On the Prediction of Downhole Drilling Tool Failures Using Game Theory

A Thesis

Presented to

The Faculty of the Department of Petroleum Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science

in Petroleum Engineering

by

Wanyi Jiang

May 2017

On the Prediction of Downhole Drilling Tool Failures Using Game Theory

Wanyi Jiang

Approved:

Chair of the Committee,
Dr. Guan Qin, Professor,
Petroleum Engineering

Committee Members:

Dr. Robello Samuel, Adjunct Professor,
Petroleum Engineering

Dr. Cumaraswamy Vipulanandan Professor,
Civil and Environmental Engineering

Dr. Suresh K. Khator,
Associate Dean,
Cullen College of Engineering

Dr. Mohamed Soliman,
Professor and Chair,
Petroleum Engineering

Acknowledgements

My sincere thanks go to Dr. Robello Samuel for his insightful advice and guidance. Before we started this project, I worked with Dr. Samuel on the application of Ant Colony Optimization algorithm to find the optimal set of drilling parameters. Encouraged by the progress on that project, Dr. Samuel suggested the use of Game Theory for downhole tool failures. Although we have experienced difficulties in understanding the fundamental principles of Game Theory and how it could be effectively applied to solve downhole drilling problems, Dr. Samuel patiently advised me to plough through this daunting task. The bearing fruit is summarized in this thesis. Even though still at its infant stage, Game Theory can be of great potential in finding optimal strategies to balance drilling cost and downhole tool failure risk.

I would thank Dr. Zhengchun Liu for his generous help and particularly the sharing of his thesis as a template, which saved me a lot of time. I would also thank Mr. Min Liu, Mr. Ping Yi, Mr. Yuanhang Liu, and Ms. Chun Zhang for their tremendous help to my drilling research projects and this thesis. I am grateful to my wife Jing Li for her support and encouragement.

On the Prediction of Downhole Drilling Tool Failures Using Game Theory

An Abstract

of a

Thesis

Presented to

The Faculty of the Department of Petroleum Engineering

University of Houston

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science

in Petroleum Engineering

by

Wanyi Jiang

May 2017

Abstract

Complex and expensive downhole tools have been employed increasingly in modern drilling. Downhole tool failures include motor failures, bit failures, and MWD failures. Any of these failures may result in reduction of rate of penetration (ROP), repair or replacement of the damaged tools, wasted trips, and rig downtime, and thus add substantially to the drilling cost. The causes of downhole tool failures are compound, embracing material, quality, design, and operating conditions. It is possible to monitor operating conditions and adjust operating parameters to avoid downhole tool failures and mitigate damages to downhole tools. In this thesis, game theory is considered for the prediction of downhole tool failures.

Game theory is a branch of mathematics for decision making in the conflict of interests. A game theory model has four important components: players, information, actions, and payoff. An appropriate approach, i.e., a two-player non-zero-sum game, has been established for downhole tool failure. Namely, one player holds interest of the drilling time free of tool failures while the other cares the probability of tool failure. The information includes all pertinent data to calculate the drilling time and the probability of tool failure. The first player may take actions such as drilling operations to maximize his payoff, e.g., minimal drilling time, while the second player tries to reduce the probability of tool failures as much as possible.

We utilized Game Theory Explorer to solve the two-player non-zero-sum game. Based on the game theory model, two payoff tables in terms of ROP and tool reliability, respectively, have been constructed and the optimal strategies have been found.

Table of Contents

List of Figures

List of Tables

Nomenclature

a	Bit constant
a_c	Chip holddown permeability coefficient, psi$^{-b_c}$
a_i	Fitting parameter
A_{abr}	Relative abrasiveness
A_v	Ratio of jet velocity to return velocity
b	bit constant
b_s	Stress confinement coefficient
c	Bit constant
c_c	Chip holddown permeability coefficient
C	Tool cost, dollar
d_{bit}	Bit diameter, in
d_n	Nozzle diameter, in
F_j	Jet impact force, lbf
F_{jm}	Modified jet impact force, lbf
$f_c(P_e)$	Chip holddown function, lbf
L	Lateral vibration
$MTBF$	Mean time between failures, hr
$MTTF$	Mean time to failure, hr
N	Rotary speed, rev/min
P_e	Differential pressure, psi
p_f	Probability of failure
Q	Mud flow rate, gal/min
ROP	Rate of penetration, ft/hr

RPM	Revolution per minute, rev/min
S	Rock strength, kPa
S_0	Unconfined rock strength, kPa
T	Temperature, °F
T_f	Time to failure, hr
t	Time, hour
V_f	Return velocity, ft/s
V_n	Nozzle velocity, ft/s
W	Weight on bit, lbf
W_c	Wear coefficient
W_f	Wear function
x	Reliability
$\bar{x}$	Stress variable
β	Shape parameter
γ	Fluid specific gravity
λ	Hazard rate
$\eta(\bar{x})$	Scale parameter
μ	Viscosity, cP
ρ	Fluid density, lbm/gal
ΔBG	Relative Bit tooth wear

Chapter 1 Introduction

1.1 Drilling Cost and Downhole Tool Failure

The recent downturn in oil and gas industry accompanies a severe decline in drilling activities. For example, active drill rigs in U.S. and Canada have dropped to about 1/5 in May 2016 as compared to the peak in 2014 (Figure 1-1). The cost reduction becomes vital for drilling companies to survive. Current oversupply of crude oil and natural gas does not change the general trend that hydrocarbon reservoirs are depleting fast, and the remaining reservoirs become more and more difficult to access. As a result, well drilling demands increasingly complicated technologies for current and future hydrocarbon extraction. Horizontal drilling replaces vertical drilling as the dominant well trajectory in North America. Directional drilling and multilateral wells are more commonly used than ever. Along with the advance in drilling technologies, many complex and expensive downhole tools become necessary for a successful drill job. Innovative technologies such as rig automation, advanced downhole tools, real-time monitoring, and managed pressures emerge and become more and more prevalent. Meanwhile, downhole conditions can be much harsher than before. Higher pressure, higher temperature, potentially more excessive vibration and shock require more reliable downhole tools. Downhole tools and drilling operations should be optimized in a synergic way to minimize downhole tool failure without incurring much additional nonproductive time cost.

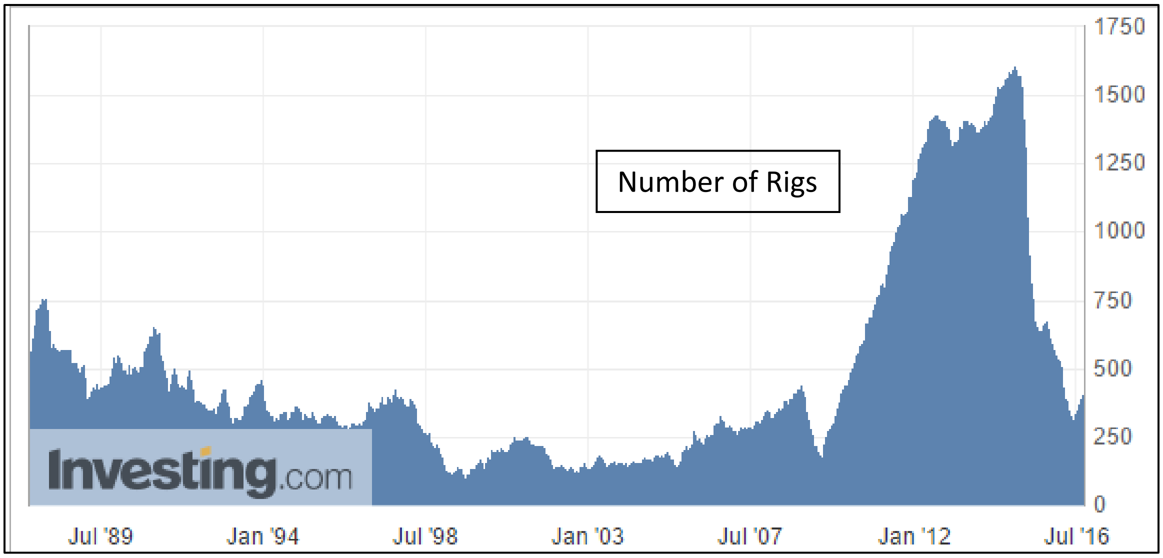

Figure 1-1 The historical chart for US rig counts by Baker Hughes (taken from http://www.investing.com/economic-calendar/baker-hughes-u.s.-rig-count-1652 on 9/11/2016).

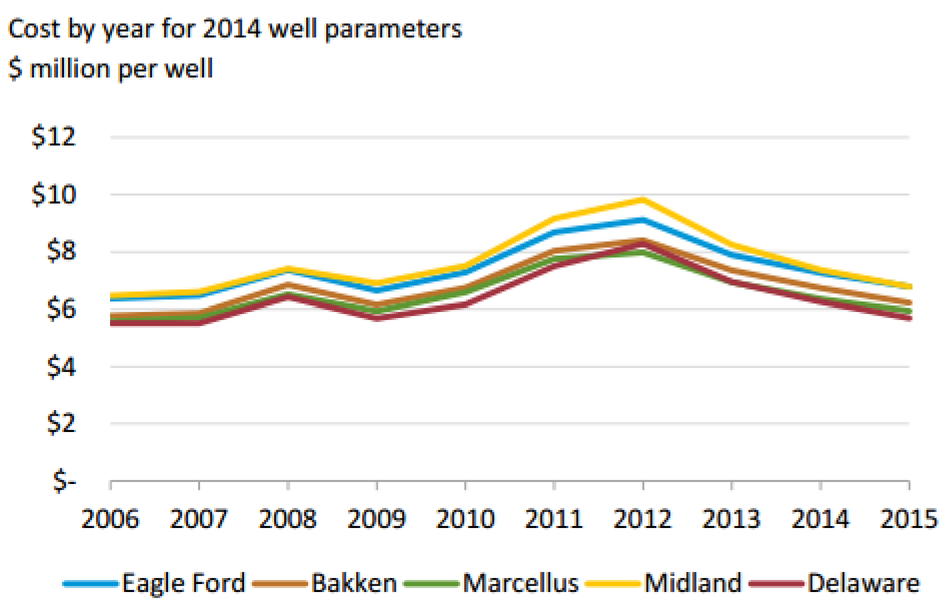

Figure 1-2 Average drilling and completion cost per well by IHS Oil and Gas Upstream Cost Study commissioned by U.S. Energy Information Administration (EIA).

The aim of this thesis is to find the optimal strategy for both drilling cost reduction and minimization of downhole tool failure. In general, the best strategy should weigh the risks of all possible downhole problems, which may not be attainable in practice. For example, the top sources of downhole problems include downhole equipment failure, downhole cement problems, stuck pipe, lost circulation, wellbore instability, well control failure. A comprehensive survey of only top downhole problems is beyond the scope of this thesis. Consequently, we focus on only a few representative downhole tool failures to find a viable methodology, which can be potentially extended to a broad use for general downhole tool failure as well as other drilling problems.

There are many available measures to minimize the probability of downhole tool failure by using advanced tools, control devices, and measures to tamper torsional vibrations (Rajnauth and Jagai, 2012). While some of the measures could be cost-effective, others may be too prohibitive for all but exploratory wells. A compromise must be made between minimizing the probability of downhole tool failure and reducing overall drilling cost. When there are multiple strategic choices, each with advantages and disadvantages, it could be difficult to weigh all factors and find the best strategy. However, with the aid of computer technologies and data management, it is possible to resolve this issue in a systematic and reliable manner. Herein it is proposed to evaluate the drilling cost together with the reliability of downhole tools. For simple cases, the best strategy may be found straightforwardly and intuitively, particularly when the actions taken to minimize drilling time do not adversely affect the probability of tool failures. But in real and complex situations, the optimal choice may not be obvious and game theory is used to find the optimal strategies.

1.2 GAME THEORY

Generally, game theory is viewed as the establishment of mathematical models for rational decision makers to maximize their own awards against competitors with conflicting interests (Myerson, 1991). A game model includes four critical components:

- Players
- Information
- Actions
- payoff

Each rational player is trying to take the action that maximizes his payoff. When there is only one player, the game theory is equivalent to a decision or an optimization problem. For example, it is an optimization problem when the target is to minimize only the drilling time, or only the probability of downhole tool failure. When both the drilling time and downhole tool failure are considered, a game theory model is appropriate as there are conflicting interests, particularly when a saving in drilling time increases the probability of downhole tool failure. The net payoff to all players could be zero or non-zero, and the corresponding games are called zero-sum and non-zero-sum respectively. Since the payoff in terms of drilling time is different from the payoff in terms of downhole tool failure probability, a non-zero-sum game model would be an ideal tool. If a strategy exists for a player to maximize his payoff no matter what actions other players would take, this strategy is dominating. When there is a dominating strategy for each player, these dominating strategies form a pure (or deterministic) strategy solution to the game problem. It is also possible that each player may take multiple strategies each with a given probability to maximize his payoff. And such a solution is called a mixed strategy. Nash proved that in a finite game, there exists at least one mixed strategy solution that each player cannot increase his payoff by unilaterally changing his strategy. Such solution is called Nash equilibrium (NE). Our goal of this study is to construct a game theory model for downhole tool failure. By solving the game theory model, the obtained NE will indicate the best possible strategy to minimize downhole tool failure.

