MECHANICAL EXPRESSION,
STRESSES AT CAKE BOUNDARIES
AND NEW CP CELL

A Dissertation

Presented to

the Faculty of the Department of Chemical Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy in Chemical Engineering

By

Hemant Madhav Risbud

December 1974

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ABSTRACT

Expression or squeezing operation under constant pressure was analyzed taking into account the medium resistance. A nonlinear partial integro-differential equation representing the squeezing operation was solved using numerical methods. A computer program was developed to calculate transient pressure profiles and the cake thickness as a function of time. The problem was solved in two different coordinate systems.

Apparatuses were developed to measure the stress distribution on the boundaries of filter cakes compacted under mechanical pressure. It was discovered that for thin cakes the stress distributions on the bottom and at top are bell- shaped and not flat profiles as assumed by investigators in this field.

An improved compression-permeability cell was developed. In this apparatus, the hydraulic pressure profile was measured inside the cake at the center and at the wall. The filtration resistance was determined from slopes of the curved profiles. Previous calculations have been made on the assumption of linearity of hydraulic pressure.

TABLE OF CONTENTS

CHAPTER | Page

CHAPTER Page

CHAPTER Page

CHAPTER Page

LIST OF FIGURES

Figure Page

CHAPTER 1

ANALYSIS OF EXPRESSION (SQUEEZING) OPERATION UNDER CONSTANT PRESSURE

1.1 INTRODUCTION

Expression or squeezing is an important operation in solid-liquid separations. In this process, mechanical force is used to expel liquid from a filter cake which is retained by means of a porous septum. By application of squeezing, the liquid content of compressible filter cakes as well as settled sludge deposits can be reduced considerably. In disposal of sewage sludge, removal of a portion of the water is important to both disposal by landfill and incineration. In the chemical process industries it is desirable to eliminate as much liquid as possible by inexpensive methods because drying is a relatively expensive process.

In spite of the importance of the squeezing or expression operations, attempts to carry out a rigorous mathematical analysis have begun only recently. Analytical methods to predict the time for required squeezing are of value in designing equipment. Consolidation and squeezing are similar. Terzaghi's (1948) theory of consolidation has been long used in soil mechanics. This theory is crude because the coefficient of consolidation is assumed constant during the consolidation process. In recent work on settling of thick slurries (Shirato, Kato, Kobayashi and Sakazaki 1970) it was demonstrated that the consolidation coefficient of sediments can vary a hundred -fold during settling. Holley (1970) used a numerical method to analyze settling of soil under its own weight with free drainage on top. In this situation, no medium resistance is involved. It is shown later that the medium resistance cannot always be neglected. Therefore, in general, his method cannot be directly applied to squeezing operations. In addition, computer time for his method was excessive (several hours). Shirato, Murase, Negawa and Moridera (1971) did an analysis of expression under constant as well as variable pressure using Gill's modification of the Runge-Kutta method. However the medium resistance was neglected. In this chapter expression under constant pressure is analyzed considering the medium resistance. An attempt is made to overcome the shortcomings of previous investigations.

1.2 PHYSICS OF THE EXPRESSION OPERATION

Filter cake considered in this analysis can be obtained from filtration or from settling under gravity. In a compressible filter cake resulting from filtration, porosity increases away from the supporting medium. In a thin cake formed by settling, porosity is approximately constant.

A filter cake saturated with liquid rests on a medium of resistance R_m as shown in figure (1-1). Distance x is measured from the medium cake interface. Cake thickness L is a function of time only. Mechanical pressure p_1 (in addition to the atmospheric pressure) is applied by a nonporous piston on top at time t=0. In a small initial transient period, the piston and solid particles accelerate downward

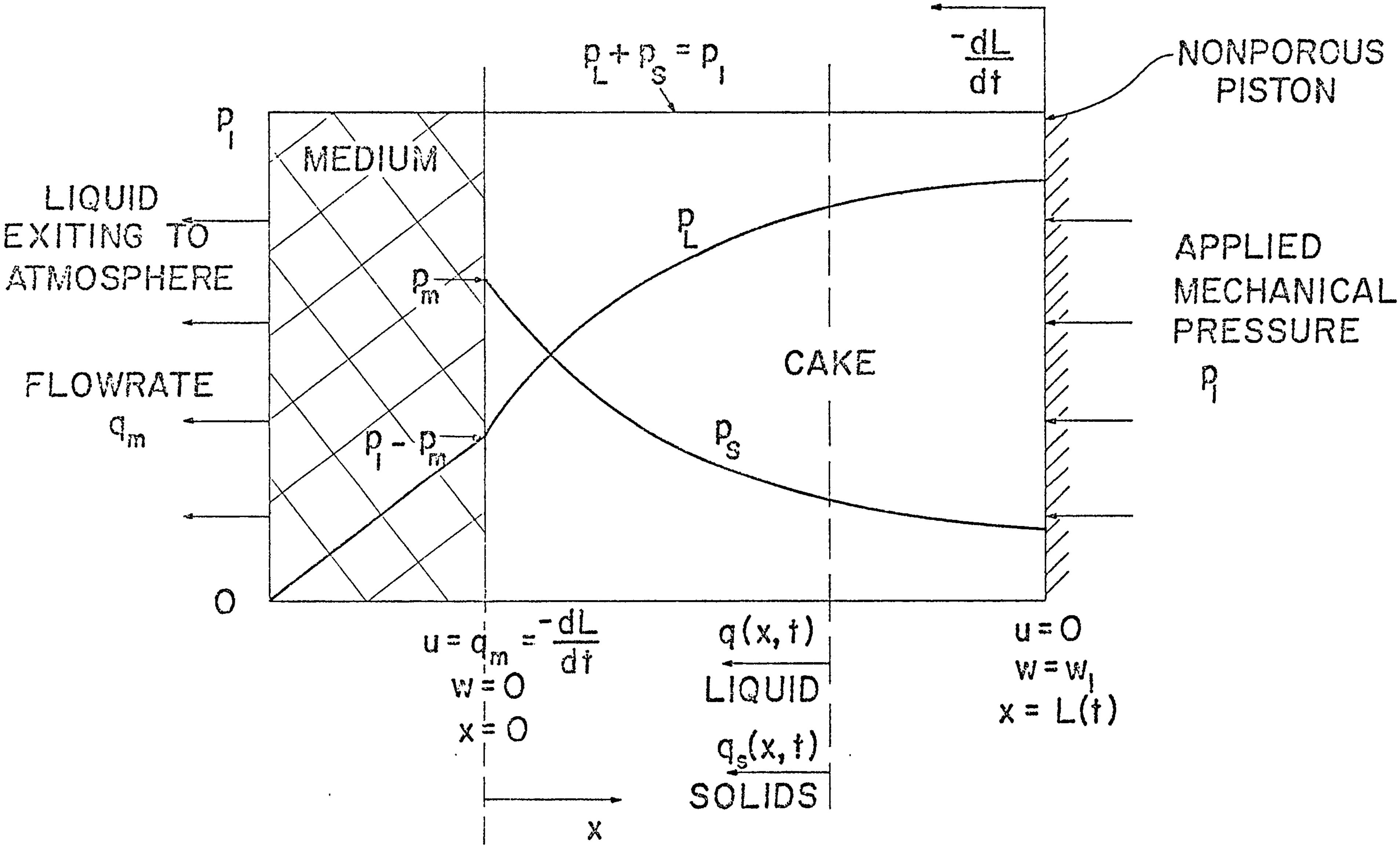


Fig. 1-1 Filter cake, medium and porous piston

as liquid exits through the medium. Then the piston reaches a steady downward velocity for a short while, and later as the system approaches equilibrium, the velocity decreases. It is assumed that the initial acceleration time period is small and its effects are negligible. A precise momentum analysis of the initial stage of consolidation under constant pressure is difficult to make because initial acceleration is not negligible when a force is suddenly applied. It has been assumed that the acceleration at the beginning is rapidly damped out. In the problem analyzed, drainage of the liquid is confined to one side. This analysis can be easily modified to fit a situation where drainage is provided on both sides. In the latter situation, a fictitious cross section at half the cake thickness can be looked upon as a nonporous piston. Pressure profiles are symmetrical about that cross section. The cake thickness to be used in the equations developed later, would be half the actual cake thickness if drainage were on both sides.

1.3 MATHEMATICAL ANALYSIS

1.3.1 ASSUMPTIONS

In order to analyze the problem, the following assumptions are made:

1) Porosity ε and the specific cake resistance α (Ruth 1946) are isotropic properties of a compressible material.

2) ε and α are unique functions of solid compressive pressure p_s (Tiller 1953) and the relationship is established

instantaneously.

3) There is no wall friction and cake structure is uniform over any cross section.

4) Gravitational forces are negligible.

5) Acceleration effects at the onset of squeezing disappear after a short time period and have little effect on the solution to the problem.

6) All solid particles are retained by the medium and total dry solid mass per unit area, w_1 is constant throughout the process.

7) Particle size is small in comparison with the geom--etry of the filter cake and the variation of liquid pressure over the surface of a particle is negligible in comparison with the liquid pressure itself, and liquid pressure is the same on surfaces of all particles in a cross section.

8) There is point contact between particles.

1.3.2 MACROSCOPIC MOMENTUM BALANCE

As velocities of liquid and solids in a filter cake are small, momentum terms are negligible in comparison with force terms. It is necessary to make only a force balance.

In Tiller's (1953) analysis of force balance on a filter cake, particles could not be sectioned. In the following analysis, a slightly different approach is used, permitting sectioning of particles.

Consider the filter cake between the piston and cross section BB, as shown in Fig. (1-2). The area of cross section is A. A free body diagram method is used to make a force

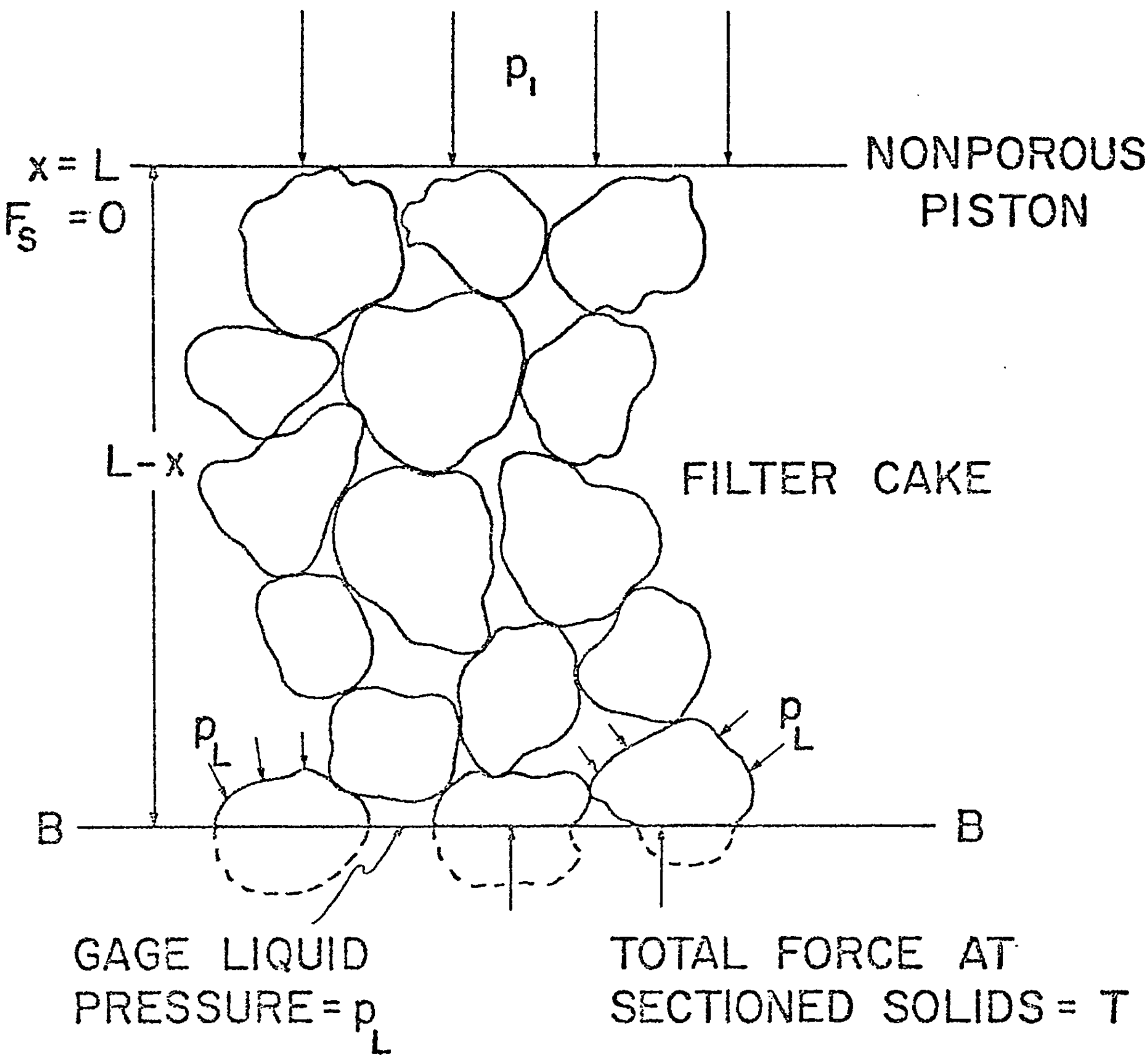


Fig. 1-2 Macroscopic force balance on a compacted filter cake

balance. Force applied by the piston is $-p_1A$. The negative sign indicates that the force is downwards. This is in addition to the force $(-p_aA)$ applied by atmospheric pressure p_a. Therefore total force applied from top is $-(p_a+p_1)A$. Gage liquid pressure at section BB (as well as over the surfaces of all particles sectioned by BB) is denoted by p_L. Absolute liquid pressure at BB is (p_L+p_a). Liquid force in section BB is $(p_L+p_a)A\epsilon$, where ϵ is the porosity at BB. Total force of reaction at solid surface sectioned by BB is denoted by T. Making an x directional macroscopic force balance from section BB to the top of the cake yields

$$\underbrace{T + (p_L+p_a)\,A\,\varepsilon}_{\text{force on the bottom}} \quad - \quad \underbrace{(p_a+p_1)\,A}_{\text{force on the top}} = 0 \qquad (1\text{-}1)$$

Form drag on a particle is the net component of forces exerted normal to the particle surface (due to liquid pressure). Skin drag on a particle is the net component of viscous forces exerted tangential to the particle surface. Total drag force is the sum of form drag and skin drag. Net component of all normal forces on one side of a particle sectioned in the middle is much larger than form drag on the whole particle. This is because form drag on the whole particle is the difference between forces of comparable magnitude on two halves. Therefore it is important to consider the net component of normal forces on particles sectioned by BB. The skin drag on the sectioned particles is negligible in comparison with the net component of normal forces. Consider an x directional force balance on the layer of particles sectioned by BB. Force transmitted by the layer above to the layer under consideration is denoted by F_s. Form drag on the portion above BB of the layer under consideration is $(p_L + p_a)A(1-\varepsilon)$. As mentioned previously, p_L is constant over the surface of sectioned particles. The force balance yields

$$\begin{array}{ccc} \text{Force from top} & & \text{Force on the bottom} \\ F_s + (p_L + p_a)\, A\, (1-\varepsilon) & - & T & = 0 \end{array} \qquad (1\text{-}2)$$

Substituting Eqn. (1-2) in (1-1) leads to

$$F_s + (p_L + p_a)\, A - (p_1 + p_a)\, A = 0 \qquad (1\text{-}3)$$

The quantity F_s/A is defined as the solids compressive pres--sure p_s at section BB. Therefore

$$p_s + p_L = p_1 \qquad (1\text{-}4)$$

This relationship is same as that derived by Tiller (1953) without considering sectioning of solids. Eqn. (1-4) will be used in later parts of this chapter.

1.3.3 EQUATIONS FOR THE TRANSIENT RESPONSE

Porosity ε is a function of both x and t. Dry solids mass per unit area from x=0 to x is denoted by w, and the differential of w is given by

$$dw = (1 - \varepsilon)\rho_s\, dx \qquad (1\text{-}5)$$

In Eqn. (1-5) ρ_s is the true density of solids. Integrating (1-5) from x=0 to x yields,

$$w = \int_0^x (1-\varepsilon)\rho_s dx \qquad (1\text{-}6)$$

This functional relationship is valid at any time t. The problem of obtaining cake thickness as a function of time can be accomplished either by using (x,t) or (w,t) coordinates. Solution in the (x,t) system is easier for physical understand--ing, but it was found that the computer time is less for the (w,t) system.

The superficial liquid velocity q at any cross-section is less than the average pore velocity which is q/ε. The superficial solids velocity at any cross section is q_s. As the solids occupy fraction $(1-\varepsilon)$ of total cross sectional area, the actual average velocity of solids is $q_s/(1-\varepsilon)$. The relative average velocity of liquid with respect to solids is $q/\varepsilon - q_s/(1-\varepsilon)$, and is denoted by u'. The superficial relative velocity u is given by

$$u = q - \frac{\varepsilon}{(1-\varepsilon)} q_s \qquad (1\text{-}7)$$

Quantity q_m is the liquid velocity at the medium. Considering a liquid volume balance from x=0 to x=x

$$\begin{array}{ccccc} \text{inlet at } x & - & \text{exit at } x=0 & = & \text{accumulation in } (0,x) \\ q & - & q_m & = & \dfrac{\partial}{\partial t}\displaystyle\int_0^x \varepsilon \, dx \end{array} \qquad (1\text{-}8)$$

Differentiating both sides with respect to x,

$$\left(\frac{\partial q}{\partial x}\right)_t = \left(\frac{\partial \varepsilon}{\partial t}\right)_x \qquad (1\text{-}9)$$

Making a total volume balance from x=0 to x=x (const. densities)

rate of incoming volume $= q + q_s$

rate of outgoing volume $= q_m$ (at $x=0$, $q_s=0$ and $q=q_m$)

therefore $q + q_s = q_m$ (1-10)

Consider $\varepsilon = \varepsilon(w,t)$

$$d\varepsilon = \left(\frac{\partial \varepsilon}{\partial w}\right)_t dw + \left(\frac{\partial \varepsilon}{\partial t}\right)_w dt \qquad (1\text{-}11)$$

Dividing both sides by dt and holding x constant,

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_x = \left(\frac{\partial \varepsilon}{\partial w}\right)_t \left(\frac{\partial w}{\partial t}\right)_x + \left(\frac{\partial \varepsilon}{\partial t}\right)_w \tag{1-12}$$

Differentiating both sides of Eqn. (1-6) with respect to t while holding x constant,

$$\left(\frac{\partial w}{\partial t}\right)_x = - \rho_s \left(\frac{\partial \varepsilon}{\partial t}\right)_x \tag{1-13}$$

Substituting for $\left(\frac{\partial \varepsilon}{\partial t}\right)_x$ using Eqn. (1-9)

$$\left(\frac{\partial w}{\partial t}\right)_x = - \rho_s \int_0^x \left(\frac{\partial q}{\partial x}\right)_t dx = - \rho_s \left(q \right)_0^x \tag{1-14}$$

Since $q=q_m$ at $x=0$, the above equation yields,

$$\left(\frac{\partial w}{\partial t}\right)_x = \rho_s (q_m - q) \tag{1-15}$$

Substituting Eqn. (1-15) in Eqn. (1-12)

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_x = \left(\frac{\partial \varepsilon}{\partial w}\right)_t \rho_s (q_m - q) + \left(\frac{\partial \varepsilon}{\partial t}\right)_w \tag{1-16}$$

Using Eqn. (1-5) and (1-10) in Eqn. (1-16) one obtains

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_w = \left(\frac{\partial \varepsilon}{\partial t}\right)_x - \left(\frac{\partial \varepsilon}{\partial x}\right)_t \frac{q_s}{(1-\varepsilon)} \tag{1-17}$$

Differentiating both sides of Eqn. (1-7) with respect to x while holding t constant,

$$\left(\frac{\partial u}{\partial x}\right)_t = \left(\frac{\partial q}{\partial x}\right)_t - q_s \frac{\partial}{\partial x} \left(\frac{\varepsilon}{1-\varepsilon}\right)_t - \left(\frac{\varepsilon}{1-\varepsilon}\right) \left(\frac{\partial q_s}{\partial x}\right)_t \tag{1-18}$$

As q_m is not a function of x in Eqn. (1-10), differentiating both sides with respect to x while holding t constant yields

$$\left(\frac{\partial q_s}{\partial x}\right)_t = -\left(\frac{\partial q}{\partial x}\right)_t \qquad (1\text{-}19)$$

Substituting Eqn. (1-19) in Eqn. (1-18) and simplifying,

$$\left(\frac{\partial u}{\partial x}\right)_t = \frac{1}{1-\varepsilon}\left(\frac{\partial q}{\partial x}\right)_t - \frac{1}{(1-\varepsilon)^2}\left(\frac{\partial \varepsilon}{\partial x}\right)_t q_s \qquad (1\text{-}20)$$

Using Eqn. (1-9) in Eqn. (1-20) and simplifying,

$$\left(\frac{\partial u}{\partial x}\right)_t = \frac{1}{(1-\varepsilon)}\left[\left(\frac{\partial \varepsilon}{\partial t}\right)_x - \left(\frac{\partial \varepsilon}{\partial x}\right)_t \frac{q_s}{(1-\varepsilon)}\right] \qquad (1\text{-}21)$$

Substituting Eqn. (1-17) in (1-21),

$$\left(\frac{\partial u}{\partial x}\right)_t = \frac{1}{(1-\varepsilon)}\left(\frac{\partial \varepsilon}{\partial t}\right)_w \qquad (1\text{-}22)$$

Dividing both sides of Eqn. (1-22) by $\rho_s(1-\varepsilon)$, and using Eqn. (1-5)

$$\left(\frac{\partial u}{\partial w}\right)_t = \frac{1}{\rho_s(1-\varepsilon)^2}\left(\frac{\partial \varepsilon}{\partial t}\right)_w \qquad (1\text{-}23)$$

Integrating Eqn. (1-23) with respect to w,

$$(u)_0^w = \int_0^w \frac{1}{\rho_s(1-\varepsilon)^2}\left(\frac{\partial \varepsilon}{\partial t}\right)_w dw \qquad (1\text{-}24)$$

At w=0, or at x=0, $q_s=0$ and $q=q_m$, therefore

$$\text{at } w=0, \quad u = q - \frac{\varepsilon}{1-\varepsilon} q_s = q_m$$

Using value of u in Eqn. (1-24)

$$u = \frac{1}{\rho_s}\int_0^w \frac{1}{(1-\varepsilon)^2}\left(\frac{\partial \varepsilon}{\partial t}\right)_w dw + q_m \qquad (1\text{-}25)$$

Consider a volume balance over the entire cake. Rate of change of total volume of the cake per unit area is dL/dt. As the volume of solids remains constant in the cake, the volume of liquid exiting through the medium is equal to change in total volume of the cake. Exiting liquid flow rate per unit area is q_m; therefore

$$q_m = - \frac{dL}{dt} \tag{1-26}$$

Substituting Eqn. (1-26) in Eqn. (1-25) produces

$$u = \frac{1}{\rho_s} \int_0^w \frac{1}{(1-\varepsilon)^2} \left(\frac{\partial \varepsilon}{\partial t}\right)_w dw - \frac{dL}{dt} \tag{1-27}$$

The Ruth form of the Darcy equation for flow through a compressible filter cake is,

$$u = \frac{1}{\mu\alpha} \left(\frac{\partial p_L}{\partial w}\right)_t \tag{1-28}$$

In Eqn. (1-28) μ is the viscosity of liquid and p_L is liquid pressure. As only the derivative of p_L is present in Eqn. (1-28), p_L can be either gage or absolute pressure. In the following solution p_L is taken as gage pressure for the sake of convenience. Combining Eqns. (1-27) and (1-28)

$$\left(\frac{\partial p_L}{\partial w}\right)_t = \frac{\mu\alpha}{\rho_s} \left[\int_0^w \frac{1}{(1-\varepsilon)^2} \left(\frac{\partial \varepsilon}{\partial t}\right)_w dw - \rho_s \frac{dL}{dt} \right] \tag{1-29}$$

The quantities α and ε are functions of solid compressive drag pressure p_s as obtained from compression permeability cell test. Either two parameter fractional power functions in p_s (Tiller and Cooper 1962) or polynomials in p_s can be used to approximate α Vs p_s and ε Vs p_s data. Although polynomial relationships possess a larger number of parameters than fractional power relationships, the approximation is closer to the experimental data. Also a computer requires less time to calculate a polynomial of fourth order than a fractional power of a number. The polynomial also better approximates slope at p_s=0. The following polynomial relationships are used:

$$\alpha = \alpha_0 \ (1 + A_1 p_s + A_2 p_s^2 + A_3 p_s^3 + A_4 p_s^4) \qquad (1\text{-}30)$$

$$\varepsilon = E_0 + E_1 p_s + E_2 p_s^2 + E_3 p_s^3 + E_4 p_s^4 \qquad (1\text{-}31)$$

The fractional exponent forms are preferred when analytical formulas are needed. These relationships can be substituted in Eqn. (1-29). It is necessary to eliminate p_L so that p_s and L will be the only two remaining dependent variables. Substituting Eqn. (1-4) in Eqn. (1-29) and noting that p_1 is a constant,

$$\left(\frac{\partial p_s}{\partial w}\right)_t = - \frac{\mu\alpha}{\rho_s}\left[\int_0^w \frac{1}{(1-\varepsilon)^2}\left(\frac{\partial \varepsilon}{\partial t}\right)_w dw - \rho_s \frac{dL}{dt}\right] \qquad (1\text{-}32)$$

This is same as,

$$\left(\frac{\partial p_s}{\partial w}\right)_t = - \frac{\mu\alpha}{\rho_s}\left[\int_0^w \frac{1}{(1-\varepsilon)^2}\left(\frac{d\varepsilon}{dp_s}\right)\left(\frac{\partial p_s}{\partial t}\right)_w dw - \rho_s \frac{dL}{dt}\right] \qquad (1\text{-}33)$$

Differentiating Eqn. (1-31) with respect to p_s,

$$\frac{d\varepsilon}{dp_s} = E_1 + 2E_2p_s + 3E_3p_s^2 + 4E_4p_s^3 \qquad (1\text{-}34)$$

L_1 is defined as the cake thickness if the entire cake were at zero solids compressive pressure, with $\varepsilon = E_0$. Therefore w_1 and L_1 are related by

$$w_1 = L_1(1-E_0)\ \rho_s \qquad (1\text{-}35)$$

It is preferable to write the equations describing system response in a dimensionless form. Therefore the following quantities are defined:

Dimensionless solids compressive pressure $\phi \equiv p_s/p_1$ (1-36)

Dimensionless cake thickness $\Lambda \equiv L/L_1$ (1-37)

Dimensionless time $\tau \equiv \mu t/(\rho_s L_1^2)$ (1-38)

Dimensionless dry solids mass $W \equiv w/w_1$ (1-39)

Equation (1-33) can be written in the form

$$\frac{dp_s}{d\phi}\left(\frac{\partial\phi}{\partial W}\right)_\tau \frac{dW}{dw} = -\frac{\mu\alpha}{\rho_s}\left[\int_0^W \frac{1}{(1-\varepsilon)^2}\left(\frac{d\varepsilon}{d\phi}\right)\frac{d\phi}{dp_s}\frac{dp_s}{d\phi}\left(\frac{\partial\phi}{\partial\tau}\right)_W \frac{d\tau}{dt}\, w_1 dW - \rho_s \frac{dL}{d\Lambda}\frac{d\Lambda}{d\tau}\frac{d\tau}{dt}\right] \qquad (1\text{-}40)$$

Substituting Eqns. (1-36) to (1-39) in Eqn. (1-40) and simplifying yields

$$\left(\frac{\partial\phi}{\partial W}\right)_\tau = -\frac{\mu^2\alpha_0 w_1}{p_1\rho_s L_1}\left(\frac{\alpha}{\alpha_0}\right)\left[\int_0^W \frac{1}{(1-\varepsilon)^2}\left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial\tau}\right)_W \frac{w_1}{\rho_s L_1}\, dW - \frac{d\Lambda}{d\tau}\right] \qquad (1\text{-}41)$$

Utilizing Eqn. (1-35),

$$\frac{\mu^2 \alpha_0 w_1}{p_1 \rho_s L_1} = \frac{\mu^2 \alpha_0}{p_1} (1-E_0) \qquad (1\text{-}42)$$

Dimensionless number B is defined as follows

$$B \equiv \frac{\mu^2 \alpha_0}{p_1} (1-E_0) \qquad (1\text{-}43)$$

Rearranging Eqn. (1-35), it is seen that

$$\frac{w_1}{\rho_s L_1} = 1 - E_0 \qquad (1\text{-}44)$$

Combining Eqns. (1-42) and (1-43) and substituting the result along with Eqn. (1-44) in Eqn. (1-41) leads to

$$\left(\frac{\partial \phi}{\partial W}\right)_\tau = - B\left(\frac{\alpha}{\alpha_0}\right)\left[\int_0^W \frac{(1-E_0)}{(1-\varepsilon)^2} \left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial \phi}{\partial \tau}\right)_W dW - \frac{d\Lambda}{d\tau}\right] \qquad (1\text{-}45)$$

Combining Eqns. (1-30) and (1-36)

$$\alpha/\alpha_0 = 1 + A_1 p_1 \phi + A_2 p_1^2 \phi^2 + A_3 p_1^3 \phi^3 + A_4 p_1^4 \phi^4 \qquad (1\text{-}46)$$

Combining Eqns. (1-31) and (1-36)

$$(1-\varepsilon) = 1 - E_0 - E_1 p_1 \phi - E_2 p_1^2 \phi^2 - E_3 p_1^3 \phi^3 - E_4 p_1^4 \phi^4 \qquad (1\text{-}47)$$

The derivative $d\varepsilon/d\phi$ can be written as,

$$\frac{d\varepsilon}{d\phi} = \frac{d\varepsilon}{dp_s} \frac{dp_s}{d\phi} \qquad (1\text{-}48)$$

Substituting Eqns. (1-34) and (1-36) in Eqn.(1-48) and simplifying yields,

$$\frac{d\varepsilon}{d\phi} = E_1 p_1 + 2E_2 p_1^2 \phi + 3E_3 p_1^3 \phi^2 + 4E_4 p_1^4 \phi^3 \qquad (1\text{-}49)$$

Eqns. (1-46), (1-47) and (1-49) can be substituted in Eqn. (1-45) so that ϕ and Λ will be the only remaining dependent variables.

1.3.4 BOUNDARY CONDITIONS

Two boundary conditions are needed, one at the medium and the other at the piston. Quantity p_m is defined as p_s at the medium. From Eqn. (1-4), gage liquid pressure at the medium is $p_1 - p_m$ which is a function of time. The liquid exits to atmosphere from the medium. Therefore gage liquid pressure at the exit is zero, and liquid pressure drop across the medium is $p_1 - p_m$. The supporting medium resistance R_m is defined such that,

$$\frac{p_1 - p_m}{\mu R_m} = q_m \qquad (1\text{-}50).$$

Define the quantity $\phi_0 \equiv p_m/p_1$ (1-51)

Substituting Eqns. (1-50) and (1-51) in Eqn. (1-26) and simplifying yields

$$\phi_0 - 1 = \frac{\mu R_m}{p_1} \frac{dL}{dt} \qquad (1\text{-}52)$$

This equation can be written as

$$\phi_0 - 1 = \frac{\mu R_m}{p_1} \frac{dL}{d\Lambda} \frac{d\Lambda}{d\tau} \frac{d\tau}{dt} \qquad (1\text{-}53)$$

Substituting Eqns. (1-36),(1-37) and (1-38) in (1-53)

$$\phi_0 - 1 = \frac{\mu^2 R_m}{p_1 \rho_s L_1} \left(\frac{d\Lambda}{d\tau}\right) \tag{1-54}$$

The dimensionless number C is defined as

$$C \equiv \frac{\alpha_0 W_1}{R_m} \tag{1-55}$$

C is the ratio of total cake resistance, at $p_s=0$, to the medium resistance. Dividing Eqn. (1-42) by (1-55) yields

$$\frac{B}{C} = \frac{\mu^2 R_m}{p_1 \rho_s L_1} \tag{1-56}$$

Substituting Eqn. (1-56) in (1-54)

$$\frac{d\Lambda}{d\tau} = \frac{C}{B} (\phi_0 - 1) \tag{1-57}$$

Putting W=0 in Eqn. (1-45) and simplifying,

$$\frac{d\Lambda}{d\tau} = \left[\frac{1}{B(\alpha/\alpha_0)} \left(\frac{\partial \phi}{\partial W}\right)_\tau\right]_{W=0} \tag{1-58}$$

Combining Eqns. (1-57) and (1-58)

$$C(\phi_0 - 1) = \left[\frac{1}{(\alpha/\alpha_0)} \left(\frac{\partial \phi}{\partial W}\right)_\tau\right]_{W=0} \tag{1-59}$$

This is the <u>first boundary condition</u>.

At the piston, the liquid and solids move with the same velocity which equals that of the piston; and the relative velocity u between liquid and solids is zero.

Substituting u=0 in Eqn. (1-28), it can be seen that the liquid pressure gradient at the piston is zero. Utilizing Eqn. (1-4) and the condition described above,

$$\left(\frac{\partial p_s}{\partial w}\right)_t = 0 \qquad \text{at } w=w_1 \qquad (1\text{-}60)$$

Transforming this equation into a dimensionless form by using Eqn. (1-36) and (1-39)

$$\left(\frac{\partial \phi}{\partial W}\right)_\tau = 0 \qquad \text{at } W=1 \qquad (1\text{-}61)$$

This is the <u>second boundary condition</u>.

1.3.5 INITIAL CONDITION

If the initial cake has resulted by settling due to gravity and if the cake thickness is restricted to a few inches, p_s can be taken to be zero initially in accordance with assumption 4). Therefore the initial condition would be

$$\phi = 0 \qquad \text{at} \quad \tau = 0 \qquad (1\text{-}62)$$

However, if the filter cake resulting at the end of filtration is to be squeezed, the initial ϕ Vs W profile must be available. In this investigation, the problem is solved with the initial condition given by Eqn. (1-62).

If the filter cake has resulted from filtration, initial profile would be between ϕ=0 and ϕ=1. Therefore time required to squeeze this filter cake would be less than the time calculated from the following analysis.

It was found impossible to make a numerical scheme work with $\phi=0$ at all points inside a cake and it was necessary to initiate the solution with small positive values of ϕ. This was done by assuming a ϕ profile as shown in Fig. (1-10) at $\tau=0$. This profile monotonically decreases with W and also satisfies the second boundary condition. Dimensionless cake thickness Λ is very close to 1 for this profile. Time correc- -tion $\Delta\tau_{cor.}$ for the assumed initial ϕ profile is obtained by calculating $d\Lambda/d\tau$ at the start of the solution and extrap- -olating Λ for the assumed profile back to $\Lambda=1$, which is at the true initial profile given by Eqn. (1-62). Quantity $\Delta\tau_{cor.}$ is very small in comparison with the values of τ involved in the numerical solution.

1.3.6 CAKE THICKNESS FOR ANY ϕ PROFILE

From a knowledge of the porosity and ϕ relationship, cake thickness for any ϕ profile can be calculated as follows. Eqn. (1-5) yields,

$$dx = \frac{dw}{(1-\varepsilon)\rho_s} \tag{1-63}$$

Integrating this equation with limits for x from 0 to L and limits for w from 0 to w_1,

$$L = \int_0^{w_1} \frac{dw}{(1-\varepsilon)\rho_s} \tag{1-64}$$

Transforming from w to W by means of Eqn. (1-39)

$$L = \frac{w_1}{\rho_s}\int_0^1 \frac{dW}{(1-\varepsilon)} \tag{1-65}$$

Substituting Eqn. (1-35) in (1-65) and simplifying,

$$L = L_1 \ (1-E_0)\int_0^I \frac{dW}{(1-\varepsilon)} \qquad (1\text{-}66)$$

Substituting Eqn. (1-37) in Eqn. (1-66),

$$\Lambda = (1-E_0)\int_0^I \frac{dW}{1-\varepsilon} \qquad (1\text{-}67)$$

$(1-\varepsilon)$ is known as a function of ϕ from Eqn. (1-47) and Λ can be evaluated.

1.3.7 EQUILIBRIUM CAKE THICKNESS

L_∞ and Λ_∞ are defined as L and Λ at equilibrium respectively. At equilibrium, $p_s=p_1$ at every point inside the cake, ε is uniform throughout, and is denoted by ε_{p_1}. Using this condition for equilibrium in Eqn. (1-67)

$$\Lambda_\infty = (1-E_0)/(1-\varepsilon_{p_1}) \qquad (1\text{-}68)$$

1.3.8 CONVERTING THE ϕ VS W PROFILE INTO A ϕ VS X PROFILE

It is necessary to convert ϕ Vs W profile to ϕ Vs dimensionless distance profiles for a better understanding. Dimensionless distance X is defined as

$$X = x/L_1 \qquad (1\text{-}69)$$

Integrating Eqn. (1-63) with limits for x being 0 to x and limits for w being 0 to w.

$$x = \int_0^w \frac{dw}{(1-\varepsilon)\rho_s} \qquad (1\text{-}70)$$

Transforming from w to W by means of Eqn. (1-39),

$$x = \frac{w_1}{\rho_s}\int_0^W \frac{dW}{(1-\varepsilon)} \qquad (1\text{-}71)$$

Substituting Eqns. (1-35) and (1-69) in (1-71) and simplifying ,

$$X = (1-E_0)\int_0^W \frac{dW}{(1-\varepsilon)} \qquad (1\text{-}72)$$

Utilizing Eqn. (1-72), ϕ Vs W profile can be converted to ϕ Vs X profile.

1.3.9 ANTICIPATED TRANSIENT RESPONSE BASED ON A QUALITATIVE PHYSICAL REASONING

As an increasing quantity of liquid is squeezed out, solids have to bear the applied load to a greater extent and p_s rises with time throughout the cake. As liquid flows toward the septum, it exerts drag on the solids, and p_s increases monotonically from the piston to the septum. As squeezing proceeds, the rate of squeezing slows down and the cake thickness Vs time curve is of an exponentially decreasing type. When equilibrium is reached, $p_s=p_1$ everywhere inside the cake; and a flat profile $\phi=1$ would result. It will be seen later that results of numerical calculations agree with this qualitatively anticipated response.

1.3.10 FORMULATION OF DIFFERENCE EQUATIONS

Total dimensionless dry solid mass per unit area is divided into N equal parts. Each division ΔW being equal to 1/N. Attempt should be made to use as large $\Delta\tau$ as possible to save on computer time. Optimum $\Delta\tau$ for each system consid--ered has to be obtained by scanning through a wide range (10^{-8} to 10^{-3} for example). Subscript indices i and j will be used for W and τ coordinates respectively. There are a number of ways in which a derivative can be represented by a numerical difference method. It was found that three point formula for differentiation with respect to W and two point forward difference formula for differentiation with respect to τ yielded an adequate formulation. The derivatives are represented as follows (Von Rosenberg 1969)

$$\left(\frac{\partial\phi}{\partial W}\right)_{i,j} = \frac{-\phi_{i-1,j} + \phi_{i+1,j}}{2\Delta W} \qquad (1\text{-}73)$$

$$\left(\frac{\partial\phi}{\partial W}\right)_{0,j} = \frac{-3\phi_{0,j} + 4\phi_{1,j} - \phi_{2,j}}{2\Delta W} \qquad (1\text{-}74)$$

$$\left(\frac{\partial\phi}{\partial W}\right)_{N,j} = \frac{\phi_{N-2,j} - 4\phi_{N-1,j} + 3\phi_{N,j}}{2\,\Delta W} \qquad (1\text{-}75)$$

Eqns. (1-74) and (1-75) are for end points.

$$\left(\frac{\partial\phi}{\partial\tau}\right)_{i,j} = \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta\tau} \qquad (1\text{-}76)$$

$$\left(\frac{d\Lambda}{d\tau}\right)_{j} = \frac{\Lambda_{j+1} - \Lambda_{j}}{\Delta\tau} \qquad (1\text{-}77)$$

Defining quantity G as follows,

$$G \equiv \int_0^W \frac{(1-E_0)}{(1-\varepsilon)^2}\left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial\tau}\right)_W dW \qquad (1\text{-}78)$$

It follows from Eqn. (1-78) that $\qquad G_{0,j} = 0 \qquad (1\text{-}79)$

Using the trapezoidal rule for integration in defining Eqn. (1-79)

$$G_{i,j} = G_{i-1,j} + \frac{\Delta W(1-E_0)}{2}\left[\frac{1}{(1-\varepsilon)^2_{i-1,j}}\left(\frac{d\varepsilon}{d\phi}\right)_{i-1,j}\left(\frac{\partial\phi}{\partial\tau}\right)_{i-1,j}\right.$$

$$\left. + \frac{1}{(1-\varepsilon)^2_{i,j}}\left(\frac{d\varepsilon}{d\phi}\right)_{i,j}\left(\frac{\partial\phi}{\partial\tau}\right)_{i,j}\right] \qquad (1\text{-}80)$$

Substituting Eqn. (1-76) in Eqn. (1-80) and simplifying,

$$G_{i,j} = G_{i-1,j} + \frac{\Delta W(1-E_0)}{2\ \Delta\tau}\left[\left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{i-1,j}(\phi_{i-1,j+1} - \phi_{i-1,j})\right.$$

$$\left. + \left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{i,j}(\phi_{i,j+1} - \phi_{i,j})\right] \qquad (1\text{-}81)$$

Putting i=1 in Eqn. (1-81) and substituting Eqn. (1-79)

$$G_{1,j} = \frac{\Delta W(1-E_0)}{2\ \Delta\tau}\left[\left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{0,j}(\phi_{0,j+1} - \phi_{0,j})\right.$$

$$\left. + \left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{1,j}(\phi_{1,j+1} - \phi_{1,j})\right] \qquad (1\text{-}82)$$

Substituting Eqns (1-73), (1-78) and (1-81) in Eqn. (1-45)

$$\frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\ \Delta W} = -B\left(\frac{\alpha}{\alpha_0}\right)_{i,j}\left[G_{i-1,j} + \frac{\Delta W(1-E_0)}{2\Delta\tau}\left\{\frac{1}{(1-\varepsilon)^2_{i-1,j}}\cdot\right.\right.$$

$$\left(\frac{d\varepsilon}{d\phi}\right)_{i-1,j}(\phi_{i-1,j+1} - \phi_{i-1,j}) + \frac{1}{(1-\varepsilon)^2_{i,j}}\cdot$$

$$\left.\left.\left(\frac{d\varepsilon}{d\phi}\right)_{i,j}(\phi_{i,j+1} - \phi_{i,j})\right\} - \left(\frac{d\Lambda}{d\tau}\right)_j\right] \qquad (1\text{-}83)$$

Upon simplification Eqn. (1-83) gives,

$$- \frac{2\ \Delta\tau}{\Delta W(1-E_0)} \left[\frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\ \Delta WB(\alpha/\alpha_0)_{i,j}} + G_{i-1,j} - \left(\frac{d\Lambda}{d\tau}\right)_j \right]$$

$$= \frac{1}{(1-\varepsilon)^2_{i-1,j}} \left(\frac{d\varepsilon}{d\phi}\right)_{i-1,j} (\phi_{i-1,j+1} - \phi_{i-1,j})$$

$$+ \frac{1}{(1-\varepsilon)^2_{i,j}} \left(\frac{d\varepsilon}{d\phi}\right)_{i,j} (\phi_{i,j+1} - \phi_{i,j}) \qquad (1-84)$$

Solving for $\phi_{i,j+1}$ yields,

$$\phi_{i,j+1} = \phi_{i,j} + \left\{\frac{(1-\varepsilon)^2}{d\varepsilon/d\phi}\right\}_{i,j} \left[\frac{2\ \Delta\tau}{\Delta W(1-E_0)} \left\{\frac{\phi_{i-1,j} - \phi_{i+1,j}}{2\Delta WB(\alpha/\alpha_0)_{i,j}} - G_{i-1,j} + \left(\frac{d\Lambda}{d\tau}\right)_j\right\} - \left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{i-1,j} \cdot (\phi_{i-1,j+1} - \phi_{i-1,j}) \right] \qquad (1-85)$$

This equation is directly used in the computer program when

$2 \leqslant i \leqslant N-1$.

Putting i=1 in Eqn. (1-85) and substituting Eqn. (1-79) results in

$$\underset{(i=1)}{\phi_{1,j+1}} = \phi_{1,j} + \left\{\frac{(1-\varepsilon)^2}{d\varepsilon/d\phi}\right\}_{1,j} \left[\frac{2\ \Delta\tau}{\Delta W(1-E_0)} \left\{\frac{\phi_{0,j} - \phi_{2,j}}{2\Delta WB(\alpha/\alpha_0)_{1,j}} + \left(\frac{d\Lambda}{d\tau}\right)_j\right\} - \left\{\frac{d\varepsilon/d\phi}{(1-\varepsilon)^2}\right\}_{0,j} (\phi_{0,j+1} - \phi_{0,j}) \right] \qquad (1-86)$$

At i=N, W=1, and boundary condition 2) is applied by setting j+1 instead of j in Eqn. (1-75) and then combining with Eqn. (1-61). After solving for $\phi_{N,j+1}$, there results

$$\underset{(i=N)}{\phi_{N,j+1}} = (4\phi_{N-1,j+1} - \phi_{N-2,j+1})/3 \qquad (1\text{-}87)$$

Thus it is seen that if the ϕ profile is known at instant j, the ϕ profile at instant j+1 can be calculated using Eqns. (1-85),(1-86) and (1-87) provided $\phi_{i=0,j+1}$ is known.

Boundary condition 1) can be applied by substituting Eqn. (1-75) at instant j+1 in Eqn. (1-59). Simplification yields

$$2C\Delta W(\alpha/\alpha_0)_{\substack{0,j+1\\ i=0}} (\phi_{0,j+1} - 1) + 3\phi_{0,j+1} - 4\phi_{1,j+1} + \phi_{2,j+1} = 0 \qquad (1\text{-}88)$$

$\phi_{i=0,j+1}$ has to be obtained by a predictor-corrector method. At the start of the solution, $\phi_{0,1} = \phi_{0,0}$ is used as the first guess. Later, as the solution proceeds, the following equation is used to calculate the first predicted value.

$$\phi_{i=0,j+1} = \phi_{0,j} + (\phi_{0,j} - \phi_{0,j-1}) \qquad (1\text{-}89)$$

$\phi_{1,j+1}$ and $\phi_{2,j+1}$ are calculated using this first predicted $\phi_{0,j+1}$ (Eqns. (1-85) and (1-86)). Then $\phi_{0,j+1}$, $\phi_{1,j+1}$ and $\phi_{2,j+1}$ are substituted in Equation (1-88). The left hand side of Eqn. (1-88) is called the residue. If its absolute value is less than a certain specified small positive number (10^{-5} for example) the equation is said to be satisfied, and the correct value of $\phi_{0,j+1}$ is obtained. Otherwise a correction has to be made to the assumed value of $\phi_{0,j+1}$. It was found that the following formula for correction enables one to obtain the correct solution in less than six trials.

$$(\phi_{i=0,j+1})_{\text{corrected}} = (\phi_{0,j+1})_{\text{assumed}} - k_0(\text{Residue with the assumed } \phi_{0,j+1}) \tag{1-90}$$

The quantity k_0 is a constant, which can vary in the range 0.12 to 0.16. The best value of k depends upon the system parameters, and has to be obtained by scanning through the range 0.12 to 0.16. With this value of k, solution can be obtained in two trials. However the computer time for the trial and error solution of $\phi_{0,j+1}$ is small in comparison with other operations in the numerical scheme; and a value of k=0.14 can be used in every case without increasing computer time significantly.

It is necessary to develop formulas for calculating Λ. Rearranging Eqn. (1-79),

$$\Lambda_{j+1} = \Lambda_j + (d\Lambda/d\tau)_j \, \Delta\tau \tag{1-91}$$

The quantity Λ can be calculated at any time by using Eqn. (1-67) and Simpson's rule of integration gives,

$$\Lambda_j = (1-E_0)\,\frac{\Delta W}{3}\left[\frac{1}{(1-\varepsilon)_{0,j}} + \frac{4}{(1-\varepsilon)_{1,j}} + \frac{2}{(1-\varepsilon)_{2,j}} + \frac{4}{(1-\varepsilon)_{3,j}} + \ldots + \ldots\ldots + \frac{2}{(1-\varepsilon)_{N-2,j}} + \frac{4}{(1-\varepsilon)_{N-1,j}} + \frac{1}{(1-\varepsilon)_{N,j}}\right] \tag{1-92}$$

$(1-\varepsilon)$ is known as a function of ϕ from Eqn. (1-49).

For every system considered, two separate runs are made on the computer. In one run, Eqn. (1-92) is used to calculate Λ at each time step; and in the other run Eqn. (1-91) is used. Values of Λ by these two methods should be very close (within 0.1 % for example) at every time step, if the solution is correct. It was found that for certain ranges of ΔW and $\Delta\tau$ these values checked. When ΔW and $\Delta\tau$ were chosen outside these ranges, the values did not check.

1.3.11 ERROR ANALYSIS

Eqn. (1-45) is a nonlinear partial integrodifferential equation. A rigorous error analysis and relating optimum ΔW and $\Delta\tau$ to the system parameters would be a formidable task and was beyond the scope of this investigation. Therefore simple numerical experiments were performed to check the errors. Checking Λ by two separate methods as described before was a part of this numerical experiment. The numerical scheme does not work with an initial profile ϕ=0. and an arbitrary profile had to be assumed as discussed under the heading 'initial condition'. The initial profile is chosen according to the following equation

$$\phi_{i,0} = \phi_{low} + (\phi_{high} - \phi_{low})(1 - i\Delta W)^M \qquad (1\text{-}93)$$

ϕ_{low} and ϕ_{high} and integer M have to be specified. It was found that as the solution advances, the ϕ profile can be approximated with powers M in the range of 2 to 12. Therefore M in this range was used to specity initial profiles also. It should be made certain that Λ for the initial profile assumed is 0.97 or more. Typical values are ϕ_{high} =0.05, ϕ_{low}= 0.005. It was found that as long as an initial profile is in a region close to W axis, it gives the same transient solution provided a time correction is made as discussed in the section entitled 'initial condition'. It was also discovered that there is a region in the $\Delta\tau$,ΔW plane in which the numerical scheme works for a given physical system. One of such regions is sketched in Fig. (1-3). The boundaries could not be defined sharply as no analytic formulas for convergence were available. There are a number of ways in which this numerical scheme does not work. The ϕ profile can oscillate with W, or ϕ can become negative or the profile can rise upto a certain level and start falling. In the numerical scheme used,$\Delta\tau$ cannot be increased independently of ΔW. As the ϕ profile approaches equilibrium it would save on the computer time if ΔW is made larger, $\Delta\tau$ can be increased by the same factor.

1.4 ALTERNATE APPROACHES TO THE PROBLEM

There are several ways in which the problem considered can be solved. Some of them which were tried besides the one discussed so far are described below. The best method is the one which yields the fastest solution on the computer and which is applicable for a variety of materials under widely differing conditions.

1.4.1 INTEGRODIFFERENTIAL EQUATION IN (x,t) COORDIANTES

As discussed before, the problem can be formulated

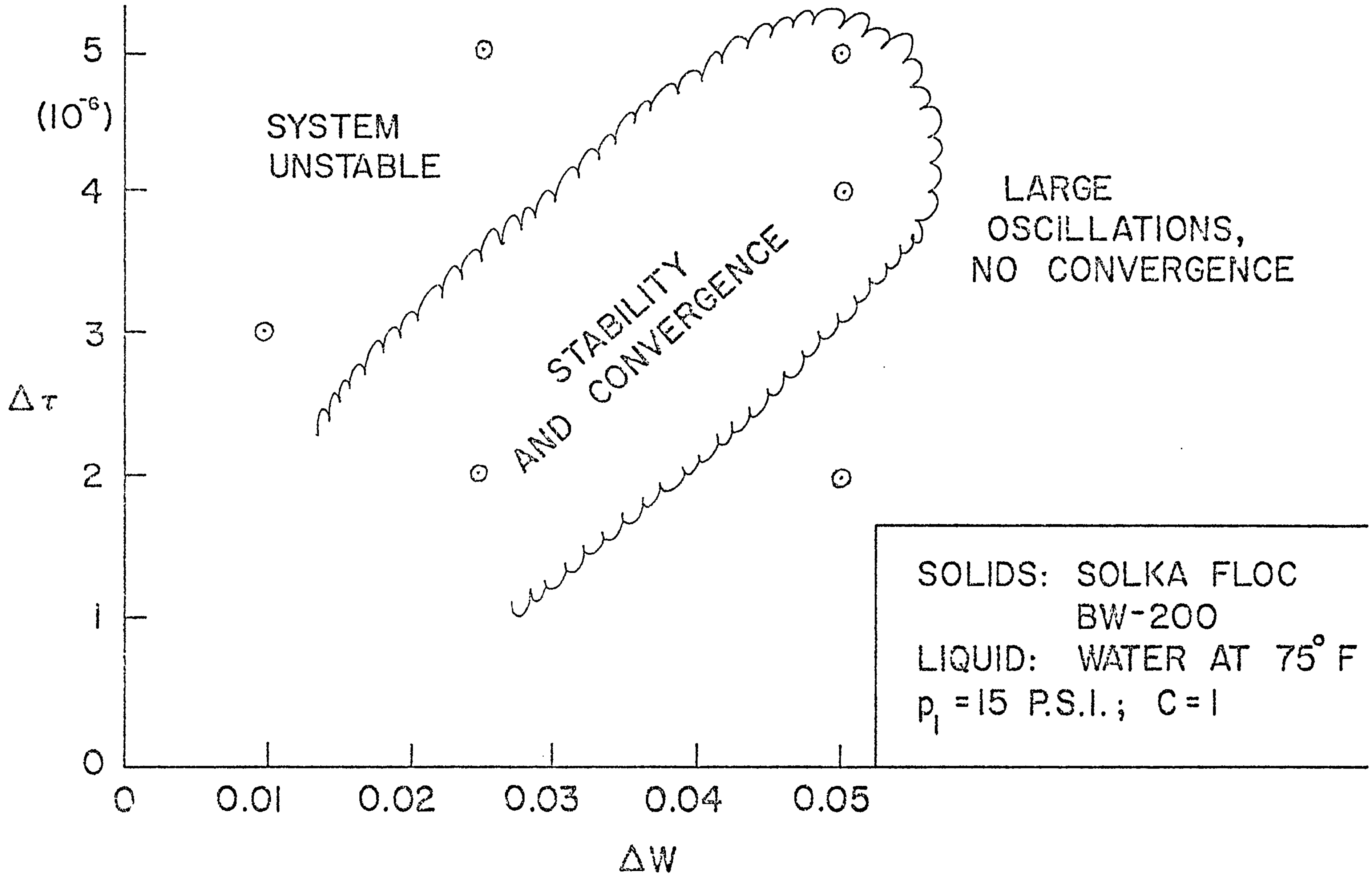


Fig. 1-3 Region of stability and convergence in $(\Delta W,\Delta\tau)$ plane.

in (x,t) coordinates system.

