimum	Maximum	Type of Layout	Cross Chart	Appendix E Page No.
600	334	484	3 x 4	Hilliers	68
700	333	471	3 x 4	Hilliers	69
720	319	481	3 x 4	Hilliers	70
3000	321	474	3 x 4	Hilliers	71
700	537	725	3 x 4	Flores	72
2160	536	749	3 x 4	Flores	73
720	3554	4830	"L" Shape	Flores	74
720	65334	75810	"U" Shape	Flores	75

Table I

Sampling Results

B. DYNAMIC PROGRAMMING RESULTS

The computer programs run for the dynamic programming procedure produced a substancial amount of output. To decrease the amount of output that could be examined only condensed results appear in this study. In Appendix E pages 76 through 84 the results are shown in more detail. It should be noted that the numbers circled are the cells or departments that were interchanged.

To offer a better perspective of the results, they are presented in Table II. In this dynamic programming analysis only the minimum results are shown since coverage or range is unimportant.

DYNAMIC PROGRAMMING RESULTS

Initial Value	VALUES OF SUCEEDING IMPROVEMENTS										Type of Layout	Cross Chart
I=0	1	2	3	4	5	6	7	8	9	10		
416	342	332	322	315	311						3 x 4	Hillier
450	360	334	322	316	301	293					3 x 4	Hillier
424	362	336	330	324	315	306	305	302			3 x 4	Hillier
4090	3924	3788	3645	3527	3467	3405	3347	3311	3268		"L" Shaped	Flores

TABLE II

CHAPTER VI

POSSIBLE EXTENTIONS AND CONCLUSION

The methods have several advantages. The principal one is that both sampling and dynamic programming apply to whatever shape of layout is being analyzed.

Apparently a restriction to the solution is that the departments should be equal in size. This difficulty, however, is not as restrictive as it seems. For example, the smallest departments can be combined, if they are related, to form a larger one of the standard size. It is also permissible to utilize only the larger ones and afterwards squeeze the smaller ones in. Still another way is to break the layout departments into smaller ones assigning them their share of the traffic from the other work centers and use a very large weight between these broken-up departments to insure their staying together.

Another procedure suggested by Hillier[25] is to create dummy work centers with zero traffic to and from them. Once a layout is found, then the real departments can expand to occupy the dummy space.

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[25] Frederich S. Hillier, "Quantitative Tools for Plant Layout Analysis", Journal of Industrial Engineering, Jan.-Feb. 1963, pp. 33-40.

Sampling at Random is a general and useful statistical technique. Layout problems are typical of the problems that can be attacked with this statistical tool, but others, such as a solution quoted by Purdue University on the traveling salesman problem, can be solved too.[26]

To conclude, the problem is characterized by the large number of possible solutions. The allocation of departments to a facility, when a previous analysis has resulted in a cross chart that reveals the future (forecasted) weight units plus a correction factor due to relationships between departments has been analyzed.

Due to the large number of possibilities computer programs were used to generate random layouts or do all possible two cell interchanges.

The values found do not guarantee an absolute optimum, but do assure in a probabilistic sense that a layout has been found which is better than a significant percent of the total values of the population.

In the case of the dynamic programming approach, the minimum value found is always a local minimum and may be an absolute minimum. At present there is no way to differentiate between the two.

The results found yielded consistently better answers by using the dynamic programming method.

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[26] S. Reiter and G. Sherman, Discrete Optimizing, Purdue University Institute Paper No. 37.

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APPENDIX

APPENDIX A

I. Feasibility of a solution.

Assume that the following array represents the optimum (minimum) solution to an arrangement.

2	4	3
5	1	6

Note that the relationships of distance that arise between department 2 and all the rest are as follows:

Between department and department the distance is:

department	department	distance
2	4	1
2	5	1
2	1	2
2	3	2
2	6	3

Completing the analysis to all the departments an array can be filled up similar to the cross chart.

Departments

	1	2	3	4	5	6	
1	0	2	2	1	1	1	$\sum R_1 = 7$
2	2	0	2	1	1	3	$\sum R_2 = 9$
3	2	2	0	1	3	1	$\sum R_3 = 9$
4	1	1	1	0	2	2	$\sum R_4 = 7$
5	1	1	3	2	0	2	$\sum R_5 = 9$
6	1	3	1	2	2	0	$\sum R_6 = 9$

For a 3 x 4 arrangement it would be as follows:

6	3	7	2
10	1	8	5
4	11	9	12

DEPARTMENTS

DEPARTMENTS	1	2	3	4	5	6	7	8	9	10	11	12	
1	0	3	1	2	2	2	2	1	2	1	1	3	$\sum R_1 = 20$
2	3	0	2	5	1	3	1	2	3	4	4	2	$\sum R_2 = 30$
3	1	2	0	3	3	1	1	2	3	2	2	4	$\sum R_3 = 24$
4	2	5	3	0	4	2	4	3	2	1	1	3	$\sum R_4 = 30$
5	2	1	3	4	0	4	2	1	2	3	3	1	$\sum R_5 = 26$
6	2	3	1	2	4	0	2	3	4	1	3	5	$\sum R_6 = 30$
7	2	1	1	4	2	2	0	1	2	3	3	3	$\sum R_7 = 24$
8	1	2	2	3	1	2	1	0	1	2	2	2	$\sum R_8 = 20$
9	2	3	3	2	2	4	2	1	0	3	1	1	$\sum R_9 = 24$
10	1	4	2	1	3	1	3	2	3	0	2	4	$\sum R_{10} = 26$
11	1	4	2	1	3	3	3	2	1	2	0	2	$\sum R_{11} = 24$
12	3	2	4	3	1	5	3	2	1	4	2	0	$\sum R_{12} = 30$

It can be seen, that in a 2 x 3 arrangement, two rows add up to 7 and 4 to 9 and in a 3 x 4 arrangement 2 of them add to 20, four to 24, two to 25, and four to 30.

Sometimes a particular solution may check if the additions check with the values found above, if this is the case further tests are necessary.

