A STUDY OF THE AUTOFRETTAGE PROCESS

A Thesis

Presented to

the Faculty of the Department of Civil Engineering

The University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

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by

Genn Shi Shaw

January, 1967

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ABSTRACT

Over the past few decades, modern industrial processes have required higher and higher operating fluid pressures, and use of these high pressures have necessitated development of the autofrettage procedure. This procedure consists of subjecting a thick-walled tube or vessel to such high internal pressure that plastic yielding in circumferential tension occurs on the inside for a significant portion of the wall thickness. When the autofrettaging pressure is released, residual compressive circumferential stresses exist inside the wall (and for a significant portion of the wall thickness). These stresses are of oppostite sign to those caused by subjection of the tube or vessel to subsequent working pressures. The inside circumferential stresses are then much lower than they would be had the tube or vessel never been autofrettaged. Use of the autofrettage procedure thus permits the use of higher pressures and insures a longer fatigue life than may be obtained by any other method.

This thesis presents a thorough study of the theory and practice of the autofrettaging procedure, and also presents a comparison of the redidual strains obtained with electric resistance strain gages during typical routine autofrettaging of high pressure tubes with the residual strains predicted by use of formulas derived for autofrettaging procedures.

CONTENTS

LIST OF FIGURES AND TABLE

NOTATIONS

a	Axial distance
e	Strain
f_r	Radial stress
f_t	Tangential (circumferential) stress
f_a	Axial (longitudinal) stress
$\Delta f_r, \Delta f_t, \Delta f_a$	Changes in stresses due to unloading
f_r^*, f_t^*, f_a^*	Residual stresses
f_r', f_t', f_a'	Changes in residual stresses due to boring
f_r'', f_t'', f_a''	Remaining stresses after boring
f_s	Shear stress
$f_{y.p.}$	Yield strength in pure tension
$f_{s.y.}$	Yield strength in simple shear
$f_{t.s.}$	Ultimate strength in pure tension
f_{rm}	Radial stress measured from shear plane
f_{tm}	Tangential stress measured from shear plane
f_{sr}	Component of radial stress along shear plane
f_{st}	Component of tangential stress along shear plane
p	Internal pressure
p^*	Autofrettage pressure
p'	Elastic break-down (first overstrain) pressure
p''	Bursting pressure
Δp	Unloading pressure
p_o	Pressure after unloading
u	Displacement
u_i, u_c	Displacement at R_i, R_c, respectively
Δu	Change of displacement due to unloading

A	Cross-sectional area of the bore following a machine cut
A_o	Initial cross-section area of cylinder including the bore
C, C_1, C_2	Integral constants
E	Young's modulus of elasticity
F_t, F_a	Fictitious surface stresses on the tube
G	Shear modulus
K	Radius ratio (R_o / R_i)
P	Loading pressure
R	Variable radial distance
R_i	Radius of inner surface of cylinder
R_o	Radius of outer surface of cylinder
R_c	Radial distance of elastic-plastic interface caused by autofrettage pressure
R_j	Radial distance of elastic-plastic interface caused in unloading process
R'	Radius of the bore following a machine cut
S	Stress
T	Torque
ϵ_r	Radial strain
ϵ_t	Tangential (circumferential) strain
ϵ_a	Axial (longitudinal) strain
$\Delta\epsilon_r$	Change of strain due to unloading
ϵ^*	Residual strain
ϵ_t', ϵ_a'	Residual strains measured from outside wall of tube during boring-out process
θ	Variable circular angle
μ	Poisson's ratio
ρ	Density
ϕ	Angle between shear plane and cross-section through cylinder axis
ψ	Unit circular twist

CHAPTER I

INTRODUCTION

The word "autofrettage" comes from the French language, and means "self-hooping." It is a type of cold treatment for pressure vessels or gun barrels which induces prestress and increases the strength in such thick-walled cylinders.

The autofrettage procedure involves the use of such high internal pressure inside a thick-walled cylinder that plastic flow occurs in circumferential tension in the inner portion of the wall thickness. After the internal pressure is released, the outer portion (which remains elastic), tends to resume its original size. The inner portion, been permanently deformed, resists this action, thus producing two different kinds of tangential residual stresses; a compressive stress near the bore, and a tensile stress near the outer surface. The strength of the cylinder is thus effectively increased. It makes the cylinder safe when subjected to much greater working pressures without increasing the wall thickness.

The autofrettaging procedure for manufacturing gun barrels has the great advantage of economy of time, labor, and material. It cuts down the cost about 25-40 per cent from that of the conventional wire-wrapping or built-up procedure. The efficiency of the gun barrel also is increased, as the procedure raises the elastic limit of the metal and produces a gun barrel which will withstand a higher pressure than a built-up or a wire-wrapped gun of the same caliber and weight. (14)

In addition to being used for making gun barrels, the autofrettage procedure is also being used today for making high pressure vessels and tubes in the industrial field.

Fig. 1-1 shows the distribution of a tangential (circumferential) stress in shell of tube before and after autofrettaging.

Ductile Metals (mild steel and low alloys) were originally considered as the material for thick-walled tubes to be treated by the autofrettaging process. After autofrettaging, these metals are apparently stronger, harder, and less ductile than before. (14) Today, such steels as those of the 4340 type are routinely autofrettaged.

In 1911, Cook found that the bursting pressure of a thick-walled tube of ductile metal is far greater than its elastic breakdown pressure at layers near the bore. (5) (See Fig. 1-2). Later, the autofrettage procedure was used for manufacturing gun barrels, but a difficulty of dimensional instability arose and the procedure was developed slowly. The problem is described as in Fig. 1-3. If we steadily raise the internal pressure in a tube, line OA shows that the tube still remains elastic. After the pressure increases above point A, the inner layers are plastically strained, beginning at the bore. When the pressure reaches point B, the autofrettage pressure, we reduce the pressure to zero, and the curve falls down to C. If we leave the tube under zero pressure for a certain time, it may change its dimensions slightly to C' because of "elastic afterworking." Then if we increase pressure again, the curve starts at C', nearly straight, as for the first time, up to D, where it bends over again. The problem is that it is difficult to make BC

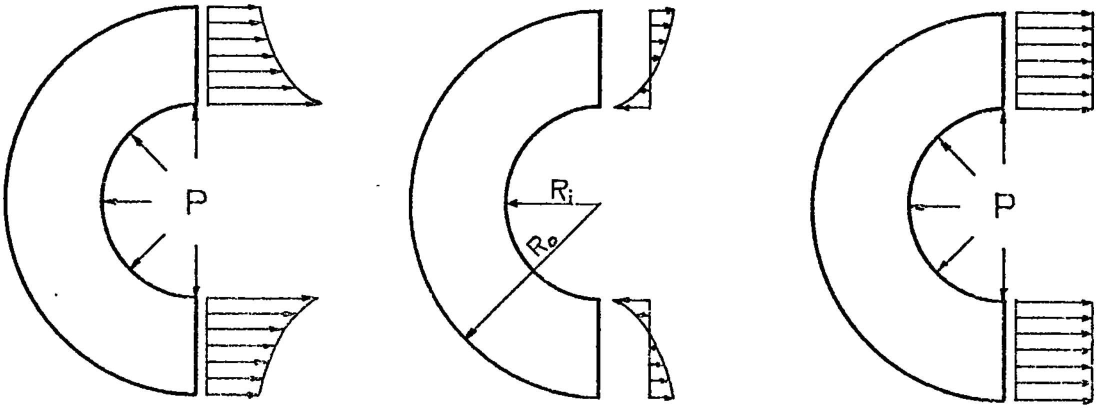


Fig. 1-1 Stress Distribution in Shells Under Load

1. Fig. (1-1a): Stress distribution under internal pressure "p" for a non-autofrettaged shell.

2. Fig. (1-1b): Residual stress distribution in an autofrettaged shell under zero pressure.

3. Fig. (1-1c): Stress distribution under internal pressure "p" for an ideal autofrettaged shell.

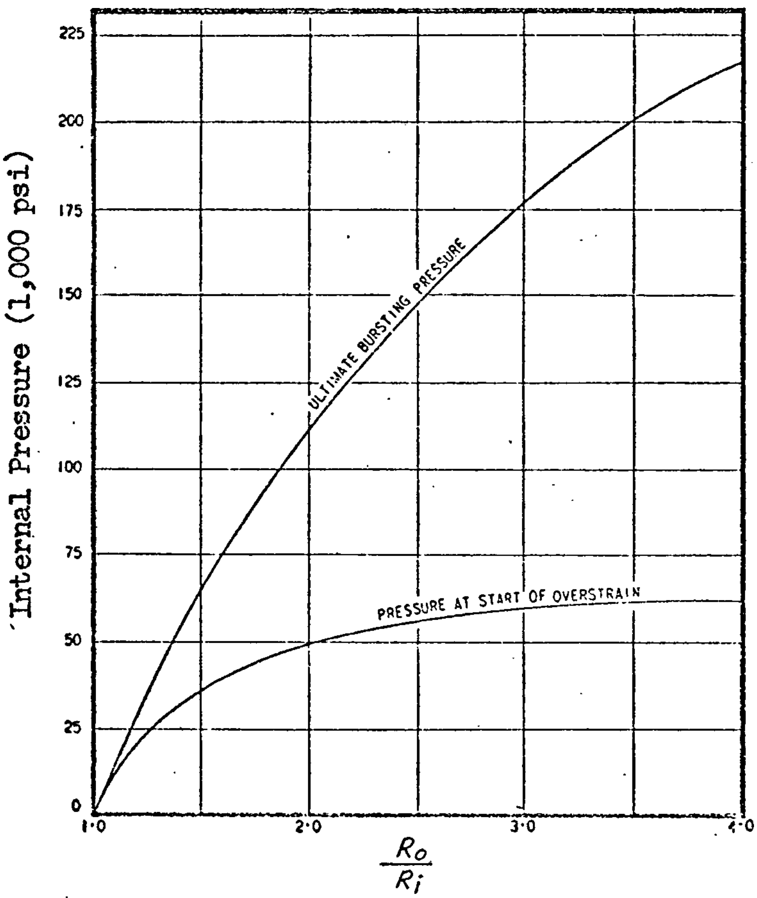


Fig. 1-2 Difference Between Pressure of Elastic Breakdown
And Bursting Pressure In A
Ductile Cylinder
(See Ref. 21.)

and C'D straight and coincident. In 1930, Macrae found that a low-temperature treatment after tube autofrettaging could solve this problem. (18)

Fig. 1-4 shows an autofretted tube subjected to an appropriate heat treatment, and the dimensional change at CC' is very small. If we now repeat the process, a much straighter line appears, and it is almost coincident with BC. Also, the point D is little higher than point B. The pressure increment represented by BD was called by Macrae the "elastic gain."

A very important point made analytically by Langenberg in 1925 is that cylinders of the same diameter ratio but of different sizes under the same proportionate overstrain have the same strengthening effect. (17). This implies that scale factors are unimportant, and that design can be based on the "principle of similarity."

In addition to the traditional internal-pressure autofrettage method discussed above, there is a new mechanical method of autofrettaging which has been developed since 1962. It will be discussed below:

The conventional autofrettage procedure involves use of internal hydrostatic pressure of sufficient magnitude to obtain the desired overstrain. This type of procedure has been used for several years in gun tube and pressure-vessel construction. Currently, pressure up to 200,000 psi are being used in the autofrettage of gun tube and pressure vessels with yield strength levels of 160,000 to 190,000 and 240,000 to 250,000 psi, respectively.

Autofrettaging by the above technique is for all practical purposes limited to pressures not exceeding 200,000 psi. To extend the

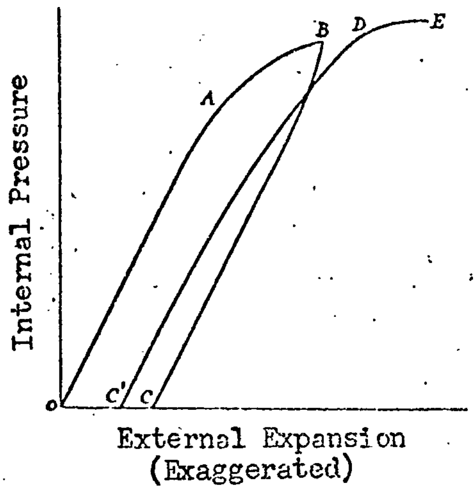


Fig. 1-3 Changes of Outside Diameter In Autofrettaged Cylinder Without Heat Treating

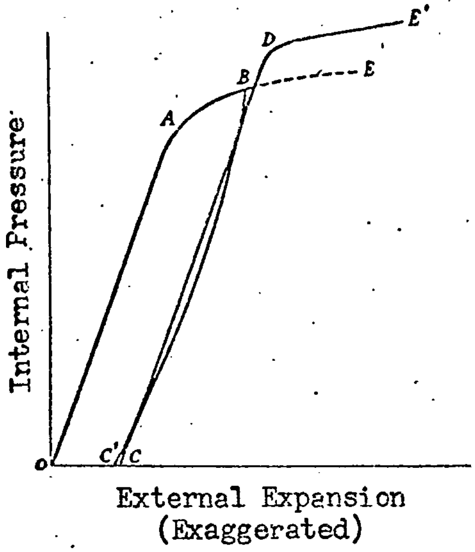


Fig. 1-4 Changes of Outside Diameter In Autofrettaged Cylinder With Heat Treating

use of the autofrettage procedure to higher pressure applications, and to eliminate the many problems encountered in the use of pressures in the range of 150,000 to 200,000 psi on a large-scale basis, a new autofrettage process has been developed by Davison, Barton, Reiner and Kendall in 1962. (9)