1.3 Game Theory Applications

Initially game theory was developed for card games (e.g., the Waldegrave problem, Bellhouse, 2007) and then applied to economics (Cournot, 1838). Game theory has later been extended to a great variety of problems where competition and conflicts of interest among different entities exist, such as war, politics, sociology, psychology, evolution, and biology.

Oil and gas industries are no exception to the use of game theory in terms of economics (Bratvold and Begg, 2009; Schitka, 2014). Oil price as driven by demand and supply, as well as other factors, has been extensively studied by game theory (Bratvold and Koch, 2011). The strategy of Organization of the Petroleum Exporting Countries (OPEC) has been modeled and predicted by game theory (see section 4.1.3 for detailed discussion). For individual oil and gas companies, game theory can be used for decision making in drilling activities (Frederick and Lieberman, 2001). Recently, game theory has also been applied to consider cost reduction in drilling (Ajimoko, 2016). Players in the game can be non-cooperative, e.g., OPEC versus non-OPEC oil-exporting countries, or cooperative, such as the allocation of profit among land-owners as discussed by Schitka (2014), and the joint-venture strategies in drilling (Ajimoko, 2016).

1.4 Game Theory for Downhole Tool Failure

Prediction of downhole tool failure has been challenging. Tool-life is not only dependent on drilling conditions at the current drill run, but also on cumulative usage. By tracking the tool's history and simulating the cumulative damage, a probabilistic method was proposed recently to predict the lifetime for downhole electronics (Kale et al., 2014; Carter-Journet et al., 2014). Since one of the major causes of downhole tool failure is excessive vibrations, the magnitude and duration of excessive vibrations, along with other critical parameters such as the temperature, are evaluated in the tool-life prediction models. To reduce the probability of tool failure, vibration suppression has been extensively studied and widely used (e.g. Sotomayor et al., 1997; Kriesels, 1999; Karkoub et al., 2009; Rajnauth and Jagai, 2012; Shor et al., 2015). While it is of paramount importance to establish a reliable approach for the tool-life prediction, further use of the predicted results may require a game theory model to simulate the outcomes and find the best strategies. For example, Carter–Journet et al. (2014) proposed tool assessment and sparing optimization based on their probabilistic tool-life model. The study gave risk criteria without justification. A decision analysis based on game theory would be promising for finding the optimal risk criteria for downhole tool failure. Examples for drilling activity can be found in the book *Introduction to Operations Research* by Frederick and Lieberman (2001). In the sparing optimization algorithm, the swapping of tool component between two tool assets did not improve the combined reliability defined as the probability that none of the assets would fail. The optimization algorithm may be useful with an additional objective function of drilling cost. In all, game theory could be considered to bridge the gap between the probabilistic prediction of downhole tool reliability and the optimal reduction of tool failure as well as reduction of total drilling cost.

This thesis is organized as follows. Chapter 2 gives a survey on downhole tool failure with an emphasis on the leading causes. The drilling cost is described in Chapter 3. Details are given in Chapter 4 for game theory and algorithms. In Chapter 5, game theory models for downhole tool failure and results of case studies are given. Chapter 6 concludes the study.

Chapter 2 Downhole Tool Failure

A comprehensive evaluation of all downhole problems should be considered for drilling. However, due to the limited scope of this study, only a few downhole tools are selectively considered. Downhole problems, such as stuck pipe, lost circulation, wellbore instability, or well control failure, will not be discussed. Problems related to casing, cementing, or coiled-tubing drilling tools are excluded. We refer to the book *Downhole Drilling Tool–Theory and Practice for Engineers and Students* (Samuel, 2007) for an exhaustive introduction to downhole drilling tools which also encompass discussion of common problems, causes, and preventive measures.

2.1 DOWNHOLE TOOLS

Some of the downhole tools are generic, but others may only be used for specific drilling conditions (Samuel, 2007). Important and common downhole tools are briefly listed in the following section, and their possible wears and failures are concisely mentioned. Tools used to reduce vibration and improve performance are given in section 2.2. A diagram of a typical drilling rig is provided in Figure 2-1. Downhole tools may be assembled together to a bottomhole assembly (BHA) (Figure 2-2).

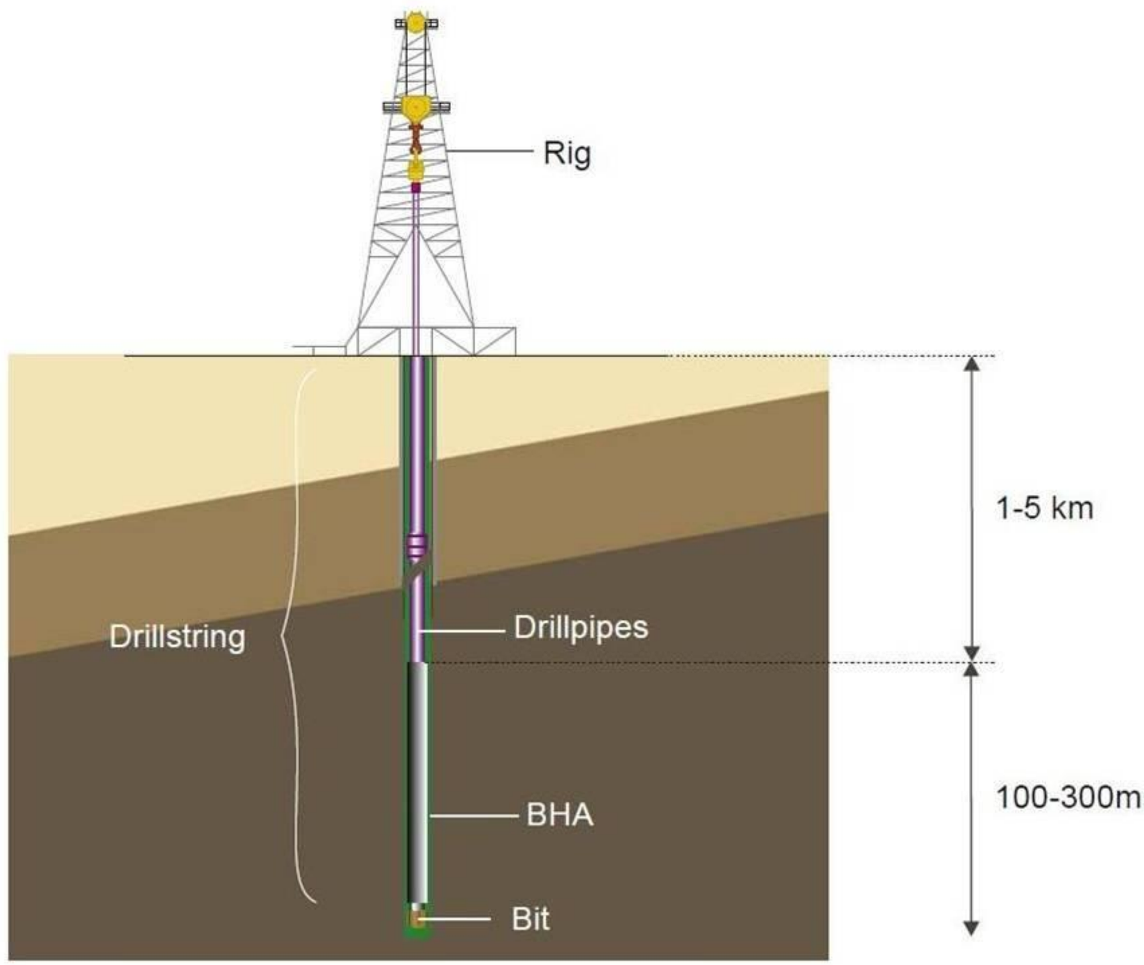

Figure 2-1 Drilling rig (taken from http://www.abdn.ac.uk/engineering/research/modeling-and-analysis-of-bha-and-drillstring-vibrations-149.php accessed Sep. 18, 2016).

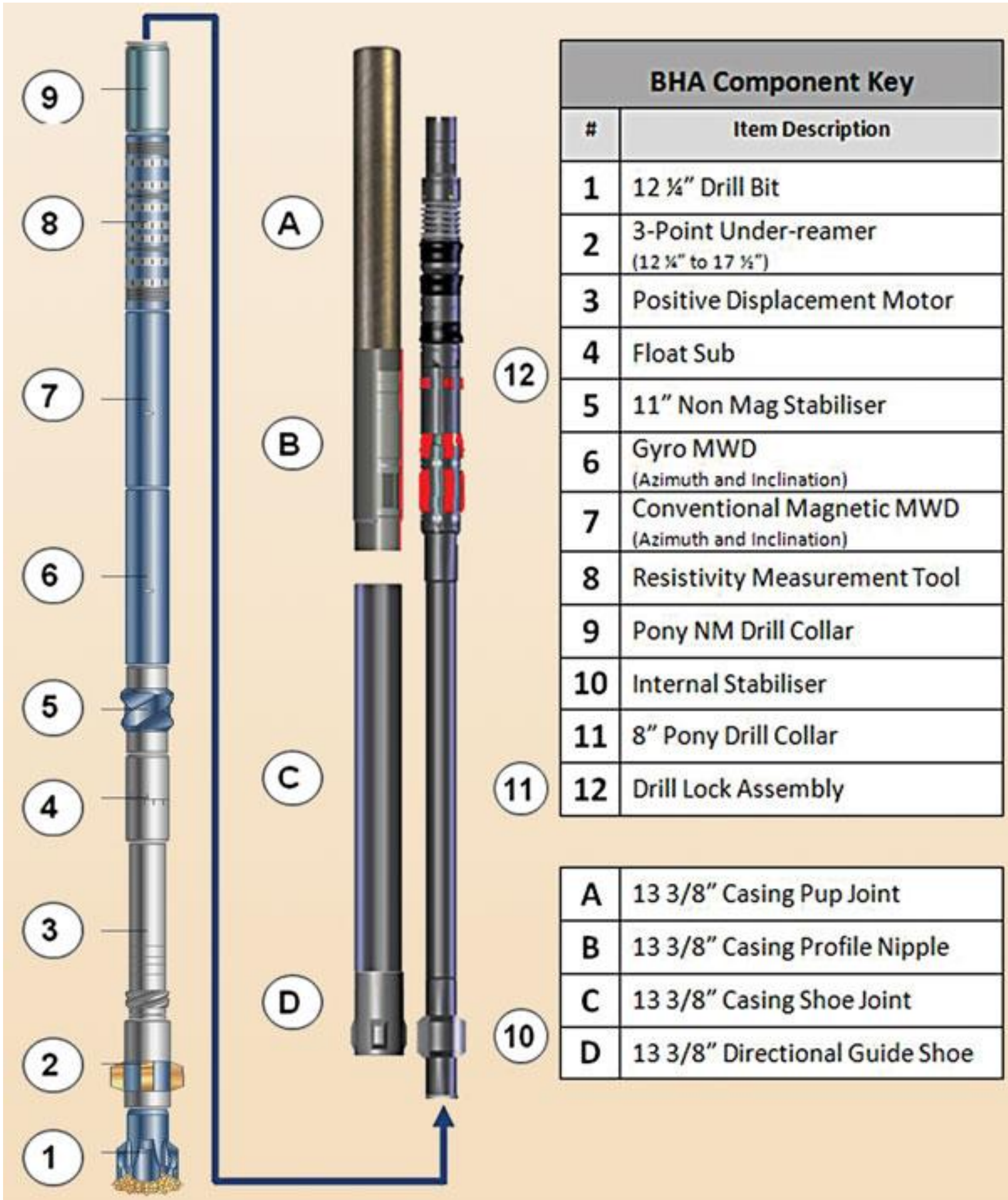

Figure 2-2 An example of bottomhole assembly (taken from http://www.drillingcontractor.org/wp-content/uploads/2010/07/cwd-web08.jpg accessed September 18, 2016).

2.1.1 Downhole drilling tools

Downhole drilling tools are a collection of tools including drill bits, hole-enlarging tools, retractable rock cutting tools, and drill pipes. Different types and configurations for these tools are available.

Drill bits

Rotary drill bits include roller cone bits, polycrystalline diamond compact (PDC) bits, thermally stable polycrystalline (TSP) bits, diamond bits, and roller cone rock bits (milled tooth bits and insert bits). There are also fixed cutter bits and drillable bits.

A number of problems may occur to drill bits during a drilling run. The dull bit grading can be an excellent indication of the remaining life of the drill bits, except for the drillable bits which may not be pulled out. Usually the grading will give the amount of teeth wear and bearing, as well as the status of the seal. The dull characteristics give additional information on the wear or failure.

Hole-enlarging tools

These tools include hole-openers, underreamers, bicenter bits, ream-while-drilling (RWD) tools and enlarge-while-drilling (EWD) tools. Hole-enlarging tools may cause severe vibrations due to mass imbalance. As a result, the tool cutters may be worn unevenly and the reamers' arm may fail. Proper stabilization and other measures can be used to reduce the vibration.

Retractable rock cutting tools

In this category we have retractable/expandable drillbits, retractable RVA-type diamond reamer, and expandable RRB-type underreamer. Retractable rock cutting tools save the trips for the replacement of worn-out drill bits, which can take a lot of time for deep wells.