1.4.1.1 <u>Equations describing the transient response:</u>

Combining Eqns. (1-7) and (1-10) leads to

$$u = \frac{q}{1-\varepsilon} - \frac{\varepsilon\, q_m}{1-\varepsilon} \qquad (1\text{-}94)$$

Substituting for q, using Eqn. (1-8), and simplifying yields

$$u = \frac{1}{(1-\varepsilon)} \frac{\partial}{\partial t}\int_0^x \varepsilon\, dx - q_m \qquad (1\text{-}95)$$

Substituting Eqn. (1-26) in (1-95) and differentiating under the integral sign yields

$$u = \frac{1}{1-\varepsilon}\int_0^x \left(\frac{\partial\varepsilon}{\partial t}\right)_x dx - \frac{dL}{dt} \qquad (1\text{-}96)$$

Substituting Eqns. (1-28) and (1-4) in Eqn. (1-96) and simplifying yields

$$\frac{1}{\mu\alpha}\left(\frac{\partial p_s}{\partial w}\right)_t = -\left[\frac{1}{1-\varepsilon}\int_0^X \left(\frac{\partial\varepsilon}{\partial t}\right)_x dx - \frac{dL}{dt}\right] \qquad (1\text{-}97)$$

Changing from dW to dx with the help of Eqn. (1-5) at constant t and substitution in Eqn. (1-97) gives

$$\left(\frac{\partial p_s}{\partial x}\right)_t = -\mu\alpha\rho_s\left[\int_0^X \left(\frac{\partial\varepsilon}{\partial t}\right)_x dx - (1-\varepsilon)\frac{dL}{dt}\right] \qquad (1\text{-}98)$$

It is more convenient to transform Eqn. (1-98) in a dimensionless form. Using Eqn. (1-69), Eqn. (1-98) can also be written as follows,

$$\frac{dp_s}{d\varphi}\left(\frac{\partial\varphi}{\partial X}\right)_\tau L_1 = -\mu\left(\frac{\alpha}{\alpha_0}\right)\alpha_0\rho_s\left[\int_0^X \frac{d\varepsilon}{d\phi}\left(\frac{\partial\phi}{\partial\tau}\right)_X \frac{d\tau}{dt} L_1 dx - (1-\varepsilon)\frac{dL}{d\Lambda}\frac{d\Lambda}{d\tau}\frac{d\tau}{dt}\right] \tag{1-99}$$

Quantities $(\partial\phi/\partial\tau)_X$ and $(\partial\phi/\partial\tau)_W$ are not the same and therefore subscripts should not be dropped. Substituting Eqns. (1-36), (1-37) and (1-38) in (1-99) and simplifying yields

$$\left(\frac{\partial\phi}{\partial X}\right)_\tau = \frac{-\mu^2\alpha_0}{p_1}\left(\frac{\alpha}{\alpha_0}\right)\left[\int_0^X\left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial\tau}\right)_X dX - (1-\varepsilon)\frac{d\Lambda}{d\tau}\right] \tag{1-100}$$

Substituting for $\mu^2\alpha_0/p_1$ using Eqn. (1-43) leads to

$$\left(\frac{\partial\phi}{\partial X}\right)_\tau = \frac{-B}{(1-E_0)}\left(\frac{\alpha}{\alpha_0}\right)\left[\int_0^X\left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial\tau}\right)_X dX - (1-\varepsilon)\frac{d\Lambda}{d\tau}\right] \tag{1-101}$$

This is the required system equation in dimensionless form.

1.4.1.2 <u>Boundary conditions</u> : Putting X=0 in Eqn. (1-101) and simplifying,

$$\frac{d\Lambda}{d\tau} = \left\{\frac{(1-E_0)}{B(\alpha/\alpha_0)(1-\varepsilon)}\left(\frac{\partial\phi}{\partial X}\right)\right\}_{X=0} \tag{1-102}$$

Combining Eqns. (1-57) and (1-102)

$$\frac{C(\varphi_0-1)}{(1-E_0)} = \left\{\frac{1}{(\alpha/\alpha_0)(1-\varepsilon)}\left(\frac{\partial\phi}{\partial X}\right)\right\}_{X=0} \tag{1-103}$$

This is the first boundary condition.

Combining Eqns. (1-5),(1-60) and noting that $w=w_1$ at $x=L$,

$$\left(\frac{\partial p_s}{\partial x}\right) = 0 \quad \text{at } x = L \tag{1-104}$$

Substituting Eqns. (1-69),(1-36) and (1-37) in Eqn. (1-104)

$$\left(\frac{\partial \phi}{\partial X}\right) = 0 \quad \text{at } X = \Lambda \tag{1-105}$$

This is the second boundary condition. Λ is a function of time and not a constant. Therefore this boundary condition is more complicated than the one described by Eqn. (1-61).

1.4.1.3 Initial condition : Initial condition is the same as given in W,τ coordinates. Again, the numerical scheme does not work for $\phi = 0$, and the solution is initiated by assuming a ϕ profile. A time correction is made as previously described.

1.4.1.4 Cake thickness at any ϕ profile : From the knowledge of ε Vs ϕ relationship, cake thickness can be obtained as follows.

In Eqn. (1-6) $w = w_1$ when $x = L$, therefore

$$w_1 = \rho_s \int_0^L (1-\varepsilon)dx \tag{1-106}$$

A dimensionless distance Y is defined as follows,

$$Y \equiv x/L \tag{1-107}$$

Substituting Eqn. (1-107) in (1-106) and combining with Eqn. (1-35) and cancelling ρ_s on both sides produces

$$L_1(1-E_0) = \int_0^1 (1-\varepsilon)LdY \tag{1-108}$$

L is constant in the integration process; therefore, taking it outside the integral sign, using Eqn. (1-37) and solving for Λ yields

$$\Lambda = (1-E_0)/\int_0^1 (1-\varepsilon)dY \tag{1-109}$$

$(1-\varepsilon)$ is known as a function of ϕ from Eqn. (1-47) and Λ can be calculated from the ϕ Vs Y profile.

1.4.1.5 <u>Conversion from ϕ Vs X to ϕ Vs W</u> : The ϕ Vs X profile can be converted to ϕ Vs W profile for the purpose of making comparison between (X,τ) and (W,τ) coordinate schemes. Substituting Eqn. (1-69) in Eqn. (1-6) and noting that ρ_s and L_1 are constants,

$$w = \rho_s L_1 \int_0^X (1-\varepsilon)\, dX \tag{1-110}$$

Dividing Eqn. (1-110) by (1-35) and using (1-39)

$$W = \frac{1}{(1-E_0)} \int_0^X (1-\varepsilon)\, dX \tag{1-111}$$

$(1-\varepsilon)$ is a function of ϕ given by Eqn. (1-47).

1.4.1.6 <u>Formulation of difference equations</u> : In the (X,τ) coordinate system if length of each X step, denoted by ΔX is fixed, the total number of steps N will go on decreasing because the cake thickness decreases with time. The number of X divisions lost in each time step should be very small as compared to N. As at least one X step has to be dropped at every time step, N has to be of the order of several hundred. Computer time is directly proportional to N, and would be excessively large. To avoid this difficulty, N is fixed and ΔX is allowed to decrease as squeezing proceeds. Again, optimum $\Delta\tau$ and N have to be found by scanning through wide ranges.

Subscript indices k and j will be used for X and τ divisions respectively. Shrinking ΔX is illustrated in Fig. (1-4) in a highly exagerated form. Only 10 X divisions are considered for simplicity. $X_{k,j+1}$ is less than $X_{k,j}$ and is related to change in ΔX with time as

$$X_{k,j} - X_{k,j+1} = k\ (\Delta X_j - \Delta X_{j+1}) \qquad (1\text{-}112)$$

The subscript on ΔX shows the corresponding time. ΔX_j is given by

$$\Delta X_j = \Lambda_j / N \qquad (1\text{-}113)$$

It should be understood that although ΔX is changing, the X coordinate of a point fixed in space does not change with time. The equation for ϕ at abscissa $X_{k,j}$ and at time interval j+1 can be derived in a manner similar to that in the (W,τ) coordinate system as discussed later. These ϕ values are denoted as $\tilde{\phi}_{k,j+1}$. From these values, ϕ at $X_{k,j+1}$ can be calculated as shown in Fig. (1-5). The variable $X_{k,j+1}$ is shown much smaller than $X_{k,j}$ for better understanding. Point S has coordinates $(X_{k,j}, \tilde{\phi}_{k,j+1})$. Point $(X_{k,j+1}, \phi_{k,j+1})$ is very close to S and can be well approximated as lying on the tangent to ϕ_{j+1} profile at S. $\phi_{k,j+1}$ can be obtained by

$$\phi_{k,j+1} = \tilde{\phi}_{k,j+1} + \left(\frac{\partial\phi}{\partial X}\right)_{X_{k,j+1},\tau_{j+1}} (X_{k,j+1} - X_{k,j}) \qquad (1\text{-}114)$$

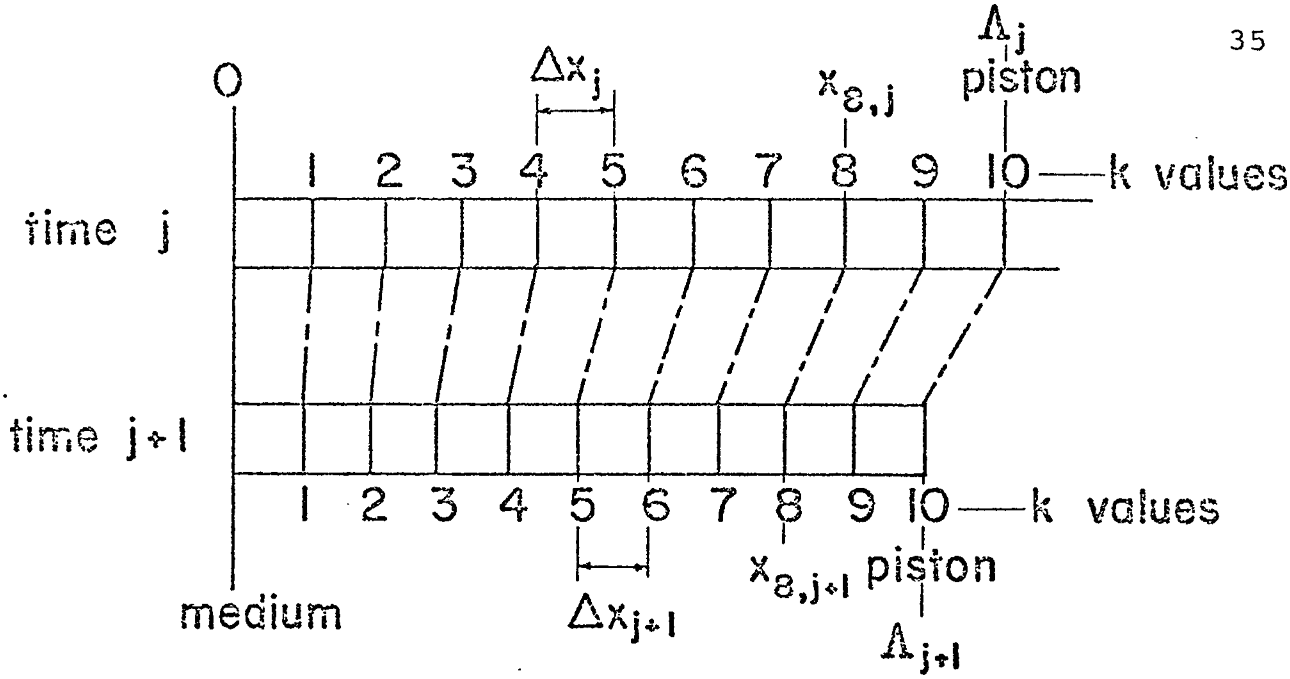


Fig. 1-4 Shrinking ΔX with constant N

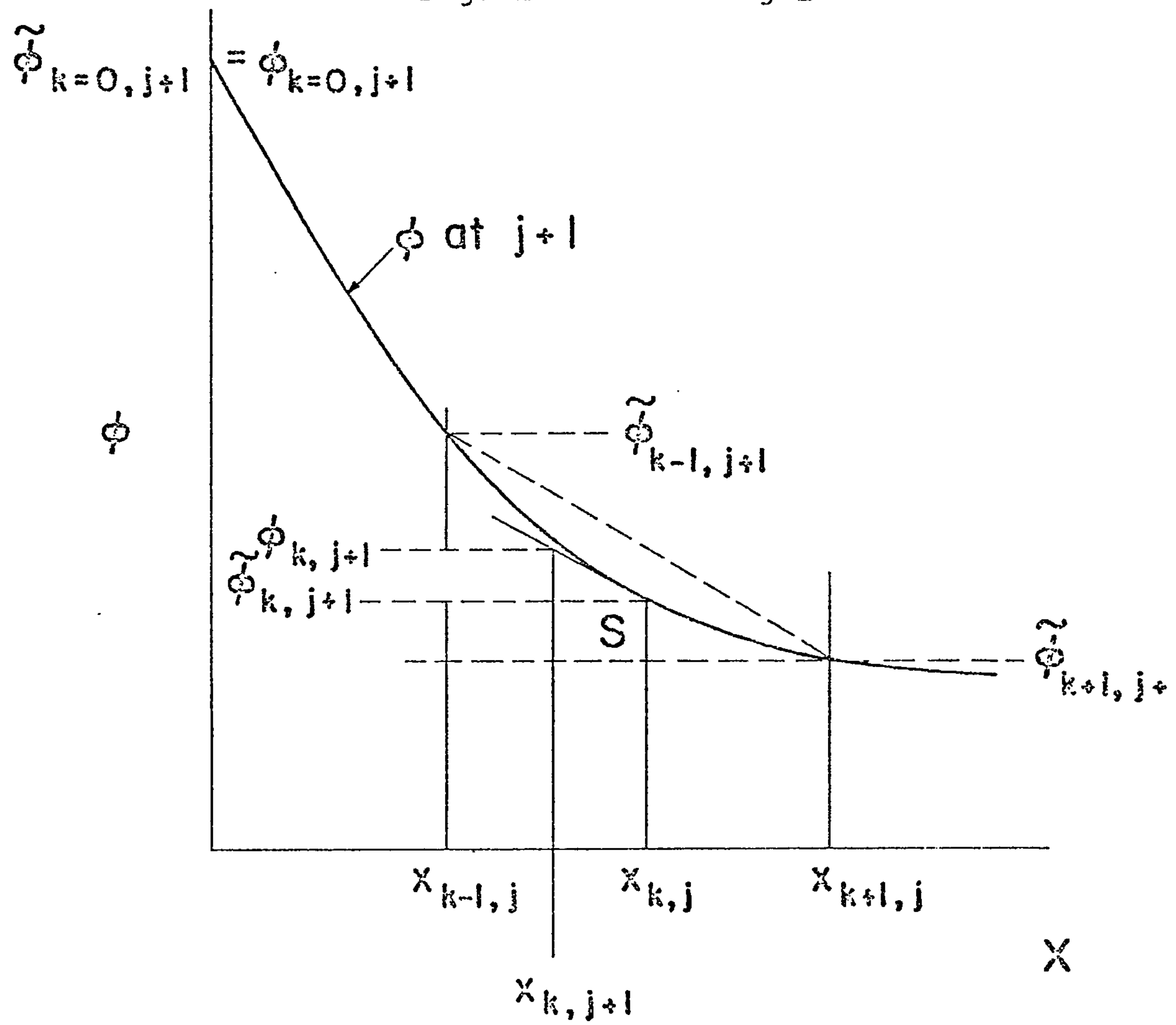


Fig. 1-5 Explanation of obtaining $\phi_{k,j+1}$ from $\tilde{\phi}$ profile

The X derivative of ϕ can be approximated as,

$$\left(\frac{\partial\phi}{\partial X}\right)_{X_{k,j+1},\tau_{j+1}} = \frac{\tilde{\phi}_{k+1,j+1} - \tilde{\phi}_{k-1,j+1}}{2\ \Delta X_j} \qquad (1\text{-}115)$$

Substituting Eqn. (1-112) and (1-114) in (1-115) gives

$$\phi_{k,j+1} = \tilde{\phi}_{k,j+1} - \frac{\tilde{\phi}_{k+1,j+1} - \tilde{\phi}_{k-1,j+1}}{2\ \Delta X_j}\, k(\Delta X_j - \Delta X_{j+1}) \qquad (1\text{-}116)$$

Equation (1-116) can be directly used in the computer program for $2 \leqslant k \leqslant (N-1)$. Putting k=1 in (1-116) and noting that $\tilde{\phi}_{k=0,j+1}$ is the same as $\phi_{k=0,j+1}$

$$\phi_{1,j+1} = \tilde{\phi}_{1,j+1} - \frac{\tilde{\phi}_{2,j+1} - \phi_{k=0,j+1}}{2\ \Delta X_j}\,(\Delta X_j - \Delta X_{j+1}) \qquad (1\text{-}117)$$

Thus it is seen that once $\tilde{\phi}_{k,j+1}$ are known, $\phi_{k,j+1}$ can be calculated. Next it is necessary to derive formulas for $\tilde{\phi}_{k,j+1}$. There are a number of ways in which a derivative can be represented by a difference method. It was discovered that three point formula for differentiation with respect to X and two point forward differnece formula for differentiation with respect to τ yielded a formulation which was adequate. The derivatives are represented as follows,

$$\left(\frac{\partial\phi}{\partial X}\right)_{k,j} = \frac{-\phi_{k-1,j} + \phi_{k+1,j}}{2\ \Delta X} \qquad (1\text{-}118)$$

$$\left(\frac{\partial\phi}{\partial X}\right)_{0,j} = \frac{-3\phi_{k=0,j} + 4\phi_{1,j} - \phi_{2,j}}{2\ \Delta X} \qquad (1\text{-}119)$$

$$\left(\frac{\partial\phi}{\partial X}\right)_{N,j} = \frac{\phi_{N-2,j} - 4\,\phi_{N-1,j} + 3\,\phi_{k=N,j}}{2\,\Delta X} \qquad (1\text{-}120)$$

$$\left(\frac{\partial\phi}{\partial\tau}\right)_{k,j} = \frac{\tilde{\phi}_{k,j+1} - \phi_{k,j}}{\Delta\tau} \qquad (1\text{-}121)$$

Defining F as

$$F_{k,j} \equiv \int_0^X \left(\frac{d\varepsilon}{d\phi}\right) \left(\frac{\partial\phi}{\partial\tau}\right)_X dX \qquad (1\text{-}122)$$

It follows from Eqn. (1-122) that

$$F_{0,j} = 0 \qquad (1\text{-}123)$$

Using the trapezoidal rule for integration in defining Eqn. (1-122)

$$F_{k,j} = F_{k-1,j} + \frac{\Delta X}{2}\left[\left(\frac{d\varepsilon}{d\phi}\right)_{k-1,j}\left(\frac{\partial\phi}{\partial\tau}\right)_{k-1,j} + \left(\frac{d\varepsilon}{d\phi}\right)_{k,j}\left(\frac{\partial\phi}{\partial\tau}\right)_{k,j}\right] \qquad (1\text{-}124)$$

Substituting Eqn. (1-121) in (1-124) and simplifying

$$F_{k,j} = F_{k-1,j} + \frac{\Delta X}{2\Delta\tau}\left[\left(\frac{d\varepsilon}{d\phi}\right)_{k-1,j}(\tilde{\phi}_{k-1,j+1} - \phi_{k-1,j}) + \left(\frac{d\varepsilon}{d\phi}\right)_{k,j}(\tilde{\phi}_{k,j+1} - \phi_{k,j})\right] \qquad (1\text{-}125)$$

Putting k=1 in Eqn. (1-125) and substituting Eqn. (1-123),

$$F_{1,j} = \frac{\Delta X}{2\Delta\tau}\left[\left(\frac{d\varepsilon}{d\phi}\right)_{0,j}(\phi_{0,j+1} - \phi_{0,j}) + \left(\frac{d\varepsilon}{d\phi}\right)_{1,j}(\tilde{\phi}_{1,j+1} - \phi_{1,j})\right] \qquad (1\text{-}126)$$

Substituting Eqns. (1-118), (1-122) and (1-125) in (1-101) the resulting equation for point k,j leads to

$$\frac{\phi_{k+1,j} - \phi_{k-1,j}}{2\,\Delta X} = -\frac{B}{(1-E_0)}\left(\frac{\alpha}{\alpha_0}\right)_{k,j}\left[F_{k-1,j} + \frac{\Delta X}{2\Delta\tau}\cdot\left\{\left(\frac{d\varepsilon}{d\phi}\right)_{k-1,j}(\tilde{\phi}_{k-1,j+1} - \phi_{k-1,j}) + \left(\frac{d\varepsilon}{d\phi}\right)_{k,j}\cdot(\tilde{\phi}_{k,j+1} - \phi_{k,j})\right\} - (1-\varepsilon)_{k,j}\left(\frac{d\Lambda}{d\tau}\right)_j\right] \qquad (1\text{-}127)$$

Upon simplification this yields,

$$-\frac{2\Delta\tau}{\Delta X}\left[\frac{\phi_{k+1,j} - \phi_{k-1,j}}{2\Delta X(B/(1-E_0))(\alpha/\alpha_0)_{k,j}} + F_{k-1,j} - (1-\varepsilon)_{k,j}\left(\frac{d\Lambda}{d\tau}\right)_j\right]$$

$$= \left(\frac{d\varepsilon}{d\varphi}\right)_{k-1,j}(\tilde{\phi}_{k-1,j+1} - \phi_{k-1,j}) + \left(\frac{d\varepsilon}{d\phi}\right)_{k,j}(\tilde{\phi}_{k,j+1} - \phi_{k,j}) \qquad (1\text{-}128)$$

Solving for $\tilde{\phi}_{k,j+1}$,

$$\tilde{\phi}_{k,j+1} = \phi_{k,j} + \frac{1}{(d\varepsilon/d\phi)_{k,j}}\left[\frac{2\Delta\tau}{\Delta X}\left\{\frac{\phi_{k-1,j} - \phi_{k+1,j}}{2\Delta X(B/(1-E_0))(\alpha/\alpha_0)_{k,j}} - F_{k-1,j} + (1-\varepsilon)_{k,j}\left(\frac{d\Lambda}{d\tau}\right)_j\right\} - \left(\frac{d\varepsilon}{d\phi}\right)_{k-1,j}\cdot(\tilde{\phi}_{k-1,j+1} - \phi_{k-1,j})\right] \qquad (1\text{-}129)$$

This equation is directly used in the computer program when

$$2 \leqslant k \leqslant N-1$$

Putting k=1 in Eqn. (1-129) and substituting Eqn. (1-123),

$$\underset{(k=1)}{\tilde{\phi}_{1,j+1}} = \phi_{1,j} + \frac{1}{(d\varepsilon/d\phi)_{1,j}}\left[\frac{2\Delta\tau}{\Delta X}\left\{\frac{\phi_{0,j} - \phi_{2,j}}{2\Delta X(B/(1-E_0)(\alpha/\alpha_0)_{1,j}} + (1-\varepsilon)_{1,j}\left(\frac{d\Lambda}{d\tau}\right)_j\right\} - \left(\frac{d\varepsilon}{d\phi}\right)_{0,j}(\phi_{0,j+1} - \phi_{0,j})\right] \quad (1\text{-}130)$$

The second boundary condition is applied by setting j+1 instead of j in Eqn. (1-120) and then combining it with Eqn. (1-105). Then solving for $\phi_{N,j+1}$ yields,

$$\underset{(k=N)}{\tilde{\phi}_{N,j+1}} = (4\tilde{\phi}_{N-1,j+1} - \tilde{\phi}_{N-2,j+1})/3 \quad (1\text{-}131)$$

If the ϕ profile is known at instant j, the j+1 values can be calculated utilizing Eqn. (1-129), (1-130) and (1-131), provided $\phi_{k=0,j+1}$ is known. Eqn. (1-119) can be used for the ϕ profile at time instant j+1. Using $\tilde{\phi}_{k,j+1}$'s and ΔX_j in (1-119) and noting that $\tilde{\phi}_{0,j+1}$ is the same as $\phi_{0,j+1}$,

$$\left(\frac{\partial\phi}{\partial X}\right)_{0,j+1} = \frac{-3\phi_{0,j+1} + 4\tilde{\phi}_{1,j+1} - \tilde{\phi}_{2,j+1}}{2\ \Delta X_j} \quad (1\text{-}132)$$

The first boundary condition can be applied by using Eqn. (1-132) in (1-103) at time instant j+1. Simplification yields,

$$\underset{k=0}{2(C/(1-E_0))\Delta X_j(\alpha/\alpha_0)_{0,j+1}(1-\varepsilon)_{0,j+1}(\phi_{0,j+1} - 1)} + 3\phi_{0,j+1} - 4\tilde{\phi}_{1,j+1} - \tilde{\phi}_{2,j+1} = 0 \quad (1\text{-}133)$$

$\phi_{k=0,j+1}$ is obtained by the predictor- corrector method as discussed in solution of the problem in (W,τ) coordinates. Once the correct $\phi_{k=0,j+1}$ is obtained, $\tilde{\phi}_{k,j+1}$'s are calculated. Then using Eqns. (1-116) and (1-117), $\phi_{k,j+1}$'s are calculated. The variable Λ_{j+1} can be obtained by Eqn. (1-91). Another way to calculate Λ at any time is using Eqn. (1-109) in which Simpson's rule of integration gives, (Note that $\Delta Y=1/N$)

$$\Lambda_j = 3N(1-E_0)\Big/\Big[\underset{k=0}{(1-\varepsilon)_{0,j}} + 4(1-\varepsilon)_{1,j} + 2(1-\varepsilon)_{2,j} + 4(1-\varepsilon)_{3,j} + \ldots + 2(1-\varepsilon)_{N-2,j} + 4(1-\varepsilon)_{N-1,j} + (1-\varepsilon)_{N,j}\Big] \tag{1-134}$$

$(1-\varepsilon)$ is known as a function of ϕ from Eqn. (1-47).

Again as discussed in (W, τ) coordinate formulation, two separate runs are made on the computer. In one run Eqn. (1-91) is used to calculate Λ at every time step and in the other run Eqn. (1-134) is used. Λ obtained by these two methods should check at every time step.

1.4.2 SECOND ORDER PARTIAL DIFFERENTIAL EQN. IN (x,t)

1.4.2.1 Equations describing the transient response

<u>response</u> : The integro-differential Eqn. (1-98) can be converted into a second order partial differential equation. Rearranging Eqn. (1-98),

$$\frac{1}{\mu\rho_s\alpha}\left(\frac{\partial p_s}{\partial x}\right)_t = -\int_0^x\left(\frac{\partial\varepsilon}{\partial t}\right)_x dx + (1-\varepsilon)\frac{dL}{dt} \tag{1-135}$$

Differentiating both sides of (1-135) with respect to x

$$-\frac{1}{\alpha^2}\left(\frac{\partial\alpha}{\partial x}\right)_t \frac{1}{\mu\rho_s}\left(\frac{\partial p_s}{\partial x}\right)_t + \frac{1}{\mu\rho_s\alpha}\left(\frac{\partial^2 p_s}{\partial x^2}\right)_t = -\left(\frac{\partial\varepsilon}{\partial t}\right)_x - \left(\frac{\partial\varepsilon}{\partial x}\right)_t \frac{dL}{dt} \tag{1-136}$$

It is necessary to convert Eqn. (1-136) into a dimensionless form. Using Eqn. (1-69), (1-136) can be written as

$$-\frac{1}{\alpha^2}\left(\frac{d\alpha}{d\phi}\right)\left(\frac{\partial\phi}{\partial X}\right)_\tau \frac{dX}{dx}\frac{1}{\mu\rho_s}\frac{dp_s}{d\phi}\left(\frac{\partial\phi}{\partial X}\right)_\tau\frac{dX}{dx} + \frac{1}{\mu\rho_s\alpha}\frac{\partial}{\partial X}\left[\frac{dp_s}{d\phi}\left(\frac{\partial\phi}{\partial X}\right)_\tau\frac{dX}{dx}\right]\frac{dX}{dx}$$

$$= -\left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial\tau}\right)_X\frac{d\tau}{dt} - \left(\frac{d\varepsilon}{d\phi}\right)\left(\frac{\partial\phi}{\partial X}\right)_\tau\frac{dX}{dx}\frac{dL}{d\Lambda}\frac{d\Lambda}{d\tau}\frac{d\tau}{dt} \tag{1-137}$$

Substituting Eqns. (1-36), (1-37) and (1-38) in Eqn. (1-137) and rearranging

$$\frac{1}{(\alpha/\alpha_0)^2}\frac{d(\alpha/\alpha_0)}{d\phi}\left(\frac{\partial\phi}{\partial X}\right)_\tau^2 + \frac{1}{(\alpha/\alpha_0)}\left(\frac{\partial^2\phi}{\partial X^2}\right)_\tau = -\left(\frac{d\varepsilon}{d\phi}\right)\left[\left(\frac{\partial\phi}{\partial\tau}\right)_X + \left(\frac{\partial\phi}{\partial X}\right)_\tau\frac{d\Lambda}{d\tau}\right]\cdot\left(\frac{\mu\alpha_0^2}{p_1}\right) \tag{1-138}$$

Using Eqn. (1-43) in (1-138),

$$\frac{1}{(\alpha/\alpha_0)^2}\frac{d(\alpha/\alpha_0)}{d\phi}\left(\frac{\partial\phi}{\partial X}\right)_\tau^2 + \frac{1}{(\alpha/\alpha_0)}\left(\frac{\partial^2\phi}{\partial X^2}\right)_\tau = \frac{-B}{(1-E_0)}\left(\frac{d\varepsilon}{d\phi}\right)\left[\left(\frac{\partial\phi}{\partial\tau}\right)_X + \left(\frac{\partial\phi}{\partial X}\right)_\tau\frac{d\Lambda}{d\tau}\right] \tag{1-139}$$

This is the required equation in dimensionless form. The initial and boundary conditions are the same as described by

Eqns. (1-103) and (1-105). Differentiating Eqn. (1-46) with respect to ϕ,

$$\frac{d(\alpha/\alpha_0)}{d\phi} = A_1p_1 + 2A_2p_1^2\phi + 3A_3p_1^3\phi^2 + 4A_4p_1^4\phi^3 \tag{1-140}$$

Eqn. (1-140) along with (1-46) and (1-49) can be substituted in Eqn. (1-139), so that ϕ and Λ will be the only dependent variables.

1.4.2.2 <u>Formulation of difference equations</u> : As in case of the integro- differential equation, subscript indices k and j are used for X and τ coordinates. The following formula is necessary in addition to Eqns. (1-118) through (1-121).

$$\left(\frac{\partial^2\phi}{\partial X^2}\right)_{k,j} = \frac{\phi_{k+1,j} - 2\phi_{k,j} + \phi_{k-1,j}}{(\Delta X)^2} \tag{1-141}$$

Considering Eqn. (1-139) at point k,j and substituting Eqns. (1-118), (1-121) and (1-141) there results

$$\frac{1}{(\alpha/\alpha_0)^2_{k,j}}\left(\frac{d(\alpha/\alpha_0)}{d\phi}\right)_{k,j}\left(\frac{\phi_{k+1,j} - \phi_{k-1,j}}{2\,\Delta X}\right)^2 + \frac{1}{(\alpha/\alpha_0)_{k,j}} \cdot$$

$$\frac{(\phi_{k+1,j} - 2\phi_{k,j} + \phi_{k-1,j})}{(\Delta X)^2} = \frac{-B}{(1-E_0)}\left(\frac{d\varepsilon}{d\phi}\right)_{k,j}\left[\frac{\tilde{\phi}_{k,j+1} - \phi_{k,j}}{\Delta\tau} + \right.$$

$$\left. + \frac{\phi_{k+1,j} - \phi_{k-1,j}}{2\,\Delta X}\left(\frac{d\Lambda}{d\tau}\right)_j\right] \tag{1-142}$$

Solving for $\tilde{\phi}_{k,j+1}$ yields,

$$\tilde{\phi}_{k,j+1} = \phi_{k,j} - \frac{\Delta\tau}{(B/(1-E_0)(d\varepsilon/d\phi)_{k,j}} \left[\left(\frac{d(\alpha/\alpha_0)}{d\phi} \right)_{k,j} \cdot \right.$$

$$\left. \left\{ \frac{\phi_{k+1,j} - \phi_{k-1,j}}{2\Delta X(\alpha/\alpha_0)_{k,j}} \right\}^2 + \frac{\phi_{k+1,j} - 2\phi_{k,j} + \phi_{k-1,j}}{(\alpha/\alpha_0)_{k,j} \; (\Delta X)^2} \right]$$

$$- \frac{\Delta\tau}{2\Delta X} \left(\frac{d\Lambda}{d\tau} \right)_j (\phi_{k+1,j} - \phi_{k-1,j}) \qquad (1\text{-}143)$$

This equation is directly used in the computer program when $2 \leqslant k \leqslant N-1$.

Writing Eqn. (1-143) for k=1,

$$\tilde{\phi}_{1,j+1} = \phi_{1,j} - \frac{\Delta\tau}{(B/(1-E_0))(d\varepsilon/d\phi)_{1,j}} \left[\left(\frac{d(\alpha/\alpha_0)}{d\phi} \right)_{1,j} \cdot \right.$$

$$\left. \left\{ \frac{\phi_{2,j} - \phi_{0,j}}{2\Delta X(\alpha/\alpha_0)_{1,j}} \right\}^2 + \frac{\phi_{2,j} - 2\phi_{1,j} + \phi_{0,j}}{(\alpha/\alpha_0)_{1,j} \; (\Delta X)^2} \right]$$

$$- \frac{\Delta\tau}{2\Delta X} \left(\frac{d\Lambda}{d\tau} \right)_j (\phi_{2,j} - \phi_{0,j}) \qquad (1\text{-}144)$$

Eqns. (1-133) and (1-131) account for first and second boundary conditions. $\phi_{k,j+1}$'s can be obtained in a manner very similar to the solution of integrodifferential equation in X,τ coordinates and Λ at every time step can be checked by the two different methods.

1.5 COMPARISON WITH EXPERIMENTAL DATA

Haynes' (1968) experimental data were used to compare with theoretical calculations. It is necessary to analyze his method of measurements before attempting to check whether theoretical predictions agree with his data.

1.5.1 ANALYSIS OF HAYNES' EXPERIMENTAL DATA

He used a 2" inside diameter cylinder to hold the cake. The piston was inserted from top and weight was applied on top of the piston, at instant t=0. Schematic force balance diagram for the piston and the weight is shown in Fig. (1-6). Quantity m is the total mass on the top including mass of the piston.

$$\text{Gravitational force} = m\,g$$
$$\text{Force of reaction by the cake} = p_1\,A$$
$$\text{Total downward force} = mg - p_1A$$

Quantity a is the downward acceleration of the piston.

$$m\,a = m\,g - p_1\,A \qquad (1\text{-}145)$$

or

$$p_1A = m\,(g - a) \qquad (1\text{-}146)$$

It is seen from Eqn. (1-146) that the effective force on solids is not mg but m(g-a).

1.5.2 TIME CORRECTION FOR EXPERIMENTAL DATA

As the piston starts moving downwards, it accelerates, reaches a maximum downward velocity and then deccelerates as the compaction continues further. For a typical experimental L Vs t curve shown in Fig. (1-7) it can be seen that the maximum velocity is reached at M_V. In the analysis of expression operation a constant p_1A has been assumed. Therefore in order to compare theoretical predictions with the experimental data, a tangent to the curve at M_V is projected backwards until it reaches $L=L_1$. The time coordinate

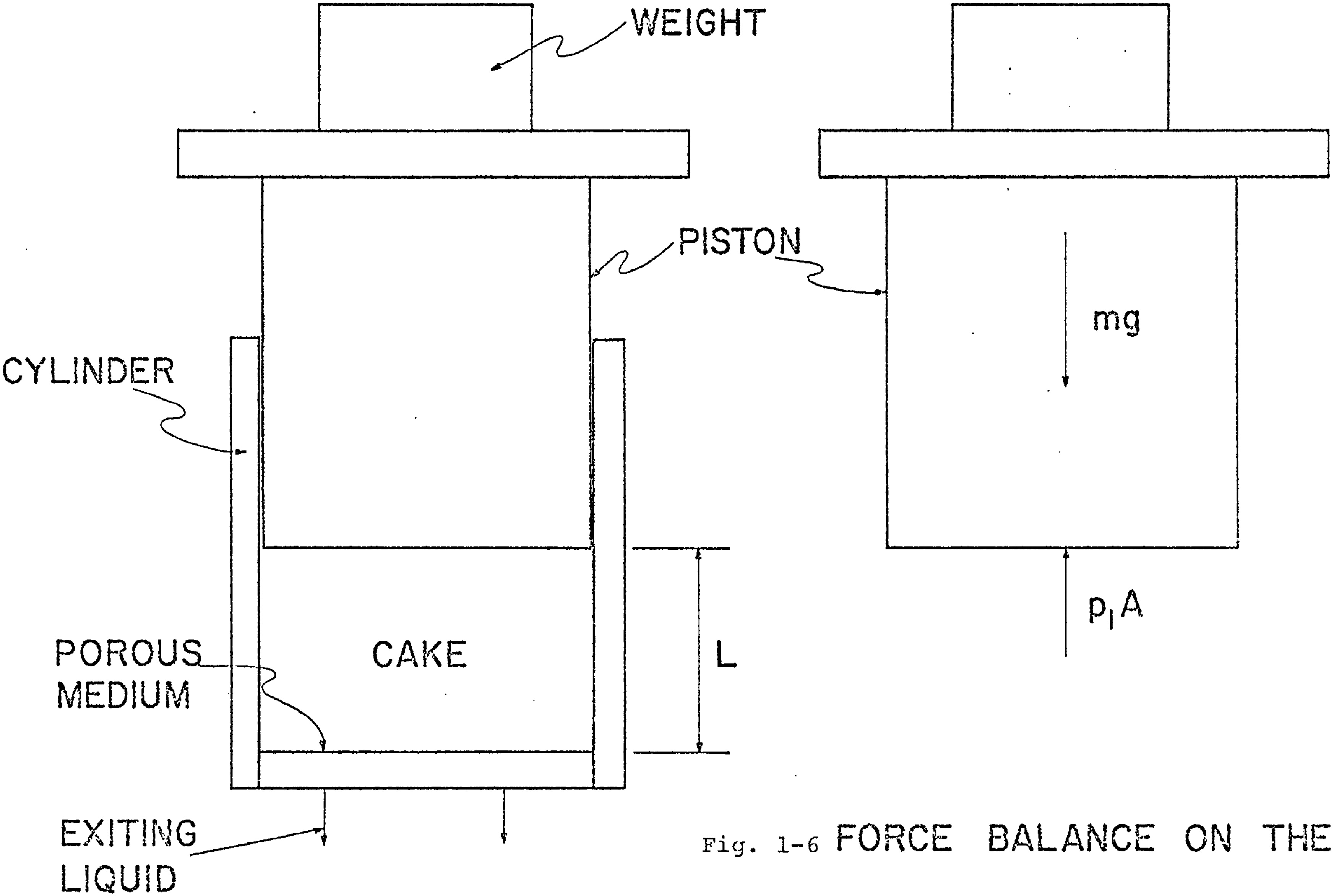


Fig. 1-6 FORCE BALANCE ON THE PISTON

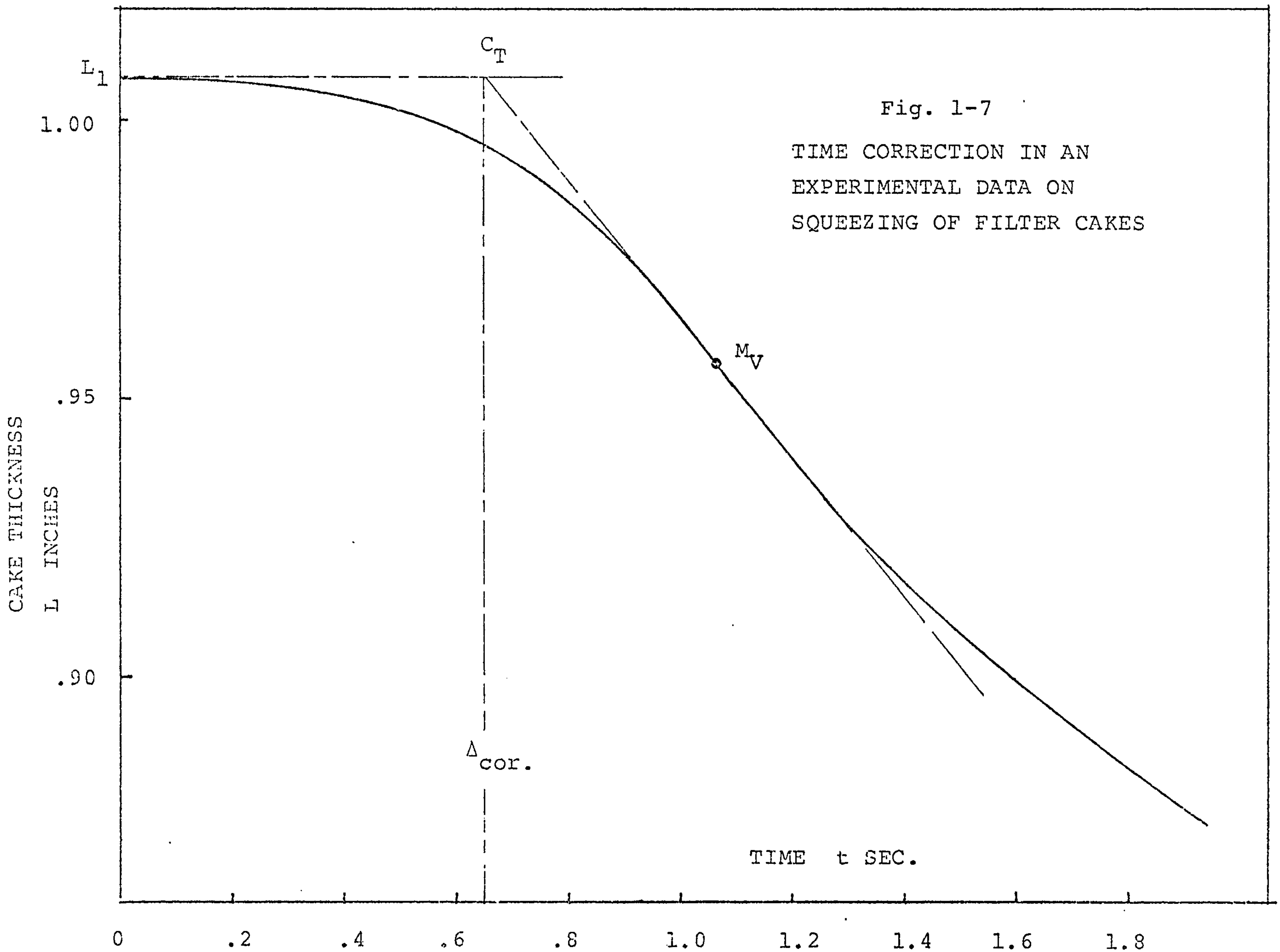


Fig. 1-7

TIME CORRECTION IN AN EXPERIMENTAL DATA ON SQUEEZING OF FILTER CAKES of this point C_T is $\Delta_{cor.}$ and is taken as the starting point for squeezing. Time t' has to be corrected according to the equation

$$t = t' - \Delta_{cor.} \qquad (1\text{-}147)$$

In Fig. (1-7) $\Delta_{cor.}$ is 0.65 sec.

1.5.3 CALCULATION OF DIMENSIONLESS QUANTITIES

Drainage is on one side only, therefore L_1 to be used in the numerical scheme is the same as experimentally observed cake thickness before squeezing starts. Dimensionless variables τ and Λ are calculated as shown in Table 1-1. Also a new dimensionless variable U defined by Shirato et al (1967) as follows, is calculated.

$$U \equiv (L_1 - L)/(L_1 - L_\infty) \qquad (1\text{-}148)$$

U is 0 at the start and 1 at the end of squeezing, regardless of the pressure applied or the material that is squeezed. Substituting Eqn. (1-37) in (1-148),

$$U = (1-\Lambda)/(1-\Lambda_\infty) \qquad (1\text{-}149)$$

There is considerable wall friction in 2" diameter cell for 1" initial cake thickness, and L_∞ obtained is much larger than that corresponding to p_1. To overcome this problem, average equilibrium porosity ε_{p_1} is calculated as follows, and effective p_1 corresponding to ε_{p_1} is used for theoretical calculations. Solving Eqn. (1-68) for ε_{p_1} yields

TABLE 1-1

Calculation of Λ, U and τ for an experimental data on squeezing of filter cakes

Solids: Solka floc BW-200; Liquid: Water at 75°F;
(Source: p 223, Haynes' Ph.D. thesis, University of Houston, 1968)

L_1 = 1.065 inch = 0.08658 ft.;

Applied mechanical pressure = 100. p.s.i.

$\mu = 6.72 \times 10^{-4}$ lbm/(ft.sec) ; $\rho_s = 95.8$ lbm/ft^3

$$\tau = \frac{t}{\rho_s L_1^2} = \frac{6.72 \times 10^{-4} \times t}{95.8 \times (.08658)^2} = 0.927 \times 10^{-3} \times t$$

$$\varepsilon_{p_1} = 1 - \frac{1 - E_0}{\Lambda_\infty} = 1 - \frac{1 - .833}{.557} = 0.700$$

Effective p_1 = 49.5 p.s.i.

t sec. (corrected)	τ	L inch	$\Lambda = \frac{L}{L_1}$	$U = \frac{L_1 - L}{L_1 - L_\infty}$
0	0	1.065	1.000	0
0.15	$0.139(10^{-3})$	1.030	0.968	0.074
0.35	0.325	0.988	0.929	0.164
0.55	0.510	0.950	0.894	0.244
0.75	0.695	0.920	0.865	0.308
1.15	1.066	0.870	0.818	0.414
1.55	1.437	0.830	0.780	0.499
2.35	2.178	0.772	0.725	0.622
3.35	3.105	0.722	0.679	0.728
4.35	4.032	0.686	0.645	0.805
6.35	5.886	0.650	0.610	0.881
9.35	8.667	0.635	0.597	0.913
14.35	13.302	0.623	0.585	0.938
19.35	17.937	0.619	0.582	0.947
296.35	-	0.594	0.557	1.000

$$\varepsilon_{p_1} = 1 - (1-E_0)/\Lambda_\infty \qquad (1\text{-}150)$$

For the data shown in Table(1-1), applied pressure is 100 p.s.i. but effective p_1 corresponding to $\varepsilon_{p_1} = 0.7$ is only 49.5 p.s.i.

Computer printout for Solka floc BW-200 is displayed in Table (1-2). Parameter C was calculated to be 1.0 from the knowledge of R_m (1.6×10^{10} /ft) in Haynes' apparatus. However, the R_m value was not reported in his thesis. Table (1-3) describes the time correction $\Delta\tau_{cor.}$ for computer printout due to the initial ϕ profile chosen. Also U is calculated at various values of τ.

1.6 COMPARISON BETWEEN SQUEEZING TIMES FOR TWO DIFFERNET SOLIDS UNDER IDENTICAL CONDITIONS

Independent variable τ contains ρ_s which would be different for two solids. Therefore it is necessary to use τS_g as a variable representing time, where S_g is the specific gravity of solids. The density of water at 4°C is denoted by ρ_0. From Eqn. (1-38) it is evident that,

$$\tau S_g = \mu t/(\rho_0 L_1^2) \qquad (1\text{-}151)$$

The parameter containing R_m is C. Because of the particular definition of C, its value is different for two different solids even if R_m is identical. Combining Eqn. (1-35) and (1-55),

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHAO=0.1200E 11 A1=0.2886E-01 A2=0.4122E-03 A3=-.2468E-06 E0=0.8333E 00 E1=-.4730E-02
E2=0.5195E-04 E3=-.2238E-06 KMAX=40000 MU=0.6720E-03 P1= 49.5 C=0.1000E 01
DELT=0.2000E-06 N= 40 PHIGH= 1.00 PLOW= 0.10

TABULATION OF DIMENSIONLESS QUANTITIES:

Table 1-2

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.0	.98890	.02020	.01675	.01366	.01093	.00857	.00657	.00493	.00366	.00275	.00220	.00202
.39998E-04	.97959	.11786	.04542	.01914	.01175	.00885	.00682	.00517	.00384	.00261	.00122	.00018
.79996E-04	.97084	.15942	.07788	.03545	.01726	.01028	.00725	.00538	.00391	.00261	.00150	.00084
.11999E-03	.96245	.19122	.10539	.05384	.02673	.01413	.00856	.00582	.00409	.00279	.00181	.00125
.15999E-03	.95437	.21784	.12942	.07194	.03804	.02001	.01114	.00683	.00451	.00307	.00211	.00160
.19999E-03	.94653	.24106	.15090	.08924	.05009	.02725	.01493	.00859	.00531	.00350	.00245	.00193
.23999E-03	.93892	.26181	.17043	.10563	.06232	.03534	.01968	.01111	.00657	.00416	.00288	.00231
.27994E-03	.93151	.28062	.18837	.12115	.07447	.04393	.02517	.01432	.00832	.00511	.00347	.00281
.31990E-03	.92428	.29789	.20499	.13585	.08641	.05281	.03121	.01810	.01054	.00638	.00427	.00348
.35985E-03	.91722	.31385	.22050	.14982	.09807	.06181	.03764	.02237	.01320	.00801	.00534	.00438
.39980E-03	.91032	.32872	.23505	.16310	.10941	.07084	.04435	.02703	.01627	.01000	.00671	.00555
.43976E-03	.90356	.34262	.24875	.17576	.12041	.07983	.05124	.03201	.01971	.01233	.00840	.00704
.47971E-03	.89695	.35569	.26170	.18785	.13109	.08873	.05826	.03725	.02347	.01500	.01042	.00884
.51967E-03	.89045	.36801	.27398	.19943	.14144	.09752	.06535	.04271	.02752	.01800	.01277	.01099
.55962E-03	.88409	.37967	.28566	.21052	.15148	.10618	.07249	.04834	.03184	.02131	.01546	.01345
.59957E-03	.87783	.39074	.29679	.22117	.16121	.11470	.07963	.05412	.03640	.02492	.01846	.01627
.63953E-03	.87169	.40126	.30743	.23142	.17067	.12309	.08679	.06002	.04117	.02880	.02179	.01940
.67948E-03	.86565	.41130	.31762	.24129	.17987	.13133	.09393	.06603	.04615	.03295	.02541	.02285
.71943E-03	.85971	.42090	.32740	.25083	.18881	.13944	.10105	.07213	.05131	.03736	.02932	.02659
.75939E-03	.85387	.43009	.33680	.26005	.19753	.14743	.10816	.07832	.05663	.04199	.03351	.03063
.79934E-03	.84811	.43892	.34587	.26898	.20605	.15530	.11525	.08458	.06212	.04685	.03797	.03495
.83930E-03	.84245	.44742	.35463	.27766	.21436	.16306	.12232	.09091	.06776	.05193	.04267	.03952
.87925E-03	.83687	.45562	.36311	.28610	.22251	.17072	.12937	.09732	.07354	.05720	.04761	.04435
.91920E-03	.83137	.46354	.37134	.29432	.23050	.17829	.13641	.10378	.07946	.06266	.05277	.04942
.95916E-03	.82595	.47122	.37933	.30236	.23835	.18579	.14345	.11031	.08550	.06829	.05814	.05469
.99911E-03	.82061	.47868	.38713	.31022	.24607	.19322	.15049	.11690	.09166	.07410	.06371	.06018
.10391E-02	.81535	.48594	.39474	.31793	.25369	.20059	.15752	.12356	.09794	.08007	.06947	.06587
.10790E-02	.81015	.49302	.40218	.32550	.26121	.20793	.16457	.13028	.10434	.08619	.07541	.07175
.11190E-02	.80503	.49994	.40948	.33296	.26865	.21522	.17163	.13705	.11084	.09245	.08152	.07781
.11589E-02	.79998	.50671	.41665	.34031	.27601	.22248	.17870	.14389	.11744	.09885	.08779	.08404
.11989E-02	.79499	.51337	.42371	.34757	.28332	.22972	.18579	.15080	.12414	.10539	.09421	.09041

TABLE 1-2 (CONTD.) SOLIDS PRESSURES PHI(I)

TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.12388E-02	.79007	.51991	.43067	.35475	.29058	.23694	.19291	.15776	.13095	.11205	.10077	.09694
.12788E-02	.78522	.52635	.43754	.36187	.29781	.24417	.20006	.16479	.13785	.11883	.10748	.10363
.13187E-02	.78044	.53270	.44434	.36894	.30500	.25139	.20723	.17188	.14483	.12573	.11431	.11044
.13587E-02	.77571	.53897	.45107	.37595	.31217	.25861	.21444	.17904	.15192	.13274	.12127	.11739
.13986E-02	.77105	.54518	.45775	.38293	.31932	.26584	.22169	.18625	.15908	.13985	.12835	.12446
.14386E-02	.76646	.55133	.46438	.38989	.32647	.27310	.22898	.19354	.16634	.14707	.13554	.13163
.14786E-02	.76192	.55743	.47098	.39682	.33361	.28037	.23631	.20088	.17367	.15439	.14285	.13893
.15185E-02	.75745	.56348	.47754	.40373	.34076	.28765	.24368	.20829	.18108	.16180	.15025	.14635
.15585E-02	.75304	.56950	.48408	.41063	.34791	.29497	.25110	.21576	.18858	.16931	.15776	.15385
.15984E-02	.74869	.57548	.49059	.41753	.35508	.30232	.25856	.22329	.19615	.17690	.16536	.16146
.16384E-02	.74440	.58143	.49709	.42442	.36225	.30969	.26606	.23088	.20380	.18458	.17306	.16916
.16783E-02	.74017	.58735	.50358	.43132	.36945	.31710	.27362	.23854	.21152	.19234	.18084	.17695
.17183E-02	.73600	.59326	.51005	.43822	.37666	.32454	.28122	.24626	.21932	.20018	.18871	.18483
.17582E-02	.73189	.59914	.51652	.44513	.38389	.33201	.28887	.25403	.22718	.20810	.19667	.19280
.17982E-02	.72784	.60500	.52299	.45204	.39115	.33952	.29657	.26186	.23511	.21610	.20471	.20087
.18381E-02	.72385	.61084	.52945	.45897	.39843	.34706	.30431	.26976	.24311	.22417	.21282	.20899
.18781E-02	.71992	.61667	.53590	.46591	.40573	.35464	.31210	.27770	.25117	.23231	.22101	.21719
.19180E-02	.71605	.62249	.54236	.47285	.41305	.36226	.31994	.28571	.25929	.24052	.22926	.22547
.19580E-02	.71224	.62829	.54881	.47981	.42040	.36991	.32782	.29376	.26748	.24879	.23759	.23382
.19980E-02	.70848	.63407	.55527	.48678	.42777	.37759	.33574	.30187	.27572	.25713	.24599	.24224
.24974E-02	.66646	.70490	.63554	.57454	.52147	.47601	.43788	.40689	.38290	.36581	.35557	.35214
.29968E-02	.63318	.77113	.71283	.66092	.61529	.57586	.54257	.51538	.49425	.47917	.47011	.46709
.34962E-02	.60785	.82976	.78325	.74132	.70407	.67159	.64398	.62129	.60359	.59091	.58329	.58075
.39946E-02	.58937	.87857	.84344	.81140	.78267	.75741	.73579	.71792	.70392	.69386	.68781	.68578
.44882E-02	.57642	.91674	.89156	.86839	.84744	.82889	.81293	.79967	.78924	.78173	.77720	.77569
.49818E-02	.56766	.94484	.92761	.91164	.89712	.88420	.87302	.86371	.85636	.85106	.84786	.84679
.54754E-02	.56191	.96445	.95309	.94251	.93285	.92422	.91673	.91047	.90553	.90195	.89979	.89906
.59690E-02	.55822	.97754	.97025	.96345	.95721	.95163	.94677	.94271	.93949	.93716	.93575	.93528
.64626E-02	.55588	.98600	.98142	.97713	.97319	.96966	.96658	.96400	.96196	.96048	.95958	.95927
.69562E-02	.55440	.99136	.98851	.98584	.98338	.98118	.97926	.97764	.97636	.97544	.97487	.97468
.74498E-02	.55347	.99469	.99293	.99128	.98977	.98841	.98722	.98622	.98542	.98485	.98450	.98438
.79434E-02	.55287	.99675	.99567	.99466	.99372	.99288	.99215	.99154	.99105	.99069	.99048	.99040

TABLE 1-3

Time correction for results of the numerical scheme :

Solids material: Solka floc BW-200 ; Liquid: Water at 75 °F

p_1 = 49.5 p.s.i. ; C = 1.

