A STUDY OF THE DEVELOPMENT OF THE CONCEPT OF QUANTITY BY SCALOGRAM ANALYSIS

A Dissertation

Presented to

the Faculty of the Department of Psychology

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

James Jerome McRoy

January 1967

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ABSTRACT

A developmental analysis was made of the sequential ordering of eleven quantity tasks selected on the basis of Piaget's theory of intelligence. The following set of tasks, in the hypothesized order of difficulty, comprised the test series: (1) subtraction/addition of discontinuous substance; (2) addition/subtraction of discontinuous substance; (3) gross comparisons; (4) intensive comparisons; (5) extensive comparisons; (6) conservation of continuous substance; (7) conservation of weight; (8) transitivity of weight; (9) conservation of occupied volume; (10) conservation of displacement volume; (11) calculation of volume.

Green's (1956) method of scalogram analysis was the principal analytic technique for evaluating the data. Scoring criteria adapted from Smedslund (1961) were found to be highly reliable.

The sample consisted of 100 children, ages five, six, eight, ten and twelve years; they were of average intelligence, members of the Caucasian race, with an equal number of children of each sex in each age group. The five age groups did not differ significantly in intelligence.

In addition to the quantity test series each subject was administered the Peabody Picture Vocabulary Test.

The hypothesis that the components of the concept of quantity conform to a hierarchy of increasing difficulty and reflect the

properties Guttman (1950) has described for an ordinal scale was not accepted. However, the data are considered adequate to support a claim for the existence of what Guttman describes as a _quasiscale_. The hypothesis of an invariant, sequential relationship among subtraction-addition, addition/subtraction and conservation of discontinuous substance was also accepted. The prediction that the infra-logical operation of conservation of weight takes genetic precedence over the logical operation of transitivity of weight was also confirmed. The hypothesis that there is an invariant order of increasing difficulty involved in making gross, intensive and extensive comparisons of discontinuous substance was given only partial support with the major discrepancy being a reversal in the predicted order of difficulty for gross and intensive comparisons. Likewise, only partial support was available for the prediction of an invariant, sequential relationship among the component concepts of conservation of substance, weight and volume. Whereas the volume concept emerged as the most advanced of the three components, no relationship was found between the other two. No support was given to the theoretical inference of developmental priority for conservation of discontinuous over continuous substance, nor was the prediction upheld that there is a systematic relationship among the separate components of the volume concept distinguished in Piaget's theory. The fact that conservation of displacement volume was acquired significantly earlier than the other two components of the volume concept is consistent with other validational evidence. The predictions of overall differences in total scale scores for age but not for sex were confirmed. A curvilinear relationship between chronological age and total scale score was obtained, and this finding was interpreted as support for the conclusion that the scale points represent a quasi-developmental scale. Moreover, it was suggested, on the basis of the latter findings, that Piaget's stage of formal operational thought may begin at an earlier age than twelve years. Other areas of theoretical significance were also discussed in relation to the findings.

An outline of some of the general implications of the study was made, and suggestions were offered of some possible applications in the field of education.

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CHAPTER I

INTRODUCTION

Inherent in most psychological definitions of conceptualization is the notion that an organism is capable of responding selectively to equivalent aspects of stimuli and of actively disregarding any non-equivalent aspects of the same stimuli. In short, concepts serve as an adaptive mechanism through which we cope with events. Moreover, stimulation is reciprocal: environmental sensations stimulate the organism and these same sensations become identified, labelled and integrated. It is presumably through his increased ability to discriminate and to generalize that the human organism acquires concepts. In the process of this development, the individual becomes increasingly emancipated from reliance upon sensory and perceptual experience and is able to approach problems in the environment in a conceptual way.

Psychological development refers to behavioral changes that occur through time. A fundamental question over which developmental theorists are divided is whether development always follows the same general pattern or, expressed more formally, whether it proceeds in a sequential, invariant order. There are those who assert that it does (Goldstein and Scheerer, 1941; Werner, 1948; Piaget, 1952*a*, 1954, 1960*b*, 1961; Inhelder and Piaget, 1958; Tanner and Inhelder, 1960; Laurendeau and Pinard, 1962; Kohlberg, 1963). These stage-dependent theories agree with the principle that for the child to arrive at stage B in his development, he must first have reached and passed through stage A. The sequence is fixed; attainment of stage B presupposes mastery of stage A. Moreover, those subscribing to this position are also generally in accord with the view that the age of appearance and duration of stages will vary as a function of hereditary potential and experience (Tanner and Inhelder, 1960).

The opposite point of view places a heavier emphasis upon learning experiences with the result that children can be expected to progress at varied rates through different sequences of stages as a consequence of differential experience (Ausubel, 1957; Estes, 1956; Hunt, 1961; Sears, 1958).

It is with a view toward clarifying some of the issues that are an outgrowth of this basic controversy in developmental psychology that the present investigation was undertaken. More specifically it will be the thesis of this study that, with respect to the concept of quantity, the notion of sequential, invariant stages in development is a useful and defensible, descriptive construct.

In order to bring the problem of the present study into sharper focus, it is necessary to outline a portion of the theory which generated the problem and against which the results of the investigation will be evaluated.

I. THE GENERAL THEORY OF PIAGET

It is generally acknowledged by students of cognition that the greatest single repository of observations and theory related to the ontogenesis of intelligence is the work of the eminent Swiss psychologist, Jean Piaget. For more than 40 years, Piaget and his associates at the center of Genetic Epistemology in Geneva, Switzerland have been studying the development of intellectual functions and logic in children. Nevertheless, the importance of this contribution to the description of developmental aspects of children's thinking is only beginning to penetrate the United States (Flavell, 1963; Wallach, 1963). The reasons for this neglect are not difficult to uncover. They include: (1) the latency or absence of translation of the bulk of Piaget's writings combined with a relative dearth of secondary source material; (2) an unhappy coupling of mathematics and philosophy with developmental psychology; (3) a disproportionate balance between data and theory in favor of an emphasis upon the latter; and (4) habitual shortcomings of experimental design and data analysis particularly in the work prior to World War II.

Although it would be presumptuous to assume that what follows constitutes either an adequate or a complete exposition of the theory, an attempt will be made to provide a schematic description of its most essential constructs, particularly those aspects of Piaget's system which pertain directly to the development of the concept of quantity.

Unlike most of his contemporaries in psychology, Piaget begins his inquiry, not from the standpoint of child-as-learner, but with a primary concern with the structure of knowledge--that is, the material to be learned. In the Geneva theory, the growth of knowledge is not regarded as a mere stockpiling of information. Rather, intelligence is based on the activity of the child, a special case of adaptation. Piaget (1964) has recently explained this creative interaction between the organism and the environment as illustrated in the following excerpt.

> Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy, or image, of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorised action which modifies the object of knowledge. For instance, an operation would consist of joining objects in a class, to construct a classification. Or an operation would consist of ordering, or putting things in a series. Or an operation would consist of counting, or of measuring. In other words, it is a set of actions modifying the object, and enabling the knower to get at the structures of the transformation (p. 8).

Bruner (1965), in his presidential address to the American Psychological Association, has adopted essentially the same stance with respect to the growth of intelligence.

Piaget (1950, 1952a) employs two terms, borrowed from biology, which denote two complementary processes involved in learning, namely, assimilation and accommodation. Assimilation is a concept corresponding to inner organization whereby the individual internalizes certain aspects of the environment which in turn become organized within classes or groups. The process is analogous to that of the assimilation of food which ultimately becomes part of the blood. Piaget (1950) refers to assimilation as ". . . the action of the organism on surrounding objects, insofar as this action depends on previous behavior involving the same or similar objects" (p. 7).

These "previous behaviors" to which Piaget refers are generalizable behaviors identified in his theory as structures or schemata. Accommodation, the process complementary to assimilation, operates as adjustments are made to new assimilations. Variations in the environment act upon the individual, not by merely eliciting a fixed response, but rather by modifying the schema affecting them. This modified schema results in an altered behavior toward the environment. According to Hunt (1961), "This is the epigenesis of mind" (p. 113).

The notion of the orderly progression of stages is basic to Piaget's system. He derives, in a sequence of age-related periods or stages, the abstract intelligence of adults from the sensorimotor coordinations of infants. Piaget repeatedly asserts that while the age-related stages are invariant in their sequence, the rate of a child's progression through the stages can be expected to vary as a consequence of the interactive effects of maturation and environment.

The Piagetian theory of the acquisition of concepts or schemata differs from the traditional reinforcement learning theory in that the former holds that there is no existential separation between drives and cognitive structures (Greco and Piaget, 1959). For Piaget the appearance of new schemata gives rise to new drives which exercise those schemata. Moreover, novel stimulation would be expected to lead to the differentiation of new schemata which in turn give rise to new drives in a continuous progression. The reasoning is admittedly circular, but the view shows an interesting congruence with recent attempts by learning theorists to conceptualize cognitive behavior in terms of fractional anticipatory response mechanisms (cf. Berlyne, 1960*a*; Hull, 1943; Osgood, 1953).

The first period of intellectual development, the *sensorimotor*, extends through six stages from birth until the child is between approximately 18 months and two years old. During this period the child begins with undifferentiated views of himself and the environment, then gradually begins to differentiate himself from the environment. In the process he also learns that the environment functions on a basis of certain physical properties, such as space, time, object permanence and causality (Piaget, 1954).

The second major period, the *preoperational*, is comprised of two stages and spans the gap between the ages of two and seven years. Piaget employs the term, preoperational, to imply that while the child for the first time is capable of functioning symbolically, the thought processes are not yet reversible. The concept of <u>reversibility</u> plays a central role in the description of distinctions among stages. In the logician's INRC group (in which each operation has two distinct opposites), two forms of reversibility are distinguished: inversion (negation), and reciprocity (Piaget, 1953).

During this period the child becomes capable of distinguishing between signs and their significates with language playing a more predominant role. The child can be observed to categorize on the basis of single characteristics of objects but is unable to classify the several dimensions of stimuli simultaneously. For example, he cannot take into account an object having width and height simultaneously. Toward the close of this period the child begins to utilize numbers and to order things in terms of quantity. In short, he gives the first evidence of disregarding non-equivalent aspects of stimuli. Piaget (1964) describes the typical experience of children at this stage as:

> . . . an experience of the action of the subject, and not an experience of objects themselves. It is an experience which is necessary before there can be operations. Once the operations have been attained, this experience is no longer needed and the coordinations of actions can take place by themselves in the form of deduction and construction for abstract structures (p. 13).

The period of <u>concrete</u> operations, Piaget's third period in the development of intelligence, covers the span of years from seven to eleven. The term, <u>operations</u>, refers to the child's internalized cognitive structures. The term, concrete, indicates the extent to which the child's thinking, however logical and systematic it may be, is still bound to direct experience. Although the child at this stage no longer requires manipulation of objects in order to comprehend their relationships, his reasoning is dependent on direct experiences he has had with the objects. In situations where direct experience is lacking, the concrete operational child reasons by analogy to something he has experienced.

During this period the child acquires such logical operations as reversibility, classification (organization of objects into hierarchies of classes), seriation (arrangement of items along continua of increasing values), transitivity (if $A > B$, and $B > C$, then $A > C$) and conservation (certain attributes of an object remain invariant across transformations in other attributes). The child's ability to perform operations such as these is viewed as critical to the existence of conceptual behavior (cf. Piaget, 1950, 1957; Hunt, 1961).

In several of Piaget's publications (Piaget and Inhelder, 1941, 1956; Piaget, 1950; Apostel, Mays, Morf, and Piaget, 1957), a distinction is made between logical and infralogical operations. Flavell (1963) summarizes the critical distinguishing properties of logical operations as follows:

(1) They bear on sets of discrete, discontinuous objects.
(2) Their operation is independent of the spatio-temporal

> proximity, or lack of it, of the objects they deal with. (3) They do not require any actual modification of their objects, neither alteration of their structure nor modification in the sense of changing their spatial or temporal location. (p. 196).

Infralogical operations can be thought of as formally similar to and developmentally contemporaneous with logical ones but which possess attributes essentially opposite to those cited above. Hence, the fundamental distinction is that infralogical operations are basically spatio-temporal and continuous in a way that logical operations are not (Piaget and Inhelder, 1941).

It is during the period of concrete operations that the child is becoming increasingly emancipated from the perceptual dominance of the environment and more capable of approximating adult conceptual behavior.

The final period of intellectual development in this succession of Piagetian stages is that of formal operations. The period extends from eleven to fifteen years of age. The most important general property of formal-operational thought concerns the hypothetical and deductive procedures of logical thought (Inhelder and Piaget, 1958).

Whereas the concrete operational child could handle only a single variable at a time, the formal operational child can handle combinations of variables in a systematic order. The child at this level can employ the calculus of proportions in the solution of scientific problems. Moreover, he can use combinatorial analyses based on the logical structures which mathematicians call latices (Piaget, 1953; Inhelder and Piaget, 1958). In sum, formal thought implies a generalized orientation toward problem solving, the hypothetical and logical justification.

II. THE THEORY PERTAINING TO THE CONCEPT OF QUANTITY

Since 1935 several studies have emerged from the Geneva tradition which bear on one or another aspect of the genetic development of concepts of quantity (Apostel, Mays, Morf, and Piaget, 1957; Fischer, 1955; Inhelder, 1936; Piaget, 1960a; Piaget and Szeminska, 1939; Szeminska, 1935). The early papers in this series are largely of historical interest, since their contents have for the most part been incorporated into the systematic book on the subject by Piaget and Inhelder, *Le Développement des Quantités Chez L'Enfant* (1941). Perhaps the best-known segment of Piaget's work on the concepts of quantity concerns the acquisition of conservation of substance (the quantity of some basic unit that cannot be compressed), weight (the downward force of an object), and volume (the space occupied by an object).

In this particular program of research, Piaget marshals evidence in support of the assumptions that each type of quantity concept reflects a similar developmental trend and that the discovery of conservation earmarks the final stage of this development. The pattern of this sequence is: (1) absence of conservation; (2) an empirically founded, "on-off" variety of conservation in which the child tentatively hypothesizes conservation for some transformations but denies it for others; and (3) an assertion of conservation across all transformations of the type of quantity in question.

Moreover, Piaget and Inhelder (1941) report that discoveries of conservation follow an invariant age-related order of acquisition (horizontal décalage). Despite the apparent similarity among the tasks used in this study (the celebrated "sausage experiments"), conservation of substance, weight and volume are not acquired simultaneously. Although no statistics are reported in the early studies (Piaget and Inhelder, 1941), Piaget later assigned different tests to the age level at which the per cent passing was 75 (Piaget, 1951). Interpreters of Piaget (e.g., Elkind, 1961c; Flavell, 1963) make the assumption that the same criterion underlies his assignment of conservation of substance to the ages seven to eight; the conservation of weight to ages, nine to ten; and the conservation of volume to ages, eleven to twelve.

In his classic investigation of the concept of quantity (Piaget and Inhelder, 1941), Piaget confronts the child with two clay balls identical in size, shape and weight. After the child has agreed on their equality, Piaget shapes one of the balls into a "sausage" and asks the child for a judgment concerning one of the three types of quantity. He also asks the child to explain the basis for his judgment that the "sausage" is more, less or the same amount (weight, volume) as the ball.

The three stages enroute to the attainment of conservation of quantity are each tied to rather specific theoretical constructs which underlie the major cognitive changes (Piaget, 1950, 1952b, 1957; Inhelder and Piaget, 1958).

Nonconserving children at the first stage have only a general or gross perception of quantity. Gross quantities are single perceived relations between objects (length or width) which are not coordinated with each other. In the "sausage experiments" these children fail to conserve because they perceive the "sausage" as being different from the ball. When required to explain the basis for their judgment, they do so in terms of a single dimension which is not related to changes in other attributes.

Children at the second stage have a differentiated impression of quantity, but it is found to be uneasily maintained in the face of perceptual impressions of size differences. If the comparison ball is only slightly elongated, it will be judged to contain the same amount of clay as the standard; but if greatly extended, then the child lapses once again into nonconservation. These children give "on-off" type conservation responses because to their differentiated impression the "sausage" is both more (in length) and less (in width) than the ball, and they are unable to resolve the contradiction. The perception of quantity relations taken two by two (length and width) is a more complex type of quantity which Piaget refers to as intensive. The ability to make correct intensive comparisons requires what Piaget calls logical multiplication.

Later still, a stage is reached when there are no longer any breakdowns of conservation in the face of extreme shape differences. Moreover, the child is capable to making successful intensive comparisons of quantity. Most characteristic of this stage, however, is the capacity of the child to accurately judge unit relations between objects (X is half of Y, X is twice Y, etc.). This most complex type of quantity judgment Piaget calls extensive. According to Piaget the equation of differences results in the formation of ratios and fixed units and underlies abstract quantity and number concepts (Piaget, 1952*b*). Here the child explains that the comparison stimulus contains the same amount of clay as the standard because what the "sausage" loses in thickness it gains in length. Similar response transitions are postulated for weight and volume.

It is important now to consider the theoretical propositions concerning the cognitive achievements which underlie conservation of substance, weight and volume.

Wallach (1963) believes there are three such achievements essential in acquiring conservation of substance.

> (1) an ability to take account of the joint effect of change in two perceived aspects of the material rather

> than being limited to considering only one aspect at a time, with the result that compensatory changes can be noted; (2) the development of an "atomic" theory of matter - a conception that matter consists of small particles that simply change their positions with respect to one another when shape transformations occur; and (3) an ability to hypothesize that a reverse change of the transformed shape back into the original could be performed (p. 248-249).

The typical arguments which concrete operational children use to support their judgments of conservation of substance are summarized by Hunt (1961).

> (1) nothing has been added or taken away (operation of identity and reversibility); (2) even though the ball has been lengthened, what it has gained in length it has lost in thickness (combination of relations with reversibility); (3) the ball has only been lengthened and it would be easy to roll it back into a ball (simple reversibility) (p. 228).

