

AN ANALOG COMPUTER STUDY
OF MULTIVARIABLE MODEL SIMPLIFICATION

A Thesis
Presented to
the Faculty of the Department of Electrical Engineering
University of Houston

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Electrical Engineering

by
Terry Edward Edgerton
December 1970

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ABSTRACT

There are several methods of simplifying a high order system with lower order one. These methods are discussed briefly and a technique developed by Chen and Shieh is chosen as the most versatile one.

The Chen-Shieh technique involves performing a Second Cauer continued fraction expansion of a system's transfer function and then discarding the higher frequency terms. The higher frequency terms are those last determined by the dividing procedures of the continued fraction.

General techniques are developed for single input-single output systems and two examples were modeled and then simplified. Next, the Chen-Shieh method is extended to multi input/output systems. The basic matrix form of the second Cauer continued fraction expansion is developed. A general analog program block is also developed.

An example of a two input-two output system is presented and the techniques that were derived are used to simplify the system.

The results of all of the systems simplified by the Chen-Shieh method and the extension of their method to multi input-output system were excellent.

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CHAPTER I

INTRODUCTION

I. THE VARIOUS TECHNIQUES

In classical techniques, there are several methods for simplifying a high order system with a lower order one. Namely:

- (1) The Nixon method [1] - by ignoring certain coefficients.
- (2) The Dominant Roots method [2] - by ignoring certain roots.
- (3) The K. Chen method [3] - by ignoring certain parts of a Bode diagram.

In the modern techniques, there are also several methods for reducing a high dimensional state space into a lower dimensional one. Namely:

- (4) The Davison method [4] - emphasizing on dominant eigenvalues.
- (5) The Marshall method [5] - emphasizing on the steady state values.
- (6) The Chen and Shieh method [6] - emphasizing on the later part of a transient curve.

II. THE MOST FLEXIBLE TECHNIQUE

In the six methods mentioned, the Chen and Shieh method [6] is the most flexible because, by using it one can either work in the state space or work in the transfer function (s) domain.

The Chen and Shieh method [6] is essentially the second Cauer form of the continued fraction expansion; however, the interpretation of the second cauer form is quite different from the one used in circuit theory. The method can be summarized as follows:

- A. Put the denominator and the numerator of the transfer function into ascending order.
- B. Expand the transfer function into continued fraction form.
- C. Discard certain quotients.
- D. Invert the simplified continued fraction into a transfer function which is the required simplified transfer function.

The procedure is extremely simple and the results are extremely satisfactory; however, Chen and Shieh's method [6] is limited to single input - single output systems. In other words, the method needs to be extended to multi-input/multi-output systems so that the method will be more versatile. This paper will use the analog computer to investigate the multi-input/multi-output cauer form. Theoretically, this is a natural extension of the scalar cauer form.

CHAPTER 11

DISCUSSION OF THE VARIOUS SIMPLIFICATION TECHNIQUES

The six existing simplification methods can be regrouped into two schools.

(A) the school based on dominant roots and (B) the school based on fractional form. Among the methods mentioned in Chapter 1, (2), (4) and (5) belong to (A); while (1), (3) and (6) belong to (B). The group (A) is good for certain systems and the group (B) is good for other systems. As far as control systems are concerned, group (B) seems to be more powerful.

1. THE METHOD BASED ON DOMINANT ROOTS

Group (A) can be best illustrated by a simple example.. Consider the transfer function

$$F(s) = \frac{s^3 + 52s^2 + 102s + 100}{s^5 + 35s^4 + 396s^3 + 1538s^2 + 3048s + 1872} \quad (1)$$

Which factor as follows

$$F(s) = \frac{(s+50)(s+1+j1)(s+1-j1)}{(s+1)(s+2+j2)(s+2-j2)(s+15+j3)(s+15-j3)} \quad (1A)$$

The pole-zero form of the transfer function is shown in (1A) and can be expressed in the S-plane as shown in Figure 1. In the pole-

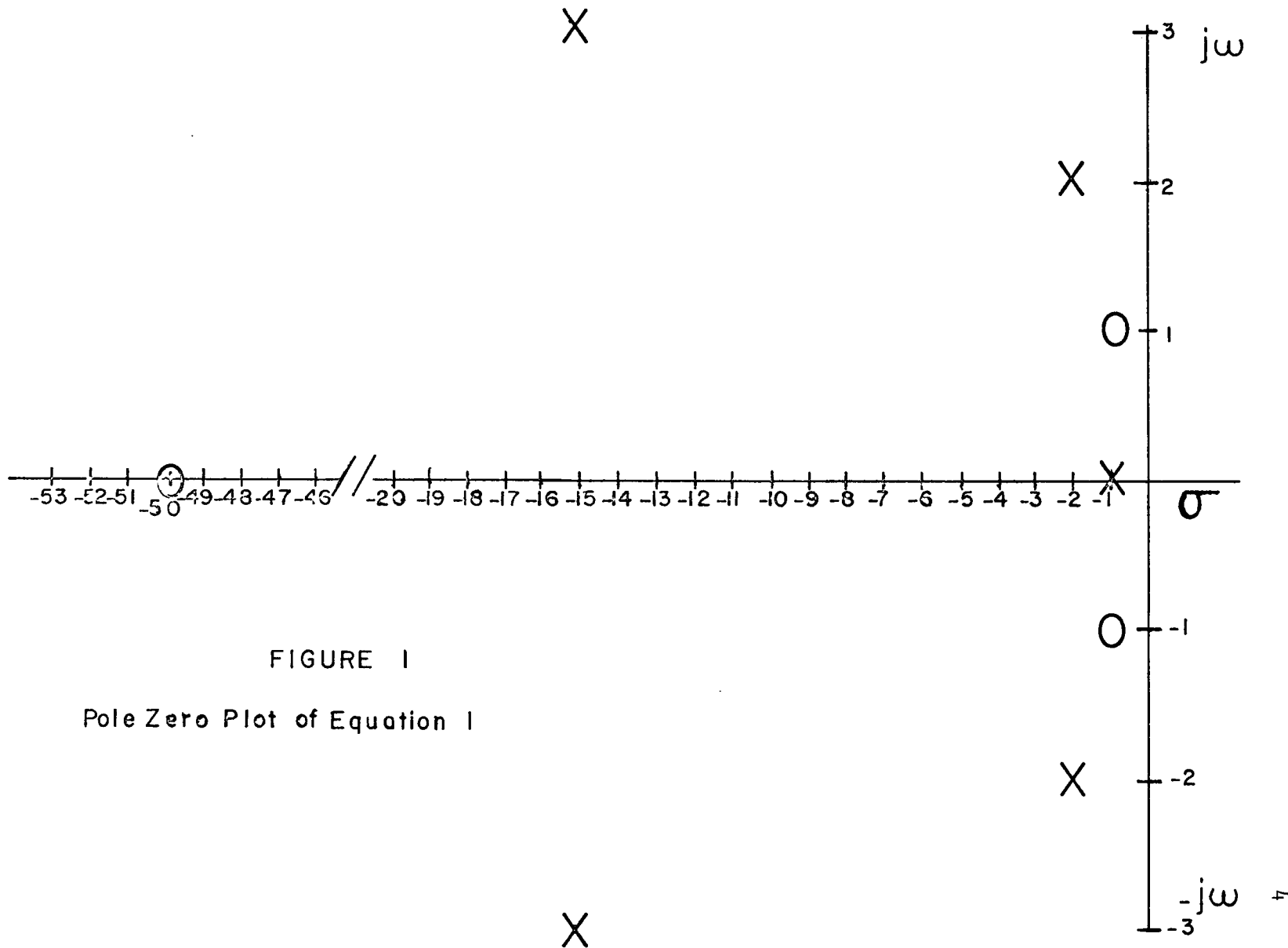


FIGURE 1

Pole Zero Plot of Equation 1

zero configuration, one knows that the poles near the imaginary axis have the most influence on the system's behavior. Based on this well known knowledge, one can divide the poles into three groups and write the transfer function as follows:

$$F(s) = \frac{1}{s+1} \cdot \frac{s^2 + 2s + 2}{s^2 + 4s + 8} \cdot \frac{s + 50}{s^2 + 30s + 234} \quad (1B)$$

The corresponding block diagram is shown in Figure 2. It is straight forward to change the block diagram into an analog computer program as shown in Figure 3.

When we study the behavior of the system using the dominant root method, one simulates the transfer function by one of the following:

$$F(s) = \frac{1}{s+1} \cdot .05342 \quad (2)$$

$$F(s) = \frac{1}{s+1} \cdot \frac{s^2 + 2s + 2}{s^2 + 4s + 8} \cdot .2137 \quad (2A)$$

$$F(s) = \frac{1}{s+1} \cdot \frac{s^2 + 2s + 2}{s^2 + 4s + 8} \cdot \frac{s + 50}{s^2 + 30s + 234} \quad (2B)$$

The two approximations (2 and 2A) can be realized by disconnecting, first the last stage parts and then disconnecting the second stage of the analog computer program. The three different responses are shown in Figure 4.

This method is not very satisfactory in many senses. Generally the approximations have poor transient response. It is well known that in control system analysis one of the most important considerations is

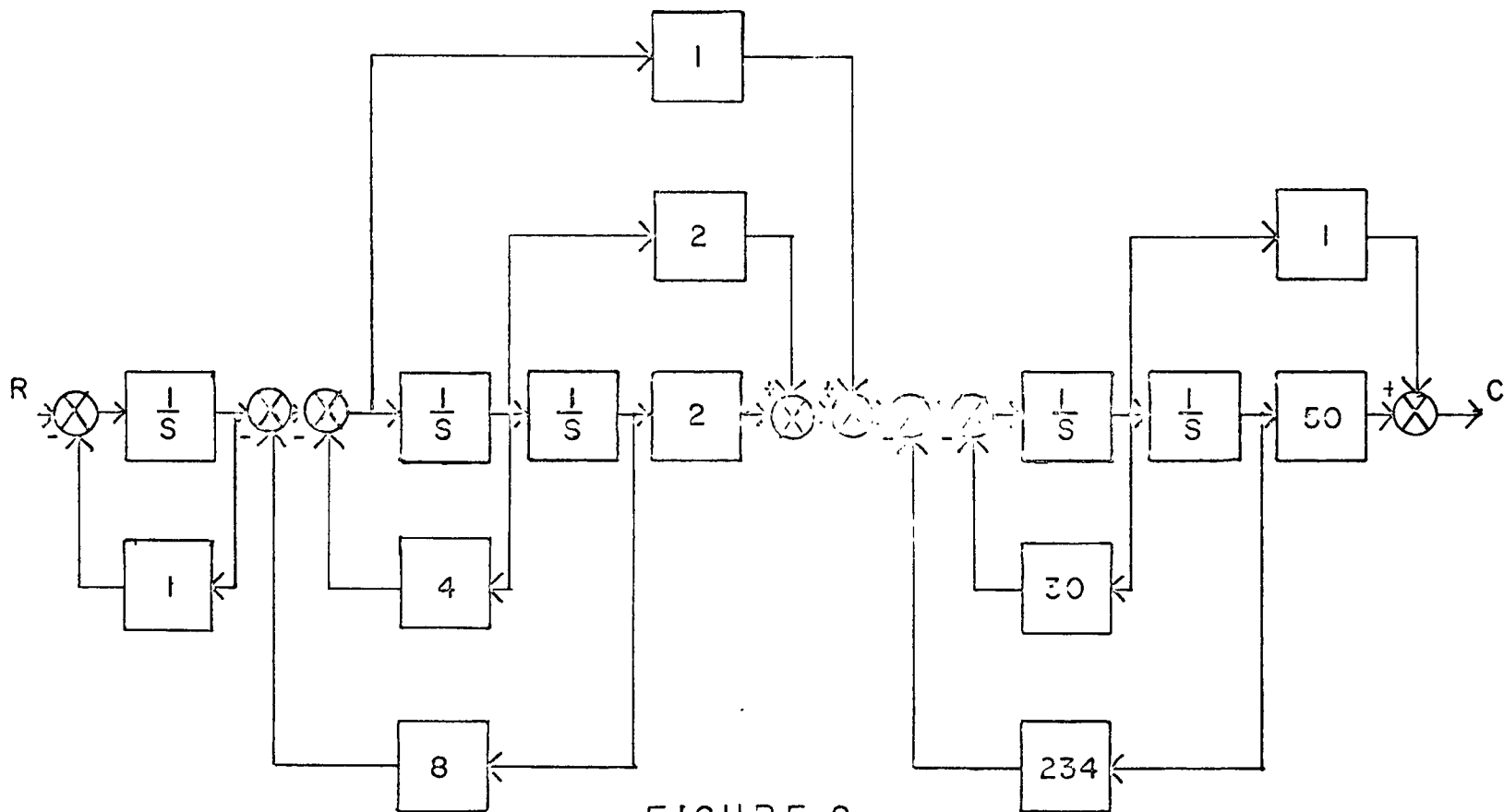


FIGURE 2
Block Diagram of Equation 1.