Λ_∞ = .553 $1 - \Lambda_\infty = 0.447$

$$\frac{d\Lambda}{d\tau_\tau} = -\frac{0.9889 - 0.9796}{4 \times 10^{-5}} = 2.325 \times 10^{2}$$

(refer to the computer printout in Table 1-3)

$$\Delta\tau_{cor.} = \frac{1 - 0.9889}{2.325 \times 10^{2}} = 0.04774 \times 10^{-3}$$

$$\tau = \tau' + \Delta\tau_{cor.}$$

τ' (10^{-3})	τ (10^{-3}) corrected	Λ	$U = \frac{1 - \Lambda}{1 - \Lambda_\infty}$
0.1999	0.2477	0.94653	0.1196
0.3998	0.4475	0.91032	0.2006
0.9991	1.0469	0.82061	0.4013
1.3986	1.4463	0.77105	0.5122
1.9980	2.0457	0.70848	0.6522
2.9968	3.0445	0.63318	0.8206
3.9946	4.0423	0.58937	0.9186
4.9818	5.0295	0.56766	0.9672
5.9213	5.9690	0.55822	0.9883
7.8957	7.9434	0.55287	1.0000

$$C = \frac{\alpha_0 L_1 (1-E_0) \rho_s}{R_m} \qquad (1\text{-}152)$$

C values for two different solids when R_m is the same are denoted by C_1 and C_2. Utilizing Eqn. (1-152) and noting that $S_g = \rho_s/\rho_0$

$$\frac{C_2}{C_1} = \frac{(\alpha_0 (1-E_0) S_g)_2}{(\alpha_0 (1-E_0) S_g)_1} \qquad (1\text{-}153)$$

Values of Λ_∞ for two different solids would be different and comparison between Λ's at different τS_g values would be meaningless. However U varies from 0 to 1 during the squeezing process for every solid material and a comparison between the U values is useful. Following example illustrates the comparison between two solids.

Liquid : Water at 75 °F

Solids 1 (Talc) : $\alpha_0 = 5.8 \times 10^{10}$ ft/lbm, E_0=0.858, S_g=2.7

Solids 2 (Solka floc BW-200) : $\alpha_0 = 1.2 \times 10^{10}$ ft/lbm, E_0 =0.833, S_g= 1.52

The comparison is to be made for $C_1 = 1$. Applying Eqn. (1-108) to evaluate C_2 ,

$$C_2 = 1 \times \frac{1.2 \times 10^{10} \times (1-0.833) \times 1.52}{5.8 \times 10^{10} \times (1-0.858) \times 2.7} = 0.14$$

The computer printouts on this example can be found in the appendix. Using these results, and after correcting τ values as discussed before, τS_g and (1-U) values are calculated and plotted in Fig. (1-8).

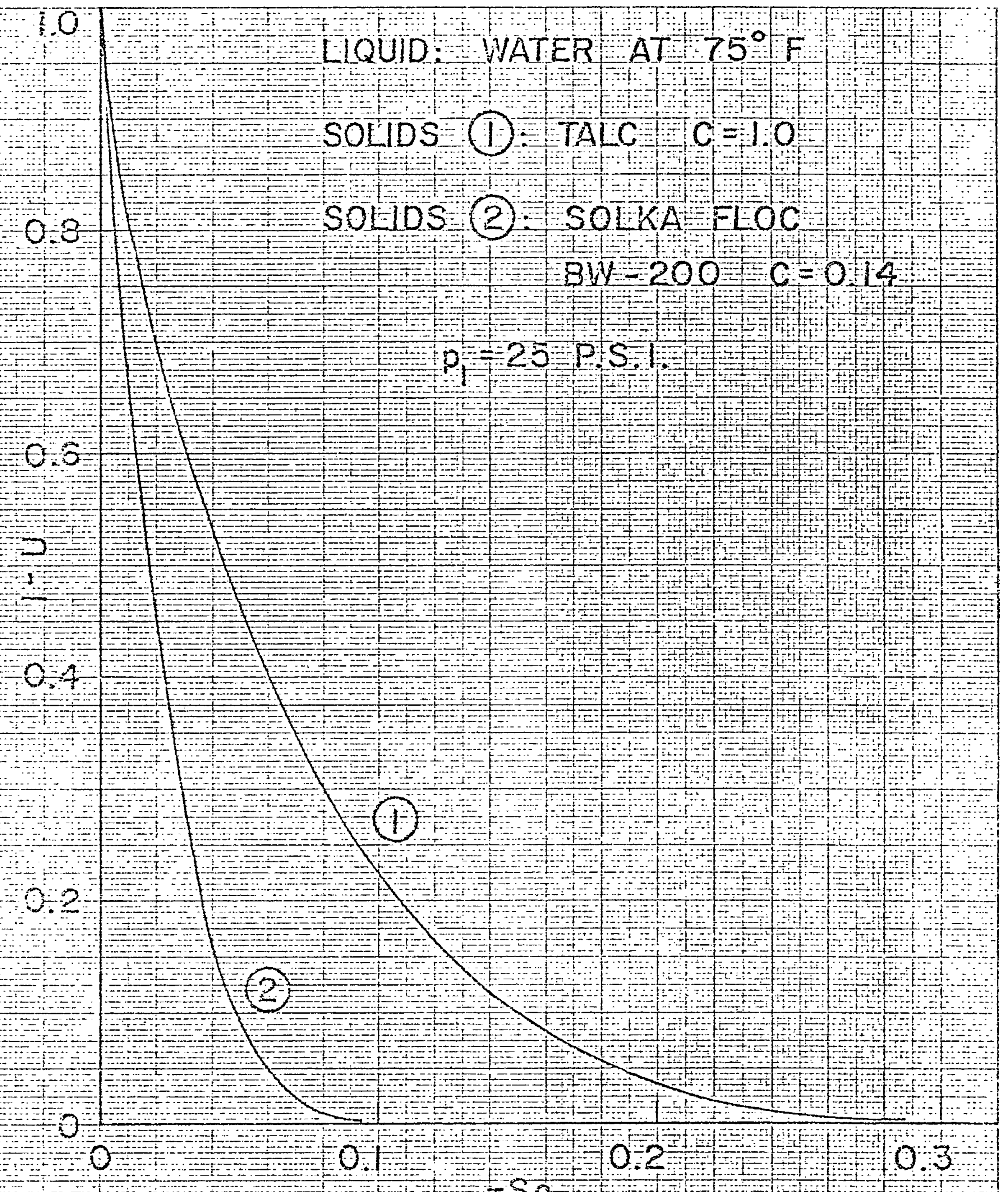


Fig. 1-8 Comparison between squeezing times for Solka floc BW-200 and Talc

1.7 DISCUSSION OF THE RESULTS AND CONCLUSIONS.

Solution to the problem in W,τ coordinates is the best of three different methods attempted. Computer time was the least for this method and also the program worked for as wide ranges of p_1 and C as in other two methods. As parameters p_1 and C increase, computer time increases because smaller ΔW and $\Delta\tau$ have to be used to keep the system stable. For C values above 30, medium resistance R_m is negligible in comparison with the cake resistance; and under these circumstances a quite elegant solution has already been obtained by Shirato et al (1971). For C values of 1 or below, R_m is high and in this situation the solution in W,τ coordinates is fast enough to reach within 1% of final equilibrium on an IBM 360 system in less than 40 minutes for p_1 as high as 100 p.s.i. Therefore it is necessary to make attempts to find a faster solution in case of moderate values of C.

It was found that by stopping the solution after a reasonable number (10,000 for example) of time steps, feeding the ϕ profile obtained to the next computer solution with increased ΔW and $\Delta\tau$, and by repeating this process computer time can be saved.

Computer programs were run for easily filtrable material Solka floc BW-200 and moderately resistant material Talc. A typical computer printout is shown in Table 1-2. Other printouts are displayed in the appendix.

For each material, the important parameters that need to be varied are p_1 and C. Initial cake thickness L_1 is not an independent parameter as it appears in dimensionless time τ, as can be seen from Eqn. (1-38). Variable τ is inversely proportional to the square of L_1. If τ is known, t can be calculated for any L_1.

Profiles ϕ Vs X and ϕ Vs W for Solka floc are plotted in Fig. (1-9) and (1-10) respectively, for various values of corrected τ. These are the results of computerized calculations in (X,τ) and (W,τ) coordinates respectively. It is seen that initially the profiles rise at a rapid rate and later their rate of advance slows down as equilibrium is approached. The ϕ Vs W profiles in Fig. (1-10) when converted to ϕ Vs X profiles using Eqn. (1-72), agreed with ϕ Vs X profiles in Fig. (1-9)

It can be seen from Fig. (1-11) that Λ Vs τ curve varies a lot with change in C.For C value below 20, medium resistance R_m is important. As C is increased above 20, R_m gradually becomes negligible in comparison with the total cake resistance.

In Figures (1-12) and (1-13) Λ Vs τ and (1-U) Vs τ for computer calculations as well as experimental data on Solka floc BW-200 are plotted respectively. In later periods of squeezing, theoretically calculated curves tend to approach the final equilibrium faster than the experimental results. The same trend was observed by Shirato et al (1971). This is probably because the so called secondary consolidation (Matsumoto and Takashima 1969) is neglected in the derivation.

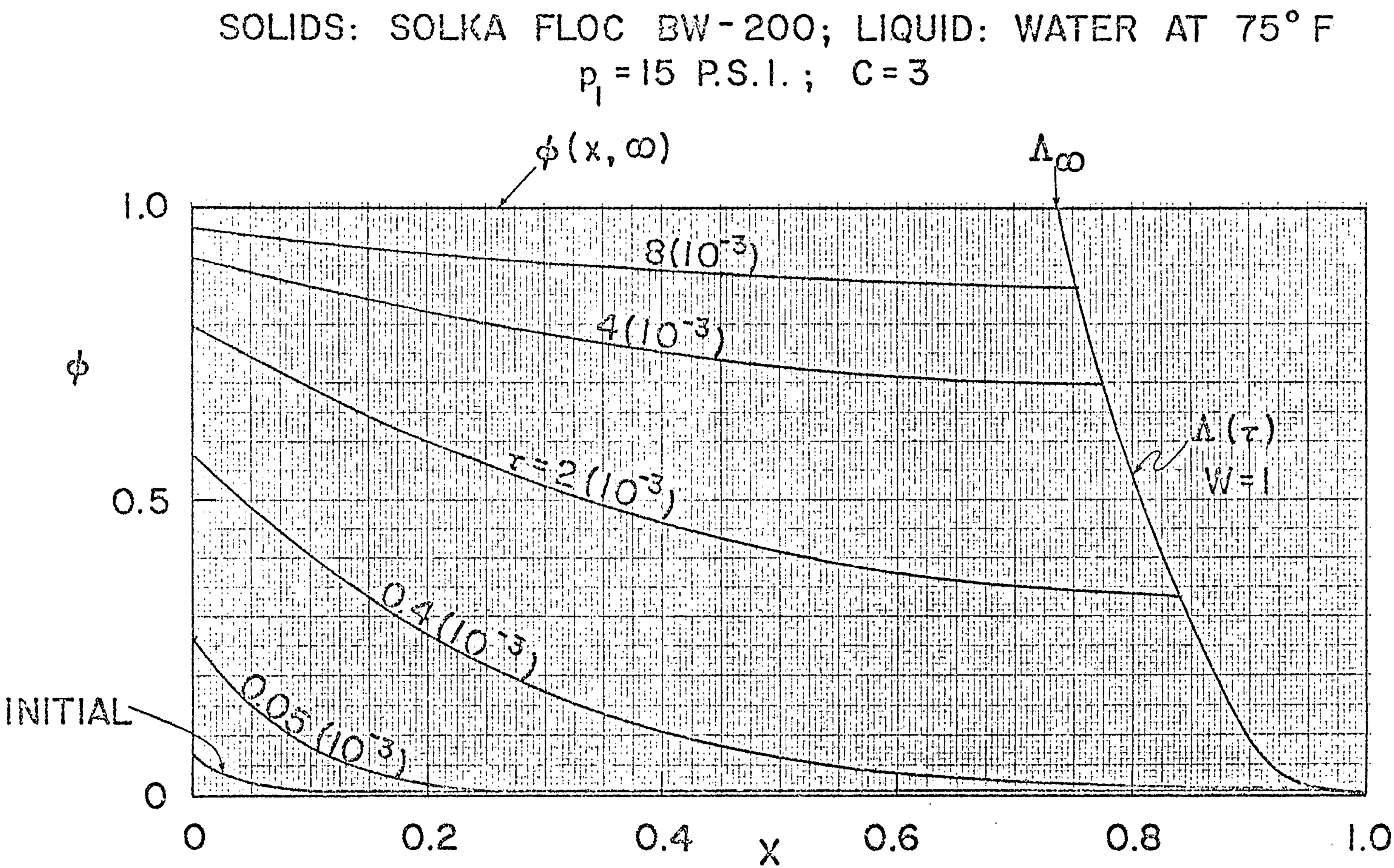


Fig. 1-9 ϕ Vs X for Solka floc BW-200

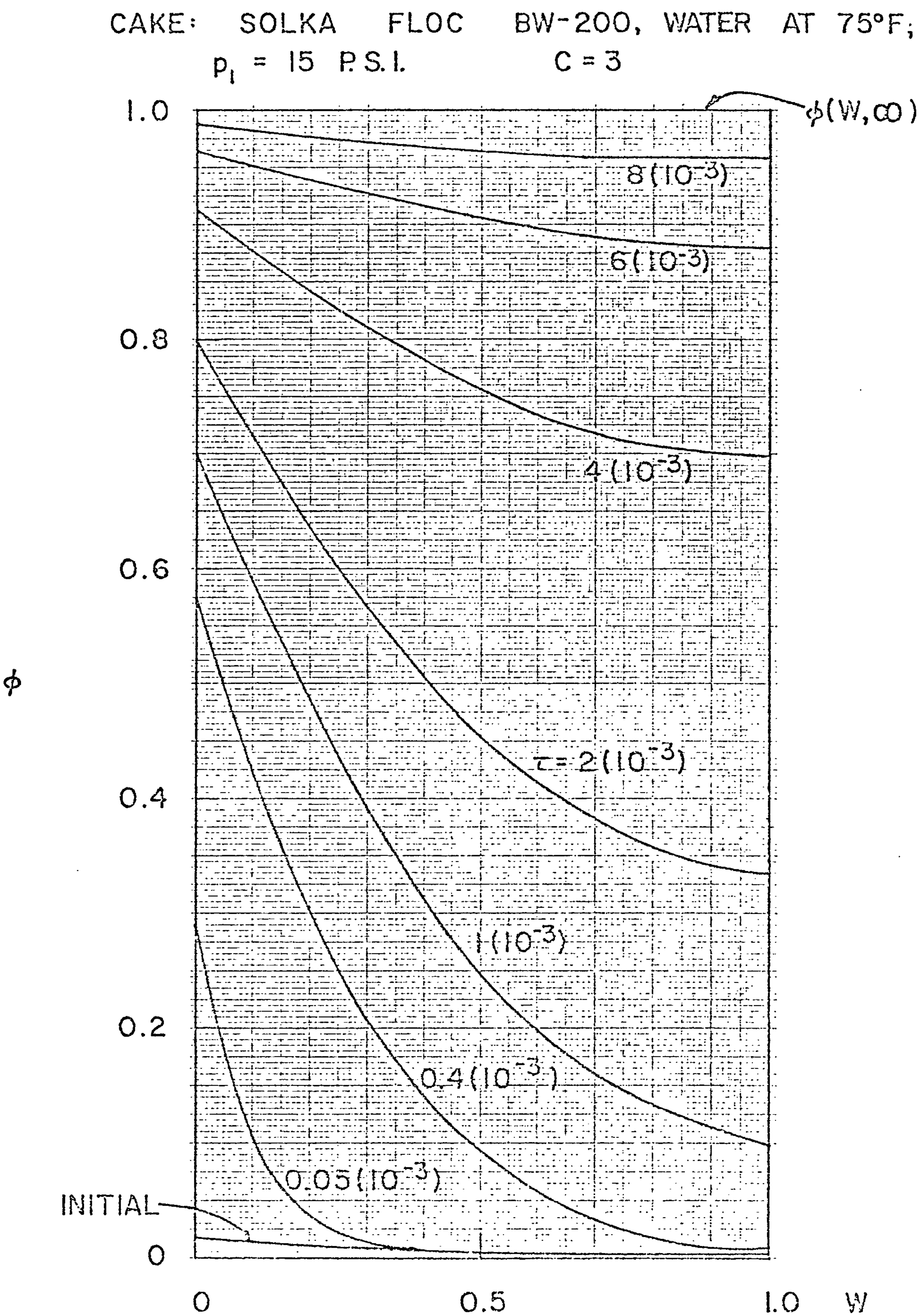


Fig. 1-10 φ Vs W for Solka floc BW-200

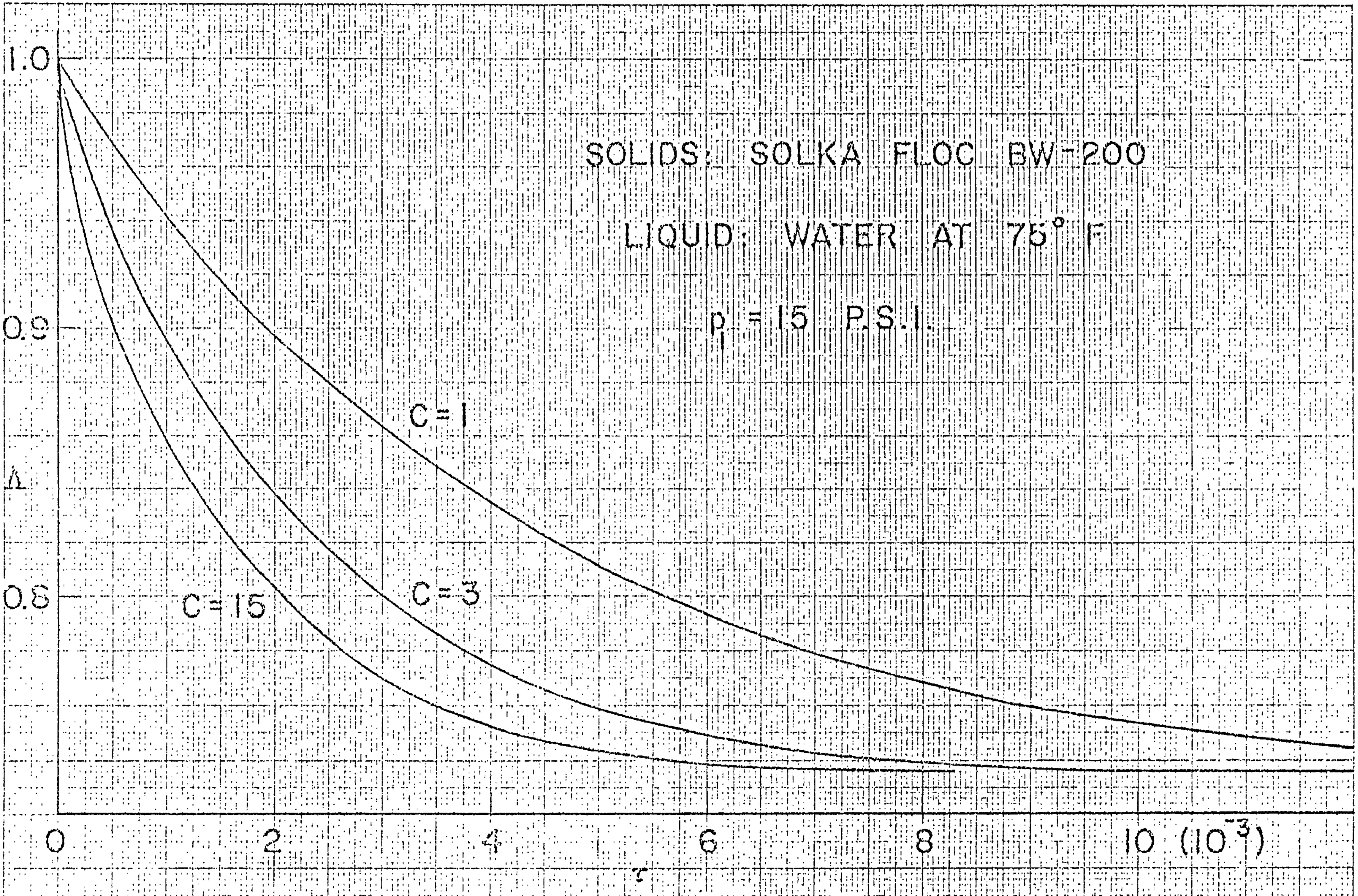


Fig. 1-11 ϕ Vs τ with C as a parameter for Solka floc BW-200

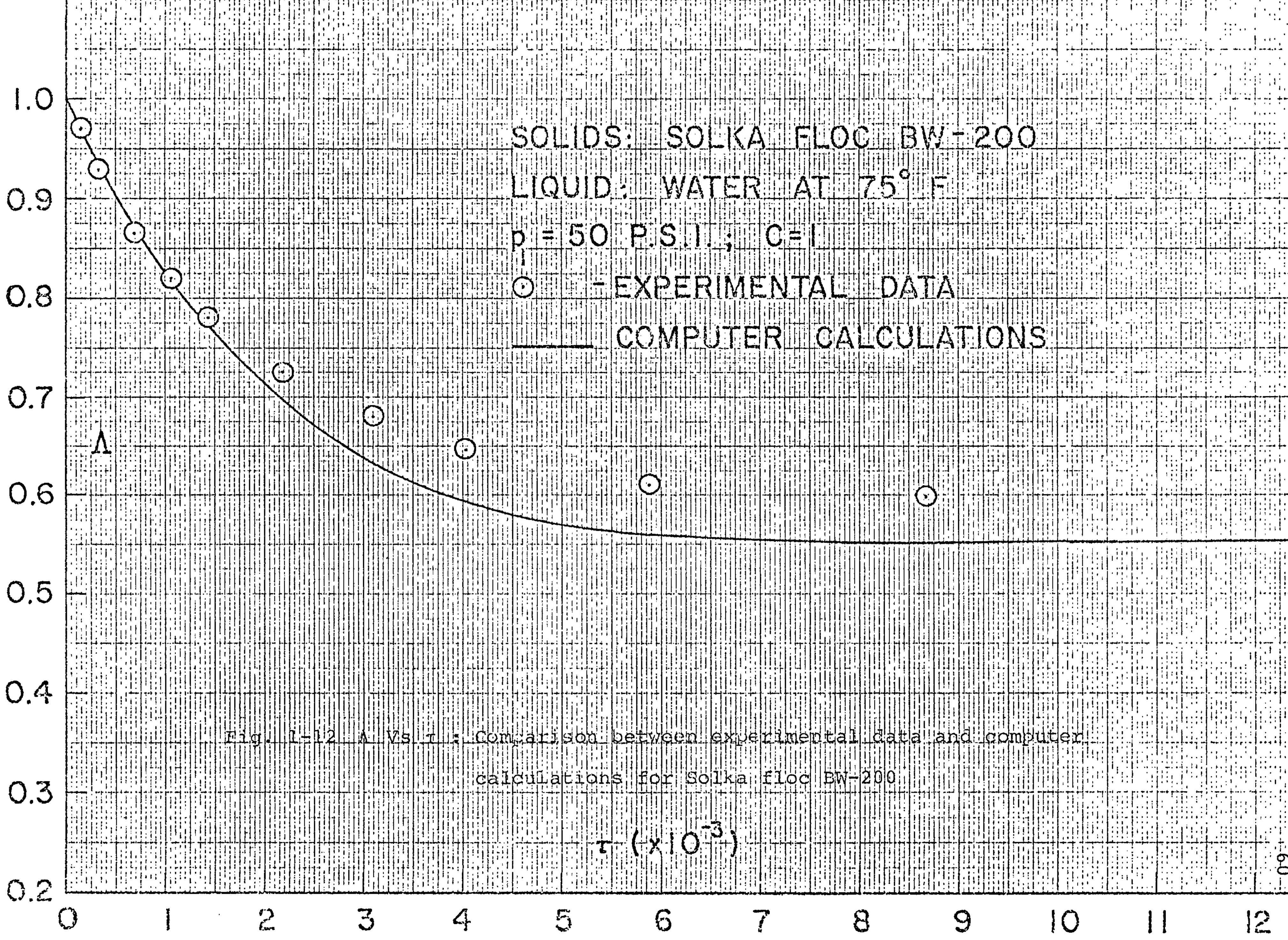


Fig. 1-12 Λ Vs τ : Comparison between experimental data and computer calculations for Solka floc BW-200

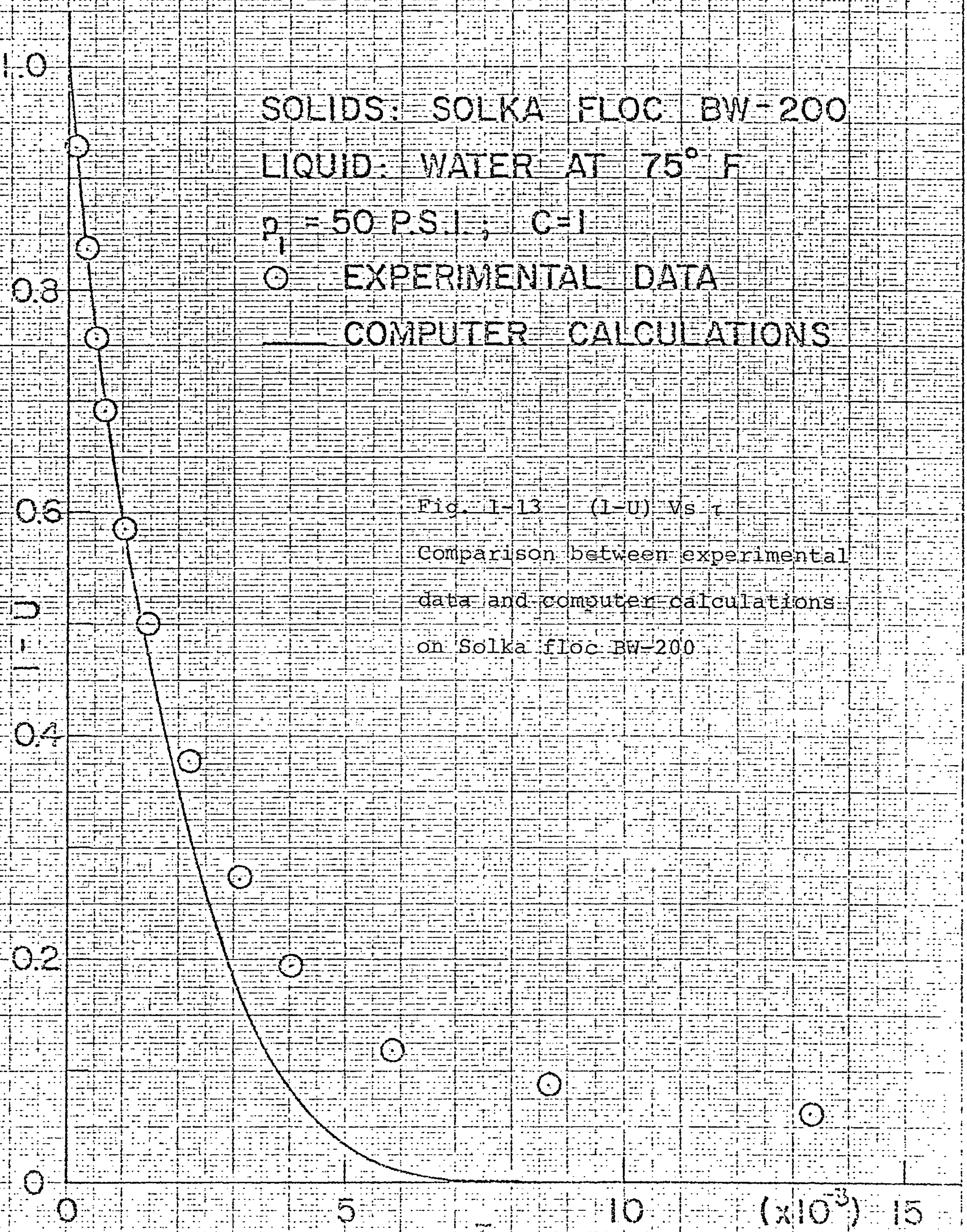


Fig. 1-13 (1-U) Vs τ
Comparison between experimental data and computer calculations on Solka floc BW-200

Time required for squeezing a moderately resistant material Talc, to a certain extent is nearly three times that for easily filtrable material Solka floc BW-200 under identical conditions as can be seen from Fig. (1-8).

The apparatus used by Haynes (1966) to obtain experimental data on squeezing, is not recommended. In this device considerable wall friction is present and also gravitational method of loading does not yield constant pressure in initial part of squeezing as discussed under "Analysis of Haynes' experimental data".

1.8 RECOMMENDATIONS FOR THE FUTURE WORK

In order to obtain better experimental data for the purpose of comparing with analytical solutions on squeezing, it is necessary to develop an apparatus in which wall friction would be negligible and applied pressure will be constant, right from the beginning. Probably a cell with a large cross section and hydraulically operated diaphragm to squeeze the cake will fulfill these requirements. It is better to have drainage only on one side because to achieve a certain effec--tive cake thickness, the actual thickness required is half of what it would be if drainage were on both sides. Thus unnecessary wall friction due to additional cake thickness is eliminated.

It was realized at the end of the investigation that because of the particular way in which τ and C are defined, it is difficult to make comparisons between two different solids for squeezing time under identical conditions. It would have been better to define them as $\tau \equiv t/(\rho_0 L_1^2)$ and $C \equiv R_m/(p_1 \rho_0 L_1)$. S_g would appear in other dimensionless parameters. An additional parameter S_g will have to be specified in the computer programs but the comparison between two solids will be easier to make. Therefore it is recommended that these modified definitions of τ and C be tried in future investigations.

The numerical method should be studied further to obtain faster solutions. In the present scheme $\Delta\tau$ and ΔW have to be obtained by trying several different values in wide ranges. It would be more convenient if optimum $\Delta\tau$ and ΔW could be related to the system parameters, so that merely by punching three data cards, entire numerical solution could be obtained in one computer run.

The Crank-Nicoulson method should be tried because it is stable for any ratio of $\Delta\tau$ to ΔW and as the solution approaches equilibrium, $\Delta\tau$ can be increased continuously without having to increase ΔW every time.

The second order partial diffenential equation in W,τ coordinates should be attempted as is done in X,τ coordinates in this investigation.

In solution of the problem of this investigation, an assumption has been made that during sqeezing the initial acceleration period is short and its effects are negligible. This assumption should be checked by a thorough analysis.

CHAPTER 2

STRESS DISTRIBUTION AT THE BOUNDARIES OF A CAKE IN A COMPRESSION PERMEABILITY (CP) CELL

2.1 INTRODUCTION

Liquid flowing through a filter cake exerts drag on the particles. If the cake is compressible, drag forces deform the cake structure, (but not necessarily the particles) and the flow resistance is increased. The extent of deformation increases in the direction of flow because drag forces accumulate. Solid compressive pressure p_s at any point in a cake is defined as the total cummulative drag force at that cross section divided by the cross sectional area. Local values of porosity ε and specific cake resistance α are functions of p_s and these relationships can be obtained through use of the Compression -Permeability (CP) cell (Ruth, 1946). Average porosity and overall resistance can be calculated by integrating local values over the cake thickness.

A filter cake is mechanically compacted between two porous plates inside the cylinder of the CP cell. The bottom plate is stationary and the top plate is attached to a movable piston. It is assumed that the mechanical pressure applied via the piston as well as p_s resulting from drag forces in a filter cake produce the same effects. Liquid is flowed through the cake at a low pressure drop. Measurements of flowrate and pressure drop are used to calculate total flow resistance. If mechanical pressure inside a compacted cake were uniform, the measurements of the total resistance to flow and the total cake volume would yield the desired relationships of α Vs p_s and ε Vs p_s. Taylor (1942) indicated that the force transmitted to the bottom of a mechanically compacted particulate bed is less than the applied force from top. Since then investigators have raised questions regarding the validity of the conventional CP- cell testing. Knowledge of stresses inside a compacted filter cake is desirable in order to obtain correct relationships between α, ε and p_s. Barber (1972) made photoelastic studies which yield stress and strain inside a bed of large particles (diameter of about 1 cm.), but for a filter cake which contains fine particles (upto 200 microns dia.) there is no published technique available to measure internal stress distribution.

Stress distribution in a compacted filter cake could be calculated by treating the filter cake as a continuum. However one has to make assumptions about the relationships between shear stress ($\tau_{r,z}$), normal radial stress (σ_r) and normal vertical stress (σ_z). These relationships can be checked only at the cake boundaries with currently available experimental techniques and not inside the cake. The cake surface is always in contact with a container. Therefore the relationships between $\tau_{r,z}$, σ_r and σ_z at the cake surface would be different from those inside the cake. Also the filter cake consists of particles of diameter up to a few hundred microns and the continuum approach is questionable. A significant contribution to the particulate concept of forces in solid particle beds was presented by Rowe (1962) in his study of stress dilatancy relationship for an assembly of uniform sized, cohesionless, two dimensional particle system. Glastonbury and Bratel (1966) proposed a two-dimensional model for determination of pressures due to the weight of monosized cylindrical particles in regular packing. Experimental knowledge of stress distribution at the boundaries of a CP cell is the key to understanding the behaviour of the cake. Lu (1968) developed an apparatus to measure the stress distribution on the bottom and the side walls of a compacted filter cake of cylindrical geometry. In the side walls, he used movable plugs which were in contact with load cells. The bottom was made of concentric rings of porous stainless steel, supported by vertical load cells. The sensitivity of the horizontal load cells was poor; and in each experiment they had to be aligned carefully so that the movable plugs were flush with the inside of the cylinder. Because of poor sensitivity and alignment difficulties, accuracy and reproducibility of measurements were not good. Also Lu could not make in place calibration by hydraulic means. The present investigation was undertaken to overcome the shortcomings of Lu's apparatus, and the following objectives were established :

1) Development of an apparatus in which accurate measurements can be made of stress distribution on side, bottom and the top surface.

2) Determination of stress distribution as a function of material, cake thickness, and load.

3) As a more specific objective investigation of uniformity of loading under the piston at top of the cake.

2.2 STRESS MEASUREMENT ON THE BOTTOM AND WALL

The apparatus consisted of a 4" i.d., 1/2" wall thickness 9" tall cylinder with a 3/4" thick bottom plate made of stainless steel 301. Pressure transducers of 0.25" effective diameter, range 0-130 p.s.i. were glued in the cylinder wall and in the bottom plate. All transducers were flush with the inside of the cell so that the stress patterns would not be affected by proturbances. Transducers were calibrated in situ by hydraulic means. During the experiment, slurry was poured into the cylinder and compacted by applying mechanical pressure with a piston. A porous plate attached to the piston rested on the top of the cake and provided drainage for the expelled liquid. Stress distributions were measured for applied pressures of 40 and 80 p.s.i.

2.3 STRESS MEASUREMENT AT THE TOP

The apparatus was the same as that in section 2.2

except that the positions of the porous drainage plate and nonporous plate with transducers, were interchanged. Drainage was provided at the bottom. Stress distributions at the top and wall were measured for applied pressures of 40 and 80 p.s.i.

The details of apparatuses and procedures can be found in sections 4.2 and 4.3 in the appendix.

2.4 DISCUSSION OF THE RESULTS AND CONCLUSIONS

2.4.1 STRESS DISTRIBUTION AT BOTTOM

Figures (2-1) and (2-2) are dimensionless plots of P_{VB}/P_0 Vs r/R for Solka floc BW-200, where P_{VB} is the vertical pressure on the bottom, P_0 the average applied pres--sure on top, r the radial distance from the center and R the inside radius of the cell (2"). These curves are plotted with L/D as a parameter where L is the total cake thickness and D is the diameter of the cell (4"). For all these curves P_{VB}/P_0 is a maximum at the center (r/R=0) and a minimum at the wall. It is seen that as the cake thickness decreases, pressure at the center P_{VB0} becomes more than the average pressure P_0 applied on top. The ratio P_{VB0}/P_0 Vs L/D for Solka floc BW-200 is plotted in Fig. (2-3) with P_0 as a parameter. It is seen that P_0 has a small effect on the P_{VB0}/P_0 Vs L/D relationship. The dimensionless plot of P_{VB}/P_{VB0} Vs r/R obtained from raw data for P_0= 80 p.s.i. is shown in

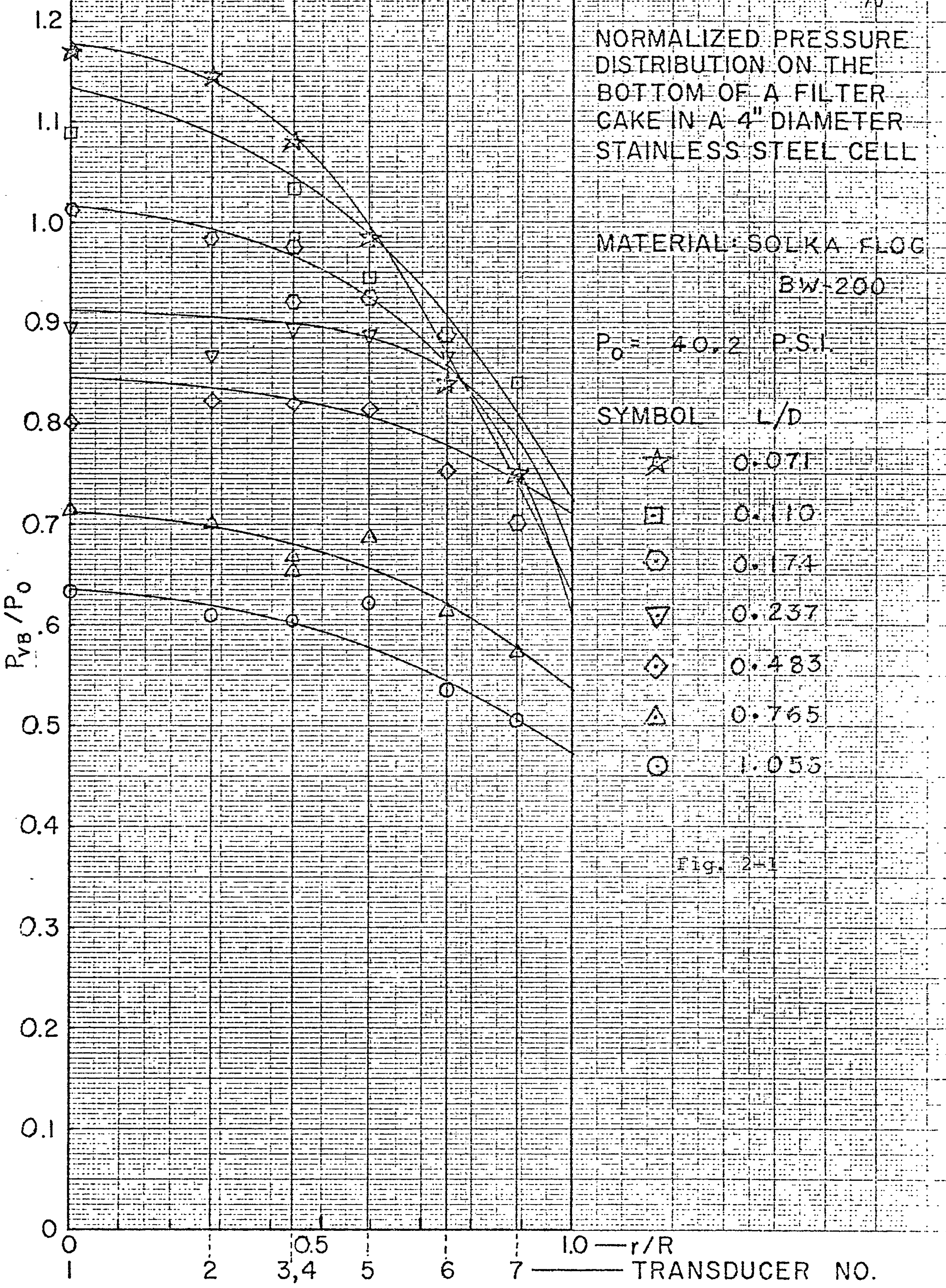

NORMALIZED PRESSURE DISTRIBUTION ON THE BOTTOM OF A FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL
MATERIAL: SOLKA FLOC BW-200
P_o = 40.2 P.S.I.
SYMBOL L/D
0.071
0.110
0.174
0.237
0.483
0.765
1.055
P_{VB}/P_o
1.2
1.1
1.0
0.9
0.8
0.7
.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1.0 — r/R
1
2
3,4
5
6
7 — TRANSDUCER NO.

Fig. 2-1

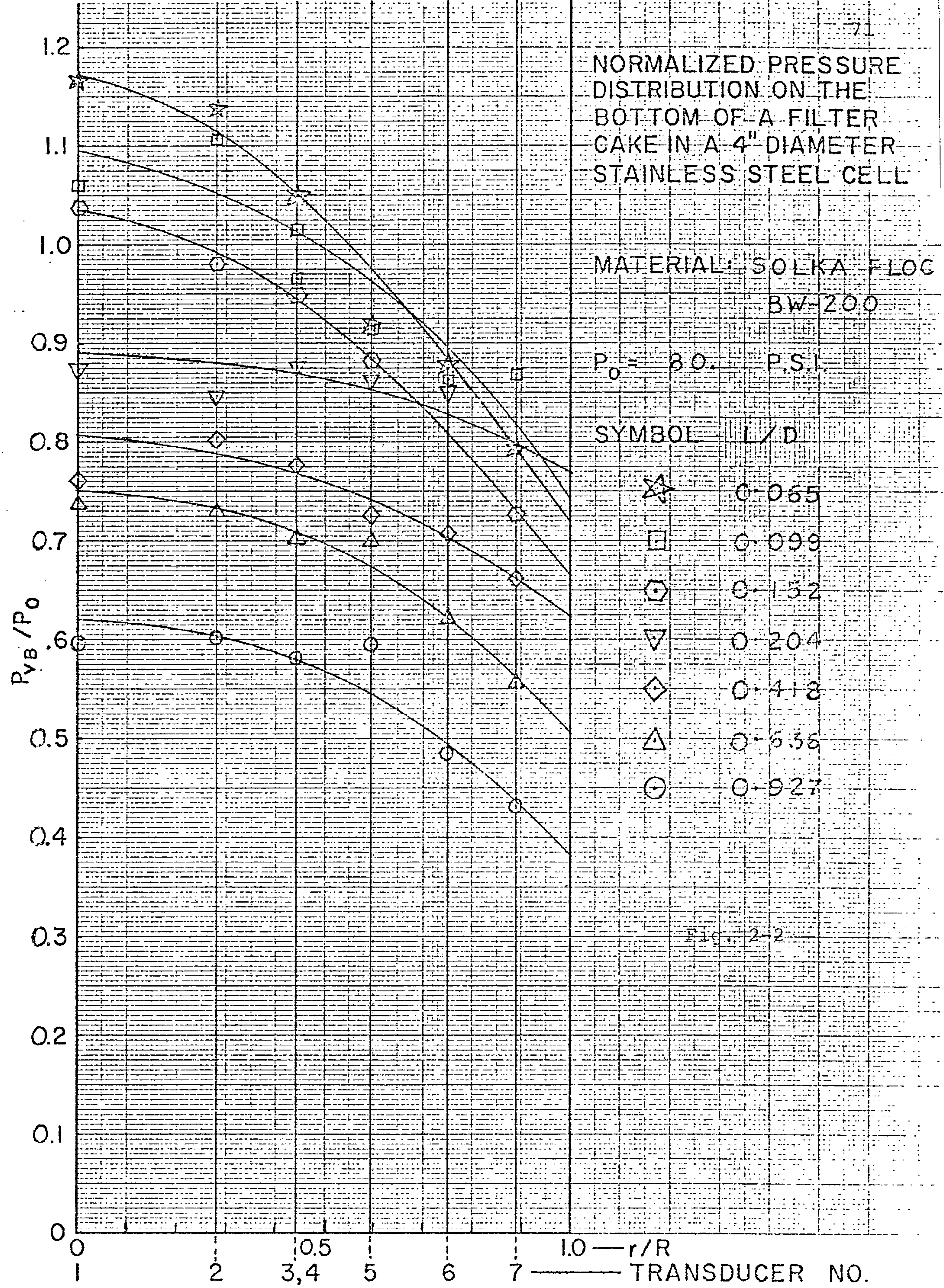

NORMALIZED PRESSURE DISTRIBUTION ON THE BOTTOM OF A FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL
MATERIAL: SOLKA FLOC BW-200
P_o = 80. P.S.I.
SYMBOL L/D
0.065
0.099
0.152
0.204
0.418
0.636
0.927
P_{VB}/P_o
1.2
1.1
1.0
0.9
0.8
0.7
.60
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1.0 — r/R
1
2
3,4
5
6
7 — TRANSDUCER NO.

Fig. 2-2


SOLIDS: SOLKA FLOC BW-200
LIQUID: WATER AT 75° F
SYMBOL
P_0 P.S.I.
40.2
80.0
$\frac{P_{VBO}}{P_0}$
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
L/D
Fig. 2-3

Fig. (2-4). If the values of P_{VB} are obtained from smoothed curves in Fig. (2-2), the plot of P_{VB}/P_{VB0} can be approximated by a parabola.

The ratio of F_{VB}, the total force transmitted to the bottom, to the total applied force on top F_0 is calculated as follows.

$$F_{VB} = \int_0^R P_{VB}\, 2\pi r dr \qquad (2\text{-}1)$$

Defining $\zeta = r/R$ (2-2)

and utilizing Eqn. (2-2) in Eqn. (2-1) ,

$$F_{VB} = 2\pi R^2 P_0 \int_0^1 (P_{VB}/P_0)\zeta\, d\zeta \qquad (2\text{-}3)$$

$$F_0 = \pi R^2 P_0 \qquad (2\text{-}4)$$

Dividing Eqn. (2-3) by Eqn. (2-4)

$$F_{VB}/F_0 = 2 \int_0^1 (P_{VB}/P_0)\zeta\, d\zeta \qquad (2\text{-}5)$$

F_{VB}/F_0 was calculated by graphical integration. It was found that F_{VB}/F_0 was as high as 0.8 for L/D = 0.065 but was always less than 1. As cake thickness decreases, the P_{VB}/P_0 profile shifts downwards, and total transmitted force becomes smaller as a result of increased wall friction. The trend of P_{VB} distribution was the same at both applied pressures 40.2 p.s.i. and 80 p.s.i.

For materials $CaCO_3$, Talcum powder, Kaolin NF a similar trend of stress distributions was observed. The data for

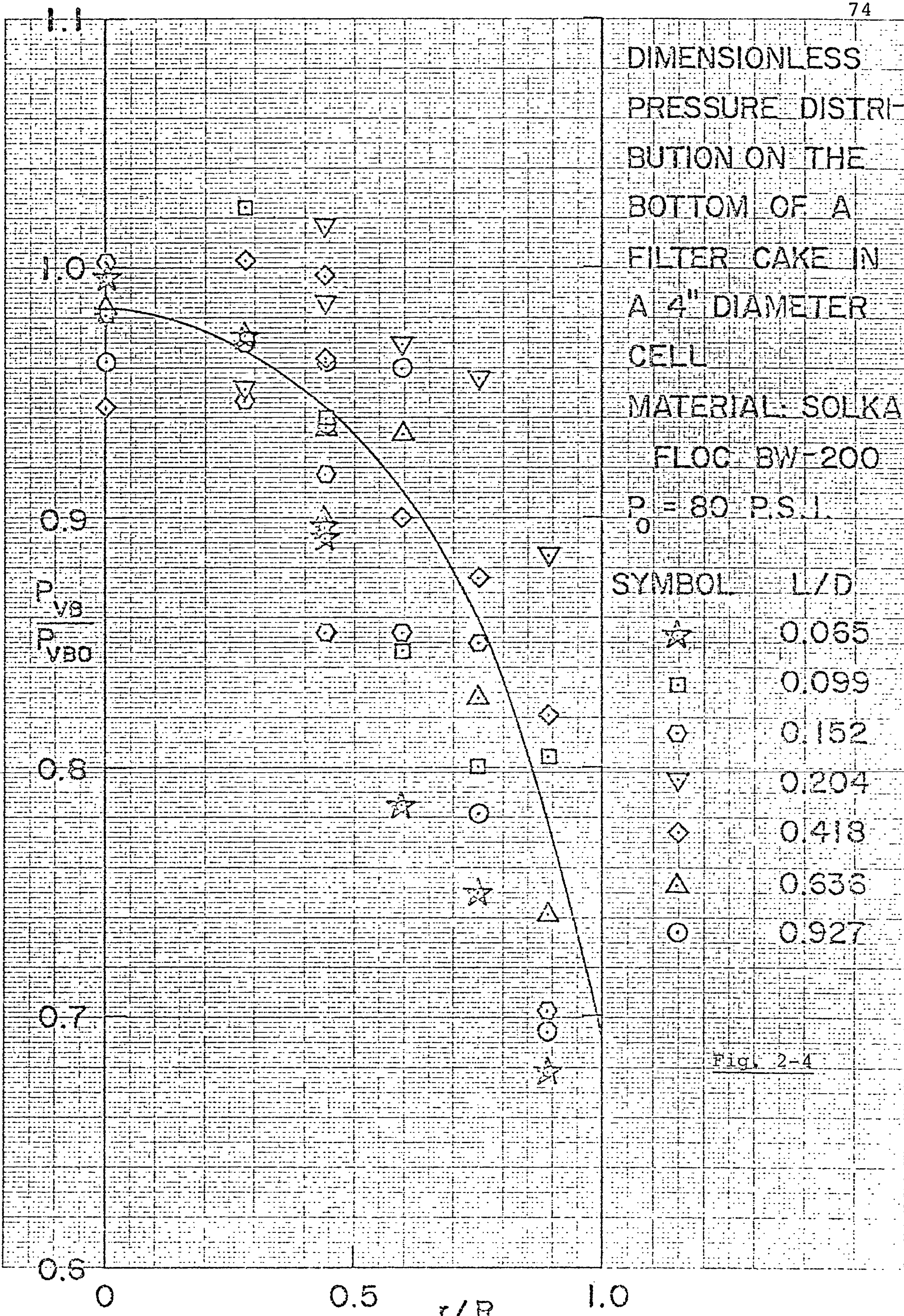

DIMENSIONLESS PRESSURE DISTRIBUTION ON THE BOTTOM OF A FILTER CAKE IN A 4" DIAMETER CELL
MATERIAL: SOLKA FLOC BW-200
P_0 = 80 P.S.I.
SYMBOL L/D
0.065
0.099
0.152
0.204
0.418
0.636
0.927
1.1
1.0
0.9
0.8
0.7
0.6
$\frac{P_{VB}}{P_{VBO}}$
0
0.5
1.0
r/R

Fig. 2-4

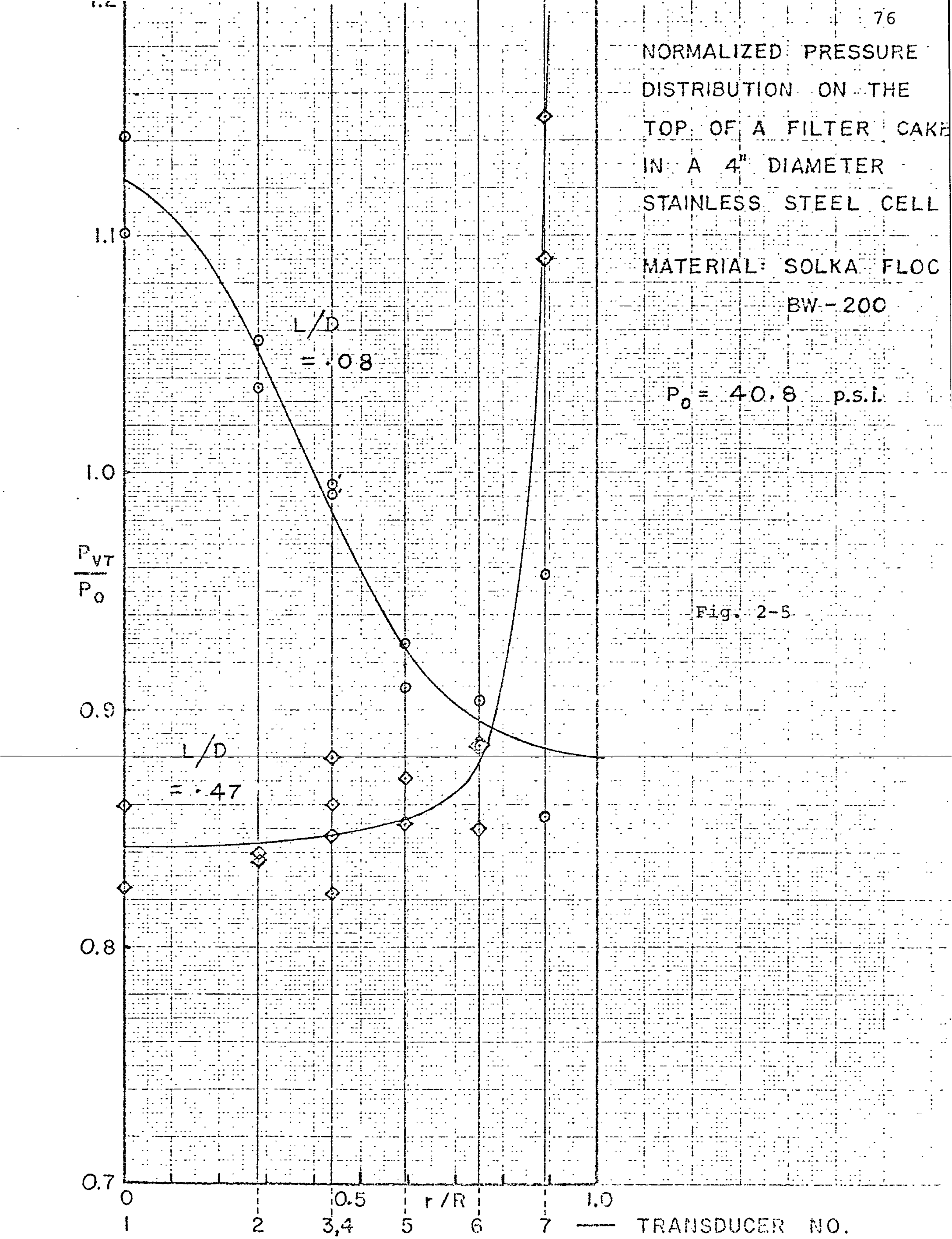
incompressible material Polystyrene beads (0.5 mm dia.) were completely scattered and are not reported.