Although the exact implications of the assumption are not clear, it is assumed in the theory that some property of the system of operations of addition and subtraction is a necessary condition for the occurrence of the concept of conservation of substance (Piaget and Inhelder, 1941).

At this point in the development of the theory pertaining to quantity, it is appropriate that attention be drawn to the distinction between continuous and discontinuous substances. Although this distinction has been implicit throughout Piaget's work (e.g., Piaget and Szeminska, 1952), its importance has been stressed in a recent publication (Tanner and Inhelder, 1960). Continuous substances (e.g., water, plasticene) lack the property of discreteness whereas discontinuous substances (e.g., blocks, chips) possess such a property. Since, theoretically, conservation of quantity presupposes the reversible operations of addition and subtraction and hence, an understanding of numerical correspondence, it would seem reasonable to infer that discrete material would be conserved earlier than continuous material. If each of the levels of conceptual invariance (i.e., substance, weight and volume) is based on a particular development of logical operations and these operations are organized in a hierarchical sequence, then the type of material used to test for the attainment of the levels of invariance should not affect the sequence of their attainment even though discontinuous would precede continuous material in age of attainment.

After invariance of substance across shape changes is achieved, the typical seven to nine year old child still believes that weight must vary with shape. Children of this age, although unable to formulate a concept of density or specific gravity, try to reconcile some of the contradictions inherent in explanations based on absolute weight and volume. The theory (Inhelder and Piaget, 1958) holds that children typically make ". . . a double entry classification with reference to weight and volume which gives four possibilities: the small light objects, the small non-light objects; the large light, and the large heavy" (p. 30). The child of this age, according to Piaget, is beginning to revise his notion of weight and ". . . to place the concept of absolute weight . . . in opposition to a new concept of weight perceived as relative to the matter under consideration" (p. 30). Typically, by the age of nine to ten, children conserve the weight of objects regardless of their shapes. This new acquisition results from the ability of the child to use the four subclasses of objects mentioned above by means of logical multiplication. In so doing, he acknowledges the presence of contradiction and gives evidence of approaching the concepts of density (weight in proportion to volume) and specific gravity (proportional relation between the density of an object and the density of the medium in which the object is placed (e.g., water).

Earlier it was noted that one of the logical operations acquired during the period of concrete operations is that of seriation, the ability to order a number of objects according to some discriminable, common property. A closely related concrete operation which enables seriation to appear is transitivity. For example, if A is heavier than B, and B is heavier than C, then A is heavier than C. Given concrete transitivity of weight, the child is able to arrange items in a series along particular continua. Concrete transitivity, involving inferences from actual observations, should not be confused with formal transitivity which permits an individual to make inferences from verbally stated, hypothetical premises. According to Smedslund (1963b), Piagetian theory assumes that concrete transitivity results from "an internal reorganization in the direction of minimum uncertainty and maximum economy. This reorganization is presumably initiated by repeated uncertainties (problems), and is guided by internal experiences of compatibility and contradiction" (p. 251). Conservation and transitivity appear between the ages of five and seven, depending on the particular content involved (cf. Braine, 1959; Kessen and Kuhlman, 1962; Kooistra, 1963; Smedslund, 1960).

The final link in this developmental sequence of quantity concepts is conservation of volume. Whereas compensation of quantity can be justified by simultaneously taking account of two variables (additive compensation), compensation of volume requires for its justification that three variables be accounted for simultaneously. This requires the logic of multiplicative compensation which, in turn, implies the concept or schema of proportionality. When volume is conserved by multiplicative compensation of dimensions, Inhelder and Piaget (1958) assert, for example, that it is a consequence of the doubling of one of the sides which is compensated by taking one-half of the product of the other two sides. Thus, conservation of volume is contingent upon the operation of proportionality. Inhelder (1953) has observed that "proportions are themselves operations applied to operations, or operations to the power of two" (p. 85). The theory would hold, then, that with proportions comes conservation of volume which depends upon logical multiplication and the capacity to reason with hypothetical propositions.

In a more recent work (Piaget, Inhelder and Szeminska, 1960), Piaget has been concerned with elaborating upon the concept of volume and relating it to the ontogenetic order of appearance of spatial concepts. Three categories of space are distinguished: topological, projective and Euclidean space. These are listed in the developmental order of their appearance as cited by Piaget (1954). Topological relationships include: (1) proximity (a point belongs to each of its neighbors); (2) separation (elements which are separated); (3) order (a synthesis of proximity and separation); (4) enclosure (space contained within the boundary surfaces of a solid); (5) continuity and discontinuity (Piaget and Inhelder, 1956). Projective space refers to the perceptually invariant features of objects even when the point from which the object was viewed changes (object constancy). This type of space deals with the problem of locating objects relative to one another in accordance with principles of perspective, or projective systems or of coordinate axes. Euclidean space involves the conservation of angularity, rectangularity, parallelism, curvilinearity and distances. Flavell (1963) has pointed out that, in the history of science, man's study of space proceeded in the order of Euclidean (300 b.c.), projective (late seventeenth century) and topological (nineteenth century). Similarly, Tuddenham (1966) has observed that:

> Taken in sequence (Euclidean, projective, topological), each level is more general, i.e., involves fewer axioms than the preceding, and the entire sequence might theoretically be expected to have developed in the opposite order. Now the curious part is that the sequence of acquisitions of mental operations by children follows not the historical sequence, but the theoretical (Piaget's) sequence (p. 216).

Piaget's research on spatial concepts has shown that discrimination of topological properties is made during the preoperational period and becomes integrated into stable operations around the age of seven. It is the concept of interior volume, or the space contained within the boundary surfaces of a solid, that becomes subject to conservation at this age. This elementary concept of volume is essentially the same as conservation of substance, the concept used in Piaget's earlier work (Piaget and Inhelder, 1941). It is only when the child is about nine or ten that he is able to handle projective and Euclidean space concepts. In more recent work on the problem (Piaget, Inhelder and Szeminska, 1960), Piaget develops further his notion of the developmental priority of topological over Euclidean conceptions of space. The recognition of a coordinated Euclidean space in which transformations from one dimension to another can be easily achieved is now held to be essential to a full understanding of the concept of volume. Moreover, the measurement of volume (multiplication of three linear dimensions) and the concept of displacement volume depend on the recognition that a solid can be regarded as an infinite series of contiguous plane sections. Piaget and Inhelder (1956) assume that this cognitive ability depends on an understanding of infinity and continuity, and they associate this ability with formal operational thought. Therefore, it is only at about the age of eleven to twelve that children achieve a "true," intuitive grasp of the volume concept, and this concept is assuméd to occur relatively independently of formal instruction. This mature understanding of volume involves at least three sequential and interdependent components; (1) the ability to understand that the space which an object occupies in relation to the surrounding external medium (e.g., water) remains invariant across transformations (*conservation of occupied volume*); (2) the ability to accurately predict complementary or displacement volume, i.e., the change in the level of the water in a container as a function of transformations of the object (*conservation of displacement volume*); (3) the ability to calculate the volume of a cuboid by multiplying length, height and breadth (*calculation of volume*).

III. ALTERNATIVE THEORETICAL MODELS

In the past few years there have been several attempts to order Piaget's empirical findings on the basis of underlying, theoretical dimensions other than the ones which Piagetian theory indicates.

Smedslund (1961*a*) has evaluated three alternative interpretations of developmental, cognitive phenomena in addition to Piaget's assimilation-accommodation (equilibration) model. The alternative positions include: (1) nativism; (2) maturation theory; and (3) learning theory. Concerning the first of these Smedslund observes that " . . . the well established findings of Piaget and his collaborators in a variety of fields seem to have excluded nativism as a serious possibility" (1961a, p. 13). He concludes that the available evidence is against a position like nativism which suggests that logical structure follows automatically when a child possesses all the relevant information necessary for a correct solution. While conceding that maturation theory will accommodate the facts of pre-logical behavior and the absence of evidence for direct learning, Smedslund cites two major obstacles for a maturation theory interpretation. It fails to account for (1) the time lags (décalages) between the separate acquisitions of conservation and (2) the accelerating and retarding effects of various environments. Apostel (1959) and Berlyne (1960b) have both argued in support of some variant of a learning theoretical account for conservation. Smedslund notes, however, that Berlyne ". . . introduces so many new assumptions into the Hullian framework that it becomes almost indistinguishable from the equilibration theory (of Piaget)" (1961a, p. 15).

Smedslund (1961e) has developed a cognitive-conflict model to account for changes in conceptual behavior. This model bears close similarity to Festinger's cognitive-dissonance theory and has proved useful in generating training procedures. Stevenson (1962) has tried to analyze age changes in cognitive behavior on the basis of S-R theory, modified by an emphasis on the need for sensory stimulation. Braine (1962) has suggested that there may be a change within the parameters of stimulus generalization, whereas McLaughlin (1963) has argued that most of the developmental phenomena Piaget describes can be accounted for on the basis of increasing memory span. In support of his argument, McLaughlin analyzes performance at each of Piaget's stages in terms of the number of items whose retention is required for the solution of the problem. Wohlwill (1962) has proposed that there is decreasing dependence of behavior on information from the immediate stimulus field with increases in age. He identifies three aspects of this progression from perception to conception: (1) the amount of redundant information decreases; (2) the amount of irrelevant information that can be tolerated increases; and (3) the spatial and temporal separation over which the total information contained in the stimulus field can be integrated increases. This formulation abandons the assumption of discontinuous stages. Of the alternative theoretical models cited, Wohlwill's would seem to lend itself best to research on the functional relationships between antecedent conditions and developmental change.

In the judgment of the writer, it is still too early to consider discarding Piaget's equilibration model in favor of one of these alternative dimensions. Therefore, the hypotheses for empirical test to be outlined in the next chapter will be based upon propositions contained within or research generated by Piagetian theory.

CHAPTER II

REVIEW OF THE LITERATURE AND STATEMENT OF THE PROBLEM

Piaget's research on the development of quantitative concepts in children has been the focus of a considerable number of validational studies over the past five years. This replication and refinement of the original work has been concentrated primarily in the United States, Great Britain and Scandinavia. In general, studies that have attempted to replicate Piaget's experiments dealing with aspects of the quantity concept have tended to support his findings regarding the sequence of stages (Almy, 1966; Sigel, 1964; Wallach, 1963).

The research to be reviewed here will be organized around two recurring questions related to any conceptual content area in Piaget's system. These problems include: (1) the developmental sequence of acquisitions within a content area; and (2) factors which regulate the transition from one stage to another.

I. VALIDITY OF RELATIONSHIPS AMONG ABILITIES WITHIN STAGES

Operations of Addition and Subtraction

Earlier it was noted that some property of the system of operations of addition and subtraction is a necessary condition for the occurrence of conservation. According to Piaget and Inhelder

(1941), this property is the so-called reversibility of operations but the exact implications of this are not spelled out. Moreover, it was inferred from related aspects of the theory that conservation involving discrete or discontinuous material should be achieved earlier than conservation of continuous substance.

Wohlwill (1960); translating Piaget's theory of the development of number into behavior theory terms, and employing a matching-from-sample technique, designed a series of tasks paralleling Piaget's experiments on the concept of number. Swiss children between the ages of four and seven years were used in this validational study, one of two such investigations to first apply scalogram analysis to a conceptual content area in Piaget's system. While admitting that his data fell short of meeting some of the requirements considered to be essential in the application of scalogram analysis, Wohlwill interprets his findings as demonstrating a relatively uniform developmental sequence. Of particular interest here is his observation that ". . . success on the addition-subtraction trials appeared to be virtually a prerequisite for the manifestation of conservation" (p. 364).

Feigenbaum (1961) administered several forms of a test for conservation of discontinuous substance to 146 children, four to seven years of age. Among his principal findings were: (1) a developmental shift away from reliance on perceptual impression toward the use of logical and arithmetic procedures; and (2) a slight tendency for performance to vary with task parameters, especially in the younger age groups (e.g., reducing the number of pieces makes the task a little easier).

Smedslund (1961*f*) provides evidence from his work on conservation of substance which suggests that the child's ability to attain conservation in tasks involving discontinuous substances regularly precedes conservation of continuous substances. Although the findings reveal that very few of his subjects (age range: approximately four and one-half to seven years) failed to conform to this sequence, Smedslund unfortunately provides no statistical analyses of the reliability of this trend.

In the same year of Wohlwill's (1960) study, Mannix (1960) applied scalogram analysis to a series of Piagetian tasks (Piaget and Szeminska, 1952) dealing primarily with aspects of the number concept to 48 English children. Although his research design led to a spuriously high index of reproducibility, Mannix's results for two quantity tasks support the notion that conservation of discontinuous substance precedes conservation of continuous substance in the sequence of acquisitions.

In addition to the variable, continuous *vs.* discontinuous material, Smedslund has also been interested in the operations of addition and subtraction, since both theory and previous research (Wohlwill, 1960; Smedslund, 1961*b*) have suggested that they are of fundamental importance in the acquisition of conservation. He reports that both addition-subtraction and conservation appear to be acquired earlier with discontinuous than with continuous substance (1961_f_).

In the most recent of this series of studies involving children of approximately the same age range, Smedslund (1962) has extended his inquiry into the relationship between conservation of discontinuous substance and the operations of addition-subtraction. In the first of two experiments comprising this study, he discovered that conservation was closely related to a task involving the subtraction of a piece and a subsequent addition of the same piece (subtraction-addition sequence), followed by an addition of a piece and a subsequent subtraction of the same piece (addition-subtraction sequence). In the second experiment, Smedslund tested several items involving various combinations of subtraction-addition and addition-subtraction sequences and found three of the items statistically significant in their relation to conservation. Conservation was defined here by: (1) a correct response or prediction; and (2) a "symbolic" explanation (one referring to previous events in the item). In these two experiments in which he uses a type of scalogram analysis, Smedslund (1962) concludes that subtraction-addition, addition-subtraction, and conservation of discontinuous substance seem to form a genetic sequence in this order. These results are interpreted as strengthening Piaget's hypothesis that a concept of conservation reflects a complete reversibility of the operations of addition and subtraction.

Gross, Intensive and Extensive Comparisons of Quantity

In what appears to be the only attempt to systematically replicate that aspect of Piaget's (1952b) theory which deals with the three stages of perceived quantity en route to conservation, Elkind (1961b) presents evidence that success in comparing quantity develops in three, age-related, hierarchically ordered, stages. These stages have been identified as gross, intensive and extensive comparisons, and they presumably earmark steps in the development of an abstract quantity concept. Using cross-sectional samples of children between the ages of four and seven, Elkind also confirmed Piaget's suggestion that comparisons of continuous quantity are more difficult at each chronological age level than comparisons of discontinuous quantity. Thus, while the majority of the four year group succeeded with gross and intensive comparisons, the six and seven year group were successful with all three types of quantity.

Conservation of Substance, Weight and Volume

In a second experiment, Elkind (1961c) administered Piaget's tests for conservation of continuous substance, weight and volume to 175 children between the ages of five and eleven years. Each type of conservation was clearly age dependent, and these norms are offered in support of Piaget's assertion that the normal genetic order for age of acquisition is substance first, then weight and finally volume.

In another investigation in this series, Elkind (1961a) used the same three conservation tasks as group tests with 469 children ranging in age from twelve to 18 years. The principal finding here was that the modal age separation (décalage) between the first two (substance and weight) and the third type of conservation may be considerably greater than Piaget had postulated. Whereas approximately 75 per cent of Elkind's subjects had acquired conservation of substance and weight between the ages of eight and nine (1961c), this criterion was not met in the case of volume until the age of 15 years. In his test for conservation of volume, Elkind (1961a, 1961c) substituted the terms "volume" and "same room or space" for the terms "amount" and "weight" as used in the other two conservation tests. In hindsight he explained his failure to replicate Piaget's findings for the volume concept ". . . as due to the fact that Piaget used a somewhat different procedure in his test for conservation of volume. Piaget had his subjects say whether the ball and the sausage would displace the same amount of water" (Elkind, 1961c, p. 224). As a check on this possibility, Elkind (1961c) used this procedure with some of his subjects following completion of the experiment and reported that ". . . conservation seemed easier to discover by means of the displacement problem" (p. 224).

The best available evidence of the normal rate of development for conservation of substance and weight is based on the age norms reported by Vinh-Bang (1959). Extrapolation from Vinh-Bang's data leads one to expect a rate of development in the period from five to seven years of approximately one to one and one-half per cent a month for the concepts of conservation of substance and conservation of weight.

Smedslund (1961*a*) reports that a large scale standardization by Vinh-Bang (in preparation) has provided reliable and specific information on the transition ages for conservation of substance and weight in the population of Geneva. In this study approximately 1500 children between the ages of four and twelve years of age were given a battery of 30 tests, including those for conservation of substance and weight. Using the 50 per cent level as a criterion, Vinh-Bang's data places conservation of substance at seven and one-half years and conservation of weight at eight years.

Smedslund (1961*b*), using 135 Norwegian children between five and one-half and seven years of age, has also tested Piaget and Inhelder's (1941) hypothesis that conservation of continuous substance invariably precedes conservation of weight. His data reveal a highly significant relationship between the two conceptual abilities in the predicted direction, but contain no direct evidence for an invariant sequential relation.