Drill pipes and drill collars

Drill pipes are usually heavy weight drill pipes (HWDP), which include aluminum and titanium drill pipes. Drill collars can be spiral or square based on the shape. Pipes are connected by tool joints. Drill collars are the major source of vibrations, which if severe may lead to the failure of downhole tools. Other sources of drill pipe failures include buckling, fatigue, and corrosion.

2.1.2 Downhole motors

There are a number of representative downhole motor types: positive displacement motors (PDM), downhole turbines, and electrodrill motor (EDM). PDM is commonly used. One possible failure for PDM and downhole turbines is bypass valve clogging. Downhole turbines may also fail on bearing.

2.1.3 Deflecting tools

Deflecting tools are used to correct undesired wellbore deviation. In directional drilling, horizontal drilling, and extended-reach drilling, deflecting tools are used to achieve desirable wellbore deviation. Downhole deviation tools include bent subs, double-bend assembly, stabilizers and stabilizer gauge, whipsticks, kick pad (offset pad), eccentric stabilizer, offset stabilizer.

2.1.4 Drill stem testing tools

Drill stem testing tools are used to obtain formation and reservoir parameters. A series of drill stem testing tools are available, including downhole test tools, reciprocating test tools, slip-joint safety valve, volume-pressure balanced slip joints, reverse-circulating subs, space-out, full-opening drill stem testing, and special tools for deep wells.

2.1.5 Downhole measurement tools

Measuring While Drilling (MWD)

MWD means measurements made downhole with electronmechanical devices. For example, as shown in Figure 2-2, item 6 is a gyro sensor, item 7 is a conventional MWD device for azimuth and inclination, and item 8 is a resistivity measuring tool. An MWD system includes a battery (or a turbine) as power source, mud-pulse telemetry (or low-frequency electromagnetic transmission), and directional sensors.

Logging While Drilling (LWD)

LWD refers to wireline-quality formation measurements made while drilling. There are mainly five different types, namely electromagnetic logging, logging while drilling induction tools, acoustic logging, nuclear magnetic resonance (NMR) logging, and nuclear logging.

2.1.6 Downhole tools for specific purposes

Stand-off devices are used to reduce the eccentricity ratio. Rotary subs refer to a short piece of pipe used for connections between parts of drilling assembly. Circulating subs/pot collars are used to achieve optimum annular velocity throughout the wellbore.

Downhole blowout preventer is a wellbore pressure control device that prevent the formation pressure to go above a packer in the device. Coring tools are used to obtain core samples.

2.2 Tool Failure Causes and Prevention

2.2.1 Causes of tool failure

One primary cause of tool failure is the normal wear of tools (Figure 2-3). For example, the drill bits will be worn out after being used for a certain period of time. Excessive vibration may cause failures of drill pipe and downhole tools. Fatigue and corrosion are also important factors to tool failures. Other factors include electrical overstress, mechanic stress, manufacture defects. Improper drilling operational parameters or poor equipment design may result in pre-mature failures.

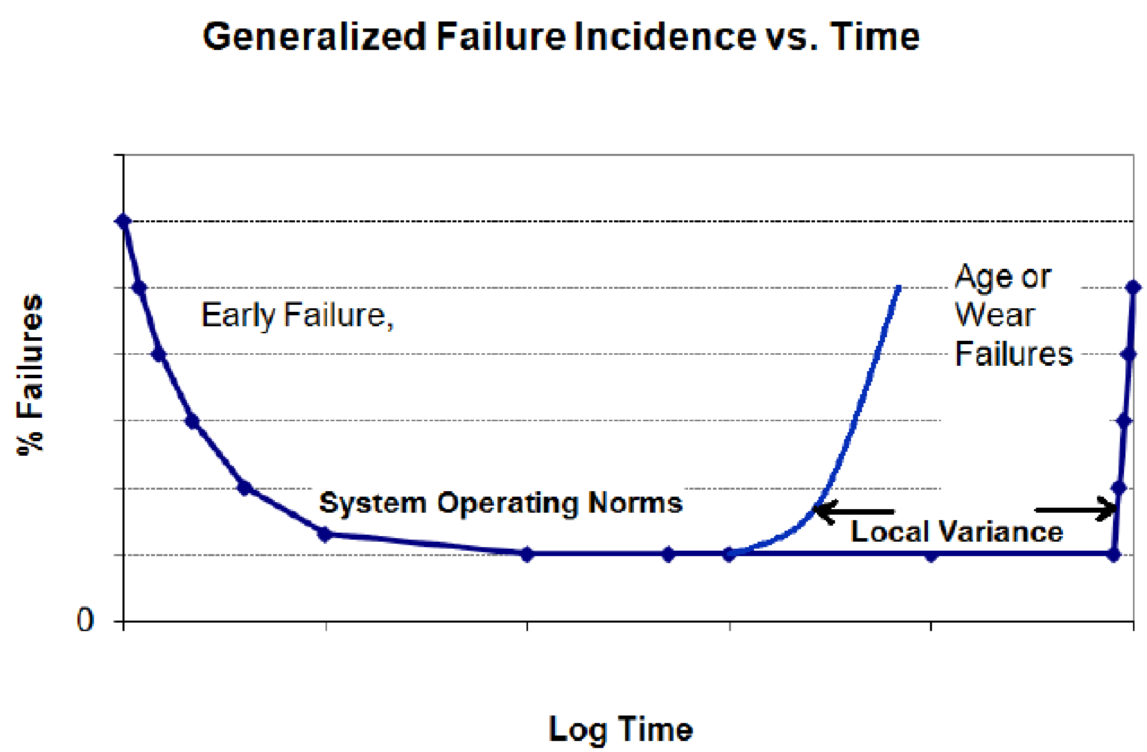

Figure 2-3 Failures over time (King 2010).

2.2.2 Vibration

Excessive vibration is one of the most hazardous factors to downhole tools. There are three principal vibration modes: axial, lateral, and stick-slip. Axial vibration is the vibration along drill string due to the bouncing of drill bit. Axial vibration may cause damages to drill bit and downhole assembly components. Lateral vibration refers to the vibration occurring transversely to the axis of drill string. It is probably the most common reason for downhole tool failure. Stick-slip is the torsional vibration, which is regarded as the most damaging mode of vibrations.

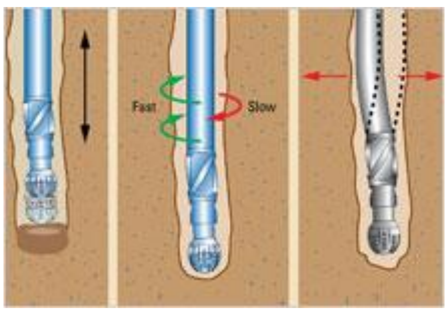

Figure 2-4 Modes of vibration: axial (bit bounce, axial acceleration), torsional (stick-slip, tangential acceleration), and lateral (taken from http://www.slb.com/resources/case_studies/drilling_system/eliminate_drillstring_vibration_underreaming_ops_sakhalin.aspx accessed September 25, 2016).

2.2.3 Reduction of vibration and drag

As mentioned above, excessive vibrations may cause severe problems and pre-mature tool failures. Control devices are used to mitigate vibrations. These devices include conventional stabilizers, stabilizers in BHA, reamers, key seat wipers, bumper subs in drill string, shock subs, hydraulic thrusters, harmonic isolation tools, vibration isolation barriers, steady scouts. Excess drag force may be reduced by using mechanical friction-reduction tools and tractors/crawlers.

2.2.4 Hole-cleaning

Hole-cleaning refers to the removal of drilled cuttings. Inadequate hole-cleaning may cause a series of drilling problems. Hole-cleaning tools include mechanical hole-cleaning device (MHCD), cuttings bed impeller, and circulating subs/port collars.

2.2.4 Electronic failure indicators

For electronic tools, failure precursors, such as shifts and variation in temperature, voltage, current, resistance, and impedance, can be detected (Pecht et al., 1997； Pecht et al., 1999). Another approach is the use of sacrificial circuits such as fuses, canaries, circuit breakers, and self-diagnostics sensors (Mishra and Pecht, 2002).

2.3 PROBABILITY OF FAILURE

2.3.1 Mean time between failures (MTBF)

Mean time between failures (MTBF) is the average time elapsed between two consecutive failures of a tool. The tool is assumed to be repaired after a failure. MTBF is calculated as

$$MTBF = \frac{\sum_{i=1}^{N_{failure}}(t_{fi}-t_{ui})}{N_{failure}}, \tag{2-1}$$

where $N_{failure}$ is the number of failures, t_{fi} is the start time of *i*th failure, and t_{ui} is the start time before *i*th failure.

2.3.2 Mean time to failure (MTTF)

MTTF is similar to MTBF except that failed system is replaced instead of being repaired. A few examples are given in the following table. There are two types of failures based on severity. Critical failures are those in which the drilling operation has to be stopped and the tool must be replaced. Non-critical failures are those in which the drilling operation can be continued with undesirable performance or loss of data.

Table 2-1. MTTF for a few examples of well equipment (King, 2010).

item	MTTF (years) (critical failure)	MTTF/years (non-critical failure)
Christmas tree	250	50
SCSSV wireline insert	83	17
SCSSV	111	
GLV	50	10
Upper completion	100	
Lower completion	33	

2.3.3 Probability density function

MTTF or MTBF can be expressed in terms of probability density function (PDF) $f(t)$,

$$MTTF = \int_0^\infty t f(t) dt, \tag{2-2}$$

with

$$\int_0^\infty f(t) dt = 1. \tag{2-3}$$

The expected lifetime or probability of failure for a given time period of usage can be obtained by the respective PDF for the downhole tool of interest. Numerical models are used to evaluate PDF for specific tools.

2.4 TOOL-LIFE PREDICTION MODEL

The tool-life prediction model can be based on the underlying physical mechanism. Modeling process can be done with highly accelerated life tests and highly accelerated stress tests. Due to difference in failure model of tests as compared to field and insufficiency in the governing equation, the predicted life may be off in orders of magnitude from the actual one.

Alternatively, models can be established on field data. Three different models are introduced to evaluate the relationship between expected tool-life and stress variables: generalized log-linear (GLL), proportional hazard (PH), and cumulative damage (CD) (Kale et al., 2014). Stress variables include temperature, vibrations. Kale et al. (2014) have developed lifetime prediction models for downhole electronic tools by an iteratively reweighted maximum likelihood algorithm.

2.4.1 Generalized log-linear model

The PDF for generalized log-linear model (GLL) with a Weibull distribution is calculated by

$$f(t,\bar{x}) = \frac{\beta}{\eta(\bar{x})}\left(\frac{t}{\eta(\bar{x})}\right)^{\beta-1} e^{-\left(\frac{t}{\eta(\bar{x})}\right)^{\beta}}, \tag{2-4}$$

where t is time, β is the shape parameter, and the scale parameter $\eta(\bar{x})$ represents the stress function,

$$\eta(\bar{x}) = e^{a_0+\sum_{i=1}^{n} a_i x_i+\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} a_{i,j} x_i x_j}, \tag{2-5}$$

where a_i is the ith fitting parameter, and x_i is the ith stress variable. The PDF for an exponential distribution can be obtained by setting $\beta = 1$. For a lognormal distribution,

$$f(t,\bar{x}) = \frac{\beta}{t\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{ln(t)-\eta(\bar{x})}{\sigma}\right)^{\beta}}. \tag{2-6}$$

The stress parameters $\bar{x} = \{T, L, S, RPM, L \times T, S \times T, L \times S, S \times RPM\}$ are T for temperature, L for lateral vibration, S for stick-slip (torsional vibration), *RPM* for revolutions per minute. The coupling of two variables are also considered.

2.4.2 Proportional hazard model

In a proportional hazard model, the instantaneous hazard rate of a tool is defined,

$$\lambda(t,\bar{x}) = \frac{f(t,\bar{x})}{R(t,\bar{x})} = \lambda_0(t)\eta(\bar{x}), \tag{2-7}$$

where $f(t,\bar{x})$ is the PDF of time *t* and stress variable $\bar{x}$, $R(t,\bar{x})$ is the reliability function, $\lambda_0(t)$ is the hazard rate function of time only, and $\eta(\bar{x})$ is the stress function,

$$\eta(\bar{x}) = e^{\sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{i,j} x_i x_j}. \tag{2-8}$$

2.4.3 Cumulative damage model

Based on Miner's rule, the damage fraction is calculated as a linear damage sum at each different stress level,

$$p = \sum_{i=1}^{n} \frac{t_i}{T_{fi}}, \tag{2-9}$$

where t_i is the number of cycles accumulated at stress σ_i and T_{fi} is the time to failure at stress σ_i.

2.4.4 Downhole tool-life prediction

Carter-Journet et al. (2014a, 2014b) have applied the life prediction models (Kale et al., 2014) to downhole tools, particularly electronic tools, for optimization and decision making in drilling operations. Unlike lifetime models based on underlying physics or the test data (e.g., accelerated tests), Carter-Journet et al. developed the models based on real field data. The variables in their models include drilling hours, temperature, and vibrations. Parameters are obtained by training with field data. Distribution types are compared and the one fitting the best is selected for each tool. The developed probabilistic models are applied to reduce maintenance costs, optimizing drilling operations, improve reliability, and minimize tool failure risk (Figure 2-5, Carter-Journet et al., 2014b).