2.4.2 STRESS DISTRIBUTION AT TOP

P_{VT}/P_0 Vs r/R is plotted in Figures (2-5) and (2-6) where P_{VT} is vertical pressure on the top of a filter cake. For thin cakes ($L/D < 0.1$) shape of the curve is similar to that of P_{VB} distribution. For thicker cakes P_{VT}/P_0 is nearly constant in the central region and rises sharply at the wall. P_{VT}/P_0 Vs r/R for different L/D 's is illustrated in Fig. (2-7) without showing data points. The stress distributions at top and bottom for thin and thick cakes are shown in Fig. (2-8). For L/D = 0.065 the gap between P_{VT}/P_0 and P_{VB}/P_0 is small and does not change with r/R. For L/D = 0.62 the gap widens with increasing r/R. This phenomena is probably due to the wall friction.

2.4.3 STRESS DISTRIBUTION AT THE WALL

The horizontal pressure normal on the wall is denoted by P_H. Dimensionless plots of P_H/P_0 Vs z/D are shown in Figures (2-9), (2-10) for Solka floc BW-200 and in Figures (2-11), (2-12) for Calcium Carbonate. Quantity z is the vertical distance from the bottom and D is inside diameter (4") of the cell. These data are somewhat scattered. The curves have to be drawn by taking an average of a large number of data points. P_H/P_0 increases from bottom to top and also increases as cake is made thinner.



NORMALIZED PRESSURE DISTRIBUTION ON THE TOP OF A FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL
MATERIAL: SOLKA FLOC BW-200
P0 = 40.8 p.s.i.
L/D = .08
L/D = .47
PVT/P0
1.1
1.0
0.9
0.8
0.7
0
0.5
r/R
1.0
1
2
3,4
5
6
7
TRANSDUCER NO.

Fig. 2-5

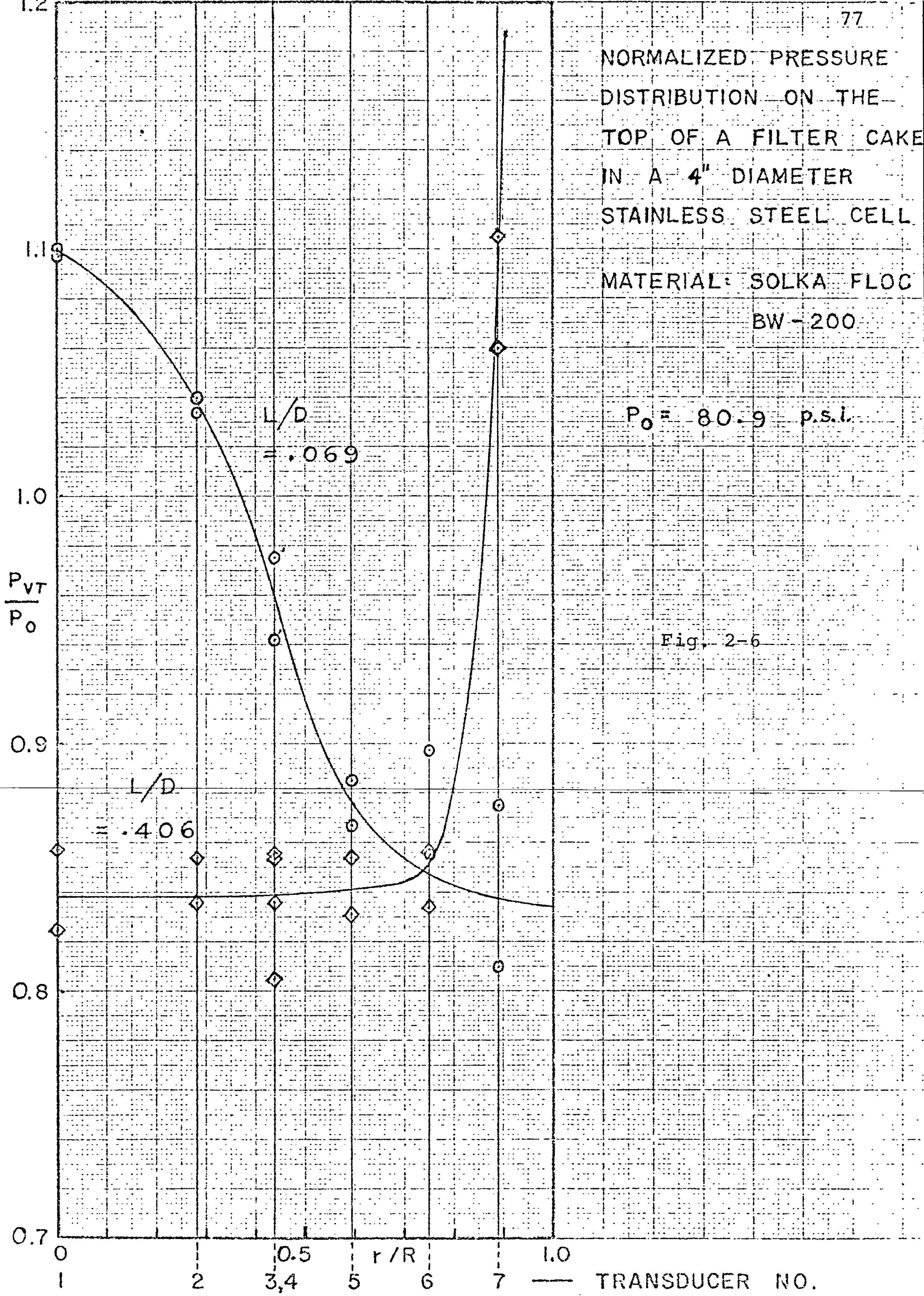
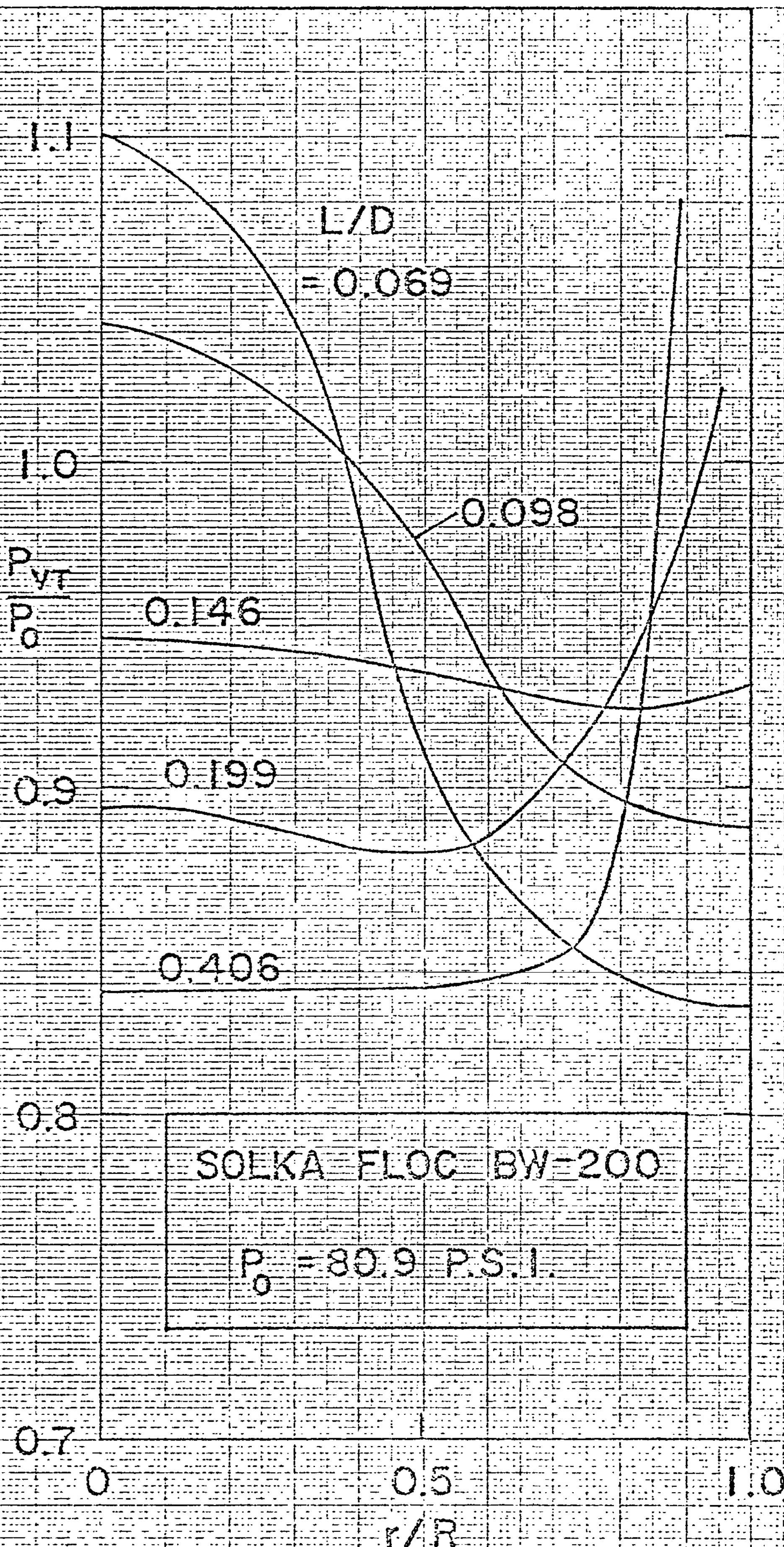

NORMALIZED PRESSURE DISTRIBUTION ON THE TOP OF A FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL
MATERIAL: SOLKA FLOC BW-200
P_o = 80.9 p.s.i.
L/D = .069
L/D = .406
$\frac{P_{VT}}{P_o}$
1.2
1.1
1.0
0.9
0.8
0.7
0
0.5
r/R
1.0
1
2
3,4
5
6
7
TRANSDUCER NO.

Fig. 2-6



1.1
1.0
0.9
0.8
0.7
$\frac{P_{VT}}{P_o}$
L/D
= 0.069
0.098
0.146
0.199
0.406
SOLKA FLOC BW-200
P_o = 80.9 P.S.I.
0
0.5
1.0
r/R

Fig. 2-7

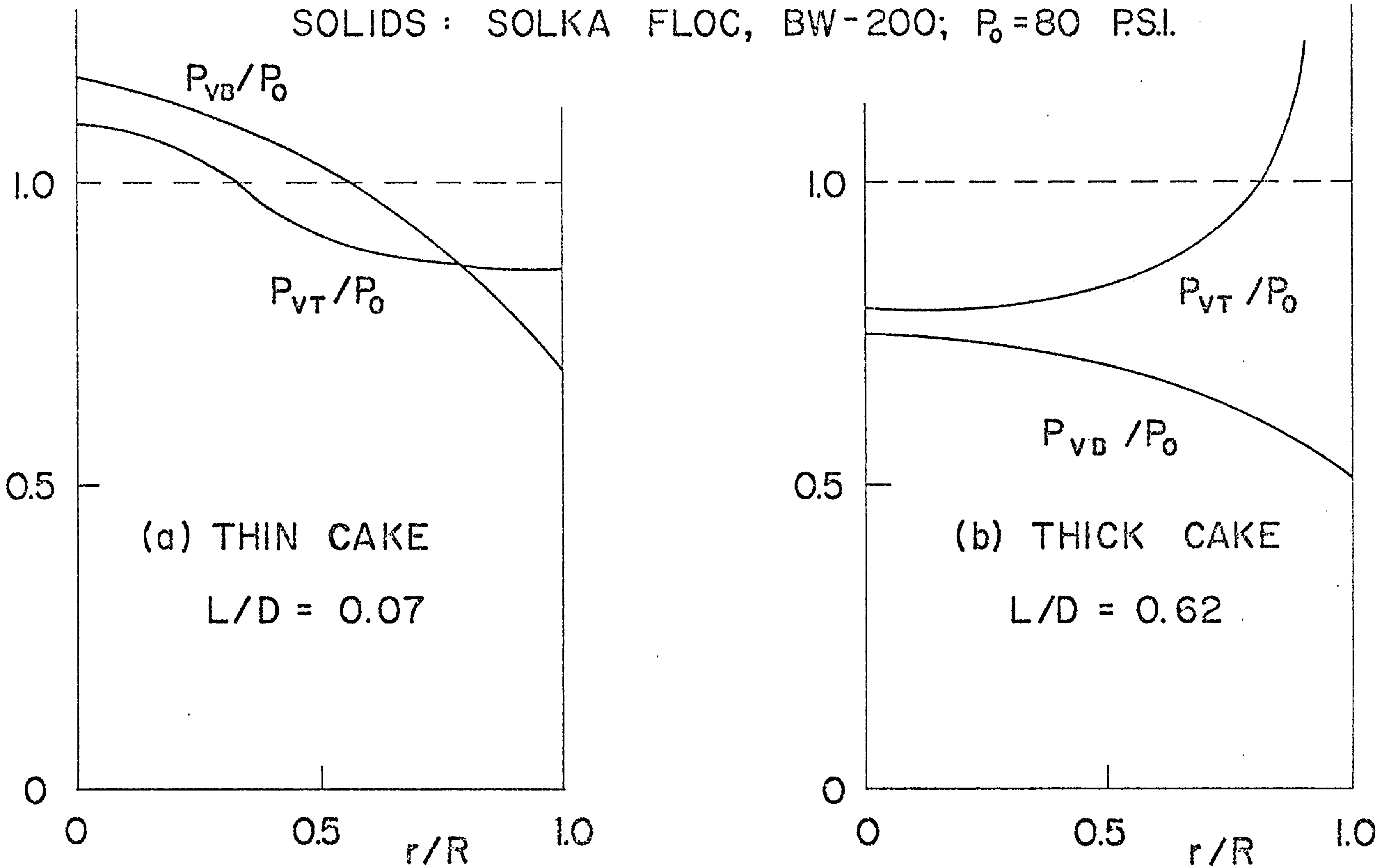


Fig. 2-8 P_{VB}/P_0 and P_{VT}/P_0 Vs r/R for thin and thick cakes.

Fig. 2-9 NORMALIZED DISTRIBUTION OF PRESSURE EXERTED BY A COMPACTED FILTER CAKE ON THE WALL OF A 4" DIAMETER STAINLESS STEEL CELL

MATERIAL: SOLKA FLOC BW-200; P_a = 40.2 P.S.I.; S = 0.206, L/D = 1.056

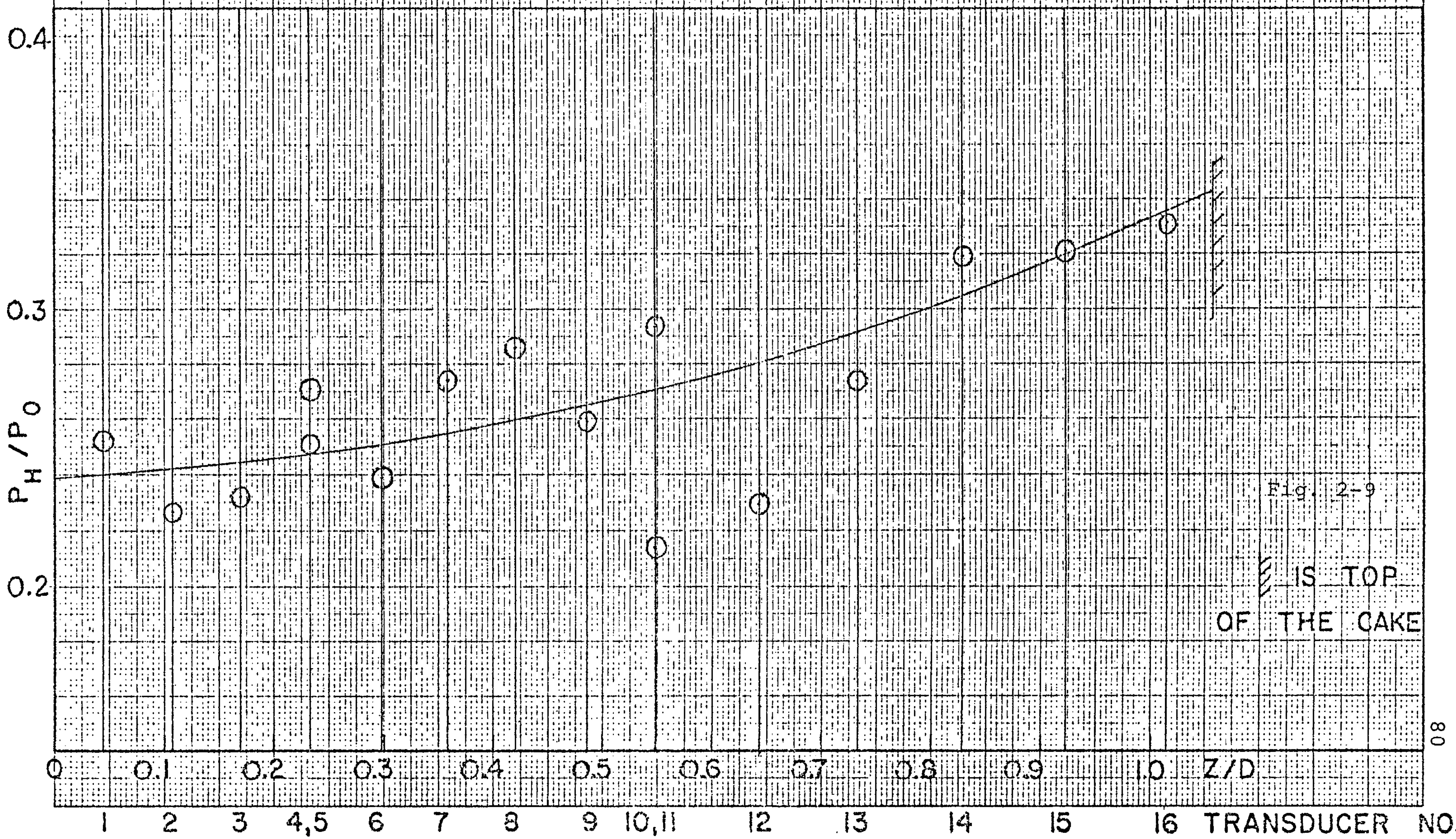
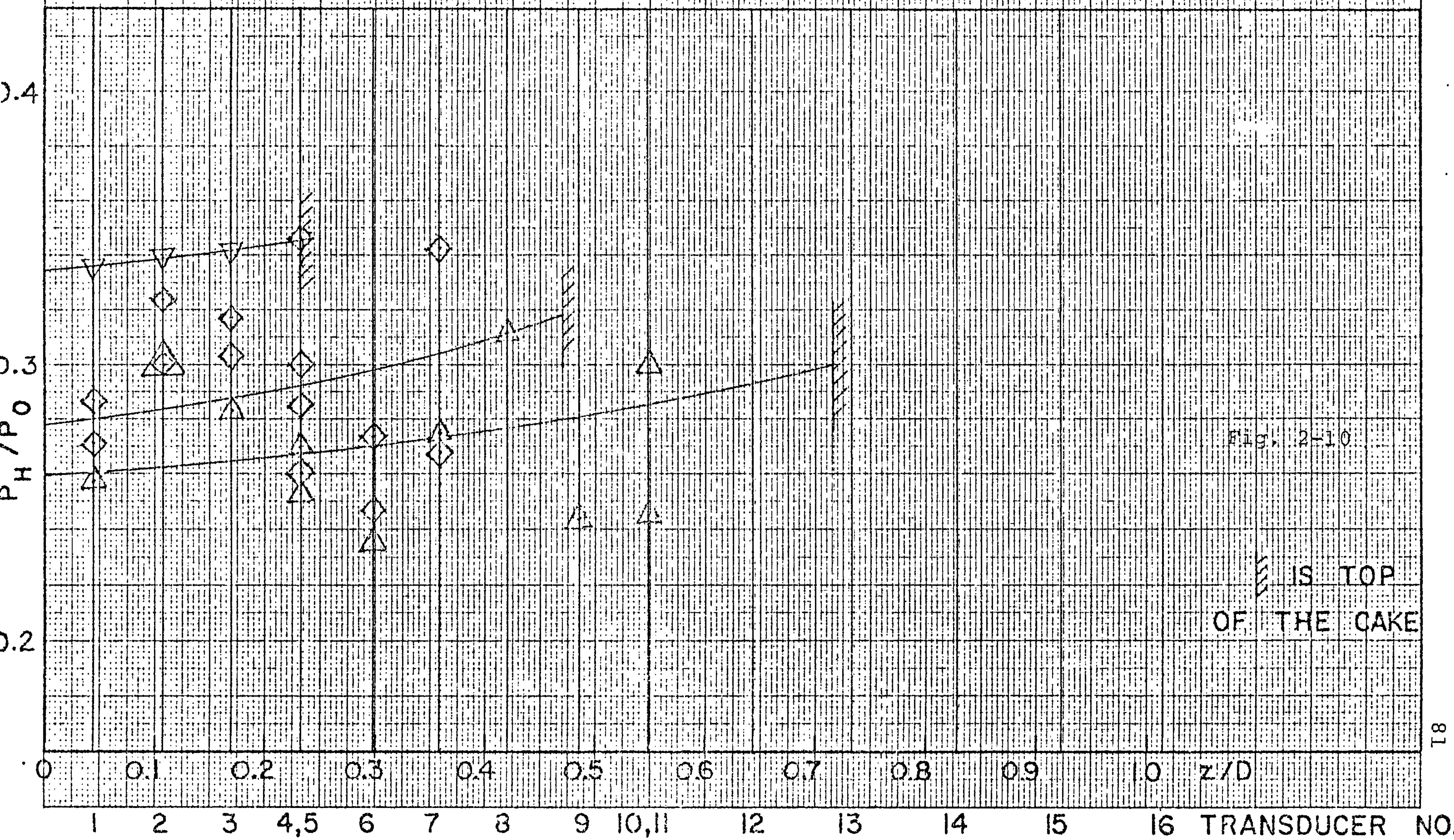


Fig.2-10 NORMALIZED DISTRIBUTION OF PRESSURE EXERTED BY A COMPACTED FILTER CAKE ON THE WALL OF A 4" DIAMETER STAINLESS STEEL CELL

MATERIAL: SOLKA FLOC BW-200, P_o = 40.2 P.S.I.;



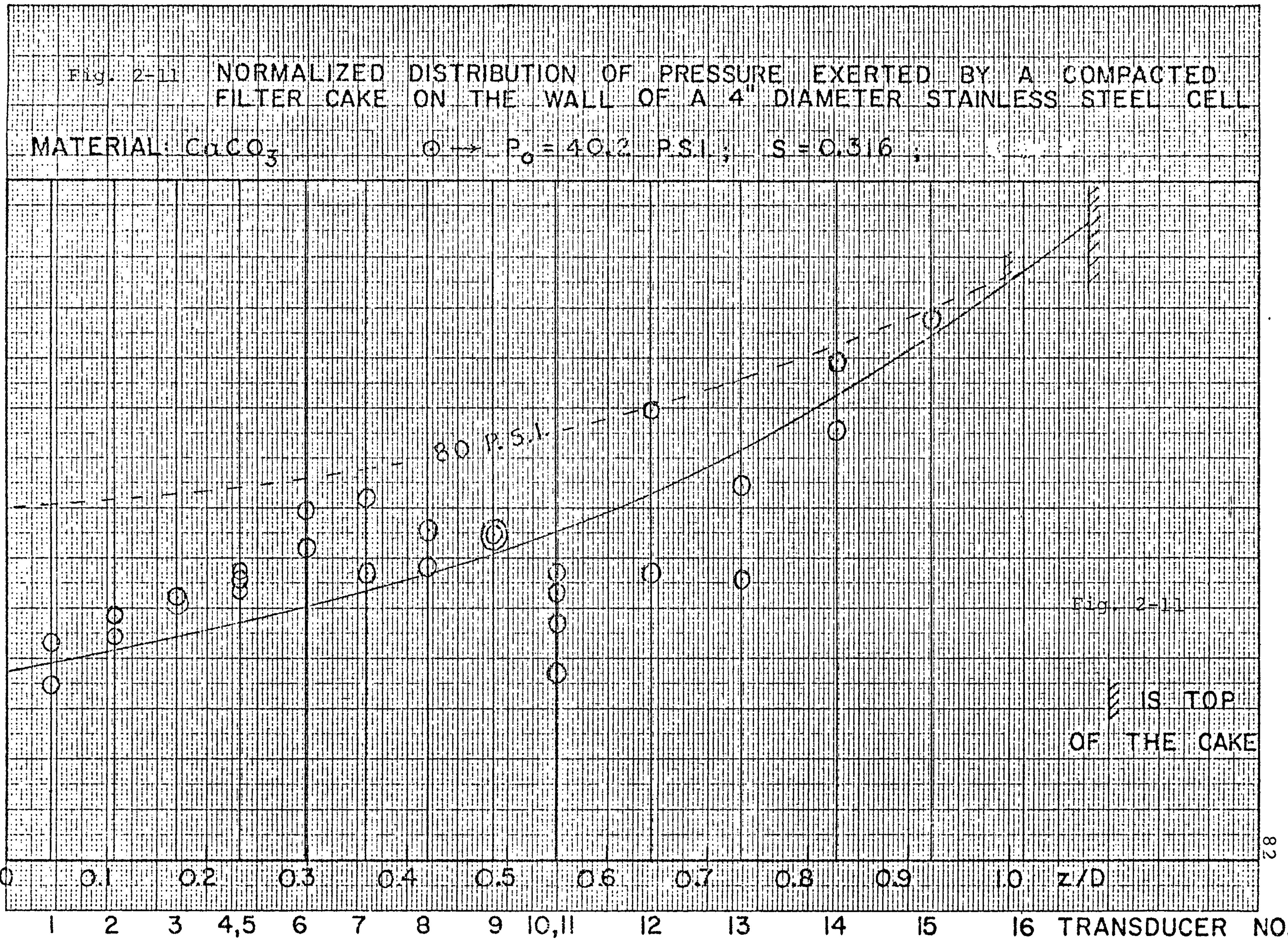
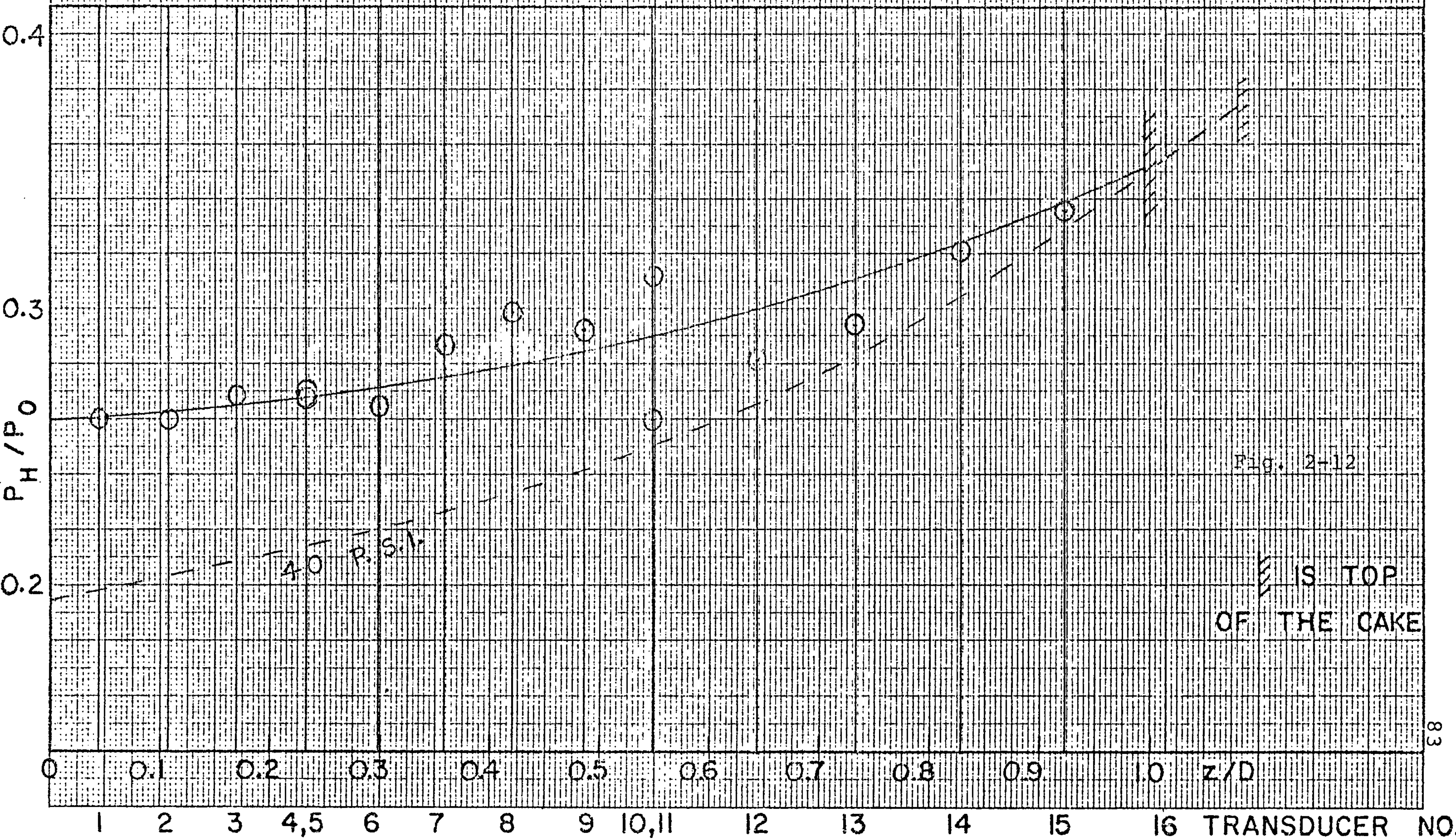


Fig. 2-11 NORMALIZED DISTRIBUTION OF PRESSURE EXERTED BY A COMPACTED FILTER CAKE ON THE WALL OF A 4" DIAMETER STAINLESS STEEL CELL
MATERIAL: CaCO3
⊙ → Po = 40.2 P.S.I.; S = 0.316 ;
80 P.S.I.
PH/Po
0.3
0.2
IS TOP OF THE CAKE
Fig. 2-11
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 Z/D
1 2 3 4,5 6 7 8 9 10,11 12 13 14 15 16 TRANSDUCER NO.

Fig. 2-12 NORMALIZED DISTRIBUTION OF PRESSURE EXERTED BY A COMPACTED FILTER CAKE ON THE WALL OF A 4" DIAMETER STAINLESS STEEL CELL

MATERIAL: $CaCO_3$ ○ → P_o = 80.0 P.S.I.; S = 0.316;



It is concluded after scrutinizing all experimental data that there is no way of obtaining a uniform stress distribution in a filter cake compacted using a conventional CP cell. If cake thickness is large, wall friction makes stresses nonuniform. If the cake is very thin, wall friction is small but frictional stresses on top and bottom surfaces leads to nonuniform stress distribution. In case of Solka floc BW-200 there exists an optimum cake thickness ($L/D \simeq 0.2$, $D=4"$) where stress distribution is nearest to uniform. For other materials, such an optimum cake thickness probably would be different. If a conventional CP cell is used to measure filter cake properties, this optimum cake thickness is recommended.

2.4.4 REPRODUCIBILITY OF THE DATA

The in situ calibration curves for pressure transducers are shown in appendix (chapter 4). Those curves are reproducible with in an accuracy of $\pm$ 1% of the average value. Therefore it can be concluded that the apparatus is capable of measuring stress distributions on the compacted filter cake surfaces with the same accuracy . All the experimental data are tabulated in appendix. The mean reproducibility of vertical stress distribution is within $\pm$ 5% of the average value. However, under identical experimental conditions, the stress at any one particular point on a cake surface may vary by as much as $\pm$ 10% of the average value. The accuracy of stress measurement is within $\pm$ 1%

and nonreproducibility of the stress distribution is probably due to the lack of reproducibility of the cake structure.

2.4.5 CYLINDRICAL SYMMETRY OF STRESS DISTRIBUTIONS

It was observed from the data that in 80% of the total 57 observations on different materials, the two transducers (NO. 3 and 4) at the same radial positions but at different angular locations, registered vertical pressures within 4% of each other. In the remaining 20% of the observations the maximum difference between readings of TR3 and TR4 was 25%. It would be infered that the cake was not uniformly deposited in 20% of the incidences. On the cylinder wall, the stress distributions are too scattered (Fig. (2-9), (2-10), (2-11), (2-12)) to enable one to draw any conclusion about cylindrical symmetry.

2.4.6 THE RATIO k_0 OF P_H TO P_V AT THE WALL

Lu (1968) indicates that the ratio of lateral to vertical stress donoted by k_0 is the same at top and bottom and is independent of the cake thickness. He assumed that P_{VT} is P_0 at every value of r and calculated k_0 on top with this assumption. From the data obtained in this investigation (Figures 2-5 and 2-10) it is seen that P_H at the top of the cake decreases and P_{VT} at the wall increases as cake is made thicker. Therefore k_0 at the top has to decrease as the cake thickness increases. For Solka floc BW-200 at P_0=40.8 p.s.i., k_0 on top is 0.45 for L/D= 0.08 but less than 0.23 (exact value of k_0 on top cannot be calculated due to steep nature of P_{VT}/P_0 Vs r/R near r/R=1) for L/D =1.05. The author questions the usefulness of the concept of k_0 because k_0 is highly dependent on L/D.

2.5 RECOMMENDATIONS FOR THE FUTURE WORK

It is desirable to investigate the effects of change in cell diameter and presence of a vertical probe at the center as far as stress distributions are concerned. In the present investigation only two values of parameter P_0 were chosen. Effect of P_0 on the stress distribution should be studied, as the data on P_{VB0}/P_0 Vs L/D has a direct application in calculations on the improved CP cell (Chapter 3).

Empirical relationships should be developed to represent the observed stress distributions. A three dimensional mathematical analysis of stress distributions inside compacted filter cakes would be useful.

CHAPTER 3

DETERMINATION OF FLOW RESISTANCE FROM HYDRAULIC PRESSURE PROFILES

3.1 INTRODUCTION

Since invention of the CP cell by Ruth (1946), many investigators have tried to correlate values of specific cake resistance α, as measured in a CP cell to the resistance of a cake during filtration. Grace (1953) and Shirato (1968) sucessfully correlated their CP cell and filtration data. Kottwitz and Boylan (1958) were partially successful in the correlation. Lu (1968), Haynes (1966) and Nieto (1967) found that values of α were profoundly affected by experimental techniques. Taylor (1942) first pointed out that the solid compressive pressure p_s inside a consolidometer is nonuniform due to wall friction. This was proved by observing that the transmitted force at the bottom is considerably less than the applied force from the top. In the CP cell p_s is not constant over a cross section perpendicular to the axis of the cylinder. Nonuniformity of p_s is discussed in section 2.4, and also by Lu (1968). In order to make a correction for the error arising out of wall friction, three approaches have been taken by investigators in this field.

a) Some investigators (Rawling 1963, Lu 1968) tried to eliminate the effect of wall friction by measuring α for different cake thicknesses L at a given applied pressure and then extrapolating α Vs L to zero thickness. Extrapolation is difficult because of steep slope of the α Vs L curve near the origin. However, this would have been a valid approach if the medium resistance R_m were known accurately and the cake became uniform as L approached zero. In a convential CP cell the pressure drop measurement includes the cake and the top and bottom septa. R_m is considered to be the same as that of a clean medium. This is incorrect as will be shown in section 3.7. When a filter cake is in contact with a medium, fine particles migrate into the medium and increase R_m. As the cake thickness L is decreased, R_m becomes more significant in comparison with cake resistance, and an error in R_m can lead to gross inaccuracies in calculated values of α for thin cakes. As discussed in section 2.4.1, p_s becomes increasingly nonuniform as L approaches zero. Because of these two reasons the first approach to make a correction for the wall friction is not valid.

b) In the second approach, developed by Haynes, Lu and Tiller (1972), unidimensional variation of p_s due to wall friction is considered and a formula is developed to relate experimentally measured α to the corrected α. In this method variation of p_s over a cross section is ignored and also calculation of α requires knowledge of R_m.

c) The third approach (Shirato, Aragaki, Mori, Sawamoto 1968) is to calculate the internal p_s distribution while accounting for cohesive forces and wall friction in case of a reasonable cake thickness (L/D ≈ 0.2). Then volume average of p_s defined by the following equation and denoted by $\bar{p}_s$ is computed.

$$\bar{p}_s = \frac{2}{R^2 L} \int_0^L \int_0^R p_s(r,z)\; r\, dr\, dz \qquad (3\text{-}1)$$

In this equation, r is radial distance from the center, z is vertical distance from the bottom and R is the inside radius of the CP cell. Experimentally measured α is reported as a function of $\bar{p}_s$. Due to the consideration of varying p_s over a cross section, this approach is better than the second method. However, experimentally measured average α is not merely a volume average of α and also α is not a linear func--tion of p_s. Therefore the third approach also has weaknesses.

Quantity α can also be calculated by measuring the total resistance at different values of w. Combined liquid pressure drop across the cake and the media (one at the top of the cake and the other at the bottom) is denoted by Δp_{LT}. This quantity is related to α and R_m (resistance of a single medium) by

$$\frac{\Delta p_{LT}}{\mu q} = \alpha w + 2R_m \qquad (3\text{-}2)$$

Where these R_m represents the resistance of the top and bottom media. The curve $\Delta p_{LT}/(\mu q)$ Vs w will not be a straight line because the wall friction will cause α to vary with w. As w approaches zero, the effect of wall friction diminishes and slope at w = 0 is the desired value of α. In this method, calculation of α does not require knowledge of R_m. However variation of p_s over a cross section is ignored. Porosity ε and α are unique functions of p_s and they also vary over a cross section of the cake. As a result liquid flow in the CP cell is not unidirectional. It is important to experimentally verify the nonunidirectionality of flow.

Lu (1968) used a conventional CP cell to study the effect of initial slurry concentration on α Vs p_s data. He measured α at a fixed p_s at several time intervals. In some experiments he allowed the liquid to flow through the cake continuously. In the remaining experiments there was no flow except at the time of measurement of liquid pressure drop. He used R_m of a clean medium to calculate α. When a filter cake is in contact with the medium, R_m would be much more than that of a clean medium. Due to erroneous values of R_m, his α values are not correct. In the improved CP cell developed in this investigation, liquid pressure profile inside the cake is measured, and α can be calculated independently of R_m. In order to overcome the shortcomings of a conventional CP cell, a new technique was developed. The liquid pressure (p_L) profile is measured inside a filter cake using a center probe with narrow (0.01" wide) horizontal slit type openings connected to a pressure measurement system. Also p_L profile on the cake wall is measured by similar slits on the cylinder wall. Slits are placed at an interval of 1/8" The method for calculation of α from the knowledge of p_L profile is explained later. Following were set up as objectives of the investigation .

1) To improve the accuracy of measurement of α as a function of p_s.

2) To check the reproducibility of α Vs p_s data.

3) To check the directionality of flow.

4) To measure the medium resistance R_m, when cake is in contact with the medium.

5) To study the effect of initial slurry concentration on α Vs p_s data.

3.2 THEORY OF CP CELL

Although liquid flow is not strictly unidirectional in a CP cell as discussed in the introduction, the deviation from unidirectionality is assumed not to be large. The analytical expression for the flow pattern in a CP cell is not available and assuming unidirectional flow permits derivation of an equation for α. Flow in a CP cell can be regarded as a special case of solid- liquid flow discussed in chapter 1. Solids are stationary in a CP cell during measurement of α, and the cake is at equilibrium as far as the stress distribution is concerned. It is necessary to derive an equation for α in terms of the hydraulic pressure gradient. Substituting Eqn. (1-5) and $q_s = 0$ in Eqn. (1-28)

$$q = \frac{1}{\mu\alpha(1-\varepsilon)\rho_s}\left(\frac{\partial p_L}{\partial x}\right)_t \qquad (3-3)$$

As p_L is not a function of t at equilibrium in a CP cell,

$(\partial p_L/\partial x)_t$ can be replaced by dp_L/dx. Upon rearrangement, Eqn. (3-3) yields,

$$\alpha = \frac{1}{\mu\rho_s\,(1-\varepsilon)}\;\frac{(dp_L/dx)}{q} \qquad (3\text{-}4)$$

In a CP cell $q_s = 0$, and it is evident from Eqn. (1-10) that the average liquid flow rate q, through a given cross section is constant. In the improved CP cell, p_L Vs x profile is measured; therefore dp_L/dx can be calculated at any value of x in a cake. But p_s is known only at the top and at the bottom. Therefore α should be calculated either at the top or at the bottom. P_0 the average applied pressure on top is increased in steps in order to measure α and ε at different p_s values. As p_s increases, the top of the filter cake moves downwards. In the improved CP cell, p_L is measured at fixed slits. The top slit is usually far below the top of a cake and dp_L/dx cannot be calculated at the cake top without a large extrapolation. However, the bottom of a cake does not move as P_0 is increased and the bottom slit is always at a distance of 0.04" from the cake bottom. Therefore dp_L/dx can be estimated more accurately at the cake bottom.

On the bottom, p_s decreasss from the center towards the wall. From results shown in chapter 2, p_s at the bottom center as well as at the wall can be obtained for different cake thicknesses and for different P_0's. The stress measurement on the bottom in chapter 2 was not done with a probe at the center. But because of lack of information, results of chapter 2 are used to calculate p_s at the desired point on the bottom.

It is easier to obtain p_s at the bottom center (denoted by P_{VB0}) than at the wall because the p_s profile is flat at the center and decreases sharply at the wall. Although dp_L/dx can be found accurately at the bottom center as well as at the bottom part of the wall, measurement of p_L at the wall requires dismantling of tubing connections at the end of every run. Therefore α is calculated at the bottom center. It is necessary to have a knowledge of ε Vs p_s relationship to calculate α. In the improved CP cell only average porosity, ε_{av}, can be measured. It is calculated as follows.

$$M_s = (1-\varepsilon_{av})\ L\ A\ \rho_s \tag{3-5}$$

where, M_s is the total dry solids mass in CP cell, L is the cake thickness, A is the area of cross section, and ρ_s is the true density of solids. Solving Eqn. (3-5) for ε_{av},

$$\varepsilon_{av} = 1 - \frac{M_s}{LA\rho_s} \tag{3-6}$$

In a CP cell, average p_s over a cross section decreases towards the bottom in an approximately exponential manner. P_T is the average vertical pressure transmitted to the bottom. According to Shirato (1968), the logarithmic mean of P_0 and P_T denoted by $\bar{P}_{LM}$ and calculated by

$$\bar{P}_{LM} = \frac{P_0 - P_T}{\ln(P_0/P_T)} \tag{3-7}$$

can be used for the average pressure. Quantity $\bar{P}_{LM}$ is regarded as the value of p_s corresponding to $\varepsilon = \varepsilon_{av}$. The medium resistance R_m is defined as,

$$R_m = \frac{\Delta p_L}{\mu q} \tag{3-8}$$

where Δp_L is the hydraulic pressure drop across the medium.

3.3 A BRIEF DESCRIPTION OF THE APPARATUS

(The detailed description can be found in the appendix) A filter cake is compacted between porous plates covered with filter papers in a cylinder of 2" i.d., 2.5" o.d. and 9" height. Fig. (3-1) is a simplified schematic diagram of the main apparatus. Top and bottom porous plates are denoted by PP1 and PP2 respectively. Plate PP1 is attached to the piston, and PP2 is attached to the floating bottom (FB) which rests on a force transducer. Mechanical pressure P_0 of range 2- 125 p.s.i.is applied to the cake by means of the piston, and water is flowed through under a pressure drop of less than 0.5 p.s.i. Flow rate is adjusted to the desired value by using a rotameter and a fine metering valve fitted at the end of the water passage through the system. A reservoir of

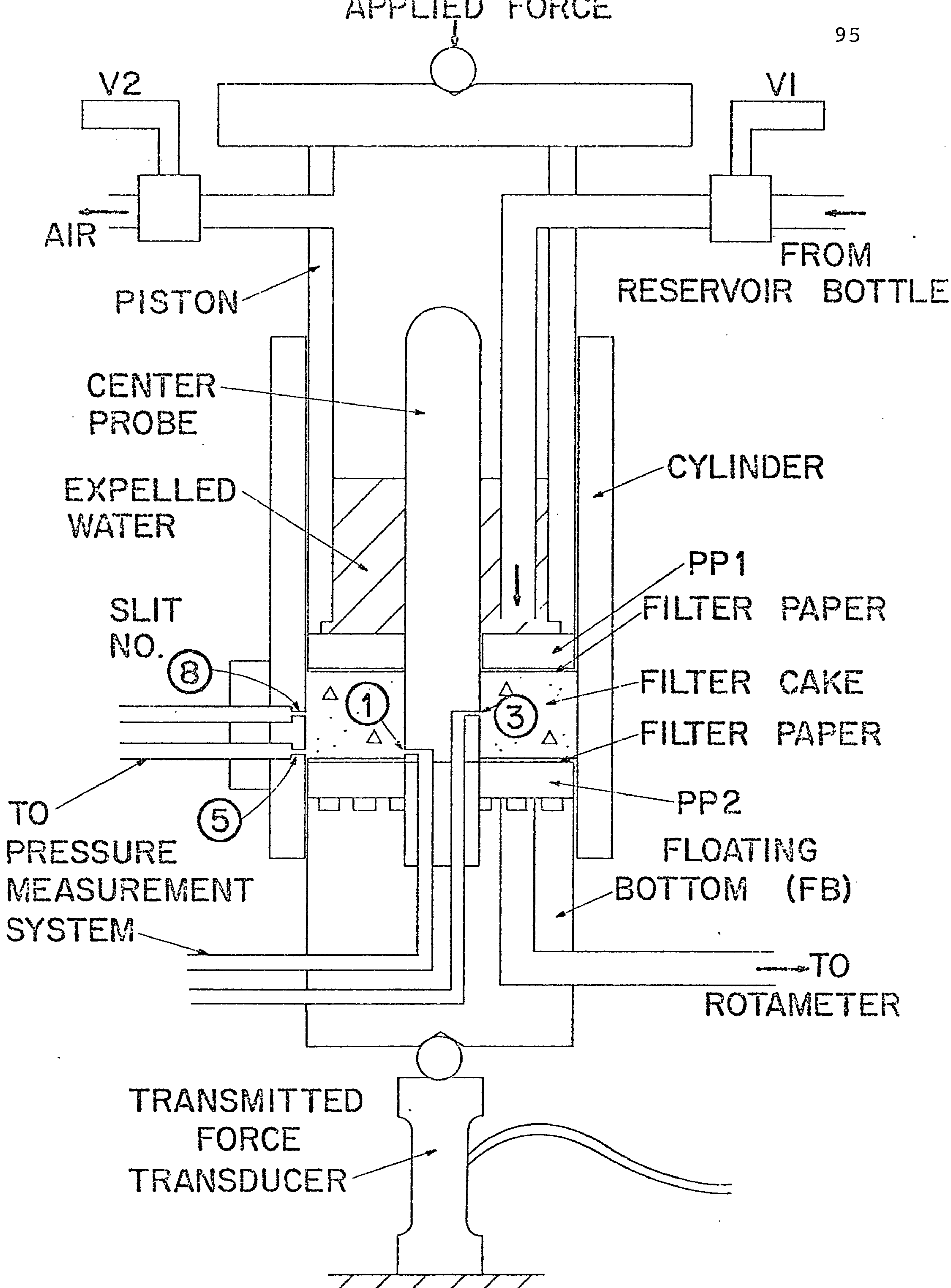


Fig. 3-1 Simplified schematic diagram of the new CP cell

cross-sectional area of about 0.5 sq. ft. provides a constant water pressure at PP1. The FB has a 1/2" diameter probe centrally located. There are four slits in the probe numbered 1 to 4 starting from the bottom. Slit No. 1 is 0.04" above PP1. A vertical tubular channel of 1/8" dia. starts from each slit and bends at right angles to exit from the FB. The wall of the cylinder was constructed with five slits opening inside. These slits are numbered from 5 to 9 starting from the bottom. Two of these slits, No. 6 and No. 7, are diametrally opposite and at the same height. Four channels from FB and five slits on the cylinder are connected to a hydraulic pres--sure transducer through switching valves. Slit No. 1 (on the probe) and slit No. 5 (on the cylinder) are brought in level, by proper adjustments.

3.4 PROCEDURE IN BRIEF

(The detailed procedure can be found in the appendix) The channels for liquid pressure measurement are purged of air with water. Removal of air from the liquid pressure measurement system improves the speed of response. A slurry of the desired concentration is prepared and poured into the cylinder and squeezed with the piston at known P_0 to form a filter cake. Expelled water enters the piston and displaces air which is present up to valve V2. After equilibrium of transmitted force is established, water is flowed at the desired rate by opening appropriate valves. Hydraulic pressures are measured at the center probe and at the cylinder wall by connecting one channel at a time to the hydraulic pressure transducer by means of switching valves. For each mechanically applied pressure several different flow rates are used. Hydraulic pressure on each slit is measured with reference to the situation when there is no flow through the cake. By making use of hydraulic pressure at each slit, the hydraulic pressure profile is calculated with reference to slit No. 1. In other words slit No. 1 is always regarded at zero pressure.

After completing measurement for one value of P_0, additional pressure is applied and the measurement procedure is repeated.

3.5 CALCUALTIONAL PROCEDURE

3.5.1 CALCULATION OF α

This is illustrated with the following data set.

Solids : Solka floc BW-200 Liquid : Water at 75°F

P_0 = 75.4 p.s.i. P_T = 50.2 p.s.i. s = 0.085

ρ_s = 94.78 lbm/ft^3 $\mu = 6.72 \times 10^{-4}$ lbm/(ft sec)

L = 0.0525 ft. (0.63 inches), M = 0.03858 lb (17.5 gms)

The cylinder has an inside diameter of 2" and the probe has a diameter of 0.5". Therefore the cross sectional area A is

$\pi/4\ (2^2 - 0.5^2) = 2.9452$ inch^2 = 0.02045 ft^2

Applying formula (3-6) $\varepsilon_{av} = 0.621$

Utilizing Eqn. (3-7), $\bar{P}_{LM}$ = 61.96 p.s.i.

Thus ε Vs p_s can be calculated for each experimental run, and the resulting relationship is plotted in Fig. (3-2).

Total flow rate Q through the cake is measured in ml/min. This should be converted to ft^3/sec.

$$Q\ (ft^3/sec) = \overset{ml/min}{Q} \left[\times \overset{ft^3/ml}{\frac{1}{28.3167\times10^3}} \right] \left[\times \overset{min/sec}{\frac{1}{60}} \right]$$

$$Q\ (ft^3/sec) = 5.8858 \times 10^{-7}\ Q\ (ml/min)$$

It is necessary to calculate q which is flow rate per unit cross-sectional area.

$$q\ (ft/sec) = Q/A = \frac{5.8858 \times 10^{-7}}{0.02045}\ Q\ \ (ml/min)$$

$$= 2.8781 \times 10^{-5}\ Q\ (ml/min) \qquad (3\text{-}9)$$

In Fig. 3-3 hydraulic pressure profiles measured by the centerprobe as well as the side wall slits are plotted. For the purpose of calculating α, only centerprobe measurements are used. By drawing tangents at the origin to each curve, $(dp_L/dx)_{x=0}$ is measured and plotted against q in Fig. (3-4).