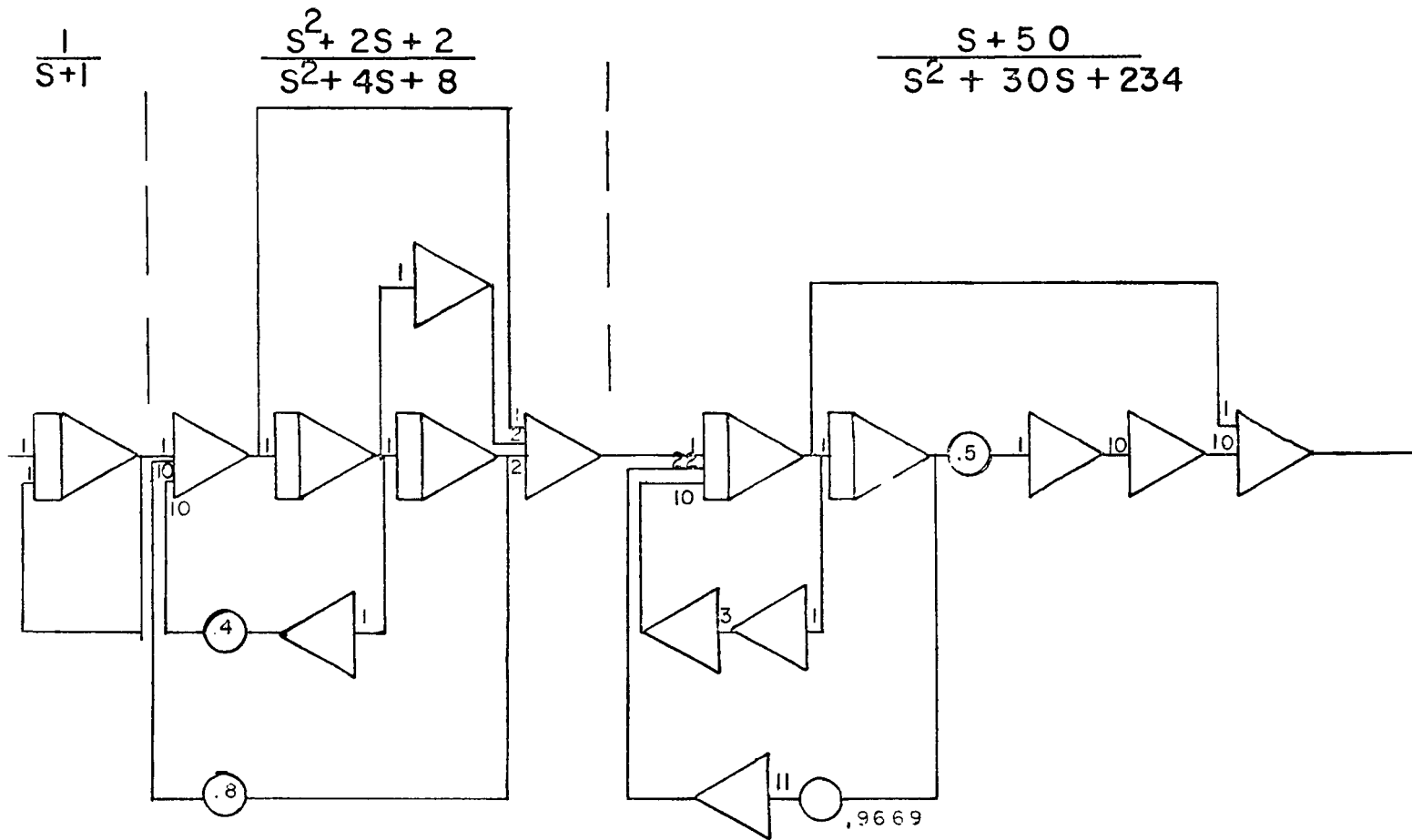


FIGURE 3

Analog Program for Equation 1

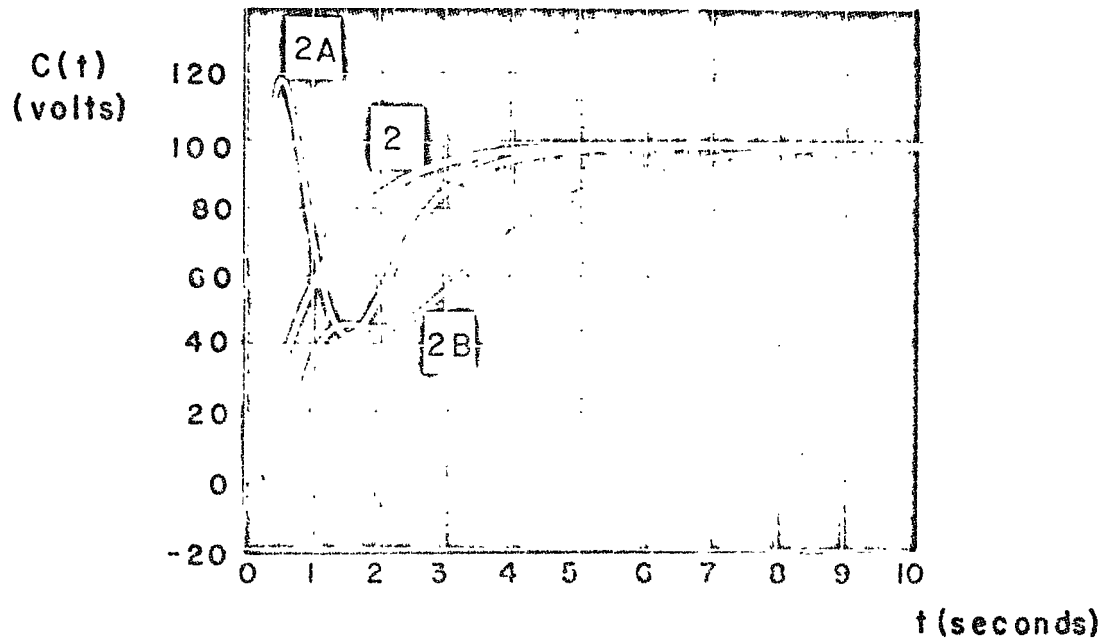


FIGURE 4

Transient Response for Equations

2, 2A, and 2B

transient response.

By observing the response curves of Figure 4 one can quickly see that the approximation, using equation 2, is not even close to the total, but it is much better than the approximation using equation 2A. It is interesting to note that equation 2A was a higher order approximation than equation 2.

This example readily points to the pitfalls of the pole-zero reduction techniques.

II. THE METHOD BASED ON THE FRACTIONAL FORM

K. Chen [3] was one of the first to realize that the dominant roots method has shortcomings. He uses a completely different approach by considering a transfer function as a feedback system and then attempting to simplify the open loop transfer function. In other words, he decomposes the system into a feedback system of the form

$$F(s) = \frac{1}{1 + G(s)} \quad (3)$$

where $G(s)$ is the open loop transfer function. $G(j\omega)$ is then plotted in Bode diagram. K. Chen considered that any poles above +15db or below -15db can be ignored. K. Chen felt the importance of the feedback loop, but he restricted himself to systems with unity feedback. Consequently, his approach is only good for the unity feedback system.

CHAPTER III

DISCUSSION OF THE SECOND CAUER CONTINUED FRACTION FORM

C. F. Chen and Shieh [6] extended and modified K. Chen's work [3] by the following procedure. Consider a general transfer function

$$T(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n} \quad (4)$$

First of all, the two polynomials of the transfer function should be arranged in ascending order as

$$T(s) = \frac{b_n + b_{n-1}s + \dots + b_1 s^{n-1}}{a_n + a_{n-1}s + \dots + s^n} \quad (4A)$$

then expanded into a continued fraction as follows:

$$T(s) = \frac{1}{H_1 + \frac{1}{\frac{H_2}{s} + \frac{1}{H_3 + \frac{1}{\frac{H_4}{s} + \dots + \frac{1}{\frac{H_{2n}}{s}}}}}} \quad (4B)$$

Next truncate (4B) by discarding certain quotients. For example,

$$T(s) \approx \frac{1}{H_1 + \frac{1}{\frac{H_2}{s}}} = \frac{H_2}{H_1 H_2 + s} \quad (5)$$

This is the simplest reduction of (4B).

Or, one can approximate (4) by

$$T(s) \approx \frac{1}{H_1 + \frac{1}{\frac{H_2}{s} + \frac{1}{H_3 + \frac{1}{\frac{H_4}{s}}}}} =$$

(6)

$$\frac{H_2 H_3 H_4 + (H_2 + H_4)s}{H_1 H_2 H_3 H_4 + [H_1 (H_2 + H_4) + H_3 H_4]s + s^2}$$

1. DEVELOPMENT OF THE SECOND CAUER FORM

The form of a general transfer function is

$$\frac{C}{R} = \frac{G}{1 + GH} \quad (7)$$

which, after rearranging, is

$$\frac{C}{R} = \frac{1}{H + \frac{1}{G}} \quad (7A)$$

Equation (4B) is a natural generalization of (7A). The block diagram for (7A) is presented in Figure 5.

Now, letting $G = -1/s$ and $H = -a$ produces the transfer function

$$F(s) = \frac{C}{R} = \frac{-1}{s + a} \quad (8)$$

which has the analog program shown in Figure 6.

To complete the basics in converting fraction expansion form

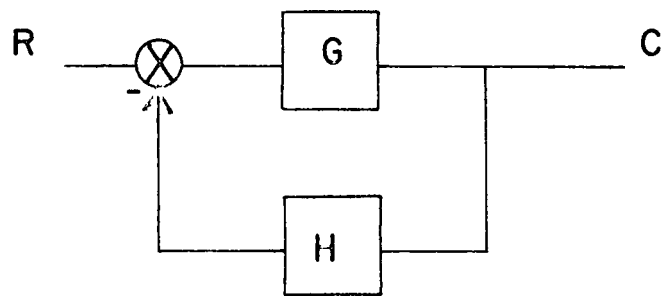


FIGURE 5

Block Diagram of Equation 7

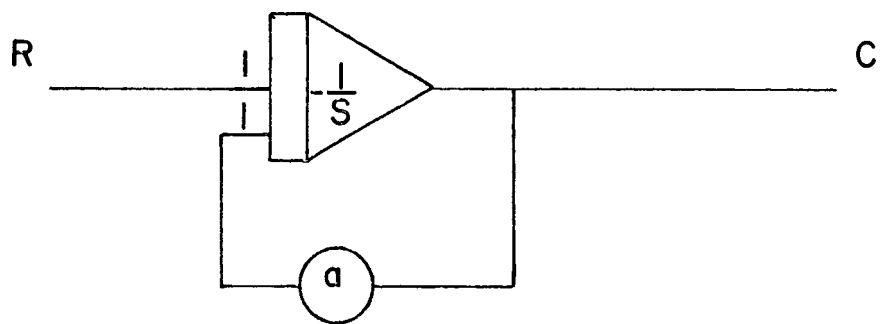


FIGURE 6

Analog Program for Equation 7

to an analog program, the last block is a feed-forward loop. This can be illustrated with the transfer function

$$\frac{C}{R} = \frac{G_1 + F_1}{1 + (G_1 + F_1)H} \quad (9)$$

which, after rearranging, is

$$\frac{C}{R} = \frac{1}{H + \frac{1}{G_1 + F_1}} \quad (10)$$

The block diagram of equation 9 is shown in Figure 7.

Again, let $G = -1/s$ and $H = -a$, and let $F_1 = -F(s)$ then the analog program is as shown in Figure 8.

II. IMPLEMENTATION OF THE SECOND CAUER FORM

In general, the form of the second Caier continued fraction is

$$\frac{C}{R} = F(s) = \cfrac{1}{H_1 + \cfrac{1}{\cfrac{H_2}{s} + \cfrac{1}{H_3 + \cfrac{1}{\cfrac{H_4}{s} + \cfrac{1}{H_5 + \ddots \cfrac{1}{\cfrac{H_{2n}}{s}}}}}}} \quad (11)$$

The block diagram for equation (11) is shown in Figure 9.