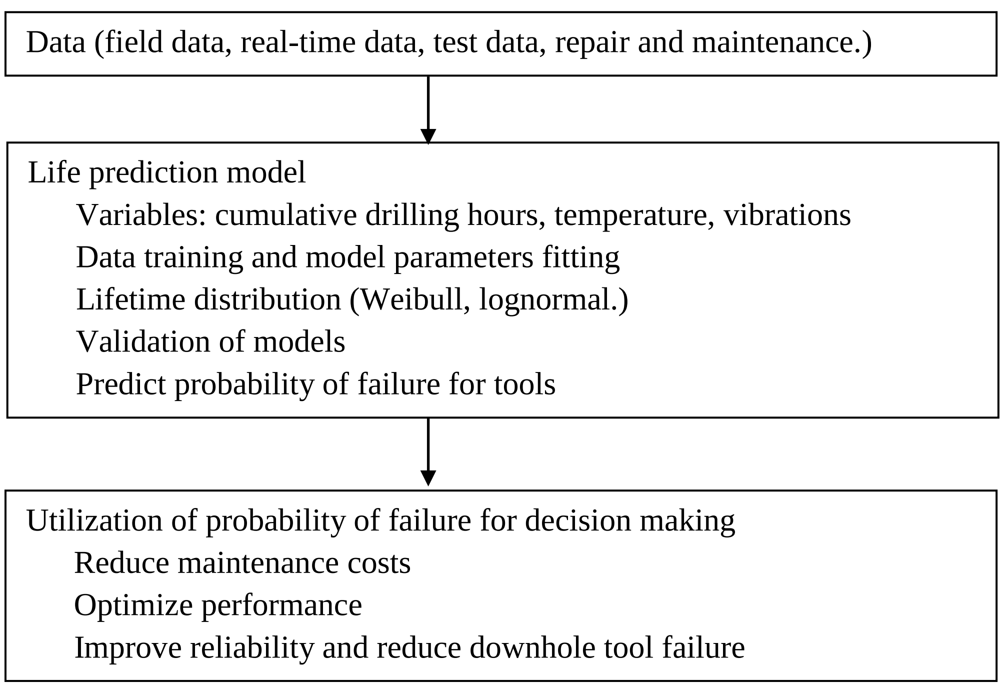

Figure 2-5 Methodology for the downhole tool-life prediction and utilization.

2.4.5 Tool component management

Carter-Journet et al. (2014a) provided an optimization approach for selecting the best possible sub-components to achieve the best overall reliability. The reliability of each tool set is calculated as

$$x_i = \prod_{j=1}^{j=n_i} \left(1 - {p_f}_i^j\right), \tag{2-10}$$

where n_i is the total number of tools and/or tool components, p_f is the probability of failure, and j is the index of tool or tool component.

For an example with two sets, the optimization problem is to maximize the sum of reliabilities subject to constraints:

$$Objective\text{: } maximize\ x_1 + x_2 = \prod_{j=1}^{j=n_i} \left(1 - {p_f}_1^j\right) + \prod_{j=1}^{j=n_i} \left(1 - {p_f}_2^j\right),$$

$$Constraints\text{: } x_1 \geq\ b_1, x_2 \geq\ b_2,$$

where b_1 and b_2 are the reliability thresholds for each tool set respectively. This optimization approach is unfortunately very limited due to the objective function. Using this approach, the swapping of tools with identical function will always resulting the maximization of reliability of the tool set with a higher initial reliability. This can be easily found in the optimization results by Carter-Journet et al. (2014a) by comparing Table 2-3 to Table 2-2. Among the components that have a different risk value between the two assets, only those with a higher risk in asset 1 as compared to asset 2 (component 5, 7, 8, 10, 17, 18, 19) are swapped.

Table 2-2 Calculated risk of tool components in two assets (Carter-Journet et al., 2014a).

Part Name Asset1	Risk of Asset 1	Part Name Asset2	Risk of Asset 2
Asset1-01	5×10^{-4}	*Asset2-01*	5×10^{-4}
Asset1-02	5×10^{-4}	*Asset2-02*	5×10^{-4}
Asset1-03	5×10^{-4}	*Asset2-03*	0.025
Asset1-04	5×10^{-4}	*Asset2-04*	5×10^{-4}
Asset1-05	0.051	*Asset2-05*	5×10^{-4}
Asset1-06	5×10^{-4}	*Asset2-06*	5×10^{-4}
Asset1-07	0.076	*Asset2-07*	0.038
Asset1-08	0.001	*Asset2-08*	5×10^{-4}
Asset1-09	5×10^{-4}	*Asset2-09*	0.029
Asset1-10	0.031	*Asset2-10*	5×10^{-4}
Asset1-11	5×10^{-4}	*Asset2-11*	5×10^{-4}
Asset1-12	5×10^{-4}	*Asset2-12*	5×10^{-4}
Asset1-13	5×10^{-4}	*Asset2-13*	0.061
Asset1-14	5×10^{-4}	*Asset2-14*	0.061
Asset1-15	5×10^{-4}	*Asset2-15*	5×10^{-4}
Asset1-16	5×10^{-4}	*Asset2-16*	5×10^{-4}
Asset1-17	0.030	*Asset2-17*	0.0287
Asset1-18	0.030	*Asset2-18*	0.0287
Asset1-19	0.030	*Asset2-19*	0.0287
System Reliability	**0.774**		**0.735**

Table 2-3 Swapping of components between two sets to maximize sum of system reliability (Carter-Journet et al., 2014a).

Part Name Asset 1	Risk of Asset 1	Part Name Asset 2	Risk of Asset 2	Comments
Asset1-01	5×10-4	*Asset2-01*	5×10-4	
Asset1-02	5×10-4	*Asset1-02*	5×10-4	
Asset1-03	5×10-4	*Asset2-03*	0.025	
Asset1-04	5×10-4	*Asset2-04*	5×10-4	
Asset2-05	5×10-4	*Asset1-05*	0.051	Swap
Asset1-06	5×10-4	*Asset2-06*	5×10-4	
Asset2-07	0.038	*Asset1-07*	0.076	Swap
Asset2-08	5×10-4	*Asset1-08*	0.00055	Swap
Asset1-09	5×10-4	*Asset2-09*	0.03	
Asset2-10	5×10-4	*Asset1-10*	0.03	Swap
Asset1-11	5×10-4	*Asset2-11*	5×10-4	
Asset1-12	5×10-4	*Asset2-12*	5×10-4	
Asset1-13	5×10-4	*Asset2-13*	0.061	
Asset1-14	5×10-4	*Asset2-14*	0.061	
Asset1-15	5×10-4	*Asset2-15*	5×10-4	
Asset1-16	5×10-4	*Asset2-16*	5×10-4	
Asset2-17	0.0287	*Asset1-17*	0.03	Swap
Asset2-18	0.0287	*Asset1-18*	0.03	Swap
Asset2-19	0.0287	*Asset1-19*	0.03	Swap
System Reliability	**0.881**		**0.646**	

Chapter 3 Well Drilling Cost

3.1 Historical Cost Analysis

Well drilling cost has been actively studied. For example, a recent cost analysis of oil and gas well drilling was given by Lukawski et al. (2014). Well drilling cost depends many factors, among which the total measured depth is the primary one (Figure 3-1). Other critical factors include geological formation, penetration rates, amount of casing strings, frequency of drilling string failures, and critical downhole drilling tool failures. Drilling is so complex that no single parameter can accurately describe the drilling cost. For example, an 11-parameter model was employed to construct a correlation between cost and field characteristics (Kaiser, 2007). As a result, the historical cost of well drilling and completion has been evaluated in the form of a composite index such as Cornell Energy Index (CEI).

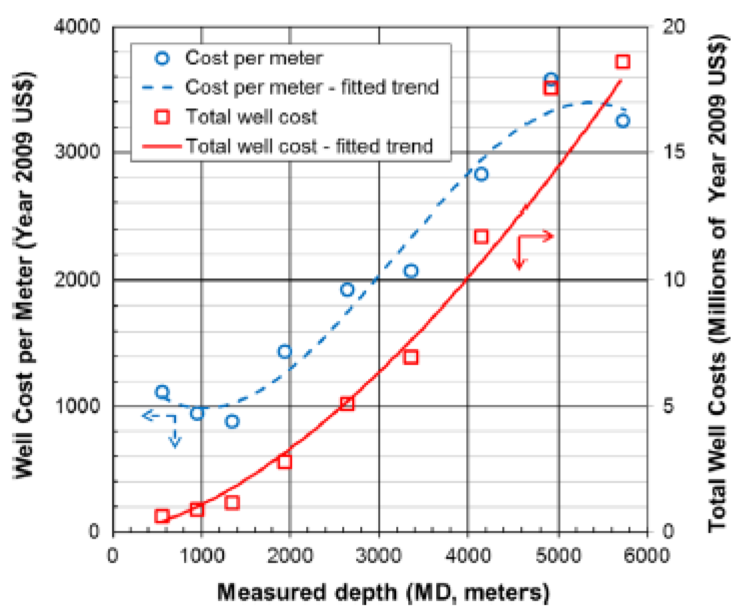

Figure 3-1 Drilling and completion costs of US onshore oil and gas wells in 2009 (Lukawski et al., 2014). Arrows indicates the respective coordinates.

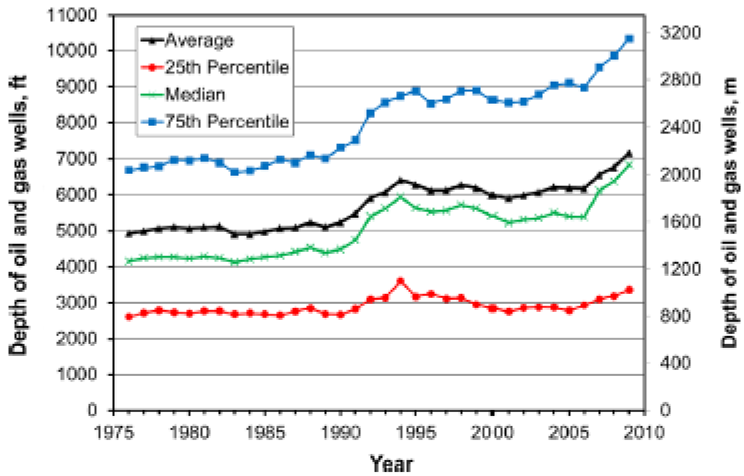

Figure 3-2 Measured depth of oil and gas wells as reported by API JAS (Lukawski et al., 2014).

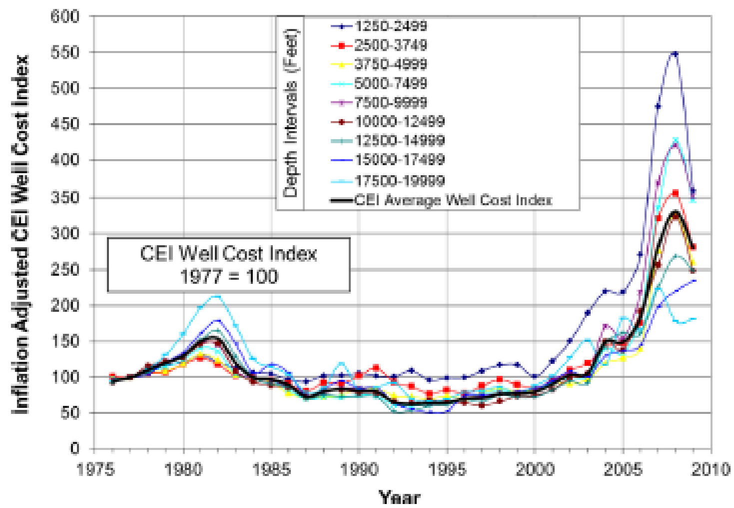

Figure 3-3 Adjusted CEI well cost index for 1975 – 2010 (Lukawski et al., 2014).

The average drilling depth has been increasing in the past years (Figure 3-2). The general trend of well cost in terms of adjusted CEI has been increasing (Figure 3-3). Consequently, it becomes of vital importance to reduce the drilling cost with a careful planning and accurate well drilling cost estimation.

3.2 Well Drilling Cost Estimation

Overall well cost includes drilling cost (or hole-making cost) and other services such as casings, mud, cementing, logging, coring, site preparation, transportation, and well completions (Azar and Samuel, 2007). To obtain an accurate estimation of drilling cost, it is important to evaluate bit run performance. Drill bits with a better performance may have a higher unit price. Overall drilling cost per unit depth should be accessed to find the best choice. The drill time can be estimated by analyzing previous bit run performance.

Cost of other services is relatively easier to estimate prior to drilling. Similar to the selection of drill bits, cost of other services may also be pertinent to the probability of downhole tool failures.

3.3 Rate of Penetration

Drilling cost can be estimated as a function of drilling time, which can be predicted by estimating rate of penetration (ROP, i.e., the depth drilled per unit time). Improving ROP reduces drill time and thus drilling cost. Investigations have been done to optimize ROP by using shuffled frog leaping algorithms (Yi et al., 2014) and ant colony optimization (Jiang and Samuel, 2016). In these studies, ROP is either obtained by the modified Warren model (Warren, 1987) or by artificial neural networks. ROP is considered as a function of bit specifics, weight on bit (W), rotation rate (N), mud flow rate (Q), and the rock properties (e.g., rock abrasiveness). Bit wear is a function of drilled depth and drilling operations.