In three separate replication studies using English children between seven and ten years of age, Lovell and Ogilvie (1960, 1961a, 1961b) have studied the extent of sequential development of the concepts of continuous substance, weight and volume. In these three investigations, 322, 364 and 191 children, respectively, were individually tested in accordance with Piaget's flexible "méthode clinique." In the case of conservation of substance and weight, the 75 per cent level for age of acquisition was located at about the age of nine and one-half for substance and ten and one-half for weight. Lovell and Ogilvie conclude that conservation of substance arises earlier than that of weight since ". . . quantity is under immediate visual perception whereas weight is not" (1961b, p. 144). Moreover, their evidence tends to show that while the group of operations (e.g., reversibility) proposed by Piaget may be necessary they are not sufficient conditions for attainment of conservation of substance and weight. Lovell and Ogilvie's (1961b) research on the concept of volume was modelled after the work by Piaget, Inhelder and Szeminska (1960). Taking their evidence as a whole it seems as if the concept of physical volume (which, in this study, included tests for interior, occupied and displacement volume) develops slowly during this age range. More specifically, these investigators found that whereas 80 to 90 per cent of the children between the age of ten and eleven conserved occupied volume, only about three-quarters of the children at this age could conserve displacement volume. This evidence is interpreted as support for the theory as regards stages, but insofar as developments within stages are concerned (viz., sequence of acquisitions) the data failed to substantiate theoretical expectations. Lovell and Ogilvie (1961b) supply no information concerning the extent of this variability.

The only study in the literature to deal with the most advanced aspect of the volume concept, calculation of volume, is one combining an initial pilot study with a subsequent replication on 40 English children, twelve to thirteen years of age (Lunzer, 1960b). In addition to the test for calculation of volume, Lunzer also included one for conservation of displacement volume, both procedures following closely on those used by Piaget (Piaget, Inhelder and Szeminska, 1960). Lunzer's results show that while 58 per cent of the children conserved displacement volume, only 30 per cent passed the test for calculation of volume. When compared with Lovell and Ogilvie's (1961b) data, the task used by Lunzer to test for conservation of displacement volume would appear to be somewhat more difficult. Lunzer found no significant degree of relationship between these two aspects of the volume concept; however, there is a slight suggestion of sequential dependency between the two tasks. It is possible that a true relationship was obscured by the small sample used in this study. It was noted earlier that Piagetian theory holds that these two acquisitions depend upon an understanding of infinity and continuity (Piaget, Inhelder and Szeminska, 1960). Lunzer's results give no support to this contention.

Other disconfirming evidence comes from a validational study by Hyde (1959) who found no strong support for the substance-weight-volume décalage. In this cross-sectional and cross-cultural study involving six to eight year old children, she found a number of children who departed from the predicted sequence. In evaluating Hyde's data, Flavell (1963) has noted that ". . . careful scrutiny of this study does not suggest any immediate explanation for the discrepancy in results" (p. 387). Again, there is evidence here to support the notion that conservation is to some extent contingent upon type of material to be conserved.

Transitivity of Weight

Piaget and Inhelder (1941) have hypothesized that logical (e.g., transitivity) and infralogical (e.g., conservation) operations are organized synchronously for any given conceptual content area where they are applicable. Their data for transitivity of weight revealed wide inter-subject variability in age of mastery for a variety of procedures and materials. While Piaget and Inhelder report no average age associated with any form of transitive inference, inspection of their data suggests that transitivity of weight in the case of three objects equal in volume might occur somewhere around eight to nine years of age (p. 271-280).

A study by Smedslund (1959) has shown practically a complete absence of elementary transitivity of weight in a group of Norwegian children between five and seven years. This finding was again obtained in a later study (1963a). Prior to the latter study, however, data were obtained which confirmed an earlier finding (Smedslund, 1959) of a certain parallel increment of conservation and transitivity of weight, but which definitely contradicted the hypothesis that these two changes are causally interrelated (1961b). On the basis of an absence of correlation (with or without age variance partialled out) between the two acquisitions, Smedslund concludes that the organization of logical operations (reflected in transitivity) and of infralogical operations (reflected in conservation) does not occur simultaneously in a subject. Smedslund (1961b) tentatively hypothesized that, while transitivity is probably achieved later than conservation of weight, it is not likely that its rate of development exceeds that of conservation of substance and weight. While no age developmental norms are available for transitivity of weight, Smedslund (1963b), in his most recent paper on the topic, reports that 44 per cent of a group of 143 children between the ages of seven and one-half and nine years of age (median age: eight years, seven months) had acquired concrete transitivity of weight.

Lovell and Ogilvie (1961a), in their study of English children between seven and ten years of age, report that most of those who conserved weight could also perform the logical operation of transitivity. At the same time, there were apparently many nonconservers who were also capable of transitivity which suggests that logical operations in themselves do not ensure conservation.

II. FACTORS INVOLVED IN TRANSITIONS

Co-equal in importance with a knowledge of the normal, genetic sequence of development of a conceptual content area is an understanding of those factors which influence the course of such development. Research questions related to the latter problem include the extent to which transitions from one conceptual ability to another or from one stage to another are facilitated or impeded by such situational variables as: (1) type of material used to test for levels of invariance; (2) training procedures designed to accelerate the acquisition of a particular concept; (3) linguistic factors; and (4) level of general intelligence as inferred from other measures of mental ability.

Type of Material

In regard to the many types of material he used, Piaget (1952*b*) concluded that success in comparing a given type of quantity of a certain type of material did not necessarily generalize to all materials. One approach to the understanding of observed situational variability is to look for broader classes of variables that will subsume groups of different materials. It has been noted earlier that Piaget has classified materials into continuous and discontinuous ones. Empirical support for the genetic precedence of discontinuous over continuous material comes from scalogram analyses (Mannix, 1960; Smedslund, 1961f; Uzgiris, 1964), and cross-sectional (Almy, 1966; Elkind, 1961a) and longitudinal (Almy, 1966) investigations. However, several studies (Beard, 1957; Carpenter, 1955; Hyde, 1959; Lovell and Ogilvie, 1960, 1961a; Lunzer, 1956) have shown that children who are conservers of a particular concept with continuous substance (e.g., plasticene) do not inevitably exhibit conservation of the same concept when confronted with a different type of material of the same class (e.g., liquid). Uzgiris (1964) has expressed the problem in this way:

> Since each of the levels of conceptual invariance (e.g., substance, weight, volume) is based on a particular development of logical operations and these operations are organized in a hierarchical sequence, then the type of material used to test for the attainment of the levels of invariance should not affect the sequence of their attainment, although there might be some variation in the time of attainment of a particular concept in the sequence with different types of materials (p. 832).

Uzgiris systematically investigated the effect of varying the materials used to test for the conservation of substance, weight and volume on the observed sequential attainment of these concepts. Based on a large sample of six to eleven year old children, her results support the claim that these three levels of conceptual invariance are attained in the same sequence with a wide variety of materials. These findings, however, do not suggest that an individual's position on the conservation sequence is constant across materials. Although the individual differences were neither large nor systematic, there was no single material on which all children were either accelerated or retarded.

In general, then, the literature pertaining to the type of material used to test for level of conceptual invariance supports Piaget's theory of sequential intellectual development, particularly as the theory has been applied to the concept of quantity.

Effects of Training

A sizeable amount of research effort has been invested during recent years in attempts to accelerate the understanding of Piagetian concepts in young children. It is manifest that the question of what is involved in the development of conservation will be clarified if knowledge is gained concerning the effectiveness of various training procedures. In this connection, it should be noted that Piaget (1964) has cautioned that the ultimate test of the effectiveness of training is to be found in the duration and generalization of the concept. A number of studies on the effects of training have been reported including several experimental procedures generated by some of the alternative theoretical models outlined earlier in this paper. Among these attempts to teach children to conserve quantitative concepts and to perform logical operations upon them (e.g., transitivity) are those of Beilin, 1964, 1965; Beilin and Franklin, 1962; Bruner, 1964; Bruner and Kenny (in press); Smedslund, 1959, 1961 a, b, c, d, e, f, 1962, 1963a, b; and Wohlwill and Lowe, 1962. The evidence accumulated from these investigations strongly implies a refractoriness to the permanent effects of training in young children who are clearly nonconservers. Flavell (1963) has interpreted these rather consistent failures as reflecting ". . . deep developmental reality about these structures, and in this sense the learning studies confer a degree of backhanded validity to Piaget's previous assertions that they are, in fact, real existents which exert weight in the young child's intellectual life" (p. 377). Even in instances where short-term effects of training have been observed (e.g., Smedslund, 1961c; Wohlwill and Lowe, 1962), conclusions should be tempered by Werner's (1948) admonition that a child's observed success in producing concepts does not necessarily imply that the underlying intellectual process is accurately reflected. Moreover, the ability to conserve represents only one dimension, albeit a pervasive one, of the child's developing intellectual power. There is, as Wohlwill (1964) points out, a possibility of ". . . accelerating the process of cognitive development with respect to one particular concept at the expense of the breadth or generality of learning or transfer to other later concepts" (p. 100). In any event, the evidence weighs against the possibility that conceptual transitions can be accelerated by any short-term manipulation.

Linguistic Factors

It is clearly conceivable that language ability may serve as a mechanism influencing the absolute ages at which the types of conservation of quantity are exhibited initially. Indeed, it has been largely through the use of verbal diagnostic techniques that Piaget and other investigators have tested for the presence of conservation. However, it has been possible to arrange non-verbal conservation training and testing procedures which minimize the effects of language as, for example, in the matching-from-sample technique. Here the child is rewarded for selecting that one of several stimuli which is the same as the sample in some respect. In instances where non-verbal techniques have been employed to test for conservation (cf. Braine, 1959; Wohlwill, 1960; Wohlwill and Lowe, 1962), the results suggest that the particular age norms reported by Piaget in connection with various cognitive attainments may often be reduced by one or two years. However, these studies have also been consistent in showing that whereas non-verbal training leads to some improvement on non-verbal tests for conservation, there is no corresponding transfer of non-verbal training to verbal conservation tests. Smedslund (1961b, c), on the basis of his work on the learning of conservation of weight, also argues for the very limited, "nonconceptual" nature of such learning acquired by means of non-verbal training.

Wohlwill (1964) takes the view that it is probably impossible to design experiences that will shift a child who is at the height of a perceptually dominated stage to a different kind of reasoning. Rather, as the ". . . language and other mediational processes such as observing responses . . ." (p. 98) develop, the perceptual approach dissipates. Zimles (1963) holds a similar view.

Bruner and his students have recently conducted a series of experiments designed to explore the role of language in relation to quantitative concepts (Bruner, 1964). In one of these investigations, Bruner used a technique derived from a method employed by Piaget to study the conservation of an amount of liquid. Bruner notes that the language used in this task by five, six and seven year old children could be classified in three different linguistic modes and that these linguistic categories are directly related to performance on conceptual tasks. He also reports that Inhelder and Sinclair (personal communication, 1963) have obtained comparable results in conservation tasks. Based on these and other related findings, Bruner suggests that improvement in language and/or activation of pre-existing language habits should improve performance on conservation tasks. In a direct test of the latter proposition, Frank (in Bruner, 1964) administered the classic conservation tests involving liquid in beakers of various sizes as pretests to children between the ages of four and seven. In the second phase of the experiment the child was permitted to observe the original equivalence of the water in two beakers of identical appearance, but when the liquid was poured from one of these into a beaker of a different size, the third beaker (but not the pouring process) was obscured from the child's view by a screen. When the child was then asked whether the amount in the third beaker would be less, more, or the same compared with that in the original beaker, the number of correct judgments greatly increased. However, when the screen was removed, all of the four-year-olds changed their minds. 70 per cent of the five-year-olds retained their conserving responses which stood in sharp contrast to the 20 per cent of this age group who had conserved on the pretest.

A related experiment by Nair (1963) exploring the arguments children use in solving a conservation task lends further support to Bruner's conclusion that ". . . if a child is to succeed in the conservation task, he must have some internalized verbal formula that shields him from the overpowering appearance of visual displays. . ." (1964, p. 7). While Bruner's research seems to clearly implicate language as a factor involved in acquiring conservation, his data fail to meet Piaget's requirements of duration and generalizability of a concept.

Level of General Intelligence

In considering the kind of relationships that one might reasonably expect to find between children's performance on concept attainment tasks, and their performance on conventional intelligence tests, the following two points are relevant.

First, insofar as Piagetian tasks are tests of concept formation, the correlations with intelligence are likely to be low. In this connection Vinacke (1952) comments on

> . . . the apparently low correlation of "concept test" scores with intelligence-test scores and their relatively high correlation with training experience, of which one important component seems to be vocabulary. Whether the latter relationship depends upon chronological age or upon amount and kind of training has not been clearly worked out (p. 120).

Second, Piaget's theory of intelligence differs in important respects from current psychometric notions underlying test construction. A score on a standard intelligence test (e.g., Stanford-Binet) is derived from passing sub-tests which participate in the score on a compensatory basis but are statistically associated with age. As Hunt (1961) has observed,

> . . . if the order or the appearance of the structures is fixed, and if the presence of a later one always implies the presence of those which have appeared earlier, the result is a natural scale of intellectual development and intelligence with the properties Guttman (1950) has described for ordinal scales (p. 256).

Inhelder has claimed that a far greater degree of generalization is possible with an ordinal scale of the type described above than is possible with the Binet-type scales where the ordering derives from the average age at which the item is passed (Tanner and Inhelder, 1960).

While these divergent views are not easily reconcilable, it might be expected that most of the literature just reviewed would contain information appropriate to this issue (viz., other measures of mental ability). On the contrary, surprisingly few authors provide any evidence on the relationship between these two types of performance or on the comparability of different age groups with respect to general intelligence.

Beard (1960) reports a correlation of .38 between MA and scores on a concept test, consisting in part of Piagetian tasks. The correlation with CA was .16. He concludes that ". . . development in concept formation depends more on increase in mental age than chronological age, contrary to what Piaget usually implies in his discussion of stages" (p. 21).

Elkind (1961a), in his study of volume conservation, reports a highly significant correlation of .31 between I.Q. (Kuhlmann-Anderson) and scores on the volume task. This low but positive coefficient is consistent with the results of another of his studies of quantitative thinking (1961b) where the correlation was .43 (WISC, full-scale).

In the most systematic attempt to date to determine relationships between other measures of mental ability and performance on Piagetian tasks involving quantitative reasoning, Almy (1966) obtained her highest correlation (.53) between conservation and a test of mathematical concepts (New York Inventory) for a group of middle class children. Commenting on data from a longitudinal study, Almy notes that "So far as the relationship of the conservation measure to the others is concerned, the number of moderate or substantial correlations suggests that information about a child's progress in conservation does have relevance for his instruction" (p. 103). In spite of substantial relationships with other measures of cognitive behavior, Almy found that for both middle and lower class children, the variable that best predicts ability to conserve is chronological age.

Inhelder (1944) has tested feebleminded children with Piaget's tests for substance, weight and volume. Although no statistical indices are available, Inhelder reports that the sequence of acquisitions exists in an invariant order but that the modal ages of attainment for each aspect of the quantity concept are later compared with norms for normal children.

Two British studies, in which children with subnormal intelligence were included, report correlations between performance on Piagetian tasks dealing with quantity and number and other measures of mental ability. Mannix (1960) obtained correlations with MA of .61 and CA of .52. Hood (1962) found that none of the children with an MA (stanford-Binet) of less than five years conserved. An MA of five to six marked the point at which some began to conserve, while those who had an MA of eight to nine or better were clearly conserving.

In the only investigation related to this problem which involves children of superior intelligence (average Stanford-Binet

I.Q. of 135), Kooistra (1963) obtained results using MA as a criterion which were comparable to previous findings in which CA was the criterion. When the children's explanations of the various conservation procedures were classified according to CA, the ages at which 50 per cent or more of the responses indicated that conservation had been achieved were: age five for substance, age six for weight, and age seven for volume. Only seven of the 96 children in Kooistra's experiment showed any deviation from the substance-weight-volume sequence.

Perhaps the safest generalization to be made from the evidence to date regarding the relationship between the two major variables considered here is that to some extent bright children progress at a faster rate through the stages of intellectual development described by Piaget. The crucial element may be verbal ability. In any event, it seems clear that any future study of the normal genetic sequence of conceptual development must at least control for the level of general intelligence of children of differing ages.

III. THE PROBLEM AND HYPOTHESES

Statement of the Problem

A recurring question in connection with conceptual content areas (e.g., number, quantity, space) is that of the developmental sequence of acquisitions within an area. Ausubel (1963) has recently done an excellent job of pointing out the essential irrelevance of most of the research which has addressed this problem with cross-sectional samples. Because Piaget's own experimental treatment of this problem is unquestionably open to many methodological criticisms, it will be the principal aim of this investigation to ascertain the normal genetic order of some of the components assumed to be involved in the acquisition of the concept of quantity.

The present research starts from the premise that in order to assess the extent to which psychological processes of a cognitive nature are related, it is necessary to investigate the development of these processes in the same subjects. Methodologically, this is the only defensible procedure, and it must remain a serious criticism of much of Piaget's experimental technique, both in his own work and in that of others, that he is willing to argue to the relatedness of different processes (e.g., substance, weight and volume) from their tendency to appear at about the same age but in different subjects.

Hypotheses

In order to clarify the sequential order of development of the components of the concept of quantity the following hypotheses were tested.

1. The arithmetic operations of addition and subtraction (Tasks I and II) take genetic precedence over conservation (Task V).

2. The sequence of subtraction-addition (Task I) occurs earlier in development than that of addition-subtraction (Task II).

3. There is an invariant order of increasing difficulty involved in making gross (Task III), intensive (Task IV) and extensive (Task V) comparisons of discontinuous substance.

4. The ability to conserve discontinuous substance (Task V) is achieved earlier in development than the ability to conserve continuous substance (Task VI).

5. There is an invariant, sequential relationship among the component concepts of conservation of continuous substance (Task VI), weight (Task VII) and volume (Tasks X, XI and XII).

6. The infra-logical operation of conservation of weight (Task VII) takes genetic precedence over the logical operation of transitivity of weight (Task VIII).

7. There is an invariant, sequential relationship among the component concepts of conservation of occupied volume (Task X), conservation of displacement volume (Task XI) and calculation of volume (Task XII).

8. The components of the concept of quantity (Tasks I - XII) conform to a hierarchy of increasing difficulty and reflect the properties (e.g., invariance) Guttman (1950) has described for an ordinal scale.

9. There is a positively-accelerated, linear increase, within the age range studied, in total scale scores associated with changes in chronological age among subjects of average intellectual ability.