Inspect the 6 x 6 array. Notice that if we add term to term additions performed row 2 and 6, and rows 3 and 5, they add up to 3 on each pair and that rows 1 and 4 add up to one in common terms and to 3 in the others. If the particular solution that satisfied the sum of rows also satisfies the check just mentioned. The solution is feasible. Similar rules can be found for the 3 x 4 lattice.

II. Examples and Value of Absolute Minimum

EXAMPLE 1. Type 2 x 3 array

Cross-Chart

Departments	1	2	3	4	5	6
1	0	4	3	7	9	2
2		0	1	4	7	3
3			0	6	4	8
4				0	2	6
5					0	5
6						0

Column (1) is formed by arranging the values in descending order. Attaching the values that could correspond to the distance (as found in pagel), column (2) and multiplying column (3) can also be formed.

Cross-chart Values (1)	Distance (2)	Product (1)·(2)=(3)	(1)	(2)	(3)	(1)	(2)	(3)
9	1	9	6	1	6	3	2	6
8	1	8	5	1	5	3	2	6
7	1	7	4	2	8	2	2	4
7	1	7	4	2	8	2	3	6
6	1	6	4	2	8	1	3	3
		37			35			25

The Minimum Absolute Value is Found by adding column (3).

$P = 97$

Using the values of column (2) in the 6 x 6 array.

0	2	2	1	1	3	$\sum R_1 = 9$
2	0	3	2	1	2	$\sum R_2 = 10$
2	3	0	1	2	1	$\sum R_3 = 9$
1	2	1	0	2	1	$\sum R_4 = 7$
1	1	2	2	0	1	$\sum R_5 = 7$
3	2	1	1	1	0	$\sum R_6 = 8$

The solution of 97 is not a feasible one. (See page 65 for minimum feasible solution).

EXAMPLE 2. Type 2 x 2 array - Rectangular

<u>Cross-chart</u>

0	25	16	8	41	33
25	0	83	56	0	65
16	83	0	32	43	13
8	56	32	0	25	17
41	0	43	25	0	29
33	65	13	17	29	0

Cross-chart Values (1)	Distance (2)	Product (3)	(1)	(2)	(3)	(1)	(2)	(3)
83	1	83	33	1	33	17	2	34
65	1	65	32	1	32	16	2	32
56	1	56	29	2	58	13	2	26
43	1	43	25	2	50	8	3	24
41	1	41	25	2	50	0	3	0

The minimum absolute value is found by adding the columns (3) = 627.

Forming the array again we find

0	2	2	3	1	1	$\sum R_1 = 9$
2	0	1	1	3	1	$\sum R_2 = 8$
2	1	0	1	1	2	$\sum R_3 = 7$
3	1	1	0	2	2	$\sum R_4 = 9$
1	3	1	2	0	2	$\sum R_5 = 9$
1	1	2	2	2	0	$\sum R_6 = 8$

The minimum solution of 627 is not feasible.

EXAMPLE 3. Type 3 x 4 array - Rectangular

Cross-chart

(see page 11)

Cross-Chart Values (1)	Distance (2)	Product (1)·(2) = (3)	(1)	(2)	(3)	(1)	(2)	(3)
10	1	10	3	2	6	1	3	3
10	1	10	3	2	6	0	3	0
10	1	10	3	2	6	0	3	0
10	1	10	2	2	4	0	3	0
10	1	10	2	2	4	0	3	0
6	1	6	2	2	4	0	3	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	3	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	2	2	4	0	4	0
5	1	5	1	2	2	0	4	0
5	1	5	1	2	2	0	4	0
5	2	10	1	3	3	0	4	0
5	2	10	1	3	3	0	4	0
4	2	8	1	3	3	0	4	0
4	2	8	1	3	3	0	5	0
4	2	8	1	3	3	0	5	0

The minimum absolute value is found by adding columns (3).

column (3) = 243

The matrix is

0	1	2	2	2	5	3	1	2	3	3	3	$\Sigma R_1 = 27$
1	0	2	4	2	2	2	3	2	1	4	4	$\Sigma R_2 = 27$
2	2	0	4	4	4	4	1	1	2	2	2	$\Sigma R_3 = 28$
2	4	4	0	1	2	2	1	4	3	1	1	$\Sigma R_4 = 25$
2	2	4	1	0	1	3	3	3	1	3	3	$\Sigma R_5 = 26$
5	2	4	2	1	0	1	2	3	1	2	5	$\Sigma R_6 = 28$
3	2	4	2	3	1	0	1	2	2	2	2	$\Sigma R_7 = 24$
1	3	1	1	3	2	1	0	3	3	1	3	$\Sigma R_8 = 22$
2	2	1	4	3	3	2	3	0	3	1	1	$\Sigma R_9 = 25$
3	1	2	3	1	1	2	3	3	0	2	3	$\Sigma R_{10} = 24$
3	4	2	1	3	2	2	1	1	2	0	2	$\Sigma R_{11} = 23$
3	4	2	1	3	5	2	3	1	3	2	0	$\Sigma R_{12} = 29$

The solution of 243 is not feasible.

APPENDIX B

AXIS OF SYMMETRY

Observing the possible layouts that can be made in a certain arrangement the following becomes apparent:

Suppose a 2 x 3 lattice is to be used for the layout of 6 departments and one of the arrangements is

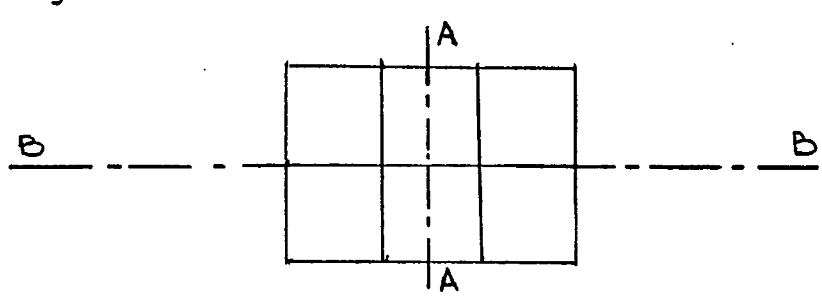
5	4	2
3	1	6

it is apparent that if another arrangement exists such as this:

3	1	6
5	4	2

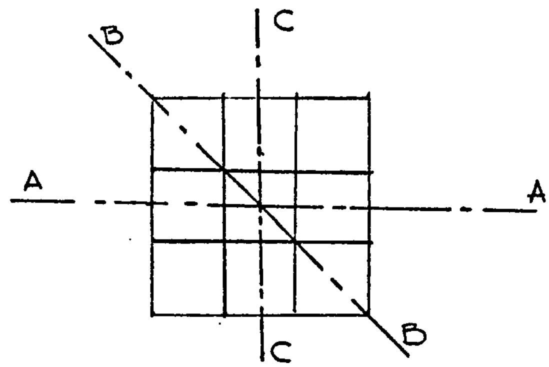
the evaluation of the layout produces the same value since the relationships between the individual departments do not change.

It was empirically deduced that a rectangular distribution such as the one just mentioned above will have 2 x (number of symmetry axis) forms which will produce identical results regardless of the values in the cross chart. Thus,



For every arrangement like (1) there will be 3 others that will produce like results or 2 to the power of (number of axis of symmetry).

In the case of a square lattice, the number increases to 8 since we have



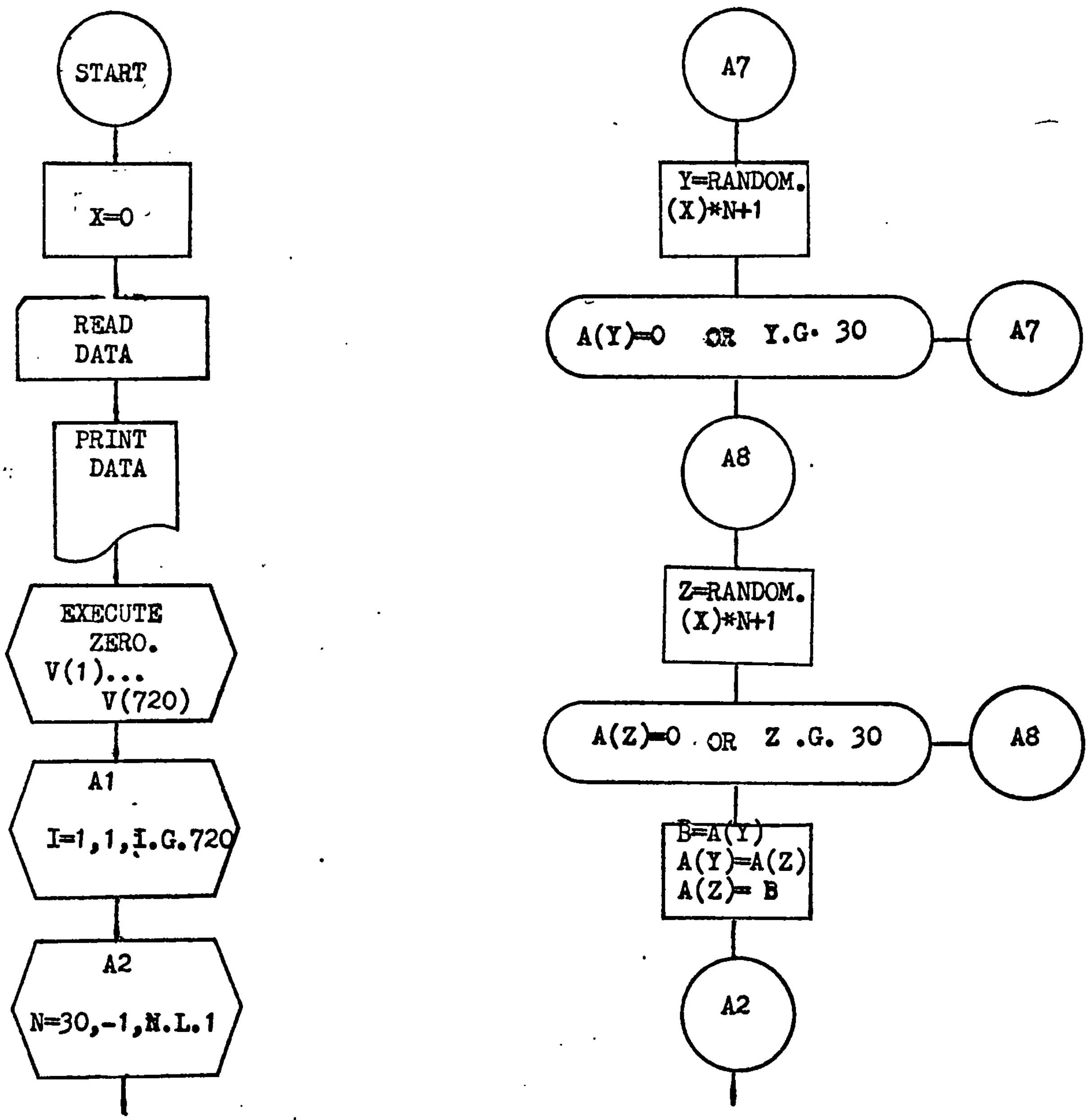
2 to the power of (3 axis of symmetry) = 8 like-value layouts.

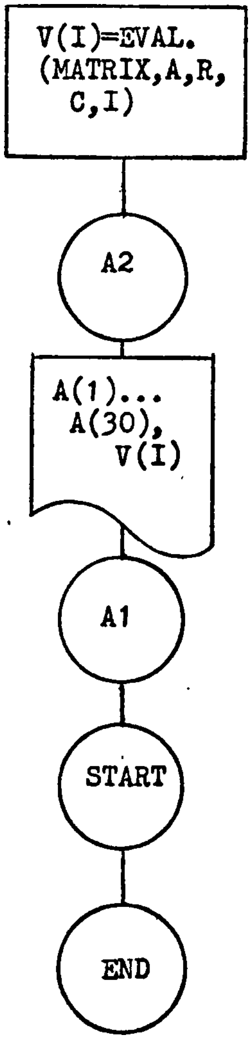
In the case of the "U" shaped layout there will be only 2 and in the case of the "L" shaped layout also only 2. Since there is only one axis of symmetry.