Warren model

The roller-cone bit model for ROP calculation was initially proposed by Warren (Warren 1984) and later modified by Hareland et al. (1993) (Rampersad et al., 1994).

$$\boldsymbol{ROP} = \boldsymbol{W_f}\left[\boldsymbol{f_c(P_e)}\left(\frac{\boldsymbol{aS^2 {d_{bit}}^3}}{\boldsymbol{N\,W^2}} + \frac{\boldsymbol{b}}{\boldsymbol{N\,d_{bit}}}\right) + \frac{\boldsymbol{c\gamma\mu\,d_{bit}}}{\boldsymbol{F_{jm}}}\right]^{-1}, \tag{3-1}$$

where *a*, *b*, *c* are bit coefficients specific to the drill bit of selection, *S* is rock compressive strength, *N* is rotation rate, *W* is weight on bit, γ is mud specific density, μ is viscosity of mud, d_{bit} is the diameter of drill bit, other parameters are defined as

$$W_f = 1 - \frac{\Delta_{\mathrm{BG}}}{8}, \tag{3-2}$$

and Δ_{BG} is the change in the bit tooth wear,

$$\Delta_{\mathrm{BG}} = W_{\mathrm{c}} \sum_{i=1}^{\mathrm{n}} W_i N_i A_{abr,i} S_i, \tag{3-3}$$

where $A_{abr,i}$ is the relative abrasiveness for rock and it is a dimensionless number ranging from 0 to 1.

Confined rock compressive strength is correlated to unconfined one by

$$S = S_0\left(1 + a_s {P_e}^{b_s}\right), \tag{3-4}$$

where a_s and b_s are correlated parameters obtained by training data set.

The chip hold down function is given by

$$f_c(P_e) = c_c + a_c(P_e - 120)^{b_c}, \tag{3-5}$$

where a_c, b_c, and c_c are lithology-dependent constant, P_e is approximated by mud hydraulic static pressure for impermeable formation, F_{jm} is modified jet impact force and

$$F_{jm} = \left(1 - {A_v}^{-0.22}\right)F_j, \tag{3-6}$$

where jet impact force

$$F_j = 0.000516\,\rho Q \bar{v}_{exit}\,, \tag{3-7}$$

and

$$A_v = \frac{V_n}{V_f} = \frac{0.15{d_{bit}}^2}{3{d_n}^2}. \tag{3-8}$$

Artificial neural network

Artificial neural network (ANN) is an intelligent algorithm for the regression fitting to non-linear function. A distinct feature of ANN is that no explicit mathematical form is required. However, a training set of data is needed. Backpropagation neural network with Bayesian regularization (BR) has been successfully applied to predict oil-gas drilling cost by virtue of the powerful ability of self-learning, self-adaptive, and nonlinear mapping (Liu, et al., 2010; Zhao, et al 2011). Jiang and Samuel (2016) applied BR-NN as implemented in MATLAB® to the prediction of ROP. The input parameters are W, N, Q, depth, and gamma ray, and the output is ROP (Figure 3-4).

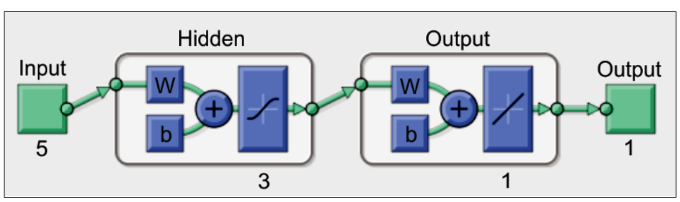

Figure 3-4. Illustration of ANN with three hidden layers (Jiang and Samuel, 2016).

Chapter 4 Game Theory

4.1 Introduction to Game Theory

4.1.1 History of game theory

The earliest record of mathematical game theory dated back in 1713. Waldegrave gave a solution now called minimax mixed strategy to the two-player version of card game le Her (the Waldegrave problem) (Bellhouse, 2007). Later, more notably, Cournot provided the strategy solution for pricing and production in the analysis of duopoly, i.e., only two producers in one market (Cournot, 1838). In Cournot's theory, each producer chooses the amount to produce in order to maximize their own profit. However, the best production output for one producer relies also on the output of the other. A Cournot equilibrium is achieved when each producer maximizes its profits given the other producer choosing rationally a production output to maximize its profits too.

In 1913, Zermelo proved that the optimal chess strategy is strictly determined (Screpanti and Zamagni, 2005). Borel suggested a formal theory of games in 1921. Seven years later, Neumann published "On the Theory of Games of Strategy", which has been viewed as the onset of game theory as a unique field (Neumann, 1928). In 1938, Borel proved a minimax theorem for two-person zero-sum matrix games with symmetric pay-off matrix. In 1944 Neumann and Morgenstern laid the fundamentals for modern game theory in the monumental book *Theory of Games and Economic Behavior*, in which the basis terminology and exemplary problems are still used now (Neumann and Morgenstern, 1944). They analyzed the special case of zero-sum games and showed that a mixed-strategy equilibrium will exist for any zero-sum game with a finite set of actions.

In 1951, Nash extended the proof to non-zero-sum non-cooperative games. He defined a mixed-strategy equilibrium for any game with a finite set of actions and proved that at least one mixed-strategy equilibrium must exist in such a game. In 1994, the Nobel Prize for economics was awarded to Harsanyi, Nash, and Selten for "for their pioneering analysis of equilibria in the theory of non-cooperative games".

In the 1950's and 1960's, game theory has been found in extensive studies and applications for war and politics. The first mathematical discussion of prisoner's dilemma appeared, and further studies were pursued for possible applications to global nuclear strategy. Since the 1970's, game theory has been widely applied in economics, sociology, psychology, evolution, and biology.

Table 4-1. History of game theory.

1713	Waldegrave: the minimax mixed strategy to the two-player version of card game le Her (the Waldegrave problem).
1838	Cournot: analysis of duopoly.
1913	Zermelo: the optimal chess strategy is strictly determined.
1921	Borel: formal game theory.
1928	Neumann: theory of Parlor games.
1938	Borel: minimax theorem for two-person zero-sum matrix games with symmetric pay-off matrix.
1944	Neumann and Morgenstern: *Theory of Games and Economic Behavior* strategy equilibrium for zero sum game.
1951	Nash: strategy equilibrium (Nash Equilibrium) for non-zero sum game.
1994	Harsanyi, Nash, and Selten: Nobel prize.

4.1.2 Terminology and definition

Player, Information, Action, and Payoff

A game model consists of four critical components: players, the information each player knows at each decision point, actions each player can take, and payoffs to each player for each outcome (Rasmusen, 2007). A game typically involves more than one player. When there is only one player, the game problem turns into a decision problem, or frequently, an optimization problem, i.e., maximization or minimization of the target value of the object function. When there are more than two players, the complexity of game theory increases substantially. In this study, we focuses on games with two players and investigate the use of a two-player game model for downhole tool problems. Action is also called strategy and these two terms are exchangeable in this thesis.

Cooperative vs Non-cooperative Games

Players may form coalition or work cooperatively with each other and the study of these games are called cooperative game theory. In contrast, non-cooperative games are those where each player take actions to their best interest independently.

Rational Player

One key assumption of game theory is that players are all rational, i.e., they will always choose an action that gives the maximal payoff. Non-cooperative games with rational players are to be considered in this study.

Zero-sum vs Non Zero-sum Games

Zero-sum games are those where one player's payoff is at the expense of other players and the net sum of payoffs to all players is zero. When the payoffs do not add up to zero, the game is called non-zero-sum or general-sum games.

Strategic vs Extensive Forms

There are two typical representations of games. Strategic form lists each player's strategies (actions) and payoffs resulting from their actions. Strategic form is often used for simultaneous actions. In the case of two-player games, strategic form can be written as a bi-matrix. Each matrix contains the values of payoff (also called utility) for each player respectively. Matrix rows corresponds to different actions by player one and columns are for different actions of player two. Extensive form is a game tree, where each node represents a state of the game and each edge corresponds to one player's action. Extensive form is appropriate for games where players are take a series of actions one after another.

Dominating, Pure and Mixed Strategy

A strategy dominates other strategies of a player if it always provides a better payoff to that player, regardless what strategies other players choose. It is called weakly dominating strategies if the strategy is always at least as good. If each player has a dominating strategy, the dominating strategies form a pure strategy equilibrium for the game. However, a pure strategy equilibrium may not exist for many games. A mixed strategy is that a player takes a random strategy with a given probability. A pure strategy can be viewed as a mixed strategy with a probability of one.

Nash Equilibrium

A Nash equilibrium is a combination of strategies that each player cannot increase his payoff by unilaterally changing his strategy. Nash (1951) proved that at least one mixed-strategy equilibrium must exist in a game with a finite set of actions. Numerous algorithms have been developed to find Nash equilibrium for games (Savani and Stengel, 2014).

4.1.3 Examples

The Oil Producer's Dilemma

One of the most known game examples is the prisoner's dilemma. We start with an analogous one regarding oil producers (Bratvold and Koch, 2011) and will discuss the original prisoner's dilemma in the next section. Let us suppose there are two oil producing countries that dominate the oil production in the world. This is a simplified but reasonable assumption since the oil producing countries can be divided into OPEC and non-OPEC countries. The oil price is dependent on the total production of the two countries (Table 4-2). Each country will produce to maximize oil revenues. Table 4-3 shows the outcomes with different production strategies. In the parenthesis, the first value is the payoff to Country A and the second is the payoff to Country B. When the two countries produce only 10 MMbbl per day each, there are only 20 MMbbl oil supply per day on market and due to the high demand, the oil price would be $120 per bbl, and each will receive $1.2 billion as revenue. However, if one maintains a production of 10 MMbbl per day while the other increases its production to 20 MMbbl per day, the total supply is now 30 MMbbl per day, which drives down the oil price to $75 per bbl. The one with increased production will get $1.5 billion as revenue, a boost from $1.2 billion; while the one maintaining production gets a less amount of only $750 million. If both countries produce 20 MMbbl per day each, the oil is so over-supplied that the price drops to $40, and each will receive only $800 million as revenue.

Table 4-2 The correlation of oil supply and oil price.

Oil supply (per day)	20 MMbbl	30 MMbbl	40 MMbbl
Oil price (per bbl)	\$120	\$75	\$40

Table 4-3 Payoff table for two oil producing countries.

US\$ in millions, production per day		Country B	
		10 MMbbl	20 MMbbl
Country A	10 MMbbl	(\$1200, \$1200)	(\$750, \$1500)
	20 MMbbl	(\$1500, \$750)	**(\$800, \$800)**

Intuitively, it seems that both countries would like to enter an agreement to produce 10 MMbbl per day, to maintain a relatively revenue. This may happen in *cooperative* games, and it was the aim of OPEC to maintain oil price at a high level by setting a production limit to each member country. However, the reality is that each country would like to produce to maximize its revenue and act in a *non-cooperative* manner. Which production is better for Country A depends on how much Country B produce. If Country B produces 10 MMbbl per day, Country A will choose to produce 20 MMbbl per day as the revenue would be \$500 million more than a production of 10 MMbbl per day. If Country B produces 20 MMbbl per day, Country A will also choose to produce 20 MMbbl per day as the revenue would be \$50 million more than a production of 10 MMbbl per day. Similarly, Country B will also choose to produce 20 MMbbl as the dominating strategy.

In history, Saudi Arabia, the largest producer in OPEC, had followed the agreement and cut the production. However, other members in OPEC broke the agreement and increased the production, and Saudi Arabia lose its market share. This explained why, in

2014, when oil price started to crash, Saudi Arabia, along with all other oil producers, tried to increase productions to defend their market shares.

Prisoner's Dilemma

Two prisoners are held suspect of a certain crime. However, there is no judicial evidence for this crime unless one of the prisoners would testify against the other. If only one prisoner testifies, the one who testifies will be rewarded with immunity of prosecution, which is indicated as payoff 3. At the same time, the other prisoner will serve a long prison sentence (payoff 0). When both testify, each will receive a less severe sentence (payoff 1). When none testifies, each will be prosecuted with a minor charge (payoff 2). Table 4-4 gives payoffs to each prisoner depending on if they would "keep silent" or "testify". In the Prisoner's Dilemma game, "testify" is the dominating strategy. The analysis is similar to the above one for Oil Producer's Dilemma. If Prisoner B keeps silent, Prisoner A will choose to testify to go off without prosecution (payoff 3) instead of a minor charge. If Prisoner B testify, Prisoner A will choose to testify too as punishment is less (payoff 1) comparing to the long prison sentence by keeping silent (payoff 0). Similarly, Country B will also choose to testify as the dominating strategy.

Table 4-4 Payoff table for two prisoners.

		Prisoner B	
		keep silent	testify
Prisoner A	keep silent	(2, 2)	(0, 3)
	testify	(3, 0)	**(1, 1)**

Coin Flipping: Head or Tail?