10. There are significant differences in total scale scores among five age groups of average intellectual ability.

11. There are no significant differences between sexes of similar ages and intellectual ability in total scale scores.

CHAPTER III

THE METHOD AND PROCEDURE

The general methodological approach and specific procedures employed in the present research are set forth in detail in the following discussion.

I. SUBJECTS

A total of 100 children comprised the sample. The sample consisted of 20 children, ten boys and ten girls, in each of the following five age groups: (1) five-year-old kindergarteners; (2) six-year-old first-graders; (3) eight-year-old third-graders; (4) ten-year-old fifth-graders; (5) twelve-year-old seventh-graders. All children were drawn from Sun Valley, a middle-class, residential suburb of Houston, Texas, were selected on the basis of average intelligence, and were members of the Caucasian race. The kindergarten children attended an Episcopal parochial day school and the remainder were students at local public elementary and junior high schools in Houston, Texas.

The average estimated intelligence for the entire sample was 101.11. The age of the sample averaged around the mid-point of each year for each of the five age groups. Appendix A-2 shows the means for chronological and mental age for both males and females at each age level. An analysis of these means is given in Appendix A-1. No significant differences in intelligence were obtained: (1) between the sexes within each age group; (2) among the same sex subjects between each age group; or (3) among combined male and female subjects between each age group. In addition, no significant differences in age between the sexes were found.

In general, a high level of rapport was maintained with the children--the game aspect of the tasks apparently held their interest throughout the experimental session which lasted from about 40 minutes to an hour. In no case were any signs of behavior problems displayed in the test situation and, in post-test interviews with the mothers, it was established that no child had a recorded history of psychological evaluation or psychiatric diagnosis.

II. MATERIALS

The apparatus employed in this study consisted of the Peabody Picture Vocabulary Test (Form A) and a set of specially constructed test objects used in the quantity tasks.

The Peabody Picture Vocabulary Test (PPVT) is an individually-administered test of verbal intelligence designed for use with children between the ages of two and 18 years. Among the advantages of the PPVT cited by the author are the following: (1) the test has high interest value and, hence, is helpful in establishing good rapport with children; (2) extensive specialized preparation is not required for its administration; (3) it can be administered in ten to 15 minutes; (4) scoring is completely objective and can be accomplished in one or two minutes; (5) it is a power test without time limits; and (6) no oral response is required of the child (Dunn, 1965). Raw scores on the PPVT can be converted to intelligence quotients, mental ages and percentiles. Product moment correlations between PPVT and 1960 Stanford-Binet mental age scores have been reported as ranging from .82 to .86 with a median of .83. Median correlations between PPVT intelligence quotients and 1937 Stanford Binet and Full Scale WISC intelligence scores are .71 and .61, respectively (Dunn, 1965, p. 33). Moreover, the PPVT apparently correlates about equally well with language arts, social studies and mathematics achievement, with median correlation values in the .50's. Based upon these results, the use of the PPVT as an overall index of intellectual level would appear to be justified even though some of the variance it shares with other tests includes the common chronological ages of the subjects on whom the scores are based.

The quantity test was comprised of twelve tasks which were either duplications or modifications of existing items used in other validational studies of the quantity concept. In some cases, more than one task involved the same materials. A detailed description of the test materials is provided in Appendix B of this paper.

III. PROCEDURE

General Methodological Approach

Traditionally, the problem of the sequential development of conceptual and other types of behavior has been approached by one or the other of two major research designs in developmental psychology. The normative, cross-sectional study is, by far, the more common of the two designs. However, since the mere determination of the modal age at which a concept is acquired affords no information concerning the extent to which the transitional components of that concept conform to an orderly sequence, cross-sectional designs are of only limited usefulness. The longitudinal study, on the other hand, is clearly the preferred design, but it is also limited by such practical disadvantages as time, expense, loss of subjects, etc.

Another type of technique, which provides somewhat similar information concerning developmental sequences as the longitudinal design, but which circumvents its disadvantages, is scalogram analysis. Several investigations using this technique have already been made in the conceptual areas of space, logical judgment and moral judgment (Peel, 1959), quantity (Smedslund, 1962; Uzgiris, 1964), number (Dodwell, 1961; Mannix, 1960; Wohlwill, 1960) and an area closely related to that of number, the concept of money (Schuessler and Strauss, 1950; Strauss and Schuessler, 1951).

In the context of Piaget's system, scalogram analysis can be used to ascertain the extent to which a set of responses, interpreted as representing different stages of development within a conceptual content area, form a genuine developmental progression: responses on level B presuppose responses on genetically more immature A, responses on level C presupposing responses on A and B, and so on. An analysis of success-failure patterns on tasks designed to tap abilities in the hypothesized genetic sequence should reveal whether the tasks constitute a scalable set. To date, the use of this technique in attempts to validate aspects of Piaget's theory has resulted in more than a modest degree of success.

The application of scalogram analysis has been predominantly in the area of attitude measurement. Here, the investigator typically selects a set of items, often on a seemingly intuitive basis, and subsequently discards those items that do not appear to be scalable. The predictive power of the technique is patently greater where there exists a theoretical and/or empirical basis for selecting the tasks relevant to a conceptual area. Piaget's theory, together with a sizeable amount of related research on the quantity concept, provide an objective orientation for the present investigation, insofar as the selection of tasks is concerned.

In a particularly thoughtful analysis of the potential applications of scalogram analysis to developmental problems together with the assumptions underlying such applications, Wohlwill (1960) has made the following observations:

> It is a unique feature of scalogram analysis - as of latent structure analysis, of which the former is but a special case, cf. Torgerson, 1958 - that it scales both stimuli (i.e., the items) and subjects simultaneously, on the basis of the same criterion: the response patterns observed. This feature is clearly a double-edged sword: it presents a clear advantage, in so far as it obviates the necessity for an independent measurement device for scaling the items; on the other hand, it places very definite limitations on the logically justifiable interpretations that can be made on the basis of a determination of scalability in a given set of items (p. 368-369).

In further discussion of the issues outlined above, Wohlwill admits that while interpretation of the results of scalogram analysis presents no special difficulty in the realm of attitude measurement, it does pose a problem for the developmental researcher who infers a unitary developmental process (e.g., equilibration) from the demonstrated scalability of a set of tasks. Since the aim of the present study extends beyond the determination of an ordinal scale to the inference of an underlying developmental process, it is necessary to consider any special requirements this objective places on this type of investigation. Wohlwill (1960) has remarked that ". . . such an inference clearly demands two supplementary steps: first, an independent validation, on internal grounds, of the psychological dimensions defined by the items, and second, the identification of the ranking of the subjects on this scale with psychological development" (p. 369). According to Wohlwill's analysis, the first requirement is met if the psychological significance of the tasks is reasonably well understood on the basis of previous research with those tasks. With respect to the second requirement, Wohlwill makes the following recommendation: "Short of a developmental follow-up, this assertion (viz., developmental sequence) can be verified only by a determination of a substantial correlation between some independent criterion of development (e.g., CA) and the scale type (total score) attained on this set of problems. . . . The validity of this assumption (viz., developmental sequence) rests on the adequacy of sampling. . . ." (p. 370). Therefore, the relationship between total scale score and one criterion of development, namely, chronological age, will be evaluated in the present study.

Test Series

All children were tested individually in a well-lighted, air-conditioned conference room of the St. Barnabas Episcopal Day School. The children were seated across the table from the examiner, and all the materials used in the study were visible on the table. After the name, age, birthdate and grade level had been recorded, the PPVT was administered and scored in accordance with the directions specified in the manual (Dunn, 1959). If the results of this screening revealed that the child was of average intelligence (i.e., a score between 90 and 109), the examiner proceeded to the second phase of the test series. Otherwise, the child was thanked for his cooperation and dismissed.

The major part of the experimental test series was administered immediately after the completion of the PPVT, and the following is a description of the tasks in the hypothesized order of difficulty. All responses made by the child were recorded by the examiner, verbatim as far as possible. (It should be noted that the order of administration followed a different sequence for two reasons: (1) to insure that, as far as possible, the order conformed to that used in other studies from which the tasks were drawn; and (2) to minimize the possible effects of transfer of training between particular tasks. The order of administration was as follows: IV, III, V, I-II, VI, VIII, IX, VII, X, XI, XII.)

Addition-subtraction of discontinuous substance (I-II). Reference was made earlier in this paper to Smedslund's (1962) finding of a high degree of relationship between three items involving various combinations of (I) subtraction/addition (-/+) and (II) addition/subtraction (+/-) sequences, and conservation of discontinuous substance, defined by a correct response and a "symbolic" explanation (one referring to earlier events in the item). These three items are used for the first two tasks in this series in strict conformity to Smedslund's procedures for administration.

Two piles of 48 pieces of colored linoleum are placed on the table. The task was introduced by the examiner in the following manner: "There is just as much in this pile as in that one (pointing to each pile in turn). I have made the two piles containing the same amount of linoleum. Remember that!" The examiner then performed the appropriate additions and subtractions, always describing what he was doing, e.g., "Now I take one piece away from this pile." After each addition or subtraction the standard question was asked: "Do you think there is more in this pile, just as much in both of them, or more in this pile (pointing)." After some repetitions the question was abbreviated to: "What do you think now?"

The following is the set of three items used in the first two tasks. In the notation below, L refers to the left pile and R to the right pile, + implies that one piece is added and - implies that one piece is taken away from a pile.

(1)	Blue Pieces:	-L	+L	+R	-R
(2)	Yellow Pieces:	+L	-L	-L	+L
(3)	Black Pieces:	-L	+L	+L	-L

The first item involving blue pieces was used as a practice trial, and performance was scored only on the counterbalanced sequences of addition and subtraction operations involving the yellow and black pieces. For a child to have passed either of these two tasks, he must have succeeded on both of the last two items involving the appropriate sequence of operations.

Gross, intensive and extensive comparisons of discontinuous substance (III, IV, V). In view of the very high inter-item correlations repeatedly found in tests of conservation of discontinuous substance (Elkind, 1961b; Smedslund, 1959, 1961b, 1961f) only three items for each of these three components of the quantity concept were used. The items employed in these three tasks were slight modifications of procedures devised by Elkind (1961b). These modifications included increasing the number of pieces across the three items within each task and requiring a "symbolic" explanation in the case of tasks III and V, the latter change being more in line with Smedslund's procedures.

Twenty-four red, linoleum strips were placed on the table in front of the child. The examiner introduced the tasks with the following instructions: "Let's pretend these sticks are candies (later, with the sticks arranged lengthwise, as 'railroad cars'; then 'candies' again)." The examiner then took six strips (later, seven; then, eight) and arranged them perpendicularly to the edge of the table and at varied distances from one another. The examiner then instructed the child to ". . . take just as many candies ('railroad cars') as I have. Take the same number of candies as me"(Intensive Comparisons, IV).

After the child made his response, the examiner shortened both rows by placing the sides of the strips flush with each other. In the event that the child had taken more or less than the required number of strips, this demonstration was expected to make the error apparent to him. In the event of an error, the examiner instructed the child to "Make them the same." After the two rows had been made equal in number of strips, the examiner asked: "Do we both have the same number of candies ('railroad cars')?" After the child had responded, the examiner asked for an explanation: "Why do you think so?" (Gross Comparisons, III).

The examiner then spread his row of strips apart so that the absolute length of his row was approximately twice that of the child's row which was left undisturbed. This arrangement gave the appearance of shorter length to the child's row, although the number of strips remained equal in both rows. The standard question was again put to the child: "Do we have the same number of candies ('railroad cars')?" Next, he was asked: "Why do you think so?" (Extensive Comparisons, V).

A criterion of two correct comparisons out of a total of three was required for passing each of the three component tasks involved in conservation of discontinuous substance. In the cases of tasks III and V, a "correct comparison" involved both a correct response to the standard question and a "symbolic" explanation. Smedslund (1961*f*, 1962) defines a "symbolic" explanation as one which directly or indirectly refers to previous events in the same test item such as a reference to the initial equality of two amounts, the fact that nothing has been added or taken away, and the fact that the transformed object(s) can be restored to its (their) original shape. Adherence to this definition was maintained throughout the present study in the scoring of protocols, except in Task III (gross comparisons), where more leniency was permitted in evaluating the explanations. For Task III, which involved simply two identical perceptual displays, explanations that directly or indirectly referred to observable features of the display were also scored as correct.

Conservation of Continuous Substance (VI). The tasks used to test for conservation of continuous substance and of weight are taken from Smedslund (1961b). They also closely resemble those used by Lovell and Ogilvie (1960, 1961a).

Two balls of plasticene of identical physical dimensions were presented to the child with the instructions: "There is just as much clay in this ball as in that one (pointing to each ball in turn). I have made the two balls containing the same amount of clay. Remember that!" The examiner then changed one of the balls into a different form (alternating from left to right in the series), commenting upon the operation by saying: "Now I change this one into a (ring, triangle, cup, cross)." After each transformation the following standard question was asked: "Do you think the (ring, triangle, cup, cross) contains more or the same amount as or less clay than the ball?" Following the child's response to the standard question, the examiner inquired in a neutral but interested way: "Why do you think so?"

The following is the set of four items comprising the task for conservation of continuous substance.

(1) Two orange balls, one changed into a ring

(2) Two green balls, one changed into a triangle

(3) Two yellow balls, one changed into a cup

(4) Two blue balls, one changed into a cross

A criterion of three correct responses with "symbolic" explanations defined successful performance on this task.

Conservation of Weight (VII). The procedure and criterion for passing this task were identical to those used in the conservation of continuous substance task, except that here the examiner referred to weight instead of amount. Although material of the same colors was used in these items as was used in Task VI, colors and transformed objects were interchanged. Below is the set of four items making up the task for conservation of weight.

(1) Two green balls, one changed into a cup

(2) Two blue balls, one changed into a ring

(3) Two orange balls, one changed into a cross

(4) Two yellow balls, one changed into a triangle

Transitivity of Weight for Three Objects with Identical Volumes (VIII). The items comprising this task were borrowed from Smedslund (1963*a*).

A balance was placed on the table in front of the child. Several objects were weighed in order to familiarize the child with the functioning of the scales and to insure that the child could accurately discriminate equalities and differences in the weights of objects. The examiner gave three such practice trials and corrected any errors before proceeding with the test items. Two of these trials involved inequalities of weights (with the position of the heavier weight reversed on the third trial) interspersed with an equality comparison. In each case, the examiner asked the child the standard question which is included in the instructions below.

The examiner introduced the test series by presenting the first set of three plasticene objects with instructions to: "Weigh the red ball and the green sausage (and similarly for the objects cited under (a) in the other items)." After the child made the appropriate response, the examiner asked: "Which one weighs more or do they weigh the same?" The examiner repeated the child's judgment and instructed him to: "Now weigh the green sausage and the yellow cake (and similarly for the objects cited under (b) in the other items)." Following the child's response, the examiner asked: "Which one weighs more or do they weigh the same?" Finally, the two objects cited under (c) in each item were placed together on the table (the child was not permitted to touch them) and the standard question was asked: "Do you think this one weighs more, do you think they weigh the same, or do you think that one weighs more (pointing to each object in turn)?" After the child had answered, the examiner asked: "Why do you think so?"

The following is the set of three items used in this task. In the notation below, = means "weighs the same as" and > means "weighs more than."

(1) Red ball, green sausage, yellow cake

(a) Red ball = green sausage

(b) Green sausage = yellow cake

(c) Red ball and yellow cake

(2) Yellow, blue and brown balls

(a) Yellow ball > blue ball

(b) Blue ball > brown ball

(c) Yellow ball and brown ball

(3) Red, brown and orange balls

(a) Red ball = brown ball

(b) Orange ball > brown ball

(c) Red ball and orange ball

For a child to have succeeded on this task he must have given correct responses on (c) with "symbolic" explanations for two of the three items.

<u>Transitivity of Weight for Three Objects with Inversely Correlated Volumes (IX)</u>. The procedure and criterion for passing this task were identical to those used in the preceeding task, except

that here the volumes of the three objects making up an item were not held constant. In addition, no practice trials were given.

Below is the set of three items making up this task which is modelled after a set of items used by Smedslund (1959). For the three objects cited in each item, the volumes increase by the order of one-half, respectively. The "weight-notation" is identical to that used above.

(1) Red block, green ball, yellow sausage

(a) Red block = green ball

(b) Green ball = yellow sausage

(c) Red block and yellow sausage

(2) Green, blue and orange balls

(a) Green ball > blue ball

(b) Blue ball > orange ball

(c) Green ball and orange ball

(3) Yellow, blue and brown blocks

(a) Yellow block = blue block

(b) Blue block > brown block

(c) Yellow block and brown block

It should be noted at this point that the reason for including two tasks (VIII and IX) for transitivity of weight was to determine whether, for children with unstable concepts of transitivity, there would be a tendency for such children to be misled by the presence of perceptually prominent but misleading cues for weight. Specifically, it was expected that fewer children would pass Task IX than would pass Task VIII, and that no children would succeed on IX who had not passed VIII. In point of fact, analysis of the data on these two tasks revealed just the opposite relationship; namely, that Task IX proved to be significantly easier than Task VIII ($X^2 = 9.09$, $p < .001$, 1 df). A separate analysis of the items of Task IX indicated that there was no systematic tendency for objects of greater volume to be judged heavier at any age level. One interpretation of this outcome is that these results simply reflect positive transfer. Whether the same results would have obtained had the differences in volumes been greater among the three objects in each item is a moot point. It should be recalled that these two tasks were administered successively in the test series. Smedslund (1961b) has noted that for tasks that are similar (e.g., conservation of continuous substance and of weight) ". . . there is a lot of 'transfer'. . . when they are given in immediate succession" (p. 81). Even though the similarity is apparently less for the transitivity than for the conservation tasks, nevertheless, it is probably substantial enough to have warranted a separation between the transitivity tasks. Based on the assumption that transfer effects were operative, it was decided to pool the data for the two tasks and to treat transitivity of weight as a single task (VIII). A criterion of four correct responses with "symbolic" explanations with a maximum of six defined successful performance on this revised task.