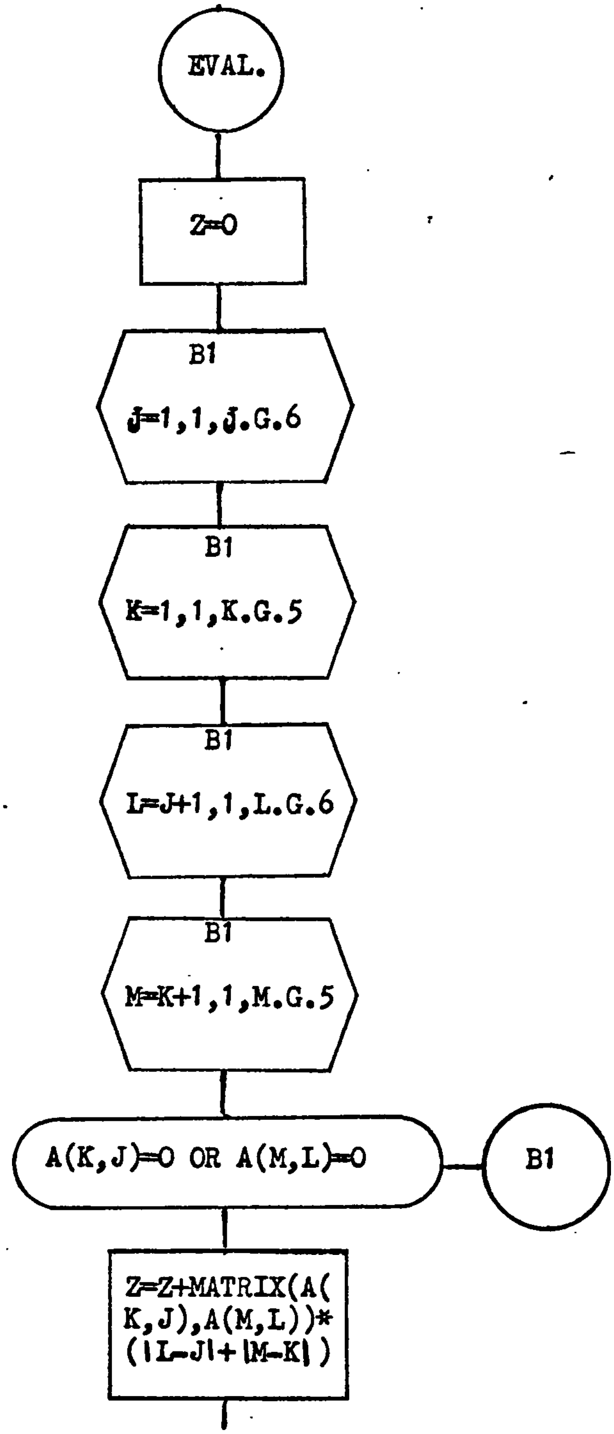
APPENDIX C

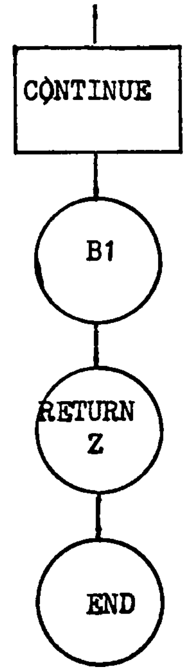
In order to exemplify the type of computer program used, the following examples will be presented. These programs were used to evaluate the "L" shaped layout. The procedural language used was MAD (Michigan Algorithm Decoder) for the IBM 709-7090 systems.