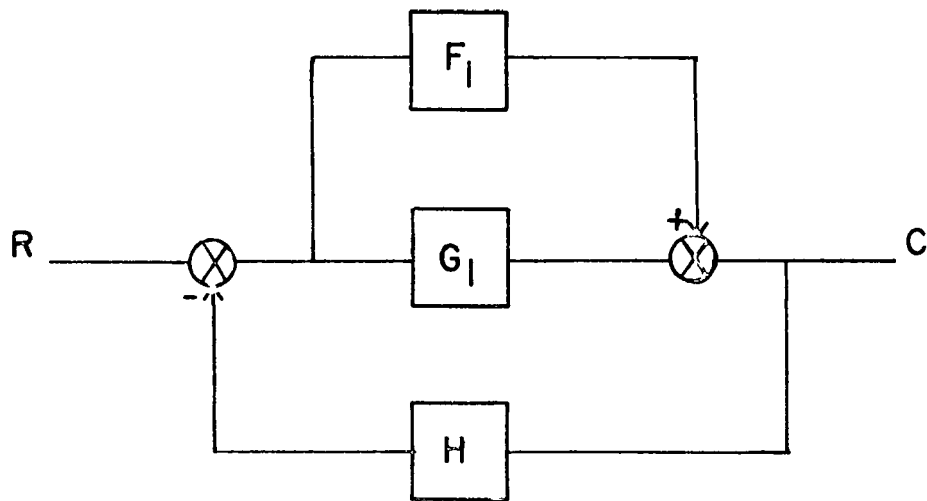


FIGURE 7

Block Diagram of Equation 9

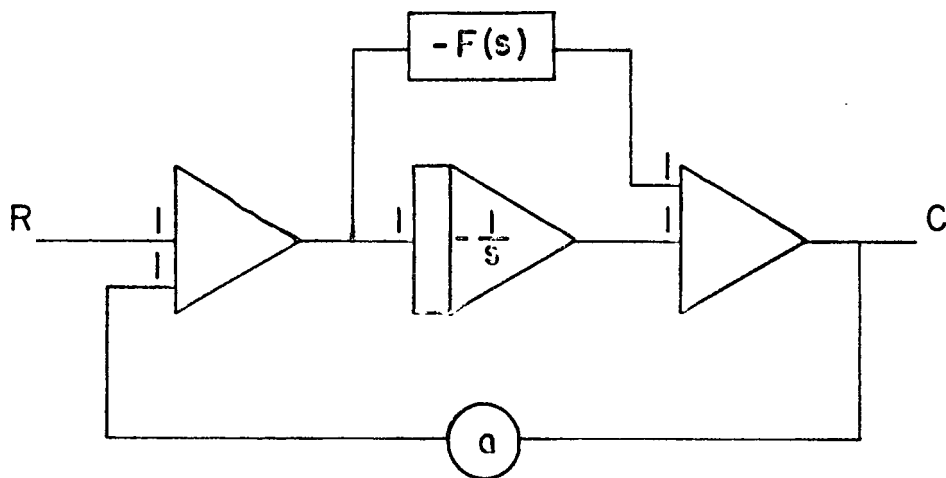


FIGURE 8

Analog Program for Equation 9

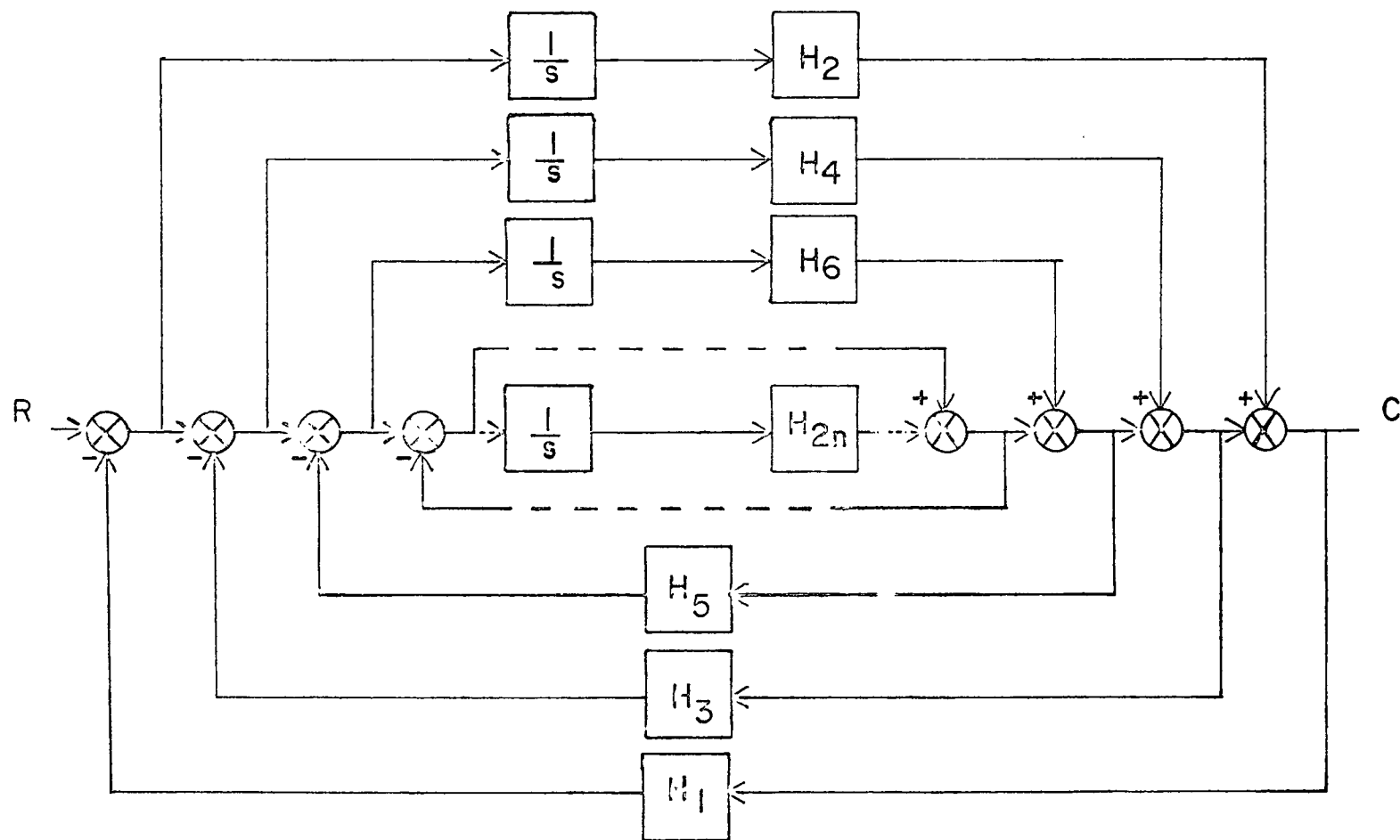


FIGURE 9

Block Diagram of Equation 11

The analog program through the $H6/s$ term is shown in Figure 10.

H terms greater than 1 require amplifiers in cascade with the integrators. Negative H terms require an odd number of cascaded amplifiers not counting the integrators. If an H term requires amplification but does not have a negative sign, then an even number of amplifiers in cascade are required. A completely general analog program is presented in appendix A.

Once the system is programmed on the analog computer it is a simple task to simplify the model as presented by Chen and Shieh [6], who state that "the most dominant term is $H1$ and the second influence term is $H2/s$. As the quotients in the continued fraction descend lower they are less important on the performance of the system." With this in mind, all one has to do is remove the inner most loops of the system (while observing the transient response) until the system response is degraded past a desired accuracy; and then replace the last loop removed.

Using the analog computer for this task affords distinct advantages over digital computer compilation or other methods of system response calculations. One advantage is an intimate feel for the system under examination. No one can deny the advantage of actually seeing the response, of their system model, traced out on some display device, and then being able to simplify the system by unpatching a loop, and then watching the simplified model response.

CHAPTER IV

PRESENTATION OF SINGLE INPUT SINGLE OUTPUT EXAMPLES

Using the techniques outlined in the foregoing chapter, two example systems were analyzed. The first is a practical example [7].

$$T_1(s) = \quad (12)$$

$$\frac{1441.53s^3 + 78319.s^2 + 525286.125s + 607693.25}{s^7 + 112.04s^6 + 3755.92s^5 + 39736.73s^4 + 363650.56s^3 + 759894.19s^2 + 683656.25s + 617497.375}$$

by arranging the polynomial in ascending order and then expanding into the continual fraction, $T_1(s)$ can be expressed as

$$T_1(s) = \quad (13)$$

$$\begin{array}{r} \frac{1}{1.016 + \frac{1}{4.054 + \frac{1}{s - 0.0666 + \frac{1}{-3.803 + \frac{1}{s - 10.27 + \frac{1}{-.2572 + \frac{1}{s - 42.77 + \frac{1}{-.2591 + \frac{1}{s + 29.01 + \frac{1}{.2695 + \frac{1}{s - 15320. + \frac{1}{-.0052 + \frac{1}{s + 47760. + \frac{1}{.0007}}}}}}}}}}}}}}}}}}}} \end{array}$$

The analog program for equation (13) is shown in Figure 11.

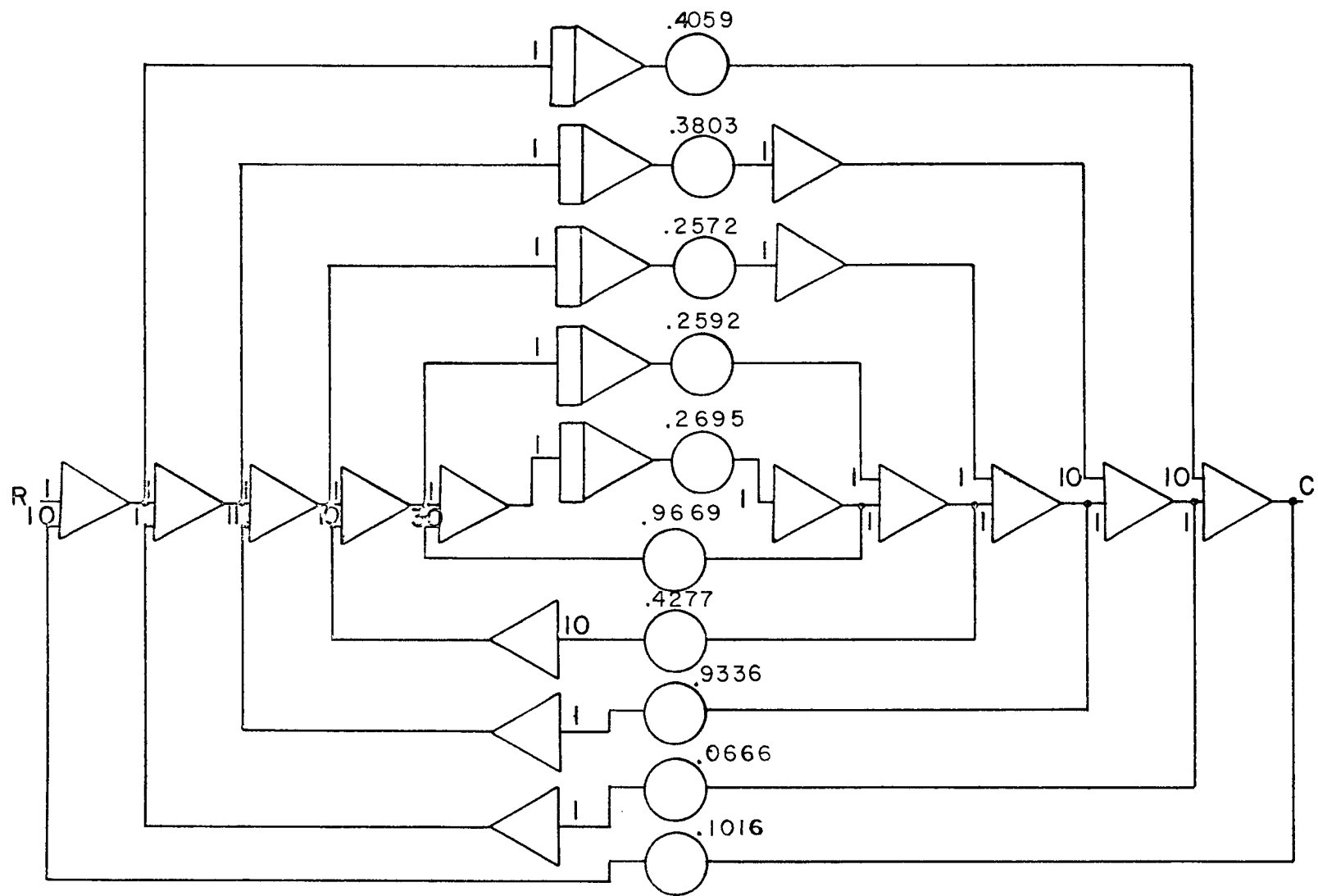


FIGURE II
Analog Program for First Example

The analog program was implemented on a Hybrid System SS-100 (100 volt analog computer). The input was a step function of 70 volts. This function is easily implemented with the program as shown in Figure 11; the input (R) is patched to a potentiometer set at .7000 and the potentiometer is patched to Reference (+100V). Thus, whenever the integrators are switched from IC to OP, the system will see a step function of 70 volts because the integrators are in series with every feed forward section of the program. The system was implemented only through the $\frac{H_{10}}{s}$ term because the H_{11} and H_{13} terms require gains that are impractical to implement on an analog computer. However, as shown in Figures 12 and 13, the omission of these terms did not impair the system modeling. As seen in Figure 12, one cannot distinguish between the third, fourth, and fifth order system approximations. Only in Figure 13 (an expanded view of the origin) can one see a difference between the third, fourth and fifth order approximations.

The region around the origin of a transient response curve is dominated by the higher frequency characteristics of the system. Therefore, in leaving off the H_{11} through $H_{14/s}$ terms, one has only altered the higher frequency characteristics of the system. Therein lies the beauty of the second Cauer form; one has at his disposal the tool which only reduces the upper frequency characteristics of a system and then only to the degree one desires. The power of this tool is seen when the results obtained are compared to the results

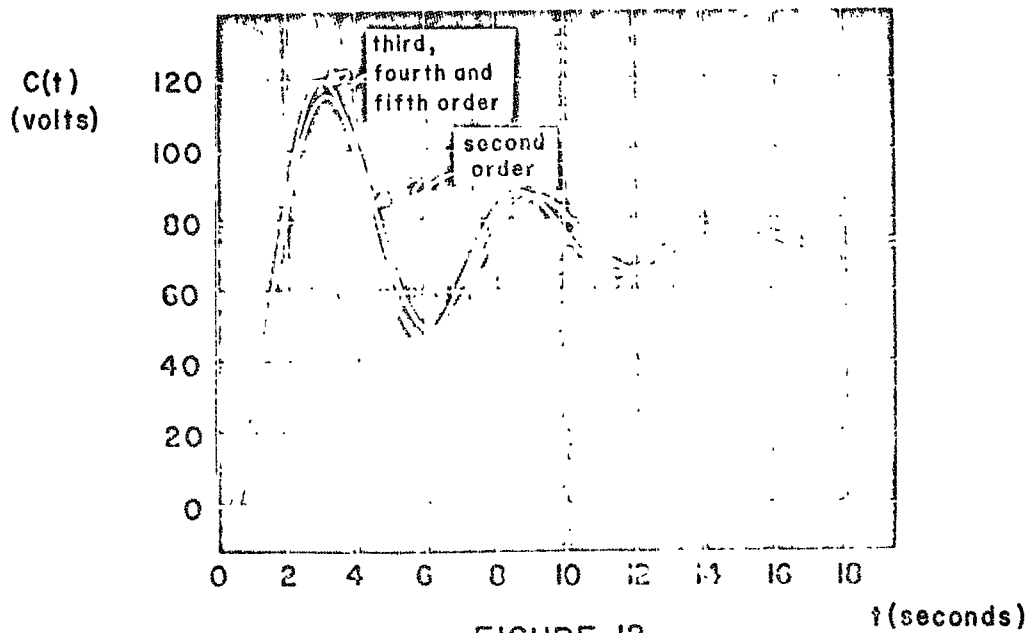


FIGURE 12

Response Curves for first Example

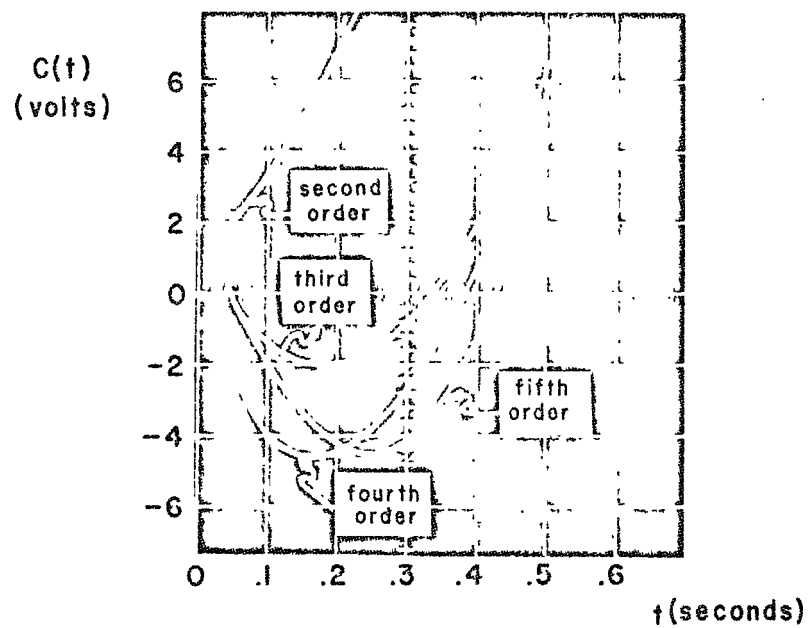


FIGURE 13

Response Curves for first Example

obtained using other techniques such as the pole-zero reduction technique.