Conservation of Occupied Volume (X). The first two items employed in this task are identical to those used by Lovell and Ogilvie (1961*b*), whereas the third item simply involves an extension of the logical possibilities inherent in the original task.

(1) A pint can and a gallon can were placed on the table in front of the child with their unit designations clearly in view. The examiner introduced the task by informing the child that: "This can will hold one gallon of water and this can will hold one pint of water." The examiner then filled the pint can with water from a beaker which was also present on the table. Following this demonstration, he presented twelve blocks arranged as a 2 x 3 x 2 inch cube and informed the child that: "Before we fill this can (gallon can) with water we are going to put some bricks in like this." The examiner proceeded to place the arrangement of blocks in the gallon can and then asked: "If we now fill this can to the top do we still get the same amount of water in as before, or do the bricks make a difference?" After the child had given his reply, the examiner inquired: "How do you know?" Without filling the can with water, the blocks were then removed and placed unaltered beside the container.

(2) Next, the examiner presented another set of twelve blocks arranged this time as a 1 x 2 x 6 inch cube. The following question was put to the child: "Suppose we put this block (1 x 2 x 6) into the gallon can. Are we able to get as much water in the can now as we could with this block (2 x 3 x 2) in the can (pointing to each

arrangement in turn)?" After the child had responded, the examiner asked: "How do you know?" Again, without filling the can, the blocks were removed and placed unaltered beside the container. The blocks of the 2 x 3 x 2 inch cube were returned to the pile of blocks at the side of the table.

(3) Finally, the examiner presented a third set of twelve blocks arranged this time as a 1 x 3 x 4 inch cube. The child was asked the standard question: "Suppose we put this block (1 x 3 x 4) into the gallon can. Are we able to get as much water in the can now as we could with this block (1 x 2 x 6) in the can (pointing to each arrangement in turn)?" Following the child's answer, he was again asked: "How do you know?"

For a child to have succeeded on this task, he must have given correct responses on all three items together with "symbolic" explanations.

Conservation of Displacement Volume (XI). Below is the set of three items making up this task which was borrowed from Lunzer (1960b).

(1) A gallon can, half-full of water, was placed on the table in front of the child, and the water level was marked by an elastic band around the outside of the container. The child was then shown a 3 x 3 x 4 inch solid block by the examiner who asked: "If I put this brick into the can, will the water rise, fall or stay at the same level?" After the child responded, the examiner immersed the block

in an upright position and readjusted the elastic band so as to mark a new water level.

(2) With the block immersed in an upright position, the examiner asked: "If the brick were lying on its side, would the water rise, fall or stay at the same level?" Following the child's response, the block was removed and the elastic bank readjusted to mark the initial water level.

(3) Finally, the examiner presented a pile of 36, one inch square blocks together with the half-filled gallon can. He then inquired: "If, instead of putting the large brick into the can, I line the floor of the can with these 36 small bricks, will the water rise, fall or stay at the same level?"

Again, a child must have given correct responses on all three items to have succeeded on this task.

Calculation of Volume (XII). The final task in the hypothesized order of difficulty and the last to be administered was Lunzer's (1960*b*) revision of a task of his own construction. The only modification in Lunzer's procedure was that 64 blocks were employed in the present study rather than 48 as was used by Lunzer. This was done to reduce the number of false positives in the form of children who, given the opportunity, would use all of the blocks available in their attempt to duplicate the solid. Again, as in

Task X, the second item simply consisted of an obvious extension of the possibilities inherent in Lunzer's original task.

(1) A 4 x 4 x 3 inch solid block was placed on its side in front of the child. The position of the block was horizontal to and about one and one-half feet from the edge of the table. A pile of 64 blocks were placed between the edge of the table and the 4 x 4 x 3 inch solid block. The examiner then instructed the child as follows: "Using the small bricks, make a block this size (pointing to the large block)." In the event that a child spontaneously began to cover an entire side of the large block, the examiner cautioned "Do not use more than five blocks in measuring the sides." He then removed any blocks in excess of five from the vicinity of the large block, and the child was permitted to continue his construction. Moreover, a child was not permitted to alter the position or location of the large block.

(2) After the child had indicated that he was satisfied with his initial construction, the blocks were returned to the original pile, and the 4 x 4 x 3 inch solid block was placed on its end in the same location as before. Again, the examiner instructed: "Using the small bricks, make a block this size." The same task restrictions as before were enforced here.

As in the two previous volume tasks, a child was required to pass both items in order to succeed on this task.

Objectivity of Scoring for the Quantity Tasks

The value of any classification depends, in part, on its objectivity. Whereas, the majority of the responses in the coded protocols could be objectively categorized (e.g., "more than," "less than," "same amount as"), several tasks (viz., Tasks III, V, VI, VII, X) involved verbal responses to "open-ended" questions (e.g., "why do you think so?"). The intersubjective agreement of the nominal scale categories (i.e., pass-fail) used in scoring the protocols of the present study was determined by having two judges (undergraduate laboratory assistants) score all the explanations in the five tasks cited above, a total of 1700 items. One commonly used reliability index is the percentage of judgments on which coders agree. The percentage of agreement between the two judges was 93 per cent which dropped to 86 per cent when both judges' codings were compared with the results of the examiner's scoring of the same items. An improved index of inter-coder agreement is π, which corrects for the number of categories in the code, and the frequency with which each is used (Scott, 1955). The values of π corresponding to the percentages cited above is .93 and .73, respectively. Both coefficients are significant beyond the .01 level of confidence. Practically all the disagreement between examiner and judges involved a misunderstanding of the more flexible criterion employed in the scoring of Task III.

CHAPTER IV

RESULTS

The relevant data for testing the first seven hypotheses are given in Table 1 and Appendix C. Each value in Table 1 expresses the degree of relationship between all possible pairs of tasks. These relationships between two dichotomous variables are represented as adjusted phi coefficients. Justification for these adjustments is offered by Wert, Neidt and Ahmann (1954):

> . . . the phi coefficient may be interpreted as a coefficient of correlation whenever the assumption is not unreasonable that both dichotomies are actually variables which are normally distributed and linearly related, if indeed any relationship exists. Whenever the phi coefficient is interpreted as a coefficient of correlation, it should be recognized that it is an underestimate of the correlation that would ensue, if numerical values of each distribution were available (p. 302).

It should be noted that variations in the marginal distributions prevent any close correspondence between these estimates of relationship and the statistical significance of the relationships (estimated by the method of chi square). Even very high estimates of relationship may remain non-significant when marginal distributions are extremely lopsided and, conversely, moderate estimates of relationship may be highly significant when marginal distributions are balanced. The chi square tests of significance contained in Appendix C are corrected for continuity (Fisher-Yates) and for the correlation which is present when the frequencies being compared are

TABLE 1

INTER-TASK TETRACHORIC CORRELATION VALUES
DERIVED FROM THE PHI COEFFICIENT
(N - 100)

Tasks	X	XII	VIII	XI	V	VI	VII	III	II	IV	I
X		.552	.885**	.538***	.538***	.578***	.709***	.443***	.457***	.371***	.282***
XII			.414	.590***	.429***	.371***	.342***	.175***	.471***	.386***	.297***
VIII				.484**	.471***	.675***	.709***	.371***	.512***	.414***	.312***
XI					.282	.498***	.297	.160***	.429***	.175***	.221***
V						.709	.484	.687***	.297***	.342***	.206***
VI							.862	.414***	.578***	.552***	.471***
VII								.484***	.538***	.512***	.414***
III									.498	.523***	.356**
II										.628	.698**
IV											.709
I											

*** $p < .001$
** $p < .01$
* $p < .05$

based on the same individuals (McNemar, 1949). Of the 55 correlation coefficients contained in Table 1, all but ten are significant beyond the .01 level of confidence.

In much of Piaget's work (e.g., 1951) and, later in several replication studies (e.g., Elkind, 1961c), tests were assigned to the age level at which the per cent passing was 75. Although this type of data does not in any sense touch on the central question at issue here, viz., the invariance of a developmental sequence, the results of the present study were converted into percentages for comparisons with this criterion and are presented in Table 2.

I. HYPOTHESIS ONE: ADDITION-SUBTRACTION OPERATIONS (I-II) vs. CONSERVATION (V)

The relevant data for testing the hypothesis that the arithmetic operations of addition and subtraction take genetic precedence over conservation are given in Tables 5 and 24 of Appendix C. They show a highly significant relationship ($p < .001$) and, although this hypothesis predicts an empty square in the upper left part of the tables, it can be seen that subjects with conservation are unlikely to make errors on these arithmetic operations. Additional support for the hypothesized order of difficulty is provided in Table 2 of the text where the average percentage of correct responses is greater at each age level for the arithmetic operations than for conservation. Figure 1, showing the proportion of children

TABLE 2

PER CENT* CORRECT RESPONSES ON QUANTITY TASKS AT SUCCESSIVE AGE LEVELS
(N - 20 in each group)

	Quantity Tasks	Age Level 5	6	8	10	12
I.	Subtraction/Addition of Discontinuous Substance	50	75	100	100	100
IV.	Conservation of Discontinuous Substance (Intensive Comparisons)	47	73	95	98	98
II.	Addition/Subtraction of Discontinuous Substance	30	55	90	100	100
III.	Conservation of Discontinuous Substance (Gross Comparisons)	61	86	84	95	90
VII.	Conservation of Weight	18	42	73	82	86
VI.	Conservation of Continuous Substance	22	34	70	92	86
V.	Conservation of Discontinuous Substance (Extensive Comparisons)	30	57	75	81	86
XI.	Conservation of Displacement Volume	57	63	80	82	85
VIII.	Transitivity of Weight	18	27	35	69	74
XII.	Calculation of Volume	2	15	27	60	60
X.	Conservation of Occupied Volume	29	26	46	67	80

*Total Possible Correct Responses

Task VIII:	240	Tasks IV and XI:	60
Tasks VI and VII:	160	Task XII:	40
Tasks III, X, and V:	120	Tasks I and II:	20

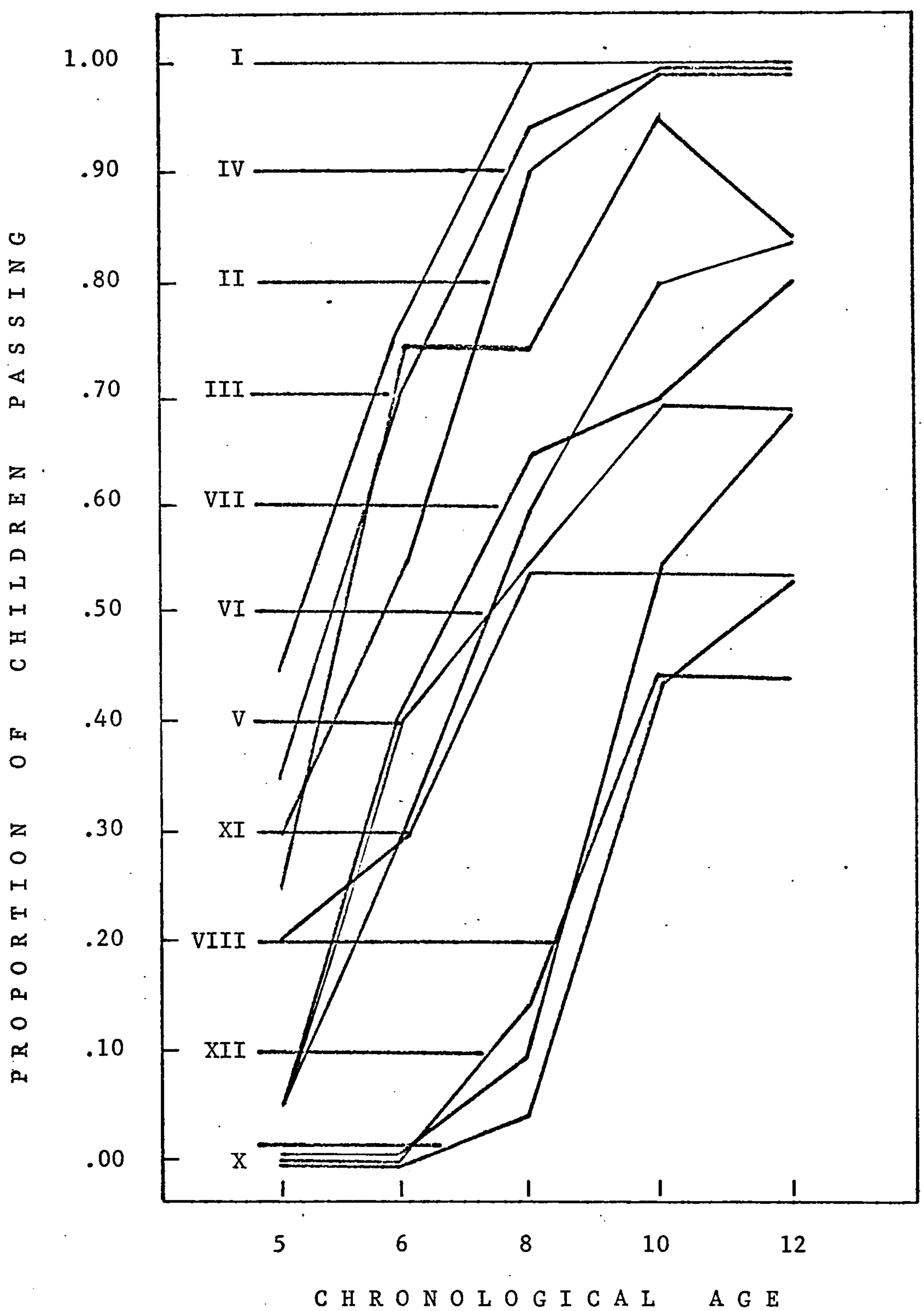

FIGURE 1

PROPORTION OF CHILDREN PASSING EACH TASK
ACROSS FIVE AGE GROUPS

passing each task at successive age levels, is also consistent with the trend of the results in Table 2. Therefore, it is concluded that conservation and the arithmetic operations of addition and subtraction form an approximate genetic sequence.

II. HYPOTHESIS TWO: SUBTRACTION/ADDITION (I) vs. ADDITION/SUBTRACTION (II)

The hypothesis that the sequence of subtraction/addition occurs earlier in development than that of addition/subtraction is accepted. Support for this claim can be found in Table 9 of Appendix C where a significant ($p < .01$) relationship exists between these two sequences of arithmetic operations. Table 2 and Figure 1 of the text also show that for the three youngest age groups, the subtraction/addition is clearly the easier of the two tasks, but that this difference disappears by the age of ten years. Thus, it is concluded that the arithmetic operations of subtraction/addition and addition/subtraction form an invariant developmental link in the concept of quantity.

III. HYPOTHESIS THREE: GROSS (III) vs. INTENSIVE (IV) vs. EXTENSIVE (V) COMPARISONS

The data of the present study failed to confirm the hypothesis that there is an invariant order of increasing difficulty involved in making gross, intensive and extensive comparisons of discontinuous quantity. Whereas the data contained in Tables 15, 18 and 32 of Appendix C reveal a highly significant relationship ($p < .001$) among these three tasks, the obtained order of difficulty as shown in Figure 1 is different from that hypothesized. Intensive comparisons were significantly easier to make than gross comparisons which, in turn, were easier than extensive comparisons. If per cent correct responses on each task are considered, Table 2 shows that the hypothesized order of difficulty holds for the two youngest age groups, but that the obtained order is the case with the older subjects. Therefore, it is concluded that, whereas there is evidence for some sequential dependency among the three tasks comprising this series, there is no support for the claim of an invariant order of increasing difficulty for gross, intensive and extensive comparisons, at least not in so far as the tasks used are valid tests of such comparisons.

IV. HYPOTHESIS FOUR: CONSERVATION OF DISCONTINUOUS SUBSTANCE (V) vs. CONSERVATION OF CONTINUOUS SUBSTANCE (VI)

The hypothesis that conservation of discontinuous substance is achieved earlier in development than the ability to conserve continuous substance is not accepted. Table 45 of Appendix C reveals no significant relationship between these two conservation tasks predicted to be adjacent in order of difficulty in the series of quantity tasks.

V. HYPOTHESIS FIVE: CONSERVATION OF CONTINUOUS SUBSTANCE (VI) vs. CONSERVATION OF WEIGHT (VII) vs. CONSERVATION OF VOLUME (X, XI, XII)

The present study affords only partial support for the hypothesis that there is an invariant, sequential relationship among the component concepts of conservation of continuous substance, weight and volume. Two of the three volume tasks, calculation of volume and conservation of occupied volume, proved to be significantly ($p < .001$) more difficult than conservation of weight as seen in Tables 35 and 36 of Appendix C. All components of the volume concept were significantly ($p < .001$) more difficult than the concept of conservation of substance (Tables 41, 42 and 44 of Appendix C). The principal discrepancy from the predicted order was the failure to confirm a sequential relationship between conservation of substance and conservation of weight (Table 40 of Appendix C). The other departure from the hypothesized sequence was the finding of no significant relationship between conservation of weight and conservation of displacement volume (Table 38 of Appendix C). It will be seen later in this paper that, of all the tasks used in this test series, conservation of displacement volume scales least well with the test as a whole. The data contained in Table 2 and Figure 1 of the text are generally consistent with the overall trend of these results. If the 75 per cent level is taken as the criterion for age of acquisition, Table 2 shows that conservation of continuous substance and conservation of weight are not acquired until the age of ten whereas the concept of volume is acquired at about the age of twelve. Thus, it is concluded that while the concept of volume seems to be generally more difficult than the concepts of conservation of continuous substance and weight, there is no apparent relationship between the latter two components of the quantity concept.