A. For Random Layouts

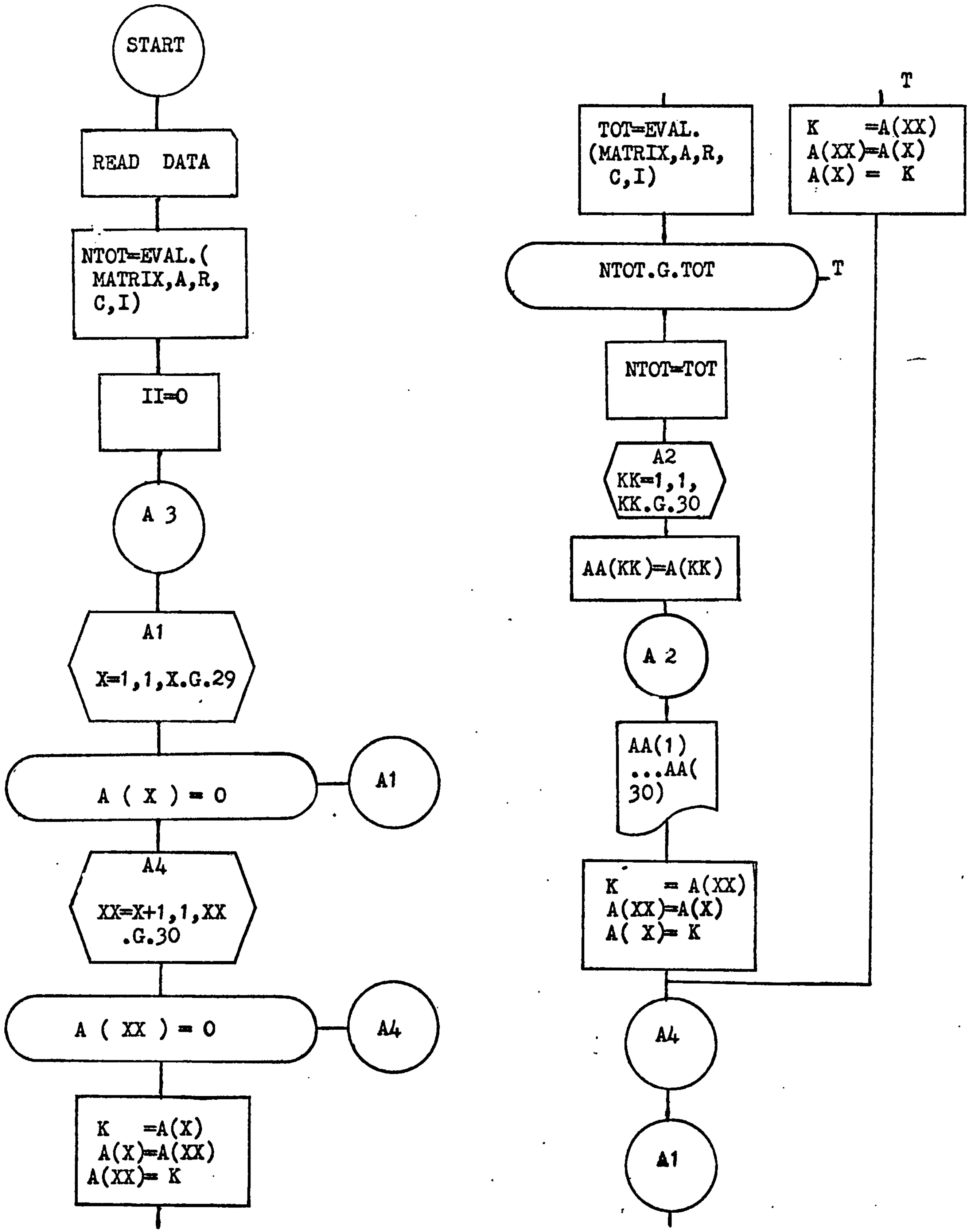


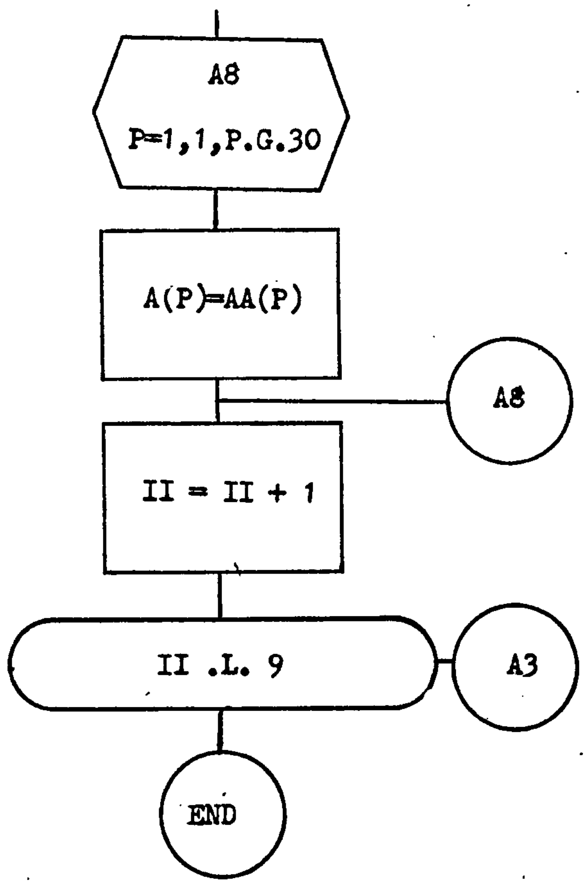

V(I)=EVAL.
(MATRIX,A,R,
C,I)
A2
A(1)...
A(30),
V(I)
A1
START
END


EVAL.
Z=0
B1
J=1,1,J.G.6
B1
K=1,1,K.G.5
B1
L=J+1,1,L.G.6
B1
M=K+1,1,M.G.5
A(K,J)=0 OR A(M,L)=0
B1
Z=Z+MATRIX(A(
K,J),A(M,L))*
(|L-J|+|M-K|)


CONTINUE
B1
RETURN
Z
END

B. DYNAMIC PROGRAMMING CASE




A8
P=1,1,P.G.30
A(P)=AA(P)
A8
II = II + 1
II .L. 9
A3
END

APPENDIX D

Tables of sample size values given a degree of assurance (ϵ) and a percentage (100β) of the population whose limits it is desired to know.

TWO TAIL TEST

ϵ	100 β (%) 99.	99.73
0.900	400	1400
0.950	480	1780
0.985	620	2280
0.995	760	2780
0.999	920	3420

ONE TAIL TEST

ϵ	100 β (%) 90.0	95.0	97.5	99.0	99.5
0.800	16	32	64	161	322
0.900	22	45	91	230	460
0.950	29	59	119	299	598
0.990	44	90	182	469	919

APPENDIX E

SAMPLE RESULTS

<u>Hypothesis</u>: That all of the numbers from 1 to 12 will appear in a cell of layout represented by array A, with equal frequency.

<u>Number of Samples</u>: 4992

<u>Method</u>:

```
       NN=12
       X=0
       THROUGH A1, FOR N=1, 1, N.G.12
A2     Y=RANDOM.(X)*NN + 1.                (Note reference 27.)
       WHENEVER Y.G.12, TRANSFER TO A2
       Z=A(Y)
       A(Y)=A(N)
A1     A(N)=Z
```

Results:

NO.	APPEARANCES	NO.	APPEARANCES	NO.	APPEARANCES
1	408	5	399	9	419
2	453	6	389	10	416
3	398	7	430	11	403
4	463	8	411	12	403

Theoretical frequency of appearance is 416

Observed χ^2 value is = 13.37

Theoretical χ^2 = 19.675 (11 degrees of freedom, .05 confidence level)

Hypothesis is accepted. Development of layouts in a random process.

---

[27] RANDOM is subroutine of the Michigan Executive System which will generate pseudo-random numbers between 0 and 1. The numbers produced before cycling (13) on a binary machine (IBM 709) is 2^{33}.