During a coil flipping game, one player will flip a coin while the other player guess which side is face up. In this case, the player flipping the coin does not have a choice of strategy, and the probability of head up is the same as tail up, each 50%. We may modify the game and suppose that Player A will put a coin on the table and cover it, Player B, who does not see the coin, guesses which side is up. The payoff is given in Table 4-5. If Player B makes a right guess, Player A loses (payoff -1) while Player B is rewarded by 1. Otherwise, if Player B makes a wrong guess, Player B's payoff is -1 and Player A is rewarded by 1. This is an example of zero-sum game since one player's gain is at the expense of the other player. Consequently, it is sufficient to have only Player A's payoff in the strategy form table (

Table 4-6). There is no dominating strategy for either player in a coin flipping game. The Nash equilibrium is a mixed strategy, 50% head up and 50% tail up for each player. More details about Nash equilibrium will be provided in following sections.

Table 4-5 Payoff table for the flipping coin game with two players.

		Player B	
		head	tail
Player A	head	(-1, 1)	(1, -1)
	tail	(1, -1)	(-1, 1)

Table 4-6 Payoff table for zero-sum flipping coin game.

		Player B	
		head	tail
Player A	head	-1	1
	tail	1	-1

Rock Paper Scissors

The payoff table for a Rock Paper Scissors game is given in Table 4-7. Since it is a zero-sum game, only payoff to Player A is given. Again, there is no dominating strategy for either player. The Nash Equilibrium is a mixed strategy, 1/3 Rock, 1/3 Paper and 1/3 Scissors for each player.

Table 4-7 Payoff for rock paper scissors game.

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0	-1	1
	Paper	1	0	-1
	Scissors	-1	1	0

4.2 Nash Equilibria Calculations

4.2.1 Minimax Theorem

The minimax theorem was first formally proposed by Neumann (1928). The name minimax arises because each player minimizes the maximum payoff possible for the other in zero-sum games. This is equivalent to each player maximize his/her own minimum payoff (i.e., minimize his/her maximum loss). In game theory, more precisely, zero-sum game theory, the minimax theorem indicates that the minimax solution of a zero-sum game is the strategy equilibrium (i.e., Nash equilibrium) when the minimal gain for player one is the same as the maximum loss for player two in the case of a two-player game. Neumann's minimax theorem is stated as below.

Let $\boldsymbol{X} \subset \mathbb{R}^n$ and $\boldsymbol{Y} \subset \mathbb{R}^m$ be compact convex sets. If $f: \boldsymbol{X} \times \boldsymbol{Y} \to \mathbb{R}$ is a continuous function that is convex-concave, i.e.,

$f(\cdot, y): \boldsymbol{X} \to \mathbb{R}$ is convex for fixed y, and

$f(x, \cdot): \boldsymbol{Y} \to \mathbb{R}$ is concave for fixed x.

Then we have that

$$\min_{x \in \boldsymbol{X}} \max_{y \in \boldsymbol{Y}} f(x, y) = \max_{y \in \boldsymbol{Y}} \min_{x \in \boldsymbol{X}} f(x, y). \tag{4-1}$$

4.2.2 Pure Strategy Equilibrium

When the Nash Equilibrium is a pure strategy, the equilibrium can be obtained by the minimax theorem. The solution is illustrated by one following example of two-player zero sum game. The strategy form is given in Table 4-8.

Table 4-8 Payoff for a two-player zero sum game.

		Player B			
		B1	B2	B3	B4
Player A	A1	7	2	5	**1**
	A2	**2**	**2**	3	4
	A3	5	$\boxed{3}$	4	4
	A4	3	2	**1**	6

As we can find from Table 4-8, the minimum payoffs for Player A are 1, 2, 3 and 1 when Player A plays A1, A2, A3 and A4 respectively. Among the three possible payoff, Player A would choose action A3 to maximize the payoff.

Table 4-9 Payoff for a two-player zero sum game.

		Player B			
		B1	**B2**	B3	B4
Player A	A1	**7**	2	**5**	1
	A2	2	2	3	4
	A3	5	$\boxed{\underline{\mathbf{3}}}$	4	4
	A4	3	2	1	**6**

As we can find from Table 4-9, the minimum payoffs for Player B are -7, -3, -5 and -6 when Player B plays B1, B2, B3 and B4 respectively. Among the three possible payoff, Player B would choose action B2 to maximize the payoff.

Table 4-10 Payoff for a two-player zero sum game.

		Player B			
		B1	**B2**	B3	B4
Player A	A1	**7**	2	5	1
	A2	2	2	3	4
	A3	5	$\boxed{\underline{\mathbf{3}}}$	4	4
	A4	3	2	1	6

Finally, the minimax solution to this game is A3 and B2. And since the payoff 3 is the maximum payoff to player A given B2, and maximum payoff to player B given A3, in other words, either player does not gain more by changing his/her own strategy unilaterally. This minimax solution is a Nash equilibrium.

4.2.3 Mixed Strategy Equilibria

Here is another example that shows that minimax solution of a pure strategy is not a Nash equilibrium.

Table 4-11 Payoff for a two-player zero sum game.

		Player B		
		B1	B2	B3
Player A	A1	3	**-2**	2
	A2	**-1**	0	4
	A3	**-4**	-3	1

As we can find from Table 4-11, the minimum payoffs for Player A are -2, -1, and -4 when Player A plays A1, A2, A3 respectively. Among the three possible payoffs, Player A would choose action A2 to maximize its payoff.

Table 4-12 Payoff for a two-player zero sum game.

		Player B		
		B1	**B2**	B3
Player A	A1	**3**	-2	2
	A2	-1	$\boxed{\underline{\mathbf{0}}}$	**4**
	A3	-4	-3	1

As we can find from

Table 4-12, the minimum payoffs for Player B are -3, 0, and -4 when Player B plays B1, B2, B3 respectively. Among the three possible payoffs, Player B would choose action B2 to maximize its payoff.

Table 4-13 Payoff for a two-player zero sum game.

		Player B		
		B1	**B2**	B3
Player A	A1	3	-2	2
	A2	-1	$\boxed{\mathbf{0}}$	4
	A3	-4	-3	1

The minimax solution is A2 and B2. However, this is not a stable solution. For example, if Player B knows Player A will choose A2, then Player B will change to B1 to gain 1; then if Player A realizes Player B will choose B1, Player A will choose A1 to gain 3; and then Player B will choose B2; and then Player A will choose A2. Eventually both players will realize the difficulty of making a choice. As a result, a pure strategy equilibrium does not exist for this game.

As discussed above, the minimax approach does not give a stable solution when a pure strategy equilibrium does not exist. A simple examples is presented in the following section to illustrate the methods to solve for a mixed strategy equilibrium.

2 × 2 Game Mixed Strategy Equilibrium

Zero-sum games are relatively easier to solve as compared to non-zero sum games. Again two-player games are used to illustrate the algorithm to solve for Nash equilibrium.

Table 4-14 Payoff for a two-player zero-sum 2 × 2 game.

		Player B	
		B1	B2
Player A	A1	a	b
	A2	c	d

For a zero-sum game with a 2 × 2 payoff matrix, the probability for the mixed strategy equilibrium (i.e., Nash equilibrium) can be obtained by:

A1: $(d-c)/(a-b-c+d)$

A2: $(a-b)/(a-b-c+d)$

B1: $(d-b)/(a-b-c+d)$

B2: $(a-c)/(a-b-c+d)$

Using the above formula, it can be found that for the coin flipping game as mentioned in section 4.1.3, the probability of head and tail is 0.5 respectively for both players. For games with a larger than 2 × 2 payoff matrix, many algorithms were suggested for solutions. One way is to reduce the payoff matrix to a 2 × 2 one by eliminating dominating rows and/or columns (Williams 1966). However, this approach may not work for an arbitrary large payoff matrix.

4.3 Algorithms for Two-Player Games

A lot of numerical algorithms have been developed to solve complicated games. The computing complexity increases substantially with the number of players. Here our discussion is limited to two-player games only. First we will introduce the generally accept method, namely Simplex, for two-player zero-sum games, then Game Theory Explorer for two-player non-zero-sum games, which will be used for our case studies, is discussed.

4.3.1 Zero-Sum Games

Two-player zero-sum game problem can be transformed into a linear programming problem and then solve it by the Simplex algorithm.

Linear Programming

Consider an arbitrary finite two-player zero-sum game with m × n payoff matrix $\boldsymbol{A}$. The mixed strategies of players A and B are denoted respectively by

$X = \{ \boldsymbol{p} = (p_1, \cdots, p_m)^T : p_i \geq 0 \text{ for } i = 1, \cdots, m \text{ and } \sum_{i=1}^{m} p_i = 1\},$

$Y = \{\boldsymbol{q} = (q_1, \cdots, q_m)^T, q_j \geq 0 \text{ for } j = 1, \cdots, n \text{ and } \sum_{j=1}^{n} q_j = 1\}.$

If player A uses strategy $\boldsymbol{p}$, and player B uses strategy $\boldsymbol{q}$, the average payoff for Player A will be

$$\sum_{i=1}^{m} \sum_{j=1}^{n} p_i a_{ij} q_j = \boldsymbol{p^T A q}. \tag{4-2}$$

By the minimax theorem, the value of the game is defined by

$$V = \min_{q \in Y} \max_{p \in X} \boldsymbol{p^T A q} = \max_{p \in X} \min_{q \in Y} \boldsymbol{p^T A q}. \tag{4-3}$$

Note the invariance of the minimax strategies under change of location and scale for the payoff matrix, i.e., the minimax strategies are the same for the game having matrix

$\boldsymbol{A} = (a_{ij})$ and the game having matrix $\boldsymbol{A}' = (a'_{ij})$ with $a'_{ij} = ca_{ij} + b$ where $c > 0$. The respective game value $V' = cV + b$.

Let us consider player A first. Player A wants to choose $\boldsymbol{p}$ to maximize his payoff,

Maximize $\min\limits_{1 \le j \le n} \sum_{i=1}^{m} p_i a_{ij}$

subject to the constraints

$$p_i \ge 0 \text{ for } i = 1, \cdots, m \text{ and } \sum_{i=1}^{m} p_i = 1. \tag{4-4}$$

This nonlinear objective function can be transformed into a linear program by introducing a new variable,

$$v \le \min_{1 \le j \le n} \sum_{i=1}^{m} p_i a_{ij} \tag{4-5}$$

And the problem now becomes: choose *v* and $\boldsymbol{p}$ to

maximize *v*,

subject to the constraints

$v \le \sum_{i=1}^{m} p_i a_{i1}$

$\vdots$

$v \le \sum_{i=1}^{m} p_i a_{in}$

$\sum_{i=1}^{m} p_i = 1$

$$p_i \ge 0 \text{ for } i = 1, \cdots, m. \tag{4-6}$$

Similarly, the linear program can be done for Player B, choose w and q to

minimize w,

subject to the constraints

$w \geq \sum_{j=1}^{n} a_{1j} q_j$

$\vdots$

$w \geq \sum_{j=1}^{n} a_{mj} q_j$

$\sum_{j=1}^{n} q_j = 1$

$q_j \geq 0$ for $j = 1, \cdots, n$. (4-7)

The duality theorem in linear programming implies the minimax theorem. The above linear program can be further simplified. Suppose $v > 0$ and let $x_i = p_i / v$ (Note: in case $v \leq 0$, the payoff matrix can be shifted by a positive number according to the invariance of mixed strategies as mentioned above.), then

$\sum_{i=1}^{m} p_i = 1$ can be transformed into $\sum_{i=1}^{m} x_i = 1/v$.

The original linear program for player A now becomes

minimize $\sum_{i=1}^{m} x_i$,

subject to the constraints

$1 \leq \sum_{i=1}^{m} x_i a_{i1}$

$\vdots$

$1 \leq \sum_{i=1}^{m} x_i a_{in}$

$x_i \geq 0$ for $i = 1, \cdots, m$. (4-8)

The game value for the original problem is $v = 1/\sum_{i=1}^{m} x_i$ and the optimal strategy for player A is $p_i = v x_i$ for $i = 1, \cdots, m$.

For player B, let $y_j = q_j/w$ and the problem becomes

maximize $\sum_{j=1}^{n} y_j$,

subject to the constraints

$1 \geq \sum_{j=1}^{n} a_{1j}y_j$

$\vdots$

$1 \geq \sum_{j=1}^{n} a_{mj}y_j$

$y_j \geq 0$ for $j = 1, \cdots, n$. (4-9)

Simplex

The Simplex algorithm by Williams (1966), called the pivot method, is introduced below.

Step 1: Matrix ***A*** is obtained by adding a constant to the payoff matrix so that all elements are non-negative.

Step 2: Augment the matrix ***A*** by appending one more row of -1, one more column of 1, and zero to the additional diagonal element. Add player A's strategies on the left of the first column, and player B's strategies on the top of the first row. See below for an example:

Table 4-15. Augmented payoff matrix.

	y_1	y_2	$\cdots$	y_3	
x_1	a_{11}	a_{11}	$\cdots$	a_{11}	1
x_2	a_{11}	a_{11}	$\cdots$	a_{11}	1
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
x_3	a_{11}	a_{11}	$\cdots$	a_{11}	1
	-1	-1	$\cdots$	-1	

Step 3: Select the pivot $a(p,q)$ with the following properties,

i) $a(m+1,q) < 0$;

ii) $a(p,q) > 0$;

iii) The pivot row p must be chosen to give the smallest of the ratios $a(p, n + 1)/a(p, q)$ among all positive pivots for that column.