In the pole-zero reduction technique one can not limit the effect of simplification to just the upper frequencies; on the contrary, as indirectly shown by Figure 4, the effect is spread over quite a large range of frequencies.

The second example demonstrates the simplification technique of a system which is not easily reduced by conventional methods. [6]

The transfer function

$$T_2(s) = \frac{s^2 + 15s + 50}{s^4 + 5s^3 + 33s^2 + 79s + 50} \quad (14)$$

can be factored into

$$T_2(s) = \frac{(s + 5)(s + 10)}{(s + 1)(s + 2)(s + 1 + j4.9)(s + 1 - j4.9)} \quad (15)$$

The problem with using existing pole-zero reduction techniques on this system is readily seen; three poles have the same real part.

Expanding $T_2(s)$ into continued fraction expansion one has (16)

$$T_2(s) = 1 + \frac{1}{\frac{.7812}{s} + \frac{1}{-6.400 + \frac{1}{\frac{-.7463}{s} + \frac{1}{-4.611 + \frac{1}{\frac{18.02}{s} + \frac{1}{.0089 + \frac{1}{-18.05}}}}}}}}}$$

The analog program for equation 16 is presented in Figure 14.

The second example response curves were obtained in the same manner as used in the previous example except that the input was changed to a step function of 60 volts.

As shown in Figures 15 and 16, the reduction on the system transfer function of the second example was a success. In Figure 16 one can not even see a difference in the two curves because of the width of the oscilloscope trace.

The simplification of this system could only be performed from a fourth order to a second order system because the original system has a pair of complex conjugate poles and the simplification to a third order yields a condition where the system has a pole in the right half of the s -plane.

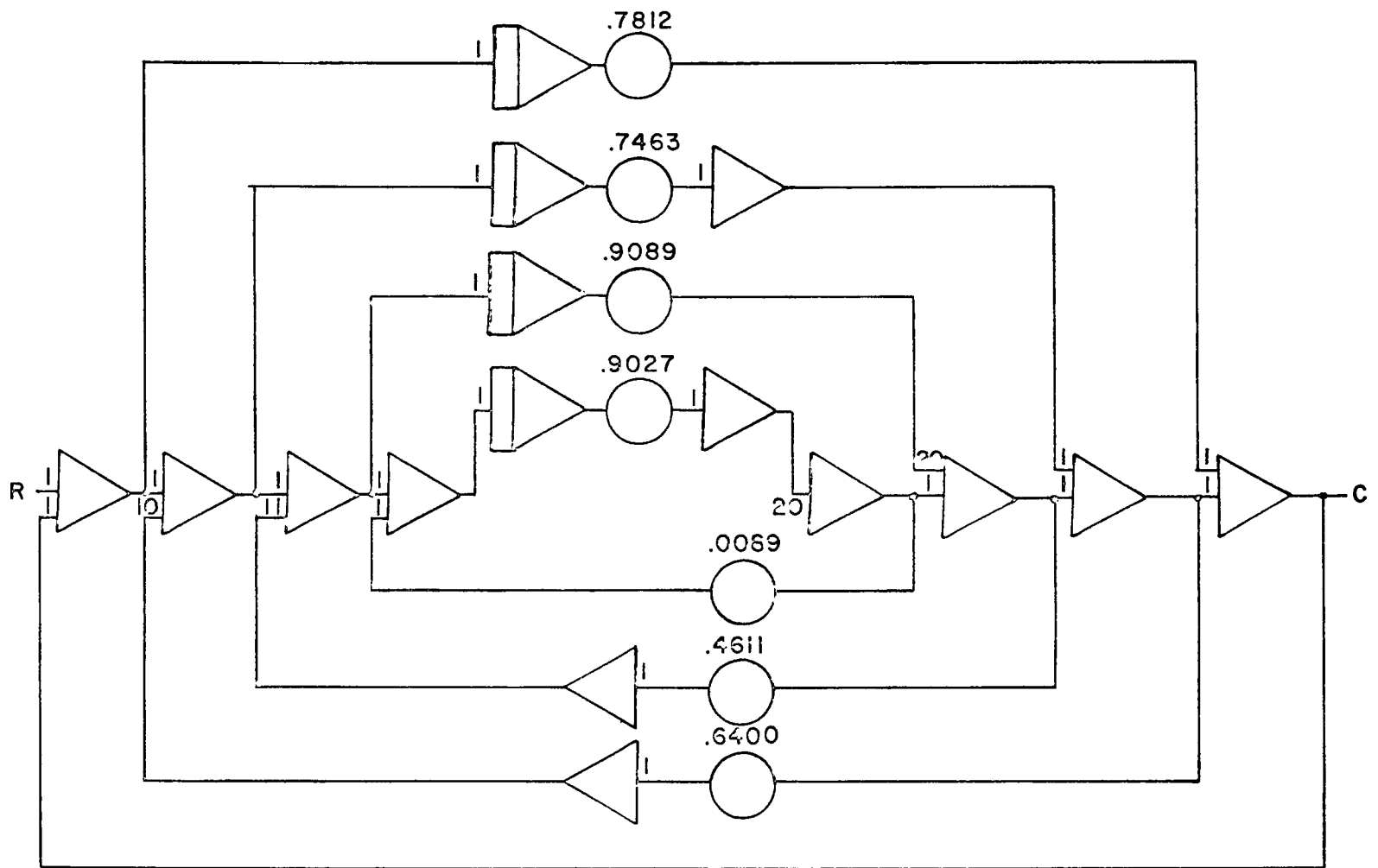
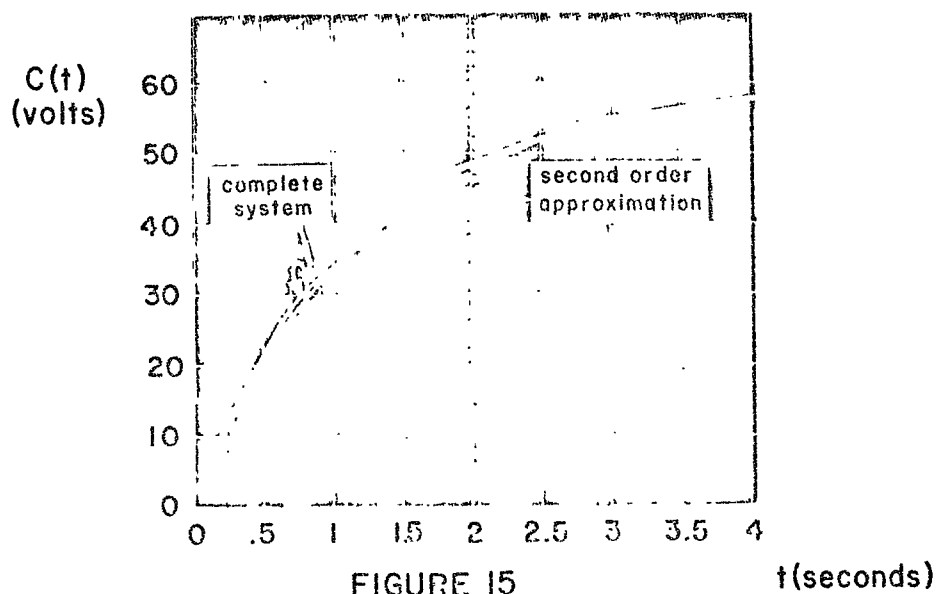
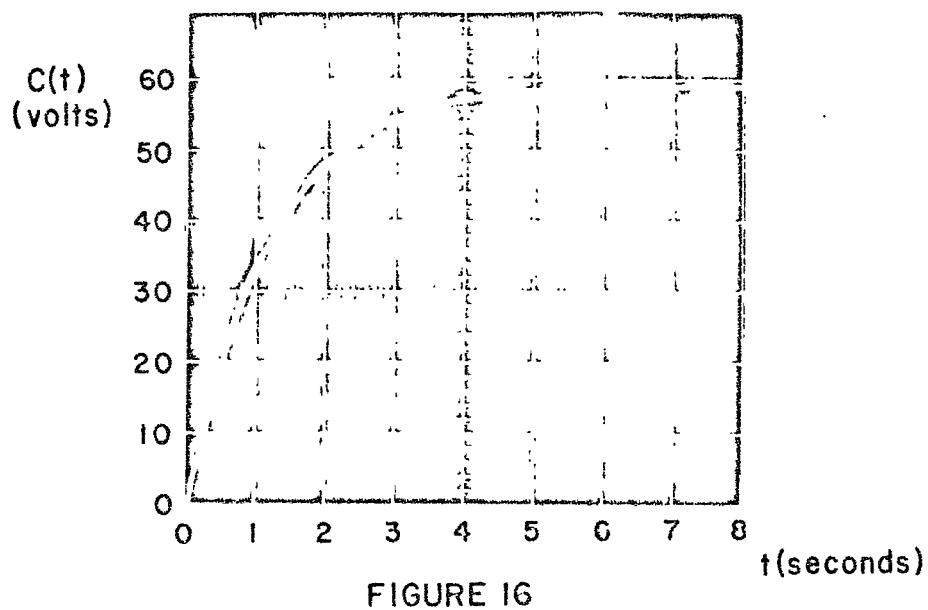


FIGURE 14
Analog Program for Second Example



Response Curves of the Second Example



Response Curves of the Second Example

CHAPTER V

THE CAUER MODEL FOR MULTIVARIABLE SYSTEMS

The techniques developed for the single input, single output system in the previous section can be directly extended to the multi-variable case. All one has to do is consider that the input and the output are two vectors.

1. DEVELOPMENT OF THE CAUER MODEL

One writes the transfer matrix of the overall system as follows:

$$[C] = [T(s)] [R] \quad (17)$$

where $[T(s)]$ is a square matrix and each element of the transfer matrix is a transfer function. Without loss of generality, one can use a two by two matrix as an example.

$$[T(s)] = \begin{bmatrix} \frac{B_{11}(s)}{\Delta(s)} & \frac{B_{12}(s)}{\Delta(s)} \\ \frac{B_{21}(s)}{\Delta(s)} & \frac{B_{22}(s)}{\Delta(s)} \end{bmatrix} \quad (18)$$

$$\text{where } \Delta(s) = A_n + A_{n-1}s + \dots + s^n \quad (19)$$

$$\text{and } B_{ij}(s) = b_{ijn} + b_{ij(n-1)}s + \dots + b_{ij1}s^{n-1} \quad (20)$$

Taking the inverse of equation (18) and then separating the constant

from the remaining parts, one has

$$[T(S)] = [H_1 + S[.]^{-1}]^{-1} \quad (21)$$

where $[H_1]$ is a constant matrix and the remaining parts have a common factor S . Then, after factoring out the S , one repeats the process.

Finally, one has

$$[T(S)] = [H_1 + S[H_2 + S[H_3 + \dots S[H_{n-1} + S[H_n]^{-1}]^{-1} \dots]^{-1}]^{-1} \quad (22)$$

Equation (22) is the general matrix Cauer form.

The crudest approximation can be obtained by keeping only two terms $[H_1]$ and $[H_2]$ and discarding all other matrices.

$$[T(S)]^1 = [H_1 + S[H_2]^{-1}]^{-1} \quad (23)$$

There is only one "S" involved in the matrix; however, even this simple first order approximation can express the exact steady state of the system shown in (17).

A better model can be obtained by keeping two more terms; one has

$$[T(S)]^{11} = [H_1 + S[H_2 + S[H_2 + S[H_4]^{-1}]^{-1}]^{-1} \quad (24)$$

The model shown in equation (24) is quite good for approximating most systems. Of course, if we take more and more terms and we get better and better approximations; however, in practice equation (24) is usually sufficient. This conclusion is a natural extension of the single input, single output case and will be demonstrated by the use of an example in the next chapter.

The block diagram representation of equation (24) can be expressed as shown in Figure 17.

The general matrix Cauer formula [equation (22)] is also easily expressed by a block diagram. See Figure 18. If one disconnects all the inner loops in figure 18, except $[H_1]$ and $[H_2]$, one has the simplified model corresponding to (23). If one also keeps $[H_3]$ and $[H_4]$ and disconnects their inner loops, then one has a better simplified model, corresponding to equation 24.