This new technique uses the mechanical advantage of a wedge to produce the desired bore enlargement, thus drastically reducing the pressure requirements to obtain a given amount of overstrain. This swaging method of autofrettage consists basically of passing an over-sized swaging tool (mandrel) through the bore of cylinder to produce the desired permanent enlargement. Three different methods of forcing the mandrel were tested, (I) the mandrel was mechanically pushed by means of a ram and hydraulic press, (II) an overhead crane pulled the mandrel through a long cylinder, and (III) the mandrel was pushed by applying hydraulic pressure directly to the end of the mandrel. This work indicated that 170,000 psi yield strength could be induced by 60,000 psi pressure of pushing or pulling a mandrel. (See Fig. 1-5)

To determine the effectiveness of the swaging method of autofrettage, Sachs' boring-out technique (27) for determining the distribution and magnitude of residual stresses was used. Because of the difference in the nature of the stress condition responsible for the inducement of the overstrain in the swaging method as compared to the direct pressure process, the resultant residual stress distribution will be somewhat different. Fig. 1-6 shows experimental curves of stress distribution through the cylinder as well as curves from a theoretical solution (conventional method). It is evident from Fig. 1-6 that a substantial longitudinal residual stress exists in cylinders autofrettaged

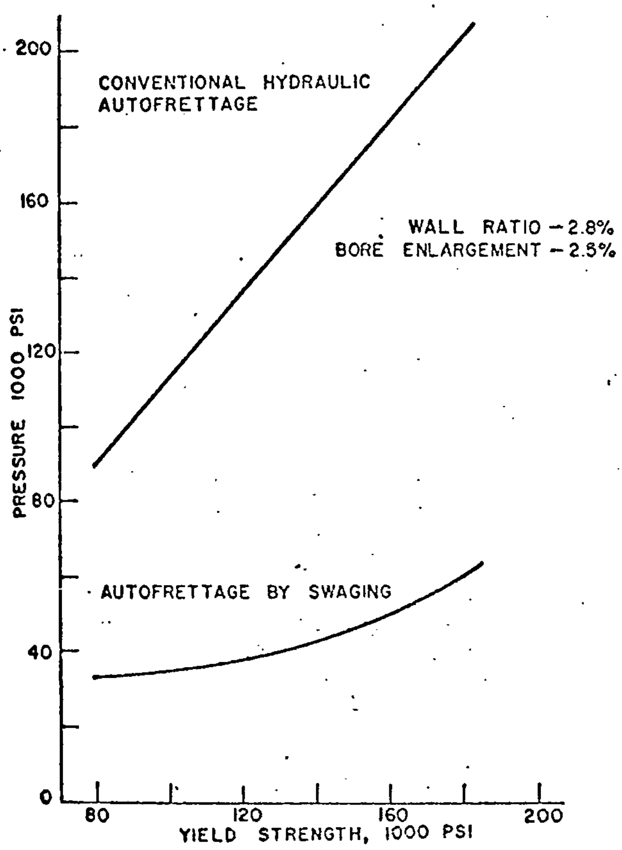


Fig. 1-5 Hydraulic Pressure vs. Yield Strength
(See Ref. 9.)

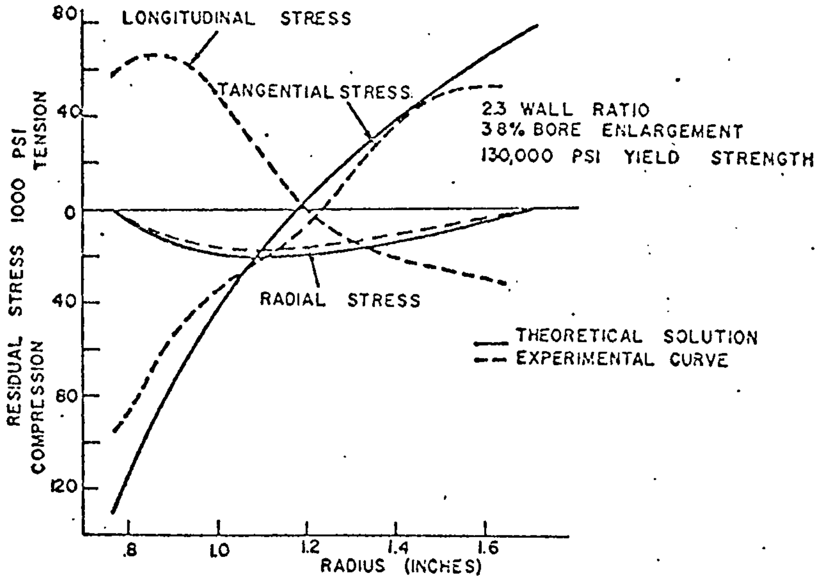


Fig. 1-6 Residual Stresses in Mechanical-Push Swaged Specimen (See Ref. 9.)

by swaging. This stress increases in magnitude with increased amounts of enlargement and accompanies a proportionate decrease in the tangential residual stress.

Miscellaneous Methods of Autofrettaging

There are also other methods for inducing the effects of the autofrettaging procedure in a thick-walled cylinder. These are the compound (built-up) cylinder technique, the wire-wrapped cylinder technique and the thermal method. The first two are the older methods for making gun tubes. The modern autofrettage method produces a tube which will withstand a higher pressure than a built-up or a wire-wrapped tube of same size and weight. The thermal method is merely a suggestion in a Ph.D. thesis; the work has never been developed. A short discussion of all of these will now be given as follows:

Compound Cylinder Technique

Desirable residual stresses can be obtained by reinforcing the core tube by shrinking other tubes over it. The procedure is to heat the outer cylinder to a temperature about 250° F and cool the inner cylinder in liquid oxygen simultaneously. Then the inner cylinder is slipped into the outer cylinder and the two permitted to shrink together. The advantage of a compound vessel is the reduction of wall thickness of each component cylinder as compared with the equivalent monobloc cylinder.

Wire-Wrapping Technique:

This technique involves wrapping wire under tension on a central tube. The wire puts the metal of the tube under compression, giving the effect of autofrettaging, and thus the tube can be subjected to greater internal pressure. The wire usually is high-tensile strap or wire of square section.

Thermal Method:

The work of Voorhees on the creep of vessels subjected simultaneously to high-temperature and high-pressure service conditions suggests the use of creep to prestress thermally a thick-walled vessel. (32) Voorhees reports that a creep of 1% is sufficient to produce stress equalization under the operating pressure. If a vessel has the pressure on it raised to the operating pressure and then is heated slowly and uniformly until a 1% strain from creep has resulted, the stress distribution across the vessel wall should become nearly uniform, approaching the ideal-prestressed condition shown in "c" of Fig. 1-1. If the pressure is released and the vessel cooled with sufficient rapidity to prevent additional creep, residual stresses will result in the shell. Upon placing the vessel in service at the same pressure as that used in prestressing, but at a temperature below that producing creep, the ideal-prestressed condition will be approached for the loaded condition. Although this method of prestressing appears to be very promising for monobloc vessels, there is no known report of the use of this method.

CHAPTER II

PRINCIPLES OF AUTOFRETTAGING

During the past fifty years, the conventional theory for autofrettaging predictions has been based on tensile test data. The advantage of using tensile test data is that it is easier to obtain such design data, which is generally prepared by steel suppliers. Tensile test machines are also available in most laboratories. This is the reason why this theory is the only one used for comparison with experimental work later in this thesis. The disadvantage of using the tensile test data theory is that it is a great deal of work to obtain a complete solution.

Since 1945, Manning (19) has developed the theory for autofrettaging prediction based on torsion-test data. Today many high pressure vessels are fabricated of high strength steel which usually fails by shear, so that shear data from torsion tests should be used in the design of such vessels rather than tensile data. The advantage of using torsion-test data is that it will save quite much arithmetic. The disadvantage is that torsion test equipment is not very available in many laboratories.

Both of these two kinds of theories will be presented as follows:

Theoretical Predictions Used For Tensile Test Data

Theoretical predictions of autofrettage pressure, stresses, and strains presented here are summarized from the derivation developed by Prager and Hodger (26) and Hoffman and Sachs (15). This derivation is presented in full in Appendix A.

1. Prediction of the Autofrettage Pressure

The autofrettage pressure is obtained using the following expression:

$$p^* = \frac{f_{y.p.}}{\sqrt{3}}\left(1 - \frac{R_c^2}{R_o^2} - 2\ln\frac{R_i}{R_c}\right) \qquad (2\text{-}1)$$

where $f_{y.p.}$ = yield limit in simple tension, psi.

p^* = autofrettage pressure, psi.

R_i = inside radius of shell, in.

R_o = outside radius of shell, in.

R_c = radial distance of elastic-plastic interface, in.

The elastic-plastic interface R_c for optimum prestressing with the autofrettage procedure recommended by Manning (20) shall be located at the geometric mean radius which means that the weight of elastic region is to the weight of plastic region as the overall diameter is to the bore.

$$\frac{\pi(R_o^2 - R_c^2)L\rho}{\pi(R_c^2 - R_i^2)L\rho} = \frac{2R_o}{2R_i}$$

$$R_c^2 = R_o R_i$$

$$R_c = \sqrt{R_o R_i} \qquad \text{or} \qquad R_c = R_i\sqrt{K} \qquad (2\text{-}2)$$

If the autofrettage pressure is known first, R_c can be obtained by solving the following equation, which is obtained from Eq. (2-1)

$$\frac{R_c^2}{R_o^2} - 2\ln\frac{R_c}{R_o} - 1 = 2\ln\frac{R_o}{R_i} - \frac{p^*}{\frac{f_{y.p.}}{\sqrt{3}}} \qquad (2\text{-}3)$$

Figure (2-1) shows the left-hand side of Eq. (2-3) versus $\frac{R_c}{R_o}$; the right-hand side can be evaluated from the data. Thus,

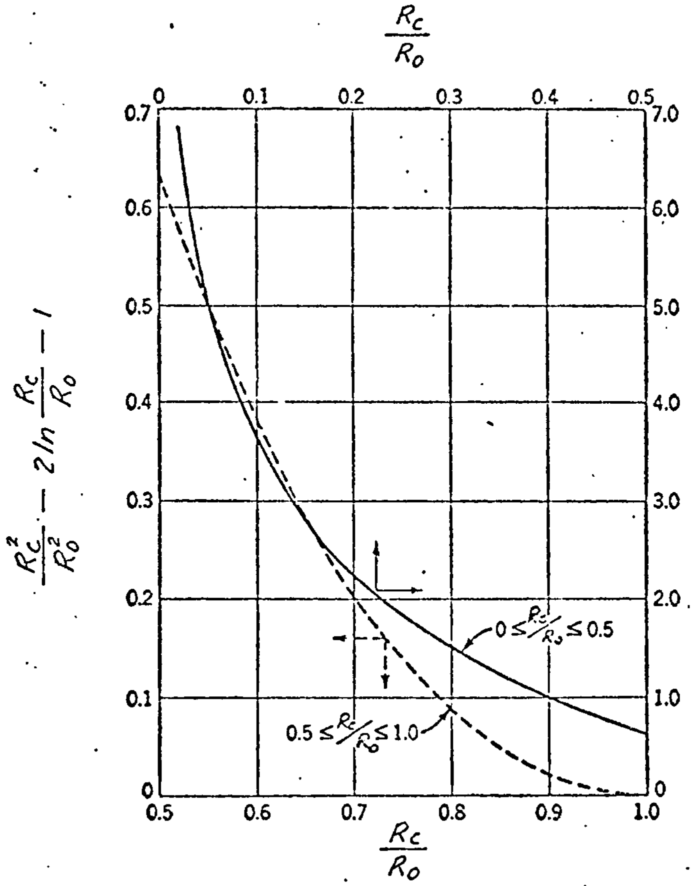


Fig. 2-1 Graph of $\frac{R_c^2}{R_o^2} - 2\ln\frac{R_c}{R_o} - 1$

(See Ref. 26.)

the value of $\frac{R_c}{R_o}$ corresponding to a given value of $\frac{p^*}{\frac{f_{y.p.}}{\sqrt{3}}}$ and $\frac{R_o}{R_i}$ can be read directly from Fig. 2-1.

The upper and lower limits of the autofrettage pressure are gained from Eq. (2-1) when

$R_c = R_i$ and when $R_c = R_o$ respectively,

$$p' = \frac{f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_i^2}{R_o^2}\right) \tag{2-4}$$

$$p'' = \frac{2f_{y.p.}}{\sqrt{3}} \left(\ln \frac{R_o}{R_i}\right) \tag{2-5}$$

where p' = elastic break-down pressure, psi.

p'' = bursting pressure, psi.

In the actual case for ordinary metals, the ultimate tensile strength is appreciably higher than the yield strength and the stress at bursting will lie between the yield and ultimate strength. Faupel (11) has proposed the modified equation below.

$$p'' = \frac{2f_{y.p.}}{\sqrt{3}} \left(\ln \frac{R_o}{R_i}\right)\left(2 - \frac{f_{y.p.}}{f_{t.s.}}\right) \tag{2-6}$$

where $f_{t.s.}$ = ultimate strength in tension, psi.