VI. HYPOTHESIS SIX: CONSERVATION OF WEIGHT (VII) vs. TRANSITIVITY OF WEIGHT (VIII)

The hypothesis that the infralogical operations of conservation of weight take genetic precedence over the logical operations of transitivity of weight is accepted. Table 37 of Appendix C reveals a highly significant ($p < .001$) and an almost perfect sequential relationship between these two components of the concept of quantity. The results shown in Table 2 and Figure 1 are also consistent with this finding. Therefore, it is concluded that conservation of weight and transitivity of weight form an approximate genetic sequence.

VII. HYPOTHESIS SEVEN: CONSERVATION OF OCCUPIED VOLUME (X) vs. CONSERVATION OF DISPLACEMENT VOLUME (XI) vs. CALCULATION OF VOLUME (XII)

The hypothesis that there is an invariant, sequential relationship among the component concepts of conservation of occupied volume, conservation of displacement volume and calculation of volume is not accepted. The results show that conservation of occupied volume and calculation of volume are both significantly ($p < .001$) more difficult than conservation of displacement volume (Tables 50 and 51 of Appendix C), although there is no evidence of perfect sequential relationships. This trend is consistently reflected in Table 2 and Figure 1 across all age groups. Moreover, contrary to expectation, calculation of volume was not found to be significantly more difficult than conservation of occupied volume (Table 55 of Appendix C). Hence, the conclusion is drawn that no invariant sequence exists in the order predicted for these components of the volume concept.

VIII. HYPOTHESIS EIGHT: SCALOGRAM ANALYSIS OF THE QUANTITY TEST SERIES (I-XII)

The hypothesis that the components of the concept of quantity conform to a hierarchy of increasing difficulty and reflect the properties (e.g., invariance) Guttman (1950) has described for an ordinal scale is not accepted. However, the data are adequate to support a claim for the existence of what Guttman (1950) describes as a _quasiscale_.

Table 3 presents the eleven tasks of the set in order of difficulty in terms of the proportion of subjects passing each task. Specific discrepancies from the hypothesized order have already been

TABLE 3

PROPORTION OF SUBJECTS PASSING EACH TASK AND REPRODUCIBILITY INDEX (Rep_i) OF EACH TASK WITH THE TEST (N - 100)

		Proportion of Subjects Passing Each Task	Rep_i
I.	Subtraction/Addition of Discontinuous Substance	.85	.90
IV.	Conservation of Discontinuous Substance (Intensive Comparisons)	.80	.88
II.	Addition/Subtraction of Discontinuous Substance	.75	.89
III.	Conservation of Discontinuous Substance (Gross Comparisons)	.71	.83
VII.	Conservation of Weight	.52	.83
VI.	Conservation of Continuous Substance	.51	.88
V.	Conservation of Discontinuous Substance (Extensive Comparisons)	.48	.82
XI.	Conservation of Displacement Volume	.43	.71
VIII.	Transitivity of Weight	.27	.87
XII.	Calculation of Volume	.20	.87
X.	Conservation of Occupied Volume	.18	.88

noted above. The rank order correlation between the predicted and observed orders is .89 ($p < .01$). For the scalogram analysis, the scores on each of the eleven separate tasks were collapsed into a single dichotomous judgment of either pass or fail based upon the criteria cited earlier in this paper. The results of this analysis can be seen in Tables 3 and 4 of the text. Of the twelve scale points, slightly over one-third of the 100 subjects in the present study gave "perfect" scale patterns. In their discussion of some of the criticism (Green, 1954; Maxwell, 1959; Torgerson, 1958) directed at Guttman's original method of scalogram analysis, White and Saltz (1957) present several alternative indices for estimating the scalability of a set of items. One of these is Green's (1954, 1956) index of consistency (I) which provides an estimate of reproducibility (rep) equivalent to that obtainable by Guttman's method. Green's technique, while less laborious than that of Guttman, makes comparisons only between patterns for adjacent pairs of items. In the case of approximately perfect scales (or quasiscales), ". . . errors of order higher than two occur very infrequently, particularly when the number of items is not extremely large. Hence, they can be ignored in the estimation of rep with very little loss in precision" (Torgerson, 1958, p. 327). Green's index of consistency is simply the ratio of the difference between the observed and the expected (i.e., chance) reproducibility to the maximum value of this difference (i.e., for a perfectly scalable set of items). It can be

TABLE 4

SCALOGRAM ANALYSIS
(N - 100)

Scale Point	X	XII	VIII	XI	V	VI	VII	III	II	IV	I	Scale Type: No. of S's	Non-Scale Type: No. of S's	Chron. Age (in months) Median	Chron. Age (in months) Range
11	+	+	+	+	+	+	+	+	+	+	+	6	0	144.5	128-154
10	-	+	+	+	+	+	+	+	+	+	+	1	4	127.0	120-145
9	-	-	+	+	+	+	+	+	+	+	+	2	3	145.0	127-155
8	-	-	-	+	+	+	+	+	+	+	+	6	12	122.0	78-155
7	-	-	-	-	+	+	+	+	+	+	+	4	8	102.5	72-155
6	-	-	-	-	-	+	+	+	+	+	+	1	6	129.0	98-150
5	-	-	-	-	-	-	+	+	+	+	+	2	10	97.5	60-121
4	-	-	-	-	-	-	-	+	+	+	+	4	5	79.5	66-153
3	-	-	-	-	-	-	-	-	+	+	+	1	5	66.0	63-107
2	-	-	-	-	-	-	-	-	-	+	+	3	7	66.5	64-81
1	-	-	-	-	-	-	-	-	-	-	+	2	3	66.2	64-77
0	-	-	-	-	-	-	-	-	-	-	-	5	0	65.8	60-66
n												37	63		

Reproducibility = 0.91
Index of Consistency = 0.49

seen in Table 4 that the scale _rep_ is 0.91. This value just surpasses the _rep_ of 0.90 which is currently cited as a criterion for scalability. Stouffer et. al. (1952) have pointed out, however, that, with a ten-item scale having a _rep_ of 0.90, as many as 90 per cent of the subjects might have nonscale patterns. Green's index of consistency (I) for the present data is 0.49. Since the sampling distribution of I is unknown, a clear interpretation of the obtained value of I is impossible. Green (1956) suggests that a value of the index equal to .5 or more corresponds approximately to a "scale" as distinguished from a "quasiscale." Using Green's approximation of the sampling error of I, it was found that the difference between the observed and expected reproducibilities was highly significant (C.R. = 10.00). Although the coefficient of reproducibility remains the primary criterion for scalability, Guttman (1950) employs a number of auxiliary criteria to insure that the obtained value of _rep_ is not spuriously high. These criteria include: (1) at least ten items should be used; (2) few, if any, items should have more than 80 per cent of the subjects in their most popular category (pass-fail); (3) the pattern of errors should be random; (4) each item should contain more non-error than error; and (5) the individual items should all have reproducibilities of 0.85 or more. The present data fail to meet the last two requirements in this set of auxiliary criteria. Only seven of the eleven individual item _reps_ exceed the above criterion (Table 3). This index (rep_i) indicates the degree to which a particular task scales with the set as a whole, in terms of its power to discriminate subjects with a relatively higher total score from subjects with a relatively lower total score. Of the four tasks whose individual reps failed to meet this criterion for scalability, three just missed the 0.85 value (Tasks III, V and VII). As was noted earlier, the task for conservation of displacement volume (XI) fell far short of the criterion for satisfactory scalability with a rep_i of 0.71. Guttman concludes that if all the criteria are met by a particular set of data, the area is scalable and the properties of a perfect scale may be attributed to the area. It is clear then, from an inspection of Tables 3 and 4, that the present set of tasks do not in fact represent a single scalable dimension, at least not within the limits of Guttman's definition. Although there is more error in a quasiscale than in a scale, the error is unsystematic. Hence, the quasiscale retains many of the desirable properties of a scale. It is always possible that "scalability" may be increased sufficiently to meet the scale criteria by revising or discarding some of the present set of items.

IX. HYPOTHESIS NINE: RELATIONSHIP BETWEEN CHRONOLOGICAL AGE AND TOTAL SCALE SCORES

The hypothesis of a positively-accelerated, linear increase in total scale scores associated with changes in chronological age (CA) is not accepted. Instead, the best description of this relationship is a curvilinear correlation (cf. Table 5; Figure 2). Figure 2 shows a steady rise in median scale scores up to the age of ten years where the curve assumes a plateau between ten and twelve. The relevance of scale type (total score) to development is indicated by its correlation (Eta) with CA of .30 (Table 5). Eta is preferable to a Pearson r as an index of this relationship for two reasons: (1) the present scale cannot be assumed to reflect the properties of an equal-interval scale, and (2) significant departure from linearity of regression. This correlation is probably as high as should be expected, given the wide variability in rates of growth of children in the age range studied here. Hence, it is inferred that the scale points can be equated with a developmental sequence.

X. HYPOTHESIS TEN: RELIABILITY OF AGE GROUP DIFFERENCES IN TOTAL SCALE SCORES

The hypothesis that there are significant differences in total scale scores among the five groups is accepted. An over-all Chi-Square analysis (Castellan, 1965) of total scale scores revealed a significant main effect due to age level (Table 6). When separate comparisons are made between adjacent age groups in terms of total scale scores, only the difference between age groups eight and ten is significant (Table 7). These results are consistent with the finding of a significant curvilinear relationship between age and total scale scores. Therefore, it is concluded that, within the age range

TABLE 5

ANALYSIS OF VARIANCE AND CORRELATION VALUES FOR AGE GROUP SCORES ON TOTAL SCALE SCORES (N - 100)

Total Scale Score	r	p value	eta	p value	Linearity of Regression (F ratio)	p value
Chronological Age	.77	.001	.30	.01	14.91	.001

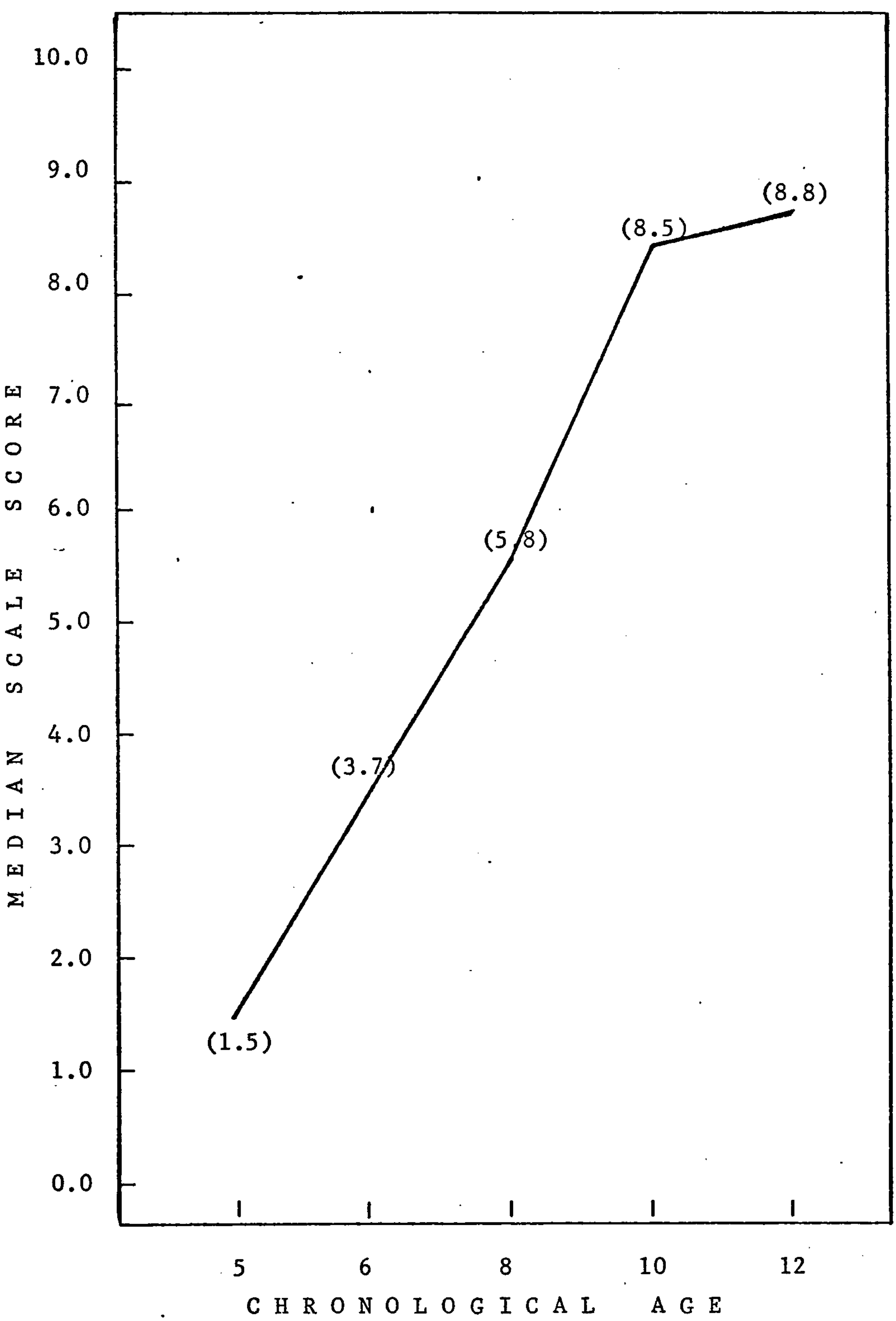

FIGURE 2

MEDIAN SCALE SCORES ACROSS FIVE AGE GROUPS

TABLE 6

CHI-SQUARE ANALYSIS FOR SEX AND AGE
GROUP TOTAL SCALE SCORES

Source of Variation	Degrees of Freedom	X^2 Values	p Values
Within Males	4	27.86	.001
Within Females	4	28.66	.001
Males X Females	1	.64	NS
Total	5	56.80	.001

TABLE 7

CHI-SQUARE ANALYSIS OF DIFFERENCES BETWEEN
AGE GROUPS ON TOTAL SCALE SCORE

Source of Variation	Degrees of Freedom	X^2 Values	p Values
Age 5 X Age 6	1	1.60	NS
Age 6 X Age 8	1	3.60	NS
Age 8 X Age 10	1	4.90	.05
Age 10 X Age 12	1	.40	NS
Total	4	10.50	.05

studied, there is a reliable increase in total scale scores up to the age of ten years followed by a sharp drop in rate of change between ten and twelve years of age.

XI. HYPOTHESIS ELEVEN: RELIABILITY OF SEX DIFFERENCES IN TOTAL SCALE SCORES

The hypothesis that there are no significant differences between male and female groups in total scale scores was upheld. The Chi-Square comparisons (Table 6) of separate groups (males x females) revealed no significant differences. Although not expressly hypothesized, no interaction was obtained between the age and sex variables. Thus, it is concluded that sex is not a relevant variable in terms of performance on this set of quantity concept tasks.

CHAPTER V

DISCUSSION

In general, the results of the present study support the thesis that with respect to the concept of quantity, the notion of sequential, invariant stages in development is a useful and defensible, descriptive construct. This conclusion, however, should not be interpreted as meaning that each of the tasks represents a reliable and well-ordered increment in conceptual development. The failure to obtain significant differences between adjacent tasks in the obtained order of difficulty combined with the marginal scalability indices of a few of the tasks makes such an inference untenable. Nevertheless, the close parallel between the obtained order of the test series and that predicted from Piagetian theory, the amount of response-invariance within the data as measured by scalogram analysis, the identification of the scale points with an independent criterion of development and the reliable differences between age groups in total scale scores are offered as evidence in support of the contention that the test series, taken as a whole, reflects a developmental sequence in the acquisition of the quantity concept. Since Piaget's equilibration theory, on which this investigation was based, was given only partial support by the data, a fuller discussion of the relationships between the present findings and related research should serve to illuminate areas of theoretical significance.

I. COMPONENTS OF THE CONCEPT OF QUANTITY

Relationship Among Arithmetic Operations of Addition-Subtraction and Conservation of Continuous and Discontinuous Substance

Equilibration theory assumes that conservation of quantity results from an understanding of numerical correspondence and an organization of the reversible operations of addition and subtraction; adding implies more of a quantity, subtracting implies less of a quantity, no adding and no subtracting must imply no change, i.e., conservation (Piaget, 1957). The results concerning a genetic sequence for the operations of subtraction/addition, addition/subtraction and conservation of discontinuous substance defined by a correct response and a "symbolic" explanation confirm the findings of three earlier studies (Smedslund, 1961_f_, 1962, Wohlwill, 1960), and lend support to this aspect of equilibration theory.

On the other hand, there is no direct support in the present data for the inference that children rarely acquire conservation of continuous quantities before they have acquired discontinuous quantities. The negative results of a direct test of this hypothesis conflict with the observations of several authors (viz., Elkind, 1961_b_; Mannix, 1960; Smedslund, 1961; Uzgiris, 1964) that conservation of substance is first achieved with discontinuous materials and only later with the continuous. Except for Elkind (1961_b_), however, no statistical evidence of the reliability of these observations is given in the literature.

Relationship Among Gross, Intensive and Extensive Comparisons of Discontinuous Substance

Piaget (1952*b*) is clear in postulating three stages in the child's impression of quantity en route to conservation, namely, gross, intensive and extensive comparisons. Characteristic of each of these stages are the number and type of relations between objects which a child is required to judge. It has already been noted that Elkind (1961*b*) has offered the results of a cross-sectional study in support of Piaget's finding of three, age-related hierarchically ordered stages of success in comparing quantity. Before attempting to reconcile the results of the present study with those of Elkind, it should be pointed out that in the tasks used by Elkind no attempt was made to elicit from a child an explanation for his response to a standard test question. Such rigid adherence to objective non-verbal procedures in what purports to be a validational study is not only at variance with Piaget's use of the *méthode clinique* but, more importantly, it runs the serious risk of failing to detect "false positives," viz., children who have a pseudo-concept of conservation or one which will not generalize across similar but different situations. The latter difficulty has been widely discussed (Braine, 1959, 1962; Bruner, 1964; Smedslund, 1961*b*, *c*, *d*, *e*, *f*; Werner, 1948).