II. DEVELOPMENT OF A CAUER MODEL ANALOG PROGRAM FOR MULTIVARIABLE SYSTEMS

The next step is to build a basic analog computer program block for the two input two output system; this block is presented in Figure 19.

The derivation of this program was quite simple. If one examines only the circuit containing the first three amplifiers (A_1 , A_2 and A_3) and the first integrator (I_1), then it is clear that the program is the same as the program used for the single input/output system. The only difference is the cross-coupling terms (H_{n12} and H_{m12}) which were added for the two input - two output system.

The interconnection of basic block diagrams for two input - two output system is presented in Figure 20.

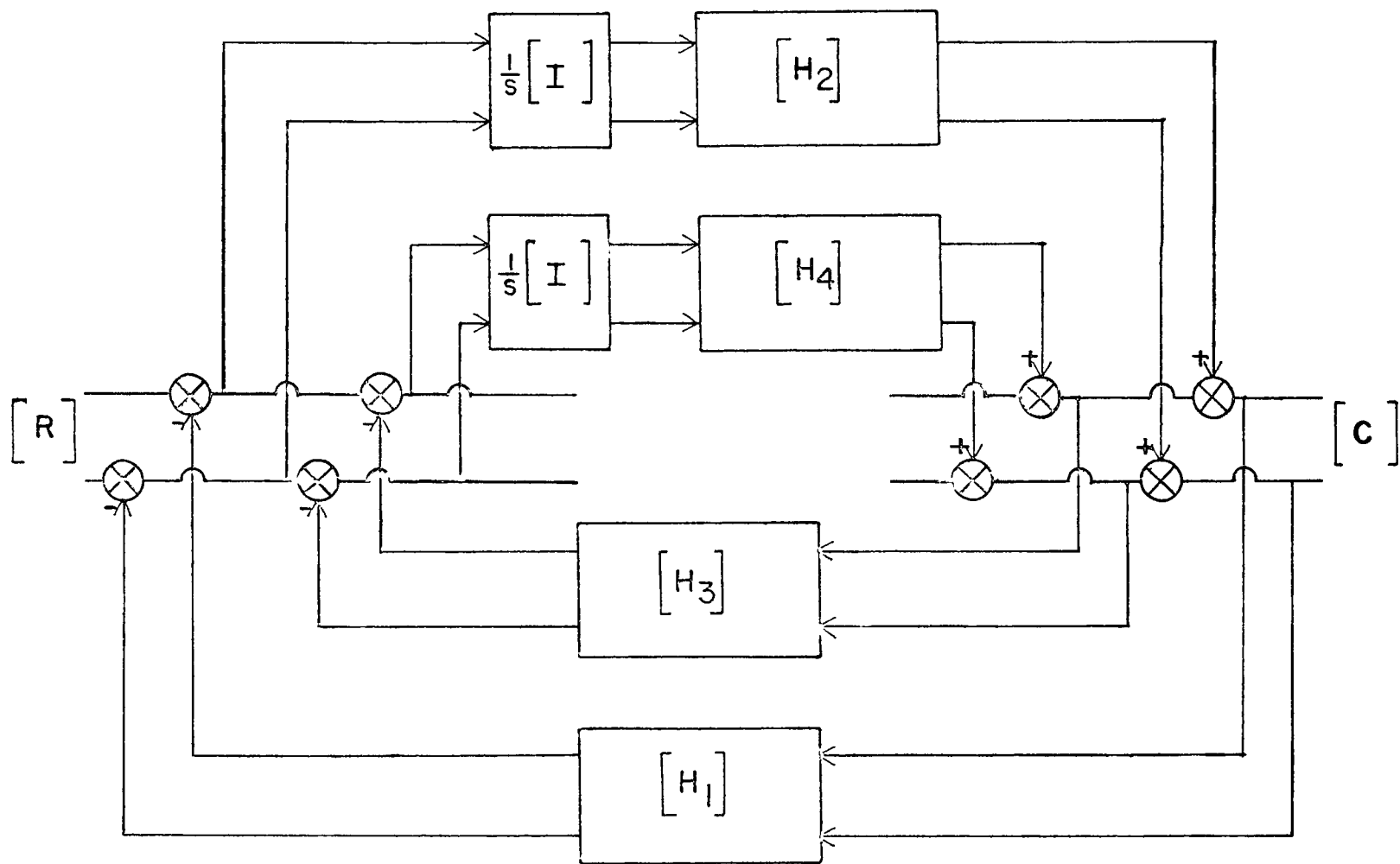


FIGURE 17

Block Diagram of Equation 24

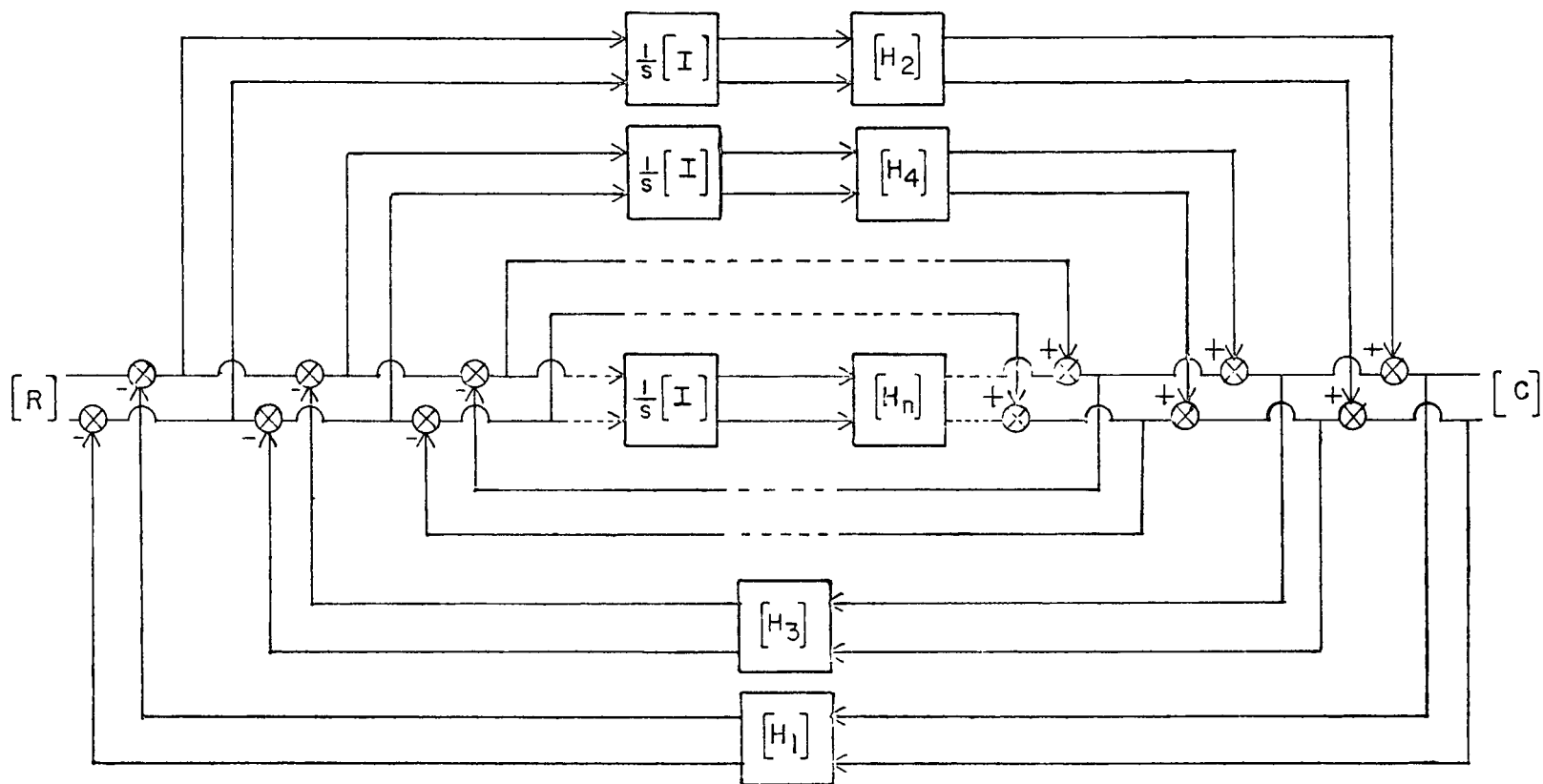


FIGURE 18

Block Diagram of Equation 22

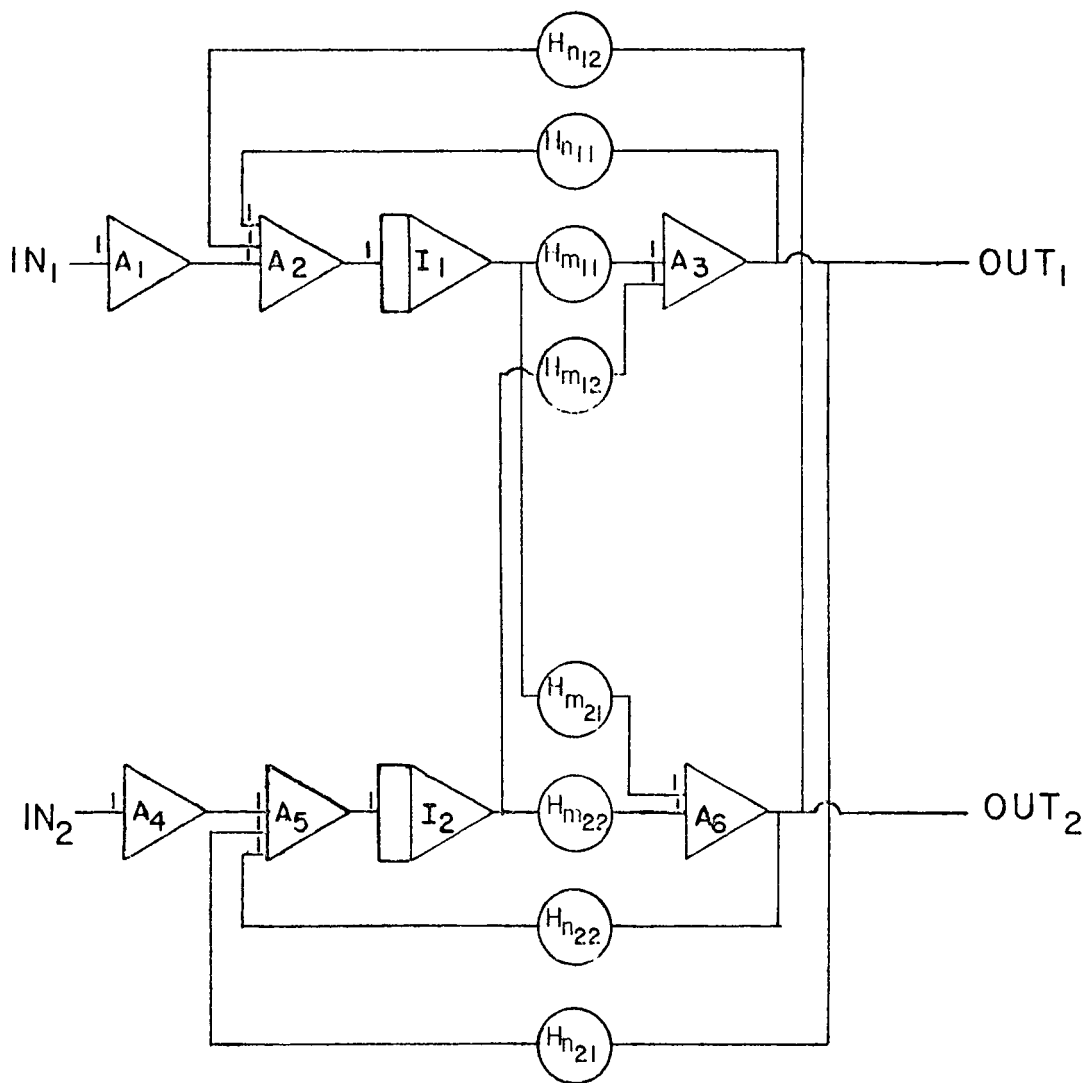


FIGURE 19

Basic Program Block for Two Input/Output System

note: n is any
odd number
and m is the next
even number

CHAPTER VI

PRESENTATION OF TWO INPUT-TWO OUTPUT EXAMPLE

Example 3 is a two input-two output system which has the transfer function

$$[T(s)] = \frac{1}{\Delta(s)} \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} \quad (25)$$

$$\text{where } T_{11}(s) = \quad (25A)$$

$$\begin{aligned} & -5.3350(s^7 + 469.26s^6 + 32141.s^5 + 375100.s^4 + 764560.s^3 + 3586600.s^2 + 8801500.s \\ & \quad + 4763000. \end{aligned}$$

$$T_{21}(s) = \quad (25B)$$

$$\begin{aligned} & 7.7620(s^7 + 332.01s^6 + 22322.s^5 + 252910s^4 + 377990.s^3 + 1805,000s^2 + 5,088,600s \\ & \quad + 2,864,400. \end{aligned}$$

$$T_{12}(s) = \quad (25C)$$

$$\begin{aligned} & 5.6698(s^7 + 521.14s^6 + 37192.s^5 + 540,450.s^4 + 813230.s^3 + 183200.s^2 + 5,651,000.s \\ & \quad + 3,361,200. \end{aligned}$$

$$T_{22}(s) = \quad (25D)$$

$$\begin{aligned} & -3.9271(s^7 + 353.00s^6 + 45952.s^5 + 791,180.s^4 + 979,940.s^3 + 1,399,000.s^2 + 6,259,800.s \\ & \quad + 4,044,000. \end{aligned}$$

$$\begin{aligned} \text{and } \Delta(s) = & s^8 + 581.58s^7 + 43543.s^6 + 667,870.s^5 + 3,562,300.s^4 + 12,832,000.s^3 \\ & + 24,944,000.s^2 + 21,288,000.s + 6,362,700. \end{aligned} \quad (25E)$$

The methods described in Chapter V for obtaining the Second Cauer matrix form are now applied to the system defined by equation (25).