2. Determination of the Autofrettage-pressure Stresses:

(a) For the plastic zone, where $R_i \leq R \leq R_c$

$$f_r = -\frac{f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_c^2}{R_o^2} - 2\ln \frac{R}{R_c}\right) \tag{2-7}$$

$$f_t = \frac{f_{y.p.}}{\sqrt{3}} \left(1 + \frac{R_c^2}{R_o^2} + 2\ln \frac{R}{R_c}\right) \tag{2-8}$$

$$f_a = \frac{f_{Y.P.}}{\sqrt{3}}\left(\frac{R_c^2}{R_o^2} + 2\ln\frac{R}{R_c}\right) \tag{2-9}$$

where f_r = autofrettage radial stress, psi

f_t = autofrettage tangential stress, psi

f_a = autofrettage longitudinal stress, psi

(b) For the elastic zone, where $R_c \leq R \leq R_o$

$$f_r = \frac{f_{Y.P.}}{\sqrt{3}}\left(\frac{R_c^2}{R_o^2} - \frac{R_o^2}{R^2}\right) \tag{2-10}$$

$$f_t = \frac{f_{Y.P.}}{\sqrt{3}}\left(\frac{R_c^2}{R_o^2} + \frac{R_o^2}{R^2}\right) \tag{2-11}$$

$$f_a = \frac{f_{Y.P.}}{\sqrt{3}}\left(\frac{R_c^2}{R_o^2}\right) \tag{2-12}$$

3. Changes in Stresses as a Result of Unloading the Autofrettage Pressure:

(a) Cylinders with unloading stresses within the elastic region:

when $\frac{R_o}{R_i} \leq 2.22$ (See Fig. A-3) $\quad p' \leq \Delta p < p''$

or $\frac{R_o}{R_i} > 2.22 \quad \Delta p \leq 2p'$

The changes in stresses in the shell are:

$(R_i < R < R_o)$

$$\Delta f_r = -\Delta p\left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(1 - \frac{R_o^2}{R^2}\right) \tag{2-13}$$

$$\Delta f_t = -\Delta p\left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(1 + \frac{R_o^2}{R^2}\right) \tag{2-14}$$

$$\Delta f_a = -\Delta p \left(\frac{R_i^2}{R_o^2 - R_i^2} \right) \tag{2-15}$$

where $\Delta p = p^* - p_o$

p^* = autofrettage pressure, psi.

p_o = pressure after unloading, psi. (normally zero)

(b) Cylinders with unloading stresses beyond the elastic region:

when $\frac{R_o}{R_i} > 2.22$ $\qquad 2p' \leq \Delta p < p''$

$$\Delta p = \frac{2 f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_j^2}{R_o^2} - 2 \ln \frac{R_i}{R_j} \right) \tag{2-16}$$

where R_j = radial distance of elastic-plastic interface caused in unloading process, in.

R_j can be determined by solving Eq.(2-3) for $R_c = R_j$,

$p^* = \Delta p$ and $\frac{f_{y.p.}}{\sqrt{3}} = \frac{2 f_{y.p.}}{\sqrt{3}}$, with the aid of Fig. 2-1.

(1) for the plastic zone : $R_i \leq R \leq R_j$

$$\Delta f_r = \frac{2 f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_j^2}{R_o^2} - 2 \ln \frac{R}{R_j} \right) \tag{2-17}$$

$$\Delta f_t = -\frac{2 f_{y.p.}}{\sqrt{3}} \left(1 + \frac{R_j^2}{R_o^2} + 2 \ln \frac{R}{R_j} \right) \tag{2-18}$$

$$\Delta f_a = -\frac{2 f_{y.p.}}{\sqrt{3}} \left(\frac{R_j^2}{R_o^2} + 2 \ln \frac{R}{R_j} \right) \tag{2-19}$$

(2) for the elastic zone : $R_j \leq R \leq R_o$

$$\Delta f_r = -\frac{2 f_{Y.P.}}{\sqrt{3}} \cdot \frac{R_j^2}{R_o^2}\left(1 - \frac{R_o^2}{R^2}\right) \tag{2-20}$$

$$\Delta f_t = -\frac{2 f_{Y.P.}}{\sqrt{3}} \cdot \frac{R_j^2}{R_o^2}\left(1 + \frac{R_o^2}{R^2}\right) \tag{2-21}$$

$$\Delta f_a = -\frac{2 f_{Y.P.}}{\sqrt{3}} \left(\frac{R_j^2}{R_o^2}\right) \tag{2-22}$$

4. Determination of Residual Autofrettage Stresses:

$$\underline{f_r^* = f_r + \Delta f_r} \tag{2-23}$$

where f_r^* = residual radial stress, psi.

$f_r = f_r$ from Eq. (2-7) or Eq. (2-10)

$\Delta f_r = \Delta f_r$ from Eq. (2-13) or Eq. (2-17) or Eq. (2-20)

$$\underline{f_t^* = f_t + \Delta f_t} \tag{2-24}$$

where f_t^* = residual tangential stress, psi.

$f_t = f_t$ from Eq. (2-8) or Eq. (11)

$\Delta f_t = \Delta f_t$ from Eq. (2-14) or Eq. (2-18) or Eq. (2-21)

$$\underline{f_a^* = f_a + \Delta f_a} \tag{2-25}$$

where f_a^* = residual axial stress, psi.

$f_a = f_a$ from Eq. (2-9) or Eq. (2-12)

$\Delta f_a = \Delta f_a$ from Eq. (2-15) or Eq. (2-19) or Eq. (2-21)

5. Calculations of Unit Radial Strains:

(a) The unit radial strain in both the elastic and plastic regions under autofrettage pressure

$$\epsilon_r = \frac{-f_{Y.P.}}{\sqrt{3}}\left(\frac{R_c^2}{2GR^2}\right) \tag{2-26}$$

where $G = \dfrac{E}{2(1+\mu)}$

E = modulus of elasticity of material, psi.

μ = Poisson's ratio

(b) Changes in unit radial strain as a result of unloading the autofrettage pressure

(1) for $\dfrac{R_o}{R_i} \leq 2.22$ $\quad p' \leq \Delta p < p''$

or $\dfrac{R_o}{R_i} > 2.22$ $\quad \Delta p \leq 2p'$

$$\Delta\epsilon_r = \frac{\Delta p}{2G}\left(\frac{R_i^2 R_o^2}{(R_o^2 - R_i^2)R^2}\right) \tag{2-27}$$

(2) for

$\dfrac{R_o}{R_i} > 2.22$ $\quad 2p' \leq \Delta p < p''$

$$\Delta\epsilon_r = -\frac{f_{y.p.}}{\sqrt{3}}\left(\frac{R_j^2}{GR^2}\right) \tag{2-28}$$

(c) Unit residual strain

$$\underline{\epsilon^* = \epsilon_r + \Delta\epsilon_r} \tag{2-29}$$

where $\epsilon_r = \epsilon_r$ From Eq. (2-26)

$\Delta\epsilon_r = \Delta\epsilon_r$ From Eq. (2-27) or Eq. (2-28)

<u>Theoretical Predictions Used For Torsion-test Data</u>

According to Manning, two assumptions are involved; (I) the cross-sectional area remains constant under strain, (II) the relation between maximum shear stress and maximum shear strain is the same in a cylinder as in a specimen for torsion test (19, 20, 21). The first assumption permits the calculation of the shear strain at any point in the wall of the vessel and the second assumption may permit the calculation of shear stress from torsion data. Equation (A-7) is a statement of the first assumption, or:

$$f_t - f_r = \frac{df_r}{dR} R \qquad \text{(A-7)}$$

The stresses on an infinitesimal element of the cross-section of cylinder are shown in Fig. 2-2 (a). f_{tm} and f_{rm} are the tangential and radial stresses measured from the m-m plane. These stresses will be defined as follows:

$$f_{tm} = f_t \cos \phi$$

$$f_{rm} = f_r \cos \left(\frac{\pi}{2} - \phi\right) = f_r \sin \phi$$

The components of these stresses on the m-m plane, f_{st} and f_{sr}, will be (See Fig. 2-2 (b).)

$$f_{st} = f_{tm} \sin \phi = f_t \cos \phi \sin \phi = \frac{f_t}{2} \sin 2\phi$$

$$f_{sr} = f_{rm} \cos \phi = f_r \sin \phi \cos \phi = \frac{f_r}{2} \sin 2\phi ,$$

and the shear stress alone m-m plane will be:

$$f_s = f_{st} - f_{sr} = \frac{f_t - f_r}{2} (\sin 2\phi)$$

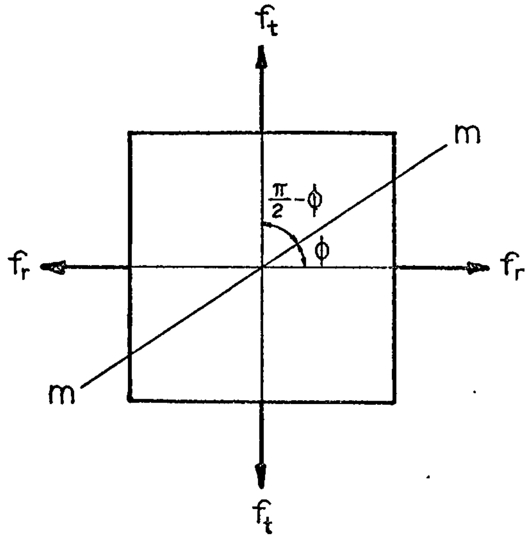
When $\phi = 45°$ the maximum (yielding) shear stress occurs:

$$f_{s.y.} = \frac{f_t - f_r}{2}. \qquad \text{(2-30)}$$

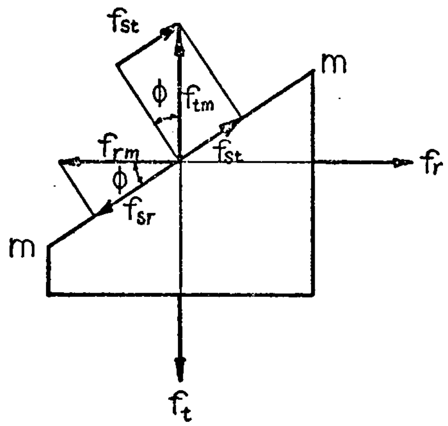
Substituting Eq. (2-30) into Eq. (A-7)

$$\frac{df_r}{dR} + \frac{f_r - f_t}{R} = 0 ,$$ we then have

$$df_r = \frac{2 f_{s.y.}}{R} dR . \qquad \text{(2-31)}$$



(a)



(b)

Fig. 2-2 Forces in an Element Under Tensile Stresses

In Eq. (2-31), the value of R should refer to the strained condition, so that it will be replaced by (R + u) where u is the radial displacement at point R caused by the straining. Now this equation can be expressed as follows:

$$f_r = -p^* + 2\int_{R_i + u_i}^{R+u} \frac{f_{s.y.}}{(R+u)}\, d(R+u) \tag{2-32}$$

where u = shift of point at radius R as a result of strain induced by pressure p^*.

u_i = shift of point at radius R_i as a result of strain induced by pressure p^*.

p^* = internal autofrettage pressure.

The shear stresses, $f_{s.y.}$, in Eq. (2-32) can be known from the following equation given by Nadai (24):

$$f_s = \frac{1}{\pi R_i^3}\left(\psi \frac{dT}{d\psi} + 3T\right) \tag{2-33}$$

where T = torque

ψ = unit angular twist.

According to the first assumption, the following relation is true:

$$\pi R^2 - \pi R_i^2 = \pi (R+u)^2 - \pi (R_i + u_i)^2 \qquad \text{or}$$

$$2Ru + u^2 = 2R_i u_i + u_i^2 \ . \tag{2-34}$$

Substituting $f_a = \dfrac{f_t + f_r}{2}$ (See Eq. A-11) into von Mises' equation of yielding condition: (31)

$$(f_r - f_t)^2 + (f_t - f_a)^2 + (f_a - f_r)^2 = 2 f_{Y.P.}^2$$

Eq. (A-27) yields $\qquad f_t - f_r = \frac{2}{\sqrt{3}} f_{Y.P.}$.

For $\quad f_{S.Y.} = \frac{f_t - f_r}{2}$, $\qquad$ We then have

$$f_{S.Y.} = \frac{f_{Y.P.}}{\sqrt{3}} \tag{2-35}$$

For $\quad R = R_i$, $\quad f_{S.Y.} = \frac{f_{Y.P.}}{\sqrt{3}}$, $\qquad$ Eq. (2-26) becomes

$$\frac{u_i}{R_i} = \epsilon_r = -f_{S.Y.} \frac{R_c^2}{2 G R_i^2}$$

$$u_i = -f_{S.Y.} \frac{R_c^2}{2 G R_i} \tag{2-36}$$

In Eq. (2-34), R, R_i and u_i are now known.

u can be evaluated easily. Then, Eq. (2-32) can also be solved.

Substituting the values of radial stresses, f_r , into Eq. (2-30), the values of tangential stress, f_t , can also be obtained.

The total axial strain will be

$$E \epsilon_a = f_a - \mu \ (f_t + f_r) \tag{2-37}$$

from Eq. (A-7)

$$f_t = R \frac{df_r}{dR} + f_r \ , \tag{2-38}$$

substitute Eq. (2-38) in Eq. (2-37)

$$f_a = E \epsilon_a + 2 \mu f_r + \mu R \frac{df_r}{dR} . \tag{2-39}$$

The total axial load P on any section is:

$$P = 2 \pi \int_{R_i}^{R_o} R \, dR \, f_a \ . \tag{2-40}$$

Substituting for f_a from Eq. (2-39) and taking ϵ_a as constant across the whole cylinder wall, it follows that, for a cylinder under internal pressure p^* (when $R = R_i$, $f_r = -p^*$; when $R = R_o$, $f_r = 0$),

$$P = 2\pi\mu p^* R_i^2 + \pi E (R_o^2 - R_i^2)\,\epsilon_a \,. \tag{2-41}$$

For a cylinder with closed ends

$P = \pi R_i^2 p^*$ and hence

$$E\epsilon_a = \frac{(1-2\mu)p^*}{(R_o^2/R_i^2 - 1)} \tag{2-42}$$

Substituting Eq. (2-42) in Eq. (2-37), then we have

$$f_a = p^* \frac{(1-2\mu)}{(R_o^2/R_i^2 - 1)} + \mu\,(f_t + f_r)\,,$$

for $f_t = 2\, f_{s.y.} + f_r$ (see Eq. (2-30).)

$$f_a = p^* \frac{(1-2\mu)}{(R_o^2/R_i^2 - 1)} + 2\mu\,(f_{s.y.} + f_r)\,. \tag{2-43}$$

Hence axial stress values can be calculated from Eq. (2-43).