Hypothesis: That all of the numbers from 1 to 12 will appear in a cell of a layout represented by array A with equal frequency.

Number of Samples: 4992

Method:

```
      X=0
      THROUGH A1, FOR N=12, -1, N.L.1
A2    Y=RANDOM.(X)*N + 1
      WHENEVER Y.G.12, TRANSFER TO A2
      Z = A(Y)
      A(Y) = A(N)
A1    A(N) = Z
```

Results:

NO.	APPEARANCES	NO.	APPEARANCES	NO.	APPEARANCES
1	427	5	432	9	423
2	429	6	399	10	418
3	407	7	393	11	422
4	403	8	431	12	408

Theoretical frequency of occurrence = 416

Observed χ^2 value is = 4.788

Theoretical χ^2 = 19.675 (11 degrees of freedom, .05 confidence level)

Hypothesis is accepted. Development of layouts is a random process.

Note: This was the method that was used throughout the study.

Title: Normalization of the evaluations of the layouts.

Cross-chart - Hillier's

Sample size: 200

Method:
1. Generate a sample of 200.
2. Calculate parameters of the values.
3. Transform the values of the samples by arithmetic operations.
4. Return to 2.

Parameters used in the transformation:

1. Powers of 1, 2, 1/3, 2/3, 2, 3

Results:

Power of	$\bar{X}$	σ	σ^2	Skewness	Kurtosis
	PARAMETERS				
1	813.5	76.1	5790.7	- 5.96	13.9
2	6.7×10^5	9.4×10^4	8.9×10^9	- 1.5	13.9
1/3	9.3	.69	.47	-12.4	168.2
2/3	87.1	7.1	50.6	- 9.0	111.7
2	6.7×10^5	9.4×10^4	8.9×10^9	- 1.5	13.9
3	5.5×10^8	1.1×10^8	1.2×10^{16}	-.199	5.4

Note: As can be seen only when raised to the power of 3 do the values approximate the normal distribution.

Title: Solution by ennumeration.

Type of layout: Two by three - Rectangular area.

Cross-chart:

		Departments					
		1	2	3	4	5	6
Departments	1	0	4	3	7	9	2
	2		0	1	4	7	3
	3			0	6	4	8
	4				0	2	6
	5					0	5
	6						0

Method: All of the 720 (6!) possible combinations were evaluated.

Results:

Minimum Value and Layout

Minimum Value = 104

1	5	2
4	3	6

Maximum Value and Layout

Maximum Value = 136

6	4	5
1	2	3

Title: Hillier's Cross Chart

DEPARTMENTS

DEPARTMENTS		1	2	3	4	5	6	7	8	9	10	11	12
	1	0	5	2	4	1	0	0	6	2	1	1	1
	2		0	3	0	2	2	2	0	4	5	0	0
	3			0	0	0	0	0	5	5	2	2	2
	4				0	5	2	2	10	0	0	5	5
	5					0	10	0	0	0	5	1	1
	6						0	5	1	1	5	4	0
	7							0	10	5	2	3	3
	8								0	0	0	5	0
	9									0	0	10	10
	10										0	5	0
	11											0	2
	12												0

Title: Cross-Chart of "L" Shaped Layout

DEPARTMENTS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0	15	21	0	1	5	9	13	29	0	27	0	3	7	4	15	0	17
2	15	0	6	0	4	3	10	12	4	6	9	6	6	2	7	30	8	0
3	21	6	0	0	3	8	0	6	7	9	4	9	6	0	6	2	7	8
4	0	0	0	0	6	26	20	28	8	7	7	7	0	3	3	0	4	4
5	1	4	3	6	0	28	5	9	7	18	3	18	6	3	20	0	4	0
6	5	3	8	26	28	0	3	0	10	7	5	7	3	7	0	21	19	22
7	9	10	0	20	5	3	0	0	8	10	0	10	23	25	7	6	0	3
8	13	12	6	28	9	0	0	0	17	6	9	6	0	25	3	7	0	1
9	29	4	7	8	7	10	8	7	0	6	4	6	2	20	0	6	7	0
10	0	0	1	0	8	15	11	0	27	5	5	5	0	4	1	4	7	8
11	27	9	4	7	3	5	0	9	4	0	0	0	9	4	8	4	0	6
12	0	6	9	7	18	7	10	6	6	10	0	0	21	0	11	3	25	26
13	3	6	6	0	6	3	23	0	2	21	9	21	0	3	0	2	6	25
14	7	2	0	3	3	7	25	25	20	0	4	0	3	0	5	0	9	3
15	4	7	6	3	20	0	7	3	0	11	8	11	0	5	0	1	5	28
16	15	30	2	0	0	21	6	7	6	3	4	3	2	0	1	0	21	8
17	0	8	7	4	4	19	0	0	7	25	0	25	6	9	5	21	0	17
18	17	0	8	4	0	22	3	1	0	26	6	26	25	3	28	8	17	0

Title: First sampling.

Sample size: = 600

Cross-chart: Hillier's, as shown on page 66.

Results:

Minimum Values and Layout

Value = 334

10	1	11	9
6	8	3	7
5	4	2	12

Maximum Value and Layout

Value = 484

7	10	4	9
5	11	2	8
3	12	1	6

Title: Sampling

Sample size: = 700

Cross-chart: Hillier's, as shown on page 66.

Results:

Minimum Value and Layout

Value = 333

3	4	6	5
1	8	11	10
2	12	9	7

Maximum Value and Layout

Value = 471

6	2	11	8
9	4	3	12
1	7	5	10

Title: Sampling

Sample size: = 720

Cross-chart: Hillier's, as shown on page 66.

Results:

Minimum Value and Layout

Value = 319

9	1	2	5
12	11	4	6
3	8	7	10

Maximum Value and Layout

Value = 481

9	10	1	7
5	2	12	4
8	11	6	3

Title: Sampling

Sample size: = 3000

Cross-chart: Hillier's, as shown on page 66.