In part, due to these considerations, but also because the task for extensive comparisons (V) was to serve as a basis for testing the hypothesis that conservation of discontinuous substance precedes conservation of continuous substance (VI), a decision was made to require an explanation of the child's response to the standard question in Task V. Then, as an indirect test of the influence of language on the predicted order of the sequence (viz., III, IV, V), a similar requirement was set in the case of Task III (gross comparisons). Task IV (intensive comparisons) was retained in its original form (Elkind, 1961*b*) as a task involving numerical correspondences without verbal justifications. From this, one might expect that if the tasks are valid measures of these three types of comparisons, and if a language criterion does tend to increase the difficulty level of a task, that the ordering of Tasks III and IV would be opposite that predicted from theory. On the other hand, if the validity of the stages were granted, and if correct responses (i.e., judgments) were highly correlated with explanations, then one would expect a confirmation of the predicted ordering of the tasks. There is some evidence in Smedslund (1961*b*) to support the latter assumption. In any case, the results of the present study failed to confirm the hypothesized ordering of the tasks, excepting Task V which was reliably more difficult than Tasks III and IV. If the 75 per cent level (total number of correct responses including explanations) were used as evidence for the development of stage sequences as Elkind has done in another investigation (1961c), then the results of the present study confirm theoretical expectations (Table 2). That such evidence is irrelevant to the theoretical postulate of sequential invariance in development has already been affirmed. The present findings do not lend themselves to easy interpretation. Inspection of the data suggests that correct responses (i.e., judgments) and satisfactory explanations are not highly correlated at the youngest age levels. Many of the younger children gave justifications such as "I don't know" or "It just is" in response to requests for explanations, even though they were able to count (e.g., Task IV) and make correct judgments of equality (e.g., Task III). Pratoomraj and Johnson (1966) have recently reported no significant differences between conservation responses (i.e., judgments) and explanations at any age level between four and seven years. They do not, unfortunately, analyze differences with respect to type of explanation (i.e., "symbolic," "perceptual," etc.), even though they do classify explanations in a way that would seem to permit such an analysis. These authors conclude, however, that the type of response (judgment vs. explanation) ". . . is of little relevance except at the age when many children begin the transition from nonconservation to conservation" (Pratoomraj and Johnson, 1966, p. 352). The present data suggest that some factor associated with language can operate to influence the ordering of stage sequences and that such a factor may have served to reverse the predicted difficulty levels of Tasks III and IV. At the same time, this aspect of the theory may be in error--the present data do not permit a clear test of this point. By way of commentary on the basic tasks devised by Elkind (1961b) and used here, it is not manifestly clear what the two dimensions to be judged are in Task IV, dimensions which must be present if the task is to qualify as a test of intensive comparisons. Rather, the task bears close similarity to a non-verbal task ("Extension") used by Wohlwill (1960) in his study of the development of the number concept and to Piaget's test for numerical correspondence (1952b). By contrast, Tasks III and V meet their respective requirements for gross and extensive comparisons in a much more convincing way.

Relationship Among the Concepts of Conservation of Continuous Substance, Weight and Volume

If the relationships among the tasks representing conservation of continuous substance (VI), weight (VII) and volume (X) are considered, the present findings confirm only the prediction that the concept of volume is, genetically, the latest in this set of conceptual acquisitions. Successful performance on these three tasks was defined by correct responses coupled with "symbolic" explanations. Moreover, the data support Piaget's (1951) claim (based on the criterion of the 75 per cent level) that the age of acquisition of the concept of volume is around eleven to twelve years (Table 2). This finding of advanced status for the volume concept over those of substance and weight is consistent with the results of Elkind (1961a, c), Lovell and Ogilvie (1961b) Lunzer (1960b) and Uzgiris (1964). It is partly at variance with Hyde's (1959) findings of no substance-weight-volume décalage. Whereas the present data reveal that very few children conserved volume who did not conserve substance and weight, no corresponding evidence for an invariant, sequential relationship between conservation of continuous substance and conservation of weight was obtained. Although a high degree of association was found between these two conceptual abilities (Table I), there was no difference in the age at which they were acquired. These results are consistent with those of Hyde (1959) and Smedslund (1961b) in which no direct evidence for sequentiality was obtained. Smedslund (1961b) did, however, find conservation of weight to be significantly more advanced, genetically, than conservation of continuous substance as did Elkind (1961c), Lovell and Ogilvie (1960, 1961a), Uzgiris (1964) and Vinh-Bang (1959). Using the criterion of the 75 per cent level (Table 2), the present findings agree with those of Elkind (1961c), Lovell and Ogilvie (1960, 1961a) and Piaget (1951) in assigning conservation of substance and weight to the age range between eight and ten years. The failure of the present study to confirm the predicted age décalage for conservation of substance and weight may have been due, in part, to transfer effects operating between two tasks given in close succession (Smedslund, 1961b), even though efforts were made to minimize such effects. Lovell and

Ogilvie, for example, used different samples to test for conservation of substance (1960) and conservation of weight (1961a). Another possibility is that differences in results may reflect differences in testing procedures. Whereas a reasonably explicit and reliable criterion was set for evidence of conservation in the present study, some investigators (e.g., Elkind, 1961c; Lovell and Ogilvie, 1960, 1961a) are not clear concerning the basis for their evaluations of childrens' explanations. The importance of a careful assessment of language when attempting to identify conceptual behaviors has already been emphasized (Pratoomraj and Johnson, 1966).

Relationship Between the Concepts of Conservation of Weight and Transitivity of Weight

From the suggestive evidence in Smedslund (1961b), it was hypothesized in the present study that transitivity is a later conceptual acquisition than conservation in any given content area. This prediction runs counter to the theoretical postulate that logical (e.g., transitivity) and infralogical (e.g., conservation) operations are organized synchronously for any conceptual content area where they are applicable (Piaget and Inhelder, 1941). Similarly, this empirical hypothesis is opposed to Piaget and Inhelder's non-empirically based suggestion that both operations applied to weight are achieved by the age of nine years. The present findings give strong support to Smedslund's suggestion that these two types of operations are not organized simultaneously in a subject. Indeed, the results confirm an almost perfect sequential relationship between the two conceptual abilities. This finding conflicts with that of Lovell and Ogilvie (1961a), although no statistical analysis nor statement of criteria for transitivity is provided in their investigation. Using Piaget's criterion, acquisition of transitivity of weight does not appear until around the age of twelve years (Table 2). This suggests that an age *décalage* of about two years separates the two types of conceptual operations in this content area. Thus, the results of the present study lend no support to the theoretical notion that logical and infralogical operations are contemporary with respect to development. Rather, the findings argue in favor of a genetic distinction between the two sets of operations. Additional research is needed to establish the reliability of this relationship across other content areas where these operations are applicable.

Relationship Among the Concepts of Conservation of Occupied Volume, Conservation of Displacement Volume and Calculation of Volume

While the results have shown rather clearly that the volume concept is, genetically, the most advanced of all the components of the concept of quantity, they fail to confirm a sequential relationship among the theoretical components of conservation of occupied volume, conservation of displacement volume and calculation of volume.

Surprisingly, conservation of displacement volume was found to be significantly easier than either of the other two volume tasks. This particular finding, however, corresponds rather nicely with the results of two other investigations in which children are asked to predict the change in the water level when an object in each of two shapes is immersed in a container of water (Lunzer, 1960b; Piaget and Inhelder, 1941). In these studies the percentage of children who conserve displacement volume is considerably higher than has been observed by investigators who have asked children which shape would occupy the greater space (Elkind, 1961a, c; Uzgiris, 1964). In the present study, 55 per cent of the twelve-year-olds conserved displacement volume as compared with 58 per cent in Lunzer's (1960b) study from which the task was drawn. It is interesting in this respect to note Lovell and Ogilvie's (1961b) observation that only 19 per cent of their ten to eleven year old children who conserved displacement volume, stated spontaneously, as a reason for equal rise in water level for objects of different shape, the fact that the two objects "take up the same room." It may well be, as Uzgiris (1964, p. 839) has pointed out, that tests of conservation of displacement volume ". . . may be tapping the conceptualization of volume at a level more concrete (less abstract). . ." than do tests for conservation of occupied volume. The weight of existing evidence suggests that this is probably the case. Although the present results have shown that conservation of occupied volume and calculation of volume are not significantly different in level of difficulty, only 45 per cent of the twelve year olds succeeded on the calculation of volume task compared with 55 per cent for conservation of occupied volume. Of those twelve-year-olds who succeeded on calculation of volume, only ten per cent attempted to "measure" the solid. The present results, then, provide little support for Piaget's claim that an intuitive grasp of the measurement of volume, independent of formal instruction, is the final, sequential link in the development of the concept of volume. Likewise, the assumption that an understanding of infinity and continuity is essential to the concept of displacement volume and that these abilities are associated with the period of formal operational thought (Piaget and Inhelder, 1956) are not supported in the present data. These particular tenets of the theory were also disconfirmed by Lunzer (1960b). On the other hand, the notion of the developmental priority of topological (conservation of substance or the concept of interior volume) over Euclidean (Tasks X, XI, XII) conceptions of space is supported by the present study.

II. SCALOGRAM ANALYSIS AS AN INDEX OF DEVELOPMENTAL SEQUENCE

Scalogram Analysis

Scalogram analysis has been shown to be uniquely suited to the problem of validating aspects of Piaget's system which relate to developmental sequence. The results of the present study have not been completely consistent with those of other investigations into the concepts of quantity and number (viz., Dodwell, 1961; Mannix, 1960; Smedslund, 1962; Uzgiris, 1964; Wohlwill, 1960) in obtaining low but satisfactory indices of scalability. Only the claim of what has previously been described as a quasiscale is warranted by the present data. An attempt was made to conform to the requirements for experimental design established by Guttman and others, and to the logical and empirical desiderata for a meaningful application of scalogram analysis to the determination of developmental sequence. These criteria have been irregularly met in previous studies in this area. The evidence from scalogram analyses in the area of quantity interlocks nicely with the results of a factor analytic study of conceptual tasks (including some Piagetian tasks of the kind used in the present study) which included a quantity factor (Beard, 1960). The close correspondence between the hypothesized order of the test series and that actually obtained would seem to strengthen the case for a quasi-developmental sequence which is reflected in the response patterns.

Independent Criterion of Development

The low but significant, positive correlation between total scale score and chronological age is consistent with the results of other studies of the development of conceptual behaviors (Almy, 1966; Beard, 1960; Elkind, 1961a, b; Mannix, 1960; Wohlwill, 1960). This evidence is interpreted as meaning that with increasing age a change in the direction of a higher scale type will be effected. Moreover, the present findings of curvilinear relationships between conceptual behaviors in the quantity area and age variables suggest that children acquire the concept of quantity at an earlier age than Piagetian theory suggests. If, as equilibration theory maintains, the period of formal operational thought marks the point at which the most advanced components of the quantity concept are acquired, then the results of this analysis suggest that this state begins earlier than twelve years of age.

Sex Differences

The results of this investigation revealed no sex differences in total scale scores in all age groups under study and thus support other studies based on Piaget's work which show sex differences to be insignificant (Braine, 1959; Case and Collison, 1962; Danzinger, 1957; Dodwell, 1962; Kooistra, 1964; Laurendeau and Pinard, 1962; Pratoomraj and Johnson, 1966).

III. IMPLICATIONS OF THE STUDY AND APPLICATIONS TO EDUCATION

Implications for a Theory of Intelligence

The specification of a developmental sequence is, at best, a first descriptive step in the analysis of the processes involved in conceptual behavior. By demonstrating that relationships among the components of a conceptual content area follow a sequential and, to some extent, an invariant order, the study has contributed to a fuller understanding of cognitive processes. More specifically, the study has shown that the gradual emergence and stabilization of increasingly more superordinate structures form the basis for an abstract representation of the concept of quantity. The finding that increasing chronological age is associated with increased success with quantity tasks highlights the importance of maturational factors. On the other hand, the argument that the ordering of success on the tasks is dependent on a child's experience is not without substance. Indeed, the importance of linguistic factors has been stressed as a potentially relevant variable in conceptual development. Further investigation is needed to verify the nature of the development of each of the components of the quantity concept and the processes that underlie their eventual association into a coherent system. Piaget's theory of stages has served as the model for this validational study. The results have confirmed some tenets of the system, failed of support for others and suggested revisions in a few aspects of the theory. Hunt has summarized some of the more important implications of this kind of approach to the problem of validating Piaget's system:

> If Piaget's successive structures prove to have a fixed order, a very useful ordinal scale of intelligence would result. The time between successive landmarks of transition could then provide an inverse index of the capacity of various kinds of child-environment interaction to foster intellectual development. Comparing average times

> between various pairs of successive landmarks for groups of children being reared under differing conditions should gradually yield new understanding of the factors in child-development interaction, both socio-familial and physical, that hamper and foster intellectual development. With the improvement in such understanding, it should become feasible more nearly to maximize the intellectual potential of children (p. 257).

Applications to Education

There are at least three ways in which the results of investigations of the kind undertaken here may be applied to problems in education. The first concerns the relevance of scalogram analysis as a research technique and the validational studies of Piaget's work in general to the area of test construction. One of the implications of this research is that it could provide valuable diagnostic instruments for the assessment of a child's readiness for various types of instruction with a view toward such practical educational goals as grade placement or assignment to remedial training programs. This type of application of Piagetian tasks has not been initiated yet on any sizeable scale, nor has Piaget, himself, evinced any interest in this extension of his work. Flavell (1963), however, sounds a note of optimism regarding this direction of applied work: ". . . if the various types of responses children give to a set of Piagetian tasks in a given content area do in fact show good age scalability, i.e., are not simply an agenetic hodgepodge attributable to individual differences, then it makes sense to think about making developmental scales out of them, scales which would possess Pinard's desideratum of having both feet solidly planted in a theory of intellectual development." (p. 364)

A second application concerns the ordering of the curricula in the context of descriptive developmental findings. For example, the present data suggest that it may be more efficient to expose children to the concepts of interior and displacement volume before presenting the more abstract concepts of physical volume. In fact, teaching a child the principles of displacement volume may well focus his attention on conservation of occupied volume and three-dimensional space concepts, provided a good teaching method is used. It would seem certain, however, that a child who has not attained a well-developed concept of physical volume cannot employ operational thinking in relation to Archimedes' Principle, or problems of density, for example. In such problems as these, his level of thinking may lead to errors of various kinds. Furthermore, in older children, one might have to steer the subject away from a concept of volume which relates only to occupied contents of objects, one which does not relate occupied space to the spatial surround.

The third way in which research of the type under discussion can be applied to problems deals with the methods by which the child ought to be taught, once the curriculum content has been ordered. Certainly before any serious attempt can be made to produce cognitive change in a child, a description of the developmental sequence of acquisitions in the average child should be clarified. Smedslund has been especially active on both these experimental fronts, and his own theory of "cognitive conflict" which he adapted from the work of Berlyne (1960) has met with some success in predicting cognitive change. A recent statement by Bruner (1960) accurately reflects the ambition and goals of many researchers interested in this third application of descriptive developmental research: ". . . any subject can be taught effectively in some intellectually honest form to any child at any stage of development if the most basic ideas underlying the subject can be translated into the child's way of seeing things." (p. 33) The type of research represented here suggests a first step toward the solution of the problem of translation.

CHAPTER VI

SUMMARY

This investigation was concerned with validating some aspects of Jean Piaget's theory of intelligence that deal with the developmental process by which children arrive at an abstract concept of quantity. Basic to his theory is the thesis that a sequential, invariant set of stages in development is a meaningful, descriptive construct. This process was traced through a set of Piagetian tasks assumed to require increasingly more superordinate structures, the objective being to ascertain the extent to which success on these tasks follows an ordered developmental sequence.

The following set of tasks, in the hypothesized order of difficulty, comprised the test series: (I) subtraction/addition of discontinuous substance; (II) addition/subtraction of discontinuous substance; (III) gross comparisons; (IV) intensive comparisons; (V) extensive comparisons; (VI) conservation of continuous substance; (VII) conservation of weight; (VIII-IX) transitivity of weight; (X) conservation of occupied volume; (XI) conservation of displacement volume; (XII) calculation of volume. The scoring of a majority of the tasks involved an evaluation of verbal explanations. Smedslund's (1961*b*) criteria were used and found to be highly communicable.

These tasks combined with the Peabody Picture Vocabulary Test were individually administered to 100 elementary school children of the Caucasian race selected on the basis of average intelligence. Ten boys and ten girls were included in each of five age-grade groups: (a) five-year old kindergartners; (b) six-year-old first-graders; (c) eight-year-old third-graders; (d) ten-year-old fifth-graders; and (e) twelve-year-old seventh-graders. No differences in mean intelligence scores were obtained among the five age groups, among the same sex categories across the five age groups, or between the sexes within each age group.

The principal analytic technique for the evaluation of the data was that of scalogram analysis. Green's (1956) method of summary statistics which yields a coefficient of reproducibility and an associated index of consistency was the type of scalogram analysis used in the study. The results of this analysis, together with a close correspondence between the predicted and observed order of the test series, were interpreted as justifying the assumption of a quasiscale of conceptual complexity and a consistent developmental process in the conceptualization of quantity.