CHAPTER III

DETERMINATION OF RESIDUAL STRESSES

The autofrettage procedure causes residual stresses and dimensional changes which may be used as indications of the effectiveness of the procedure. The simplest measurement which may be made is that of the permanent increase in the diameter of the autofrettaged tube or vessel, and a rather accurate method is the boring-out method developed by Sachs (27) in 1927. In the boring-out procedure, an autofrettaged thick-walled cylinder is aligned in a lathe, and concentric layers of metal are machined from the bore in successive steps. The axial and tangential strains are recorded for each successive cut (usually about 0.007 to 0.05 in, measured on the diameter). Also, after each cut the bore diameter will be measured. These data on the strains and bore diameters can be substituted into the following equations developed by Sachs (for the derivations, see Appendix B.). Then we can obtain the residual stresses.

$$f_a^* = \frac{E}{1-\mu^2}\left[(A_0 - A)\frac{d\alpha}{dA} - \alpha\right] \tag{3-1}$$

$$f_r^* = \frac{E}{1-\mu^2}\left[\frac{A_0 - A}{2A}\right]\beta \tag{3-2}$$

$$f_t^* = \frac{E}{1-\mu^2}\left[(A_0 - A)\frac{d\beta}{dA} - \left(\frac{A_0 + A}{2A}\right)\beta\right] \tag{3-3}$$

where A_0 = initial cross-section area of cylinder including the bore, in^2

A = cross-section area of the bore following a machine cut, in^2

$\alpha = (\epsilon_a' + \mu \epsilon_t')$, in./ in.

$\beta = (\epsilon_t' + \mu \epsilon_a')$, in./ in.

ϵ_a' = axial strain, in./ in.

ϵ_t' = circumferential strain, in./ in.

E = Young's modulus of elasticity, lb./ in.^{2}

μ = Poisson's ratio

When an autofrettaged cylinder is bored out in stages, changes in dimensions of the cylinder will result from the removal of the overstrained inner core. When all the overstrained material has been removed, subsequent machining should produce no further change in dimensions, since the elastic outer shell will have resumed the dimension it had prior to the autofrettage treatment. The dimensional changes just mentioned are accompanied by strain changes on the outside of the tube or vessel, and these changes in strain are easily measured using electric resistance strain gages. The usual way is to mount strain gages in the longitudinal and tangential directions on the surface of cylinder. For greater accuracy, the strain gage installations may be duplicated. This procedure is so simple, and so accurate, that strain gages are often used today instead of the boring-out procedure. (Both may be used together until the desired correlation is obtained.) It is well known that the Sachs method of residual stress measurement is subject to errors because of the difficulty of measuring dimensional changes with sufficient accuracy, and because of modification of the residual stresses by the machining operations. Special attention must thus be given to the machining operations (must be done slowly, carefully, and with plenty of coolant), and to the diameter or other measurements taken during the procedure. The specimen must always be permitted to cool to the same temperature each time before taking any measurements of dimensions.

CHAPTER IV

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESIDUAL STRAINS

One original objective of the work of this thesis was to build autofrettaging equipment, and to use this equipment in obtaining experimental values of strains which could be compared with corresponding theoretical values. Such equipment was actually developed, and a special cylinder made and instrumented with electric resistance strain gages, and a typical autofrettaging procedure followed. Release of the "autofrettaging" pressure revealed a very discouraging situation--there was no evidence, either in the strain gage readings nor the diameter measurements of any residual stress (strain) in the cylinder--proof that no autofrettaging occurred during the test. After considerable checking of the theoretical computations, it was concluded that the test cylinder, which was thought to be mild steel was an alloy steel of considerably higher strength than assumed. As the pressure pump, pipe, and connections would have been overstressed by the use of higher pressures, the test work was discontinued.

As autofrettaging is a rather common procedure today, it was easy to obtain the results of successful work such as that described above, and values of strains obtained during a typical autofrettaging procedure were used to permit comparison of theoretical and experimental values of residual strain. Figs. 4-1 and 4-2 show the comparison between the theoretical and experimental stresses for two different assumptions (110,000 psi, and 107,000 psi) for the yield strength of the material. The exact value could not be obtained from the available records, thus the two assumptions were used to show the effect of using these different values. (The computations necessary to obtain the values for plotting Figs. 4-1 and 4-2 are presented as Appendix C.)

From Fig. 4-1 it is seen that the experimental and theoretical value of the stresses at the autofrettaging pressure is almost the same assuming the yield strength to be 107,300 psi, whereas the theoretical value is considerably different from the experimental value when an assumed value of 110,000 psi is used. The opposite phenomenon holds for the residual stresses upon release of the autofrettaging pressure. These comments point up the necessity of knowing accurately the properties of any materials intended to be used for autofrettaging work.

Fig. 4-2 presents curves similar in pattern to those of Fig. 4-1, although the check between theoretical and experimental values is not as close as for Fig. 4-1.

Fig. 4-3 was obtained from data from the same test producing the data for Figs. 4-1 and 4-2, using an assumed yield strength of 110,000 psi. The curves show the distribution of the theoretical residual stresses upon release of the autofrettaging pressure. The values were obtained from formulas presented herein, and the curves are presented merely to show their shape, so neither the computations nor a tabulation of the curve values are presented herein.

The results given in Figs. 4-1, 4-2, and 4-3 cannot be used in a quantitative manner, because the exact properties of the high pressure tubes from which the strain data were obtained were not available. In actual practice, procedures have been worked out to obtain very close correlation between desired and actual values of residual strain in autofrettaging procedures. As the primary objective of this thesis was to present and discuss the theoretical aspects of the autofrettaging procedure, further experimental work was not done.

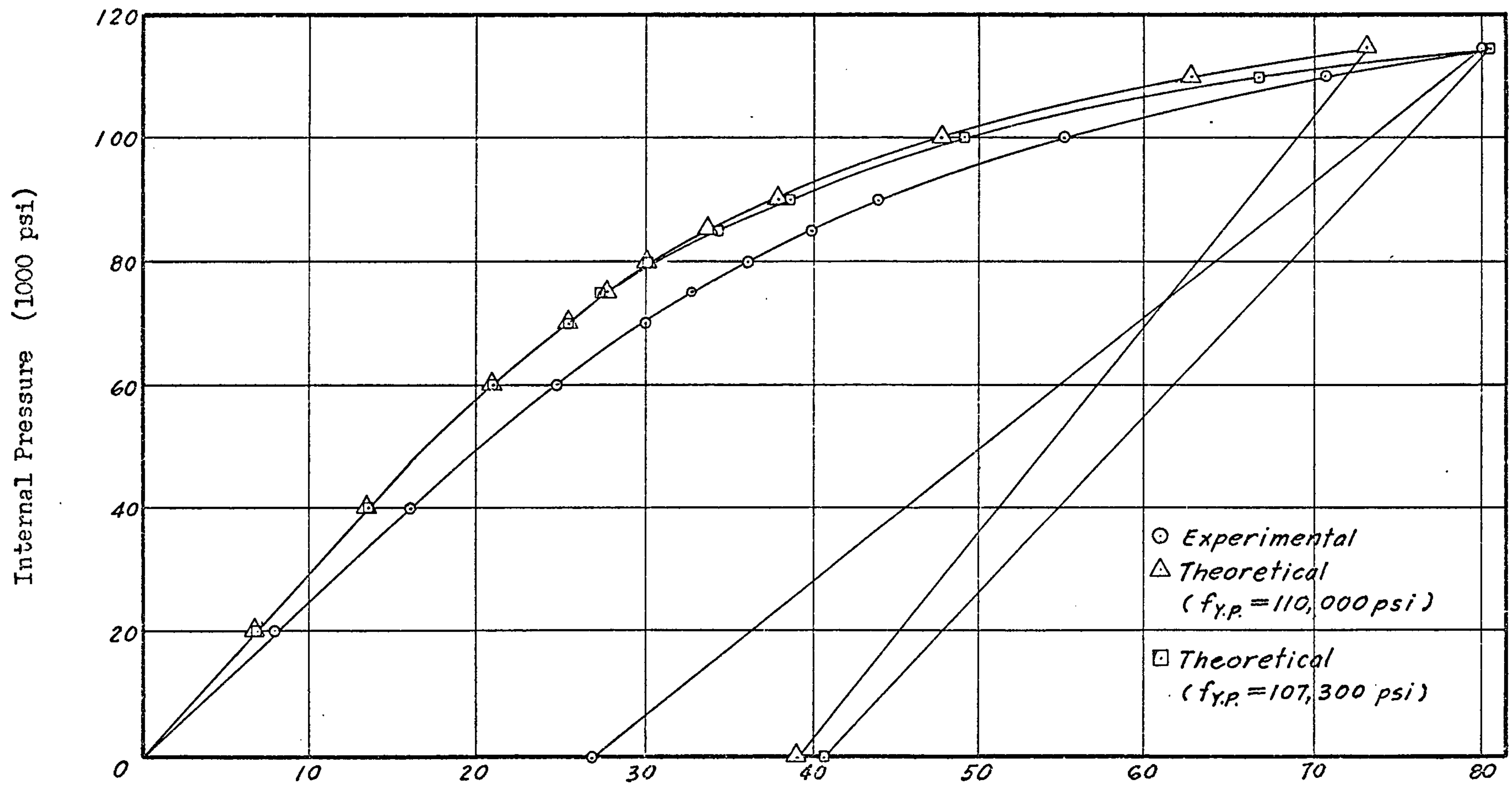


Fig. 4-1 Tangential Stresses (1000 psi)

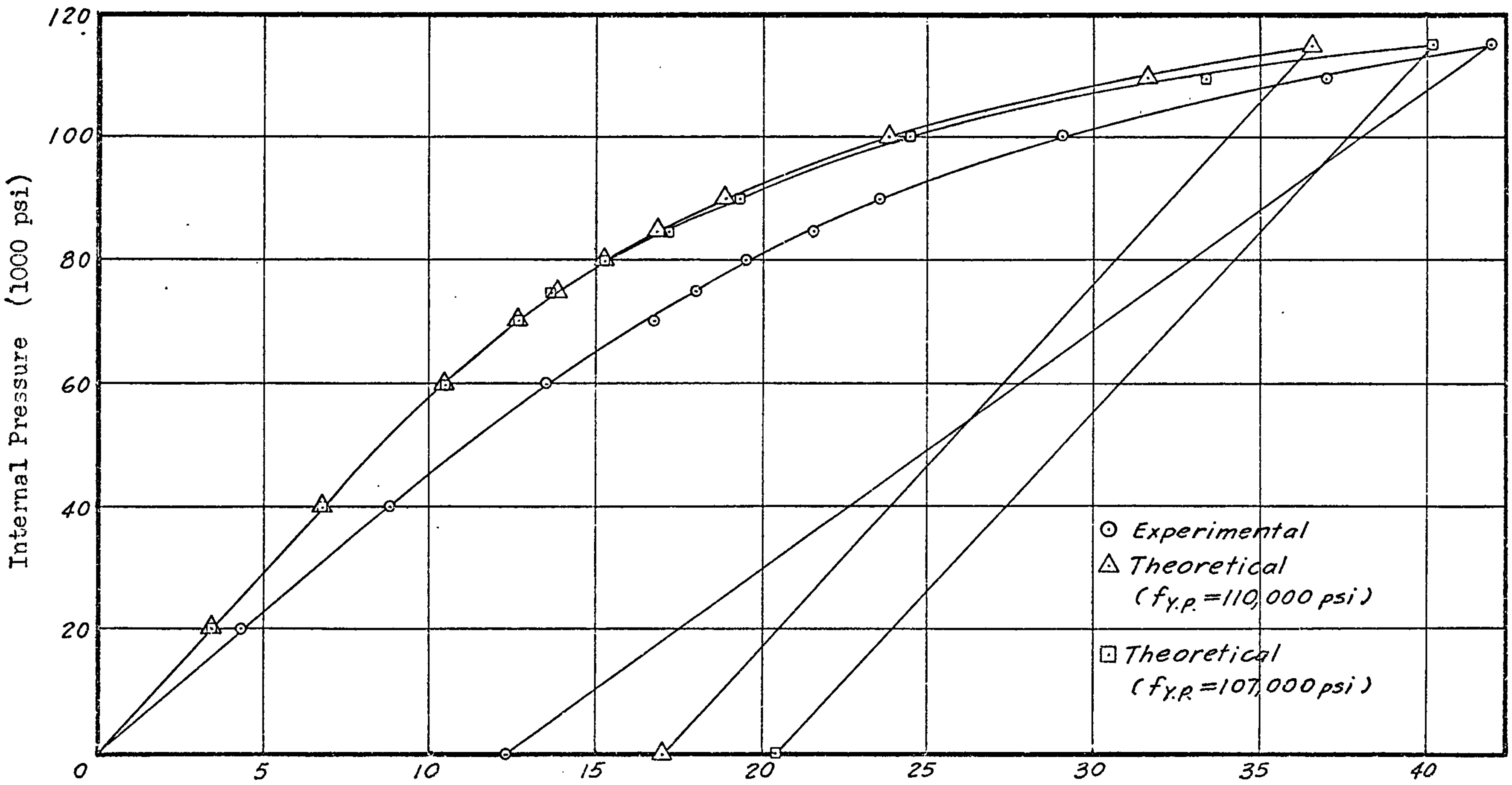


Fig. 4-2 Axial Stresses (1000 psi)