Results:

Minimum Value and Layout

Value = 321

12	9	2	3
8	4	5	1
7	11	6	10

Maximum Value and Layout

Value = 474

1	7	4	9
12	5	3	6
11	2	10	8

<u>Title</u>: Sampling

<u>Sample size</u>: = 700

<u>Type of Layout</u>: 3 x 4 - Rectangular area.

<u>Cross-chart</u>:

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	5	2	4	1	6	0	6	2	1	1	1
2		0	3	0	2	2	2	0	4	5	0	9
3			0	5	2	3	4	5	5	2	2	2
4				0	5	2	2	10	7	3	5	5
5					0	10	1	4	2	5	1	1
6						0	5	1	1	5	4	0
7							0	10	5	2	3	3
8								0	15	6	5	0
9									0	11	10	10
10										0	5	21
11											0	0
12												0

<u>Results</u>:

<u>Minimum Value and Layout</u>

Value = 537

3	8	4	11
5	9	10	12
6	1	7	2

<u>Maximum Value and Layout</u>

Value = 725

4	6	11	12
3	7	1	8
10	2	9	5

<u>Title</u>: Sampling

<u>Sample size</u>: 2160

<u>Cross-chart</u>: As shown on page 72.

<u>Results</u>:

<u>Minimum Value and Layout</u>

Value = 536

4	8	11	5
9	7	3	6
12	10	2	1

<u>Maximum Value and Layout</u>

Value = 749

8	5	7	10
2	11	1	4
6	12	3	9

Title: Sampling

Sample size: 720

Type of Layout: "L" shaped - 18 departments.

Cross-chart: As shown on page 67.

Results:

Minimum Value and Layout

Value = 3554

15	7	6	5	12	16
4	18	2	13	17	3
11	1				
9	10				
8	4				

Maximum Value and Layout

Value = 4830

17	4	16	8	6	7
3	13	11	15	12	9
5	10				
2	14				
1	18				

Title: Sampling

Sample Size: 720

Type of Layout: "U" shaped layout - 30 departments

Cross-chart: As shown on page 85.

Results:

Minimum Value and Layout

Value = 65334

25	2	9	24	13	19
14	7	17	29	4	16
30	23				
15	21				
11	6				
3	8	26	22	20	12
10	28	18	27	5	1

Maximum Value and Layout

Value = 75810

15	17	4	10	1	3
5	6	24	23	26	21
28	14				
2	19				
22	27				
30	25	20	8	29	16
11	9	8	12	7	13

Title: Dynamic Programming

Computer: IBM 709

Cross-chart: Hillier's, as shown on page 66.

Method: Starting from an initial configuration, we will try to improve on the solution by making two cell interchanges.

Results:

a. Initial Configuration

1	2	3	4
5	6	7	8
9	10	11	12

Value = g_0 = 416

b. First Improvement

1	2	3	4
5	6	7	8
10	9	11	12

Value = g_1 = 342

c. Second Improvement

2	1	3	4
5	6	7	8
10	9	11	12

Value = g_2 = 332

d. Third Improvement

2	1	3	8
5	6	7	4
10	9	11	12

Value = g_3 = 322

e. Fourth Improvement

2	1	3	8
5	6	7	4
10	11	9	12

Value = g_4 = 315

No further improvement was possible with two cell interchanges.

f. Utilizing three cell interchanges a further improvement was found.

2	3	8	1
5	6	7	4
10	11	9	12

Value = g_5 = 311

Title: Dynamic Programming

Computer: IBM 709

Cross-chart: Hillier's, as shown on page 66.

Method: See previous result.

Results:

a. Initial configuration (random)

1	5	3	12
9	6	7	10
8	4	11	2

Value = g_0 = 450

b. First Improvement

1	5	3	12
7	6	9	10
8	4	11	2

Value = g_1 = 360

c. Second Improvement

1	5	3	2
7	6	9	10
8	4	11	3

Value = g_2 = 334

d. Third Improvement

1	5	10	2
7	6	9	3
8	4	11	12

Value = g_3 = 322

e. Fourth Improvement

6	5	10	2
7	1	9	3
8	4	11	12

Value = g_4 = 316

f. Fifth Improvement

6	5	10	2
7	12	9	3
8	4	11	1

Value = g_5 = 301

g. Sixth Improvement

6	5	10	2
7	11	9	3
8	4	12	1

Value = g_6 = 293

No further improvement was found with three cell interchanges.

Title: Dynamic Programming

Computer: IBM 709

Cross-chart: Hillier's

Method: See previous result.

Results:

a. Initial Configuration (random)

6	1	10	5
12	9	4	7
11	8	3	2

Value = g_o = 424

b. First Improvement

6	7	10	5
12	9	4	1
11	8	3	2

Value = g_1 = 362

c. Second Improvement

6	11	10	5
12	9	4	1
7	8	3	2

Value = g_2 = 336

d. Third Improvement

6	11	5	10
12	9	4	1
7	8	3	2

Value = g_3 = 330

e. Fourth Improvement

6	11	5	10
12	9	4	2
7	8	3	1

Value = g_4 = 324

f. Fifth Improvement

6	11	5	10
12	9	3	2
7	8	4	1

Value = g_5 = 318

No further improvement was found by interchanging two cells.

Interchanging three cells we have:

g. Sixth Improvement

9	11	5	10
12	3	6	2
7	8	4	1

Value = g_6 = 315

h. Seventh Improvement

12	11	5	10
9	7	6	2
3	8	4	1

Value = g_7 = 306

No further improvement was found by interchanging three cells.

Interchanging two cells again we find:

i. Eighth Improvement

9	11	5	10
12	7	6	2
3	8	4	1

Value = g_8 = 305

j. Nineth Improvement

9	11	6	10
12	7	5	2
3	8	4	1

Value = g_9 = 302

No further improvement was found with two cell interchanges.

Title: Dynamic Programming

Computer: IBM 709

Cross-chart: As shown on page 67.

Method: Starting from an initial configuration, we will try to improve on the solution by making two cell interchanges.