The result is the second Cauer matrix equation (26)

$$[T(s)] = \left[\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} + s \left[\begin{pmatrix} -2 & 1 \\ 1.5 & -.5 \end{pmatrix} + s \left[\begin{pmatrix} 25 & 20 \\ 4 & 1 \end{pmatrix} + s \left[\begin{pmatrix} -2 & 2 \\ 2.5 & -1 \end{pmatrix} + s \left[\begin{pmatrix} 1 & 10 \\ 3 & 7 \end{pmatrix} + s \left[\begin{pmatrix} -.2 & .4 \\ .3 & -.1 \end{pmatrix} + s \left[\begin{pmatrix} 1 & 100 \\ 20 & 10 \end{pmatrix} + s \left[\begin{pmatrix} -.1 & .2 \\ .3 & -.2 \end{pmatrix} \right] -1 \right] -1 \right] -1 \right] -1 \right] -1 \right] -1 \right] -1 \right]^{-1}$$

Once equation (26) is found, it is a simple matter to plug the coefficients into the general analog program blocks shown in figure 19 and then interconnect the blocks as shown in figure 20. The analog program for the third example is shown in figure 21.

Page 1 of figure 21 contains the H_1 and H_2 terms which are

$$[H_1] = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad (27)$$

$$[H_2] = \begin{bmatrix} -2 & 1 \\ 1.5 & -.5 \end{bmatrix} \quad (28)$$

Page 2 of figure 21 contains the H_3 and H_4 terms which are

$$[H_3] = \begin{bmatrix} 25 & 20 \\ 4 & 1 \end{bmatrix} \quad (29)$$

$$[H_4] = \begin{bmatrix} -2 & 2 \\ 2.5 & -1 \end{bmatrix} \quad (30)$$

Page 3 of figure 21 contains the H_5 and H_6 terms which are

$$[H_5] = \begin{bmatrix} 1 & 10 \\ 3 & 7 \end{bmatrix} \quad (31)$$

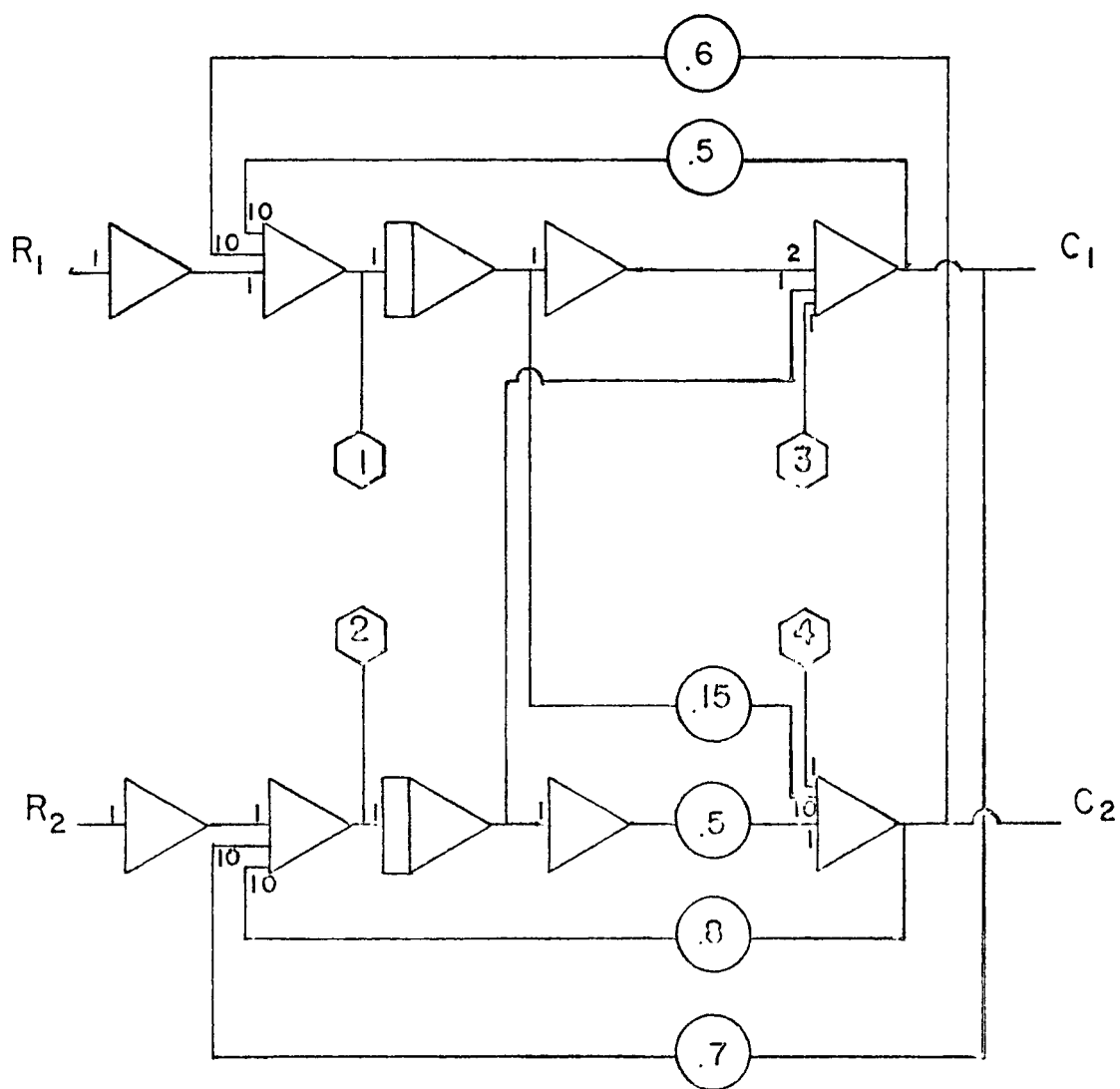


FIGURE 21

Analog Program for Third Example (equation 26)

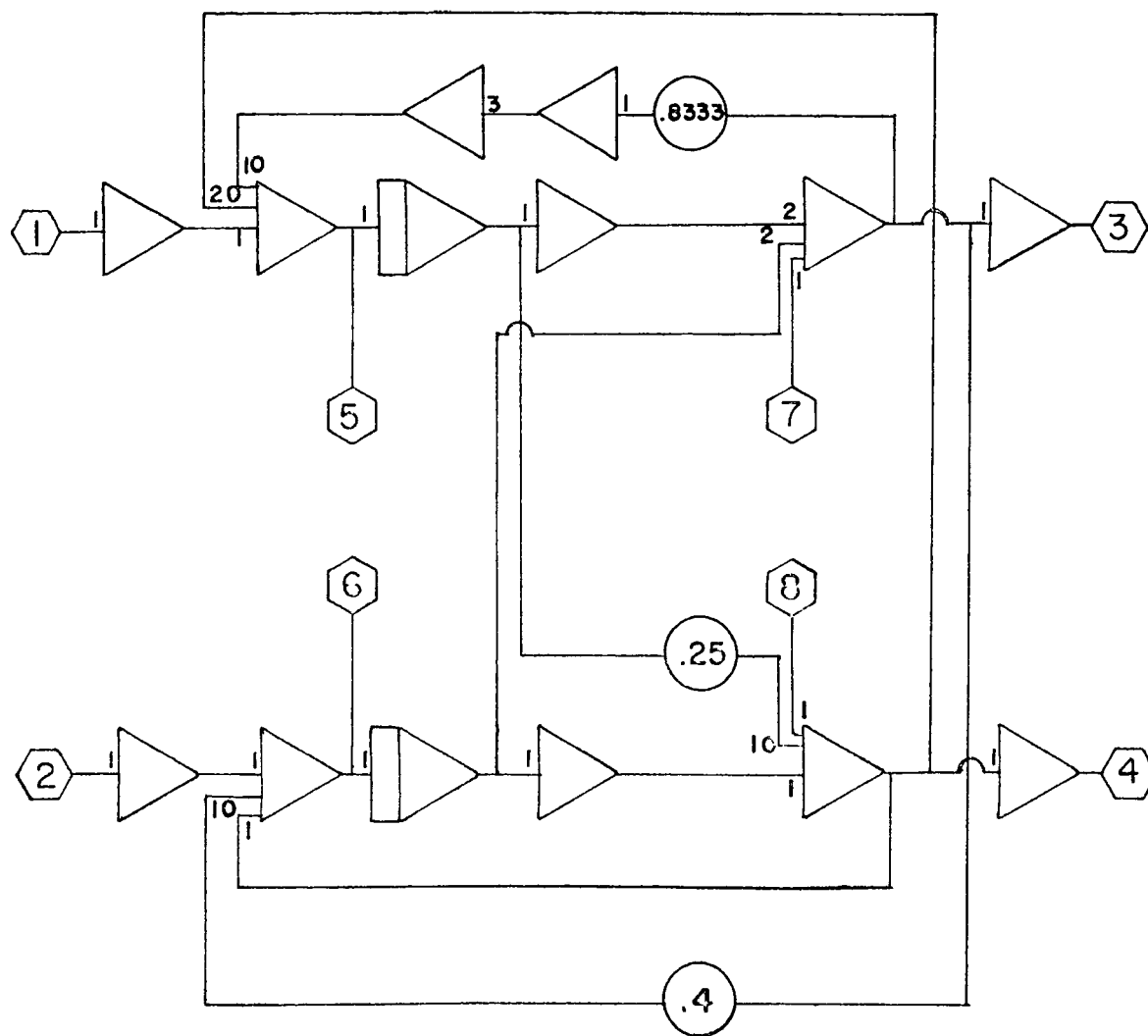


FIGURE 21 (continued)

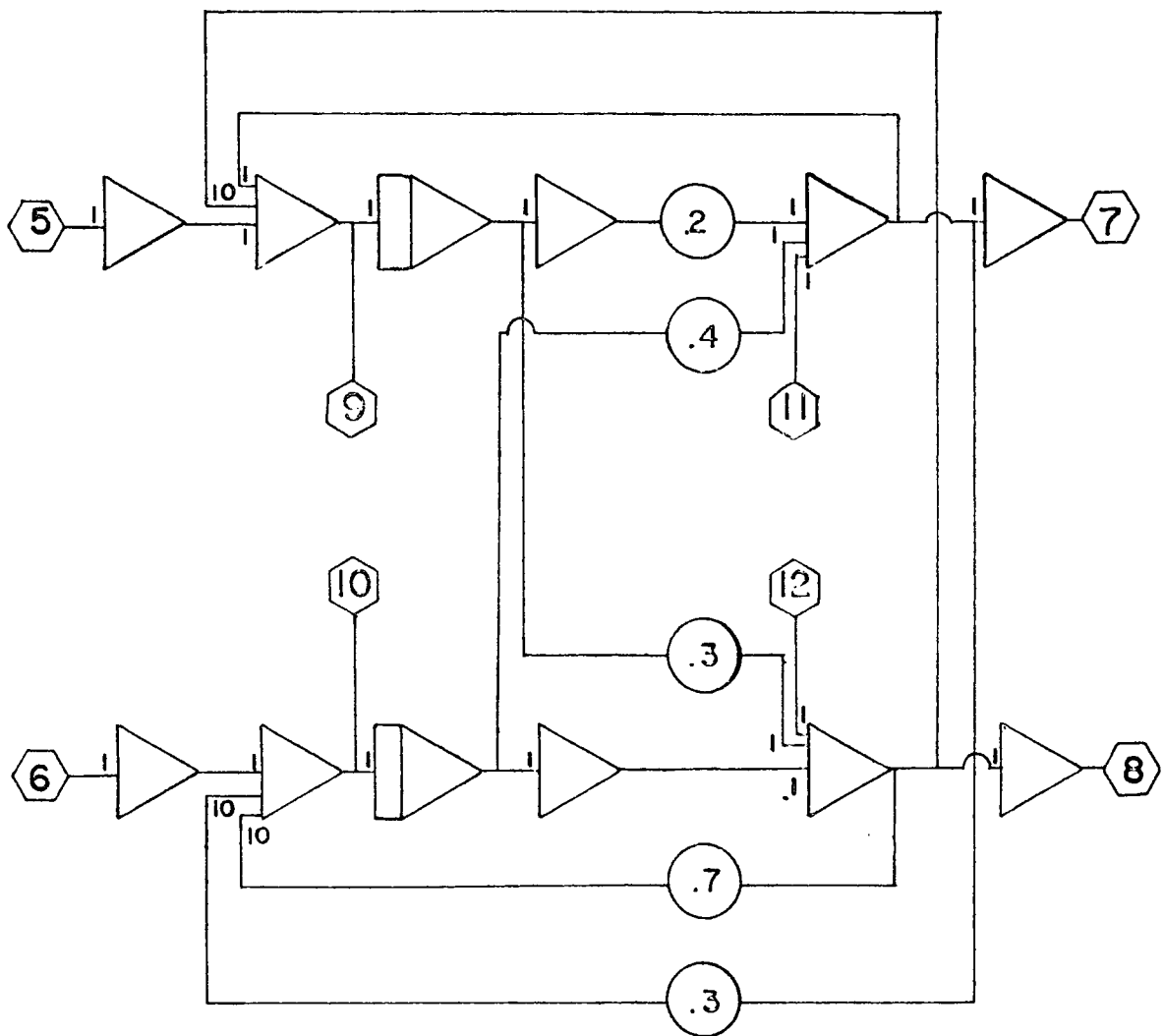


FIGURE 21 (continued)