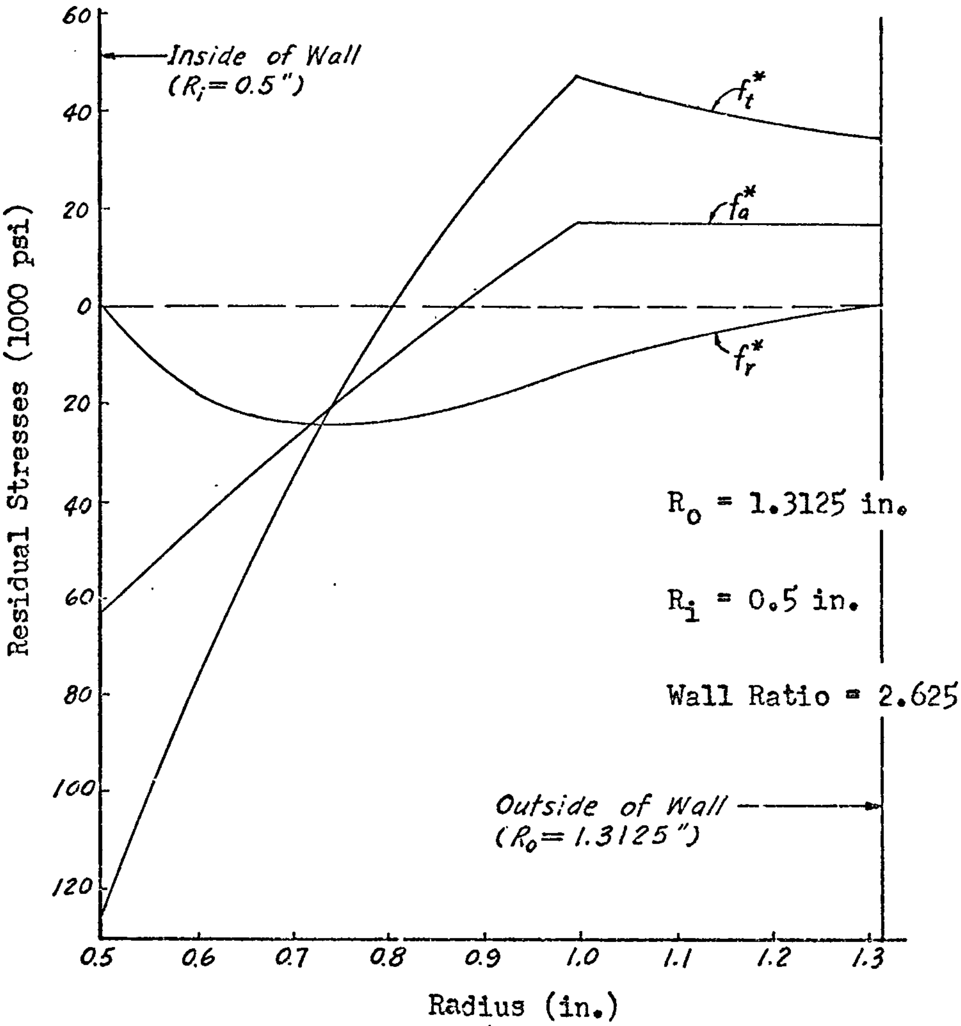


Fig. 4-3

Prediction of Distribution of Residual Stress

CHAPTER V

CONCLUSIONS

The primary objective of this thesis was to obtain all the available references pertinent to the autofrettaging procedure, and to study, evaluate, and correlate the theoretical and practical aspects of this process. This has been done, and certain conclusions may be drawn.

Of the several methods of autofrettaging, only the conventional method involving the use of internal pressures appears to be of practical value today.

There are actually only two different methods for predicting the stresses and strains resulting from the autofrettaging procedure. These are based on tension-test and torsion-test data, and give predicted values which agree very closely.

The most practical way of checking on the effectiveness of an autofrettaging procedure is to use electric resistance strain gages on the outside of the tube or vessel, and to measure the residual strains upon removal of the autofrettaging pressure. Diameter measurements may also be used in a similar manner, but the actual change in diameter is often very small, and great care must be taken in making such measurements.

The conventional method of autofrettaging produces a remarkable improvement in working stress conditions and in the fatigue life of high pressure tubes and vessels, and the procedure is very widely used today. Relatively few engineers are acquainted with the method, however, and much of the work is done at the present time is considered to be of a proprietary nature. The benefits of economy and safety derived from the procedure are very great, and its use will no doubt increase greatly in the future.

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A general theory for the elastic design of a compound cylinder made by shrinking together many concentric cylinders is described. Each cylinder may have entirely different strength and operate at any temperature under any pressure, internal or external. The only assumptions, in addition to the usual homogeneity and isotropy in each member, are that they have the same Young's modulus, although even different elastic moduli could be accommodated, and that they are not so thin that stability becomes a problem.

2. Brace W. F., "Effect of Pressure on Electric-resistance Strain Gages," Experimental Mechanics, vol. 4, no. 7, July 1964, PP. 212-216.

Electric resistance strain gage behavior under hydrostatic pressure is observed by attaching gage to variety of materials of known linear compressibility; simple theory is developed to explain pressure effect and predict its value.

3. Brasier R. I. and Dove R. C., "Use of Electric-resistance Strain Elements in Three-dimensional Stress Analysis," Experimental Mechanics, vol. 1, no. 6, June 1961, PP. 186-191.

The application of this new 3-D technique to evaluate the stress distribution in a thick-walled cylindrical pressure vessel is discussed. Correlation between experimental data and calculated values is given.

4. Brownell and Young, "Process Equipment Design," Wiley, New York, 1959. (Chapter 14; High-pressure Monobloc Vessels Design)

Several criteria for the failure of nonprestressed monobloc vessels are presented and are compared both theoretically and with rupture data; this is followed by a discussion of the behavior of such vessels at elevated temperatures. The advantages of prestressing and methods of design using autofrettage are then considered.

5. Cook G., and Robertson, A., "The Strength of Thick Hollow Cylinders under Internal Pressure," Engineering, v. 92 (1911) p. 787.

6. Crossland B., "The Design of Thick-Walled Closed-Ended Cylinders Based on Torsion Data," Welding Research Council Bulletin Series, no. 94, February 1964.

Review of theoretical analysis for determining pressure contained by thick-walled closed-ended cylinder at onset of yielding and during subsequent plastic deformation up to maximum or "ultimate" pressure which cylinder will withstand; analysis requires shear stress-strain data which can be derived from either torsion or tension tests, results of bursting tests are considered in relation to values predicted for torsion and tension data, and advantage and disadvantages of each are considered; torsion test data are preferred for many reasons.

7. Crossland B. and Burns D. J., "Behavior of Compound Steel Cylinders Subjected to Internal Pressure," Proc. Inst. Mech. Engrs., vol. 175, no. 27, 1961.

The theoretical treatment is briefly discussed and a theory for the pressure-expansion curve for a partially plastic duplex or two-component vessel is given. Pressure tests on two duplex vessels are reported. It is concluded that considerable and unpredictable axial stresses are set up when the two-component cylinders are shrunk together. Apparently, however, these do not seriously influence the yield pressure computed on the assumption that no axial stresses are induced during shrinking. In the partially plastic region there is good agreement between the experimental values and the theoretical pressure-expansion curve. The ultimate pressure and experimental pressure-expansion curves at large strains are in good agreement with values predicted for monobloc cylinders of the same overall diameter ratio.

8. Davidson T. E., Barton C. S., Reiner A. N. and Kendall D. P., "The Autofrettage Principle, as Applied to High Strength, Light Weight Gun Tubes," Watervliet Arsenal, Watervliet, N.Y., Report No. WVT-RI-5907, Oct. 1959.

Developed and described are data and design criteria and the process to be utilized for the application of the autofrettage principle to high strength gun tubes. The developed data, which is based on a nominal yield strength of 165,000 pounds per square inch, is compared to various thick-wall cylinder theories.
Empirical data and relationships are presented for the pressure-strain curve, the one hundred per cent overstrain or optimum condition, permanent enlargement ratio, longitudinal strain and other phenomena associated with the overstraining of thick-wall cylinders of intermediate diameter ratio.

Strain 'stabilization', the hysteresis loop effect, effects of machining, and reverse yielding are briefly analyzed and discussed.
A typical experimentally determined residual stress distribution is presented and compared to that predicted from theory based on various yield criteria.
The results obtained from the testing of miniature specimens in the diameter ratio range of 1.4 to 2.4 are complimented by the autofrettage of a series of four high strength lightweight 90mm gun tubes of various design safety factors. The design technique for autofrettage of gun tubes along with an example based on one of the above 90mm tubes is presented.

9. Davidson T. E., Barton C. S., Reiner A. N. and Kendall D. P., "New Approach to the Autofrettage of High-strength Cylinders," Experimental Mechanics, vol. 2, no. 2, 1962, pp. 33-40.

Three methods of forcing an oversize mandrel through the tube to plastically deform the walls are investigated. An engineering analysis is made investigating such factors as per cent enlargement and elastic recovery at the bore, the ratio of pressure required for pushing the mandrel to the yield strength of the mandrel, the effectes of various lubricants on the frictional forces involved, and the induced three-dimensional stress in the cylinder walls.

10. Davidson T. E., Kendall D. P. and Reiner A. N., "Residual Stresses in Thick-walled Cylinders Resulting from Mechanically Induced Overstrain," Experimental Mechanics, Nov. 1963, pp. 253-262.

Purpose of investigation was to determine the residual stress distribution as a function of magnitude of overstrain and diameter ratio, and the reyielding characteristics of cylinders autofrettaged by sliding-wedge techniques.

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Data obtained from nearly 100 static cylinder tests under internal pressure have been examined statistically, and design formula proposed for elastic-breakdown pressure and bursting pressure. The formula are reliable within ± 15 per cent of the observed value on a 90 per cent certainty basis.

12. Faupel J. H. and Furbeck A. R. "Influence of Residual Stress on Behavior of Thick-wall Close-end Cylinders" Trans. ASME, v. 75 (1953), pp. 345-354.

Results obtained from macro residual-stress tests, internal-pressure tests, and mechanical property tests on Cr-Ni-Mo-V steel, SAE 3320, SAE 4340, and SAE 1045 steel cylinders are presented. The presence of micro residual stresses appears to influence only initial yielding at the bore of cylinders; the effect of this residual stress can be allowed for in the formula used to predict the internal pressure at initial yielding (overstrain) and bursting. Finally a method is suggested for predicting the pressure-strain curve for heavy-wall cylinders from an analysis of tensile-test data.

13. Franklin G. J. and Morrison J. L. M., "Autofrettage of Cylinders: Prediction of Pressure / External Expansion Curves and Calculation of Residual Stresses," Pros. Inst. Mech. Engrs., 175, (1961).

One of the assumptions used in the calculations has an important effect on the values of the residial axial stresses; moreover the axial dimensional changes during the boring-out process are so small that great accuracy cannot be claimed for the values measured. To this extent the results are inconclusive. It is, however, shown that both the dimensional changes and the shear stress distribution (from which the radial and hoop stress distribution can, of course, be deduced) at any stage in the process can be calculated with satisfactory accuracy by the relatively simple approximate methods described.

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Autofrettage defined as art of inducing elasticity in tube at pressures which otherwise cause overstrain, overstraining tube by internal pressure, and then annealing it at low temperature; in present state, autofrettage should be restricted to mild steel and low alloy steels used for guns, method of applying autofrettage involved, special charts, number of which are presented, and their derivation and preparation described.

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29. Tien J. K. and Gorden R. B., "Use of Electrical-Resistance Strain Gages under Hydrostatic Pressure," ASME - paper 64-WA/PT-20 for meeting Nov. 29 - Dec. 4, 1964,

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APPENDIX A

Derivations of Equations of Autofrettage Pressure, Stresses and Strains

In the problem which is to be discussed here, the ends of the thick-walled cylinder are restrained from motion in the axial direction and the axial strain ϵ_a vanishes throughout the tube and at all times; moreover, all stresses and strains are independent of tangential and axial directions.

In order to simplify the mathematical treatment of our problem, we assume the material of cylinder is incompressible in both the elastic and the plastic ranges, thus

$$\epsilon_r + \epsilon_t + \epsilon_a = 0$$

Since $\epsilon_a = 0$, therefore $\epsilon_t + \epsilon_r = 0$ (A-1)

Fig. A-1 shows an arbitrary point in the cross-section of a cylinder. Its dimensions are $dR \times R d\theta$. After a radially directed displacement u, we then have $\epsilon_r = \frac{du}{dR}$, and

$$\epsilon_t = \frac{u}{R} .$$

from Eq.(A-1) we have

$$\frac{du}{dR} + \frac{u}{R} = 0 \qquad \text{(A-2)}$$

which can be integrated by separating the variables and yields the solution

$$u = \frac{C}{R} \qquad \text{(A-3)}$$

when $R = R_i$

$$u_i = \frac{C}{R_i} \qquad or \qquad C = u_i R_i \tag{A-3a}$$

Using Eq. (A-3) and Eq. (A-3a), the three components of strain may be expressed as:

$$\epsilon_r = \frac{du}{dR} = -\frac{u_i R_i}{R^2} \tag{A-4}$$

$$\epsilon_t = \frac{u}{R} = \frac{u_i R_i}{R^2} \tag{A-5}$$

$$\epsilon_a = 0 \tag{A-6}$$

From Fig. A-2, Differential Equation of Equilibrium can be obtained:

$$(f_r + \frac{df_r}{dR} dR)(R+dR)d\theta da - f_r R d\theta da - f_t dR da d\theta = 0$$

$$\frac{df_r}{dR} + \frac{f_r - f_t}{R} = 0 \tag{A-7}$$

The elastic stress-strain relation, combined with the incompressibility condition $(\epsilon_a = 0,\ \epsilon_r + \epsilon_t = 0)$, is expressed as follows:

The relationships between the instantaneous states of stress and strain at a point of a solid body within the elastic range are expressed by the "generalized Hook's law." (23) This law can be simply written as follows:

$$S_r = 2Ge_r \qquad S_t = 2Ge_t \qquad S_a = 2Ge_a \tag{A-8}$$

where $S_r = f_r - S \qquad S_t = f_t - S \qquad S_a = f_a - S$

$$e_r = \epsilon_r - e \qquad e_t = \epsilon_t - e \qquad e_a = \epsilon_a - e$$

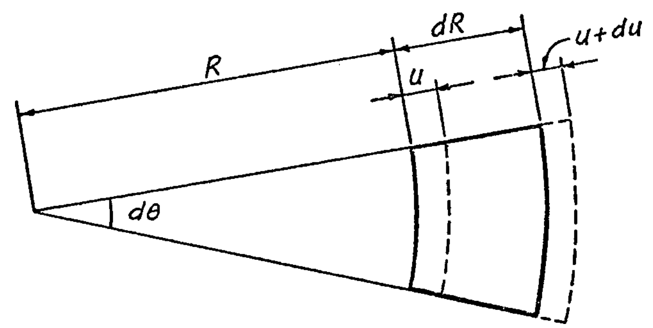

Fig. A-1 Displacement of a Volume Element

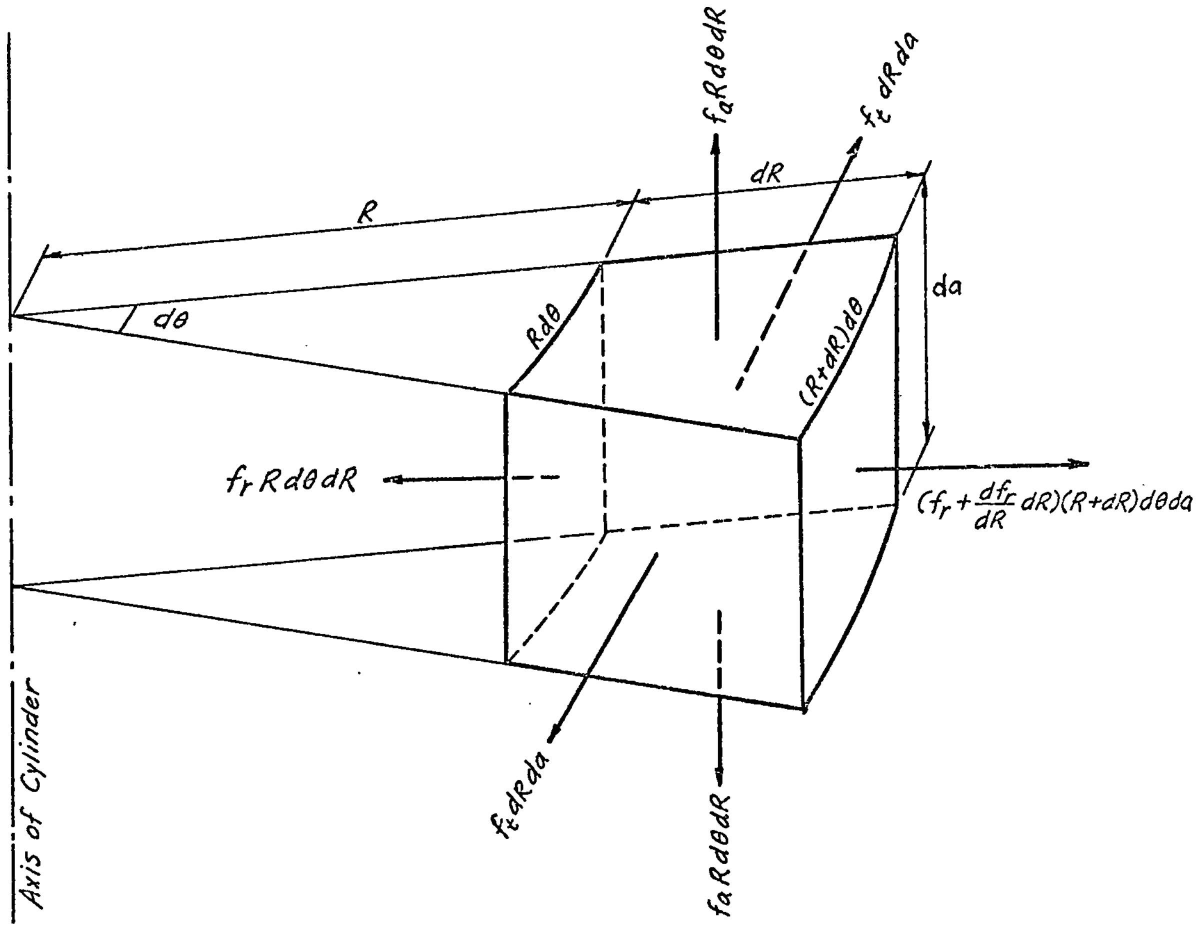

Fig. A-2 Forces on Volume Element

where $s = \frac{1}{3}(f_r + f_t + f_a)$ $\qquad e = \frac{1}{3}(\epsilon_r + \epsilon_t + \epsilon_a)$

From Eq. (A-8)

$$\frac{2f_r - f_t - f_a}{3} = 2G\,\frac{2\epsilon_r - \epsilon_t - \epsilon_a}{3}$$

$$\frac{2f_t - f_a - f_r}{3} = 2G\,\frac{2\epsilon_t - \epsilon_a - \epsilon_r}{3} \qquad \text{(A-9)}$$

$$\frac{2f_a - f_r - f_t}{3} = 2G\,\frac{2\epsilon_a - \epsilon_r - \epsilon_t}{3}$$

for $\epsilon_a = 0$ $\quad \epsilon_t = -\epsilon_r$, these relationships become

$$2f_r - f_t - f_a = 6G\epsilon_r \qquad \text{(A-10a)}$$

$$2f_t - f_a - f_r = -6G\epsilon_r \qquad \text{(A-10b)}$$

$$2f_a - f_r - f_t = 0 \qquad \text{(A-10c)}$$

From Eq. (A-10c) $f_a = \dfrac{f_r + f_t}{2}$, $\qquad$ (A-11)

substituting into Eqs. (A-10a) and (A-10b) yields

$$f_r - f_t = 4G\epsilon_r .$$

From Eq. (A-4)

$$f_r - f_t = -4G\frac{u_i R_i}{R^2} \qquad \text{(A-12)}$$

substituting Eq. (A-12) into Eq. (A-7), it yields

$$\frac{df_r}{dR} - \frac{4Gu_i R_i}{R^3} = 0 . \qquad \text{(A-13)}$$

Integrating Eq. (A-13), we have

$$f_r = C_1 - \frac{2Gu_i R_i}{R^2} \qquad \text{(A-14)}$$

when R = R_i

$$f_r = -p = C_1 - \frac{2Gu_i}{R_i}, \text{ and} \tag{A-15}$$

when R = R_o

$$f_r = 0 = C_1 - \frac{2Gu_iR_i}{R_o^2}. \tag{A-16}$$

Solving Eq. (A-15) and Eq. (A-16) simultaneously, we can get

$$C_1 = p\frac{R_i^2/R_o^2}{1-R_i^2/R_o^2}, \text{ and} \tag{A-17}$$

$$u_i = \frac{p}{2G}\cdot\frac{R_i}{1-R_i^2/R_o^2}. \tag{A-18}$$

Substituting Eq. (A-17) and Eq. (A-18) into Eqs. (A-14), (A-12) and (A-11), yields the following equations respectively:

$$f_r = p\frac{R_i^2/R_o^2 - R_i^2/R^2}{1-R_i^2/R_o^2} \tag{A-19}$$

$$f_t = p\frac{R_i^2/R_o^2 + R_i^2/R^2}{1-R_i^2/R_o^2} \tag{A-20}$$

$$f_a = p\frac{R_i^2/R_o^2}{1-R_i^2/R_o^2} \tag{A-21}$$

The von Mises condition of yielding for plane strain will be as follows: (31)

$$(f_r - f_t)^2 + (f_t - f_a)^2 + (f_a - f_r)^2 = 2 f_{y.p.}^2 \qquad \text{(A-22)}$$

By substituting f_r, f_t, f_a from Eqs. (A-19), (A-20) and (A-21), this relationship becomes

$$p \frac{R_i^2/R^2}{1 - R_i^2/R_o^2} = \frac{f_{y.p.}}{\sqrt{3}}$$

When $R = R_i$, the inner surface begins yielding at a value of the internal pressure

$$p' = \frac{f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_i^2}{R_o^2}\right) \qquad \text{(A-23)}$$

Substituting p' for p and R_c for R_i in Eqs. (A-19), (A-20) and (A-21), we obtain equations of <u>stresses in elastic zone</u> as follows:

$$f_r = \frac{f_{y.p.}}{\sqrt{3}} \left(\frac{R_c^2}{R_o^2} - \frac{R_c^2}{R^2}\right) \qquad \text{(A-24)}$$

$$f_t = \frac{f_{y.p.}}{\sqrt{3}} \left(\frac{R_c^2}{R_o^2} + \frac{R_c^2}{R^2}\right) \qquad \text{(A-25)}$$

$$f_a = \frac{f_{y.p.}}{\sqrt{3}} \left(\frac{R_c^2}{R_o^2}\right) \qquad \text{(A-26)}$$

By substituting Eq. (A-11) into Eq. (A-22), the von Mises yield condition becomes

$$f_t - f_r = \frac{2 f_{y.p.}}{\sqrt{3}} \qquad \text{(A-27)}$$

Substituting Eq. (A-27) into Eq. (A-7), then we have

$$\frac{df_r}{dR} - \frac{2 f_{y.p.}}{\sqrt{3}\, R} = 0 \text{ , and}$$

integrating it, we may have

$$f_r = \frac{2 f_{y.p.}}{\sqrt{3}} \ln R + C_2 \text{ ,} \qquad \text{(A-28)}$$

when R = R_c

$$(f_r)_{R=R_c} = \frac{2 f_{y.p.}}{\sqrt{3}} \ln R_c + C_2 \text{ .}$$

And this value must equal, with negative sign, the pressure at the inner surface of the elastic portion of the tube as expressed by Eq. (A-23) ($R_i = R_c$)

$$\frac{2 f_{y.p.}}{\sqrt{3}} \ln R_c + C_2 = -\frac{f_{y.p.}}{\sqrt{3}}\left(1 - \frac{R_c^2}{R_o^2}\right)$$

$$C_2 = -\frac{f_{y.p.}}{\sqrt{3}}\left(1 - \frac{R_c^2}{R_o^2} + 2 \ln R_c\right) \qquad \text{(A-29)}$$

Entering C_2 into Eqs. (A-28), (A-27), and (A-11), we have the equations for <u>stresses in plastic zone</u> as follows:

$$f_r = -\frac{f_{y.p.}}{\sqrt{3}}\left(1 - \frac{R_c^2}{R_o^2} - 2 \ln \frac{R}{R_c}\right) \qquad \text{(A-30)}$$

$$f_t = \frac{f_{y.p.}}{\sqrt{3}}\left(1 + \frac{R_c^2}{R_o^2} + 2 \ln \frac{R}{R_c}\right) \qquad \text{(A-31)}$$

$$f_a = \frac{f_t + f_r}{2} = \frac{f_{y.p.}}{\sqrt{3}} \left(\frac{R_c^2}{R_o^2} + 2 \ln \frac{R}{R_c} \right) \tag{A-32}$$

We can get the autofrettage pressure from Eq. (A-30), when $R = R_i$, $f_r = - p^*$ thus

$$p^* = \frac{f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_c^2}{R_o^2} - 2 \ln \frac{R_i}{R_c} \right) \tag{A-33}$$

When $R_c = R_i$ and $R_c = R_o$ in Eq. (A-33), we can determine break-down pressure and bursting pressure respectively.

$$p' = \frac{f_{y.p.}}{\sqrt{3}} \left(1 - \frac{R_i^2}{R_o^2} \right) \tag{A-34}$$

$$p'' = \frac{2 f_{y.p.}}{\sqrt{3}} \left(\ln \frac{R_o}{R_i} \right) \tag{A-35}$$

The method of computing the changes in stresses resulting from unloading the autofrettage pressure will depend upon whether or not the unloading stresses exceed the compressive yield strength of the material in Fig. (A-3). Vessels will have unloading stresses within the elastic region if

$$K = \frac{R_o}{R_i} \leq 2.22 \tag{A-36}$$

$$p' \leq \Delta p < p'' \tag{A-37}$$

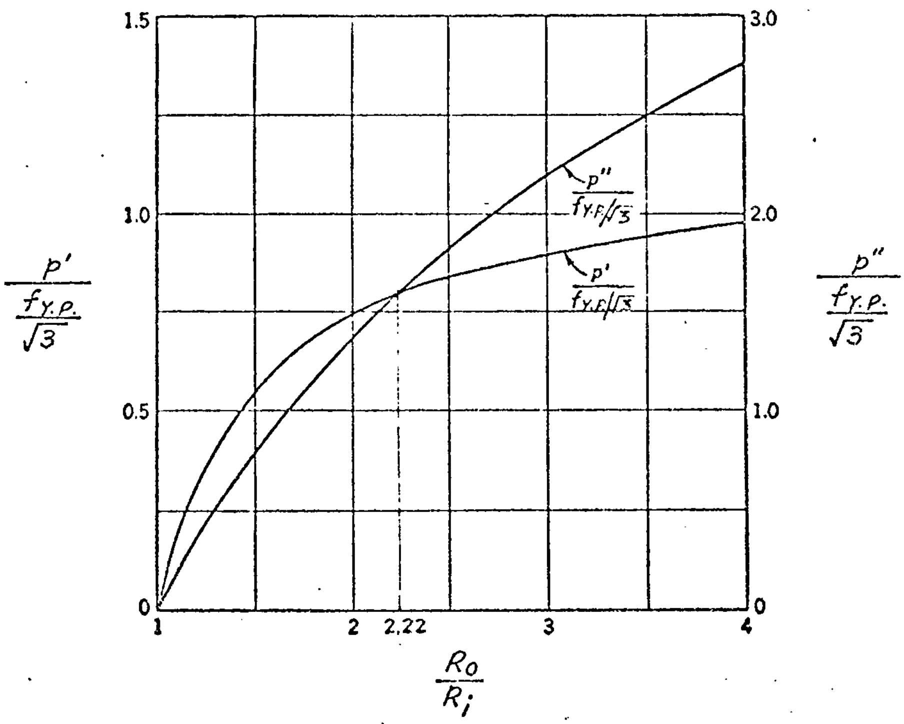

Fig. A-3 Limiting Pressures
(See Ref. 26.)