The liquid velocity q varies over the cross sectional area. But because of lack of information, average value of q measured by the rotameter is used to make calculations. When there is no flow through the cake, hydraulic pressure on each slit is considered to be zero. Therefore hydraulic pressure on each slit with reference to slit No. 1 is also zero. Thus it is automatically assumed that the quantity dp_L/dx

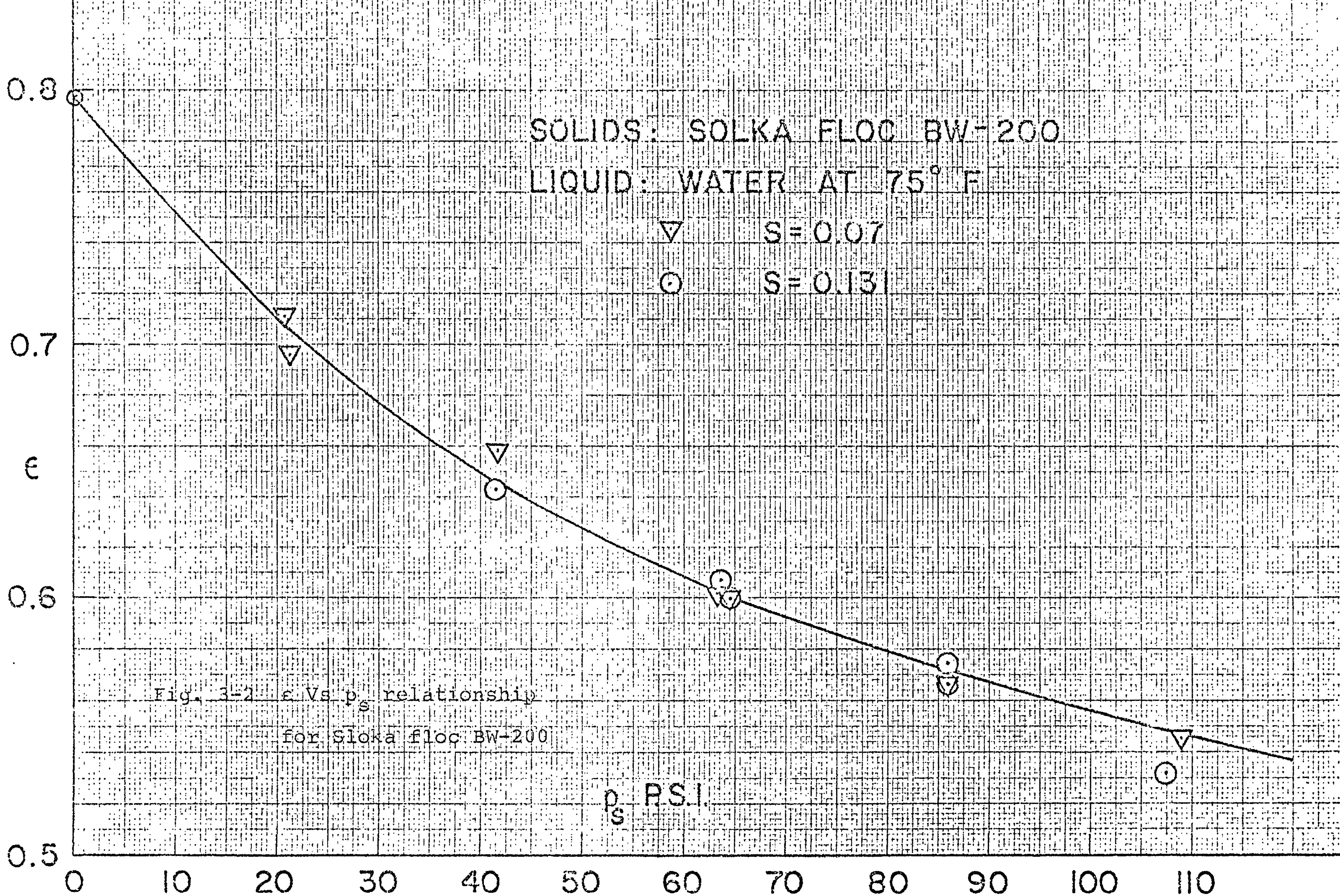


Fig. 3-2 ε Vs p_s relationship for Sloka floc BW-200

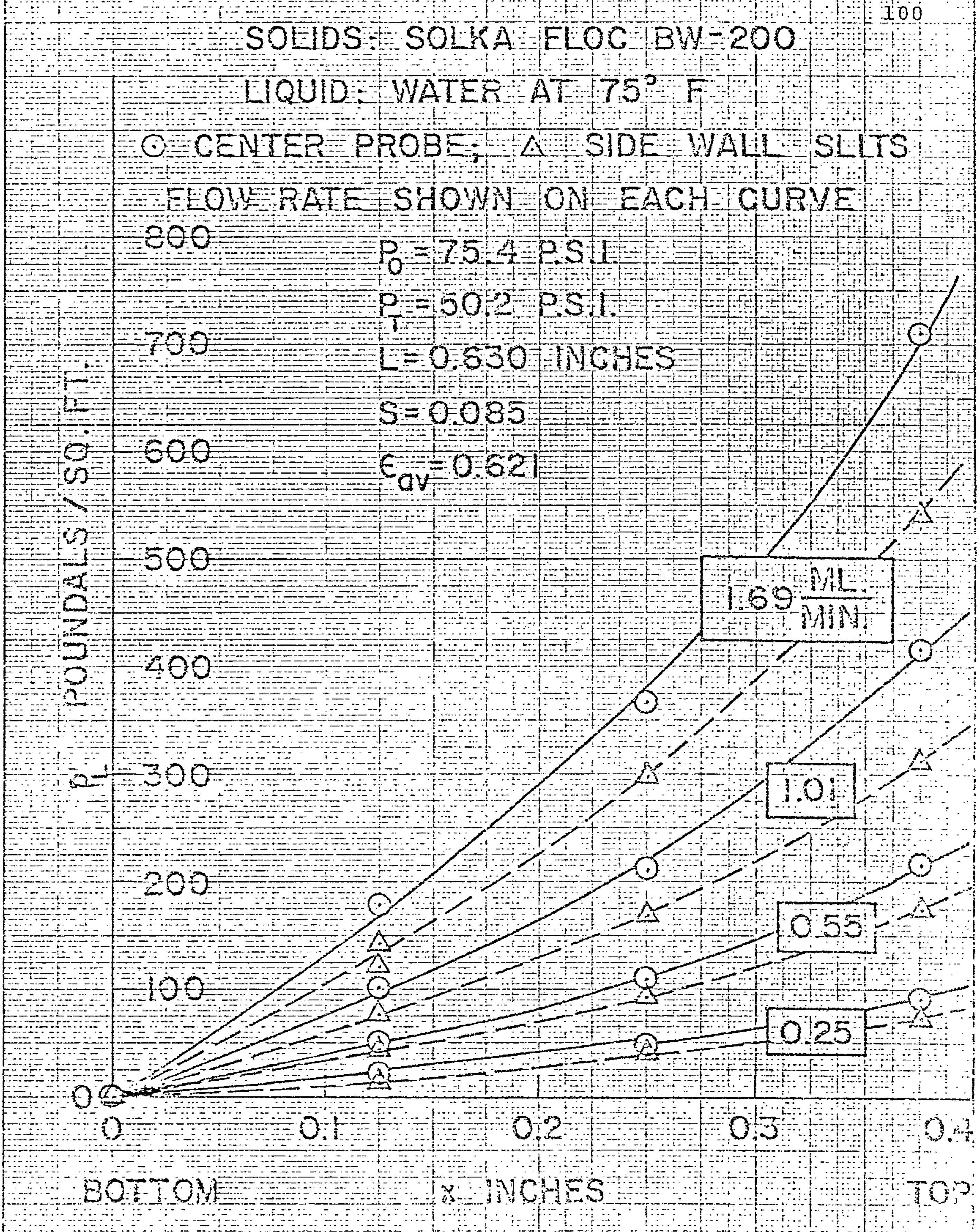


Fig. 3-3 Hydraulic pressure profiles in a 0.63" thick cake

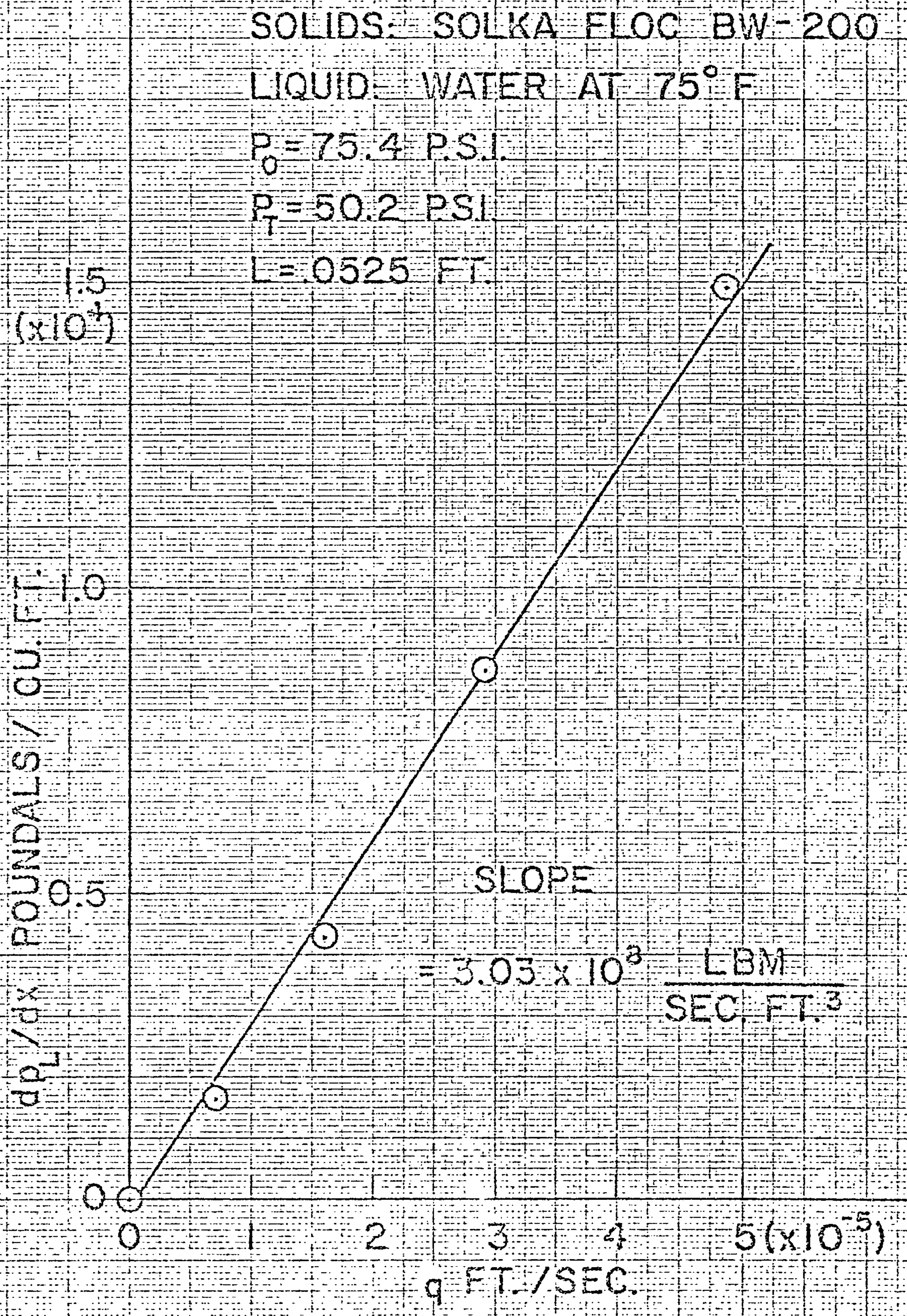


Fig. 3-4 $(dp_L/dx)_{x=0}$ Vs q

is zero when q is zero. A straight line is drawn through these four data points and the origin. Slope of this line is 3.033×10^8 lbm/(sec. ft^2) and is to be used in Eqn. (3-3) to calculate α. It is necessary to know p_s at x=0 on the probe.

Fig. (2-3) is the only available information for this purpose. Presence of the probe is neglected in order to calculate p_s, and it is assumed that Fig. (2-3) is also valid for 2" diameter cell. Also it is necessary to make linear extrapolations for P_0's different from 40 and 80 p.s.i. For L/D = 0.315 (L = 0.63 inches). $P_{VB0}/P_0 = 0.83$ at P_0 = 75.4 p.s.i. Therefore required p_s is 0.83 x 75.4 = 62.58 p.s.i. Porosity ε=0.604 at this p_s is obtained from Fig. (3-2). Substituting all relevant values in Eqn. (3-4),

$$\alpha = \frac{1}{6.72 \times 10^{-4} \times 94.78 \times (1-0.604)} \times 3.033 \times 10^{8}$$

$$= 1.203 \times 10^{10} \ \text{ft/lbm} \quad \text{at } p_s = 62.58 \text{ p.s.i.}$$

In this manner, α for every experimental run is calculated and is plotted as a function of p_s with slurry concentration s as a parameter in Fig. (3-5).

3.5.2 CALCULATION OF R_m

When the top of a cake is close to slit No. 4, the difference between reference reading on slit No. 4, (when there is no flow) and the reading on slit No. 4 when there is flow

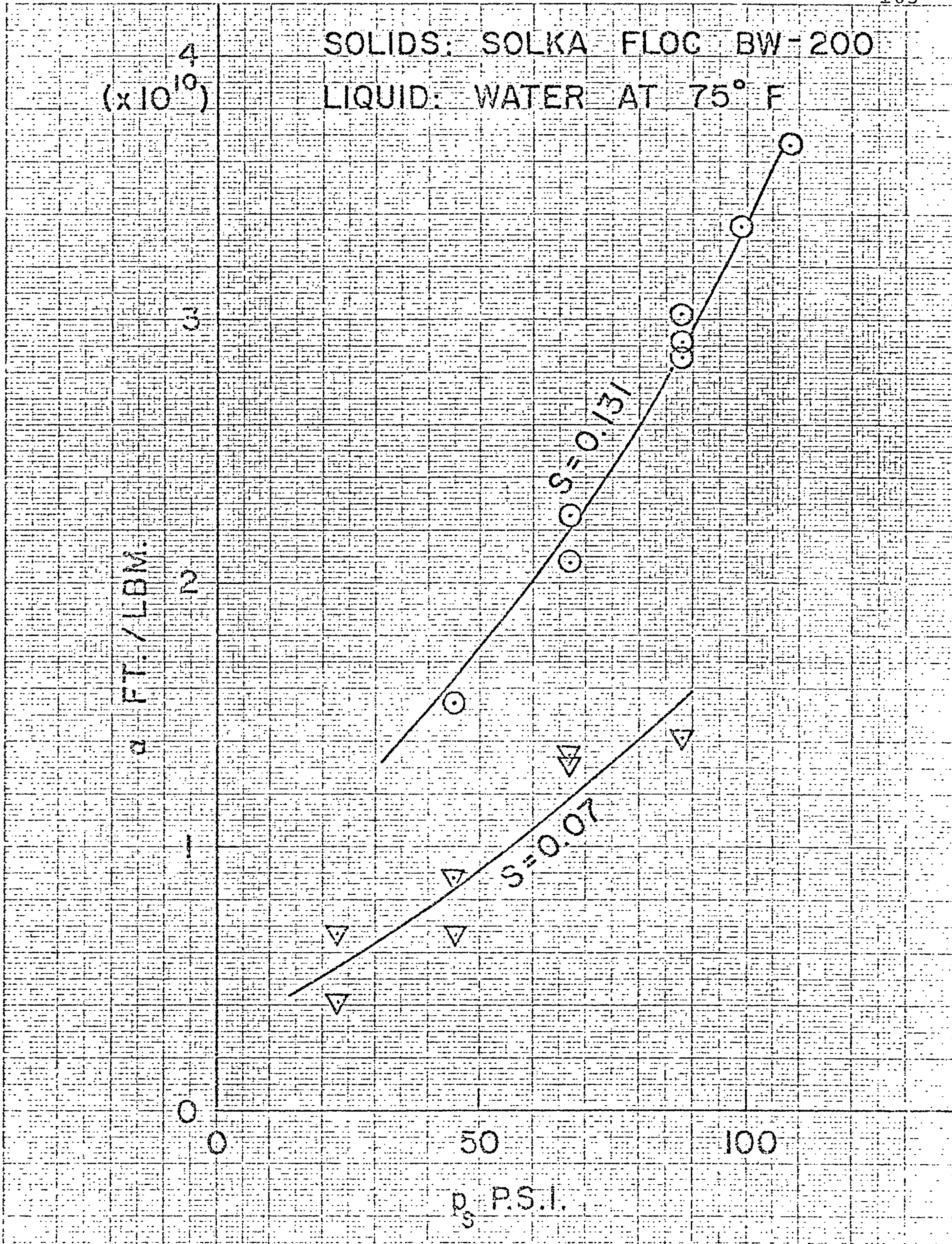


Fig. 3-5 α Vs p_s with s as a parameter for Solka floc is essentially due to pressure drop across PP1. Slit No. 4 is at 0.415" from the cake bottom and at 0.375" from slit NO. 1. Considering Fig. (3-6) it is seen that the thickness of filter cake between slit No. 4 and PP1 is (0.433- 0.415) or 0.018". In Fig. (3-6), extrapolating the pressure profile on the center probe for Q = 1.01 ml/min (9 units on rotometer) to x= 0.393" (0.375" + 0.018"), it is seen that pressure drop across the top 0.018" of cake is 40 poundals/ft^2. The detailed experimental data relevant to Fig. (3-7) are illustrated in Table (4-1) in the Appendix. The difference in readings on slit No. 4 due to a flow of 1.01 ml/min. is 1874 microstrains, or 2151 poundals/ft^2, out of which 50 poundals/ft^2 is through the cake between slit No. 4 and PP1. Therefore pressure drop across PP1 alone is Δp_L = 2101 poundals/ft^2. Applying Eqn. (3-9), $q = 2.8781 \times 10^{-5} \times 1.01 = 2.9069 \times 10^{-5}$ ft/sec. Substituting all relevant quantities in Eqn. (3-8),

$$R_m = \frac{2101}{6.72 \times 10^{-4} \times 2.9069 \times 10^{-5}} = 1.076 \times 10^{11} \text{ /ft.}$$

In this manner R_m can be calculated when PP1 is close to slit No. 4. R_m is in the range of 10^{10} to 10^{11} /ft.

3.6 DISCUSSION OF THE RESULTS AND CONCLUSIONS

The measurements of hydraulic pressure profile in a filter cake are repeatable with in 3% of the average value, as can be seen from majority of the data presented in the appendix. All data points on α Vs p_s curve for s = 0.131, are

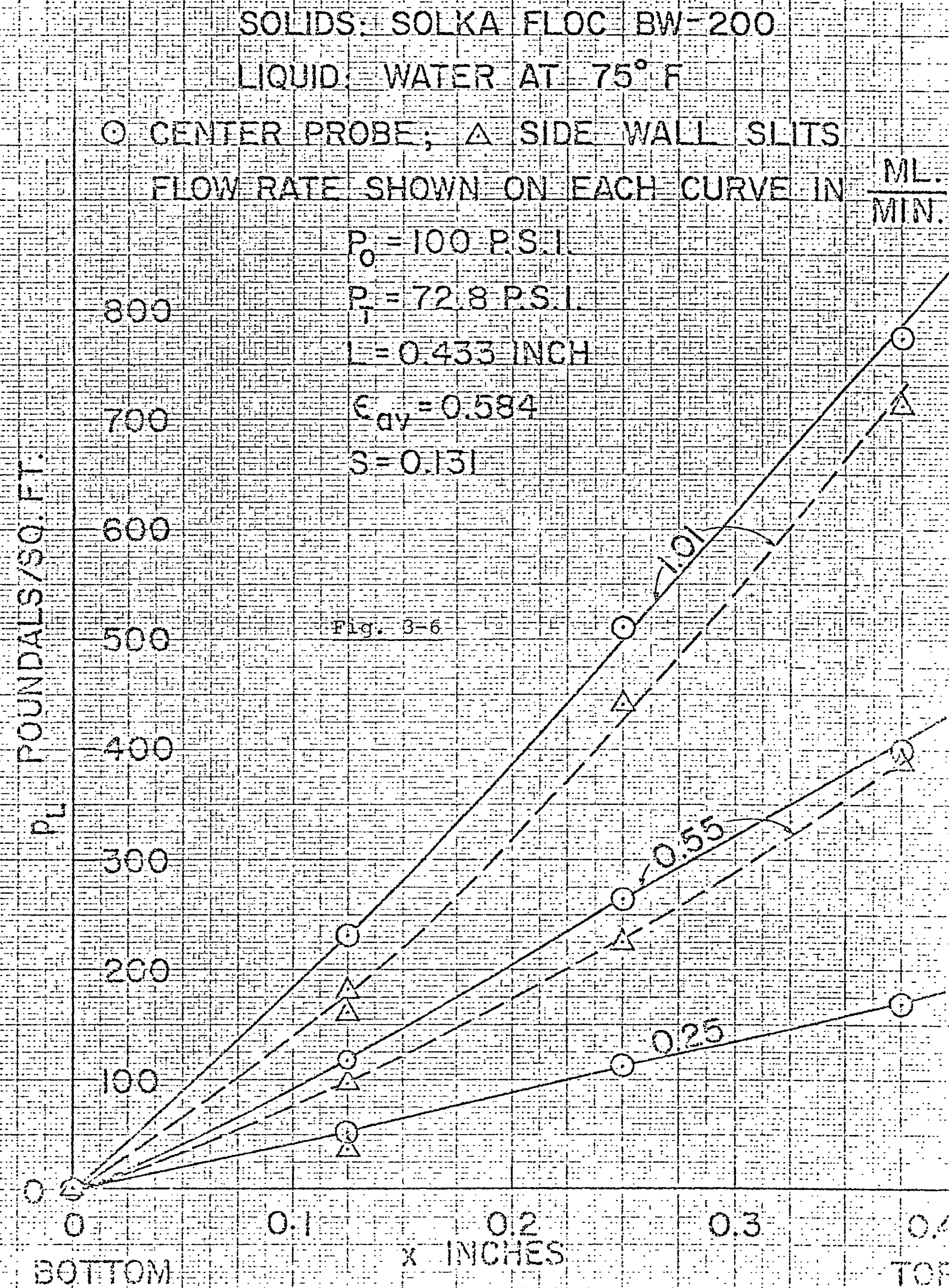

SOLIDS: SOLKA FLOC BW-200
LIQUID: WATER AT 75° F
⊙ CENTER PROBE; △ SIDE WALL SLITS
FLOW RATE SHOWN ON EACH CURVE IN ML./MIN.
P0 = 100 P.S.I.
P1 = 72.8 P.S.I.
L = 0.433 INCH
εav = 0.584
S = 0.131
PL POUNDALS/SQ. FT.
800
700
600
500
400
300
200
100
0
1.01
0.55
0.25
0
0.1
0.2
0.3
0.4
X INCHES
BOTTOM
TOP

Fig. 3-6

within 3% of a smooth curve. The reproducibility of α Vs p_s is not that good for s = 0.07.

The slurry concentration s has a marked effect on α Vs p_s data but no effect on ε Vs p_s data. A possible explanation can be given as follows. At a lower s, liquid volume is more for the same mass of solids and the depth of slurry poured in the cylinder is greater. As mentioned previously, same stratification is inevitable and greater the depth of slurry, more effective is the stratification. As a result, there is a larger deficit of fine particles near the bottom where α is measured. Finer the particles in a cake structure, higher is the α value. Thus α value is considerably less for a lower s. However the porosity ε measured is the average value and is apparently unaffected by a small stratification.

R_m obtained in this investigation is much higher (10^{10} - 10^{11} /ft) than that reported by Lu(1968) (10^9- 10^{10} /ft). This is because he measured R_m of a clean medium only. In this investigation R_m is measured when cake is in contact with the medium.

It can be seen from Figures 3-3, 3-6 and also from the data tabulated in the appendix that slits 6 and 7 which are at the same distance from slit No. 5, registered hydraulic pressures within 5% of each other. This fact tends to verify the cylindrical symmetry of the flow. In other words hydraulic pressure inside a cake does not vary with angle θ, but varies with r and z in a cylindrical coordinate system.

While making pressure measurements on the slits, it was verified that the difference between pressures registered by slits 1 and 5 is less than 2% of the pressure drop from slit Nos. 4 to 1. Slit 1 is on the center probe and slit No. 5 is on the wall. Both slit No. 1 and 5 are at 0.04" from the bottom. From Fig. 3-3 it is seen that for a cake thickness of 0.63" the distance between the center probe readings and the side wall readings becomes wider from x=0 to x=0.375". In Fig. (3-6) the cake thickness is 0.433" and top of the cake is close to x=0.375". The distance between the center probe readings and the side wall readings narrows near the cake top. Profiles drawn by using other data in the appendix support this fact. From this information the flow field inside a cake in a CP cell can be speculated to be as shown in Fig. (3-7). Flow is unidirectional at the top and at the bottom and deviates from unidirectionality in the middle. After making the first hydraulic pressure measurements in some experimental runs, the cake was allowed to remain under the same P_0 for several hours without liquid flow. After the end of that period, liquid flow was started. It was found that p_L at the top slit increased between 10% to 25% but dp_L/dx at x=0 did not change significantly and thus the measured α was the same. In one run, a continuous flow was maintained for 35 minutes. The hydraulic pressure profile did not change.

3.7 RECOMMENDATIONS FOR FURTHER WORK

In order to prepare a cake with identical particle size

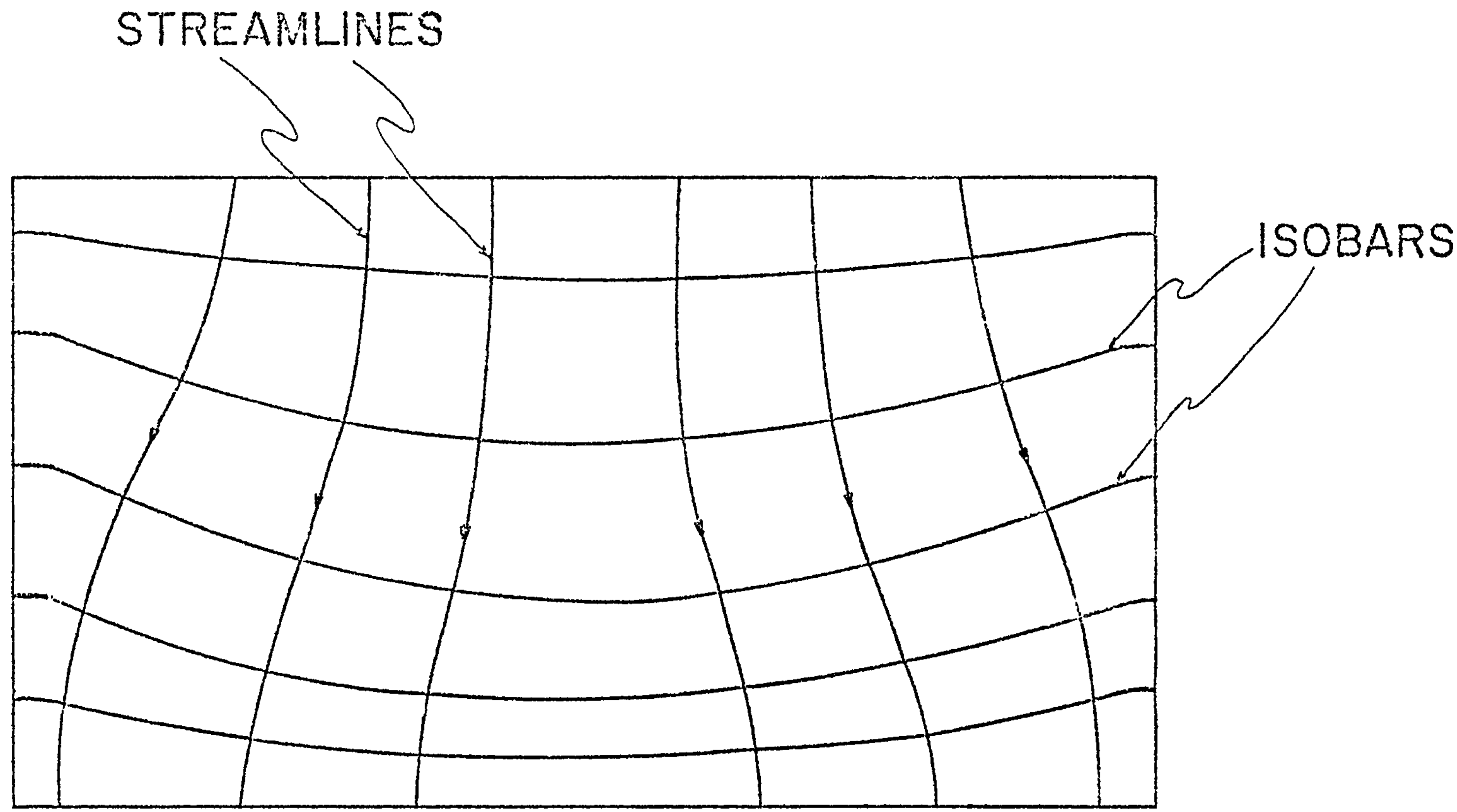


Fig. 3-7 SPECULATED FLOW FIELD IN A COMPACTED FILTER CAKE

distribution everywhere inside, (stratification changes the size distribution from one region inside the cake to another) it is better to lay the cake by Cooper's caking piston method. His caking piston should be modified to fit the new CP cell.

Presently developed apparatus works well only with easily filtrable materials like Solka floc BW-200. More resistant materials like $CaCO_3$ clog the slits and the system response is considerably slowed down. In order to overcome this problem the author has designed an improved probe (detailed diagrams can be found in section 4.4.3 in appendix). Particles are stopped by tiny filter paper strips at the probe surface and they don't enter the slits. A thin stainless steel casing with slits on it covers the probe. Slits on the casing match the slits on the probe. The author encourages the future investigators to use this type of probe.

It is desirable to develop a 4" diameter CP cell with about 1" diameter replaceable probes having eight or more slits. Each such replaceable probe will have different spacing of slits.

This idea of centerprobe can be extended to measure hydraulic pressure profile in a cake during filtration.

CHAPTER 4

APPENDIX

```
C***********************************************************************
C       NUMERICAL SOLUTION FOR SQUEEZING OF FILTER CAKE                *
C                                                                      *
C       INTEGRODIFFERENTIAL EQUATION WITH POLYNOMIALS RELATIONSHIPS IN *
C       POROSITY (EPS) VS SOLID COMPRESSIVE PRESSURE AND SPECIFIC CAKE *
C       RESISTANCE(ALPHA) VS SOLID COMPRESSIVE PRESSURE                *
C                                                                      *
C       DIMENSIONLESS MASS OF DRY SOLIDS AND DIMENSIONLESS TIME ARE    *
C       INDEPENDANT VARIABLES                                          *
C***********************************************************************
      DIMENSION PHI1(900), PHI2(900),NAME(20)
      REAL INTEG,MU,LEN1
      READ(5,11) (NAME(I),I=1,20),ALPHAO,A1,A2,A3,E0,E1,E2,E3
   11 FORMAT(20A4,/(8E10.4))
  300 READ(5,13,END=400) C,DELT,KMAX,N,MU,P1,PHIGH,PLOW
   13 FORMAT(F10.3,E10.4,2I10,E10.4,3F10.3)
      WRITE(6,12) (NAME(I),I=1,20),ALPHAO,A1,A2,A3,E0,E1,E2,E3,KMAX,
     1 MU,P1,C,DELT,N,PHIGH,PLOW
   12 FORMAT(1H1,//,6X,9HMATERIAL:,20A4,//,6X,7HALPHAO=,E10.4,2X,3HA1=,
     1 E10.4,2X,3HA2=,E10.4,2X,3HA3=,E10.4,3X,3HE0=,E10.4,2X,3HE1=,
     2 E10.4,/6X,3HE2=,E10.4,3X,3HE3=,E10.4,3X,5HKMAX=,I5,3X,3HMU=,
     3 E10.4, 3X,3HP1=,F5.1,3X,2HC=,E10.4,/6X,5HDELT=,E10.4,3X,2HN=,I3,
     4 3X,6HPHIGH=,F5.2,3X,5HPLOW=,F5.2)
      A=1.-E0
      B=MU**2*ALPHAO/(P1*32.2*144.)*A
      FN=FLOAT(N)
      DELW=1./FN
      DEWAT=DELW*A/(2.*DELT)
      DETAW=1./DEWAT
      CBYB=C/B
      TODEWB=2.*DELW*B
      TOCW=2.*C*DELW
C   UNITS OF P1 IN THE FOLLOWING EQUATIONS SHOULD BE IN P.S.I.
      A1P1=A1*P1
      A2P1SQ=A2*P1**2
      A3P1CU=A3*P1**3
      E1P1=E1*P1
      E2P1SQ=E2*P1**2
      E3P1CU=E3*P1**3
      PHIO1=PHIGH/P1
      NM1=N-1
      NM2=N-2
      W=0
      DO 10 I=1,NM1
      W=W+DELW
      PHI1(I)=PLOW/P1+(PHIO1-PLOW/P1)*(1.-W)**2
   10 CONTINUE
      PHI1(N)=PLOW/P1
      INCR=N/10
      WRITE(6,14) (J,J=INCR,N,INCR)
   14 FORMAT(/,9X,'TABULATION OF DIMENSIONLESS QUANTITIES:',//,46X,
     1 'SOLIDS PRESSURES  PHI(I)',/,8X,4HTIME,5X,6HLENGTH,2X,5H(   0),
     2 10(2X,1H(,I3,1H)),/)
      K=0
```

```
      M=0
      PHIO3=PHIO1
      TIME=0
      INTEG=1./(A-E1P1*PHIO1-E2P1SQ*PHIO1**2-E3P1CU*PHIO1**3)
      DO 20 I=1,NM1,2
   20 INTEG=INTEG+4./(A-E1P1*PHI1(I)-E2P1SQ*PHI1(I)**2-
     1 E3P1CU*PHI1(I)**3)
      DO 30 I=2,NM2,2
   30 INTEG=INTEG+2./(A-E1P1*PHI1(I)-E2P1SQ*PHI1(I)**2
     1  -E3P1CU*PHI1(I)**3)
      INTEG=INTEG+1./(A-E1P1*PHI1(N)-E2P1SQ*PHI1(N)**2-E3P1CU*
     1  PHI1(N)**3)
      LEN1=A*DELW/3.*INTEG
  100 IF(K.GT.KMAX) GO TO 300
      IF(K/200.LT.M) GO TO 200
      WRITE(6,16) TIME,LEN1,PHIO1,(PHI1(I),I=INCR,N,INCR)
      IF(PHI1(N).GE.0.990) GO TO 300
      M=M+1
  200 H=CBYB*(PHIO1-1.)
      PHIO2=PHIO1+(PHIO1-PHIO3)
      ITR=1
   40 DEDPI1=E1P1+2.*E2P1SQ*PHIO1+3.*E3P1CU*PHIO1**2
      DEDPI2=E1P1+2.*E2P1SQ*PHI1(1)+3.*E3P1CU*PHI1(1)**2
      ONMEP1=A-E1P1*PHIO1-E2P1SQ*PHIO1**2-E3P1CU*PHIO1**3
      ONMEP2=A-E1P1*PHI1(1)-E2P1SQ*PHI1(1)**2-E3P1CU*PHI1(1)**3
      DEPIO1=DEDPI1/ONMEP1**2
      DEPIO2=DEDPI2/ONMEP2**2
      ALBYAO=1.+A1P1*PHI1(1)+A2P1SQ*PHI1(1)**2+A3P1CU*PHI1(1)**3
      PHIWBA=(PHIO1-PHI1(2))/(TODEWB*ALBYAO)
      PHI2(1)=PHI1(1)+(DETAW*(PHIWBA+H)-DEPIO1*(PHIO2-PHIO1))/DEPIO2
      G=DEWAT*(DEPIO1*(PHIO2-PHIO1)+DEPIO2*(PHI2(1)-PHI1(1)))
      DEPIO1=DEPIO2
      DEDPI2=E1P1+2.*E2P1SQ*PHI1(2)+3.*E3P1CU*PHI1(2)**2
      ONMEP2=A-E1P1*PHI1(2)-E2P1SQ*PHI1(2)**2-E3P1CU*PHI1(2)**3
      DEPIO2=DEDPI2/ONMEP2**2
      ALBYAO=1.+A1P1*PHI1(2)+A2P1SQ*PHI1(2)**2+A3P1CU*PHI1(2)**3
      PHIWBA=(PHI1(1)-PHI1(3))/(TODEWB*ALBYAO)
      PHI2(2)=PHI1(2)+(DETAW*(PHIWBA-G+H)-DEPIO1*(PHI2(1)
     1  -PHI1(1)))/DEPIO2
      ALBYAO=1.+A1P1*PHIO2+A2P1SQ*PHIO2**2+A3P1CU*PHIO2**3
      TOCWAP=TOCW*ALBYAO*(PHIO2-1.)
      FUNPHI=3.*PHIO2-4.*PHI2(1)+PHI2(2)
      RSDUE=TOCWAP+FUNPHI
      IF(ABS(RSDUE).LE.1.0E-05) GO TO 50
      ITR=ITR+1
      IF(ITR.GT.20) GO TO 300
      PHIO2=PHIO2-0.134*RSDUE
      GO TO 40
   50 G=DEWAT*(DEPIO1*(PHI2(1)-PHI1(1))+DEPIO2*(PHI2(2)-PHI1(2)))+G
      DO 60 I=3,NM1
      DEPIO1=DEPIO2
      DEDPI2=E1P1+2.*E2P1SQ*PHI1(I)+3.*E3P1CU*PHI1(I)**2
      ONMEP2=A-E1P1*PHI1(I)-E2P1SQ*PHI1(I)**2-E3P1CU*PHI1(I)**3
      DEPIO2=DEDPI2/ONMEP2**2
```

```
      ALBYAO=1.+A1P1*PHI1(I)+A2P1SQ*PHI1(I)**2+A3P1CU*PHI1(I)**3
      PHIWBA=(PHI1(I-1)-PHI1(I+1))/(TODEWB*ALBYAO)
      PHI2(I)=PHI1(I)+(DETAW*(PHIWBA-G+H)-DEPIO1*(PHI2(I-1)-
     1   PHI1(I-1)))/DEPIO2
      G=DEWAT*(DEPIO1*(PHI2(I-1)-PHI1(I-1))+DEPIO2*(PHI2(I)-PHI1(I)))+G
   60 CONTINUE
      DO 70 I=1,NM1
   70 PHI1(I)=PHI2(I)
      PHI1(N)=(4.*PHI1(NM1)-PHI1(NM2))/3.
      TIME=TIME+DELT
      LEN1=LEN1+DELT*H
      PHIO3=PHIO1
      PHIO1=PHIO2
      K=K+1
      GO TO 100
  400 STOP
   16 FORMAT(6X,E10.5,1X,F6.5,11(1X,F6.5))
      END
```

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHA0=0.1200E 11   A1=0.2886E-01   A2=0.4122E-03   A3=-.2468E-06   EC=0.8333E 00   E1=-.4730E-02
E2=0.5195E-04   E3=-.2238E-06   KMAX=30000   MU=0.6720E-03   P1= 15.0   C=0.1000E 01
DELT=0.2000E-05   N= 20   PHIGH= 0.20   PLOW= 0.02

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(2)	(4)	(6)	(8)	(10)	(12)	(14)	(16)	(18)	(20)
.0	.99774	.01333	.01105	.00901	.00721	.00565	.00433	.00325	.00241	.00181	.00145	.00133
.59996E-03	.96187	.31509	.24251	.18175	.13245	.09378	.06453	.04331	.02864	.01922	.01402	.01238
.11999E-02	.93253	.40793	.34108	.28177	.23010	.18602	.14937	.11995	.09746	.08166	.07229	.06920
.17998E-02	.90674	.47388	.41235	.35690	.30770	.26488	.22854	.19877	.17559	.15903	.14911	.14582
.23998E-02	.88377	.53100	.47467	.42364	.37812	.33828	.30430	.27631	.25442	.23874	.22931	.22618
.29997E-02	.86332	.58368	.53249	.48601	.44446	.40803	.37689	.35120	.33109	.31666	.30798	.30511
.35996E-02	.84522	.63271	.58657	.54460	.50702	.47404	.44582	.42252	.40427	.39117	.38329	.38069
.41991E-02	.82931	.67803	.63677	.59918	.56549	.53588	.51053	.48959	.47317	.46138	.45429	.45195
.47982E-02	.81539	.71948	.68287	.64948	.61951	.59315	.57055	.55188	.53723	.52671	.52038	.51829
.53972E-02	.80331	.75699	.72476	.69531	.66886	.64556	.62558	.60905	.59608	.58677	.58117	.57932
.59962E-02	.79286	.79060	.76241	.73664	.71345	.69301	.67546	.66094	.64955	.64136	.63643	.63480
.65952E-02	.78388	.82042	.79594	.77352	.75333	.73552	.72022	.70755	.69760	.69045	.68615	.68473
.71943E-02	.77619	.84667	.82552	.80614	.78866	.77324	.75998	.74900	.74037	.73416	.73042	.72919
.77933E-02	.76964	.86958	.85141	.83474	.81971	.80642	.79500	.78552	.77808	.77273	.76950	.76844
.83923E-02	.76408	.88944	.87391	.85965	.84677	.83538	.82559	.81746	.81107	.80648	.80371	.80280
.89913E-02	.75937	.90656	.89334	.88118	.87020	.86049	.85213	.84519	.83974	.83581	.83345	.83267
.95904E-02	.75539	.92123	.91001	.89969	.89037	.88211	.87501	.86911	.86447	.86113	.85912	.85846
.10189E-01	.75204	.93375	.92426	.91553	.90764	.90065	.89463	.88963	.88569	.88286	.88116	.88059
.10788E-01	.74922	.94438	.93638	.92902	.92235	.91645	.91136	.90714	.90382	.90143	.89999	.89951
.11387E-01	.74686	.95339	.94666	.94046	.93484	.92987	.92559	.92203	.91923	.91721	.91600	.91561
.11986E-01	.74488	.96099	.95534	.95013	.94542	.94124	.93764	.93464	.93229	.93059	.92956	.92922
.12585E-01	.74322	.96739	.96265	.95828	.95433	.95082	.94780	.94529	.94331	.94189	.94104	.94075
.13185E-01	.74183	.97277	.96880	.96514	.96183	.95889	.95636	.95426	.95260	.95141	.95069	.95045
.13784E-01	.74067	.97727	.97396	.97090	.96813	.96567	.96355	.96179	.96040	.95940	.95880	.95860
.14383E-01	.73970	.98105	.97828	.97572	.97341	.97135	.96958	.96811	.96695	.96612	.96561	.96545
.14982E-01	.73890	.98420	.98189	.97976	.97783	.97611	.97463	.97340	.97243	.97174	.97132	.97118
.15581E-01	.73822	.98684	.98491	.98313	.98152	.98009	.97886	.97783	.97702	.97644	.97608	.97596
.16180E-01	.73766	.98904	.98743	.98595	.98461	.98341	.98238	.98153	.98085	.98037	.98008	.97998
.16779E-01	.73719	.99088	.98954	.98830	.98718	.98619	.98533	.98462	.98405	.98365	.98340	.98332
.17378E-01	.73679	.99241	.99129	.99026	.98933	.98850	.98778	.98719	.98672	.98638	.98618	.98611
.17977E-01	.73647	.99368	.99275	.99190	.99112	.99043	.98983	.98934	.98895	.98867	.98849	.98844
.18576E-01	.73619	.99474	.99397	.99326	.99261	.99204	.99154	.99113	.99080	.99056	.99042	.99037

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHA0=0.1200E 11 A1=0.2886E-01 A2=0.4122E-03 A3=-.2468E-06 E0=0.8333E 00 E1=-.4730E-02
E2=0.5195E-04 E3=-.2238E-06 KMAX=30000 MU=0.6720E-03 P1= 15.0 C=0.3000E 01
DELT=0.2000E-05 N= 20 PHIGH= 0.20 PLOW= 0.02

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(2)	(4)	(6)	(8)	(10)	(12)	(14)	(16)	(18)	(20)
.0	.99774	.01333	.01105	.00901	.00721	.00565	.00433	.00325	.00241	.00181	.00145	.00133
.39998E-03	.94546	.57245	.42159	.29783	.20098	.12906	.07860	.04523	.02452	.01265	.00677	.00518
.79994E-03	.91106	.67065	.54594	.43565	.34035	.26017	.19478	.14348	.10526	.07902	.06380	.05891
.11999E-02	.88327	.72447	.61663	.51884	.43173	.35581	.29141	.23872	.19783	.16872	.15134	.14564
.15999E-02	.85969	.76394	.66958	.58311	.50522	.43651	.37751	.32865	.29030	.26275	.24618	.24073
.19998E-02	.83944	.79732	.71493	.63904	.57031	.50939	.45684	.41314	.37873	.35393	.33900	.33409
.23998E-02	.82209	.82682	.75541	.68938	.62938	.57606	.52994	.49153	.46124	.43938	.42622	.42189
.27997E-02	.80732	.85299	.79161	.73466	.68279	.63658	.59656	.56317	.53682	.51780	.50634	.50257
.31997E-02	.79481	.87602	.82368	.77498	.73052	.69084	.65642	.62767	.60496	.58856	.57868	.57544
.35996E-02	.78429	.89606	.85176	.81044	.77263	.73883	.70947	.68493	.66552	.65150	.64305	.64028
.39994E-02	.77550	.91332	.87607	.84125	.80933	.78075	.75589	.73509	.71864	.70674	.69957	.69722
.43988E-02	.76818	.92805	.89691	.86774	.84096	.81695	.79605	.77854	.76468	.75466	.74861	.74664
.47982E-02	.76212	.94051	.91461	.89031	.86797	.84792	.83044	.81579	.80419	.79579	.79073	.78907
.51975E-02	.75711	.95098	.92953	.90938	.89083	.87416	.85963	.84743	.83777	.83077	.82655	.82517
.55969E-02	.75299	.95973	.94203	.92538	.91004	.89625	.88421	.87411	.86609	.86029	.85679	.85565
.59962E-02	.74961	.96699	.95243	.93873	.92609	.91471	.90478	.89644	.88982	.88503	.88214	.88118
.63956E-02	.74684	.97300	.96105	.94980	.93942	.93007	.92189	.91503	.90958	.90564	.90325	.90247
.67949E-02	.74457	.97795	.96817	.95895	.95044	.94277	.93607	.93044	.92597	.92273	.92077	.92013
.71943E-02	.74273	.98202	.97403	.96649	.95953	.95325	.94777	.94315	.93949	.93684	.93524	.93471
.75936E-02	.74122	.98535	.97883	.97268	.96700	.96187	.95738	.95361	.95062	.94845	.94714	.94671
.79930E-02	.73999	.98808	.98277	.97775	.97311	.96893	.96527	.96220	.95975	.95798	.95691	.95656
.83923E-02	.73899	.99030	.98598	.98190	.97812	.97471	.97173	.96922	.96723	.96579	.96492	.96463
.87917E-02	.73817	.99212	.98860	.98528	.98221	.97943	.97701	.97497	.97334	.97216	.97145	.97121
.91910E-02	.73751	.99360	.99074	.98804	.98554	.98328	.98131	.97965	.97833	.97737	.97679	.97660
.95904E-02	.73697	.99480	.99248	.99028	.98825	.98642	.98481	.98346	.98239	.98161	.98114	.98098
.99897E-02	.73653	.99578	.99389	.99211	.99046	.98897	.98767	.98657	.98570	.98506	.98468	.98455
.10389E-01	.73617	.99657	.99504	.99359	.99225	.99104	.98998	.98909	.98838	.98787	.98756	.98746
.10788E-01	.73588	.99722	.99598	.99480	.99372	.99273	.99187	.99115	.99057	.99016	.98990	.98982
.11188E-01	.73565	.99774	.99674	.99578	.99490	.99410	.99340	.99281	.99234	.99200	.99180	.99173

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHA0=0.1200E 11 A1=0.2886E-01 A2=0.4122E-03 A3=-.2468E-06 E0=0.8333E 00 E1=-.4730E-02
E2=0.5195E-04 E3=-.2238E-06 KMAX= 3000 MU=0.6720E-03 P1= 15.0 C=0.3000E 01
DELT=0.5000E-06 N= 60 PHIGH= 0.20 PLOW= 0.02

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(6)	(12)	(18)	(24)	(30)	(36)	(42)	(48)	(54)	(60)
.0	.99774	.01333	.01105	.00901	.00721	.00565	.00433	.00325	.00241	.00181	.00145	.00133
.49999E-04	.98850	.28503	.10569	.03140	.01075	.00617	.00456	.00346	.00259	.00188	.00133	.00117
.99998E-04	.98078	.37079	.18930	.08291	.03223	.01264	.00613	.00388	.00279	.00208	.00167	.00159
.15000E-03	.97385	.42724	.25042	.13180	.06240	.02733	.01191	.00581	.00343	.00242	.00197	.00189
.20000E-03	.96748	.46925	.29816	.17454	.09379	.04657	.02189	.01027	.00523	.00317	.00238	.00221
.24999E-03	.96155	.50250	.33707	.21169	.12391	.06764	.03474	.01717	.00857	.00469	.00313	.00275
.29998E-03	.95596	.52983	.36972	.24421	.15203	.08907	.04927	.02602	.01347	.00723	.00449	.00377
.34997E-03	.95066	.55290	.39771	.27294	.17802	.11012	.06466	.03625	.01973	.01085	.00667	.00549
.39996E-03	.94561	.57276	.42210	.29856	.20200	.13043	.08037	.04742	.02712	.01554	.00979	.00809
.44995E-03	.94078	.59011	.44362	.32159	.22412	.14984	.09605	.05919	.03542	.02123	.01388	.01165
.49994E-03	.93613	.60547	.46283	.34245	.24459	.16830	.11151	.07131	.04445	.02782	.01894	.01618
.54993E-03	.93165	.61919	.48012	.36146	.26358	.18585	.12665	.08363	.05405	.03522	.02491	.02166
.59991E-03	.92732	.63156	.49581	.37890	.28127	.20251	.14141	.09603	.06411	.04332	.03173	.02804
.64990E-03	.92313	.64280	.51014	.39498	.29779	.21837	.15577	.10845	.07453	.05205	.03932	.03523
.69989E-03	.91906	.65308	.52332	.40988	.31330	.23348	.16974	.12084	.08524	.06130	.04760	.04317
.74988E-03	.91511	.66254	.53551	.42378	.32791	.24792	.18333	.13318	.09619	.07103	.05649	.05176
.79987E-03	.91126	.67130	.54684	.43679	.34172	.26176	.19658	.14543	.10732	.08114	.06591	.06092
.84986E-03	.90751	.67945	.55743	.44903	.35484	.27506	.20950	.15761	.11861	.09160	.07577	.07057
.89985E-03	.90384	.68708	.56738	.46060	.36735	.28788	.22213	.16970	.13000	.10233	.08604	.08069
.94984E-03	.90027	.69426	.57677	.47159	.37932	.30028	.23449	.18171	.14149	.11331	.09665	.09115
.99982E-03	.89678	.70104	.58568	.48206	.39081	.31229	.24660	.19362	.15304	.12448	.10754	.10194
.10498E-02	.89336	.70748	.59416	.49209	.40189	.32396	.25849	.20545	.16465	.13581	.11866	.11299
.10998E-02	.89001	.71362	.60227	.50172	.41259	.33533	.27018	.21719	.17628	.14728	.12998	.12426
.11498E-02	.88673	.71949	.61005	.51100	.42297	.34644	.28169	.22885	.18793	.15884	.14146	.13570
.11998E-02	.88353	.72513	.61755	.51998	.43306	.35730	.29302	.24043	.19959	.17048	.15307	.14728
.12498E-02	.88038	.73057	.62479	.52868	.44290	.36795	.30421	.25193	.21123	.18218	.16477	.15897
.12998E-02	.87730	.73583	.63181	.53715	.45250	.37840	.31524	.26334	.22286	.19391	.17654	.17076
.13497E-02	.87427	.74092	.63863	.54540	.46190	.38867	.32615	.27467	.23446	.20566	.18837	.18262
.13997E-02	.87130	.74588	.64527	.55346	.47111	.39878	.33693	.28592	.24602	.21742	.20022	.19450
.14497E-02	.86839	.75070	.65175	.56134	.48015	.40873	.34759	.29709	.25755	.22917	.21209	.20641
.14997E-02	.86554	.75541	.65809	.56907	.48903	.41856	.35814	.30819	.26902	.24089	.22396	.21830

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHAO=0.1200E 11   A1=0.2886E-01   A2=0.4122E-03   A3=-.2468E-06   E0=0.8333E 00   E1=-.4730E-02
E2=0.5195E-04   E3=-.2238E-06   KMAX=30000   MU=0.6720E-03   P1= 15.0   C=0.3000E 01
DELT=0.2000E-05   N= 20   PHIGH= 0.50   PLOW= 0.05

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(2)	(4)	(6)	(8)	(10)	(12)	(14)	(16)	(18)	(20)
.0	.99439	.03333	.02763	.02253	.01803	.01413	.01083	.00813	.00603	.00453	.00363	.00333
.39998E-03	.94303	.57850	.42919	.30628	.20964	.13742	.08629	.05208	.03055	.01801	.01170	.00996
.79994E-03	.90905	.67435	.55077	.44133	.34662	.26679	.20155	.15023	.11191	.08555	.07023	.06530
.11999E-02	.88156	.72731	.62042	.52343	.43698	.36158	.29757	.24516	.20447	.17548	.15817	.15249
.15999E-02	.85822	.76638	.67287	.58717	.50993	.44178	.38324	.33475	.29669	.26933	.25289	.24747
.19998E-02	.83819	.79949	.71790	.64272	.57463	.51425	.46216	.41885	.38473	.36015	.34534	.34047
.23998E-02	.82103	.82876	.75808	.69271	.63331	.58050	.53483	.49678	.46677	.44513	.43209	.42780
.27997E-02	.80642	.85471	.79399	.73765	.68633	.64059	.60098	.56793	.54184	.52302	.51167	.50794
.31997E-02	.79406	.87752	.82578	.77762	.73366	.69441	.66036	.63193	.60946	.59323	.58345	.58024
.35996E-02	.78368	.89736	.85358	.81274	.77538	.74196	.71294	.68867	.66949	.65562	.64727	.64453
.39994E-02	.77499	.91444	.87764	.84324	.81171	.78347	.75891	.73836	.72209	.71034	.70325	.70093
.43988E-02	.76777	.92900	.89825	.86945	.84301	.81929	.79865	.78135	.76766	.75776	.75179	.74983
.47982E-02	.76179	.94131	.91574	.89176	.86970	.84990	.83265	.81819	.80673	.79844	.79344	.79180
.51975E-02	.75685	.95165	.93048	.91060	.89229	.87584	.86149	.84946	.83992	.83301	.82885	.82748
.55969E-02	.75279	.96028	.94282	.92640	.91127	.89765	.88577	.87580	.86790	.86217	.85872	.85758
.59962E-02	.74945	.96745	.95309	.93957	.92710	.91588	.90608	.89785	.89132	.88659	.88374	.88280
.63956E-02	.74672	.97338	.96160	.95050	.94026	.93103	.92297	.91620	.91083	.90694	.90459	.90382
.67949E-02	.74449	.97826	.96862	.95953	.95114	.94357	.93696	.93141	.92700	.92380	.92187	.92124
.71943E-02	.74266	.98227	.97440	.96697	.96010	.95391	.94850	.94395	.94034	.93773	.93615	.93563
.75936E-02	.74118	.98556	.97913	.97307	.96747	.96241	.95799	.95428	.95132	.94919	.94789	.94747
.79930E-02	.73996	.98825	.98301	.97807	.97350	.96938	.96577	.96274	.96033	.95858	.95752	.95718
.83923E-02	.73898	.99045	.98618	.98216	.97844	.97508	.97214	.96966	.96770	.96627	.96541	.96512
.87917E-02	.73817	.99223	.98877	.98549	.98246	.97973	.97733	.97532	.97372	.97256	.97186	.97163
.91910E-02	.73752	.99369	.99087	.98821	.98575	.98352	.98158	.97994	.97864	.97769	.97712	.97693
.95904E-02	.73699	.99488	.99259	.99042	.98842	.98661	.98503	.98370	.98264	.98187	.98141	.98125
.99897E-02	.73656	.99584	.99398	.99222	.99060	.98913	.98784	.98676	.98590	.98528	.98490	.98477
.10389E-01	.73620	.99662	.99512	.99369	.99237	.99117	.99013	.98925	.98855	.98805	.98774	.98764
.10788E-01	.73592	.99726	.99604	.99488	.99381	.99284	.99199	.99128	.99071	.99030	.99005	.98997
.11188E-01	.73568	.99778	.99678	.99584	.99497	.99419	.99350	.99292	.99246	.99212	.99192	.99185