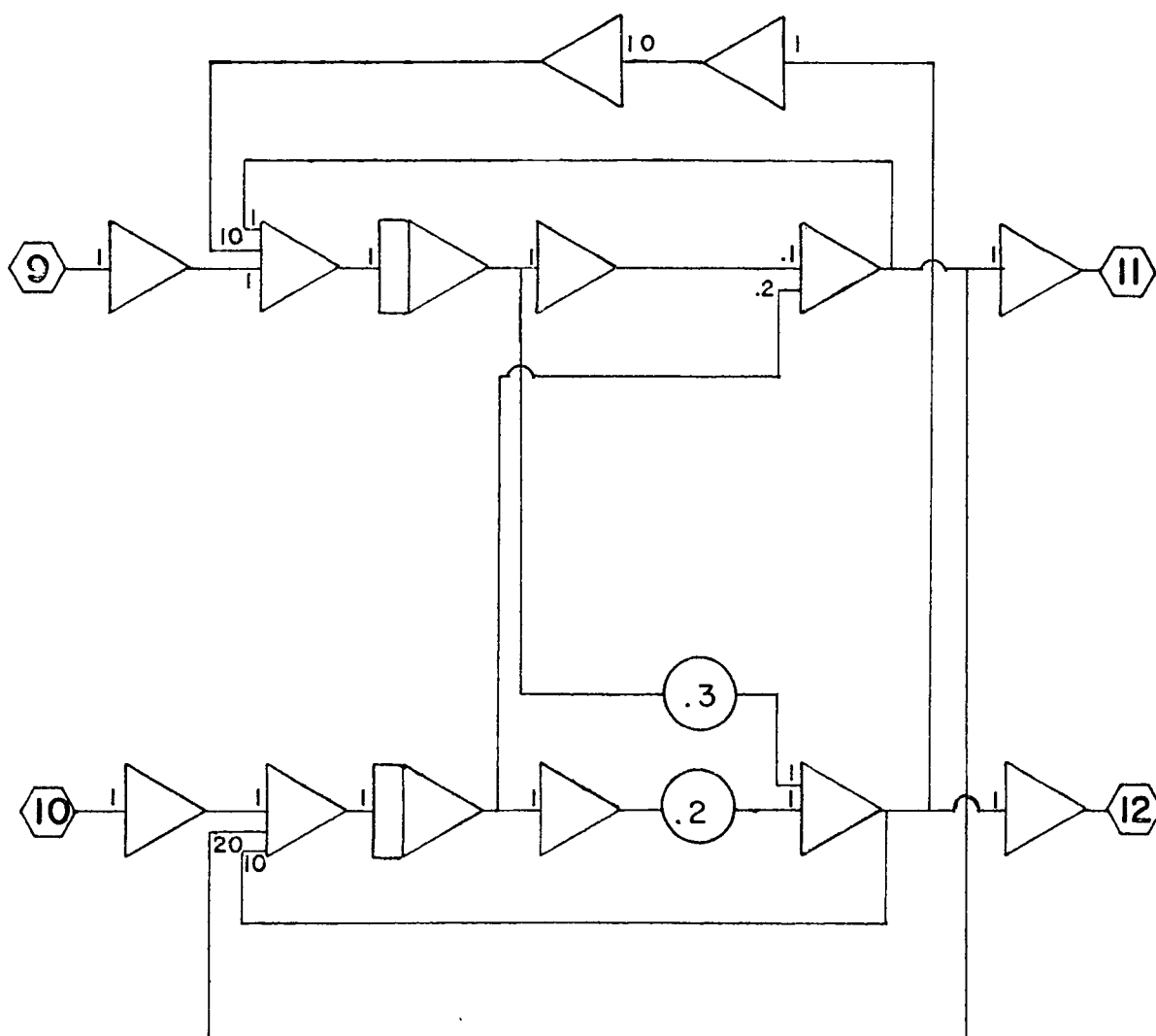


FIGURE 21 (continued)

$$[H_6] = \begin{bmatrix} -.2 & .4 \\ .3 & -.1 \end{bmatrix} \quad (32)$$

Page 4 of figure 21 contains the H_7 and H_8 terms which are

$$[H_7] = \begin{bmatrix} 1 & 100 \\ 20 & 10 \end{bmatrix} \quad (33)$$

$$[H_8] = \begin{bmatrix} -.1 & .2 \\ .3 & -.2 \end{bmatrix} \quad (34)$$

The program blocks are connected by patching the like numbered points together. Once the analog program was set up and debugged, the results as shown in figure 22 thru figure 28 were obtained. In the figures, H_8 represents the entire function, H_6 represents an approximation through the H_6 term and H_4 represents an approximation through the H_4 term. No results are presented for an approximation through the H_2 term because a first order solution for this high order system is of no interest.

In addition to the results presented, a test run of the system was compared to digital computer generated data. The analog program response's overall accuracy was within .5% with the majority of the data within .1% accuracy. In most cases an accuracy of 5% would be sufficient because one is not interested in accuracy, but relative degradation of each approximation as compared to the original system.

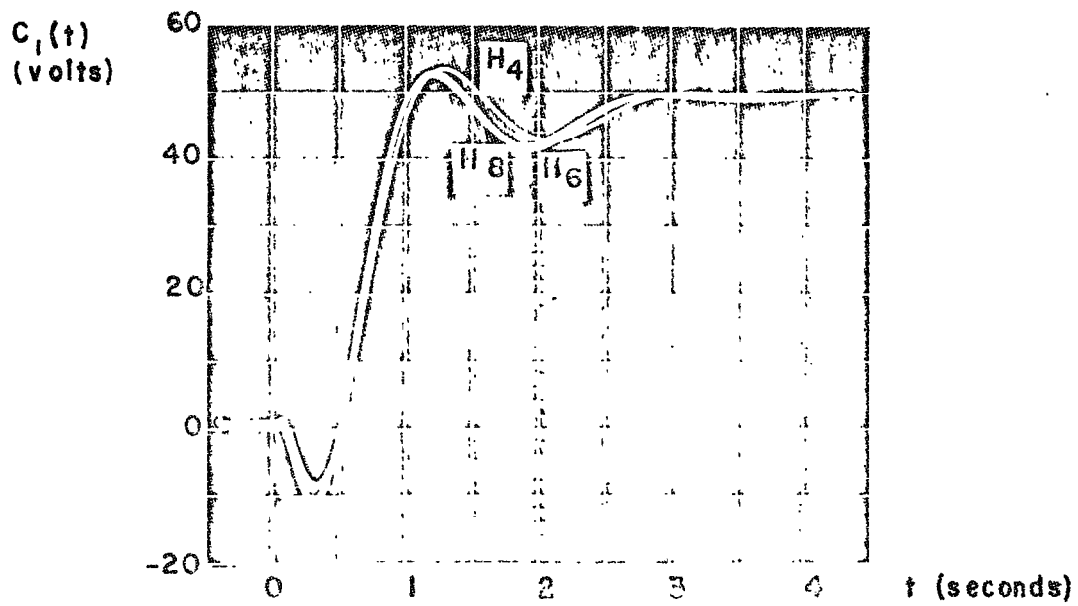


FIGURE 22

C_1 Responses for Example 3 ($R_1 = R_2 = -\frac{50}{s}$)

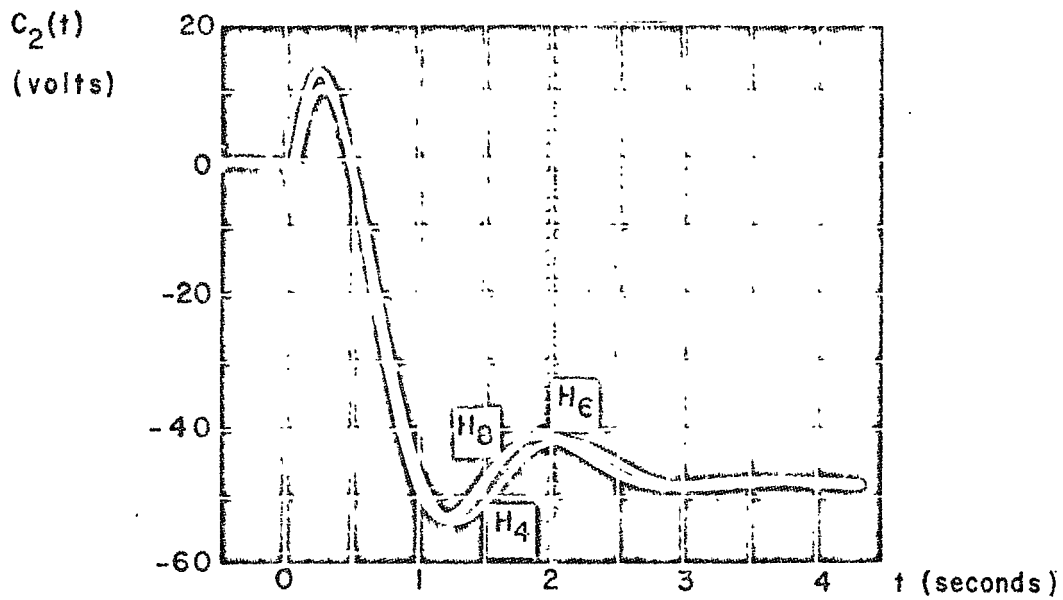


FIGURE 23

C_2 Responses for Example 3 ($R_1 = R_2 = -\frac{50}{s}$)

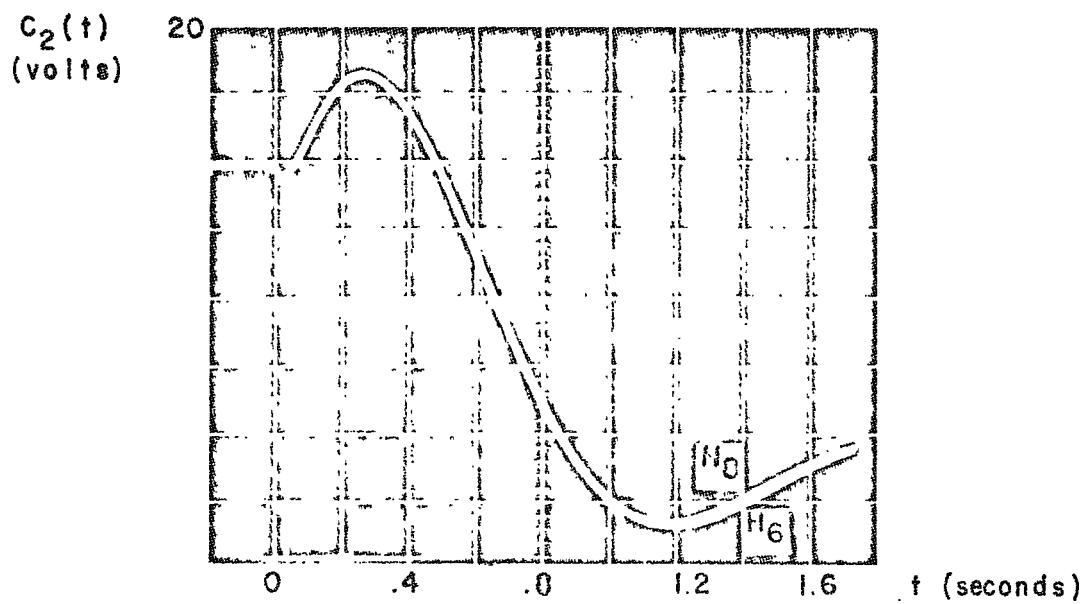


FIGURE 24

C_2 Responses for Example 3 ($R_1 = R_2 = -\frac{50}{s}$)

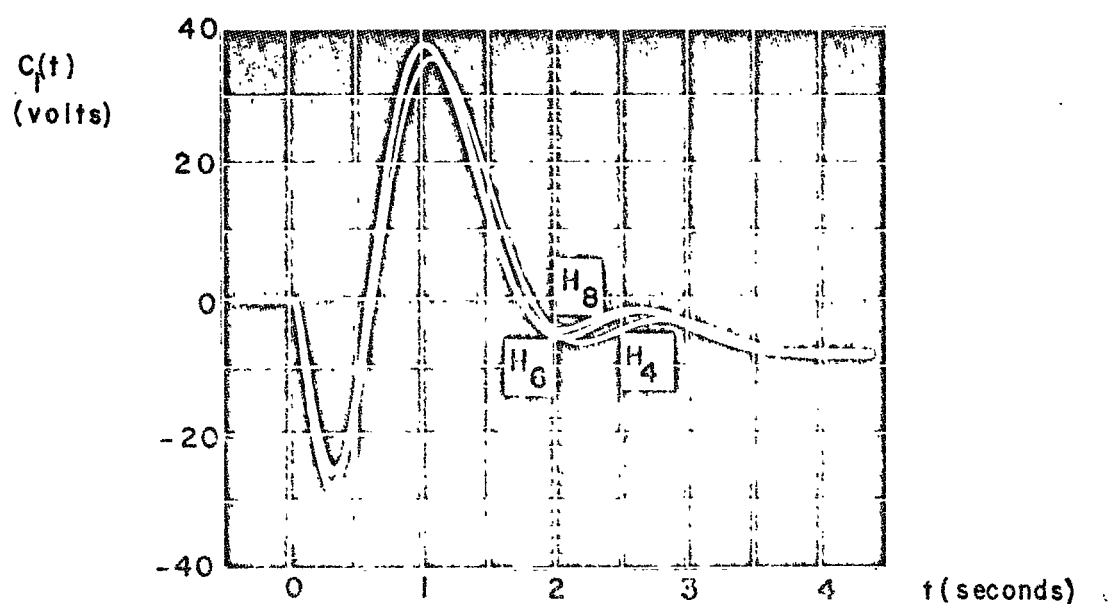


FIGURE 25

C_1 Responses for Example 3 ($R_1 = -\frac{50}{s}$, $R_2 = -\frac{70}{s}$)