Fig. A-3 shows p' and p" vs. $\frac{R_o}{R_i}$, the scale used for p' being twice that used for p". The abscissa of the point of intersection of the curves gives the value of $\frac{R_o}{R_i}$ (= 2.22) for which the equality sign holds in next equation

$$p'' = 2p'$$

or if

$$K = \frac{R_o}{R_i} > 2.22 \qquad \text{(A-38)}$$

$$\Delta p \leq 2p' \qquad \text{(A-39)}$$

where $\Delta p = p^* - p_o \qquad (p_o = 0)$

for $p = -\Delta p$ from Eqs. (A-19), (A-20), (A-21)

we have <u>stress equations for the whole wall of cylinder:</u>

$$\Delta f_r = -\Delta p \left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(1 - \frac{R_o^2}{R^2}\right) \qquad \text{(A-40)}$$

$$\Delta f_t = -\Delta p \left(\frac{R_i^2}{R_o^2 - R_i^2}\right)\left(1 + \frac{R_o^2}{R^2}\right) \qquad \text{(A-41)}$$

$$\Delta f_a = -\Delta p \left(\frac{R_i^2}{R_o^2 - R_i^2}\right) \qquad \text{(A-42)}$$

Vessels will have loading stresses beyond the compressive yield strength if

$$K = \frac{R_o}{R_i} > 2.22 \qquad \text{(A-43)}$$

$$2p' \leq \Delta p < p'' \qquad \text{(A-44)}$$

The elastic range for unloading from a plastic state and subsequent loading in the opposite sense equals twice the elastic range for the original loading.

In accordance with the doubling of the elastic range for the unloading process, $f_{y.p.}$ must now be replaced by 2 $f_{y.p.}$ For a given $\Delta p > 2p'$ we must therefore determine R_j by solving

$$\frac{R_j^2}{R_o^2} - 2\ln\frac{R_j}{R_o} - 1 = 2\ln\frac{R_o}{R_i} - \frac{\Delta p}{2f_{Y.P.}/\sqrt{3}} \qquad \text{(A-45)}$$

with the aid of Fig. (2-1) (See chapter 2.)

We then have the changes in autofrettage-pressure stresses due to unloading:

If $R_i \leq R \leq R_j$

for $-2f_{Y.P.} = f_{Y.P.}$, $R_j = R_c$,

from Eqs. (A-30), (A-31) and (A-32), the changes of stresses in plastic zone will be

$$\Delta f_r = \frac{2f_{Y.P.}}{\sqrt{3}}\left(1 - \frac{R_j^2}{R_o^2} - 2\ln\frac{R}{R_j}\right) \qquad \text{(A-46)}$$

$$\Delta f_t = \frac{-2f_{Y.P.}}{\sqrt{3}}\left(1 + \frac{R_j^2}{R_o^2} + 2\ln\frac{R}{R_j}\right) \qquad \text{(A-47)}$$

$$\Delta f_a = \frac{-2f_{Y.P.}}{\sqrt{3}}\left(\frac{R_j^2}{R_o^2} + 2\ln\frac{R}{R_j}\right) \qquad \text{(A-48)}$$

and from Eqs. (A-24), (A-25) and (A-26) the changes of stresses in elastic zone will be

$$\Delta f_r = -\frac{2 f_{Y.P.}}{\sqrt{3}} \left(\frac{R_j^2}{R_o^2} - \frac{R_j^2}{R^2} \right) \tag{A-49}$$

$$\Delta f_t = -\frac{2 f_{Y.P.}}{\sqrt{3}} \left(\frac{R_j^2}{R_o^2} + \frac{R_j^2}{R^2} \right) \tag{A-50}$$

$$\Delta f_a = -\frac{2 f_{Y.P.}}{\sqrt{3}} \left(\frac{R_j^2}{R_o^2} \right) \tag{A-51}$$

The unit radial strain in both the elastic and plastic regions under autofrettage pressure will be derived as follow:

Substituting $p = p'$ and $R_i = R_c$ into the Eq. (A-18), the displacement of the point of the plastic front is obtained as

$$u_c = \frac{p'}{2G} \left(\frac{R_c}{1 - R_c^2 / R_o^2} \right) \tag{A-52}$$

From Eq. (A-23)

$$p' = \frac{f_{Y.P.}}{\sqrt{3}} \left(1 - \frac{R_i^2}{R_o^2} \right),$$ for $R_i = R_c$, substitute into Eq. (A-52)

$$u_c = \frac{f_{Y.P.}}{\sqrt{3}} \cdot \frac{R_c}{2G}$$

From Eq. (A-4), then we have

$$\epsilon_r = -\frac{u_c R_c}{R^2} = -\frac{f_{Y.P.}}{\sqrt{3}} \left(\frac{R_c^2}{2 G R^2} \right) \tag{A-53}$$

where
$$G = \frac{E}{2\,(1+\mu)}$$

Upon unloading of the autofrettage pressure a change in unit radial strain $\Delta\epsilon_r$ will occur.

If the conditions are same as Eqs. (A-36), (A-37), (A-38) and (A-39), for p = $-\Delta$p, Eq. (A-18) will be

$$\Delta u_i = -\frac{\Delta p}{2G}\left(\frac{R_i}{1-R_i^2/R_o^2}\right)$$

from Eq. (A-4),

$$\Delta\epsilon_r = -\frac{\Delta u_i R_i}{R^2} = \frac{\Delta p}{2G}\left(\frac{R_i^2}{(1-R_i^2/R_o^2)R^2}\right)$$

$$\Delta\epsilon_r = \frac{\Delta p}{2G}\left(\frac{R_i^2 R_o^2}{(R_o^2-R_i^2)R^2}\right) \qquad \text{(A-54)}$$

If the conditions are same as Eqs. (A-43) and (A-44),

for $\quad p = -\Delta p \quad or \quad f_{y.p.} = -2f_{y.p.} \,, \quad R_c = R_j \,,$

from Eq. (A-53),

$$\Delta\epsilon_r = -\frac{2f_{y.p.}}{\sqrt{3}}\left(\frac{R_j^2}{2GR^2}\right) = -\frac{f_{y.p.}}{\sqrt{3}}\left(\frac{R_j^2}{GR^2}\right) \qquad \text{(A-55)}$$

APPENDIX B

Derivation of the Sachs Boring-out Method

Consider a tube (see Fig. B-1) with an inside diameter $2R_i$ and an outside diameter $2R_o$. The longitudinal, tangential, and radial residual stresses at an arbitrary point at a distance R from the axis are f_a^*, f_t^*, and f_r^*, respectively. The original cross sectional area corresponding to the outside diameter of the tube is A_o or R_o^2. After boring out the hole to a radius R' (corresponding to a cross section $A = R'^2$), the surface strains at the outside surface are measured as ϵ_a' (axial strain) and ϵ_t' (tangential strain). The stresses f_a^*, f_t^*, and f_r^* originally present in the cylinder at radius R may be considered as being the sum of the remaining stresses after boring out to a radius R and the stress that was removed in the boring-out process. Denote the remaining stresses by f_a'', f_t'', and f_r'' and the stresses removed by the boring as f_a', f_t', and f_r'. Then

$$f_a^* = f_a' + f_a'' \qquad \text{(B-1)}$$

$$f_t^* = f_t' + f_t'' \qquad \text{(B-2)}$$

$$f_r^* = f_r' + f_r'' \qquad \text{(B-3)}$$

(1) Calculation of the stresses Remaining after Boring

Since the radial stress at outside surface is zero

$$f_r'' = 0 \qquad \text{(B-4)}$$

and Eq. (B-3) becomes

$$f_r^* = f_r' \qquad \text{(B-5)}$$

For the purpose of simplicity in relating the measured surface strains to the unknown stresses within the metal, fictitious surface stresses F_a and F_t will be introduced. These are related to the measured surface strains, according to the theory of elasticity, by the relation

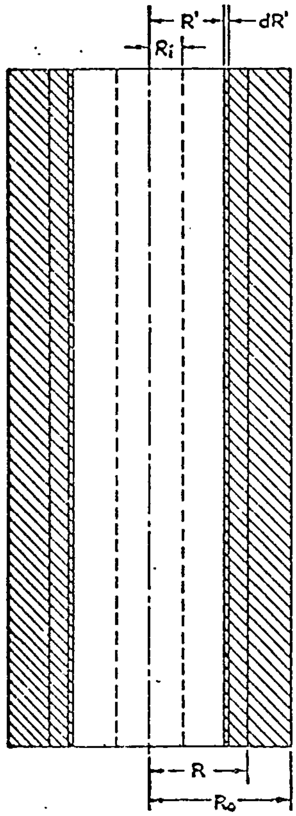

Fig. B-1 The Cross-section of Tube Under a Boring-out Process

$$E\,\epsilon_a' = F_a - \mu\,F_t \tag{B-6}$$

$$E\,\epsilon_t' = F_t - \mu\,F_a \tag{B-7}$$

where E is the modulus of elasticity and μ is Poisson's ratio.

Solving for the surface stresses yields the equations

$$F_a = \frac{E}{1-\mu^2}\,(\epsilon_a' + \mu\,\epsilon_t') \tag{B-8}$$

$$F_t = \frac{E}{1-\mu^2}\,(\epsilon_t' + \mu\,\epsilon_a') \tag{B-9}$$

To determine the remaining axial stress, f_a'', in the surface of the bore after boring out a cross section A, assume that axial stress introduced into the remaining cylinder by boring out an additional cross section dA is uniformly distributed over the remaining cylinder of the cross section. Then the change in the average axial stress in the remaining cylinder will be the same as the change in the outside surface stress dF_a. Then by equilibrium of axial forces

$$f_a''\,dA = dF_a\,(A_o - A) \tag{B-10}$$

The small change dF_a in the surface stress corresponds to small changes in the measured strains, which are determined from Eqs. (B-8) and (B-9) as follows

$$dF_a = \frac{E}{1-\mu^2}\,(d\epsilon_a' + \mu\,d\epsilon_t') \tag{B-11}$$

$$dF_t = \frac{E}{1-\mu^2}\,(d\epsilon_t' + \mu\,d\epsilon_a') \tag{B-12}$$

Substituting Eq. (B-11) into Eq. (B-10) and solve for f_a'' yields the desired relation

$$f_a'' = \frac{E}{1-\mu^2} \cdot (A_0 - A) \; \frac{d\epsilon_a' + \mu \, d\epsilon_t'}{dA} \tag{B-13}$$

The value of f_t'' as a function of the measured strains may be determined in a similar manner by finding the effect on the surface stresses of boring out an additional thin shell of cross section dA from the inner surface that results from boring out to a cross section A. Such a thin shell before removal by boring was subject to a tangential stress f_t'' , and internal pressure of zero, and an external pressure which is the same as the radial stress at its outside surface (before boring out). By the formula for a thin-walled tube:

$$df_r'' = f_t'' \frac{dR}{R} \tag{B-14}$$

The effect on the measured strains of boring out the thin shell is due to the removal of the radial stress df_r'' at a radius R + dR and consequently may be considered as due to an internal pressure of f_r''. By the theory of elasticity the tangential stress in the outside surface of a cylinder subjected to an internal pressure p

$$f_t = \frac{2R_i^2}{R_o^2 - R_i^2} \, p \tag{B-15}$$

for $f_t = dF_t$, $R_i = R$ and $p = df_r''$

$$dF_t = \frac{2R^2}{R_o^2 - R^2} \, df_r'' \tag{B-16}$$

Combining Eq.(B-16) with Eq.(B-14) and Eq.(B-12) and solving for f_t''

$$f_t'' = (A_0 - A)\,\frac{E}{1-\mu^2}\cdot\frac{d\epsilon_t' + \mu\, d\epsilon_a'}{dA} \tag{B-17}$$

Thus Eqs. (B-13) and (B-17) determine the desired remaining surface stresses after boring out.

(2) Calculation of the Changes in Stress due to Boring

In order to determine the changes in stress f_r', f_t', and f_a' at a radius R due to boring out the hole from a radius R_1 to a radius R, consider the effect at radius R caused by boring out a layer dR' at a radius R' (some arbitrary point between R_1 and R), see Fig. B-1. Then any changes caused by boring out from R_1 to R will be the summation by integration of the changes caused by boring out the layer dR'.

Assuming, as previously, that any change in longitudinal stress caused by boring out is uniformly distributed over the remaining cross section:

$$df_a' = -\,dF_a \tag{B-18}$$

for an element dR' bored out, and likewise:

$$f_a' = -\,F_a \tag{B-19}$$

for the entire quantity A bored out.

Substituting Eq. (B-8) in Eq. (B-19) yields:

$$f_a' = -\frac{E}{1-\mu^2}\,(\epsilon_a' + \mu\,\epsilon_t') \tag{B-20}$$

The negative sign in Eq. (B-18) is due to the fact that F_a is a stress that causes changes in measured strains, while f_a' is a stress the removal of which causes the same measured strains. Thus f_a' and F_a must be of opposite signs.