In addition, separate chi square and correlational analyses were made of performance on all possible combinations of tasks in order to test several specific hypotheses concerning the invariance of developmental sequences. The hypothesis of an invariant, sequential relationship among subtraction/addition (I), addition/subtraction

(II) and conservation of discontinuous substance (V) was accepted. The prediction that the infra-logical operation of conservation of weight (VII) takes genetic precedence over the logical operation of transitivity of weight (VIII) was also confirmed. The hypothesis that there is an invariant order of increasing difficulty involved in making gross (III), intensive (IV) and extensive (V) comparisons of discontinuous substance was given only partial support with the major discrepancy being a reversal in the predicted order of difficulty for Tasks III and IV. Likewise, only partial support was available for the prediction of an invariant, sequential relationship among the component concepts of conservation of continuous substance (VI), weight (VII) and volume (X, XI, XII). Whereas the volume concept emerged as the most advanced of the three components, no relationship was found between Tasks VI and VII. No support was given to the theoretical inference that the ability to conserve discontinuous (V) substance is achieved earlier in development than the ability to conserve continuous substance (VI) nor was the prediction upheld that there is a systematic relationship among the separate components of the volume concept (X, XI, XII) distinguished in Piaget's theory. The fact that conservation of displacement volume (XI) was acquired significantly earlier than the other two components of the volume concept (X, XII) coincided with the results of other investigations into the concept of volume.

Chi-square analyses of total scale scores confirmed the hypothesis of an overall difference associated with age. The prediction of no significant difference in total scale scores between sexes was upheld. Contrary to the hypothesis of a positively-accelerated, linear increase in total scale scores associated with changes in chronological age, curvilinear relationships were obtained, and these findings were interpreted as additional support for the conclusion that the scale points represent a developmental scale. Moreover, it was suggested, on the basis of the latter findings, that Piaget's stage of formal operational thought may begin at an earlier age than twelve years. Other areas of theoretical significance highlighted by certain of the results of the study were also cited.

An outline of some of the implications of the study for a general theory of intelligence was made, with particular emphasis assigned to the importance of assessing the role of linguistic factors in future developmental studies of conceptual behavior. In addition, several suggestions were made of possible ways in which this type of research might be applied to practical problems in the field of education.

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APPENDICES

APPENDIX A-1

t TEST COMPARISONS OF MEAN INTELLIGENCE AND AGE FOR FIVE AGE GROUPS (N-100)

Variable	Age 5 vs. Age 6 *t*	Age 6 vs. Age 8 *t*	Age 8 vs. Age 10 *t*	Age 10 vs. Age 12 *t*	
Intelligence					
Total group, inter-age (38*df*)	1.41	.49	1.15	1.05	
Same Sex: inter-age, Female (18 *df*)	.12	2.13	2.17	.46	
Same Sex: inter-age, Male (18 *df*)	2.40	1.21	.15	.98	
	Age 5 *t*	Age 6 *t*	Age 8 *t*	Age 10 *t*	Age 12 *t*
Between Sexes: Intra-Age (18 *df*)	.29	2.08	1.32	.68	.04
Age					
Between Sexes: Intra-Age (18 *df*)	.09	1.71	.10	1.45	1.64

Note: No group difference was significantly ($<$ .01) greater than chance.

APPENDIX A-2

MEAN CHRONOLOGICAL AND MENTAL AGE SCORES
FOR FIVE AGE GROUPS
(N-100)

	Age 5		Age 6		Age 8		Age 10		Age 12	
	Male M	Female M	Male M	Female M	Male M	Female M	Male M	Female M	Male M	Female M
Chronological Age	64.5	64.4	78.7	76.6	100.6	100.8	124.5	127.1	151.3	148.7
Mental Age	71.8	67.8	76.4	79.2	101.9	100.2	128.4	133.3	161.4	157.1

APPENDIX B

DESCRIPTION OF TEST MATERIALS

1. Twenty-four strips of red, linoleum tile (1 1/4" x 1/4")

2. Three sets (blue, yellow, black) of forty-eight, 1/2 cm. squares of linoleum tile

3. Four pairs (orange, green, yellow, blue) of plasticene balls (36 gm. ea.)

4. Seven weighted, plasticene objects of equivalent volumes (red ball, 30 gm.; brown ball, 30 gm.; green "sausage," 30 gm.; yellow "cake," 30 gm.; blue ball, 41 gm.; orange ball, 42 gm.; yellow ball, 44 gm.)

5. Three sets of three weighted objects with volumes increased by the order of 1/2 within each set respectively (red, 1 in. sq. block, 130 gm.; green ball, 130 gm.; yellow "sausage," 130 gm.) - (green, 1 in. diam. ball, 140 gm.; blue ball, 110 gm.; orange ball, 80 gm.) - (yellow, 1 in. sq. block, 44 gm.; blue, 1 1/2 in. sq. block, 44 gm.; brown, 2 1/4 in. sq. block, 40 gm.)

6. One double-tray, single-beam balance (Ohaus Scale Corp.)

7. One pint (568 ml.) and one gallon (4543 ml.) containers (green) with inscribed unit designations

8. Sixty-four, green, wooden blocks (2.22 sq. cm.)

9. One beaker of water

10. Two green, wooden blocks (4" x 4" x 3" and 3" x 3" x 4")

APPENDIX C-1

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Occupied Volume	Pass	0	18
	Fail	15	67

$X^2 = 67.00$, $p < .001$, 1 df

APPENDIX C-2

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CALCULATION OF VOLUME
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Calculation of Volume	Pass	0	20
	Fail	15	65

$X^2 = 65.00$, $p < .001$, 1 df

APPENDIX C-3

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND TRANSITIVITY OF WEIGHT
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Transitivity of Weight	Pass	0	27
	Fail	15	58

$X^2 = 58.00$, $p < .001$, 1 df

APPENDIX C-4

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISPLACEMENT VOLUME (N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Displacement Volume	Pass	3	40
	Fail	12	45

$X^2 = 36.75$, $p < .001$, 1 df

APPENDIX C-5

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (Extensive Comparisons) (N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	4	44
	Fail	11	41

$X^2 = 30.42$, $p < .001$, 1 df

APPENDIX C-6

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF CONTINUOUS SUBSTANCE
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Continuous Substance	Pass	0	51
	Fail	15	34

$X^2 = 34.00$, $p < .001$, 1 df

APPENDIX C-7

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF WEIGHT
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Weight	Pass	1	51
	Fail	14	34

$X^2 = 31.11$, $p < .001$, 1 df

APPENDIX C-8

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE
(Gross Comparisons)
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Discontinuous Substance (Gross Comparisons)	Pass	6	65
	Fail	9	20

$X^2 = 7.54$, $p < .01$, 1 df

APPENDIX C-9

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION AND ADDITION/
SUBTRACTION OF DISCONTINUOUS SUBSTANCE
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Addition/Subtraction	Pass	2	73
	Fail	13	12

$X^2 = 8.64$, $p < .01$, 1 df

APPENDIX C-10

RELATIONSHIP BETWEEN SUBTRACTION/ADDITION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE
(Intensive Comparisons)
(N - 100)

		Subtraction/Addition	
		Fail	Pass
Conservation of Discontinuous Substance (Intensive Comparisons)	Pass	4	76
	Fail	11	9

$X^2 = 2.77$, N.S., 1 *df*

APPENDIX C-11

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF OCCUPIED VOLUME (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Occupied Volume	Pass	0	18
	Fail	20	62

$X^2 = 62.00$, $p < .001$, 1 df

APPENDIX C-12

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CALCULATION OF VOLUME (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Calculation of Volume	Pass	0	20
	Fail	20	60

$X^2 = 60.00$, $p < .001$, 1 df

APPENDIX C-13

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND TRANSITIVITY OF WEIGHT (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Transitivity of Weight	Pass	0	27
	Fail	20	53

$X^2 = 53.00$, $p < .001$, 1 *df*

APPENDIX C-14

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF DISPLACEMENT VOLUME (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Displacement Volume	Pass	6	37
	Fail	14	43

$X^2 = 27.94$, $p < .001$, 1 *df*

APPENDIX C-15

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS)
(N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	4	44
	Fail	16	36

$X^2 = 25.60$, $p < .001$, 1 *df*

APPENDIX C-16

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF CONTINUOUS SUBSTANCE (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Continuous Substance	Pass	1	50
	Fail	19	30

$X^2 = 27.13$, $p < .001$, 1 *df*

APPENDIX C-17

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF WEIGHT (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Weight	Pass	2	50
	Fail	18	30

$X^2 = 24.50$, $p < .001$, 1 df

APPENDIX C-18

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Conservation of Discontinuous Substance (Gross Comparisons)	Pass	7	64
	Fail	13	16

$X^2 = 35.22$, $p < .001$, 1 df

APPENDIX C-19

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (INTENSIVE COMPARISONS) AND ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE (N - 100)

		Conservation of Discontinuous Substance (Intensive Comparisons)	
		Fail	Pass
Addition/Subtraction	Pass	7	68
	Fail	13	12

X^2 = 1.89, N.S., 1 df

APPENDIX C-20

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Occupied Volume	Pass	0	18
	Fail	25	57

$X^2 = 57.00$, $p < .001$, 1 df

APPENDIX C-21

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CALCULATION OF VOLUME
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Calculation of Volume	Pass	0	20
	Fail	25	55

$X^2 = 55.00$, $p < .001$, 1 df

APPENDIX C-22

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND TRANSITIVITY OF WEIGHT
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Transitivity of Weight	Pass	0	27
	Fail	25	48

$X^2 = 48.00$, $p < .001$, 1 df

APPENDIX C-23

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISPLACEMENT VOLUME
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Displacement Volume	Pass	4	39
	Fail	21	36

$X^2 = 25.60$, $p < .001$, 1 df

APPENDIX C-24

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) (N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	6	37
	Fail	19	38

$X^2 = 23.27$, $p < .001$, 1 df

APPENDIX C-25

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF CONTINUOUS SUBSTANCE
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Continuous Substance	Pass	3	48
	Fail	22	27

$X^2 = 18.58$, $p < .001$, 1 df

APPENDIX C-26

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF WEIGHT
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Weight	Pass	4	48
	Fail	21	27

$X^2 = 17.06$, $p < .001$, 1 *df*

APPENDIX C-27

RELATIONSHIP BETWEEN ADDITION/SUBTRACTION OF DISCONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE
(Gross Comparisons)
(N - 100)

		Addition/Subtraction	
		Fail	Pass
Conservation of Discontinuous Substance (Gross Comparisons)	Pass	11	60
	Fail	14	15

X^2 = .62, N.S., 1 df

APPENDIX C-28

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE
(GROSS COMPARISONS) AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Conservation of Occupied Volume	Pass	1	17
	Fail	28	54

$X^2 = 51.07$, $p < .001$, 1 df

APPENDIX C-29

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND CALCULATION OF VOLUME
(N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Calculation of Volume	Pass	4	16
	Fail	25	55

$X^2 = 44.08$, $p < .001$, 1 df

APPENDIX C-30

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND TRANSITIVITY OF WEIGHT (N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Transitivity of Weight	Pass	3	24
	Fail	26	47

$X^2 = 38.72$, $p < .001$, 1 df

APPENDIX C-31

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND CONSERVATION OF DISPLACEMENT VOLUME (N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Conservation of Displacement Volume	Pass	10	33
	Fail	19	38

$X^2 = 16.33$, $p < .001$, 1 df

APPENDIX C-32

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) (N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	2	46
	Fail	27	25

$X^2 = 19.59$, $p < .001$, 1 df

APPENDIX C-33

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND CONSERVATION OF CONTINUOUS SUBSTANCE (N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Conservation of Continuous Substance	Pass	8	43
	Fail	21	28

$X^2 = 11.11$, $p < .001$, 1 *df*

APPENDIX C-34

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (GROSS COMPARISONS) AND CONSERVATION OF WEIGHT (N - 100)

		Conservation of Discontinuous Substance (Gross Comparisons)	
		Fail	Pass
Conservation of Weight	Pass	7	45
	Fail	22	26

$X^2 = 10.94$, $p < .001$, 1 *df*

APPENDIX C-35

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Conservation of Weight	
		Fail	Pass
Conservation of Occupied Volume	Pass	2	16
	Fail	50	32

$X^2 = 26.47$, $p < .001$, 1 df

APPENDIX C-36

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND CALCULATION OF VOLUME
(N - 100)

		Conservation of Weight	
		Fail	Pass
Calculation of Volume	Pass	6	14
	Fail	42	38

$X^2 = 23.27$, $p < .001$, 1 df

APPENDIX C-37

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND TRANSITIVITY OF WEIGHT
(N - 100)

		Conservation of Weight	
		Fail	Pass
Transitivity of Weight	Pass	3	24
	Fail	45	28

$X^2 = 20.16$, $p < .001$, 1 df

APPENDIX C-38

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND CONSERVATION OF DISPLACEMENT VOLUME
(N - 100)

		Conservation of Weight	
		Fail	Pass
Conservation of Displacement Volume	Pass	16	27
	Fail	32	25

$X^2 = 1.98$, N.S., 1 *df*

APPENDIX C-39

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS)
(N - 100)

		Conservation of Weight	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	15	33
	Fail	33	19

$X^2 = .47$, N.S., 1 df

APPENDIX C-40

RELATIONSHIP BETWEEN CONSERVATION OF WEIGHT AND CONSERVATION OF CONTINUOUS SUBSTANCE
(N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Conservation of Weight	Pass	9	43
	Fail	40	8

$X^2 = 0.00$, N.S., 1 df

APPENDIX C-41

RELATIONSHIP BETWEEN CONSERVATION OF CONTINUOUS SUBSTANCE AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Conservation of Occupied Volume	Pass	3	15
	Fail	46	36

$X^2 = 27.92$, $p < .001$, 1 df

APPENDIX C-42

RELATIONSHIP BETWEEN CONSERVATION OF CONTINUOUS SUBSTANCE
AND CALCULATION OF VOLUME
(N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Calculation of Volume	Pass	6	14
	Fail	43	37

$X^2 = 22.34$, $p < .001$, 1 df

APPENDIX C-43

RELATIONSHIP BETWEEN CONSERVATION OF CONTINUOUS SUBSTANCE AND TRANSITIVITY OF WEIGHT
(N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Transitivity of Weight	Pass	4	23
	Fail	45	28

$X^2 = 18.00$, $p < .001$, 1 df

APPENDIX C-44

RELATIONSHIP BETWEEN CONSERVATION OF CONTINUOUS SUBSTANCE AND CONSERVATION OF DISPLACEMENT VOLUME
(N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Conservation of Displacement Volume	Pass	13	30
	Fail	36	21

$X^2 = 18.82$, $p < .001$, 1 df

APPENDIX C-45

RELATIONSHIP BETWEEN CONSERVATION OF CONTINUOUS SUBSTANCE AND CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) (N - 100)

		Conservation of Continuous Substance	
		Fail	Pass
Conservation of Discontinuous Substance (Extensive Comparisons)	Pass	11	37
	Fail	38	14

X^2 = .36, N.S., 1 *df*

APPENDIX C-46

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) AND CONSERVATION OF OCCUPIED VOLUME (N - 100)

		Conservation of Discontinuous Substance (Extensive Comparisons)	
		Fail	Pass
Conservation of Occupied Volume	Pass	4	14
	Fail	48	34

$X^2 = 23.68$, $p < .001$, 1 df

APPENDIX C-47

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) AND CALCULATION OF VOLUME (N - 100)

		Conservation of Discontinuous Substance (Extensive Comparisons)	
		Fail	Pass
Calculation of Volume	Pass	6	14
	Fail	46	34

$X^2 = 19.60$, $p < .001$, 1 df

APPENDIX C-48

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) AND TRANSITIVITY OF WEIGHT (N - 100)

		Conservation of Discontinuous Substance (Extensive Comparisons)	
		Fail	Pass
Transitivity of Weight	Pass	8	19
	Fail	44	29

$X^2 = 11.92$, $p < .001$, 1 df

APPENDIX C-49

RELATIONSHIP BETWEEN CONSERVATION OF DISCONTINUOUS SUBSTANCE (EXTENSIVE COMPARISONS) AND CONSERVATION OF DISPLACEMENT VOLUME (N - 100)

		Conservation of Discontinuous Substance (Extensive Comparisons)	
		Fail	Pass
Conservation of Displacement Volume	Pass	18	25
	Fail	34	23

$X^2 = .61$, N.S., 1 *df*

APPENDIX C-50

RELATIONSHIP BETWEEN CONSERVATION OF DISPLACEMENT VOLUME AND CONSERVATION OF OCCUPIED VOLUME (N - 100)

		Conservation of Displacement Volume	
		Fail	Pass
Conservation of Occupied Volume	Pass	5	13
	Fail	52	30

$X^2 = 17.86$, $p < .001$, 1 df

APPENDIX C-51

RELATIONSHIP BETWEEN CONSERVATION OF DISPLACEMENT VOLUME AND CALCULATION OF VOLUME
(N - 100)

		Conservation of Displacement Volume	
		Fail	Pass
Calculation of Volume	Pass	5	15
	Fail	52	28

$X^2 = 16.03$, $p < .001$, 1 df

APPENDIX C-52

RELATIONSHIP BETWEEN CONSERVATION OF DISPLACEMENT VOLUME AND TRANSITIVITY OF WEIGHT
(N - 100)

		Conservation of Displacement Volume	
		Fail	Pass
Transitivity of Weight	Pass	9	18
	Fail	48	25

$X^2 = 7.52$, $p < .01$, 1 df

APPENDIX C-53

RELATIONSHIP BETWEEN TRANSITIVITY OF WEIGHT AND
CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Transitivity of Weight	
		Fail	Pass
Conservation of Occupied Volume	Pass	3	15
	Fail	70	12

$X^2 = 6.67$, $p < .01$, 1 df

APPENDIX C-54

RELATIONSHIP BETWEEN TRANSITIVITY OF WEIGHT AND
CALCULATION OF VOLUME
(N - 100)

		Calculation of Volume	
		Fail	Pass
Transitivity of Weight	Pass	16	11
	Fail	64	9

$X^2 = 1.96$, N.S., 1 df

APPENDIX C-55

RELATIONSHIP BETWEEN CALCULATION OF VOLUME
AND CONSERVATION OF OCCUPIED VOLUME
(N - 100)

		Calculation of Volume	
		Fail	Pass
Conservation of Occupied Volume	Pass	9	9
	Fail	71	11

X^2 = .20, N.S., 1 df