Step 4: Pivot the matrix as follows,

i) Replace each entry, $a(i, j)$, not in the row or column of the pivot by $a(i, j) - a(p, j) \cdot a(i, q)/a(p, q)$.

ii) Replace each entry in the pivot row, except for the pivot, by its value divided by the pivot value;

iii) Replace each entry in the pivot column, except for the pivot, by the negative of its value divided by the pivot value

iv) Replace the pivot value by its reciprocal.

Symbolically, the pivot operation can be described by

p	r
c	q

$\longrightarrow$

$1/p$	r/p
$-c/p$	$q - (rc/p)$

Step 5: Exchange the label on the left of the pivot row with the label on the top of the pivot column.

Step 6: If there are any negative numbers remaining in the lower border row, go back to Step 3.

Step 7: Otherwise, a solution is obtained:

i) The game value (the average payoff of player A) v is the reciprocal of the number in the lower right corner. If any constant has been added to the payoff matrix in Step 1, the same value should be subtracted here.

ii) Player A's optimal strategy: Those variables of player A that end up on the left side receive probability zero. Those that end up on the top receive the value of the bottom entry in the same column divided by the lower right corner.

iii) Player B's optimal strategy: Those variables of Player B that end up on the top receive probability zero. Those that end on the left receive the value of the right edge in the same row divided by the lower right corner.

4.3.2 Non-zero-Sum Games

Many game theory algorithms for two-player non-zero-sum games have been developed, for example, Gambit (McKelvey, McLennan, and Turocy, 2010), GamePlan (Langlois, 2015) and XGame (Belhaiza, Mve, and Audet, 2010), as well as Game Theory Explorer (GTE) by Savani and Stengel (2015). Particularly, GTE is one robust and reliable algorithm with the open source code available to public. Our program is based on the source codes of GTE.

Game Theory Explorer

GTE can be accessed via the website http://www.gamethoeryexplorer.org. Users do not have to download the source codes or install them on a local computer. The graphical user interface (GUI) makes it readily available to any users who are not familiar with computer techniques. Both strategic and extensive forms can be created via GUI.

There are also some restriction on GTE via the web GUI. The complexity of the game to be solved is limited. The equilibrium computation is often intensive and the computation capability may be constrained as compared to a local installation. It may also be inconvenient if any user want to solve a series of games. Consequently, we adapted the source codes and implemented a local version with an Excel interface.

4.4 GAME THEORY IMPLEMENTATION

The two-player zero-sum game can be solved by the Simplex algorithm as highlighted in Section 4.3.1. One example of the computer implementation in C++ can be found on http://marioslapseofreason.blogspot.com/2010/12/finding-nash-equilibrium-in-two-person.html. A copy of the source code with modification is given in Appendix I.

However, as we explained earlier, our target problem is to find the optimal downhole tool failure with minimal drilling cost, and the expected payoffs for the two fictitious players do not sum to zero. As a result, we implemented the computer program for two-player non-zero-sum games based on GTE. An excel interface was created for our case study.

On the master sheet "Master" (Figure 4-1), there are four buttons:

- Click "Load Input Data" to read the input data from a text file to the excel sheet "Input";
- Click "Generate Input File" to generate input files for the game solver;
- Click "Calculate Nash Equilibrium" to compute Nash Equilibria;
- Click "Show Results" to extract solution to excel sheet "Output".

Note “Load Input Data” is not necessary if the payoff table is already given in the excel sheet “Input”.

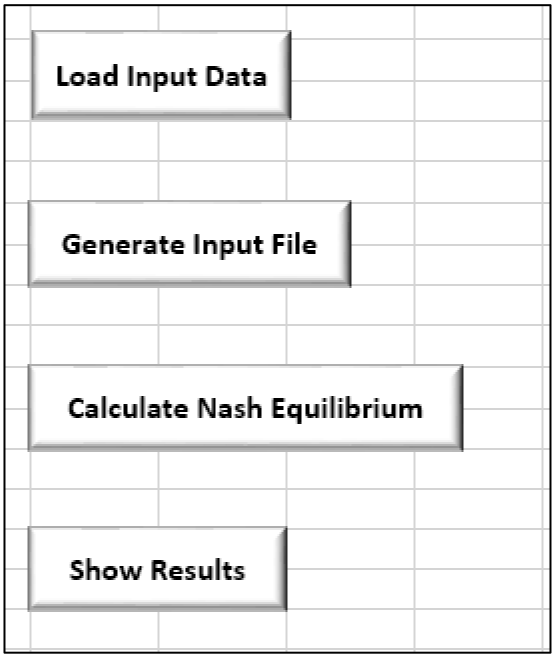

Figure 4-1 The master sheet interface for the game solver.

If the payoff table is given in a plain ACSII text file, the data should be given in the following format (Figure 4-2):

First line: the number of rows and the number of columns separated by a space.

Second line: blank

Payoff table for player I.

A blank line after the payoff table for player I.

Payoff table for player II.

All numbers must be given in the format of integer or fractions.

```
4 5

48        50        44        45        43
55        62        53        69        67
64        58        58        64        62
72        60        56        59        58

942/1000        940/1000        920/1000        907/1000        879/1000
947/1000        944/1000        925/1000        930/1000        918/1000
938/1000        922/1000        928/1000        894/1000        895/1000
906/1000        880/1000        866/1000        871/1000        866/1000
```

Figure 4-2 Payoff table for players I and II in plain text format.

Payoff for player I

					player II		strategy		
		1	2	3	4	5	6	7	8
	1	48	50	44	45	43			
	2	55	62	53	69	67			
	3	64	58	58	64	62			
	4	72	60	56	59	58			
player I	5								
	6								
strategy	7								
	8								

Figure 4-3 Payoff table for player I in sheet "Input".

Payoff for player II

					player II		strategy		
		1	2	3	4	5	6	7	8
	1	0.942	0.94	0.92	0.907	0.879			
	2	0.947	0.944	0.925	0.93	0.918			
	3	0.938	0.922	0.928	0.894	0.895			
	4	0.906	0.88	0.866	0.871	0.866			
player I	5								
	6								
strategy	7								
	8								

Figure 4-4 Payoff table for player II in sheet "Input".

Once the computation is done and by clicking “Show Results”, the optimal mixed strategies for the two players are given in sheet “Output”.

Nash Equilibria:			1					
		Payoff		Strategy				
Player 1 (Row):		72.0000		0.000	0.000	0.000	1.000	
Player 2 (Column):		0.9060		1.000	0.000	0.000	0.000	0.000

Figure 4-5 An example of the results of the payoff and optimal strategies for each player.

Chapter 5 Game Theory Model and Case Study

5.1 Game Theory Model

5.1.1 Decision analysis model

A risk model predicts the probability of tool failure. If the cost of replacing a tool with high risk is known and the cost of the tool failure can be estimated, the risk criteria can be rationally determined by decision analysis. Suppose a tool component replacement cost is C^j and the incurred cost is C if a tool fails, the payoff table is generated (Table 5-1). The game model can be viewed as one player games. The replacement is economically practical only if the incremental payoff is non-negative.

$$C\left(\frac{x}{1-p_f{}^j} - x\right) - C^j \geq 0\ . \tag{5-1}$$

And thus the risk criteria to replace the tool component can be established by

$$p_f{}^j \geq \frac{C^j}{C^j + xC}\ . \tag{5-2}$$

Table 5-1 The Payoff table for tool assets with and without tool component replacement.

<table>
<tr><td rowspan="2"></td><td colspan="2">No failure</td><td colspan="2">Failure</td></tr>
<tr><td>Payoff</td><td>Probability</td><td>Payoff</td><td>Probability</td></tr>
<tr><td>No replacement</td><td>0</td><td>x</td><td>$-C$</td><td>$1-x$</td></tr>
<tr><td rowspan="2">Replacement</td><td>0</td><td>$\frac{x}{1-p_f{}^j}$</td><td>$-C$</td><td>$1-\frac{x}{1-p_f{}^j}$</td></tr>
<tr><td colspan="4">$-C^j$</td></tr>
</table>

5.1.2 Two player non-zero-sum game model

When minimizing the downhole tool failure is not necessarily improving drilling time, a two-player game theory model is an appropriate choice to simulate the outcome. When proper probabilistic lifetime models are available for each downhole tool, it is possible to the tool manufacturer (or provider) to evaluate the risk levels for different combination of tools by swapping tools, maintaining services, and replacing high risk tools. The risk of a subsequent drill run can be further evaluated with different operation parameters, and the risk threshold can serve as a constraint on the operation parameters. At the same time, the tool user estimates drilling time with these different sets of operation parameters. A payoff table can be constructed with manufacturer minimize risks for downhole tool failures while user maximize drilling distance per unit time. The payoff could also be equivalently expressed as overall reliability for downhole tools for manufacturer and average ROP for user.

Table 5-2 Two-player non-zero-sum game for downhole tool failure.

Player A	Manufacturer
Strategy	Downhole tool maintenance
Payoff	Overall reliability of downhole tools (Based on the risk model of each tool as a function of cumulative drill hours, temperature, lateral and torsional vibration.)
Player B	User
Strategy	Drilling operations
Payoff	Drilling distance per unit time (or ROP) (As a function of WOB, RPM, Q, depth, and rock properties)

In general, the workflow chart can be summarized as follows:

1. Identify two players, i.e., entities with conflicting interest.
2. Find the strategies for each player.
3. Obtain the payoff table with player A's strategy in rows and player B's strategy in columns. The payoff of player A should be listed in the first table, and that of Player B be listed in the second table.
4. Use GTE to solve the two-player non-zero-sum game to find the Nash equilibrium.
5. Analysis of the Nash equilibrium and locate the strategy that minimize downhole tool failure.

The payoff table can be generated for the planned next drilling run. Play A's strategies are different selections of tools, different combinations of tool components, and maintenance. Player B's strategies are different choices of drilling operations. For each combination of the two players' strategies, the overall reliability and average ROP are calculated, respectively. The overall reliability is obtained from the probability of failure for each downhole tool (Kale et al., 2014) as a function of cumulative drilling hours, temperature, lateral and torsional vibrations, RPM. Average ROP can be calculated either as a simple function of RPM (Carter-Journet et al., 2014a) or by using a sophisticated model (see Section 3.3) of WOB, RPM, Q, depth, and rock properties. With the payoff tables, the game theory solver GTE is used to find the Nash equilibrium, which might be helpful for the optimal tool risk management and drilling operations.

5.2 CASE STUDY

5.2.1 Optimization constraints

Decision analysis can be extended to find optimization constraints. Swapping tool components (see section 2.4.5) can be evaluated with the consideration of cost. Suppose two assets will be used next for drilling and if tool failure occurs, a cost of C_1 and C_2 will be incurred respectively. The cost of swapping is C_S. The payoff table is given in Table 5-3. The incremental payoff with swapping can be obtained by,

$$C_1(x_{s1} - x_1) + C_2(x_{s2} - x_2) - C_S,$$

where x is the reliability of a tool set.

For the example (reproduced in Table 2-2 and Table 2-3) given by Carter–Journet et al. (2014b), the cost of swapping tool components must meet the constraint,

$$C_S \leq 0.107C_1 - 0.089C_2. \quad (5\text{-}3)$$

Table 5-3 The Payoff table for tool assets with and without swapping components.

Swapping		No failure		Failure	
		Payoff	Probability	Payoff	Probability
No	Asset 1	0	x_1	$-C_1$	$1 - x_1$
	Asset 2	0	x_2	$-C_2$	$1 - x_2$
Yes	Asset 1	0	x_{s1}	$-C_1$	$1 - x_{s1}$
	Asset 2	0	x_{s2}	$-C_2$	$1 - x_{s2}$
	Swapping	$-C_S$			

Another strategy is the choice of vibration suppression tools, which mitigate the vibrations and reduce the tool failure risk. At the same time, the vibration control device has a cost. Note the vibration reduction is also beneficial to ROP, but only the effects on downhole tool failure risk is discussed here. By adding a vibration control device with a cost of C_R and an improvement of reliability to x_R, the payoff table is given in Table 5-4. The improvement in reliability must meet the condition,

$$x_R \geq \frac{C_R}{C} + x, \tag{5-4}$$

or the cost of control device should be

$$C_R \leq (x_R - x)C. \tag{5-5}$$

Table 5-4 The Payoff table for tool assets with and without vibration control device.

	No failure		Failure	
	Payoff	Probability	Payoff	Probability
No vibration control	0	x	$-C$	$1 - x$
Vibration control	0	x_R	$-C$	$1 - x_R$
	$-C_R$			

5.2.2 Game theory optimization

The decision analysis above helps to identify the risk criteria, optimize maintenance schedule, and determine the range of operation parameters. When risk criteria are met and there are different strategies at disposal, it can be helpful to include the drilling cost (e.g. in terms of drilling time or ROP) to find the best tool strategy and optimize the operation parameters.

Based on the work by Carter–Journet et al. (2014a), payoff tables for the drilling cost in terms of distance drilled per hour and the reliability of downhole tools are estimated in Table 5-5 and Table 5-6, respectively. The reliability of downhole tools is changing with the drill hours. However, the distance drilled per hour (i.e., ROP) is constant at a given RPM.

Table 5-5 Drilling distance per hour (ROP).

		Downhole tool set selection				
		B1	B2	B3	B4	B5
RPM	50	43	43	43	43	43
	60	51	51	51	51	51
	70	59	59	59	59	59
	80	**70**	70	70	70	70

Table 5-6 Reliability of downhole tools.