To determine f_t', consider the effect at radius R of boring out a shell dR'. This has the effect of imposing an internal pressure equal to the radial stress at R' + dR' upon the cylinder extending from radius R' + dR' to radius R_o. The changes in the tangential stress f_t' at R is desired. The stress within the wall of a tube is given by the theory of elasticity as:

$$f_t = \frac{(R^2 + R_o^2)R_i^2}{R^2(R_o^2 - R_i^2)} p \tag{B-21}$$

Applying to the situation at hand

$$f_t = df_t', \quad R_i = (R' + dR') \quad and \quad p = df^*_{r(R'+dR')}$$

$$df_t' = \frac{(R^2 + R_o^2)(R' + dR')^2}{R^2\left[R_o^2 - (R'+dR')^2\right]} df_{r(R'+dR')}^* \tag{B-22}$$

where $df^*_{r(R'+dR')}$ was the radial stress at R' + dR before boring out the shell dR'. The same boring out process produces a corresponding change in the surface stress F_t. This can be found by applying Eq. (B-21) to the point where R = R_o:

$$F_t = \frac{2R_i^2}{R_o^2 - R_i^2} p \tag{B-23}$$

For the cylinder of I.D. = 2 (R' + dR') and O.D. = $2R_o$, becomes:

$$dF_t = \frac{2(R'+dR')^2}{R_o^2 - (R'+dR')^2} df^*_{r(R'+dR')} \tag{A-24}$$

Eliminating $df^*_{r(R'+dR')}$ between Eq. (B-22) and Eq. (B-24), replacing F_t by Eq.(B-12), solving for df_t', integrating, and changing the sign:

$$f_t' = -\frac{E}{1-\mu^2}\cdot\frac{A_0+A}{2A}\,(\epsilon_t' + \mu\,\epsilon_a') \tag{B-25}$$

The change in sign is made for the same reason as in Eq. (B-18).

The radial stress f_r' is determined in the same way as f_t', i.e., by considering the effect at radius R of boring out a shell dR'. For this purpose, the radial stress in the wall of a tube is required. This is given by the theory of elasticity as:

$$f_r = \frac{(R^2-R_o^2)\,R_i^2}{R^2\,(R_o^2-R_i^2)}\,p \tag{A-26}$$

in which the notation is the same as in Eq. (B-21). Applying to the cylinder of inside radius R' + dR' and outside radius R_o, the change in radial stress at R is found to be:

$$df_r' = \frac{(R^2-R_o^2)(R'+dR')^2}{R^2\left[R_o^2-(R'+dR')^2\right]}\,df^*_{r(R'+dR')} \tag{B-27}$$

considering that the removal of the layer dR' is equivalent to applying an internal pressure to the remaining cylinder equivalent to the radial stress at R' + dR'. Eliminating df_r' between Eqs. (B-24) and (B-27), replacing dF_t by Eq. (B-12), solving for $df_r{}^*{}_{(R'+dR)}$, integrating, and changing sign as before:

$$f_r' = \frac{E}{1-\mu^2}\cdot\frac{A_0-A}{2A}\,(\epsilon_t' + \mu\,\epsilon_a') \tag{B-28}$$

Combining Eqs. (B-13), (B-17), (B-20), (B-25) and (B-28) according to Eqs. (B-1), (B-2), and (B-5),

$$f_a^* = \frac{E}{1-\mu^2}\left[(A_o - A)\,\frac{d\epsilon_a' + \mu\, d\epsilon_t'}{dA} - (\epsilon_a' + \mu\,\epsilon_t')\right] \quad \text{(B-29)}$$

$$f_t^* = \frac{E}{1-\mu^2}\left[(A_o - A)\,\frac{d\epsilon_t' + \mu\, d\epsilon_a'}{dA} - \frac{A_o + A}{2A}(\epsilon_t' + \mu\,\epsilon_a')\right] \quad \text{(B-30)}$$

$$f_r^* = \frac{E}{1-\mu^2}\left[\frac{A_o - A}{2A}\,(\epsilon_t' + \mu\,\epsilon_a')\right] \quad \text{(B-31)}$$

which are the desired relations. These may also be written as:

$$f_a^* = \frac{E}{1-\mu^2}\left[(A_o - A)\,\frac{d\alpha}{dA} - \alpha\right]$$

$$f_t^* = \frac{E}{1-\mu^2}\left[(A_o - A)\,\frac{d\beta}{dA} - \frac{A_o + A}{2A}\cdot\beta\right]$$

$$f_r^* = \frac{E}{1-\mu^2}\left[\frac{A_o - A}{2A}\cdot\beta\right]$$

where:

$$\alpha = \epsilon_a' + \mu\,\epsilon_t'$$

$$\beta = \epsilon_t' + \mu\,\epsilon_a'$$

APPENDIX C

This section of the thesis is devoted to computations necessary to obtain values for plotting Figs. 4-1 and 4-2. Most of the work is concerned with computation of the predicted stresses on the outside of the tube for various pressures during the autofrettaging procedure, and the last portion of the section contains a table of the stresses obtained from electric resistance strain gage measurements. Two-element rosettes were used, and an ordinary nomograph used to compute the corresponding stresses corresponding to the measured strains.

Calculations of Predicting Stresses on the Outside Wall of Cylinder During the Autofrettage Process

1. Given data:

   O.D. = 2 5/8" $\qquad R_o = 1.3125"$

   I.D. = 1" $\qquad R_i = 0.5"$

   Wall thickness = 0.8125"

   Wall ratio $K = \dfrac{R_o}{R_i} = 2.625$

   Material: AISI 4335 Alloy Steel

   Yield strength in tension $f_{y.p.} = 110{,}000$ psi

   Ultimate strength in tension $f_{t.s.} = 125{,}000$ psi

2. Assume autofrettage pressure $p^* = 114{,}500$ psi.

3. Elastic-plastic interface, R_c, can be obtained from

$$p^* = \frac{f_{y.p.}}{\sqrt{3}}\left(1 - \frac{R_c^2}{R_o^2} - 2\ln\frac{R_i}{R_c}\right) \tag{2-1}$$

or from

$$\frac{R_c^2}{R_o^2} - 2\ln\frac{R_c}{R_o} - 1 = 2\ln\frac{R_o}{R_i} - \frac{p^*}{f_{Y.P}/\sqrt{3}} \qquad (2\text{-}3)$$

with the aid of Fig. 2-1 to find out $\frac{R_c}{R_o}$.

$$\frac{R_c^2}{R_o^2} - 2\ln\frac{R_c}{R_o} - 1 = 2\ln 2.625 - \frac{114{,}500}{\frac{110.000}{1.732}} = 0.128$$

$$\frac{R_c}{R_o} = 0.759 \; ; \qquad R_c = 0.759\,(1.3125) = 0.996''$$

4. Elastic break-down pressure :

$$p' = \frac{f_{Y.P}}{\sqrt{3}}\left(1 - \frac{R_i^2}{R_o^2}\right) = \frac{110{,}000}{1.732}\left[1 - \left(\frac{0.5}{1.3125}\right)^2\right] = 54{,}300 \text{ psi}$$

5. Bursting pressure:

$$p'' = \frac{2 f_{Y.P.}}{\sqrt{3}}\left[\ln\frac{R_o}{R_i}\right] = \frac{2 \times 110{,}000}{1.732}(\ln 2.625) = 122{,}500 \text{ psi}$$

6. Stresses prediction during loading process:

(a) When internal pressure $p < p'$, stresses according to Láme's theory (4) will be:

$$f_t = \left[\frac{R_i^2}{R_o^2 - R_i^2} + \frac{R_o^2 R_i^2}{R^2(R_o^2 - R_i^2)}\right] p$$

$$f_a = \left[\frac{R_i^2}{R_o^2 - R_i^2}\right] p$$

when $R = R_o$,

$$f_t = 2\left[\frac{R_i^2}{R_o^2 - R_i^2}\right] p$$

$$f_a = \frac{f_t}{2}$$

when p = 20,000 psi : $\quad f_t = 6{,}780 \text{ psi} \; ; \qquad f_a = 3{,}390 \text{ psi}$

when p = 40,000 psi:

$$f_t = 13,560 \text{ psi} \quad ; \qquad f_a = 6,780 \text{ psi}$$

(b) When internal pressure p > p', stresses will be

$$f_t = \frac{f_{Y.P.}}{\sqrt{3}} \left[\frac{R_c^2}{R_o^2} + \frac{R_c^2}{R^2} \right] \tag{2-11}$$

$$f_a = \frac{f_{Y.P.}}{\sqrt{3}} \left[\frac{R_c^2}{R_o^2} \right] \tag{2-12}$$

when R = R_o

$$f_t = \frac{2 f_{Y.P.}}{\sqrt{3}} \left[\frac{R_c^2}{R_o^2} \right]$$

$$f_a = \frac{f_t}{2}$$

$$f_t = \frac{2 \times 110,000}{1.732} \times \frac{R_c^2}{(1.3125)^2} = 73,140 \times R_c^2$$

when p = 60,000 psi, R_c = 0.533":

f_t = 21,000 psi ; f_a = 10,500 psi

when p = 70,000 psi, R_c = 0.587":

f_t = 25,400 psi ; f_a = 12,700 psi

when p = 75,000 psi, R_c = 0.613"

f_t = 27,700 psi ; f_a = 13,850 psi

when p = 80,000 psi, R_c = 0.643"

f_t = 30,500 psi ; f_a = 15,250 psi

when $p = 85{,}000$ psi , $R_c = 0.676''$

$f_t = 33{,}750$ psi ; $f_a = 16{,}875$ psi

when $p = 90{,}000$ psi , $R_c = 0.717''$

$f_t = 37{,}800$ psi ; $f_a = 18{,}900$ psi

when $p = 100{,}000$ psi, $R_c = 0.804''$

$f_t = 47{,}700$ psi ; $f_a = 23{,}850$ psi

when $p = 110{,}000$ psi , $R_c = 0.923''$

$f_t = 62{,}750$ psi ; $f_a = 31{,}375$ psi

when $p = p^* = 114{,}500$ psi, $R_c = 0.996''$

$f_t = 73{,}100$ psi ; $f_a = 36{,}550$ psi

7. Changes of stresses due to unloading process :

$K = 2.625 > 2.22$

pressure unloaded :

$\Delta p = p^* - p_o$, for $p_o = 0$, $\Delta p = p^* = 114{,}500$ psi

$2p' < \Delta p < p$

the elastic-plastic interface caused during unloading process R_j can be obtained from

$$\Delta p = \frac{2 f_{Y.P.}}{\sqrt{3}} \left[1 - \frac{R_j^2}{R_o^2} - 2 \ln \cdot \frac{R_i}{R_j} \right] \qquad (2\text{-}16)$$

or from

$$\frac{R_j^2}{R_o^2} - 2 \ln \frac{R_j}{R_o} - 1 = 2 \ln \frac{R_o}{R_i} - \frac{\Delta p}{2 f_{Y.P.}/\sqrt{3}}$$

with the aid of Fig. 2-1 to find out $\frac{R_j}{R_o}$

$$\frac{R_j^2}{R_o^2} - 2\ln\frac{R_j}{R_o} - 1 = 2(\ln 2.625) - \frac{114,500}{\frac{2\times 110,000}{1.732}} = 1.028$$

$$\frac{R_j}{R_o} = 0.392 \; ; \qquad R_j = 0.392 \times 1.3125'' = 0.515''$$

$$\Delta f_t = \frac{-2 f_{y.p.}}{\sqrt{3}} \left[\frac{R_j^2}{R_o^2} + \frac{R_j^2}{R} \right] \tag{2-21}$$

$$\Delta f_a = \frac{-2 f_{y.p.}}{\sqrt{3}} \left[\frac{R_j^2}{R_o^2} \right] \tag{2-22}$$

when $R = R_o$

$$\Delta f_t = \frac{-4 f_{y.p.}}{\sqrt{3}} \left[\frac{R_j^2}{R_o^2} \right]$$

$$\Delta f_a = \frac{\Delta f_t}{2}$$

$$\Delta f_t = \frac{-4 \times 110,000}{1.732} \cdot \left[\frac{0.515}{1.3125} \right]^2 = -39,100 \text{ psi}$$

$$\Delta f_a = -19,550 \text{ psi}$$

8. Residual Stresses :

$$f_t^* = f_{t\,(p=p^*)} + \Delta f_t = 73,100 - 39,100 = 34,000 \text{ psi}$$

$$f_a^* = f_{a\,(p=p^*)} + \Delta f_a = 36,550 - 19,550 = 17,000 \text{ psi}$$

Item	Description	INTERNAL PRESSURE (1000 psi)												
		0	20	40	60	70	75	80	85	90	100	110	114.5	0
A	Reading-Gage 1 (axial)	11,000	11,065	11,130	11,205	11,245	11,270	11,295	11,325	11,350	11,430	11,550	11,630	11,175
B	Reading-Gage 2 (tangential)	11,000	11,225	11,460	11,705	11,855	11,940	12,030	12,145	12,265	12,590	13,055	13,350	11,820
C	Reading-Gage 3 (axial)	11,000	11,065	11,130	11,205	11,250	11,270	11,290	11,320	11,345	11,415	11,510	11,570	11,125
D	Reading-Gage 4 (tangential)	11,000	11,220	11,445	11,680	11,820	11,900	11,980	12,085	12,195	12,500	12,915	13,155	11,685
E	Axial Strain-Gage 1 (microin./in.)	0	65	130	205	245	270	295	325	350	430	550	630	175
F	Tan. Strain-Gage 2 (microin./in.)	0	225	460	705	855	940	1030	1145	1265	1590	2055	2350	820
G	Axial Strain-Gage 3 (microin./in.)	0	65	130	205	250	270	290	320	345	415	510	570	125
H	Tan. Strain-Gage 4 (microin./in.)	0	220	445	680	820	900	980	1085	1195	1500	1915	2155	685
$I=\frac{E+G}{2}$	Ave. Axial Strain (microin./in.)	0	65	130	205	257.5	270	292.5	322.5	347.5	422.5	530	600	150
$J=\frac{F+H}{2}$	Ave. Axial Strain (microin./in.)	0	222.5	452.5	692.5	837.5	920	1005	1115	1230	1545	1985	2250	752.5
K*	Axial Stress (psi)	0	4.35	8.77	13.60	16.80	18.00	19.60	21.65	23.60	29.20	37.10	42.03	12.39
L**	Tangential Stress (psi)	0	8.01	16.20	24.90	30.10	33,00	36.10	40.00	44.00	55.20	70.80	80.11	26.29

* $K=\frac{30\times10^6}{1-(0.3)^2}(I+0.3J)$

** $L=\frac{30\times10^6}{1-(0.3)^2}(J+0.3I)$

Table. C-1 Readings of Strain Gages and Calculations of Experimental Stresses