MATERIAL: SOLKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHAO=0.1200E 11 A1=0.2886E-01 A2=0.4122E-03 A3=-.2468E-06 E0=0.8333E 00 E1=-.4730E-02
E2=0.5195E-04 E3=-.2238E-06 KMAX=30000 MU=0.6720E-03 P1= 15.0 C=0.1500E 02
DELT=0.1000E-05 N= 40 PHIGH= 0.20 PLOW= 0.02

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.0	.99773	.01333	.01105	.00901	.00721	.00565	.00433	.00325	.00241	.00181	.00145	.00133
.29999E-03	.92694	.88322	.64608	.44813	.29266	.17897	.10231	.05499	.02840	.01497	.00919	.00784
.59992E-03	.89326	.91688	.74154	.58483	.44870	.33423	.24156	.16983	.11738	.08211	.06199	.05555
.89985E-03	.86737	.93242	.78757	.65493	.53593	.43170	.34307	.27053	.21429	.17431	.15045	.14257
.11998E-02	.84583	.94283	.81913	.70452	.60032	.50769	.42764	.36102	.30851	.27066	.24782	.24023
.14997E-02	.82751	.95128	.84509	.74599	.65527	.57408	.50347	.44437	.39755	.36365	.34315	.33632
.17996E-02	.81192	.95860	.86784	.78267	.70432	.63393	.57250	.52094	.48001	.45032	.43235	.42636
.20996E-02	.79870	.96504	.88798	.81533	.74825	.68778	.63489	.59041	.55505	.52938	.51382	.50864
.23995E-02	.78757	.97066	.90568	.84418	.78720	.73570	.69056	.65253	.62226	.60028	.58695	.58250
.26994E-02	.77826	.97551	.92108	.86937	.82132	.77779	.73956	.70731	.68162	.66294	.65161	.64783
.29994E-02	.77050	.97967	.93432	.89110	.85085	.81431	.78216	.75501	.73335	.71760	.70804	.70485
.32993E-02	.76408	.98319	.94559	.90968	.87614	.84565	.81878	.79607	.77793	.76473	.75671	.75404
.35992E-02	.75877	.98616	.95512	.92540	.89761	.87229	.84996	.83105	.81595	.80495	.79827	.79605
.38992E-02	.75441	.98863	.96310	.93861	.91567	.89475	.87627	.86061	.84810	.83898	.83344	.83159
.41987E-02	.75083	.99069	.96976	.94964	.93077	.91354	.89831	.88540	.87508	.86755	.86297	.86145
.44982E-02	.74790	.99240	.97527	.95879	.94332	.92918	.91667	.90606	.89757	.89138	.88761	.88636
.47977E-02	.74551	.99380	.97982	.96635	.95370	.94212	.93188	.92318	.91622	.91115	.90806	.90703
.50972E-02	.74356	.99495	.98356	.97258	.96225	.95279	.94442	.93731	.93162	.92747	.92494	.92410
.53967E-02	.74197	.99590	.98662	.97768	.96926	.96156	.95473	.94893	.94428	.94089	.93883	.93814
.56963E-02	.74068	.99667	.98913	.98186	.97501	.96874	.96318	.95845	.95467	.95190	.95022	.94966
.59958E-02	.73963	.99730	.99117	.98527	.97970	.97460	.97008	.96624	.96316	.96091	.95954	.95908
.62953E-02	.73878	.99781	.99284	.98804	.98352	.97938	.97571	.97259	.97008	.96826	.96714	.96677
.65948E-02	.73809	.99822	.99419	.99030	.98664	.98328	.98029	.97776	.97573	.97424	.97334	.97303
.68943E-02	.73752	.99856	.99530	.99214	.98917	.98644	.98402	.98197	.98032	.97911	.97838	.97813
.71938E-02	.73706	.99883	.99619	.99363	.99123	.98901	.98705	.98539	.98405	.98307	.98247	.98227
.74933E-02	.73669	.99905	.99691	.99485	.99289	.99110	.98951	.98816	.98708	.98628	.98580	.98564
.77929E-02	.73639	.99923	.99750	.99583	.99425	.99280	.99151	.99041	.98953	.98889	.98850	.98837
.80924E-02	.73614	.99938	.99798	.99662	.99534	.99416	.99312	.99223	.99152	.99100	.99068	.99058

MATERIAL: SULKA FLOC BW-200 (SOURCE: PH.D. THESIS BY LU (1968) CHE. U OF HOUSTON P204)

ALPHA0=0.1200E 11 A1=0.2886E-01 A2=0.4122E-03 A3=-.2468E-06 E0=0.8333E 00 E1=-.4730E-02
E2=0.5195E-04 E3=-.2238E-06 KMAX=30000 MU=0.6720E-03 P1= 25.0 C=0.1400E 00
DELT=0.2000E-05 N= 20 PHIGH= 0.20 PLOW= 0.02

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(2)	(4)	(6)	(8)	(10)	(12)	(14)	(16)	(18)	(20)
.0	.99904	.00800	.00283	.00129	.00090	.00082	.00080	.00080	.00080	.00080	.00080	.00080
.19998E-02	.96539	.09606	.08320	.07177	.06174	.05310	.04583	.03993	.03536	.03211	.03016	.02951
.39994E-02	.93381	.14719	.13458	.12330	.11334	.10471	.09741	.09143	.08678	.08346	.08146	.08080
.59962E-02	.90407	.19933	.18704	.17604	.16634	.15792	.15080	.14497	.14043	.13719	.13524	.13459
.79930E-02	.87623	.25280	.24089	.23022	.22081	.21265	.20574	.20008	.19568	.19254	.19065	.19002
.99897E-02	.85034	.30723	.29574	.28547	.27639	.26852	.26186	.25641	.25217	.24914	.24732	.24671
.11986E-01	.82640	.36215	.35116	.34132	.33263	.32509	.31871	.31349	.30942	.30652	.30477	.30419
.13983E-01	.80444	.41701	.40657	.39722	.38897	.38180	.37574	.37077	.36690	.36414	.36248	.36192
.15980E-01	.78445	.47124	.46141	.45260	.44482	.43807	.43234	.42766	.42401	.42140	.41984	.41931
.17977E-01	.76638	.52423	.51505	.50683	.49956	.49325	.48790	.48352	.48011	.47767	.47621	.47572
.19973E-01	.75018	.57536	.56688	.55928	.55255	.54671	.54176	.53771	.53455	.53229	.53093	.53048
.21970E-01	.73578	.62408	.61632	.60936	.60320	.59785	.59331	.58960	.58670	.58463	.58339	.58297
.23967E-01	.72308	.66938	.66285	.65654	.65096	.64611	.64199	.63862	.63599	.63411	.63298	.63260
.25964E-01	.71196	.71236	.70606	.70040	.69539	.69104	.68734	.68432	.68195	.68027	.67925	.67891
.27961E-01	.70231	.75125	.74566	.74064	.73619	.73232	.72904	.72634	.72424	.72274	.72184	.72153
.29957E-01	.69399	.78640	.78149	.77707	.77316	.76975	.76687	.76450	.76265	.76133	.76053	.76027
.31954E-01	.68686	.81779	.81351	.80967	.80626	.80329	.80077	.79870	.79709	.79594	.79524	.79501
.33951E-01	.68080	.84549	.84180	.83848	.83553	.83297	.83079	.82900	.82761	.82661	.82601	.82581
.35948E-01	.67567	.86968	.86652	.86367	.86115	.85895	.85708	.85555	.85435	.85350	.85298	.85282
.37944E-01	.67134	.89061	.88792	.88550	.88335	.88147	.87988	.87858	.87756	.87683	.87639	.87624
.39941E-01	.66772	.90856	.90629	.90424	.90242	.90083	.89949	.89838	.89752	.89690	.89653	.89640
.41938E-01	.66469	.92384	.92193	.92020	.91867	.91734	.91621	.91527	.91455	.91402	.91371	.91360
.43935E-01	.66216	.93677	.93516	.93372	.93244	.93132	.93037	.92959	.92898	.92854	.92828	.92819
.45931E-01	.66007	.94763	.94630	.94509	.94402	.94309	.94230	.94165	.94114	.94078	.94056	.94049
.47928E-01	.65833	.95674	.95563	.95463	.95374	.95296	.95230	.95176	.95134	.95103	.95085	.95078
.49925E-01	.65689	.96433	.96341	.96258	.96184	.96120	.96065	.96020	.95985	.95960	.95945	.95939
.51922E-01	.65570	.97062	.96986	.96917	.96856	.96803	.96758	.96721	.96692	.96671	.96658	.96654
.53918E-01	.65471	.97583	.97520	.97464	.97413	.97369	.97332	.97301	.97277	.97260	.97249	.97245
.55915E-01	.65389	.98013	.97961	.97915	.97873	.97837	.97806	.97781	.97761	.97747	.97738	.97735
.57912E-01	.65322	.98369	.98326	.98288	.98254	.98224	.98199	.98178	.98161	.98150	.98142	.98140
.59909E-01	.65266	.98662	.98627	.98595	.98567	.98543	.98522	.98505	.98491	.98481	.98475	.98473

MATERIAL: TALC C

ALPHA0=0.5800E 11 A1=0.3406E 00 A2=-.4100E-02 A3=0.2498E-04 E0=0.8575E 00 E1=-.8204E-02
E2=0.1356E-03 E3=-.7397E-06 KMAX=90000 MU=0.6720E-03 P1= 25.0 C=0.1000E 01
DELT=0.5000E-06 N=100 PHIGH= 1.00 PLOW= 0.10

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(10)	(20)	(30)	(40)	(50)	(60)	(70)	(80)	(90)	(100)
.0	.97791	.04000	.03316	.02704	.02164	.01696	.01300	.00976	.00724	.00544	.00436	.00400
.24999E-03	.97097	.14632	.04969	.02878	.02203	.01732	.01339	.01017	.00766	.00568	.00354	.00042
.49994E-03	.96454	.19612	.07407	.03626	.02361	.01775	.01370	.01047	.00785	.00555	.00311	.00031
.74988E-03	.95844	.23518	.09688	.04640	.02716	.01882	.01412	.01073	.00797	.00546	.00294	.00040
.99982E-03	.95262	.26811	.11802	.05738	.03207	.02073	.01483	.01103	.00807	.00544	.00294	.00062
.12498E-02	.94703	.29680	.13773	.06858	.03779	.02334	.01592	.01147	.00823	.00550	.00304	.00094
.14997E-02	.94166	.32226	.15619	.07975	.04398	.02649	.01739	.01210	.00849	.00564	.00323	.00130
.17497E-02	.93648	.34514	.17354	.09076	.05044	.03005	.01920	.01295	.00888	.00587	.00349	.00171
.19996E-02	.93146	.36590	.18992	.10156	.05706	.03389	.02129	.01402	.00942	.00620	.00381	.00216
.22495E-02	.92660	.38485	.20540	.11212	.06377	.03795	.02363	.01530	.01012	.00664	.00420	.00266
.24995E-02	.92188	.40227	.22006	.12242	.07049	.04216	.02616	.01677	.01096	.00718	.00468	.00319
.27494E-02	.91729	.41834	.23398	.13245	.07721	.04649	.02886	.01840	.01195	.00784	.00523	.00378
.29994E-02	.91282	.43323	.24721	.14222	.08390	.05090	.03169	.02018	.01308	.00861	.00585	.00443
.32493E-02	.90846	.44708	.25980	.15172	.09054	.05538	.03464	.02208	.01432	.00947	.00656	.00514
.34993E-02	.90420	.46001	.27181	.16097	.09711	.05989	.03768	.02411	.01568	.01044	.00735	.00592
.37492E-02	.90004	.47211	.28327	.16996	.10361	.06443	.04079	.02623	.01714	.01150	.00823	.00680
.39990E-02	.89598	.48346	.29422	.17870	.11004	.06899	.04397	.02843	.01869	.01266	.00919	.00772
.42486E-02	.89199	.49413	.30469	.18720	.11638	.07355	.04720	.03072	.02033	.01390	.01023	.00872
.44982E-02	.88809	.50420	.31473	.19548	.12263	.07810	.05048	.03307	.02206	.01522	.01134	.00982
.47478E-02	.88427	.51370	.32435	.20353	.12880	.08265	.05379	.03549	.02386	.01663	.01254	.01095
.49974E-02	.88051	.52271	.33359	.21136	.13487	.08719	.05714	.03797	.02572	.01810	.01382	.01220
.52470E-02	.87683	.53124	.34246	.21899	.14086	.09171	.06051	.04049	.02765	.01965	.01516	.01351
.54966E-02	.87320	.53936	.35100	.22642	.14676	.09621	.06390	.04306	.02965	.02127	.01657	.01487
.57462E-02	.86964	.54708	.35922	.23366	.15258	.10069	.06731	.04567	.03169	.02295	.01805	.01630
.59958E-02	.86614	.55444	.36715	.24072	.15830	.10515	.07074	.04832	.03379	.02469	.01960	.01779
.62454E-02	.86270	.56147	.37480	.24760	.16395	.10958	.07417	.05101	.03594	.02648	.02120	.01935
.64950E-02	.85931	.56819	.38219	.25432	.16951	.11399	.07762	.05373	.03814	.02834	.02287	.02097
.67446E-02	.85597	.57463	.38933	.26088	.17498	.11837	.08108	.05648	.04038	.03024	.02459	.02265
.69942E-02	.85267	.58081	.39624	.26729	.18038	.12272	.08454	.05925	.04266	.03219	.02636	.02437
.72437E-02	.84943	.58674	.40293	.27355	.18571	.12705	.08802	.06206	.04499	.03420	.02818	.02613
.74933E-02	.84623	.59244	.40942	.27967	.19096	.13136	.09150	.06489	.04735	.03625	.03005	.02797
.77429E-02	.84308	.59792	.41571	.28566	.19614	.13563	.09498	.06775	.04974	.03833	.03196	.02984

(continued) SOLIDS PRESSURES PHI(I)

TIME	LENGTH	(0)	(10)	(20)	(30)	(40)	(50)	(60)	(70)	(80)	(90)	(100)
.79925E-02	.83996	.60321	.42182	.29152	.20124	.13988	.09846	.07062	.05217	.04046	.03392	.03172
.82421E-02	.83688	.60832	.42776	.29726	.20629	.14410	.10195	.07352	.05463	.04263	.03593	.03373
.84917E-02	.83385	.61325	.43354	.30288	.21126	.14830	.10544	.07644	.05712	.04484	.03798	.03569
.87413E-02	.83085	.61801	.43916	.30840	.21617	.15248	.10893	.07937	.05964	.04708	.04006	.03774
.89909E-02	.82789	.62263	.44464	.31381	.22103	.15663	.11242	.08233	.06220	.04936	.04218	.03980
.92405E-02	.82497	.62710	.44998	.31912	.22582	.16076	.11591	.08530	.06477	.05167	.04433	.04190
.94901E-02	.82207	.63143	.45519	.32433	.23056	.16486	.11941	.08828	.06738	.05401	.04652	.04404
.97397E-02	.81922	.63564	.46028	.32945	.23524	.16894	.12290	.09128	.07000	.05637	.04874	.04624
.99893E-02	.81639	.63973	.46525	.33448	.23987	.17300	.12639	.09430	.07265	.05877	.05100	.04844
.10239E-01	.81360	.64371	.47011	.33943	.24445	.17704	.12989	.09733	.07533	.06120	.05329	.05068
.10488E-01	.81083	.64758	.47486	.34430	.24898	.18105	.13338	.10037	.07802	.06366	.05560	.05297
.10738E-01	.80810	.65135	.47951	.34909	.25347	.18505	.13687	.10343	.08074	.06614	.05795	.05524
.10988E-01	.80539	.65503	.48407	.35381	.25791	.18903	.14036	.10649	.08348	.06865	.06032	.05758
.11237E-01	.80271	.65861	.48854	.35845	.26231	.19299	.14386	.10957	.08624	.07118	.06272	.05994
.11487E-01	.80006	.66211	.49292	.36304	.26667	.19693	.14735	.11267	.08901	.07374	.06515	.06234
.11736E-01	.79744	.66553	.49722	.36755	.27098	.20085	.15083	.11577	.09181	.07631	.06760	.06477
.11986E-01	.79484	.66887	.50144	.37200	.27526	.20475	.15433	.11888	.09462	.07892	.07009	.06720
.12236E-01	.79227	.67214	.50559	.37640	.27950	.20864	.15781	.12200	.09745	.08154	.07259	.06968
.12485E-01	.78973	.67535	.50966	.38074	.28370	.21251	.16130	.12514	.10031	.08419	.07512	.07217
.12735E-01	.78720	.67848	.51367	.38502	.28787	.21637	.16479	.12829	.10317	.08686	.07767	.07468
.12984E-01	.78470	.68155	.51761	.38925	.29200	.22021	.16828	.13144	.10606	.08955	.08024	.07718
.13234E-01	.78223	.68457	.52149	.39343	.29611	.22404	.17176	.13460	.10896	.09226	.08283	.07975
.13484E-01	.77978	.68752	.52531	.39756	.30018	.22785	.17525	.13778	.11187	.09498	.08545	.08232
.13733E-01	.77735	.69042	.52907	.40165	.30422	.23165	.17873	.14096	.11480	.09773	.08808	.08492
.13983E-01	.77495	.69327	.53278	.40569	.30823	.23543	.18221	.14414	.11774	.10049	.09073	.08756
.14232E-01	.77256	.69607	.53643	.40969	.31221	.23920	.18570	.14734	.12070	.10327	.09341	.09021
.14482E-01	.77020	.69882	.54003	.41365	.31617	.24296	.18918	.15055	.12367	.10607	.09612	.09287
.14732E-01	.76786	.70152	.54359	.41756	.32010	.24671	.19266	.15376	.12666	.10889	.09883	.09555
.14981E-01	.76554	.70418	.54710	.42144	.32401	.25044	.19614	.15699	.12966	.11173	.10157	.09824
.15231E-01	.76324	.70680	.55056	.42528	.32789	.25416	.19962	.16022	.13267	.11458	.10432	.10095
.15480E-01	.76096	.70938	.55398	.42909	.33175	.25788	.20310	.16345	.13570	.11744	.10708	.10370
.15730E-01	.75870	.71192	.55736	.43285	.33558	.26158	.20658	.16670	.13873	.12033	.10987	.10648
.15980E-01	.75646	.71442	.56070	.43659	.33939	.26527	.21006	.16995	.14179	.12323	.11268	.10923
.16229E-01	.75424	.71689	.56400	.44029	.34318	.26895	.21354	.17321	.14485	.12614	.11550	.11204
.16479E-01	.75204	.71932	.56726	.44397	.34695	.27262	.21702	.17647	.14792	.12907	.11834	.11485
.16728E-01	.74986	.72171	.57049	.44761	.35069	.27628	.22050	.17974	.15101	.13201	.12120	.11767
.16978E-01	.74769	.72408	.57368	.45122	.35442	.27993	.22398	.18302	.15411	.13497	.12408	.12053
.17228E-01	.74554	.72641	.57684	.45480	.35813	.28357	.22745	.18631	.15722	.13795	.12697	.12338
.17477E-01	.74342	.72871	.57996	.45836	.36182	.28720	.23093	.18960	.16034	.14094	.12987	.12628

(continued)

SOLIDS PRESSURES PHI(I)

TIME	LENGTH	(0)	(10)	(20)	(30)	(40)	(50)	(60)	(70)	(80)	(90)	(100)
.17727E-01	.74130	.73099	.58305	.46188	.36549	.29082	.23441	.19290	.16347	.14394	.13280	.12916
.17976E-01	.73921	.73323	.58612	.46539	.36914	.29444	.23788	.19620	.16661	.14696	.13573	.13208
.18226E-01	.73714	.73545	.58915	.46886	.37278	.29805	.24136	.19952	.16977	.14999	.13869	.13499
.18475E-01	.73508	.73764	.59215	.47231	.37640	.30164	.24483	.20283	.17294	.15303	.14165	.13792
.18725E-01	.73304	.73981	.59513	.47574	.38000	.30523	.24831	.20616	.17611	.15609	.14463	.14088
.18975E-01	.73101	.74195	.59807	.47914	.38358	.30881	.25178	.20948	.17929	.15916	.14762	.14387
.19224E-01	.72900	.74407	.60099	.48252	.38715	.31239	.25526	.21282	.18249	.16224	.15064	.14685
.19474E-01	.72701	.74616	.60389	.48588	.39071	.31595	.25873	.21616	.18569	.16534	.15367	.14985
.19723E-01	.72504	.74823	.60676	.48922	.39424	.31951	.26221	.21950	.18891	.16844	.15671	.15287
.19973E-01	.72307	.75028	.60960	.49253	.39777	.32306	.26568	.22285	.19213	.17156	.15976	.15590
.20223E-01	.72113	.75230	.61242	.49582	.40128	.32661	.26915	.22621	.19536	.17470	.16283	.15896
.20472E-01	.71920	.75431	.61522	.49910	.40477	.33014	.27263	.22957	.19861	.17784	.16591	.16202
.20722E-01	.71729	.75630	.61800	.50235	.40825	.33368	.27610	.23294	.20186	.18100	.16901	.16509
.20971E-01	.71539	.75826	.62075	.50559	.41172	.33720	.27957	.23631	.20512	.18417	.17212	.16817
.21221E-01	.71351	.76020	.62348	.50880	.41517	.34071	.28304	.23968	.20839	.18735	.17524	.17128
.21471E-01	.71164	.76213	.62619	.51199	.41861	.34422	.28651	.24306	.21167	.19054	.17837	.17438
.21720E-01	.70979	.76404	.62888	.51517	.42204	.34773	.28999	.24645	.21496	.19374	.18152	.17753
.21970E-01	.70795	.76593	.63155	.51833	.42545	.35122	.29345	.24984	.21825	.19696	.18468	.18066
.22219E-01	.70613	.76780	.63420	.52148	.42885	.35471	.29693	.25324	.22156	.20018	.18785	.18381
.22469E-01	.70432	.76966	.63683	.52460	.43224	.35820	.30040	.25664	.22487	.20342	.19104	.18697
.22719E-01	.70253	.77149	.63944	.52771	.43562	.36167	.30386	.26004	.22819	.20666	.19423	.19016
.22968E-01	.70074	.77331	.64203	.53080	.43898	.36514	.30733	.26345	.23152	.20992	.19744	.19334
.23218E-01	.69898	.77512	.64460	.53387	.44234	.36861	.31080	.26686	.23486	.21318	.20065	.19656
.23467E-01	.69722	.77691	.64716	.53693	.44568	.37207	.31427	.27028	.23820	.21646	.20389	.19977
.23717E-01	.69549	.77868	.64970	.53997	.44901	.37552	.31773	.27370	.24155	.21975	.20713	.20301
.23967E-01	.69376	.78044	.65222	.54300	.45233	.37897	.32120	.27712	.24491	.22305	.21039	.20623
.24216E-01	.69205	.78218	.65472	.54601	.45563	.38241	.32466	.28055	.24828	.22635	.21365	.20948
.24466E-01	.69035	.78391	.65721	.54901	.45893	.38584	.32813	.28398	.25165	.22967	.21692	.21274
.24715E-01	.68867	.78563	.65968	.55199	.46221	.38927	.33159	.28741	.25503	.23299	.22021	.21601
.24965E-01	.68700	.78733	.66213	.55496	.46549	.39269	.33505	.29085	.25842	.23633	.22350	.21929
.25215E-01	.68534	.78901	.66457	.55791	.46875	.39610	.33851	.29429	.26181	.23967	.22681	.22259
.25464E-01	.68370	.79069	.66700	.56085	.47200	.39952	.34197	.29774	.26522	.24302	.23013	.22589
.25714E-01	.68206	.79235	.66940	.56377	.47524	.40292	.34543	.30118	.26862	.24638	.23346	.22922
.25963E-01	.68045	.79400	.67180	.56668	.47848	.40632	.34889	.30464	.27203	.24976	.23680	.23255
.26213E-01	.67884	.79563	.67417	.56958	.48170	.40971	.35235	.30809	.27546	.25314	.24015	.23588
.26463E-01	.67725	.79725	.67654	.57247	.48491	.41310	.35580	.31155	.27888	.25653	.24351	.23922
.26712E-01	.67567	.79886	.67889	.57534	.48811	.41648	.35925	.31501	.28231	.25992	.24687	.24259
.26962E-01	.67409	.80046	.68122	.57819	.49130	.41985	.36270	.31847	.28575	.26332	.25025	.24595
.27211E-01	.67254	.80204	.68354	.58104	.49448	.42322	.36616	.32193	.28919	.26674	.25364	.24933
.27461E-01	.67099	.80362	.68585	.58387	.49765	.42659	.36960	.32540	.29264	.27016	.25704	.25271

MATERIAL: TALC C

ALPHA0=0.5800E 11 A1=0.3406E 00 A2=-.4100E-02 A3=0.2498E-04 E0=0.8575E 00 E1=-.8204E-02
E2=0.1356E-03 E3=-.7392E-06 KMAX=70000 MU=0.6720E-03 P1= 25.0 C=0.1000E 01
DELT=0.2000E-05 N= 40 PHIGH= 0.0 PLOW= 0.0

TABULATION OF DIMENSIONLESS QUANTITIES:

		SOLIDS PRESSURES PHI(I)										
TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.0	.73220	.74400	.60100	.48200	.38720	.31240	.25530	.21280	.18250	.16220	.15060	.14690
.99992E-03	.72438	.75284	.61317	.49667	.40208	.32725	.26958	.22649	.19566	.17514	.16346	.15968
.19998E-02	.71681	.76066	.62410	.50948	.41582	.34129	.28353	.24013	.20884	.18785	.17581	.17190
.29997E-02	.70948	.76817	.63470	.52205	.42944	.35528	.29746	.25375	.22208	.20072	.18843	.18443
.39994E-02	.70238	.77543	.64504	.53439	.44289	.36917	.31135	.26741	.23541	.21376	.20125	.19720
.49978E-02	.69551	.78247	.65513	.54650	.45617	.38296	.32523	.28112	.24887	.22696	.21428	.21015
.59962E-02	.68884	.78929	.66496	.55838	.46928	.39667	.33910	.29489	.26243	.24031	.22748	.22330
.69946E-02	.68239	.79589	.67455	.57004	.48222	.41028	.35294	.30871	.27609	.25381	.24086	.23663
.79930E-02	.67614	.80230	.68391	.58149	.49500	.42380	.36676	.32257	.28986	.26744	.25438	.25012
.89913E-02	.67008	.80852	.69306	.59274	.50763	.43722	.38056	.33647	.30371	.28119	.26805	.26376
.99897E-02	.66421	.81456	.70200	.60379	.52011	.45055	.39433	.35040	.31764	.29506	.28186	.27755
.10988E-01	.65853	.82044	.71074	.61466	.53243	.46379	.40807	.36435	.33165	.30904	.29580	.29147
.11986E-01	.65303	.82615	.71929	.62534	.54461	.47693	.42177	.37832	.34571	.32311	.30985	.30551
.12985E-01	.64771	.83172	.72766	.63584	.55664	.48997	.43542	.39230	.35983	.33727	.32401	.31966
.13983E-01	.64255	.83714	.73585	.64617	.56852	.50291	.44902	.40628	.37399	.35150	.33826	.33391
.14982E-01	.63756	.84243	.74387	.65632	.58025	.51575	.46257	.42024	.38818	.36580	.35260	.34826
.15980E-01	.63274	.84758	.75172	.66631	.59184	.52847	.47605	.43419	.40239	.38014	.36700	.36268
.16978E-01	.62807	.85260	.75942	.67614	.60329	.54109	.48946	.44811	.41662	.39453	.38146	.37717
.17977E-01	.62355	.85749	.76696	.68580	.61458	.55358	.50280	.46200	.43084	.40895	.39598	.39170
.18975E-01	.61919	.86227	.77434	.69530	.62573	.56596	.51604	.47583	.44505	.42338	.41052	.40628
.19973E-01	.61497	.86693	.78157	.70463	.63673	.57821	.52920	.48962	.45924	.43781	.42508	.42088
.20972E-01	.61089	.87147	.78866	.71381	.64757	.59033	.54226	.50333	.47339	.45223	.43964	.43549
.21970E-01	.60696	.87591	.79559	.72283	.65827	.60232	.55522	.51697	.48750	.46663	.45420	.45009
.22969E-01	.60316	.88024	.80239	.73169	.66880	.61417	.56805	.53053	.50154	.48099	.46873	.46468
.23967E-01	.59949	.88446	.80904	.74039	.67918	.62588	.58077	.54398	.51552	.49530	.48323	.47924
.24965E-01	.59595	.88857	.81555	.74894	.68940	.63743	.59336	.55734	.52941	.50954	.49767	.49375
.25964E-01	.59254	.89258	.82192	.75732	.69945	.64883	.60581	.57057	.54320	.52371	.51205	.50819
.26962E-01	.58925	.89650	.82815	.76554	.70934	.66007	.61811	.58368	.55689	.53778	.52634	.52255
.27961E-01	.58608	.90031	.83424	.77360	.71906	.67115	.63026	.59665	.57046	.55175	.54054	.53682
.28959E-01	.58303	.90403	.84020	.78150	.72861	.68205	.64225	.60947	.58389	.56559	.55462	.55098
.29957E-01	.58010	.90765	.84601	.78924	.73798	.69278	.65407	.62213	.59718	.57931	.56858	.56502

(continued) SOLIDS PRESSURES PHI(I)

TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.30956E-01	.57727	.91117	.85170	.79681	.74717	.70333	.66571	.63463	.61031	.59288	.58240	.57892
.31954E-01	.57455	.91460	.85724	.80422	.75619	.71369	.67717	.64695	.62327	.60628	.59607	.59268
.32952E-01	.57194	.91794	.86265	.81147	.76502	.72386	.68844	.65909	.63606	.61952	.60957	.60625
.33951E-01	.56943	.92119	.86793	.81854	.77367	.73384	.69951	.67103	.64865	.63257	.62289	.61967
.34949E-01	.56702	.92435	.87307	.82545	.78212	.74361	.71037	.68276	.66105	.64543	.63602	.63289
.35948E-01	.56470	.92741	.87807	.83220	.79039	.75318	.72103	.69429	.67324	.65808	.64894	.64590
.36946E-01	.56248	.93039	.88294	.83877	.79846	.76255	.73147	.70559	.68520	.67051	.66165	.65870
.37944E-01	.56035	.93327	.88768	.84518	.80634	.77170	.74168	.71667	.69694	.68271	.67413	.67128
.38943E-01	.55831	.93607	.89229	.85141	.81403	.78063	.75167	.72751	.70844	.69468	.68637	.68361
.39941E-01	.55635	.93879	.89676	.85748	.82151	.78935	.76143	.73812	.71970	.70640	.69837	.69569
.40939E-01	.55448	.94142	.90110	.86338	.82880	.79785	.77096	.74848	.73071	.71787	.71011	.70753
.41938E-01	.55269	.94396	.90531	.86911	.83589	.80613	.78024	.75859	.74146	.72907	.72159	.71909
.42936E-01	.55097	.94642	.90939	.87467	.84278	.81418	.78928	.76844	.75194	.74001	.73279	.73038
.43935E-01	.54933	.94880	.91334	.88006	.84946	.82200	.79808	.77804	.76216	.75067	.74372	.74140
.44933E-01	.54776	.95110	.91716	.88529	.85595	.82960	.80663	.78737	.77211	.76106	.75437	.75213
.45931E-01	.54627	.95332	.92086	.89034	.86224	.83697	.81493	.79644	.78178	.77116	.76473	.76259
.46930E-01	.54484	.95546	.92443	.89523	.86833	.84412	.82299	.80525	.79118	.78098	.77480	.77274
.47928E-01	.54347	.95752	.92788	.89996	.87422	.85104	.83079	.81379	.80029	.79051	.78458	.78260
.48926E-01	.54217	.95951	.93120	.90453	.87991	.85773	.83835	.82206	.80913	.79975	.79407	.79217
.49925E-01	.54093	.96142	.93441	.90893	.88541	.86420	.84566	.83007	.81768	.80870	.80326	.80144
.50923E-01	.53975	.96326	.93749	.91318	.89071	.87045	.85272	.83781	.82596	.81736	.81215	.81041
.51922E-01	.53862	.96503	.94046	.91727	.89583	.87647	.85954	.84529	.83396	.82573	.82075	.81908
.52920E-01	.53755	.96673	.94332	.92121	.90075	.88228	.86611	.85250	.84168	.83382	.82905	.82746
.53918E-01	.53653	.96836	.94607	.92500	.90549	.88788	.87245	.85945	.84912	.84161	.83706	.83554
.54917E-01	.53556	.96992	.94870	.92864	.91005	.89326	.87854	.86615	.85629	.84912	.84478	.84332
.55915E-01	.53463	.97141	.95123	.93213	.91443	.89843	.88441	.87259	.86319	.85635	.85221	.85083
.56913E-01	.53375	.97285	.95365	.93548	.91864	.90340	.89004	.87879	.86982	.86331	.85936	.85804
.57912E-01	.53292	.97422	.95598	.93870	.92267	.90818	.89546	.88474	.87620	.87000	.86623	.86497
.58910E-01	.53213	.97554	.95820	.94178	.92654	.91275	.90065	.89045	.88232	.87642	.87283	.87162
.59909E-01	.53137	.97679	.96033	.94472	.93024	.91713	.90562	.89592	.88819	.88257	.87916	.87802
.60907E-01	.53066	.97799	.96236	.94754	.93378	.92133	.91039	.90117	.89382	.88847	.88523	.88415
.61905E-01	.52998	.97913	.96431	.95024	.93717	.92534	.91495	.90619	.89920	.89412	.89104	.89000
.62898E-01	.52933	.98022	.96616	.95281	.94041	.92918	.91931	.91099	.90435	.89953	.89660	.89562
.63881E-01	.52872	.98127	.96793	.95527	.94351	.93285	.92348	.91558	.90928	.90470	.90191	.90098
.64865E-01	.52814	.98226	.96962	.95761	.94646	.93635	.92746	.91996	.91399	.90964	.90700	.90611
.65848E-01	.52759	.98320	.97123	.95985	.94927	.93969	.93126	.92415	.91848	.91435	.91185	.91101
.66832E-01	.52707	.98410	.97276	.96198	.95196	.94287	.93488	.92814	.92276	.91885	.91647	.91568
.67815E-01	.52658	.98496	.97422	.96401	.95451	.94590	.93834	.93194	.92685	.92314	.92089	.92013

(continued) SOLIDS PRESSURES PHI(I)

TIME	LENGTH	(0)	(4)	(8)	(12)	(16)	(20)	(24)	(28)	(32)	(36)	(40)
.68799E-01	.52611	.98577	.97560	.96594	.95695	.94879	.94162	.93557	.93074	.92723	.92509	.92437
.69782E-01	.52567	.98654	.97692	.96777	.95926	.95154	.94475	.93902	.93445	.93112	.92909	.92842
.70765E-01	.52525	.98728	.97818	.96952	.96147	.95416	.94773	.94230	.93797	.93482	.93290	.93226
.71749E-01	.52485	.98797	.97937	.97118	.96356	.95664	.95056	.94542	.94132	.93834	.93653	.93592
.72732E-01	.52448	.98864	.98050	.97275	.96555	.95901	.95325	.94839	.94451	.94169	.93998	.93940
.73716E-01	.52412	.98926	.98157	.97425	.96743	.96125	.95581	.95121	.94754	.94487	.94325	.94271
.74699E-01	.52378	.98986	.98259	.97567	.96923	.96338	.95823	.95389	.95042	.94789	.94636	.94584
.75683E-01	.52346	.99042	.98355	.97701	.97093	.96540	.96054	.95643	.95315	.95076	.94931	.94882
.76666E-01	.52316	.99096	.98447	.97829	.97254	.96732	.96272	.95883	.95574	.95348	.95211	.95164
.77650E-01	.52287	.99146	.98534	.97950	.97407	.96913	.96479	.96112	.95819	.95606	.95476	.95432
.78633E-01	.52261	.99194	.98615	.98065	.97551	.97085	.96675	.96328	.96051	.95850	.95728	.95687
.79617E-01	.52235	.99239	.98693	.98173	.97688	.97248	.96861	.96533	.96272	.96082	.95966	.95927
.80600E-01	.52211	.99283	.98767	.98276	.97818	.97403	.97037	.96728	.96481	.96301	.96192	.96155
.81584E-01	.52187	.99323	.98837	.98373	.97941	.97549	.97204	.96912	.96679	.96509	.96405	.96371
.82567E-01	.52166	.99361	.98902	.98465	.98057	.97687	.97361	.97086	.96866	.96705	.96608	.96575
.83551E-01	.52145	.99398	.98965	.98552	.98167	.97818	.97510	.97250	.97043	.96892	.96800	.96769
.84534E-01	.52126	.99432	.99024	.98634	.98271	.97942	.97652	.97406	.97210	.97068	.96981	.96952
.85518E-01	.52108	.99464	.99079	.98712	.98370	.98059	.97785	.97553	.97369	.97234	.97152	.97124
.86501E-01	.52090	.99495	.99132	.98785	.98462	.98169	.97911	.97692	.97518	.97391	.97314	.97288
.87485E-01	.52073	.99524	.99181	.98855	.98550	.98274	.98030	.97824	.97660	.97540	.97467	.97442
.88468E-01	.52058	.99551	.99228	.98920	.98633	.98372	.98143	.97948	.97793	.97680	.97611	.97588
.89452E-01	.52043	.99577	.99272	.98982	.98711	.98466	.98249	.98066	.97919	.97813	.97748	.97727
.90435E-01	.52028	.99601	.99314	.99041	.98785	.98554	.98349	.98177	.98039	.97938	.97877	.97857
.91419E-01	.52015	.99624	.99354	.99096	.98855	.98637	.98444	.98281	.98151	.98057	.97999	.97979
.92402E-01	.52003	.99646	.99391	.99148	.98921	.98715	.98534	.98380	.98258	.98168	.98114	.98096
.93385E-01	.51991	.99667	.99426	.99197	.98984	.98789	.98618	.98474	.98358	.98274	.98222	.98205
.94369E-01	.51980	.99686	.99459	.99244	.99042	.98859	.98698	.98562	.98453	.98373	.98325	.98308
.95352E-01	.51968	.99704	.99491	.99287	.99098	.98925	.98773	.98645	.98542	.98467	.98422	.98406
.96336E-01	.51958	.99721	.99520	.99329	.99150	.98987	.98844	.98723	.98627	.98556	.98513	.98498
.97319E-01	.51949	.99737	.99548	.99367	.99199	.99046	.98911	.98797	.98706	.98639	.98599	.98585
.98303E-01	.51940	.99753	.99574	.99404	.99246	.99101	.98974	.98867	.98781	.98718	.98680	.98667
.99286E-01	.51932	.99767	.99599	.99439	.99289	.99153	.99034	.98932	.98852	.98792	.98756	.98744
.10027E 00	.51923	.99781	.99622	.99471	.99331	.99203	.99090	.98994	.98918	.98863	.98829	.98817
.10125E 00	.51914	.99793	.99644	.99502	.99370	.99249	.99143	.99053	.98981	.98928	.98896	.98885
.10224E 00	.51905	.99805	.99665	.99531	.99406	.99292	.99192	.99108	.99040	.98991	.98960	.98950
.10322E 00	.51900	.99817	.99684	.99558	.99441	.99334	.99239	.99160	.99096	.99049	.99021	.99011

4.2 STRESS MEASUREMENT ON THE BOTTOM AND WALL (DETAILS)

An apparatus was built to measure vertical stresses on the bottom and horizontal stresses on the sidewall of a compacted filter cake.

4.2.1 DESCRIPTION OF THE APPARATUS

The apparatus consisted of a 4" i.d., 1/2" wall thickness cylinder (Figure 4-1) with a 3/4" thick bottom plate made of Stainless Steel 301. Figure (4-2) is a schematic diagram of the entire apparatus. Miniature pressure transducers (zero displacement type) of 0.25" effective diameter, range 0 - 130 p.s.i. and with a $\pm$ 0.5% accuracy were constructed and glued in place with epoxy adhesive. All transducers were flush with the inside of the cell so that the stress patterns would not be affected by proturbances. The transducers (TR) on the wall and bottom were arranged in a spiral to avoid weakening at any one cross section. The spiral arrangement also enabled measurements to be distributed over the cake. On the bottom plate TR No. 3 and 4 were at the same radius to check cylindrical symmetry. On the cylinder wall, TR No. 4 and 5 were at the same level as were TR No. 10 and 11.

The piston has a diameter approximately 0.015" less than that of the cylinder. The gap between the piston and the cylinder is filled by three O-rings thereby reducing the friction considerably. However, the rubber O-rings are compressible and the piston is liable to tilt slightly while inside the cylinder. It has to be straightened by hand during the experiment. The piston is hollow with 1/4" wall thickness. A 3/8" thick porous stainless steel plate is attached with screws to the bottom end

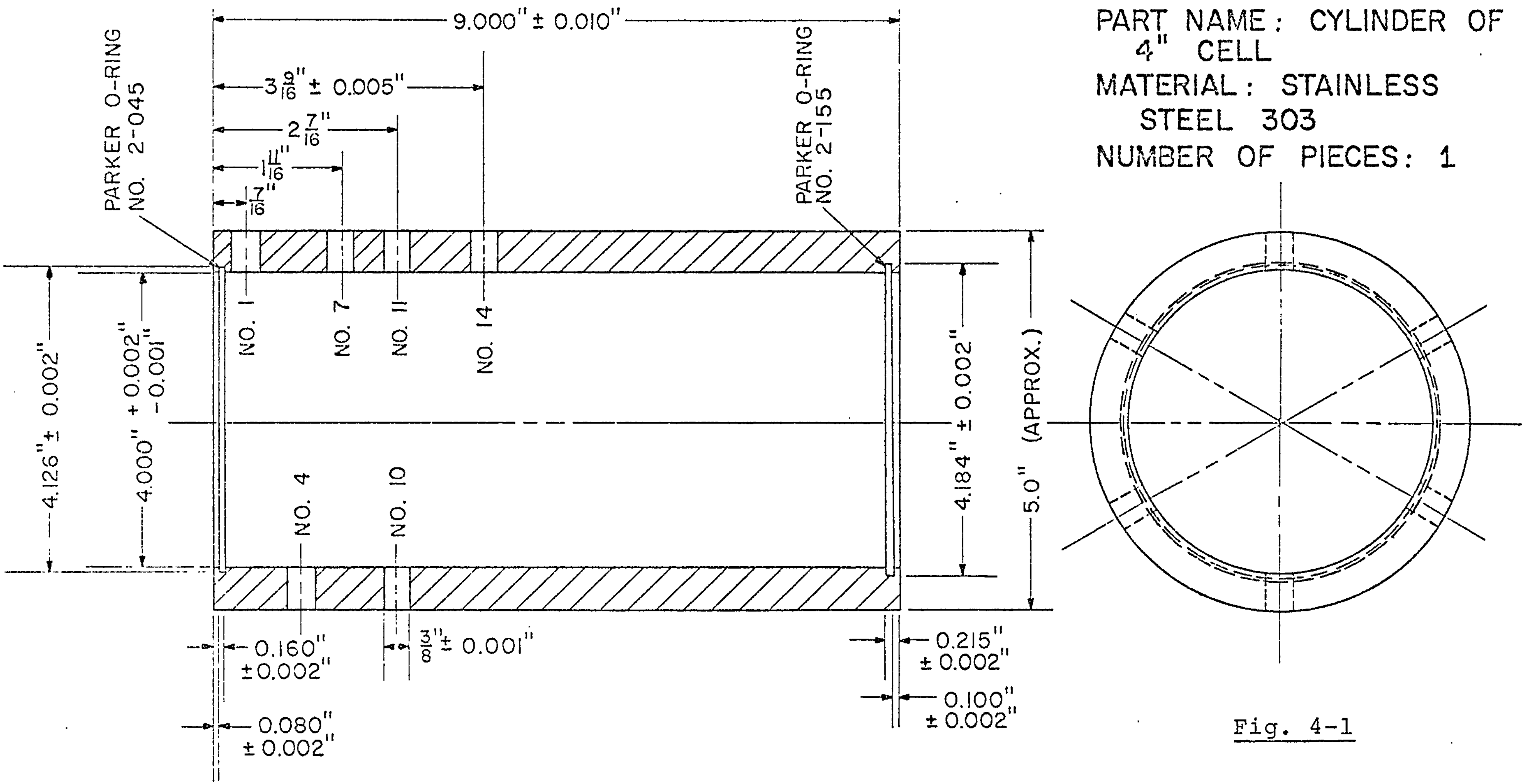


Fig. 4-1

HOLE NO.	1	2	3	4	5	6	7	8	9	10 11	12	13	14	15	16
±0.005" DISTANCE FROM BOTTOM EDGE →	$\frac{7}{16}$"	$\frac{11}{16}$"	$\frac{15}{16}$"	$1\frac{3}{16}$"	$1\frac{3}{16}$"	$1\frac{7}{16}$"	$1\frac{11}{16}$"	$1\frac{15}{16}$"	$2\frac{3}{16}$"	$2\frac{7}{16}$" $2\frac{7}{16}$"	$2\frac{13}{16}$"	$3\frac{3}{16}$"	$3\frac{9}{16}$"	$3\frac{15}{16}$"	$4\frac{5}{16}$"
ANGLE θ°	0	60	120	180	240	300	0	60	120	180 0	240	300	0	60	120

NOTE: ALL HOLES ARE $\frac{3}{8}$" ±0.001" DIA.

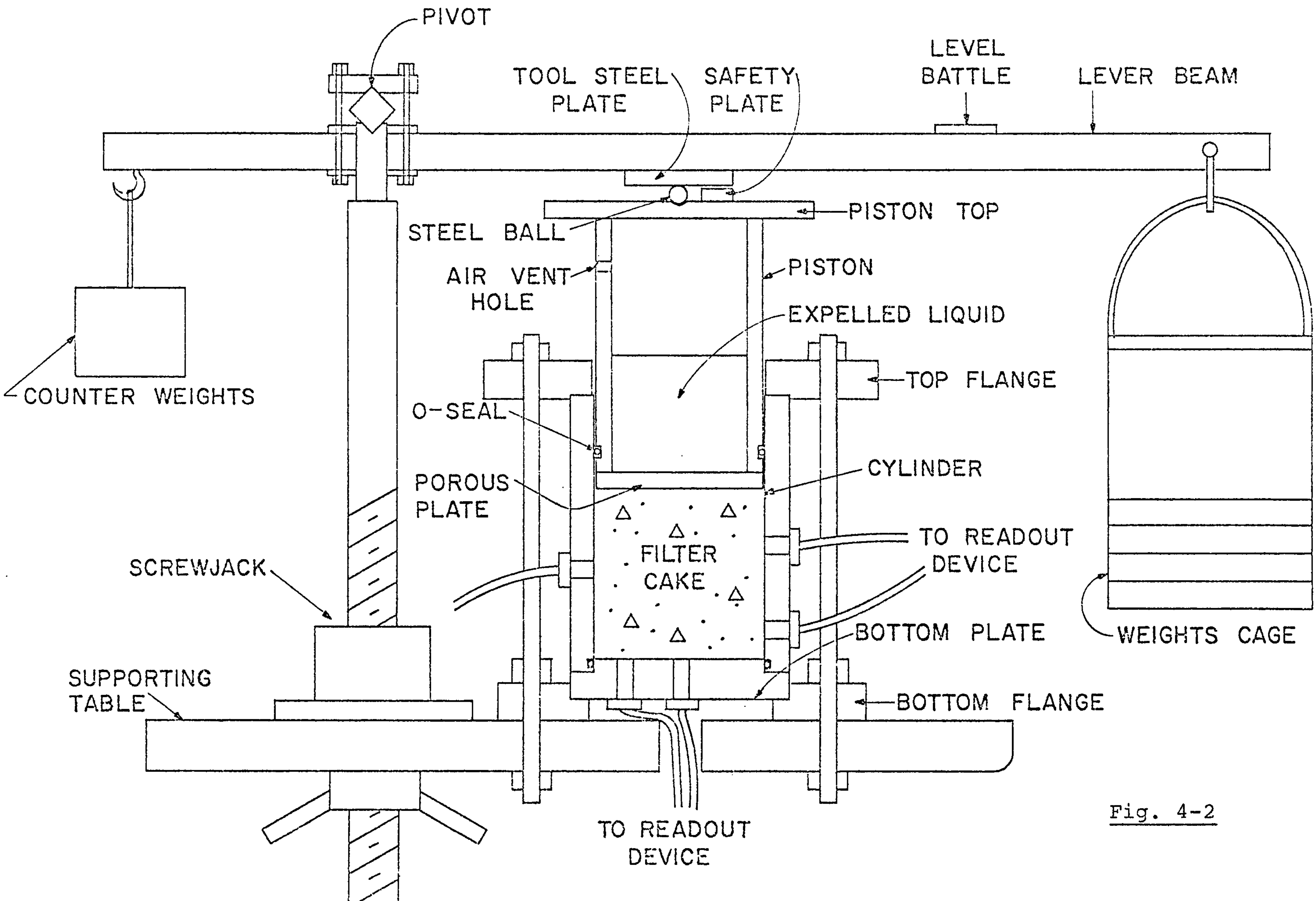

PIVOT
TOOL STEEL PLATE
SAFETY PLATE
LEVEL BATTLE
LEVER BEAM
PISTON TOP
STEEL BALL
AIR VENT HOLE
PISTON
EXPELLED LIQUID
COUNTER WEIGHTS
TOP FLANGE
O-SEAL
CYLINDER
POROUS PLATE
FILTER CAKE
TO READOUT DEVICE
SCREWJACK
BOTTOM PLATE
WEIGHTS CAGE
SUPPORTING TABLE
BOTTOM FLANGE
TO READOUT DEVICE

Fig. 4-2

of the piston. The author suggests that adhesive should not be used to attach the porous plate to the piston, because, the adhesive spreads inside the porous structure and blocks part of the flow path. Three small machine screws of 1/16" diameter can be utilized with minimal blocking of the porous structure. The porous plate attached to the piston rests on the cake top and provides drainage for the liquid(water) during the compaction process. As compaction proceeds, liquid rises inside the piston and displaces the air which passes through a vent.

The cylinder and bottom plate material should have high modulus of elasticity; otherwise, when pressure is applied they deform, thereby laterally stressing the transducers. The transducer diaphragms should be subjected to normal stresses only. Lateral stresses cause additional strain on the diaphragms and incorrect pressure readings are registered. During the early stages of this investigation, Plexiglass was used as the cell material. Although the deformation of the cell was not enough to change the cake geometry significantly, the lateral stressing of transducers produced erroneous measurements. Thereafter parts which contained transducers were made of Stainless Steel 301. (Modulus of elasticity for Stainless Steel 301 is 28×10^6 p.s.i. Vs 0.3×10^6 p.s.i. for Plexiglass.)

The top of the piston is made of 1" thick Plexiglass plate. Mechanical force is applied via a hard steel ball restrained firmly in a pit at the center of the top plate. The plate will bend excessively if its thickness is less than 1/2". A lever mechanism with a leverage of 5 is used to apply the mechanical force on the piston. The lever beam is raised and lowered by means of a screw jack. The beam rotates about the vertical axis making it possible to insert or remove the piston without changing the position of the jack. A tool steel plate tied to the mild steel beam, rests on the ball. Essentially a point contact is provided. Because of slight play in the screw jack mechanism, the distance between the axis of screw jack and top center of the piston may change by a few hundredths of an inch with increased loading on the lever beam. By pressing the piston sideways with one hand while holding the beam firmly in the other, the beam can be made to slide over the hard steel ball to accomodate the change in distance described above. The line of action of force exerted by the beam on the ball can be kept vertical provided the beam is leveled. Without this tool steel piece, the hard steel ball makes an indentation which locks the beam at that particular position and prevents necessary adjustments. As a result, the beam pulls the piston sideways.

It should be made certain that the beam cannot tilt sideways around its major axis (Figure 4-3), as it may fall off the ball. A safety plate should be placed below the beam close to the ball. The gap between this safety plate and the beam is around 1/32". In the event the beam slides off the ball of 3/8" dia., it rests on the safety plate and does not damage the apparatus. Also the counter weights on the beam could be dislodged leading to a serious accident.

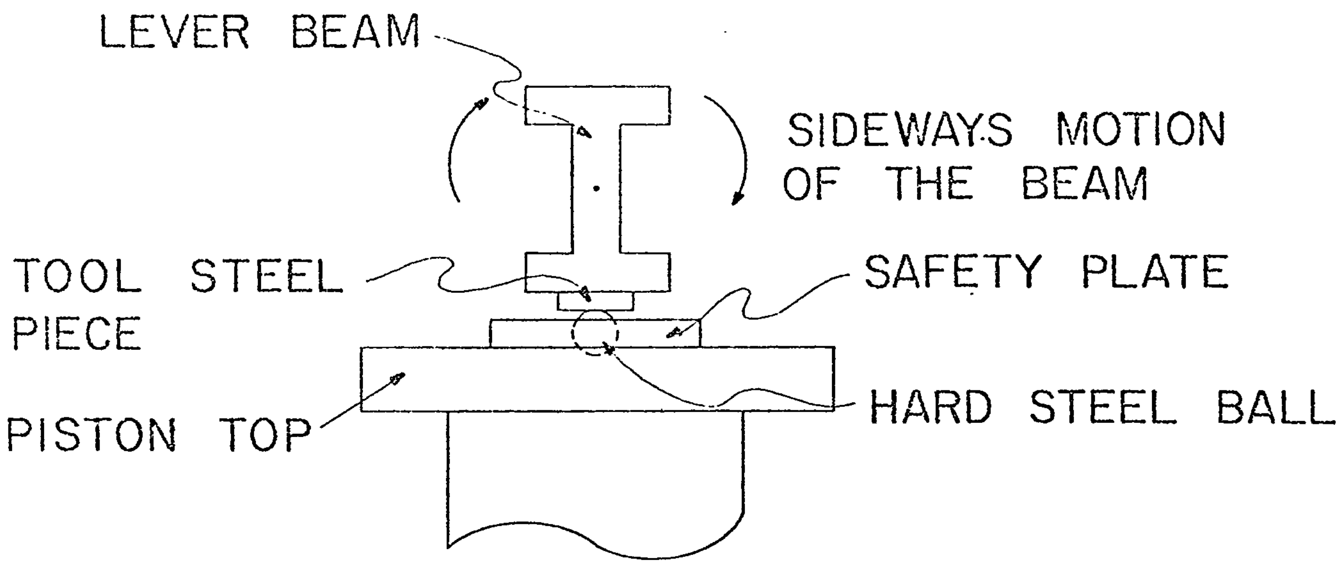


Fig. 4-3 End view of the loading mechanism.