		Downhole tool set selection				
		B1	B2	B3	B4	B5
RPM	50	0.73	0.70	0.65	0.58	0.49
	60	0.72	0.65	0.60	0.53	0.44
	70	0.60	0.54	0.48	0.41	0.36
	80	**0.32**	0.28	0.25	0.24	0.22

By using the GTE game theory solver (see Section 4.4 for details) for two-player non-zero-sum games, two sets of solutions are found. One set of solution is that ROP is maximized to 70 ft/hr. The optimal reliability of downhole tools is 0.32. This reflects that the drilling operation team is prone to maximize ROP, regardless the downhole tool failure risk.

In reality, ROP may not be monotonically increasing with the drilling operation parameters. And even though the tool failure risk is lower at a smaller RPM (or smaller ROP), the additional hours of operation for the same total drilled distance would be longer, which increase the tool failure risk as a result of a longer usage. To simulate a complicated case, five different downhole tool strategies were considered. The drilling distance per unit time or ROP is listed in Table 5-7. The overall reliability of the tool sets is given in Table 5-8.

Table 5-7 Drilling distance per unit time, ROP.

		Downhole tool set selection				
		B1	B2	B3	B4	B5
RPM	50	43	50	44	45	43
	60	51	60	53	69	67
	70	**59**	**65**	58	64	62
	80	**60**	**62**	56	59	58

Table 5-8 Reliability of downhole tools.

		Downhole tool set selection				
		B1	B2	B3	B4	B5
RPM	50	0.954	0.940	0.920	0.907	0.879
	60	0.947	0.935	0.925	0.930	0.918
	70	**0.938**	**0.922**	0.928	0.894	0.895
	80	**0.906**	**0.912**	0.866	0.871	0.866

By using the GTE game theory solver (see Section 4.4 for the procedure) for two-player non-zero-sum games, it is found that the Nash equilibrium solution is a mixed strategy.

Table 5-9 Nash Equilibrium solution for downhole tool failure.

RPM selection	50	60	70	80		Payoff
	0.00%	0.00%	27.3%	72.7%		60.5
Downhole tool set selection	B1	B2	B3	B4	B5	
	75%	25%	0%	0%	0%	0.9147

The mixed strategy suggest optimal drilling operations with 27.3% of time at 70 RPM and 72.7% 80 RPM. Since our simulation use discrete RPM values, this suggest an optimal RPM is between 70 and 80, likely around 77. The optimal downhole tool set selection is to choose B1 with a probability of 75% and B2 with a probability of 25%.

5.3 Discussion

Risk of downhole tool failure can be modeled in terms of probability. Based on the risk model, the overall reliability of downhole tools for a drilling job can be evaluated as probability that no single tool or tool component fails. A risk model is helpful to identify high risk tool and tool component, and thus improve the efficiency in maintenance and reduce the risk. Risk criteria can be established based on the cost of replacement, the probability of failure, and the incurred extra cost if a failure occurs.

For a given set of downhole tools, a slower ROP normally means less vibrations, and thus less consumption of tool-life; but a slower ROP also means longer drill time, which adds to consumption of tool-life and drilling cost. Control devices can be used to mitigate vibration and reduce risk of tool failure, and thus improve the reliability, but at the same time, the ROP may be affected and the drilling cost would be higher. Given a risk model for downhole tool failure, the reliability of downhole tools can be evaluated for possible drilling operations. Simultaneously, ROP can also be estimated. Arguably tool vendors expect the least consumption of tool-life, which may be evaluated as the change in reliability before and after a drill job. However, the drilling crew would maximize ROP regardless of the wear on tools. When there are multiple competitive choices for both parties, a non-cooperative two-player game theory model may help identify the best choice by looking at the Nash equilibrium solutions.

The suggested approach is not limited to downhole tools. As long as a risk model is available and the cost for different scenario can be evaluated, decision analysis and game theory model can be applied.

Chapter 6 Conclusions and Future Work

Based on our study, conclusions were drawn as follows:

- It is the first time, to the best of our knowledge, that game theory has been used for drilling operations.
- Game theory can be an effective tool to resolve decision making problems in drilling operations and downhole tool management.
- We proposed a possible use of game theory for the optimal management of downhole tools.
- Decision analysis may help to identify the risk criteria for downhole tool failure.
- As demonstrated in the example for complicated situations, game theory is capable of finding optimal strategies for both downhole tool management and drilling operations.

Future work includes the application of this methodology to realistic situations, the extension of this methodology to drilling automation and a broad spectrum of drilling tools and equipment, and a consideration of overall drilling cost in a cooperative manner.

References

Ajimoko, O. O. (Tayo). 2016. "Application of Game Theory for Optimizing Drilling Cost Reduction Programmes," OTC-26598-MS. Offshore Technology Conference Asia, 22-25 March, Kuala Lumpur, Malaysia, doi:10.4043/26598-MS

Azar, J. J., Samuel, G. R. 2007, Drilling Engineering, PennWell, USA.

Belhaiza, S. J., Mve, A. D., and Audet C. 2010. XGame-Solver Software. http://faculty.kfupm.edu.sa/MATH/slimb/XGame-Solver-Webpage/index.htm

Bellhouse, D. 2007. "The Problem of Waldegrave," *Journal Électronique d'Histoire des Probabilités et de la Statistique* 3(2): 1-12.

Bratvold, R. B. and Begg, S. H. 2009. "Would You Know a Good Decision if You Saw One?" SPE The Way Ahead, SPE-0209-021-TWA. 5 (2): 21 -23. doi:10.2118/0209-021-TWA.

Bratvold, R. B. and Koch, F. 2011. "Game Theory in the Oil and Gas Industry. Society of Petroleum Engineers," SPE-0111-018-TWA, SPE The Way Ahead, 7(1): 18 -20. doi:10.2118/0111-018-TWA.

Carter-Journet, K., Kale, A., Falgout, T., Heuermann-Kuehn, L. 2014a. "Drilling Optimization: Utilizing Lifetime Prediction to Improve Drilling Performance and Reduce Downtime," SPE-170270, SPE Deepwater Drilling and Completions Conference, Galveston, Texas, 10-11 September.

Carter-Journet, K., Kale, A., Zahng, D., Pradeep, E., Falgout, T., Heuermann-Kuehn, L. 2014b. "Estimating Probability of Failure for Drilling Tools with Life Prediction," SPE-171517, SPE Asia Pacific Oil&Gas Conference and Exhibition, Adelaide, Australia, 14-16 October.

Cournot, A. A. Recherches sur les principes mathématiques de la théorie des richesses (Researches into the Mathematical Principles of the Theory of Wealth, 1838.

Frederick S. H. Lieberman, G. J. 2001. "Decision Analysis," Introduction to Operations Research. 7th ed. McGraw-Hill Higher Education Chapter 15. pp 749 – 794.

Hareland, G., and Hobercock, L. L. 1993 "Use of Drilling Parameters to Predict In-Situ Stress Bounds," SPE 25727 presented at the Drilling Conference in Amsterdam, 22-24 February.

Jiang, W., Samuel, R. 2016 “Optimization of Rate of Penetration in a Convoluted Drilling Framework using Ant Colony Optimization,” IADC/SPE Drilling Conference and Exhibition, 1-3 March, Fort Worth, Texas, SPE-178847-MS. http://dx.doi.org/10.2118/178847-MS.

Kaiser, M. J. 2007. “A Survey of Drilling Cost and Complexity Estimation Models.” *Int. J. Pet. Sci. Technol*. 1: 1-22.

Kale, A. A., Carter-Journet, K., Heuermann-Kuehn, L., Falgout, T., Zurche, D. 2014. “A Probabilistic Approach to Reliability and Life Prediction of Electronic in Drilling and Evaluation Tools,” https://www.phmsociety.org/sites/phmsociety.org/files/phm_submission/2014/phmc_14_054.pdf.

Karkoub, M., Abdel-Magid, Y. L., Balachandran, B. 2009. “Drill-String Torsional Vibration Suppression Using GA Optimized Controllers,” *J. Can. Petro. Tech.* 48(12): 32–38.

King, G. E. 2010. “Reliability of Downhole Equipment,” http://gekengineering.com/Downloads/Free_Downloads/Reliability_of_Downhole_Equipment1.pdf.

Kriesels, P. C., Keultjes, W. J. G., Dumont, P., Huneidi, I., Owoeye, O. O., Hartmann, R. A., 1999. “Cost Savings through an Integrated Approach to Drillstring Vibration Control,” SPE/IADC 57555. SPE/IADC Middle East Drilling Technology Conference, Abu Dhabi, UAE, 8-10 November.

Langlois, J.-P. 2006. GamePlan, a Windows application for representing and solving games. http://userwww.sfsu.edu/langlois/.

Liu, T., Zhao, Y., and Ma G., 2010. Forecast of Oil-Gas Drilling Cost Based on BP Neural Network, *J. Xi’an Shiyou Univ.*, 25 (1): 87–90.

Lukawski, M. Z., Anderson, B. J., Augustine, C., Capuano Jr. L. E., Beckers, K. F., Livesay, B., Tester, J. W., 2014. “Cost Analysis of Oil, Gas, and Geothermal Well Drilling”, *J. Pet. Sci. Eng.* 118: 1-14.

McKelvey, R. D., McLennan, A. M., and Turocy, T. L. 2015. Gambit: Software Tools for Game Theory, Version 15.1.0. http://www.gambit-project.org.

Mishra, S. and Pecht, M. 2002. In-situ Sensors for Product Reliability Monitoring, *Proceedings of SPIE*, Vol. 4755, 2002, pp. 10-19. Doi: 10.1117/12.462807.

Myerson, Roger B. 1991. Game Theory: Analysis of Conflict, Harvard University Press, p.1. Chapter-preview links, pp.vii–xi.

Nash, J. 1951. “Noncooperative games,” *Annals of Mathematics*, 54: 286-295.

Neumann, J. v. 1928. “Zur Theorie der Gesellschaftsspiele,” Mathematische Annalen 100 (1): 295–320, doi:10.1007/BF01448847 English translation: Tucker, A. W.; Luce, R. D., eds. (1959),"On the Theory of Games of Strategy", Contributions to the Theory of Game 4, pp. 13–42.

Pecht, M., Lall, P., Hakim, E. 1997. Influence of Temperature on Microelectronics and System Reliability, CRC Press, New York, NY.

Pecht, M., Radojcic, R., Rao, G., 1999. Guidebook for Managing Silicon Chip Reliability, CRC Press, Boca Raton, FL.

Rajnauth, J., Jagai, T. 2012. “Reduce Torsional Vibration and Improve Drilling Operations,” *Int. J. App. Sci. Tech.* 2(7): 109-123.

Rampersad, P.R., Hareland, G. and Boonyapaluk, P. 1994. “Drilling Optimization Using Drilling Data and Available Technology,” Paper SPE 27034 presented at the III Latin American/ Caribbean Petroleum Engineering Conference, Buenos Aires, Argentina, 27-29 April 1994.

Rasmusen, E. 2007. Games and Information. 4th ed.

Samuel, R. G. 2007. Downhole Drilling Tool – Theory and Practice for Engineers and Students, Gulf Publishing Company.

Savani, R., von Stengel, B. 2015. “Game Theory Explorer – Software for the Applied Game Theorist,” *Computational Management Science*, 12: 5-33.

Schitka, B. B. 2014. “Applying game theory to oil and gas unitization agreements: how to resolve mutually beneficial, yet competitive situations,” *J. World Energy Law Bus.* 7(6): 572-581.

Screpanti, E., Zamagni, S. 2005. An Outline of the History of Economic Thought (2nd ed.). Oxford University Press.

Shor, R. J., Dykstra, M. W., Hoffmann, O. J., Coming, M. 2015. “For Better or Worse: Applications of the Transfer Matrix Approach for Analyzing Axial and Torsional Vibration,” SPE/IADC Drilling Conference and Exhibition, London, United Kingdom, 17–19 March.

Sotomayor, G. P. G., Placido, J. C., Cunha, J. C. 1997. “Drill String Vibration: How to Identify and Suppress,” SPE 39002. The Fifth Latin American and Caribbean Petroleum Engineering Conference and Exhibition. Rio de Janeiro, Brazil, 30 August – 3 September.

Warren, T.M. 1987. “Penetration Rate Performance of Roller Cone Bits,” *SPE Drill Eng*, 2 (1): 9–18. SPE-13259-PA. http://dx.doi.org/10.2118/13259-PA.

Williams, J.D 1966. The Compleat Strategyst: Being a Primer on the Theory of Games of Strategy, McGraw Hill Book Company.

Yi, P., Kumar, A., and Samuel, R. 2014. “Real-Time Rate of Penetration Optimization Using the Shuffled Frog Leaping Algorithm (SFLA),” Presented at the SPE Intelligent Energy Conference & Exhibition, Utrecht, The Netherlands, 1–3 April. SPE-167824-MS. http://dx.doi.org/10.2118/167824-MS.

Zhao, Y., Zhao S., and Liu, T. 2011. “Bayesian Regularization BP Neural Network Model for Predicting Oil-Gas Drilling Cost,” Presented at International Conference on Business Management and Electronic Information, Guangzhou, China, 13–15 May. http://dx.doi.org/10.1109/ICBMEI.2011.5917952.