Results:

a. Initial Congifuration

1	2	3	4	5	6
7	8	9	10	11	12
13	14				
15	16				
17	18				

g_o = value = 4090

b. First Improvement

1	2	3	4	5	6
7	8	9	10	17	12
13	14				
15	16				
11	18				

g_1 = value = 3924

c. Second Improvement

1	18	3	4	5	6
7	8	9	10	17	12
13	14				
15	16				
11	12				

g_2 = 3788

d. Third Improvement

15	18	3	4	5	6
7	8	9	10	17	12
13	14				
1	16				
11	2				

$g_3 = 3645$

e. Fourth Improvement

15	18	3	12	5	6
7	8	9	10	17	4
13	14				
1	16				
11	2				

$g_4 = 3527$

f. Fifth Improvement

15	18	13	12	5	6
7	8	9	10	17	14
3	14				
11	16				
11	2				

$g_5 = 3467$

g. Sixth Improvement

15	18	13	12	5	6
7	14	9	10	17	4
3	8				
1	16				
11	2				

$g_6 = 3405$

h. Seventh Improvement

15	18	13	12	5	6
9	14	7	10	17	4
3	8				
1	16				
11	2				

$g_7 = 3347$

i. Eighth Improvement

15	18	13	12	5	17
9	14	7	10	6	4
3	8				
1	16				
11	2				

$g_8 = 3311$

j. Ninth Improvement

15	18	12	13	17	5
9	14	7	17	6	4
3	8				
1	16				
11	2				

$g_9 = 3268$

Cross-Chart - "U" Shaped Layout

0	10	28	34	61	61	91	0	36	88	4	22	19	16	78	3	93	78	23	15	58	57	48	61	36	18	88	9	12	85
10	0	53	40	2	95	35	26	77	46	37	61	93	21	95	97	69	4	61	85	21	15	2	87	98	10	47	22	67	27
28	53	0	13	13	92	66	99	47	24	49	57	74	32	25	43	62	17	10	97	11	69	84	99	63	22	32	98	76	25
34	40	13	0	12	25	63	44	67	33	45	30	63	47	61	41	80	49	46	0	54	57	68	18	4	70	61	26	38	15
61	2	13	12	0	46	80	43	82	13	59	57	72	34	8	66	4	47	85	41	56	13	29	80	40	56	65	43	96	20
61	95	92	25	46	0	86	92	31	6	93	74	69	89	18	83	8	90	85	80	62	78	69	60	47	21	6	17	98	85
91	35	66	63	80	86	0	32	53	8	15	71	58	56	61	43	50	80	92	56	75	44	18	68	64	12	97	78	34	21
0	26	99	44	43	92	32	0	3	86	47	82	4	89	94	21	10	68	79	96	87	66	72	84	5	53	92	41	82	52
36	77	47	67	82	31	53	3	0	9	66	7	99	97	73	13	56	91	88	45	80	49	79	22	66	8	90	33	72	15
88	46	24	33	13	6	8	86	9	0	99	0	57	12	31	96	85	72	91	77	37	34	11	27	10	59	33	87	72	73
4	37	49	45	59	93	15	47	66	99	0	79	20	85	59	72	88	49	12	79	38	47	41	99	59	51	11	0	47	82
22	61	57	30	57	74	71	82	7	0	79	0	0	36	53	27	18	0	20	37	65	71	73	14	0	87	96	96	21	43
19	93	74	63	72	69	58	4	99	57	20	0	0	97	68	0	2	64	83	44	30	24	86	99	52	10	83	4	32	74
16	21	32	47	34	89	56	89	97	12	85	36	97	0	84	47	4	99	83	81	74	99	0	77	8	7	23	14	1	50
78	95	25	61	8	18	61	94	73	31	59	53	68	84	0	49	84	92	0	11	61	6	49	79	90	57	0	96	44	66
3	97	43	41	66	83	43	21	13	96	72	27	0	47	49	0	99	0	43	46	39	52	45	28	92	0	17	19	43	62
93	69	62	80	4	8	50	10	56	85	88	18	2	4	84	99	0	94	53	0	68	72	99	29	27	0	85	41	0	40
78	4	17	49	47	90	80	68	91	72	49	0	64	99	92	0	94	0	38	57	79	37	2	0	46	80	8	90	53	47
23	61	10	46	85	85	92	79	88	91	12	20	83	83	0	43	53	38	0	19	35	18	71	0	59	32	87	0	96	40
15	85	97	0	41	80	56	96	45	77	79	37	44	81	11	46	0	57	19	0	52	10	86	73	52	31	0	83	44	16
58	21	11	54	56	62	75	87	80	37	38	65	30	74	61	39	68	79	35	52	0	15	21	74	68	36	22	92	34	0
57	15	69	57	13	78	44	66	49	34	47	71	24	99	6	52	72	37	18	10	15	0	34	63	0	30	31	84	56	48
48	2	84	68	29	69	18	72	79	11	41	73	86	0	49	45	99	2	71	86	21	34	0	0	58	27	26	43	16	9
61	87	99	18	80	60	68	84	22	27	99	14	99	77	79	28	29	0	0	73	74	63	0	0	65	87	8	8	89	42
36	98	63	4	40	47	64	5	66	10	59	0	52	8	90	92	27	46	59	52	68	0	58	65	0	16	25	14	14	32
18	10	22	70	56	21	12	53	8	59	51	87	10	7	57	0	0	80	32	31	36	30	27	87	16	0	40	12	89	88
88	47	32	61	65	6	97	92	90	33	11	96	83	23	0	17	85	8	87	0	22	31	26	8	25	40	0	58	19	68
9	22	98	26	43	17	78	41	33	87	0	96	4	14	96	19	41	90	0	83	92	84	43	8	14	12	58	0	70	53
12	67	76	38	96	98	34	82	72	72	47	21	32	1	44	43	0	53	96	44	34	56	16	89	14	89	19	70	0	40
85	27	25	15	20	85	21	52	15	73	82	43	74	50	66	62	40	47	40	16	0	48	9	42	32	88	68	53	40	0