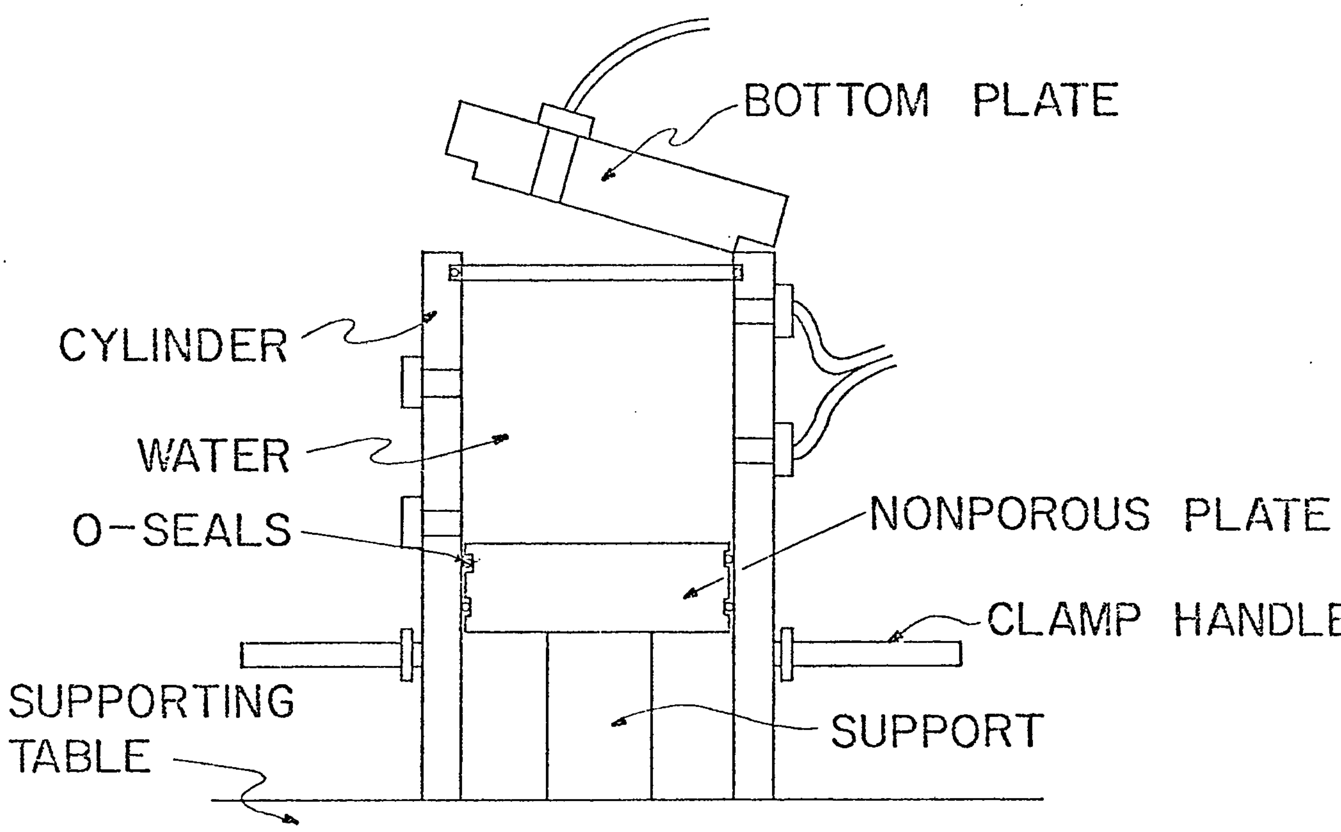


Fig. 4-4 Elimination of air while filling the cell with water

Before placing the cylinder on top of the bottom plate, the plate should be carefully leveled with the help of a level. The axis of the cylindrical cell should be vertical, otherwise the piston rubs against the wall, and a large frictional force is created. O-ring seals were used to prevent leakage and also to help the piston remain vertical.

There were 16 transducers on the wall and 7 on the bottom plate. A 'Straincert' TN8 model strain indicator was used as a readout device. Wires of the bottom transducers were connected directly to seven channels of the strain indicator. Wires of wall pressure transducers were connected to the 8 th channel via a switching box. All transducers were of quarterbridge circuit type, with 120 Ohm resistance.

4.2.2 CALIBRATION OF THE TRANSDUCERS

Calibration is accomplished by filling the cell with water (without trapping an appreciable amount of air) using a nonporous floating plate on top. This plate has floating type O-seals. Mechanical force is applied on the floating plate by the piston.

1) The cylinder of the cell is taken apart and kept inverted on a flat surface (the table top).

2) A support of required height is put inside as shown in Figure (4-4).

3) The nonporous floating plate (made of Plexiglass) is inserted until it touches the support.

4) Water is poured inside upto the brim.

5) The bottom plate, with transducers, is held by hand.

6) First a corner of the plate is made to touch the water

level and then the plate is slowly rested on the water. This prevents air from getting trapped inside.

7) The cylinder is tilted slightly, thus separating the nonporous plate from its support. The plate is given room to move.

8) The bottom plate is then pressed inside.

9) The cell is inverted back to its regular position and slowly rested on the supporting flange. It should be made certain that the bottom transducer wires going through the hole in the supporting table do not get trapped between the supporting flange and the bottom plate.

10) The top flange is put on the cylinder and the nuts are tightened.

11) Initial readings of wall pressure transducers are measured using rotary switches on the switching box. Initial readings of bottom pressure transducers are set to zero by changing bridge resistance with a turning knob on each channel.

12) The piston is inserted from the top and pressed until it rests on the nonporous plate.

13) The hard steel ball is placed on top of the piston and the lever is rotated horizontally so that center-line of the beam rests on the ball.

14) Weights are put in the cage attached to the long end of the lever beam which is leveled.

(The floating seals cannot be made too tight in the interest of reducing friction. Therefore some leakage is inevitable. If air is trapped inside the cell, it can escape more easily than water causing the piston to sink. Also as the load is increased, air is compressed making it necessary to level the beam at intervals.)

15) It should be made certain that the piston top is horizontal, by checking with a level. This assures that the piston is in vertical position.

(The weight in the cage is known. The leverage ratio is 5:1, and the inside diameter of the cell is 4". From this, applied pressure can be calculated. Weight of the piston should be taken in the account.)

16) For every applied pressure, output of each transducer is measured on the readout device and zero reading is subtracted to obtain the net output. About 10 nearly equidistant pressures are chosen in the desired range of 0-100 p.s.i.

17) Transducers output is also recorded during step-by-step unloading operation.

In figures (4-5 A,B,C) calibration curves for wall pressure transducers are shown. Those calibrations were checked by another method described below.

The cell was completely filled with water. A flange with a O-seal and with a 1/4" tube fitting was bolted on top of the cell and a connection was made to a dead weight tester by means of a tube filled with oil. Desired pressures were applied by putting known weights on the pan of the dead weight tester and calibrations were made as before. During the readout process the floating pan of the dead weight tester was rotated to reduce the friction.

There was a small difference between the two sets of calibrations which was due to friction between the piston

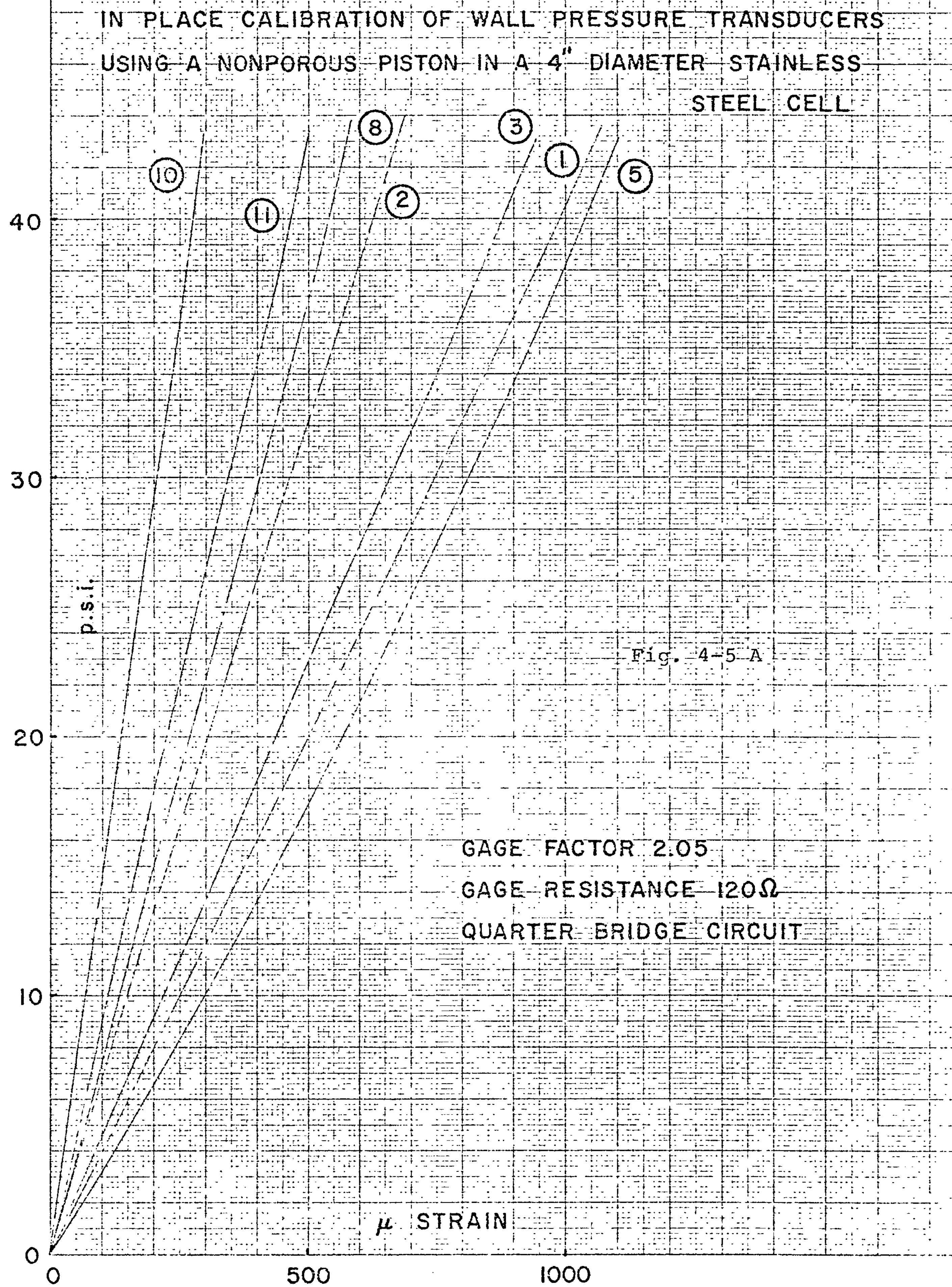

IN PLACE CALIBRATION OF WALL PRESSURE TRANSDUCERS
USING A NONPOROUS PISTON IN A 4" DIAMETER STAINLESS
STEEL CELL
10
11
8
2
3
1
5
40
30
20
10
0
p.s.i.
GAGE FACTOR 2.05
GAGE RESISTANCE 120Ω
QUARTER BRIDGE CIRCUIT
μ STRAIN
0
500
1000

Fig. 4-5 A

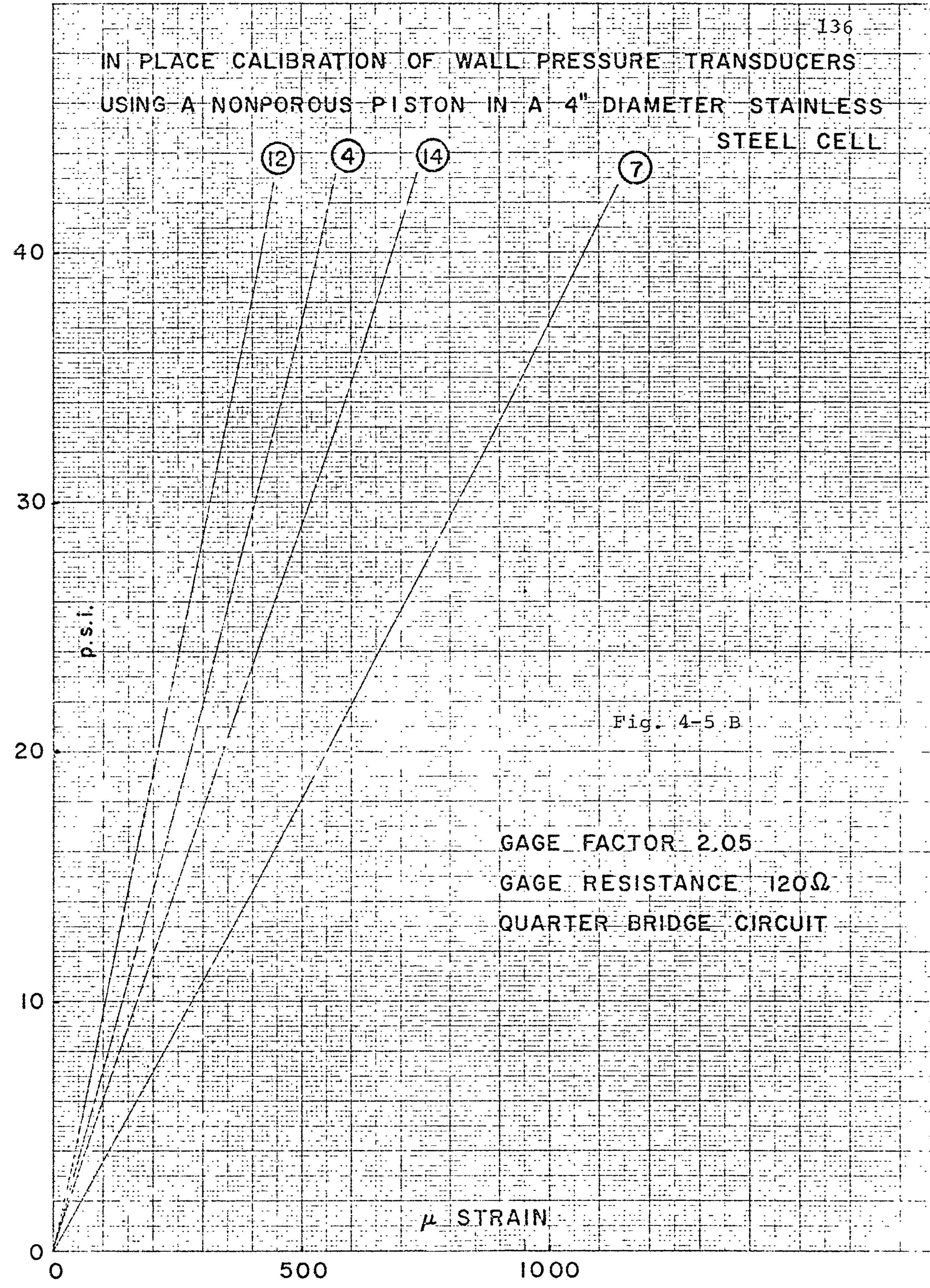

IN PLACE CALIBRATION OF WALL PRESSURE TRANSDUCERS
USING A NONPOROUS PISTON IN A 4" DIAMETER STAINLESS
STEEL CELL
12
4
14
7
p.s.i.
40
30
20
10
0
GAGE FACTOR 2.05
GAGE RESISTANCE 120Ω
QUARTER BRIDGE CIRCUIT
μ STRAIN
0
500
1000

Fig. 4-5 B

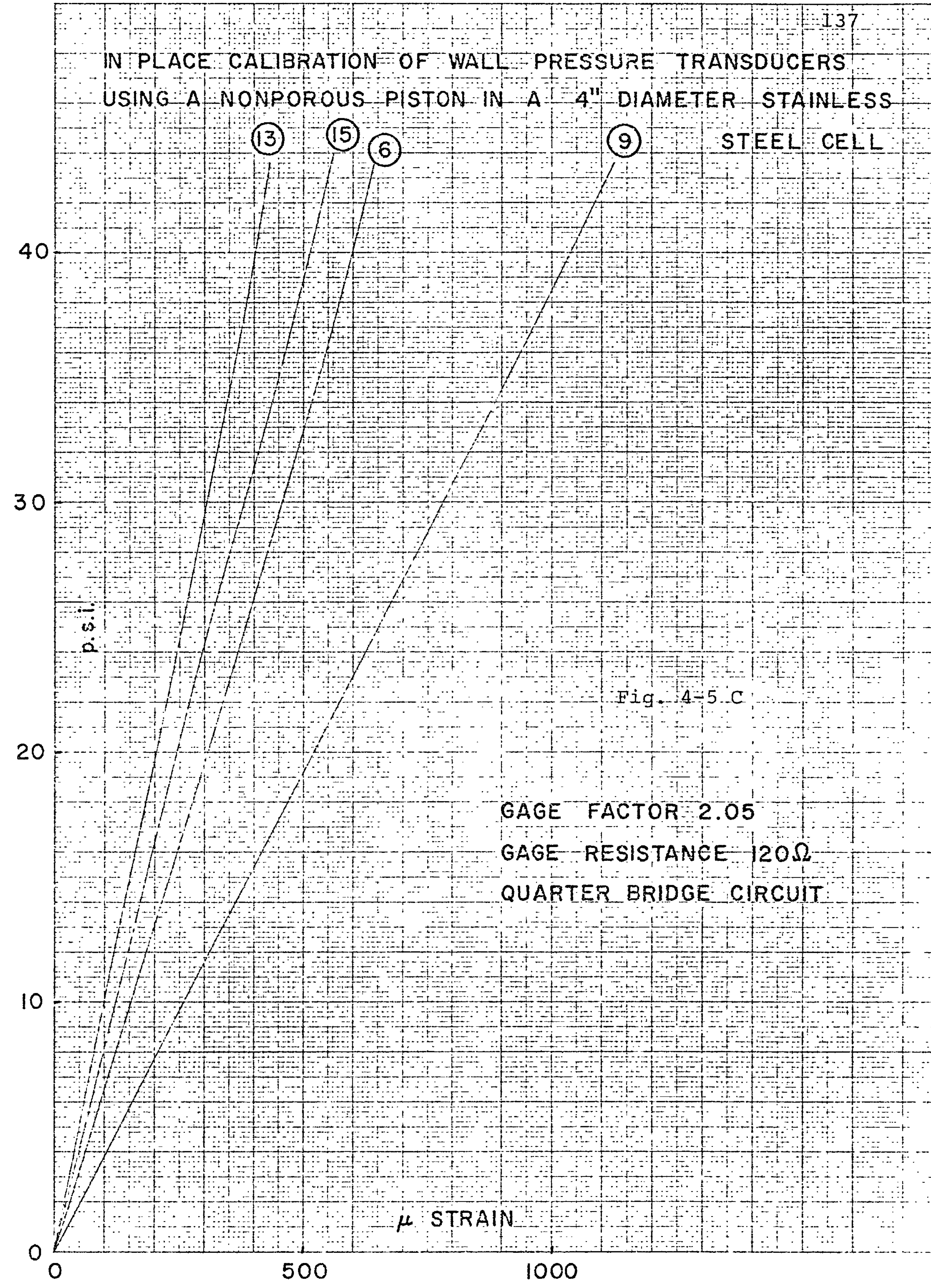

IN PLACE CALIBRATION OF WALL PRESSURE TRANSDUCERS USING A NONPOROUS PISTON IN A 4" DIAMETER STAINLESS STEEL CELL
13
15
6
9
p.s.i.
40
30
20
10
0
0
500
1000
μ STRAIN
GAGE FACTOR 2.05
GAGE RESISTANCE 120Ω
QUARTER BRIDGE CIRCUIT

Fig. 4-5 C

and the cylinder of the cell. The friction was equivalent to an average value of 1 p.s.i. and varied from 0.5 to 1.5 p.s.i. The friction was not correlated to the applied pressure and a constant value of 1.0 p.s.i. was deducted from the mechanical pressure applied by the piston.

4.2.3 EXPERIMENTAL PROCEDURE

1) Slurry of a predetermined concentration is prepared in a large beaker. Stirring is done by hand to prevent particle degradation.

2) The cell is greased on the inside (except on the porous plate) with silicone grease to reduce friction at the cake boundaries. The grease layer on the cylinder wall should be thin or excess grease will accumulate at the corners. To minimize accumulation, after grease is applied by hand, a nonporous disc with O-ring having a close tolerance is passed through.

3) Slurry is stirred again and poured in the cell. Initial readings of wall pressure transducers are recorded. Initial readings of bottom pressure transducers are set to zero. Initial readings can drift by a few microstrains (less than 0.5 p.s.i.) over a period of a few days and should be recorded in every experiment. The slurry is allowed to settle for about 10 minutes.

4) A machine-cut Whatman No. 2 filter paper of 3.96" diameter (when dry) is gently laid on the top liquid surface of the slurry in such a way that no air is trapped between the filter paper and the liquid. When wet, the filter paper expands to a diameter of 3.996". Whatman No. 4 paper retains

Solka floc particles, for finer materials like Talc,Whatman No. 2 paper is preferred.

5) The porous piston is inserted from the top. Sound of air passing through the porous plate can be heard until the latter reaches the liquid level. After that the piston is allowed to sink a little further until resistance to pressing increases. At that point the piston is considered to have contacted the top of the settled cake. The settled cake surface is not at level, and rotating the piston levels it. The hard steel ball is put on top of the piston, and lever beam is engaged.

6) Required weight is placed in the cage, and the cake is allowed to be squeezed. The beam is leveled constantly during the first few minutes as initial squeezing is quite rapid. After the rate of squeezing slows down, piston top should be horizontal. If not, leveling should be done by pressing the piston sideways while holding the beam firmly by one hand. This guarantees that the piston will be in a vertical position.

7) After that the apparatus is left undisturbed for about 40 minutes. Then readings of a few bottom pressure transducers are recorded. The cake bottom is the farthest from the drainage surface and is the last region to reach equilibrium. The beam is leveled again; and after an additional 20 minutes, readings are taken on the same bottom transducers as before. If the readings do not differ by more than two microstrains, (less than 0.2 p.s.i.) it can be assumed that equilibrium has been reached, and readings on all

transducers can be recorded. Otherwise the procedure is repeated at 20 minute intervals until equilibrium is established.

8) The gap between the top flange and the cylinder is measured with Vernier calipers. The difference between this reading and the reading when there is no cake inside, is the cake thickness.

9) More weights are added and steps 6) through 8) are repeated.

10) When the experiment is over, the weights are removed. The beam is rotated horizontally and taken off the piston top. The piston is pulled out. Water is siphoned out.

11) The top flange is removed and the cylinder is lifted. The cake comes along with the cylinder which is placed on a large piece of paper. If the cake is thin, it can be pushed out with hand. A thick cake has to be broken with a knifelike device. The device used for this purpose should not scratch the cylinder wall or ruin the TR diaphragms.

12) At the termination of the experiment the cell is cleaned with soap and water, rinsed and wiped dry with a towel.

13) The porous plate is backwashed with water and air after a few runs in order to remove the solid particles which have lodged inside the porous plate.

4.2.4 TABULATION OF THE DATA ON BOTTOM AND WALL

The measurement of stress distribution was carried out at applied pressures of 40 and 80 p.s.i. with different quantities of solids. Data for bottom and wall pressure distribution are tabulated on the following pages.

NORMALIZED VERTICAL PRESSURE DISTRIBUTION ON THE BOTTOM OF A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

(Inside of the cell is greased with Silicone grease.)

			Dimensionless radius r/R						
			0	.281	.438	.438	.594	.750	.890
L/D	$\varepsilon_{av.}$	s	P_{VB}/P_0 values are tabulated below						
			Solids: Solka floc BW-200; Liquid:Water; P_0=40.2						
.070		.123	1.170	1.145	1.145	1.080	0.984	0.840	0.750
.115		.167	1.275	1.230	1.160	1.152	1.060	0.910	0.747
.110		.156	1.090	1.152	0.985	1.033	0.945	0.860	0.840
.110		.152	1.290	1.255	1.035	1.182	0.923	0.913	-
(.112)		.143	1.160	1.260	1.037	1.100	1.020	0.898	0.667
.174		.167	1.013	0.985	0.921	0.975	0.925	0.890	0.702
.238		.167	0.895	0.867	0.925	0.895	0.888	0.865	0.750
.240		.167	0.823	0.778	0.876	0.892	0.868	0.921	0.745
.484		.167	0.801	0.824	0.839	0.821	0.815	0.752	0.740
.481		.167	0.855	0.760	0.750	0.767	0.821	0.715	0.690
.765		.167	0.715	0.701	0.654	0.667	0.687	0.614	0.572
.893		.189	0.672	0.652	0.645	0.635	0.637	0.628	0.535
1.056		.206	0.632	0.610	0.605	0.605	0.622	0.535	0.505
			Solids: Solka floc BW-200; Liquid: Water; P_0=80.						
.065		.123	1.166	1.138	1.050	1.043	0.919	0.878	0.794
.100		.167	0.987	1.196	0.962	1.120	0.959	0.887	0.743
.152		.167	1.038	0.981	0.884	0.950	0.884	0.880	0.728
.204		.167	0.874	0.847	0.905	0.877	0.863	0.850	0.788
.206		.167	0.800	0.816	0.850	0.860	0.823	0.870	0.748
.419		.167	0.762	0.809	0.804	0.778	0.726	0.708	0.663
.414		.167	0.844	0.867	0.750	0.760	0.812	0.800	0.662
.636		.167	0.738	0.730	0.675	0.702	0.700	0.621	0.557
.763		.189	0.688	0.680	0.675	0.652	0.635	0.615	0.503
.927		.206	0.597	0.602	0.581	0.597	0.595	0.485	0.431

NORMALIZED VERTICAL PRESSURE DISTRIBUTION ON THE BOTTOM OF A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

(Inside of the cell is greased with Silicone grease.)

			Dimensionless radius r/R						
			0	.281	.438	.438	.594	.750	.890
L/D	$\varepsilon_{av.}$	s	P_{VB}/P_0 values are tabulated below						
			Solids: $CaCO_3$; Sp.gr.=2.398; Liquid: Water at 75 °F; P_0= 40.2 p.s.i.						
.081		.185	1.050	1.095	1.033	1.075	0.959	1.120	0.715
.084		.167	1.123	1.210	1.270	1.045	1.110	1.060	0.505
1.098		.316	0.784	0.776	0.776	0.746	0.751	0.647	0.764
1.176		.333	0.808	0.746	0.776	0.771	0.779	0.759	0.779
			Solids: $CaCO_3$; Liquid: Water at 75 °F; P_0= 80.						
.074		.185	1.010	1.028	0.935	1.000	0.884	1.016	0.753
.074		.167	1.063	1.016	1.138	0.960	0.969	0.979	0.619
1.019		.316	0.653	0.640	0.634	0.615	0.591	0.581	0.558
1.019		.316	0.660	0.628	0.594	0.581	0.534	0.520	0.550
1.098		.333	0.644	0.607	0.603	0.615	0.590	0.579	0.569
			Solids: Talcum powder; Liquid: Water; P_0=40.2						
.044		.143	1.115	1.134	1.131	0.973	1.015	1.129	0.894
.115		-	1.175	1.189	1.160	1.089	0.952	1.060	0.922
			Solids: Talcum powder; Liquid: Water; P_0= 60.2						
.109		-	1.040	1.061	1.049	0.980	0.905	0.965	0.873
			Solids: Talcum powder; Liquid: Water; P_0= 80.						
.038		.143	1.115	1.134	1.131	0.973	1.015	1.129	0.894
.102		-	1.085	1.078	1.039	0.921	0.875	0.965	0.905

NORMALIZED DISTRIBUTION OF HORIZONTAL PRESSURE EXERTED NORMAL TO THE WALL BY A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

Material	Solka floc BW-200; Sp. Gr.= 1.519														
P_0 p.s.i.	40.7														
L/D	.124	.167	.174	.231	.231	.470	.471	.716	.115	.110	.110	.110	.174	.237	.238
ε_{av}	.678	.642	.656	.654	.654	.660	.661	.665							
s	.102	.162	.173	.162	.154	.167	.167	.188	.167	.156	.152	.143	.167	.167	.167
z/D ↓	Values of P_H/P_0 are tabulated below. z = distance from the bottom. z/D ⩽ L/D														
0.047	.322	.339	.290	.342	.336	.271	.287	.258	.390	.433	.351	.366	.363	.386	.396
0.109		.430	.430	.381	.339	.300	.324	.300					.351	.403	.366
0.172				.341	.341	.317	.304	.284						.423	.376
0.234						.346	.300	.253							
0.234						.260	.285	.270							
0.297						.247	.273	.236							
0.360						.343	.268	.275							
0.422							.351	.312							
0.484								.244							
0.547								.246							
0.547								.300							
0.641															
0.734															
0.828															
0.922															
1.016															

NORMALIZED DISTRIBUTION OF HORIZONTAL PRESSURE EXERTED NORMAL TO THE WALL BY A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

Material	Solka floc BW-200; Sp. Gr.=1.519						Solka floc BW-200								
P_0 p.s.i.	40.7						80.9								
L/D	.483	.481	.765	.893	1.056	1.056	.148	.148	.199	.406	.406	.621	.100	.099	.100
ε_{av}							.594	.594	.598	.606	.606	.614			
s	.167	.167	.167	.189	.206	.206	.162	.173	.154	.167	.167	.188	.167	.156	.152
z/D ↓	Values of P_H/P_0 are tabulated below. z = distance from the bottom. $z/D \leqslant L/D$														
0.047	.346	.331	.284	.276	.252	.224	.297	.291	.329	.285	.306	.271	.343	.281	.301
0.109	.381	.356	.303	.311	.227	.242	.373	.420	.349	.304	.349	.323			
0.172	.351	.341	.299	.305	.232	.239			.359	.325	.298	.315			
0.234	.301	.343	.284	.294	.271	.222				.304	.307	.307			
0.234	.289	.341	.286	.274	.251	.204				.291	.309	.296			
0.297	.319	.326	.294	.276	.239	.202				.263	.279	.302			
0.360	.371	.298	.316	.286	.274	.236				.312	.296	.310			
0.422		.361	.328	.294	.286	.244						.347			
0.484			.340	.276	.259	.226						.300			
0.547			.289	.288	.214	.177						.402			
0.547			.343	.334	.294	.207						.460			
0.641				.306	.229	.254									
0.734				.304	.274	.212									
0.828				.348	.319	.284									
0.922					.321	.376									
1.016					.331										

NORMALIZED DISTRIBUTION OF HORIZONTAL PRESSURE EXERTED NORMAL TO THE WALL BY A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

Material	Solka floc BW-200; Sp.Gr.= 1.519									$CaCO_3$ Sp. Gr.= 2.398					
P_0 p.s.i.	80.9									40.2					
L/D	.152	.204	.206	.418	.413	.636	.763	.927	.933	.081	.084	.700	.694	.879	1.058
ε_{av} s	.167	.167	.167	.167	.167	.167	.189	.206	.206	.185	.167	.286	----	.333	.316
z/D ↓	Values of P_H/P_0 are tabulated below. z = distance from the bottom. $z/D \leq L/D$														
0.047	.328	.352	.345	.315	.286	.255	.252	.228	.230	.460	.405	.341	.306	.217	.259
0.109	.370	.358	.385	.355	.324	.291	.305	.225	.260			.259	.279	.264	.247
0.172		.341	.340	.300	.317	.268	.285	.214	.256			.284	.288	.291	.281
0.234				.288	.291	.275	.286	.233	.243			.276	.308	.294	.271
0.234				.291	.292	.266	.255	.265	.211			.310	.296	.311	.306
0.297				.304	.288	.294	.258	.246	.233			.299	.298	.269	.272
0.360				.298	.302	.310	.270	.233	.263			.299	.298	.286	.237
0.422						.296	.249	.303	.259			.266	.341	.306	.249
0.484						.336	.272	.234	.220			.269	.308	.259	.284
0.547						.306	.286	.181	.183			.301	.304	.336	.324
0.547						.338	.334	.200	.234			.264	.284	.319	.212
0.641							.268	.253	.231			.326	.334	.324	.251
0.734							.350	.270	.295					.301	.294
0.828								.313							.299
0.922															
1.016															

NORMALIZED DISTRIBUTION OF HORIZONTAL PRESSURE EXERTED NORMAL TO THE WALL BY A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

Material P_0 p.s.i.	$CaCO_3$ 40.2			$CaCO_3$ 80.0								$CaCO_3$ (greaseless wall) 40.2			
L/D	1.048	1.097	1.176	.645	.645	.810	.969	.979	.993	1.019	1.097	1.078	1.078	1.078	1.078
ε_{av} s	.316	.316	.333	.286	.286	.333	.316	.316	.316	.316	.333	.316	.316	.316	.316
z/D ↓	Values of P_H/P_0 are tabulated below. z = distance from the bottom. z/D ≤ L/D														
0.047	.314	.361	.323	.293	.261	.245	.227	.244	.260	.239	.264	.207	.189	.239	.147
0.109	.314	.331	.348	.246	.262	.260	.205	.261	.260	.220	.265	.209	.217	.217	.154
0.172	.333	.338	.353	.260	.294	.268	.235	.264	.269	.235	.269	.222	.224	.239	.187
0.234	.301	.336	.358	.270	.287	.278	.209	.270	.268	.225	.274	.207	.232	.254	.197
0.234	.308	.331	.343	.275	.260	.275	.215	.258	.269	.250	.260	.314	.234	.242	.201
0.297	.308	.323	.351	.278	.308	.252	.211	.285	.266	.231	.269	.259	.244	.262	.192
0.360	.299	.346	.376	.270	.312	.285	.232	.285	.287	.266	.285	.234	.264	.264	.174
0.422	.334	.343	.346	.287	.321	.312	.231	.312	.299	.248	.304	.251	.236	.274	.199
0.484	.358	.354	.368	.280	.307	.286	.241	.282	.292	.268	.289	.249	.249	.259	.189
0.547	.381	.294	.294	.288	.274	.255	.217	.256	.260	.178	.286	.194	.226	.234	.204
0.547	.271	.361	.348	.286	.202	.270	.154	.315	.312	.256	.299	.234	.214	.239	.254
0.641	.371	.316	.346	.243		.306	.220	.244	.282	.210	.299	.234	.299	.306	.224
0.734	.371	.334	.336			.270	.205	.339	.294	.234	.304	.269	.231	.262	.179
0.828	.366	.363	.458				.288	.392	.321	.295	.358	.319	.291	.324	.266
0.922	.348	.400	.351						.336	.294	.334	.419	.336	.319	.294
1.016	.289	.371	.400								.296		.229	.306	

NORMALIZED DISTRIBUTION OF HORIZONTAL PRESSURE EXERTED NORMAL TO THE WALL BY A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

Material P_0 p.s.i.	$CaCO_3$ (greaseless wall) 80.0				Talcum Powder		Kaolin NF 40.2		Kaolin NF 80.0	
					40.2	80.0				
L/D	.989	.989	.989	.989	.115	.102	.515	.630	.483	.586
ε_{av} s	.316	.316	.316	.316	(.14)	(.14)	.251	.286	.251	.286
z/D ↓	Values of P_H/P_0 are tabulated below. z = distance from the bottom. $z/D \leqslant L/D$									
0.047	.212	.183	.214	.184	.749	.648	.408	.403	.441	.464
0.109	.238	.196	.216	.203			.455	.443	.507	.468
0.172	.244	.214	.237	.219			.520	.448	.550	.490
0.234	.231	.215	.239	.221			.455	.443	.508	.493
0.234	.231	.220	.227	.215			.453	.408	.488	.450
0.297	.279	.246	.246	.233			.483	.485	.530	.523
0.360	.250	.235	.246	.251			.518	--	.570	--
0.422	.275	.243	.260	.232			.568	.821	.605	--
0.484	.252	.234	.258	.240			.518	.535		.538
0.547	.272	.220	.272	.250				.447		
0.547	.294	.193	.272	.262				.373		
0.641	.270	.250	.285	.235						
0.734	.275	.195	.295	.260						
0.828	.315	.284	.304	.284						
0.922	.362	.308	.320	.330						
1.016										

4.3 STRESS MEASUREMENT AT THE TOP OF THE CAKE (DETAILS)

4.3.1 DESCRIPTION OF THE APPARATUS

Figure (4-6) illustrates a schematic diagram of the apparatus which is a modification of the earlier cell. The bottom plate containing transducers was machined to replace the porous plate on the piston. In the new apparatus, drainage is provided at the bottom by placing a porous stainless steel plate. The expelled liquid exits the cell through a needle valve VB and then through a tube. The transducer wires come out from a hole on the piston wall near the top. The piston has an air vent which can be opened or closed by means of shut off valve. The piston should always be kept inverted when it is outside the cell so that TR diaphragms will not be damaged. The top TR wires are connected directly to seven channels of the readout device. Apart from these changes the apparatus was exactly same as before.

4.3.2 CALIBRATION OF THE TRANSDUCERS

The previous calibrations of transducers on the cylinder wall were used and no recalibrations were necessary. However the transducers on the top plate were recalibrated because of possible changes resulting from stresses induced during machining. It was found that the calibrations had not changed more than 2 %.

The cell is greased and the cylinder is placed on the bottom plate. The end of the drainage tube is connected to a water tap and valve VB as well as the tap are opened to let water enter the cell. Water drives out air trapped in the porous

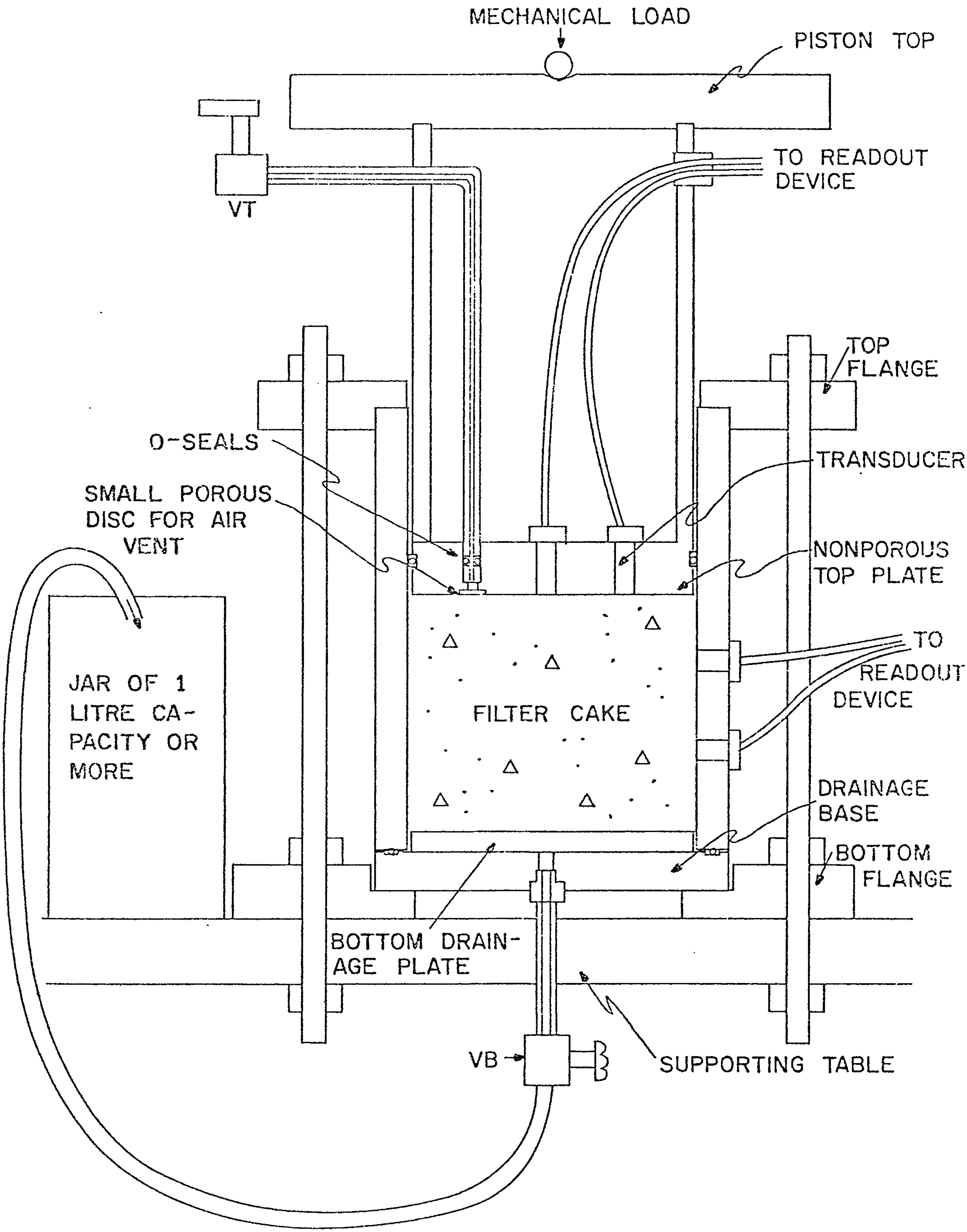


Fig. 4-6 Stress measurement at the top of a cake : schematic diagram of the apparatus.

plate. Valve VB is closed when the water level is slightly above the topmost pressure transducer on the wall. The flange is placed on the cylinder, adjusted concentrically to the cylinder, and the nuts tightened. The reading of the top plate transducers were adjusted to zero by turning channel knobs as before. Valve VT is opened and the piston is inserted from the top. Air inside passes through the small porous disc (acting as vent) on the top plate, and then goes through the stainless steel tubing fitted inside the piston with an O-seal and finally exits from the piston through valve VT. When the piston reaches the water level, the sound of exiting air stops. After pressing the piston until water drips out, valve VT is closed. Then there is no air inside the cell. Pressure is applied in exactly the same manner as described before, and the calibrations are recorded. After calibrations are over, VT and VB are opened and the piston is pulled out. Water inside is allowed to drain. Figures (4-7 A,B) display the calibration curves for top pressure transducers.

4.3.3 PROCEDURE FOR STRESS MEASUREMENT

Preparation of slurry and greasing inside of the cell are done as described in section 4.2.3 and the cell is assembled. The end of the drainage tube exiting VB is connected to a water tap. Valve VB is opened, and water flows into the cell. Air inside the porous plate is forced out. Water tap is closed. The tube end is disconnected from the water tap and water inside the cell is allowed to drain through the tube end until a small layer is left above the porous plate. VB is closed. End of the tube is raised above the cell level and kept in a flask of

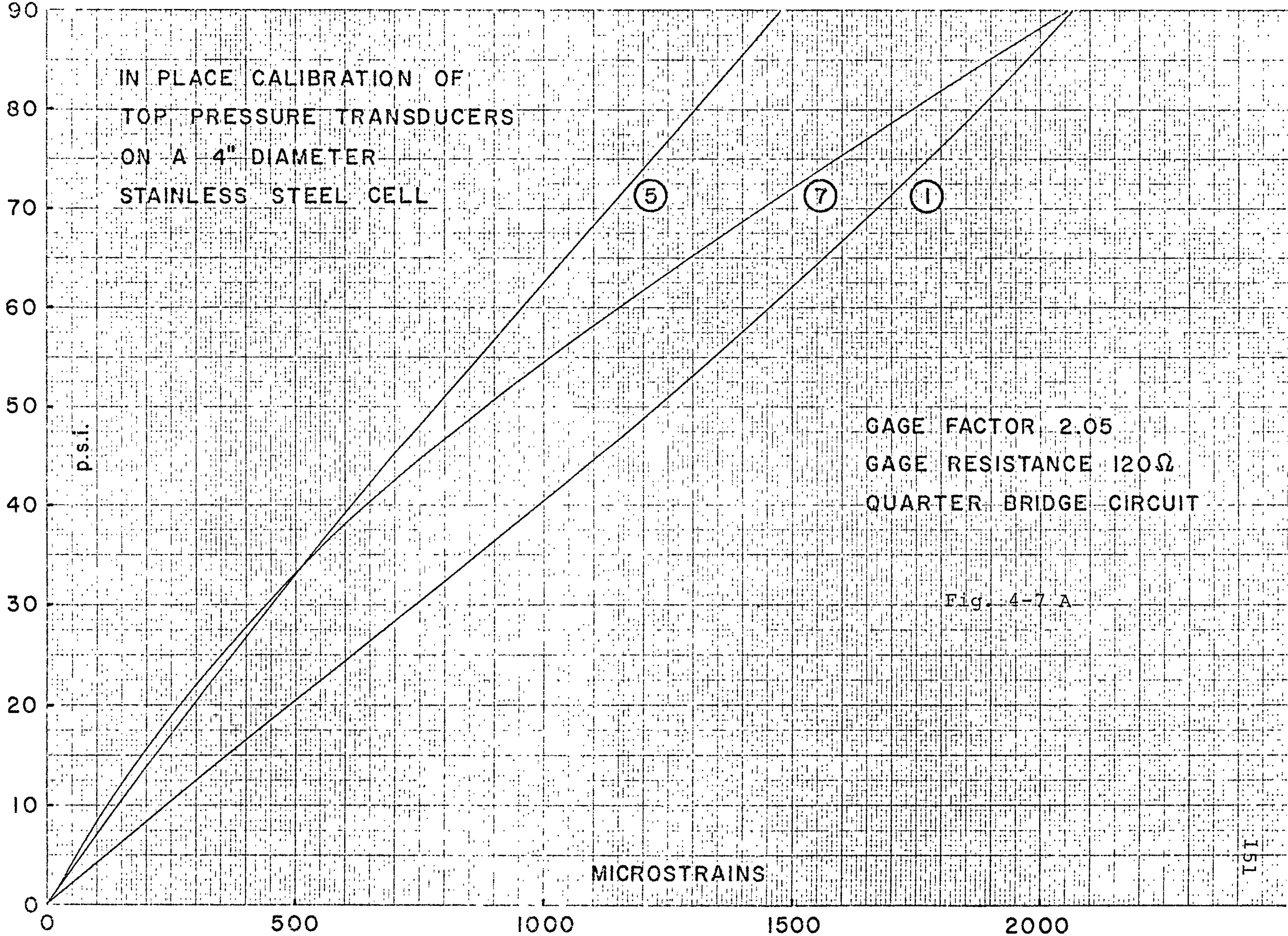
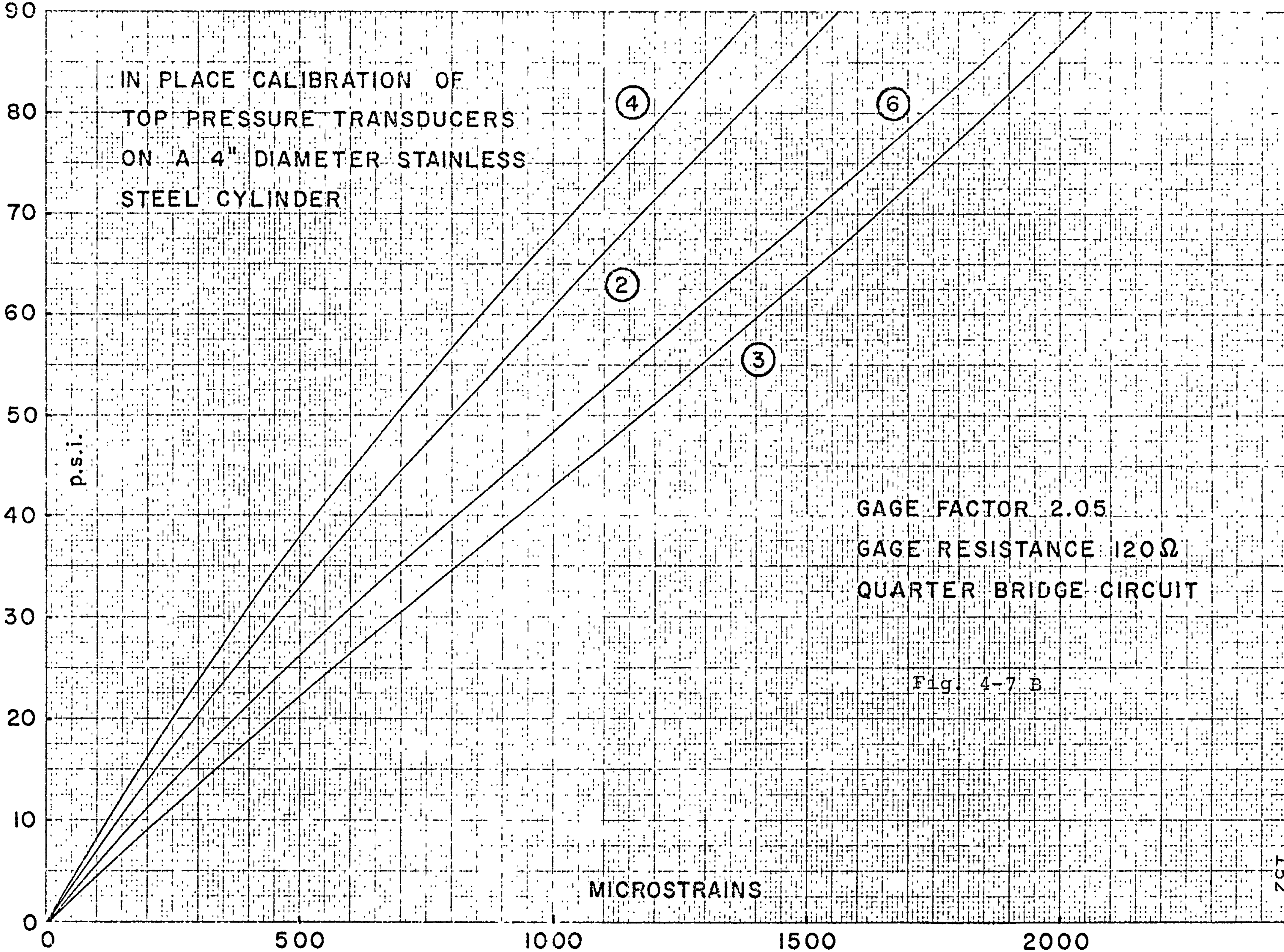

IN PLACE CALIBRATION OF
TOP PRESSURE TRANSDUCERS
ON A 4" DIAMETER
STAINLESS STEEL CELL
GAGE FACTOR 2.05
GAGE RESISTANCE 120Ω
QUARTER BRIDGE CIRCUIT
p.s.i.
MICROSTRAINS
5
7
1
0
10
20
30
40
50
60
70
80
90
500
1000
1500
2000

Fig. 4-7 A


IN PLACE CALIBRATION OF
TOP PRESSURE TRANSDUCERS
ON A 4" DIAMETER STAINLESS
STEEL CYLINDER
GAGE FACTOR 2.05
GAGE RESISTANCE 120 Ω
QUARTER BRIDGE CIRCUIT
p.s.i.
MICROSTRAINS
4
2
6
3
0
10
20
30
40
50
60
70
80
90
500
1000
1500
2000
Fig. 4-7 B

a capacity of one litre or more. This procedure guarantees that no air will go in during squeezing and the cake will remain saturated with water. Water expelled from the cake does not exceed one liter. A circular cut filter paper is placed on the bottom plate in the manner previously described. The edges of the paper are smoothened out with a spatula to assure a good fit.

Slurry is stirred again and poured into the cell. Initial readings of wall TR's are recorded as described before. Initial readings of top TR's are set to zero when the piston is outside the cell. Valve VT is opened, and the piston is inserted in the cell. After the sound of exiting air stops, the piston is pressed a little bit further to ensure that no air is left between the piston and the slurry level. VB is opened and the piston is allowed to sink a little bit further. Then the piston is rotated, VT is closed, the hard steel ball is placed on the top center of the piston and the lever beam is engaged on it. After this the procedure is same as in section 4.2.3, except that instead of measuring stresses on the bottom, stresses on the top of the cake are measured.

After the experiment is over, the weights in the cage are removed and the beam is disengaged. Valves VT and VB are opened. The piston is pulled out and kept inverted on the table. The cell is dismantled, the cake is discharged and the cell is cleaned as described in section 4.2.3. The porous plate on the bottom and the air vent are cleaned free of particles. Pressurised air tap can be connected to VT for backwashing with air.

4.3.4 TABULATION OF DATA FOR TOP

The stress measurement was carried out at applied pressures 40 and 80 p.s.i., and for different quantities of solids. Data on stress distribution at top are tabulated on the next page.

4.4 DETAILS OF THE APPARATUS FOR MEASUREMENT OF HYDRAULIC PRESSURE PROFILE THROUGH A COMPACTED FILTER CAKE

4.4.1 THE MAIN APPARATUS

A filter cake is compacted between two porous plates in a cylinder of 2" i.d., 2.5" o.d. and 9" height. Most of the apparatus is made out of plexiglass. Figure (4-8) is a schematic diagram of the apparatus. The piston is hollow inside. The Plexiglass plate is glued onto the top of the piston. At the top center there is a pit for containing a hard steel ball. Mechanical force is applied on this ball by means of a lever mechanism. The piston has a porous stainless steel plate PP1 fitted with screws on the end having a grooved surface to distribute water uniformly to PP1. The cylinder has five 0.01" wide slits placed at intervals of 0.125". Two of these slits are diametrally opposite and at the same height. Tube pieces of 1/8" o.d. are glued in holes in the cylinder wall to make connections with the slits. The cylinder rests on a flange fixed to the supporting table. This flange is carefully leveled. On top of the cylinder, there is a second flange. The cylinder is locked in its position during the experiment.

The floating bottom FB rests on a force transducer which measures the vertical force transmitted to FB by the cake.

NORMALIZED VERTICAL PRESSURE DISTRIBUTION ON THE TOP OF A COMPACTED FILTER CAKE IN A 4" DIAMETER STAINLESS STEEL CELL.

(Inside of the cell is greased with Silicone grease.)