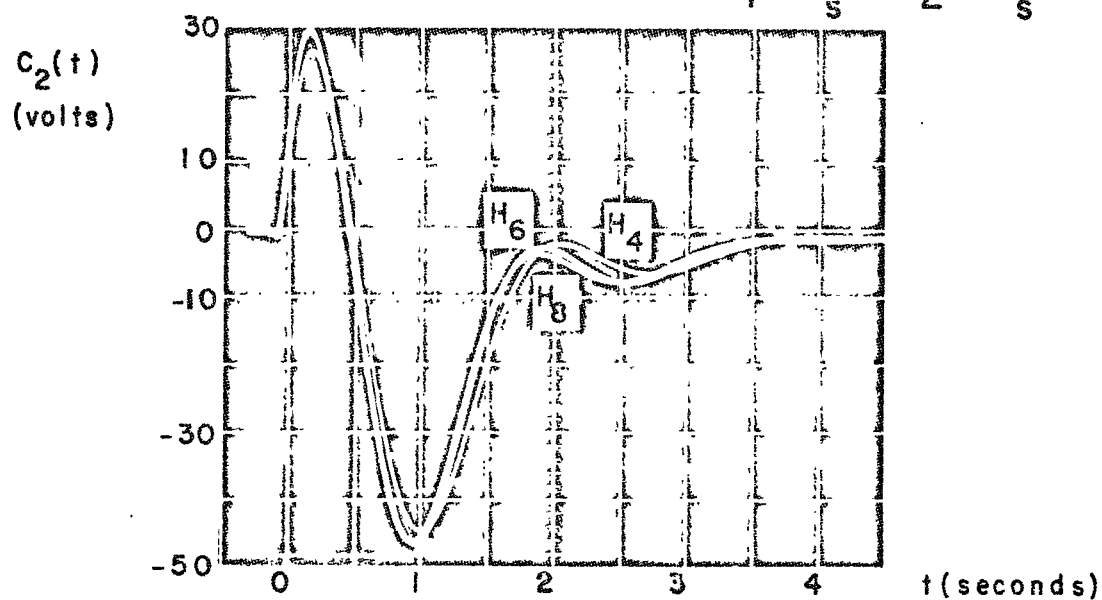


FIGURE 26

C_2 Responses for Example 3 ($R_1 = -\frac{50}{s}$, $R_2 = -\frac{70}{s}$)

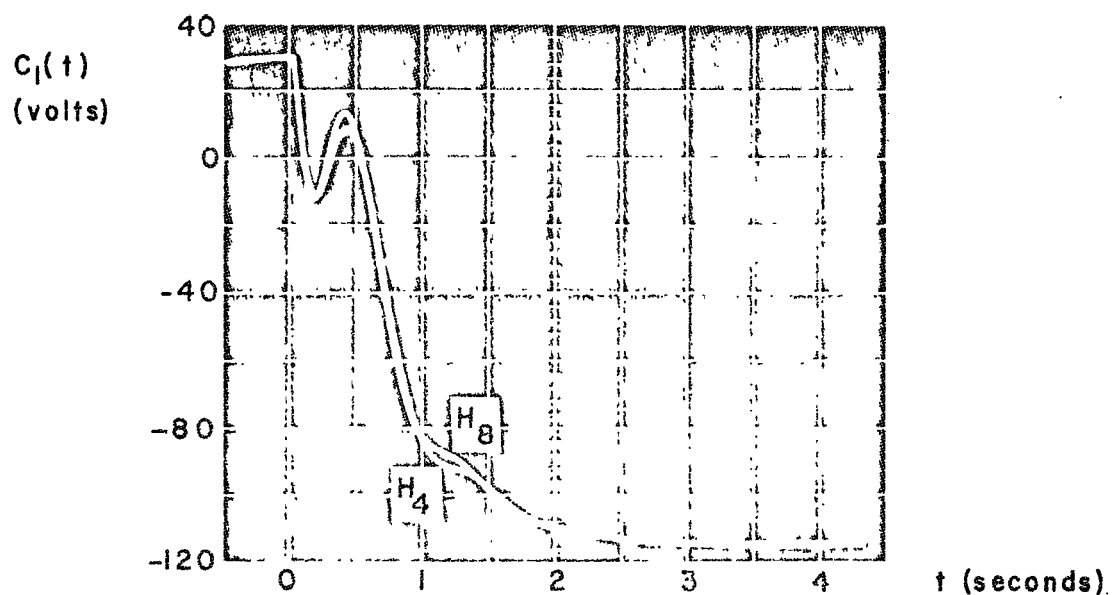


FIGURE 27

C_1 Responses for Example 3 ($R_1 = -28.4$, $R_2 = -\frac{50}{s}$)

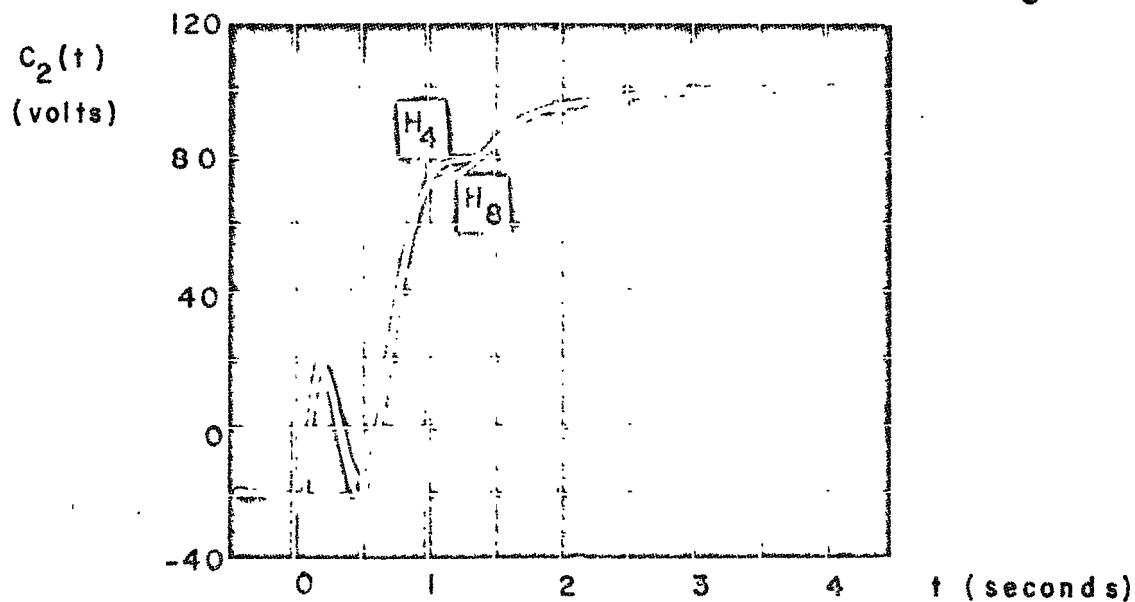


FIGURE 28

C_2 Responses for Example 3 ($R_1 = -28.4$, $R_2 = -\frac{50}{s}$)

Figures 22 and 23 present the C_1 and C_2 output responses for -50 volt steps on both the R_1 and R_2 inputs. Figure 24 presents an expanded view of the C_2 output as shown in Figure 23 except the H_4 approximation is not shown. The results as presented by these figures are extremely good. Only in Figure 24 can one see the difference the H_6 approximation has on the response. Even the H_4 approximation is within 5% of the original system.

C_1 and C_2 responses to unequal step inputs are presented in Figures 25 and 26. The R_1 input has a step of -50 volts applied, while the R_2 input has a step of -70 volts applied. The results again are excellent; however, one can see a difference in the responses of the original system (H_8) and the H_6 approximation more clearly than in Figures 22 and 23.

The last set of responses (Figures 27 and 28) were run to determine if any cross-coupling degradation could be caused by the approximation techniques. To accomplish this, the R_1 input had a -28.4 volt step applied and the system was allowed to reach a steady state ($t = 0$ on the figures). At $t=0$ in figures 27 and 28, a step of -50 volts was applied to R_2 . This procedure allows one to isolate the effects of the cross-coupling terms. The complete system (H_8) was plotted and the system approximation was performed through the H_4 term. The results are self evident. There was no cross-coupling degradation from the

approximation techniques. Again, the response of the approximated system are extremely close to the original system.

CHAPTER VII

CONCLUSIONS

An analog study of the matrix Cauer model has been presented. It is a natural extension of Chen-Shieh [6] technique for model simplification. The advantages of the method are that one has several approximations after once performing the expansion and the accuracies of the approximations are excellent. The simplest model is an approximation of the first order (H_1 and H_2) only; however, in practice, H_1 through H_4 would be as simple an approximation as one will use. When one takes more and more terms of the expansion, better and better results are obtained.

Multiple input/output systems are important in practice and difficult in analysis and they are almost impossible to design theoretically. This thesis offers a decomposition method to simplify the problem and to make the simulation practicable.

In introducing the matrix Cauer model, analog computer techniques were evolved for the scalar Cauer model. These techniques, as well as matrix model techniques, should be helpful to the designer from the standpoint of ease of simulation. Once the general programs are patched it is a simple task to plug in the values for a system and view the model's responses.

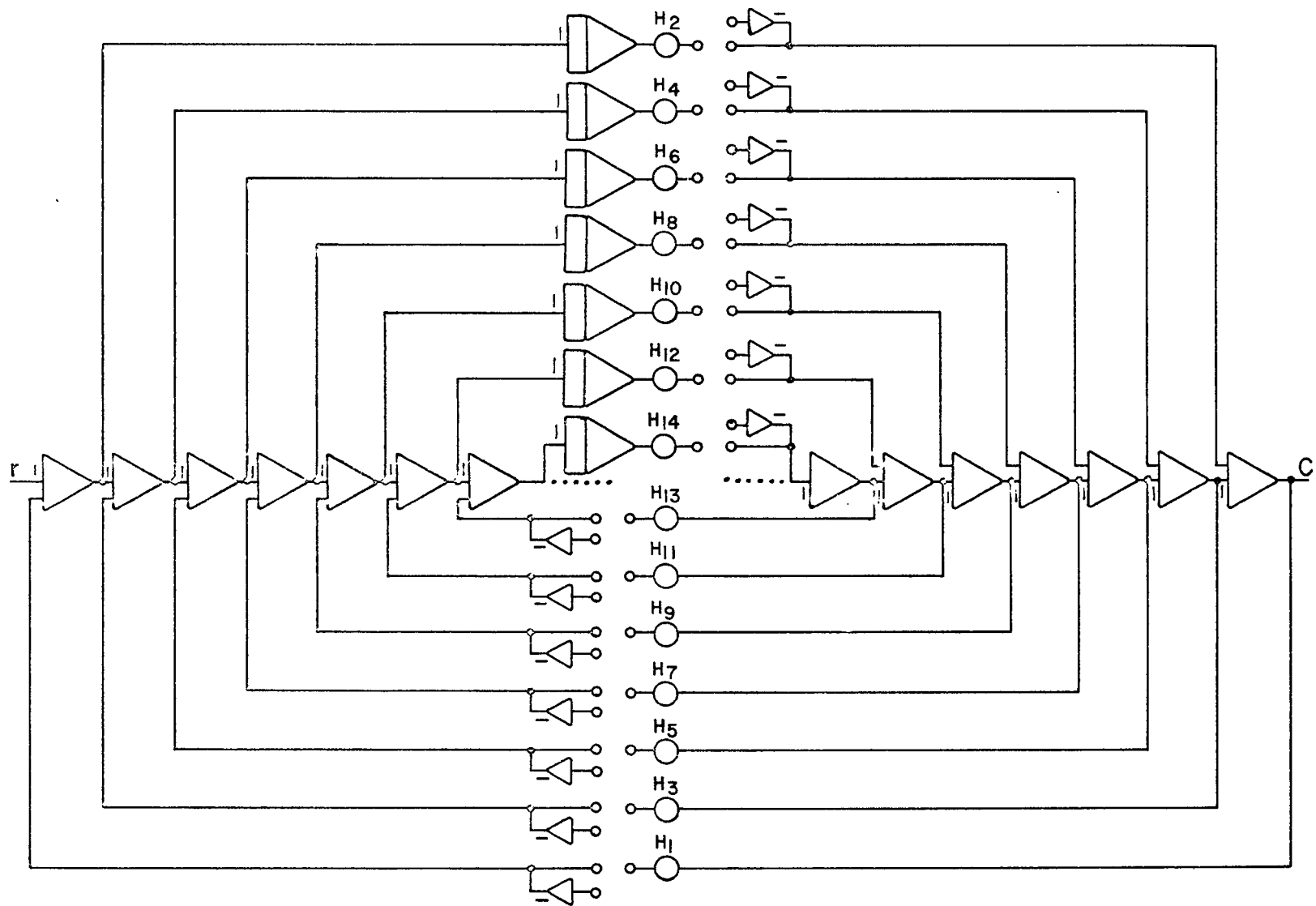
The analog computer programs generated using the second Cauer form are well suited to "canned program" environments. This is evidenced by

the lack of need to scale the examples because the second Cauer form inner loops are only active during transient conditions; therefore, if the outer loop is within the capabilities of the analog computer, scaling will probably not be needed. If the inner loops overload, the input amplitude can be reduced.

Hopefully, this paper will help take the analog computer out of the hands of the professional analog programmer and put it where it has always belonged --- in the hands of the engineer.

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APPENDIX A

General Analog Program for Scalar Cauer Form