			Dimensionless radius r/R						
			0	.281	.438	.438	.594	.750	.890
L/D	$\varepsilon_{av.}$	s	P_{VT}/P_0 values are tabulated below						
			Solids: Solka floc BW-200; Sp.gr.= 1.519						
			Liquid: Water at 75 °F; P_0= 40.7 p.s.i.						
.080	.649	(.123)	1.101	1.036	0.920	1.069	0.909	0.904	0.855
.079	.645	–	1.142	1.056	0.876	1.105	0.928	0.885	0.958
.079	.645	–	1.118	1.092	0.973	1.120	0.830	0.950	0.885
.124	.678	.102	1.055	1.010	0.957	0.970	0.971	0.895	0.927
.167	.642	.162	0.893	0.975	0.932	1.008	0.938	0.923	0.923
.174	.656	.173	0.953	0.920	0.933	0.975	1.027	0.852	0.982
.231	.654	.162	0.865	0.865	0.859	0.810	0.860	0.876	0.986
(.231)	–	.154	0.859	0.872	0.847	0.872	0.815	0.884	0.999
.470	.660	.167	0.825	0.840	0.823	0.860	0.871	0.885	1.150
.471	.661	.167	0.860	0.837	0.847	0.880	0.852	0.850	1.090
.716	.665	.188	0.808	0.761	0.828	0.830	0.800	1.037	1.070
			Solids: Solka floc BW-200; Sp. gravity = 1.519						
			Liquid: Water at 75 °F; P_0= 80.87 p.s.i.						
(.070)	–	–	1.097	1.034	0.898	1.052	0.885	0.898	0.810
(.070)	–	–	1.100	1.040	0.830	1.054	0.867	0.855	0.875
.069	.594	–	1.088	1.076	0.952	1.082	0.824	0.940	0.851
.098	.594	.102	1.047	1.010	0.944	0.964	0.969	0.900	0.894
.148	.594	.162	0.870	0.940	0.910	0.990	0.914	0.898	0.890
.148	.594	.173	0.931	0.916	0.918	0.958	1.014	0.840	0.920
.200	.598	.154	0.889	0.898	0.866	0.891	0.844	0.872	0.970
.406	.606	.167	0.825	0.854	0.805	0.856	0.854	0.856	1.105
.406	.606	.167	0.857	0.836	0.836	0.855	0.831	0.834	1.060
.621	.614	.188	0.810	0.775	0.835	0.806	0.795	0.985	1.052

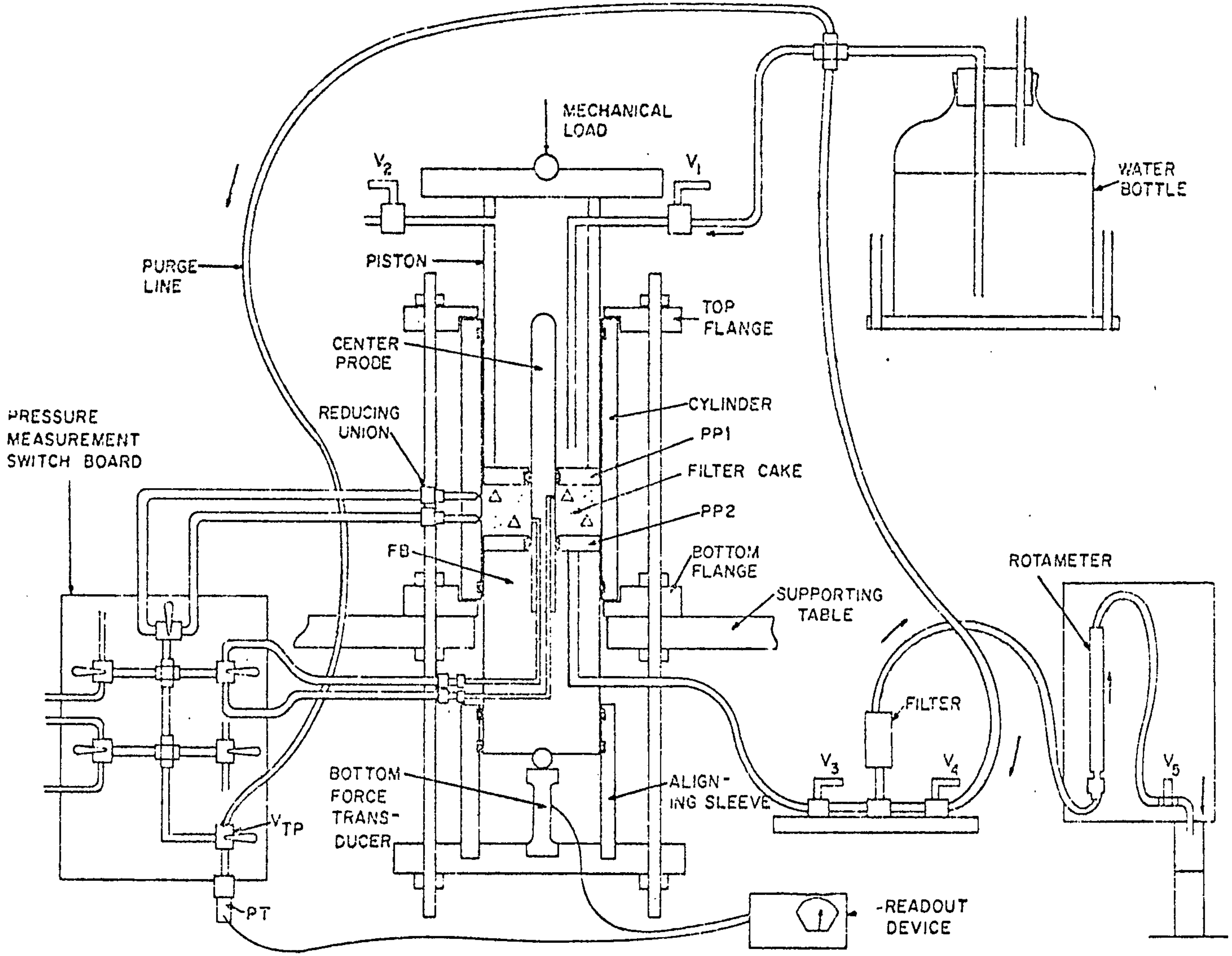


Fig. 4-8 Schematic diagram of the new CP cell

A ball-socket type contact is provided to make the line of action of the transmitted force coincide with the axis. FB has a 1/2" diameter probe centrally glued. A porous stainless steel plate is attached with screws on the grooved side of FB. These grooves (fig. 4-9) collect the water passing through the porous plate and carry it to a 1/4" o.d. tube exiting from FB. The center probe is the most important part of FB. The tip of the probe is rounded so that it can enter the central holes in PP1 and PP2 easily. The probe has four 0.01" wide slits placed at intervals of 0.125". The first slit is 0.04" above the porous plate. A vertical tubular chanel starts from each slit, then bends at right angles to open on the wall of FB. A 1/8" o.d. stainless steel tube is glued in each of these 4 holes. Two pins are provided on FB such that when it is pulled up, these pins touch the supporting base before channel tubes do. FB cannot be pulled up further and any damage to channel tubes is avoided.

Four tubes coming out of FB and five tubes coming out of the cylinder are connected by means of reducing unions and 1/4" o.d. copper tubing to Imperial Hi duty shutoff valves fixed on a board (fig. 4-8). These valves are connected to a Statham pressure transducer PT. All tube fittings are dismantlable. By selecting the proper position of the switching lever of the appropriate valve, any desired channel can be connected to the pressure transducer, keeping all other channels separate. The slits and corresponding channels are numbered 1 to 4 on the center probe and 5 to 9 on the side wall. Slits number

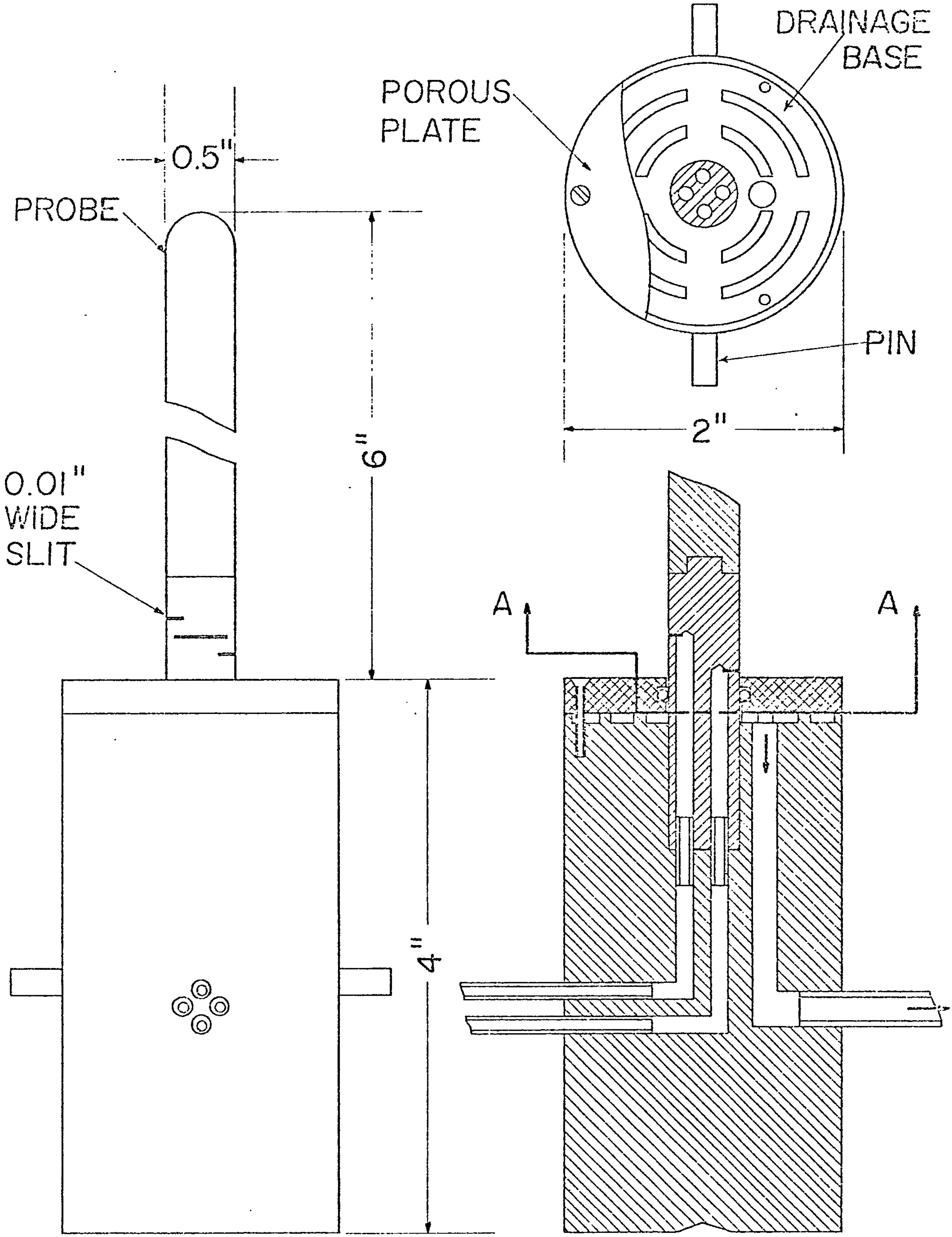


Fig. 4-9 Details of the floating bottom and the center probe.

1 and 5 are the closest to the bottom. The plate supporting the transmitted force transducer has four nuts below it. By adjusting these nuts, the top of FB can be leveled as well as moved up and down. By making this adjustment, slit 1 should be brought to the same level as slit 5. Slits 6 and 7 on the wall are at the same level.

A purge line is provided which removes air from the channels. It also helps cleaning of the slits by washing the slurry particles out. No air bubbles should remain in the channels for the following reasons. Water inside channels exerts hydroststic head on PT, and air bubbles affect the head. Also air is compressible and slows response of the pressure measuring system. Just above PT, a three way valve V_{TP} is provided (fig. 4-8). At 9 O'clock position, it connects only PT to the channels. At 12 O'clock position it connects only purge line to channels and disconnects PT.

A large number of O-ring seals are provided in this apparatus. Their purpose is twofold. One is to prevent leakage and the other is to aid alignment without causing excessive friction. Alignment of the force transducer, FB, the cylinder and the piston is extremely important. Without good alignment, the line of action of the compressive force is not coincident with the axis of the cylinder. These off centered forces cause moments which jam the piston as well as FB and affect the transmitted force.

The piston has two openings on its wall near the top, with tubing pieces glued in them. Water enters through shut off valve V1 (fig. 4-8). Air inside the piston can exit through shut off valve V2 during squeezing. Water flows downwards through the cake and passes through valve V3, a rotometer and exits through a fine metering valve V5. Some fine particles escape the cake and pass through the small gap between the filter paper and the cylinder. Some of those particles pass through PP2 and diposit on inside of the rotometer and the fine metering valve. In order to prevent fowling of these sensitive instruments, a filter is installed between V3 and the rotometer. Valves V3 and V4 are Imperial Hi-duty shut off valves. It is important that these valves be of high quality as a very small leakage can result in incorrect measurement of flow rate through the cake, particularly when flow rate is less than 1 ml/min. Valve V4 is used for backwashing PP2 as explained later. A bottle of large capacity is used to provide constant head. The bottle is connected to valves V_{TP} (for purge line), V4 and V1 by siphoning action.

Both plates PP1 and PP2 are made of porous stainless steel of 40 microns average pore diameter and 1/4" plate thickness. During machining, oil penetrates the porous structure. Before assembling these plates in the apparatus, they are kept submerged in acetone for a few hours. Resulting oil acetone mixture is discarded. The procedure is repeated. Then water is passed through the plates to wash out the acetone.

4.4.2 ACCESSORIES

4.4.2.1 Excess grease wiper: This consists of a 1.998" diameter Delrin rod fitted with a long handle.

4.4.2.2 Filter papers : Annulii of o.d. 1.98" and i.d. 0.5" made out of Whatman No. 4 paper (or equivalent) are used.

4.4.2.3 PAPER PUSHING TUBE: A tube of 0.75" i.d. and 1.5" o.d. is used to push the annular filter paper disc over the center probe.

4.4.2.4 READOUT DEVICE FOR TRANSDUCERS: Both the bottom force transducer and the pressure transducer are connected to 'Straincert TN8' strain indicator. The resolution of the readout device is one microstrain. Calibration relationship for hydraulic pressure transducer and bottom force transducer are linear. One microstrain output is equivalent to 1.148 poundals/ft^2 of hydraulic pressure and 0.094 p.s.i. of bottom (transmitted) pressure, at a gage factor of 2.05. The frictional force experienced by FB is equivalent to a constant 2 p.s.i. which should be added to the transmitted pressure.

4.4.2.5 SLIT CLEANER STRIP: A 0.01" thick, 1/2" wide, 2" long brass shimstock piece is bent in L shape. This strip is used to clean the slits.

4.4.3 IMPROVED CENTER PROBE

As mentioned in section 3.8 details of the improved center probe are shown in figure (4-10). The drawing is an enlargement by a scale of two. The O-rings separate slits into individual compartments to prevent short circuiting of the water flow next to the probe. The casing for the probe and the porous plate are displayed in figure (4-11). After removing air from the channels by purging with water, a 0.04" wide

PART NAME: NEW CENTER PROBE
MATERIAL: PLEXIGLASS
NUMBER OF PIECES: 1

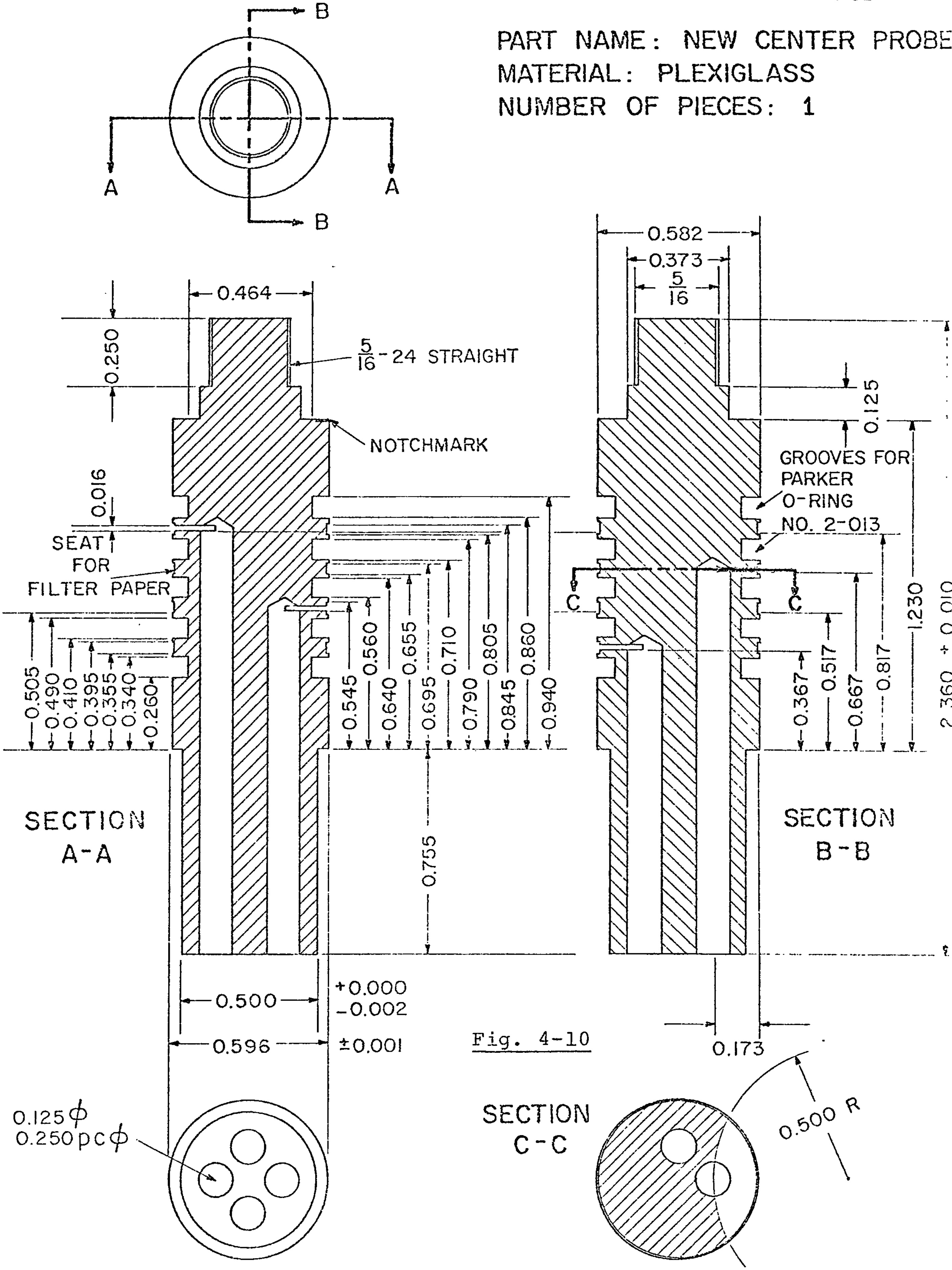


Fig. 4-10

NOTE: DIMENSIONAL TOLERANCE IS ±0.002 UNLESS STATED OTHERWISE. ALL DIMENSIONS ARE IN INCHES.

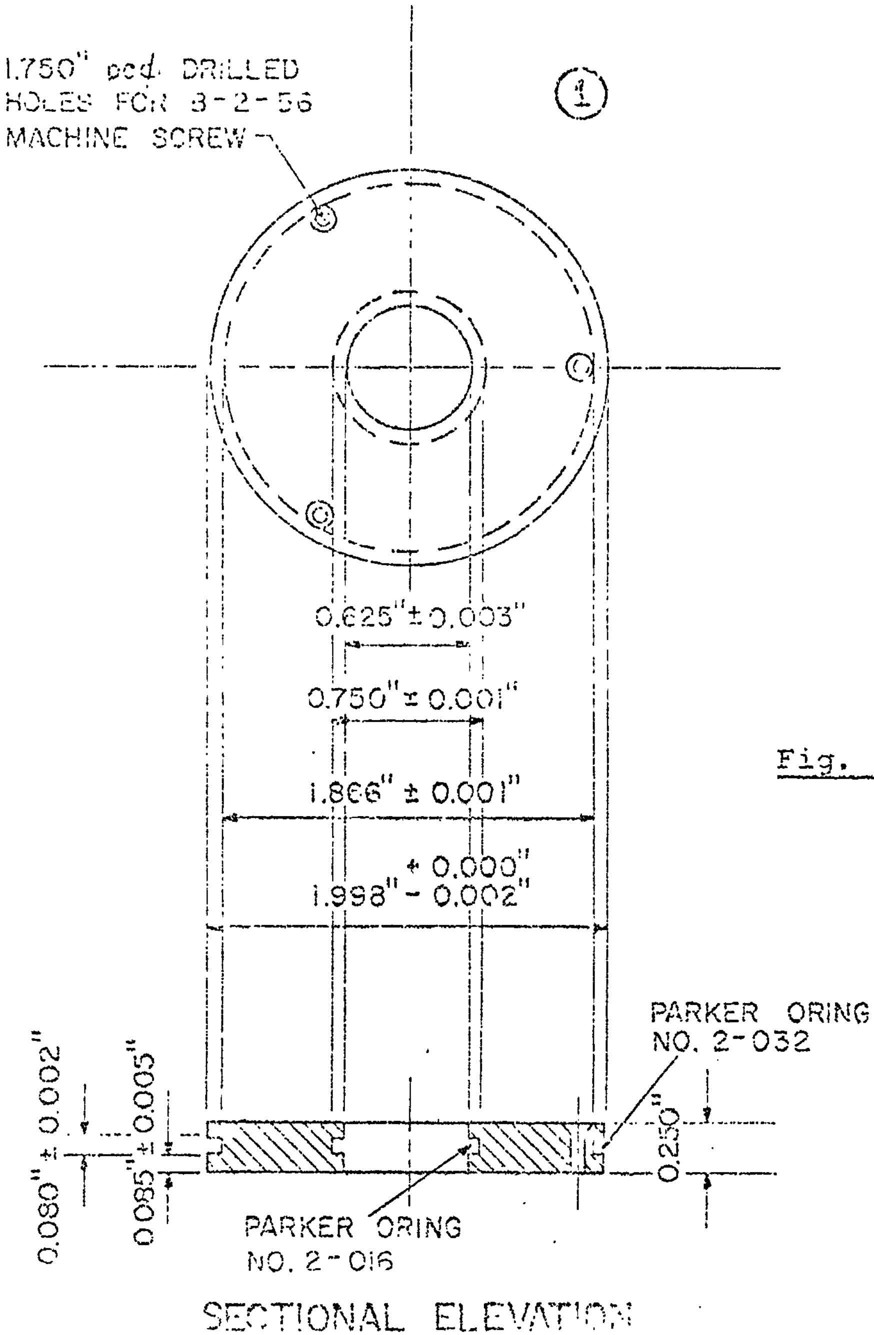
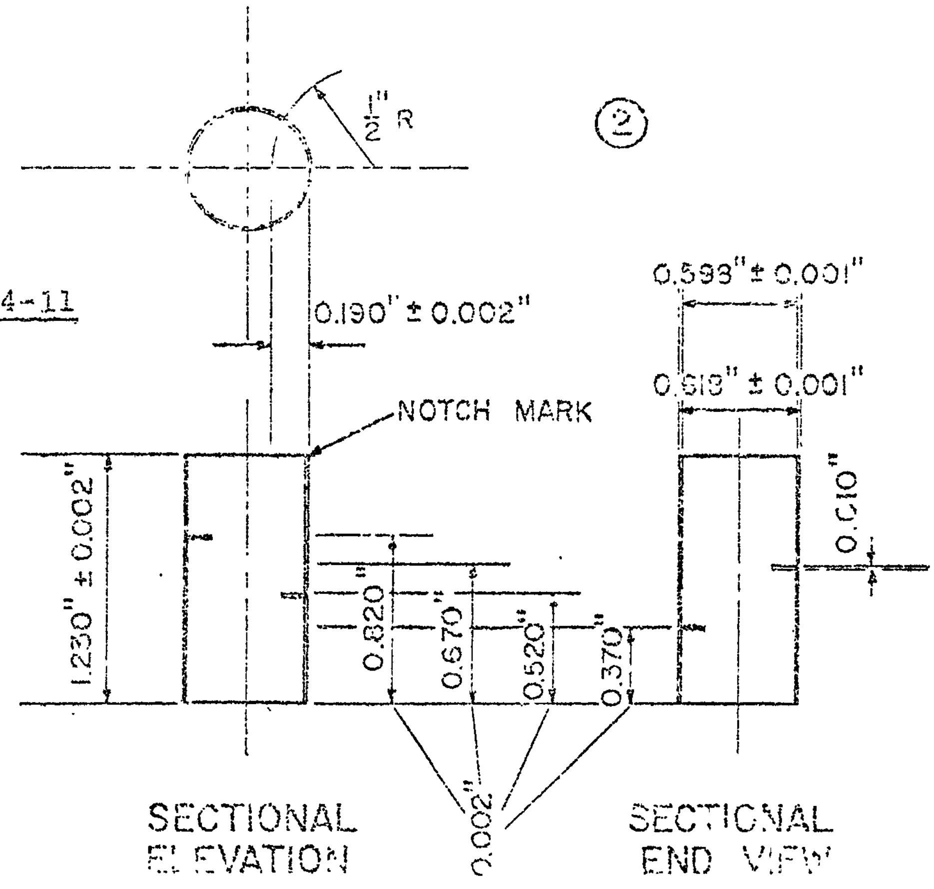


FIG. NO.	PART NAME	MATERIAL	NO. OF PIECES
1	POROUS PLATE	POROUS STAINLESS STEEL 40μ GRADE	2
2	CENTER PROBE CASING	STAINLESS STEEL TUBING	1

Fig. 4-11



strip of Whatman No. 2 (or equivalent) filter paper is wound around the probe and fitted in the seat around each slit. The casing is slid gently over the probe, taking care not to tear off filter paper. Alignment of notchmarks on the probe and on the casing assures alignment of slits. Filter paper prevents particles from going inside slits on the probe.

4.5 DETAILED EXPERIMENTAL PROCEDURE FOR MEASUREMENT OF HYDRAULIC PRESSURE PROFILE THROUGH A COMPACTED FILTER CAKE

4.5.1 PREPARATION OF THE SLURRY

The quantity of solids should be such that the cake thickness lies between 0.415 and 0.65 inches. If cake theckness is less than 0.415", the top slits are ineffective, as they will be above the top surface of the cake. For larger cake thickness the pressure drop from top to bottom slit is small as compared to the pressure drop across the entire cake. Slurry concentration affects the specific cake resistance α, as discussed before. Desired quantity of solids is taken in a jar (or beaker) and a quantity of distilled (or demineralized) water is added to the powder until easily stirable slurry is formed. It is necessary to save some water for washing the beaker, rod etc., so that when all washings are poured into the cylinder of the apparatus, the required concentration is obtained. If slurry is very thick, it is necessary to apply vacuum to remove the air bubbles.

4.5.2 SETTING UP THE CELL

It is necessary to clean the entire cell. To accomplish this,

4.5.2.1) The probe is wiped with a moist towel to remove adhering particles. Then the probe is wiped with a sponge dipped in a soap solution. Again moist towel is used to wipe off the traces of soap. Slits on the probe are cleaned with the cleaner strip. Then these slits are purged with water by setting valve V_{TP} at 12 O'clock position and opening appropriate valves on the board. All solid particles inside are washed off. Valves V3 and V4 are opened for a short time interval to backwash porous plate PP2. Water coming up from PP2 carries solid particles and is wiped off. This is repeated to enhance cleaning. Finally both PP2 and the probe are wiped with a moist towel. A small amount of grease is applied on the probe and wiped upwards with a dry towel. Any grease entering the slits is removed by the cleaner strip. Slits are again purged with water to make certain that grease does not obstruct passage of water and air is removed from the probe channels.

4.5.2.2) The piston and the cylinder are washed with soap and water and rinsed. To clean porous plate PP1, valve V2 on the piston is closed, V1 is opened and thumb is placed on the central hole in PP1, and the piston is held in its normal position. Water is allowed to flow through PP1 while a finger is rubbed on the plate to dislodge all solid particles. The groove of the bottom flange where cylinder rests, is cleaned free of solid particles.

4.5.2.3) The slits on the inside wall of the cylinder are cleaned with the cleaner strip. Compressed air is blown through openings of reducing unions that are fitted on the tubes coming out of the cylinder wall. This operation blows out all remaining solid particles. The cylinder is washed and wiped dry. Grease is applied on inside and then the wiper is run through and rotated simultaniously. Grease collected on the wiper surface is wiped off with a dry towel. Wall slits are recleaned with the cleaner strip, and again compressed air is blown through.

4.5.2.4) The cylinder is placed in its position. Top flange is placed on it and bolted. Tubing connections are made by turning caps on the reducing unions tight. Slits on cylinder walls are purged with water. This removes air inside those channels. Now the lever of valve V_{TP} is brought to 6 O'clock or 3 O'clock position. This isolates PT, purge line and the channels. Also all valves on the board are closed. Valves V3 and V4 are opened again until some water comes up from PP2. This water contains final traces of solid particles and is siphoned out. About 1/8" thick layer of water is allowed to remain on PP2 and V3 and V4 are closed. Water adhering to the probe and cylinder is gently blotted off by a piece of filter paper. One annular filter paper is placed on the tip of the probe, and pushed in all the way to PP2 with the pusher tube. It shound be made certain that there are no air bubbles trapped between the paper and PP2. The edges of the paper are straightened by using a spatula.

4.5.3 PREPARATION OF FILTER CAKE

4.5.3.1) The slurry is restirred and poured into the cylinder. The beaker and the stirrer are washed with the extra water. Settling of the slurry should be minimized.

(Stratification occurs upon settling and fine particles come to the top. Alpha is highly dependant upon particle size distribution.) However some settling is inevitable.

4.5.3.2) A second piece of annular filter paper is placed on the tip of the probe and pushed in until it reaches the slurry. It is made certain that valve V1 of the piston is closed and V2 is open.

4.5.3.3) The piston is inserted in the cylinder and pushed in gently until it touches the filter paper. Then the piston is rotated to level the cake surface. At this point it is important to check that valve V_{TP} is in 6 O'clock or 3 O'clock position. Otherwise the diaphragm may burst upon loading.

4.5.3.4) The hard steel ball is placed on the piston top, and the beam is lowered by screw jack mechanism. After placing the required load in the cage, the cake is squeezed. The beam is leveled from time to time. After 10 minutes, reading of bottom force transducer is recorded and the apparatus is allowed to sit idle for 30 minutes. By this time the transmitted force reaches an equilibrium value.

4.5.4 MEASUREMENTS IN THE CP CELL TEST

4.5.4.1) The transmitted force is checked against the previous reading. If readings do not agree, more time is allowed for squeezing. The thickness of the cake is measured with Vernier calipers. (A dial gage could also be used for this purpose.) Now it is necessary to apply liquid pressure on the cake and record zero readings of all channels.

4.5.4.2) V2 is closed and V1 is opened. Lever of V_{TP} is set at 9 O'clock position and channel 4 is turned on. Zero reading of channel 4 is recorded. In this manner zero readings of channels on the probe are recorded on one chart (Table 4-1) and those of channels on the wall are recorded on another.

4.5.4.3) V3 is opened and a predetermined flow is set up by turning the knob of V5. Temperature of the outflowing liquid is measured with a thermometer. (Viscosity of the liquid at this temperature can be obtained from a liquid properties handbook.) The reading for every channel decreases with time. (This is apparently because of change in medium resistance with time) Accuracy of measurement of pressure is more important near the bottom where α is to be calculated. Readings are taken from top to bottom and again back to top. Change in reading is almost linear with time for any slit. Therefore arithmetic average of forward and backward readings is recorded as final reading. For channels 1 and 5 there is no error due to changes with time. For channels 4 and 9 errors are the maximum. After readings for all channels are recorded on corresponding charts, the flow rate is checked again on the rota meter to make certain that it is not changed during the process of taking readings on the channels. If flowrate changes, that set of readings is discarded and the procedure is repeated. In this manner channel readings are recorded for three or four different flow rates.

4.5.4.4) V5 is gently closed. It is important that V5 is not tightened to avoid damages to the delicate valve stem. V3 is closed and zero readings for all channels are checked. All of these readings decrease by the same amount. (This decrease is less than 50 microstrains and is due to changes in the level

TABLE 4-1

Measurement of Hydraulic pressure profiles through a compacted filter cake in a 2" diameter CP cell with a 0.5" diameter center probe

Solids: Solka floc BW-200; Weight: 13.5 grams.

Liquid: Water at 75 °F; Volume: 90 ml.

P_0 = 100 p.s.i.; P_T reading = 757 microstrains; P_T = 75.2 p.s.i.

Reference gap = 3.894 inches; Flange gap = 4.327 inches;

L = 0.433 inches.

Hydraulic pressure transducer output is tabulated below in microstrains. 1 microstrain = 1.148 poundals/ft^2.

(Quantities in heavy bordered rectangles are the readings on slits with reference to slit No. 1)

Slit No.	Reference reading at Q=0	Rotameter reading: 5			Rotameter reading: 9		
		4→1 / 1←4	Average	Aveg. - ref.	4→1 / 1←4	Average	Aveg. - ref.
1	4768	4378	4378	390	2125	2125	2643
2	4771	4430 / 4426	4428	343	2366 / 2322	2344	2427
Δ_{12}				47			216
3	4778	4492 / 4482	4487	291	2638 / 2546	2592	2186
Δ_{13}				99			457
4	4769	4554 / 4537	4546	223	2965 / 2824	2995	1874
Δ_{14}				167			769

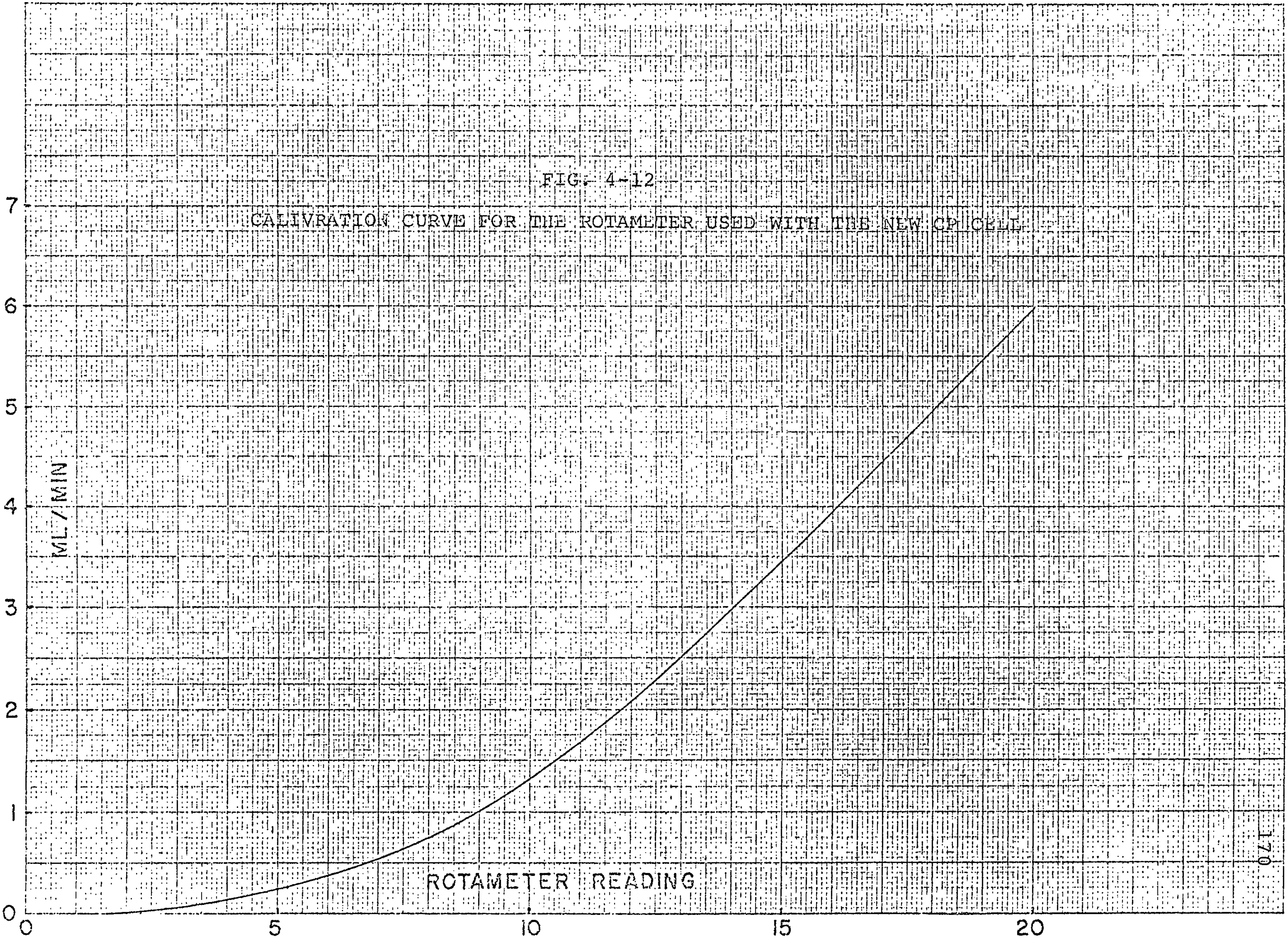


FIG. 4-12
CALIVRATION CURVE FOR THE ROTAMETER USED WITH THE NEW CP CELL
ML / MIN
ROTAMETER READING
0
1
2
3
4
5
6
7
0
5
10
15
20

in overhead bottle. At this point, measurements of the channel readings are complete.

4.5.4.5) The lever of V_{TP} is set at 6 or 3 O'clock position to isolate PT, system and purge line. V1 is closed and V2 is opened. More load is added in the cage as in step 4.5.3.4 and the procedure from there onwards is repeated. For one chosen quantity of solids, about four different mechanical pressures are applied. The entire p_s range cannot be covered with one quantity of solids, because the change in L is too large for L to be between 0.415" and 0.65". For Solka floc BW-200, 7.7 grams of solids should be used for the p_s range of 2 to 25 p.s.i. and 13.5 grams for 25 to 125 p.s.i..

4.5.4.6) After the experiment is over, it is made certain that V_{TP} lever is at 6 or 3 O'clock position, all channels are turned off, V1 and V3 are closed and V2 is open. The lever beam is raised and the piston is pulled out. The piston is kept inverted on the supporting table. Top flange on the cylinder is removed after taking the nuts off. The reducing unions on the cylinder wall are dismantled. Two wrenches are used for this purpose, so that epoxy joint between cylinderwall and 1/8" o.d. tubing will not be stressed. One of the pins on FB is held firmly by left hand while the cylinder is pulled out. Thus the floating bottom is not allowed to move and develop stresses on the channel tube joints. The cylinder should be tilted sideways and pulled up but not rotated, so that tube joints on the wall are not damaged.

4.5.5 REFERENCE GAP BETWEEN THE TOP FLANGE AND THE PISTON

It is necessary to measure this gap only once when there are two filter papers inside but no cake. By subtracting this reference gap from every gap reading during CP cell testing, the cake thickness is calculated.

4.5.6 SIMPLIFIED EXPERIMENTAL PROCEDURE

Measurement of p_L at the wall is not necessary for calculation of α, as the latter is calculated using p_L only at the center. For this purpose, a plain cylinder without any slits on the wall is used and tubing connections to the side walls are removed. The experimental procedure is modified accordingly.

4.5.7 TABULATION OF THE DATA

The data collected is on Solka floc BW-200 and is displayed on following pages.

HYDRAULIC PRESSURE PROFILES INSIDE A COMPACTED FILTER CAKE IN A 2" DIA. CP CELL WITH 0.5" DIA. CENTER PROBE

Material: Solka floc BW-200 Liquid: Water

P_0= applied mechanical solid compressive pressure in p.s.i.
P_T= transmitted " " " " "
L = cake thickness in inches; s= slurry concentration in mass of dry solids per unit mass of slurry that is used to prepare a filter cake in the CP cell; x= distance in inches from the bottom slit.
Q= total flow rate in ml/min.; p_L= liquid pressures in poundals/sq. ft. with reference to liquid pressure at x= 0.
ε = average porosity of the filter cake ($\varepsilon_{av.}$)

	Q	x .125	x .250	x .375
	$\frac{ml}{min.}$	p_L poundals/ft.^{2}		
P_0=4.8	3.00	17	36	51
P_T=4.50	3.02	14	31	47
s=.074	3.98	17	41	67
L=.78	5.00	25	55	85
ε=.817	6.00	30	68	99
	6.00	26	64	101
P_0=11.1	3.00	34	80	137
P_T=9.8	3.95	41	102	175
s=.074	4.97	53	132	224
L=.63	6.00	64	164	278
ε=.773				
P_0=11.9	3.00	29	80	146
------	3.98	34	105	189
s=.074	5.00	47	134	237
L=.62	6.00	54	160	282
ε=.769				

	Q	x .125	x .250	x .375
	$\frac{ml.}{min.}$	p_L poundals/ft.^{2}		
P_0=25.1	0.75	18	40	69
P_T=16.5	1.34	31	71	124
s=.081	2.10	49	113	199
(L=.862)	2.10	45	109	189
	3.00	67	161	285
	3.98	192	323	490
P_0=25.1	0.75	21	45	73
P_T=16.5	1.34	33	73	134
s=.081	2.10	51	117	210
L=.862	2.10	49	116	204
ε=.723	3.00	77	177	315
	3.98	100	233	305
P_0=50.1	0.39	16	40	77
P_T=31.6	0.75	22	71	141
s =.081	1.34	55	133	270
L =.760	2.10	88	214	432
ε =.686				

Hydraulic pressure profiles (continued)

	Q $\frac{\text{ml}}{\text{min.}}$	x inches .125	.250	.375
		p_L poundals/ft.^{2}		
P_0=75.4	0.25	17	46	86
P_T=52.0	0.55	37	87	191
s =.08	0.55	39	92	195
L=.690	1.01	70	169	367
ε=.654	1.69	115	276	618
P_0=100	0.25	18	53	125
P_T=69.2	0.55	53	134	329
s =.08	0.55	51	130	313
L =.646	1.01	100	250	608
ε =.631	1.01	100	250	608
P_0=125.5	0.25	38	95	209
P_T= 91.0	0.55	84	209	502
s =.08	1.01	155	388	954
L =.606				
ε =.607				
after 5 hrs. of no flow	0.25	55	113	246
	1.01	153	411	1041
P_0=125.5	0.25	73	181	315
P_T= 90.0	0.55	166	402	714
s =.131				
L =(.61)				
P_0=25.1	0.75	18	46	90
P_T=17.8	1.34	34	86	169
s =.07	2.10	55	110	269
L =.606	3.00	84	183	411
ε =.696				

	Q $\frac{\text{ml}}{\text{min.}}$	x inches .125	.250	.375
		p_L poundals/ft.^{2}		
P_0=50.1	0.75	36	103	205
P_T=36.0	1.34	63	181	357
s =.07	2.10	109	304	594
L =.520				
ε =.646				
P_0=75.4	0.55	53	178	284
P_T=54.4	1.01	92	316	542
s =.07	1.01	99	325	551
L =.461				
ε =.600				
after 35 min. of continuous flow	1.69	161	533	937
	1.01	98	329	534
P_0=100.	0.25	26	88	164
P_T=73.1	0.55	65	227	398
s =.07	1.01	126	441	788
L =.433				
ε =.575				
P_0=125.4				
P_T=94.0				
s =.131				
L =.406				
ε =.546				
P_0=75.4	0.39	57	118	179
P_T=53.1	0.75	122	246	381
s =.131	1.34	219	459	754
L =.516				
ε =.643				

Hydraulic pressure profiles (continued)

	Q ml/min.	Center probe			Sidewall slits			
		x inches						
		.125	.250	.375	.125	.125	.250	.375
		p_L poundals/ft.^{2}						
P_0=75.4	0.39	63	129	202	40	52	119	196
P_T=54.7	0.75	124	269	391	88	93	228	379
s =.131	1.34	220	482	711	164	168	414	643
L =.461								
ε =.600								
P_0= 100	0.25	59	130	179	29	55	87	135
P_T= 73.1	0.55	133	295	437	90	110	214	351
s = .131	1.01	245	571	924	169	207	434	667
L = .425								
ε = .567								
(after 11 hrs. without flow)	0.55	137	340	537	76	127	259	405
P_0= 100.	0.25	53	115	172	40	47	117	214
P_T= 72.8	0.55	121	266	401	100	100	228	396
s = .131	0.55				96	98	242	385
L =.433 ε =.584	1.01	232	512	774	183	166	443	713
P_0= 50.1	0.75	68	148	227	33	38	122	186
P_T= 33.9	1.34	137	284	439	71	68	215	317
s = .131	2.10	212	452	727	108	95	341	514
L = .516								
ε = .643								
P_0= 100.	0.25	54	114	192	29	29	92	145
P_T= 72.2	0.55	126	273	437	71	67	201	298
s = .131	1.01	248	525	883	133	106	417	614
L = .433								
ε =.575								

Hydraulic pressure profiles (continued)

	Q $\frac{ml}{min.}$	Center probe .125	Center probe .250	Center probe .375	Sidewall slits .125	Sidewall slits .125	Sidewall slits .250	Sidewall slits .375
		x inches; p_L poundals/ft.^{2}						
P_0 = 125.5	0.25	77	225	270	48	78	139	232
P_T = 91.1	0.55	184	455	661	104	134	321	537
s = .131	1.01	340	821	1379				
L = .394								
ε = .532								
(after 15 hrs) without flow	0.55	180	451	823				
(after 22 hrs)	0.55	196	521	894	70	119	388	856
P_0 = 25.1	1.34	38	87	144	36	44	63	110
P_T = 16.8	2.10	62	142	239	63	77	109	191
s = .070	3.00	96	216	366	92	113	163	281
L = .622 ε = .711	3.98	155	286	503	132	150	231	406
P_0 = 50.1	0.75	55	199	219	23	34	72	129
P_T = 34.5	1.34	86	188	357	56	75	141	257
s = .070	2.10	131	302	551	95	111	228	396
L = .528	3.00				147	168	352	590
ε = .659	3.00				147	177	362	585
P_0 = 75.4	0.39	40	101	201	24	32	71	117
P_T = 52.8	0.75	76	191	377	54	69	146	243
s = .070	1.34	138	344	661	101	122	272	447
L = .461	2.10	222	557	1060	144	172	432	677
ε = .609								
(after 10 hrs without flow	1.34	146	385	724	63	142	280	453

Hydraulic pressure profiles (continued)

	Q ml/min.	Center probe .125	Center probe .250	Center probe .375	Sidewall slits .125	Sidewall slits .125	Sidewall slits .250	Sidewall slits .375
		x inches; p_L poundals/ft.^{2}						
P_0=25.1	1.34	36	75	114	34	38	69	101
P_T=16.0	2.10	61	122	187	57	63	111	165
s =.085	2.10				60	68	114	163
L =.839	2.10				49	60	108	152
ε =.715	3.00	86	117	278	83	95	163	248
	3.98	124	249	388	111	126	227	325
P_0=50.1	0.39	23	46	83	16	21	39	56
P_T=32.1	0.75	41	90	158	36	41	78	121
s =.085	1.34	78	160	284	70	78	139	215
L =.709	2.10	126	255	456	106	123	222	336
ε =.663								
P_0=75.4	0.25	22	47	92	15	21	39	75
P_T=50.2	0.55	52	110	216	37	45	93	174
s =.085	1.01	102	210	417	67	79	169	313
L =.630	1.69	177	366	711	118	142	298	542
ε =.621	1.69				114	137	292	515
P_0=100.0	0.25	34	77	169	20	26	53	106
P_T=71.2	0.55	86	184	383	52	72	135	256
s =.085 L =.579 ε =.587	1.01	124	281	530	102	133	265	490
P_0=125.5	0.25	42	104	220	23	32	61	139
P_T=91.2	0.55	102	245	522	67	85	169	329
s =.085 L =.524 ε =.544	1.01	199	468	999	129	162	341	657
(after 14 hrs. without flow)	0.55	108	304	651	70	94	220	416

NOTATION

a = downward acceleration of the piston, ft/sec^2

A = cross sectional area of a filter cake that is squeezed, ft^2

A_1 = constant in eqn.1-30 /p.s.i.

A_2 = " " $/(p.s.i.)^2$

A_3 = " " $/(p.s.i.)^3$

A_4 = " " $/(p.s.i.)^4$

B = $\mu^2\alpha_0(1-E_0)/p_1$ (eqn. 1-43) --

C = $\alpha_0 w_1/R_m$ (eqn. 1-55) --

D = $2R$, ft.

E_0 = constant in eqn. 1-31 --

E_1, E_2, E_3, E_4 = constants in eqn. 1-31 --

$F_{k,j}$ = quantity defined by eqn. 1-122 --

F_s = particle to particle transmission force over a cross sectional area of a filter cake, poundals

F_{VB} = total vertical corce transmitted to the bottom, poundals

F_o = total vertical force applied on top of a filter cake, poundals

g = gravitational acceleration, ft/sec^2

$G_{i,j}$ = quantity defined by eqn. 1-78 --

i = index for W interval

j = index for τ interval

k = index for X interval

k_o = correcting constant in eqn. 1-90

L = thickness of a filter cake, ft.

L_1 = cake thickness if entire cake is at $p_s=0$, ft.

L_∞ = cake thickness at equilibrium at the end of squeezing, ft.

m = total mass of piston on top of the cake, lbm.

M = power index in eqn. 1-93

M_s = total mass of dry solids in a filter cake, lbm

N = total number of W or X intervals

p_a = atmospheric pressure, poundals/ ft^2

p_L = liquid pressure in a filter cake. In chapter 1, p_L is the gage liquid pressure and in chapter 3, p_L is the liquid pressure with reference to slit No. 1, poundals/ ft^2

p_m = p_s at the medium, poundals/ft^2

p_s = solid compressive pressure, poundals/ft^2

$\bar{p}_s$ = volume average of p_s, poundals/ft^2

p_1 = constant mechanical pressure applied on a filter cake that is squeezed, poundals/ft^2
(For the sake of convinience p_1 is entered in p.s.i. in the computer input.)

P_H = horizontal p_s normal on the wall of a compacted filter cake, poundals/ft^2

$\bar{P}_{LM}$ = logarithmic mean of P_0 and P_T, poundals/ft^2

P_V = vertical p_s in a filter cake, poundals/ft^2

P_{VB} = vertical p_s on the bottom of a filter cake, poundals/ft^2

P_{VB0} = vertical p_s at the bottom center of a filter cake, poundals/ft^2

P_{VT} = vertical p_s at the top of a filter cake, poundals/ft^2

P_0 = $F_0/(\pi R^2)$, poundals/ft^2

q = superficial velocity of liquid in a filter cake, ft/sec

q_m = q at the medium, ft/sec

q_s = superficial velocity of solids in a filter cake, ft/sec

Q = total flow rate through the cross sectional area of a filter cake, ft^3/sec

r = radial distance from the center of a filter cake, ft

R = inside radius of the cell for measurement of stress distribution on boundaries of a filter cake, ft

R_m = medium resistance, /ft

s = mass fraction of dry solids in a slurry, --

S_g = specific gravity of solids in a filter cake, --

t = time from the start of squeezing, chapter 1, sec

t' = uncorrected time in experimental data on squeezing, sec

T = total force at sectioned solids surface in a cross sectional area of a filter cake, poundals

u = superficial relative velocity of liquid with respect to solids in a filter cake, ft/sec

u' = actual relative velocity of liquid with respect to solids in squeezing operation, ft/sec

U $\equiv$ $(L_1-L)/(L_1-L_\infty)$ eqn. 1-148, --

w = mass of dry solids per unit cross sectional area of a filter cake from septum to x, lbm/ft^2

w_1 = total dry solid mass per unit cross sectional area in a filter cake, lbm/ft^2

W = dimensionless dry solids mass (eqn. 1-39) --

x = distance measured towards the top of a filter cake. In chapter 1, x is from the cake-medium interface. In chapter 3, x is from slit No. 1, ft

X = x/L_1 (eqn. 1-69)

Y = x/L (eqn. 1-107)

z = vertical distance from the bottom of a cake, ft

Greek Letters

α = specific cake resistance, ft/lbm

α = constant in eqn. 1-30, also is at p_s=0, ft/lbm

$\Delta_{cor.}$ = time correction for experimental data on squeezing, sec.

Δp_L = liquid pressure drop across the medium, $poundals/ft^2$

ΔW = W step used in the numerical scheme, --

$\Delta\tau$ = step " " " " --

$\Delta\tau_{cor.}$ = time correction for the initial profile assumed, --

ε = porosity at a point in a filter cake, --

$\varepsilon_{av.}$ = average porosity of a filter cake, --

ε_{p_1} = at $p_s = p_1$, --

ζ = r/R (eqn. 2-2) --

θ = angle in cylindrical coordinate system, radians

Λ = dimensionless cake thickness, (eqn. 1-37), --

μ = viscosity of liquid, lbm/(ft sec)

ρ_s = density of solids, lbm/ft^3

ρ_0 = density of water at 4°C, lbm/ft^3

σ_r = normal radial stress in a filter cake, poundal/ft^2

σ_z = normal vertical stress in a filter cake, poundal/ft^2

τ = dimensionless time (eqn. 1-38), --

τ_{rz} = shear stress in z direction in a plane perpendicular to radial vector, poundals/ft^2

ϕ = dimensionless p_s (eqn. 1-36) --

ϕ_{high} = a constant in eqn. 1-93 to predict the initial profile, --

$\phi_{i,j}$ = ϕ at W interval i and τ interval j, --

$\phi_{k,j}$ = ϕ at X interval k and τ interval j, --

ϕ_{low} = a constant in eqn. 1-93 to predict the initial profile, --

ϕ_0 = ϕ at the medium, --

$\tilde{\phi}_{k,j}$ = ϕ at $X_{k,j}$ and at time interval j+1, figure 1-5, --

ABBREVIATIONS AND SYMBOLS

CP cell : compression permeability cell

FB : floating bottom in the improved CP cell

PP1 : porous plate attached to the piston of improved CP cell

PP2 : porous plate attached to FB of the improved CP cell

PT : hydraulic pressure transducer in the improved CP cell

TR : pressure transducer to measure stresses on boundaries of a filter cake

VB : valve on the bottom of the apparatus for measuring stress on top of a cake, fig. 4-6

VT : valve on the piston of the apparatus mentioned above

V_{TP} : valve to connect purge line or PT to the improved CP cell.

V1 : water inlet valve on the piston of improved CP cell

V2 : air vent valve on " " " "

V3 : exit valve for liquid leaving during the improved CP cell testing

V4 : valve for backwashing PP2

V5 : fine metering valve for final exit of liquid

(V_{TP} through V5 : refer to fig. 4-8)

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